### Solving Linear Equations - Distance, Rate and Time

# Objective: Solve distance problems by creating and solving a linear equation.

An application of linear equations can be found in distance problems. When solving distance problems we will use the relationship rt = d or rate (speed) times time equals distance. For example, if a person were to travel 30 mph for 4 hours. To find the total distance we would multiply rate times time or (30)(4) = 120. This person travel a distance of 120 miles. The problems we will be solving here will be a few more steps than described above. So to keep the information in the problem organized we will use a table. An example of the basic structure of the table is blow:

	Rate	Time	Distance
Person 1			
Person 2			

 Table 1. Structure of Distance Problem

The third column, distance, will always be filled in by multiplying the rate and time columns together. If we are given a total distance of both persons or trips we will put this information below the distance column. We will now use this table to set up and solve the following example

#### Example 1.

Two joggers start from opposite ends of an 8 mile course running towards each other. One jogger is running at a rate of 4 mph, and the other is running at a rate of 6 mph. After how long will the joggers meet?

	Rate	Time	Distance
Jogger 1			
Jogger 2			

The basic table for the joggers, one and two  $% \left( {{{\left[ {{{{\rm{b}}} \right]}_{{\rm{c}}}}_{{\rm{c}}}}} \right)_{{\rm{c}}}} \right)$ 

	Rate	Time	Distance
Jogger 1	4		
Jogger 2	6		

	Rate	Time	Distance
Jogger 1	4	t	
Jogger 2	6	t	

	Rate	Time	Distance
Jogger 1	4	t	4t
Jogger 2	6	t	6t
			8

$$4t + 6t = 8$$
$$10t = 8$$
$$\overline{10} \ \overline{10}$$
$$t = \frac{4}{5}$$

We are given the rates for each jogger. These are added to the table

We only know they both start and end at the same time. We use the variable t for both times

The distance column is filled in by multiplying rate by time

We have **total distance**, 8 miles, under distance The distance column gives equation by adding Combine like terms, 4t + 6tDivide both sides by 10 Our solution for  $t, \frac{4}{5}$  hour (48 minutes)

As the example illustrates, once the table is filled in, the equation to solve is very easy to find. This same process can be seen in the following example

#### Example 2.

Bob and Fred start from the same point and walk in opposite directions. Bob walks 2 miles per hour faster than Fred. After 3 hours they are 30 miles apart. How fast did each walk?

	Rate	Time	Distance
Bob		3	
Fred		3	

The basic table with given times filled in Both traveled 3 hours

	Rate	Time	Distance
Bob	r+2	3	
Fred	r	3	

Bob walks 2 mph faster than Fred We know nothing about Fred, so use r for his rate Bob is r + 2, showing 2 mph faster

	Rate	Time	Distance
Bob	r+2	3	3r+6
Fred	r	3	3r
			30

3r + 6 + 3r = 30				
6r + 6 = 30				
	-6 - 6			
	6r = 24			
	$\overline{6}$ $\overline{6}$			
	r = 4			
	Rate			
Bob	4 + 2 = 6			
Fred	4			

Distance column is filled in by multiplying rate by Time. Be sure to distribute the 3(r+2) for Bob.

Total distance is put under distanceThe distance columns is our equation, by addingCombine like terms 3r + 3rSubtract 6 from both sidesThe variable is multiplied by 6Divide both sides by 6Our solution for rTo answer the question completely we plug 4 in for

To answer the question completely we plug 4 in for r in the table. Bob traveled 6 miles per hour and Fred traveled 4 mph

Some problems will require us to do a bit of work before we can just fill in the cells. One example of this is if we are given a total time, rather than the individual times like we had in the previous example. If we are given total time we will write this above the time column, use t for the first person's time, and make a subtraction problem, Total -t, for the second person's time. This is shown in the next example

#### Example 3.

Two campers left their campsite by canoe and paddled downstream at an average speed of 12 mph. They turned around and paddled back upstream at an average rate of 4 mph. The total trip took 1 hour. After how much time did the campers turn around downstream?

