Chapter 6

Matrix Calculations

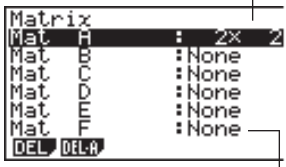
26 matrix memories (Mat A through Mat Z) plus a Matrix Answer Memory (MatAns), make it possible to perform the following matrix operations.

- Addition, subtraction, multiplication
- Scalar product calculations
- Determinant calculations
- Matrix transposition
- Matrix inversion
- Matrix squaring
- Raising a matrix to a specific power
- Absolute value, integer part extraction, fractional part extraction, maximum integer calculations
- Matrix modification using matrix commands

6-1 Before Performing Matrix Calculations

In the Main Menu, select the **MAT** icon and press [EXE] to enter the Matrix Mode and display its initial screen.

2 (row) × 2 (column) matrix



Not dimension preset

[F1] (DEL) ..... Delete specific matrix

[F2] (DEL•A) .... Delete all matrices

- The maximum matrix dimension (size) is 255 (rows) × 255 (columns).

■ About Matrix Answer Memory (MatAns)

The calculator automatically store matrix calculation results in Matrix Answer Memory. Note the following points about Matrix Answer Memory.

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- Whenever you perform a matrix calculation, the current Matrix Answer Memory contents are replaced by the new result. The previous contents are deleted and cannot be recovered.
- Inputting values into a matrix does not affect Matrix Answer Memory contents.

■ Creating a Matrix

To create a matrix, you must first define its dimensions (size) in the MATRIX list. Then you can input values into the matrix.

●To specify the dimensions of a matrix

Example **To create a 2-row × 3-column matrix in the area named Mat B**

Highlight Mat B.

[▼]

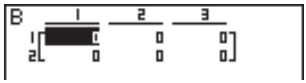
Specify the number of rows.

[2] [EXE]

Specify the number of columns.

[3]

[EXE]



- All of the cells of a new matrix contain the value 0.
- If "Mem ERROR" remains next to the matrix area name after you input the dimensions, it means there is not enough free memory to create the matrix you want.

●To input cell values

Example **To input the following data into Matrix B :**

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Select Mat B.

(▼)

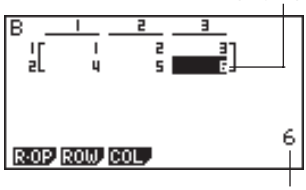
Highlighted cell (up to six digits can be displayed)

[EXE]

[1] [EXE] [2] [EXE] [3] [EXE]

[4] [EXE] [5] [EXE] [6] [EXE]

(Data is input into the highlighted cell. Each time you press [EXE], the highlighting move to the next cell to the right.)



Value in currently highlighted cell

- Displayed cell values show positive integers up to six digits, and negative integers up to five digits (one digit used for negative sign). Exponential values are shown with up to two digits for the exponent. Fractional values are not displayed.
- You can see the entire value assigned to a cell by using the cursor keys to move the highlighting to the cell whose value you want to view.
- The amount of memory required for a matrix is ten bytes per cell. This means that a 3×3 matrix requires 90 bytes of memory ($3 \times 3 \times 10 = 90$).

■ Deleting Matrices

You can delete either a specific matrix or all matrices in memory.

●To delete a specific matrix

1. While the MATRIX list is on the display, use (▲) and (▼) to highlight the matrix you want to delete.
2. Press [F1] (DEL).

[F1] (DEL)

3. Press F1 (YES) to delete the matrix or F6 (NO) to abort the operation without deleting anything.

   • The indicator “None” replaces the dimensions of the matrix you delete.

●To delete all matrices

1. While the MATRIX list is on the display, press F2 (DEL•A).

   F2 (DEL•A)

2. Press F1 (YES) to delete all matrices in memory or F6 (NO) to abort the operation without deleting anything.

   • The indicator “None” is shown for all the matrices.

6-2 Matrix Cell Operations

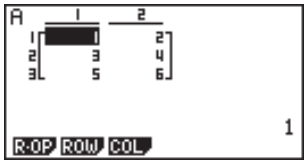
You can perform any of the following operations involving the cells of a matrix on the display.

- Row swapping, scalar product, addition
- Row deletion, insertion, addition
- Column deletion, insertion, addition

Use the following procedure to prepare a matrix for cell operations.

1. While the MATRIX list is on the display, use ▲ and ▼ to highlight the name of the matrix you want to use.
2. Press [EXE].

