Instructor Solutions Manual

to accompany

Applied Linear Statistical Models

Fifth Edition

Michael H. Kutner
Emory University
Christopher J. Nachtsheim
University of Minnesota
John Neter
University of Georgia
William Li
University of Minnesota

2005
McGraw-Hill/Irwin
Chicago, IL
Boston, MA

PREFACE

This Solutions Manual gives intermediate and final numerical results for all end-of-chapter Problems, Exercises, and Projects with computational elements contained in *Applied Linear Statistical Models*, 5th edition. This Solutions Manual also contains proofs for all Exercises that require derivations. No solutions are provided for the Case Studies.

In presenting calculational results we frequently show, for ease in checking, more digits than are significant for the original data. Students and other users may obtain slightly different answers than those presented here, because of different rounding procedures. When a problem requires a percentile (e.g. of the t or F distributions) not included in the Appendix B Tables, users may either interpolate in the table or employ an available computer program for finding the needed value. Again, slightly different values may be obtained than the ones shown here.

We have included many more Problems, Exercises, and Projects at the ends of chapters than can be used in a term, in order to provide choice and flexibility to instructors in assigning problem material. For all major topics, three or more problem settings are presented, and the instructor can select different ones from term to term. Another option is to supply students with a computer printout for one of the problem settings for study and class discussion and to select one or more of the other problem settings for individual computation and solution. By drawing on the basic numerical results in this Manual, the instructor also can easily design additional questions to supplement those given in the text for a given problem setting.

The data sets for all Problems, Exercises, Projects and Case Studies are contained in the compact disk provided with the text to facilitate data entry. It is expected that the student will use a computer or have access to computer output for all but the simplest data sets, where use of a basic calculator would be adequate. For most students, hands-on experience in obtaining the computations by computer will be an important part of the educational experience in the course.

While we have checked the solutions very carefully, it is possible that some errors are still present. We would be most grateful to have any errors called to our attention. Errata can be reported via the website for the book: http://www.mhhe.com/KutnerALSM5e. We acknowledge with thanks the assistance of Lexin Li and Yingwen Dong in the checking of Chapters 1-14 of this manual. We, of course, are responsible for any errors or omissions that remain.

Michael H. Kutner
Christopher J. Nachtsheim
John Neter
William Li

Contents

Chapter 1

LINEAR REGRESSION WITH ONE PREDICTOR VARIABLE

1.1. No

1.2. $Y = 300 + 2X$, functional

1.5. No

1.7. a. No
b. Yes, .68

1.8. Yes, no

1.10. No

1.12. a. Observational

1.13. a. Observational

1.18. No

1.19. a. $\beta_0 = 2.11405$, $\beta_1 = 0.03883$, $\hat{Y} = 2.11405 + .03883X$
c. $\hat{Y}_h = 3.27895$
d. $\beta_1 = 0.03883$

1.20. a. $\hat{Y} = -0.5802 + 15.0352X$
d. $\hat{Y}_h = 74.5958$

1.21. a. $\hat{Y} = 10.20 + 4.00X$
b. $\hat{Y}_h = 14.2$
c. 4.0
d. $(\bar{X}, \bar{Y}) = (1, 14.2)$

1.22. a. $\hat{Y} = 168.600000 + 2.034375X$

b. $\hat{Y}_h = 249.975$

c. $\beta_1 = 2.034375$

1.23. a.

i:	1	2	...	119	120
e_i:	0.9676	1.2274	...	-0.8753	-0.2532

Yes

b. $MSE = 0.388$, $\sqrt{MSE} = 0.623$, grade points

1.24. a.

i:	1	2	...	44	45
e_i:	-9.4903	0.4392	...	1.4392	2.4039

$\sum e_i^2 = 3416.377$

Min $Q = \sum e_i^2$

b. $MSE = 79.45063$, $\sqrt{MSE} = 8.913508$, minutes

1.25. a. $e_1 = 1.8000$

b. $\sum e_i^2 = 17.6000$, $MSE = 2.2000$, σ^2

1.26. a.

i:	1	2	3	4	5	6
e_i:	-2.150	3.850	-5.150	-1.150	.575	2.575

i:	7	8	9	10	11	12
e_i:	-2.425	5.575	3.300	.300	1.300	-3.700

i:	13	14	15	16
e_i:	.025	-1.975	3.025	-3.975

Yes

b. $MSE = 10.459$, $\sqrt{MSE} = 3.234$, Brinell units

1.27. a. $\hat{Y} = 156.35 - 1.19X$

b. (1) $b_1 = -1.19$, (2) $\hat{Y}_h = 84.95$, (3) $e_8 = 4.4433$,

(4) $MSE = 66.8$

1.28. a. $\hat{Y} = 20517.6 - 170.575X$

b. (1) $b_1 = -170.575$, (2) $\hat{Y}_h = 6871.6$, (3) $e_{10} = 1401.566$,

(4) $MSE = 5552112$

1.31. No, no

1.32. Solving (1.9a) and (1.9b) for b_0 and equating the results:

$$\frac{\sum Y_i - b_1 \sum X_i}{n} = \frac{\sum X_i Y_i - b_1 \sum X_i^2}{\sum X_i}$$

and then solving for b_1 yields:

$$b_1 = \frac{n\sum X_i Y_i - \sum X_i \sum Y_i}{n\sum X_i^2 - (\sum X_i)^2} = \frac{\sum X_i Y_i - \dfrac{\sum X_i \sum Y_i}{n}}{\sum X_i^2 - \dfrac{(\sum X_i)^2}{n}}$$

1.33. $Q = \sum(Y_i - \beta_0)^2$

$$\frac{dQ}{d\beta_0} = -2\sum(Y_i - \beta_0)$$

Setting the derivative equal to zero, simplifying, and substituting the least squares estimator b_0 yields:

$\sum(Y_i - b_0) = 0$ or $b_0 = \bar{Y}$

1.34. $E\{b_0\} = E\{\bar{Y}\} = \dfrac{1}{n}\sum E\{Y_i\} = \dfrac{1}{n}\sum \beta_0 = \beta_0$

1.35. From the first normal equation (1.9a):

$$\sum Y_i = nb_0 + b_1 \sum X_i = \sum(b_0 + b_1 X_i) = \sum \hat{Y}_i \text{ from (1.13)}$$

1.36. $\sum \hat{Y}_i e_i = \sum(b_0 + b_1 X_i)e_i = b_0 \sum e_i + b_1 \sum X_i e_i = 0$ because $\sum e_i = 0$ from (1.17) and $\sum X_i e_i = 0$ from (1.19).

1.38. (1) 76, yes; (2) 60, yes

1.39. a. Applying (1.10a) and (1.10b) to $(5, \bar{Y}_1)$, $(10, \bar{Y}_2)$ and $(15, \bar{Y}_3)$, we obtain:

$$b_1 = \frac{\bar{Y}_3 - \bar{Y}_1}{10} \qquad b_0 = \frac{4\bar{Y}_1 + \bar{Y}_2 - 2\bar{Y}_3}{3}$$

Using (1.10a) and (1.10b) with the six original points yields the same results.

b. Yes

1.40. No

1.41. a. $Q = \sum(Y_i - \beta_1 X_i)^2$

$$\frac{dQ}{d\beta_1} = -2\sum(Y_i - \beta_1 X_i)X_i$$

Setting the derivative equal to zero, simplifying, and substituting the least squares estimator b_1 yields:

$$b_1 = \frac{\sum Y_i X_i}{\sum X_i^2}$$

b. $L = \prod_{i=1}^{n} \dfrac{1}{(2\pi\sigma^2)^{1/2}} \exp\left[-\dfrac{1}{2\sigma^2}(Y_i - \beta_1 X_i)^2\right]$

It is more convenient to work with $\log_e L$:

$$\begin{aligned}\log_e L &= -\frac{n}{2}\log_e(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum(Y_i - \beta_1 X_i)^2 \\ \frac{d\log_e L}{d\beta_1} &= \frac{1}{\sigma^2}\sum(Y_i - \beta_1 X_i)X_i\end{aligned}$$

Setting the derivative equal to zero, simplifying, and substituting the maximum likelihood estimator b_1 yields:

$\sum(Y_i - b_1 X_i)X_i = 0$ or $b_1 = \frac{\sum Y_i X_i}{\sum X_i^2}$

Yes

c. $E\{b_1\} = E\left\{\frac{\sum Y_i X_i}{\sum X_i^2}\right\} = \frac{1}{\sum X_i^2}\sum X_i E\{Y_i\}$

$$= \frac{1}{\sum X_i^2}\sum X_i(\beta_1 X_i) = \beta_1$$

1.42. a. $L(\beta_1) = \prod_{i=1}^{6} \frac{1}{\sqrt{32\pi}} \exp[-\frac{1}{32}(Y_i - \beta_1 X_i)^2]$

b. $L(17) = 9.45 \times 10^{-30}$, $L(18) = 2.65 \times 10^{-7}$, $L(19) = 3.05 \times 10^{-37}$

$\beta_1 = 18$

c. $b_1 = 17.928$, yes

d. Yes

1.43. a. Total population: $\hat{Y} = -110.635 + 0.0027954X$

Number of hospital beds: $\hat{Y} = -95.9322 + 0.743116X$

Total personal income: $\hat{Y} = -48.3948 + .131701X$

c. Total population: $MSE = 372,203.5$

Number of hospital beds: $MSE = 310,191.9$

Total personal income: $MSE = 324,539.4$

1.44. a. Region 1: $\hat{Y} = -1723.0 + 480.0X$

Region 2: $\hat{Y} = 916.4 + 299.3X$

Region 3: $\hat{Y} = 401.56 + 272.22X$

Region 4: $\hat{Y} = 396.1 + 508.0X$

c. Region 1: $MSE = 64,444,465$

Region 2: $MSE = 141,479,673$

Region 3: $MSE = 50,242,464$

Region 4: $MSE = 514,289,367$

1.45. a. Infection risk: $\hat{Y} = 6.3368 + .7604X$

Facilities: $\hat{Y} = 7.7188 + .0447X$

X-ray: $\hat{Y} = 6.5664 + .0378X$

c. Infection risk: $MSE = 2.638$

Facilities: $MSE = 3.221$

X-ray: $MSE = 3.147$

1.46. a. Region 1: $\hat{Y} = 4.5379 + 1.3478X$

Region 2: $\hat{Y} = 7.5605 + .4832X$

Region 3: $\hat{Y} = 7.1293 + .5251X$

Region 4: $\hat{Y} = 8.0381 + .0173X$

c. Region 1: $MSE = 4.353$

Region 2: $MSE = 1.038$

Region 3: $MSE = .940$

Region 4: $MSE = 1.078$

1.47. a. $L(\beta_0, \beta_1) = \prod_{i=1}^{6} \frac{1}{\sqrt{32\pi}} \exp[-\frac{1}{32}(Y_i - \beta_0 - \beta_1 X_i)^2]$

b. $b_0 = 1.5969$, $b_1 = 17.8524$

c. Yes

Chapter 2

INFERENCES IN REGRESSION AND CORRELATION ANALYSIS

2.1. a. Yes, $\alpha = .05$

2.2. No

2.4. a. $t(.995; 118) = 2.61814$, $.03883 \pm 2.61814(.01277)$, $.00540 \leq \beta_1 \leq .07226$

b. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $t^* = (.03883 - 0)/.01277 = 3.04072$. If $|t^*| \leq 2.61814$, conclude H_0, otherwise H_a. Conclude H_a.

c. 0.00291

2.5. a. $t(.95; 43) = 1.6811$, $15.0352 \pm 1.6811(.4831)$, $14.2231 \leq \beta_1 \leq 15.8473$

b. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $t^* = (15.0352 - 0)/.4831 = 31.122$. If $|t^*| \leq 1.681$ conclude H_0, otherwise H_a. Conclude H_a. P-value= 0+

c. Yes

d. H_0: $\beta_1 \leq 14$, H_a: $\beta_1 > 14$. $t^* = (15.0352 - 14)/.4831 = 2.1428$. If $t^* \leq 1.681$ conclude H_0, otherwise H_a. Conclude H_a. P-value= .0189

2.6. a. $t(.975; 8) = 2.306$, $b_1 = 4.0$, $s\{b_1\} = .469$, $4.0 \pm 2.306(.469)$,

$2.918 \leq \beta_1 \leq 5.082$

b. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $t^* = (4.0 - 0)/.469 = 8.529$. If $|t^*| \leq 2.306$ conclude H_0, otherwise H_a. Conclude H_a. P-value= .00003

c. $b_0 = 10.20$, $s\{b_0\} = .663$, $10.20 \pm 2.306(.663)$, $8.671 \leq \beta_0 \leq 11.729$

d. H_0: $\beta_0 \leq 9$, H_a: $\beta_0 > 9$. $t^* = (10.20 - 9)/.663 = 1.810$. If $t^* \leq 2.306$ conclude H_0, otherwise H_a. Conclude H_0. P-value= .053

e. H_0: $\beta_1 = 0$: $\delta = |2 - 0|/.5 = 4$, power $= .93$

H_0: $\beta_0 \leq 9$: $\delta = |11 - 9|/.75 = 2.67$, power $= .78$

2.7. a. $t(.995; 14) = 2.977$, $b_1 = 2.0344$, $s\{b_1\} = .0904$, $2.0344 \pm 2.977(.0904)$,

$1.765 \leq \beta_1 \leq 2.304$

b. H_0: $\beta_1 = 2$, H_a: $\beta_1 \neq 2$. $t^* = (2.0344 - 2)/.0904 = .381$. If $|t^*| \leq 2.977$ conclude H_0, otherwise H_a. Conclude H_0. P-value$= .71$

c. $\delta = |.3|/.1 = 3$, power $= .50$

2.8. a. H_0: $\beta_1 = 3.0$, H_a: $\beta_1 \neq 3.0$. $t^* = (3.57 - 3.0)/.3470 = 1.643$, $t(.975; 23) = 2.069$. If $|t^*| \leq 2.069$ conclude H_0, otherwise H_a. Conclude H_0.

b. $\delta = |.5|/.35 = 1.43$, power $= .30$ (by linear interpolation)

2.10. a. Prediction

b. Mean response

c. Prediction

2.12. No, no

2.13. a. $\hat{Y}_h = 3.2012$, $s\{\hat{Y}_h\} = .0706$, $t(.975;\ 118) = 1.9803$, $3.2012 \pm 1.9803(.0706)$, $3.0614 \leq E\{Y_h\} \leq 3.3410$

b. $s\{\text{pred}\} = .6271$, $3.2012 \pm 1.9803(.6271)$, $1.9594 \leq Y_{h(\text{ new})} \leq 4.4430$

c. Yes, yes

d. $W^2 = 2F(.95; 2, 118) = 2(3.0731) = 6.1462$, $W = 2.4792$, $3.2012 \pm 2.4792(.0706)$, $3.0262 \leq \beta_0 + \beta_1 X_h \leq 3.3762$, yes, yes

2.14. a. $\hat{Y}_h = 89.6313$, $s\{\hat{Y}_h\} = 1.3964$, $t(.95;\ 43) = 1.6811$, $89.6313 \pm 1.6811(1.3964)$, $87.2838 \leq E\{Y_h\} \leq 91.9788$

b. $s\{\text{pred}\} = 9.0222$, $89.6313 \pm 1.6811(9.0222)$, $74.4641 \leq Y_{h(\text{new})} \leq 104.7985$, yes, yes

c. $87.2838/6 = 14.5473$, $91.9788/6 = 15.3298$, $14.5473 \leq$ Mean time per machine ≤ 15.3298

d. $W^2 = 2F(.90;\ 2,\ 43) = 2(2.4304) = 4.8608$, $W = 2.2047$, $89.6313 \pm 2.2047(1.3964)$, $86.5527 \leq \beta_0 + \beta_1 X_h \leq 92.7099$, yes, yes

2.15. a. $X_h = 2$: $\hat{Y}_h = 18.2$, $s\{\hat{Y}_h\} = .663$, $t(.995;\ 8) = 3.355$, $18.2 \pm 3.355(.663)$, $15.976 \leq E\{Y_h\} \leq 20.424$

$X_h = 4$: $\hat{Y}_h = 26.2$, $s\{\hat{Y}_h\} = 1.483$, $26.2 \pm 3.355(1.483)$, $21.225 \leq E\{Y_h\} \leq 31.175$

b. $s\{\text{pred}\} = 1.625$, $18.2 \pm 3.355(1.625)$, $12.748 \leq Y_{h(\text{new})} \leq 23.652$

c. $s\{\text{predmean}\} = 1.083$, $18.2 \pm 3.355(1.083)$, $14.567 \leq \bar{Y}_{h(\text{new})} \leq 21.833$, $44 = 3(14.567) \leq$ Total number of broken ampules $\leq 3(21.833) = 65$

d. $W^2 = 2F(.99; 2, 8) = 2(8.649) = 17.298$, $W = 4.159$

$X_h = 2$: $18.2 \pm 4.159(.663)$, $15.443 \leq \beta_0 + \beta_1 X_h \leq 20.957$

$X_h = 4$: $26.2 \pm 4.159(1.483)$, $20.032 \leq \beta_0 + \beta_1 X_h \leq 32.368$

yes, yes

2.16. a. $\hat{Y}_h = 229.631$, $s\{\hat{Y}_h\} = .8285$, $t(.99;\ 14) = 2.624$, $229.631 \pm 2.624(.8285)$, $227.457 \leq E\{Y_h\} \leq 231.805$

b. $s\{\text{pred}\} = 3.338$, $229.631 \pm 2.624(3.338)$, $220.872 \leq Y_{h(\text{new})} \leq 238.390$

c. $s\{\text{predmean}\} = 1.316$, $229.631 \pm 2.624(1.316)$, $226.178 \leq \bar{Y}_{h(\text{new})} \leq 233.084$

d. Yes, yes

e. $W^2 = 2F(.98; 2, 14) = 2(5.241) = 10.482$, $W = 3.238$, $229.631 \pm 3.238(.8285)$, $226.948 \leq \beta_0 + \beta_1 X_h \leq 232.314$, yes, yes

2.17. Greater, H_0: $\beta_1 = 0$

2.20. No

2.21. No

2.22. Yes, yes

2.23. a.

Source	SS	df	MS
Regression	3.58785	1	3.58785
Error	45.8176	118	0.388285
Total	49.40545	119	

b. $\sigma^2 + \beta_1^2 \sum(X_i - \bar{X})^2$, σ^2, when $\beta_1 = 0$

c. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $F^* = 3.58785/0.388285 = 9.24$, $F(.99; 1, 118) = 6.855$. If $F^* \leq 6.855$ conclude H_0, otherwise H_a. Conclude H_a.

d. $SSR = 3.58785$, 7.26% or 0.0726, coefficient of determination

e. +0.2695

f. R^2

2.24. a.

Source	SS	df	MS
Regression	76,960.4	1	76,960.4
Error	3,416.38	43	79.4506
Total	80,376.78	44	

Source	SS	df	MS
Regression	76,960.4	1	76,960.4
Error	3,416.38	43	79.4506
Total	80,376.78	44	
Correction for mean	261,747.2	1	
Total, uncorrected	342,124	45	

b. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $F^* = 76,960.4/79.4506 = 968.66$, $F(.90; 1, 43) = 2.826$. If $F^* \leq 2.826$ conclude H_0, otherwise H_a. Conclude H_a.

c. 95.75% or 0.9575, coefficient of determination

d. +.9785

e. R^2

2.25. a.

Source	SS	df	MS
Regression	160.00	1	160.00
Error	17.60	8	2.20
Total	177.60	9	

b. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $F^* = 160.00/2.20 = 72.727$, $F(.95; 1, 8) = 5.32$. If $F^* \leq 5.32$ conclude H_0, otherwise H_a. Conclude H_a.

c. $t^* = (4.00 - 0)/.469 = 8.529$, $(t^*)^2 = (8.529)^2 = 72.7 = F^*$

d. $R^2 = .9009$, $r = .9492$, 90.09%

2.26. a.

Source	SS	df	MS
Regression	5,297.5125	1	5,297.5125
Error	146.4250	14	10.4589
Total	5,443.9375	15	

b. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$, $F^* = 5,297.5125/10.4589 = 506.51$, $F(.99; 1, 14) = 8.86$.If $F^* \leq 8.86$ conclude H_0, otherwise H_a. Conclude H_a.

c.

i:	1	2	3	4	5	6
$Y_i - \hat{Y}_i$:	-2.150	3.850	-5.150	-1.150	.575	2.575
$\hat{Y}_i - \bar{Y}$:	-24.4125	-24.4125	-24.4125	-24.4125	-8.1375	-8.1375

i:	7	8	9	10	11	12
$Y_i - \hat{Y}_i$:	-2.425	5.575	3.300	.300	1.300	-3.700
$\hat{Y}_i - \bar{Y}$:	-8.1375	-8.1375	8.1375	8.1375	8.1375	8.1375

i:	13	14	15	16
$Y_i - \hat{Y}_i$:	.025	-1.975	3.025	-3.975
$\hat{Y}_i - \bar{Y}$:	24.4125	24.4125	24.4125	24.4125

d. $R^2 = .9731$, $r = .9865$

2.27. a. H_0: $\beta_1 \geq 0$, H_a: $\beta_1 < 0$. $s\{b_1\} = 0.090197$,
$t^* = (-1.19 - 0)/.090197 = -13.193$, $t(.05; 58) = -1.67155$.
If $t^* \geq -1.67155$ conclude H_0, otherwise H_a. Conclude H_a.
P-value= 0+

c. $t(.975; 58) = 2.00172$, $-1.19 \pm 2.00172(.090197)$, $-1.3705 \leq \beta_1 \leq -1.0095$

2.28. a. $\hat{Y}_h = 84.9468$, $s\{\hat{Y}_h\} = 1.05515$, $t(.975; 58) = 2.00172$,
$84.9468 \pm 2.00172(1.05515)$, $82.835 \leq E\{Y_h\} \leq 87.059$

b. $s\{Y_{h(\text{new})}\} = 8.24101$, $84.9468 \pm 2.00172(8.24101)$, $68.451 \leq Y_{h(\text{new})} \leq 101.443$

c. $W^2 = 2F(.95; 2, 58) = 2(3.15593) = 6.31186$, $W = 2.512342$,
$84.9468 \pm 2.512342(1.05515)$, $82.296 \leq \beta_0 + \beta_1 X_h \leq 87.598$, yes, yes

2.29. a.

i:	1	2	...	59	60
$Y_i - \hat{Y}_i$:	0.823243	-1.55675	...	-0.666887	8.09309
$\hat{Y}_i - \bar{Y}$:	20.2101	22.5901	...	-14.2998	-19.0598

b.

Source	SS	df	MS
Regression	11,627.5	1	11,627.5
Error	3,874.45	58	66.8008
Total	15,501.95	59	

c. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $F^* = 11,627.5/66.8008 = 174.0623$, $F(.90;\ 1, 58) = 2.79409$. If $F^* \leq 2.79409$ conclude H_0, otherwise H_a. Conclude H_a.

d. 24.993% or .24993

e. $R^2 = 0.750067$, $r = -0.866064$

2.30. a. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $s\{b_1\} = 41.5743$,
$t^* = (-170.575 - 0)/41.5743 = -4.1029$, $t(.995; 82) = 2.63712$.
If $|t^*| \leq 2.63712$ conclude H_0, otherwise H_a. Conclude H_a.
P-value = 0.000096

b. $-170.575 \pm 2.63712(41.5743)$, $-280.2114 \leq \beta_1 \leq -60.9386$

2.31. a.

Source	SS	df	MS
Regression	93,462,942	1	93,462,942
Error	455,273,165	82	5,552,112
Total	548,736,107	83	

b. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $F^* = 93,462,942/5,552,112 = 16.8338$, $F(.99; 1, 82) = 6.9544$. If $F^* \leq 6.9544$ conclude H_0, otherwise H_a. Conclude H_a. $(t^*)^2 = (-4.102895)^2 = 16.8338 = F^*$. $[t(.995; 82)]^2 = (2.63712)^2 = 6.9544 = F(.99; 1, 82)$. Yes.

c. $SSR = 93,462,942$, 17.03% or 0.1703

d. -0.4127

2.32. a. Full: $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$, reduced: $Y_i = \beta_0 + \varepsilon_i$

b. (1) $SSE(F) = 455,273,165$, (2) $SSE(R) = 548,736,107$,
(3) $df_F = 82$, (4) $df_R = 83$,
(5) $F^* = [(548,736,107 - 455,273,165)/1] \div [455,273,165/82] = 16.83376$, (6) If $F^* \leq F(.99; 1, 82) = 6.95442$ conclude H_0, otherwise H_a.

c. Yes

2.33. a. H_0: $\beta_0 = 7.5$, H_a: $\beta_0 \neq 7.5$

b. Full: $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$, reduced: $Y_i - 7.5 = \beta_1 X_i + \varepsilon_i$

c. Yes, $df_R - df_F = (n-1) - (n-2) = 1$

2.36 Regression model

2.38. No

2.39. a. Normal, mean $\mu_1 = 50$, standard deviation $\sigma_1 = 3$

b. Normal, mean $E\{Y_2|Y_1 = 55\} = 105.33$, standard deviation $\sigma_{2|1} = 2.40$

c. Normal, mean $E\{Y_1|Y_2 = 95\} = 47$, standard deviation $\sigma_{1|2} = 1.80$

2.40. (1) No, (2) no, (3) yes

2.41. No

2.42. b. .95285, ρ_{12}

c. $H_0 : \rho_{12} = 0$, $H_a : \rho_{12} \neq 0$. $t^* = (.95285\sqrt{13})/\sqrt{1-(.95285)^2} = 11.32194$, $t(.995; 13) = 3.012$. If $|t^*| \leq 3.012$ conclude H_0, otherwise H_a. Conclude H_a.

d. No

2.43. a. $H_0 : \rho_{12} = 0$, $H_a : \rho_{12} \neq 0$. $t^* = (.61\sqrt{82})/\sqrt{1-(.61)^2} = 6.9709$,

$t(.975; 82) = 1.993$. If $|t^*| \leq 1.993$ conclude H_0, otherwise H_a. Conclude H_a.

b. $z' = .70892$, $\sigma\{z'\} = .1111$, $z(.975) = 1.960$, $.70892 \pm 1.960(.1111)$, $.49116 \leq \zeta \leq .92668$, $.455 \leq \rho_{12} \leq .729$

c. $.207 \leq \rho_{12}^2 \leq .531$

2.44. a. $H_0 : \rho_{12} = 0$, $H_a : \rho_{12} \neq 0$. $t^* = (.87\sqrt{101})/\sqrt{1-(.87)^2} = 17.73321$, $t(.95; 101) = 1.663$. If $|t^*| \leq 1.663$ conclude H_0, otherwise H_a. Conclude H_a.

b. $z' = 1.33308$, $\sigma\{z'\} = .1$, $z(.95) = 1.645$, $1.33308 \pm 1.645(.1)$, $1.16858 \leq \zeta \leq 1.49758$, $.824 \leq \rho_{12} \leq .905$

c. $.679 \leq \rho_{12}^2 \leq .819$

2.45. a. $z' = 1.18814$, $\sigma\{z'\} = .0833$, $z(.995) = 2.576$, $1.18814 \pm 2.576(.0833)$,

$.97356 \leq \zeta \leq 1.40272$, $.750 \leq \rho_{12} \leq .886$.

b. $.563 \leq \rho_{12}^2 \leq .785$

2.46. a. 0.9454874

b. H_0 : There is no association between Y_1 and Y_2
H_a : There is an association between Y_1 and Y_2
$t^* = \dfrac{0.9454874\sqrt{13}}{\sqrt{1-(0.9454874)^2}} = 10.46803$. $t(0.995, 13) = 3.012276$. If $|t^*| \leq 3.012276$, conclude H_0, otherwise, conclude H_a. Conclude H_a.

2.47. a. -0.866064,

b. $H_0 : \rho_{12} = 0$, $H_a : \rho_{12} \neq 0$. $t^* = (-0.866064\sqrt{58})/\sqrt{1-(-0.866064)^2} = -13.19326$, $t(.975; 58) = 2.00172$. If $|t^*| \leq 2.00172$ conclude H_0, otherwise H_a. Conclude H_a.

c. -0.8657217

d. H_0 : There is no association between X and Y

H_a : There is an association between X and Y

$t^* = \dfrac{-0.8657217\sqrt{58}}{\sqrt{1-(-0.8657217)^2}} = -13.17243$. $t(0.975,\ 58) = 2.001717$. If $|t^*| \leq 2.001717$, conclude H_0, otherwise, conclude H_a. Conclude H_a.

2.48. a. -0.4127033

b. $H_0 : \rho_{12} = 0$, $H_a : \rho_{12} \neq 0$. $t^* = (-0.4127033\sqrt{82})/\sqrt{1-(-0.4127033)^2} = -4.102897$, $t(.995; 82) = 2.637123$. If $|t^*| \leq 2.637123$ conclude H_0, otherwise H_a. Conclude H_a.

2.49. a. -0.4259324

b. H_0 : There is no association between X and Y

H_a : There is an association between X and Y

$t^* = \dfrac{-0.4259324\sqrt{58}}{\sqrt{1-(-0.4259324)^2}} = -4.263013$. $t(0.995,\ 80) = 2.637123$. If $|t^*| \leq 2.637123$, conclude H_0, otherwise, conclude H_a. Conclude H_a.

2.50.
$$\begin{aligned}\sum k_i X_i &= \sum \left(\frac{X_i - \bar{X}}{\sum (X_i - \bar{X})^2} \right) X_i \\ &= \sum \frac{(X_i - \bar{X})(X_i - \bar{X})}{\sum (X_i - \bar{X})^2} \quad \text{because } \sum \frac{(X_i - \bar{X})\bar{X}}{\sum (X_i - \bar{X})^2} = 0 \\ &= \frac{\sum (X_i - \bar{X})^2}{\sum (X_i - \bar{X})^2} = 1\end{aligned}$$

2.51.
$$\begin{aligned}E\{b_0\} &= E\{\bar{Y} - b_1 \bar{X}\} \\ &= \frac{1}{n} \sum E\{Y_i\} - \bar{X} E\{b_1\} \\ &= \frac{1}{n} \sum (\beta_0 + \beta_1 X_i) - \bar{X} \beta_1 \\ &= \beta_0 + \beta_1 \bar{X} - \bar{X} \beta_1 = \beta_0\end{aligned}$$

2.52.
$$\begin{aligned}\sigma^2\{b_0\} &= \sigma^2\{\bar{Y} - b_1 \bar{X}\} \\ &= \sigma^2\{\bar{Y}\} + \bar{X}^2 \sigma^2\{b_1\} - 2\bar{X}\sigma\{\bar{Y}, b_1\} \\ &= \frac{\sigma^2}{n} + \bar{X}^2 \frac{\sigma^2}{\sum (X_i - \bar{X})^2} - 0 \\ &= \sigma^2 \left[\frac{1}{n} + \frac{\bar{X}^2}{\sum (X_i - \bar{X})^2} \right]\end{aligned}$$

2.53. a. $$L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2} (Y_i - \beta_0 - \beta_1 X_i)^2 \right] g(X_i)$$

b. Maximum likelihood estimators can be found more easily by working with $\log_e L$:

$$\log_e L = -\frac{n}{2}\log_e(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum(Y_i - \beta_0 - \beta_1 X_i)^2 + \sum\ \log_e g(X_i)$$

$$\frac{\partial\ \log_e L}{\partial\beta_0} = \frac{1}{\sigma^2}\sum(Y_i - \beta_0 - \beta_1 X_i)$$

$$\frac{\partial\ \log_e L}{\partial\beta_1} = \frac{1}{\sigma^2}\sum(Y_i - \beta_0 - \beta_1 X_i)(X_i)$$

$$\frac{\partial\ \log_e L}{\partial\sigma^2} = -\frac{n}{2}\left(\frac{1}{\sigma^2}\right) + \frac{1}{2}\sum(Y_i - \beta_0 - \beta_1 X_i)^2\left(\frac{1}{\sigma^4}\right)$$

Setting each derivative equal to zero, simplifying, and substituting the maximum likelihood estimators b_0, b_1, and $\hat{\sigma}^2$ yields:

(1) $\sum Y_i - nb_0 - b_1\sum X_i = 0$

(2) $\sum Y_i X_i - b_0\sum X_i - b_1\sum X_i^2 = 0$

(3) $\dfrac{\sum(Y_i - b_0 - b_1 X_i)^2}{n} = \hat{\sigma}^2$

Equations (1) and (2) are the same as the least squares normal equations (1.9), hence the maximum likelihood estimators b_0 and b_1 are the same as those in (1.27).

2.54. Yes, no

2.55. $SSR = \sum(\hat{Y}_i - \bar{Y})^2 = \sum[(b_0 + b_1 X_i) - \bar{Y}]^2$
$= \sum[(\bar{Y} - b_1\bar{X}) + b_1 X_i - \bar{Y}]^2$
$= b_1^2\sum(X_i - \bar{X})^2$

2.56. a. $E\{MSR\} = 1,026.36$, $E\{MSE\} = .36$

b. $E\{MSR\} = 90.36$, $E\{MSE\} = .36$

2.57. a. $Y_i - 5X_i = \beta_0 + \varepsilon_i$, $n - 1$

b. $Y_i - 2 - 5X_i = \varepsilon_i$, n

2.58. If $\rho_{12} = 0$, (2.74) becomes:

$$f(Y_1, Y_2) = \frac{1}{2\pi\sigma_1\sigma_2}\exp\left\{-\frac{1}{2}\left[\left(\frac{Y_1 - \mu_1}{\sigma_1}\right)^2 + \left(\frac{Y_2 - \mu_2}{\sigma_2}\right)^2\right]\right\}$$

$$= \frac{1}{\sqrt{2\pi}\sigma_1}\exp\left[-\frac{1}{2}\left(\frac{Y_1 - \mu_1}{\sigma_1}\right)^2\right]\cdot\frac{1}{\sqrt{2\pi}\sigma_2}\exp\left[-\frac{1}{2}\left(\frac{Y_2 - \mu_2}{\sigma_2}\right)^2\right]$$

$$= f_1(Y_1)\cdot f_2(Y_2)$$

2.59. a. $$L = \prod_{i=1}^{n}\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1 - \rho_{12}^2}}\times\exp\{-\frac{1}{2(1 - \rho_{12}^2)}[(\frac{Y_{i1} - \mu_1}{\sigma_1})^2$$
$$-2\rho_{12}(\frac{Y_{i1} - \mu_1}{\sigma_1})(\frac{Y_{i2} - \mu_2}{\sigma_2}) + (\frac{Y_{i2} - \mu_2}{\sigma_2})^2]\}$$

Maximum likelihood estimators can be found more easily by working with $\log_e L$:

$$\log_e L = -n\ \log_e 2\pi - n\ \log_e \sigma_1 - n\ \log_e \sigma_2 - \frac{n}{2}\log_e(1-\rho_{12}^2)$$
$$-\frac{1}{2(1-\rho_{12}^2)}\sum_{i=1}^{n}[(\frac{Y_{i1}-\mu_1}{\sigma_1})^2 - 2\rho_{12}(\frac{Y_{i1}-\mu_1}{\sigma_1})(\frac{Y_{i2}-\mu_2}{\sigma_2})$$
$$+(\frac{Y_{i2}-\mu_2}{\sigma_2})^2]$$
$$\frac{\partial\ \log_e L}{\partial \mu_1} = \frac{1}{\sigma_1^2(1-\rho_{12}^2)}\sum(Y_{i1}-\mu_1) - \frac{\rho_{12}}{\sigma_1\sigma_2(1-\rho_{12}^2)}\sum(Y_{i2}-\mu_2)$$
$$\frac{\partial\ \log_e L}{\partial \mu_2} = \frac{1}{\sigma_2^2(1-\rho_{12}^2)}\sum(Y_{i2}-\mu_2) - \frac{\rho_{12}}{\sigma_1\sigma_2(1-\rho_{12}^2)}\sum(Y_{i1}-\mu_1)$$
$$\frac{\partial\ \log_e L}{\partial \sigma_1} = -\frac{n}{\sigma_1} + \frac{1}{(1-\rho_{12})^2}\left[\frac{\sum(Y_{i1}-\mu_1)^2}{\sigma_1^3} - \rho_{12}\frac{\sum(Y_{i1}-\mu_1)(Y_{i2}-\mu_2)}{\sigma_1^2\sigma_2}\right]$$
$$\frac{\partial\ \log_e L}{\partial \sigma_2} = -\frac{n}{\sigma_2} + \frac{1}{(1-\rho_{12})^2}\left[\frac{\sum(Y_{i2}-\mu_2)^2}{\sigma_2^3} - \rho_{12}\frac{\sum(Y_{i1}-\mu_1)(Y_{i2}-\mu_2)}{\sigma_1\sigma_2^2}\right]$$
$$\frac{\partial\ \log_e L}{\partial \rho_{12}} = \frac{n\rho_{12}}{1-\rho_{12}^2} + \frac{1}{1-\rho_{12}^2}\sum\left(\frac{Y_{i1}-\mu_1}{\sigma_1}\right)\left(\frac{Y_{i2}-\mu_2}{\sigma_2}\right) - \frac{\rho_{12}}{(1-\rho_{12}^2)^2}$$
$$\times\sum\left[\left(\frac{Y_{i1}-\mu_1}{\sigma_1}\right)^2 - 2\rho_{12}\left(\frac{Y_{i1}-\mu_1}{\sigma_1}\right)\left(\frac{Y_{i2}-\mu_2}{\sigma_2}\right) + \left(\frac{Y_{i2}-\mu_2}{\sigma_2}\right)^2\right]$$

Setting the derivatives equal to zero, simplifying, and substituting the maximum likelihood estimators $\hat{\mu}_1$, $\hat{\mu}_2$, $\hat{\sigma}_1$, $\hat{\sigma}_2$, and $\hat{\rho}_{12}$ yields:

$$(1)\qquad \frac{1}{\hat{\sigma}_1}\sum(Y_{i1}-\hat{\mu}_1) - \frac{\hat{\rho}_{12}}{\hat{\sigma}_2}\sum(Y_{i2}-\hat{\mu}_2) = 0$$
$$(2)\qquad \frac{1}{\hat{\sigma}_2}\sum(Y_{i2}-\hat{\mu}_2) - \frac{\hat{\rho}_{12}}{\hat{\sigma}_1}\sum(Y_{i1}-\hat{\mu}_1) = 0$$
$$(3)\qquad \frac{\sum(Y_{i1}-\hat{\mu}_1)^2}{\hat{\sigma}_1^2} - \hat{\rho}_{12}\frac{\sum(Y_{i1}-\hat{\mu}_1)(Y_{i2}-\hat{\mu}_2)}{\hat{\sigma}_1\hat{\sigma}_2} - n(1-\hat{\rho}_{12}^2) = 0$$
$$(4)\qquad \frac{\sum(Y_{i2}-\hat{\mu}_2)^2}{\hat{\sigma}_2^2} - \hat{\rho}_{12}\frac{\sum(Y_{i1}-\hat{\mu}_1)(Y_{i2}-\hat{\mu}_2)}{\hat{\sigma}_1\hat{\sigma}_2} - n(1-\hat{\rho}_{12}^2) = 0$$
$$(5)\qquad n\hat{\rho}_{12}(1-\hat{\rho}_{12}^2) + (1+\hat{\rho}_{12}^2)\sum\left(\frac{Y_{i1}-\hat{\mu}_1}{\hat{\sigma}_1}\right)\left(\frac{Y_{i2}-\hat{\mu}_2}{\hat{\sigma}_2}\right)$$
$$-\hat{\rho}_{12}\sum\left[\left(\frac{Y_{i1}-\hat{\mu}_1}{\hat{\sigma}_1}\right)^2 + \left(\frac{Y_{i2}-\hat{\mu}_2}{\hat{\sigma}_2}\right)^2\right] = 0$$

Solving equations (1) and (2) yields:

$$\hat{\mu}_1 = \bar{Y}_1 \qquad\qquad \hat{\mu}_2 = \bar{Y}_2$$

Using these results in equations (3), (4), and (5), it will be found that the maximum likelihood estimators are:

$$\hat{\mu}_1 = \bar{Y}_1 \qquad\qquad \hat{\mu}_2 = \bar{Y}_2 \qquad\qquad \hat{\sigma}_1 = \sqrt{\frac{\sum(Y_{i1}-\bar{Y}_1)^2}{n}}$$
$$\hat{\sigma}_2 = \sqrt{\frac{\sum(Y_{i2}-\bar{Y}_2)^2}{n}} \qquad\qquad \hat{\rho}_{12} = \frac{\sum(Y_{i1}-\bar{Y}_1)(Y_{i2}-\bar{Y}_2)}{[\sum(Y_{i1}-\bar{Y}_1)^2]^{\frac{1}{2}}[\sum(Y_{i2}-\bar{Y}_2)^2]^{\frac{1}{2}}}$$

b. $\hat{\alpha}_{1|2} = \hat{\mu}_1 - \hat{\mu}_2 \hat{\rho}_{12} \dfrac{\hat{\sigma}_1}{\hat{\sigma}_2}$

$$= \bar{Y}_1 - \bar{Y}_2 \left[\frac{\sum(Y_{i1} - \bar{Y}_1)(Y_{i2} - \bar{Y}_2)}{[\sum(Y_{i1} - \bar{Y}_1)^2]^{\frac{1}{2}} [\sum(Y_{i2} - \bar{Y}_2)^2]^{\frac{1}{2}}} \right] \left[\frac{\sqrt{\sum(Y_{i1} - \bar{Y}_1)^2/n}}{\sqrt{\sum(Y_{i2} - \bar{Y}_2)^2/n}} \right]$$

$$= \bar{Y}_1 - \bar{Y}_2 \left[\frac{\sum(Y_{i1} - \bar{Y}_1)(Y_{i2} - \bar{Y}_2)}{\sum(Y_{i2} - \bar{Y}_2)^2} \right]$$

$$\hat{\beta}_{12} = \hat{\rho}_{12} \frac{\hat{\sigma}_1}{\hat{\sigma}_2}$$

$$= \left[\frac{\sum(Y_{i1} - \bar{Y}_1)(Y_{i2} - \bar{Y}_2)}{[\sum(Y_{i1} - \bar{Y}_1)^2]^{\frac{1}{2}} [\sum(Y_{i2} - \bar{Y}_2)^2]^{\frac{1}{2}}} \right] \left[\frac{\sqrt{\sum(Y_{i1} - \bar{Y}_1)^2/n}}{\sqrt{\sum(Y_{i2} - \bar{Y}_2)^2/n}} \right]$$

$$= \frac{\sum(Y_{i1} - \bar{Y}_1)(Y_{i2} - \bar{Y}_2)}{\sum(Y_{i2} - \bar{Y}_2)^2}$$

$$\hat{\sigma}^2_{1|2} = \hat{\sigma}^2_1(1 - \hat{\rho}^2_{12})$$

$$= \frac{\sum(Y_{i1} - \bar{Y}_1)^2}{n} \left[1 - \frac{[\sum(Y_{i1} - \bar{Y}_1)(Y_{i2} - \bar{Y}_2)]^2}{\sum(Y_{i1} - \bar{Y}_1)^2 \sum(Y_{i2} - \bar{Y}_2)^2} \right]$$

$$= \frac{\sum(Y_{i1} - \bar{Y}_1)^2}{n} - \frac{[\sum(Y_{i1} - \bar{Y}_1)(Y_{i2} - \bar{Y}_2)]^2}{n \sum(Y_{i2} - \bar{Y}_2)^2}$$

c. The equivalence is shown by letting Y_{i1} and Y_{i2} in part (b) be Y_i and X_i, respectively.

2.60. Using regression notation and letting

$$\sum(X_i - \bar{X})^2 = (n-1)s^2_X$$

and

$$\sum(Y_i - \bar{Y})^2 = (n-1)s^2_Y,$$

we have from (2.84) with $Y_{i1} = Y_i$ and $Y_{i2} = X_i$

$$b_1 = r_{12} \frac{s_Y}{s_X} \text{ since } b_1 = \left[\frac{\sum(Y_i - \bar{Y})^2}{\sum(X_i - \bar{X})^2} \right]^{\frac{1}{2}} r_{12}$$

$$SSE = \sum(Y_i - \bar{Y})^2 - \frac{[\sum(X_i - \bar{X})(Y_i - \bar{Y})]^2}{\sum(X_i - \bar{X})^2}$$

$$= (n-1)s^2_Y - r^2_{12}(n-1)s^2_Y = (n-1)s^2_Y(1 - r^2_{12})$$

$$s^2\{b_1\} = \frac{(n-1)s^2_Y(1 - r^2_{12})}{n-2} \div (n-1)s^2_X = \frac{s^2_Y(1 - r^2_{12})}{(n-2)s^2_X}$$

Hence:

$$\frac{b_1}{s\{b_1\}} = r_{12} \frac{s_Y}{s_X} \div \frac{s_Y \sqrt{1 - r^2_{12}}}{\left(\sqrt{n-2}\right) s_X} = \frac{\left(\sqrt{n-2}\right) r_{12}}{\sqrt{1 - r^2_{12}}} = t^*$$

2.61. $$\frac{SSR(Y_1)}{SSTO} = \frac{\left[\frac{\Sigma(Y_{i1}-\bar{Y}_1)(Y_{i2}-\bar{Y}_2)}{\Sigma(Y_{i1}-\bar{Y}_1)^2}\right]^2 \left[\Sigma(Y_{i1}-\bar{Y}_1)^2\right]}{\Sigma(Y_{i2}-\bar{Y}_2)^2}$$

$$= \frac{[\sum(Y_{i1}-\bar{Y}_1)(Y_{i2}-\bar{Y}_2)]^2}{\sum(Y_{i1}-\bar{Y}_1)^2\sum(Y_{i2}-\bar{Y}_2)^2}$$

$$\frac{SSR(Y_2)}{SSTO} = \frac{\left[\frac{\sum(Y_{i2}-\bar{Y}_2)(Y_{i1}-\bar{Y}_1)}{\sum(Y_{i2}-\bar{Y}_2)^2}\right]^2 \left[\sum(Y_{i2}-\bar{Y}_2)^2\right]}{\sum(Y_{i1}-\bar{Y}_1)^2}$$

$$= \frac{[\sum(Y_{i2}-\bar{Y}_2)(Y_{i1}-\bar{Y}_1)]^2}{\sum(Y_{i1}-\bar{Y}_1)^2\sum(Y_{i2}-\bar{Y}_2)^2}$$

2.62. Total population: $R^2 = 0.884067$

Number of hospital beds: $R^2 = 0.903383$

Total personal income: $R^2 = 0.898914$

2.63. Region 1: $480.0 \pm 1.66008(110.1)$, $297.2252 \leq \beta_1 \leq 662.7748$

Region 2: $299.3 \pm 1.65936(154.2)$, $43.42669 \leq \beta_1 \leq 555.1733$

Region 3: $272.22 \pm 1.65508(70.34)$, $155.8017 \leq \beta_1 \leq 388.6383$

Region 4: $508.0 \pm 1.66543(359.0)$, $-89.88937 \leq \beta_1 \leq 1105.889$

2.64. Infection rate: $R^2 = .2846$

Facilities: $R^2 = .1264$

X-ray: $R^2 = .1463$

2.65. Region 1: $1.3478 \pm 2.056(.316)$, $.6981 \leq \beta_1 \leq 1.9975$

Region 2: $.4832 \pm 2.042(.137)$, $.2034 \leq \beta_1 \leq .7630$

Region 3: $.5251 \pm 2.031(.111)$, $.2997 \leq \beta_1 \leq .7505$

Region 4: $.0173 \pm 2.145(.306)$, $-.6391 \leq \beta_1 \leq .6737$

2.66. a. $E\{Y_h\} = 36$ when $X_h = 4$, $E\{Y_h\} = 52$ when $X_h = 8$, $E\{Y_h\} = 68$ when $X_h = 12$, $E\{Y_h\} = 84$ when $X_h = 16$, $E\{Y_h\} = 100$ when $X_h = 20$

c. $E\{b_1\} = 4$, $\sigma\{b_1\} = \sqrt{\frac{25}{160}} = .3953$

d. Expected proportion is .95

Chapter 3

DIAGNOSTICS AND REMEDIAL MEASURES

3.3. b.and c.

i:	1	2	3	...	118	119	120
$\hat{Y}_i$:	2.92942	2.65763	3.20121	...	3.20121	2.73528	3.20121
e_i:	0.967581	1.22737	0.57679	...	0.71279	-0.87528	-0.25321

d.

Ascending order:	1	2	3	...	119	120
Ordered residual:	-2.74004	-1.83169	-1.24373	...	0.99441	1.22737
Expected value:	-1.59670	-1.37781	-1.25706	...	1.37781	1.59670

H_0: Normal, H_a: not normal. $r = 0.97373$. If $r \geq .987$ concludeH_0, otherwise H_a. Conclude H_a.

e. $n_1 = 65$, $\bar{d}_1 = 0.43796$, $n_2 = 55$, $\bar{d}_2 = 0.50652$, $s = 0.417275$, $t^*_{BF} = (0.43796 - 0.50652)/0.417275\sqrt{(1/65) + (1/55)} = -0.89674$, $t(.995; 18) = 2.61814$. If $|t^*_{BF}| \leq 2.61814$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

3.4.c and d.

i:	1	2	...	44	45
$\hat{Y}_i$:	29.49034	59.56084	...	59.56084	74.59608
e_i:	-9.49034	0.43916	...	1.43916	2.40392

e.

Ascending order:	1	2	...	44	45
Ordered residual:	-22.77232	-19.70183	...	14.40392	15.40392
Expected value:	-19.63272	-16.04643	...	16.04643	19.63272

H_0: Normal, H_a: not normal. $r = 0.9891$. If $r \geq .9785$ conclude H_0, otherwise H_a. Conclude H_0.

g. $SSR^* = 15,155$, $SSE = 3416.38$, $X^2_{BP} = (15,155/2) \div (3416.38/45)^2 = 1.314676$, $\chi^2(.95; 1) = 3.84$. If $X^2_{BP} \leq 3.84$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

3.5. c.

i:	1	2	3	4	5	6	7	8	9	10
e_i:	1.8	-1.2	-1.2	1.8	-.2	-1.2	-2.2	.8	.8	.8

e.

Ascending Order:	1	2	3	4	5	6	7	8	9	10
Ordered residual:	-2.2	-1.2	-1.2	-1.2	-.2	.8	.8	.8	1.8	1.8
Expected value:	-2.3	-1.5	-1.0	-.6	-.2	.2	.6	1.0	1.5	2.3

H_0: Normal, H_a: not normal. $r = .961$. If $r \geq .879$ conclude H_0, otherwise H_a. Conclude H_0.

g. $SSR^* = 6.4$, $SSE = 17.6$, $X^2_{BP} = (6.4/2) \div (17.6/10)^2 = 1.03$, $\chi^2(.90; 1) = 2.71$. If $X^2_{BP} \leq 2.71$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

Yes.

3.6.a and b.

i:	1	2	3	4	5	6
e_i:	-2.150	3.850	-5.150	-1.150	.575	2.575
$\hat{Y}_i$:	201.150	201.150	201.150	201.150	217.425	217.425

i:	7	8	9	10	11	12
e_i:	-2.425	5.575	3.300	.300	1.300	-3.700
$\hat{Y}_i$:	217.425	217.425	233.700	233.700	233.700	233.700

i:	13	14	15	16
e_i:	.025	-1.975	3.025	-3.975
$\hat{Y}_i$:	249.975	249.975	249.975	249.975

c. and d.

Ascending order:	1	2	3	4	5	6
Ordered residual:	-5.150	-3.975	-3.700	-2.425	-2.150	-1.975
Expected value	-5.720	-4.145	-3.196	-2.464	-1.841	-1.280
e_i^*:	-1.592	-1.229	-1.144	-.750	-.665	-.611

Ascending order:	7	8	9	10	11	12
Ordered residual:	-1.150	.025	.300	.575	1.300	2.575
Expected value:	-.755	-.250	.250	.755	1.280	1.841
e_i^*:	-.356	.008	.093	.178	.402	.796

Ascending order:	13	14	15	16
Ordered residual:	3.025	3.300	3.850	5.575
Expected value:	2.464	3.196	4.145	5.720
e_i^*:	.935	1.020	1.190	1.724

H_0: Normal, H_a: not normal. $r = .992$. If $r \geq .941$ conclude H_0, otherwise H_a. Conclude H_0. $t(.25; 14) = -.692$, $t(.50; 14) = 0$, $t(.75; 14) = .692$

Actual: 4/16 7/16 11/16

e. $n_1 = 8$, $\bar{d}_1 = 2.931$, $n_2 = 8$, $\bar{d}_2 = 2.194$, $s = 1.724$, $t^*_{BF} = (2.931 - 2.194)/1.724\sqrt{(1/8) + (1/8)} = .86$, $t(.975; 14) = 2.145$. If $|t^*_{BF}| \leq 2.145$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

3.7. b and c.

i:	1	2	...	59	60
e_i:	0.82324	-1.55675	...	-0.66689	8.09309
$\hat{Y}_i$:	105.17676	107.55675	...	70.66689	65.90691

d.

Ascending order:	1	2	...	59	60
Ordered residual:	-16.13683	-13.80686	...	13.95312	23.47309
Expected value:	-18.90095	-15.75218	...	15.75218	18.90095

H_0: Normal, H_a: not normal. $r = 0.9897$. If $r \geq 0.984$ conclude H_0, otherwise H_a. Conclude H_0.

e. $SSR^* = 31,833.4$, $SSE = 3,874.45$,

$X^2_{BP} = (31,833.4/2) \div (3,874.45/60)^2 = 3.817116$, $\chi^2(.99;1) = 6.63$. If $X^2_{BP} \leq 6.63$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant. Yes.

3.8. b and c.

i:	1	2	...	83	84
e_i:	591.964	1648.566	...	621.141	28.114
$\hat{Y}_i$:	7895.036	6530.434	...	6359.859	7553.886

d.

Ascending order:	1	2	...	83	84
Ordered residual:	-5278.310	-3285.062	...	4623.566	6803.265
Expected value:	-5740.725	-4874.426	...	4874.426	5740.725

H_0: Normal, H_a: not normal. $r = 0.98876$. If $r \geq 0.9854$ conclude H_0, otherwise H_a. Conclude H_0.

e. $n_1 = 8$, $\bar{d}_1 = 1751.872$, $n_2 = 76$, $\bar{d}_2 = 1927.083$, $s = 1327.772$,

$t^*_{BF} = (1751.872 - 1927.083)/1327.772\sqrt{(1/8) + (1/76)} = -0.35502$, $t(.975; 82) = 1.98932$. If $|t^*_{BF}| \leq 1.98932$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

3.10. b. 4, 4

3.11. b. $SSR^* = 330.042$, $SSE = 59.960$, $X^2_{BP} = (330.042/2) \div (59.960/9)^2 = 3.72$, $\chi^2(.95;1) = 3.84$. If $X^2_{BP} \leq 3.84$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

3.13. a. H_0: $E\{Y\} = \beta_0 + \beta_1 X$, H_a: $E\{Y\} \neq \beta_0 + \beta_1 X$

b. $SSPE = 2797.66$, $SSLF = 618.719$, $F^* = (618.719/8) \div (2797.66/35) = 0.967557$, $F(.95; 8, 35) = 2.21668$. If $F^* \leq 2.21668$ conclude H_0, otherwise H_a. Conclude H_0.

3.14. a. H_0: $E\{Y\} = \beta_0 + \beta_1 X$, H_a: $E\{Y\} \neq \beta_0 + \beta_1 X$. $SSPE = 128.750$,

$SSLF = 17.675$, $F^* = (17.675/2) \div (128.750/12) = .824$, $F(.99; 2, 12) = 6.93$. If $F^* \leq 6.93$ conclude H_0, otherwise H_a. Conclude H_0.

