## ACI 318-14 RC-PN EXAMPLE 001

## Slab Punching Shear Design

## Problem Description

The purpose of this example is to verify slab punching shear design in ETABS.
The numerical example is a flat slab that has three 24 -foot-long spans in each direction, as shown in Figure 1.


Figure 1: Flat Slab For Numerical Example
program name: ETABS
REVISION NO.: $\quad \underline{ }$
The slab overhangs the face of the column by 6 inches along each side of the structure. The columns are typically 12 inches wide by 36 inches long, with the long side parallel to the Y-axis. The slab is typically 10 inches thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 150 pcf and an f'c of 4000 psi. The dead load consists of the self weight of the structure plus an additional 20 psf . The live load is 80 psf .

## Technical Features of ETABS Tested

- Calculation of punching shear capacity, shear stress, and D/C ratio.


## Results Comparison

Table 1 shows the comparison of the ETABS punching shear capacity, shear stress ratio, and D/C ratio with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this example.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

| Method | Shear Stress <br> (ksi) | Shear Capacity <br> (ksi) | D/C ratio |
| :---: | :---: | :---: | :---: |
| ETABS | 0.1930 | 0.158 | 1.22 |
| Calculated | 0.1930 | 0.158 | 1.22 |

Computer File: ACI 318-14 RC-PN Ex001.EDB

## Conclusion

The ETABS results show an exact comparison with the independent results.

## Hand Calculation

Hand Calculation for Interior Column Using ETABS Method
$\mathrm{d}=[(10-1)+(10-2)] / 2=8.5^{\prime \prime}$
Refer to Figure 2.

$$
\mathrm{b}_{0}=44.5+20.5+44.5+20.5=130 "
$$



Figure 2: Interior Column, Grid B-2 in ETABS Model

$$
\begin{aligned}
& \gamma_{V 2}=1-\frac{1}{1+\left(\frac{2}{3}\right) \sqrt{\frac{44.5}{20.5}}}=0.4955 \\
& \gamma_{V 3}=1-\frac{1}{1+\left(\frac{2}{3}\right) \sqrt{\frac{20.5}{44.5}}}=0.3115
\end{aligned}
$$

The coordinates of the center of the column $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ are taken as $(0,0)$.

## Software Verification

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program name: ETABS
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The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear, as identified in Figure 2.

| Item | Side 1 | Side 2 | Side 3 | Side 4 | Sum |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{x}_{2}$ | -10.25 | 0 | 10.25 | 0 | N.A. |
| $\mathrm{y}_{2}$ | 0 | 22.25 | 0 | -22.25 | N.A. |
| L | 44.5 | 20.5 | 44.5 | 20.5 | $\mathrm{~b}_{0}=130$ |
| d | 8.5 | 8.5 | 8.5 | 8.5 | N.A. |
| Ld | 378.25 | 174.25 | 378.25 | 174.25 | 1105 |
| $\mathrm{Ldx}_{2}$ | -3877.06 | 0 | 3877.06 | 0 | 0 |
| $\mathrm{Ldy}_{2}$ | 0 | 3877.06 | 0 | -3877.06 | 0 |

$$
\begin{aligned}
& x_{3}=\frac{\sum L d x_{2}}{L d}=\frac{0}{1105}=0^{\prime \prime} \\
& y_{3}=\frac{\sum L d y_{2}}{L d}=\frac{0}{1105}=0^{\prime \prime}
\end{aligned}
$$

The following table is used to calculate $\mathrm{I}_{\mathrm{XX}}, \mathrm{I}_{\mathrm{YY}}$ and $\mathrm{I}_{\mathrm{XY}}$. The values for $\mathrm{I}_{\mathrm{XX}}, \mathrm{I}_{\mathrm{YY}}$ and $\mathrm{I}_{\mathrm{XY}}$ are given in the "Sum" column.

| Item | Side 1 | Side 2 | Side 3 | Side 4 | Sum |
| :---: | :---: | :---: | :---: | :---: | :---: |
| L | 44.5 | 20.5 | 44.5 | 20.5 | N.A. |
| d | 8.5 | 8.5 | 8.5 | 8.5 | N.A. |
| $\mathrm{x}_{2}-\mathrm{x}_{3}$ | -10.25 | 0 | 10.25 | 0 | N.A. |
| $\mathrm{y}_{2}-\mathrm{y}_{3}$ | 0 | 22.25 | 0 | -22.25 | N.A. |
| Parallel to | Y-Axis | X -axis | Y-Axis | X-axis | N.A. |
| Equations | $5 \mathrm{~b}, 6 \mathrm{~b}, 7$ | $5 \mathrm{a}, 6 \mathrm{a}, 7$ | $5 \mathrm{~b}, 6 \mathrm{~b}, 7$ | $5 \mathrm{a}, 6 \mathrm{a}, 7$ | N.A. |
| $\mathrm{I}_{\mathrm{XX}}$ | 64696.5 | 86264.6 | 64696.5 | 86264.6 | 301922.3 |
| $\mathrm{I}_{\mathrm{YY}}$ | 39739.9 | 7151.5 | 39739.9 | 7151.5 | 93782.8 |
| $\mathrm{I}_{\mathrm{XY}}$ | 0 | 0 | 0 | 0 | 0 |

