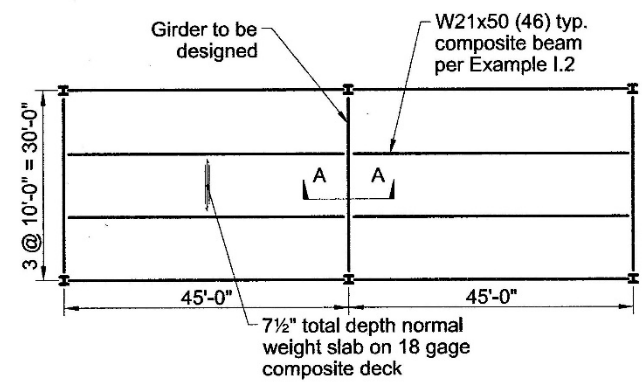
AISC-360-10 Example 002

COMPOSITE GIRDER DESIGN

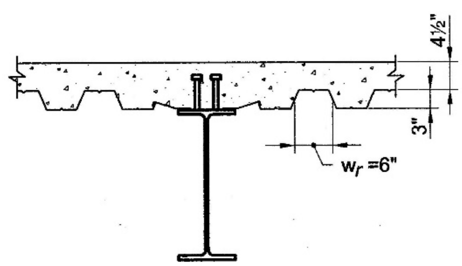
EXAMPLE DESCRIPTION

The design is checked for the composite girder shown below. The deck is 3 in. deep with 4 ½″ normal weight (145 pcf) concrete cover with a compressive strength of 4 ksi. The girder will not be shored during construction. The applied loads are the weight of the structure, a 25 psf construction live load, a 10 psf superimposed dead load and a 100 psf non-reducible service line load.

GEOMETRY, PROPERTIES AND LOADING



Plan



Section A-A

Member Properties	Loading	Geometry
W24x76	P = 36K (Dead Load)	Span, L = 45 ft
E = 29000 ksi	P = 4.5K (SDL)	
F_y = 50 ksi	P = 45K (Live Load)	

TECHNICAL FEATURES OF ETABS TESTED

Composite beam design, including:

- Selection of steel section, camber and shear stud distribution
- Member bending capacities, at construction and in service
- Member deflections, at construction and in service

RESULTS COMPARISON

Independent results are referenced from Example I.2 from the AISC Design Examples, Version 14.0.

Output Parameter	ETABS	Independent	Percent Difference
Pre-composite M_u (k-ft)	622.3	622.3	0.00%
Pre-composite $\Phi_b M_n$ (k-ft)	677.2	677.2	0.00%
Pre-composite Deflection (in.)	1.0	1.0	0.00%
Required Strength M_u (k-ft)	1216.3	1216.3	0.00%
Full Composite $\Phi_b M_n$ (k-ft)	1480.1	1480.1	0.00%
Partial Composite $\Phi_b M_n$ (k-ft)	1267.3	1267.3	0.00%
Shear Stud Capacity Q_n	21.54	21.54	0.00%
Shear Stud Distribution	26, 3, 26	26, 3, 26	0.00%
Live Load Deflection (in.)	0.63	0.55	12.7%
Required Strength V_u (kip)	122.0	122.0	0.00%
ΦV_n (k)	315.5	315.5	0.00%

COMPUTER FILE: AISC-360-10 EXAMPLE 002.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results. The live load deflection differs more markedly because of a difference in methodology. In the AISC example, the live load deflection is computed based on a lower bound value of the beam moment of inertia, whereas in ETABS, it is computed based on the approximate value of the beam moment of inertia derived from Equation (C-I3-6) from the *Commentary on the AISC Load and Resistance Factor Design Specification – Second Edition*.

HAND CALCULATION

Properties:

Materials:

ASTM A572 Grade 50 Steel

E = 29,000 ksi, F_y = 50 ksi, w_{steel} = 490 pcf

4000 psi normal weight concrete

E_c = 3,644 ksi, f_c' = 4 ksi, w_{concrete} = 145 pcf

Section:

W24x76

d = 23.9 in, b_f = 8.99 in, t_f = 0.68 in, t_w = 0.44 in

A_{steel} = 22.4 in^2, I_{steel} = 2100 in^4

Deck:

t_c =4 ½ in., h_r = 3 in., s_r =12 in., w_r = 6 in.