3.15. a. $\hat{Y} = 2.57533 - 0.32400X$

b. H_0: $E\{Y\} = \beta_0 + \beta_1 X$, H_a: $E\{Y\} \neq \beta_0 + \beta_1 X$. $SSPE = .1575$, $SSLF = 2.7675$, $F^* = (2.7675/3) \div (.1575/10) = 58.5714$, $F(.975; 3, 10) = 4.83$. If $F^* \leq 4.83$ conclude H_0, otherwise H_a. Conclude H_a.

3.16. b.

λ:	-.2	-.1	0	.1	.2
SSE:	.1235	.0651	.0390	.0440	.0813

c. $\hat{Y}' = .65488 - .19540X$

e.

i:	1	2	3	4	5	6	7	8
e_i:	-.051	.058	.007	-.083	-.057	.035	.012	.086
$\hat{Y}_i'$:	-1.104	-1.104	-1.104	-.713	-.713	-.713	-.322	-.322
Expected value:	-.047	.062	.000	-.086	-.062	.035	.008	.086

i:	9	10	11	12	13	14	15
e_i:	.046	.018	-.008	-.039	-.006	-.050	.032
$\hat{Y}_i'$:	-.322	.069	.069	.069	.459	.459	.459
Expected value:	.047	.017	-.017	-.026	-.008	-.035	.026

f. $\hat{Y} = \text{antilog}_{10}(.65488 - .19540X) = 4.51731(.63768)^X$

3.17. b.

λ:	.3	.4	.5	.6	.7
SSE:	1099.7	967.9	916.4	942.4	1044.2

c. $\hat{Y}' = 10.26093 + 1.07629X$

e.

i:	1	2	3	4	5
e_i:	-.36	.28	.31	-.15	.30
$\hat{Y}_i'$:	10.26	11.34	12.41	13.49	14.57
Expected value:	-.24	.14	.36	-.14	.24

i:	6	7	8	9	10
e_i:	-.41	.10	-.47	.47	-.07
$\hat{Y}_i'$:	15.64	16.72	17.79	18.87	19.95
Expected value:	-.36	.04	-.56	.56	-.04

f. $\hat{Y} = (10.26093 + 1.07629X)^2$

3.18. b. $\hat{Y} = 1.25470 - 3.62352X'$

d.

i:	1	2	3	...	110	111
e_i:	-1.00853	-3.32526	1.64837	...	-0.67526	0.49147
$\hat{Y}_i$:	15.28853	12.12526	10.84163	...	12.12526	15.28853
Expected value:	-0.97979	-3.10159	1.58857	...	-0.59149	0.36067

e. $\hat{Y} = 1.25470 - 3.62352\sqrt{X}$

3.21. $\sum\sum(Y_{ij}-\hat{Y}_{ij})^2 = \sum\sum\left[(Y_{ij}-\bar{Y}_j)+(\bar{Y}_j-\hat{Y}_{ij})\right]^2$
$= \sum\sum(Y_{ij}-\bar{Y}_j)^2 + \sum\sum(\bar{Y}_j-\hat{Y}_{ij})^2 + 2\sum\sum(Y_{ij}-\bar{Y}_j)(\bar{Y}_j-\hat{Y}_{ij})$

Now,$\sum\sum(Y_{ij}-\bar{Y}_j)(\bar{Y}_j-\hat{Y}_{ij})$
$= \sum\sum Y_{ij}\bar{Y}_j - \sum\sum\bar{Y}_j^2 - \sum\sum Y_{ij}\hat{Y}_{ij} + \sum\sum\bar{Y}_j\hat{Y}_{ij}$
$= \sum_j n_j\bar{Y}_j^2 - \sum_j n_j\bar{Y}_j^2 - \sum_j \hat{Y}_{ij}n_j\bar{Y}_j + \sum_j n_j\bar{Y}_j\hat{Y}_{ij} = 0$

since $\hat{Y}_{ij} = b_0 + b_1X_j$ is independent of i.

3.22. $E\{MSPE\} = E\left\{\dfrac{\sum\sum(Y_{ij}-\bar{Y}_j)^2}{n-c}\right\} = \dfrac{1}{n-c}\sum E\{(n_j-1)s_j^2\}$
$= \dfrac{1}{n-c}\sum E\{\sigma^2\chi^2(n_j-1)\} = \dfrac{\sigma^2}{n-c}\sum(n_j-1) = \sigma^2$

3.23. Full: $Y_{ij} = \mu_j + \varepsilon_{ij}$, reduced: $Y_{ij} = \beta_1X_j + \varepsilon_{ij}$
$df_F = 20-10 = 10$, $df_R = 20-1 = 19$

3.24. a. $\hat{Y} = 48.66667 + 2.33333X$

i:	1	2	3	4	5	6	7	8
e_i:	2.6667	-.3333	-.3333	-1.0000	-4.0000	-7.6667	13.3333	-2.6667

b. $\hat{Y} = 53.06796 + 1.62136X$

c. $\hat{Y}_h = 72.52428$, $s\{\text{pred}\} = 3.0286$, $t(.995;5) = 4.032$, $72.52428 \pm 4.032(3.0286)$, $60.31296 \le Y_{h(\text{new})} \le 84.73560$, yes

3.27. b. $\hat{Y} = 6.84922 + .60975X$

$X_h = 6.5$: $\hat{Y}_h = 10.81260$, $s\{\text{pred}\} = 1.2583$, $t(.975;109) = 1.982$, $10.81260 \pm 1.982(1.2583)$, $8.31865 \le Y_{h(\text{new})} \le 13.30655$

$X_h = 5.9$: $\hat{Y}_h = 10.44675$, $s\{\text{pred}\} = 1.2512$, $10.44675 \pm 1.982(1.2512)$, $7.96687 \le Y_{h(\text{new})} \le 12.92663$

Yes

3.29. a.

	Median	
Band	X	Y
1	2	23.5
2	4	57
3	5	81.5
4	7	111

b. $F(.90;2,43) = 2.43041$, $W = 2.204727$

$X_h = 2$: $29.4903 \pm 2.204727(2.00609)$, $25.067 \le E\{Y_h\} \le 33.913$

$X_h = 4$: $59.5608 \pm 2.204727(1.43307)$, $56.401 \le E\{Y_h\} \le 62.720$

$X_h = 5$: $74.5961 \pm 2.204727(1.32983)$, $71.664 \le E\{Y_h\} \le 77.528$

$X_h = 7$: $104.667 \pm 2.204727(1.6119)$, $101.113 \leq E\{Y_h\} \leq 108.221$

No

c.

Neighborhood	X_c	$\hat{Y}_c$
1	2	27.000
2	3	43.969
3	4	60.298
4	5	77.905
5	6	93.285
6	7	107.411

3.30. a.

	Median	
Band	X	Y
1	0.5	116.5
2	2.5	170.0
3	4.5	226.5
4	6.5	291.5
5	8.5	384.5

b.

Neighborhood	X_c	$\hat{Y}_c$
1	1	131.67
2	2	158.33
3	3	187.00
4	4	210.33
5	5	245.33
6	6	271.67
7	7	319.00

c. $F(.95; 2, 8) = 4.46$, $W = 2.987$

$X_h = 1$: $124.061 \pm 2.987(7.4756)$, $101.731 \leq E\{Y_h\} \leq 146.391$

$X_h = 2$: $156.558 \pm 2.987(6.2872)$, $137.778 \leq E\{Y_h\} \leq 175.338$

$X_h = 3$: $189.055 \pm 2.987(5.3501)$, $173.074 \leq E\{Y_h\} \leq 205.036$

$X_h = 4$: $221.552 \pm 2.987(4.8137)$, $207.174 \leq E\{Y_h\} \leq 235.931$

$X_h = 5$: $254.049 \pm 2.987(4.8137)$, $239.671 \leq E\{Y_h\} \leq 268.428$

$X_h = 6$: $286.546 \pm 2.987(5.3501)$, $270.565 \leq E\{Y_h\} \leq 302.527$

$X_h = 7$: $319.043 \pm 2.987(6.2872)$, $300.263 \leq E\{Y_h\} \leq 337.823$

Yes

Chapter 4

SIMULTANEOUS INFERENCES AND OTHER TOPICS IN REGRESSION ANALYSIS

4.1. No, no

4.2. 90 percent

4.3.
a. Opposite directions, negative tilt

b. $B = t(.9875; 43) = 2.32262$, $b_0 = -0.580157$, $s\{b_0\} = 2.80394$, $b_1 = 15.0352$, $s\{b_1\} = 0.483087$

$-0.580157 \pm 2.32262(2.80394)$ $\quad -7.093 \leq \beta_0 \leq 5.932$

$15.0352 \pm 2.32262(0.483087)$ $\quad 13.913 \leq \beta_1 \leq 16.157$

c. Yes

4.4.
a. Opposite directions, negative tilt

b. $B = t(.9975; 8) = 3.833$, $b_0 = 10.2000$, $s\{b_0\} = .6633$, $b_1 = 4.0000$, $s\{b_1\} = .4690$

$10.2000 \pm 3.833(.6633)$ $\quad 7.658 \leq \beta_0 \leq 12.742$

$4.0000 \pm 3.833(.4690)$ $\quad 2.202 \leq \beta_1 \leq 5.798$

4.5.
a. $B = t(.975; 14) = 2.145$, $b_0 = 168.6000$, $s\{b_0\} = 2.6570$, $b_1 = 2.0344$, $s\{b_1\} = .0904$

$168.6000 \pm 2.145(2.6570)$ $\quad 162.901 \leq \beta_0 \leq 174.299$

$2.0344 \pm 2.145(.0904)$ $\quad 1.840 \leq \beta_1 \leq 2.228$

b. Negatively, no

4.6.
a. $B = t(.9975; 14) = 2.91839$, $b_0 = 156.347$, $s\{b_0\} = 5.51226$, $b_1 = -1.190$,$s\{b_1\} = 0.0901973$

$156.347 \pm 2.91839(5.51226)$ $\quad 140.260 \leq \beta_0 \leq 172.434$

$-1.190 \pm 2.91839(0.0901973)$ $\quad -1.453 \leq \beta_1 \leq -0.927$

b. Opposite directions

c. No

4.7. a. $F(.90; 2, 43) = 2.43041$, $W = 2.204727$

$X_h = 3$: $44.5256 \pm 2.204727(1.67501)$ $40.833 \leq E\{Y_h\} \leq 48.219$

$X_h = 5$: $74.5961 \pm 2.204727(1.32983)$ $71.664 \leq E\{Y_h\} \leq 77.528$

$X_h = 7$: $104.667 \pm 2.204727(1.6119)$ $101.113 \leq E\{Y_h\} \leq 108.221$

b. $F(.90; 2, 43) = 2.43041$, $S = 2.204727$; $B = t(.975;\ 43) = 2.01669$; Bonferroni

c. $X_h = 4$: $59.5608 \pm 2.01669(9.02797)$ $41.354 \leq Y_{h(\text{new})} \leq 77.767$

$X_h = 7$: $104.667 \pm 2.01669(9.05808)$ $86.3997 \leq Y_{h(\text{new})} \leq 122.934$

4.8. a. $F(.95; 2, 8) = 4.46$, $W = 2.987$

$X_h = 0$: $10.2000 \pm 2.987(.6633)$ $8.219 \leq E\{Y_h\} \leq 12.181$

$X_h = 1$: $14.2000 \pm 2.987(.4690)$ $12.799 \leq E\{Y_h\} \leq 15.601$

$X_h = 2$: $18.2000 \pm 2.987(.6633)$ $16.219 \leq E\{Y_h\} \leq 20.181$

b. $B = t(.99167; 8) = 3.016$, yes

c. $F(.95; 3, 8) = 4.07$, $S = 3.494$

$X_h = 0$: $10.2000 \pm 3.494(1.6248)$ $4.523 \leq Y_{h(\text{new})} \leq 15.877$

$X_h = 1$: $14.2000 \pm 3.494(1.5556)$ $8.765 \leq Y_{h(\text{new})} \leq 19.635$

$X_h = 2$: $18.2000 \pm 3.494(1.6248)$ $12.523 \leq Y_{h(\text{new})} \leq 23.877$

d. $B = 3.016$, yes

4.9. a. $B = t(.9833; 14) = 2.360$

$X_h = 20$: $209.2875 \pm 2.360(1.0847)$ $206.727 \leq E\{Y_h\} \leq 211.847$

$X_h = 30$: $229.6312 \pm 2.360(0.8285)$ $227.676 \leq E\{Y_h\} \leq 231.586$

$X_h = 40$: $249.9750 \pm 2.360(1.3529)$ $246.782 \leq E\{Y_h\} \leq 253.168$

b. $F(.90; 2, 14) = 2.737$, $W = 2.340$, no

c. $F(.90; 2, 14) = 2.737$, $S = 2.340$, $B = t(.975;\ 14) = 2.145$

$X_h = 30$: $229.6312 \pm 2.145(3.3385)$ $222.470 \leq Y_{h(\text{new})} \leq 236.792$

$X_h = 40$: $249.9750 \pm 2.145(3.5056)$ $242.455 \leq Y_{h(\text{new})} \leq 257.495$

4.10. a. $F(.95; 2, 58) = 3.15593$, $W = 2.512342$

$X_h = 45$: $102.797 \pm 2.512342(1.71458)$ $98.489 \leq E\{Y_h\} \leq 107.105$

$X_h = 55$: $90.8968 \pm 2.512342(1.1469)$ $88.015 \leq E\{Y_h\} \leq 93.778$

$X_h = 65$: $78.9969 \pm 2.512342(1.14808)$ $76.113 \leq E\{Y_h\} \leq 81.881$

b. $B = t(.99167; 58) = 2.46556$, no

c. $B = 2.46556$

$X_h = 48$: $99.2268 \pm 2.46556(8.31158)$ $78.734 \leq Y_{h(\text{new})} \leq 119.720$

$X_h = 59$: $86.1368 \pm 2.46556(8.24148)$ $65.817 \leq Y_{h(\text{new})} \leq 106.457$

$X_h = 74$: $68.2869 \pm 2.46556(8.33742)$ $\quad 47.730 \leq Y_{h(\text{new})} \leq 88.843$

d. Yes, yes

4.12. a. $\hat{Y} = 18.0283X$

c. H_0: $\beta_1 = 17.50$, H_a: $\beta_1 \neq 17.50$. $MSE = 20.3113$, $s\{b_1\} = .07948$, $t^* = (18.0283-17.50)/.07948 = 6.65$, $t(.99; 11) = 2.718$. If $|t^*| \leq 2.718$ conclude H_0, otherwise H_a. Conclude H_a.

d. $\hat{Y}_h = 180.283$, $s\{\text{pred}\} = 4.576$, $180.283 \pm 2.718(4.576)$, $167.845 \leq Y_{h(\text{new})} \leq 192.721$

4.13. a.

i:	1	2	3	4	5	6
e_i:	1.802	-3.340	10.717	-2.283	-2.396	-4.708

i:	7	8	9	10	11	12
e_i:	-.849	6.292	-.510	-3.283	2.887	-1.170

No

b. H_0: $E\{Y\} = \beta_1 X$, H_a: $E\{Y\} \neq \beta_1 X$. $SSLF = 40.924$, $SSPE = 182.500$, $F^* = (40.924/8) \div (182.500/3) = .084$, $F(.99; 8, 3) = 27.5$. If $F^* \leq 27.5$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .997$

4.14. a. $\hat{Y} = 0.121643X$

b. $s\{b_1\} = 0.00263691$, $t(.975; 19) = 1.9801$, $0.121643 \pm 1.9801(0.00263691)$, $0.116 \leq \beta_1 \leq 0.127$

c. $\hat{Y}_h = 3.64929$, $s\{\hat{Y}_h\} = 0.0791074$, $3.64929 \pm 1.9801(0.0791074)$, $3.493 \leq E\{Y_h\} \leq 3.806$

4.15. b.

i:	1	2	...	119	120
e_i:	1.3425	2.1820	...	-0.0863	-0.4580

No

c. H_0: $E\{Y\} = \beta_1 X$, H_a: $E\{Y\} \neq \beta_1 X$. $SSLF = 23.3378$, $SSPE = 39.3319$, $F^* = (23.3378/20) \div (39.3319/99) = 2.93711$, $F(.995; 20, 99) = 2.22939$. If $F^* \leq 2.22939$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0.0002$

4.16. a. $\hat{Y} = 14.9472X$

b. $s\{b_1\} = 0.226424$, $t(.95; 44) = 1.68023$, $14.9472 \pm 1.68023(0.226424)$, $14.567 \leq \beta_1 \leq 15.328$

c. $\hat{Y}_h = 89.6834$, $s\{\text{pred}\} = 8.92008$, $89.6834 \pm 1.68023(8.92008)$, $74.696 \leq Y_{h(\text{new})} \leq 104.671$

4.17. b.

i:	1	2	...	44	45
e_i:	-9.89445	0.21108	...	1.2111	2.2639

No

c. H_0: $E\{Y\} = \beta_1 X$, H_a: $E\{Y\} \neq \beta_1 X$. $SSLF = 622.12$, $SSPE = 2797.66$, $F^* = (622.12/9) \div (2797.66/35) = 0.8647783$, $F(.99; 9, 35) = 2.96301$. If $F^* \leq 2.96301$ conclude H_0, otherwise H_a. Conclude H_0. P-value = 0.564

4.18. No

4.19. a. $\hat{X}_{h(\text{new})} = 33.11991$, $t(.95; 118) = 1.657870$, $s\{\text{pred}X\} = 16.35037$,

$33.11991 \pm 1.657870(16.35037)$, $6.013 \leq X_{h(\text{new})}) \leq 60.227$

b. No, $0.297453 > .1$

4.20. a. $\hat{X}_{h(\text{new})} = 34.1137$, $t(.995; 14) = 2.977$, $s\{\text{pred}X\} = 1.6610$,

$34.1137 \pm 2.977(1.6610)$, $29.169 \leq X_{h(\text{new})} \leq 39.058$

b. Yes, $.0175 < .1$

4.21. Yes, no

4.22. Let $\bar{A}_3$ denote the event that statement 3 is correct and $\bar{B}$ the event $\bar{A}_1 \cap \bar{A}_2$. Then by (4.2a):

$$P(\bar{B} \cap \bar{A}_3) = P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3) \geq 1 - 2\alpha - \alpha = 1 - 3\alpha$$

4.23. From (4.13) it follows at once that:

$$\sum X_i(Y_i - b_1 X_i) = \sum X_i e_i = 0$$

4.24. From Exercise 1.41c, we have that $E\{b_1\} = \beta_1$. Hence:

$$E\{\hat{Y}\} = E\{b_1 X\} = X E\{b_1\} = \beta_1 X = E\{Y\}.$$

4.25. $\sigma^2\{\hat{Y}_h\} = \sigma^2\{b_1 X_h\} = X_h^2 \sigma^2\{b_1\} = X_h^2(\sigma^2 / \sum X_i^2)$; hence, $s^2\{\hat{Y}_h\} = X_h^2(MSE/\sum X_i^2)$.

4.26. a. $B = t(.9875; 438) = 2.24913$, $b_0 = -110.635$, $s\{b_0\} = 34.7460$,

$b_1 = 0.00279542$, $s\{b_1\} = 0.0000483694$

$-110.635 \pm 2.24913(34.7460)$ $\quad -188.783 \leq \beta_0 \leq -32.487$

$0.00279542 \pm 2.24913(0.00004837)$ $\quad 0.00269 \leq \beta_1 \leq 0.0029$

b. Yes

c. $F(.90; 2, 438) = 2.31473$, $W = 2.151618$;
$B = t(.9833; 438) = 2.13397$;
Bonferroni

d. $X_h = 500$: $-109.237 \pm 2.13397(34.7328)$ $\; -183.356 \leq E\{Y_h\} \leq -35.118$

$X_h = 1,000$: $-107.839 \pm 2.13397(34.7196)$ $\quad -181.930 \leq E\{Y_h\} \leq -33.748$

$X_h = 5,000$: $-96.6577 \pm 2.13397(34.6143)$ $\quad -170.524 \leq E\{Y_h\} \leq -22.792$

4.27. a. $B = t(.975; 111) = 1.982$, $b_0 = 6.3368$, $s\{b_0\} = .5213$, $b_1 = .7604$, $s\{b_1\} = .1144$

$6.3368 \pm 1.982(.5213) \qquad 5.304 \leq \beta_0 \leq 7.370$

$0.7604 \pm 1.982(.1144) \qquad 0.534 \leq \beta_1 \leq 0.987$

b. No

c. $F(.95; 2, 111) = 3.08$, $W = 2.482$; $B = t(.99375; 111) = 2.539$; Working-Hotelling

d. $X_h = 2$: $7.858 \pm 2.482(.3098) \qquad 7.089 \leq E\{Y_h\} \leq 8.627$

$X_h = 3$: $8.618 \pm 2.482(.2177) \qquad 8.078 \leq E\{Y_h\} \leq 9.158$

$X_h = 4$: $9.378 \pm 2.482(.1581) \qquad 8.986 \leq E\{Y_h\} \leq 9.770$

$X_h = 5$: $10.139 \pm 2.482(.1697) \qquad 9.718 \leq E\{Y_h\} \leq 10.560$

Chapter 5

MATRIX APPROACH TO SIMPLE LINEAR REGRESSION ANALYSIS

5.1. (1) $\begin{bmatrix} 2 & 7 \\ 3 & 10 \\ 5 & 13 \end{bmatrix}$ (2) $\begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$ (3) $\begin{bmatrix} 23 & 24 & 1 \\ 36 & 40 & 2 \\ 49 & 56 & 3 \end{bmatrix}$ (4) $\begin{bmatrix} 13 & 17 & 22 \\ 20 & 26 & 34 \\ 27 & 35 & 46 \end{bmatrix}$

(5) $\begin{bmatrix} 9 & 26 \\ 26 & 76 \end{bmatrix}$

5.2. (1) $\begin{bmatrix} 5 & 9 \\ 11 & 11 \\ 10 & 8 \\ 6 & 12 \end{bmatrix}$ (2) $\begin{bmatrix} -1 & -7 \\ -5 & -1 \\ 0 & 6 \\ 2 & 4 \end{bmatrix}$ (3) $\begin{bmatrix} 58 & 80 \end{bmatrix}$

(4) $\begin{bmatrix} 14 & 22 & 11 & 8 \\ 49 & 54 & 20 & 26 \\ 71 & 82 & 32 & 38 \\ 76 & 80 & 28 & 40 \end{bmatrix}$ (5) $\begin{bmatrix} 63 & 94 \\ 55 & 73 \end{bmatrix}$

5.3. (1) $\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} - \begin{bmatrix} \hat{Y}_1 \\ \hat{Y}_2 \\ \hat{Y}_3 \\ \hat{Y}_4 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}$

(2) $\begin{bmatrix} X_1 & X_2 & X_3 & X_4 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$

5.4. (1) 503.77 (2) $\begin{bmatrix} 5 & 0 \\ 0 & 160 \end{bmatrix}$ (3) $\begin{bmatrix} 49.7 \\ -39.2 \end{bmatrix}$

5.5. (1) 1,259 (2) $\begin{bmatrix} 6 & 17 \\ 17 & 55 \end{bmatrix}$ (3) $\begin{bmatrix} 81 \\ 261 \end{bmatrix}$

5.6. (1) 2,194 (2) $\begin{bmatrix} 10 & 10 \\ 10 & 20 \end{bmatrix}$ (3) $\begin{bmatrix} 142 \\ 182 \end{bmatrix}$

5.7. (1) 819,499 (2) $\begin{bmatrix} 16 & 448 \\ 448 & 13,824 \end{bmatrix}$ (3) $\begin{bmatrix} 3,609 \\ 103,656 \end{bmatrix}$

5.8. a. Yes

b. 2

c. 0

5.9. a. Yes

b. Yes

c. 2

d. 0

5.10. $\mathbf{A}^{-1} = \begin{bmatrix} -.1 & .4 \\ .3 & -.2 \end{bmatrix}$ $\mathbf{B}^{-1} = \begin{bmatrix} .10870 & -.08696 & .10870 \\ .34783 & .02174 & -.15217 \\ -.23913 & .14130 & .01087 \end{bmatrix}$

5.11. $\begin{bmatrix} .33088 & -.15441 & -.03676 \\ .13971 & -.19853 & .09559 \\ -.26471 & .32353 & .02941 \end{bmatrix}$

5.12. $\begin{bmatrix} .2 & 0 \\ 0 & .00625 \end{bmatrix}$

5.13. $\begin{bmatrix} 1.34146 & -.41463 \\ -.41463 & .14634 \end{bmatrix}$

5.14. a. $\begin{bmatrix} 4 & 7 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 25 \\ 12 \end{bmatrix}$

b. $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 4.5 \\ 1 \end{bmatrix}$

5.15. a. $\begin{bmatrix} 5 & 2 \\ 23 & 7 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 8 \\ 28 \end{bmatrix}$

b. $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$

5.16. $\begin{bmatrix} \hat{Y}_1 \\ \hat{Y}_2 \\ \hat{Y}_3 \\ \hat{Y}_4 \\ \hat{Y}_5 \end{bmatrix} = \bar{Y} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + b_1 \begin{bmatrix} X_1 - \bar{X} \\ X_2 - \bar{X} \\ X_3 - \bar{X} \\ X_4 - \bar{X} \\ X_5 - \bar{X} \end{bmatrix}$

5.17. a. $\begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix}$

b. $\mathbf{E}\left\{\begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix}\right\} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} E\{Y_1\} \\ E\{Y_2\} \\ E\{Y_3\} \end{bmatrix} = \begin{bmatrix} E\{Y_1\}+E\{Y_2\}+E\{Y_3\} \\ E\{Y_1\}-E\{Y_2\} \\ E\{Y_1\}-E\{Y_2\}-E\{Y_3\} \end{bmatrix}$

c. $\boldsymbol{\sigma}^2\{\mathbf{W}\} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} \sigma^2\{Y_1\} & \sigma\{Y_1,Y_2\} & \sigma\{Y_1,Y_3\} \\ \sigma\{Y_2,Y_1\} & \sigma^2\{Y_2\} & \sigma\{Y_2,Y_3\} \\ \sigma\{Y_3,Y_1\} & \sigma\{Y_3,Y_2\} & \sigma^2\{Y_3\} \end{bmatrix}$

$$\times \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Using the notation σ_1^2 for $\sigma^2\{Y_1\}$, σ_{12} for $\sigma\{Y_1,Y_2\}$, etc., we obtain:

$\sigma^2\{W_1\} = \sigma_1^2+\sigma_2^2+\sigma_3^2+2\sigma_{12}+2\sigma_{13}+2\sigma_{23}$

$\sigma^2\{W_2\} = \sigma_1^2+\sigma_2^2-2\sigma_{12}$

$\sigma^2\{W_3\} = \sigma_1^2+\sigma_2^2+\sigma_3^2-2\sigma_{12}-2\sigma_{13}+2\sigma_{23}$

$\sigma\{W_1,\,W_2\} = \sigma_1^2-\sigma_2^2+\sigma_{13}-\sigma_{23}$

$\sigma\{W_1,\,W_3\} = \sigma_1^2-\sigma_2^2-\sigma_3^2-2\sigma_{23}$

$\sigma\{W_2,\,W_3\} = \sigma_1^2+\sigma_2^2-2\sigma_{12}-\sigma_{13}+\sigma_{23}$

5.18. a. $\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix}$

b. $\mathbf{E}\left\{\begin{bmatrix} W_1 \\ W_2 \end{bmatrix}\right\} = \begin{bmatrix} \frac{1}{4}[E\{Y_1\}+E\{Y_2\}+E\{Y_3\}+E\{Y_4\}] \\ \frac{1}{2}[E\{Y_1\}+E\{Y_2\}-E\{Y_3\}-E\{Y_4\}] \end{bmatrix}$

c. $\boldsymbol{\sigma}^2\{\mathbf{W}\} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \sigma^2\{Y_1\} & \sigma\{Y_1,Y_2\} & \sigma\{Y_1,Y_3\} & \sigma\{Y_1,Y_4\} \\ \sigma\{Y_2,Y_1\} & \sigma^2\{Y_2\} & \sigma\{Y_2,Y_3\} & \sigma\{Y_2,Y_4\} \\ \sigma\{Y_3,Y_1\} & \sigma\{Y_3,Y_2\} & \sigma^2\{Y_3\} & \sigma\{Y_3,Y_4\} \\ \sigma\{Y_4,Y_1\} & \sigma\{Y_4,Y_2\} & \sigma\{Y_4,Y_3\} & \sigma^2\{Y_4\} \end{bmatrix}$

$$\times \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} \end{bmatrix}$$

Using the notation σ_1^2 for $\sigma^2\{Y_1\}, \sigma_{12}$ for $\sigma\{Y_1,Y_2\}$, etc., we obtain:

$\sigma^2\{W_1\} = \frac{1}{16}(\sigma_1^2++\sigma_2^2+\sigma_3^2+\sigma_4^2+2\sigma_{12}+2\sigma_{13}+2\sigma_{14}+2\sigma_{23}+2\sigma_{24}+2\sigma_{34})$

$\sigma^2\{W_2\} = \frac{1}{4}(\sigma_1^2+\sigma_2^2+\sigma_3^2+\sigma_4^2+2\sigma_{12}-2\sigma_{13}-2\sigma_{14}-2\sigma_{23}-2\sigma_{24}+2\sigma_{34})$

$\sigma\{W_1,\,W_2\} = \frac{1}{8}(\sigma_1^2+\sigma_2^2-\sigma_3^2-\sigma_4^2+2\sigma_{12}-2\sigma_{34})$

5.19. $\begin{bmatrix} 3 & 5 \\ 5 & 17 \end{bmatrix}$

5.20. $\begin{bmatrix} 7 & -4 \\ -4 & 8 \end{bmatrix}$

5.21. $5Y_1^2 + 4Y_1Y_2 + Y_2^2$

5.22. $Y_1^2 + 3Y_2^2 + 9Y_3^2 + 8Y_1Y_3$

5.23. a. (1) $\begin{bmatrix} 9.940 \\ -.245 \end{bmatrix}$ (2) $\begin{bmatrix} -.18 \\ .04 \\ .26 \\ .08 \\ -.20 \end{bmatrix}$ (3) 9.604 (4) .148

(5) $\begin{bmatrix} .00987 & 0 \\ 0 & .000308 \end{bmatrix}$ (6) 11.41 (7) .02097

c. $\begin{bmatrix} .6 & .4 & .2 & 0 & -.2 \\ .4 & .3 & .2 & .1 & 0 \\ .2 & .2 & .2 & .2 & .2 \\ 0 & .1 & .2 & .3 & .4 \\ -.2 & 0 & .2 & .4 & .6 \end{bmatrix}$

d. $\begin{bmatrix} .01973 & -.01973 & -.00987 & .00000 & .00987 \\ -.01973 & .03453 & -.00987 & -.00493 & .00000 \\ -.00987 & -.00987 & .03947 & -.00987 & -.00987 \\ .00000 & -.00493 & -.00987 & .03453 & -.01973 \\ .00987 & .00000 & -.00987 & -.01973 & .01973 \end{bmatrix}$

5.24. a. (1) $\begin{bmatrix} .43902 \\ 4.60976 \end{bmatrix}$ (2) $\begin{bmatrix} -2.8781 \\ -.0488 \\ .3415 \\ .7317 \\ -1.2683 \\ 3.1219 \end{bmatrix}$ (3) 145.2073 (4) 20.2927

(5) $\begin{bmatrix} 6.8055 & -2.1035 \\ -2.1035 & .7424 \end{bmatrix}$ (6) 18.878 (7) 6.9290

b. (1) -2.1035 (2) 6.8055 (3) .8616

c. $\begin{bmatrix} .366 & -.146 & .024 & .195 & .195 & .366 \\ -.146 & .658 & .390 & .122 & .122 & -.146 \\ .024 & .390 & .268 & .146 & .146 & .024 \\ .195 & .122 & .146 & .171 & .171 & .195 \\ .195 & .122 & .146 & .171 & .171 & .195 \\ .366 & -.146 & .024 & .195 & .195 & .366 \end{bmatrix}$

d. $\begin{bmatrix} 3.217 & .742 & -.124 & -.990 & -.990 & -1.856 \\ .742 & 1.732 & -1.980 & -.619 & -.619 & .742 \\ -.124 & -1.980 & 3.712 & -.742 & -.742 & -.124 \\ -.990 & -.619 & -.742 & 4.207 & -.866 & -.990 \\ -.990 & -.619 & -.742 & -.866 & 4.207 & -.990 \\ -1.856 & .742 & -.124 & -.990 & -.990 & 3.127 \end{bmatrix}$

5.25. a. (1) $\begin{bmatrix} .2 & -.1 \\ -.1 & .1 \end{bmatrix}$ (2) $\begin{bmatrix} 10.2 \\ 4.0 \end{bmatrix}$ (3) $\begin{bmatrix} 1.8 \\ -1.2 \\ -1.2 \\ 1.8 \\ -.2 \\ -1.2 \\ -2.2 \\ .8 \\ .8 \\ .8 \end{bmatrix}$

(4) $\begin{bmatrix} .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\ .1 & .2 & 0 & .2 & -.1 & .1 & .2 & .1 & 0 & .2 \\ .1 & 0 & .2 & 0 & .3 & .1 & 0 & .1 & .2 & 0 \\ .1 & .2 & 0 & .2 & -.1 & .1 & .2 & .1 & 0 & .2 \\ .1 & -.1 & .3 & -.1 & .5 & .1 & -.1 & .1 & .3 & -.1 \\ .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\ .1 & .2 & 0 & .2 & -.1 & .1 & .2 & .1 & 0 & .2 \\ .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\ .1 & .0 & .2 & 0 & .3 & .1 & 0 & .1 & .2 & 0 \\ .1 & .2 & 0 & .2 & -.1 & .1 & .2 & .1 & .0 & .2 \end{bmatrix}$

(5) 17.60 (6) $\begin{bmatrix} .44 & -.22 \\ -.22 & .22 \end{bmatrix}$ (7) 18.2 (8) .44

b. (1) .22 (2) $-.22$ (3) .663

c. $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .1 & -.1 & .1 & -.2 & 0 & .1 & 0 & -.1 & .1 \\ 0 & -.1 & .1 & -.1 & .2 & 0 & -.1 & 0 & .1 & -.1 \\ 0 & .1 & -.1 & .1 & -.2 & 0 & .1 & 0 & -.1 & .1 \\ 0 & -.2 & .2 & -.2 & .4 & 0 & -.2 & 0 & .2 & -.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .1 & -.1 & .1 & -.2 & 0 & .1 & 0 & -.1 & .1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.1 & .1 & -.1 & .2 & 0 & -.1 & 0 & .1 & -.1 \\ 0 & .1 & -.1 & .1 & -.2 & 0 & .1 & 0 & -.1 & .1 \end{bmatrix}$

5.26. a. (1) $\begin{bmatrix} .675000 & -.021875 \\ -.021875 & .00078125 \end{bmatrix}$ (2) $\begin{bmatrix} 168.600000 \\ 2.034375 \end{bmatrix}$

$$(3)\ \begin{bmatrix} 201.150 \\ 201.150 \\ 201.150 \\ 201.150 \\ 217.425 \\ 217.425 \\ 217.425 \\ 217.425 \\ 233.700 \\ 233.700 \\ 233.700 \\ 233.700 \\ 249.975 \\ 249.975 \\ 249.975 \\ 249.975 \end{bmatrix}$$

$$(4)\ \begin{bmatrix} .175 & .175 & .175 & \cdots & -.050 & -.050 & -.050 \\ .175 & .175 & .175 & \cdots & -.050 & -.050 & -.050 \\ .175 & .175 & .175 & \cdots & -.050 & -.050 & -.050 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ -.050 & -.050 & -.050 & \cdots & .175 & .175 & .175 \\ -.050 & -.050 & -.050 & \cdots & .175 & .175 & .175 \\ -.050 & -.050 & -.050 & \cdots & .175 & .175 & .175 \end{bmatrix}$$

$$(5)\ 146.425 \qquad (6)\ \begin{bmatrix} 7.0598 & -.2288 \\ -.2288 & .008171 \end{bmatrix} \qquad (7)\ 11.1453$$

b. (1) 7.0598 (2) $-.2288$ (3) .0904

$$\text{c.}\quad \begin{bmatrix} .825 & -.175 & -.175 & \cdots & .050 & .050 & .050 \\ -.175 & .825 & -.175 & \cdots & .050 & .050 & .050 \\ -.175 & -.175 & .825 & \cdots & .050 & .050 & .050 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ .050 & .050 & .050 & \cdots & .825 & -.175 & -.175 \\ .050 & .050 & .050 & \cdots & -.175 & .825 & -.175 \\ .050 & .050 & .050 & \cdots & -.175 & -.175 & .825 \end{bmatrix}$$

5.27.
$$\mathbf{E}\left\{\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}\right\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \mathbf{0}$$

5.28. Let

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$$

Then by (5.60) $\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} = \sum X_i Y_i / \sum X_i^2$.

5.29 $\mathbf{E}\{\mathbf{b}\} = \mathbf{E}\{(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}\} = (\mathbf{X}'\mathbf{X})^{-\mathbf{1}}\mathbf{X}'\mathbf{E}\{\mathbf{Y}\}$

$$= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{X}\boldsymbol{\beta} = \boldsymbol{\beta}$$

5.30. $\hat{Y}_h = \mathbf{X}_h'\mathbf{b}$ is a scalar, hence it equals its transpose. By (5.32) then, $\mathbf{X}_h'\mathbf{b} = (\mathbf{X}_h'\mathbf{b})' = \mathbf{b}'\mathbf{X}_h$.

5.31. $\boldsymbol{\sigma}^2\{\hat{\mathbf{Y}}\} = \mathbf{H}\boldsymbol{\sigma}^2\{\mathbf{Y}\}\mathbf{H}'$ [by (5.46)]

$$= \mathbf{H}\sigma^2\mathbf{I}\mathbf{H} \qquad \text{(since } \mathbf{H} \text{ is symmetric)}$$

$$= \sigma^2\mathbf{H} \qquad \text{(since } \mathbf{H}\mathbf{H} = \mathbf{H}\text{)}$$

Chapter 6

MULTIPLE REGRESSION – I

6.1. a. $\mathbf{X} = \begin{bmatrix} 1 & X_{11} & X_{11}X_{12} \\ 1 & X_{21} & X_{21}X_{22} \\ 1 & X_{31} & X_{31}X_{32} \\ 1 & X_{41} & X_{41}X_{42} \end{bmatrix}$ $\boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$

b. $\mathbf{X} = \begin{bmatrix} 1 & X_{11} & X_{12} \\ 1 & X_{21} & X_{22} \\ 1 & X_{31} & X_{32} \\ 1 & X_{41} & X_{42} \end{bmatrix}$ $\boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$

6.2. a. $\mathbf{X} = \begin{bmatrix} X_{11} & X_{12} & X_{11}^2 \\ X_{21} & X_{22} & X_{21}^2 \\ X_{31} & X_{32} & X_{31}^2 \\ X_{41} & X_{42} & X_{41}^2 \\ X_{51} & X_{52} & X_{51}^2 \end{bmatrix}$ $\boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$

b. $\mathbf{X} = \begin{bmatrix} 1 & X_{11} & \log_{10} X_{12} \\ 1 & X_{21} & \log_{10} X_{22} \\ 1 & X_{31} & \log_{10} X_{32} \\ 1 & X_{41} & \log_{10} X_{42} \\ 1 & X_{51} & \log_{10} X_{52} \end{bmatrix}$ $\boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$

6.5. a.

$$\begin{array}{c} Y \\ X_1 \\ X_2 \end{array} \begin{bmatrix} 1.000 & .892 & .395 \\ & 1.000 & .000 \\ & & 1.000 \end{bmatrix}$$

b. $b_0 = 37.650$, $b_1 = 4.425$, $b_2 = 4.375$, $\hat{Y} = 37.650 + 4.425X_1 + 4.375X_2$

c&d.

i:	1	2	3	4	5	6	7	8
e_i:	$-.10$	.15	-3.10	3.15	$-.95$	-1.70	-1.95	1.30
Expected Val.:	$-.208$	.208	-3.452	2.661	$-.629$	-1.533	-2.052	1.533

i:	9	10	11	12	13	14	15	16
e_i:	1.20	-1.55	4.20	2.45	-2.65	-4.40	3.35	.60
Expected Val.:	1.066	-1.066	4.764	2.052	-2.661	-4.764	3.452	.629

e. $SSR^* = 72.41$, $SSE = 94.30$, $X^2_{BP} = (72.41/2) \div (94.30/16)^2 = 1.04$, $\chi^2(.99; 2) = 9.21$. If $X^2_{BP} \leq 9.21$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

f. H_0: $E\{Y\} = \beta_0 + \beta_1 X_1 + \beta_2 X_2$, H_a: $E\{Y\} \neq \beta_0 + \beta_1 X_1 + \beta_2 X_2$. $MSLF = 7.46$, $MSPE = 7.125$, $F^* = 7.46/7.125 = 1.047$, $F(.99; 5, 8) = 6.63$. If $F^* \leq 6.63$ conclude H_0, otherwise H_a. Conclude H_0.

6.6. a. H_0: $\beta_1 = \beta_2 = 0$, H_a: not all $\beta_k = 0$ $(k = 1, 2)$. $MSR = 936.350, MSE = 7.254$, $F^* = 936.350/7.254 = 129.083$, $F(.99; 2,\ 13) = 6.70$. If $F^* \leq 6.70$ conclude H_0, otherwise H_a. Conclude H_a.

b. P-value $= 0+$

c. $s\{b_1\} = .301$, $s\{b_2\} = .673$, $B = t(.9975; 13) = 3.372$

$4.425 \pm 3.372(.301)$ $\quad\quad$ $3.410 \leq \beta_1 \leq 5.440$

$4.375 \pm 3.372(.673)$ $\quad\quad$ $2.106 \leq \beta_2 \leq 6.644$

6.7. a. $SSR = 1,872.7$, $SSTO = 1,967.0$, $R^2 = .952$

b. .952, yes.

6.8. a. $\hat{Y}_h = 77.275$, $s\{\hat{Y}_h\} = 1.127$, $t(.995; 13) = 3.012$, $77.275 \pm 3.012(1.127)$,

$73.880 \leq E\{Y_h\} \leq 80.670$

b. $s\{\text{pred}\} = 2.919$, $77.275 \pm 3.012(2.919)$, $68.483 \leq Y_{h(\text{new})} \leq 86.067$

6.9. c.
$$\begin{array}{c} Y \\ X_1 \\ X_2 \\ X_3 \end{array} \begin{bmatrix} 1.0000 & .2077 & .0600 & .8106 \\ & 1.0000 & .0849 & .0457 \\ & & 1.0000 & .1134 \\ & & & 1.0000 \end{bmatrix}$$

6.10. a. $\hat{Y} = 4149.89 + 0.000787X_1 - 13.166X_2 + 623.554X_3$

b&c.

i:	1	2	...	51	52
e_i:	-32.0635	169.2051	...	-184.8776	64.5168
Expected Val.:	-24.1737	151.0325	...	-212.1315	75.5358

e. $n_1 = 26$, $\bar{d}_1 = 145.0$, $n_2 = 26$, $\bar{d}_2 = 77.4$, $s = 81.7$,
$t^*_{BF} = (145.0 - 77.4)/[81.7\sqrt{(1/26) + (1/26)}] = 2.99$, $t(.995; 50) = 2.67779$. If $|t^*_{BF}| \leq 2.67779$ conclude error variance constant, otherwise error variance not constant. Conclude error variance not constant.

6.11. a. H_0: $\beta_1 = \beta_2 = \beta_3 = 0$, H_a: not all $\beta_k = 0$ $(k = 1,\ 2,3)$. $MSR = 725,535$, $MSE = 20,531.9$, $F^* = 725,535/20,531.9 = 35.337$, $F(.95; 3, 48) = 2.79806$. If $F^* \leq 2.79806$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$.

b. $s\{b_1\} = .000365$, $s\{b_3\} = 62.6409$, $B = t(.9875; 48) = 2.3139$

$0.000787 \pm 2.3139(.000365)$ $\quad\quad$ $-.000058 \leq \beta_1 \leq 0.00163$

$623.554 \pm 2.3139(62.6409)$ $\quad\quad$ $478.6092 \leq \beta_3 \leq 768.4988$

c. $SSR = 2,176,606$, $SSTO = 3,162,136$, $R^2 = .6883$

6.12. a. $F(.95; 4, 48) = 2.56524$, $W = 3.2033$; $B = t(.995; 48) = 2.6822$

X_{h1}	X_{h2}	X_{h3}		
302,000	7.2	0:	$4292.79 \pm 2.6822(21.3567)$	$4235.507 \leq E\{Y_h\} \leq 4350.073$
245,000	7.4	0:	$4245.29 \pm 2.6822(29.7021)$	$4165.623 \leq E\{Y_h\} \leq 4324.957$
280,000	6.9	0:	$4279.42 \pm 2.6822(24.4444)$	$4213.855 \leq E\{Y_h\} \leq 4344.985$
350,000	7.0	0:	$4333.20 \pm 2.6822(28.9293)$	$4255.606 \leq E\{Y_h\} \leq 4410.794$
295,000	6.7	1:	$4917.42 \pm 2.6822(62.4998)$	$4749.783 \leq E\{Y_h\} \leq 5085.057$

b.Yes, no

6.13. $F(.95; 4, 48) = 2.5652$, $S = 3.2033$; $B = t(.99375; 48) = 2.5953$

X_{h1}	X_{h2}	X_{h3}		
230,000	7.5	0:	$4232.17 \pm 2.5953(147.288)$	$3849.913 \leq Y_{h(\text{new})} \leq 4614.427$
250,000	7.3	0:	$4250.55 \pm 2.5953(146.058)$	$3871.486 \leq Y_{h(\text{new})} \leq 4629.614$
280,000	7.1	0:	$4276.79 \pm 2.5953(145.134)$	$3900.124 \leq Y_{h(\text{new})} \leq 4653.456$
340,000	6.9	0:	$4326.65 \pm 2.5953(145.930)$	$3947.918 \leq Y_{h(\text{new})} \leq 4705.382$

6.14. a. $\hat{Y}_h = 4278.37$, $s\{\text{predmean}\} = 85.82262$, $t(.975; 48) = 2.01063$,

$4278.37 \pm 2.01063(85.82262)$, $4105.812 \leq \bar{Y}_{h(\text{new})} \leq 4450.928$

b. $12317.44 \leq$ Total labor hours≤ 13352.78

6.15. b.

$$\begin{array}{c} Y \\ X_1 \\ X_2 \\ X_3 \end{array} \begin{bmatrix} 1.000 & -.7868 & -.6029 & -.6446 \\ & 1.000 & .5680 & .5697 \\ & & 1.000 & .6705 \\ & & & 1.000 \end{bmatrix}$$

c. $\hat{Y} = 158.491 - 1.1416X_1 - 0.4420X_2 - 13.4702X_3$

d&e.

i:	1	2	...	45	46
e_i:	.1129	-9.0797	...	-5.5380	10.0524
Expected Val.:	-0.8186	-8.1772	...	-5.4314	8.1772

f. No

g. $SSR^* = 21,355.5$, $SSE = 4,248.8$, $X^2_{BP} = (21,355.5/2) \div (4,248.8\ /46)^2 = 1.2516$, $\chi^2(.99; 3) = 11.3449$. If $X^2_{BP} \leq 11.3449$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

6.16. a. H_0: $\beta_1 = \beta_2 = \beta_3 = 0$, H_a: not all $\beta_k = 0$ $(k = 1, 2, 3)$.

$MSR = 3,040.2$, $MSE = 101.2$, $F^* = 3,040.2/101.2 = 30.05$, $F(.90; 3, 42) = 2.2191$. If $F^* \leq 2.2191$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0.4878$

b. $s\{b_1\} = .2148$, $s\{b_2\} = .4920$, $s\{b_3\} = 7.0997$, $B = t(.9833; 42) = 2.1995$

$-1.1416 \pm 2.1995(.2148)$ $\quad - 1.6141 \leq \beta_1 \leq -0.6691$

$-.4420 \pm 2.1995(.4920)$ $\quad - 1.5242 \leq \beta_2 \leq 0.6402$

$-13.4702 \pm 2.1995(7.0997)$ $\quad - 29.0860 \leq \beta_3 \leq 2.1456$

c. $SSR = 9{,}120.46$, $SSTO = 13{,}369.3$, $R = .8260$

6.17. a. $\hat{Y}_h = 69.0103$, $s\{\hat{Y}_h\} = 2.6646$, $t(.95; 42) = 1.6820$, $69.0103 \pm 1.6820(2.6646)$, $64.5284 \leq E\{Y_h\} \leq 73.4922$

b. $s\{\text{pred}\} = 10.405$, $69.0103 \pm 1.6820(10.405)$, $51.5091 \leq Y_{h(\text{new})} \leq 86.5115$

6.18. b.

$$\begin{array}{c} Y \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \left[\begin{array}{rrrrr} 1.0000 & -.2503 & .4138 & .0665 & .5353 \\ & 1.0000 & .3888 & -.2527 & .2886 \\ & & 1.0000 & -.3798 & .4407 \\ & & & 1.0000 & .0806 \\ & & & & 1.0000 \end{array}\right]$$

c. $\hat{Y} = 12.2006 - .1420X_1 + .2820X_2 + 0.6193X_3 + 0.0000079X_4$

d&e.

i:	1	2	...	80	81
e_i:	-1.0357	-1.5138	...	-2.0302	$-.9068$
Expected Val.:	-1.1524	-1.5857	...	-1.9321	-1.0407

f. No

g. $n_1 = 40$, $\bar{d}_1 = 0.8696$, $n_2 = 41$, $\bar{d}_2 = 0.7793$, $s = 0.7357$, $t^*_{BF} = (0.8696 - 0.7793)/0.7357\sqrt{(1/40) + (1/41)} = 0.5523$, $t(.975; 79) = 1.9905$. If $|t^*_{BF}| \leq 1.9905$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

6.19. a. H_0: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$, H_a: not all $\beta_k = 0$ $(k = 1, 2, 3, 4)$. $MSR = 34.5817$ $MSE = 1.2925$, $F^* = 34.5817/1.2925 = 26.7557$, $F(.95; 4, 76) = 2.4920$. If $F^* \leq 2.4920$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

b. $s\{b_1\} = .02134$, $s\{b_2\} = .06317$, $s\{b_3\} = 1.08681$, $s\{b_4\} = .00000138$, $B = t(.99375; 76) = 2.5585$

$-.1420 \pm 2.5585(.02134)$ $\quad -.1966 \leq \beta_1 \leq -.0874$

$.2820 \pm 2.5585(.06317)$ $\quad .1204 \leq \beta_2 \leq .4436$

$.6193 \pm 2.5585(1.08681)$ $\quad -2.1613 \leq \beta_3 \leq 3.3999$

$.0000079 \pm 2.5585(.00000138)$ $\quad .0000044 \leq \beta_1 \leq .0000114$

c. $SSR = 138.327$, $SSTO = 236.5576$, $R^2 = .5847$

6.20. $F(.95; 5, 76) = 2.3349$, $W = 3.4168$; $B = t(.99375; 76) = 2.5585$

X_{h1}	X_{h2}	X_{h3}	X_{h4}		
5	8.25	0	250,000:	$15.7981 \pm 2.5585(.2781)$	$15.087 \leq E\{Y_h\} \leq 16.510$
6	8.50	.23	270,000:	$16.0275 \pm 2.5585(.2359)$	$15.424 \leq E\{Y_h\} \leq 16.631$
14	11.50	.11	300,000:	$15.9007 \pm 2.5585(.2222)$	$15.332 \leq E\{Y_h\} \leq 16.469$
12	10.25	0	310,000:	$15.8434 \pm 2.5585(.2591)$	$15.180 \leq E\{Y_h\} \leq 16.506$

6.21. $t(.975; 76) = 1.9917$

X_{h1}	X_{h2}	X_{h3}	X_{h4}		
4	10.0	0.10	80,000:	$15.1485 \pm 1.9917(1.1528)$	$12.852 \leq Y_{h(\text{new})} \leq 17.445$
6	11.5	0	120,000:	$15.5425 \pm 1.9917(1.1535)$	$13.245 \leq Y_{h(\text{new})} \leq 17.840$
12	12.5	.32	340,000:	$16.9138 \pm 1.9917(1.1946)$	$14.535 \leq Y_{h(\text{new})} \leq 19.293$

85 percent

6.22. a. Yes

b. No, yes, $Y_i' = \log_e Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2}^2 + \varepsilon_i'$, where $\varepsilon_i' = \log_e \varepsilon_i$

c. Yes

d. No, no

e. No, yes, $Y_i' = \log_e(Y_i^{-1} - 1) = \beta_0 + \beta_1 X_{i1} + \varepsilon_i$

6.23. a. $Q = \sum(Y_i - \beta_1 X_{i1} - \beta_2 X_{i2})^2$

$$\frac{\partial Q}{\partial \beta_1} = -2\sum(Y_i - \beta_1 X_{i1} - \beta_2 X_{i2})X_{i1}$$

$$\frac{\partial Q}{\partial \beta_2} = -2\sum(Y_i - \beta_1 X_{i1} - \beta_2 X_{i2})X_{i2}$$

Setting the derivatives equal to zero, simplifying, and substituting the least squares estimators b_1 and b_2 yields:

$$\sum Y_i X_{i1} - b_1 \sum X_{i1}^2 - b_2 \sum X_{i1}X_{i2} = 0$$

$$\sum Y_i X_{i2} - b_1 \sum X_{i1}X_{i2} - b_2 \sum X_{i2}^2 = 0$$

and:

$$b_1 = \frac{\sum Y_i X_{i2} \sum X_{i1}X_{i2} - \sum Y_i X_{i1} \sum X_{i2}^2}{\left(\sum X_{i1}X_{i2}\right)^2 - \sum X_{i1}^2 \sum X_{i2}^2}$$

$$b_2 = \frac{\sum Y_i X_{i1} \sum X_{i1}X_{i2} - \sum Y_i X_{i2} \sum X_{i1}^2}{\left(\sum X_{i1}X_{i2}\right)^2 - \sum X_{i1}^2 \sum X_{i2}^2}$$

b. $L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(Y_i - \beta_1 X_{i1} - \beta_2 X_{i2})^2\right]$

It is more convenient to work with $\log_e L$:

$$\log_e L = -\frac{n}{2}\log_e(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum(Y_i - \beta_1 X_{i1} - \beta_2 X_{i2})^2$$

$$\frac{\partial \log_e L}{\partial \beta_1} = \frac{1}{\sigma^2}\sum(Y_i - \beta_1 X_{i1} - \beta_2 X_{i2})X_{i1}$$

$$\frac{\partial \log_e L}{\partial \beta_2} = \frac{1}{\sigma^2}\sum(Y_i - \beta_1 X_{i1} - \beta_2 X_{i2})X_{i2}$$

Setting the derivatives equal to zero, simplifying, and substituting the maximum likelihood estimators b_1 and b_2 yields the same normal equations as in part (a), and hence the same estimators.