	Rate	Time	Distance
Down	12		
Up	4		

		1	
	Rate	Time	Distance
Down	12	t	
Up	4	1-t	

Basic table for down and upstream Given rates are filled in

Total time is put above time column As we have the total time, in the first time we have t, the second time becomes the subtraction, total -t

	Rate	Time	Distance	1
Down	12	t	12t	=
Up	4	1-t	4-4t	
			12t = 4 -	4t
		-	+4t +	4t

 $\frac{16t = 4}{16}$ 

 $t = \frac{1}{4}$ 

Distance column is found by multiplying rate
by time. Be sure to distribute $4(1-t)$ for
upstream. As they cover the same distance,
= is put after the down distance
${\rm With equal sign, distance colum is equation}$
$\operatorname{Add}4t\operatorname{to}\operatorname{both}\operatorname{sides}\operatorname{so}\operatorname{variable}\operatorname{is}\operatorname{only}\operatorname{on}\operatorname{one}\operatorname{side}$
Variable is multiplied by 16
Divide both sides by 16
Our solution, turn around after $\frac{1}{4}$ hr (15 min)

Another type of a distance problem where we do some work is when one person catches up with another. Here a slower person has a head start and the faster person is trying to catch up with him or her and we want to know how long it will take the fast person to do this. Our startegy for this problem will be to use t for the faster person's time, and add amount of time the head start was to get the slower person's time. This is shown in the next example.

#### Example 4.

Mike leaves his house traveling 2 miles per hour. Joy leaves 6 hours later to catch up with him traveling 8 miles per hour. How long will it take her to catch up with him?

	Rate	Time	Distance
Mike	2		
Joy	8		

	Rate	Time	Distance
Mike	2	t+6	
Joy	8	t	

	Rate	Time	Distance	1
Mike	2	t+6	2t+12	=
Joy	8	t	8t	

$$2t + 12 = 8t$$

$$-2t - 2t$$

$$12 = 6t$$

 $\overline{\mathbf{6}}$   $\overline{\mathbf{6}}$  Divide both sides by 6

Mike's distance

2 = t Our solution for t, she catches him after 2 hours

Now the distance column is the equation

Subtract 2t from both sides

The variable is multiplied by 6

Basic table for Mike and Joy The given rates are filled in

Joy, the faster person, we use t for time Mike's time is t + 6 showing his 6 hour head start

Distance column is found by multiplying the rate by time. Be sure to distribute the 2(t+6) for Mike

As they cover the **same distance**, = is put after

World View Note: The 10,000 race is the longest standard track event. 10,000 meters is approximately 6.2 miles. The current (at the time of printing) world record for this race is held by Ethiopian Kenenisa Bekele with a time of 26 minutes, 17.53 second. That is a rate of 12.7 miles per hour!

As these example have shown, using the table can help keep all the given information organized, help fill in the cells, and help find the equation we will solve. The final example clearly illustrates this.

#### Example 5.

On a 130 mile trip a car travled at an average speed of 55 mph and then reduced its speed to 40 mph for the remainder of the trip. The trip took 2.5 hours. For how long did the car travel 40 mph?

	Rate	Time	Distance
Fast	55		
Slow	40		

		2.5	
	Rate	Time	Distance
Fast	55	t	
Slow	40	2.5-t	

2.5 Time

t

2.5 - t

Fast

Slow

Distance

55t

 $\frac{100-40t}{130}$ 

15t + 100 = 130

-100 - 100

Time

2

2.5 - 2 = 0.5

15t = 30

 $\overline{15}$   $\overline{15}$ 

t = 2

55t + 100 - 40t = 130

Rate

55

40

Fast

Slow

Basic table for fast and slow speeds The given rates are filled in

Total time is put above the time column As we have total time, the first time we have tThe second time is the subtraction problem 2.5 - t

Distance column is found by multiplying rate by time. Be sure to distribute 40(2.5 - t) for slow

**Total distance** is put under distance The distance column gives our equation by adding Combine like terms 55t - 40tSubtract 100 from both sides The variable is multiplied by 30 Divide both sides by 15

Our solution for t.