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$



[F1] (R•OP) ..... Row calculation menu
[F2] (ROW)..... Row operation menu
[F3] (COL) ..... Column operation menu

All of the following examples use Matrix A recalled by the above operation.

■ Row Calculations

The following menu appears whenever you press [F1] (R•OP) while a recalled matrix is on the display.

[F1](R•OP)

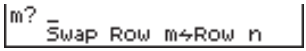
[F1] (Swap) ..... Row swap
[F2] (xRw) ..... Scalar product for a specific row
[F3] (xRw+) ..... Addition of scalar product of specific row to another row
[F4] (Rw+) ..... Addition of contents of specific row to another row

●To swap two rows

Example **To swap rows two and three of the following matrix :**

$$\textbf{Matrix A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \\ \mathbf{5} & \mathbf{6} \end{bmatrix}$$

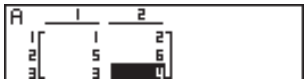
F1 (R•OP) F1 (Swap)



Input the number of the rows you want to swap.

2 EXE

3 EXE



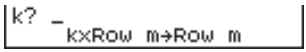
●To calculate the scalar product of a row

Example **To calculate the scalar product of row 2 of the following matrix by 4 :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

F1 (R•OP)

F2 (×Rw)

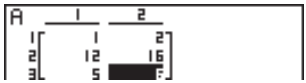


Input multiplier value.

4 EXE

Specify row number.

2 EXE



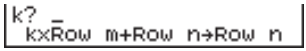
●To calculate the scalar product of a row and add the result to another row

Example **To calculate the scalar product of row 2 of the following matrix by 4 and add the result to row 3 :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

F1 (R•OP)

F3 (×Rw+)



Input multiplier value.

4 EXE

Specify number of row whose scalar product should be calculated.

2 EXE

Specify number of row where result should be added.

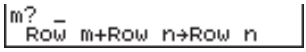
3 EXE

●To add two rows together

Example **To add row 2 to row 3 of the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

[F1](R•OP)
[F4](Rw+)

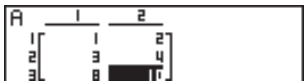


Specify number of row to be added.

[2] [EXE]

Specify number of row to be added to.

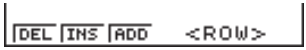
[3] [EXE]



■ Row Operations

The following menu appears whenever you press [F2] (ROW) while a recalled matrix is on the display.

[F2] (ROW)



[F1] [F2] [F3]

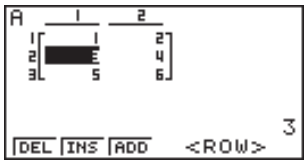
[F1] (DEL) ..... Delete row
[F2] (INS) ..... Insert row
[F3] (ADD) ..... Add row

●To delete a row

Example **To delete row 2 of the following matrix :**

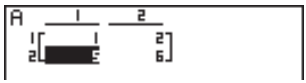
$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

[F2](ROW)[▼]



[F1]

[F1](DEL)

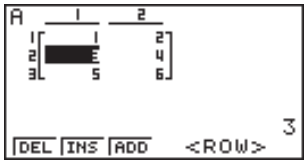


●To insert a row

Example **To insert a new row between rows one and two of the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

F2(ROW)▼



F2

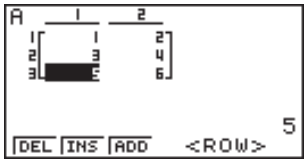
F2(INS)

●To add a row

Example **To add a new row below row 3 of the following matrix :**

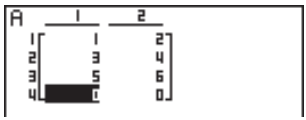
$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

F2(ROW)▼▼



F3

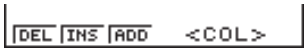
F3(ADD)



■ Column Operations

The following menu appears whenever you press F3 (COL) while a recalled matrix is on the display.

