

The Praxis™ Study Companion

Mathematics: Content Knowledge

0061/5061



Welcome to *The Praxis™ Study Companion*

Prepare to Show What You Know

You have gained the knowledge and skills you need for your teaching career. Now you are ready to demonstrate your abilities by taking a *Praxis™* test.

Using *The Praxis Study Companion* is a smart way to prepare for the test so you can do your best on test day. This guide can help keep you on track and make the most efficient use of your study time.

The Study Companion contains practical information and helpful tools, including:

- An overview of the tests
- Specific information on the *Praxis* test you are taking
- A template study plan
- Practice questions and explanations of correct answers
- Test-taking tips and strategies
- Frequently asked questions
- Links to more detailed information

So where should you start? Begin by reviewing this guide in its entirety and note those sections that you need to revisit. Then you can create your own personalized study plan and schedule based on your individual needs and how much time you have before test day.

Keep in mind that study habits are individual. There are many different ways to successfully prepare for your test. Some people study better on their own, while others prefer a group dynamic. You may have more energy early in the day, but another test taker may concentrate better in the evening. So use this guide to develop the approach that works best for you.

Your teaching career begins with preparation. Good luck!

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1. Know What to Expect

Familiarize yourself with the Praxis tests so you know what to expect

Which test should I take?

Each state or agency that uses the *Praxis* tests sets its own requirements for which test or tests you must take for the teaching area you wish to pursue.

Before you register for a test, confirm your state or agency's testing requirements at www.ets.org/praxis/states.

How are the *Praxis* tests given?

Praxis tests are given in both computer and paper formats. **Note:** Not all *Praxis* tests are offered in both formats.

Should I take the computer- or paper-delivered test?

You should take the test in whichever format you are most comfortable. Some test takers prefer taking a paper-and-pencil test, while others are more comfortable on a computer. Please note that not all tests are available in both formats. To help you decide, watch the [What to Expect on Test Day video](#) for computer-delivered tests.

If I'm taking more than one *Praxis* test, do I have to take them all in the same format?

No. You can take each test in the format in which you are most comfortable.

Is there a difference between the subject matter covered on the computer-delivered test and the paper-delivered test?

No. The computer-delivered test and paper-delivered test cover the same content.

Where and when are the *Praxis* tests offered?

You can select the test center that is most convenient for you. The *Praxis* tests are administered through an international network of test centers, which includes some universities, high schools, Prometric® Testing Centers, and other locations throughout the world.

Testing schedules depend on whether you are taking computer-delivered tests or paper-delivered tests. See the *Praxis* Web site for more detailed test registration information at www.ets.org/praxis/register.

2. Familiarize Yourself with Test Questions

Become comfortable with the types of questions you'll find on the Praxis tests

The *Praxis* tests include two types of questions — **multiple-choice** (for which you select your answers from a list of choices) and **constructed-response** (for which you write a response of your own). You may be familiar with these question formats from taking other standardized tests. If not, familiarize yourself with them so you don't spend time during the test figuring out how to answer them.

Understanding Multiple-Choice Questions

Many multiple-choice questions begin with the phrase “which of the following.” Take a look at this example:

Which of the following is a flavor made from beans?

- (A) Strawberry
- (B) Cherry
- (C) Vanilla
- (D) Mint

How would you answer this question?

All of the answer choices are flavors. Your job is to decide which of the flavors is the one made from beans.

Try following these steps to select the correct answer.

- 1) **Limit your answer to one of the choices given.** You may know that chocolate and coffee are also flavors made from beans, but they are not listed. Rather than thinking of other possible answers, focus only on the choices given (“which of the following”).
- 2) **Eliminate incorrect answers.** You may know that strawberry and cherry flavors are made from fruit and that mint flavor is made from a plant. That leaves vanilla as the only possible answer.
- 3) **Verify your answer.** You can substitute “vanilla” for the phrase “which of the following” and turn the question into this statement: “Vanilla is a flavor made from beans.” This will help you be sure that your answer is correct. If you're still uncertain, try substituting the other choices to see if they make sense. You may want to use this technique as you answer multiple-choice questions on the practice tests.

Try a more challenging example

The vanilla bean question is pretty straightforward, but you'll find that more challenging questions have a similar structure. For example:

Entries in outlines are generally arranged according to which of the following relationships of ideas?

- (A) Literal and inferential
- (B) Concrete and abstract
- (C) Linear and recursive
- (D) Main and subordinate

You'll notice that this example also contains the phrase "which of the following." This phrase helps you determine that your answer will be a "relationship of ideas" from the choices provided. You are supposed to find the choice that describes how entries, or ideas, in outlines are related.

Sometimes it helps to put the question in your own words. Here, you could paraphrase the question in this way: "How are outlines usually organized?" Since the ideas in outlines usually appear as main ideas and subordinate ideas, the answer is (D).

QUICK TIP: Don't be intimidated by words you may not understand. It might be easy to be thrown by words like "recursive" or "inferential." Read carefully to understand the question and look for an answer that fits. An outline is something you are probably familiar with and expect to teach to your students. So slow down, and use what you know.

Watch out for multiple-choice questions containing "NOT," "LEAST," and "EXCEPT"

This type of question asks you to select the choice that does not fit. You must be very careful because it is easy to forget that you are selecting the negative. This question type is used in situations in which there are several good solutions or ways to approach something, but also a clearly wrong way.

How to approach questions about graphs, tables, or reading passages

When answering questions about graphs, tables, or reading passages, provide only the information that the questions ask for. In the case of a map or graph, you might want to read the questions first, and then look at the map or graph. In the case of a long reading passage, you might want to go ahead and read the passage first, marking places you think are important, and then answer the questions. Again, the important thing is to be sure you answer the questions as they refer to the material presented. So read the questions carefully.

How to approach unfamiliar formats

From time to time, new question formats are developed to find new ways of assessing knowledge. The latest tests may include audio and video components, such as a movie clip or animation, instead of the more traditional map or reading passage. Other tests may allow you to zoom in on details of a graphic or picture. Tests may also include interactive questions that take advantage of technology to assess knowledge and skills. They can assess knowledge more than standard multiple-choice questions can. If you see a format you are not familiar with, **read the directions carefully**. They always give clear instructions on how you are expected to respond.

For most questions, you will respond by clicking an oval to select a single answer from a list of options. Other questions may ask you to respond in the following ways:

- **Typing in an entry box.** When the answer is a number, you may be asked to enter a numerical answer or, if the test has an on-screen calculator, you may need to transfer the calculated result from the calculator to the entry box. Some questions may have more than one place to enter a response.
- **Clicking check boxes.** You may be asked to click check boxes instead of an oval when more than one choice within a set of answers can be selected.
- **Clicking parts of a graphic.** In some questions, you will select your answers by clicking on a location (or locations) on a graphic such as a map or chart, as opposed to choosing your answer from a list.
- **Clicking on sentences.** In questions with reading passages, you may be asked to choose your answers by clicking on a sentence (or sentences) within the reading passage.
- **Dragging and dropping answer choices into targets on the screen.** You may be asked to select answers from a list of options and drag your answers to the appropriate location in a table, paragraph of text or graphic.
- **Selecting options from a drop-down menu.** You may be asked to choose answers by selecting options from a drop-down menu (e.g., to complete a sentence).

Remember that with every question you will get clear instructions on how to respond. See the [Praxis Computer-delivered Testing Demonstration](#) on the *Praxis* website to learn more about *Praxis* tests and to see examples of some of the types of questions you may encounter.

QUICK TIP: Don't make the questions more difficult than they are. Don't read for hidden meanings or tricks. There are no trick questions on *Praxis* tests. They are intended to be serious, straightforward tests that accurately assess your knowledge.

Understanding Constructed-Response Questions

Constructed-response questions require you to demonstrate your knowledge in a subject area by providing in-depth explanations on particular topics. Essay and problem solving are types of constructed-response questions.

For example, an essay question might present you with a topic and ask you to discuss the extent to which you agree or disagree with the opinion stated. You must support your position with specific reasons and examples from your own experience, observations, or reading.

Take a look at a few sample essay topics:

- "Celebrities have a tremendous influence on the young, and for that reason, they have a responsibility to act as role models."
- "We are constantly bombarded by advertisements—on television and radio, in newspapers and magazines, on highway signs, and the sides of buses. They have become too pervasive. It's time to put limits on advertising."
- "Advances in computer technology have made the classroom unnecessary, since students and teachers are able to communicate with one another from computer terminals at home or at work."

A problem-solving question might ask you to solve a mathematics problem such as the one below and show how you arrived at your solution:

- a) In how many different ways can 700 be expressed as the product of two positive integers? Show how you arrived at your answer.
- b) Among all pairs of positive integers whose product is 700, which pair has the maximum greatest common divisor? Explain how you arrived at your answer.

Keep these things in mind when you respond to a constructed-response question

- 1) **Answer the question accurately.** Analyze what each part of the question is asking you to do. If the question asks you to describe or discuss, you should provide more than just a list.
- 2) **Answer the question completely.** If a question asks you to do three distinct things in your response, you should cover all three things for the best score. Otherwise, no matter how well you write, you will not be awarded full credit.
- 3) **Answer the question that is asked.** Do not change the question or challenge the basis of the question. You will receive no credit or a low score if you answer another question or if you state, for example, that there is no possible answer.
- 4) **Give a thorough and detailed response.** You must demonstrate that you have a thorough understanding of the subject matter. However, your response should be straightforward and not filled with unnecessary information.
- 5) **Reread your response.** Check that you have written what you thought you wrote. Be sure not to leave sentences unfinished or omit clarifying information.

QUICK TIP: You may find that it helps to circle each of the details of the question in your test book or take notes on scratch paper so that you don't miss any of them. Then you'll be sure to have all the information you need to answer the question.

For tests that have constructed-response questions, more detailed information can be found in "4. Learn About Your Test" on page 11.

Understanding Computer-Delivered Questions

Questions on computer-delivered tests are interactive in the sense that you answer by selecting an option or entering text on the screen. If you see a format you are not familiar with, read the directions carefully. The directions always give clear instructions on how you are expected to respond.

Interactive question types may ask you to respond by:

- Typing in an entry box, particularly for a constructed-response question.
- Clicking an oval answer option for a multiple-choice question.
- Clicking on sentences. In questions with reading passages, you may be asked to choose your answer by clicking on a sentence or sentences within the reading passage.

Perhaps the best way to understand computer-delivered questions is to view the [Computer-delivered Testing Demonstration](#) on the Praxis Web site to learn how a computer-delivered test works and see examples of some types of questions you may encounter.

3. Understand Your Scores

Understand how tests are scored and how to interpret your test scores

Of course, passing the *Praxis* test is important to you so you need to understand what your scores mean and what your state requirements are.

What are the score requirements for my state?

States, institutions, and associations that require the tests set their own passing scores. Visit www.ets.org/praxis/states for the most up-to-date information.

If I move to another state, will my new state accept my scores?

The *Praxis Series* tests are part of a national testing program, meaning that they are required in more than one state for licensure. The advantage of a national program is that if you move to another state that also requires *Praxis* tests, you can transfer your scores. Each state has specific test requirements and passing scores, which you can find at www.ets.org/praxis/states.

How do I know whether I passed the test?

Your score report will include information on passing scores for the states you identified as recipients of your test results. If you test in a state with automatic score reporting, you will receive passing score information for that state.

A list of states and their passing scores for each test are available online at www.ets.org/praxis/states.

What your *Praxis* scores mean

You received your score report. Now what does it mean? It's important to interpret your score report correctly and to know what to do if you have questions about your scores.

Visit http://www.ets.org/s/praxis/pdf/sample_score_report.pdf to see a sample score report.