Shear Connectors:

d = ¾ in, h =4 ½ in, F_u = 65 ksi

Design for Pre-Composite Condition:

Construction Required Flexural Strength:

$$w = A_{\text{steel}} \cdot w_{\text{steel}} = \left(\frac{22.4}{144}\text{sq.ft.}\right) \bullet 490 \text{ pcf} = 76.2 \text{ plf}$$

$$P_D = \left[(45 \text{ ft})(10 \text{ ft})(75 \text{ psf}) + (50 \text{ plf})(45 \text{ ft})\right](0.001 \text{ kip / lb}) = 36 \text{ kips}$$

$$P_L = \left[(45 \text{ ft})(10 \text{ ft})(25 \text{ psf})\right](0.001 \text{ kip/lb}) = 11.25 \text{ kips}$$

$$M_u = \frac{1.2wL^2}{8} + \left(1.2P_D + 1.6P_L\right)\frac{L}{3}$$

$$= 1.2\frac{76.2 \bullet 30^2}{8} + \left(1.2 \bullet 36 + 1.6 \bullet 11.25\right)\frac{30}{3} = 622.3 \text{ kip-ft}$$

Moment Capacity:

L_b = 10 ft

L_p = 6.78 ft

Lr = 19.5 ft

$\Phi_b BF$ = 22.6 kips

$\Phi_b M_{px}$ = 750 kip-ft

C_b = 1.0

$$\Phi_b M_n = C_b\left[\Phi_b M_{px} - \Phi_b BF(L_b - L_p)\right]$$

$$= 1.0\left[750 - 22.6 \bullet (10 - 6.78)\right] \qquad = 677.2 \text{ kip-ft}$$

Pre-Composite Deflection:

$$\Delta_{nc} = \frac{P_D L^3}{28EI} + \frac{5w_D L^4}{384EI} = \frac{36.0 \bullet 360^3}{28 \bullet 29,000 \bullet 2,100} + \frac{5 \bullet \frac{0.0762}{12} \bullet 360^4}{384 \bullet 29,000 \bullet 2,100} = 1.0$$

$\text{Camber} = 0.8 \bullet \Delta_{nc} = 0.8 \text{ in.}$ which is rounded down to ¾ in.

Design for Composite Flexural Strength:

Required Flexural Strength:

$$P_D = \left[(45 \text{ ft})(10 \text{ ft})(75 + 10\text{psf}) + (50 \text{ plf})(45 \text{ ft})\right](0.001 \text{ kip/lb}) = 40.5 \text{ kips}$$

$$P_L = \left[(45 \text{ ft})(10 \text{ ft})(100 \text{ psf})\right](0.001 \text{ kip/lb}) = 45 \text{ kips}$$

$$M_u = \frac{1.2wL^2}{8} + (1.2P_D + 1.6P_L)\frac{L}{3}$$

$$= \frac{1.2 \bullet 76.22 \bullet 30^2}{8} + (1.2 \bullet 40.5 + 1.6 \bullet 45)\frac{30}{3} = 1216.3 \text{ kip-ft}$$

Full Composite Action Available Flexural Strength:

Effective width of slab:

$$b_{\text{eff}} = \frac{30.0 \text{ ft}}{8} = 7.5 \text{ ft} = 90 \text{ in.}$$

Resistance of steel in tension:

$$C = P_y = A_s \bullet F_y = 22.4 \bullet 50 = 1{,}120 \text{ kips} \ \textbf{controls}$$

Resistance of slab in compression

$$A_c = b_{\text{eff}} \bullet t_c + (b_{\text{eff}}/2) \bullet h_r = (7.5 \bullet 12) \bullet 4.5 + \frac{7.5 \bullet 12}{2} \bullet 3 = 540 \text{ in}^2$$

$$C = 0.85 \bullet f'_c A_c = 0.85 \bullet 4 \bullet 540 = 1836 \text{ kips}$$

Depth of compression block within slab:

$$a = \frac{C}{0.85 \bullet b_{eff} \bullet f'_c} = \frac{1{,}120}{0.85 \bullet (7.5 \bullet 12) \bullet 4} = 3.66 \text{ in.}$$

Moment resistance of composite beam for full composite action:

$$d_1 = (t_c + h_r) - \frac{a}{2} = (4.5 + 3) - \frac{3.66}{2} = 5.67 \text{ in.}$$

$$\Phi M_n = \Phi\left(C \bullet d_1 + P_y \bullet \frac{d}{2}\right)$$

$$= 0.9 \bullet \left(1{,}120 \bullet 5.67/12 + 1{,}120 \bullet \frac{23.9/12}{2}\right) = 1480.1 \text{ kip-ft}$$

<u>Partial Composite Action Available Flexural Strength:</u>

Assume 50% composite action:

$$C = 0.5 \bullet P_y = 560 \text{ kips}$$

Depth of compression block within slab

$$a = \frac{C}{0.85 \bullet b_{eff} \bullet f'_c} = \frac{560}{0.85 \bullet (7.5 \bullet 12) \bullet 4} = 1.83 \text{ in.}$$

$$d_1 = (t_c + h_r) - \frac{a}{2} = (4.5 + 3) - \frac{1.83}{2} = 6.58 \text{ in.}$$

