Chapter 3

Solve, Differential/Quadratic Differential, Integration, Maximum/Minimum Value, and Σ Calculations

3-1 Function Analysis Menu

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3-1 Function Analysis Menu

The following describes the items that are available in the menus you use when performing Solve, differential/ quadratic differential, integration, maximum/minimum value, and Σ calculations.

When the option menu is on the display, press F4 (CALC) to display the function analysis menu.

AC OPTN F4 (CALC)

F1 (Solve) ..... Used in Solve calculations

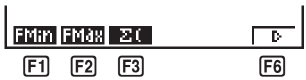
F2 (d/dx) .... Used in differential calculations

F3 (d^2/dx^2) ..... Used in quadratic differential calculations

F4 ($\int dx$) .... Used in integration calculations

F6 (▷) .... Previous menu

F6 (▷)



F1 (FMin) ...... Used in minimum calculations

F2 (FMax) ..... Used in maximum calculations

F3 (Σ() .... Used in Σ calculations

F6 (▷) .... Previous menu

3-2 Solve Calculations

P.64

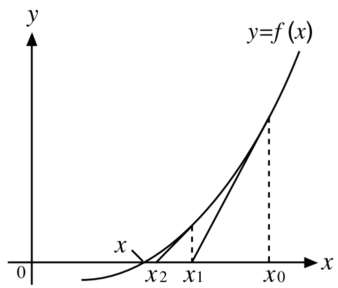
To solve calculations, first display the function analysis menu, and then input the values shown in the formula below to determine root x values in the function $f(x)$.

[F1] (Solve) $f(x)$ [,] n [,] a [,] b [)]

Initial estimate value (n)
Lower limit (a)
Upper limit (b)

With Solve calculations, the root of a function is determined using Newton's method.

•Newton's Method



This method is based on the assumption that $f(x)$ can be approximated by a linear expression within a very narrow range.

First, a starting value (predicted value) x_0 is given. Using this starting value as a base, approximate value x_1 is obtained, and then the left side and right side calculation results are compared. Next, approximate value x_1 is used as the initial value to calculate the next approximate value x_2. This procedure is repeated until the difference between the left side and right side calculated values is less than some minute value.

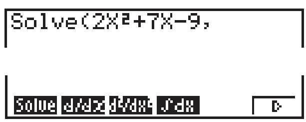
•To perform solve calculations

Example **To calculate the value of root x in the following formula when the initial estimated value is $n = 1$, the lower limit is $a = 0$, and the upper limit is $b = 1$:**

$2x^2 + 7x - 9 = 0$

Input the function $f(x)$.

[AC] [OPTN] [F4] (CALC) [F1] (Solve)
[2] [X,θ,T] [x^2] [+] [7] [X,θ,T] [−] [9] [,]



[F1]

Input initial estimated value n.

[1] [,]

```
Solve(2X²+7X-9,1,
```

Input lower limit a and upper limit b.

[0] [,] [1] [)]

```
Solve(2X²+7X-9,1,0,1)
```

[EXE]

```
Solve(2X²+7X-9,1,0,1)
                     1
```

- In the function $f(x)$, only X can be used as a variable in expressions. Other variables (A through Z, r, θ) are treated as constants, and the value currently assigned to that variable is applied during the calculation.
- Input of the closing parenthesis, lower limit a and upper limit b can be omitted.
- Roots obtained using Solve may include errors.

- Since Solve uses Newton's method, the following can sometimes occur.
  —Certain initial estimated values can make it impossible to obtain roots. In this case, try inputting another value that you assume to be near the root and perform the calculation again.
  —The calculator may be unable to obtain a root, even though a root exists.
- Due to certain idiosyncrasies of Newton's method, roots for the following types of functions tend to be difficult to calculate.
  —Periodic functions (i.e. sin x = 0)
  —Functions whose graph produce sharp slopes (i.e. $e^x = 0$, $^1/_x = 0$)
  —Discontinuous functions (i.e. $\sqrt{x} = 0$)

- Note that you cannot use a Solve, differential, quadratic differential, integration, maximum/minimum value or Σ calculation expression inside of a Solve calculation term.