To access *Understanding Your Praxis Scores*, a document that provides additional information on how to read your score report, visit www.ets.org/praxis/scores/understand.

Put your scores in perspective

Your score report indicates:

- Your score and whether you passed
- The range of possible scores
- The raw points available in each content category
- The range of the middle 50 percent of scores on the test
- Your Recognition of Excellence (ROE) Award status, if applicable (found at www.ets.org/praxis/scores/understand/roe)

If you have taken the same test or other tests in *The Praxis Series* over the last 10 years, your score report also lists the highest score you earned on each test taken.

Content category scores and score interpretation

On many of the *Praxis* tests, questions are grouped into content categories. To help you in future study or in preparing to retake the test, your score report shows how many “raw points” you earned in each content category. Compare your “raw points earned” with the maximum points you could have earned (“raw points available”). The greater the difference, the greater the opportunity to improve your score by further study.

Score scale changes

ETS updates *Praxis* tests on a regular basis to ensure they accurately measure the knowledge and skills that are required for licensure. Updated tests cover the same content as the previous tests. However, scores might be reported on a different scale, so requirements may vary between the new and previous versions. All scores for previous, discontinued tests are valid and reportable for 10 years.

These resources may also help you interpret your scores:

- *Understanding Your Praxis Scores* (PDF), found at www.ets.org/praxis/scores/understand
- *The Praxis Series Passing Scores* (PDF), found at www.ets.org/praxis/scores/understand
- State requirements, found at www.ets.org/praxis/states

4. Learn About Your Test

Learn about the specific test you will be taking

Mathematics: Content Knowledge (0061/5061)

Test at a Glance			
Test Name	Mathematics: Content Knowledge		
Test Code	0061	5061	
Time	2 hours	2 hours	
Number of Questions	50	50	
Format	Multiple-choice questions; Graphing calculator required	Multiple-choice questions; On-screen graphing calculator available or bring your own; the test may include nonscored research sections that consist of the following question types: multiple-choice questions—select one or more answers, numeric entry questions, drag-and-drop questions, and text completion questions	
Test Delivery	Paper delivered	Computer delivered	
	Content Categories	Approximate Number of Questions	Approximate Percentage of Examination
	I. Algebra and Number Theory	8	16%
	II. Measurement Geometry Trigonometry	3 5 4	6% 10% 8%
III. Functions Calculus	8 6	16% 12%	
IV. Data Analysis and Statistics Probability	5–6 2–3	10–12% 4–6%	
V. Matrix Algebra Discrete Mathematics	4–5 3–4	8–10% 6–8%	
	Process Categories	Distributed Across Content Categories	
	Mathematical Problem Solving Mathematical Reasoning and Proof Mathematical Connections Mathematical Representation Use of Technology		

About This Test

The *Praxis* Mathematics: Content Knowledge test is designed to assess the mathematical knowledge and competencies necessary for a beginning teacher of secondary school mathematics. Examinees have typically completed a bachelor's program with an emphasis in mathematics or mathematics education.

The examinee will be required to understand and work with mathematical concepts, to reason mathematically, to make conjectures, to see patterns, to justify statements using informal logical arguments, and to construct simple proofs. Additionally, the examinee will be expected to solve problems by integrating knowledge from different areas of mathematics, to use various representations of concepts, to solve problems that have several solution paths, and to develop mathematical models and use them to solve real-world problems.

This test may contain some questions that will not count toward your score.

The test is not designed to be aligned with any particular school mathematics curriculum, but it is intended to be consistent with the recommendations of national studies on mathematics education, such as the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (2000) and the National Council for Accreditation of Teacher Education (NCATE) *Program Standards for Initial Preparation of Mathematics Teachers* (2003).

Graphing calculators without QWERTY (typewriter) keyboards are required for this test.

Some questions require the use of a calculator. The minimum capabilities required of the calculator are described in the section on graphing calculators. Because many test questions may be solved in more than one way, examinees should decide first how to solve each problem and then decide whether to use a calculator.

On the test day, examinees taking the paper-delivered test should bring a calculator they are comfortable using. Examinees taking the computer-delivered test may bring a graphing calculator or use the on-screen graphing calculator provided.

For computer-delivered tests, selected notations, formulas, and definitions are in the Math Reference tab and available to you on the screen throughout the test. For paper-delivered tests, selected notations, formulas, and definitions are printed in the test book. They are also provided in chapter 8 of this Study Companion.

Graphing Calculators

If you are taking the **paper-delivered test**, you must bring to the examination a graphing calculator with the built-in capability to

1. produce the graph of a function within an arbitrary viewing window;
2. find the zeros of a function;
3. compute the derivative of a function numerically;
4. compute definite integrals numerically.

If you are taking the **computer-delivered test**, an on-screen graphing calculator is provided or you may bring your own graphing calculator as described above.

Computers, calculators with QWERTY (typewriter) keyboards, and electronic writing pads are NOT allowed when taking the test.

Unacceptable machines also include the following:

- Powerbooks and portable/handheld computers
- Pocket organizers
- Electronic writing pads or pen-input/stylus-driven devices (e.g., Palm, PDAs, Casio Class Pad 300, etc.)
- Devices with QWERTY keyboards (e.g., TI-92 PLUS, Voyage 200, etc.)
- Cell-phone calculators

Praxis Graphing Calculator Policy

Test administration staff will clear the memory of all graphing calculators both before and after the test administration.

We recommend that you

- back up any important information in your calculator's memory, including applications, before arriving at the test site;
- know how to clear the memory on the approved calculator that you plan to use during the test.

Note: Instructions on how to back up and clear the memory of calculators can be found on various calculator websites.

On-Screen Graphing Calculator

An on-screen graphing calculator is provided for the computer-delivered test. Please consult the [Praxis Calculator Use web page](#) for further information.

You are expected to know how and when to use the graphing calculator since it will be helpful for some questions. You are expected to become familiar with its functionality before taking the test. To practice using the calculator, [download the 30-day trial version and view tutorials on how to use it](#). The calculator may be used to perform calculations (e.g., exponents, roots, trigonometric values, logarithms), to graph and analyze functions, to find numerical solutions to equations, and to generate a table of values for a function.

Using Your Calculator

Take time to [download the 30-day trial version of the calculator](#). View the tutorials on the website. Practice with the calculator so that you are comfortable using it on the test.

There are only some questions on the test for which a calculator is helpful or necessary. First, decide how you will solve a problem, then determine if you need a calculator. For many questions, there is more than one way to solve the problem. Don't use the calculator if you don't need to; you may waste time.

Sometimes answer choices are rounded, so the answer that you get might not match the answer choices in the question. Since the answer choices are rounded, plugging the choices into the question might not produce an exact answer.

Don't round any intermediate calculations. For example, if the calculator produces a result for the first step of a solution, keep the result in the calculator and use it for the second step. If you round the result from the first step and the answer choices are close to each other, you might choose the incorrect answer.

Read the question carefully so that you know what you are being asked to do. Sometimes a result from the calculator is NOT the final answer. If an answer you get is not one of the choices in the question, it may be that you didn't answer the question being asked. Read the question again. It might also be that you rounded at an intermediate step in solving the problem.

Think about how you are going to solve the question before using the calculator. You may only need the calculator in the final step or two. Don't use it more than necessary.

Check the calculator modes (degree versus radian, floating decimal versus scientific notation) to see that these are correct for the question being asked.

Make sure that you know how to perform the basic arithmetic operations and calculations (e.g., exponents, roots, trigonometric values, logarithms). Your test may involve questions that require you to do some of the following: graph functions and analyze the graphs, find zeros of functions, find points of intersection of graphs of functions, find minima/maxima of functions, find numerical solutions to equations, and generate a table of values for a function.

Mathematics Content Descriptions

Representative descriptions of the topics covered in the content categories for the Mathematics: Content Knowledge test follow. Because the assessment is designed to measure the ability to integrate knowledge of mathematics, answering any question may involve more than one competency and may involve competencies from more than one content area.

I. Algebra and Number Theory

- A. Demonstrate an understanding of the structure of the natural, integer, rational, real, and complex number systems and the ability to perform basic operations (+, −, ×, and ÷) on numbers in these systems
- B. Compare and contrast properties (e.g., closure, commutativity, associativity, distributivity) of number systems under various operations
- C. Demonstrate an understanding of the properties of counting numbers (e.g., prime, composite, prime factorization, even, odd, factors, multiples)
- D. Solve ratio, proportion, percent, and average (including arithmetic mean and weighted average) problems
- E. Work with algebraic expressions, formulas, and equations; add, subtract, and multiply polynomials; divide polynomials; add, subtract, multiply, and divide algebraic fractions; perform standard algebraic operations involving complex numbers, radicals, and exponents, including fractional and negative exponents
- F. Solve and graph systems of equations and inequalities, including those involving absolute value
- G. Interpret algebraic principles geometrically
- H. Recognize and use algebraic representations of lines, planes, conic sections, and spheres
- I. Solve problems in two and three dimensions (e.g., distance between two points, the coordinates of the midpoint of a line segment)

II. Measurement

- A. Make decisions about units and scales that are appropriate for problem situations involving measurement; use unit analysis
- B. Analyze precision, accuracy, and approximate error in measurement situations
- C. Apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations

Geometry

- D. Solve problems using relationships of parts of geometric figures (e.g., medians of triangles, inscribed angles in circles) and among geometric figures (e.g., congruence, similarity) in two and three dimensions
- E. Describe relationships among sets of special quadrilaterals, such as the square, rectangle, parallelogram, rhombus, and trapezoid
- F. Solve problems using the properties of triangles, quadrilaterals, polygons, circles, and parallel and perpendicular lines
- G. Solve problems using the properties of circles, including those involving inscribed angles, central angles, chords, radii, tangents, secants, arcs, and sectors
- H. Understand and apply the Pythagorean theorem and its converse
- I. Compute and reason about perimeter, area/surface area, or volume of two- or three-dimensional figures or of regions or solids that are combinations of these figures
- J. Solve problems involving reflections, rotations, and translations of geometric figures in the plane

Trigonometry

- K. Define and use the six basic trigonometric functions using degree or radian measure of angles; know their graphs and be able to identify their periods, amplitudes, phase displacements or shifts, and asymptotes
- L. Apply the law of sines and the law of cosines
- M. Apply the formulas for the trigonometric functions of $\frac{x}{2}$, $2x$, x , $x + y$, and $x - y$; prove trigonometric identities
- N. Solve trigonometric equations and inequalities
- O. Convert between rectangular and polar coordinate systems

III. Functions

- A. Demonstrate understanding of and ability to work with functions in various representations (e.g., graphs, tables, symbolic expressions, and verbal narratives) and to convert flexibly among them
- B. Find an appropriate family of functions to model particular phenomena (e.g., population growth, cooling, simple harmonic motion)
- C. Determine properties of functions and their graphs, such as domain, range, intercepts, symmetries, intervals of increase or decrease, discontinuities, and asymptotes
- D. Use the properties of trigonometric, exponential, logarithmic, polynomial, and rational functions to solve problems
- E. Determine the composition of two functions; find the inverse of a one-to-one function in simple cases and know why only one-to-one functions have inverses
- F. Interpret representations of functions of two variables, such as three-dimensional graphs, level curves, and tables

Calculus

- G. Demonstrate understanding of what it means for a function to have a limit at a point; calculate limits of functions or determine that the limit does not exist; solve problems using the properties of limits
- H. Understand the derivative of a function as a limit, as the slope of a curve, and as a rate of change (e.g., velocity, acceleration, growth, decay)
- I. Show that a particular function is continuous; understand the relationship between continuity and differentiability
- J. Numerically approximate derivatives and integrals
- K. Use standard differentiation and integration techniques
- L. Analyze the behavior of a function (e.g., find relative maxima and minima, concavity); solve problems involving related rates; solve applied minima-maxima problems
- M. Demonstrate understanding of and ability to use the Mean Value Theorem and the Fundamental Theorem of Calculus
- N. Demonstrate understanding of integration as a limiting sum that can be used to compute area, volume, distance, or other accumulation processes
- O. Determine the limits of sequences and simple infinite series