To answer the question we plug 2 in for tThe car traveled 40 mph for 0.5 hours (30 minutes)



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## 1.10 Practice - Distance, Rate, and Time Problems

- 1. A is 60 miles from B. An automobile at A starts for B at the rate of 20 miles an hour at the same time that an automobile at B starts for A at the rate of 25 miles an hour. How long will it be before the automobiles meet?
- 2. Two automobiles are 276 miles apart and start at the same time to travel toward each other. They travel at rates differing by 5 miles per hour. If they meet after 6 hours, find the rate of each.
- 3. Two trains travel toward each other from points which are 195 miles apart. They travel at rate of 25 and 40 miles an hour respectively. If they start at the same time, how soon will they meet?
- 4. A and B start toward each other at the same time from points 150 miles apart. If A went at the rate of 20 miles an hour, at what rate must B travel if they meet in 5 hours?
- 5. A passenger and a freight train start toward each other at the same time from two points 300 miles apart. If the rate of the passenger train exceeds the rate of the freight train by 15 miles per hour, and they meet after 4 hours, what must the rate of each be?
- 6. Two automobiles started at the same time from a point, but traveled in opposite directions. Their rates were 25 and 35 miles per hour respectively. After how many hours were they 180 miles apart?
- 7. A man having ten hours at his disposal made an excursion, riding out at the rate of 10 miles an hour and returning on foot, at the rate of 3 miles an hour. Find the distance he rode.

- 8. A man walks at the rate of 4 miles per hour. How far can he walk into the country and ride back on a trolley that travels at the rate of 20 miles per hour, if he must be back home 3 hours from the time he started?
- 9. A boy rides away from home in an automobile at the rate of 28 miles an hour and walks back at the rate of 4 miles an hour. The round trip requires 2 hours. How far does he ride?
- 10. A motorboat leaves a harbor and travels at an average speed of 15 mph toward an island. The average speed on the return trip was 10 mph. How far was the island from the harbor if the total trip took 5 hours?
- 11. A family drove to a resort at an average speed of 30 mph and later returned over the same road at an average speed of 50 mph. Find the distance to the resort if the total driving time was 8 hours.
- 12. As part of his flight trainging, a student pilot was required to fly to an airport and then return. The average speed to the airport was 90 mph, and the average speed returning was 120 mph. Find the distance between the two airports if the total flying time was 7 hours.
- 13. A, who travels 4 miles an hour starts from a certain place 2 hours in advance of B, who travels 5 miles an hour in the same direction. How many hours must B travel to overtake A?
- 14. A man travels 5 miles an hour. After traveling for 6 hours another man starts at the same place, following at the rate of 8 miles an hour. When will the second man overtake the first?
- 15. A motorboat leaves a harbor and travels at an average speed of 8 mph toward a small island. Two hours later a cabin cruiser leaves the same harbor and travels at an average speed of 16 mph toward the same island. In how many hours after the cabin cruiser leaves will the cabin cuiser be alongside the motorboat?
- 16. A long distance runner started on a course running at an average speed of 6 mph. One hour later, a second runner began the same course at an average speed of 8 mph. How long after the second runner started will the second runner overtake the first runner?
- 17. A car traveling at 48 mph overtakes a cyclist who, riding at 12 mph, has had a 3 hour head start. How far from the starting point does the car overtake the cyclist?
- 18. A jet plane traveling at 600 mph overtakes a propeller-driven plane which has had a 2 hour head start. The propeller-driven plane is traveling at 200 mph. How far from the starting point does the jet overtake the propeller-driven

plane?

