3. Confidence Interval (INTR)

A confidence interval is a range (interval) that includes the population mean value.

A confidence interval that is too broad makes it difficult to get an idea of where the population value (true value) is located. A narrow confidence interval, on the other hand, limits the population value and makes it possible to obtain reliable results. The most commonly used confidence levels are 95% and 99%. Raising the confidence level broadens the confidence interval, while lowering the confidence level narrows the confidence level, but it also increases the chance of accidently overlooking the population value. With a 95% confidence interval, for example, the population value is not included within the resulting intervals 5% of the time.

When you plan to conduct a survey and then t test and Z test the data, you must also consider the sample size, confidence interval width, and confidence level. The confidence level changes in accordance with the application.

1-Sample Z Interval calculates the confidence interval for an unknown population mean when standard deviation is known.

2-Sample Z Interval calculates the confidence interval for the difference between two population means when the standard deviations of two samples are known.

1-Prop Z Interval uses the number of data to calculate the confidence interval for an unknown proportion of successes .

2-Prop Z Interval uses the number of data items to calculate the confidence interval for the difference between the propotion of successes in two populations .

1-Sample t Interval calculates the confidence interval for an unknown population mean when the population standard deviation is unknown .

2-Sample t Interval calculates the confidence interval for the difference between two population means when both population standard deviations are unknown.

On the initial STAT Mode screen, press F4 (INTR) to display the confidence interval menu, which contains the following items.

- F4 (INTR) 1 (Z) ... Z intervals (p.33)
  2 (T)... t intervals (p.38)

There is no graphing for confidence interval functions.

●General Confidence Interval Precautions

Inputting a value in the range of 0 ≦ C-Level<1 for the C-Level setting sets you value you input. Inputting a value in the range of 1 ≦ C-Level<100 sets a value equivalent to your input divided by 100.

Inputting a value of 100 or greater, or a negative value causes an error (Ma ERROR).

■ Z Interval

•1-Sample Z Interval

1-Sample Z Interval calculates the confidence interval for an unknown population mean when standard deviation is known.

The following is the confidence interval.

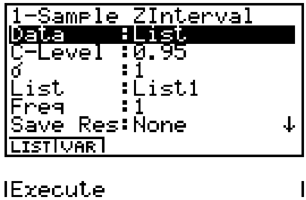
$$Left = \bar{x} - Z\left(\frac{\alpha}{2}\right)\frac{\sigma}{\sqrt{n}}$$

$$Right = \bar{x} + Z\left(\frac{\alpha}{2}\right)\frac{\sigma}{\sqrt{n}}$$

However, α is not the confidence level itself. The value 100 (1-α) % is the confidence level. When the confidence level is 95%, for example, inputting 0.95 produces 1 – 0.95 = 0.05 = α.

Perform the following key operation from the statistical data list.

F4 (INTR)
1 (Z)
1 (1-Smpl)



The following shows the meaning of each item in the case of list data specification.

Data .... data type
C-Level .... confidence level (0 ≦ C-Level < 1)
σ .... population standard deviation (σ > 0)
List .... list whose contents you want to use as sample data
Freq .... sample frequency
Save Res .... list for storage of calculation results (None or List 1 to 20)
Execute .... executes a calculation

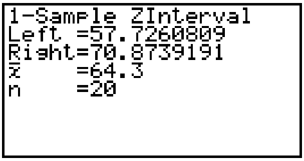
The following shows the meaning of parameter data specification items that are different from list data specification.

$\bar{x}$ .... mean of sample
n .... size of sample (positive integer)

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

- [F1] (CALC) ... Performs the calculation.

Calculation Result Output Example



Left .... interval lower limit (left edge)
Right .... interval upper limit (right edge)
$\bar{x}$ .... mean of sample
$x\sigma_{n-1}$ .... sample standard deviation
(Displayed only for Data: List Setting)
n .... size of sample

• 2-Sample *Z* Interval

2-Sample *Z* Interval calculates the confidence interval for the difference between two population means when the standard deviations of two samples are known.
The following is confidence interval. The value 100 (1-α) % is the confidence level.