F3 (COL)



F1 F2 F3

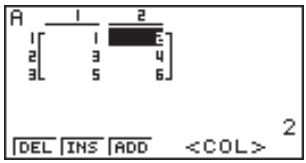
F1 (DEL) ..... Delete column
F2 (INS) ..... Insert column
F3 (ADD) ..... Add column

●To delete a column

Example **To delete column 2 of the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

F3 (COL) ▶



F1

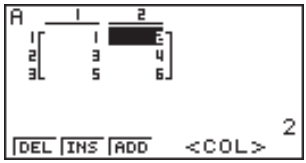
F1 (DEL)

●To insert a column

Example **To insert a new column between columns 1 and 2 of the following matrix :**

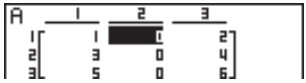
$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

F3 (COL) ▶



F2

F2(INS)

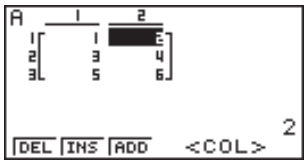


●To add a column

Example **To add a new column to the right of column 2 of the following matrix :**

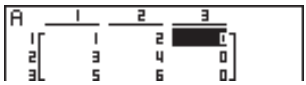
$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

F3(COL)▶



F3

F3(ADD)



6-3 Modifying Matrices Using Matrix Commands

In addition to using the MATRIX list to create and modify a matrix, you can also use matrix commands to input data and create a matrix without actually displaying it.

●To display the matrix commands

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1. From the Main Menu, select the **RUN** icon and press [EXE].
2. Press [OPTN] to display the option menu.
3. Press [F2] (MAT) to display the matrix operation menu.

[OPTN] [F2] (MAT)

The following describes only the matrix command menu items that are used for creating matrices and inputting matrix data.

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- [F1] (Mat) ..... Mat command (matrix specification)
- [F2] (M→L) ..... Mat→List command (assign contents of selected column to list file)
- [F5] (Aug) ..... Augment command (link two matrices)
- [F6] (▷) ..... Next menu

[F6] (▷)

- [F1] (Iden) ..... Identity command (identity matrix input)
- [F2] (Dim) ..... Dim command (dimension check)
- [F3] (Fill) ..... Fill command (identical cell values)
- [F6] (▷) ..... Previous menu

■ Matrix Data Input Format

The following shows the format you should use when inputting data to create a matrix using the matrix operation menu's Mat command.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

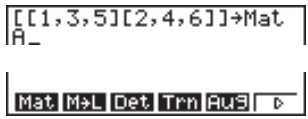
= [[a_{11}, a_{12}, ..., a_{1n}] [a_{21}, a_{22}, ..., a_{2n}] .... [a_{m1}, a_{m2}, ..., a_{mn}]]
→ Mat [letter A through Z]

- The maximum value of both *m* and *n* is 255.

Example 1 **To input the following data as Matrix A :**

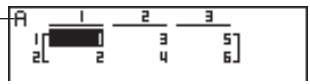
$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

OPTN F2 (MAT)
SHIFT [SHIFT [1 , 3 , 5
SHIFT] SHIFT [2 , 4 , 6
SHIFT] SHIFT] → F1 (Mat) ALPHA A



EXE

Matrix name



- An error (Mem ERROR) occurs if memory becomes full as you are inputting data.
- You can also use the above format inside a program that inputs matrix data.

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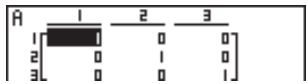
●To input an identity matrix

Use the matrix operation menu's Identity command (F1) to create an identity matrix.

Example 2 **To create a 3 × 3 identity matrix as Matrix A**

OPTN F2 (MAT)
F6 (▷) F1 (Iden) 3 →
Number of rows/columns
F6 (▷) F1 (Mat) ALPHA A

EXE



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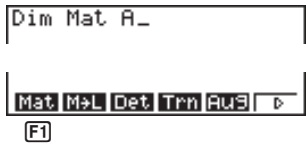
●To check the dimensions of a matrix

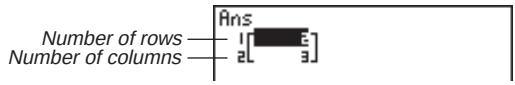
Use the matrix operation menu's Dim command (F2) to check the dimensions of an existing matrix.

Example 3 **To check the dimensions of Matrix A, which was input in Example 1**

OPTN F2 (MAT)
F6 (▷) F2 (Dim) F6 (▷) F1 (Mat)
ALPHA A



[EXE]



The display shows that Matrix A consists of two rows and three columns.

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■ Modifying Matrices Using Matrix Commands

You can also use matrix commands to assign values to and recall values from an existing matrix, to fill in all cells of an existing matrix with the same value, to combine two matrices into a single matrix, and to assign the contents of a matrix column to a list file.

●To assign values to and recall values from an existing matrix

Use the following format with the matrix operation menu's Mat command ([F1]) to specify a cell for value assignment and recall.