IV. Data Analysis and Statistics

- A. Organize data into a suitable form (e.g., construct a histogram and use it in the calculation of probabilities)
- B. Choose and apply appropriate measures of central tendency (e.g., population mean, sample mean, median, mode) and dispersion (e.g., range, population standard deviation, sample standard deviation, population variance, sample variance) to describe and compare data sets; recognize when to use sample statistics or population parameters
- C. Analyze data from specific situations to determine what type of function (e.g., linear, quadratic, exponential) would most likely model that particular phenomenon; use the regression feature of the calculator to determine curve of best fit; interpret the regression coefficients, correlation, and residuals in context
- D. Understand and apply normal distributions and their characteristics (e.g., mean, standard deviation)
- E. Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference
- F. Understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each
- G. Know the characteristics of well-designed studies, including the role of randomization in surveys and experiments

Probability

- H. Understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases
- I. Understand the concepts of conditional probability and independent events; understand how to compute the probability of a compound event
- J. Compute and interpret the expected value of random variables in simple cases (e.g., fair coins, expected winnings, expected profit)
- K. Use simulations to construct empirical probability distributions and to make informal inferences about the theoretical probability distribution

XII. Matrix Algebra

- A. Understand vectors and matrices as systems that have some of the same properties as the real number system (e.g., identity, inverse, and commutativity under addition and multiplication)
- B. Scalar multiply, add, subtract, and multiply vectors and matrices; find inverses of matrices
- C. Use matrix techniques to solve systems of linear equations
- D. Use determinants to reason about inverses of matrices and solutions to systems of equations
- E. Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, and matrices

Discrete Mathematics

- F. Solve basic problems that involve counting techniques, including the multiplication principle, permutations, and combinations; use counting techniques to understand various situations (e.g., number of ways to order a set of objects, to choose a subcommittee from a committee, to visit n cities)
- G. Find values of functions defined recursively and understand how recursion can be used to model various phenomena; translate between recursive and closed-form expressions for a function
- H. Determine whether a binary relation on a set is reflexive, symmetric, or transitive; determine whether a relation is an equivalence relation
- I. Use finite and infinite arithmetic and geometric sequences and series to model simple phenomena (e.g., compound interest, annuity, growth, decay)
- J. Understand the relationship between discrete and continuous representations and how they can be used to model various phenomena
- K. Use difference equations, vertex-edge graphs, trees, and networks to model and solve problems

Mathematical Process Categories

In addition to knowing and understanding the mathematics content explicitly described in the Mathematics Content Descriptions section, entry-level mathematics teachers must also be able to think mathematically; that is, they must have an understanding of the ways in which mathematical content knowledge is acquired and used. Answering questions on this assessment may involve one or more of the processes described below, and all of the processes may be applied to any of the content topics.

Mathematical Problem Solving

- A. Solve problems that arise in mathematics and those involving mathematics in other contexts
- B. Build new mathematical knowledge through problem solving
- C. Apply and adapt a variety of appropriate strategies to solve problems

Mathematical Reasoning and Proof

- A. Select and use various types of reasoning and methods of proof
- B. Make and investigate mathematical conjectures
- C. Develop and evaluate mathematical arguments and proofs

Mathematical Connections

- A. Recognize and use connections among mathematical ideas
- B. Apply mathematics in context outside of mathematics
- C. Demonstrate an understanding of how mathematical ideas interconnect and build on one another

Mathematical Representation

- A. Select, apply, and translate among mathematical representations to solve problems
- B. Use representations to model and interpret physical, social, and mathematical phenomena
- C. Create and use representations to organize, record, and communicate mathematical ideas

Use of Technology

- A. Use technology appropriately as a tool for problem solving and analysis
- B. Use technology as an aid to understanding mathematical ideas

5. Determine Your Strategy for Success

Set clear goals and deadlines so your test preparation is focused and efficient

Effective *Praxis* test preparation doesn't just happen. You'll want to set clear goals and deadlines for yourself along the way. Otherwise, you may not feel ready and confident on test day. A helpful resource is the [Strategies for Success video](#), which includes tips for preparing and studying, along with tips for reducing test anxiety.

1) Learn what the test covers.

You may have heard that there are several different versions of the same test. It's true. You may take one version of the test and your friend may take a different version a few months later. Each test has different questions covering the same subject area, but both versions of the test measure the same skills and content knowledge.

You'll find specific information on the test you're taking in "4. Learn About Your Test" on page 11, which outlines the content categories that the test measures and what percentage of the test covers each topic. Visit www.ets.org/praxis/testprep for information on other *Praxis* tests.

2) Assess how well you know the content.

Research shows that test takers tend to overestimate their preparedness—this is why some test takers assume they did well and then find out they did not pass.

The *Praxis* tests are demanding enough to require serious review of likely content, and the longer you've been away from the content, the more preparation you will most likely need. If it has been longer than a few months since you've studied your content area, make a concerted effort to prepare.

3) Collect study materials.

Gathering and organizing your materials for review are critical steps in preparing for the *Praxis* tests. Consider the following reference sources as you plan your study:

- Did you take a course in which the content area was covered? If yes, do you still have your books or your notes?
- Does your college library have a good introductory college-level textbook in this area?
- Does your local library have a high school-level textbook?

Study guides are available for purchase for many *Praxis* tests at www.ets.org/praxis/testprep. Each guide provides a combination of test preparation and practice, including sample questions and answers with explanations.

4) Plan and organize your time.

You can begin to plan and organize your time while you are still collecting materials. Allow yourself plenty of review time to avoid cramming new material at the end. Here are a few tips:

- Choose a test date far enough in the future to leave you plenty of preparation time at www.ets.org/praxis/register/centers_dates.
- Work backward from that date to figure out how much time you will need for review.
- Set a realistic schedule—and stick to it.

5) Practice explaining the key concepts.

Praxis tests with constructed-response questions assess your ability to explain material effectively. As a teacher, you'll need to be able to explain concepts and processes to students in a clear, understandable way. What are the major concepts you will be required to teach? Can you explain them in your own words accurately, completely, and clearly? Practice explaining these concepts to test your ability to effectively explain what you know.

6) Understand how questions will be scored.

Scoring information can be found in "3. Understand Your Scores" on page 9.

7) Develop a study plan.

A study plan provides a road map to prepare for the *Praxis* tests. It can help you understand what skills and knowledge are covered on the test and where to focus your attention. Use the study plan template on page 23 to organize your efforts.

And most important—get started!

Would a Study Group Work for You?

Using this guide as part of a study group

People who have a lot of studying to do sometimes find it helpful to form a study group with others who are working toward the same goal. Study groups give members opportunities to ask questions and get detailed answers. In a group, some members usually have a better understanding of certain topics, while others in the group may be better at other topics. As members take turns explaining concepts to one another, everyone builds self-confidence.

If the group encounters a question that none of the members can answer well, the group can go to a teacher or other expert and get answers efficiently. Because study groups schedule regular meetings, members study in a more disciplined fashion. They also gain emotional support. The group should be large enough so that multiple people can contribute different kinds of knowledge, but small enough so that it stays focused. Often, three to six members is a good size.

Here are some ways to use this guide as part of a study group:

- **Plan the group's study program.** Parts of the study plan template, beginning on page 21 can help to structure your group's study program. By filling out the first five columns and sharing the worksheets, everyone will learn more about your group's mix of abilities and about the resources, such as textbooks, that members can share with the group. In the sixth column ("Dates I will study the content"), you can create an overall schedule for your group's study program.
- **Plan individual group sessions.** At the end of each session, the group should decide what specific topics will be covered at the next meeting and who will present each topic. Use the topic headings and subheadings in the Test at a Glance table on page 11 to select topics, and then select practice questions, beginning on page 31.
- **Prepare your presentation for the group.** When it's your turn to present, prepare something that is more than a lecture. Write two or three original questions to pose to the group. Practicing writing actual questions can help you better understand the topics covered on the test as well as the types of questions you will encounter on the test. It will also give other members of the group extra practice at answering questions.

- **Take the practice test together.** The idea of the practice test is to simulate an actual administration of the test, so scheduling a test session with the group will add to the realism and may also help boost everyone's confidence. Remember, complete the practice test using only the time that will be allotted for that test on your administration day.
- **Learn from the results of the practice test.** Score one another's answer sheets. For tests that contain constructed-response questions, look at the Sample Test Questions section, which also contain sample responses to those questions and shows how they were scored. Then try to follow the same guidelines that the test scorers use.
- **Be as critical as you can.** You're not doing your study partner(s) any favors by letting them get away with an answer that does not cover all parts of the question adequately.
- **Be specific.** Write comments that are as detailed as the comments about the sample responses. Indicate where and how your study partner(s) are doing an inadequate job of answering the question. Writing notes in the margins of the answer sheet may also help.
- **Be supportive.** Include comments that point out what your study partner(s) got right.

Then plan one or more study sessions based on aspects of the questions on which group members performed poorly. For example, each group member might be responsible for rewriting one paragraph of a response in which someone else did an inadequate job.

Whether you decide to study alone or with a group, remember that the best way to prepare is to have an organized plan. The plan should set goals based on specific topics and skills that you need to learn, and it should commit you to a realistic set of deadlines for meeting those goals. Then you need to discipline yourself to stick with your plan and accomplish your goals on schedule.

6. Develop Your Study Plan

Develop a personalized study plan and schedule

Planning your study time is important because it will help ensure that you review all content areas covered on the test. Use the sample study plan below as a guide. It shows a plan for the *Praxis I® Pre-Professional Skills Test: Reading* test. Following that is a study plan template that you can fill out to create your own plan. Use the “Learn about Your Test” and “Topics Covered” information beginning on page 11 to help complete it.

Use this worksheet to:

- 1. Define Content Areas:** List the most important content areas for your test as defined in the Topics Covered section.
- 2. Determine Strengths and Weaknesses:** Identify your strengths and weaknesses in each content area.
- 3. Identify Resources:** Identify the books, courses, and other resources you plan to use for each content area.
- 4. Study:** Create and commit to a schedule that provides for regular study periods.