6.24. a. $Q = \sum(Y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i1}^2 - \beta_3 X_{i2})^2$

$$\frac{\partial Q}{\partial \beta_0} = -2\sum(Y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i1}^2 - \beta_3 X_{i2})$$

$$\frac{\partial Q}{\partial \beta_1} = -2\sum(Y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i1}^2 - \beta_3 X_{i2})X_{i1}$$

$$\frac{\partial Q}{\partial \beta_2} = -2\sum(Y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i1}^2 - \beta_3 X_{i2})X_{i1}^2$$

$$\frac{\partial Q}{\partial \beta_3} = -2\sum(Y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i1}^2 - \beta_3 X_{i2})X_{i2}$$

Setting the derivatives equal to zero, simplifying, and substituting the least squares estimators b_0, b_1, b_2, and b_3 yields the normal equations:

$$\sum Y_i - nb_0 - b_1 \sum X_{i1} - b_2 \sum X_{i1}^2 - b_3 \sum X_{i2} = 0$$

$$\sum Y_i X_{i1} - b_0 \sum X_{i1} - b_1 \sum X_{i1}^2 - b_2 \sum X_{i1}^3 - b_3 \sum X_{i1} X_{i2} = 0$$

$$\sum Y_i X_{i1}^2 - b_0 \sum X_{i1}^2 - b_1 \sum X_{i1}^3 - b_2 \sum X_{i1}^4 - b_3 \sum X_{i1}^2 X_{i2} = 0$$

$$\sum Y_i X_{i2} - b_0 \sum X_{i2} - b_1 \sum X_{i1} X_{i2} - b_2 \sum X_{i1}^2 X_{i2} - b_3 \sum X_{i2}^2 = 0$$

b. $$L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-\frac{1}{2\sigma^2}(Y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i1}^2 - \beta_3 X_{i2})^2]$$

or

$$\log_e L = -\frac{n}{2}\log_e(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum(Y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i1}^2 - \beta_3 X_{i2})^2$$

6.25. Fit $Y_i' = \beta_0 + \beta_1 X_{i1} + \beta_3 X_{i3} + \varepsilon_i$, where $Y_i' = Y_i - 4X_{i2}$

6.26. For regression model (6.1), $R^2 = 1 - \dfrac{SSE(X_1, X_2)}{SSTO}$

When regressing Y_i on $\hat{Y}_i$, $SSTO$ remains unchanged and the fitted regression equation:

$$\hat{Y}_i^* = b_0^* + b_1^* \hat{Y}_i$$

has coefficients $b_0^* = 0$, $b_1^* = 1$ because:

$$\begin{aligned} b_1^* &= \frac{\sum(\hat{Y}_i - \bar{Y})(Y_i - \bar{Y})}{\sum(\hat{Y}_i - \bar{Y})^2} = \frac{\sum(\hat{Y}_i - \bar{Y})[(Y_i - \hat{Y}_i) + (\hat{Y}_i - \bar{Y})]}{\sum(\hat{Y}_i - \bar{Y})^2} \\ &= \frac{\sum(\hat{Y}_i - \bar{Y})[e_i + (\hat{Y}_i - \bar{Y})]}{\sum(\hat{Y}_i - \bar{Y})^2} = 1 \end{aligned}$$

since $\sum e_i \hat{Y}_i = 0$ and $\sum e_i \bar{Y} = 0$ by (1.20) and (1.17).

$$b_0^* = \bar{Y} - b_1^* \bar{Y} = 0$$

Hence $\hat{Y}_i^* = \hat{Y}_i$ and $SSE(\hat{Y}) = \sum(Y_i - \hat{Y}_i^*)^2 = \sum(Y_i - \hat{Y}_i)^2 = SSE(X_1, X_2)$, and:

$$r^2 = 1 - \frac{SSE(\hat{Y})}{SSTO} = 1 - \frac{SSE(X_1, X_2)}{SSTO} = R^2$$

6.27. a. $$\begin{bmatrix} 33.93210 \\ 2.78476 \\ -.26442 \end{bmatrix}$$

b. $\begin{bmatrix} -2.6996 \\ -1.2300 \\ -1.6374 \\ -1.3299 \\ -.0900 \\ 6.9868 \end{bmatrix}$

c. $\begin{bmatrix} .2314 & .2517 & .2118 & .1489 & -.0548 & .2110 \\ .2517 & .3124 & .0944 & .2663 & -.1479 & .2231 \\ .2118 & .0944 & .7044 & -.3192 & .1045 & .2041 \\ .1489 & .2663 & -.3192 & .6143 & .1414 & .1483 \\ -.0548 & -.1479 & .1045 & .1414 & .9404 & .0163 \\ .2110 & .2231 & .2041 & .1483 & .0163 & .1971 \end{bmatrix}$

d. 3,009.926

e. $\begin{bmatrix} 715.4711 & -34.1589 & -13.5949 \\ -34.1589 & 1.6617 & .6441 \\ -13.5949 & .6441 & .2625 \end{bmatrix}$

f. 53.8471

g. 5.4247

6.28. b. Model I:

$$\begin{array}{c} Y \\ X_1 \\ X_2 \\ X_3 \end{array} \begin{bmatrix} 1.0000 & 0.9402 & 0.0781 & 0.9481 \\ & 1.0000 & 0.1731 & 0.9867 \\ & & 1.0000 & 0.1271 \\ & & & 1.0000 \end{bmatrix}$$

Model II:

$$\begin{array}{c} Y \\ X_1 \\ X_2 \\ X_3 \end{array} \begin{bmatrix} 1.0000 & 0.4064 & -0.0031 & 0.9481 \\ & 1.0000 & 0.0292 & 0.3162 \\ & & 1.0000 & -0.0227 \\ & & & 1.0000 \end{bmatrix}$$

c. Model I: $\hat{Y} = -13.3162 + 0.000836618X_1 - 0.065523X_2 + 0.094132X_3$

Model II: $\hat{Y} = -170.574 + 0.0961589X_1 + 6.33984X_2 + 0.126566X_3$

d. Model I: 0.902643

Model II: 0.911749

6.29. a. Region 1: $\hat{Y} = -26,140 + 16.34X_1 + 0.3834X_2 + 291.1X_3$

Region 2: $\hat{Y} = 63,104.1209 + 2.5883X_1 + 3.6022X_2 - 854.5493X_3$

Region 3: $\hat{Y} = 56,929.3851 + 0.3065X_1 + 4.8955X_2 - 800.3958X_3$

Region 4: $\hat{Y} = 37,720 - 0.9915X_1 + 3.627X_2 - 489.0X_3$

c.

	MSE	R^2
Region 1:	8.0728×10^8	0.831
Region 2:	1.4017×10^8	0.9392
Region 3:	1.9707×10^8	0.8692
Region 4:	2.1042×10^8	0.9713

6.30. b. Model I:

$$\begin{array}{c} Y \\ X_1 \\ X_2 \\ X_3 \end{array} \begin{bmatrix} 1.000 & .189 & .533 & .356 \\ & 1.000 & .001 & -.040 \\ & & 1.000 & .413 \\ & & & 1.000 \end{bmatrix}$$

Model II:

$$\begin{array}{c} Y \\ X_1 \\ X_2 \\ X_3 \end{array} \begin{bmatrix} 1.000 & .409 & .533 & .356 \\ & 1.000 & .360 & .795 \\ & & 1.000 & .413 \\ & & & 1.000 \end{bmatrix}$$

c. Model I: $\hat{Y} = 1.38646 + .08371X_1 + .65845X_2 + .02174X_3$

Model II: $\hat{Y} = 6.46738 + .00302X_1 + .64771X_2 - .00929X_3$

d. Model I: .3448

Model II: .3407

6.31. a. Region 1: $\hat{Y} = -3.34958 + .11695X_1 + .05824X_2 + .00151X_3 + .00661X_4$

Region 2: $\hat{Y} = 2.29154 + .00474X_1 + .05803X_2 + .00117X_3 + .01502X_4$

Region 3: $\hat{Y} = -.14386 + .03085X_1 + .10228X_2 + .00411X_3 + .00804X_4$

Region 4: $\hat{Y} = 1.56655 + .03524X_1 + .04033X_2 - .00066X_3 + .01279X_4$

	MSE	R^2
Region 1:	1.022	.4613
Region 2:	1.212	.4115
Region 3:	.937	.6088
Region 4:	.954	.0896

Chapter 7

MULTIPLE REGRESSION – II

7.1. (1) 1 (2) 1 (3) 2 (4) 3

7.3. a. $SSR(X_1) = 1,566.45$, $SSR(X_2|X_1) = 306.25$, $SSE(X_1, X_2) = 94.30$, df: 1, 1, 13.

b. H_0: $\beta_2 = 0$, H_a: $\beta_2 \neq 0$. $SSR(X_2|X_1) = 306.25$, $SSE(X_1, X_2) = 94.30$, $F^* = (306.25/1) \div (94.30/13) = 42.219$, $F(.99; 1, 13) = 9.07$. If $F^* \leq 9.07$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+.

7.4. a. $SSR(X_1) = 136,366$, $SSR(X_3|X_1) = 2,033,566$, $SSR(X_2|X_1, X_3) = 6,674$, $SSE(X_1, X_2, X_3) = 985,530$, df: 1, 1, 1,48.

b. H_0: $\beta_2 = 0$, H_a: $\beta_2 \neq 0$. $SSR(X_2|X_1, X_3) = 6,674$, $SSE(X_1, X_2, X_3) = 985,530$, $F^* = (6,674/1) \div (985,530/48) = 0.32491$, $F(.95; 1, 17) = 4.04265$. If $F^* \leq 4.04265$ conclude H_0, otherwise H_a. Conclude H_0. P-value = 0.5713.

c. Yes, $SSR(X_1)+SSR(X_2|X_1) = 136,366+5,726 = 142,092$, $SSR(X_2)+SSR(X_1|X_2) = 11,394.9 + 130,697.1 = 142,092$.

Yes.

7.5. a. $SSR(X_2) = 4,860.26$, $SSR(X_1|X_2) = 3,896.04$, $SSR(X_3|X_2, X_1) = 364.16$, $SSE(X_1, X_2, X_3) = 4,248.84$, df: 1, 1, 1, 42

b. H_0: $\beta_3 = 0$, H_a: $\beta_3 \neq 0$. $SSR(X_3|X_1, X_2) = 364.16$, $SSE(X_1, X_2, X_3) = 4,248.84$, $F^* = (364.16/1)\div(4,248.84/42) = 3.5997$, $F(.975; 1, 42) = 5.4039$. If $F^* \leq 5.4039$ conclude H_0, otherwise H_a . Conclude H_0. P-value = 0.065.

7.6. H_0: $\beta_2 = \beta_3 = 0$, H_a: not both β_2 and $\beta_3 = 0$. $SSR(X_2, X_3|X_1) = 845.07$, $SSE(X_1, X_2, X_3) = 4,248.84$, $F^* = (845.07/2)\div(4,248.84/42) = 4.1768$, $F(.975; 2, 42) = 4.0327$. If $F^* \leq 4.0327$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0.022.

7.7. a. $SSR(X_4) = 40.5033$, $SSR(X_1|X_4) = 42.2746$, $SSR(X_2|X_1, X_4) = 27.8575$, $SSR(X_3|X_1, X_2, X_4) = 0.4195$, $SSE(X_1, X_2, X_3, X_4) = 98.2306$, df: 1, 1, 1, 1, 76.

b. H_0: $\beta_3 = 0$, H_a: $\beta_3 \neq 0$. $F^* = (0.42/1) \div (98.2306/76) = 0.3249$, $F(.99; 1, 76) = 6.9806$. If $F^* \leq 6.9806$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .5704.

7.8. H_0: $\beta_2 = \beta_3 = 0$, H_a: not both β_2 and $\beta_3 = 0$. $SSR(X_2, X_3|X_1, X_4) = 28.277$, $SSE(X_1, X_2, X_3, X_4) = 98.2306$, $F^* = (28.277/2) \div (98.2306/76) = 10.9388$, $F(.99; 2, 20) = 4.8958$. If $F^* \leq 4.8958$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+.

7.9. H_0: $\beta_1 = -1.0$, $\beta_2 = 0$; H_a: not both equalities hold. Full model: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \varepsilon_i$, reduced model: $Y_i + X_{i1} = \beta_0 + \beta_3 X_{i3} + \varepsilon_i$. $SSE(F) = 4,248.84$, $df_F = 42$, $SSE(R) = 4,427.7$, $df_R = 44$, $F^* = [(4427.7 - 4248.84)/2] \div (4,248.84/42) = .8840$, $F(.975; 2, 42) = 4.0327$. If $F^* \leq 4.0327$ conclude H_0, otherwise H_a. Conclude H_0.

7.10. H_0: $\beta_1 = -.1$, $\beta_2 = .4$; H_a: not both equalities hold. Full model: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \varepsilon_i$, reduced model: $Y_i + .1X_{i1} - .4X_{i2} = \beta_0 + \beta_3 X_{i3} + \beta_4 X_{i4} + \varepsilon_i$. $SSE(F) = 98.2306$, $df_F = 76$, $SSE(R) = 110.141$, $df_R = 78$, $F^* = [(110.141 - 98.2306)/2] \div (98.2306/76) = 4.607$, $F(.99; 2, 76) = 4.89584$. If $F^* \leq 4.89584$ conclude H_0, otherwise H_a. Conclude H_0.

7.11. a. $R^2_{Y1} = .550$, $R^2_{Y2} = .408$, $R^2_{12} = 0$, $R^2_{Y1|2} = .929$, $R^2_{Y2|1} = .907$, $R^2 = .958$

7.12. $R^2_{Y1} = .796$, $R^2_{Y2} = .156$, $R^2_{12} = 0$, $R^2_{Y1|2} = .943$, $R^2_{Y2|1} = .765$, $R^2 = .952$

7.13. $R^2_{Y1} = .0431$, $R^2_{Y2} = .0036$, $R^2_{12} = .0072$, $R^2_{Y1|2} = 0.0415$, $R^2_{Y2|1} = 0.0019$, $R^2_{Y2|13} = .0067$ $R^2 = .6883$

7.14. a. $R^2_{Y1} = .6190$, $R^2_{Y1|2} = .4579$, $R^2_{Y1|23} = .4021$

b. $R^2_{Y2} = .3635$, $R^2_{Y2|1} = .0944$, $R^2_{Y2|13} = .0189$

7.15. $R^2_{Y4} = .2865$, $R^2_{Y1} = .0626$, $R^2_{Y1|4} = .2505$, $R^2_{14} = .4652$, $R^2_{Y2|14} = .2202$, $R^2_{Y3|124} = .0043$, $R^2 = .5848$

7.16. a. $\hat{Y}^* = .89239X_1^* + .39458X_2^*$

c. $s_Y = 11.45135$, $s_1 = 2.30940$, $s_2 = 1.03280$, $b_1 = \dfrac{11.45135}{2.30940}(.89239) = 4.425$, $b_2 = \dfrac{11.45135}{1.03280}(.39458) = 4.375$, $b_0 = 81.7500 - 4.425(7) - 4.375(3) = 37.650$.

7.17. a. $\hat{Y}^* = .17472X_1^* - .04639X_2^* + .80786X_3^*$

b. $R^2_{12} = .0072$, $R^2_{13} = .0021$,$R^2_{23} = .0129$

c. $s_Y = 249.003$, $s_1 = 55274.6$, $s_2 = .87738$, $s_3 = .32260$ $b_1 = \dfrac{249.003}{55274.6}(.17472) = .00079$, $b_2 = \dfrac{249.003}{.87738}(-.04639) = -13.16562$, $b_3 = \dfrac{249.003}{5.32260}(.80786) = 623.5572$, $b_0 = 4363.04 - .00079(302,693) + 13.16562(7.37058) - 623.5572(0.115385) = 4149.002$.

7.18. a. $\hat{Y}^* = -.59067X_1^* - .11062X_2^* - .23393X_3^*$

b. $R^2_{12} = .32262$, $R^2_{13} = .32456$, $R^2_{23} = .44957$

c. $s_Y = 17.2365$, $s_1 = 8.91809$, $s_2 = 4.31356$, $s_3 = .29934$, $b_1 = \dfrac{17.2365}{8.91809}(-.59067) = -1.14162$, $b_2 = \dfrac{17.2365}{4.31356}(-.11062) = -.44203$, $b_3 = \dfrac{17.2365}{.29934}(-.23393) = -13.47008$, $b_0 = 61.5652 + 1.14162(38.3913) + .44203(50.4348) + 13.47008(2.28696) = 158.4927$

7.19. a. $\hat{Y}^* = -.547853X_1^* + .423647X_2^* + .0484614X_3^* + .502757X_4^*$

c. $s_Y = 1.71958$, $s_1 = 6.63278$, $s_2 = 2.58317$, $s_3 = .13455$,$s_4 = 109099$, $b_1 = \frac{1.71958}{6.63278}(-.547853) = -.14203$, $b_2 = \frac{1.71958}{2.58317}(.423647) = .28202$, $b_3 = \frac{1.71958}{.13455}(.0484614) = .61934$, $b_4 = \frac{1.71958}{109099}(.502757) = 7.9243 \times 10^{-6}$, $b_0 = 15.1389 + .14203(7.8642) - .28202(9.68815) - .61934(.08099) - 7.9243 \times 10^{-6}(160633) = 12.20054$.

7.21. b. The line of fitted values when $.5X_1 - X_2 = -5$.

7.24. a. $\hat{Y} = 50.775 + 4.425X_1$

c. Yes, $SSR(X_1) = 1,566.45$, $SSR(X_1|X_2) = 1,566.45$

d. $r_{12} = 0$

7.25. a. $\hat{Y} = 4079.87 + 0.000935X_2$

c. No, $SSR(X_1) = 136,366$, $SSR(X_1|X_2) = 130,697$

d. $r_{12} = .0849$

7.26. a. $\hat{Y} = 156.672 - 1.26765X_1 - 0.920788X_2$

c. No, $SSR(X_1) = 8,275.3$, $SSR(X_1|X_3) = 3,483.89$

No, $SSR(X_2) = 4,860.26$, $SSR(X_2|X_3) = 708$

d. $r_{12} = .5680$, $r_{13} = .5697$, $r_{23} = .6705$

7.27. a. $\hat{Y} = 14.3613 - .11447X_1 + .00001X_4$

c. No, $SSR(X_4) = 67.7751$, $SSR(X_4|X_3) = 66.8582$

No, $SSR(X_1) = 14.8185$, $SSR(X_1|X_3) = 13.7744$

d. $r_{12} = .4670$, $r_{13} = .3228$, $r_{23} = .2538$

7.28. a. (1) $SSR(X_1, X_5) - SSR(X_1)$ or $SSE(X_1) - SSE(X_1, X_5)$

(2) $SSR(X_1, X_3, X_4) - SSR(X_1)$ or $SSE(X_1) - SSE(X_1, X_3, X_4)$

(3) $SSR(X_1, X_2, X_3, X_4) - SSR(X_1, X_2, X_3)$

or $SSE(X_1, X_2, X_3) - SSE(X_1, X_2, X_3, X_4)$

b. $SSR(X_5|X_1, X_2, X_3, X_4)$, $SSR(X_2, X_4|X_1, X_3, X_5)$

7.29. a. $SSR(X_1) + SSR(X_2, X_3|X_1) + SSR(X_4|X_1, X_2, X_3)$

$= SSR(X_1) + [SSR(X_1, X_2, X_3) - SSR(X_1)]$

$+[SSR(X_1, X_2, X_3, X_4) - SSR(X_1, X_2, X_3)]$

$= SSR(X_1, X_2, X_3, X_4)$

b. $SSR(X_2, X_3) + SSR(X_1|X_2, X_3) + SSR(X_4|X_1, X_2, X_3)$

$= SSR(X_2, X_3) + [SSR(X_1, X_2, X_3) - SSR(X_2, X_3)]$

$+[SSR(X_1, X_2, X_3, X_4) - SSR(X_1, X_2, X_3)] = SSR(X_1, X_2, X_3, X_4)$

7.30. a. $\hat{Y} = 68.625 + 4.375X_2$

i:	1	2	3	4	5	6
e_i:	-13.3750	-13.1250	-16.3750	-10.1250	-5.3750	-6.1250

i:	7	8	9	10	11	12
e_i:	-6.3750	-3.1250	5.6250	2.8750	8.6250	6.8750

i:	13	14	15	16
e_i:	10.6250	8.8750	16.6250	13.8750

b. $\hat{X}_1 = 7$

i:	1	2	3	4	5	6	7	8
e_i:	-3	-3	-3	-3	-1	-1	-1	-1

i:	9	10	11	12	13	14	15	16
e_i:	1	1	1	1	3	3	3	3

c. $r = .971 = r_{Y1|2}$

7.31. (1) $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i$

(2) $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_4\sqrt{X_{i3}} + \varepsilon_i$

(3) $Y_i' = Y_i - 5(X_{i1} + X_{i2}) = \beta_0 + \beta_3 X_{i1}X_{i2} + \beta_4\sqrt{X_{i3}} + \varepsilon_i$

(4) $Y_i' = Y_i - 7\sqrt{X_{i3}} = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}X_{i2} + \varepsilon_i$

7.32. (1) $Y_i = \beta_0 + \beta_2 X_{i2} + \varepsilon_i$

(2) $Y_i = \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \varepsilon_i$

(3) $Y_i' = Y_i - 5X_{i1}^2 = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i$

(4) $Y_i' = Y_i - 10 = \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \varepsilon_i$

(5) $Y_i = \beta_0 + \beta_c(X_{i1} + X_{i2}) + \beta_3 X_{i1}^2 + \varepsilon_i$, where $\beta_c = \beta_1 = \beta_2$

7.33. *Let*: $y_i = Y_i - \bar{Y}$

$$x_{i1} = X_{i1} - \bar{X}_1$$
$$x_{i2} = X_{i2} - \bar{X}_2$$

Then: $SSR(X_1) = \dfrac{(\sum x_{i1}y_i)^2}{\sum x_{i1}^2} = \sum y_i^2 r_{Y1}^2$ by (1.10a), (2.51) and (2.84)

$$SSE(X_1) = \sum y_i^2 - \frac{(\sum x_{i1}y_i)^2}{\sum x_{i1}^2} = \sum y_i^2(1 - r_{Y1}^2)$$

$SSR(X_1, X_2) = b_1 \sum x_{i1}y_i + b_2 \sum x_{i2}y_i$ by (2.43) and $\sum y_i = 0$

Further:

$$b_1 = \frac{\dfrac{\sum x_{i1}y_i}{\sum x_{i1}^2} - \left[\dfrac{\sum y_i^2}{\sum x_{i1}^2}\right]^{1/2} r_{Y2}r_{12}}{1 - r_{12}^2} \quad \text{by (7.56)}$$

and similarly:

$$b_2 = \frac{\dfrac{\sum x_{i2}y_i}{\sum x_{i2}^2} - \left[\dfrac{\sum y_i^2}{\sum x_{i2}^2}\right]^{1/2} r_{Y1}r_{12}}{1 - r_{12}^2}$$

Substituting these expressions for b_1 and b_2 into $SSR(X_1,\, X_2)$, we obtain after some simplification:

$$SSR(X_1,\, X_2) = \frac{1}{1 - r_{12}^2}\left[\sum y_i^2 r_{Y1}^2 + \sum y_i^2 r_{Y2}^2 - 2\sum y_i^2 r_{Y1}r_{Y2}r_{12}\right]$$

Now by (7.36) and (7.2b), we have:

$$r_{Y2|1}^2 = \frac{SSR(X_1,\, X_2) - SSR(X_1)}{SSE(X_1)}$$

Substituting the earlier expressions into the above, we obtain after some simplifying:

$$r_{Y2|1}^2 = \frac{1}{\sum y_i^2(1 - r_{Y1}^2)(1 - r_{12}^2)}[\sum y_i^2 r_{Y1}^2 + \sum y_i^2 r_{Y2}^2 - 2\sum y_i^2 r_{Y1}r_{Y2}r_{12}$$
$$- (1 - r_{12}^2)\sum y_i^2 r_{Y1}^2]$$

After some further simplifying, we obtain:

$$r_{Y2|1}^2 = \frac{(r_{Y2} - r_{12}r_{Y1})^2}{(1 - r_{Y1}^2)(1 - r_{12}^2)}$$

7.34. a. (1) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} .7420 \\ .6385 \end{bmatrix}$

(3) $\begin{bmatrix} .7420 \\ .6385 \end{bmatrix}$ (4) $\begin{bmatrix} .0083 & 0 \\ 0 & .0083 \end{bmatrix}$

b. From (7.53), $b_1^* = \dfrac{1.069}{7.745}(5.375) = .742$

$$b_2^* = \frac{.5345}{7.745}(9.250) = .638$$

7.35. From (7.45), we have:

$$Y_i^* = \beta_1^* X_{i1}^* + \beta_2^* X_{i2}^* + \varepsilon_i^*$$

$$\frac{1}{\sqrt{n-1}}\left(\frac{Y_i - \bar{Y}}{s_Y}\right) = \beta_1^*\frac{1}{\sqrt{n-1}}\left(\frac{X_{i1} - \bar{X}_1}{s_1}\right) + \beta_2^*\frac{1}{\sqrt{n-1}}\left(\frac{X_{i2} - \bar{X}_2}{s_2}\right) + \varepsilon_i^*$$

Simplifying, we obtain:

$$Y_i = (\bar{Y} - \beta_1^*\frac{s_Y}{s_1}\bar{X}_1 - \beta_2^*\frac{s_Y}{s_2}\bar{X}_2) + \beta_1^*\frac{s_Y}{s_1}X_{i1} + \beta_2^*\frac{s_Y}{s_2}X_{i2} + \sqrt{n-1}s_Y\varepsilon_i^*$$

Hence:

$$\beta_1^* \frac{s_Y}{s_1} = \beta_1 \qquad \beta_2^* \frac{s_Y}{s_2} = \beta_2$$

7.36. $\mathbf{X}^*\mathbf{Y} = \begin{bmatrix} \Sigma X_{i1}^* Y_i^* \\ \Sigma X_{i2}^* Y_i^* \end{bmatrix} = \begin{bmatrix} \dfrac{\Sigma(X_{i1} - \bar{X}_1)(Y_i - \bar{Y})}{(n-1)s_1 s_Y} \\ \dfrac{\Sigma(X_{i2} - \bar{X}_2)(Y_i - \bar{Y})}{(n-1)s_2 s_Y} \end{bmatrix} = \begin{bmatrix} r_{Y1} \\ r_{Y2} \end{bmatrix}$

7.37. a. $R^2_{Y3|12} = .02883$, $R^2_{Y4|12} = .00384$, $R^2_{Y5|12} = .55382$, $R^2_{Y6|12} = .00732$

b. X_5, yes.

c. Full model: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_5 X_{i5} + \varepsilon_i$. H_0: $\beta_5 = 0$, H_a: $\beta_5 \neq 0$. $SSR(X_5|X_1, X_2) = 78,070,132$, $SSE(X_1, X_2, X_5) = 62,896,949$, $F^* = (78,070,132/1) \div (62,896,949/436) = 541.1801$, $F(.99; 1, 137) = 6.69336$. If $F^* \leq 6.69336$ conclude H_0, otherwise H_a. Conclude H_a. No.

7.38. a. $R^2_{Y3|12} = .01167$, $R^2_{Y4|12} = .13620$, $R^2_{Y5|12} = .03737$, $R^2_{Y6|12} = .03639$

b. X_4, yes.

c. Full model: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_4 X_{i4} + \varepsilon_i$. H_0: $\beta_4 = 0$, H_a: $\beta_4 \neq 0$. $SSR(X_4|X_1, X_2) = 37.89858$, $SSE(X_1, X_2, X_4) = 240.35163$, $F^* = (37.89858/1) \div (240.35163/109) = 17.187$, $F(.95; 1, 109) = 3.93$. If $F^* \leq 3.93$ conclude H_0, otherwise H_a. Conclude H_a. No.

Chapter 8

MODELS FOR QUANTITATIVE AND QUALITATIVE PREDICTORS

8.4. a. $\hat{Y} = 82.9357 - 1.18396x + .0148405x^2$, $R^2 = .76317$

b. H_0: $\beta_1 = \beta_{11} = 0$, H_a: not both β_1 and $\beta_{11} = 0$. $MSR = 5915.31$, $MSE = 64.409$, $F^* = 5915.31/64.409 = 91.8398$, $F(.95; 2, 57) = 3.15884$. If $F^* \leq 3.15884$ conclude H_0, otherwise H_a. Conclude H_a.

c. $\hat{Y}_h = 99.2546$, $s\{\hat{Y}_h\} = 1.4833$, $t(.975; 57) = 2.00247$, $99.2546 \pm 2.00247(1.4833)$, $96.2843 \leq E\{Y_h\} \leq 102.2249$

d. $s\{\text{pred}\} = 8.16144$, $99.2546 \pm 2.00247(8.16144)$, $82.91156 \leq Y_{h(\text{new})} \leq 115.5976$

e. H_0: $\beta_{11} = 0$, H_a: $\beta_{11} \neq 0$. $s\{b_{11}\} = .00836$, $t^* = .0148405/.00836 = 1.7759$, $t(.975; 57) = 2.00247$. If $|t^*| \leq 2.00247$ conclude H_0, otherwise H_a. Conclude H_0. Alternatively, $SSR(x^2|x) = 203.1$, $SSE(x, x^2) = 3671.31$, $F^* = (203.1/1) \div (3671.31/57) = 3.15329$, $F(.95; 1, 57) = 4.00987$. If $F^* \leq 4.00987$ conclude H_0, otherwise H_a. Conclude H_0.

f. $\hat{Y} = 207.350 - 2.96432X + .0148405X^2$

g. $r_{X,X^2} = .9961$, $r_{x,x^2} = -.0384$

8.5. a.

i:	1	2	3	...	58	59	60
e_i:	-1.3238	-4.7592	-3.8091	...	-11.7798	-.8515	6.22023

b. H_0: $E\{Y\} = \beta_0 + \beta_1 x + \beta_{11}x^2$, H_a: $E\{Y\} \neq \beta_0 + \beta_1 x + \beta_{11}x^2$. $MSLF = 62.8154$, $MSPE = 66.0595$, $F^* = 62.8154/66.0595 = 0.95$, $F(.95; 29, 28) = 1.87519$. If $F^* \leq 1.87519$ conclude H_0, otherwise H_a. Conclude H_0.

c. $\hat{Y} = 82.92730 - 1.26789x + .01504x^2 + .000337x^3$

H_0: $\beta_{111} = 0$, H_a: $\beta_{111} \neq 0$. $s\{b_{111}\} = .000933$, $t^* = .000337/.000933 = .3612$, $t(.975; 56) = 2.00324$. If $|t^*| \leq 2.00324$ conclude H_0, otherwise H_a. Conclude H_0. Yes. Alternatively, $SSR(x^3|x, x^2) = 8.6$, $SSE(x, x^2, x^3) = 3662.78$, $F^* = (8.6/1) \div (3662.78/56) = .13148$, $F(.95; 1, 56) = 4.01297$. If $F^* \leq 4.01297$ conclude H_0, otherwise H_a. Conclude H_0. Yes.

8.6. a. $\hat{Y} = 21.0942 + 1.13736x - .118401x^2$, $R^2 = .81434$

b. H_0: $\beta_1 = \beta_{11} = 0$, H_a: not all $\beta_k = 0$ ($k = 1, 11$). $MSR = 523.133$, $MSE = 9.9392$, $F^* = 523.133/9.9392 = 52.6333$, $F(.99; 2, 24) = 5.6136$. If $F^* \leq 5.6136$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $F(.99; 3, 24) = 5.04$, $W = 3.7622$; $B = t(.99833; 24) = 3.25756$

$\underline{X}$

10: $20.6276 \pm 3.25756(1.8945)$ $\quad 14.45615 \leq E\{Y_h\} \leq 26.79905$

15: $11.5142 \pm 3.25756(4.56694)$ $\quad -3.36288 \leq E\{Y_h\} \leq 26.39128$

20: $-3.5192 \pm 3.25756(8.50084)$ $\quad -31.2112 \leq E\{Y_h\} \leq 24.1728$

d. $s\{\text{pred}\} = 5.54942$, $t(.995; 24) = 2.79694$, $11.5142 \pm 2.79694(5.54942)$, $-4.0072 \leq Y_{h(\text{new})} \leq 27.0356$

e. H_0: $\beta_{11} = 0$, H_a: $\beta_{11} \neq 0$. $s\{b_{11}\} = .02347$, $t^* = -.118401/.02347 = -5.04478$, $t(.995; 24) = 2.79694$. If $|t^*| \leq 2.79694$ conclude H_0, otherwise H_a. Conclude H_a. Alternatively, $SSR(x^2|x) = 252.989$, $SSE(x, x^2) = 238.541$, $F^* = (252.989/1) \div (238.541/24) = 25.4536$, $F(.99; 1, 24) = 7.82287$. If $F^* \leq 7.82287$ conclude H_0, otherwise H_a. Conclude H_a.

f. $\hat{Y} = -26.3254 + 4.87357X - .118401X^2$

8.7. a.

i:	1	2	...	26	27
e_i:	3.96746	-1.42965	...	2.10202	-2.43692

b. H_0: $E\{Y\} = \beta_0 + \beta_1 x + \beta_{11}x^2$, H_a: $E\{Y\} \neq \beta_0 + \beta_1 x + \beta_{11}x^2$. $MSLF = 6.65396$, $MSPE = 13.2244$, $F^* = 6.65396/13.2244 = 0.50316$, $F(.99; 12, 12) = 4.15526$. If $F^* \leq 4.15526$ conclude H_0, otherwise H_a. Conclude H_0.

8.8. a. $\hat{Y} = 10.1893 - .181775x_1 + .0141477x_1^2 + .314031X_2 + .000008X_4$

b. .5927

c. H_0: $\beta_{11} = 0$, H_a: $\beta_{11} \neq 0$. $s\{b_{11}\} = .005821$, $t^* = .0141477/.005821 = 2.43046$, $t(.975; 76) = 1.99167$. If $|t^*| \leq 1.99167$ conclude H_0, otherwise H_a. Conclude H_a.

d. $\hat{Y}_h = 17.2009$, $s\{\hat{Y}_h\} = .37345$, $t(.975;\ 76) = 1.99167$, $17.2009 \pm 1.99167(.37345)$, $16.45711 \leq E\{Y_h\} \leq 17.94469$

e. $\hat{Y} = 12.4938 - .404296x_1 + .0141477x_1^2 + .314031X_2 + .000008X_4$

8.9. a. $X_2 = 3$: $E\{Y\} = 37 + 7.5X_1$

$X_2 = 6$: $E\{Y\} = 49 + 12X_1$

8.10. a. $X_1 = 1$: $E\{Y\} = 21 + X_2$

$X_1 = 4$: $E\{Y\} = 42 - 11X_2$

8.11. a. $\hat{Y} = 27.150 + 5.925X_1 + 7.875X_2 - .500X_1X_2$

b. H_0: $\beta_3 = 0$, H_a: $\beta_3 \neq 0$. $MSR(X_1X_2|X_1, X_2) = 20.0000$, $MSE = 6.1917$, $F^* = 20.0000/6.1917 = 3.23$, $F(.95; 1, 12) = 4.75$. If $F^* \leq 4.75$ conclude H_0, otherwise H_a. Conclude H_0.

8.13. $E\{Y\} = 25.3 + .20X_1$ for mutual firms,

$E\{Y\} = 13.2 + .20X_1$ for stock firms.

8.15. b. $\hat{Y} = -0.92247 + 15.0461X_1 + .75872X_2$

c. $s\{b_2\} = 2.77986$, $t(.975;\ 42) = 2.01808$, $.75872 \pm 2.01808(2.77986)$, $-4.85126 \leq \beta_2 \leq 6.3687$

e.

i:	1	2	...	44	45
$X_{i1}X_{i2}$:	2	0	...	0	0
e_i:	-9.92854	.73790	...	1.73790	2.69176

8.16. b. $\hat{Y} = 2.19842 + .03789X_1 - .09430X_2$

c. $H_0 : \beta_2 = 0$, $H_a : \beta_2 \neq 0$. $s\{b_2\} = .11997$, $t^* = -.09430/.11997 = -.786$, $t(.995; 117) = 2.6185$. If $|t^*| \leq 2.6185$ conclude H_0, otherwise H_a. Conclude H_0.

d.

i:	1	2	...	119	120
$X_{i1}X_{i2}$:	0	14	...	16	0
e_i:	.90281	1.25037	...	-.85042	-.31145

8.17. No

8.18. $E\{Y\} = 25 + .30X_1$ for mutual firms,

$E\{Y\} = 12.5 + .35X_1$ for stock firms.

8.19. a. $\hat{Y} = 2.81311 + 14.3394X_1 - 8.14120X_2 + 1.77739X_1X_2$

b. $H_0 : \beta_3 = 0$, $H_a : \beta_3 \neq 0$. $s\{b_3\} = .97459$, $t^* = 1.77739/.97459 = 1.8237$, $t(.95; 41) = 1.68288$. If $|t^*| \leq 1.68288$ conclude H_0, otherwise H_a. Conclude H_a. Alternatively, $SSR(X_1X_2|X_1, X_2) = 255.9$, $SSE(X_1, X_2, X_1X_2) = 3154.44$, $F^* = (255.9/1) \div (3154.44/\ 41) = 3.32607$, $F(.90; 1, 41) = 2.83208$. If $F^* \leq 2.83208$ conclude H_0, otherwise H_a. Conclude H_a.

8.20. a. $\hat{Y} = 3.22632 - .00276X_1 - 1.64958X_2 + .06224X_1X_2$

b. $H_0 : \beta_3 = 0$, $H_a : \beta_3 \neq 0$. $s\{b_3\} = .02649$, $t^* = .06224/.02649 = 2.3496$, $t(.975; 116) = 1.9806$. If $|t^*| \leq 1.9806$ conclude H_0, otherwise H_a. Conclude H_a. Alternatively, $SSR(X_1X_2|X_1, X_2) = 2.07126$, $SSE(X_1, X_2, X_1X_2) = 45.5769$, $F^* = (2.07126/1) \div (45.5769/116) = 5.271665$, $F(.95; 1, 116) = 3.9229$. If $F^* \leq 3.9229$ conclude H_0, otherwise H_a. Conclude H_a.

8.21. a. Hard hat: $E\{Y\} = (\beta_0 + \beta_2) + \beta_1X_1$

Bump cap: $E\{Y\} = (\beta_0 + \beta_3) + \beta_1X_1$

None: $E\{Y\} = \beta_0 + \beta_1X_1$

b. (1) $H_0 : \beta_3 \geq 0$, $H_a : \beta_3 < 0$; (2) $H_0 : \beta_2 = \beta_3$, $H_a : \beta_2 \neq \beta_3$

8.22. $E\{Y\} = \beta_0 + \beta_1X_1$ Tool models M1

$E\{Y\} = (\beta_0 + \beta_2) + (\beta_1 + \beta_5)X_1$ Tool models M2

$E\{Y\} = (\beta_0 + \beta_3) + (\beta_1 + \beta_6)X_1$ Tool models M3

$E\{Y\} = (\beta_0 + \beta_4) + (\beta_1 + \beta_7)X_1$ Tool models M4

8.24. b. $\hat{Y} = -126.905 + 2.7759X_1 + 76.0215X_2 - 1.10748X_1X_2$,

$H_0: \beta_2 = \beta_3 = 0$, H_a : not both $\beta_2 = 0$ and $\beta_3 = 0$. $SSR(X_2, X_1X_2|\ X_1) = 566.15$, $SSE(X_1, X_2, X_1X_2) = 909.105$, $F^* = (369.85/2) \div (909.105/60) = 12.2049$, $F(.95; 2, 60) = 3.15041$. If $F^* \leq 3.15041$ conclude H_0, otherwise H_a. Conclude H_a.

c. $\hat{Y} = -126.9052 + 2.7759X_1$ for noncorner lots

$\hat{Y} = -50.8836 + 1.6684X_1$ for corner lots

8.25. a. $\hat{Y} = 4295.72 + .000903x_1 - (1.5767 \times 10^{-9})x_1^2 + 614.393X_3 - .000188x_1X_3 + (1.8076 \times 10^{-9})x_1^2X_3$

b. $H_0: \beta_2 = \beta_4 = \beta_5 = 0$, H_a : not all $\beta_2 = 0$,$\beta_4 = 0$ and $\beta_5 = 0$. $SSR(x_1^2, x_1X_3, x_1^2X_3|x_1, X_3) = 1442$, $SSE(x_1, x_1^2, X_3, x_1X_3, x_1^2X_3) = 990762$, $F^* = (1442/3) \div (990762/46) = .02232$, $F(.95; 3, 46) = 2.8068$. If $F^* \leq 2.80681$ conclude H_0, otherwise H_a. Conclude H_0.

Set 1

8.29.

$$\begin{array}{c} X \\ X^2 \\ X^3 \end{array} \begin{bmatrix} 1 & .990 & .966 \\ & 1 & - \\ & & 1 \end{bmatrix} \qquad \begin{array}{c} x \\ x^2 \\ x^3 \end{array} \begin{bmatrix} 1 & .379 & .904 \\ & 1 & - \\ & & 1 \end{bmatrix}$$

Set 2

$$\begin{array}{c} X \\ X^2 \\ X^3 \end{array} \begin{bmatrix} 1 & .970 & .929 \\ & 1 & - \\ & & 1 \end{bmatrix} \qquad \begin{array}{c} x \\ x^2 \\ x^3 \end{array} \begin{bmatrix} 1 & .846 & .89 \\ & 1 & - \\ & & 1 \end{bmatrix}$$

8.30. $\dfrac{dE\{Y\}}{dx} = \beta_1 + 2\beta_{11}x$

$$\frac{d^2E\{Y\}}{dx^2} = 2\beta_{11}$$

8.31. a.
$$\begin{aligned}\hat{Y} &= b_0 + b_1x + b_{11}x^2 \\ &= b_0 + b_1(X - \bar{X}) + b_{11}(X - \bar{X})^2 \\ &= b_0 + b_1X - b_1\bar{X} + b_{11}X^2 + b_{11}\bar{X}^2 - 2b_{11}X\bar{X} \\ &= (b_0 - b_1\bar{X} + b_{11}\bar{X}^2) + (b_1 - 2b_{11}\bar{X})X + b_{11}X^2\end{aligned}$$

Hence:

$$b_0' = b_0 - b_1\bar{X} + b_{11}\bar{X}^2$$
$$b_1' = b_1 - 2b_{11}\bar{X}$$
$$b_{11}' = b_{11}$$

b. $\mathbf{A} = \begin{bmatrix} 1 & -\bar{X} & \bar{X}^2 \\ 0 & 1 & -2\bar{X} \\ 0 & 0 & 1 \end{bmatrix}$ $\quad\quad\quad$ $\boldsymbol{\sigma}^2\{\mathbf{b}\} = \begin{bmatrix} \sigma_0^2 & \sigma_{01} & \sigma_{02} \\ \sigma_{01} & \sigma_1^2 & \sigma_{12} \\ \sigma_{02} & \sigma_{12} & \sigma_2^2 \end{bmatrix}$

where $\sigma_0^2 = \sigma^2\{b_0\}$, $\sigma_{01} = \sigma\{b_0, b_1\}$, etc. for the regression coefficients in the transformed x variables.

The variance-covariance matrix of the regression coefficients in the original X variables, $\mathbf{A}\,[\boldsymbol{\sigma}^2\{\mathbf{b}\}]\,\mathbf{A}'$, then yields:

$$\sigma^2\{b_0'\} = \sigma_0^2 - 2\bar{X}\sigma_{01} + 2\bar{X}^2\sigma_{02} + \bar{X}^2\sigma_1^2 - 2\bar{X}^3\sigma_{12} + \bar{X}^4\sigma_2^2$$
$$\sigma^2\{b_1'\} = \sigma_1^2 - 4\bar{X}\sigma_{12} + 4\bar{X}^2\sigma_2^2$$
$$\sigma^2\{b_2'\} = \sigma_2^2$$
$$\sigma\{b_0', b_1'\} = \sigma_{01} - 2\bar{X}\sigma_{02} + 3\bar{X}^2\sigma_{12} - \bar{X}\sigma_1^2 - 2\bar{X}^3\sigma_2^2$$
$$\sigma\{b_0', b_2'\} = \sigma_{02} - \bar{X}\sigma_{12} + \bar{X}^2\sigma_2^2$$
$$\sigma\{b_1', b_2'\} = \sigma_{12} - 2\bar{X}\sigma_2^2$$

8.32. When X_i are equally spaced, $\sum x_i^3 = 0$; hence (8.4) becomes:

$$\sum Y_i = nb_0 + b_{11}\sum x_i^2$$
$$\sum x_i Y_i = b_1 \sum x_i^2$$
$$\sum x_i^2 Y_i = b_0 \sum x_i^2 + b_{11} \sum x_i^4$$

8.33. a. $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i1}^2 + \beta_3 X_{i2} + \beta_4 x_{i1} X_{i2} + \beta_5 x_{i1}^2 X_{i2} + \beta_6 X_{i3} + \beta_7 x_{i1} X_{i3} + \beta_8 x_{i1}^2 X_{i3} + \varepsilon_i$

b. (1) $H_0: \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0$

H_a: not all $\beta_k = 0$ $(k = 3, ..., 8)$

$SSE(R) = SSE(x_1, x_1^2)$

$$F^* = \frac{SSE(R) - SSE(F)}{6} \div \frac{SSE(F)}{n-9}$$

If $F^* \leq F(.99; 6, n-9)$ conclude H_0, otherwise H_a.

(2) $H_0: \beta_3 = \beta_6 = 0$, H_a: not both $\beta_3 = 0$ and $\beta_6 = 0$

$SSE(R) = SSE(x_1, x_1^2, x_1X_2, x_1^2X_2, x_1X_3, x_1^2X_3)$

$$F^* = \frac{SSE(R) - SSE(F)}{2} \div \frac{SSE(F)}{n-9}$$

If $F^* \leq F(.99; 2, n-9)$ conclude H_0, otherwise H_a.

(3) $H_0: \beta_4 = \beta_5 = \beta_7 = \beta_8 = 0$, H_a: not all $\beta_k = 0$ $(k = 4, 5, 7, 8)$

$SSE(R) = SSE(x_1, x_1^2, X_2, X_3)$

$$F^* = \frac{SSE(R) - SSE(F)}{4} \div \frac{SSE(F)}{n-9}$$

If $F^* \leq F(.99; 4, n-9)$ conclude H_0, otherwise H_a.

8.34. a. $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \varepsilon_i$

b. Commercial: $E\{Y\} = (\beta_0 + \beta_2) + \beta_1 X_1$

Mutual savings: $E\{Y\} = (\beta_0 + \beta_3) + \beta_1 X_1$

Savings and loan: $E\{Y\} = (\beta_0 - \beta_2 - \beta_3) + \beta_1 X_1$

8.35. a. Let $n_2 = n - n_1$ and define:

$$\mathbf{X} = \left[\begin{array}{cc} 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ \hline 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{array}\right] \begin{array}{l} \\ \leftarrow n_1 \\ \\ \\ \leftarrow n_2 \\ \\ \end{array} \qquad \mathbf{Y} = \left[\begin{array}{c} Y_{11} \\ \vdots \\ Y_{n_1 1} \\ \hline Y_{12} \\ \vdots \\ Y_{n_2 2} \end{array}\right]$$

$$\bar{Y}_1 = \frac{\sum Y_{i1}}{n_1} \qquad \bar{Y}_2 = \frac{\sum Y_{i2}}{n_2} \qquad \bar{Y} = \frac{\sum\sum Y_{ij}}{n_1 + n_2}$$

Then:

$$\mathbf{X}'\mathbf{X} = \begin{bmatrix} n & n_2 \\ n_2 & n_2 \end{bmatrix} \qquad \mathbf{X}'\mathbf{Y} = \begin{bmatrix} n\bar{Y} \\ n_2\bar{Y}_2 \end{bmatrix}$$

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} \frac{1}{n_1} & -\frac{1}{n_1} \\ -\frac{1}{n_1} & \frac{1}{n_1} + \frac{1}{n_2} \end{bmatrix}$$

b. $\mathbf{b} = \begin{bmatrix} \bar{Y}_1 \\ \bar{Y}_2 - \bar{Y}_1 \end{bmatrix}$

c. $SSR = n_1\bar{Y}_1^2 + n_2\bar{Y}_2^2 - n\bar{Y}^2$

$SSE = \sum\sum Y_{ij}^2 - n_1\bar{Y}_1^2 - n_2\bar{Y}_2^2$

8.36. a. $\hat{Y} = 999.912 + .00296x - 3.29518 \times 10^{-11}x^2$

b. $R^2 = .8855$ for second-order model; $R^2 = .6711$ for first-order model.

c. H_0: $\beta_{11} = 0$, H_a: $\beta_{11} \neq 0$. $s\{b_{11}\} = 1.400396 \times 10^{-11}$, $t^* = -3.29518 \times 10^{-11}/1.400396 \times 10^{-11} = -2.353$, $t(.975;\ 437) = 1.9654$. If $|t^*| \leq 1.9654$ conclude H_0, otherwise H_a. Conclude H_a. Alternatively, $SSR(x^2|x) = 2,039,681$, $SSE(x, x^2) = 160,985,454$, $F^* = (2,039,681/1) \div (160,985,454/437) = 5.5368$, $F(.95; 1, 437) = 3.8628$. If $F^* \leq 3.8628$ conclude H_0, otherwise H_a. Conclude H_a.

8.37. a. $\hat{Y} = .056288 + 0.000004585x_1 - .000088x_3 + 2.6982 \times 10^{-12}x_1^2 + .00016293x_3^2 + 8.3337 \times 10^{-7}x_1x_3$, $R^2 = .2485$

b. H_0: $\beta_{11} = \beta_{33} = \beta_{13} = 0$, H_a: not all $\beta_k = 0$ $(k = 11, 33, 13)$.

$SSR(x_1^2, x_3^2, x_1x_3|x_1, x_3) = .005477$, $SSE(x_1, x_3, x_1^2, x_3^2, x_1x_3) = .246385$, $F^* = (.005477/3) \div (.246385/437) = 3.2381$, $F(.99; 1, 437) = 3.8267$. If $F^* \leq 3.8267$ conclude H_0, otherwise H_a. Conclude H_0.

c. $\hat{Y} = .0584998 + 2.9419 \times 10^{-8}x_1 - 5.5765 \times 10^{-7}x_2 + .00068244x_3 - 3.3559 \times 10^{-15}x_1^2$, $R^2 = .1444$

8.38. a. $\hat{Y} = 150.07921 + 7.06617x + .10116x^2$

b. $R^2 = .6569$ for second-order model; $R^2 = .6139$ for first-order model.

c. H_0: $\beta_{11} = 0$, H_a: $\beta_{11} \neq 0$. $s\{b_{11}\} = .02722$, $t^* = .10116/.02722 = 3.716$, $t(.995; 110) = 2.621$. If $|t^*| \leq 2.621$ conclude H_0, otherwise H_a. Conclude H_a. Alternatively, $SSR(x^2|x) = 93,533.252$, $SSE(x, x^2) = 745,203.642$, $F^* = (93,533.252/1) \div (745,203.642/\ 110) = 13.807$, $F(.99; 1, 110) = 6.871$. If $F^* \leq 6.871$ conclude H_0, otherwise H_a. Conclude H_a.

8.39. a. $\hat{Y} = -207.5 + .0005515X_1 + .107X_2 + 149.0X_3 + 145.5X_4 + 191.2X_5$

b. $b_3 - b_4 = 3.5$, $s\{b_3 - b_4\} = 1.68$, $t(.95; 434) = 1.6484$, $3.5 \pm 1.6484(1.68)$, $0.730688 \leq \beta_3 - \beta_4 \leq 6.2693$

c. $H_0 : \beta_3 = \beta_4 = \beta_5 = 0$, H_a : not all $\beta_k = 0$ $(k = 3, 4, 5)$. $SSR(X_3, X_4, X_5|X_1, X_2) = 1,873,626$, $SSE(X_1, X_2, X_3, X_4, X_5) = 139,093,455$, $F^* = (1,873,626/3) \div (139,093,455/434) = 1.9487$, $F(.90; 3, 434) = 2.09645$. If $F^* \leq 2.09645$ conclude H_0, otherwise H_a. Conclude H_0. $P-$value=.121.

8.40. a. $\hat{Y} = .85738 + .28882X_1 - .01805X_2 + .01995X_3 + .28782X_4$

b. $s\{b_4\} = .30668$, $t(.99;\ 108) = 2.361$, $.28782 \pm 2.361(.30668)$, $-.476 \leq \beta_4 \leq 1.012$

c. $\hat{Y} = .99413 + .26414X_1 - .02283X_2 + .02429X_3 - 5.69520X_4 + .15576X_2X_4 - .02406X_3X_4$

$H_0 : \beta_5 = \beta_6 = 0$, H_a : not both $\beta_5 = 0$ and $\beta_6 = 0$. $SSR(X_2X_4, X_3X_4\ |X_1, X_2, X_3, X_4) = 5.1964$, $SSE(X_1, X_2, X_3, X_4, X_2X_4, X_3X_4) = 122.0468$, $F^* = (5.1964/2) \div (122.0468/106) = 2.257$, $F(.90; 2, 106) = 2.353$. If $F^* \leq 2.353$ conclude H_0, otherwise H_a. Conclude H_0.

8.41. a. $\hat{Y} = 2.0478 + .10369X_1 + .04030X_2 + .00660X_3 - .020761X_4 + 2.14999X_5 + 1.19033X_6 + .63348X_7$

b. $H_0 : \beta_2 = 0$, $H_a : \beta_2 \neq 0$. $s\{b_2\} = .01430$, $t^* = .04030/.01430 = 2.818$, $t(.975; 105) = 1.983$. If $|t^*| \leq 1.983$ conclude H_0, otherwise H_a. Conclude H_a. Alternatively, $SSR(X_2|X_1, X_3, X_4, X_5, X_6, X_7) = 15.52782$, $SSE(X_1, X_2, X_3, X_4, X_5, X_6, X_7) = 205.3634$, $F^* = (15.52782/1) \div (205.3634/105) = 7.9392$, $F(.95; 1, 105) = 3.932$. If $F^* \leq 3.932$ conclude H_0, otherwise H_a. Conclude H_a.

c. $s\{b_5\} = .46152$, $s\{b_6\} = .43706$, $s\{b_7\} = .42755$, $B = t(.99167; 105) = 2.433$

$$2.14999 \pm 2.443(.46152) \qquad 1.0225 \leq \beta_5 \leq 3.2775$$
$$1.19033 \pm 2.443(.43706) \qquad .1226 \leq \beta_6 \leq 2.2581$$
$$.63348 \pm 2.443(.42755) \qquad -.4110 \leq \beta_7 \leq 1.6780$$

8.42. a. $\hat{Y} = 3.0211 - .247X_1 - .000097X_2 + .4093X_3 + .124X_4 - .01324X_5\{1999\} - .1088X_5\{2001\} - .08306X_5\{2002\}$

b. $\hat{Y} = 2.38 - 0.453x_1 - 0.000144x_2 + 0.00016x_1x_2 + 0.92x_1^2 + 0.000001x_2^2 + 0.394X_3 + 0.115X_4 + 0.012X_5\{1999\} - 0.101X_5\{2001\} - 0.0581X_5\{2002\}$

$H_0 : \beta_3 = \beta_4 = \beta_5 = 0$, H_a : not all $\beta_k = 0$ $(k = 3, 4, 5)$. $SSE(R) = .65424, df_R = 28$, $SSE(F) = .62614, df_F = 25, MSE(F) = .02505$ $F^* = .37392$, $F(.95; 3, 30) = 2.9223$. If $F^* \leq 2.9223$ conclude H_0, otherwise H_a. Conclude H_0.

c. $H_0 : \beta_2 = \beta_5 = \beta_6 = \beta_7 = 0$, H_a : not all $\beta_k = 0$ $(k = 2, 5, 6, 7)$. $SSE(R) = .71795, df_R = 32$, $SSE(F) = .65424, df_F = 28, MSE(F) = .02337$, $F^* = .68154$, $F(.95; 4, 28) = 2.71408$. If $F^* \leq 2.71408$ conclude H_0, otherwise H_a. Conclude H_0.