From the ETABS output at Grid B-2:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{U}}=189.45 \mathrm{k} \\
& \gamma_{V 2} M_{U 2}=-156.39 \mathrm{k}-\mathrm{in} \\
& \gamma_{V 3} M_{U 3}=91.538 \mathrm{k}-\mathrm{in}
\end{aligned}
$$

## Software Verification <br> program name: ETABS <br> REVISION NO.: <br> 0

At the point labeled A in Figure 2, $\mathrm{x}_{4}=-10.25$ and $\mathrm{y}_{4}=22.25$, thus:

$$
\begin{gathered}
v_{U}=\frac{189.45}{130 \bullet 8.5}-\frac{156.39[93782.8(22.25-0)-(0)(-10.25-0)]}{(301922.3)(93782.8)-(0)^{2}}- \\
\frac{91.538[301922.3(-10.25-0)-(0)(22.25-0)]}{(301922.3)(93782.8)-(0)^{2}}
\end{gathered}
$$

$v_{U}=0.1714-0.0115-0.0100=\mathbf{0 . 1 4 9 9} \mathbf{k s i}$ at point A

At the point labeled B in Figure 2, $x_{4}=10.25$ and $y_{4}=22.25$, thus:

$$
\begin{gathered}
v_{U}=\frac{189.45}{130 \bullet 8.5}-\frac{156.39[93782.8(22.25-0)-(0)(10.25-0)]}{(301922.3)(93782.8)-(0)^{2}}- \\
\frac{91.538[301922.3(10.25-0)-(0)(22.25-0)]}{(301922.3)(93782.8)-(0)^{2}}
\end{gathered}
$$

$v_{U}=0.1714-0.0115+0.0100=\mathbf{0 . 1 6 9 9} \mathbf{k s i}$ at point B

At the point labeled C in Figure 2, $x_{4}=10.25$ and $y_{4}=-22.25$, thus:

$$
\begin{gathered}
v_{U}=\frac{189.45}{130 \bullet 8.5}-\frac{156.39[93782.8(-22.25-0)-(0)(10.25-0)]}{(301922.3)(93782.8)-(0)^{2}}- \\
\frac{91.538[301922.3(10.25-0)-(0)(-22.25-0)]}{(301922.3)(93782.8)-(0)^{2}}
\end{gathered}
$$

$v_{U}=0.1714+0.0115+0.0100=\mathbf{0 . 1 9 3 0} \mathbf{k s i}$ at point C

At the point labeled $D$ in Figure 2, $x_{4}=-10.25$ and $y_{4}=-22.25$, thus:

$$
\begin{gathered}
v_{U}=\frac{189.45}{130 \bullet 8.5}-\frac{156.39[93782.8(-22.25-0)-(0)(-10.25-0)]}{(301922.3)(93782.8)-(0)^{2}}- \\
\frac{91.538[301922.3(-10.25-0)-(0)(-22.25-0)]}{(301922.3)(93782.8)-(0)^{2}} \\
v_{U}=0.1714+0.0115-0.0100=\mathbf{0 . 1 7 2 9} \mathbf{~ k s i} \text { at point } \mathrm{D}
\end{gathered}
$$

## Software Verification

## REVISION NO.:

0
Point C has the largest absolute value of $v_{u}$, thus $\mathrm{v}_{\text {max }}=0.1930 \mathrm{ksi}$

The shear capacity is calculated based on the smallest of ACI 318-14 equations 11-34, $11-35$ and 11-36 with the $b_{0}$ and $d$ terms removed to convert force to stress.

$$
\begin{aligned}
& \varphi v_{C}=\frac{0.75\left(2+\frac{4}{36 / 12}\right) \sqrt{4000}}{1000}=0.158 \text { ksi in accordance with equation 11-34 } \\
& \varphi v_{C}=\frac{0.75\left(\frac{40 \bullet 8.5}{130}+2\right) \sqrt{4000}}{1000}=0.219 \text { ksi in accordance with equation 11-35 } \\
& \varphi v_{C}=\frac{0.75 \bullet 4 \bullet \sqrt{4000}}{1000}=0.190 \text { ksi in accordance with equation 11-36 }
\end{aligned}
$$

Equation 11-34 yields the smallest value of $\phi v_{C}=0.158 \mathrm{ksi}$ and thus this is the shear capacity.

$$
\text { Shear } \text { Ratio }=\frac{v_{U}}{\varphi v_{C}}=\frac{0.193}{0.158}=1.22
$$

