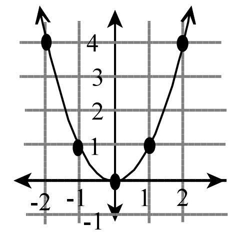
Quadratic Equation/Relation: a degree two equation
Degree 2: highest exponent is two

Graph of quadratic equation:
Shape is called a PARABOLA
Ex: $y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4



Standard form of the quadratic equation: $y = ax^2 + bx + c$

examples: $y = 3x^2 - 2x + 5$ $y = x^2$ $y = -2x^2 + x$ $y = 3x^2 - 5$

Vertex: the "turning" point of graph. It is the lowest(minimum) or the highest(maximum) point on the parabola

Finding vertex, if equation is in standard form:

1) calculate $\frac{-b}{2a}$ (this is x-coordinate of vertex, ie $x = \frac{-b}{2a}$)

2)sub in calculated value for "x" and solve for y(this is the y coordinate of the vertex)

Axis of symmetry: the vertical line that passes through the vertex of a parabola that shows reflective symmetry(how the left and right side are mirror images in line of symmetry)

> *eq. of axis of sym.*
> *x = avg. of x co-ordinates of mirror points*
> *(and x co-ordinate of vertex)*

Equation will be: x = "x value of vertex" or $x = \frac{-b}{2a}$ (when in standard form $y = ax^2 + bx + c$)

How each coefficient ("a" and "b") or constant ("c") affects graph, when $y = ax^2 + bx + c$

"a" - i) if $a > 0$, then graph opens up (vertex is a min.) ii) if $a < 0$, then graph opens down (vertex is a max.)

"b" - if it is changed , then it affects the line of symmetry (and affects vertex)

"c" - if it is changed , then it affects the y-intercept [is the y intercept (0,c)](and affects vertex)

Domain: x is an element of the real numbers $D = \{x \mid x \in R\}$ ...set notation $(-\infty, \infty)$... interval notation

Range: i) $y \geq$ y-value of the vertex, if graph turns up (if $a > 0$) $R = \{y \mid y \geq y_v, y \in R\}$ .... OR $[y_v, \infty)$

ii) $y \leq$ y-value of the vertex, if graph turns down (if $a < 0$) $R = \{y \mid y \leq y_v, y \in R\}$ ....OR $(-\infty, y_v]$

(Where "y_v" is the y value of the vertex)

Zero Product Rule: If ab = 0 then a = 0 or b= 0 (if a product is zero, then a factor must equal zero)

Mirror Points: two points that are equidistant from the axis of symmetry of a parabola (will have same y co-ordinates)

Partial Factors: Factoring the "x" parts of standard form to find the "mirror point" of the y-intercept (and y-intercept)

ex) $y = x^2 + 4x - 2$

$y = (x^2 + 4x) - 2$ What makes the x (factored) parts = 0? (Set this part equal to 0, and solve)

$y = x(x+4) - 2$ So x = 0 or x = -4, make the "x part" = 0

$$x(x+4) = 0$$
$$x = 0 \text{ or } x + 4 = 0$$
$$x = 0 - 4$$
$$x = -4$$

SO, if (0, -2) is a point , so will (- 4 , -2)

Graphing using standard form:

1) find y-int. (0 , c)
2) partial factor the "x stuff" [factoring out a GCF of x is enough]
3) set partial factors = 0, and solve for "x" this gives "x co-ordinate of y-int(which we knew) and the x coordinate of mirror point. ... i.e. (0, c) and (other "x" , c)
4) average these x values....this gives axis of sym (x = avg) AND the x-coordinate of vertex (avg , y)
5) sub in "x" of vertex and find y (write vertex point (x , y))
6) plot points (y-int , mirror of y-int, and vertex) and sketch parabola

---

In general: $y = ax^2 + bx + c$
$y = (ax^2 + bx) + c$
$y = x(ax + b) + c$

$$x(ax+b) = 0$$
$$x = 0 \text{ or } ax + b = 0$$
$$ax = -b$$
$$x = \frac{-b}{a}$$

y-int (0 , c) , mirror point $\left(\frac{-b}{a}, c\right)$

When equation is in standard form

Explanation of why the formulas for axis of symmetry and finding x-coordinate of vertex works(how derived or made)

Using this to find axis of symmetry: x = average of the x-coordinates of any mirror points .....use "0" and "$\frac{-b}{a}$"

$$x = avg. = \frac{sum}{2} = \frac{0 + \frac{-b}{a}}{2} = \frac{\frac{-b}{a}}{2} = \frac{-b}{a} \div 2 = \frac{-b}{a} \times \frac{1}{2} = \frac{-b}{2a}$$