Praxis Test Name: Praxis I Pre-Professional Skills Test: Reading
Praxis Test Code(s): 0710
Test Date: 11/15/12

Content covered	Description of content	How well do I know the content? (scale 1–5)	What resources do I have/need for the content?	Where can I find the resources I need?	Dates I will study the content	Date completed
Literal Comprehension						
Main Ideas	Identify summaries or paraphrases of main idea or primary purpose of reading selection	2	Middle school English text book	College library, middle school teacher	9/15/12	9/15/12
Supporting Ideas	Identify summaries or paraphrases of supporting ideas and specific details in reading selection	2	Middle school English text book	College library, middle school teacher	9/17/12	9/17/12
Organization	Identify how reading selection is organized in terms of cause/effect and compare/contrast	3	Middle and high school English text book	College library, middle and high school teachers	9/20/12	9/21/12
Organization	Identify key transition words/phrases in reading selection and how used	4	Middle and high school English text book	College library, middle and high school teachers	9/25/12	9/26/12
Vocabulary in Context	Identify meanings of words as used in context of reading selection	3	Middle and high school English text book, dictionary	College library, middle and high school teachers	9/25/12	9/27/12

(continued on next page)

Content covered	Description of content	How well do I know the content? (scale 1–5)	What resources do I have/need for the content?	Where can I find the resources I need?	Dates I will study the content	Date completed
Critical and Inferential Comprehension						
Evaluation	Determine whether evidence strengthens, weakens, or is relevant to arguments in reading selection	5	High school text book, college course notes	College library, course notes, high school teacher, college professor	10/1/12	10/1/12
Evaluation	Determine role that an idea, reference, or piece of information plays in author's discussion/argument	5	High school text book, college course notes	College library, course notes, high school teacher, college professor	10/1/12	10/1/12
Evaluation	Determine if information presented is fact or opinion	4	High school text book, college course notes	College library, course notes, high school teacher, college professor	10/1/12	10/1/12
Evaluation	Identify relationship among ideas presented in reading selection	2	High school text book, college course notes	College library, course notes, high school teacher, college professor	10/1/12	10/1/12
Inferential Reasoning	Draw inferences/implications from directly stated content of reading selection	3	High school text book, college course notes	College library, course notes, high school teacher, college professor	10/8/12	10/8/12
Inferential Reasoning	Determine logical assumptions on which argument or conclusion is based	2	High school text book, college course notes	College library, course notes, high school teacher, college professor	10/8/12	10/8/12
Inferential Reasoning	Determine author's attitude toward materials discussed in reading selection	1	High school text book, college course notes	College library, course notes, high school teacher, college professor	10/15/12	10/17/12
Generalization	Recognize or predict ideas/situations that are extensions of, or similar to, what has been presented in reading selection	2	High school text book, college course notes	College library, course notes, high school teacher, college professor	10/22/12	10/24/12
Generalization	Draw conclusions from materials presented in reading selection	3	High school text book, college course notes	College library, course notes, high school teacher, college professor	10/24/12	10/24/12
Generalization	Apply ideas presented in a reading selection to other situations	3	High school text book, college course notes	College library, course notes, high school teacher, college professor	10/27/12	10/27/12

My Study Plan

Use this worksheet to:

- 1. Define Content Areas:** List the most important content areas for your test as defined in the Learn about Your Test and Topics Covered sections.
- 2. Determine Strengths and Weaknesses:** Identify your strengths and weaknesses in each content area.
- 3. Identify Resources:** Identify the books, courses, and other resources you plan to use for each content area.
- 4. Study:** Create and commit to a schedule that provides for regular study periods.

Praxis Test Name: _____

Praxis Test Code: _____

Test Date: _____

Content covered	Description of content	How well do I know the content? (scale 1–5)	What resources do I have/need for the content?	Where can I find the resources I need?	Dates I will study the content	Date completed

(continued on next page)

Content covered	Description of content	How well do I know the content? (scale 1-5)	What resources do I have/need for the content?	Where can I find the resources I need?	Dates I will study the content	Date completed

7. Review Smart Tips for Success

Follow test-taking tips developed by experts

Learn from the experts. Take advantage of the following answers to questions you may have and practical tips to help you navigate the *Praxis* test and make the best use of your time.

Should I Guess?

Yes. Your score is based on the number of questions you answer correctly, with no penalty or subtraction for an incorrect answer. When you don't know the answer to a question, try to eliminate any obviously wrong answers and then guess at the correct one. Try to pace yourself so that you have enough time to carefully consider every question.

Can I answer the questions in any order?

Yes. You can go through the questions from beginning to end, as many test takers do, or you can create your own path. Perhaps you will want to answer questions in your strongest area of knowledge first and then move from your strengths to your weaker areas. On computer-delivered tests, you can use the "Skip" function to skip a question and come back to it later. There is no right or wrong way. Use the approach that works best for you.

Are there trick questions on the test?

No. There are no hidden meanings or trick wording. All of the questions on the test ask about subject matter knowledge in a straightforward manner.

Are there answer patterns on the test?

No. You might have heard this myth: the answers on multiple-choice tests follow patterns. Another myth is that there will never be more than two questions with the same lettered answer following each other. Neither myth is true. Select the answer you think is correct based on your knowledge of the subject.

Can I write in the test booklet or, for a computer-delivered test, on the scratch paper I am given?

Yes. You can work out problems right on the pages of the booklet or scratch paper, make notes to yourself, mark questions you want to review later or write anything at all. Your test booklet or scratch paper will be destroyed after you are finished with it, so use it in any way that is helpful to you. But make sure to mark your answers on the answer sheet or enter them on the computer.

Smart Tips for Taking the Test

- 1. For a paper-delivered test, put your answers in the right bubbles.** It seems obvious, but be sure that you fill in the answer bubble that corresponds to the question you are answering. A significant number of test takers fill in a bubble without checking to see that the number matches the question they are answering.
- 2. Skip the questions you find extremely difficult.** Rather than trying to answer these on your first pass through the test, leave them blank and mark them in your test booklet. Pay attention to the time as you answer the rest of the questions on the test, and try to finish with 10 or 15 minutes remaining so that you

can go back over the questions you left blank. Even if you don't know the answer the second time you read the questions, see if you can narrow down the possible answers, and then guess.

- 3. Keep track of the time.** Bring a watch to the test, just in case the clock in the test room is difficult for you to see. Keep the watch as simple as possible—alarms and other functions may distract others or may violate test security. If the test center supervisor suspects there could be an issue with your watch, they will ask you to remove it, so simpler is better! You will probably have plenty of time to answer all of the questions, but if you find yourself becoming bogged down in one section, you might decide to move on and come back to that section later.
- 4. Read all of the possible answers before selecting one.** Then reread the question to be sure the answer you have selected really answers the question. Remember, a question that contains a phrase such as “Which of the following does NOT ...” is asking for the one answer that is NOT a correct statement or conclusion.
- 5. Check your answers.** If you have extra time left over at the end of the test, look over each question and make sure that you have answered it as you intended. Many test takers make careless mistakes that they could have corrected if they had checked their answers.
- 6. Don't worry about your score when you are taking the test.** No one is expected to answer all of the questions correctly. Your score on this test is not analogous to your score on the *GRE*® or other similar-looking (but in fact very different) tests. It doesn't matter on the *Praxis* tests whether you score very high or barely pass. If you meet the minimum passing scores for your state and you meet the state's other requirements for obtaining a teaching license, you will receive a license. In other words, what matters is meeting the minimum passing score. You can find passing scores for all states that use *The Praxis Series* tests at http://www.ets.org/s/praxis/pdf/passing_scores.pdf or on the Web site of the state for which you are seeking certification/licensure.
- 7. Use your energy to take the test, not to get angry at it.** Getting angry at the test only increases stress and decreases the likelihood that you will do your best. Highly qualified educators and test development professionals, all with backgrounds in teaching, worked diligently to make the test a fair and valid measure of your knowledge and skills. Your state painstakingly reviewed the test before adopting it as a licensure requirement. The best thing to do is concentrate on answering the questions.

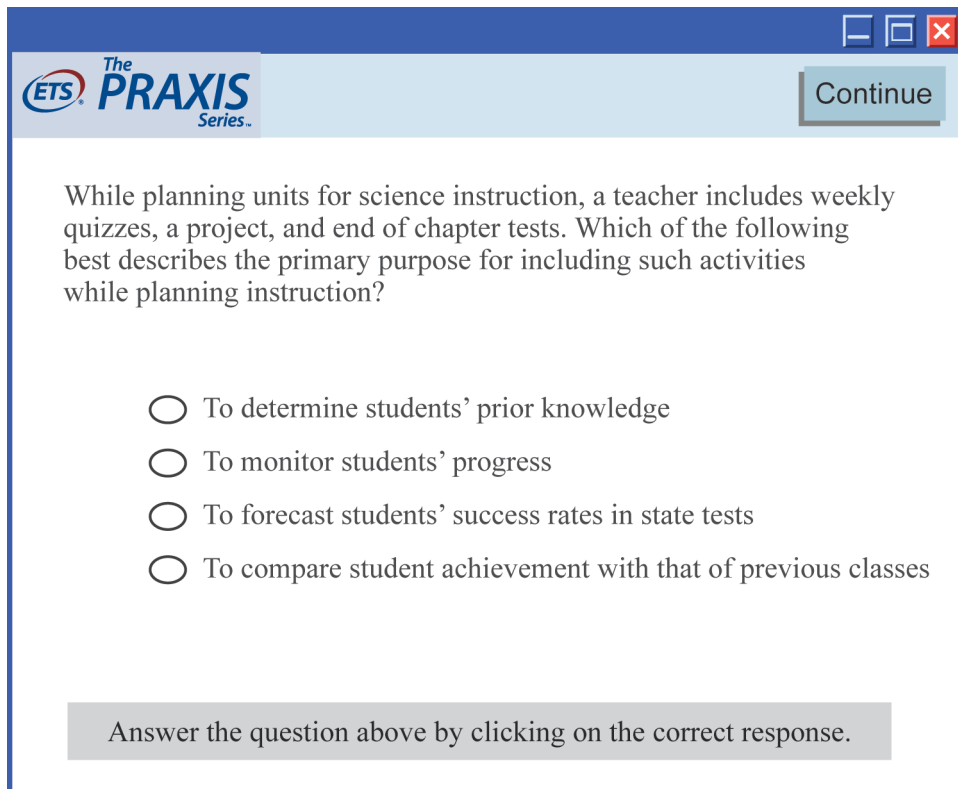
8. Practice with Sample Test Questions

Answer practice questions and find explanations for correct answers

Sample Test Questions

This test is available via computer delivery and paper delivery. Other than the delivery method, there is no difference between the tests. The scope of the test content is the same for both test codes.

To illustrate what the computer-delivered test looks like, the following sample question shows an actual screen used in a computer-delivered test. For the purposes of this guide, sample questions are provided as they would appear in a paper-delivered test.



The PRAXIS Series

Continue

While planning units for science instruction, a teacher includes weekly quizzes, a project, and end of chapter tests. Which of the following best describes the primary purpose for including such activities while planning instruction?

- To determine students' prior knowledge
- To monitor students' progress
- To forecast students' success rates in state tests
- To compare student achievement with that of previous classes

Answer the question above by clicking on the correct response.

Selected Notations, Definitions, and Formulas (as provided with the test)

NOTATIONS

(a, b)	$\{x: a < x < b\}$
$[a, b)$	$\{x: a \leq x < b\}$
$(a, b]$	$\{x: a < x \leq b\}$
$[a, b]$	$\{x: a \leq x \leq b\}$
$\text{gcd}(m, n)$	greatest common divisor of two integers m and n
$\text{lcm}(m, n)$	least common multiple of two integers m and n
$[x]$	greatest integer m such that $m \leq x$
$m \equiv k \pmod{n}$	m and k are congruent modulo n (m and k have the same remainder when divided by n , or equivalently, $m - k$ is a multiple of n)
f^{-1}	inverse of an invertible function f ; (not to be read as $\frac{1}{f}$)
$\lim_{x \rightarrow a^+} f(x)$	right-hand limit of $f(x)$; limit of $f(x)$ as x approaches a from the right
$\lim_{x \rightarrow a^-} f(x)$	left-hand limit of $f(x)$; limit of $f(x)$ as x approaches a from the left
\emptyset	the empty set
$x \in S$	x is an element of set S
$S \subset T$	set S is a proper subset of set T
$S \subseteq T$	either set S is a proper subset of set T or $S = T$
\bar{S}	complement of set S ; the set of all elements not in S that are in some specified universal set
$T \setminus S$	relative complement of set S in set T , i.e., the set of all elements of T that are not elements of S
$S \cup T$	union of sets S and T
$S \cap T$	intersection of sets S and T

DEFINITIONS

A relation \mathfrak{R} on a set S

reflexive if $x \mathfrak{R} x$ for all $x \in S$

symmetric if $x \mathfrak{R} y \Rightarrow y \mathfrak{R} x$ for all $x, y \in S$

transitive if $(x \mathfrak{R} y \text{ and } y \mathfrak{R} z) \Rightarrow x \mathfrak{R} z$ for all $x, y, z \in S$

antisymmetric if $(x \mathfrak{R} y \text{ and } y \mathfrak{R} x) \Rightarrow x = y$ for all $x, y \in S$

An equivalence relation is a reflexive, symmetric, and transitive relation.