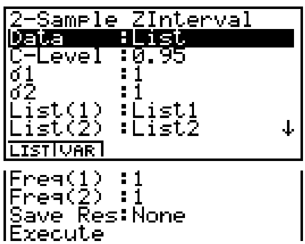
$$Left = (\bar{x}_1 - \bar{x}_2) - Z\left(\frac{\alpha}{2}\right)\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$Right = (\bar{x}_1 - \bar{x}_2) + Z\left(\frac{\alpha}{2}\right)\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$\bar{x}_1$: mean of sample 1
$\bar{x}_2$: mean of sample 2
σ_1 : population standard deviation of sample 1
σ_2 : population standard deviation of sample 2
n_1 : size of sample 1
n_2 : size of sample 2

Perform the following key operation from the statistical data list.

[F4] (INTR)
[1] (Z)
[2] (2-Smpl)



The following shows the meaning of each item in the case of list data specification.

Data .... data type
C-Level .... confidence level ($0 \leqq$ C-Level < 1)
σ_1 .... population standard deviation of sample 1 ($\sigma_1 > 0$)
σ_2 .... population standard deviation of sample 2 ($\sigma_2 > 0$)
List(1) .... list whose contents you want to use as data of sample 1
List(2) .... list whose contents you want to use as data of sample 2
Freq(1) .... frequency of sample 1
Freq(2) .... frequency of sample 2
Save Res .... list for storage of calculation results (None or List 1 to 20)
Execute .... executes a calculation

The following shows the meaning of parameter data specification items that are different from list data specification.

```
x̄1        :0
n1        :0
x̄2        :0
n2        :0
```

$\bar{x}_1$ .... mean of sample 1
n_1 .... size (positive integer) of sample 1
$\bar{x}_2$ .... mean of sample 2
n_2 .... size (positive integer) of sample 2

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

- F1 (CALC) ... Performs the calculation.

Calculation Result Output Example

```
2-Sample ZInterval
Left =6.30341903
Right=25.696581
x̄1    =418
x̄2    =402
n1    =40
n2    =50
```

Left .... interval lower limit (left edge)
Right .... interval upper limit (right edge)
$\bar{x}_1$ .... mean of sample 1
$\bar{x}_2$ .... mean of sample 2
$x_1\sigma_{n-1}$ .... standard deviation of sample 1
(Displayed only for Data: List Setting)
$x_2\sigma_{n-1}$ .... standard deviation of sample 2
(Displayed only for Data: List Setting)
n_1 .... size of sample 1
n_2 .... size of sample 2

•1-Prop Z Interval

1-Prop Z Interval uses the number of data to calculate the confidence interval for an unknown proportion of successes.

The following is the confidence interval. The value 100 (1-α) % is the confidence level.

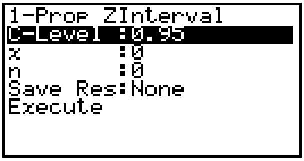
$$Left = \frac{x}{n} - Z\left(\frac{\alpha}{2}\right)\sqrt{\frac{1}{n}\left(\frac{x}{n}\left(1-\frac{x}{n}\right)\right)}$$

$$Right = \frac{x}{n} + Z\left(\frac{\alpha}{2}\right)\sqrt{\frac{1}{n}\left(\frac{x}{n}\left(1-\frac{x}{n}\right)\right)}$$

n : size of sample
x : data

Perform the following key operation from the statistical data list.

F4 (INTR)
1 (Z)
3 (1-Prop)



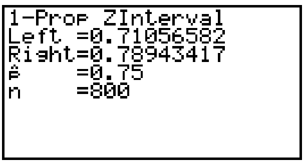
Data is specified using parameter specification. The following shows the meaning of each item.

C-Level .... confidence level (0 ≦ C-Level < 1)
x .... data (0 or positive integer)
n .... size of sample (positive integer)
Save Res .... list for storage of calculation results (None or List 1 to 20)
Execute .... executes a calculation

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

- F1 (CALC) ... Performs the calculation.

Calculation Result Output Example



Left .... interval lower limit (left edge)
Right .... interval upper limit (right edge)
$\hat{p}$ .... estimated sample proportion
n .... size of sample

• 2-Prop Z Interval

2-Prop Z Interval uses the number of data items to calculate the confidence interval for the defference between the proportion of successes in two populations.

