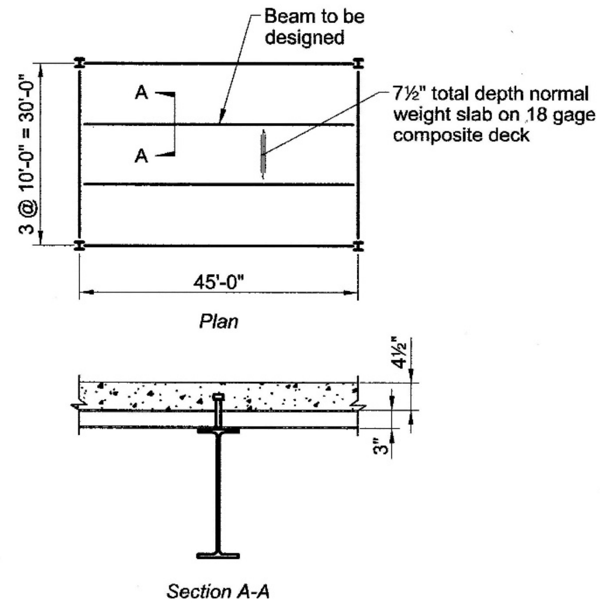
# AISC-360-10 Example 001

## COMPOSITE GIRDER DESIGN

### EXAMPLE DESCRIPTION

A typical bay of a composite floor system is illustrated below. Select an appropriate ASTM A992 W-shaped beam and determine the required number of ¾ in.-diameter steel headed stud anchors. The beam will not be shored during construction. To achieve a two-hour fire rating without the application of spray applied fire protection material to the composite deck, 4 ½ in. of normal weight (145 lb/ft$^3$) concrete will be placed above the top of the deck. The concrete has a specified compressive strength, $f'_c = 4$ ksi.

### GEOMETRY, PROPERTIES AND LOADING




| Member Properties | Loading | Geometry |
|---|---|---|
| W21x50 | w = 800 plf (Dead Load) | Span, L = 45 ft |
| $E$ = 29000 ksi | w = 250 plf (Construction) | |
| $F_y$ = 50 ksi | w = 100 plf (SDL) | |
| | w = 1000 plf (Live Load) | |

## TECHNICAL FEATURES OF ETABS TESTED

Composite beam design, including:

- Selection of steel section, camber and shear stud distribution
- Member bending capacities, at construction and in service
- Member deflections, at construction and in service

## RESULTS COMPARISON

Independent results are referenced from Example I.1 from the AISC Design Examples, Version 14.0.

| Output Parameter | ETABS | Independent | Percent Difference |
|---|---|---|---|
| Pre-composite $M_u$ (k-ft) | 344.2 | 344.2 | 0.00% |
| Pre-composite $\Phi_b M_n$ (k-ft) | 412.5 | 412.5 | 0.00% |
| Pre-composite Deflection (in.) | 2.6 | 2.6 | 0.00% |
| Required Strength $M_u$ (k-ft) | 678.3 | 678.4 | 0.01% |
| Full Composite $\Phi_b M_n$ (k-ft) | 937.1 | 937.1 | 0.00% |
| Partial Composite $\Phi_b M_n$ (k-ft) | 763.2 | 763.2 | 0.00% |
| Shear Stud Capacity $Q_n$ | 17.2; 14.6 | 17.2; 14.6 | 0.00% |
| Shear Stud Distribution | 46 | 46 | 0.00% |
| Live Load Deflection (in.) | 1.34 | 1.26 | 6.0% |
| Required Strength $V_u$ (kip) | 60.3 | 60.3 | 0.00% |
| $\Phi V_n$ (k) | 237.1 | 237.1 | 0.00% |

## COMPUTER FILE: AISC-360-10 EXAMPLE 001.EDB

## CONCLUSION

The ETABS results show an acceptable comparison with the independent results. The live load deflection differs due to a difference in methodology. In the AISC example, the live load deflection is computed based on a lower bound value of the beam moment of inertia, whereas in ETABS, it is computed based on the approximate value of the beam moment of inertia derived from Equation (C-I3-6) from the *Commentary on the AISC Load and Resistance Factor Design Specification – Second Edition.*

## HAND CALCULATION

**Properties:**

Materials:

ASTM A572 Grade 50 Steel

$E$ = 29,000 ksi, $F_y$ = 50 ksi, $w_{steel}$ = 490 pcf

4000 psi normal weight concrete

$E_c$ = 3,644 ksi, $f_c' = 4$ ksi, $w_{concrete}$ = 145 pcf

Section:

W21x50

$d$ = 20.8 in, $b_f$ = 6.53 in, $t_f$ = 0.535 in, $t_w$ = 0.38 in, k = 1.04 in

$A_{steel}$ = 14.7 in$^2$, $S_{steel}$ = 94.6 in$^3$, $Z_{steel}$ = 110 in$^3$, $I_{steel}$ = 984 in$^4$

Deck:

$t_c$ =4 ½ in., $h_r$ = 3 in., $s_r$ =12 in., $w_r$ = 6 in.