3-3 Differential Calculations

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- To perform differential calculations, first display the function analysis menu, and then input the values shown in the formula below.

$\boxed{\text{F2}}(d/dx)\ f(x)\ \boxed{,}\ \underbrace{a}_{\text{Point for which you want to determine the derivative}}\ \boxed{,}\ \underbrace{\Delta x}_{\text{Increase/decrease of } x}\ \boxed{)}$

$$d/dx\ (\ f(x),\ a,\ \Delta x) \Rightarrow \frac{d}{dx} f(a)$$

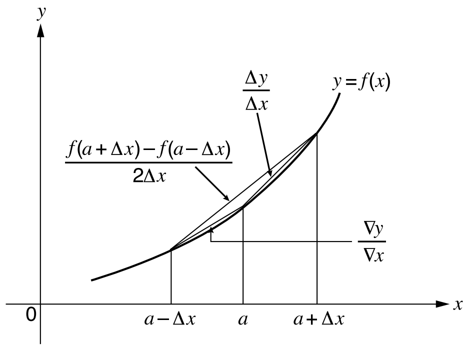
The differentiation for this type of calculation is defined as:

$$f'(a) = \lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

In this definition, *infinitesimal* is replaced by a *sufficiently small* Δx, with the value in the neighborhood of $f'(a)$ calculated as:

$$f'(a) \fallingdotseq \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

In order to provide the best precision possible, this unit employs central difference to perform differential calculations. The following illustrates central difference.



The slopes of point a and point $a + \Delta x$, and of point a and point $a - \Delta x$ in function $y = f(x)$ are as follows:

$$\frac{f(a + \Delta x) - f(a)}{\Delta x} = \frac{\Delta y}{\Delta x}\ ,\ \frac{f(a) - f(a - \Delta x)}{\Delta x} = \frac{\nabla y}{\nabla x}$$

In the above, $\Delta y/\Delta x$ is called the forward difference, while $\nabla y/\nabla x$ is the backward difference.To calculate derivatives, the unit takes the average between the value of $\Delta y/\Delta x$ and $\nabla y/\nabla x$, thereby providing higher precision for derivatives.

This average, which is called the *central difference*, is expressed as:

$$f'(a) = \frac{1}{2}\left(\frac{f(a+\Delta x)-f(a)}{\Delta x} + \frac{f(a)-f(a-\Delta x)}{\Delta x}\right)$$

$$= \frac{f(a+\Delta x)-f(a-\Delta x)}{2\Delta x}$$

•To perform a differential calculation

Example **To determine the derivative at point x = 3 for the function $y = x^3 + 4x^2 + x - 6$, when the increase/decrease of x is defined as Δx = 1E – 5**

Input the function $f(x)$.

[AC] [OPTN] [F4] (CALC) [F2] (d/dx)
[X,θ,T] [^] [3] [+] [4] [X,θ,T] [x^2]
[+] [X,θ,T] [–] [6] [,]

```
d/dx(X^3+4X²+X-6,_
```

Input point $x = a$ for which you want to determine the derivative.

[3] [,]

```
d/dx(X^3+4X²+X-6,3,_
```

Input Δx, which is the increase/decrease of x.

[1] [EXP] [(–)] [5] [)]

```
d/dx(X^3+4X²+X-6,3,1E
-5)_
```

[EXE]

```
d/dx(X^3+4X²+X-6,3,1E
-5)
                   52
```

- In the function $f(x)$, only X can be used as a variable in expressions. Other variables (A through Z, r, θ) are treated as constants, and the value currently assigned to that variable is applied during the calculation.
- Input of Δx and the closing parenthesis can be omitted. If you omit Δx, the calculator automatically uses a value for Δx that is appropriate for the derivative value you are trying to determine.
- Discontinuous points or sections with drastic fluctuation can adversely affect precision or even cause an error.