Mat X [*m*, *n*]

X ..... matrix name (A through Z, or Ans)

m ..... row number

n ..... column number

Example 1 **Assign 10 to the cell at row 1, column 2 of the following matrix :**

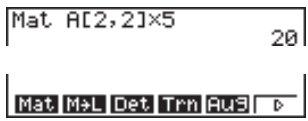
$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

[1] [0] [→] [OPTN] [F2] (MAT) [F1] (Mat)
[ALPHA] [A] [SHIFT] [(] [1] [,] [2] [SHIFT] [)]
[EXE]

[F1]

Example 2 **Multiply the value in the cell at row 2, column 2 of the above matrix by 5**

[OPTN] [F2] (MAT) [F1] (Mat)
[ALPHA] [A] [SHIFT] [(] [2] [,] [2] [SHIFT] [)]
[×] [5] [EXE]



[F1]

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●To fill a matrix with identical values and to combine two matrices into a single matrix

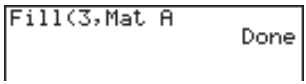
Use the matrix operation menu's Fill command (F3) to fill all the cells of an existing matrix with an identical value and the Augment command (F5) to combine two existing matrices into a single matrix.

Example 1 **To fill all of the cells of Matrix A with the value 3**

OPTN F2 (MAT)
F6 (▷) F3 (Fill) 3 ,
Filler value
F6 (▷) F1 (Mat) ALPHA A

F1

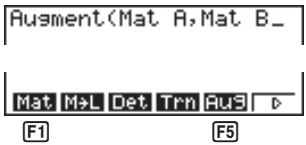
EXE



Example 2 **To combine the following two matrices :**

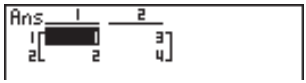
$$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

OPTN F2 (MAT)
F5 (Aug) F1 (Mat) ALPHA A ,
F1 (Mat) ALPHA B



F1 F5

EXE



- The two matrices you combine must have the same number of rows. An error (Ma ERROR) occurs if you try to combine two matrices that have different numbers of rows.

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●To assign the contents of a matrix column to a list file

Use the following format with the matrix operation menu's Mat→List command (F2) to specify a column and a list file.

Mat → List (Mat X, *m*) → List *n*
X = matrix name (A through Z, or Ans)
m = column number
n = list number

Example **To assign the contents of column 2 of the following matrix to list file 1 :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

[OPTN] [F2](MAT)
[F2](M→L)[F1](Mat)
[ALPHA][A][,][2][)][→]
Column number
[OPTN][F1](LIST)[F1](List)[1][EXE]

[F1]

You can use Matrix Answer Memory to assign the results of the above matrix input and edit operations to a matrix variable. To do so, use the following syntax.

- Fill (n, Mat α) → Mat β
- Augment (Mat α, Mat β) → Mat γ

In the above, α, β, and γ are variable names A through Z, and n is any value. The above does not affect the contents of Matrix Answer Memory.

6-4 Matrix Calculations

Use the matrix command menu to perform matrix calculation operations.

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●To display the matrix commands

1. From the Main Menu, select the **RUN** icon and press [EXE].
2. Press [OPTN] to display the option menu.
3. Press [F2] (MAT) to display the matrix command menu.

[OPTN] [F2] (MAT)

The following describes only the matrix commands that are used for matrix arithmetic operations.

[F1] (Mat) ..... Mat command (matrix specification)
[F3] (Det) ..... Det command (determinant command)
[F4] (Trn) ..... Trn command (transpose matrix command)
[F6] (▷) ..... Next menu

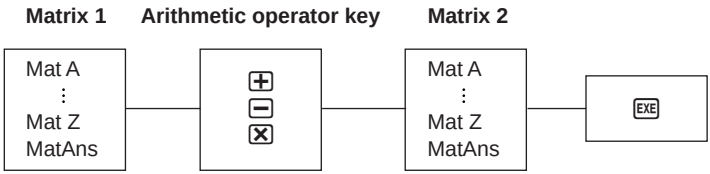
[F6] (▷)

[F1] (Iden) ..... Identity command (identity matrix input)
[F6] (▷) ..... Previous menu

All of the following examples assume that matrix data is already stored in memory.

■ Matrix Arithmetic Operations

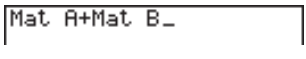
The following is the format for matrix arithmetic operations.