The following is the confidence interval. The value 100 (1-α) % is the confidence level.

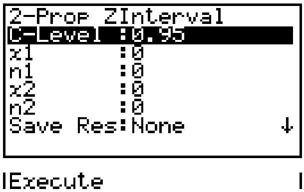
$$Left = \frac{x_1}{n_1} - \frac{x_2}{n_2} - Z\left(\frac{\alpha}{2}\right)\sqrt{\frac{\frac{x_1}{n_1}\left(1-\frac{x_1}{n_1}\right)}{n_1} + \frac{\frac{x_2}{n_2}\left(1-\frac{x_2}{n_2}\right)}{n_2}}$$

$$Right = \frac{x_1}{n_1} - \frac{x_2}{n_2} + Z\left(\frac{\alpha}{2}\right)\sqrt{\frac{\frac{x_1}{n_1}\left(1-\frac{x_1}{n_1}\right)}{n_1} + \frac{\frac{x_2}{n_2}\left(1-\frac{x_2}{n_2}\right)}{n_2}}$$

n_1, n_2 : size of sample
x_1, x_2 : data

Perform the following key operation from the statistical data list.

F4 (INTR)
1 (Z)
4 (2-Prop)



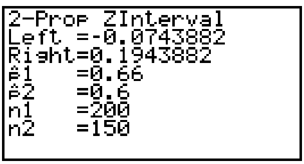
Data is specified using parameter specification. The following shows the meaning of each item.

C-Level .... confidence level ($0 \leqq$ C-Level < 1)
x_1 .... data value ($x_1 \geqq 0$) of sample 1
n_1 .... size (positive integer) of sample 1
x_2 .... data value ($x_2 \geqq 0$) of sample 2
n_2 .... size (positive integer) of sample 2
Save Res .... list for storage of calculation results (None or List 1 to 20)
Execute .... executes a calculation

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

- F1 (CALC) ... Performs the calculation.

Calculation Result Output Example



Left .... interval lower limit (left edge)
Right .... interval upper limit (right edge)
$\hat{p}_1$ .... estimated sample propotion for sample 1
$\hat{p}_2$ .... estimated sample propotion for sample 2
n_1 .... size of sample 1
n_2 .... size of sample 2

■ *t* Interval

● 1-Sample *t* Interval

1-Sample *t* Interval calculates the confidence interval for an unknown population mean when the population standard deviation is unknown.

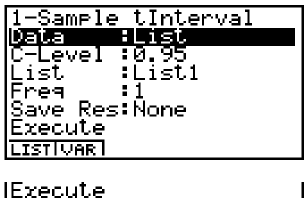
The following is the confidence interval. The value 100 (1-α) % is the confidence level.

$$Left = \bar{x} - t_{n-1}\left(\frac{\alpha}{2}\right)\frac{x\sigma_{n-1}}{\sqrt{n}}$$

$$Right = \bar{x} + t_{n-1}\left(\frac{\alpha}{2}\right)\frac{x\sigma_{n-1}}{\sqrt{n}}$$

Perform the following key operation from the statistical data list.

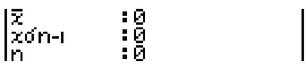
[F4] (INTR)
[2] (T)
[1] (1-Smpl)



The following shows the meaning of each item in the case of list data specification.

Data .... data type
C-Level .... confidence level (0 ≦ C-Level < 1)
List .... list whose contents you want to use as sample data
Freq .... sample frequency
Save Res .... list for storage of calculation results (None or List 1 to 20)
Execute .... executes a calculation

The following shows the meaning of parameter data specification items that are different from list data specification.



$\bar{x}$ .... mean of sample

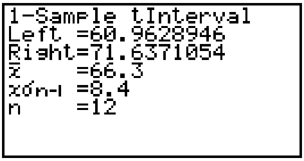
$x\sigma_{n-1}$ .... sample standard deviation ($x\sigma_{n-1} \geqq 0$)

n .... size of sample (positive integer)

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

- F1 (CALC) ... Performs the calculation.