Applications of Differential Calculations

- Differentials can be added, subtracted, multiplied and divided with each other.

Example $\frac{d}{dx} f(a) = f'(a),\ \frac{d}{dx} g(a) = g'(a)$

Therefore:

$f'(a) + g'(a),\ f'(a) \times g'(a)$

- Differential results can be used in addition, subtraction, multiplication, and division, and in functions.

Example $2 \times f'(a)$, $\log(f'(a))$

- Functions can be used in any of the terms ($f(x)$, a, Δx) of a differential.

Example $\frac{d}{dx}(\sin x + \cos x, \sin 0.5)$

- Note that you cannot use a Solve, differential, quadratic differential, integration, maximum/minimum value or Σ calculation expression inside of a differential calculation term.

- Pressing AC during calculation of a differential (while the cursor is not shown on the display) interrupts the calculation.
- Always perform trigonometric differentials using radians (Rad Mode) as the angle unit.

3-4 Quadratic Differential Calculations

P.64

After displaying the function analysis menu, you can input quadratic differentials using either of the two following formats.

[F3] (d^2/dx^2) $f(x)$ [,] a [,] n [)]

Final boundary (n = 1 to 15) — n

Differential coefficient point — a

$$\frac{d^2}{dx^2}\,(f(x), a, n) \Rightarrow \frac{d^2}{dx^2}\,f(a)$$

Quadratic differential calculations produce an approximate differential value using the following second order differential formula, which is based on Newton's polynomial interpretation.

$$f''(x) = \frac{-f(x-2h) + 16\,f(x-h) - 30\,f(x) + 16\,f(x+h) - f(x+2h)}{12h^2}$$

In this expression, values for "sufficiently small increments of x" are sequentially calculated using the following formula, with the value of m being substituted as m = 1, 2, 3 and so on.

$$h = \frac{1}{5^m}$$

The calculation is finished when the value of $f''(x)$ based on the value of h calculated using the last value of m, and the value of $f''(x)$ based on the value of h calculated using the current value of m are identical before the upper n digit is reached.

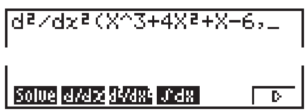
- Normally, you should not input a value for n. It is recommended that you only input a value for n when required for calculation precision.
- Inputting a larger value for n does not necessarily produce greater precision.

●To perform a quadratic differential calculation

Example **To determine the quadratic differential coefficient at the point where $x = 3$ for the function $y = x^3 + 4x^2 + x - 6$**
Here we will use a final boundary value of n = 6.

Input the function $f(x)$.

[AC] [OPTN] [F4] (CALC) [F3] (d^2/dx^2)
[X,θ,T] [^] [3] [+] [4] [X,θ,T] [x^2] [+]
[X,θ,T] [−] [6] [,]



[F3]

Input 3 as point a, which is differential coefficient point.

[3] [,]

```
d²/dx²(X^3+4X²+X−6,3,
_
```

Input 6 as n, which is final boundary.

[6] [)]

```
d²/dx²(X^3+4X²+X−6,3,
6)_
```

[EXE]

```
d²/dx²(X^3+4X²+X−6,3,
6)
                   26
```

- In the function $f(x)$, only X can be used as a variable in expressions. Other variables (A through Z, r, θ) are treated as constants, and the value currently assigned to that variable is applied during the calculation.
- Input of the final boundary value n and the closing parenthesis can be omitted.
- Discontinuous points or sections with drastic fluctuation can adversely affect precision or even cause an error.

