Prerequisite Skills

This lesson requires the use of the following skills:

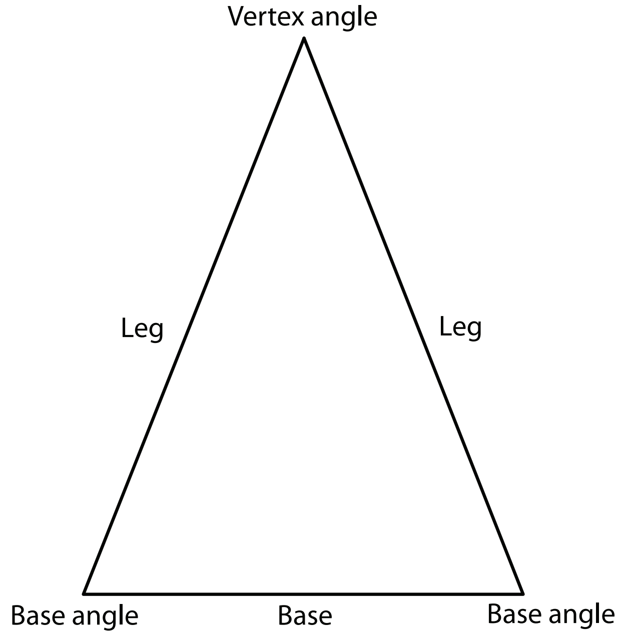
- classifying triangles
- identifying and using vertical angles, supplementary angles, and complementary angles to find unknown angle measures
- applying the Triangle Sum Theorem and the Exterior Angle Theorem to find unknown measures of triangles
- justifying congruence of triangles
- writing various forms of proofs

Introduction

Isosceles triangles can be seen throughout our daily lives in structures, supports, architectural details, and even bicycle frames. Isosceles triangles are a distinct classification of triangles with unique characteristics and parts that have specific names. In this lesson, we will explore the qualities of isosceles triangles.

Key Concepts

- Isosceles triangles have at least two congruent sides, called **legs**.
- The angle created by the intersection of the legs is called the **vertex angle**.
- Opposite the vertex angle is the **base** of the isosceles triangle.
- Each of the remaining angles is referred to as a **base angle**. The intersection of one leg and the base of the isosceles triangle creates a base angle.

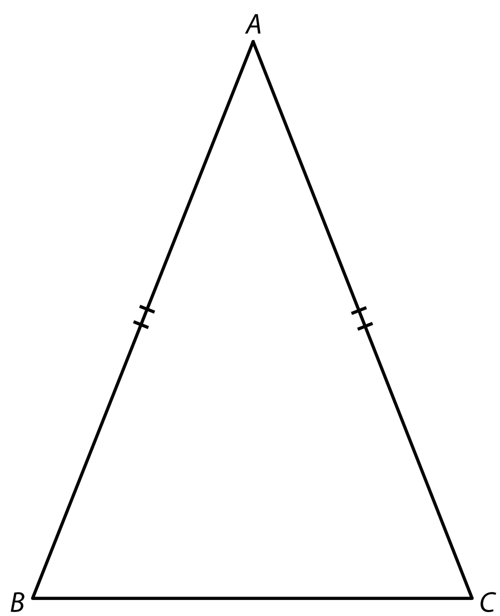


- The following theorem is true of every isosceles triangle.

Theorem

Isosceles Triangle Theorem

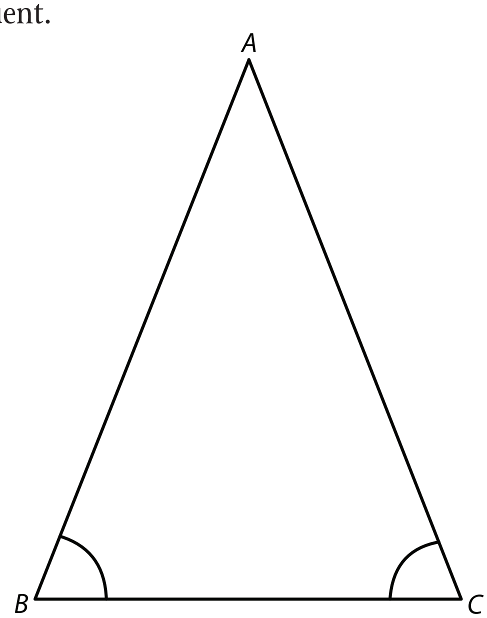
If two sides of a triangle are congruent, then the angles opposite the congruent sides are congruent.