Chapter 9

BUILDING THE REGRESSION MODEL I: MODEL SELECTION AND VALIDATION

9.9.

Variables in Model	R_p^2	AIC_p	C_p	$PRESS_p$
None	0	262.916	88.16	13,970.10
X_1	.6190	220.529	8.35	5,569.56
X_2	.3635	244.131	42.11	9,254.49
X_3	.4155	240.214	35.25	8,451.43
X_1, X_2	.6550	217.968	5.60	5,235.19
X_1, X_3	.6761	215.061	2.81	4,902.75
X_2, X_3	.4685	237.845	30.25	8,115.91
X_1, X_2, X_3	.6822	216.185	4.00	5,057.886

9.10. b.

$$\begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{bmatrix} 1 & .102 & .181 & .327 \\ & 1 & .519 & .397 \\ & & 1 & .782 \\ & & & 1 \end{bmatrix}$$

c. $\hat{Y} = -124.3820 + .2957X_1 + .0483X_2 + 1.3060X_3 + .5198X_4$

9.11. a.

Subset	$R_{a,p}^2$
X_1, X_3, X_4	.9560
X_1, X_2, X_3, X_4	.9555
X_1, X_3	.9269
X_1, X_2, X_3	.9247

9.12.

Subset	SBC_p
X_3, X_5, X_6	-126.601
X_5	-124.970
$X_3, X_5, X_6, X_8 = 2001$	-124.969

Note: Variable numbers for predictors are those in the appendix.

9.13. b.

$$\begin{array}{c} X_1 \\ X_2 \\ X_3 \end{array} \begin{bmatrix} 1 & .653 & -.046 \\ & 1 & -.423 \\ & & 1 \end{bmatrix}$$

c. $\hat{Y} = 87.1875 - .5645X_1 - .5132X_2 - .0720X_3$

9.14. a.

Subset	$R^2_{a,p}$
x_1, x_2, x_1^2, x_2^2	.75067
x_1, x_2, x_1x_2	.75066
x_1, x_2, x_1x_2, x_2^2	.74156

9.15. b.

$$\begin{array}{c} X_1 \\ X_2 \\ X_3 \end{array} \begin{bmatrix} 1 & .468 & -.089 \\ & 1 & .068 \\ & & 1 \end{bmatrix}$$

c. $\hat{Y} = 120.0473 - 39.9393X_1 - .7368X_2 + .7764X_3$

9.16. a.

Subset	$R^2_{a,p}$
$x_1, x_2, x_3, x_3^2, x_1x_2$	.8668
$x_1, x_2, x_3, x_2^2, x_3^2, x_1x_2$	.8652
$x_1, x_2, x_3, x_3^2, x_1x_2, x_1x_3$	.8638

9.17. a. X_1, X_3

b. .10

c. X_1, X_3

d. X_1, X_3

9.18. a. X_1, X_3, X_4

9.19 a. x_1, x_2, x_3, x_1x_2

b. $R^2_{a,p} = .8615$

9.20. X_3, X_5, X_6 in appendix.

9.21. $PRESS = 760.974$, $SSE = 660.657$

9.22. a.

$$\begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{bmatrix} 1 & .011 & .177 & .320 \\ & 1 & .344 & .221 \\ & & 1 & .871 \\ & & & 1 \end{bmatrix}$$

b.

	Model-building data set	Validation data set
b_0:	-127.596	-130.652
$s\{b_0\}$:	12.685	12.189
b_1:	.348	.347
$s\{b_1\}$:	.054	.048
b_3:	1.823	1.848
$s\{b_3\}$:	.123	.122
MSE:	27.575	21.446
R^2:	.933	.937

c. $MSPR = 486.519/25 = 19.461$

d. $\hat{Y} = -129.664 + .349X_1 + 1.840X_3$, $s\{b_0\} = 8.445$, $s\{b_1\} = .035$, $s\{b_3\} = .084$

9.23. a. $PRESS = 5,102.494$, $SSE = 1,680.465$

9.24. $X_i = 10$: $[E\{\hat{Y}\} - E\{Y\}]^2 = (-55)^2$, $[\hat{Y} - E\{\hat{Y}\}]^2 = (-47)^2$

$X_i = 20$: $[E\{\hat{Y}\} - E\{Y\}]^2 = (-705)^2$, $[\hat{Y} - E\{\hat{Y}\}]^2 = (-97)^2$

9.25. b.

$$\begin{array}{c} X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_{10} \\ X_{11} \\ X_{12} \end{array} \begin{bmatrix} 1 & .025 & -.101 & .161 & -.198 & -.172 & -.236 & -.164 \\ & 1 & .448 & .334 & .490 & .501 & .530 & .453 \\ & & 1 & .195 & .168 & .204 & .239 & .240 \\ & & & 1 & .067 & .086 & .060 & .128 \\ & & & & 1 & .990 & .909 & .764 \\ & & & & & 1 & .904 & .729 \\ & & & & & & 1 & .707 \\ & & & & & & & 1 \end{bmatrix}$$

Note: Variable numbers for predictor variables are those in the data set description.

c.

Subset	C_p
X_3, X_6, X_{10}	3.81
X_3, X_6, X_{10}, X_{11}	3.86
X_3, X_6, X_7, X_{10}	4.27

9.26. b.

$$\begin{array}{c} X_4 \\ X_6 \\ X_7 \\ X_8 \\ X_9 \\ X_{11} \\ X_{12} \\ X_{13} \\ X_{14} \\ X_{15} \\ X_{16} \end{array} \begin{bmatrix} 1 & -.063 & .016 & .040 & .021 & -.113 & -.145 & .169 & .181 & -.190 & .078 \\ & 1 & -.599 & .174 & .113 & .245 & .486 & -.025 & -.222 & .078 & .116 \\ & & 1 & -.023 & .054 & -.240 & -.359 & -.003 & .189 & -.028 & -.023 \\ & & & 1 & .921 & .056 & .264 & .033 & -.056 & .312 & .934 \\ & & & & 1 & -.100 & .059 & .173 & .034 & .145 & .891 \\ & & & & & 1 & .722 & -.753 & -.671 & .585 & .087 \\ & & & & & & 1 & -.466 & -.551 & .748 & .238 \\ & & & & & & & 1 & .513 & -.649 & -.052 \\ & & & & & & & & 1 & -.379 & -.023 \\ & & & & & & & & & 1 & .347 \\ & & & & & & & & & & 1 \end{bmatrix}$$

Note: Variable numbers for predictor variables are those in the data set description.

c.

Subset	SBC_p
X_6, X_9, X_{13}, X_{14}	3407.16
$X_6, X_8, X_9, X_{13}, X_{14}, X_{15}$	3407.41
$X_6, X_9, X_{13}, X_{14}, X_{15}$	3408.09

9.27. a.

	Model-building data set	Validation data set
b_0:	.6104	.6189
$s\{b_0\}$:	.0888	.1248
b_3:	.00388	.00399
$s\{b_3\}$:	.00163	.00211
b_6:	.00117	.00152
$s\{b_6\}$:	.000419	.000437
b_{10}:	.000293	.000157
$s\{b_{10}\}$:	.0000456	.0000622
MSE:	.00305	.00423
R^2:	.519	.293

b. $MSPR = .258271/56 = .00461$

c. $\hat{Y}' = .6272 + .00353X_3 + .00143X_6 + .000236X_{10}$, $s\{b_0\} = .0738$, $s\{b_3\} = .00129$, $s\{b_6\} = .000297$, $s\{b_{10}\} = .0000374$, where $Y' = \log_{10} Y$.

9.28. a.

	Model-building data set	Validation data set
b_0:	243.680	3015.63
$s\{b_0\}$:	1322.82	1189.63
b_6:	122.507	34.3137
$s\{b_6\}$:	41.1906	34.2984
b_9:	.578662	.221509
$s\{b_9\}$:	.075844	.057344
b_{13}:	296.117	269.557
$s\{b_{13}\}$:	34.3417	39.0049
b_{14}:	-224.020	-128.343
$s\{b_{14}\}$:	77.1406	70.4556
MSE:	4,816,124	4,484,316
R^2:	.463	.284

b. $MSPR = \dfrac{2,259,424,814}{220} = 10,270,113$

Chapter 10

BUILDING THE REGRESSION MODEL II: DIAGNOSTICS

10.5. a.

i:	1	2	3	...	6	7	8
$e(Y \mid X_1)$:	-4.475	4.525	-7.475	...	2.675	-6.325	5.675
$e(X_2 \mid X_1)$:	-1	1	-1	...	1	-1	1
$e(Y \mid X_2)$:	-13.38	-13.13	-16.38	...	-6.125	-6.375	-3.125
$e(X_1 \mid X_{2y})$:	-3	-3	-3	...	-1	-1	-1

i:	9	10	11	...	14	15	16
$e(Y \mid X_1)$:	-3.175	2.825	$-.175$	...	$-.025$	-1.025	4.975
$e(X_2 \mid X_1)$:	-1	1	-1	...	1	-1	1
$e(Y \mid X_2)$:	5.625	2.875	8.625	...	8.875	16.625	13.875
$e(X_1 \mid X_2)$:	1	1	1	...	3	3	3

c. $\hat{Y}(X_1) = 50.775 + 4.425X_1$, $\hat{X}_2(X_1) = 3$

$[Y - \hat{Y}(X_1)] = 4.375[X_2 - \hat{X}_2(X_1)]$, $\hat{Y} = 37.650 + 4.425X_1 + 4.375X_2$

10.6. a. $\hat{Y} = 3995.48 + .00091916X_1 + 12.1205X_2$

b.

i:	1	2	...	51	52
$e(Y \mid X_1)$:	-101.811	108.842	...	-279.061	-11.1165
$e(X_2 \mid X_1)$:	$-.205$	-1.205	...	$.312$	$.414$
$e(Y \mid X_2)$:	-95.621	152.904	...	-184.865	-27.843
$e(X_1 \mid X_2)$:	$4,036.66$	$32,043.7$	...	$106,600$	$-12,742.4$

d. $\hat{Y}(X_1) = 4079.87 + .000935X_1$, $\hat{X}_2(X_1) = 6.96268 + .00000135X_1$

$[Y - \hat{Y}(X_1)] = 12.1205[X_2 - \hat{X}_2(X_1)]$, $\hat{Y} = 3995.48 + .000919X_1 + 12.1205X_2$

10.7. a.

i:	1	2	...	45	46
$e(Y \mid X_1, X_2)$:	1.671	-11.680	...	-1.967	13.179
$e(X_3 \mid X_1, X_2)$:	$-.116$	$.193$	...	$-.265$	$-.232$
$e(Y \mid X_1, X_3)$:	$.589$	-7.218	...	-7.365	9.808
$e(X_2 \mid X_1, X_3)$:	-1.077	-4.213	...	4.134	$.554$
$e(Y \mid X_2, X_3)$:	-12.537	-9.734	...	-4.086	12.696
$e(X_1 \mid X_2, X_3)$:	11.081	$.573$	...	-1.272	-2.316

10.8. a.

i:	1	2	...	80	81
$e(Y \mid X_1, X_2)$:	$-.630$	-1.768	...	$-.129$	-1.068
$e(X_3 \mid X_1, X_2)$:	$-.039$	$.179$	...	$-.016$	$.016$
$e(Y \mid X_1, X_3)$:	-2.085	-2.75065	...	$.316$	$-.240$
$e(X_2 \mid X_1, X_3)$:	-3.491	-1.155	...	$.905$	1.950
$e(Y \mid X_2, X_3)$:	$-.242$	-3.259	...	$-.259$	-1.490
$e(X_1 \mid X_2, X_3)$:	-2.405	8.596	...	1.437	3.198

10.9. a&g.

i:	1	2	3	4	5	6
t_i:	$-.041$	$.061$	-1.361	1.386	$-.367$	$-.665$
D_i:	.0002	.0004	.1804	.1863	.0077	.0245

i:	7	8	9	10	11	12
t_i:	$-.767$	$.505$	$.465$	$-.604$	1.823	$.978$
D_i:	.0323	.0144	.0122	.0204	.1498	.0510

i:	13	14	15	16
t_i:	-1.140	-2.103	1.490	$.246$
D_i:	.1318	.3634	.2107	.0068

$t(.9969; 12) = 3.31$. If $|t_i| \leq 3.31$ conclude no outliers, otherwise outliers. Conclude no outliers.

c. $2p/n = 2(3)/16 = .375$, no

d. $\mathbf{X}'_{\text{new}} = \begin{bmatrix} 1 & 10 & 3 \end{bmatrix}$

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 1.2375 & -.0875 & -.1875 \\ & .0125 & 0 \\ & 0 & .0625 \end{bmatrix}$$

$h_{\text{new,new}} = .175$, no extrapolation

e.

		DFBETAS			
	DFFITS	b_0	b_1	b_2	D
Case 14:	-1.174	$.839$	$-.808$	$-.602$	$.3634$

f. .68%

10.10. a&f.

i:	1	2	...	51	52
t_i:	$-.224$	1.225	...	-1.375	$.453$
D_i:	.0003	.0245	...	.0531	.0015

$t(.9995192; 47) = 3.523$. If $|t_i| \leq 3.523$ conclude no outliers, otherwise outliers. Conclude no outliers.

b. $2p/n = 2(4)/52 = .15385$. Cases 3, 5, 16, 21, 22, 43, 44, and 48.

c. $\mathbf{X}'_{\text{new}} = [\ 1 \quad 300,000 \quad 7.2 \quad 0\ ]$

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 1.8628 & -.0000 & -.1806 & .0473 \\ & .0000 & -.0000 & -.0000 \\ & & .0260 & -.0078 \\ & & & .1911 \end{bmatrix}$$

$h_{\text{new, new}} = .01829$, no extrapolation

d.

		DFBETAS				
	DFFITS	b_0	b_1	b_2	b_3	D
Case 16:	−.554	−.2477	−.0598	.3248	−.4521	.0769
Case 22:	.055	.0304	−.0253	−.0107	.0446	.0008
Case 43:	.562	−.3578	.1338	.3262	.3566	.0792
Case 48:	−.147	.0450	−.0938	.0090	−.1022	.0055
Case 10:	.459	.3641	−.1044	−.3142	−.0633	.0494
Case 32:	−.651	.4095	.0913	−.5708	.1652	.0998
Case 38:	.386	−.0996	−.0827	.2084	−.1270	.0346
Case 40:	.397	.0738	−.2121	.0933	−.1110	.0365

e. Case 16: .161%, case 22: .015%, case 43: .164%, case 48: .042%, case 10: .167%, case 32: .227%, case 38: .152%, case 40: .157%.

10.11. a&f.

i:	1	2	...	45	46
t_i:	.0116	−.9332	...	−.5671	1.0449
D_i:	.000003	.015699	...	.006400	.024702

$t(.998913; 41) = 3.27$. If $|t_i| \leq 3.27$ conclude no outliers, otherwise outliers. Conclude no outliers.

b. $2p/n = 2(4)/46 = .1739$. Cases 9, 28, and 39.

c. $\mathbf{X}'_{\text{new}} = [\ 1 \quad 30 \quad 58 \quad 2.0\ ]$

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 3.24771 & .00922 & -.06793 & -.06730 \\ & .00046 & -.00032 & -.00466 \\ & & .00239 & -.01771 \\ & & & .49826 \end{bmatrix}$$

$h_{\text{new, new}} = .3267$, extrapolation

d.

		DFBETAS				
	DFFITS	b_0	b_1	b_2	b_3	D
Case 11:	.5688	.0991	−.3631	−.1900	.3900	.0766
Case 17:	.6657	−.4491	−.4711	.4432	.0893	.1051
Case 27:	−.6087	−.0172	.4172	−.2499	.1614	.0867

e. Case 11: 1.10%, case 17: 1.32% , case 27: 1.12%.

10.12. a&f.

i:	1	2	...	80	81
t_i:	−.9399	−1.3926	...	−1.9232	−.8095
D_i:	.0117	.0308	...	.0858	.0046

$t(.999938; 75) = 4.05$. If $|t_i| \leq 4.05$ conclude no outliers, otherwise outliers. Conclude no outliers.

b. $2p/n = 2(5)/81 = .1235$. Cases 3, 8, 53, 61, and 65.

c. $\mathbf{X}'_{\text{new}} = [\ 1 \quad 10 \quad 12 \quad .05 \quad 350,000\ ]$

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} .2584 & -.0003 & -.0251 & -.2508 & .0000 \\ & .0004 & -.0002 & .0031 & -.0000 \\ & & .0031 & .0219 & -.0000 \\ & & & .9139 & -.0000 \\ & & & & .0000 \end{bmatrix}$$

$h_{\text{new, new}} = .0402$, no extrapolation

d.

		DFBETAS					
	DFFITS	b_0	b_1	b_2	b_3	b_4	D
Case 61:	.639	−.0554	.0242	−.0076	.5457	.0038	.082
Case 8:	.116	−.0142	−.0072	.0030	.0955	.0126	.003
Case 3:	−.284	−.2318	−.1553	.2364	.1008	−.0115	.016
Case 53:	.525	−.0196	−.0240	−.0243	.4180	.0490	.055
Case 6:	−.873	.1951	−.5649	−.1767	−.6182	.4482	.137
Case 62:	.690	.2758	−.3335	−.2595	.0627	.4051	.088

e. Case 61: .300%, case 8: .054%, case 3: .192%, case 53: .235%, case 6: .556%, case 62: .417%.

10.13. a. $\hat{Y} = 1.02325 + .96569X_1 + .62916X_2 + .67603X_3$

b. H_0: $\beta_1 = \beta_2 = \beta_3 = 0$, H_a: not all $\beta_k = 0$ $(k = 1, 2, 3)$. $MSR = 127.553$, $MSE = 3.33216$, $F^* = 127.553/3.33216 = 38.28$, $F(.95; 3, 10) = 2.84$. If $F^* \leq 2.84$ conclude H_0, otherwise H_a. Conclude H_a.

c. H_0: $\beta_k = 0$, H_a: $\beta_k \neq 0$. $t(.975; 10) = 2.021$. If $|t^*| \leq 2.021$ conclude H_0, otherwise H_a.

$b_1 = .96569$, $s\{b_1\} = .70922$, $t_1^* = 1.362$, conclude H_0

$b_2 = .62916$, $s\{b_2\} = .77830$, $t_2^* = .808$, conclude H_0

$b_3 = .67603$, $s\{b_3\} = .35574$, $t_3^* = 1.900$, conclude H_0

No

d.

$$\begin{array}{c} X_1 \\ X_2 \\ X_3 \end{array} \begin{bmatrix} 1 & .9744 & .3760 \\ & 1 & .4099 \\ & & 1 \end{bmatrix}$$

10.14. a. $(VIF)_1 = (1 - .950179)^{-1} = 20.072$

$(VIF)_2 = (1 - .951728)^{-1} = 20.716$

$(VIF)_3 = (1 - .178964)^{-1} = 1.218$

b. $\hat{Y} = 3.16277 + 1.65806X_1$

10.15. b. $(VIF)_1 = 1$, $(VIF)_2 = 1$

10.16. b. $(VIF)_1 = 1.0086$, $(VIF)_2 = 1.0196$, $(VIF)_3 = 1.0144$.

10.17. b. $(VIF)_1 = 1.6323$, $(VIF)_2 = 2.0032$, $(VIF)_3 = 2.0091$

10.18. b.

$$(VIF)_1 = (1 - .193775)^{-1} = 1.2403$$
$$(VIF)_2 = (1 - .393287)^{-1} = 1.6482$$
$$(VIF)_3 = (1 - .244458)^{-1} = 1.3236$$
$$(VIF)_4 = (1 - .292147)^{-1} = 1.4127$$

10.19a,b&c.

i:	1	2	3	$\cdots$	23	24	25
e_i:	3.308	5.494	−2.525	$\cdots$	−.202	.172	2.035
$e(Y \mid X_1)$:	4.35	8.16	.53	$\cdots$	−14.52	27.41	−3.12
$e(X_3 \mid X_1)$:	.57	1.46	1.68	$\cdots$	−7.85	14.94	−2.83
$e(Y \mid X_3)$:	−2.63	−8.66	−.53	$\cdots$	1.15	−5.14	−1.72
$e(X_1 \mid X_3)$:	−17.03	−40.61	5.73	$\cdots$	3.87	−15.24	−10.79
Exp. value:	4.744	5.590	−2.724	$\cdots$	.522	1.050	2.724

H_0: normal, H_a: not normal. $r = .983$. If $r \geq .939$ conclude H_0, otherwise H_a. Conclude H_0.

d and e.

i:	1	2	3	$\cdots$	23	24	25
h_{ii}:	.071	.214	.046	$\cdots$	.074	.171	.060
t_i:	.645	1.191	−.484	$\cdots$	−.039	.035	.392

$t(.999; 21) = 3.53$. If $|t_i| \leq 3.53$ conclude no outliers, otherwise outliers. Conclude no outliers.

f.

		$DFBETAS$			
Case	$DFFITS$	b_0	b_1	b_3	D
7	−.340	−.240	−.151	.303	.040
16	.603	−.069	.152	.051	.092
18	1.000	−.464	.878	.115	.308

g. $(VIF)_1 = (VIF)_2 = 1.034$

10.20.a&b. $\hat{Y} = 134.400 - 2.133X_1 - 1.699X_2 + .0333X_1X_2$

i:	1	2	3	$\cdots$	17	18	19
e_i:	17.740	4.161	−4.616	$\cdots$	−7.061	−.582	−8.256
Exp. value:	14.564	5.789	−2.788	$\cdots$	−7.467	.000	−11.609

$r = .963$

c. $(VIF)_1 = 5.431$, $(VIF)_2 = 11.640$, $(VIF)_3 = 22.474$

d&e.

i:	1	2	3	$\cdots$	17	18	19
h_{ii}:	.276	.083	.539	$\cdots$	.144	.139	.077
t_i:	2.210	.399	−.629	$\cdots$	−.709	−.057	−.802

$t(.9987; 14) = 3.65$. If $|t_i| \leq 3.65$ conclude no outliers, otherwise outliers. Conclude no outliers.

f.

Case	$DFFITS$	$DFBETAS$ b_0	b_1	b_2	b_3	D
3	$-.680$	$-.652$	.592	.433	$-.482$	.121
7	1.749	1.454	-1.278	$-.742$	.848	.459
8	-4.780	-1.547	1.187	3.162	-3.286	4.991
15	.175	$-.016$	$-.035$	.077	$-.016$	.008

10.21. a. $(VIF)_1 = 1.305$, $(VIF)_2 = 1.300$, $(VIF)_3 = 1.024$

b&c.

i:	1	2	3	$\cdots$	32	33
e_i:	13.181	-4.042	3.060	$\cdots$	14.335	1.396
$e(Y \mid X_2, X_3)$:	26.368	-2.038	-31.111	$\cdots$	6.310	5.845
$e(X_1 \mid X_2, X_3)$:	$-.330$	$-.050$	.856	$\cdots$	.201	.111
$e(Y \mid X_1, X_3)$:	18.734	-17.470	8.212	$\cdots$	12.566	-8.099
$e(X_2 \mid X_1, X_3)$:	-7.537	18.226	-6.993	$\cdots$	2.401	12.888
$e(Y \mid X_1, X_2)$:	11.542	-7.756	15.022	$\cdots$	6.732	-15.100
$e(X_3 \mid X_1, X_2)$:	-2.111	-4.784	15.406	$\cdots$	-9.793	-21.247
Exp. value:	11.926	-4.812	1.886	$\cdots$	17.591	$-.940$

10.22. a. $\hat{Y}' = -2.0427 - .7120X_1' + .7474X_2' + .7574X_3'$, where $Y' = \log_e Y$, $X_1' = \log_e X_1$, $X_2' = \log_e(140 - X_2)$, $X_3' = \log_e X_3$

b.

i:	1	2	3	$\cdots$	31	32	33
e_i:	$-.0036$	.0005	$-.0316$	$\cdots$	$-.1487$	.2863	.1208
Exp. value:	.0238	.0358	$-.0481$	$\cdots$	$-.1703$	.2601	.1164

c. $(VIF)_1 = 1.339$, $(VIF)_2 = 1.330$, $(VIF)_3 = 1.016$

d&e.

i:	1	2	3	$\cdots$	31	32	33
h_{ii}:	.101	.092	.176	$\cdots$	.058	.069	.149
t_i:	$-.024$	.003	$-.218$	$\cdots$	$-.975$	1.983	.829

$t(.9985; 28) = 3.25$. If $|t_i| \leq 3.25$ conclude no outliers, otherwise outliers. Conclude no outliers.

f.

Case	$DFFITS$	$DFBETAS$ b_0	b_1	b_2	b_3	D
28	.739	.530	$-.151$	$-.577$	$-.187$	.120
29	$-.719$	$-.197$	$-.310$	$-.133$	.420	.109

10.23. $\hat{\mathbf{Y}} = \mathbf{Xb}$, $\hat{\mathbf{Y}}_{(i)} = \mathbf{Xb}_{(i)}$. From (10.33a), we obtain:

$$D_i = \frac{(\mathbf{Xb} - \mathbf{Xb}_{(i)})'(\mathbf{Xb} - \mathbf{Xb}_{(i)})}{pMSE} = \frac{(\mathbf{b} - \mathbf{b}_{(i)})'\mathbf{X}'\mathbf{X}(\mathbf{b} - \mathbf{b}_{(i)})}{pMSE}$$

10.24. $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}' = \mathbf{X}\mathbf{X}^{-1}(\mathbf{X}')^{-1}\mathbf{X}' = \mathbf{II} = \mathbf{I}$

$h_{ii} = 1$, $\hat{Y}_i = Y_i$

10.25. $MSE_{(i)} = \left[\frac{(n-p)SSE}{n-p} - \frac{e_i^2}{1-h_{ii}}\right] \div (n-p-1)$ from (10.25)

Substitution into (10.24a) yields (10.26):

$$t_i = e_i \left[\frac{n-p-1}{SSE(1-h_{ii}) - e_i^2}\right]^{1/2}$$

10.26. From Exercise 5.31, $\boldsymbol{\sigma}^2\{\hat{\mathbf{Y}}\} = \mathbf{H}\sigma^2$ or $\sigma^2\{\hat{Y}_i\} = \sigma^2 h_{ii}$; hence

$\sum \sigma^2\{\hat{Y}_i\} = \sigma^2 \sum h_{ii} = \sigma^2 p$ by (10.27)

10.27.a&b.

i:	57	58	59	$\cdots$	111	112	113
e_i:	$-.086$	$-.064$	$-.004$	$\cdots$	$-.049$	.086	.019
Exp. value:	$-.077$	$-.057$	$-.005$	$\cdots$	$-.043$	.084	.010

H_0: normal, H_a: not normal. $r = .990$. If $r \geq .980$ conclude H_0, otherwise H_a. Conclude H_0.

c. $(VIF)_3 = 1.065$, $(VIF)_6 = 1.041$, $(VIF)_{10} = 1.045$

$$\begin{array}{c} X_3 \\ X_6 \\ X_{10} \end{array} \begin{bmatrix} 1 & .161 & -.172 \\ & 1 & .086 \\ & & 1 \end{bmatrix}$$

Note: Variable numbers for predictor variables are those in the data set description.

d&e.

i:	57	58	59	$\cdots$	111	112	113
h_{ii}:	.055	.055	.069	$\cdots$	.042	.288	.067
t_i:	-1.617	-1.201	$-.079$	$\cdots$	$-.911$	1.889	.348

$t(.9999; 52) = 4.00$. If $|t_i| \leq 4.00$ conclude no outliers, otherwise outliers. Conclude no outliers.

f.

		$DFBETAS$				
Case	$DFFITS$	b_0	b_3	b_6	b_{10}	D
62	.116	.010	.007	$-.061$	.094	.003
75	.254	.222	$-.242$	.069	$-.066$	.016
87	$-.411$	.025	$-.031$	.022	$-.291$	.040
106	.757	$-.437$	.626	$-.400$	$-.032$	.138
112	1.200	$-.464$	.372	.051	1.132	.343

10.28.a&b.

i:	2	4	6	$\cdots$	436	438	440
e_i:	$-.794$	.323	4.615	$\cdots$	.078	.007	$-.008$
Exp. value:	-1.011	.644	1.011	$\cdots$	.249	.052	$-.010$

H_0: normal, H_a: not normal. $r = .636$. If $r \geq .982$ conclude H_0, otherwise H_a. Conclude H_a.

c. $(VIF)_6 = 1.0093$, $(VIF)_8 = 4.5906$,$(VIF)_9 = 4.2859$, $(VIF)_{13} = 1.4728$,$(VIF)_{14} = 1.1056$, $(VIF)_{15} = 1.4357$,

$$\begin{array}{c} X_6 \\ X_8 \\ X_9 \\ X_{13} \\ X_{14} \\ X_{15} \end{array} \begin{bmatrix} 1 & .174 & .113 & -.025 & -.222 & .078 \\ & 1 & .921 & .033 & -.056 & .312 \\ & & 1 & .173 & .034 & .145 \\ & & & 1 & .513 & -.649 \\ & & & & 1 & -.379 \end{bmatrix}$$

Note: Variable numbers for predictor variables are those in the data set description.

d&e.

i:	2	4	6	$\cdots$	436	438	440
h_{ii}:	.514	.090	.095	$\cdots$	.063	.041	.007
t_i:	-3.182	.926	31.797	$\cdots$	.219	.020	$-.021$

$t(.99989; 212) = 3.759$. If $|t_i| \leq 3.759$ conclude no outliers, otherwise outliers. Conclude case 6 is an outlier.

f.

		DFBETAS							
Case	*DFFITS*	b_0	b_6	b_8	b_9	b_{13}	b_{14}	b_{15}	D
2	-3.27	-2745.72	.403	$-.479$	$-.815$	1.184	$-.825$	1.188	1.467
8	$-.60$	595.06	$-.052$	.456	$-.548$	.006	$-.488$	$-.178$	.052
48	.31	134.34	$-.079$	.290	$-.271$	.088	$-.215$	.030	.014
128	$-.10$	170.25	$-.003$	$-.014$	.023	$-.093$	$-.222$	$-.038$	.001
206	.20	-399.49	$-.056$	.030	.0005	.157	$-.142$	$-.275$	.006
404	$-.12$	220.73	$-.019$	$-.011$	.018	$-.001$	$-.654$	$-.028$	.002
6	10.29	-8536.94	.274	-4.236	6.678	2.729	5.196	2.110	2.634

Chapter 11

BUILDING THE REGRESSION MODEL III: REMEDIAL MEASURES

11.6. a. $\hat{Y} = 19.4727 + 3.2689X$

i:	1	2	3	4	5	6
e_i:	5.225	4.763	-6.389	-2.162	-3.237	-5.044

i:	7	8	9	10	11	12
e_i:	2.838	1.032	6.418	-1.700	2.687	-4.431

b. $n_1 = 6$, $\bar{d}_1 = 2.821$, $n_2 = 6$, $\bar{d}_2 = 4.833$, $s = 1.572$,

$t^*_{BF} = (2.821 - 4.833)/\left(1.572\sqrt{1/6 + 1/6}\right) = -2.218$,

$t(.975; 10) = 2.228$. If $|t^*_{BF}| \leq 2.228$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

d. $\hat{s} = -.905 + .3226X$; smallest weight = .02607, case 3; largest weight = .18556, cases 4 and 7.

e. $\hat{Y} = 17.3006 + 3.4211X$

f.

	Unweighted	Weighted
$s\{b_0\}$:	5.5162	4.8277
$s\{b_1\}$:	.3651	.3703

g. $\hat{Y} = 17.2697 + 3.4234X$

11.7. a. $\hat{Y} = -5.750 + .1875X$

i:	1	2	3	4	5	6
e_i:	-3.75	5.75	-13.50	-16.25	-9.75	7.50

i:	7	8	9	10	11	12
e_i:	-10.50	26.75	14.25	-17.25	-1.75	18.50

b. $SSR^* = 123{,}753.125$, $SSE = 2{,}316.500$,

$X_{BP}^2 = (123,753.125/2)/(2,316.500/12)^2 = 1.66$, $\chi^2(.90;1) = 2.71$. If $X_{BP}^2 \leq 2.71$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

d. $\hat{v} = -180.1 + 1.2437X$

i:	1	2	3	4	5	6
weight:	.01456	.00315	.00518	.00315	.01456	.00518

i:	7	8	9	10	11	12
weight:	.00518	.00315	.01456	.00315	.01456	.00518

e. $\hat{Y} = -6.2332 + .1891X$

f.

	Unweighted	Weighted
$s\{b_0\}$:	16.7305	13.1672
$s\{b_1\}$:	.0538	.0506

g. $\hat{Y} = -6.2335 + .1891X$

11.8. b. $\hat{Y} = 31.4714 + 10.8120X_1 + 22.6307X_2 + 1.2581X_3 + 1.8523X_4$

i:	1	2	3	$\cdots$	63	64	65
e_i:	-3.2892	-3.2812	-.3274	$\cdots$	36.9093	-18.6811	-5.3643

c. $n_1 = 33$, $\bar{d}_1 = 2.7595$, $n_2 = 32$, $\bar{d}_2 = 10.1166$, $s = 6.3643$,

$t_{BF}^* = (2.7595 - 10.1166)/\left(6.3643\sqrt{1/33 + 1/32}\right) = -4.659$,

$t(.995;63) = 2.656$. If $|t_{BF}^*| \leq 2.656$ conclude error variance constant, otherwise error variance not constant. Conclude error variance not constant.

e. $\hat{s} = 2.420 + .3996X_3 + .2695X_4$

i:	1	2	3	$\cdots$	63	64	65
weight:	.0563	.0777	.0015	$\cdots$	.1484	.0941	.0035

f. $\hat{Y} = 29.4255 + 10.8996X_1 + 26.6849X_2 + 1.4253X_3 + 1.7239X_4$

g.

	Unweighted	Weighted
$s\{b_0\}$:	2.8691	1.3617
$s\{b_1\}$:	3.2183	1.4918
$s\{b_2\}$:	3.4846	1.6686
$s\{b_3\}$:	.2273	.2002
$s\{b_4\}$:	.2276	.3206

h. $\hat{Y} = 29.0832 + 11.0075X_1 + 26.8142X_2 + 1.4904X_3 + 1.6922X_4$

11.9. b. $c = .06$

c. $\hat{Y}^* = .410X_1^* + .354X_2^* + .165X_3^*$

$\hat{Y} = 21.7290 + 1.7380X_1 + .1727X_2 + .6929X_3$

11.10. a. $\hat{Y} = 3.32429 + 3.76811X_1 + 5.07959X_2$

d. $c = .07$

e. $\hat{Y} = 6.06599 + 3.84335X_1 + 4.68044X_2$

11.11. a. $\hat{Y} = 1.88602 + 15.1094X$ (47 cases)

$\hat{Y} = -.58016 + 15.0352X$ (45 cases)

b.

i:	1	2	...	46	47
u_i:	-1.4123	-.2711	...	4.6045	10.3331

smallest weights: .13016 (case 47), .29217 (case 46)

c. $\hat{Y} = -.9235 + 15.13552X$

d. 2nd iteration: $\hat{Y} = -1.535 + 15.425X$

3rd iteration: $\hat{Y} = -1.678 + 15.444X$

smallest weights: .12629 (case 47), .27858 (case 46)

11.12. a. $\hat{Y} = -193.924 + 5.248X$

b. smallest weight: .5582 (case 2)

c. $\hat{Y} = -236.259 + 5.838X$

d. 2nd iteration: $\hat{Y} = -241.577 + 5.914X$

3rd iteration: $\hat{Y} = -242.606 + 5.928X$

smallest weight: .5025 (case 2)

11.13. $Q_w = \sum \frac{1}{kX_i}(Y_i - \beta_0 - \beta_1 X_i)^2$

$$\frac{\partial Q_w}{\partial \beta_0} = -2\sum \frac{1}{kX_i}(Y_i - \beta_0 - \beta_1 X_i)$$

$$\frac{\partial Q_w}{\partial \beta_1} = -2\sum \frac{1}{k}(Y_i - \beta_0 - \beta_1 X_i)$$

Setting the derivatives equal to zero, simplifying, and substituting the least squares estimators b_0 and b_1 yields:

$$\sum \frac{Y_i}{X_i} - b_0 \sum \frac{1}{X_i} - nb_1 = 0$$

$$\sum Y_i - nb_0 - b_1 \sum X_i = 0$$

11.14. $b_{w1} = \dfrac{\sum w_i(X_i - \bar{X}_w)(Y_i - \bar{Y}_w)}{\sum w_i(X_i - \bar{X}_w)^2}$

since $\sum w_i(X_i - \bar{X}_w)(Y_i - \bar{Y}_w) = \sum w_i X_i Y_i - (\sum w_i)\bar{X}_w \bar{Y}_w$

$$= \sum w_i X_i Y_i - \frac{\sum w_i X_i \sum w_i Y_i}{\sum w_i}$$

and $\sum w_i(X_i - \bar{X}_w)^2 = \sum w_i X_i^2 - (\sum w_i)\bar{X}_w^2$

$$= \sum w_i X_i^2 - \frac{(\sum w_i X_i)^2}{\sum w_i}$$

11.17.
$$\begin{bmatrix} X_1/.3 & 0 & 0 & 0 \\ 0 & X_2/.3 & 0 & 0 \\ 0 & 0 & X_3/.3 & 0 \\ 0 & 0 & 0 & X_4/.3 \end{bmatrix}$$

11.18. $\mathbf{b}_w = (\mathbf{X}'\mathbf{W}\mathbf{X})^{-1}\mathbf{X}'\mathbf{W}\mathbf{Y}$

$$\boldsymbol{\sigma}^2\{\mathbf{b}_w\} = [(\mathbf{X}'\mathbf{W}\mathbf{X})^{-1}\mathbf{X}'\mathbf{W}](k\mathbf{W}^{-1})[(\mathbf{X}'\mathbf{W}\mathbf{X})^{-1}\mathbf{X}'\mathbf{W}]'$$

$$= k(\mathbf{X}'\mathbf{W}\mathbf{X})^{-1}\mathbf{X}'\mathbf{W}\mathbf{W}^{-1}[\mathbf{W}'\mathbf{X}(\mathbf{X}'\mathbf{W}\mathbf{X})^{-1}]$$

(since $(\mathbf{X}'\mathbf{W}\mathbf{X})^{-1}$ is symmetric)

$$= k(\mathbf{X}'\mathbf{W}\mathbf{X})^{-1}\mathbf{X}'\mathbf{I}\mathbf{W}\mathbf{X}(\mathbf{X}'\mathbf{W}\mathbf{X})^{-1}$$

(since $\mathbf{W}$ is symmetric)

$$= k(\mathbf{X}'\mathbf{W}\mathbf{X})^{-1}$$

11.19. $E\{b^R - \beta\}^2 = E\{b^R - E\{b^R\} + E\{b^R\} - \beta\}^2$

$$= E\{b^R - E\{b^R\}\}^2 + 2E\{b^R - E\{b^R\}\}[E\{b^R\} - \beta] + E\{E\{b^R\} - \beta\}^2$$

$$= \sigma^2\{b^R\} + 0 + [E\{b^R\} - \beta]^2$$

11.20. a. 38.3666

b. 38.5822, yes.

11.21. a.

X_h:	10	20	30	40	50
$E\{Y_h\}$:	120	220	320	420	520

c. Ordinary least squares: $E\{b_1\} = 10$, $\sigma^2\{b_1\} = .024$

Weighted least squares: $E\{b_1\} = 10$, $\sigma^2\{b_1\} = .01975$

11.22. a.

c	$(VIF)_1$	$(VIF)_2$	$(VIF)_3$	R^2
.000	1.6323	2.0032	2.0091	.68219
.005	1.6000	1.9506	1.9561	.68218
.010	1.5687	1.9002	1.9054	.68215
.020	1.5089	1.8054	1.8101	.68204
.030	1.4527	1.7181	1.7222	.68185
.040	1.3997	1.6374	1.6411	.68160
.050	1.3497	1.5626	1.5659	.68129

c	b_1^R	b_2^R	b_3^R
.000	−.5907	−.1106	−.2339
.005	−.5868	−.1123	−.2338
.010	−.5831	−.1140	−.2337
.020	−.5758	−.1171	−.2334
.030	−.5687	−.1200	−.2331
.040	−.5619	−.1228	−.2329
.050	−.5553	−.1253	−.2326

11.23. a. $\hat{Y} = 62.4054 + 1.5511X_1 + .5102X_2 + .1019X_3 - .1441X_4$

b.

c	$(VIF)_1$	$(VIF)_2$	$(VIF)_3$	$(VIF)_4$	R^2
.000	38.496	254.423	46.868	282.513	.9824
.002	9.844	51.695	11.346	57.092	.9823
.004	5.592	21.903	6.089	23.971	.9822
.006	4.183	12.253	4.359	13.248	.9822
.008	3.530	7.957	3.566	8.478	.9821
.020	2.456	2.108	2.323	2.015	.9819
.040	1.967	.986	1.833	.820	.9813
.060	1.674	.715	1.560	.560	.9805
.080	1.455	.591	1.360	.454	.9794
.010	1.284	.516	1.204	.396	.9783

c	b_1^R	b_2^R	b_3^R	b_4^R
.000	.6065	.5277	.0434	−.1603
.002	.5524	.3909	−.0159	−.3043
.004	.5351	.3519	−.0343	−.3452
.006	.5257	.3337	−.0439	−.3641
.008	.5193	.3233	−.0502	−.3748
.020	.4975	.3033	−.0694	−.3942
.040	.4751	.2986	−.0864	−.3958
.060	.4577	.2986	−.0984	−.3920
.080	.4429	.2992	−.1079	−.3873
.100	.4300	.2998	−.1157	−.3824

11.24. a. $\hat{Y} = 12.2138 - 0.1462X_1 + .2893X_2 + 1.4277X_3 + 0.0000X_4$

b. $\sum |Y_i - \hat{Y}_i| = 64.8315$

c. 66.9736, yes.

11.25. a. $\hat{Y} = 50.3840 - .7620x_1 - .5300x_2 - .2929x_1^2$

11.26. a. $t(.975; 10) = 2.228$, $b_{w1} = 3.4211$, $s\{b_{w1}\} = .3703$,

$3.4211 \pm 2.228(.3703)$, $2.5961 \leq \beta_1 \leq 4.2461$

11.27. a. $t(.95; 10) = 1.8125$, $b_{w1} = .18911$, $s\{b_{w1}\} = .05056$,

$.18911 \pm 1.8125(.05056)$, $.0975 \leq \beta_1 \leq .2808$

11.28. a. $\hat{Y} = 38.64062 + .33143x - .09107x^2$, $R^2 = .9474$

b. $\bar{X} = 47.5$, $b_1 = .331429$, $b_{11} = -.091071$,

$\hat{X}_{\max} = 47.5 - [.5(.331429)]/(-.091071) = 49.3196$

$\hat{Y}_h = 38.640625 + .331429(\hat{X}_{\max} - 47.5) - .091071(\hat{X}_{\max} - 47.5)^2 = 38.942$

11.29 a. First split point at $X = 57$, $SSE = 5108.14$

b. Second split point at $X = 66$, $SSE = 4148.78$

c. Third split point at $X = 47$, $SSE = 3511.66$

11.30 a. First split point at $X_1 = 37$, $SSE = 6753.62$

b. Second split point at $X_1 = 47$, $SSE = 5276.25$

c. Third split point at $X_1 = 30$, $SSE = 3948.85$

d. Fourth split point at $X_2 = 49$, for the region defined by $X_1 < 30$. $SSE = 3563.79$

Chapter 12

AUTOCORRELATION IN TIME SERIES DATA

12.1. a.

t:	1	2	3	4	5	6	7	8	9	10
ε_t:	3.5	2.8	3.1	3.1	.8	-1.1	$-.9$	-1.2	-1.0	-1.1
ε_{t-1}:	3.0	3.5	2.8	3.1	3.1	.8	-1.1	$-.9$	-1.2	-1.0

b.

t:	1	2	3	4	5	6	7	8	9	10
u_t:	.5	$-.7$	.3	0	-2.3	-1.9	.2	$-.3$	.2	$-.1$
ε_{t-1}:	3.0	3.5	2.8	3.1	3.1	.8	-1.1	$-.9$	-1.2	-1.0

12.2. Yes.

12.5. (1) $H_0 : \rho = 0$, $H_a : \rho \neq 0$. $d_L = 1.12$, $d_U = 1.45$. If $D > 1.45$ and $4 - D > 1.45$, conclude H_0, if $D < 1.12$ or $4 - D < 1.12$ conclude H_a, otherwise the test is inconclusive.

(2) $H_0 : \rho = 0$, $H_a : \rho < 0$. $d_L = 1.32$, $d_U = 1.66$. If $4 - D > 1.66$ conclude H_0, if $4 - D < 1.32$ conclude H_a, otherwise the test is inconclusive.

(3) $H_0 : \rho = 0$, $H_a : \rho > 0$. $d_L = 1.12$, $d_U = 1.45$. If $D > 1.45$ conclude H_0, if $D < 1.12$ conclude H_a, otherwise the test is inconclusive.

12.6. $H_0 : \rho = 0$, $H_a : \rho > 0$. $D = 2.4015$, $d_L = 1.29$, $d_U = 1.38$. If $D > 1.38$ conclude H_0, if $D < 1.29$ conclude H_a, otherwise the test is inconclusive. Conclude H_0.

12.7 $H_0 : \rho = 0$, $H_a : \rho > 0$. $D = 2.2984$, $d_L = 1.51$, $d_U = 1.59$. If $D > 1.59$ conclude H_0, if $D < 1.51$ conclude H_a, otherwise the test is inconclusive. Conclude H_0.

12.8. $H_0 : \rho = 0$, $H_a : \rho > 0$. $D = 2.652$, $d_L = .83$, $d_U = 1.52$. If $D > 1.52$ conclude H_0, if $D < .83$ conclude H_a, otherwise the test is inconclusive. Conclude H_0.

12.9. a. $\hat{Y} = -7.7385 + 53.9533X$, $s\{b_0\} = 7.1746$, $s\{b_1\} = 3.5197$

t:	1	2	3	4	5	6	7	8
e_t:	$-.0737$	$-.0709$	.5240	.5835	.2612	$-.5714$	-1.9127	$-.8276$

t:	9	10	11	12	13	14	15	16
e_t:	$-.6714$	.9352	1.803	.4947	.9435	.3156	$-.6714$	-1.0611

c. $H_0: \rho = 0$, $H_a: \rho > 0$. $D = .857$, $d_L = 1.10$, $d_U = 1.37$. If $D > 1.37$ conclude H_0, if $D < 1.10$ conclude H_a, otherwise the test is inconclusive. Conclude H_a.

12.10. a. $r = .5784$, $2(1 - .5784) = .8432$, $D = .857$

b. $b_0' = -.69434$, $b_1' = 50.93322$

$\hat{Y}' = -.69434 + 50.93322X'$

$s\{b_0'\} = 3.75590$, $s\{b_1'\} = 4.34890$

c. $H_0: \rho = 0$, $H_a: \rho > 0$. $D = 1.476$, $d_L = 1.08$, $d_U = 1.36$. If $D > 1.36$ conclude H_0, if $D < 1.08$ conclude H_a, otherwise the test is inconclusive. Conclude H_0.

d. $\hat{Y} = -1.64692 + 50.93322X$

$s\{b_0\} = 8.90868$, $s\{b_1\} = 4.34890$

f. $F_{17} = -1.64692 + 50.93322(2.210) + .5784(-.6595) = 110.534$, $s\{\text{pred}\} = .9508$, $t(.975; 13) = 2.160$, $110.534 \pm 2.160(.9508)$, $108.48 \leq Y_{17(\text{new})} \leq 112.59$

g. $t(.975; 13) = 2.160$, $50.93322 \pm 2.160(4.349)$, $41.539 \leq \beta_1 \leq 60.327$.

12.11. a.

ρ:	.1	.2	.3	.4	.5
SSE:	11.5073	10.4819	9.6665	9.0616	8.6710

ρ:	.6	.7	.8	.9	1.0
SSE:	8.5032	8.5718	8.8932	9.4811	10.3408

$\rho = .6$

b. $\hat{Y}' = -.5574 + 50.8065X'$, $s\{b_0'\} = 3.5967$, $s\{b_1'\} = 4.3871$

c. $H_0: \rho = 0$, $H_a: \rho > 0$. $D = 1.499$, $d_L = 1.08$, $d_U = 1.36$. If $D > 1.36$ conclude H_0, if $D < 1.08$ conclude H_a, otherwise test is inconclusive. Conclude H_0.

d. $\hat{Y} = -1.3935 + 50.8065X$, $s\{b_0\} = 8.9918$, $s\{b_1\} = 4.3871$

f. $F_{17} = -1.3935 + 50.8065(2.210) + .6(-.6405) = 110.505$, $s\{\text{pred}\} = .9467$, $t(.975; 13) = 2.160$, $110.505 \pm 2.160(.9467)$, $108.46 \leq Y_{17(\text{new})} \leq 112.55$

12.12. a. $b_1 = 49.80564$, $s\{b_1\} = 4.77891$

b. $H_0: \rho = 0$, $H_a: \rho \neq 0$. $D = 1.75$ (based on regression with intercept term), $d_L = 1.08$, $d_U = 1.36$. If $D > 1.36$ and $4 - D > 1.36$ conclude H_0, if $D < 1.08$ or $4 - D < 1.08$ conclude H_a, otherwise the test is inconclusive. Conclude H_0.

c. $\hat{Y} = .71172 + 49.80564X$, $s\{b_1\} = 4.77891$

e. $F_{17} = .71172 + 49.80564(2.210) - .5938 = 110.188$, $s\{\text{pred}\} = .9078$, $t(.975; 14) = 2.145$, $110.188 \pm 2.145(.9078)$, $108.24 \leq Y_{17(\text{new})} \leq 112.14$

f. $t(.975; 14) = 2.145$, $49.80564 \pm 2.145(4.77891)$, $39.555 \leq \beta_1 \leq 60.056$

12.13. a. $\hat{Y} = 93.6865 + 50.8801X$, $s\{b_0\} = .8229$, $s\{b_1\} = .2634$

t:	1	2	3	4	5	6	7
e_t:	-1.5552	$-.2471$	$-.1526$	$-.2078$	.3349	.6431	.2557

t:	8	9	10	11	12	13	14
e_t:	.5610	−.4949	−.6824	.0747	−.0817	−.2336	−1.0425

t:	15	16	17	18	19	20
e_t:	−.0299	.5671	.8066	.1203	.5750	.7294

c. $H_0: \rho = 0$, $H_a: \rho > 0$. $D = .974$, $d_L = .95$, $d_U = 1.15$. If $D > 1.15$ conclude H_0, if $D < .95$ conclude H_a, otherwise the test is inconclusive. The test is inconclusive.

12.14. a. $r = .3319$, $2(1 - .3319) = 1.3362$, $D = .974$

b. $b'_0 = 63.3840$, $b'_1 = 50.5470$

$\hat{Y}' = 63.3840 + 50.5470X'$

$s\{b'_0\} = .5592$, $s\{b'_1\} = .2622$

c. $H_0: \rho = 0$, $H_a: \rho > 0$. $D = 1.76$, $d_L = .93$, $d_U = 1.13$. If $D > 1.13$ conclude H_0, if $D < .93$ conclude H_a, otherwise the test is inconclusive. Conclude H_0.

d. $\hat{Y} = 94.8720 + 50.5470X$

$s\{b_0\} = .8370$, $s\{b_1\} = .2622$

f. $F_{21} = 94.8720 + 50.5470(3.625) + .3319(.7490) = 278.3535$, $s\{\text{pred}\} = .4743$, $t(.995; 17) = 2.898$, $278.3535 \pm 2.898(.4743)$, $276.98 \leq Y_{21(\text{new})} \leq 279.73$

g. $t(.995; 17) = 2.898$, $50.5470 \pm 2.898(.2622)$, $49.787 \leq \beta_1 \leq 51.307$

12.15. a.

ρ:	.1	.2	.3	.4	.5
SSE:	4.0450	3.7414	3.5511	3.4685	3.4889

ρ:	.6	.7	.8	.9	1.0
SSE:	3.6126	3.8511	4.2292	4.7772	5.5140

$\rho = .4$

b. $\hat{Y}' = 57.04056 + 50.49249X'$, $s\{b'_0\} = .53287$, $s\{b'_1\} = .27697$

c. $H_0: \rho = 0$, $H_a: \rho > 0$. $D = 1.905$, $d_L = .93$, $d_U = 1.13$. If $D > 1.13$ conclude H_0, if $D < .93$ conclude H_a, otherwise test is inconclusive. Conclude H_0.

d. $\hat{Y} = 95.0676 + 50.49249X$, $s\{b_0\} = .88812$, $s\{b_1\} = .27697$

f. $F_{21} = 95.0676 + 50.49249(3.625) + .4(.7506) = 278.403$, $s\{\text{pred}\} = .4703$, $t(.995; 17) = 2.898$, $278.403 \pm 2.898(.4703)$, $277.04 \leq Y_{21(\text{new})} \leq 279.77$

g. $t(.995; 17) = 2.898$, $50.49249 \pm 2.898(.27697)$, $49.690 \leq \beta_1 \leq 51.295$

12.16. a. $b'_1 = 50.16414$, $s\{b'_1\} = .42496$, $\hat{Y}' = 50.16414X'$

b. $H_0: \rho = 0$, $H_a: \rho \neq 0$. $D = 2.425$ (based on regression with intercept term), $d_L = .93$, $d_U = 1.13$. If $D > 1.13$ and $4 - D > 1.13$ conclude H_0, if $D < .93$ or $4 - D < .93$ conclude H_a, otherwise test is inconclusive. Conclude H_0.

c. $\hat{Y} = 95.88984 + 50.16414X$, $s\{b_1\} = .42496$

e. $F_{21} = 95.88984 + 50.16414(3.625) + 1.116 = 278.851$, $s\{\text{pred}\} = .5787$, $t(.995; 18) = 2.878$, $278.851 \pm 2.878(.5787)$, $277.19 \leq Y_{21(\text{new})} \leq 280.52$

f. $t(.995; 18) = 2.878$, $50.16416 \pm 2.878(.42496)$, $48.941 \leq \beta_1 \leq 51.387$

12.17. a. Positive

b. $\hat{Y} = -1.43484 + .17616X$, $s\{b_0\} = .24196$, $s\{b_1\} = .0016322$

c.

t:	1	2	3	4	5	6	7
e_t:	$-.0307$	$-.0664$	.0180	.1593	.0428	.0429	.0582

t:	8	9	10	11	12	13	14
e_t:	$-.0613$	$-.0969$	$-.1517$	$-.1501$	$-.0754$	$-.0249$	.1043

t:	15	16	17	18	19	20
e_t:	.1844	.1054	.0289	.0422	$-.0439$	$-.0852$

d. $H_0 : \rho = 0$, $H_a : \rho > 0$. $D = .663$, $d_L = .95$, $d_U = 1.15$. If $D > 1.15$ conclude H_0, if $D < .95$ conclude H_a, otherwise the test is inconclusive. Conclude H_a.

12.18. a. $r = .67296$, $2(1 - .67296) = .65408$, $D = .663$

b. $\hat{Y} = -.29235 + .17261X'$, $s\{b_0'\} = .17709$, $s\{b_1'\} = .00351$.

c. $H_0 : \rho = 0$, $H_a : \rho > 0$. $D = 1.364$, $d_L = .93$, $d_U = 1.13$. If $D > 1.13$ conclude H_0, if $D < .93$ conclude H_a, otherwise test is inconclusive. Conclude H_0.

d. $\hat{Y} = -.89390 + .17261X$, $s\{b_0\} = .54149$, $s\{b_1\} = .00351$

f. $F_{21} = -.89390 + .17261(181.0) + .67296(-.015405) = 30.338$, $s\{\text{pred}\} = .09155$, $t(.95; 17) = 1.740$, $30.338 \pm 1.740(.09155)$, $30.179 \leq Y_{21(\text{new})} \leq 30.497$.

g. $t(.95; 17) = 1.740$, $.17261 \pm 1.740(.00351)$, $.1665 \leq \beta_1 \leq .1787$.