Shear Connectors:

$d$ = ¾ in, $h$ =4 ½ in, $F_u$ = 65 ksi

**Design for Pre-Composite Condition:**

Construction Required Flexural Strength:

$$w_D = (10 \bullet 75 + 50) \bullet 10^{-3} = 0.800 \text{ kip/ft}$$

$$w_L = 10 \bullet 25 \bullet 10^{-3} = 0.250 \text{ kip/ft}$$

$$w_u = 1.2 \bullet 0.800 + 1.6 \bullet 0.250 = 1.36 \text{ kip/ft}$$

$$M_u = \frac{w_u \bullet L^2}{8} = \frac{1.36 \bullet 45^2}{8} = 344.25 \text{ kip-ft}$$

Moment Capacity:

$$\Phi_b M_n = \Phi_b \bullet Z_s \bullet F_y = (0.9 \bullet 110 \bullet 50)/12 = 412.5 \text{ kip-ft}$$

Pre-Composite Deflection:

$$\Delta_{nc} = \frac{5 w_D L^4}{384 EI} = \frac{5 \bullet \frac{0.800}{12} \bullet (45 \bullet 12)^4}{384 \bullet 29,000 \bullet 984} = 2.59 \text{ in.}$$

$\text{Camber} = 0.8 \bullet \Delta_{nc} = 0.8 \bullet 2.59 = 2.07$ in., which is rounded down to 2 in.

**Design for Composite Flexural Strength:**

Required Flexural Strength:

$$w_u = 1.2 \bullet 0.800 + 1.2 \bullet 0.100 + 1.6 \bullet 1 = 2.68 \text{ kip/ft}$$

$$M_u = \frac{w_u \bullet L^2}{8} = \frac{2.68 \bullet 45^2}{8} = 678.38 \text{ kip-ft}$$

Full Composite Action Available Flexural Strength:

Effective width of slab:

$$b_{\text{eff}} = \frac{10.0}{2} \bullet 2 \text{ sides} = 10.0 \text{ ft} \le \frac{45.0 \text{ ft}}{8} = 11.25 \text{ ft}$$

Resistance of steel in tension:

$$C = P_y = A_s \bullet F_y = 14.7 \bullet 50 = 735 \text{ kips} \quad \textbf{controls}$$

Resistance of slab in compression:

$$A_c = b_{\text{eff}} \bullet t_c = (10 \bullet 12) \bullet 4.5 = 540 \text{ in}^2$$

$$C = 0.85 \bullet f'_c A_c = 0.85 \bullet 4 \bullet 540 = 1836 \text{ kips}$$

Depth of compression block within slab:

$$a = \frac{C}{0.85 \bullet b_{\text{eff}} \bullet f'_c} = \frac{735}{0.85 \bullet (10 \bullet 12) \bullet 4} = 1.80 \text{ in.}$$

Moment resistance of composite beam for full composite action:

$$d_1 = (t_c + h_r) - \frac{a}{2} = (4.5 + 3) - \frac{1.80}{2} = 6.60 \text{ in.}$$

$$\Phi M_n = \Phi\left(P_y \bullet d_1 + P_y \bullet \frac{d}{2}\right) = 0.9\left(735 \bullet 6.60/12 + 735 \bullet \frac{20.8/12}{2}\right) = 937.1 \text{ kip-ft}$$

Partial Composite Action Available Flexural Strength:

Assume 50.9% composite action:

$$C = 0.509 \bullet P_y = 373.9 \text{ kips}$$

Depth of compression block within concrete slab:

$$a = \frac{C}{0.85 \bullet b_{\text{eff}} \bullet f'_c} = \frac{373.9}{0.85 \bullet (10 \bullet 12) \bullet 4} = 0.92 \text{ in.}$$

$$d_1 = \left(t_c + h_r\right) - \frac{a}{2} = (4.5 + 3) - \frac{0.92}{2} = 7.04 \text{ in.}$$

Compressive force in steel section:

$$\frac{P_y - C}{2} = \frac{735 - 373.9}{2} = 180.6 \text{ kips}$$

Steel section flange ultimate compressive force:

$$C_{flange} = b_f \bullet t_f \bullet F_y = 6.53 \bullet 0.535 \bullet 50 = 174.7 \text{ kips}$$

Steel section web (excluding fillet areas) ultimate compressive force:

$$C_{web} = (d - 2 \bullet k) \bullet t_w \bullet F_y = (20.8 - 2 \bullet 1.04) \bullet 0.38 \bullet 50 = 355.7 \text{ kips}$$