Quadratic Differential Applications

- Arithmetic operations can be performed using two quadratic differentials.

$$\frac{d^2}{dx^2} f(a) = f''(a), \quad \frac{d^2}{dx^2} g(a) = g''(a)$$

Therefore:

$$f''(a) + g''(a), \quad f''(a) \times g''(a)$$

- The result of a quadratic differential calculation can be used in a subsequent arithmetic or function calculation.

$$2 \times f''(a), \quad \log(f''(a))$$

- Functions can be used within the terms ($f(x)$, a, n) of a quadratic differential expression.

$$\frac{d^2}{dx^2}(\sin x + \cos x, \sin 0.5)$$

- Note that you cannot use a Solve, differential, quadratic differential, integration, maximum/minimum value or Σ calculation expression inside of a quadratic differential calculation term.

- Use only integers within the range of 1 to 15 for the value of final boundary n. Use of a value outside this range produces an Ma ERROR.
- You can interrupt an ongoing quadratic differential calculation by pressing the [AC] key.
- You should always specify radians (Rad) as the unit of angle unit before performing a quadratic differential calculation using trigonometric functions.

3-5 Integration Calculations

P.64

To perform integration calculations, first display the function analysis menu, and then input the values shown in the formula below.

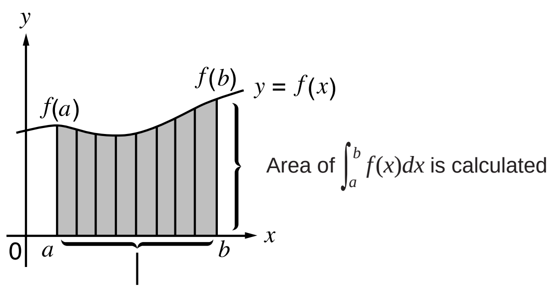
[F4] ($\int dx$) $f(x)$ [,] a [,] b [,] n [)]

n: *Number of Divisions (value for n in N = 2^n, n is an integer from 1 through 9)*

b: *End Point*

a: *Start Point*

$$\int(\, f(x),\, a,\, b,\, n) \Rightarrow \int_a^b f(x)dx,\ N = 2^n$$



N number of divisions

Integration calculations are performed by applying Simpson's Rule for the $f(x)$ function you input. This method requires that the number divisions be defined as $N = 2^n$, where the value of n is an integer in the range of 1 through 9. If you do not specify a value for n, the calculator automatically assigns a value in accordance with the integration being performed.

As shown in the illustration above, integration calculations are performed by calculating integral values from a through b for the function $y = f(x)$ where $a \leqq x \leqq b$, and $f(x) \geqq 0$*. This in effect calculates the surface area of the shaded area in the illustration.

* If $f(x) < 0$ where $a \leqq x \leqq b$, the surface area calculation produces negative values (surface area × – 1).

• To perform an integration calculation

Example **To perform the integration calculation for the function**
$$\int_1^5 (2x^2 + 3x + 4)\, dx$$

Input the function $f(x)$.

[AC] [OPTN] [F4] (CALC) [F4] ($\int dx$) [2] [X,θ,T] [x^2]
[+] [3] [X,θ,T] [+] [4] [,]

```
∫(2X²+3X+4,_
```

Input the start point and end point.

[1] [,] [5] [,]

```
∫(2X²+3X+4,1,5,_
```

Input the number of divisions.

[6] [)]

```
∫(2X²+3X+4,1,5,6)_
```

[EXE]

```
∫(2X²+3X+4,1,5,6)
             134.6666667
```

- In the function $f(x)$, only X can be used as a variable in expressions. Other variables (A through Z, r, θ) are treated as constants, and the value currently assigned to that variable is applied during the calculation.
- Input of n and the closing parenthesis can be omitted. If you omit n, the calculator automatically selects the most appropriate value.
- Calculation precision is theoretically ±1 at the least significant digit of the displayed result.

■ Application of Integration Calculation

- Integrals can be used in addition, subtraction, multiplication and division.

Example $\int_a^b f(x)\, dx + \int_c^d g(x)\, dx$

- Integration results can be used in addition, subtraction, multiplication and division, in functions.

Example $2 \times \int_a^b f(x)\, dx,\ \log\left(\int_a^b f(x)\, dx\right)$

- Functions can be used in any of the terms ($f(x)$, a, b, n) of an integral.