12.19. a.

ρ:	.1	.2	.3	.4	.5
SSE:	.1492	.1318	.1176	.1064	.09817

ρ:	.6	.7	.8	.9	1.0
SSE:	.09275	.08978	.08857	.08855	.09433

$\rho = .9$

b. $\hat{Y}' = .04644 + .16484X'$, $s\{b_0'\} = .11230$, $s\{b_1'\} = .006538$

c. $H_0 : \rho = 0$, $H_a : \rho > 0$. $D = 1.453$, $d_L = .93$, $d_U = 1.13$. If $D > 1.13$ conclude H_0, if $D < .93$ conclude H_a, otherwise test is inconclusive. Conclude H_0.

d. $\hat{Y} = .4644 + .16484X$, $s\{b_0\} = 1.1230$, $s\{b_1\} = .006538$.

f. $F_{21} = .4644 + .16484(181.0) + .9(-.03688) = 30.267$, $s\{\text{pred}\} = .09545$, $t(.95; 17) = 1.740$, $30.267 \pm 1.740(.09545)$, $30.101 \leq Y_{21(\text{new})} \leq 30.433$

g. $t(.95; 17) = 1.740$, $.16484 \pm 1.740(.006538)$, $.1535 \leq \beta_1 \leq .1762$.

12.20. a. $b_1' = .16883$, $s\{b_1'\} = .0055426$, $\hat{Y}' = .16883X'$

b. $H_0 : \rho = 0$, $H_a : \rho > 0$. $D = 1.480$ (based on regression with intercept term), $d_L = .93$, $d_U = 1.13$. If $D > 1.13$ conclude H_0, if $D < .93$ conclude H_a, otherwise test is inconclusive. Conclude H_0.

c. $\hat{Y} = -.35222 + .16883X$, $s\{b_1\} = .0055426$

e. $F_{21} = -.35222 + .16883(181.0) + .0942 = 30.300$, $s\{\text{pred}\} = .0907$, $t(.95; 18) = 1.734$, $30.300 \pm 1.734(.0907)$, $30.143 \leq Y_{21(\text{new})} \leq 30.457$

f. $t(.95; 18) = 1.734$, $.16883 \pm 1.734(.0055426)$, $.1592 \leq \beta_1 \leq .1784$

12.22.

$$\begin{aligned}
\sigma\{\varepsilon_t, \varepsilon_{t-2}\} &= E\{\varepsilon_t \varepsilon_{t-2}\} \\
&= E\{[u_t + \rho u_{t-1} + \rho^2 u_{t-2} + \rho^3 u_{t-3} + \cdots] \\
&\quad \times [u_{t-2} + \rho u_{t-3} + \rho^2 u_{t-4} + \cdots]\} \\
&= E\{[(u_t + \rho u_{t-1}) + \rho^2(u_{t-2} + \rho u_{t-3} + \cdots)] \\
&\quad \times [u_{t-2} + \rho u_{t-3} + \rho^2 u_{t-4} + \cdots]\} \\
&= E\{(u_t + \rho u_{t-1})(u_{t-2} + \rho u_{t-3} + \rho^2 u_{t-4} + \cdots)\} \\
&\quad + E\{\rho^2(u_{t-2} + \rho u_{t-3} + \rho^2 u_{t-4} + \cdots)^2\} \\
&= \rho^2 E\{u_{t-2} + \rho u_{t-3} + \rho^2 u_{t-4} + \cdots\}^2 = \rho^2 E\{\varepsilon_{t-2}^2\} \\
&= \rho^2 \sigma^2\{\varepsilon_{t-2}\} = \rho^2 \sigma^2\{\varepsilon_t\} = \rho^2 \left(\frac{\sigma^2}{1-\rho^2}\right)
\end{aligned}$$

12.23. a. $E\{Y\} = 100 - .35X$

t:	1	2	3	4	5	6
Y_t:	67.2058	61.5825	58.8570	67.2065	68.9889	73.4943

t:	7	8	9	10
Y_t:	74.8076	66.7686	62.9622	61.3573

$\hat{Y} = 96.08317 - .30839X$

b.

t:	1	2	3	4	5	6
Y_t:	65.7640	60.2590	57.7580	66.6920	69.7650	74.2510

t:	7	8	9	10
Y_t:	74.9610	67.1840	62.9510	61.5300

$\hat{Y} = 98.94338 - .34023X$

c.

t:	1	2	3	4	5	6
Y_t:	64.0819	60.9017	56.9518	67.4257	70.5170	74.0641

t:	7	8	9	10
Y_t:	74.7411	67.7152	62.2754	62.2122

$\hat{Y} = 99.45434 - .34576X$

$\varepsilon_t - \varepsilon_{t-1}$:

t:	1	2	3	4	5
$\rho = .6$:	$-.1972$	$-.3733$	$-.9750$	-2.1510	.0324
$\rho = 0$:	-1.6390	$-.2550$	$-.7510$	-1.5660	1.3230
$\rho = -.7$:	-3.3211	2.0698	-2.1999	$-.0261$	1.3413

t:	6	7	8	9	10
$\rho = .6$:	1.0054	$-.4367$	.7110	$-.3064$	.1451
$\rho = 0$:	.9860	-1.0400	.9730	$-.7330$	.3290
$\rho = -.7$:	.0471	-1.0730	1.7241	-1.9398	1.6868

d.

ρ	$\Sigma(\varepsilon_t - \varepsilon_{t-1})^2$
.6	7.579
0	11.164
-.7	32.687

12.24. $Y_t' = Y_t - \rho Y_{t-1}$

$$= \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \varepsilon_t - \rho(\beta_0 + \beta_1 X_{t-1,1} + \beta_2 X_{t-1,2} + \varepsilon_{t-1})$$

$$= \beta_0(1-\rho) + \beta_1(X_{t1} - \rho X_{t-1,1}) + \beta_2(X_{t2} - \rho X_{t-1,2}) + (\varepsilon_t - \rho\varepsilon_{t-1})$$

Since $\varepsilon_t - \rho\varepsilon_{t-1} = u_t$, we have:

$$Y_t' = \beta_0' + \beta_1' X_{t1}' + \beta_2' X_{t2}' + u_t$$

where $\beta_0' = \beta_0(1-\rho)$, $\beta_1' = \beta_1$, $\beta_2' = \beta_2$, $X_{t1}' = X_{t1} - \rho X_{t-1,1}$, and

$X_{t2}' = X_{t2} - \rho X_{t-1,2}$

12.25. a. $Y_t' = Y_t - \rho_1 Y_{t-1} - \rho_2 Y_{t-2}$

$X_t' = X_t - \rho_1 X_{t-1} - \rho_2 X_{t-2}$

b. By regressing the residuals e_t against the two independent variables e_{t-1} and e_{t-2} with no intercept term in the regression model and obtaining the two regression coefficients. The answer to Exercise 6.23a provides the explicit formulas, with Y, X_1, and X_2 replaced by e_t, e_{t-1}, and e_{t-2}, respectively.

c. By minimizing $SSE = \sum(Y_t' - b_0' - b_1' X_t')^2$ with respect to ρ_1 and ρ_2.

12.26. $Y_{n+1} = \beta_0 + \beta_1 X_{n+1} + \rho_1\varepsilon_n + \rho_2\varepsilon_{n-1} + u_{n+1}$

since $\varepsilon_{n+1} = \rho_1\varepsilon_n + \rho_2\varepsilon_{n-1} + u_{n+1}$. Therefore:

$$F_{n+1} = \hat{Y}_{n+1} + r_1 e_n + r_2 e_{n-1}$$

where r_1 and r_2 are point estimates of ρ_1 and ρ_2, respectively, obtained by either the Cochrane-Orcutt procedure or the Hildreth-Lu procedure.

12.27. c. $E\{b_1\} = 24$ even in presence of positive autocorrelation.

Chapter 13

INTRODUCTION TO NONLINEAR REGRESSION AND NEURAL NETWORKS

13.1. a. Intrinsically linear

$\log_e f(\mathbf{X}, \boldsymbol{\gamma}) = \gamma_0 + \gamma_1 X$

b. Nonlinear

c. Nonlinear

13.2. a. Intrinsically linear

$\log_e f(\mathbf{X}, \boldsymbol{\gamma}) = \gamma_0 + \gamma_1 \log_e X$

b. Intrinsically linear

$\log_e f(\mathbf{X}, \boldsymbol{\gamma}) = \log_e \gamma_0 + \gamma_1 \log_e X_1 + \gamma_2 \log_e X_2$

c. Nonlinear

13.3. b. 300, 3.7323

13.4. b. 49, 2.2774

13.5. a. $b_0 = -.5072512$, $b_1 = -0.0006934571$, $g_0^{(0)} = 0$, $g_1^{(0)} = .0006934571$, $g_2^{(0)} = .6021485$

b. $g_0 = .04823$, $g_1 = .00112$, $g_2 = .71341$

13.6. a. $\hat{Y} = .04823 + .71341\exp(-.00112X)$

City A

i:	1	2	3	4	5
$\hat{Y}_i$:	.61877	.50451	.34006	.23488	.16760
e_i:	.03123	$-.04451$	$-.00006$	.02512	.00240
Exp. value:	.04125	$-.04125$	$-.00180$	.02304	.00180

i:	6	7	8
$\hat{Y}_i$:	.12458	.07320	.05640
e_i:	.02542	$-.01320$	$-.01640$
Exp. value:	.02989	$-.01777$	$-.02304$

City B

i:	9	10	11	12	13
$\hat{Y}_i$:	.61877	.50451	.34006	.23488	.16760
e_i:	.01123	−.00451	−.04006	.00512	.02240
Exp. value:	.01327	−.00545	−.02989	.00545	.01777

i:	14	15	16
$\hat{Y}_i$:	.12458	.07320	.05640
e_i:	−.00458	.00680	−.00640
Exp. value:	−.00923	.00923	−.01327

13.7. $H_0: E\{Y\} = \gamma_0 + \gamma_2 \exp(-\gamma_1 X)$, $H_a: E\{Y\} \neq \gamma_0 + \gamma_2 \exp(-\gamma_1 X)$.

$SSPE = .00290$, $SSE = .00707$, $MSPE = .00290/8 = .0003625$,

$MSLF = (.00707 - .00290)/5 = .000834$, $F^* = .000834/.0003625 = 2.30069$, $F(.99; 5, 8) = 6.6318$. If $F^* \leq 6.6318$ conclude H_0, otherwise H_a. Conclude H_0.

13.8. $s\{g_0\} = .01456$, $s\{g_1\} = .000092$, $s\{g_2\} = .02277$, $z(.9833) = 2.128$

$.04823 \pm 2.128(.01456)$ $\qquad .01725 \leq \gamma_0 \leq .07921$

$.00112 \pm 2.128(.000092)$ $\qquad .00092 \leq \gamma_1 \leq .00132$

$.71341 \pm 2.128(.02277)$ $\qquad .66496 \leq \gamma_2 \leq .76186$

13.9. a. $g_0 = .04948$, $g_1 = .00112$, $g_2 = .71341$, $g_3 = -.00250$

b. $z(.975) = 1.96$, $s\{g_3\} = .01211$, $-.00250 \pm 1.96(.01211)$, $-.02624 \leq \gamma_3 \leq .02124$, yes, no.

13.10. a. $b_0 = .03376$, $b_1 = .454$, $g_0^{(0)} = 29.6209$, $g_1^{(0)} = 13.4479$

b. $g_0 = 28.13705$, $g_1 = 12.57445$

13.11. a. $\hat{Y} = 28.13705X/(12.57445 + X)$

b.

i:	1	2	3	4	5	6
$\hat{Y}_i$:	2.0728	2.9987	3.8611	5.4198	6.7905	8.0051
e_i:	.0272	−.4987	1.0389	.0802	.2095	.3949
Exp. value:	−.1076	−.5513	.9447	.1076	.2597	.3442

i:	7	8	9	10	11	12
$\hat{Y}_i$:	9.0890	10.5123	11.3486	12.4641	14.0268	15.3060
e_i:	.5110	−.3123	.0514	.0359	−.9268	−.7060
Exp. value:	.4390	−.3442	.0356	−.0356	−.9447	−.6983

i:	13	14	15	16	17	18
$\hat{Y}_i$:	16.3726	17.2755	18.7209	19.8267	20.7001	21.4074
e_i:	.6274	−.4755	−.1209	−.1267	.5999	.1926
Exp. value:	.6983	−.4390	−.1817	−.2597	.5513	.1817

c. No

13.12. $s\{g_0\} = .72798$, $s\{g_1\} = .76305$, $z(.975) = 1.960$

(1) $28.13705 \pm 1.960(.72798)$, $26.7102 \leq \gamma_0 \leq 29.5639$

(2) $H_0 : \gamma_1 = 20$, $H_a : \gamma_1 \neq 20$. $z^* = (12.57445 - 20)/.76305 = -9.731$.

If $|z^*| \leq 1.960$ conclude H_0, otherwise H_a. Conclude H_a.

13.13. $g_0 = 100.3401$, $g_1 = 6.4802$, $g_2 = 4.8155$

13.14. a. $\hat{Y} = 100.3401 - 100.3401/[1 + (X/4.8155)^{6.4802}]$

b.

i:	1	2	3	4	5	6	7
$\hat{Y}_i$:	.0038	.3366	4.4654	11.2653	11.2653	23.1829	23.1829
e_i:	.4962	1.9634	−1.0654	.2347	−.3653	.8171	2.1171
Expected Val.:	.3928	1.6354	−1.0519	−.1947	−.5981	.8155	2.0516

i:	8	9	10	11	12	13	14
$\hat{Y}_i$:	39.3272	39.3272	56.2506	56.2506	70.5308	70.5308	80.8876
e_i:	.2728	−1.4272	−1.5506	.5494	.2692	−2.1308	1.2124
Expected Val.:	.1947	−1.3183	−1.6354	.5981	.0000	−2.0516	1.0519

i:	15	16	17	18	19
$\hat{Y}_i$:	80.8876	87.7742	92.1765	96.7340	98.6263
e_i:	−.2876	1.4258	2.6235	−.5340	−2.2263
Expected Val.:	−.3928	1.3183	2.7520	−.8155	−2.7520

13.15. $H_0 : E\{Y\} = \gamma_0 - \gamma_0/[1 + (X/\gamma_2)^{\gamma_1}]$, $H_a : E\{Y\} \neq \gamma_0 - \gamma_0/[1 + (X/\gamma_2)^{\gamma_1}]$.

$SSPE = 8.67999$, $SSE = 35.71488$, $MSPE = 8.67999/6 = 1.4467$, $MSLF = (35.71488 - 8.67999)/10 = 2.7035$, $F^* = 2.7035/1.4467 = 1.869$, $F(.99; 10, 6) = 7.87$. If $F^* \leq 7.87$ conclude H_0, otherwise H_a. Conclude H_0.

13.16. $s\{g_0\} = 1.1741$, $s\{g_1\} = .1943$, $s\{g_2\} = .02802$, $z(.985) = 2.17$

$$100.3401 \pm 2.17(1.1741) \qquad 97.7923 \leq \gamma_0 \leq 102.8879$$
$$6.4802 \pm 2.17(.1943) \qquad 6.0586 \leq \gamma_1 \leq 6.9018$$
$$4.8155 \pm 2.17(.02802) \qquad 4.7547 \leq \gamma_2 \leq 4.8763$$

13.17. a. $b_0 = .98187$, $b_1 = .51485$, $b_2 = .29845$, $g_0^{(0)} = 9.5911$, $g_1^{(0)} = .51485$, $g_2^{(0)} = .29845$

b. $g_0 = 10.0797$, $g_1 = .49871$, $g_2 = .30199$

13.18. a. $\hat{Y} = 10.0797X_1^{.49871}X_2^{.30199}$

b.

i:	1	2	3	4	5	6
$\hat{Y}_i$:	10.0797	31.7801	100.1987	20.2039	63.7005	200.8399
e_i:	1.9203	.2199	2.8013	−.2039	−2.7005	−2.8399
Exp.val:	1.4685	.2880	2.7817	−.2880	−2.7817	−3.5476

i:	7	8	9	10	11	12
$\hat{Y}_i$:	40.4970	127.6823	402.5668	10.0797	31.7801	100.1987
e_i:	-2.4970	5.3177	3.4332	-2.0797	6.2199	-2.1987
Exp.val:	-2.0992	5.6437	3.5476	$-.8696$	7.6346	-1.4685

i:	13	14	15	16	17	18
$\hat{Y}_i$:	20.2039	63.7005	200.8399	40.4970	127.6823	402.5668
e_i:	-6.2039	-7.7005	4.1601	2.5030	.3177	-4.5668
Exp.val:	-5.6437	-7.6346	4.4559	2.0992	.8696	-4.4559

13.19. $H_0: E\{Y\} = \gamma_0 X_1^{\gamma_1} X_2^{\gamma_2}$, $H_a: E\{Y\} \neq \gamma_0 X_1^{\gamma_1} X_2^{\gamma_2}$. $F(.95; 6, 9) = 3.37$, $SSPE = 150.5$, $SSE = 263.443$, $SSLF = 112.943$, $F^* = [112.943/(15-9)] \div (150.5/9) = 1.126$. If $F^* \leq 3.37$ conclude H_0, otherwise H_a. Conclude H_0.

13.20. a. $H_0: \gamma_1 = \gamma_2$, $H_a: \gamma_1 \neq \gamma_2$. $F(.95; 1, 15) = 4.54$, $SSPE = 263.443$, $SSE = 9,331.62$, $MSPE = 263.443/15 = 17.563$, $MSLF = (9,331.62 - 263.443)/1 = 9,068.177$, $F^* = 9,068.177/17.563 = 516.327$. If $F^* \leq 4.54$ conclude H_0, otherwise H_a. Conclude H_a.

b. $s\{g_1\} = .00781$, $s\{g_2\} = .00485$, $z(.9875) = 2.24$

$$.49871 \pm 2.24(.00781) \qquad .4812 \leq \gamma_1 \leq .5162$$

$$.30199 \pm 2.24(.00485) \qquad .2911 \leq \gamma_2 \leq .3129$$

c. $\gamma_1 \neq \gamma_2$

13.21. a. $Q = \sum \{Y_i - [\gamma_0 + \gamma_2 \exp(-\gamma_1 X_i)]\}^2$

$$\frac{\partial Q}{\partial \gamma_0} = -2 \sum [Y_i - \gamma_0 - \gamma_2 \exp(-\gamma_1 X_i)]$$

$$\frac{\partial Q}{\partial \gamma_1} = 2 \sum [Y_i - \gamma_0 - \gamma_2 \exp(-\gamma_1 X_i)][\gamma_2 X_i \exp(-\gamma_1 X_i)]$$

$$\frac{\partial Q}{\partial \gamma_2} = -2 \sum [Y_i - \gamma_0 - \gamma_2 \exp(-\gamma_1 X_i)][\exp(-\gamma_1 X_i)]$$

Setting each derivative equal to zero, simplifying, and substituting the least squares estimators g_0, g_1, and g_2 yields:

$\sum Y_i - n g_0 - g_2 \sum exp(-g_1 X_i) = 0$

$g_2 \sum Y_i X_i \exp(-g_1 X_i) - g_0 g_2 \sum X_i \exp(-g_1 X_i) - g_2^2 \sum X_i \exp(-2 g_1 X_i) = 0$

$\sum Y_i \exp(-g_1 X_i) - g_0 \sum exp(-g_1 X_i) - g_2 \sum exp(-2 g_1 X_i) = 0$

b. $L(\boldsymbol{\gamma}, \sigma^2) = \dfrac{1}{(2\pi\sigma^2)^{n/2}} \exp\left\{-\dfrac{1}{2\sigma^2} \sum [Y_i - \gamma_0 - \gamma_2 \exp(-\gamma_1 X_i)]^2\right\}$

13.22. a. $Q = \sum \left(Y_i - \dfrac{\gamma_0 X_i}{\gamma_1 + X_i}\right)^2$

$$\frac{\partial Q}{\partial \gamma_0} = -2 \sum \left(Y_i - \frac{\gamma_0 X_i}{\gamma_1 + X_i}\right)\left(\frac{X_i}{\gamma_1 + X_i}\right)$$

$$\frac{\partial Q}{\partial \gamma_1} = 2 \sum \left(Y_i - \frac{\gamma_0 X_i}{\gamma_1 + X_i}\right)\left[\frac{\gamma_0 X_i}{(\gamma_1 + X_i)^2}\right]$$

Setting the derivatives equal to zero, simplifying, and substituting the least squares estimators g_0 and g_1 yields:

$$\sum \frac{Y_i X_i}{g_1 + X_i} - g_0 \sum \left(\frac{X_i}{g_1 + X_i} \right)^2 = 0$$

$$g_0 \sum \frac{Y_i X_i}{(g_1 + X_i)^2} - g_0^2 \sum \left[\frac{X_i^2}{(g_1 + X_i)^3} \right] = 0$$

b. $L(\boldsymbol{\gamma}, \sigma^2) = \dfrac{1}{(2\pi\sigma^2)^{n/2}} \exp \left[-\dfrac{1}{2\sigma^2} \sum \left(Y_i - \dfrac{\gamma_0 X_i}{\gamma_1 + X_i} \right)^2 \right]$

13.23. a. $Q = \sum (Y_i - \gamma_0 X_{i1}^{\gamma_1} X_{i2}^{\gamma_2})^2$

$$\frac{\partial Q}{\partial \gamma_0} = -2 \sum (Y_i - \gamma_0 X_{i1}^{\gamma_1} X_{i2}^{\gamma_2})(X_{i1}^{\gamma_1} X_{i2}^{\gamma_2})$$

$$\frac{\partial Q}{\partial \gamma_1} = -2 \sum (Y_i - \gamma_0 X_{i1}^{\gamma_1} X_{i2}^{\gamma_2})(\gamma_0 X_{i1}^{\gamma_1} X_{i2}^{\gamma_2} \log_e X_{i1})$$

$$\frac{\partial Q}{\partial \gamma_2} = -2 \sum (Y_i - \gamma_0 X_{i1}^{\gamma_1} X_{i2}^{\gamma_2})(\gamma_0 X_{i1}^{\gamma_1} X_{i2}^{\gamma_2} \log_e X_{i2})$$

Setting the derivatives equal to zero, simplifying, and substituting the least squares estimators g_0, g_1, and g_2 yields:

$$\sum Y_i X_{i1}^{g_1} X_{i2}^{g_2} - g_0 \sum X_{i1}^{2g_1} X_{i2}^{2g_2} = 0$$

$$g_0 \sum Y_i X_{i1}^{g_1} X_{i2}^{g_2} \log_e X_{i1} - g_0^2 \sum X_{i1}^{2g_1} X_{i2}^{2g_2} \log_e X_{i1} = 0$$

$$g_0 \sum Y_i X_{i1}^{g_1} X_{i2}^{g_2} \log_e X_{i2} - g_0^2 \sum X_{i1}^{2g_1} X_{i2}^{2g_2} \log_e X_{i2} = 0$$

b. $L(\boldsymbol{\gamma}, \sigma^2) = \dfrac{1}{(2\pi\sigma^2)^{n/2}} \exp \left[-\dfrac{1}{2\sigma^2} \sum (Y_i - \gamma_0 X_{i1}^{\gamma_1} X_{i2}^{\gamma_2})^2 \right]$

13.24. a. $E\{Y\} = E\left\{ \gamma_0 - \dfrac{\gamma_0}{1 + (X/\gamma_2)^{\gamma_1}} + \varepsilon \right\}$

$$= \gamma_0 - \frac{\gamma_0}{1 + (X/\gamma_2)^{\gamma_1}} = \gamma_0 \left[\frac{(X/\gamma_2)^{\gamma_1}}{1 + (X/\gamma_2)^{\gamma_1}} \right] = \gamma_0 \left(\frac{A}{1 + A} \right)$$

since $(X/\gamma_2)^{\gamma_1} = \exp[\gamma_1(\log_e X - \log_e \gamma_2)]$.

b. $E\{Y'\} = (1/\gamma_0)E\{Y'\} = A/(1 + A)$; hence:

$$\frac{E\{Y'\}}{1 - E\{Y'\}} = \frac{\dfrac{A}{1 + A}}{1 - \dfrac{A}{1 + A}} = A = \exp(\beta_0 + \beta_1 X')$$

c. $\log_e \left(\dfrac{Y'}{1 - Y'} \right) = \log_e \left(\dfrac{Y}{\gamma_0 - Y} \right)$, $X' = \log_e X$

d. Since $\beta_0 = -\gamma_1 \log_e \gamma_2$ or $\gamma_2 = \exp(-\beta_0/\gamma_1)$ and $\gamma_1 = \beta_1$, starting values are $g_1^{(0)} = b_1$ and $g_2^{(0)} = \exp\left(-b_0/g_1^{(0)}\right)$.

13.25.

(5, 5)	1,908.388	(35, 45)	480.747
(5, 15)	2,285.707	(35, 55)	694.863
(5, 25)	2,489.092	(35, 65)	887.306
(5, 35)	2,620.201	(45, 5)	4,551.038
(5, 45)	2,712.754	(45, 15)	782.035
(5, 55)	2,781.925	(45, 25)	127.119
(5, 65)	2,835.726	(45, 35)	66.999
(15, 5)	303.526	(45, 45)	176.620
(15, 15)	838.411	(45, 55)	336.160
(15, 25)	1,241.451	(45, 65)	504.661
(15, 35)	1,531.436	(55, 5)	8,987.574
(15, 45)	1,748.814	(55, 15)	2,191.748
(15, 55)	1,917.745	(55, 25)	631.873
(15, 65)	2,052.838	(55, 35)	179.473
(25, 5)	209.013	(55, 45)	92.431
(25, 15)	105.367	(55, 55)	145.951
(25, 25)	431.908	(55, 65)	255.430
(25, 35)	742.980	(65, 5)	14,934.461
(25, 45)	1,004.812	(65, 15)	4,315.713
(25, 55)	1,222.057	(65, 25)	1,574.725
(25, 65)	1,403.365	(65, 35)	592.257
(35, 5)	1,624.851	(65, 45)	228.178
(35, 15)	86.575	(65, 55)	124.234
(35, 25)	60.464	(65, 65)	139.613
(35, 35)	254.834		

13.26.

(1, .2, .1)	459,935	(11, .5, .7)	12,640,200
(1, .2, .4)	433,916	(11, .8, .1)	649,132
(1, .2, .7)	345,157	(11, .8, .4)	13,211,900
(1, .5, .1)	429,284	(11, .8, .7)	238,296,000
(1, .5, .4)	342,964	(21, .2, .1)	257,136
(1, .5, .7)	119,656	(21, .2, .4)	57,435.2
(1, .8, .1)	322,547	(21, .2, .7)	3,225,660
(1, .8, .4)	98,262.9	(21, .5, .1)	46,639.7
(1, .8, .7)	728,313	(21, .5, .4)	2,152,210
(11, .2, .1)	348,524	(21, .5, .7)	54,335,000
(11, .2, .4)	153,117	(21, .8, .1)	4,290,060
(11, .2, .7)	494,720	(21, .8, .4)	56,967,000
(11, .5, .1)	124,813	(21, .8, .7)	903,149,000
(11, .5, .4)	201,515		

Chapter 14

LOGISTIC REGRESSION, POISSON REGRESSION,AND GENERALIZED LINEAR MODELS

14.3. No

14.4. a. $E\{Y\} = [1 + \exp(25 - .2X)]^{-1}$

b. 125

c. $X = 150: \quad \pi = .993307149,\ \pi/(1-\pi) = 148.41316$

$X = 151: \quad \pi = .994513701,\ \pi/(1-\pi) = 181.27224$

$181.27224/148.41316 = 1.2214 = \exp(.2)$

14.5. a. $E\{Y\} = [1 + \exp(-20 + .2X)]^{-1}$

b. 100

c. $X = 125: \quad \pi = .006692851,\ \pi/(1-\pi) = .006737947$

$X = 126: \quad \pi = .005486299,\ \pi/(1-\pi) = .005516565$

$005516565/.006737947 = .81873 = \exp(-.2)$

14.6. a. $E\{Y\} = \Phi(-25 + .2X)$

b. 125

14.7. a. $b_0 = -4.80751,\ b_1 = .12508,\ \hat{\pi} = [1 + \exp(4.80751 - .12508X)]^{-1}$

c. 1.133

d. .5487

e. 47.22

14.8. a. $b_0 = -2.94964,\ b_1 = .07666,$

$\hat{\pi} = \Phi(-2.94964 + .07666X)$

b. $b_0 = -3.56532,\ b_1 = .08227,$

$\hat{\pi} = 1 - \exp(-\exp(-3.56532 + .08227X))$

14.9. a. $b_0 = -10.3089$, $b_1 = .01892$,

$\hat{\pi} = [1 + \exp(10.3089 - .01892X)]^{-1}$

c. 1.019

d. .5243

e. 589.65

14.10. a. $b_0 = -6.37366$, $b_1 = .01169$,

$\hat{\pi} = \Phi(-6.37366 + .01169X)$

b. $b_0 = -7.78587$, $b_1 = .01344$,

$\hat{\pi} = 1 - \exp(-\exp(-7.78587 + .01344X))$

14.11. a.

j:	1	2	3	4	5	6
p_j:	.144	.206	.340	.592	.812	.898

b. $b_0 = -2.07656$, $b_1 = .13585$

$\hat{\pi} = [1 + \exp(2.07656 - .13585X)]^{-1}$

d. 1.1455

e. .4903

f. 23.3726

14.12. a&b.

j:	1	2	3	4	5	6
p_j:	.112	.212	.372	.504	.688	.788

$b_0 = -2.6437$, $b_1 = .67399$

$\hat{\pi} = [1 + \exp(2.6437 - .67399X)]^{-1}$

d. 1.962

e. .4293

f. 3.922

14.13. a. $b_0 = -4.73931$, $b_1 = .067733$, $b_2 = .598632$,

$\hat{\pi} = [1 + \exp(4.73931 - .067733X_1 - .598632X_2)]^{-1}$

b. 1.070, 1.820

c. .6090

14.14. a. $b_0 = -1.17717$, $b_1 = .07279$, $b_2 = -.09899$, $b_3 = .43397$

$\hat{\pi} = [1 + \exp(1.17717 - .07279X_1 + .09899X_2 - .43397X_3)]^{-1}$

b. $\exp(b_1) = 1.0755$, $\exp(b_2) = .9058$, $\exp(b_3) = 1.5434$

c. .0642

14.15. a. $z(.95) = 1.645$, $s\{b_1\} = .06676$, $\exp[.12508 \pm 1.645(.06676)]$,

$1.015 \leq \exp(\beta_1) \leq 1.265$

b. $H_0 : \beta_1 = 0$, $H_a : \beta_1 \neq 0$. $b_1 = .12508$, $s\{b_1\} = .06676$, $z^* = .12508/.06676 = 1.8736$. $z(.95) = 1.645$, $|z^*| \leq 1.645$, conclude H_0, otherwise conclude H_a. Conclude H_a. P-value=.0609.

c. $H_0 : \beta_1 = 0$, $H_a : \beta_1 \neq 0$. $G^2 = 3.99$, $\chi^2(.90; 1) = 2.7055$. If $G^2 \leq 2.7055$, conclude H_0, otherwise conclude H_a. Conclude H_a. P-value=.046

14.16. a. $z(.975) = 1.960$, $s\{b_1\} = .007877$, $\exp[.01892 \pm 1.960(.007877)]$, $1.0035 \leq \exp(\beta_1) \leq 1.0350$

b. $H_0 : \beta_1 = 0$, $H_a : \beta_1 \neq 0$. $b_1 = .01892$, $s\{b_1\} = .007877$, $z^* = .01892/.007877 = 2.402$. $z(.975) = 1.960$, $|z^*| \leq 1.960$, conclude H_0, otherwise conclude H_a. Conclude H_a. P-value= .0163.

c. $H_0 : \beta_1 = 0$, $H_a : \beta_1 \neq 0$. $G^2 = 8.151$, $\chi^2(.95; 1) = 3.8415$. If $G^2 \leq 3.8415$, conclude H_0, otherwise conclude H_a. Conclude H_a. P-value=.004.

14.17. a. $z(.975) = 1.960$, $s\{b_1\} = .004772$, $.13585 \pm 1.960(.004772)$,

$$.1265 \leq \beta_1 \leq .1452, \qquad 1.1348 \leq \exp(\beta_1) \leq 1.1563.$$

b. $H_0 : \beta_1 = 0$, $H_a : \beta_1 \neq 0$. $b_1 = .13585$, $s\{b_1\} = .004772$, $z^* = .13585/.004772 = 28.468$. $z(.975) = 1.960$, $|z^*| \leq 1.960$, conclude H_0, otherwise conclude H_a. Conclude H_a. P-value= 0+.

c. $H_0 : \beta_1 = 0$, $H_a : \beta_1 \neq 0$. $G^2 = 1095.99$, $\chi^2(.95; 1) = 3.8415$. If $G^2 \leq 3.8415$, conclude H_0, otherwise conclude H_a. Conclude H_a. P-value= 0+.

14.18. a. $z(.995) = 2.576$, $s\{b_1\} = .03911$, $.67399 \pm 2.576(.03911)$,

$$.5732 \leq \beta_1 \leq .7747, \qquad 1.774 \leq \exp(\beta_1) \leq 2.170.$$

b. $H_0 : \beta_1 = 0$, $H_a : \beta_1 \neq 0$. $b_1 = .67399$, $s\{b_1\} = .03911$, $z^* = .67399/.03911 = 17.23$. $z(.995) = 2.576$, $|z^*| \leq 2.576$, conclude H_0, otherwise conclude H_a. Conclude H_a. P-value= 0+.

c. $H_0 : \beta_1 = 0$, $H_a : \beta_1 \neq 0$. $G^2 = 381.62$, $\chi^2(.99; 1) = 6.6349$. If $G^2 \leq 6.6349$, conclude H_0, otherwise conclude H_a. Conclude H_a. P-value= 0+.

14.19. a. $z(1-.1/[2(2)]) = z(.975) = 1.960$, $s\{b_1\} = .02806$, $s\{b_2\} = .3901$, $\exp\{20[.067733 \pm 1.960(.02806)]\}$, $1.29 \leq \exp(20\beta_1) \leq 11.64$, $\exp\{2[.5986 \pm 1.960(.3901)]\}$,$.72 \leq \exp(2\beta_2) \leq 15.28$.

b. $H_0 : \beta_2 = 0$, $H_a : \beta_2 \neq 0$. $b_2 = .5986$, $s\{b_2\} = .3901$, $z^* = .5986/.3901 = 1.53$. $z(.975) = 1.96$, $|z^*| \leq 1.96$, conclude H_0, otherwise conclude H_a. Conclude H_0. P-value= .125.

c. $H_0 : \beta_2 = 0$, $H_a : \beta_2 \neq 0$. $G^2 = 2.614$, $\chi^2(.95; 1) = 3.8415$. If $G^2 \leq 3.8415$, conclude H_0, otherwise conclude H_a. Conclude H_0. P-value= .1059.

d. $H_0 : \beta_3 = \beta_4 = \beta_5 = 0$, H_a : not all $\beta_k = 0$, for $k = 3, 4, 5$. $G^2 = 2.438$, $\chi^2(.95; 3) = 7.81$. If $G^2 \leq 7.81$, conclude H_0, otherwise conclude H_a. Conclude H_0. P-value= .4866.

14.20. a. $z(1-.1/[2(2)]) = z(.975) = 1.960$, $s\{b_1\} = .03036$, $s\{b_2\} = .03343$, $\exp\{30[.07279 \pm 1.960(.03036)]\}$, $1.49 \leq \exp(30\beta_1) \leq 52.92$, $\exp\{25[-.09899 \pm 1.960(.03343)]\}$,$.016 \leq \exp(2\beta_2) \leq .433$.

b. $H_0 : \beta_3 = 0$, $H_a : \beta_3 \neq 0$. $b_3 = .43397$, $s\{b_3\} = .52132$, $z^* = .43397/.52132 = .8324$. $z(.975) = 1.96$, $|z^*| \leq 1.96$, conclude H_0, otherwise conclude H_a. Conclude H_0. P-value$= .405$.

c. $H_0 : \beta_3 = 0$, $H_a : \beta_3 \neq 0$. $G^2 = .702$, $\chi^2(.95; 1) = 3.8415$. If $G^2 \leq 3.8415$, conclude H_0, otherwise conclude H_a. Conclude H_0.

d. $H_0 : \beta_3 = \beta_4 = \beta_5 = 0$, H_a : not all $\beta_k = 0$, for $k = 3, 4, 5$. $G^2 = 1.534$, $\chi^2(.95; 3) = 7.81$. If $G^2 \leq 7.81$, conclude H_0, otherwise conclude H_a. Conclude H_0.

14.21. a. X_1 enters in step 1;

no variables satisfy criterion for entry in step 2.

b. X_{22} is deleted in step 1; X_{11} is deleted in step 2; X_{12} is deleted in step 3; X_2 is deleted in step 4; X_1 is retained in the model.

c. The best model according to the AIC_p criterion is based on X_1 and X_2. $AIC_3 = 42.6896$.

d. The best model according to the SBC_p criterion is based on X_1. $SBC_2 = 46.2976$.

14.22. a. X_1 enters in step 1; X_2 enters in step 2;

no variables satisfy criterion for entry in step 3.

b. X_{11} is deleted in step 1; X_{12} is deleted in step 2; X_3 is deleted in step 3; X_{22} is deleted in step 4; X_1 and X_2 are retained in the model.

c. The best model according to the AIC_p criterion is based on X_1 and X_2. $AIC_3 = 111.795$.

d. The best model according to the SBC_p criterion is based on X_1 and X_2. $SBC_3 = 121.002$.

14.23.

j:	1	2	3	4	5	6
O_{j1}:	72	103	170	296	406	449
E_{j1}:	71.0	99.5	164.1	327.2	394.2	440.0
O_{j0}:	428	397	330	204	94	51
E_{j0}:	429.0	400.5	335.9	172.9	105.8	60.0

$H_0 : E\{Y\} = [1 + \exp(-\beta_0 - \beta_1 X)]^{-1}$,

$H_a : E\{Y\} \neq [1 + \exp(-\beta_0 - \beta_1 X)]^{-1}$.

$X^2 = 12.284$, $\chi^2(.99; 4) = 13.28$. If $X^2 \leq 13.28$ conclude H_0, otherwise H_a. Conclude H_0.

14.24.

j:	1	2	3	4	5	6
O_{j1}:	28	53	93	126	172	197
E_{j1}:	30.7	53.8	87.4	128.3	168.5	200.5
O_{j0}:	222	197	157	124	78	53
E_{j0}:	219.3	196.2	162.6	121.7	81.6	49.5

$H_0 : E\{Y\} = [1 + \exp(-\beta_0 - \beta_1 X)]^{-1}$,

$H_a : E\{Y\} \neq [1 + \exp(-\beta_0 - \beta_1 X)]^{-1}$.

$X^2 = 1.452$, $\chi^2(.99; 4) = 13.28$. If $X^2 \leq 13.28$ conclude H_0, otherwise H_a. Conclude H_0.

14.25. a.

Class j	$\hat{\pi}'$ Interval	Midpoint	n_j	p_j
1	-1.1 - under $-.4$	$-.75$	10	.3
2	$-.4$ - under .6	.10	10	.6
3	.6 - under 1.5	1.05	10	.7

b.

i:	1	2	3	$\cdots$	28	29	30
r_{SP_i}:	$-.6233$	1.7905	$-.6233$	$\cdots$	.6099	.5754	-2.0347

14.26. a.

Class j	$\hat{\pi}'$ Interval	Midpoint	n_j	p_j
1	-2.80 - under $-.70$	-1.75	9	.222
2	$-.70$ - under .80	.05	9	.556
3	.80 - under 2.00	1.40	9	.778

b.

i:	1	2	3	$\cdots$	25	26	27
dev_i:	$-.6817$	$-.4727$	$-.5692$	$\cdots$	1.0433	$-.8849$	.7770

14.27. a.

Class j	$\hat{\pi}'$ Interval	Midpoint	n_j	p_j
1	-3.00 - under -1.10	-2.050	11	.273
2	-1.10 - under .35	$-.375$	11	.182
3	.35 - under 3.00	1.675	11	.818

b.

i:	1	2	3	$\cdots$	31	32	33
r_{SP_i}:	$-.7584$	-1.0080	.7622	$\cdots$	$-.6014$	1.3700	$-.5532$

14.28. a.

j:	1	2	3	4	5	6	7	8
O_{j1}:	0	1	0	2	1	8	2	10
E_{j1}:	.2	.5	1.0	1.5	2.4	3.4	4.7	10.3
O_{j0}:	19	19	20	18	19	12	18	10
E_{j0}:	18.8	19.5	19.0	18.5	17.6	16.6	15.3	9.7

b. $H_0: E\{Y\} = [1 + \exp(-\beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3)]^{-1}$,
$H_a: E\{Y\} \neq [1 + \exp(-\beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3)]^{-1}$.
$X^2 = 12.116$, $\chi^2(.95; 6) = 12.59$. If $X^2 \leq 12.59$, conclude H_0, otherwise conclude H_a. Conclude H_0. P-value = .0594.

c.

i:	1	2	3	$\cdots$	157	158	159
dev_i:	$-.5460$	$-.5137$	1.1526	$\cdots$	.4248	.8679	1.6745

14.29 a.

i:	1	2	3	$\cdots$	28	29	30
h_{ii}:	.1040	.1040	.1040	$\cdots$	.0946	.1017	.1017

b.

i:	1	2	3	$\cdots$	28	29	30
ΔX_i^2:	.3885	3.2058	.3885	$\cdots$	4.1399	.2621	.2621
Δdev_i:	.6379	3.0411	.6379	$\cdots$	3.5071	.4495	.4495
D_i:	.0225	.1860	.0225	$\cdots$	.2162	.0148	.0148

14.30 a.

i:	1	2	3	$\cdots$	25	26	27
h_{ii}:	.0968	.1048	.1044	$\cdots$	.0511	.0744	.0662

b.

i:	1	2	3	$\cdots$	25	26	27
ΔX_i^2:	.2896	.1320	.1963	$\cdots$	.7622	.5178	.3774
Δdev_i:	.4928	.2372	.3445	$\cdots$	1.1274	.8216	.6287
D_i:	.0155	.0077	.0114	$\cdots$	.0205	.0208	.0134

14.31 a.

i:	1	2	3	$\cdots$	31	32	33
h_{ii}:	.0375	.0420	.0780	$\cdots$	.0507	.0375	.0570

b.

i:	1	2	3	$\cdots$	31	32	33
ΔX_i:	.5751	1.0161	.5809	$\cdots$	.3617	1.8769	.3061
Δdev_i^2:	.9027	1.4022	.9031	$\cdots$	.6087	2.1343	.5246
D_i:	.0112	.0223	.0246	$\cdots$	.0097	.0366	.0093

14.32 a.

i:	1	2	3	$\cdots$	157	158	159
h_{ii}:	.0197	.0186	.0992	$\cdots$	.0760	.1364	.0273

b.

i:	1	2	3	$\cdots$	157	158	159
ΔX_i^2:	.1340	.1775	1.4352	$\cdots$	.0795	.6324	2.7200
Δdev_i:	.2495	.3245	1.8020	$\cdots$	.1478	.9578	2.6614
D_i:	.0007	.0008	.0395	$\cdots$	.0016	.0250	.0191

14.33. a. $z(.95) = 1.645$, $\hat{\pi}'_h = .19561$, $s^2\{b_0\} = 7.05306$, $s^2\{b_1\} = .004457$, $s\{b_0, b_1\} = -.175353$, $s\{\hat{\pi}'_h\} = .39428$, $.389 \leq \pi_h \leq .699$

b.

Cutoff	Renewers	Nonrenewers	Total
.40	18.8	50.0	33.3
.45	25.0	50.0	36.7
.50	25.0	35.7	30.0
.55	43.8	28.6	36.7
.60	43.8	21.4	33.3

c. Cutoff = .50. Area = .70089.

14.34. a. $z(.975) = 1.960$, $s^2\{b_0\} = 19.1581$, $s^2\{b_1\} = .00006205$, $s\{b_0, b_1\} = -.034293$

X_h	$\hat{\pi}_h^{'}$	$s\{\hat{\pi}_h^{'}\}$	
550	.0971	.4538	$.312 \leq \pi_h \leq .728$
625	1.5161	.7281	$.522 \leq \pi_h \leq .950$

b.

Cutoff	Able	Unable	Total
.325	14.3	46.2	29.6
.425	14.3	38.5	25.9
.525	21.4	30.8	25.9
.625	42.9	30.8	37.0

c. Cutoff = .525. Area = .79670.

14.35. a. $z(.975) = 1.960$, $\hat{\pi}_h' = -.04281$, $s^2\{b_0\} = .021824$, $s^2\{b_1\} = .000072174$, $s\{b_0, b_1\} = -.0010644$, $s\{\hat{\pi}_h'\} = .0783$, $.451 \leq \pi_h \leq .528$

b.

Cutoff	Purchasers	Nonpurchasers	Total
.15	4.81	71.54	76.36
.30	11.70	45.15	56.84
.45	23.06	23.30	46.27
.60	23.06	23.30	46.27
.75	48.85	9.64	52.49

c. Cutoff = .45 (or .60). Area = .82445.

14.36. a. $\hat{\pi}_h' = -1.3953$, $s^2\{\hat{\pi}_h'\} = .1613$, $s\{\pi_h'\} = .4016$, $z(.95) = 1.645$. $L = -1.3953 - 1.645(.4016) = -2.05597$, $U = -1.3953 + 1.645(.4016) = -.73463$.
$L^* = [1 + \exp(2.05597)]^{-1} = .11345$, $U^* = [1 + \exp(.73463)]^{-1} = .32418$.

b.

Cutoff	Received	Not receive	Total
.05	4.35	62.20	66.55
.10	13.04	39.37	52.41
.15	17.39	26.77	44.16
.20	39.13	15.75	54.88

c. Cutoff = .15. Area = .82222.

14.38. a. $b_0 = 2.3529$, $b_1 = .2638$, $s\{b_0\} = .1317$, $s\{b_1\} = .0792$, $\hat{\mu} = \exp(2.3529 + .2638X)$.

b.

i:	1	2	3	$\cdots$	8	9	10
dev_i:	.6074	$-.4796$	$-.1971$	$\cdots$	.3482	.2752	.1480

c.

X_h:	0	1	2	3
Poisson:	10.5	13.7	17.8	23.2
Linear:	10.2	14.2	18.2	22.2

e. $\hat{\mu}_h = \exp(2.3529) = 10.516$

$$P(Y \leq 10 \mid X_h = 0) = \sum_{Y=0}^{10} \frac{(10.516)^Y \exp(-10.516)}{Y!}$$
$$= 2.7 \times 10^{-5} + \cdots + .1235 = .5187$$

f. $z(.975) = 1.96$, $.2638 \pm 1.96(.0792)$, $.1086 \leq \beta_1 \leq .4190$

14.39. a. $b_0 = .4895$, $b_1 = -1.0694$, $b_2 = -.0466$, $b_3 = .0095$, $b_4 = .0086$, $s\{b_0\} = .3369$, $s\{b_1\} = .1332$, $s\{b_2\} = .1200$, $s\{b_3\} = .0030$, $s\{b_4\} = .0043$,

$$\hat{\mu} = \exp(.4895 - 1.0694X_1 - .0466X_2 + .0095X_3 + .0086X_4)$$

b.

i:	1	2	3	$\cdots$	98	99	100
dev_i:	$-.4816$	$-.6328$	.4857	$\cdots$	$-.3452$	.0488	$-.9889$

c. $H_0: \beta_2 = 0$, $H_a: \beta_2 \neq 0$. $G^2 = .151$, $\chi^2(.95; 1) = 3.84$. If $G^2 \leq 3.84$ conclude H_0, otherwise H_a. Conclude H_0.

d. $b_1 = -1.0778$, $s\{b_1\} = .1314$, $z(.975) = 1.96$, $-1.0778 \pm 1.96(.1314)$, $-1.335 \leq \beta_1 \leq -.820$.

14.40. $$E\{Y\} = \frac{\exp(\beta_0 + \beta_1 X)}{1 + \exp(\beta_0 + \beta_1 X)} \left[\frac{\exp(-\beta_0 - \beta_1 X)}{\exp(-\beta_0 - \beta_1 X)}\right] = \frac{1}{1 + \exp(-\beta_0 - \beta_1 X)}$$
$$= [1 + \exp(-\beta_0 - \beta_1 X)]^{-1}$$

14.41. Formula (14.26) holds for given observations Y_1, Y_2,... Y_n. Assembling all terms with a given X value, X_j, we obtain:

$$y_{.j}(\beta_0 + \beta_1 X_j) - n_j \log_e[1 + \exp(\beta_0 + \beta_1 X_j)]$$

since there are n_j cases with X value X_j, of which $y_{.j}$ have value $Y_i = 1$. There are $\binom{n_j}{y_{.j}}$ ways of obtaining these $y_{.j}$ 1s out of n_j, all of which are equally likely. Hence, in the log-likelihood function of the $y_{.j}$, we must add $\log_e \binom{n_j}{y_{.j}}$ to the above term for given X_j :

$$\log_e \binom{n_j}{y_{.j}} + y_{.j}(\beta_0 + \beta_1 X_j) - n_j \log_e [1 + \exp(\beta_0 + \beta_1 X_j)]$$

Assembling the terms for all X_j, we obtain (14.34).

14.42. From (14.16) and (14.18), we have:

$$\pi_i = \frac{\exp(\pi')}{1+\exp(\pi')}$$

Then:

$$1-\pi = \frac{1+\exp(\pi')-\exp(\pi')}{1+\exp(\pi')} = [1+\exp(\pi')]^{-1}$$

$$\frac{\pi}{1-\pi} = \frac{\exp(\pi')}{1+\exp(\pi')} \times [1+\exp(\pi')] = \exp(\pi')$$

Solving for $\pi' = F_L^{-1}(\pi)$ by taking logarithms of both sides yields the result.

14.43. From (14.26), we obtain:

$$\frac{\partial^2 \log_e L}{\partial \beta_0^2} = -\sum_{i=1}^{n} \frac{\exp(\beta_0+\beta_1 X_i)}{[(1+\exp(\beta_0+\beta_1 X_i)]^2}$$

$$\frac{\partial^2 \log_e L}{\partial \beta_1^2} = -\sum_{i=1}^{n} \frac{X_i^2\exp(\beta_0+\beta_1 X_i)}{[1+\exp(\beta_0+\beta_1 X_i)]^2}$$

$$\frac{\partial^2 \log_e L}{\partial \beta_0 \partial \beta_1} = -\sum_{i=1}^{n} \frac{X_i\exp(\beta_0+\beta_1 X_i)}{[1+\exp(\beta_0+\beta_1 X_i)]^2}$$

Since these partial derivatives only involve the constants X_i, β_0, and β_1, the expectations of the partial derivatives are the partial derivatives themselves. Hence:

$$-E\left\{\frac{\partial^2 \log_e L}{\partial \beta_0^2}\right\} = -g_{00} \qquad -E\left\{\frac{\partial^2 \log_e L}{\partial \beta_0 \partial \beta_1}\right\} = -g_{01} = -g_{10}$$

$$-E\left\{\frac{\partial^2 \log_e L}{\partial \beta_1^2}\right\} = -g_{11}$$

and the stated matrix reduces to (14.51).

14.44.
$$\begin{bmatrix} 4.1762385 & 74.574657 \\ 74.574657 & 1,568.4817 \end{bmatrix}^{-1} = \begin{bmatrix} 1.58597 & -.075406 \\ -.075406 & .0042228 \end{bmatrix}$$

14.45. $E\{Y\} = \dfrac{\gamma_0}{1+\gamma_1\exp(\gamma_2 X)}$

Consider $\gamma_2 < 0$ and $\gamma_1 > 0$; as $X \to \infty$, $E\{Y\} = \pi \to 1$ so that

$$1 = \lim_{X\to\infty}\left[\frac{\gamma_0}{1+\gamma_1\exp(\gamma_2 X)}\right] = \gamma_0$$

Therefore, letting $\gamma_2 = -\beta_1$ and $\gamma_1 = \exp(-\beta_0)$ we have:

$$E\{Y\} = \frac{1}{1+\exp(-\beta_0-\beta_1 X)} = \frac{\exp(\beta_0+\beta_1 X)}{1+\exp(\beta_0+\beta_1 X)}$$

14.46. $E\{Y\} = [1+\exp(-\beta_0-\beta_1 X_1-\beta_2 X_2-\beta_3 X_1 X_2)]^{-1}$

$\pi'(X_1+1) = \beta_0+\beta_1(X_1+1)+\beta_2 X_2+\beta_3(X_1+1)X_2$

$\pi'(X_1) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2$

$\pi'(X_1 + 1) - \pi'(X_1) = \log_e(\text{odds ratio}) = \beta_1 + \beta_3 X_2$

Hence the odds ratio for X_1 is $\exp(\beta_1 + \beta_3 X_2)$. No.

14.47. $1 - \pi_i = \exp\left[-\exp\left(\dfrac{X_i - \gamma_0}{\gamma_1}\right)\right]$

$-\log_e(1 - \pi_i) = \exp\left(\dfrac{X_i - \gamma_0}{\gamma_1}\right)$

Hence: $\log_e[-\log_e(1 - \pi_i)] = \dfrac{X_i - \gamma_0}{\gamma_1} = -\dfrac{\gamma_0}{\gamma_1} + \dfrac{1}{\gamma_1} X_i = \beta_0 + \beta_1 X_i$

where $\beta_0 = -\dfrac{\gamma_0}{\gamma_1}$ and $\beta_1 = \dfrac{1}{\gamma_1}$.

14.48. a. $X_1 =$ age

Socioeconomic status	X_2	X_3
Upper	0	0
Middle	1	0
Lower	0	1

Sector	X_4
1	0
2	1

$b_0 = .1932$, $b_1 = .03476$, $b_2 = -1.9092$, $b_3 = -2.0940$, $b_4 = .9508$,

$b_5 = .02633$, $b_6 = .007144$, $b_7 = -.01721$, $b_8 = .004107$, $b_9 = .4145$,

where $\mathbf{X}' = (1 \quad X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_1X_2 \quad X_1X_3 \quad X_1X_4 \quad X_2X_4 \quad X_3X_4)$

b. $H_0: \beta_5 = \beta_6 = \beta_7 = \beta_8 = \beta_9 = 0$, H_a : not all equalities hold.

$G^2 = .858$, $\chi^2(.99; 5) = 15.09$. If $G^2 \leq 15.09$ conclude H_0, otherwise conclude H_a. Conclude H_0. P-value $= .973$.

c. Retain socioeconomic status and age.

14.49. a.

j:	1	2	3	4	5
O_{j1}:	6	4	13	15	16
E_{j1}:	4.6	7.3	11.7	14.8	15.7
O_{j0}:	14	16	7	5	2
E_{j0}:	15.4	12.7	8.3	5.2	2.3
n_j:	20	20	20	20	18

$H_0: E\{Y\} = [1 + \exp(-\beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3)]^{-1}$,

$H_a: E\{Y\} \neq [1 + \exp(-\beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3)]^{-1}$.