$m\angle B \cong m\angle C$

- If the Isosceles Triangle Theorem is reversed, then that statement is also true.
- This is known as the converse of the Isosceles Triangle Theorem.

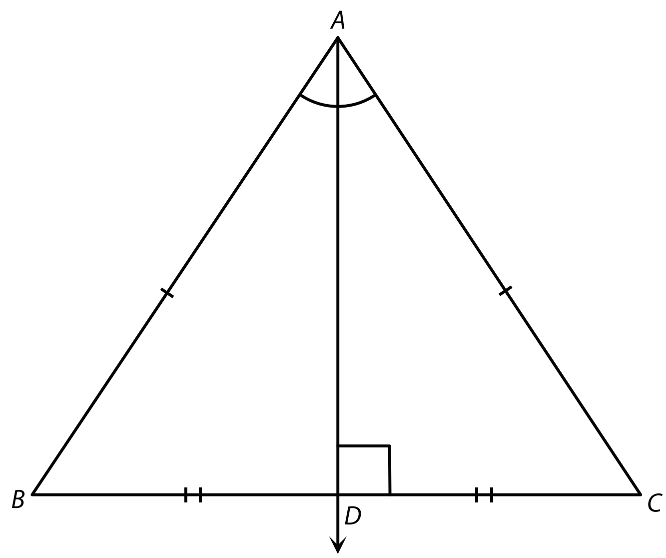
Theorem
Converse of the Isosceles Triangle Theorem If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



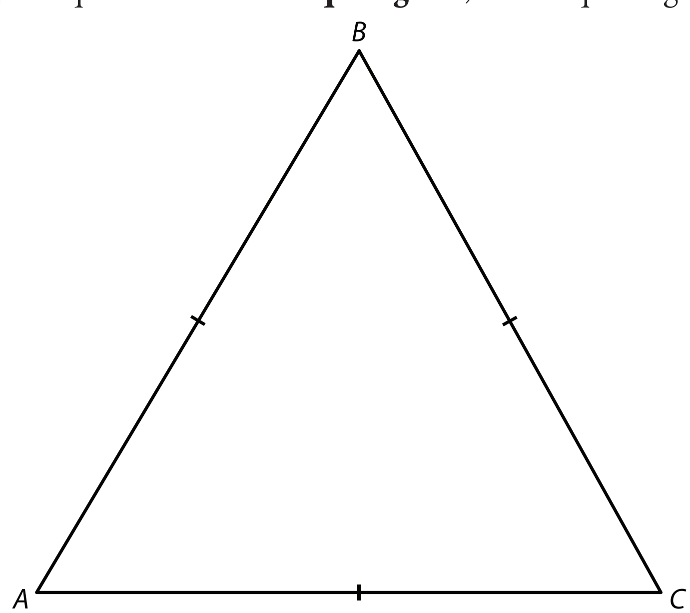
$\overline{AB} \cong \overline{AC}$

- If the vertex angle of an isosceles triangle is bisected, the bisector is perpendicular to the base, creating two right triangles.

- In the diagram that follows, D is the midpoint of $\overline{BC}$.



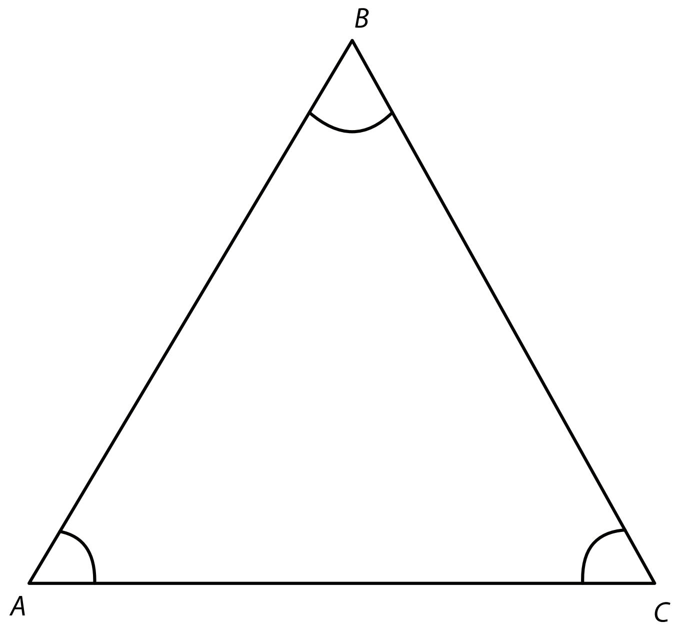
- Equilateral triangles are a special type of isosceles triangle, for which each side of the triangle is congruent.
- If all sides of a triangle are congruent, then all angles have the same measure.

