ResearchGate

See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/200744645

A Practical Guide to Spline

Book in Mathematics of Computation · January 1978

DOI: 10.2307/2006241

CITATIONS READS

4,662 21,217

1 author:



Carl R de Boor

University of Wisconsin-Madison

237 PUBLICATIONS 15,542 CITATIONS

SEE PROFILE

Carl de Boor

A Practical Guide to Splines

Revised Edition

With 32 figures



Contents

Preface	v
Notation	xv
I · Polynomial Interpolation	
Polynomial interpolation: Lagrange form Polynomial Interpolation: Divided differences and Newton form Divided difference table Example: Osculatory interpolation to the logarithm Evaluation of the Newton form	2 3 8 9 9
Example: Computing the derivatives of a polynomial in Newton form Other polynomial forms and conditions Problems	11 12 15
II · Limitations of Polynomial Approximation	
Uniform spacing of data can have bad consequences Chebyshev sites are good Runge example with Chebyshev sites Squareroot example Interpolation at Chebyshev sites is nearly optimal	17 20 22 22 24
The distance from polynomials Problems	$\frac{24}{27}$

x Contents

III ·	Piecewise	Linear	Approximation	a
-------	-----------	--------	---------------	---

Broken line interpolation Broken line interpolation is nearly optimal	
Good meshes	35
Problems	37
IV · Piecewise Cubic Interpolation	
Piecewise cubic Hermite interpolation	40
Runge example continued	41
Piecewise cubic Bessel interpolation	42
Akima's interpolation	42
Cubic spline interpolation	43
Boundary conditions	43
Problems	48
V Doot Annuarimentian Deposition of Complete Cubic Spline	
V · Best Approximation Properties of Complete Cubic Spline Interpolation and Its Error	51
•	91
Problems	56
VI · Parabolic Spline Interpolation	59
Problems	64
VII · A Representation for Piecewise Polynomial Functions	
Piecewise polynomial functions	69
The subroutine PPVALU	72
The subroutine INTERV	74
Problems	77
VIII · The Spaces $\Pi_{\langle k,\xi,\nu}$ and the Truncated Power Basis	
Example: The smoothing of a histogram by parabolic splines	79
The space $\Pi_{\langle k,\xi,\nu}$	82
The truncated power basis for $\Pi_{\leq k,\xi}$ and $\Pi_{\leq k,\xi,\nu}$	82
Example: The truncated power basis can be bad	85
Problems	86

Contents xi

IX \cdot The Representation of PP Functions by B-Splines	
Definition of a B-spline Two special knot sequences A recurrence relation for B-splines Example: A sequence of parabolic B-splines The spline space \$\frac{1}{8}_{k,t}\$	87 89 89 91 93
The polynomials in $\$_{k,t}$ The pp functions in $\$_{k,t}$ B stands for basis Conversion from one form to the other Example: Conversion to B-form	94 96 99 101 103
Problems	106
X · The Stable Evaluation of B-Splines and Splines	
Stable evaluation of B-splines The subroutine BSPLVB Example: To plot B-splines Example: To plot the polynomials that make up a B-spline Differentiation	109 109 113 114 115
The subroutine BSPLPP Example: Computing a B-spline once again The subroutine BVALUE Example: Computing a B-Spline one more time Integration	117 120 121 126 127
Problems	128
XI · The B-Spline Series, Control Points, and Knot Insertion	
Bounding spline values in terms of "nearby" coefficients Control points and control polygon Knot insertion Variation diminution Schoenberg's variation diminishing spline approximation	131 133 135 138 141
Problems	142
XII · Local Spline Approximation and the Distance from Spline	s
The distance of a continuous function from $\$_{k,t}$ The distance of a smooth function from $\$_{k,t}$ Example: Schoenberg's variation-diminishing spline approximation Local schemes that provide best possible approximation order Good knot placement	145 148 149 152

xii Contents

The subroutine NEWNOT	159
Example: A failure for NEWNOT	161
The distance from $\$_{k,n}$	163
Example: A failure for CUBSPL	165
Example: Knot placement works when used with a local scheme	167
Problems	169
XIII · Spline Interpolation	
The Schoenberg-Whitney Theorem	171
Bandedness of the spline collocation matrix	173
Total positivity of the spline collocation matrix	169
The subroutine SPLINT	175
The interplay between knots and data sites	180
Even order interpolation at knots	182
Example: A large $ I $ amplifies noise	183
Interpolation at knot averages	185
Example: Cubic spline interpolation at knot averages with good knots	
Interpolation at the Chebyshev-Demko sites	189
Optimal interpolation ·	193
Example: "Optimal" interpolation need not be "good"	197
Osculatory spline interpolation	200
Problems	204
XIV · Smoothing and Least-Squares Approximation	
The smoothing spline of Schoenberg and Reinsch	207
The subroutine SMOOTH and its subroutines	211
Example: The cubic smoothing spline	214
Least-squares approximation	220
Least-squares approximation from $\$_{k,\mathbf{t}}$	223
The subroutine L2APPR (with BCHFAC/BCHSLV)	224
L2MAIN and its subroutines	228
The use of L2APPR	232
Example: Fewer sign changes in the error than perhaps expected	232
Example: The noise plateau in the error	235
Example: Once more the Titanium Heat data	237
Least-squares approximation by splines with variable knots	239
Example: Approximation to the Titanium Heat data from \$4.9	239
Problems	-240

Contents xiii

by Collocation)11
Mathematical background The almost block diagonal character of the system of collocation	243
equations; EQBLOK, PUTIT	246
The subroutine BSPLVD	251
COLLOC and its subroutines	253
Example: A second order nonlinear two-point boundary-value	
problem with a boundary layer	258
Problems	261
XVI · Taut Splines, Periodic Splines, Cardinal Splines and the Approximation of Curves	
Lack of data	263
"Extraneous" inflection points	264
Spline in tension	264
Example: Coping with a large endslope	265
A taut cubic spline	266
Example: Taut cubic spline interpolation to Titanium Heat data	275
Proper choice of parametrization	276
Example: Choice of parametrization is important	277
The approximation of a curve	279
Nonlinear splines	280
Periodic splines	282
Cardinal splines	283
Example: Conversion to ppform is cheaper when knots are uniform	284
Example: Cubic spline interpolation at uniformly spaced sites	284
Periodic splines on uniform meshes	285
Example: Periodic spline interpolation to uniformly spaced data	
and harmonic analysis	287

XVII · Surface Approximation by Tensor Products

289

Problems

An abstract linear interpolation scheme	291
Tensor product of two linear spaces of functions	293
Example: Evaluation of a tensor product spline.	297
The tensor product of two linear interpolation schemes	297
The calculation of a tensor product interpolant	299

xiv Contents

Example: Tensor product spline interpolation	
The ppform of a tensor product spline	305
The evaluation of a tensor product spline from its ppform	305
Conversion from B-form to ppform	307
Example: Tensor product spline interpolation (continued)	309
Limitations of tensor product approximation and alternatives	310
Problems	311
Postscript on Things Not Covered	313
Appendix: Fortran Programs	
Fortran programs	315
List of Fortran programs	315
Listing of SOLVEBLOK Package	318
Bibliography	331
\mathbf{Index}	341