Depth of compression block within steel section flange

$$x = \frac{P_y - C}{2 \bullet b_f \bullet F_y} = \frac{1{,}120 - 560}{2 \bullet 8.99 \bullet 50} = 0.623 \text{ in.}$$

$$d_2 = x/2 = 0.311 \text{ in.}$$

$$M_n = C \bullet (d_1 + d_2) + P_y \bullet (d_3 - d_2)$$
$$= \left[560 \bullet (6.58 + 0.312) + 1{,}120 \bullet \left(\frac{23.9}{2} - 0.312 \right) \right] \Big/ 12 = 1{,}408 \text{ kip-ft}$$
$$\Phi M_n = 0.9 M_n = 0.9 \bullet 1{,}408 = 1{,}267.3 \text{ kip-ft}$$

Shear Stud Strength:

$$Q_n = 0.5 A_{sa} \sqrt{f'_c E_c} \le R_g R_p A_{sa} F_u$$
$$A_{sa} = \pi d_{sa}{}^2 / 4 = \pi (0.75)^2 / 4 = 0.442 \text{ in}^2$$
$$f_c' = 4 \text{ ksi}$$
$$E = w_c^{1.5} \sqrt{f_c'} = 145^{1.5} \sqrt{4} = 3{,}490 \text{ ksi}$$

R_g = 1.0 Studs welded directly to the steel shape with the slab haunch

R_p = 0.75 Studs welded directly to the steel shape

F_u = 65 ksi

$$Q_n = 0.5 \bullet 0.442^2 \sqrt{4 \bullet 3{,}490} \le 1.0 \bullet 0.75 \bullet 0.442^2 \bullet 65$$
$$= 26.1 \text{ kips } \ge 21.54 \text{ kips controls}$$

Shear Stud Distribution:

$$n = \frac{\Sigma Q_n}{Q_n}$$
$$= \frac{560}{21.54} = 26 \text{ studs from each end to nearest concentrated load point}$$

Add 3 studs between load points to satisfy maximum stud spacing requirement.

Live Load Deflection:

Modulus of elasticity ratio:

$$n = E / E_c = 29{,}000 / 3{,}644 = 8.0$$

Transformed elastic moment of inertia assuming full composite action:

Element	Transformed Area A (in^2)	Moment Arm from Centroid y (in.)	Ay (in.^{3})	Ay^2 (in,^{4})	I_0 (in.^{4})
Slab	50.9	17.2	875	15,055	86
Deck ribs	17.0	13.45	228	3,069	13
W21x50	22.4	0	0	0	2,100
	89.5		1,103	18,124	2,199

$$I_x = I_0 + Ay^2 = 2{,}199 + 18{,}124 = 20{,}323 \text{ in.}^4$$

$$\bar{y} = \frac{1{,}092}{89.5} = 12.2 \text{ in.}$$

$$I_{tr} = I_x - A \bullet \bar{y}^2 = 20{,}323 - 90.3 \bullet 12.2^2 = 6{,}831 \text{ in}^4$$

Effective moment of inertia assuming partial composite action:

$$I_{\text{equiv}} = I_s + \sqrt{\Sigma Q_n / P_y}\left(I_{tr} - I_s\right) = 2{,}100 + \sqrt{0.5}\left(6{,}831 - 2{,}100\right) = 5{,}446 \text{ in}^4$$

$$I_{\text{eff}} = 0.75 \bullet I_{\text{equiv}} = 0.75 \bullet 5{,}446 = 4{,}084 \text{ in}^4$$

$$\Delta_{LL} = \frac{P_L L^3}{28 E I_{eff}} = \frac{45.0 \bullet (30 \bullet 12)^3}{28 \bullet 29{,}000 \bullet 4{,}084} = 0.633 \text{ in.}$$

Design for Shear Strength:

Required Shear Strength:

$$P_u = 1.2 \bullet P_D + 1.6 \bullet P_L = 1.2 \bullet 40.5 + 1.6 \bullet 45 = 120.6 \text{ kip}$$

$$V_u = \frac{1.2 \bullet w \bullet L}{2} + P_u = \frac{1.2 \bullet 0.076 \bullet 30}{2} + 120.6 = 121.2 \text{ kip-ft}$$

Available Shear Strength:

$$\Phi V_n = \Phi \bullet 0.6 \bullet d \bullet t_w \bullet F_y = 1.0 \bullet 0.6 \bullet 23.9 \bullet 0.44 \bullet 50 = 315.5 \text{ kips}$$