- 19. Two men are traveling in opposite directions at the rate of 20 and 30 miles an hour at the same time and from the same place. In how many hours will they be 300 miles apart?
- 20. Running at an average rate of 8 m/s, a sprinter ran to the end of a track and then jogged back to the starting point at an average rate of 3 m/s. The sprinter took 55 s to run to the end of the track and jog back. Find the length of the track.
- 21. A motorboat leaves a harbor and travels at an average speed of 18 mph to an island. The average speed on the return trip was 12 mph. How far was the island from the harbor if the total trip took 5 h?
- 22. A motorboat leaves a harbor and travels at an average speed of 9 mph toward a small island. Two hours later a cabin cruiser leaves the same harbor and travels at an average speed of 18 mph toward the same island. In how many hours after the cabin cruiser leaves will the cabin cruiser be alongside the motorboat?
- 23. A jet plane traveling at 570 mph overtakes a propeller-driven plane that has had a 2 h head start. The propeller-driven plane is traveling at 190 mph. How far from the starting point does the jet overtake the propeller-driven plane?
- 24. Two trains start at the same time from the same place and travel in opposite directions. If the rate of one is 6 miles per hour more than the rate of the other and they are 168 miles apart at the end of 4 hours, what is the rate of each?
- 25. As part of flight traning, a student pilot was required to fly to an airport and then return. The average speed on the way to the airport was 100 mph, and the average speed returning was 150 mph. Find the distance between the two airports if the total flight time was 5 h.
- 26. Two cyclists start from the same point and ride in opposite directions. One cyclist rides twice as fast as the other. In three hours they are 72 miles apart. Find the rate of each cyclist.
- 27. A car traveling at 56 mph overtakes a cyclist who, riding at 14 mph, has had a 3 h head start. How far from the starting point does the car overtake the cyclist?
- 28. Two small planes start from the same point and fly in opposite directions. The first plan is flying 25 mph slower than the second plane. In two hours the planes are 430 miles apart. Find the rate of each plane.

- 29. A bus traveling at a rate of 60 mph overtakes a car traveling at a rate of 45 mph. If the car had a 1 h head start, how far from the starting point does the bus overtake the car?
- 30. Two small planes start from the same point and fly in opposite directions. The first plane is flying 25 mph slower than the second plane. In 2 h, the planes are 470 mi apart. Find the rate of each plane.
- 31. A truck leaves a depot at 11 A.M. and travels at a speed of 45 mph. At noon, a van leaves the same place and travels the same route at a speed of 65 mph. At what time does the van overtake the truck?
- 32. A family drove to a resort at an average speed of 25 mph and later returned over the same road at an average speed of 40 mph. Find the distance to the resort if the total driving time was 13 h.
- 33. Three campers left their campsite by canoe and paddled downstream at an average rate of 10 mph. They then turned around and paddled back upstream at an average rate of 5 mph to return to their campsite. How long did it take the campers to canoe downstream if the total trip took 1 hr?
- 34. A motorcycle breaks down and the rider has to walk the rest of the way to work. The motorcycle was being driven at 45 mph, and the rider walks at a speed of 6 mph. The distance from home to work is 25 miles, and the total time for the trip was 2 hours. How far did the motorcycle go before if broke down?
- 35. A student walks and jogs to college each day. The student averages 5 km/hr walking and 9 km/hr jogging. The distance from home to college is 8 km, and the student makes the trip in one hour. How far does the student jog?
- 36. On a 130 mi trip, a car traveled at an average speed of 55 mph and then reduced its speed to 40 mph for the remainder of the trip. The trip took a total of 2.5 h. For how long did the car travel at 40 mph?
- 37. On a 220 mi trip, a car traveled at an average speed of 50 mph and then reduced its average speed to 35 mph for the remainder of the trip. The trip took a total of 5 h. How long did the car travel at each speed?
- 38. An executive drove from home at an average speed of 40 mph to an airport where a helicopter was waiting. The executive boarded the helicopter and flew to the corporate offices at and average speed of 60 mph. The entire distance was 150 mi. The entire trip took 3 h. Find the distance from the airport to the corporate offices.



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	Answers - Distance, Rate, and Ti	me Problems
1) $1\frac{1}{3}$	14) 10	28) 95, 120
2) $25\frac{1}{2}, 20\frac{1}{2}$	15) 2	29) 180
3) 3	$16) \ 3$	30) 105, 130
<ul><li>4) 10</li></ul>	17) 48	, .
	18) 600	31) 2:15 PM
5) 30, 45	19) 6	32) 200
6) 3	20) 120	$(33) \frac{1}{3}$
7) $\frac{300}{13}$	$21) \ 36$	0
8) 10	22) 2	34) 15
9)  7	23) 570	$35) \frac{27}{4}$
10) 30	24) 24, 18	$36) \frac{1}{2}$
11) 150	$25) \ 300$	$(30) \frac{1}{2}$
12) 360	26) 8, 16	$37) \ 3, \ 2$
13) 8	27) 56	38) 90



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