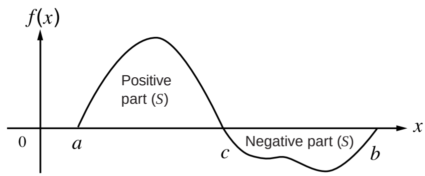
Example $\int_{\sin 0.5}^{\cos 0.5} (\sin x + \cos x)\, dx = \int (\sin x + \cos x, \sin 0.5, \cos 0.5, 5)$

- Note that you cannot use a Solve, differential, quadratic differential, integration, maximum/minimum value or Σ calculation expression inside of an integration calculation term.

- Pressing AC during calculation of an integral (while the cursor is not shown on the display) interrupts the calculation.
- Always perform trigonometric integrations using radians (Rad Mode) as the angle unit.
- This unit utilizes Simpson's rule for integration calculation. As the number of significant digits is increased, more calculation time is required. In some cases, calculation results may be erroneous even after considerable time is spent performing a calculation. In particular, when significant digits are less than 1, an ERROR (Ma ERROR) sometimes occurs.
- Integration involving certain types of functions or ranges can result in relatively large errors being generated in the values produced.

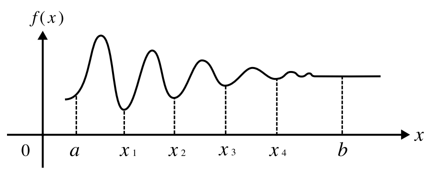
Note the following points to ensure correct integration values.

(1) When cyclical functions for integration values become positive or negative for different divisions, perform the calculation for single cycles, or divide between negative and positive, and then add the results together.



$$\int_a^b f(x)dx = \underbrace{\int_a^c f(x)dx}_{\text{Positive part }(S)} + \underbrace{\left(-\int_c^b f(x)dx\right)}_{\text{Negative part }(S)}$$

(2) When minute fluctuations in integration divisions produce large fluctuations in integration values, calculate the integration divisions separately (divide the large fluctuation areas into smaller divisions), and then add the results together.



$$\int_a^b f(x)dx = \int_a^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \ldots.. + \int_{x_4}^{b} f(x)dx$$

3-6 Maximum/Minimum Value Calculations

P.64

After displaying the function analysis menu, you can input maximum/minimum calculations using the formats below, and solve for the maximum and minimum of a function within interval $a \leqq x \leqq b$.

●Minimum Value

F6 (▷) F1 (FMin) $f(x)$, a , b , n)

Precision (n = 1 ~ 9)
End point of interval
Start point of interval

●Maximum Value

F6 (▷) F2 (FMax) $f(x)$, a , b , n)

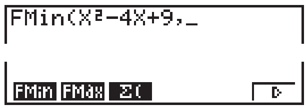
Precision (n = 1 ~ 9)
End point of interval
Start point of interval

●To perform maximum/minimum value calculations

Example 1 **To determine the minimum value for the interval defined by start point a = 0 and end point b = 3, with a precision of n = 6 for the function $y = x^2 - 4x + 9$**

Input $f(x)$.

AC OPTN F4 (CALC)
F6 (▷) F1 (FMin)
X,θ,T x^2 − 4 X,θ,T + 9 ,



F1

Input the interval a = 0, b = 3.

0 , 3 ,

Input the precision n = 6.

6)

EXE

Example 2 **To determine the maximum value for the interval defined by start point a = 0 and end point b = 3, with a precision of n = 6 for the function $y = -x^2 + 2x + 2$**

Input $f(x)$.

[AC] [OPTN] [F4](CALC)

[F6](▷) [F2](FMax)

[(-)] [X,θ,T] [x^2] [+] [2] [X,θ,T] [+] [2] [,]

```
FMax(-X²+2X+2,_

FMin FMax Σ(          ▷
```

[F2]

Input the interval a = 0, b = 3.