Steel section fillet ultimate compressive force:

$$C_{fillet} = \frac{P_y - (2 \bullet C_{flange} + C_{web})}{2} = \frac{735 - (2 \bullet 174.7 + 355.7)}{2} = 14.5 \text{ kips}$$

Assuming a rectangular fillet area, the distance from the bottom of the top flange to the neutral axis of the composite section is:

$$x = (\mathrm{k} - \mathrm{t_f}) \bullet \left[ \frac{(P_y - C)/2 - C_{flange}}{C_{fillet}} \right]$$

$$= (1.04 - 0.535) \bullet \left[ \frac{180.6 - 174.7}{14.98} \right] = 0.20 \text{ in.}$$

Distance from the centroid of the compressive force in the steel section to the top of the steel section:

$$d_2 = \frac{C_{flange} \bullet t_f / 2 + ((P_y - C)/2 - C_{flange}) \bullet (t_f + x/2)}{(P_y - C)/2}$$

$$= \frac{174.7 \bullet 0.535/2 + (180.6 - 174.7) \bullet (0.535 + 0.2/2)}{180.6} = 0.279 \text{ in.}$$

Moment resistance of composite beam for partial composite action:

$$\Phi M_n = \Phi\left[C \bullet \left(d_1 + d_2\right) + P_y \bullet \left(d_3 - d_2\right)\right]$$

$$= 0.9\left[373.9 \bullet \left(7.04 + 0.279\right) + 735 \bullet \left(\frac{20.8}{2} - 0.279\right)\right] \Big/ 12 = 763.2 \text{ kip-ft}$$

Shear Stud Strength:

From AISC Manual Table 3.21, assuming the shear studs are placed in the weak position, the strength of ¾ in.-diameter shear studs in normal weight concrete with $f_c' = 4$ ksi and deck oriented perpendicular to the beam is:

$Q_n = 17.2$ kips for one shear stud per deck flute

$Q_n = 14.6$ kips for two shear studs per deck flute

Shear Stud Distribution:

There are at most 22 deck flutes along each half of the clear span of the beam. ETABS only counts the studs in the first 21 deck flutes as the 22nd flute is potentially too close to the point of zero moment for any stud located in it to be effective. With two shear studs in the first flute, 20 in the next in the next twenty flutes, and one shear stud in the 22nd flute, in each half of the beam, there is a total of 46 shear studs on the beam, and the total force provided by the shear studs in each half span is:

$$\Sigma Q_n = 2 \bullet 14.6 + 20 \bullet 17.2 = 373.9 \text{ kip}$$

Live Load Deflection:

Modulus of elasticity ratio:

$$n = E/E_c = 29,000/3,644 = 8.0$$

Transformed elastic moment of inertia assuming full composite action:

| Element | Transformed Area $A$ (in$^2$) | Moment Arm from Centroid $y$ (in.) | $Ay$ (in.$^3$) | $Ay^2$ (in,$^4$) | $I_0$ (in.$^4$) |
|---|---|---|---|---|---|
| Slab | 67.9 | 15.65 | 1,062 | 16,620 | 115 |
| W21x50 | 14.7 | 0 | 0 | 0 | 984 |
| | 82.6 | | 1,062 | 16,620 | 1,099 |

$$I_x = I_0 + Ay^2 = 1,099 + 16,620 = 17,719 \text{ in.}^4$$

$$\bar{y} = \frac{1,062}{82.6} = 12.9 \text{ in.}$$

$$I_{tr} = I_x - A \bullet \bar{y}^2 = 17,719 - 82.6 \bullet 12.9^2 = 4,058 \text{ in}^4$$

Effective moment inertia assuming partial composite action:

$$I_{\text{equiv}} = I_s + \sqrt{\Sigma Q_n / P_y}(I_{tr} - I_s) = 984 + \sqrt{0.51}(4,058 - 984) = 3,176 \text{ in}^4$$

$$I_{\text{eff}} = 0.75 \bullet I_{\text{equiv}} = 0.75 \bullet 3,176 = 2,382 \text{ in}^4$$

$$\Delta_{LL} = \frac{5w_L L^4}{384EI_{\text{eff}}} = \frac{5 \bullet (1/12) \bullet (30 \bullet 12)^4}{384 \bullet 29,000 \bullet 2,382} = 1.34 \text{ in.}$$

**Design for Shear Strength:**

Required Shear Strength:

$$w_u = 1.2 \bullet 0.800 + 1.2 \bullet 0.100 + 1.6 \bullet 1 = 2.68 \text{ kip/ft}$$

$$V_u = \frac{w_u \bullet L}{2} = \frac{2.68 \bullet 45}{2} = 60.3 \text{ kip-ft}$$

Available Shear Strength:

$$\Phi V_n = \Phi \bullet 0.6 \bullet d \bullet t_w \bullet F_y = 1.0 \bullet 0.6 \bullet 20.8 \bullet 0.38 \bullet 50 = 237.1 \text{ kips}$$