$X^2 = 3.28$, $\chi^2(.95; 3) = 7.81$. If $X^2 \leq 7.81$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .35$.

b.

i:	1	2	3	$\cdots$	96	97	98
dev_i:	.6107	.5905	-1.4368	$\cdots$	$-.8493$	$-.7487$	-1.0750

d&e.

i:	1	2	3	$\cdots$	96	97	98
h_{ii}	.0265	.0265	.0509	$\cdots$	.0305	.0316	.0410
ΔX_i^2:	.2106	.1956	1.9041	$\cdots$	.4479	.3341	.8156
Δdev_i:	.3785	.3538	2.1613	$\cdots$	.7350	.5712	1.1891
D_i:	.0014	.0013	.0255	$\cdots$	.0035	.0027	.0087

f.

Cutoff	Savings Account	No Savings Account	Total
.45	18.5	31.8	24.5
.50	22.2	31.8	26.5
.55	22.2	22.7	22.4
.60	29.6	22.7	26.5

Cutoff = .55. Area = .766.

14.50. a.

Cutoff	Savings Account	No Savings Account	Total
.55	24.5	28.9	26.5

b.

	Model Building Data Set	Combined Data Set
b_0:	.3711	.3896
$s\{b_0\}$:	.5174	.3493
b_1:	.03678	.03575
$s\{b_1\}$:	.01393	.00961
b_2:	-1.2555	-1.1572
$s\{b_2\}$:	.5892	.4095
b_3:	-1.9040	-2.0897
$s\{b_3\}$:	.5552	.3967

c. $z(.9833) = 2.128$, $\exp[.03575 \pm 2.128(.00961)]$, $1.015 \leq \exp(\beta_1) \leq 1.058$, $\exp[-1.1572 \pm 2.128(.4095)]$, $.132 \leq \exp(\beta_2) \leq .751$, $\exp\,[-2.0897 \pm 2.128(.3967)]$, $.053 \leq \exp(\beta_3) \leq .288$

14.51. a. X_1 = age, X_2 = routine chest X-ray ratio, X_3 = average daily census, X_4 = number of nurses

$b_0 = -8.8416$, $b_1 = .02238$, $b_2 = .005645$, $b_3 = .14721$, $b_4 = -.10475$, $b_5 = .0002529$, $b_6 = -.001995$, $b_7 = .0014375$, $b_8 = -.000335$, $b_9 = .0003912$, $b_{10} = -.00000519$,

where

$$\mathbf{X}' = (1 \quad X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_1X_2 \quad X_1X_3 \quad X_1X_4 \quad X_2X_3 \quad X_2X_4 \quad X_3X_4)$$

b. $H_0 : \beta_5 = \beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = 0$,

H_a : not all equalities hold. $G^2 = 7.45$, $\chi^2(.95; 6) = 12.59$. If $G^2 \leq 12.59$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .28

c. Retain age and average daily census.

d. The best subset: X_3, X_6, X_{10}, $AIC_4 = 59.6852$;

The best subset: X_{10}, $SBC_2 = 66.963$.

14.52. a.

j:	1	2	3	4	5
O_{j1}:	0	0	1	3	13
E_{j1}:	.1	.4	.9	2.3	13.3
O_{j0}:	22	23	21	20	10
E_{j0}:	21.9	22.6	21.1	20.7	9.7
n_j:	22	23	22	23	23

$H_0 : E\{Y\} = [1 + \exp(-\beta_0 - \beta_1 X_1 - \beta_3 X_3)]^{-1}$,

$H_a : E\{Y\} \neq [1 + \exp(-\beta_0 - \beta_1 X_1 - \beta_3 X_3)]^{-1}$,

$X^2 = .872$, $\chi^2(.95; 3) = 7.81$. If $X^2 \leq 7.81$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .832$

b

i:	1	2	3	$\cdots$	111	112	113
dev_i:	$-.3166$	$-.1039$	$-.1377$	$\cdots$	$-.1402$	.1895	$-.0784$

d & e.

i:	1	2	3	$\cdots$	111	112	113
h_{ii}	.0168	.0056	.0074	$\cdots$	.0076	.0279	.0041
ΔX_i^2:	.0523	.0054	.0096	$\cdots$	.0100	.0186	.0031
Δdev_i:	.1011	.0108	.0190	$\cdots$	.0197	.0364	.0062
D_i:	.00030	.00001	.00002	$\cdots$	.00003	.00018	.000004

f.

Cutoff	Affiliation	No Affiliation	Total
.30	29.4	9.4	12.4
.40	29.4	6.3	9.7
.50	41.2	4.2	9.7
.60	52.9	2.1	9.7

Cutoff = .40. Area = .923.

g. $z(.95) = 1.645$, $\hat{\pi}'_h = .6622$, $s^2\{b_0\} = 17.1276$, $s^2\{b_1\} = .006744$, $s^2\{b_3\} = .000006687$, $s\{b_0, b_1\} = -.33241$, $s\{b_0, b_3\} = .0003495$, $s\{b_1, b_3\} = -.00004731$, $s\{\hat{\pi}'_h\} = .6193$, $-.35655 \leq \pi'_h \leq 1.68095$, $.70 \leq \pi_h/(1 - \pi_h) \leq 5.37$

14.57. a. $b_1 = \begin{bmatrix} 33.249 \\ -1.905 \\ -.046 \\ -.039 \\ .039 \\ -4.513 \\ -.088 \\ .039 \\ -.085 \end{bmatrix}, \; b_2 = \begin{bmatrix} 12.387 \\ -.838 \\ -.016 \\ -.028 \\ .016 \\ .590 \\ .00008 \\ -.009 \\ -.097 \end{bmatrix}, \; b_3 = \begin{bmatrix} 13.505 \\ -.562 \\ -.095 \\ -.010 \\ .020 \\ -.595 \\ -.011 \\ -.008 \\ -.044 \end{bmatrix}$

b. $H_0 : b_{13} = b_{23} = b_{33} = 0$;
H_a: not all $b_{k3} = 0$, for $k = 1, 2, 3$. $G^2 = 2.34$, conclude H_0.
P-value=.5049.

c. $G^2 = 10.3$, conclude H_0. P-value=.1126.

d. $NE = 1, NC = 0:$ $\quad b_1 = \begin{bmatrix} -12.840 \\ .585 \\ .108 \\ .007 \\ -.017 \\ .231 \\ .008 \\ .009 \\ .023 \end{bmatrix}$

$NE = 1, S = 0:$ $\quad b_1 = \begin{bmatrix} -14.087 \\ .754 \\ .016 \\ .026 \\ -.025 \\ .567 \\ .010 \\ .010 \\ .113 \end{bmatrix}$

$NE = 1, W = 0:$ $\quad b_1 = \begin{bmatrix} -48.020 \\ 3.014 \\ .060 \\ .012 \\ -.033 \\ 7.415 \\ .079 \\ -.038 \\ .122 \end{bmatrix}$

e&f. $NE = 1, NC = 0:$

i:	1	2	3	$\cdots$	58	59	60
Dev_i:	-1.137	.708	-1.200	$\cdots$	$-.562$	-1.406	.547
ΔX_i^2:	1.061	.306	1.205	$\cdots$	.195	2.260	.347
Δdev_i:	1.445	.523	1.591	$\cdots$	.339	2.550	.485
D_i:	.020	.003	.019	$\cdots$	.003	.085	.044

$NE = 1, S = 0:$

i:	1	2	3	$\cdots$	63	64	65
Dev_i:	$-.327$	.630	-1.153	$\cdots$	$-.528$	.696	-1.080
ΔX_i^2:	.058	.237	1.028	$\cdots$	.164	1.189	1.030
Δdev_i:	.110	.415	1.413	$\cdots$	.293	1.400	1.404
D_i:	.0003	.0021	.0103	$\cdots$	.002	.441	.035

$NE = 1, W = 0:$

i:	1	2	3	$\cdots$	42	43	44
Dev_i:	$-.3762$	$-.3152$	.0177	$\cdots$	.0000	$-.8576$	.0000
ΔX_i^2:	.0936	.0852	.0002	$\cdots$	.0000	1.1225	.0000
Δdev_i:	.1618	.1336	.0003	$\cdots$	.0000	1.4135	.0000
D_i:	.0029	.0064	.0000	$\cdots$	.0000	.1903	.0000

14.58. a. $b_1 = \begin{bmatrix} -20.8100 \\ -.0016 \\ -.5738 \\ -.2150 \\ 142.1400 \\ .3998 \\ .2751 \\ .4516 \\ .2236 \\ -.0005 \end{bmatrix}$, $b_2 = \begin{bmatrix} 28.7900 \\ -.0013 \\ -.3878 \\ -.1253 \\ 147.73 \\ -.2426 \\ .3778 \\ .1510 \\ -.6755 \\ -.0004 \end{bmatrix}$, $b_3 = \begin{bmatrix} -18.4800 \\ -.0008 \\ -.0354 \\ -.1897 \\ 93.3700 \\ .2884 \\ -.2055 \\ .2979 \\ -.4803 \\ .00008 \end{bmatrix}$

b.

Row	*Term*	$\log L(\mathbf{b})$	G^2	P-value
1	X_5/X_4	-189.129	51.074	.0000
2	X_6	-178.009	28.834	.0000
3	X_7	-166.716	6.248	.1001
4	X_{10}/X_5	-192.499	57.814	.0000
5	X_{11}	-197.042	66.900	.0000
6	X_{12}	-186.324	45.464	.0000
7	X_{13}	-168.769	10.354	.0158
8	X_{14}	-183.663	40.142	.0000
9	X_{15}	-172.189	17.194	.0006

c. $NE = 1, NC = 0:$ $\quad b_1 = \begin{bmatrix} 7.8100 \\ .0009 \\ .1208 \\ .9224 \\ -107.4200 \\ -.3536 \\ .4683 \\ -.6225 \\ 1.0985 \\ -.0002 \end{bmatrix}$

$$NE = 1, S = 0: \qquad b_1 = \begin{bmatrix} -25.3800 \\ .0015 \\ .2399 \\ -.0852 \\ -172.7000 \\ .2126 \\ -.4522 \\ -.4086 \\ 1.7355 \\ .0006 \end{bmatrix}$$

$$NE = 1, W = 0: \qquad b_1 = \begin{bmatrix} -48.7700 \\ .0054 \\ 1.9580 \\ 1.3413 \\ -457.9000 \\ .0917 \\ -.6156 \\ -.7196 \\ -.3703 \\ .0005 \end{bmatrix}$$

d&e. $NE = 1, NC = 0:$

i:	1	2	3	$\cdots$	101	102	103
Dev_i:	-1.1205	.6339	$-.0909$	$\cdots$	$-.6718$	.0464	$-.4253$
ΔX_i^2:	1.1715	6.5919	.0042	$\cdots$	.3024	.0011	.1067
Δdev_i:	1.5536	6.7712	.0083	$\cdots$	.5006	.0022	.1929
D_i:	.0400	18.8671	.000	$\cdots$	.0059	.0000	.0014

$NE = 1, S = 0:$

i:	1	2	3	$\cdots$	122	123	124
Dev_i:	.6801	$-.0030$	.0542	$\cdots$	$-.5644$	$-.4215$	$-.4334$
ΔX_i^2:	5.5413	.0000	.0015	$\cdots$	.2338	.1275	.1091
Δdev_i:	5.7437	.0000	.0029	$\cdots$	.3797	.2123	.1985
D_i:	11.2465	.0000	.0000	$\cdots$	.0083	.0047	.0012

$NE = 1, W = 0:$

i:	1	2	3	$\cdots$	87	88	89
Dev_i:	$-.2713$	.0000	$-.0011$	$\cdots$	.0713	$-.2523$	.0004
ΔX_i^2:	.0506	.0000	.0000	$\cdots$	.0027	.0795	.0000
Δdev_i:	.0867	.0000	.0000	$\cdots$	.0052	.1108	.0000
D_i:	.0018	.0000	.0000	$\cdots$	.0000	.0116	.0000

14.59. a. $b = \begin{bmatrix} 4.6970 \\ 7.5020 \\ -.0509 \\ -.0359 \\ .0061 \\ -.0710 \\ -.0051 \\ .3531 \\ -.1699 \end{bmatrix}$

b. X_3, orX_4, or X_6, or X_7, or X_8 can be dropped.

c. Drop X_6, then X_7, then X_3, and then X_4, then stop.

d. The result is as follows:

```
Variable  Value       Count
Y(1)      1              33  (Event)
          0              64
          Total          97

Logistic Regression Table
Predictor        Coef     SE Coef         Z      P
Constant        3.767       2.208      1.71 0.088
PSA          -0.03499     0.02208     -1.59 0.113
age          -0.05548     0.03507     -1.58 0.114
Capspen       -0.2668      0.1498     -1.78 0.075

Response Information
Variable  Value       Count
Y(2)      1              76  (Event)
          0              21
          Total          97

Logistic Regression Table
Predictor        Coef     SE Coef         Z      P
Constant        8.704       3.595      2.42 0.015
PSA          -0.06045     0.01944     -3.11 0.002
age          -0.08484     0.05253     -1.61 0.106
Capspen      -0.14496     0.08098     -1.79 0.073

Log-Likelihood = -32.633
Test that all slopes are zero: G = 36.086, DF = 3, P-Value = 0.000
```

e&f. $Y^{(1)}$

i:	1	2	$\cdots$	95	96	97
Dev_i:	.8018	$-.3651$	$\cdots$	$-.0233$	$-.0113$	$-.0012$
ΔX_i^2:	.4036	.1476	$\cdots$	.0003	.0001	.0000
Δdev_i:	.6673	.1431	$\cdots$	.0005	.0001	.0000
D_i:	.0065	.0026	$\cdots$	.0000	.0000	.0000

$Y^{(2)}$

i:	1	2	$\cdots$	95	96	97
Dev_i:	.1545	.3077	$\cdots$	$-.0297$	$-.0031$	$-.0005$
ΔX_i^2:	.0122	.0492	$\cdots$	.0004	.0000	.0000
Δdev_i:	.0240	.0960	$\cdots$	.0009	.0000	.0000
D_i:	.00004	.00033	$\cdots$	.00000	.00000	.00000

14.60. a. $b = \begin{bmatrix} -133.0400 \\ -123.4400 \\ .00002 \\ .0014 \\ -.5250 \\ .9014 \\ 1.1787 \\ .5412 \\ -.3977 \\ .0585 \\ .0000 \\ .4336 \end{bmatrix}$

b. X_{13}, or X_{12}, or X_8, or X_7 can be dropped.

c. Drop X_{12}, then X_{13}, then X_8, then X_7, then stop.

e&f. $Y^{(1)}$

i:	1	2	$\cdots$	520	521	522
Dev_i:	$-.4904$	$-.1791$	$\cdots$	$-.0000$	$-.0143$	$-.0000$
ΔX_i^2:	.1300	.0160	$\cdots$	.0000	.0000	.0000
Δdev_i:	.2423	.0322	$\cdots$	.0000	.0002	.0000
D_i:	.0003	.0000	$\cdots$	.0000	.0000	.0000

$Y^{(2)}$

i:	1	2	$\cdots$	520	521	522
Dev_i:	.0162	.2472	$\cdots$	$-.2214$	$-.4205$	$-.0850$
ΔX_i^2:	.0000	.0320	$\cdots$	.0250	.0940	.0040
Δdev_i:	.0003	.0616	$\cdots$	.0492	.1782	.0072
D_i:	.00000	.00007	$\cdots$	.00003	.00020	.00000

14.61. a. The estimated regression coefficents and their estimated standard deviations are as follows,

```
Poisson Regression
Coefficient Estimates
Label            Estimate          Std. Error
Constant         0.499446          0.176041
Cost             0.0000149508      2.854645E-6
Age              0.00672387        0.00296715
Gender           0.181920          0.0439932
Interventions    0.0100748         0.00380812
Drugs            0.193237          0.0126846
```

```
Complications   0.0612547        0.0599478
Comorbids      -0.000899912      0.00368517
Duration        0.000352919      0.000189870
```

b.

i:	1	2	3	$\cdots$	786	787	788
Dev_i:	.2813	1.7836	-1.0373	$\cdots$	.6562	-1.2158	$-.0544$

c. X_3, or X_8, or X_9 or X_{10} can be dropped.

d. $G^2 = 5.262$, conclude X_0, the P-value=.1536.

e. We drop X_9, then drop X_8, then stop.

Chapter 15

INTRODUCTION TO THE DESIGN OF EXPERIMENTAL AND OBSERVATIONAL STUDIES

15.7. Panel.

15.8. a. Mixed. Type of instruction is an experimental factor, and school is an observational factor.

b. Factor 1: type of instruction, two levels (standard curriculum, computer-based curriculum).

Factor 2: school, three levels.

Randomized complete blocked design.

d. Section.

15.9. a. Observational.

b. Factor: expenditures for research and development in the past three years.

Factor levels: low, moderate, and high.

c. Cross-sectional study.

d. Firm.

15.10. a. Mixed. Color of paper is experimental factor, and parking lot is an observational factor.

b. Factor 1: color of paper, three levels (blue, green, orange).

Factor 2: supermarket parking lot, four levels.

c. Randomized complete block design.

d. Car.

15.11. a. Observational.

b. Fitness status, three levels (below average, average, above average).

c. Cross-sectional study.

d. Person

15.12. a. Mixed. Applicant's eye contact is an experimental factor, and personnel officer's gender is an observational factor.

b. Factor 1: applicant's eye contact, two levels (yes, no).

Factor 2: personnel officer's gender (male, female).

c. Randomized complete blocked design.

d. Personnel officer.

15.13. a. Mixed.

b. Wheel.

c. Four rubber compounds.

d. Randomized complete blocked design.

e. Balanced incomplete blocked design.

15.14. a. Experimental.

b. Factor 1: ingredient 1, with three levels (low, medium, high).

Factor 2: ingredient 2, with three levels (low, medium, high).

There are 9 factor-level combinations.

d. Completely randomized design.

e. Volunteer.

15.15. a. Observational.

b. Factor 1: treatment duration, with 2 levels (short, long).

Factor 2: weight gain, with 3 levels (slight, moderate, substantial)

c. Cross-sectional study.

d. Patient.

15.16. a. Mixed.

b. Factor: questionnaire, with 3 levels (A, B, C).

c. Repeated measure design.

d. Subject-time combination (i.e., the different occasions when a treatment is applied to a subject).

15.17. a. Observational.

b. Factor 1: batch, with 5 levels.

Factor 2: barrel, with 4 levels (nested within batch).

c. Nested design.

d. Barrel.

15.18. a. Experimental.

b. Factor 1: poly-film thickness, with 2 levels (low, high).

Factor 2: old mixture ratio, with 2 levels (low, high).

Factor 3: operator glove type, with 2 levels (cotton, nylon).

Factor 4: underside oil coating, with 2 levels (no coating, coating).

c. Fractional factorial design.

d. 1000 moldings in a batch.

15.19. a. Randomized complete block design with four blocks and three treatments.

c. Assembler.

15.20. a. 2^3 factorial design with two replicates.

c. Rod.

15.23. a. H_0: $\bar{W} = 0$, H_a: $\bar{W} \neq 0$. $t^* = -.1915/.0112 = -17.10$, $t(.975, 19) = 2.093$. If $|t^*| > 2.093$ conclude H_0, otherwise conclude H_a. Conclude H_a. P-value = 0+.

Agree with results on page 670. They should agree.

b. H_0: $\beta_2 = \cdots = \beta_{20} = 0$, H_a: not all β_k $(k = 2, 3, \ldots, 20)$ equal zero. $F^* = [(.23586 - .023828)/(38 - 19)] \div [.023828/19] = 8.90$, $F(.95; 19, 19) = 2.17$. If $F^* > 2.17$ conclude H_0, otherwise conclude H_a. Conclude H_a. P-value = 0+.

Not of primary interest because blocking factor was used here to increase the precision.

15.24. Since $\bar{X} = \frac{n/2}{n} = 1/2$, it follows from the definition of X_i that:

$\sum(X_i - \bar{X}) = n/2(1 - 1/2)^2 + n/2(0 - 1/2)^2 = n/4$.

Then from (15.5a): $\sigma^2\{b_1\} = 4\sigma^2/n$.

Chapter 16

SINGLE-FACTOR STUDIES

16.4. b. $E\{MSTR\} = 9 + \dfrac{25(450)}{2} = 5,634$

$E\{MSE\} = 9$

16.5. b. $E\{MSTR\} = (2.8)^2 + \dfrac{100(11)}{3} = 374.507$

$E\{MSE\} = 7.84$

c. $E\{MSTR\} = (2.8)^2 + \dfrac{100(15.46)}{3} = 523.173$

16.7. b. $\hat{Y}_{1j} = \bar{Y}_{1.} = 6.87778$, $\hat{Y}_{2j} = \bar{Y}_{2.} = 8.13333$, $\hat{Y}_{3j} = \bar{Y}_{3.} = 9.20000$

c. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	.772	1.322	−.078	−1.078	.022	−.278
2	−1.433	−.033	1.267	.467	−.333	−.433
3	−.700	.500	.900	−1.400	.400	.300

i	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$
1	−.578	.822	−.878			
2	.767	−.233	.167	.567	−1.033	.267

Yes

d.

Source	SS	df	MS
Between levels	20.125	2	10.0625
Error	15.362	24	.6401
Total	35.487	26	

e. H_0: all μ_i are equal ($i = 1, 2, 3$), H_a: not all μ_i are equal. $F^* = 10.0625/.6401 = 15.720$, $F(.95; 2, 24) = 3.40$. If $F^* \leq 3.40$ conclude H_0, otherwise H_a. Conclude H_a.

f. P-value = 0+

16.8. b. $\hat{Y}_{1j} = \bar{Y}_{1.} = 29.4$, $\hat{Y}_{2j} = \bar{Y}_{2.} = 29.6$, $\hat{Y}_{3j} = \bar{Y}_{3.} = 28.0$

c. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	-1.4	-3.4	1.6	-2.4	5.6
2	4.4	$-.6$	-4.6	1.4	$-.6$
3	3.0	-3.0	-1.0	1.0	0.0

d.

Source	SS	df	MS
Between colors	7.60	2	3.80
Error	116.40	12	9.70
Total	124.00	14	

e. H_0: all μ_i are equal $(i = 1, 2, 3)$, H_a: not all μ_i are equal. $F^* = 3.80/9.70 = .392$, $F(.90; 2, 12) = 2.81$. If $F^* \leq 2.81$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .684

16.9. b. $\hat{Y}_{1j} = \bar{Y}_{1.} = 38.0$, $\hat{Y}_{2j} = \bar{Y}_{2.} = 32.0$, $\hat{Y}_{3j} = \bar{Y}_{3.} = 24.0$

c. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	-9.0	4.0	0.0	2.0	5.0
2	-2.0	3.0	7.0	-4.0	-1.0
3	2.0	8.0	-3.0	-4.0	-1.0

i	$j=6$	$j=7$	$j=8$	$j=9$	$j=10$
1	2.0	-8.0	4.0		
2	-1.0	-3.0	3.0	-3.0	1.0
3	-2.0				

Yes

d.

Source	SS	df	MS
Between treatments	672.0	2	336.00
Error	416.0	21	19.81
Total	1,088.0	23	

e. H_0: all μ_i are equal $(i = 1, 2, 3)$, H_a: not all μ_i are equal. $F^* = 336.00/19.81 = 16.96$, $F(.99; 2, 21) = 5.78$. If $F^* \leq 5.78$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

16.10. b. $\hat{Y}_{1j} = \bar{Y}_{1.} = 21.500$, $\hat{Y}_{2j} = \bar{Y}_{2.} = 27.750$, $\hat{Y}_{3j} = \bar{Y}_{3.} = 21.417$

c. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	1.500	3.500	$-.500$	.500	$-.500$	.500
2	.250	$-.750$	$-.750$	1.250	-1.750	1.250
3	1.583	-1.417	3.583	$-.417$	.583	1.583

i	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$
1	-1.500	1.500	-2.500	.500	-2.500	$-.500$
2	$-.750$	2.250	.250	$-.750$	-1.750	1.250
3	$-.417$	-1.417	-2.417	-1.417	.583	$-.417$

d.

Source	SS	df	MS
Between ages	316.722	2	158.361
Error	82.167	33	2.490
Total	398.889	35	

e. H_0: all μ_i are equal $(i = 1, 2, 3)$, H_a: not all μ_i are equal. $F^* = 158.361/2.490 = 63.599$, $F(.99; 2, 33) = 5.31$. If $F^* \leq 5.31$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

16.11. b. $\hat{Y}_{1j} = \bar{Y}_{1.} = .0735$, $\hat{Y}_{2j} = \bar{Y}_{2.} = .1905$, $\hat{Y}_{3j} = \bar{Y}_{3.} = .4600$, $\hat{Y}_{4j} = \bar{Y}_{4.} = .3655$, $\hat{Y}_{5j} = \bar{Y}_{5.} = .1250$, $\hat{Y}_{6j} = \bar{Y}_{6.} = .1515$

c. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	−.2135	.1265	−.0035	.1065	.3065
2	.2695	−.0805	−.0705	.2795	.0495
3	−.2500	.3200	−.1400	−.0100	−.2400
4	.1245	.2145	.1545	−.0755	−.0955
5	−.3150	.1450	−.0650	−.0150	.1050
6	−.1015	−.2015	.1285	.3185	−.0315

i	$j=6$	$j=7$	$j=8$	$j=9$	$j=10$
1	.0265	−.1135	−.3435	.1965	−.2835
2	−.1305	−.3105	.1395	−.1305	−.2205
3	−.1100	.0800	−.2200	.0100	.1600
4	.1845	.0345	−.2255	.1145	−.0255
5	.0250	−.1150	.0950	.1650	.0150
6	.1185	−.0715	.0185	.2785	−.2215

i	$j=11$	$j=12$	$j=13$	$j=14$	$j=15$
1	.3165	−.1435	−.0935	.2065	.0165
2	−.1405	.3395	.2295	.0995	.1695
3	.0100	.0900	.1300	.2500	−.0100
4	−.3555	−.0355	−.1855	−.2355	.1145
5	.0750	.1750	−.2350	.1450	−.3250
6	.0485	−.1415	−.0515	.0085	−.2115

i	$j=16$	$j=17$	$j=18$	$j=19$	$j=20$
1	.0565	.1865	−.0035	−.0835	−.2635
2	−.1505	−.0205	−.1705	−.0805	−.0705
3	.0200	−.0200	.0400	−.2600	.1500
4	.1745	.1445	.0545	.0845	−.1655
5	.1150	.0750	.0150	.2250	−.3050
6	−.0215	.2785	.1985	−.2415	−.1015

Yes

d.

Source	SS	df	MS
Between machines	2.28935	5	.45787
Error	3.53060	114	.03097
Total	5.81995	119	

e. H_0: all μ_i are equal ($i = 1, ..., 6$), H_a: not all μ_i are equal. $F^* = .45787/.03097 = 14.78$, $F(.95; 5, 114) = 2.29$. If $F^* \leq 2.29$ conclude H_0, otherwise H_a. Conclude H_a.

f. P-value = 0+

16.12. b. $\hat{Y}_{1j} = \bar{Y}_{1.} = 24.55$, $\hat{Y}_{2j} = \bar{Y}_{2.} = 22.55$, $\hat{Y}_{3j} = \bar{Y}_{3.} = 11.75$, $\hat{Y}_{4j} = \bar{Y}_{4.} = 14.80$, $\hat{Y}_{5j} = \bar{Y}_{5.} = 30.10$

c. e_{ij}:

i	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$
1	−.55	−.55	4.45	−4.55	−3.55
2	−4.55	−2.55	−2.55	1.45	−.55
3	−1.75	−.75	−3.75	.25	.25
4	.20	−1.80	3.20	1.20	−2.80
5	2.90	−8.10	−2.10	4.90	−1.10

i	$j = 6$	$j = 7$	$j = 8$	$j = 9$	$j = 10$
1	.45	3.45	2.45	−1.55	−3.55
2	6.45	.45	1.45	5.45	−3.55
3	−1.75	2.25	−2.75	−3.75	−.75
4	4.20	−4.80	3.20	−3.80	2.20
5	−2.10	−.10	.90	−1.10	−2.10

i	$j = 11$	$j = 12$	$j = 13$	$j = 14$	$j = 15$
1	−.55	1.45	−1.55	−.55	3.45
2	1.45	2.45	−1.55	−2.55	1.45
3	4.25	.25	6.25	2.25	1.25
4	.20	−2.80	−1.80	−1.80	−.80
5	2.90	−.10	1.90	2.90	−1.10

i	$j = 16$	$j = 17$	$j = 18$	$j = 19$	$j = 20$
1	−1.55	−1.55	2.45	1.45	.45
2	−.55	−3.55	3.45	−.55	−1.55
3	−.75	2.25	−2.75	−.75	.25
4	2.20	1.20	2.20	−.80	1.20
5	4.90	1.90	−4.10	−.10	−1.10

Yes

d.

Source	SS	df	MS
Between agents	4,430.10	4	1,107.525
Error	714.65	95	7.523
Total	5,144.75	99	

e. H_0: all μ_i are equal ($i = 1, ..., 5$), H_a: not all μ_i are equal. $F^* = 1,107.525/7.523 = 147.22$, $F(.90; 4, 95) = 2.00$. If $F^* \leq 2.00$ conclude H_0, otherwise H_a. Conclude H_a.

f. P-value = 0+

16.15. $\mu_. = 80$, $\tau_1 = -15$, $\tau_2 = 0$, $\tau_3 = 15$

16.16. $\mu_. = 7.2$, $\tau_1 = -2.1$, $\tau_2 = -.9$, $\tau_3 = .7$, $\tau_4 = 2.3$

16.17. a. $\hat{\mu}_. = 20.4725$

b. H_0: all τ_i equal zero ($i = 1, ..., 5$), H_a: not all τ_i equal zero.

No

16.18. a.

$$\mathbf{Y} = \begin{bmatrix} 7.6 \\ 8.2 \\ 6.8 \\ 5.8 \\ 6.9 \\ 6.6 \\ 6.3 \\ 7.7 \\ 6.0 \\ 6.7 \\ 8.1 \\ 9.4 \\ 8.6 \\ 7.8 \\ 7.7 \\ 8.9 \\ 7.9 \\ 8.3 \\ 8.7 \\ 7.1 \\ 8.4 \\ 8.5 \\ 9.7 \\ 10.1 \\ 7.8 \\ 9.6 \\ 9.5 \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \quad \boldsymbol{\beta} = \begin{bmatrix} \mu_. \\ \tau_1 \\ \tau_2 \end{bmatrix}$$

b.

$$\mathbf{X}\boldsymbol{\beta} = \begin{bmatrix} \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_3 \\ \mu_3 \\ \mu_3 \\ \mu_3 \\ \mu_3 \\ \mu_3 \end{bmatrix}$$

c. $\hat{Y} = 8.07037 - 1.19259X_1 + .06296X_2$, $\mu_.$ defined in (16.63)

d.

Source	SS	df	MS
Regression	20.125	2	10.0625
Error	15.362	24	.6401
Total	35.487	26	

e. H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero. $F^* = 10.0625/.6401 = 15.720$, $F(.95; 2, 24) = 3.40$. If $F^* \leq 3.40$ conclude H_0, otherwise H_a. Conclude H_a.

16.19. a.

$$\mathbf{Y} = \begin{bmatrix} 28 \\ 26 \\ 31 \\ 27 \\ 35 \\ 34 \\ 29 \\ 25 \\ 31 \\ 29 \\ 31 \\ 25 \\ 27 \\ 29 \\ 28 \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \boldsymbol{\beta} = \begin{bmatrix} \mu_. \\ \tau_1 \\ \tau_2 \end{bmatrix}$$

b.

$$\mathbf{X}\boldsymbol{\beta} = \begin{bmatrix} \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_3 \\ \mu_3 \\ \mu_3 \\ \mu_3 \\ \mu_3 \end{bmatrix}$$

c. $\hat{Y} = 29.0 + .4X_1 + .6X_2$, $\mu_.$ defined in (16.63)

d.

Source	SS	df	MS
Regression	7.60	2	3.80
Error	116.40	12	9.70
Total	124.00	14	

e. H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = 3.80/9.70 = .392$, $F(.90; 2, 12) = 2.81$. If $F^* \leq 2.81$ conclude H_0, otherwise H_a. Conclude H_0.

16.20. a.

$$\mathbf{Y} = \begin{bmatrix} 29 \\ \vdots \\ 42 \\ 30 \\ \vdots \\ 33 \\ 26 \\ \vdots \\ 22 \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} 1 & 1 & 0 \\ \vdots & \vdots & \vdots \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ \vdots & \vdots & \vdots \\ 1 & 0 & 1 \\ 1 & -\frac{8}{6} & -\frac{10}{6} \\ \vdots & \vdots & \vdots \\ 1 & -\frac{8}{6} & -\frac{10}{6} \end{bmatrix} \quad \boldsymbol{\beta} = \begin{bmatrix} \mu_. \\ \tau_1 \\ \tau_2 \end{bmatrix}$$

b.

$$\mathbf{X}\boldsymbol{\beta} = \begin{bmatrix} \mu_. + \tau_1 \\ \vdots \\ \mu_. + \tau_1 \\ \mu_. + \tau_2 \\ \vdots \\ \mu_. + \tau_2 \\ \mu_. - \frac{8}{6}\tau_1 - \frac{10}{6}\tau_2 \\ \vdots \\ \mu_. - \frac{8}{6}\tau_1 - \frac{10}{6}\tau_2 \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_1 \\ \mu_2 \\ \vdots \\ \mu_2 \\ \mu_3 \\ \vdots \\ \mu_3 \end{bmatrix}$$

c. $\hat{Y} = 32.0 + 6.0X_1 + 0.0X_2$, $\mu_.$ defined in (16.80a)

d.

Source	SS	df	MS
Regression	672	2	336.00
Error	416	21	19.81
Total	1088	23	

e. H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = 336.00/19.81 = 16.96$, $F(.99; 2, 21) = 5.78$. If $F^* \leq 5.78$ conclude H_0, otherwise H_a. Conclude H_0.

16.21. a. $\hat{Y} = 23.55556 - 2.05556X_1 + 4.19444X_2$, $\mu_.$ defined in (16.63)

b.

Source	SS	df	MS
Regression	316.722	2	158.361
Error	82.167	33	2.490
Total	398.889	35	

H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero. $F^* = 158.361/2.490 = 63.599$, $F(.99; 2, 33) = 5.31$. If $F^* \leq 5.31$ conclude H_0, otherwise H_a. Conclude H_a.

16.22. a. $\hat{Y} = 38X_1 + 32X_2 + 24X_3$

b. $\hat{Y} = 32$

c. H_0: all μ_i are equal ($i = 1, 2, 3$), H_a: not all μ_i are equal. $SSE(F) = 416$, $SSE(R) = 1,088$, $F^* = (672/2) \div (416/21) = 16.96$, $F(.99; 2, 21) = 5.78$. If $F^* \leq 5.78$ conclude H_0, otherwise H_a. Conclude H_a.

16.23. $1 - \beta \cong .878$

16.24. a. $\mu_{.} = 15.5$, $\phi = 1.58$, $1 - \beta \cong .47$

b. $1 - \beta \cong .18$

16.25. $\mu_{.} = 7.889$, $\phi = 2.457$, $1 - \beta \cong .95$

16.26. $\mu_{.} = 33.917$, $\phi = 2.214$, $1 - \beta \cong .70$

16.27. $\mu_{.} = 24$, $\phi = 6.12$, $1 - \beta > .99$

16.29. a.

$\triangle$:	10	15	20	30
n :	51	23	14	7

b.

$\triangle$:	10	15	20	30
n :	39	18	11	6

16.30. a.

σ :	50	25	20
n :	34	10	7

b.

σ :	50	25	20
n :	30	9	6

16.31. a.

λ :	20	10	5
n :	10	38	150

b.

λ :	20	10	5
n :	22	85	337

16.32. a. $\Delta/\sigma = 4.5/3.0 = 1.5$, $n = 13$

b. $\Delta/\sigma = 6.0/3.0 = 2.0$, $1 - \beta \geq .95$

c. $n = [3.6173(3.0)/1.5]^2 = 53$

16.33. a. $\Delta/\sigma = 5.63/4.5 = 1.25$, $n = 20$

b. $\phi = \dfrac{1}{4.5}\left[\dfrac{20}{3}(40.6667)\right]^{1/2} = 3.659$, $1 - \beta \geq .99$

c. $(2.0\sqrt{n})/4.5 = 2.2302$, $n = 26$

16.34. a. $\Delta/\sigma = .15/.15 = 1.0$, $n = 22$

b. $\phi = \frac{1}{.15}\left[\frac{22}{6}(.02968)\right]^{1/2} = 2.199,\ 1-\beta \geq .97$

c. $(.10\sqrt{n})/.15 = 3.1591,\ n = 23$

16.35. a. $\Delta/\sigma = 3.75/3.0 = 1.25,\ n = 22$

b. $(1.0\sqrt{n})/3.0 = 2.5997,\ n = 61$

16.36.
$$L = \prod_{i=1}^{3}\prod_{j=1}^{2}\left(\frac{1}{2\pi\sigma^2}\right)^{1/2}\exp\left[-\frac{1}{2\sigma^2}(Y_{ij}-\mu_i)^2\right]$$
$$= \frac{1}{(2\pi\sigma^2)^3}\exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{3}\sum_{j=1}^{2}(Y_{ij}-\mu_i)^2\right]$$
$$\log_e L = -3\ \log_e 2\pi - 3\ \log_e\sigma^2 - \frac{1}{2\sigma^2}\sum\sum(Y_{ij}-\mu_i)^2$$
$$\frac{\partial(\log_e L)}{\partial\mu_i} = -\frac{2}{2\sigma^2}\sum_j(Y_{ij}-\mu_i)(-1)$$

Setting the partial derivatives equal to zero, simplifying, and substituting the maximum likelihood estimators yields:

$$\sum_j(Y_{ij}-\hat{\mu}_i) = 0$$

or:

$$\hat{\mu}_i = \bar{Y}_{i.}$$

Yes

16.37.
$$t^* = \frac{\bar{Y}_{1.}-\bar{Y}_{2.}}{s\{\bar{Y}_{1.}-\bar{Y}_{2.}\}} = \frac{\bar{Y}_{1.}-\bar{Y}_{2.}}{\sqrt{\frac{n_T}{n_1 n_2}}\sqrt{\frac{\Sigma(Y_{1j}-\bar{Y}_{1.})^2+\Sigma(Y_{2j}-\bar{Y}_{2.})^2}{n_T-2}}}$$
$$F^* = \frac{n_1(\bar{Y}_{1.}-\bar{Y}_{..})^2+n_2(\bar{Y}_{2.}-\bar{Y}_{..})^2}{\left(\frac{\Sigma(Y_{1j}-\bar{Y}_{1.})^2+\Sigma(Y_{2j}-\bar{Y}_{2.})^2}{n_T-2}\right)}$$

Therefore to show $(t^*)^2 = F^*$, it suffices to show:

$$\frac{n_1 n_2}{n_T}(\bar{Y}_{1.}-\bar{Y}_{2.})^2 = n_1(\bar{Y}_{1.}-\bar{Y}_{..})^2+n_2(\bar{Y}_{2.}-\bar{Y}_{..})^2$$

Now, the right-hand side equals:

$$n_1\left[\bar{Y}_{1.}-\left(\frac{n_1\bar{Y}_{1.}+n_2\bar{Y}_{2.}}{n_T}\right)\right]^2+n_2\left[\bar{Y}_{2.}-\left(\frac{n_1\bar{Y}_{1.}+n_2\bar{Y}_{2.}}{n_T}\right)\right]^2$$
$$= n_1\left(\frac{n_T\bar{Y}_{1.}-n_1\bar{Y}_{1.}-n_2\bar{Y}_{2.}}{n_T}\right)^2+n_2\left(\frac{n_T\bar{Y}_{2.}-n_1\bar{Y}_{1.}-n_2\bar{Y}_{2.}}{n_T}\right)^2$$
$$= n_1\left[\frac{n_2}{n_T}\left(\bar{Y}_{1.}-\bar{Y}_{2.}\right)\right]^2+n_2\left[\frac{n_1}{n_T}\left(\bar{Y}_{2.}-\bar{Y}_{1.}\right)\right]^2$$
$$= \frac{n_1 n_2}{n_T}\left(\bar{Y}_{1.}-\bar{Y}_{2.}\right)^2$$

16.38. $\sum w_i\tau_i = \sum w_i(\mu_i - \mu_.) = \sum w_i\mu_i - \mu_. \sum w_i$

$= \mu_. - \mu_. = 0$

since $\sum w_i = 1$ and $\mu_. = \sum w_i\mu_i$

16.39. a. Using (6.25) and substituting $\hat{\boldsymbol{\mu}}$ for $\mathbf{b}$:

$\hat{\boldsymbol{\mu}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$

$$\mathbf{X}'\mathbf{X} = \begin{bmatrix} n_1 & & & & \\ & n_2 & & 0 & \\ & & \cdot & & \\ & & & \cdot & \\ & 0 & & \cdot & \\ & & & & n_r \end{bmatrix} \qquad (\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} n_1^{-1} & & & & \\ & n_2^{-1} & & 0 & \\ & & \cdot & & \\ & & & \cdot & \\ & 0 & & \cdot & \\ & & & & n_r^{-1} \end{bmatrix}$$

$$\mathbf{X}'\mathbf{Y} = \begin{bmatrix} Y_{1.} \\ Y_{2.} \\ \cdot \\ \cdot \\ \cdot \\ Y_{r.} \end{bmatrix} \qquad \hat{\mu} = \begin{bmatrix} \bar{Y}_{1.} \\ \bar{Y}_{2.} \\ \cdot \\ \cdot \\ \cdot \\ \bar{Y}_{r.} \end{bmatrix}$$

$SSE(F) = \sum\sum(Y_{ij} - \bar{Y}_{i.})^2 = SSE$

b. $\mathbf{X}'\mathbf{X} = n_T$, $(\mathbf{X}'\mathbf{X})^{-1} = \dfrac{1}{n_T}$, $\mathbf{X}'\mathbf{Y} = \sum\sum Y_{ij}$

$\hat{\mu}_c = \dfrac{1}{n_T}\sum\sum Y_{ij} = \bar{Y}_{..}$

$SSE(R) = \sum\sum(Y_{ij} - \bar{Y}_{..})^2 = SSTO$

16.40. a. 90, 15

b. Smallest P-value = .067

16.41. $\sum(\mu_i - \mu_.)^2 = \left(-\dfrac{\mu_2 + 1}{3}\right)^2 + \left(\dfrac{2\mu_2 - 1}{3}\right)^2 + \left(\dfrac{2 - \mu_2}{3}\right)^2$

Differentiating with respect to μ_2 yields:

$\dfrac{12}{9}\mu_2 - \dfrac{6}{9}$

Setting this derivative equal to zero and solving yields $\mu_2 = .5$.

16.42. H_0: all μ_i are equal $(i = 1, ..., 4)$, H_a: not all μ_i are equal.

Source	SS	df	MS
Between regions	13.9969	3	4.6656
Error	187.3829	109	1.7191
Total	201.3798	112	

$F^* = 4.6656/1.7191 = 2.714$, $F(.95; 3, 109) = 2.688$. If $F^* \le 2.688$ conclude H_0, otherwise H_a. Conclude H_a.

16.43. H_0: all μ_i are equal $(i = 1, ..., 4)$, H_a: not all μ_i are equal.

Source	SS	df	MS
Between ages	3.0677	3	1.02257
Error	198.3121	109	1.81938
Total	201.3798	112	

$F^* = 1.02257/1.81938 = .562$, $F(.90;\ 3,\ 109) = 2.135$. If $F^* \leq 2.135$ conclude H_0, otherwise H_a. Conclude H_0.

16.44. H_0: all μ_i are equal ($i = 1, ..., 4$), H_a: not all μ_i are equal.

Source	SS	df	MS
Between regions	.059181	3	.019727
Error	.268666	436	.000616
Total	.327847	439	

$F^* = .019727/.000616 = 32.01$, $F(.95;\ 3,\ 436) = 2.6254$. If $F^* \leq 2.6254$ conclude H_0, otherwise H_a. Conclude H_a.

16.45. H_0: all μ_i are equal ($i = 1, ..., 4$), H_a: not all μ_i are equal.

Source	SS	df	MS
Between treatments	1.6613	3	.5538
Error	.7850	32	.0245
Total	2.4463	35	

$F^* = .5538/.0245 = 22.57$, $F(.95;\ 3,\ 32) = 2.9011$. If $F^* \leq 2.9011$ conclude H_0, otherwise H_a. Conclude H_a.

16.46. c. $E\{F^*\} = \dfrac{\nu_2}{\nu_2 - 2} = 1.2$

d. Expected proportion is .95.

e. $E\{F^*\} = 117.9$; $E\{MSTR\} = 14,144$, $E\{MSE\} = 144$

f. $\phi = 8.05$, expected proportion is $1 - \beta > .99$.

16.47. a. 20, 6

b.

F^* :	.29	.59	.97	1.06	1.64	2.74
$P(F^*)$:	4/20	4/20	2/20	4/20	4/20	2/20

P-value = .10

c. $P\{F(1,\ 4) \geq 2.74\} = .17$

16.48. a.

F^* :	0	.1	.4	.98	2.0	3.85	7.71	19.60
$P(F^*)$:	8/70	18/70	12/70	14/70	8/70	6/70	2/70	2/70

H_0: $\mu_1 = \mu_2$, H_a: $\mu_1 \neq \mu_2$. $F^* = 7.71$. P-value $= P(F^* \geq 7.71) = .0571$. If P-value $\geq .10$ conclude H_0, otherwise H_a. Conclude H_a.

b&c.

$F(.90; 1, 6) = 3.78$	$F(.95; 1, 6) = 5.99$	$F(.99; 1, 6) = 13.7$
$10/70 = .143$	$4/70 = .0571$	$2/70 = .0286$

Chapter 17

ANALYSIS OF FACTOR LEVEL MEANS

17.3. a. (i) and (iii) are contrasts.

b. (i) $\hat{L} = \bar{Y}_{1.} + 3\bar{Y}_{2.} - 4\bar{Y}_{3.}$, $s^2\{\hat{L}\} = \dfrac{26MSE}{n}$

(ii) $\hat{L} = .3\bar{Y}_{1.} + .5\bar{Y}_{2.} + .1\bar{Y}_{3.} + .1\bar{Y}_{4.}$, $s^2\{\hat{L}\} = \dfrac{.36MSE}{n}$

(iii) $\hat{L} = \dfrac{(\bar{Y}_{1.} + \bar{Y}_{2.} + \bar{Y}_{3.})}{3} - \bar{Y}_{4.}$, $s^2\{\hat{L}\} = \dfrac{4MSE}{3n}$

17.4. a. $q(.90; 6, 54) = 3.765$, $F(.90; 5, 54) = 1.96$

g	T	S	B
2	2.66	3.13	$t(.975; 54) = 2.00$
5	2.66	3.13	$t(.99; 54) = 2.40$
15	2.66	3.13	$t(.99667; 54) = 2.82$

b. Refer to part (a) for S and B multiples.

17.5. a. $q(.95;\ 5,\ 20) = 4.23$, $F(.95; 4, 20) = 2.87$

g	T	S	B
2	2.99	3.39	$t(.9875; 20) = 2.42$
5	2.99	3.39	$t(.995; 20) = 2.845$
10	2.99	3.39	$t(.9975; 20) = 3.15$

b. $q(.95; 5, 95) = 3.94$, $F(.95; 4, 95) = 2.46$

g	T	S	B
2	2.79	3.14	$t(.9875; 95) = 2.28$
5	2.79	3.14	$t(.995; 95) = 2.63$
10	2.79	3.14	$t(.9975; 95) = 2.87$

17.7. $q(.99; 2, 18) = 4.07$, $F(.99; 1, 18) = 8.29$, $T = S = B = t(.995; 18) = 2.88$

17.8. a. $\bar{Y}_{1.} = 6.878$, $\bar{Y}_{2.} = 8.133$, $\bar{Y}_{3.} = 9.200$

b. $s\{\bar{Y}_{3.}\} = .327$, $t(.975; 24) = 2.064$, $9.200 \pm 2.064(.327), 8.525 \leq \mu_3 \leq 9.875$

c. $\hat{D} = \bar{Y}_{2.} - \bar{Y}_{1.} = 1.255$, $s\{\hat{D}\} = .353$, $t(.975; 24) = 2.064, 1.255 \pm 2.064(.353)$, $.526 \leq D \leq 1.984$

d. $\hat{D}_1 = \bar{Y}_{3.} - \bar{Y}_{2.} = 1.067$, $\hat{D}_2 = \bar{Y}_{3.} - \bar{Y}_{1.} = 2.322$, $\hat{D}_3 = \bar{Y}_{2.} - \bar{Y}_{1.} = 1.255$, $s\{\hat{D}_1\} = .400$, $s\{\hat{D}_2\} = .422$, $s\{\hat{D}_3\} = .353$, $q(.90; 3, 24) = 3.05$, $T = 2.157$

$1.067 \pm 2.157(.400)$	$.204 \leq D_1 \leq 1.930$
$2.322 \pm 2.157(.422)$	$1.412 \leq D_2 \leq 3.232$
$1.255 \pm 2.157(.353)$	$.494 \leq D_3 \leq 2.016$

e. $F(.90; 2, 24) = 2.54$, $S = 2.25$

$B = t(.9833; 24) = 2.257$

Yes

17.9. a. $\bar{Y}_{1.} = 29.4$, $\bar{Y}_{2.} = 29.6$, $\bar{Y}_{3.} = 28.0$, $s\{\bar{Y}_{1.}\} = s\{\bar{Y}_{2.}\} = s\{\bar{Y}_{3.}\} = \sqrt{\frac{9.7}{5}} = 1.3928$, $t(.975; 12) = 2.179$

b. $s\{\bar{Y}_{1.}\} = 1.393$, $t(.95; 12) = 1.782$, $29.40 \pm 1.782(1.393)$, $26.92 \leq \mu_1 \leq 31.88$

c. $H_0 : D = \mu_3 - \mu_2 = 0$, $H_a : D \neq 0$. $\hat{D} = -1.6$, $s\{\hat{D}\} = 1.970$,

$t^* = -1.6/1.970 = -.81$, $t(.95; 12) = 1.782$.

If $|\ t^*\ | \leq 1.782$ conclude H_0, otherwise H_a. Conclude H_0. No

17.10. a. $\bar{Y}_{1.} = 38.00$, $\bar{Y}_{2.} = 32.00$, $\bar{Y}_{3.} = 24.00$

b. $MSE = 19.81$, $s\{\bar{Y}_{2.}\} = 1.4075$, $t(.995; 21) = 2.831$, $32.00 \pm 2.831(1.4075)$, $28.02 \leq \mu_2 \leq 35.98$

c. $\hat{D}_1 = \bar{Y}_{2.} - \bar{Y}_{3.} = 8.00$, $\hat{D}_2 = \bar{Y}_{1.} - \bar{Y}_{2.} = 6.00$, $s\{\hat{D}_1\} = 2.298$, $s\{\hat{D}_2\} = 2.111$, $B = t(.9875; 21) = 2.414$

$8.00 \pm 2.414(2.298)$	$2.45 \leq D_1 \leq 13.55$
$6.00 \pm 2.414(2.111)$	$.90 \leq D_2 \leq 11.10$

d. $q(.95;\ 3,\ 21) = 3.57$, $T = 2.524$, no

e. Yes, no

f. $q(.95; 3, 21) = 3.57$

Test i	Comparison D_i	$\hat{D}_i$	$s\{\hat{D}_i\}$	q_i^*	Conclusion
1	$\mu_1 - \mu_2$	6.00	2.111	4.02	H_a
2	$\mu_1 - \mu_3$	14.00	2.404	8.24	H_a
3	$\mu_2 - \mu_3$	8.00	2.298	4.92	H_a

Group 1:	Below Average
Group 2:	Average
Group 3:	Above Average

17.11. a. $\bar{Y}_{1.} = 21.500$, $\bar{Y}_{2.} = 27.750$, $\bar{Y}_{3.} = 21.417$

b. $MSE = 2.490$, $s\{\bar{Y}_{1.}\} = .456$, $t(.995; 33) = 2.733$, $21.500 \pm 2.733(.456)$, $20.254 \leq \mu_1 \leq 22.746$

c. $\hat{D} = \bar{Y}_{3.} - \bar{Y}_{1.} = -.083$, $s\{\hat{D}\} = .644$, $t(.995; 33) = 2.733$, $-.083 \pm 2.733(.644)$, $-1.843 \leq D \leq 1.677$

d. $H_0: 2\mu_2 - \mu_1 - \mu_3 = 0$, $H_a: 2\mu_2 - \mu_1 - \mu_3 \neq 0$. $F^* = (12.583)^2/1.245 = 127.17$, $F(.99; 1, 33) = 7.47$. If $F^* \leq 7.47$ conclude H_0, otherwise H_a. Conclude H_a.

e. $\hat{D}_1 = \bar{Y}_{3.} - \bar{Y}_{1.} = -.083$, $\hat{D}_2 = \bar{Y}_{3.} - \bar{Y}_{2.} = -6.333$, $\hat{D}_3 = \bar{Y}_{2.} - \bar{Y}_{1.} = 6.250$, $s\{\hat{D}_i\} = .644$ $(i = 1, 2, 3)$, $q(.90; 3, 33) = 3.01$, $T = 2.128$

$$\begin{array}{rr}
-.083 \pm 2.128(.644) & -1.453 \leq D_1 \leq 1.287 \\
-6.333 \pm 2.128(.644) & -7.703 \leq D_2 \leq -4.963 \\
6.250 \pm 2.128(.644) & 4.880 \leq D_3 \leq 7.620
\end{array}$$

f. $B = t(.9833; 33) = 2.220$, no

17.12. a. $\bar{Y}_{1.} = .0735$, $\bar{Y}_{2.} = .1905$, $\bar{Y}_{3.} = .4600$, $\bar{Y}_{4.} = .3655$, $\bar{Y}_{5.} = .1250$, $\bar{Y}_{6.} = .1515$

b. $MSE = .03097$, $s\{\bar{Y}_{1.}\} = .0394$, $t(.975; 114) = 1.981$, $.0735 \pm 1.981(.0394)$, $-.005 \leq \mu_1 \leq .152$

c. $\hat{D} = \bar{Y}_{2.} - \bar{Y}_{1.} = .1170$, $s\{\hat{D}\} = .0557$, $t(.975; 114) = 1.981$, $.1170 \pm 1.981(.0557)$, $.007 \leq D \leq .227$

e. $\hat{D}_1 = \bar{Y}_{1.} - \bar{Y}_{4.} = -.2920$, $\hat{D}_2 = \bar{Y}_{1.} - \bar{Y}_{5.} = -.0515$, $\hat{D}_3 = \bar{Y}_{4.} - \bar{Y}_{5.} = .2405$, $s\{\hat{D}_i\} = .0557$ $(i = 1, 2, 3)$, $B = t(.9833; 114) = 2.178$

Test i	Comparison i	t_i^*	Conclusion
1	D_1	-5.242	H_a
2	D_2	$-.925$	H_0
3	D_3	4.318	H_a

f. $q(.90; 6, 114) = 3.71$, $T = 2.623$, no

17.13. a. $\bar{Y}_{1.} = 24.55$, $\bar{Y}_{2.} = 22.55$, $\bar{Y}_{3.} = 11.75$, $\bar{Y}_{4.} = 14.80$, $\bar{Y}_{5.} = 30.10$,

$s\{\bar{Y}_{i.}\} = \sqrt{\frac{7.52}{20}} = .6132$, $(i = 1, 2, 3, 4, 5)$, $t(.975; 95) = 1.985$

b.

$$\begin{array}{rlrl}
\text{Test 1:} & H_0\colon \mu_1 - \mu_2 = 0 & \text{Test 6:} & H_0\colon \mu_2 - \mu_4 = 0 \\
 & H_a\colon \mu_1 - \mu_2 \neq 0 & & H_a\colon \mu_2 - \mu_4 \neq 0 \\
\text{Test 2:} & H_0\colon \mu_1 - \mu_3 = 0 & \text{Test 7:} & H_0\colon \mu_2 - \mu_5 = 0 \\
 & H_a\colon \mu_1 - \mu_3 \neq 0 & & H_a\colon \mu_2 - \mu_5 \neq 0 \\
\text{Test 3:} & H_0\colon \mu_1 - \mu_4 = 0 & \text{Test 8:} & H_0\colon \mu_3 - \mu_4 = 0 \\
 & H_a\colon \mu_1 - \mu_4 \neq 0 & & H_a\colon \mu_3 - \mu_4 \neq 0 \\
\text{Test 4:} & H_0\colon \mu_1 - \mu_5 = 0 & \text{Test 9:} & H_0\colon \mu_3 - \mu_5 = 0 \\
 & H_a\colon \mu_1 - \mu_5 \neq 0 & & H_a\colon \mu_3 - \mu_5 \neq 0 \\
\text{Test 5:} & H_0\colon \mu_2 - \mu_3 = 0 & \text{Test 10:} & H_0\colon \mu_4 - \mu_5 = 0 \\
 & H_a\colon \mu_2 - \mu_3 \neq 0 & & H_a\colon \mu_4 - \mu_5 \neq 0
\end{array}$$

$\hat{D}_1 = 24.55 - 22.55 = 2.00$, $\hat{D}_2 = 24.55 - 11.75 = 12.80$,

$\hat{D}_3 = 24.55 - 14.80 = 9.75$, $\hat{D}_4 = 24.55 - 30.10 = -5.55$,

$\hat{D}_5 = 22.55 - 11.75 = 10.80$, $\hat{D}_6 = 22.55 - 14.80 = 7.75$,

$\hat{D}_7 = 22.55 - 30.10 = -7.55$, $\hat{D}_8 = 11.75 - 14.80 = -3.05$,

$\hat{D}_9 = 11.75 - 30.10 = -18.35$, $\hat{D}_{10} = 14.80 - 30.10 = -15.30$,

$s\{\hat{D}_i\} = .8673$ $(i = 1, ..., 10)$, $q(.90; 5, 95) = 3.54$.