Example 1 **To add the following two matrices (Matrix A + Matrix B) :**

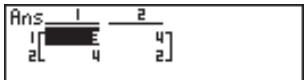
$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$$

F1 (Mat) ALPHA A +
F1 (Mat) ALPHA B



F1

EXE

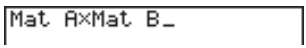


This display indicates the following result.

$$A + B = \begin{bmatrix} 3 & 4 \\ 4 & 2 \end{bmatrix}$$

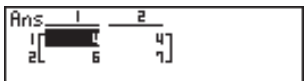
Example 2 **To multiply the two matrices in Example 1 (Matrix A × Matrix B)**

F1 (Mat) ALPHA A ×
F1 (Mat) ALPHA B



F1

EXE



This display indicates the following result.

$$A \times B = \begin{bmatrix} 4 & 4 \\ 6 & 7 \end{bmatrix}$$

- The two matrices must have the same dimensions in order to be added or subtracted. An error (Dim ERROR) occurs if you try to add or subtract matrices of different dimensions.
- For multiplication, the number of columns in Matrix 1 must match the number of rows in Matrix 2. Otherwise, an error (Dim ERROR) occurs.

- You can use an identity matrix in place of Matrix 1 or Matrix 2 in the matrix arithmetic format. Use the matrix command menu's Identity (F1) command to input the identity matrix.

Example 3 **To multiply Matrix A (from Example 1) by a 2 × 2 identity matrix**

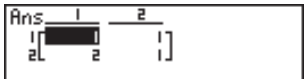
F1 (Mat) ALPHA A ×

F6 (▷) F1 (Iden) 2

Number of rows and columns.

F1

EXE

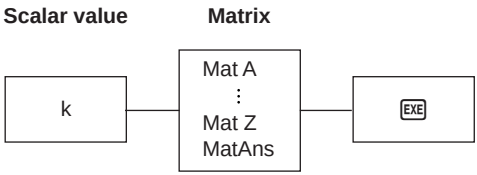


This display indicates the following result.

$$A \times E = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

■ Matrix Scalar Product

The following is the format for calculating a matrix scalar product, which multiplies the value in each cell of the matrix by the same value.



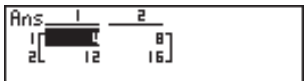
Example **Calculate the scalar product of the following matrix using a multiplier value of 4 :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

4 F1 (Mat) ALPHA A

F1

EXE



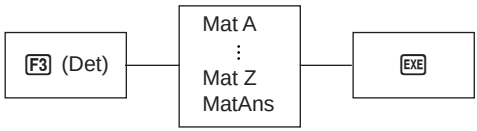
This display indicates the following result.

$$4A = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$$

■ Determinant

The following is the format for obtaining a determinant.

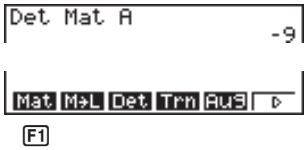
Matrix



Example **Obtain the determinant for the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & -2 & 0 \end{bmatrix}$$

F3 (Det) F1 (Mat) ALPHA A EXE



This display indicates determinant |A| = –9.

- Determinants can be obtained only for square matrices (same number of rows and columns). Trying to obtain a determinant for a matrix that is not square produces an error (Dim ERROR).

- The determinant of a 2×2 matrix is calculated as shown below.

$$|A| = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

- The determinant of a 3×3 matrix is calculated as shown below.

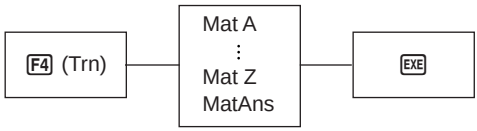
$$|A| = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

■ Matrix Transposition

A matrix is transposed when its rows become columns and its columns become rows. The following is the format for matrix transposition.

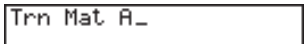
Matrix



Example **To transpose the following matrix:**

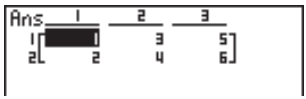
$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

F4 (Trn) F1 (Mat) ALPHA A



F1

EXE



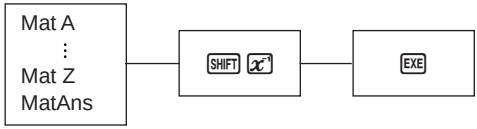
This operation produces the following result.

$$\text{A}^{t} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

■ Matrix Inversion

The following is the format for matrix inversion.