FORMULAS

Sum

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Half-angle (sign depends on the quadrant of $\frac{\theta}{2}$)

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

Range of Inverse Trigonometric Functions

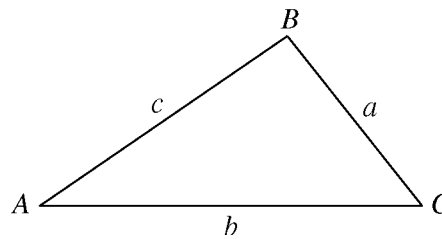
$$\sin^{-1} x \quad \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$\cos^{-1} x \quad [0, \pi]$$

$$\tan^{-1} x \quad \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Law of Cosines

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

DeMoivre's Theorem

$$(\cos \theta + i \sin \theta)^k = \cos(k\theta) + i \sin(k\theta)$$

Coordinate Transformation

Rectangular (x, y) to polar (r, θ) : $r^2 = x^2 + y^2$; $\tan \theta = \frac{y}{x}$ if $x \neq 0$

Polar (r, θ) to rectangular (x, y) : $x = r \cos \theta$; $y = r \sin \theta$

Distance from point (x_1, y_1) to line $Ax + By + C = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Volume

Sphere with radius r :

$$V = \frac{4}{3}\pi r^3$$

Right circular cone with height h and base of radius r :

$$V = \frac{1}{3}\pi r^2 h$$

Right circular cylinder with height h and base of radius r :

$$V = \pi r^2 h$$

Pyramid with height h and base of area B :

$$V = \frac{1}{3}Bh$$

Right prism with height h and base of area B :

$$V = Bh$$

Surface Area

Sphere with radius r :

$$A = 4\pi r^2$$

Right circular cone with radius r and slant height s :

$$A = \pi rs + \pi r^2$$

Differentiation

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

$$(f(g(x)))' = f'(g(x))g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \text{ if } g(x) \neq 0$$

Integration by Parts

$$\int u dv = uv - \int v du$$

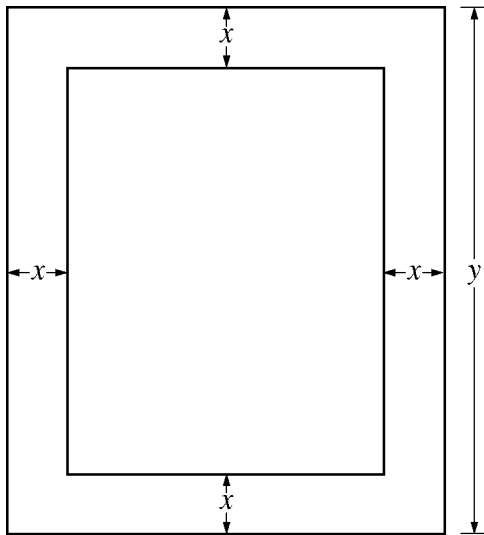
The sample questions that follow illustrate the kinds of questions on the test. They are not, however, representative of the entire scope of the test in either content or difficulty. Answers with explanations follow the questions.

Directions: Each of the questions or statements below is followed by four suggested answers or completions. Select the one that is best in each case.

Algebra and Number Theory

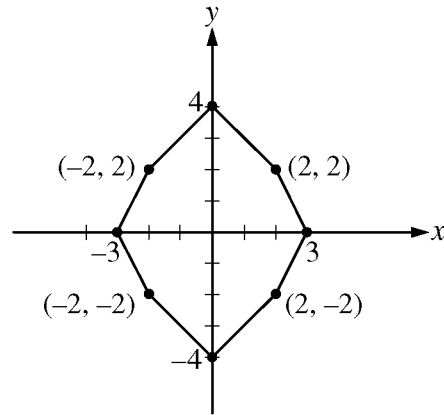
- Jerry is 50 inches tall and is growing at the rate of $\frac{1}{24}$ inch per month. Adam is 47 inches tall and is growing at the rate of $\frac{1}{8}$ inch per month. If they each continue to grow at these rates for the next four years, in how many months will they be the same height?
 - 24
 - 30
 - 36
 - 42
- What is the units digit of 33^{408} ?
 - 1
 - 3
 - 7
 - 9
- If x and y are even numbers and $z = 2x^2 + 4y^2$, then the greatest even number that must be a divisor of z is
 - 2
 - 4
 - 8
 - 16
- A taxicab driver charges a fare of \$2.00 for the first quarter-mile or less and \$0.75 for each quarter-mile after that. Which of the following equations models the fare, f , in dollars, for a ride m miles long, where m is a positive integer?
 - $f(m) = 2.00 + 0.75(m-1)$
 - $f(m) = 2.00 + 0.75\left(\frac{m}{4} - 1\right)$
 - $f(m) = 2.00 + 0.75(4m-1)$
 - $f(m) = 2.00 + 0.75(4(m-1))$
- For which of the following values of k does the equation $x^4 - 4x^2 + x + k = 0$ have four distinct real roots?
 - 2
 - 1
 - 3
 - II only
 - III only
 - II and III only
 - I, II, and III

Measurement

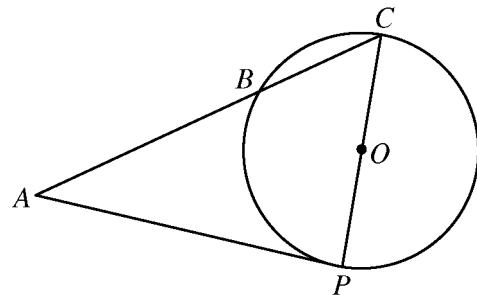


6. The inside of a rectangular picture frame measures 36 inches long and 24 inches wide. The width of the frame is x inches, as shown in the figure above. When hung, the frame and its contents cover 1,408 square inches of wall space. What is the length, y , of the frame, in inches?
- (A) 44
 (B) 40
 (C) 38
 (D) 34

Geometry



7. For how many angles θ , where $0 < \theta \leq 2\pi$, will rotation about the origin by angle θ map the octagon in the figure above onto itself?
- (A) One
 (B) Two
 (C) Four
 (D) Eight



Note: Figure not drawn to scale.

8. In the circle above with center O and radius 2, AP has length 3 and is tangent to the circle at P . If CP is a diameter of the circle, what is the length of BC ?
- (A) 1.25
 (B) 2
 (C) 3.2
 (D) 5

Trigonometry

9. If $y = 5 \sin x - 6$, what is the maximum value of y ?
- (A) -6
 (B) -1
 (C) 1
 (D) 5
10. In $\triangle ABC$ (not shown), the length of side AB is 12, the length of side BC is 9, and the measure of angle BAC is 30° . What is the length of side AC ?
- (A) 17.10
 (B) 4.73
 (C) 3.68
 (D) It cannot be determined from the information given
11. In the xy -plane, an acute angle with vertex at the origin is formed by the positive x -axis and the line with equation $y = 3x$. What is the slope of the line that contains the bisector of this angle?
- (A) 3
 (B) $\frac{3}{2}$
 (C) $\frac{\sqrt{10} + 1}{3}$
 (D) $\frac{\sqrt{10} - 1}{3}$

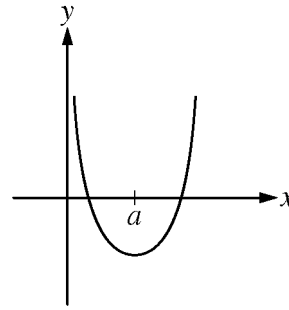
Functions

12. At how many points in the xy -plane do the graphs of $y = 4x^5 - 3x^2 - 1$ and $y = -0.4 - 0.11x$ intersect?
- (A) One
 (B) Two
 (C) Three
 (D) Four
13. If
- (i) the graph of the function $f(x)$ is the line with slope 2 and y -intercept 1
 and
 (ii) the graph of the function $g(x)$ is the line with slope -2 and y -intercept -1 ,
- which of the following is an algebraic representation of the function $y = f(g(x))$?
- (A) $y = 0$
 (B) $y = -4x - 3$
 (C) $y = -4x - 1$
 (D) $y = -(2x + 1)^2$
-

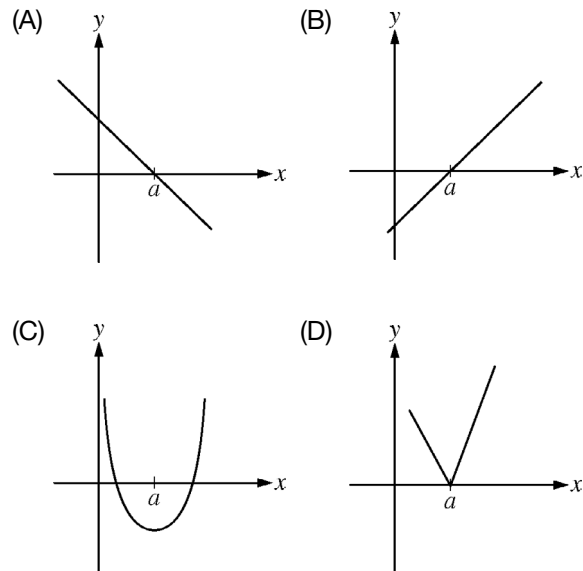
$$P(t) = 250 \cdot (3.04)^{\frac{t}{1.98}}$$

14. At the beginning of 1990, the population of rabbits in a wooded area was 250. The function above was used to model the approximate population, P , of rabbits in the area t years after January 1, 1990. According to this model, which of the following best describes how the rabbit population changed in the area?
- (A) The rabbit population doubled every 4 months.
- (B) The rabbit population tripled every 6 months.
- (C) The rabbit population doubled every 36 months.
- (D) The rabbit population tripled every 24 months.
15. If $f(x) = 3x^2$, what are all real values of a and b for which the graph of $g(x) = ax^2 + b$ is below the graph of $f(x)$ for all values of x ?
- (A) $a \geq 3$ and b is positive.
- (B) $a \leq 3$ and b is negative.
- (C) a is negative and b is positive.
- (D) a is any real number and b is negative.

Calculus



16. The figure above is a graph of a differentiable function f . Which of the following could be the graph of the first derivative of this function?



17. If $\lim_{x \rightarrow c} f(x) = 0$ and $\lim_{x \rightarrow c} g(x) = 0$ what can be concluded about the value of $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$?
- (A) The value is not finite.
- (B) The value is 0.
- (C) The value is 1.
- (D) The value cannot be determined from the information given.

18. In a certain chemical reaction, the number of grams, N , of a substance produced t hours after the reaction begins is given by $N(t) = 16t - 4t^2$, where $0 < t < 2$. At what rate, in grams per hour, is the substance being produced 30 minutes after the reaction begins?
- (A) 7
(B) 12
(C) 16
(D) 20

Data Analysis and Statistics

19. The measures of the hand spans of ninth-grade students at Tyler High School are approximately normally distributed, with a mean of 7 inches and a standard deviation of 1 inch. Of the following groups of measurements of hand span, which is expected to contain the largest number of ninth-graders?
- (A) Less than 6 inches
(B) Greater than 7 inches
(C) Between 6 and 8 inches
(D) Between 5 and 7 inches

Stem	Leaf					
9	1	3	4	5	7	
8	2	2	5	6	6	8 9
7	0	2	4	5	8	8
6	1	3	7	9		

20. The stem plot above shows the course grades that each of 22 students received in a history course. The course grade is represented by using the tens digit of each grade as a stem and the corresponding units digit as a leaf. For example, the stem 9 and the leaf 1 in the first row of the table represent a grade of 91. What was the median course grade of the 22 students?
- (A) 78
(B) 80
(C) 80.7
(D) 82

Probability

21. A two-sided coin is unfairly weighted so that when it is tossed, the probability that heads will result is twice the probability that tails will result. If the coin is to be tossed 3 separate times, what is the probability that tails will result on exactly 2 of the tosses?
- (A) $\frac{2}{9}$
(B) $\frac{3}{8}$
(C) $\frac{4}{9}$
(D) $\frac{2}{3}$

Matrix Algebra

22. The orthogonal projection of 3-space onto the xy -plane takes the point (x, y, z) onto the point $(x, y, 0)$. This transformation can be represented by the matrix equation

$$M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}, \text{ where } M \text{ is which of the}$$

following matrices?