Theorem
If a triangle is equilateral then it is **equiangular**, or has equal angles.  $\angle A \cong \angle B \cong \angle C$

- Each angle of an equilateral triangle measures 60° ($180 \div 3 = 60$).
- Conversely, if a triangle has equal angles, it is equilateral.

Theorem

If a triangle is equiangular, then it is equilateral.



$\overline{AB} \cong \overline{BC} \cong \overline{AC}$

- These theorems and properties can be used to solve many triangle problems.

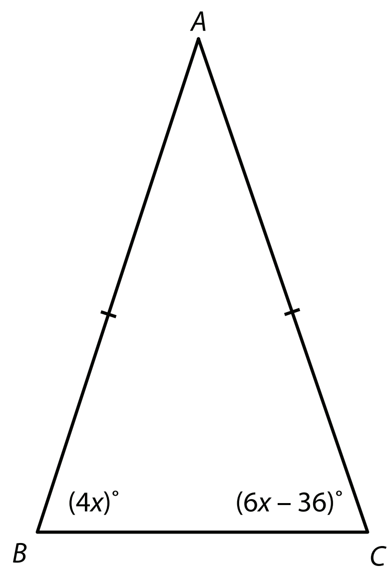
Common Errors/Misconceptions

- incorrectly identifying parts of isosceles triangles
- not identifying equilateral triangles as having the same properties of isosceles triangles
- incorrectly setting up and solving equations to find unknown measures of triangles
- misidentifying or leaving out theorems, postulates, or definitions when writing proofs

Guided Practice 1.9.2

Example 1

Find the measure of each angle of $\triangle ABC$.



1. Identify the congruent angles.

   The legs of an isosceles triangle are congruent; therefore, $\overline{AB} \cong \overline{AC}$.

   The base of $\triangle ABC$ is $\overline{BC}$.

   $\angle B$ and $\angle C$ are base angles and are congruent.

2. Calculate the value of x.

   Congruent angles have the same measure.

   Create an equation.

$m\angle B = m\angle C$	The measures of base angles of isosceles triangles are equal.
$4x = 6x - 36$	Substitute values for $m\angle B$ and $m\angle C$.
$-2x = -36$	Solve for x.
$x = 18$	

3. Calculate each angle measure.

$m\angle B = 4x = 4(18) = 72$	Substitute the value of x into the expression for $m\angle B$.
$m\angle C = 6(18) - 36 = 72$	Substitute the value of x into the expression for $m\angle C$.
$m\angle A + m\angle B + m\angle C = 180$	The sum of the angles of a triangle is 180°.
$m\angle A + 72 + 72 = 180$	Substitute the known values.
$m\angle A = 36$	Solve for $m\angle A$.

4. Summarize your findings.

$m\angle A = 36$

$m\angle B = 72$

$m\angle C = 72$

Example 2

Determine whether $\triangle ABC$ with vertices A (–4, 5), B (–1, –4), and C (5, 2) is an isosceles triangle. If it is isosceles, name a pair of congruent angles.

1. Use the distance formula to calculate the length of each side.

   Calculate the length of $\overline{AB}$.

   $$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

   $$AB=\sqrt{[(-1)-(-4)]^2+[(-4)-(5)]^2}$$ Substitute (–4, 5) and (–1, –4) for (x_1, y_1) and (x_2, y_2).

   $$AB=\sqrt{(3)^2+(-9)^2}$$ Simplify.

   $$AB=\sqrt{9+81}$$

   $$AB=\sqrt{90}=3\sqrt{10}$$

   Calculate the length of $\overline{BC}$.

   $$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

   $$BC=\sqrt{[(5)-(-1)]^2+[(2)-(-4)]^2}$$ Substitute (–1, –4) and (5, 2) for (x_1, y_1) and (x_2, y_2).