If $|q_i^*| \leq 3.54$ conclude H_0, otherwise H_a.

Test i	q_i^*	Conclusion
1	3.26	H_o
2	20.87	H_a
3	15.90	H_a
4	-9.05	H_a
5	17.61	H_a
6	12.64	H_a
7	-12.31	H_a
8	-4.97	H_a
9	-29.92	H_a
10	-24.95	H_a

Group 1	**Group 2**
Agent 3 $\bar{Y}_{3.} = 11.75$	Agent 4 $\bar{Y}_{4.} = 14.80$

Group 3	**Group 4**
Agent 1 $\bar{Y}_{1.} = 24.55$	Agent 5 $\bar{Y}_{5.} = 30.10$
Agent 2 $\bar{Y}_{2.} = 22.55$	

c. $MSE = 7.523$, $s\{\bar{Y}_{1.}\} = .6133$, $t(.95; 95) = 1.661$, $24.550 \pm 1.661(.6133)$, $23.531 \leq \mu_1 \leq 25.569$

d. $\hat{D} = \bar{Y}_{2.} - \bar{Y}_{1.} = -2.000$, $s\{\hat{D}\} = .8673$, $t(.95; 95) = 1.661$, $-2.000 \pm 1.661(.8673)$, $-3.441 \leq D \leq -.559$

e. $\hat{D}_1 = \bar{Y}_{1.} - \bar{Y}_{3.} = 12.800$, $\hat{D}_2 = \bar{Y}_{1.} - \bar{Y}_{5.} = -5.550$, $\hat{D}_3 = \bar{Y}_{3.} - \bar{Y}_{5.} = -18.350$, $s\{\hat{D}_i\} = .8673$ $(i = 1, 2, 3)$, $B = t(.9833; 95) = 2.158$

$$\begin{array}{cc}
12.800 \pm 2.158(.8673) & 10.928 \leq D_1 \leq 14.672 \\
-5.550 \pm 2.158(.8673) & -7.422 \leq D_2 \leq -3.678 \\
-18.350 \pm 2.158(.8673) & -20.222 \leq D_3 \leq -16.478
\end{array}$$

f. $q(.90; 5, 95) = 3.54$, $T = 2.503$, no

17.14. a. $\hat{L} = (\bar{Y}_{1.} + \bar{Y}_{2.})/2 - \bar{Y}_{3.} = (6.8778 + 8.1333)/2 - 9.200 = -1.6945$,

$s\{\hat{L}\} = .3712$, $t(.975; 24) = 2.064$, $-1.6945 \pm 2.064(.3712)$, $-2.461 \leq L \leq -.928$

b. $\hat{L} = (3/9)\bar{Y}_{1.} + (4/9)\bar{Y}_{2.} + (2/9)\bar{Y}_{3.} = 7.9518$, $s\{\hat{L}\} = .1540$, $t(.975; 24) = 2.064$, $7.9518 \pm 2.064(.1540)$, $7.634 \leq L \leq 8.270$

c. $F(.90;\ 2,\ 24) = 2.54$, $S = 2.254$; see also part (a) and Problem 17.8.

$$\begin{array}{cc}
1.067 \pm 2.254(.400) & .165 \leq D_1 \leq 1.969 \\
2.322 \pm 2.254(.422) & 1.371 \leq D_2 \leq 3.273
\end{array}$$

$$1.255 \pm 2.254(.353) \qquad .459 \leq D_3 \leq 2.051$$
$$-1.6945 \pm 2.254(.3712) \qquad -2.531 \leq L_1 \leq -.858$$

17.15. a. $\hat{L} = (\bar{Y}_{1.} - \bar{Y}_{2.}) - (\bar{Y}_{2.} - \bar{Y}_{3.}) = \bar{Y}_{1.} - 2\bar{Y}_{2.} + \bar{Y}_{3.} = 38.000 - 2(32.000)$ $+24.000 = -2.000$, $s\{\hat{L}\} = 3.7016$, $t(.995; 21) = 2.831$, $-2.000 \pm 2.831(3.7016)$, $-12.479 \leq L \leq 8.479$

b. $\hat{D}_1 = \bar{Y}_{1.} - \bar{Y}_{2.} = 6.000$, $\hat{D}_2 = \bar{Y}_{1.} - \bar{Y}_{3.} = 14.000$, $\hat{D}_3 = \bar{Y}_{2.} - \bar{Y}_{3.} = 8.000$, $\hat{L}_1 = \hat{D}_1 - \hat{D}_3 = -2.000$, $s\{\hat{D}_1\} = 2.1112$, $s\{\hat{D}_2\} = 2.4037$, $s\{\hat{D}_3\} = 2.2984$, $s\{\hat{L}_1\} = 3.7016$, $B = t(.99375; 21) = 2.732$

$$6.000 \pm 2.732(2.1112) \qquad .232 \leq D_1 \leq 11.768$$
$$14.000 \pm 2.732(2.4037) \qquad 7.433 \leq D_2 \leq 20.567$$
$$8.000 \pm 2.732(2.2984) \qquad 1.721 \leq D_3 \leq 14.279$$
$$-2.000 \pm 2.732(3.7016) \qquad -12.113 \leq L_1 \leq 8.113$$

c. $F(.95; 2, 21) = 3.47$, $S = 2.634$, yes

17.16. a. $\hat{L} = (\bar{Y}_{3.} - \bar{Y}_{2.}) - (\bar{Y}_{2.} - \bar{Y}_{1.}) = \bar{Y}_{3.} - 2\bar{Y}_{2.} + \bar{Y}_{1.} = 21.4167-$ $2(27.7500) + 21.500 = -12.5833$, $s\{\hat{L}\} = 1.1158$, $t(.995; 33) = 2.733$, $-12.5833 \pm 2.733(1.1158)$, $-15.632 \leq L \leq -9.534$

b. $\hat{D}_1 = \bar{Y}_{2.} - \bar{Y}_{1.} = 6.2500$, $\hat{D}_2 = \bar{Y}_{3.} - \bar{Y}_{2.} = -6.3333$, $\hat{D}_3 = \bar{Y}_{3.} - \bar{Y}_{1.} = -.0833$, $\hat{L}_1 = \hat{D}_2 - \hat{D}_1 = -12.5833$, $s\{\hat{D}_i\} = .6442$ $(i = 1, 2, 3)$, $s\{\hat{L}_1\} = 1.1158$, $F(.90; 2, 33) = 2.47$, $S = 2.223$

$$6.2500 \pm 2.223(.6442) \qquad 4.818 \leq D_1 \leq 7.682$$
$$-6.3333 \pm 2.223(.6442) \qquad -7.765 \leq D_2 \leq -4.901$$
$$-.0833 \pm 2.223(.6442) \qquad -1.515 \leq D_3 \leq 1.349$$
$$-12.5833 \pm 2.223(1.1158) \qquad -15.064 \leq L_1 \leq -10.103$$

17.17. a. $\hat{L} = (\bar{Y}_{1.} + \bar{Y}_{2.})/2 - (\bar{Y}_{3.} + \bar{Y}_{4.})/2 = (.0735 + .1905)/2 - (.4600 + .3655)/2$ $= -.28075$, $s\{\hat{L}\} = .03935$, $t(.975; 114) = 1.981$, $-.28075 \pm 1.981(.03935)$, $-.3587 \leq L \leq -.2028$

b. $\hat{D}_1 = -.1170$, $\hat{D}_2 = .0945$, $\hat{D}_3 = -.0265$, $\hat{L}_1 = -.28075$, $\hat{L}_2 = -.00625$, $\hat{L}_3 = -.2776$, $\hat{L}_4 = .1341$, $s\{\hat{D}_i\} = .0557$ $(i = 1, 2, 3)$, $s\{\hat{L}_1\} = s\{\hat{L}_2\} = .03935$, $s\{\hat{L}_3\} = s\{\hat{L}_4\} = .03408$, $B = t(.99286; 114) = 2.488$

$$-.1170 \pm 2.488(.0557) \qquad -.2556 \leq D_1 \leq .0216$$
$$.0945 \pm 2.488(.0557) \qquad -.0441 \leq D_2 \leq .2331$$
$$-.0265 \pm 2.488(.0557) \qquad -.1651 \leq D_3 \leq .1121$$
$$-.28075 \pm 2.488(.03935) \qquad -.3787 \leq L_1 \leq -.1828$$
$$-.00625 \pm 2.488(.03935) \qquad -.1042 \leq L_2 \leq .0917$$
$$-.2776 \pm 2.488(.03408) \qquad -.3624 \leq L_3 \leq -.1928$$
$$.1341 \pm 2.488(.03408) \qquad .0493 \leq L_4 \leq .2189$$

17.18. a. $\hat{L} = (\bar{Y}_{1.} + \bar{Y}_{2.})/2 - (\bar{Y}_{3.} + \bar{Y}_{4.})/2 = (24.55 + 22.55)/2 - (11.75 + 14.80)/2$

$= 10.275$, $s\{\hat{L}\} = .6133$, $t(.95;\ 95) = 1.661$, $10.275 \pm 1.661(.6133)$, $9.256 \leq L \leq 11.294$

b. $\hat{D}_1 = 2.00$, $\hat{D}_2 = -3.05$, $\hat{L}_1 = -6.55$, $\hat{L}_2 = -16.825$, $\hat{L}_3 = 10.275$, $s\{\hat{D}_i\} = .8673$ $(i = 1, 2)$, $s\{\hat{L}_i\} = .7511$ $(i = 1, 2)$, $s\{\hat{L}_3\} = .6133$, $F(.90; 4, 95) = 1.997$, $S = 2.826$

$$\begin{array}{rr}
2.00 \pm 2.826(.8673) & -.451 \leq D_1 \leq 4.451 \\
-3.05 \pm 2.826(.8673) & -5.501 \leq D_2 \leq -.599 \\
-6.55 \pm 2.826(.7511) & -8.673 \leq L_1 \leq -4.427 \\
-16.825 \pm 2.826(.7511) & -18.948 \leq L_2 \leq -14.702 \\
10.275 \pm 2.826(.6133) & 8.542 \leq L_3 \leq 12.008
\end{array}$$

c. $\hat{L} = .25\bar{Y}_{1.} + .20\bar{Y}_{2.} + .20\bar{Y}_{3.} + .20\bar{Y}_{4.} + .15\bar{Y}_{5.} = 20.4725$, $s\{\hat{L}\} = .2777$, $t(.95; 95) = 1.661$, $20.4725 \pm 1.661(.2777)$, $20.011 \leq L \leq 20.934$

17.19. a. $L_1 = \mu_1 - \mu_.$ $\quad L_2 = \mu_2 - \mu_.$

$L_3 = \mu_3 - \mu_.$ $\quad L_4 = \mu_4 - \mu_.$

$L_5 = \mu_5 - \mu_.$ $\quad L_6 = \mu_6 - \mu_.$

$\hat{L}_1 = .0735 - .2277 = -.1542$, $\hat{L}_2 = .1905 - .2277 = -.0372$

$\hat{L}_3 = .4600 - .2277 = .2323$, $\hat{L}_4 = .3655 - .2277 = .1378$

$\hat{L}_5 = .1250 - .2277 = -.1027$, $\hat{L}_6 = .1515 - .2277 = -.0762$

$$s\{\hat{L}_i\} = \sqrt{\frac{.03097}{20}\left(\frac{25}{36}\right) + \frac{.03097}{36}\left(\frac{5}{20}\right)} = .0359$$

$B = t(.99583; 114) = 2.685$

$$\begin{array}{rr}
-.1542 \pm 2.685(.0359) & .2506 \leq L_1 \leq -.0578 \\
-.0372 \pm 2.685(.0359) & -.1336 \leq L_2 \leq .0592 \\
.2323 \pm 2.685(.0359) & .1359 \leq L_3 \leq .3287 \\
.1378 \pm 2.685(.0359) & .0414 \leq L_4 \leq .2342 \\
-.1027 \pm 2.685(.0359) & -.1991 \leq L_5 \leq -.0063 \\
-.0762 \pm 2.685(.0359) & -.1726 \leq L_6 \leq .0202
\end{array}$$

Conclude not all μ_i are equal.

17.20. a. $L_1 = \mu_1 - \mu_.$ $\quad L_2 = \mu_2 - \mu_.$

$L_3 = \mu_3 - \mu_.$ $\quad L_4 = \mu_4 - \mu_.$

$L_5 = \mu_5 - \mu_.$

$\hat{L}_1 = 24.55 - 20.75 = 3.80$, $\hat{L}_2 = 22.55 - 20.75 = 1.80$

$\hat{L}_3 = 11.75 - 20.75 = -9.00$, $\hat{L}_4 = 14.80 - 20.75 = -5.95$

$\hat{L}_5 = 30.10 - 20.75 = 9.35$

$$s\{\hat{L}_i\} = \sqrt{\frac{7.5226}{20}\left(\frac{16}{25}\right) + \frac{7.5226}{25}\left(\frac{4}{20}\right)} = .5485$$

$B = t(.99; 95) = 2.366$

$$3.80 \pm 2.366(.5485) \qquad 2.502 \leq L_1 \leq 5.098$$
$$1.80 \pm 2.366(.5485) \qquad .502 \leq L_2 \leq 3.098$$
$$-9.00 \pm 2.366(.5485) \qquad -10.298 \leq L_3 \leq -7.702$$
$$-5.95 \pm 2.366(.5485) \qquad -7.248 \leq L_4 \leq -4.652$$
$$9.35 \pm 2.366(.5485) \qquad 8.052 \leq L_5 \leq 10.648$$

Conclude not all μ_i are equal.

17.21. a. $Y_{ij} = \mu_i + \epsilon_{ij}$

b.

i:	1	2	3	4	5
$\bar{Y}_{i.}$:	.0800	.1800	.5333	1.1467	2.8367

c.

Source	SS	df	MS
Treatments	15.3644	4	3.8411
Error	.1574	10	.01574
Total	15.5218	14	

d. H_0 : all μ_i are equal ($i = 1, ..., 5$), H_a : not all μ_i are equal. $F^* = 3.8411/.01574 = 244.034$, $F(.975; 4, 10) = 4.47$. If $F^* \leq 4.47$ conclude H_0, otherwise H_a. Conclude H_a.

e. $\hat{D}_1 = \bar{Y}_{1.} - \bar{Y}_{2.} = -.1000$, $\hat{D}_2 = \bar{Y}_{2.} - \bar{Y}_{3.} = -.3533$, $\hat{D}_3 = \bar{Y}_{3.} - \bar{Y}_{4.} = -.6134$. $D_4 = \bar{Y}_{4.} - \bar{Y}_{5.} = -1.6900$, $s\{\hat{D}_i\} = .1024$ ($i = 1, ..., 4$), $B = t(.99375; 10) = 3.038$

$$-.1000 \pm 3.038(.1024) \qquad -.411 \leq D_1 \leq .211$$
$$-.3533 \pm 3.038(.1024) \qquad -.664 \leq D_2 \leq -.042$$
$$-.6134 \pm 3.038(.1024) \qquad -.924 \leq D_3 \leq -.302$$
$$-1.6900 \pm 3.038(.1024) \qquad -2.001 \leq D_4 \leq -1.379$$

17.23. $n = 13$

17.24. Bonferroni, $n = 24$

17.25. Bonferroni, $n = 45$

17.26. Bonferroni, $n = 92$

17.27. a. $n = 20$, $2n = 40$, $n = 20$

b. (1) $n = 26$, $n = 26$, $n = 26$

(2) $n = 18$, $3n = 54$, $n = 18$

c. b(1)

17.28. a. $\hat{Y} = 68.66655 - .36820X$

b. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	-9.106	3.894	$-.106$	1.894	4.894
2	-1.847	3.153	7.153	-3.847	$-.847$
3	1.886	7.886	-3.114	-4.114	-1.114

i	$j=6$	$j=7$	$j=8$	$j=9$	$j=10$
1	1.894	-8.106	3.894		
2	$-.847$	-2.847	3.153	-2.847	1.153
3	-2.114				

c. $H_0: E\{Y\} = \beta_0 + \beta_1 X$, $H_a: E\{Y\} \neq \beta_0 + \beta_1 X$. $SSPE = 416.0000$, $SSLF = .4037$, $F^* = (.4037/1) \div (416.0000/21) = .020, F(.95; 1, 21) = 4.32$. If $F^* \leq 4.32$ conclude H_0, otherwise H_a. Conclude H_0.

d. No

17.29. a. $\hat{Y} = .18472 + .06199x + .01016x^2$

b. e_{ij} :

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$
1	$-.2310$	.1090	$-.0210$	.0890	.2890	.0090	$-.1310$
2	.2393	$-.1107$	$-.1007$	.2493	.0193	$-.1607$	$-.3407$
3	$-.2440$	.3260	$-.1340$	$-.0040$	$-.2340$	$-.1040$	.0860
4	.1268	.2168	.1568	$-.0732$	$-.0932$	.1868	.0368
5	$-.2969$	.1631	$-.0469$	.0031	.1231	.0431	$-.0969$
6	$-.0802$	$-.1802$	.1498	.3398	$-.0102$	.1398	$-.0502$

i	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$	$j=13$	$j=14$
1	$-.3610$	.1790	$-.3010$	.2990	$-.1610$	$-.1110$	.1890
2	.1093	$-.1607$	$-.2507$	$-.1707$	.3093	.1993	.0693
3	$-.2140$	.0160	.1660	.0160	.0960	.1360	.2560
4	$-.2232$	.1168	$-.0232$	$-.3532$	$-.0332$	$-.1832$	$-.2332$
5	.1131	.1831	.0331	.0931	.1931	$-.2169$	.1631
6	.0398	.2998	$-.2002$	.0698	$-.1202$	$-.0302$	.0298

i	$j=15$	$j=16$	$j=17$	$j=18$	$j=19$	$j=20$
1	$-.0010$	.0390	.1690	$-.0210$	$-.1010$	$-.2810$
2	.1393	$-.1807$	$-.0507$	$-.2007$	$-.1107$	$-.1007$
3	$-.0040$	.0260	$-.0140$	.0460	$-.2540$	.1560
4	.1168	.1768	.1468	.0568	.0868	$-.1632$
5	$-.3069$	.1331	.0931	.0331	.2431	$-.2869$
6	$-.1902$	$-.0002$	.2998	.2198	$-.2202$	$-.0802$

c. $H_0: E\{Y\} = \beta_0 + \beta_1 x + \beta_{11} x^2$, $H_a: E\{Y\} \neq \beta_0 + \beta_1 x + \beta_{11} x^2$. $SSPE = 3.5306$, $SSLF = .0408$, $F^* = (.0408/3) \div (3.5306/114) = .439, F(.99; 3, 114) = 3.96$. If $F^* \leq 3.96$ conclude H_0, otherwise H_a. Conclude H_0.

d. $H_0: \beta_{11} = 0$, $H_a: \beta_{11} \neq 0$. $s\{b_{11}\} = .00525$, $t^* = .01016/.00525 = 1.935, t(.995; 117) = 2.619$. If $|t^*| \leq 2.619$ conclude H_0, otherwise H_a. Conclude H_0.

17.30. With $r = 2$ and $n_i \equiv n$, $MSE = s^2$ as defined in (A.63) and

$\max(\bar{Y}_{i.} - \mu_i) - \min(\bar{Y}_{i.} - \mu_i) = (\bar{Y}_{i.} - \mu_i) - (\bar{Y}_{i'.} - \mu_{i'}) =$

$= (\bar{Y}_{i.} - \bar{Y}_{i'.}) - (\mu_i - \mu_{i'})$, $i \neq i'$.

Thus:

$$q^* = \frac{(\bar{Y}_{i.} - \bar{Y}_{i'.}) - (\mu_i - \mu_{i'})}{s/\sqrt{n}} = \sqrt{2}|t^*|$$

17.31. Working within the probability expression, we obtain:

$$\left| \frac{(\bar{Y}_{i.} - \mu_i) - (\bar{Y}_{i'.} - \mu_{i'})}{\sqrt{MSE/n}} \right| \leq q(1-\alpha; r, n_T - r) \quad \text{or}$$

$$|(\bar{Y}_{i.} - \mu_i) - (\bar{Y}_{i'.} - \mu_{i'})| \leq \left(\sqrt{MSE/n}\right) q(1-\alpha; r, n_T - r) \quad \text{or}$$

$$|(\bar{Y}_{i.} - \mu_i) - (\bar{Y}_{i'.} - \mu_{i'})| \leq s\{\hat{D}\}T$$

since $T = \frac{1}{\sqrt{2}} q(1-\alpha; r, n_T - r)$ and $\sqrt{\frac{MSE}{n}} = \frac{s\{\hat{D}\}}{\sqrt{2}}$ or

$$-s\{\hat{D}\}T \leq (\bar{Y}_{i.} - \bar{Y}_{i'.}) - (\mu_i - \mu_{i'}) \leq s\{\hat{D}\}T \quad \text{or}$$

$$(\bar{Y}_{i.} - \bar{Y}_{i'.}) - Ts\{\hat{D}\} \leq \mu_i - \mu_{i'} \leq (\bar{Y}_{i.} - \bar{Y}_{i'.}) + Ts\{\hat{D}\}$$

17.32. When $r = 2$, $S^2 = F(1-\alpha;\, 1,\, n_T - 2)$ which by (A.50b) equals $[t(1-\alpha/2; n_T - 2)]^2$.

17.33.

$$\begin{aligned}
\sigma^2\{\hat{L}_i\} &= \sigma^2\{\bar{Y}_{i.} - \sum_{h=1}^{r} \bar{Y}_{h.}/r\} \\
&= \sigma^2\{\bar{Y}_{i.}\} + \sigma^2\{\sum \bar{Y}_{h.}/r\} - 2\sigma\{\bar{Y}_{i.}, \sum \bar{Y}_{h.}/r\} \\
&= \frac{\sigma^2}{n_i} + \frac{1}{r^2}\sum(\sigma^2/n_h) - \frac{2\sigma^2}{rn_i} \\
&= \frac{1}{r^2}\sigma^2 \sum_{h\neq i}^{r}(1/n_h) + \frac{\sigma^2}{n_i} + \frac{\sigma^2}{r^2 n_i} - \frac{2\sigma^2}{rn_i} \\
&= \frac{1}{r^2}\sigma^2 \sum_{h\neq i}^{r}(1/n_h) + \frac{\sigma^2}{n_i}\left(1 + \frac{1}{r^2} - \frac{2}{r}\right) \\
&= \frac{1}{r^2}\sigma^2 \sum_{h\neq i}^{r}(1/n_h) + \frac{\sigma^2}{n_i}\left(\frac{r-1}{r}\right)^2
\end{aligned}$$

Replacing σ^2 by the estimator MSE leads to (17.49).

17.34. Given $n_1 = n_3 = n$ and $n_2 = kn$. Let $c = kn/n_T$. Then $n_1 = n_3 = (n_T - kn)/2 = n_T(1-c)/2$ and $n_2 = cn_T$. Hence:

$$\sigma^2\{\bar{Y}_{1.} - \bar{Y}_{2.}\} = \sigma^2\{\bar{Y}_{3.} - \bar{Y}_{2.}\} = \sigma^2\left[\frac{2}{n_T(1-c)} + \frac{1}{cn_T}\right]$$

Differentiating with respect to c yields:

$$\frac{2\sigma^2}{n_T}(1-c)^{-2} + \frac{\sigma^2}{n_T}(-c^{-2})$$

Setting the derivative equal to zero and solving yields $c = .4142$.

Hence, $n_2 = (.4142)n_T$ and $n_1 = n_3 = (.2929)n_T$.

(Note: This derivation treats n as a continuous variable. Since n_2 must be an even integer, appropriate rounding of the calculated sample sizes is required. For example, if $n_T = 100$, the calculated sample sizes are $n_1 = 29.29$, $n_2 = 41.42$, and $n_3 = 29.29$. The smallest variance is obtained for $n_1 = 29$, $n_2 = 42$, and $n_3 = 29$.)

17.35. $\bar{Y}_{1.} = 4.86071$, $\bar{Y}_{2.} = 4.39375$, $\bar{Y}_{3.} = 3.92703$, $\bar{Y}_{4.} = 4.38125$, $MSE = 1.7191$,

$n_1 = 28$, $n_2 = 32$, $n_3 = 37$, $n_4 = 16$,

$\hat{D}_1 = \bar{Y}_{1.} - \bar{Y}_{2.} = .46696$, $\hat{D}_2 = \bar{Y}_{1.} - \bar{Y}_{3.} = .93368$, $\hat{D}_3 = \bar{Y}_{1.} - \bar{Y}_{4.} = .47946$,

$\hat{D}_4 = \bar{Y}_{2.} - \bar{Y}_{3.} = .46667$, $\hat{D}_5 = \bar{Y}_{2.} - \bar{Y}_{4.} = .01250$, $\hat{D}_6 = \bar{Y}_{3.} - \bar{Y}_{4.} = -.45422$,

$s\{\hat{D}_1\} = .3393$, $s\{\hat{D}_2\} = .3284$, $s\{\hat{D}_3\} = .4109$, $s\{\hat{D}_4\} = .3165$, $s\{\hat{D}_5\} = .4015$, $s\{\hat{D}_6\} = .3923$, $q(.90; 4, 109) = 3.28$, $T = 2.319$

$$\begin{array}{cc}
.46696 \pm 2.319(.3393) & -.320 \leq D_1 \leq 1.254 \\
.93368 \pm 2.319(.3284) & .172 \leq D_2 \leq 1.695 \\
.47946 \pm 2.319(.4109) & -.473 \leq D_3 \leq 1.432 \\
.46667 \pm 2.319(.3165) & -.267 \leq D_4 \leq 1.201 \\
.01250 \pm 2.319(.4015) & -.919 \leq D_5 \leq .944 \\
-.45422 \pm 2.319(.3923) & -1.364 \leq D_6 \leq .456
\end{array}$$

17.36. $\bar{Y}_{1.} = .04123$, $\bar{Y}_{2.} = .05111$, $\bar{Y}_{3.} = .07074$, $\bar{Y}_{4.} = .06088$, $MSE = .000616$,

$n_1 = 103$, $n_2 = 108$, $n_3 = 152$, $n_4 = 77$,

$\hat{D}_1 = \bar{Y}_{1.} - \bar{Y}_{2.} = -.0099$, $s\{\hat{D}_1\} = .0034$

$\hat{D}_2 = \bar{Y}_{1.} - \bar{Y}_{3.} = -.0295$, $s\{\hat{D}_2\} = .0032$

$\hat{D}_3 = \bar{Y}_{1.} - \bar{Y}_{4.} = -.0196$, $s\{\hat{D}_3\} = .0037$

$\hat{D}_4 = \bar{Y}_{2.} - \bar{Y}_{3.} = -.0196$, $s\{\hat{D}_4\} = .0031$

$\hat{D}_5 = \bar{Y}_{2.} - \bar{Y}_{4.} = -.0098$, $s\{\hat{D}_5\} = .0037$

$\hat{D}_6 = \bar{Y}_{3.} - \bar{Y}_{4.} = .0099$, $s\{\hat{D}_6\} = .0035$, $q(.90; 4, 137) = 3.24$, $T = 2.291$

$-.01771 \leq D_1 \leq -.00204$, $-.03677 \leq D_2 \leq -.02225$

$-.02822 \leq D_3 \leq -.01108$, $-.02679 \leq D_4 \leq -.01247$

$-.01825 \leq D_5 \leq -.00129$, $.00191 \leq D_6 \leq .01782$

17.37. $\bar{Y}_{1.} = 2.4125$, $\bar{Y}_{2.} = 2.7375$, $\bar{Y}_{3.} = 2.4286$, $\bar{Y}_{4.} = 2.9000$, $MSE = .0245$,

$n_1 = 8$, $n_2 = 8$, $n_3 = 7$, $n_4 = 13$,

$\hat{D}_1 = \bar{Y}_{1.} - \bar{Y}_{2.} = -.3250$, $s\{\hat{D}_1\} = .0783$

$\hat{D}_2 = \bar{Y}_{1.} - \bar{Y}_{3.} = -.0161$, $s\{\hat{D}_2\} = .0810$

$\hat{D}_3 = \bar{Y}_{1.} - \bar{Y}_{4.} = -.4875$, $s\{\hat{D}_3\} = .0703$

$\hat{D}_4 = \bar{Y}_{2.} - \bar{Y}_{3.} = .3089$, $s\{\hat{D}_4\} = .0810$

$\hat{D}_5 = \bar{Y}_{2.} - \bar{Y}_{4.} = -.1625$, $s\{\hat{D}_5\} = .0703$

$\hat{D}_6 = \bar{Y}_{3.} - \bar{Y}_{4.} = -.4714, \quad s\{\hat{D}_6\} = .0734$

$q(.95; 4, 32) = 3.83$, $T = 2.708$

$-.5371 \leq D_1 \leq -.1129, \quad -.2356 \leq D_2 \leq .2035$

$-.6781 \leq D_3 \leq -.2969, \quad .0894 \leq D_4 \leq .5285$

$-.3531 \leq D_5 \leq .0281, \quad -.6703 \leq D_6 \leq -.2726$

17.38. b. Expected proportion is .95.

Chapter 18

ANOVA DIAGNOSTICS AND REMEDIAL MEASURES

18.4. a. See Problem 16.7c.

b. $r = .992$

c. t_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	.9557	1.8377	−.1010	−1.4623	.0288	−.3615
2	−1.9821	−.0426	1.7197	.6011	−.4277	−.5575
3	−.9568	.6768	1.2464	−2.0391	.5395	.4035

i	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$
1	−.7592	1.0945	−1.1728			
2	1.0009	−.2988	.2132	.7326	−1.3737	.3417

H_0: no outliers, H_a: at least one outlier. $t(.999815; 23) = 4.17$.

If $\mid t_{ij} \mid \leq 4.17$ conclude H_0, otherwise H_a. Conclude H_0.

18.5. a. See Problem 16.8c.

b. $r = .991$

d. t_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	−.4863	−1.2486	.5576	−.8516	2.3634
2	1.6992	−.2066	−1.7985	.4863	−.2066
3	1.0849	−1.0849	−.3456	.3456	.0000

H_0: no outliers, H_a: at least one outlier. $t(.99917; 11) = 4.13$.

If $|t_{ij}| \leq 4.13$ conclude H_0, otherwise H_a. Conclude H_0.

18.6. a. See Problem 16.9c.

b. $r = .990$

d. t_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	-2.3926	.9589	.0000	.4714	1.2145	.4714
2	$-.4647$	.7019	1.7354	$-.9449$	$-.2314$	$-.2314$
3	.4832	2.1280	$-.7301$	$-.9837$	$-.2405$	$-.4832$

i	$j=7$	$j=8$	$j=9$	$j=10$
1	-2.0656	.9589		
2	$-.7019$	.7019	$-.7019$	.2314

H_0: no outliers, H_a: at least one outlier. $t(.99979; 20) = 4.22$.

If $|t_{ij}| \leq 4.22$ conclude H_0, otherwise H_a. Conclude H_0.

18.7. a. See Problem 16.10c.

b. $r = .984$

d. t_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	.9927	2.4931	$-.3265$	.3265	$-.3265$	.3265
2	.1630	$-.4907$	$-.4907$	.8234	-1.1646	.8234
3	1.0497	$-.9360$	2.5645	$-.2719$	.3811	1.0497

i	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$
1	$-.9927$	.9927	-1.7017	.3265	-1.7017	$-.3265$
2	$-.4907$	1.5185	.1630	$-.4907$	-1.1646	.8234
3	$-.2719$	$-.9360$	-1.6401	$-.9360$	.3811	$-.2719$

H_0: no outliers, H_a: at least one outlier. $t(.99965; 32) = 3.75$.

If $|t_{ij}| \leq 3.75$ conclude H_0, otherwise H_a. Conclude H_0.

18.8. a. See Problem 16.11c.

b. $r = .992$

d. t_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$
1	-1.2477	.7360	$-.0203$	.6192	1.8045	.1538	$-.6601$
2	1.5815	$-.4677$	$-.4095$	1.6415	.2874	$-.7594$	-1.8287
3	-1.4648	1.8864	$-.8150$	$-.0580$	-1.4052	$-.6396$	.4648
4	.7243	1.2537	.9000	$-.4386$	$-.5551$	1.0764	.2003
5	-1.8560	.8443	$-.3775$	$-.0871$	.6105	.1451	$-.6688$
6	$-.5901$	-1.1767	.7477	1.8773	$-.1829$	.6893	$-.4153$

i	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$	$j=13$	$j=14$
1	-2.0298	1.1472	-1.6656	1.8651	$-.8355$	$-.5434$	1.2063
2	.8121	$-.7594$	-1.2892	$-.8179$	2.0053	1.3427	.5784
3	-1.2863	.0580	.9323	.0580	.5230	.7565	1.4648
4	-1.3189	.6659	$-.1480$	-2.1035	$-.2061$	-1.0823	-1.3784
5	.5522	.9616	.0871	.4357	1.0204	-1.3754	.8443
6	.1074	1.6355	-1.2952	.2816	$-.8238$	$-.2990$	.0493

i	$j=15$	$j=16$	$j=17$	$j=18$	$j=19$	$j=20$
1	.0958	.3281	1.0882	−.0203	−.4852	−1.5455
2	.9881	−.8765	−.1190	−.9940	−.4677	−.4095
3	−.0580	.1161	−.1161	.2322	−1.5246	.8736
4	.6659	1.0175	.8414	.3165	.4910	−.9646
5	−1.9168	.6688	.4357	.0871	1.3160	−1.7954
6	−1.2359	−.1248	1.6355	1.1590	−1.4141	−.5901

H_0 : no outliers, H_a : at least one outlier. $t(.9999417; 113) = 4.08$.

If $|t_{ij}| \leq 4.08$ conclude H_0, otherwise H_a. Conclude H_0.

18.9. a. See Problem 16.12c.

b. $r = .995$

d. t_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$
1	−.2047	−.2047	1.6805	−1.7195	−1.3334	.1675	1.2951
2	−1.7195	−.9534	−.9534	.5404	−.2047	2.4771	.1675
3	−.6526	−.2792	−1.4100	.0930	.0930	−.6526	.8404
4	.0744	−.6714	1.1998	.4470	−1.0479	1.5835	−1.8172
5	1.0858	−3.1711	−.7840	1.8564	−.4097	−.7840	−.0372

i	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$	$j=13$	$j=14$
1	.9157	−.5778	−1.3334	−.2047	.5404	−.5778	−.2047
2	.5404	2.0738	−1.3334	.5404	.9157	−.5778	−.9534
3	−1.0290	−1.4100	−.2792	1.6029	.0930	2.3955	.8404
4	1.1998	−1.4293	.8216	.0744	−1.0479	−.6714	−.6714
5	.3351	−.4097	−.7840	1.0858	−.0372	.7089	1.0858

i	$j=15$	$j=16$	$j=17$	$j=18$	$j=19$	$j=20$
1	1.2951	−.5778	−.5778	.9157	.5404	.1675
2	.5404	−.2047	−1.3334	1.2951	−.2047	−.5778
3	.4657	−.2792	.8404	−1.0290	−.2792	.0930
4	−.2978	.8216	.4470	.8216	−.2978	.4470
5	−.4097	1.8564	.7089	−1.5448	−.0372	−.4097

H_0 : no outliers, H_a : at least one outlier. $t(.999875; 94) = 3.81$.

If $|t_{ij}| \leq 3.81$ conclude H_0, otherwise H_a. Conclude H_0.

18.11. H_0: all σ_i^2 are equal ($i = 1, 2, 3$), H_a: not all σ_i^2 are equal.

$\tilde{Y}_1 = 6.80$, $\tilde{Y}_2 = 8.20$, $\tilde{Y}_3 = 9.55$, $MSTR = .0064815$, $MSE = .26465$,

$F^*_{BF} = .0064815/.26465 = .02$, $F(.95; 2, 24) = 3.40$. If $F^*_{BF} \leq 3.40$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .98

18.12. H_0: all σ_i^2 are equal ($i = 1, 2, 3$), H_a: not all σ_i^2 are equal.

$\tilde{Y}_1 = 40.0$, $\tilde{Y}_2 = 31.0$, $\tilde{Y}_3 = 22.5$, $MSTR = 2.96667$, $MSE = 11.30476$,

$F^*_{BF} = 2.96667/11.30476 = .26$, $F(.90; 2, 21) = 2.575$. If $F^*_{BF} \leq 2.575$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .77

18.13. a. H_0: all σ_i^2 are equal ($i = 1, 2, 3$), H_a: not all σ_i^2 are equal.

$s_1 = 1.7321$, $s_2 = 1.2881$, $s_3 = 1.6765$, $n_i \equiv 12$, $H^* = (1.7321)^2/(1.2881)^2 = 1.808$, $H(.99; 3, 11) = 6.75$. If $H^* \leq 6.75$ conclude H_0, otherwise H_a. Conclude H_0. P-value $> .05$

b. $\tilde{Y}_1 = 21.5$, $\tilde{Y}_2 = 27.5$, $\tilde{Y}_3 = 21.0$, $MSTR = .19444$, $MSE = .93434$,

$F^*_{BF} = .19444/.93434 = .21$, $F(.99; 2, 33) = 5.31$. If $F^*_{BF} \leq 5.31$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .81$

18.14. a. H_0: all σ_i^2 are equal ($i = 1, ..., 6$), H_a: not all σ_i^2 are equal.

$s_1 = .1925$, $s_2 = .1854$, $s_3 = .1646$, $s_4 = .1654$, $s_5 = .1727$, $s_6 = .1735$, $n_i \equiv 20$, $H^* = (.1925)^2/(.1646)^2 = 1.3677$, $H(.99;\ 6,\ 19) = 5.2$. If $H^* \leq 5.2$ conclude H_0, otherwise H_a. Conclude H_0. P-value $> .05$

b. $\tilde{Y}_1 = .08$, $\tilde{Y}_2 = .12$, $\tilde{Y}_3 = .47$, $\tilde{Y}_4 = .41$, $\tilde{Y}_5 = .175$, $\tilde{Y}_6 = .125$, $MSTR = .002336$, $MSE = .012336$, $F^*_{BF} = .002336/.012336 = .19$, $F(.99; 5, 114) = 3.18$. If $F^*_{BF} \leq 3.18$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .97$

18.15. a. $\bar{Y}_{1.} = 3.90$, $\bar{Y}_{2.} = 1.15$, $\bar{Y}_{3.} = 2.00$, $\bar{Y}_{4.} = 3.40$

e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$
1	.10	−.90	1.10	.10	2.10	−.90	−1.90
2	−1.15	.85	−1.15	1.85	.85	−.15	−1.15
3	0.0	−1.00	−2.00	1.00	2.00	−1.00	1.00
4	1.60	−1.40	.60	.60	2.60	1.60	−.40

i	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$	$j=13$	$j=14$
1	1.10	3.10	−2.90	−1.90	1.10	.10	3.10
2	1.85	−.15	−1.15	−1.15	−.15	−.15	−1.15
3	2.00	0.0	−2.00	−1.00	1.00	0.0	2.00
4	1.60	3.60	−.40	−2.40	−3.40	−1.40	−.40

i	$j=15$	$j=16$	$j=17$	$j=18$	$j=19$	$j=20$
1	.10	1.10	−3.90	.10	−2.90	2.10
2	−.15	1.85	−.15	.85	.85	−1.15
3	−2.00	−1.00	1.00	−2.00	0.0	2.00
4	−.40	.60	−2.40	1.60	−1.40	−.40

c. H_0: all σ_i^2 are equal ($i = 1,\ 2,\ 3,\ 4$), H_a: not all σ_i^2 are equal.

$\tilde{Y}_1 = 4$, $\tilde{Y}_2 = 1$, $\tilde{Y}_3 = 2$, $\tilde{Y}_4 = 3$, $MSTR = 1.64583$, $MSE = .96776$,

$F^*_{BF} = 1.64583/.96776 = 1.70$, $F(.90;\ 3,\ 76) = 2.157$. If $F^*_{BF} \leq 2.157$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .17$

d.

i	$\bar{Y}_{i.}$	s_i
1	3.9000	1.9708
2	1.1500	1.0894
3	2.0000	1.4510
4	3.4000	1.7889

e.

λ	SSE
-1.0	434.22
$-.8$	355.23
$-.6$	297.21
$-.4$	254.90
$-.2$	224.59
$-.1$	213.09
0	203.67

λ	SSE
.1	196.14
.2	190.35
.4	183.48
.6	182.41
.8	186.91
1.0	197.15

Yes

18.16. a. $\bar{Y}'_{1.} = 1.8714$, $\bar{Y}'_{2.} = .8427$, $\bar{Y}'_{3.} = 1.2293$, $\bar{Y}'_{4.} = 1.7471$

e'_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$
1	.129	$-.139$	.365	.129	.578	$-.139$	$-.457$
2	$-.843$	.572	$-.843$	.889	.572	.157	$-.843$
3	.185	$-.229$	-1.229	.503	.771	$-.229$	.503
4	.489	$-.333$	.253	.253	.702	.489	$-.015$

i	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$	$j=13$	$j=14$
1	.365	.774	$-.871$	$-.457$	.365	.129	.774
2	.889	.157	$-.843$	$-.843$	.157	.157	$-.843$
3	.771	.185	-1.229	$-.229$	.503	.185	.771
4	.489	.899	$-.015$	$-.747$	-1.747	$-.333$	$-.015$

i	$j=15$	$j=16$	$j=17$	$j=18$	$j=19$	$j=20$
1	.129	.365	-1.871	.129	$-.871$	.578
2	.157	.889	.157	.572	.572	$-.843$
3	-1.229	$-.229$	.503	-1.229	.185	.771
4	$-.015$	.253	$-.747$	.489	$-.333$	$-.015$

b. $r = .964$

c. H_0: all σ_i^2 are equal $(i = 1, 2, 3, 4)$, H_a: not all σ_i^2 are equal.

$\tilde{Y}_1 = 2.000$, $\tilde{Y}_2 = 1.000$, $\tilde{Y}_3 = 1.414$, $\tilde{Y}_4 = 1.732$, $MSTR = .07895$, $MSE = .20441$, $F^*_{BF} = .07895/.20441 = .39$, $F(.90;\, 3,\, 76) = 2.157$. If $F^*_{BF} \leq 2.157$ conclude H_0, otherwise H_a. Conclude H_0.

18.17. a. $\bar{Y}_{1.} = 3.5625$, $\bar{Y}_{2.} = 5.8750$, $\bar{Y}_{3.} = 10.6875$, $\bar{Y}_{4.} = 15.5625$

e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	.4375	$-.5625$	-1.5625	$-.5625$	.4375	.4375
2	1.1250	.1250	-1.8750	.1250	1.1250	-3.8750
3	1.3125	-4.6875	3.3125	1.3125	$-.6875$	-1.6875
4	.4375	-1.5625	-9.5625	3.4375	-3.5625	-5.5625

i	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$
1	$-.5625$	2.4375	1.4375	.4375	-1.5625	.4375
2	3.1250	$-.8750$	$-.8750$	3.1250	-2.8750	2.1250
3	1.3125	6.3125	-3.6875	-4.6875	1.3125	.3125
4	$-.5625$	8.4375	-5.5625	7.4375	1.4375	4.4375

i	$j=13$	$j=14$	$j=15$	$j=16$
1	.4375	-1.5625	$-.5625$	.4375
2	.1250	-1.8750	1.1250	.1250
3	-4.6875	2.3125	$-.6875$	3.3125
4	$-.5625$	2.4375	-7.5625	6.4375

c. H_0: all σ_i^2 are equal $(i = 1, 2, 3, 4)$, H_a: not all σ_i^2 are equal.

$\tilde{Y}_1 = 4.0$, $\tilde{Y}_2 = 6.0$, $\tilde{Y}_3 = 11.5$, $\tilde{Y}_4 = 16.5$, $MSTR = 37.1823$, $MSE = 3.8969$, $F^*_{BF} = 37.1823/3.8969 = 9.54$, $F(.95;\ 3,\ 60) = 2.76$. If $F^*_{BF} \leq 2.76$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d.

i	$\bar{Y}_{i.}$	s_i
1	3.5625	1.0935
2	5.8750	1.9958
3	10.6875	3.2397
4	16.5625	5.3786

e.

λ	SSE
-1.0	1,038.26
$-.8$	790.43
$-.6$	624.41
$-.4$	516.16
$-.2$	450.16
$-.1$	429.84
0	416.84

λ	SSE
.1	410.65
.2	410.92
.4	430.49
.6	476.68
.8	553.64
1.0	669.06

Yes

18.18. a. $\bar{Y}'_{1.} = .5314$, $\bar{Y}'_{2.} = .7400$, $\bar{Y}'_{3.} = 1.0080$, $\bar{Y}'_{4.} = 1.1943$

e'_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$
1	.071	$-.054$	$-.230$	$-.054$	.071	.071	$-.054$	.247
2	.105	.038	$-.138$	.038	.105	$-.439$	.214	$-.041$
3	.071	$-.230$	.138	.071	$-.008$	$-.054$	.071	.222
4	.036	$-.018$	$-.349$	.107	$-.080$	$-.153$	.010	.204

i	$j=9$	$j=10$	$j=11$	$j=12$	$j=13$	$j=14$	$j=15$	$j=16$
1	.168	.071	$-.230$	.071	.071	$-.230$	$-.054$	.071
2	$-.041$	.214	$-.263$	.163	.038	$-.138$	.105	.038
3	$-.163$	$-.230$	.071	.033	$-.230$	.106	$-.008$	.138
4	$-.153$	.186	.061	.128	.010	.085	$-.240$	.167

b. $r = .971$

c. H_0: all σ_i^2 are equal ($i = 1, 2, 3, 4$), H_a: not all σ_i^2 are equal.

$\tilde{Y}_1 = .6021$, $\tilde{Y}_2 = .7782$, $\tilde{Y}_3 = 1.0603$, $\tilde{Y}_4 = 1.2173$, $MSTR = .001214$,

$MSE = .01241$, $F^*_{BF} = .001214/.01241 = .10$, $F(.95;\ 3,\ 60) = 2.76$.

If $F^*_{BF} \leq 2.76$ conclude H_0, otherwise H_a. Conclude H_0.

18.19.

i:	1	2	3	4
s_i:	1.97084	1.08942	1.45095	1.78885
w_i:	.25745	.84257	.47500	.31250

H_0: all μ_i are equal ($i = 1, 2, 3, 4$), H_a: not all μ_i are equal.

$SSE_w(F) = 76$, $df_F = 76$, $SSE_w(R) = 118.54385$, $df_R = 79$,

$F^*_w = [(118.54385 - 76)/3] \div (76/76) = 14.18$, $F(.95; 3, 76) = 2.725$.

If $F^*_w \leq 2.725$ conclude H_0, otherwise H_a. Conclude H_a.

18.20.

i:	1	2	3	4
s_i:	1.09354	1.99583	3.23973	5.37858
w_i:	.83624	.25105	.09528	.034567

H_0: all μ_i are equal ($i = 1, 2, 3, 4$), H_a: not all μ_i are equal.

$SSE_w(F) = 60$, $df_F = 60$, $SSE_w(R) = 213.9541$, $df_R = 63$,

$F^*_w = [(213.9541 - 60)/3] \div (60/60) = 51.32$, $F(.99;\ 3,\ 60) = 4.13$.

If $F^*_w \leq 4.13$ conclude H_0, otherwise H_a. Conclude H_a.

18.23. a. H_0: all μ_i are equal ($i = 1, 2, 3, 4$), H_a: not all μ_i are equal.

$MSTR = 470.8125$, $MSE = 28.9740$, $F^*_R = 470.8125/28.9740 = 16.25$,

$F(.95; 2, 24) = 3.40$. If $F^*_R \leq 3.40$ conclude H_0, otherwise H_a. Conclude H_a.

b. P-value $= 0+$

e. $\bar{R}_{1.} = 6.50$, $\bar{R}_{2.} = 15.50$, $\bar{R}_{3.} = 22.25$, $B = z(.9833) = 2.13$

Comparison	Testing Limits	
1 and 2	$-9.00 \pm 2.13(3.500)$	-16.455 and -1.545
1 and 3	$15.75 \pm 2.13(4.183)$	-24.660 and -9.840
2 and 3	$-6.75 \pm 2.13(3.969)$	-15.204 and 1.704

Group 1	**Group 2**
Low Level $i = 1$	Moderate level $i = 2$
	High level $i = 3$

18.24. a. H_0: all μ_i are equal ($i = 1, 2, 3$), H_a: not all μ_i are equal.

$MSTR = 1,297.0000$, $MSE = 37.6667$, $F^*_R = 1,297.0000/37.6667 = 34.43$,

$F(.99; 2, 33) = 5.31$. If $F^*_R \leq 5.31$ conclude H_0, otherwise H_a. Conclude H_a.

b. P-value $= 0+$

e. $\bar{R}_{1.} = 12.792$, $\bar{R}_{2.} = 30.500$, $\bar{R}_{3.} = 12.208$, $B = z(.9833) = 2.128$

Comparison	Testing Limits	
1 and 2	$-17.708 \pm 2.128(4.301)$	-26.861 and -8.555
1 and 3	$.584 \pm 2.128(4.301)$	-8.569 and 9.737
2 and 3	$18.292 \pm 2.128(4.301)$	9.140 and 27.445

Group 1	**Group 2**
Young $i = 1$	Middle $i = 2$
Elderly $i = 3$	

18.25. b. H_0: all μ_i are equal ($i = 1, 2, 3$), H_a: not all μ_i are equal.

$MSTR = 465.6000$, $MSE = 48.7519$, $F_R^* = 465.6000/48.7519 = 9.550$,

$F(.95; 2, 27) = 3.354$. If $F_R^* \leq 3.354$ conclude H_0, otherwise H_a. Conclude H_a.

P-value $= .0007$

c. $\bar{R}_{1.} = 21.1$, $\bar{R}_{2.} = 7.9$, $\bar{R}_{3.} = 17.5$, $B = z(.99167) = 2.394$

Comparison	Testing Limits	
1 and 2	$13.2 \pm 2.394(3.937)$	3.775 and 22.625
1 and 3	$3.6 \pm 2.394(3.937)$	-5.825 and 13.025
2 and 3	$-9.6 \pm 2.394(3.937)$	-19.025 and $-.175$

Group 1	**Group 2**
Production $i = 2$	Sales $i = 1$
	Research $i = 3$

18.26. $Y_{ij} = \mu_i + \beta t_j + \epsilon_{ij}$, $t_j = 1, ..., 7$

18.27. $$\sum_{i=1}^{n_T} i = \frac{n_T(n_T+1)}{2} \qquad \sum_{i=1}^{n_T} i^2 = \frac{n_T(n_T+1)(2n_T+1)}{6}$$

$$SSTO = \sum_{i=1}^{n_T} i^2 - \left(\sum_{i=1}^{n_T} i\right)^2 / n_T$$

$$= \frac{n_T(n_T+1)(2n_T+1)}{6} - \frac{n_T^2(n_T+1)^2}{4n_T} = \frac{n_T(n_T+1)(n_T-1)}{12}$$

$SSTO/(n_T - 1) = n_T(n_T + 1)/12$

18.28. $$X_{KW}^2 = SSTR \div \frac{SSTO}{n_T - 1}$$

$$F_R^* = \frac{SSTR}{r-1} \div \frac{SSTO - SSTR}{n_T - r}$$

$$= \left(\frac{X_{KW}^2}{r-1}\right)\left(\frac{SSTO}{n_T-1}\right) \div \frac{\left[SSTO - X_{KW}^2\left(\frac{SSTO}{n_T-1}\right)\right]}{n_T - r}$$

$$= \frac{[(n_T - r)X_{KW}^2]\,SSTO}{(r-1)(n_T-1)} \div \frac{SSTO(n_T - 1 - X_{KW}^2)}{n_T - 1}$$

$= [(n_T - r)X^2_{KW}] \div [(r-1)(n_T - 1 - X^2_{KW})]$

18.29. b. $r = .994$

c. H_0: all σ_i^2 are equal ($i = 1, ..., 4$), H_a: not all σ_i^2 are equal.

$\tilde{Y}_1 = 4.85$, $\tilde{Y}_2 = 4.40$, $\tilde{Y}_3 = 4.20$, $\tilde{Y}_4 = 4.45$, $MSTR = .97716$, $MSE = .70526$,

$F^*_{BF} = .97716/.70526 = 1.39$, $F(.95;\ 3,\ 109) = 2.688$. If $F^*_{BF} \leq 2.688$ conclude H_0, otherwise H_a. Conclude H_0.

P-value = .25

18.30. b.

i	$\bar{Y}_{i.}$	s_i
1	11.08893	2.66962
2	9.68344	1.19294
3	9.19135	1.22499
4	8.11375	1.00312

c.

λ	SSE
−1.0	206.15
−.8	208.55
−.6	212.09
−.4	216.87
−.2	223.09
−.1	226.79
0	230.93

λ	SSE
.1	235.54
.2	240.65
.4	252.56
.6	267.04
.8	284.55
1.0	305.66

Yes

e. $r = .995$

f. H_0: all σ_i^2 are equal ($i = 1, ..., 4$), H_a: not all σ_i^2 are equal.

$\tilde{Y}_1 = .09332$, $\tilde{Y}_2 = .10199$, $\tilde{Y}_3 = .11111$, $\tilde{Y}_4 = .12799$, $MSTR = .00008213$,

$MSE = .00008472$, $F^*_{BF} = .00008213/.00008472 = .97$, $F(.99; 3, 109) = 3.97$.