(A) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

23. For what value of x is the matrix $\begin{pmatrix} 1 & 4 \\ x & 6 \end{pmatrix}$ NOT invertible?

(A) $-\frac{3}{2}$

(B) 0

(C) $\frac{3}{2}$

(D) 2

Discrete Mathematics

24. Given the recursive function defined by

$$f(1) = -3,$$

$$f(n) = f(n-1) - 6 \text{ for } n \geq 2,$$

what is the value of $f(4)$?

(A) -2

(B) -9

(C) -10

(D) -21

25. For lines in the plane, the relation “is perpendicular to” is

(A) reflexive but not transitive

(B) symmetric but not transitive

(C) transitive but not symmetric

(D) both symmetric and transitive

Answers to Sample Questions

Algebra and Number Theory

1. The heights in this question can be expressed as two linear equations. Jerry's height in inches, J , can be expressed as $J = 50 + \frac{1}{24}m$, where m is the number of months from now. Adam's height in inches, A , can be expressed as $A = 47 + \frac{1}{8}m$. The question asks, "in how many months will they be the same height?" This is the same as asking, "for what value of m will $J = A$?" The solution can be found by solving

$$50 + \frac{1}{24}m = 47 + \frac{1}{8}m \text{ for } m.$$

$$50 + \frac{1}{24}m = 47 + \frac{1}{8}m$$

$$50 - 47 = \left(\frac{1}{8} - \frac{1}{24}\right)m$$

$$3 = \left(\frac{3}{24} - \frac{1}{24}\right)m$$

$$3 = \frac{1}{12}m$$

$$m = 36$$

The correct answer is (C), 36 months.

2. To find the units digit of 33^{408} , it is helpful to find the first few integer powers of 33 and look for a pattern. For example,

$$33^1 = 33$$

$$33^2 = 1089$$

$$33^3 = 35,937$$

$$33^4 = 1,185,921$$

$$33^5 = 39,135,393$$

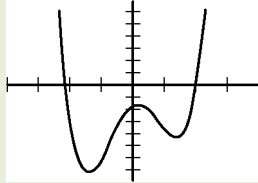
You can see that the pattern in the units digits is 3, 9, 7, 1, 3, ... and that it will continue to repeat with every four integers of the exponent. Dividing 408 by 4 yields 102 with no remainder. So the units digit of 33^{408} will be the same as the units digit of 33^4 , which is 1. The correct answer is (A).

3. Since 2 is a divisor of both $2x^2$ and $4y^2$, it follows that 2 is a divisor of z . To find out if there is a greater even number that **must** be a divisor of z , you need to consider the additional information given, which is that x and y are both even numbers. Since x and y are even numbers, they can be expressed as $x = 2m$ and $y = 2n$, respectively, where m and n can be either odd or even integers. Substituting these values for x and y in the expression for z yields $z = 2(2m)^2 + 4(2n)^2$. It follows then that $z = 8m^2 + 16n^2$ and that 8 is a divisor of z . The number 16 would also be a divisor of z if m is even, but not if m is odd. Since m and n can be either even or odd and the question asks for the largest even number that **must** be a divisor of z , the correct answer is (C), 8.

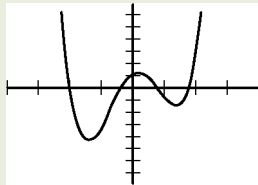
4. This question asks you to determine which of the four equations given as choices models the fare for a taxi ride of m miles, where m is a positive integer. The question states that the fare is \$2.00 for the first quarter-mile or less and \$0.75 for each quarter mile after that. You will notice by examining the answer choices that all of the choices include a constant term of 2.00 (for the first quarter-mile). Thus, the task is to model the fare for the remaining distance beyond the first quarter-mile. Since the question states that \$0.75 is charged for each quarter-mile after the first, you must determine how many **quarter-miles** the trip is. Since the trip is given as m miles (where m is an integer), the number of quarter-miles in the trip would be $4m$. The charge for the first quarter mile is \$2.00, so that would leave $4m - 1$ quarter miles to be charged at a rate of \$0.75 each. The total fare for the trip would thus be modeled by the equation $f = 2.00 + 0.75(4m - 1)$. By comparing this with the choices given, you will see that the correct answer is (C).

5. You may recall from your study of solutions to polynomial equations that a fourth-degree polynomial has at most four distinct real roots and that the roots of the equation are the x -intercepts of the graph of the equation. One way to determine for which of the three given values of k the equation will have four distinct real roots is to graph the equations using your calculator.

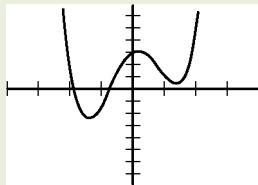
I. $x^4 - 4x^2 + x - 2 = 0$



II. $x^4 - 4x^2 + x + 1 = 0$



III. $x^4 - 4x^2 + x + 3 = 0$



Using an appropriate viewing window to see the behavior of the graphs for the three values of k clearly, you can see that the values of k given in I and III each result in the equation having only two distinct real roots. The value of k given in II results in the equation having four distinct real roots. The correct answer is (A), II only.

Measurement

6. This question requires you to use your knowledge of the area of a rectangle in order to find the outer length, y , of the picture frame described. You should recall that the area of a rectangle can be found by multiplying the length of the rectangle by the width of the rectangle. The inside dimensions of the frame are given as 36 inches long and 24 inches wide. The width of the frame is given as x inches, so that the outside dimensions of the frame would be $36 + 2x$ inches long and $24 + 2x$ inches wide. The area of the rectangle with the outside dimensions of the frame is given as 1,408 square inches. This area can then be represented as $(36 + 2x)(24 + 2x) = 1,408$. Solving this for x yields

$$(36 + 2x)(24 + 2x) = 1,408$$

$$864 + 48x + 72x + 4x^2 = 1,408$$

$$4x^2 + 120x - 544 = 0$$

$$x^2 + 30x - 136 = 0$$

$$(x + 34)(x - 4) = 0$$

$$x = -34 \text{ or } x = 4$$

Only $x = 4$ makes sense in the context of this question, so the width of the frame is 4 inches and, therefore, the outer length, y , of the frame is $36 + 2x = 36 + 2(4) = 44$. The correct answer is (A), 44 inches.

Geometry

7. The question asks you to consider rotation about the origin of the octagon in the figure and to determine for how many angles θ , where $0 < \theta \leq 2\pi$, would rotation of the octagon result in the octagon being mapped onto itself. One way to begin is to consider a single point on the octagon, such as the point $(0, 4)$, at the "top" of the octagon in the figure. This point is 4 units from the origin, so any rotation that maps the octagon onto itself would need to map this point onto a point that is also 4 units from the origin. The only other point on the octagon that is 4 units from the origin is the point $(0, -4)$. A rotation of angle $\theta = \pi$ would map the point $(0, 4)$ onto the point $(0, -4)$. You can see that the octagon is symmetric about both the x - and y -axes, so a rotation of angle $\theta = \pi$ would map all of the points of the octagon onto corresponding points of the octagon. Likewise, a rotation of angle $\theta = 2\pi$ would map the point $(0, 4)$ onto itself (and map all other points of the octagon onto themselves). No other values of θ such that $0 < \theta \leq 2\pi$ would map the octagon onto itself. Therefore, the correct answer is two, choice (B).

8. To determine the length of BC , it would be helpful to first label the figure with the information given. Since the circle has radius 2, then both OC and OP have length 2 and CP has length 4. AP is tangent to the circle at P , so angle APC is a right angle. The length of AP is given as 3. This means that triangle ACP is a 3-4-5 right triangle and AC has length 5. You should also notice that since CP is a diameter of the circle, angle CBP is also a right angle. Angle BCP is in both triangle ACP and triangle PCB and, therefore, the two triangles are similar. You can then find the length of BC by setting up a proportion between the corresponding parts of the similar triangles as follows:

$$\frac{CP}{AC} = \frac{BC}{PC}$$

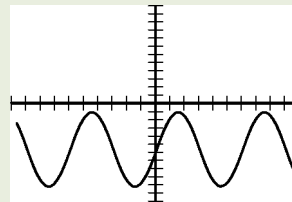
$$\frac{4}{5} = \frac{BC}{4}$$

$$BC = \frac{16}{5} = 3.2$$

The correct answer, 3.2, is (C).

Trigonometry

9. There are two ways to answer this question. You should be able to use either method. The first solution is based on reasoning about the function $f(x) = \sin x$. First, you need to recall that the maximum value of $\sin x$ is 1 and, therefore, the maximum value of $5\sin x$ is 5. The maximum value of $y = 5\sin x - 6$ is then $5 - 6 = -1$. Alternatively, you could graph the function $y = 5\sin x - 6$ and find the maximum value of y from the graph.



The maximum value is -1 , and the correct answer is (B).

10. In this question, you are given the length of two sides of a triangle and the measure of the angle opposite one of those two sides of the triangle. You are asked to find the length of the third side of the triangle. You should recall that the law of sines relates the lengths of two sides of a triangle and the sines of the angles opposite the sides. (The law of sines is included in the Notations, Definitions, and Formulas pages that are included in this document and at the beginning of each of the Content Knowledge tests.) Using the law of sines yields

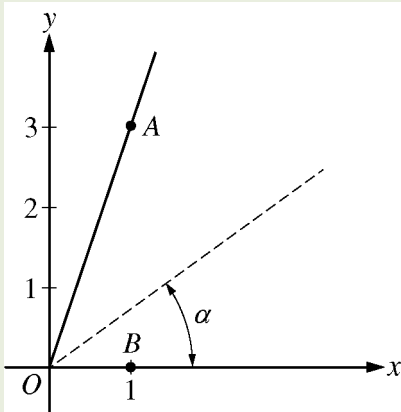
$$\frac{\sin(\angle BAC)}{\sin(\angle BCA)} = \frac{BC}{BA} \quad \text{and} \quad \frac{\sin 30^\circ}{\sin(\angle BCA)} = \frac{9}{12}$$

$$\text{Therefore, } \sin(\angle BCA) = \frac{4}{3} \sin 30^\circ = \frac{2}{3}.$$

You should recall that this is an example of the ambiguous case of the law of sines—that since the value of the sine is between 0 and 1, there are two angles between 0 and 180 degrees, one acute and one obtuse, associated with this sine and therefore there are two possible triangles with the given sides and angle measure.

The two values of the measure of $\angle BCA$ are approximately 41.8° and 138.2° . Using either the law of sines again (with $\angle BAC$ and $\angle ABC$, or with $\angle BCA$ and $\angle ABC$) or the law of cosines, you can determine that the length of side AC is either approximately 3.68 or 17.10. Since the length of side AC cannot be uniquely determined, the correct answer is (D), "It cannot be determined from the information given."

