## ACI 318-08 Example 001

SHEAR AND FLEXURAL REINFORCEMENT DESIGN OF A SINGLY REINFORCED RECTANGLE

EXAMPLE DESCRIPTION

The flexural and shear design of a rectangular concrete beam is calculated in this example.

A simply supported beam is subjected to an ultimate uniform load of 9.736 k/ft. This example is tested using the ACI 318-08 concrete design code. The flexural and shear reinforcing computed is compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING
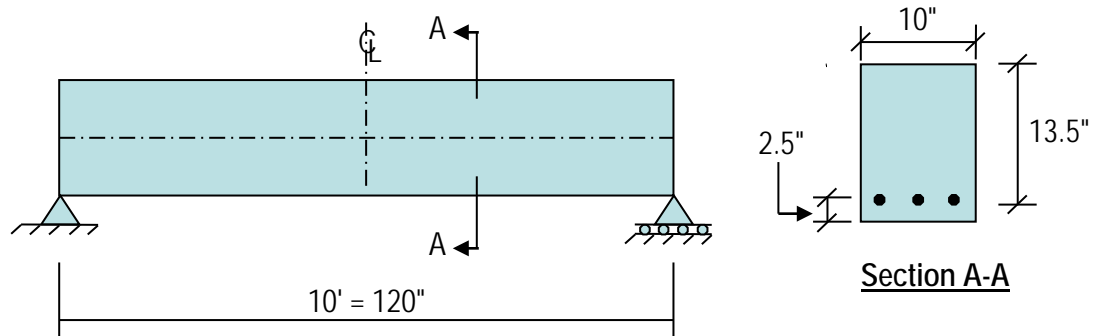
Section A-A

Material Properties
E = 3600 k/in²
v = 0.2
G = 1500 k/in²

Section Properties
d = 13.5 in
b = 10.0 in
I = 3,413 in⁴

Design Properties
$f'_c = 4$ k/in²
$f_y = 60$ k/in²

TECHNICAL FEATURES TESTED

➢ Calculation of Flexural reinforcement, A_s

➢ Enforcement of Minimum tension reinforcement, $A_{s,min}$

➢ Calculation of Shear reinforcement, A_v

➢ Enforcement of Minimum shear reinforcing, $A_{v,min}$

RESULTS COMPARISON

Independent results are hand calculated based on the equivalent rectangular stress distribution described in Example 6.1 in Notes on ACI 318-08 Building Code.

Output Parameter	ETABS	Independent	Percent Difference
Design Moment, M_u (k-in)	1460.4	1460.4	0.00%
Tension Reinf, A_s (in^2)	2.37	2.37	0.00%
Design Shear Force, V_u	37.73	37.73	0.00%
Shear Reinf, A_v/s (in^2/in)	0.041	0.041	0.00%

COMPUTER FILE: ACI 318-08 Ex001

CONCLUSION

The computed results show an exact match for the flexural and the shear reinforcing.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\varphi = 0.9, \ A_g = 160$ sq-in

$A_{s,min} = \dfrac{200}{f_y} b_w d = 0.450$ sq-in (Govern)

$\qquad = \dfrac{3\sqrt{f_c^{'}}}{f_y} b_w d = 0.427$ sq-in

$\beta_1 = 0.85 - 0.05\left(\dfrac{f_c' - 4000}{1000}\right) = 0.85$

$c_{max} = \dfrac{0.003}{0.003 + 0.005} d = 5.0625$ in

$a_{max} = \beta_1 c_{max} = 4.303$ in

Combo1

$w_u = (1.2w_d + 1.6w_l) = 9.736$ k/ft

$M_u = \dfrac{w_u l^2}{8} = 9.736 \bullet 10^2/8 = 121.7$ k-ft $= 1460.4$ k-in

The depth of the compression block is given by:

$a = d - \sqrt{d^2 - \dfrac{2|M_u|}{0.85 f_c^{'} \varphi b}} = 4.183$ in ($a < a_{max}$)

The area of tensile steel reinforcement is given by:

$A_s = \dfrac{M_u}{\varphi f_y \left(d - \dfrac{a}{2}\right)} = \dfrac{1460.4}{0.9 \bullet 60 \bullet (13.5 - 4.183/2)}$

$A_s = 2.37$ sq-in

Shear Design

The following quantities are computed for all of the load combinations:

$$\varphi \quad = \quad 0.75$$

Check the limit of $\sqrt{f'_c}$:

$$\sqrt{f'_c} \ = 63.246 \text{ psi} < 100 \text{ psi}$$

The concrete shear capacity is given by:

$$\varphi \ V_c = \quad \varphi \ 2 \ \sqrt{f'_c} \ bd \ = \ 12.807 \text{ k}$$

The maximum shear that can be carried by reinforcement is given by:

$$\varphi \ V_s = \quad \varphi \ 8 \ \sqrt{f'_c} \ bd \ = \ 51.229 \text{ k}$$

The following limits are required in the determination of the reinforcement:

$$(\varphi \ V_c/2) \qquad\qquad = \ 6.4035 \text{ k}$$

$$\varphi \ V_{\max} = \varphi \ V_c + \varphi \ V_s \ = \ 64.036 \text{ k}$$

Given V_u, V_c and $V_{\max}$, the required shear reinforcement in area/unit length for any load combination is calculated as follows:

If $V_u \leq \varphi \ (V_c/2)$,

$$\frac{A_v}{s} = 0,$$

else if $\varphi \ (V_c/2) < V_u \leq \varphi \ V_{\max}$

$$\frac{A_v}{s} \ = \ \frac{(V_u - \phi V_c)}{\phi \ f_{ys} \ d} \geq \left(\frac{A_v}{s} \right)_{\min}$$

where:

$$\left(\frac{A_v}{s} \right)_{\min} = \max \left\{ 50 \left(\frac{b_w}{f_{yt}} \right), \left(\frac{b_w}{f_{yt}} \right) \cdot \frac{3}{4} \sqrt{f'_c} \right\}$$

else if $V_u > \varphi \ V_{\max}$,

a failure condition is declared.

Combo1

$$V_u = 9.736 \cdot (5\text{-}13.5/12) = 37.727 \text{ k}$$

$$\phi\left(V_c / 2\right) = 6.4035\,k \leq V_u = 37.727\,k \leq \phi V_{\max} = 64.036\,k$$

$$\left(\frac{A_v}{s}\right)_{\min} = \max\left\{50\left(\frac{10}{60,000}\right), \left(\frac{10}{60,000}\right)\cdot\frac{3}{4}\sqrt{4,000}\right\}$$

$$\left(\frac{A_v}{s}\right)_{\min} = \max\left\{0.0083,\ 0.0079\right\} = 0.0083\frac{in^2}{in}$$

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d} = 0.041\frac{in^2}{in} = 0.492\frac{in^2}{ft}$$