Calculation Result Output Example



Left .... interval lower limit (left edge)

Right............................ interval upper limit (right edge)

$\bar{x}$ .... mean of sample

$x\sigma_{n-1}$ .... sample standard deviation

n .... size of sample

• 2-Sample *t* Interval

2-Sample *t* Interval calculates the confidence interval for the difference between two population means when both population standard deviations are unknown. The *t* Interval is applied to *t* distribution.

The following confidence interval applies when pooling is in effect. The value 100 (1-α) % is the confidence level.

$$Left = (\bar{x}_1 - \bar{x}_2) - t_{n_1+n_2-2}\left(\frac{\alpha}{2}\right)\sqrt{x_p\sigma_{n-1}{}^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

$$Right = (\bar{x}_1 - \bar{x}_2) + t_{n_1+n_2-2}\left(\frac{\alpha}{2}\right)\sqrt{x_p\sigma_{n-1}{}^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

The following confidence interval applies when pooling is not in effect. The value 100 (1-α) % is the confidence level.

$$Left = (\bar{x}_1 - \bar{x}_2) - t_{df}\left(\frac{\alpha}{2}\right)\sqrt{\left(\frac{x_1\sigma_{n-1}{}^2}{n_1} + \frac{x_2\sigma_{n-1}{}^2}{n_2}\right)}$$

$$Right = (\bar{x}_1 - \bar{x}_2) + t_{df}\left(\frac{\alpha}{2}\right)\sqrt{\left(\frac{x_1\sigma_{n-1}{}^2}{n_1} + \frac{x_2\sigma_{n-1}{}^2}{n_2}\right)}$$

$$df = \frac{1}{\dfrac{C^2}{n_1-1} + \dfrac{(1-C)^2}{n_2-1}}$$

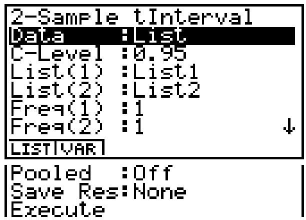
$$C = \frac{\dfrac{x_1\sigma_{n-1}{}^2}{n_1}}{\left(\dfrac{x_1\sigma_{n-1}{}^2}{n_1} + \dfrac{x_2\sigma_{n-1}{}^2}{n_2}\right)}$$

Perform the following key operation from the statistical data list.

F4 (INTR)

2 (T)

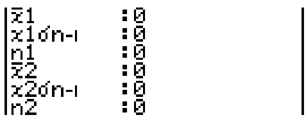
2 (2-Smpl)



The following shows the meaning of each item in the case of list data specification.

Data .... data type
C-Level .... confidence level (0 ≦ C-Level < 1)
List(1) .... list whose contents you want to use as data of sample 1
List(2) .... list whose contents you want to use as data of sample 2
Freq(1) .... frequency of sample 1
Freq(2) .... frequency of sample 2
Pooled .... pooling On (in effect) or Off (not in effect)
Save Res .... list for storage of calculation results (None or List 1 to 20)
Execute .... executes a calculation

The following shows the meaning of parameter data specification items that are different from list data specification.



$\bar{x}_1$ .... mean of sample 1

$x_1\sigma_{n-1}$ .... standard deviation ($x_1\sigma_{n-1} \geqq 0$) of sample 1

n_1 .... size (positive integer) of sample 1

$\bar{x}_2$ .... mean of sample 2

$x_2\sigma_{n-1}$ .... standard deviation ($x_2\sigma_{n-1} \geqq 0$) of sample 2

n_2 .... size (positive integer) of sample 2

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

- F1 (CALC) ... Performs the calculation.

Calculation Result Output Example

```
2-Sample tInterval
Left =-7.5088264
Right=-0.0911735
df   =7.29033011
x̄1   =80.4
x̄2   =84.2
x1σn-1=2.07364414    ↓
```

Left .... interval lower limit (left edge)

Right............................ interval upper limit (right edge)

df .... degrees of freedom

$\bar{x}_1$ .... mean of sample 1

$\bar{x}_2$ .... mean of sample 2

$x_1\sigma_{n-1}$ .... standard deviation of sample 1

$x_2\sigma_{n-1}$ .... standard deviation of sample 2

$x_p\sigma_{n-1}$ .... pooled sample standard deviation
(Displayed only when Pooled: On Setting.)

n_1 .... size of sample 1

n_2 .... size of sample 2