11. To answer this question, it might be helpful to first draw a figure such as the one shown below.



Consider the triangle OAB , where O is the origin, A is the point $(1, 3)$, and B is the point $(1, 0)$. Point A lies on the line $y = 3x$. The acute angle described in the question is the angle AOB . The question asks you to find the slope of the line that contains the angle bisector of angle AOB . Let α be the angle between the x -axis and the angle bisector of angle AOB . Then the slope of the line that contains the angle bisector of angle AOB will be equal to $\tan \alpha$. You can use the half-angle formulas in the Notations, Definitions, and Formulas that begin page 28, and also are provided when you take the Mathematics: Content Knowledge test, to find $\tan \alpha$ in terms of the sine and cosine of angle AOB . From your figure, you can see that $OB = 1$, $AB = 3$, and $OA = \sqrt{10}$.

$$\tan \alpha = \frac{\sin\left(\frac{\angle AOB}{2}\right)}{\cos\left(\frac{\angle AOB}{2}\right)} = \frac{\sqrt{\frac{1 - \cos(\angle AOB)}{2}}}{\sqrt{\frac{1 + \cos(\angle AOB)}{2}}}$$

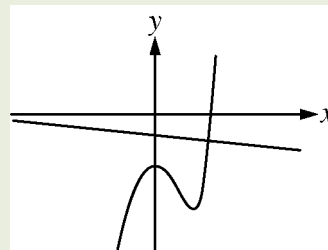
Simplifying yields

$$\begin{aligned} \frac{\sqrt{\frac{1 - \cos(\angle AOB)}{2}}}{\sqrt{\frac{1 + \cos(\angle AOB)}{2}}} &= \frac{\sqrt{1 - \cos^2(\angle AOB)}}{\sqrt{1 + \cos(\angle AOB)}} \\ &= \frac{\sin(\angle AOB)}{1 + \cos(\angle AOB)} \\ &= \frac{3}{1 + \frac{1}{\sqrt{10}}} \\ &= \frac{3}{\sqrt{10} + 1} \\ &= \frac{3(\sqrt{10} - 1)}{10 - 1} \\ &= \frac{\sqrt{10} - 1}{3} \end{aligned}$$

The correct answer is (D).

Functions

12. To answer this question, you should graph the equations on your calculator using an appropriate viewing window and then see how many points of intersection are shown. The figure below shows one view of the intersections of the two graphs.



You should also convince yourself that there are no additional points of intersection that are not visible in this viewing window. One way to do that is to verify that $y = 4x^5 - 3x^2 - 1$ has only two relative extrema, both of which are shown. (Find where $y' = 0$.) Only one point of intersection is shown in the figure above, so the correct answer is (A).

13. This question asks you to find an algebraic representation of the composition of the functions $f(x)$ and $g(x)$. First, you should write algebraic representations of the individual functions. You are given the slopes and y -intercepts of the lines that are the graphs of $f(x)$ and $g(x)$. Using the slope-intercept form of the equation of a line ($y = mx + b$, where m is the slope and b is the y -intercept) and the information given in parts (i) and (ii) yields the following functions, which have the graphs described in the question: $f(x) = 2x + 1$ and $g(x) = -2x - 1$. These functions imply that $f(g(x)) = 2(-2x - 1) + 1 = -4x - 1$. So $y = -4x - 1$, and (C) is the correct answer.

14. In this question, a model is given for the growth of the rabbit population as a function of time, t , in years. The question asks for a verbal description of the change in the rabbit population, based on the function given. You should recall the meaning of the base (growth factor) and the exponent in an exponential growth model. You should note that the function given

$P(t) = 250 \cdot (3.04)^{\frac{t}{98}} \approx 250 \cdot 3^{\frac{t}{2}}$. You can observe from this approximation (with base 3, and exponent $\frac{t}{2}$) that the population tripled every two years. Thus, the correct answer is (D), "The rabbit population tripled every 24 months."

15. This question is asking about your understanding of how changing the values of the coefficient a and y -intercept b in a quadratic function $f(x) = ax^2 + b$ affects the graph of the function. You should recall that for $a > 0$, as a decreases, the width of the parabola that is the graph of $y = ax^2$ increases, and for $a < 0$, the graph opens downward. You should also recall that as the value of b decreases, the vertex of the graph of $y = ax^2 + b$ moves in a negative direction along the y -axis. So for the graph of $g(x) = ax^2 + b$ to be below the graph of $f(x) = 3x^2$ for all values of x , a must be less than or equal to 3 and b must be negative (the vertex will be below the vertex of $f(x)$, which is at the origin). The correct answer, therefore, is (B).

Calculus

16. This question asks you to determine the possible shape of the graph of the first derivative of a differentiable function from the shape of the graph of the function. You should recall that the first derivative of the function at a point is equal to the slope of the graph of the function at that point. By inspection, you will see that, starting near $x = 0$, the slope of the graph of $f(x)$ is negative and becomes less negative as x approaches a and that the slope is 0 at $x = a$ (at the minimum value of f) and then becomes increasingly positive as x increases. Only (B) is consistent with this behavior. Therefore, (B) is the correct answer.

17. In a problem such as this, which contains the answer choice "It cannot be determined from the information given," you should be careful to base your answer on correct reasoning. If you conclude that the value can be determined, you should base your conclusion on known mathematical facts or principles; however, if you conclude that the value cannot be determined, you should support your conclusion by producing two different possible values for the limit.

You should recall that the quotient property of limits

states that if $\lim_{x \rightarrow c} f(x) = L$ and $\lim_{x \rightarrow c} g(x) = M$, and if

$M \neq 0$, then $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{M}$. However, this property

cannot be used to determine $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$ for the problem

at hand since the value of $\lim_{x \rightarrow c} g(x)$ is 0 and the quotient

property is inconclusive in this case. In fact, for this problem,

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)} = \frac{0}{0}. \text{ Note that the expression } \frac{0}{0}$$

does not represent a real number; in particular, it is not

equal to either 0 or 1. Thus, the value of $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$

cannot be determined by using the basic properties of

limits. As a result, you should suspect that, in fact,

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$$
 cannot be determined and verify your hunch

by producing examples to show that the value of the limit depends on the particular functions f and g .

In the remaining discussion, it will be assumed that $c = 0$. (It is always possible to apply a translation of c units to the two functions.) You should be aware that, although both f and g have the limit 0 as $x \rightarrow 0$, one function might be approaching 0 more quickly than the other, which would affect the value of the limit of

the quotient. Thus, if one of the functions is x and the other x^2 , then the quotient is either x or $\frac{1}{x}$, and so the

limit of the quotient is either 0 or nonexistent, respectively. The value of the limit can, in fact, be any nonzero real number b , as the functions bx and x show. Thus, answer choices (A), (B), and (C) are incorrect, and the correct answer is (D).

18. In this question, you are given a function, N , that models the production of a certain chemical reaction in grams as a function of time, t , in hours. You are asked to find the **rate** of production at 30 minutes after the reaction begins. The rate of production will be equal to the first derivative of N evaluated at 30 minutes. You should recognize that you first need to convert 30 minutes into hours and then evaluate the first

derivative of N at that value of t . Since 30 minutes

equals $\frac{1}{2}$ hour, you will need to evaluate $N' \left(\frac{1}{2} \right)$.

First, find $N'(t)$.

$$N'(t) = 16 - 8t.$$

Therefore, $N' \left(\frac{1}{2} \right) = 16 - 8 \left(\frac{1}{2} \right) = 12$. The answer is 12

grams per hour, so the correct answer is (B).

Data Analysis and Statistics

19. In this question, you will need to use your knowledge of a normally distributed set of data. In particular, you should know that approximately 68 percent of a normally distributed set of data lie within ± 1 standard deviation of the mean and that approximately 95 percent of the data lie within ± 2 standard deviations of the mean. The question asks you to identify which of the groups given in the answer choices is expected to correspond to the greatest number of ninth-graders if the hand spans of ninth-graders are approximately normally distributed with a mean of 7 inches and a standard deviation of 1 inch. You will need to evaluate each answer choice in order to determine which of the groups is largest.

(A) is the group of hand spans less than 6 inches. Since the mean hand span is 7 inches and the standard deviation is 1 inch, the group of hand spans that is less than 6 inches is the group that is more than 1 standard deviation less than the mean. The group of hand spans that is less than 7 inches includes 50 percent of the measurements. Approximately 34 percent ($\frac{1}{2}$ of 68 percent) of the measurements are between 6 inches and 7 inches (within 1 standard deviation less than the mean). So the group with hand spans less than 6 inches would be approximately equal to $50 - 34$, or 16 percent of the measurements.

(B) is the group of hand spans greater than 7 inches. Since 7 inches is the mean, approximately 50 percent of the measurements are greater than the mean.

(C) is the group of hand spans between 6 and 8 inches. This is the group that is within ± 1 standard deviation of the mean. This group contains approximately 68 percent of the measurements.

(D) is the group of hand spans between 5 and 7 inches. This group is between the mean and 2 standard deviations less than the mean. Approximately 47.5 percent ($\frac{1}{2}$ of 95 percent) of the measurements are between 5 inches and 7 inches.

Of the answer choices given, the group described in (C) is expected to contain the greatest percent of the measurements, approximately 68 percent, and would correspond to the largest number of ninth-graders, so (C) is the correct answer.

20. A stem plot such as the one shown in this question is a very useful way to display data such as these when you are interested in determining the median value of the data. The data in a stem plot is ordered, so finding the median, the middle number when the data are ordered from least to greatest or greatest to least, is straightforward. You are given the course grades received by 22 students. The median course grade would be the average of the course grades of the 11th and 12th students. You can start at either the least or greatest data entry and count in increasing (or decreasing) order along the leaves until you reach the 11th and 12th entries. In this case, both the 11th and 12th entries have a value of 82 (i.e., a stem value of 8 and a leaf value of 2). Therefore, the median course grade received by the 22 students is 82. The correct answer is (D).

Probability

21. In this question, you are asked to apply your knowledge of independent events to find the probability of tossing tails exactly 2 out of 3 times when using an unfairly weighted coin. Because each toss of the coin is an independent event, the probability of tossing heads then 2 tails, $P(HTT)$, is equal to $P(H) \cdot P(T) \cdot P(T)$, where $P(H)$ is the probability of tossing heads and $P(T)$ is the probability of tossing tails. In this case, you are given that the probability of tossing heads is twice the probability of tossing tails.

So, $P(H) = \frac{2}{3}$ and $P(T) = \frac{1}{3}$. (Out of 3 tosses, 2 would be expected to be heads and 1 would be expected to be tails.) Therefore, $P(HTT) = \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{2}{27}$. There are 3 ways in which exactly 2 of 3 tosses would be tails and each of them has an equal probability of occurring: $P(THT) = P(TTH) = P(HTT) = \frac{2}{27}$. Therefore, the total probability that tails will result exactly 2 times in 3 tosses is $3\left(\frac{2}{27}\right) = \frac{2}{9}$. The correct answer is (A).

Matrix Algebra

22. In order to answer this question, you need to consider how matrix multiplication is performed. You are asked to find a matrix, M , that when multiplied by

any matrix of the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, yields the result $\begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$. You

will notice that all of the answer choices are 3×3 matrices. You can either solve this problem for the general case or reason to the answer. First, the general solution:

$$M = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix}. \text{ Then } M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax+by+cz \\ dx+ey+fz \\ gx+hy+jz \end{pmatrix} \text{ for all}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}. \text{ Since } M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \text{ for all } \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \text{ then}$$

$$\begin{cases} ax+by+cz=y \\ dx+ey+fz=y \text{ for all } x, y, \text{ and } z. \text{ This implies } a=1, \\ gx+hy+jz=0 \end{cases}$$

$b=0$, $c=0$; and $d=0$, $e=1$, $f=0$; and

$$g=h=j=0; \text{ and, therefore, } M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The correct answer is (B). You could also reason to the answer by inspecting the answer choices given. Since

multiplying the first row of M by the matrix $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ has to

result in only the x term for all x, y , and z , the first entry in the first row must be 1 and the others 0. Likewise,

multiplying the second row of M by the matrix $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ will

result only in the y term for all x, y , and z , so the entries in the second row must be 0, 1, 0, in that order.