[0] [,] [3] [,]

```
FMax(-X²+2X+2,0,3,_
```

Input the precision n = 6.

[6] [)]

```
FMax(-X²+2X+2,0,3,6)_
```

[EXE]

- In the function $f(x)$, only X can be used as a variable in expressions. Other variables (A through Z, r, θ) are treated as constants, and the value currently assigned to that variable is applied during the calculation.
- Input of n and the closing parenthesis following the precision value can be omitted.
- Discontinuous points or sections with drastic fluctuation can adversely affect precision or even cause an error.
- Note that you cannot use a Solve, differential, quadratic differential, integration, maximum/minimum value or Σ calculation expression inside of a maximum/minimum calculation term.
- Inputting a larger value for n increases the precision of the calculation, but it also increases the amount of time required to perform the calculation.

- The value you input for the end point of the interval (b) must be greater than the value you input for the start point (a). Otherwise an Ma ERROR is generated.
- You can interrupt an ongoing maximum/minimum calculation by pressing the [AC] key.
- You can input an integer in the range of 1 to 9 for the value of n. Using any value outside this range causes an error (Arg ERROR).

3-7 Σ Calculations

To perform Σ calculations, first display the function analysis menu, and then input the values shown in the formula below.

[F6] (▷) [F3] (Σ() a_k [,] k [,] α [,] β [,] n [)]

Distance between partitions (n)
Last term of sequence {a_k} (β)
Initial term of sequence {a_k} (α)
Variable used by sequence {a_k} (k)

$$\Sigma(a_k, k, \alpha, \beta, n) = \sum_{k=\alpha}^{\beta} a_k$$

Σ calculation is the calculation of the partial sum of sequence {a_k}, using the following formula.

$$S = a_\alpha + a_{\alpha+1} + \ldots\ldots + a_\beta = \sum_{k=\alpha}^{\beta} a_k$$

■ Example Σ Calculation

Example **To calculate the following:**

$$\sum_{k=2}^{6} (k^2 - 3k + 5)$$

Use n = 1 as the distance between partitions.

Input sequence {a_k}

[AC] [OPTN] [F4] (CALC) [F6] (▷) [F3] (Σ()
[ALPHA] [K] [x^2] [−] [3] [ALPHA] [K] [+] [5] [,]

```
Σ(K²-3K+5,_
```

Input variable used by sequence {a_k}

[ALPHA] [K] [,]

```
Σ(K²-3K+5,K,_
```

Input the initial term of sequence {a_k} and last term of sequence {a_k}.

[2] [,] [6] [,]

```
Σ(K²-3K+5,K,2,6,_
```

Input n.

[1] [)]

```
Σ(K²-3K+5,K,2,6,1)_
```

EXE

```
Σ(K²-3K+5,K,2,6,1)
                55
```

- You can use only one variable in the function for input sequence $\{a_k\}$.
- Input integers only for the initial term of sequence $\{a_k\}$ and last term of sequence $\{a_k\}$.
- Input of n and the closing parentheses can be omitted. If you omit n, the calculator automatically uses $n = 1$.

■ Σ Calculation Applications

●Arithmetic operations using Σ calculation expressions

Expressions: $S_n = \sum_{k=1}^{n} a_k,\ T_n = \sum_{k=1}^{n} b_k$

Possible operations: $S_n + T_n$, $S_n - T_n$, etc.

●Arithmetic and function operations using Σ calculation results

$2 \times S_n$, $\log(S_n)$, etc.

●Function operations using Σ calculation terms (a_k, k)

$\Sigma(\sin k, k, 1, 5)$, etc.

- Note that you cannot use a Solve, differential, quadratic differential, integration, maximum/minimum value or Σ calculation expression inside of a Σ calculation term.

■ Σ Calculation Precautions

- Make sure that the value used as the final term β is greater than the value used as the initial term α. Otherwise, an Ma ERROR will occur.
- To interrupt an ongoing Σ calculation (indicated when the cursor is not on the display), press the AC key.