   $$BC=\sqrt{(6)^2+(6)^2}$$ Simplify.

   $$BC=\sqrt{36+36}$$

   $$BC=\sqrt{72}=6\sqrt{2}$$

   Calculate the length of $\overline{AC}$.

   $$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

   $$AC=\sqrt{[(5)-(-4)]^2+[(2)-(5)]^2}$$ Substitute (–4, 5) and (5, 2) for (x_1, y_1) and (x_2, y_2).

   $$AC=\sqrt{(9)^2+(-3)^2}$$ Simplify.

   $$AC=\sqrt{81+9}$$

   $$AC=\sqrt{90}=3\sqrt{10}$$

2. Determine if the triangle is isosceles.

   A triangle with at least two congruent sides is an isosceles triangle.

   $\overline{AB} \cong \overline{AC}$, so $\triangle ABC$ is isosceles.

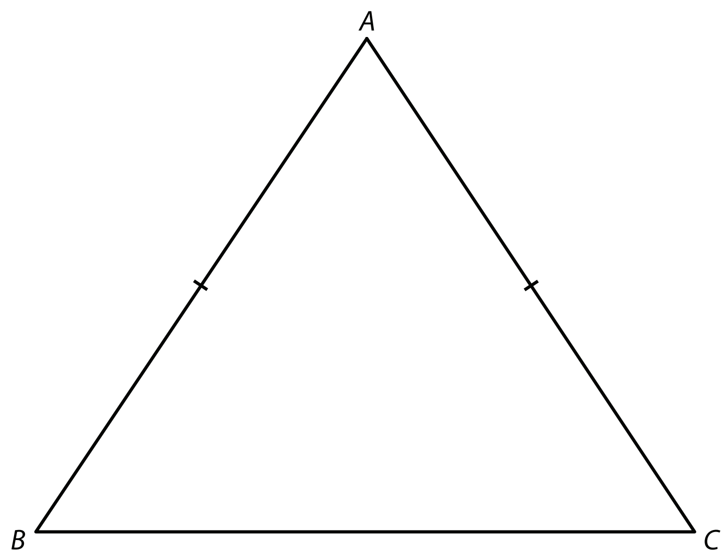
3. Identify congruent angles.

   If two sides of a triangle are congruent, then the angles opposite the sides are congruent.

   $\angle B \cong \angle C$

Example 3

Given $\overline{AB} \cong \overline{AC}$, prove that $\angle B \cong \angle C$.

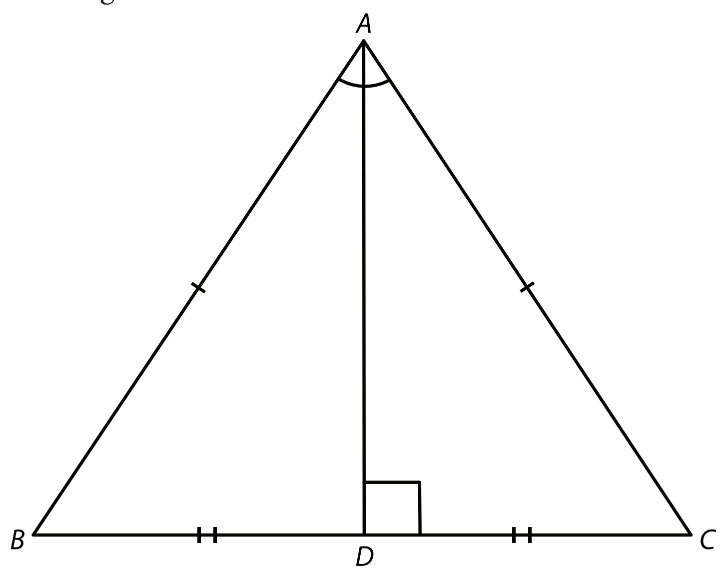


1. State the given information.

   $\overline{AB} \cong \overline{AC}$

2. Draw the angle bisector of $\angle A$ and extend it to $\overline{BC}$, creating the perpendicular bisector of $\overline{BC}$. Label the point of intersection D.

Indicate congruent sides.