Multiplying the third row of M by the matrix $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ results

in 0 for all x, y , and z , so the entries in the third row

must all be 0. Therefore, $M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and the correct answer is (B).

23. This question asks you to find the value of x for which the given matrix is NOT invertible. A matrix is not invertible if the determinant of the matrix is equal to

zero. The determinant of the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is equal to

$ad - bc$. For the matrix given in the question, the determinant is equal to $(1)(6) - (4)(x)$. This equals 0

when $6 - 4x = 0$, or $x = \frac{3}{2}$. The correct answer is (C).

Discrete Mathematics

24. Given the recursive function defined in the question, in order to find $f(4)$, you first need to find $f(2)$ and $f(3)$. ($f(1)$ is given.)

Since $f(1) = -3$ and $f(n) = f(n-1) - 6$ for $n \geq 2$, then

$$f(2) = -3 - 6 = -9$$

$$f(3) = -9 - 6 = -15$$

$$f(4) = -15 - 6 = -21$$

The correct answer is (D).

25. To answer this question, you must read each answer choice and find the statement that correctly describes the properties of the relation defined as “is perpendicular to.” You can see that each answer choice includes two of three properties: reflexivity, symmetry, or transitivity. It may be most efficient to consider each of these properties first and then find the statement that describes these properties correctly for the given relation. The definition of these properties can be found in Notation, Definitions, and Formulas page 28. These are also provided when you take the test.

A relation \mathcal{R} is reflexive if $x\mathcal{R}x$ for all x . In this case, a line cannot be perpendicular to itself, so the relation given in the question is **not** reflexive.

A relation \mathcal{R} is symmetric if $x\mathcal{R}y \Rightarrow y\mathcal{R}x$ for all x and y . In this case, if line j is perpendicular to line k , it follows that line k is perpendicular to line j . This relation **is** symmetric.

A relation \mathcal{R} is transitive if $(x\mathcal{R}y \text{ and } y\mathcal{R}z) \Rightarrow x\mathcal{R}z$ for all x , y , and z . In this case, if line j is perpendicular to line k and line k is perpendicular to line l , then lines j and l are either the same line or are parallel to each other. Thus, line j is not perpendicular to line l . So this relation is **not** transitive.

The answer that correctly describes the relation “is perpendicular to” is (B), “symmetric but not transitive.”

9. Check on Testing Accommodations

See if you qualify for accommodations that may make it easier to take the Praxis test

What if English is not my primary language?

Praxis tests are given only in English. If your primary language is not English (PLNE), you may be eligible for extended testing time. For more details, visit www.ets.org/praxis/register/accommodations/plne.

What if I cannot take the paper-based test on Saturday?

Monday is the alternate paper-delivered test day for test takers who can't test on Saturday due to:

- religious convictions
- duties as a member of the United States armed forces

Online registration is not available for Monday test takers. You must complete a registration form and provide a photocopy of your military orders or a letter from your cleric. You'll find details at www.ets.org/praxis/register/accommodations/monday_testing.

What if I have a disability or other health-related need?

The following accommodations are available for *Praxis* test takers who meet the Americans with Disabilities Act (ADA) Amendments Act disability requirements:

- Extended testing time
- Additional rest breaks
- Separate testing room
- Writer/recorder of answers
- Test reader
- Sign language interpreter for spoken directions only
- Perkins Braille
- Braille slate and stylus
- Printed copy of spoken directions
- Oral interpreter
- Audio test
- Braille test
- Large print test book (14 pt.)
- Large print answer sheet
- Listening section omitted

For more information on these accommodations, visit www.ets.org/praxis/register/disabilities.

Note: Test takers who have health-related needs requiring them to bring equipment, beverages, or snacks into the testing room or to take extra or extended breaks must request these accommodations by following the procedures described in the *Bulletin Supplement for Test Takers with Disabilities or Health-Related Needs* (PDF), which can be found at <http://www.ets.org/praxis/register/disabilities>.

You can find additional information on available resources for test takers with disabilities or health-related needs at www.ets.org/disabilities.

10. Do Your Best on Test Day

Get ready for test day so you will be calm and confident

You followed your study plan. You are prepared for the test. Now it's time to prepare for test day.

Plan to end your review a day or two before the actual test date so you avoid cramming. Take a dry run to the test center so you're sure of the route, traffic conditions, and parking. Most of all, you want to eliminate any unexpected factors that could distract you from your ultimate goal—passing the *Praxis* test!

On the day of the test, you should:

- be well rested
- wear comfortable clothes and dress in layers
- eat before you take the test and bring food with you to eat during break to keep your energy level up
- bring an acceptable and valid photo identification with you
- bring a supply of well-sharpened No. 2 pencils (at least 3) and a blue or black pen for the essay or constructed-response questions for a paper-delivered test
- be prepared to stand in line to check in or to wait while other test takers check in
- select a seat away from doors, aisles, and other high-traffic areas

You can't control the testing situation, but you can control yourself. Stay calm. The supervisors are well trained and make every effort to provide uniform testing conditions, but don't let it bother you if the test doesn't start exactly on time. You will have the necessary amount of time once it does start.

You can think of preparing for this test as training for an athletic event. Once you've trained, prepared, and rested, give it everything you've got.

What items am I restricted from bringing into the test center?

You cannot bring into the test center personal items such as:

- handbags, knapsacks, or briefcases
- water bottles or canned or bottled beverages
- study materials, books, or notes
- scrap paper
- any electronic, photographic, recording, or listening devices

Note: All cell phones, smart phones (e.g., BlackBerry® devices, iPhones®, etc.), PDAs, and other electronic, photographic, recording, or listening devices are strictly prohibited from the test center. If you are seen with such a device, you will be dismissed from the test, your test scores will be canceled, and you will forfeit your test fees. If you are seen USING such a device, the device will be confiscated and inspected. For more information on what you can bring to the test center, visit www.ets.org/praxis/test_day/bring.

Are You Ready?

Complete this checklist to determine whether you are ready to take your test.

- Do you know the testing requirements for the license or certification you are seeking in the state(s) where you plan to teach?
- Have you followed all of the test registration procedures?
- Do you know the topics that will be covered in each test you plan to take?
- Have you reviewed any textbooks, class notes, and course readings that relate to the topics covered?
- Do you know how long the test will take and the number of questions it contains?
- Have you considered how you will pace your work?
- Are you familiar with the types of questions for your test?
- Are you familiar with the recommended test-taking strategies?
- Have you practiced by working through the practice questions in this study companion or in a study guide or practice test?
- If constructed-response questions are part of your test, do you understand the scoring criteria for these items?
- If you are repeating a *Praxis* test, have you analyzed your previous score report to determine areas where additional study and test preparation could be useful?

If you answered “yes” to the questions above, your preparation has paid off. Now take the *Praxis* test, do your best, pass it—and begin your teaching career!

Appendix: Other Questions You May Have

Here is some supplemental information that can give you a better understanding of the *Praxis* tests.

What do the *Praxis* tests measure?

The *Praxis* tests measure the specific pedagogical skills and knowledge that beginning teachers need. The tests do not measure an individual's disposition toward teaching or potential for success. The assessments are designed to be comprehensive and inclusive, but are limited to what can be covered in a finite number of questions and question types. Ranging from Agriculture to World Languages, there are more than 100 *Praxis* tests, which contain multiple-choice questions, constructed-response questions, or a combination of both.

What is the difference between *Praxis* multiple-choice and constructed-response tests?

Multiple-choice tests measure a broad range of knowledge across your content area. Constructed-response tests measure your ability to provide in-depth explanations of a few essential topics in a given subject area. Content-specific *Praxis* pedagogy tests, most of which are constructed-response, measure your understanding of how to teach certain fundamental concepts in a subject area.

The tests do not measure your actual teaching ability, however. Teaching combines many complex skills that are typically measured in other ways, including classroom observation, videotaped practice, or portfolios not included in the *Praxis* test.

Who takes the tests and why?

Some colleges and universities use the *Praxis* Core Academic Skills for Educators tests (Reading, Writing, and Mathematics) to evaluate individuals for entry into teacher education programs. The assessments are generally taken early in your college career. Many states also require *Praxis* Core Academic Skills for Educators test scores as part of their teacher licensing process.

Individuals entering the teaching profession take the *Praxis* tests as part of the teacher licensing and certification process required by many states. In addition, some professional associations and organizations require *Praxis* tests for professional licensing.

Do all states require these tests?

The *Praxis Series* tests are currently required for teacher licensure in approximately 40 states and United States territories. These tests are also used by several professional licensing agencies and by several hundred colleges and universities. Teacher candidates can test in one state and submit their scores in any other state that requires *Praxis* testing for licensure. You can find details at www.ets.org/praxis/states.

What is licensure/certification?

Licensure in any area—medicine, law, architecture, accounting, cosmetology—is an assurance to the public that the person holding the license possesses sufficient knowledge and skills to perform important occupational activities safely and effectively. In the case of teacher licensing, a license tells the public that the individual has met predefined competency standards for beginning teaching practice.

Because a license makes such a serious claim about its holder, licensure tests are usually quite demanding. In some fields, licensure tests have more than one part and last for more than one day. Candidates for licensure in all fields plan intensive study as part of their professional preparation. Some join study groups, others study alone. But preparing to take a licensure test is, in all cases, a professional activity. Because it assesses the entire body of knowledge for the field you are entering, preparing for a licensure exam takes planning, discipline, and sustained effort.

Why does my state require *The Praxis Series* tests?

Your state chose *The Praxis Series* tests because they assess the breadth and depth of content—called the “domain”—that your state wants its teachers to possess before they begin to teach. The level of content knowledge, reflected in the passing score, is based on recommendations of panels of teachers and teacher educators in each subject area. The state licensing agency and, in some states, the state legislature ratify the passing scores that have been recommended by panels of teachers.

How are the tests updated to ensure the content remains current?

Praxis tests are reviewed regularly. During the first phase of review, ETS conducts an analysis of relevant state and association standards and of the current test content. State licensure titles and the results of relevant job analyses are also considered. Revised test questions are then produced following the standard test development methodology. National advisory committees may also be convened to review existing test specifications and to evaluate test forms for alignment with the specifications.

How long will it take to receive my scores?

Scores for computer-delivered tests are available faster than scores for paper-delivered tests. Scores for most computer-delivered multiple-choice tests are reported on the screen immediately after the test. Scores for tests that contain constructed-response questions or essays aren't available immediately after the test because of the scoring process involved. Official scores for computer-delivered tests are reported to you and your designated score recipients approximately two to three weeks after the test date. Scores for paper-delivered tests will be available within four weeks after the test date. See the test dates and deadlines calendar at www.ets.org/praxis/register/centers_dates for exact score reporting dates.

Can I access my scores on the Web?

All test takers can access their test scores via their *Praxis* account free of charge for one year from the posting date. This online access replaces the mailing of a paper score report.

The process is easy—simply log in to your *Praxis* account at www.ets.org/praxis and click on your score report. If you do not already have a *Praxis* account, you must create one to view your scores.

Note: You must create a *Praxis* account to access your scores, even if you registered by mail or phone.

Your teaching career is worth preparing for, so start today!
Let the *Praxis*™ *Study Companion* guide you.



To search for the *Praxis* test prep resources
that meet your specific needs, visit:

www.ets.org/praxis/testprep

To purchase official test prep made by the creators
of the *Praxis* tests, visit the ETS Store:

www.ets.org/praxis/store

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