Matrix



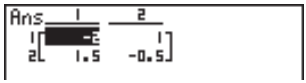
Example **To invert the following matrix :**

$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

F1 (Mat) ALPHA A SHIFT x^{-1}

F1

EXE



This operation produces the following result.

$$\mathbf{A^{-1}} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

- Only square matrices (same number of rows and columns) can be inverted. Trying to invert a matrix that is not square produces an error (Dim ERROR).
- A matrix with a value of zero cannot be inverted. Trying to invert a matrix with value of zero produces an error (Ma ERROR).
- Calculation precision is affected for matrices whose value is near zero.

- A matrix being inverted must satisfy the conditions shown below.

$$\mathbf{A\,A^{-1} = A^{-1}\,A = E} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- The following shows the formula used to invert Matrix A into inverse matrix A^{-1}.

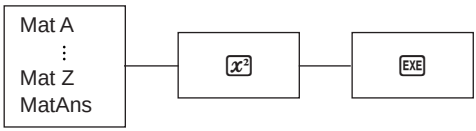
$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\mathbf{A^{-1}} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ Note that $ad - bc \neq 0$.

■ Squaring a Matrix

The following is the format for squaring a matrix.

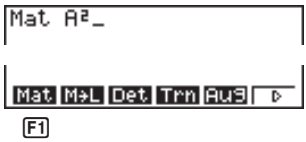
Matrix



Example **To square the following matrix :**

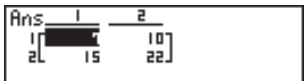
$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

F1 (Mat) ALPHA A x^2



F1

EXE

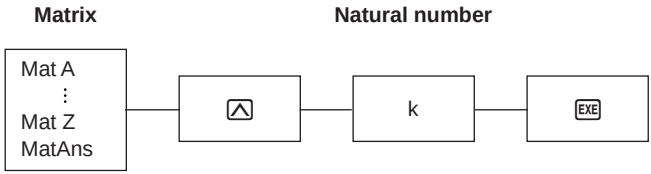


This operation produces the following result.

$$A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

■ Raising a Matrix to a Power

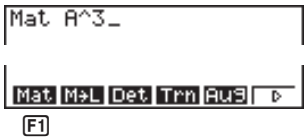
The following is the format for raising a matrix to a power.



Example **To raise the following matrix to the third power :**

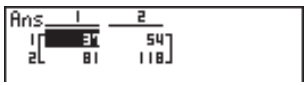
$$\text{Matrix A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

F1 (Mat) ALPHA A ^ 3



F1

EXE



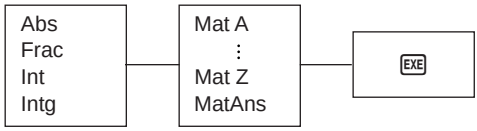
This operation produces the following result.

$$A^3 = \begin{bmatrix} 37 & 54 \\ 81 & 118 \end{bmatrix}$$

■ Determining the Absolute Value, Integer Part, Fraction Part, and Maximum Integer of a Matrix

The following is the format for using a matrix in built in functions to obtain an absolute value, integer part, fraction part, and maximum integer.

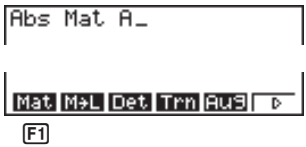
Function command **Matrix**



Example **To determine the absolute value of the following matrix :**

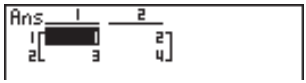
$$\text{Matrix A} = \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$$

OPTN F6 (▷) F4 (NUM) F1 (Abs)
OPTN F2 (MAT) F1 (Mat) ALPHA A



F1

EXE



This operation produces the following result.

$$\text{Abs A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

- Determinants and inverse matrices are calculated using the elimination method, so errors (such as dropped digits) may be generated.
- Matrix operations are performed individually on each cell, so calculations may required considerable time to complete.
- The calculation precision of displayed results for matrix calculations is ± 1 at the least significant digit.
- If a matrix calculation result is too large to fit into Matrix Answer Memory, an error (Mem ERROR) occurs.
- You can use the following operations to transfer Matrix Answer Memory contents to another matrix (or when Matrix Answer Memory contains a determinant to a variable).

  MatAns → Mat α

  In the above, α is a variable name A through Z. The above does not affect the contents of Matrix Answer Memory.