> *equation of axis of sym*
> $x = \frac{-b}{2a}$
> *(and x coordinate of vertex)*

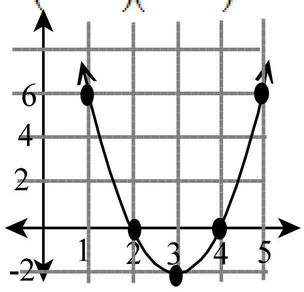
**sub in x and find y of vertex....$f\left(\frac{-b}{2a}\right)$

3 quick points if in $y = ax^2 + bx + c$ form: y-int and mirror are (o, c) *and* $\left(\frac{-b}{a}, c\right)$, vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

Factored Form of the Quadratic Function: y = a(x-r)(x-s) or f(x) = a(x-r)(x-s)

ex) $y = 2(x-2)(x-4)$

x	y
1	6
2	0
3	-2
2	0
5	6



x-intercepts (2 , 0) and (4 , 0) or x-int = 2 and 4 (assumption is y = 0)

Notice: 2 and 4 are the values subtracted in the equation

So, in the factored form "r" and "s" are the x-intercepts

Finding line of symmetry: x = avg. of x-intercepts = $\frac{r+s}{2}$

Finding the x- intercepts using factored form
1) sub in y = 0
2) apply the zero product rule to find
3) write x-intercepts as points (r , 0) and (s , 0)

Finding vertex:
1) find x-intercepts
2) average the x-intercepts $\frac{r+s}{2}$
3) sub in x and find y

vertex

$\left(\frac{r+s}{2}, f\left(\frac{r+s}{2}\right)\right)$

Graphing using factored form:
1) find x intercepts (see steps above)
2) find vertex (see steps above)
3) plot x intercepts and vertex and sketch graph

"ZEROS" = what x values make the function = "0".....the "x-intercepts" (sub y = 0 and solve for x)

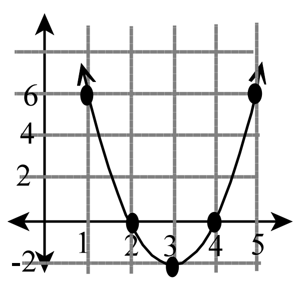
Finding equation in factored form if given graph:
[need: two x-intercepts and one other point]
1) write $y = a(x-r)(x-s)$
2) sub in x-intercepts and simplify each bracket
3) sub in any other point and calculate "a"
4) make sure you write the final version of the equation
(only variables should be x and y)

for graph above: $x-\text{int}(2,0)(4,0)$ *and pt.* $(5,6)$
1....$y = a(x-r)(x-s)$
2....$y = a(x-2)(x-4)$
3....$6 = a((5)-2)((5)-4)$..*subbed*$(5,6)$
$2 = a$
4....$y = 2(x-2)(x-4)$

Vertex Form of the Quadratic Function: $y = a(x-h)^2 + k$ or $f(x) = a(x-h)^2 + k$

$y = 2(x-3)^2 - 2$

x	y
1	6
2	0
3	-2
2	0
5	6



Vertex is (3 , -2)

3...what makes bracket (partial factor) zero...the "h"
-2... the constant added to bracket in this form...the "k"

Graphing using Vertex form(some quick methods): (don't forget you can always sub in x's and find y's, to always graph)

Method 1
1) note vertex (h , k)
2) sub in an "x" left and right of vertex to find two more points
3) plot points and sketch graph

Method 2
1) note vertex (h , k)
2) Use "a" like a slope and plot mirror points one unit left and right of vertex
3) sketch graph

x	y
h-1	k+a
h	k
h+1	k+a

Method 3
1) note vertex (h , k)
2) find y-int (sub x = 0)
3) note mirror point of y-int.
4) sketch graph

Finding equation in vertex form if given graph:
[need: vertex and one other point]
1) write $y = a(x-h)^2 + k$
2) sub in "h" and "k" of vertex (and simplify if necessary)
3) sub in any other point and calculate "a"
4) make sure you write the final version of the equation
(only variables should be x and y)

for graph above: $V:(3,-2)$ *and pt.* $(5,6)$
1....$y = a(x-h)^2 + k$
2....$y = a(x-3)^2 + -2$
$y = a(x-3)^2 - 2$
3....$6 = a(5-3)^2 - 2$..*subbed*$(5,6)$
$2 = a$
4....$y = 2(x-3)^2 - 2$

Word Problems:
1) set up an equation in standard , factored or vertex form ...READ CAREFULLY...sketching situation may help
based on using previous/application of knowledge; area , volume , cost, etc...too many possibilities to list all
2) max/min problems need a vertex to be found...see notes on finding vertex
3) when done, re-read question to see if you answered it.