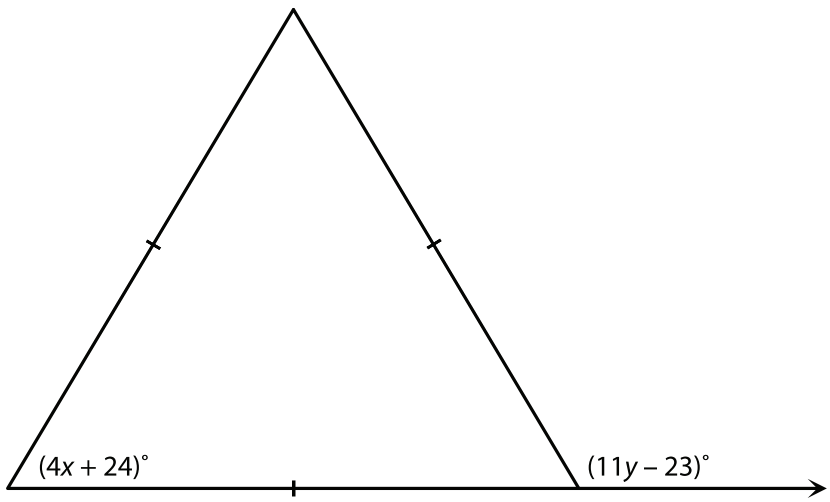
$\angle B$ and $\angle C$ are congruent corresponding parts.

3. Write the information in a two-column proof.

Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$	1. Given
2. Draw the angle bisector of $\angle A$ and extend it to $\overline{BC}$, creating a perpendicular bisector of $\overline{BC}$ and the midpoint of $\overline{BC}$.	2. There is exactly one line through two points.
3. $\overline{BD} \cong \overline{BC}$	3. Definition of midpoint
4. $\overline{AD} \cong \overline{AD}$	4. Reflexive Property
5. $\triangle ABD \cong \triangle ACD$	5. SSS Congruence Statement
6. $\angle B \cong \angle C$	6. Corresponding Parts of Congruent Triangles are Congruent

Example 4

Find the values of x and y.



1. Make observations about the figure.

   The triangle in the diagram has three congruent sides.

   A triangle with three congruent sides is equilateral.

   Equilateral triangles are also equiangular.

   The measure of each angle of an equilateral triangle is 60˚.

   An exterior angle is also included in the diagram.

   The measure of an exterior angle is the supplement of the adjacent interior angle.

2. Determine the value of x.

   The measure of each angle of an equilateral triangle is 60˚.

   Create and solve an equation for x using this information.

$4x + 24 = 60$	Equation
$4x = 36$	Solve for x.
$x = 9$	

   The value of x is 9.

3. Determine the value of y.

   The exterior angle is the supplement to the interior angle.

   The interior angle is 60˚ by the properties of equilateral triangles.

   The sum of the measures of an exterior angle and interior angle pair equals 180.

   Create and solve an equation for y using this information.

$11y - 23 + 60 = 180$	Equation
$11y + 37 = 180$	Simplify.
$11y = 143$	Solve for y.
$y = 13$	

   The value of y is 13.

Example 5

$\triangle ABC$ is equilateral. Prove that it is equiangular.

1. State the given information.

   $\triangle ABC$ is an equilateral triangle.

2. Plan the proof.

   Equilateral triangles are also isosceles triangles.

   Isosceles triangles have at least two congruent sides.

   $\overline{AB} \cong \overline{BC}$

   $\angle A$ and $\angle C$ are base angles in relation to $\overline{AB}$ and $\overline{BC}$.

   $\angle A \cong \angle C$ because of the Isosceles Triangle Theorem.

   $\overline{BC} \cong \overline{AC}$

   $\angle A$ and $\angle B$ are base angles in relation to $\overline{BC}$ and $\overline{AC}$.

   $\angle A \cong \angle B$ because of the Isosceles Triangle Theorem.

   By the Transitive Property, $\angle A \cong \angle B \cong \angle C$; therefore, $\triangle ABC$ is equiangular.

3. Write the information in a paragraph proof.

   Since $\triangle ABC$ is equilateral, $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{AC}$. By the Isosceles Triangle Theorem, $\angle A \cong \angle C$ and $\angle A \cong \angle B$. By the Transitive Property, $\angle A \cong \angle B \cong \angle C$; therefore, $\triangle ABC$ is equiangular.