If $F^*_{BF} \leq 3.97$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .41

g.

Source	SS	df	MS
Between regions	.0103495	3	.0034498
Error	.0254284	109	.0002333
Total	.0357779	112	

H_0: all μ_i are equal ($i = 1, ..., 4$), H_a: not all μ_i are equal.

$F^* = .0034498/.0002333 = 14.787$, $F(.99; 3, 109) = 3.967$.

If $F^* \leq 3.967$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

18.31. b. $r = .9154$

c. H_0: all σ_i^2 are equal ($i = 1, ..., 4$), H_a: not all σ_i^2 are equal.

$\tilde{Y}_1 = .03489$, $\tilde{Y}_2 = .04781$, $\tilde{Y}_3 = .06948$, $\tilde{Y}_4 = .05966$, $MSTR = .001001$, $MSE = .000326$, $F^*_{BF} = .001001/.000326 = 3.07$, $F(.99;\ 3,\ 436) = 3.83$. If $F^*_{BF} \leq 3.83$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .028

18.32. b. $r = .9902$

c. H_0: all σ_i^2 are equal ($i = 1, ..., 4$), H_a: not all σ_i^2 are equal.

$\tilde{Y}_1 = 2.415$, $\tilde{Y}_2 = 2.705$, $\tilde{Y}_3 = 2.480$, $\tilde{Y}_4 = 2.880$, $MSTR = .0106$, $MSE = .0085$, $F^*_{BF} = .0106/.0085 = 1.25$, $F(.95;\ 3,\ 32) = 2.90$. If $F^*_{BF} \leq 2.90$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .31

18.33. a. H_0: all μ_i are equal ($i = 1, ..., 4$), H_a: not all μ_i are equal. $MSTR = 2,582.575$,

$MSE = 1,031.966$, $F^*_R = 2,582.575/1,031.966 = 2.50$, $F(.95; 3, 109) = 2.69$. If $F^*_R \leq 2.69$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .063

c. $\bar{R}_{1.} = 69.196$, $\bar{R}_{2.} = 57.797$, $\bar{R}_{3.} = 47.189$, $\bar{R}_{4.} = 56.750$, $B = z(.99167) = 2.394$

Comparison	Testing Limits	
1 and 2	$11.399 \pm 2.394(8.479)$	-8.900 and 31.698
1 and 3	$22.007 \pm 2.394(8.207)$	2.359 and 41.655
1 and 4	$12.446 \pm 2.394(10.268)$	-12.136 and 37.028
2 and 3	$10.608 \pm 2.394(7.910)$	-8.329 and 29.545
2 and 4	$1.047 \pm 2.394(10.032)$	-22.970 and 25.064
3 and 4	$-9.561 \pm 2.394(9.803)$	-33.029 and 13.907

Group 1	**Group 2**
Region 3	Region 4
Region 4	Region 2
Region 2	Region 1

18.34. a. H_0: all μ_i are equal ($i = 1, ..., 4$), H_a: not all μ_i are equal.

$MSTR = 651,049$, $MSE = 11,802$, $F^*_R = 651,049/11,802 = 55.17$,

$F(.95; 3, 436) = 2.6254$. If $F^*_R \leq 2.6254$ conclude H_0, otherwise H_a. Conclude H_a.

P-value = 0+

c. $\bar{R}_{1.} = 120.4$, $\bar{R}_{2.} = 192.9$, $\bar{R}_{3.} = 290.7$, $\bar{R}_{4.} = 254.6$,

$n_1 = 103, n_2 = 108, n_3 = 152, n_4 = 77$, $B = z(.99583) = 2.638$

Comparison	Testing Limits	
1 and 2	$-72.5 \pm 2.638(17.51)$	-118.7 and -26.3
1 and 3	$-170.3 \pm 2.638(16.23)$	-213.1 and -127.5
1 and 4	$-134.2 \pm 2.638(19.16)$	-184.7 and -83.7
2 and 3	$-97.8 \pm 2.638(16.00)$	-140.0 and -55.6
2 and 4	$-61.7 \pm 2.638(18.97)$	-111.7 and -11.7
3 and 4	$36.1 \pm 2.638(17.79)$	-10.8 and 83.0

Group 1	**Group 2**	**Group 3**
Region 1	Region 2	Region 3
		Region 4

18.35. a. H_0: all μ_i are equal $(i = 1, ..., 4)$, H_a: not all μ_i are equal.

$MSTR = 955.5$, $MSE = 31.8$, $F_R^* = 955.5/31.8 = 30.1$,

$F(.95; 3, 32) = 2.90$. If $F_R^* \leq 2.90$ conclude H_0, otherwise H_a. Conclude H_a.

P-value = 0+

c. $\bar{R}_{1.} = 7.938$, $\bar{R}_{2.} = 22.375$, $\bar{R}_{3.} = 8.571, \bar{R}_{4.} = 27.962$,

$n_1 = 8, n_2 = 8, n_3 = 7, n_4 = 13$, $B = z(.99583) = 2.638$

Comparison	Testing Limits	
1 and 2	$-14.437 \pm 2.638(5.268)$	-28.334 and $-.541$
1 and 3	$-.633 \pm 2.638(5.453)$	-15.017 and 13.751
1 and 4	$-20.024 \pm 2.638(4.734)$	-32.513 and -7.535
2 and 3	$13.804 \pm 2.638(5.453)$	$-.580$ and 28.188
2 and 4	$-5.587 \pm 2.638(4.734)$	-18.076 and 6.902
3 and 4	$-19.391 \pm 2.638(4.939)$	-32.421 and -6.361

Group 1	**Group 2**	**Group 3**
Region 1	Region 2	Region 2
Region 3	Region 3	Region 4

18.36. Under H_0, each arrangement of the ranks 1, ..., 4 into groups of size 2 are equally likely and occur with probability $2!2!/4! = 1/6$. The values of F_R^* computed for the six arrangements are 0, .5, and 8, each occurring twice. Therefore the probability function $f(x)$ is:

x	$f(x) = P(F_R^* = x)$
0	1/3
.5	1/3
8	1/3

18.37. c. For the F distribution with $\nu_1 = 2$ degrees of freedom and $\nu_2 = 27$ degrees of freedom, the mean is:

$$\frac{\nu_2}{\nu_2 - 2} = 1.08$$

and the standard deviation is:

$$\frac{\nu_2}{\nu_2 - 2}\left[\frac{2(\nu_1 + \nu_2 - 2)}{\nu_1(\nu_2 - 4)}\right]^{1/2} = 1.17.$$

d. Expect 90% less than 2.51 and 99% less than 5.49.

Chapter 19

TWO-FACTOR ANALYSIS OF VARIANCE WITH EQUAL SAMPLE SIZES

19.1. a. 8

b. Infection risk

19.3.

$$(\alpha\beta)_{11} = \mu_{11} - (\mu_{..} + \alpha_1 + \beta_1) = 9 - (12 + 1 - 3) = -1$$
$$(\alpha\beta)_{12} = \mu_{12} - (\mu_{..} + \alpha_1 + \beta_2) = 12 - (12 + 1 - 1) = 0$$
$$(\alpha\beta)_{13} = \mu_{13} - (\mu_{..} + \alpha_1 + \beta_3) = 18 - (12 + 1 + 4) = 1$$
$$(\alpha\beta)_{21} = \mu_{21} - (\mu_{..} + \alpha_2 + \beta_1) = 9 - (12 - 1 - 3) = 1$$
$$(\alpha\beta)_{22} = \mu_{22} - (\mu_{..} + \alpha_2 + \beta_2) = 10 - (12 - 1 - 1) = 0$$
$$(\alpha\beta)_{23} = \mu_{23} - (\mu_{..} + \alpha_2 + \beta_3) = 14 - (12 - 1 + 4) = -1$$

19.4. a. $\mu_{1.} = 31$, $\mu_{2.} = 37$

b. $\alpha_1 = \mu_{1.} - \mu_{..} = 31 - 34 = -3$, $\alpha_2 = \mu_{2.} - \mu_{..} = 37 - 34 = 3$

19.5. a. $\mu_{.j} = 269$ $(j = 1, ..., 4)$, $\beta_j = \mu_{.j} - \mu_{..}$, $\beta_j = 0$ $(j = 1, ..., 4)$

c. $\log_e \mu_{ij}$:

	Factor B			
Factor A	B_1	B_2	B_3	B_4
A_1	5.5215	5.5797	5.5910	5.5947
A_2	5.6630	5.6095	5.5984	5.5947

19.7. a. $E\{MSE\} = 1.96$, $E\{MSA\} = 541.96$

19.8. a. $E\{MSE\} = 16$, $E\{MSAB\} = 952$

19.10. a. $\bar{Y}_{11.} = 21.66667$, $\bar{Y}_{12.} = 21.33333$, $\bar{Y}_{21.} = 27.83333$,

$\bar{Y}_{22.} = 27.66667$, $\bar{Y}_{31.} = 22.33333$, $\bar{Y}_{32.} = 20.50000$

b. e_{ijk}:

i	$j=1$	$j=2$
1	$-.66667$	$-.33333$
	1.33333	.66667
	-2.66667	-1.33333
	.33333	$-.33333$
	.33333	-2.33333
	1.33333	3.66667

i	$j=1$	$j=2$
2	2.16667	-1.66667
	1.16667	1.33333
	-1.83333	$-.66667$
	.16667	.33333
	$-.83333$	$-.66667$
	$-.83333$	1.33333

i	$j=1$	$j=2$
3	2.66667	2.50000
	$-.33333$	-1.50000
	.66667	$-.50000$
	-1.33333	.50000
	$-.33333$	$-.50000$
	-1.33333	$-.50000$

d. $r = .986$

19.11. b.

Source	SS	df	MS
Treatments	327.222	5	65.444
A (age)	316.722	2	158.361
B (gender)	5.444	1	5.444
AB interactions	5.056	2	2.528
Error	71.667	30	2.389
Total	398.889	35	

Yes, factor A (age) accounts for most of the total variability.

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero.

$F^* = 2.528/2.389 = 1.06$, $F(.95; 2, 30) = 3.32$.

If $F^* \leq 3.32$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .36

d. H_0: all α_i equal zero ($i = 1, 2, 3$), H_a: not all α_i equal zero.

$F^* = 158.361/2.389 = 66.29$, $F(.95; 2, 30) = 3.32$.

If $F^* \leq 3.32$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = 5.444/2.389 = 2.28$, $F(.95; 1, 30) = 4.17$.

If $F^* \leq 4.17$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .14

e. $\alpha \leq .143$

g. $SSA = SSTR$, $SSB + SSAB + SSE = SSE$, yes

19.12. a. $\bar{Y}_{11.} = 9.2$, $\bar{Y}_{12.} = 13.6$, $\bar{Y}_{21.} = 13.0$, $\bar{Y}_{22.} = 16.4$

b. e_{ijk}:

i	$j=1$	$j=2$
1	1.8	1.4
	-2.2	-1.6
	2.8	.4
	-3.2	-2.6
	.8	2.4

i	$j=1$	$j=2$
2	-1.0	-2.4
	3.0	.6
	-3.0	-3.4
	0	3.6
	1.0	1.6

d. $r = .976$

19.13. b.

Source	SS	*df*	MS
Treatments	131.75	3	43.917
A (eye contact)	54.45	1	54.45
B (gender)	76.05	1	76.05
AB interactions	1.25	1	1.25
Error	97.20	16	6.075
Total	228.95	19	

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = 1.25/6.075 = .21$, $F(.99; 1, 16) = 8.53$. If $F^* \leq 8.53$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .66

d. H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_1 and α_2 equal zero. $F^* = 54.45/6.075 = 8.96$, $F(.99; 1, 16) = 8.53$. If $F^* \leq 8.53$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .009

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero. $F^* = 76.05/6.075 = 12.52$, $F(.99; 1, 16) = 8.53$. If $F^* \leq 8.53$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .003.

e. $\alpha \leq .030$

19.14. a. $\bar{Y}_{11.} = 2.475$, $\bar{Y}_{12.} = 4.600$, $\bar{Y}_{13.} = 4.575$, $\bar{Y}_{21.} = 5.450$, $\bar{Y}_{22.} = 8.925$, $\bar{Y}_{23.} = 9.125$, $\bar{Y}_{31.} = 5.975$, $\bar{Y}_{32.} = 10.275$, $\bar{Y}_{33.} = 13.250$

b. e_{ijk}:

i	$j=1$	$j=2$	$j=3$
1	−.075	0	.225
	.225	−.400	−.075
	−.175	.300	−.175
	.025	.100	.025

i	$j=1$	$j=2$	$j=3$
2	.350	−.025	−.025
	−.250	.175	.175
	.050	−.225	−.425
	−.150	.075	.275

i	$j=1$	$j=2$	$j=3$
3	.125	−.375	.250
	−.275	.225	−.250
	−.075	.325	.050
	.225	−.175	−.050

d. $r = .988$

19.15. b.

Source	SS	*df*	MS
Treatments	373.105	8	46.638
A (ingredient 1)	220.020	2	110.010
B (ingredient 2)	123.660	2	61.830
AB interactions	29.425	4	7.356
Error	1.625	27	.0602
Total	374.730	35	

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = 7.356/.0602 = 122.19$, $F(.95; 4, 27) = 2.73$. If $F^* \leq 2.73$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. H_0: all α_i equal zero ($i = 1, 2, 3$), H_a: not all α_i equal zero. $F^* = 110.010/.0602 = 1,827.41$, $F(.95; 2, 27) = 3.35$. If $F^* \leq 3.35$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: all β_j equal zero ($j = 1, 2, 3$), H_a: not all β_j equal zero. $F^* = 61.830/.0602 = 1,027.08$, $F(.95; 2, 27) = 3.35$. If $F^* \leq 3.35$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. $\alpha \leq .143$

19.16. a. $\bar{Y}_{11.} = 59.8$, $\bar{Y}_{12.} = 47.8$, $\bar{Y}_{13.} = 58.4$, $\bar{Y}_{21.} = 48.4$, $\bar{Y}_{22.} = 61.2$,
$\bar{Y}_{23.} = 56.2$, $\bar{Y}_{31.} = 60.2$, $\bar{Y}_{32.} = 60.8$, $\bar{Y}_{33.} = 49.6$

b. e_{ijk}:

i	$j = 1$	$j = 2$	$j = 3$
1	2.2	9.2	.6
	−11.8	−2.8	−5.4
	3.2	−8.8	8.6
	−2.8	6.2	7.6
	9.2	−3.8	−11.4

i	$j = 1$	$j = 2$	$j = 3$
2	2.6	−.2	−1.2
	8.6	−3.2	1.8
	−3.4	8.8	−6.2
	1.6	4.8	12.8
	−9.4	−10.2	−7.2

i	$j = 1$	$j = 2$	$j = 3$
3	−1.2	−2.8	−2.6
	4.8	2.2	6.4
	−5.2	9.2	1.4
	−8.2	−7.8	−5.6
	9.8	−.8	.4

d. $r = .989$

19.17. b.

Source	SS	df	MS
Treatments	1,268.17778	8	158.52222
A (technician)	24.57778	2	12.28889
B (make)	28.31111	2	14.15556
AB interactions	1,215.28889	4	303.82222
Error	1,872.40000	36	52.01111
Total	3,140.57778	44	

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = 303.82222/52.01111 = 5.84$, $F(.99; 4, 36) = 3.89$. If $F^* \leq 3.89$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .001

d. H_0: all α_i equal zero ($i = 1, 2, 3$), H_a: not all α_i equal zero. $F^* = 12.28889/52.01111 = .24$, $F(.99; 2, 36) = 5.25$. If $F^* \leq 5.25$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .79

H_0: all β_j equal zero ($j = 1, 2, 3$), H_a: not all β_j equal zero. $F^* = 14.15556/52.01111 = .27$, $F(.99; 2, 36) = 5.25$. If $F^* \leq 5.25$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .76

e. $\alpha \leq .003$

19.18. a. $\bar{Y}'_{11.} = .44348$, $\bar{Y}'_{12.} = .80997$, $\bar{Y}'_{13.} = 1.10670$,

$\bar{Y}'_{21.} = .39823$, $\bar{Y}'_{22.} = .58096$, $\bar{Y}'_{23.} = .86639$

e'_{ijk}:

i	$j=1$		$j=2$		$j=3$	
1	−.44348	.03364	−.33285	−.11100	.09742	.12375
	.03364	−.44348	−.11100	−.20791	−.06531	−.20361
	−.14245	.33467	.09312	−.50894	−.15246	.38466
	.15858	.40162	.30398	−.03182	−.32855	−.50464
	−.44348	.51076	.39415	.51225	.30827	.34046

i	$j=1$		$j=2$		$j=3$	
2	−.39823	.07889	.19719	−.27993	.17500	.33773
	−.09720	.50486	.02110	.02110	.08785	−.16742
	−.09720	.30074	−.10384	.26413	.24755	.13361
	−.39823	−.39823	−.58096	.32213	−.26433	−.02130
	.30074	.20383	−.27993	.41904	.03670	−.56536

c. $r = .987$

19.19. b.

Source	SS	df	MS
Treatments	3.76217	5	.75243
A (duration)	.44129	1	.44129
B (weight gain)	3.20098	2	1.60049
AB interactions	.11989	2	.05995
Error	5.46770	54	.10125
Total	9.22987	59	

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = .05995/.10125 = .59$, $F(.95; 2, 54) = 3.17$. If $F^* \leq 3.17$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .56

d. H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_1 and α_2 equal zero. $F^* = .44129/.10125 = 4.36$, $F(.95; 1, 54) = 4.02$. If $F^* \leq 4.02$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .04

H_0: all β_j equal zero ($j = 1, 2, 3$), H_a: not all β_j equal zero. $F^* = 1.60049/.10125 = 15.81$, $F(.95; 2, 54) = 3.17$. If $F^* \leq 3.17$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. $\alpha \leq .143$

19.20. a. $\bar{Y}_{11.} = 222.00$, $\bar{Y}_{12.} = 106.50$, $\bar{Y}_{13.} = 60.50$, $\bar{Y}_{21.} = 62.25$, $\bar{Y}_{22.} = 44.75$, $\bar{Y}_{23.} = 38.75$

b. e_{ijk}:

i	$j=1$	$j=2$	$j=3$
1	18	3.5	−4.5
	−16	11.5	−.5
	−5	−3.5	7.5
	3	−11.5	−2.5

i	$j=1$	$j=2$	$j=3$
2	8.75	2.25	−1.75
	−9.25	7.25	−5.75
	5.75	−13.75	1.25
	−5.25	4.25	6.25

d. $r = .994$

19.21. b.

Source	SS	df	MS
Treatments	96,024.37500	5	19,204.87500
A (type)	39,447.04167	1	39,447.04167
B (years)	36,412.00000	2	18,206.00000
AB interactions	20,165.33333	2	10,082.66667
Error	1,550.25000	18	86.12500
Total	97,574.62500	23	

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = 10,082.66667/86.12500 = 117.07$, $F(.99; 2, 18) = 6.01$. If $F^* \leq 6.01$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_1 and α_2 equal zero. $F^* = 39,447.04167/86.12500 = 458.02$, $F(.99; 1, 18) = 8.29$. If $F^* \leq 8.29$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: all β_j equal zero ($j = 1, 2, 3$), H_a: not all β_j equal zero. $F^* = 18,206.00000/86.12500 = 211.39$, $F(.99; 2, 18) = 6.01$. If $F^* \leq 6.01$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. $\alpha \leq .030$

19.27. a. $B = t(.9975; 75) = 2.8925$, $q(.95; 5, 75) = 3.96$, $T = 2.800$

b. $B = t(.99167; 27) = 2.552$, $q(.95; 3, 27) = 3.51$, $T = 2.482$

19.28. (1) $B = t(.9972; 324) = 2.791$,

(2) $F(.975; 5, 324) = 2.606$, $S = 3.6097$

(3) $F(.95; 10, 324) = 1.86$, $S = 4.3128$

19.30. a. $s\{\bar{Y}_{11.}\} = .631$, $t(.975; 30) = 2.042$, $21.66667 \pm 2.042(.631)$, $20.378 \leq \mu_{11} \leq 22.955$

b. $\bar{Y}_{.1.} = 23.94$, $\bar{Y}_{.2.} = 23.17$

c. $\hat{D} = .77$, $s\{\hat{D}\} = .5152$, $t(.975; 30) = 2.042$, $.77 \pm 2.042(.5152)$, $-.282 \leq D \leq 1.822$

d. $\bar{Y}_{1..} = 21.50$, $\bar{Y}_{2..} = 27.75$, $\bar{Y}_{3..} = 21.42$

e. $\hat{D}_1 = \bar{Y}_{1..} - \bar{Y}_{2..} = -6.25$, $\hat{D}_2 = \bar{Y}_{1..} - \bar{Y}_{3..} = .08$, $\hat{D}_3 = \bar{Y}_{2..} - \bar{Y}_{3..} = 6.33$, $s\{\hat{D}_i\} = .631$ ($i = 1, 2, 3$), $q(.90; 3, 30) = 3.02$, $T = 2.1355$

$$\begin{array}{rr} -6.25 \pm 2.1355(.631) & -7.598 \leq D_1 \leq -4.902 \\ .08 \pm 2.1355(.631) & -1.268 \leq D_2 \leq 1.428 \\ 6.33 \pm 2.1355(.631) & 4.982 \leq D_3 \leq 7.678 \end{array}$$

f. Yes

g. $\hat{L} = -6.29$, $s\{\hat{L}\} = .5465$, $t(.976; 30) = 2.042$, $-6.29 \pm 2.042(.5465)$, $-7.406 \leq L \leq -5.174$

h. $L = .3\mu_{12} + .6\mu_{22} + .1\mu_{32}$, $\hat{L} = 25.05000$, $s\{\hat{L}\} = .4280$, $t(.975; 30) = 2.042$, $25.05000 \pm 2.042(.4280)$, $24.176 \leq L \leq 25.924$

19.31. a. $s\{\bar{Y}_{21.}\} = 1.1023$, $t(.995; 16) = 2.921$, $13.0 \pm 2.921(1.1023)$, $9.780 \leq \mu_{21} \leq 16.220$

b. $s\{\bar{Y}_{1..}\} = .7794$, $t(.995; 16) = 2.921$, $11.4 \pm 2.921(.7794)$, $9.123 \leq \mu_{1.} \leq 13.677$

c. $\bar{Y}_{.1.} = 11.1$, $\bar{Y}_{.2.} = 15.0$

d. $s\{\bar{Y}_{.1.}\} = s\{\bar{Y}_{.2.}\} = .7794$, $t(.995; 16) = 2.921$

$$11.1 \pm 2.921(.7794) \qquad 8.823 \leq \mu_{.1} \leq 13.377$$
$$15.0 \pm 2.921(.7794) \qquad 12.723 \leq \mu_{.2} \leq 17.277$$

98 percent

e. $\bar{Y}_{1..} = 11.4$, $\bar{Y}_{2..} = 14.7$

f. $\hat{D}_1 = 3.3$, $\hat{D}_2 = 3.9$, $s\{\hat{D}_i\} = 1.1023$ $(i = 1, 2)$, $B = t(.9875; 16) = 2.473$

$$3.3 \pm 2.473(1.1023) \qquad .574 \leq D_1 \leq 6.026$$
$$3.9 \pm 2.473(1.1023) \qquad 1.174 \leq D_2 \leq 6.626$$

g. Yes

19.32. a. $s\{\bar{Y}_{23.}\} = .1227$, $t(.975; 27) = 2.052$, $9.125 \pm 2.052(.1227)$, $8.873 \leq \mu_{23} \leq 9.377$

b. $\hat{D} = 2.125$, $s\{\hat{D}\} = .1735$, $t(.975; 27) = 2.052$, $2.125 \pm 2.052(.1735)$, $1.769 \leq D \leq 2.481$

c. $\hat{L}_1 = 2.1125$, $\hat{L}_2 = 3.5750$, $\hat{L}_3 = 5.7875$, $\hat{L}_4 = 1.4625$, $\hat{L}_5 = 3.6750$, $\hat{L}_6 = 2.2125$, $s\{\hat{L}_i\} = .1502$ $(i = 1, 2, 3)$, $s\{\hat{L}_i\} = .2125$ $(i = 4, 5, 6)$, $F(.90; 8, 27) = 1.90$, $S = 3.899$

$$2.1125 \pm 3.899(.1502) \qquad 1.527 \leq L_1 \leq 2.698$$
$$3.5750 \pm 3.899(.1502) \qquad 2.989 \leq L_2 \leq 4.161$$
$$5.7875 \pm 3.899(.1502) \qquad 5.202 \leq L_3 \leq 6.373$$
$$1.4625 \pm 3.899(.2125) \qquad .634 \leq L_4 \leq 2.291$$
$$3.6750 \pm 3.899(.2125) \qquad 2.846 \leq L_5 \leq 4.504$$
$$2.2125 \pm 3.899(.2125) \qquad 1.384 \leq L_6 \leq 3.041$$

d. $s\{\hat{D}_i\} = .1735$, $q(.90; 9, 27) = 4.31$, $T = 3.048$, $Ts\{\hat{D}_i\} = .529$, $\bar{Y}_{33.} = 13.250$

e.

i	j	$1/\bar{Y}_{ij.}$	$\sqrt{\bar{Y}_{ij.}}$
1	1	.404	1.573
1	2	.217	2.145
1	3	.219	2.139
2	1	.183	2.335
2	2	.112	2.987
2	3	.110	3.021
3	1	.167	2.444
3	2	.097	3.205
3	3	.075	3.640

19.33. a. $s\{\bar{Y}_{11.}\} = 3.2252$, $t(.995; 36) = 2.7195$, $59.8 \pm 2.7195(3.2252)$,

$51.029 \leq \mu_{11} \leq 68.571$

b. $\hat{D} = 12.8$, $s\{\hat{D}\} = 4.5612$, $t(.995; 36) = 2.7195$, $12.8 \pm 2.7195(4.5612)$, $.396 \leq D \leq 25.204$

c. $\hat{D}_1 = \bar{Y}_{11.} - \bar{Y}_{12.} = 12.0$, $\hat{D}_2 = \bar{Y}_{11.} - \bar{Y}_{13.} = 1.4$, $\hat{D}_3 = \bar{Y}_{12.} - \bar{Y}_{13.} = -10.6$, $\hat{D}_4 = \bar{Y}_{21.} - \bar{Y}_{22.} = -12.8$, $\hat{D}_5 = \bar{Y}_{21.} - \bar{Y}_{23.} = -7.8$, $\hat{D}_6 = \bar{Y}_{22.} - \bar{Y}_{23.} = 5.0$, $\hat{D}_7 = \bar{Y}_{31.} - \bar{Y}_{32.} = -.6$, $\hat{D}_8 = \bar{Y}_{31.} - \bar{Y}_{33.} = 10.6$, $\hat{D}_9 = \bar{Y}_{32.} - \bar{Y}_{33.} = 11.2$, $s\{\hat{D}_i\} = 4.5612$ $(i = 1, ..., 9)$, $B = t(.99167; 36) = 2.511$

$$\begin{array}{rr}
12.0 \pm 2.511(4.5612) & .547 \leq D_1 \leq 23.453 \\
1.4 \pm 2.511(4.5612) & -10.053 \leq D_2 \leq 12.853 \\
-10.6 \pm 2.511(4.5612) & -22.053 \leq D_3 \leq .853 \\
-12.8 \pm 2.511(4.5612) & -24.253 \leq D_4 \leq -1.347 \\
-7.8 \pm 2.511(4.5612) & -19.253 \leq D_5 \leq 3.653 \\
5.0 \pm 2.511(4.5612) & -6.453 \leq D_6 \leq 16.453 \\
-.6 \pm 2.511(4.5612) & -12.053 \leq D_7 \leq 10.853 \\
10.6 \pm 2.511(4.5612) & -.853 \leq D_8 \leq 22.053 \\
11.2 \pm 2.511(4.5612) & -.253 \leq D_9 \leq 22.653
\end{array}$$

d. $\bar{Y}_{...} = 55.8222$, $90\bar{Y}_{...} = 5,024$, $s\{90\bar{Y}_{...}\} = 96.7574$, $t(.995; 36) = 2.7195$,

$5,024 \pm 2.7195(96.7574)$, $4,760.87 \leq 90\mu_{..} \leq 5,287.13$

e. $L = 10\mu_{11} + 10\mu_{13} + 10\mu_{22} + 10\mu_{23} + 10\mu_{31} + 10\mu_{32} - 20\mu_{12} - 20\mu_{21} - 20\mu_{33}$,

$\hat{L} = 650$, $s\{\hat{L}\} = 136.8357$, $t(.995; 36) = 2.7195$,

$650 \pm 2.7195(136.8357)$, $277.875 \leq L \leq 1,022.125$

f.

i	j	$1/\bar{Y}_{ij.}$	$\log_{10}\bar{Y}_{ij.}$
1	1	.0167	1.777
1	2	.0209	1.679
1	3	.0171	1.766
2	1	.0207	1.685
2	2	.0163	1.787
2	3	.0178	1.750
3	1	.0166	1.780
3	2	.0164	1.784
3	3	.0202	1.695

19.34. a. $s\{\bar{Y}'_{22.}\} = .1006$, $t(.975; 54) = 2.005$,

$.58096 \pm 2.005(.1006)$, $.37926 \leq \mu_{22} \leq .78266$

b. $\hat{D} = .46816$, $s\{\hat{D}\} = .1423$, $t(.975; 54) = 2.005$,

$.46816 \pm 2.005(.1423)$, $.18285 \leq D \leq .75347$

c. $\bar{Y}'_{1..} = .78672$, $\bar{Y}'_{2..} = .61519$

$\bar{Y}'_{.1.} = .42086$, $\bar{Y}'_{.2.} = .69547$, $\bar{Y}'_{.3.} = .98655$

d. $B = t(.9875; 54) = 2.306$, $q(.95; 2, 54) = 2.84$, $T = 2.008$, $q(.95; 3, 54) = 3.41$, $T = 2.411$, $F(.90; 3, 54) = 2.20$, $S = 2.569$

e. $\hat{D}_1 = \bar{Y}'_{1..} - \bar{Y}'_{2..} = .17153$, $\hat{D}_2 = \bar{Y}'_{.1.} - \bar{Y}'_{.2.} = -.27461$,

$\hat{D}_3 = \bar{Y}'_{.1.} - \bar{Y}'_{.3.} = -.56569$, $\hat{D}_4 = \bar{Y}'_{.2.} - \bar{Y}'_{.3.} = -.29108$,

$s\{\hat{D}_1\} = .0822$, $s\{\hat{D}_i\} = .1006$ $(i = 2, 3, 4)$, $B = 2.306$

$.17153 \pm 2.306(.0822)$	$-.0180 \leq D_1 \leq .3611$
$-.27461 \pm 2.306(.1006)$	$-.5066 \leq D_2 \leq -.0426$
$-.56569 \pm 2.306(.1006)$	$-.7977 \leq D_3 \leq -.3337$
$-.29108 \pm 2.306(.1006)$	$-.5231 \leq D_4 \leq -.0591$

f. $L = .3\mu_{.1} + .4\mu_{.2} + .3\mu_{.3}$, $\hat{L} = .70041$, $s\{\hat{L}\} = .04149$, $t(.975; 54) = 2.005$, $.70041 \pm 2.005(.04149)$, $.6172 \leq L \leq .7836$, (3.142, 5.076), yes

19.35. a. $s\{\bar{Y}_{23.}\} = 4.6402$, $t(.995;\ 18) = 2.878$,

$38.75 \pm 2.878(4.6402)$, $25.3955 \leq \mu_{23} \leq 52.1045$

b. $\hat{D} = 46.00$, $s\{\hat{D}\} = 6.5622$, $t(.995; 18) = 2.878$,

$46.00 \pm 2.878(6.5622)$, $27.114 \leq D \leq 64.886$

c. $F(.95; 5, 18) = 2.77$, $S = 3.7216$, $B = t(.99583; 18) = 2.963$

d. $\hat{D}_1 = 159.75$, $\hat{D}_2 = 61.75$, $\hat{D}_3 = 21.75$, $\hat{L}_1 = 98.00$, $\hat{L}_2 = 138.00$, $\hat{L}_3 = 40.00$, $s\{\hat{D}_i\} = 6.5622$ $(i = 1, 2, 3)$, $s\{\hat{L}_i\} = 9.2804$ $(i = 1, 2, 3)$, $B = t(.99583; 18) = 2.963$

$159.75 \pm 2.963(6.5622)$	$140.31 \leq D_1 \leq 179.19$
$61.75 \pm 2.963(6.5622)$	$42.31 \leq D_2 \leq 81.19$
$21.75 \pm 2.963(6.5622)$	$2.31 \leq D_3 \leq 41.19$
$98.00 \pm 2.963(9.2804)$	$70.50 \leq L_1 \leq 125.50$
$138.00 \pm 2.963(9.2804)$	$110.50 \leq L_2 \leq 165.50$
$40.00 \pm 2.963(9.2804)$	$12.50 \leq L_3 \leq 67.50$

e. $q(.95; 6, 18) = 4.49$, $T = 3.1749$, $s\{\hat{D}\} = 6.5622$, $Ts\{\hat{D}\} = 20.834$, $\bar{Y}_{23.} = 38.75$, $\bar{Y}_{22.} = 44.75$

f. $B = t(.9875; 18) = 2.445$, $s\{\bar{Y}_{ij.}\} = 4.6402$

$44.75 \pm 2.445(4.6402)$	$33.405 \leq \mu_{22} \leq 56.095$
$38.75 \pm 2.445(4.6402)$	$27.405 \leq \mu_{23} \leq 50.095$

g.

i	j	$1/\bar{Y}_{ij.}$	$\log_{10}\bar{Y}_{ij.}$
1	1	.00450	2.346
1	2	.00939	2.027
1	3	.01653	1.782
2	1	.01606	1.794
2	2	.02235	1.651
2	3	.02581	1.588

19.36. a. $Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$, $i = 1, ..., 4$; $j = 1, 2$; $k = 1, 2$

b.

Source	SS	df	MS
Treatments	1,910.00	7	272.85714
A (moisture content)	1,581.50	3	527.16667
B (sweetness)	306.25	1	306.25000
AB interactions	22.25	3	7.41667
Error	57.00	8	7.12500
Total	1,967.00	15	

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = 7.41667/7.125 = 1.04$, $F(.99; 3, 8) = 7.59$. If $F^* \le 7.59$ conclude H_0, otherwise H_a. Conclude H_0.

d. $\hat{L} = -1.500$, $s\{\hat{L}\} = 2.669$, $t(.975; 8) = 2.306$,

$-1.500 \pm 2.306(2.669)$, $-7.655 \le L \le 4.655$

e. H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero. $F^* = 306.25/7.125 = 42.98$, $F(.99; 1, 8) = 11.3$. If $F^* \le 11.3$ conclude H_0, otherwise H_a. Conclude H_a.

19.37. $n = 21$

19.38. $\Delta/\sigma = 2$, $2n = 8$, $n = 4$

19.39. $n = 21$

19.40. $.5\sqrt{n}/.29 = 4.1999$, $n = 6$

19.41. $n = 14$

19.42. $8\sqrt{n}/9.1 = 3.1591$, $n = 13$

19.43. Using (19.4) and (19.5), we have:

$$\begin{aligned}\mu_{ij} &= \mu_{..} + \alpha_i + \beta_j \\ &= \mu_{..} + (\mu_{i.} - \mu_{..}) + (\mu_{.j} - \mu_{..}) = \mu_{i.} + \mu_{.j} - \mu_{..}\end{aligned}$$

19.44.

$$\begin{aligned}\sum_j (\alpha\beta)_{ij} &= \sum_j (\mu_{ij} - \mu_{i.} - \mu_{.j} + \mu_{..}) \\ &= b\mu_{i.} - b\mu_{i.} - b\mu_{..} + b\mu_{..} = 0\end{aligned}$$

19.45.

$$\begin{aligned}L &= \prod_{i=1}^{2}\prod_{j=1}^{2}\prod_{k=1}^{2} \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left[-\frac{1}{2\sigma^2}(Y_{ijk} - \mu_{ij})^2\right] \\ &= \frac{1}{(2\pi\sigma^2)^4} \exp\left[-\frac{1}{2\sigma^2}\sum\sum\sum(Y_{ijk} - \mu_{ij})^2\right]\end{aligned}$$

$$\log_e L = -4 \log_e(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum\sum\sum(Y_{ijk} - \mu_{ij})^2$$

$$\frac{\partial(\log_e L)}{\partial\mu_{ij}} = -\frac{2}{2\sigma^2}\sum_k (Y_{ijk} - \mu_{ij})(-1)$$

Setting the derivatives equal to zero, simplifying, and solving for the maximum likelihood estimators $\hat{\mu}_{ij}$ yields:

$$\hat{\mu}_{ij} = \frac{\sum_{k=1}^{2} Y_{ijk}}{2} = \bar{Y}_{ij.}$$

Yes

19.46.

$$Q = \sum\sum\sum(Y_{ijk} - \mu_{ij})^2$$

$$\frac{\partial Q}{\partial\mu_{ij}} = 2\sum_k (Y_{ijk} - \mu_{ij})(-1)$$

Setting the derivatives equal to zero, simplifying, and solving for the least squares estimators yields:

$$\hat{\mu}_{ij} = \frac{\sum\limits_k Y_{ijk}}{n} = \bar{Y}_{ij.}$$

19.47. $$\sum\sum\sum(\bar{Y}_{ij.} - \bar{Y}_{...})^2 = \sum\sum\sum[(\bar{Y}_{i..} - \bar{Y}_{...}) + (\bar{Y}_{.j.} - \bar{Y}_{...}) + (\bar{Y}_{ij.} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...})]^2$$
$$= \sum\sum\sum[(\bar{Y}_{i..} - \bar{Y}_{...})^2 + (\bar{Y}_{.j.} - \bar{Y}_{...})^2 + (\bar{Y}_{ij.} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...})^2$$
$$+2(\bar{Y}_{i..} - \bar{Y}_{...})(\bar{Y}_{.j.} - \bar{Y}_{...}) + 2(\bar{Y}_{i..} - \bar{Y}_{...})(\bar{Y}_{ij.} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...})$$
$$+2(\bar{Y}_{.j.} - \bar{Y}_{...})(\bar{Y}_{ij.} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...})]$$
$$= nb\sum(\bar{Y}_{i..} - \bar{Y}_{...})^2 + na\sum(\bar{Y}_{.j.} - \bar{Y}_{...})^2 + n\sum\sum(\bar{Y}_{ij.} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...})^2$$

since all summations of cross-product terms equal zero.

19.48. $E\{\hat{L}\} = E\{\sum c_j \bar{Y}_{.j.}\} = \sum c_j E\{\bar{Y}_{.j.}\} = \sum c_j \mu_{.j} = L$

$\sigma^2\{\hat{L}\} = \sigma^2\{\sum c_j \bar{Y}_{.j.}\} = \sum c_j^2 \sigma^2\{\bar{Y}_{.j.}\}$ because of independence

$$= \sum c_j^2 \frac{\sigma^2}{an} = \frac{\sigma^2}{an}\sum c_j^2$$

19.49. $\sigma^2\{\sum\sum c_{ij}\bar{Y}_{ij.}\} = \sum\sum c_{ij}^2 \sigma^2\{\bar{Y}_{ij.}\}$ because of independence

$$= \sum\sum c_{ij}^2 \frac{\sigma^2}{n} = \frac{\sigma^2}{n}\sum\sum c_{ij}^2$$

19.50. By (19.9a), $(\alpha\beta)_{11} + (\alpha\beta)_{21} = 0$; hence $(\alpha\beta)_{21} = -(\alpha\beta)_{11}$.

By (19.9b), $(\alpha\beta)_{11} + (\alpha\beta)_{12} = 0$; hence $(\alpha\beta)_{12} = -(\alpha\beta)_{11}$.

19.51. a. $\bar{Y}_{11.} = 10.05875$, $\bar{Y}_{12.} = 11.45500$, $\bar{Y}_{21.} = 9.84000$, $\bar{Y}_{22.} = 9.57250$, $\bar{Y}_{31.} = 9.68250$, $\bar{Y}_{32.} = 9.52375$, $\bar{Y}_{41.} = 8.21250$, $\bar{Y}_{42.} = 8.01500$

d. $r = .996$

19.52. b.

Source	SS	df	MS
Treatments	65.08508	7	9.29787
A (region)	56.74396	3	18.91465
B (average age)	.59676	1	.59676
AB interactions	7.74436	3	2.58145
Error	76.03013	56	1.35768
Total	141.11521	63	

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = 2.58145/1.35768 = 1.90$, $F(.95; 3, 56) = 2.77$. If $F^* \leq 2.77$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .14$

d. H_0: all α_i equal zero ($i = 1, ..., 4$), H_a: not all α_i equal zero. $F^* = 18.91465/1.35768 = 13.93$, $F(.95; 3, 56) = 2.77$. If $F^* \leq 2.77$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero. $F^* = .59676/1.35768 = .44$, $F(.95; 1, 56) = 4.01$. If $F^* \leq 4.01$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .51

e. $\alpha \leq .143$

19.53. a. $\bar{Y}_{11.} = .0359$, $\bar{Y}_{12.} = .0454$, $\bar{Y}_{21.} = .0516$, $\bar{Y}_{22.} = .0515$,
$\bar{Y}_{31.} = .0758$, $\bar{Y}_{32.} = .1015$, $\bar{Y}_{41.} = .0673$, $\bar{Y}_{42.} = .0766$

d. $r = .993$

19.54. a. $\bar{Y}_{1..} = .0406, \bar{Y}_{2..} = .0515, \bar{Y}_{3..} = .0886, \bar{Y}_{4..} = .0719$
$\bar{Y}_{.1.} = .0576, \bar{Y}_{.2.} = .0687$

b.

Source	SS	*df*	MS
A	.019171	3	.006390
B	.001732	1	.001732
AB	.001205	3	.000402
Error	.011042	48	.000230
Total	.033151	55	

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero.
$F^* = .000402/.000230 = 1.75$, $F(.99; 3, 48) = 4.22$.
If $F^* \leq 4.22$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .170

d. (i) Test for factor A (region effect)
H_0: all α_i equal zero ($i = 1, ..., 4$), H_a: not all α_i equal zero.
$F^* = .006390/.000230 = 27.79$, $F(.99; 3, 48) = 4.22$.
If $F^* \leq 4.22$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+
(ii) Test for factor B (% below poverty)
H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.
$F^* = .001732/.000230 = 7.53$, $F(.99; 1, 48) = 7.19$.
If $F^* \leq 7.19$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .009

e. $\alpha \leq .030$

19.55. a. $\bar{Y}_{11.} = 2.4386$, $\bar{Y}_{12.} = 2.4286$, $\bar{Y}_{21.} = 2.7286$, $\bar{Y}_{22.} = 2.9786$,

d. $r = .994$

19.56. a. $\bar{Y}_{1..} = 2.4336, \bar{Y}_{2..} = 2.8536$
$\bar{Y}_{.1.} = 2.5836, \bar{Y}_{.2.} = 2.7036$

b.

Source	SS	*df*	MS
A	1.2348	1	1.2348
B	.1008	1	.1008
AB	.1183	1	.1183
Error	.5341	24	.0223
Total	1.9880	27	

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero.

$F^* = .1183/.0223 = 5.32$, $F(.95; 1, 24) = 4.26$.

If $F^* \leq 4.26$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= .030$

d. (i) Test for factor A (region effect)

H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_1 and α_2 equal zero.

$F^* = 1.2348/.0223 = 55.48$, $F(.95; 1, 24) = 4.26$.

If $F^* \leq 4.26$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

(ii) Test for factor B (% below poverty)

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = .1008/.0223 = 4.53$, $F(.95; 1, 24) = 4.26$.

If $F^* \leq 4.26$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= .044$

e. $\alpha \leq .143$

19.57 b. $\hat{D}_1 = \bar{Y}_{1..} - \bar{Y}_{2..} = 1.0506$, $\hat{D}_2 = \bar{Y}_{1..} - \bar{Y}_{3..} = 1.1538$,

$\hat{D}_3 = \bar{Y}_{1..} - \bar{Y}_{4..} = 2.6431$, $\hat{D}_4 = \bar{Y}_{2..} - \bar{Y}_{3..} = .1032$,

$\hat{D}_5 = \bar{Y}_{2..} - \bar{Y}_{4..} = 1.5925$, $\hat{D}_6 = \bar{Y}_{3..} - \bar{Y}_{4..} = 1.4893$,

$s\{\hat{D}_i\} = .41196$ $(i = 1, ..., 6)$, $q(.90; 4, 56) = 3.31$, $T = 2.341$

$1.0506 \pm 2.341(.41196)$	$.0862 \leq D_1 \leq 2.0150$
$1.1538 \pm 2.341(.41196)$	$.1894 \leq D_2 \leq 2.1182$
$2.6431 \pm 2.341(.41196)$	$1.6787 \leq D_3 \leq 3.6075$
$.1032 \pm 2.341(.41196)$	$-.8612 \leq D_4 \leq 1.0676$
$1.5925 \pm 2.341(.41196)$	$.6281 \leq D_5 \leq 2.5569$
$1.4893 \pm 2.341(.41196)$	$.5249 \leq D_6 \leq 2.4537$

19.58. b. $\hat{D}_1 = \bar{Y}_{1..} - \bar{Y}_{2..} = -.0109$, $\hat{D}_2 = \bar{Y}_{1..} - \bar{Y}_{3..} = -.0480$,

$\hat{D}_3 = \bar{Y}_{1..} - \bar{Y}_{4..} = -.0313$, $\hat{D}_4 = \bar{Y}_{2..} - \bar{Y}_{3..} = -.0371$,

$\hat{D}_5 = \bar{Y}_{2..} - \bar{Y}_{4..} = -.0204$, $\hat{D}_6 = \bar{Y}_{3..} - \bar{Y}_{4..} = .0167$,

$s\{\hat{D}_i\} = .005732$ $(i = 1, ..., 6)$, $q(.95; 4, 48) = 3.79$, $T = 2.680$

$-.0109 \pm 2.680(.0057)$	$-.0262 \leq D_1 \leq .0043$
$-.0480 \pm 2.680(.0057)$	$-.0633 \leq D_2 \leq -.0328$
$-.0313 \pm 2.680(.0057)$	$-.0466 \leq D_3 \leq -.0161$
$-.0371 \pm 2.680(.0057)$	$-.0524 \leq D_4 \leq -.0219$
$-.0204 \pm 2.680(.0057)$	$-.0356 \leq D_5 \leq -.0052$
$.0167 \pm 2.680(.0057)$	$.0015 \leq D_6 \leq .0320$

Chapter 20

TWO-FACTOR STUDIES – ONE CASE PER TREATMENT

20.1. 0

20.2. b.

Source	SS	df	MS
Location	37.0050	3	12.3350
Week	47.0450	1	47.0450
Error	.3450	3	.1150
Total	84.3950	7	

H_0: all α_i equal zero ($i = 1, ..., 4$), H_a: not all α_i equal zero.

$F^* = 12.3350/.1150 = 107.26$, $F(.95; 3, 3) = 9.28$. If $F^* \leq 9.28$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .0015

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = 47.0450/.1150 = 409.09$, $F(.95; 1, 3) = 10.1$. If $F^* \leq 10.1$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .0003. $\alpha \leq .0975$

c. $\hat{D}_1 = \bar{Y}_{1.} - \bar{Y}_{2.} = 18.95 - 14.55 = 4.40$, $\hat{D}_2 = \bar{Y}_{1.} - \bar{Y}_{3.} = 18.95 - 14.60 = 4.35$, $\hat{D}_3 = \bar{Y}_{1.} - \bar{Y}_{4.} = 18.95 - 18.80 = .15$, $\hat{D}_4 = \bar{Y}_{2.} - \bar{Y}_{3.} = -.05$, $\hat{D}_5 = \bar{Y}_{2.} - \bar{Y}_{4.} = -4.25$, $\hat{D}_6 = \bar{Y}_{3.} - \bar{Y}_{4.} = -4.20$, $\hat{D}_7 = \bar{Y}_{.1} - \bar{Y}_{.2} = 14.30 - 19.15 = -4.85$, $s\{\hat{D}_i\} = .3391$ ($i = 1, ..., 6$), $s\{\hat{D}_7\} = .2398$, $B = t(.99286; 3) = 5.139$

$$
\begin{array}{rr}
4.40 \pm 5.139(.3391) & 2.66 \leq D_1 \leq 6.14 \\
4.35 \pm 5.139(.3391) & 2.61 \leq D_2 \leq 6.09 \\
.15 \pm 5.139(.3391) & -1.59 \leq D_3 \leq 1.89 \\
-.05 \pm 5.139(.3391) & -1.79 \leq D_4 \leq 1.69 \\
-4.25 \pm 5.139(.3391) & -5.99 \leq D_5 \leq -2.51 \\
-4.20 \pm 5.139(.3391) & -5.94 \leq D_6 \leq -2.46 \\
-4.85 \pm 5.139(.2398) & -6.08 \leq D_7 \leq -3.62
\end{array}
$$

20.3. a. $\hat{\mu}_{32} = \bar{Y}_{3.} + \bar{Y}_{.2} - \bar{Y}_{..} = 14.600 + 19.150 - 16.725 = 17.025$

b. $s^2\{\hat{\mu}_{32}\} = .071875$

c. $s\{\hat{\mu}_{32}\} = .2681$, $t(.975; 3) = 3.182$, $17.025 \pm 3.182(.2681)$, $16.172 \leq \mu_{32} \leq 17.878$

20.4. $\hat{D} = (-4.13473)/(18.5025)(11.76125) = -.019$, $SSAB^* = .0786, SSRem^* = .2664$.

H_0: $D = 0$, H_a: $D \neq 0$. $F^* = (.0786/1) \div (.2664/2) = .59$, $F(.975; 1, 2) = 38.5$.

If $F^* \leq 38.5$ conclude H_0, otherwise H_a. Conclude H_0.

20.5. b.

Source	SS	df	MS
Type of group	1.125	1	1.125
Size of group	318.375	3	106.125
Error	6.375	3	2.125
Total	325.875	7	

H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_1 and α_2 equal zero. $F^* = 1.125/2.125 = .53$, $F(.99; 1, 3) = 34.1$. If $F^* \leq 34.1$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .52

H_0: all β_j equal zero ($j = 1, ..., 4$), H_a: not all β_j equal zero. $F^* = 106.125/2.125 = 49.94$, $F(.99; 3, 3) = 29.5$. If $F^* \leq 29.5$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .005. $\alpha \leq .0199$

c. $\hat{D}_1 = \bar{Y}_{.2} - \bar{Y}_{.1} = 22.5 - 16.5 = 6.0$, $\hat{D}_2 = \bar{Y}_{.3} - \bar{Y}_{.2} = 30.0 - 22.5 = 7.5$, $\hat{D}_3 = \bar{Y}_{.4} - \bar{Y}_{.3} = 32.5 - 30.0 = 2.5$, $s\{\hat{D}_i\} = 1.4577$ ($i = 1, 2, 3$), $B = t(.99167; 3) = 4.857$

$$
\begin{array}{ll}
6.0 \pm 4.857(1.4577) & -1.08 \leq D_1 \leq 13.08 \\
7.5 \pm 4.857(1.4577) & .42 \leq D_2 \leq 14.58 \\
2.5 \pm 4.857(1.4577) & -4.58 \leq D_3 \leq 9.58
\end{array}
$$

d. No, $q(.95; 4, 3) = 6.82$, $T = 4.822$

20.6. a. $\hat{\mu}_{14} = \bar{Y}_{1.} + \bar{Y}_{.4} - \bar{Y}_{..} = 25.750 + 32.500 - 25.375 = 32.875$

b. $s^2\{\hat{\mu}_{14}\} = 1.3281$

c. $s\{\hat{\mu}_{14}\} = 1.1524$, $t(.995; 3) = 5.841$, $32.875 \pm 5.841(1.1524)$, $26.144 \leq \mu_{14} \leq 39.606$

20.7. $\hat{D} = (-8.109375)/(.28125)(159.1875) = -.1811$, $SSAB^* = 1.4688$,

$SSRem^* = 4.9062$. H_0: $D = 0$, H_a: $D \neq 0$. $F^* = (1.4688/1) \div (4.9062/2) = .60$,

$F(.99; 1, 2) = 98.5$. If $F^* \leq 98.5$ conclude H_0, otherwise H_a. Conclude H_0.

20.8. b.

Source	SS	df	MS
Humidity	2.12167	2	1.06083
Temperature	202.20000	3	67.40000
Error	6.58500	6	1.09750
Total	210.90667	11	

H_0: all α_i equal zero ($i = 1, 2, 3$), H_a: not all α_i equal zero.

$F^* = 1.06083/1.09750 = .97$, $F(.975; 2, 6) = 7.26$. If $F^* \leq 7.26$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .43

H_0: all β_j equal zero ($j = 1, ..., 4$), H_a: not all β_j equal zero.

$F^* = 67.40000/1.09750 = 61.41$, $F(.975; 3, 6) = 6.60$. If $F^* \leq 6.60$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $\hat{D}_1 = \bar{Y}_{.2} - \bar{Y}_{.1} = 15.30 - 14.90 = .40$, $\hat{D}_2 = \bar{Y}_{.3} - \bar{Y}_{.2} = 20.70 - 15.30 = 5.40$, $\hat{D}_3 = \bar{Y}_{.4} - \bar{Y}_{.3} = 24.83 - 20.70 = 4.13$, $s\{\hat{D}_i\} = .8554$ $(i = 1, 2, 3)$, $B = t(.99167; 6) = 3.2875$

$$\begin{array}{rr} .40 \pm 3.2875(.8554) & -2.41 \leq D_1 \leq 3.21 \\ 5.40 \pm 3.2875(.8554) & 2.59 \leq D_2 \leq 8.21 \\ 4.13 \pm 3.2875(.8554) & 1.32 \leq D_3 \leq 6.94 \end{array}$$

d. Yes

20.9. a. $\hat{\mu}_{23} = \bar{Y}_{2.} + \bar{Y}_{.3} - \bar{Y}_{..} = 19.325 + 20.700 - 18.933 = 21.092$

b. $s^2\{\hat{\mu}_{23}\} = .54875$

c. $s\{\hat{\mu}_{23}\} = .7408$, $t(.99;\ 6) = 3.143$, $21.092 \pm 3.143(.7408)$, $18.764 \leq \mu_{23} \leq 23.420$, (2.66%, 4.12%)

20.10. $\hat{D} = (-8.27113)/(.5304)(67.4000) = -.2314$, $SSAB^* = 1.9137$, $SSRem^* = 4.6713$.

H_0: $D = 0$, H_a: $D \neq 0$. $F^* = (1.9137/1) \div (4.6713/5) = 2.05$,

$F(.995; 1, 5) = 22.8$. If $F^* \leq 22.8$ conclude H_0, otherwise H_a. Conclude H_0.

20.11. $SSA = b\sum(\bar{Y}_{i.} - \bar{Y}_{..})^2$, $SSB = a\sum(\bar{Y}_{.j} - \bar{Y}_{..})^2$

20.12.

$$(\alpha\beta)_{ij} = A + B\alpha_i + C\beta_j + D\alpha_i\beta_j + E\alpha_i^2 + F\beta_j^2 \quad (1)$$

Averaging (1) over i yields:

$$\overline{(\alpha\beta)}_{.j} = A + C\beta_j + E\sum\alpha_i^2/a + F\beta_j^2 = 0 \quad (2)$$

because $\sum\alpha_i = 0$ and $\sum_i(\alpha\beta)_{ij} = 0$. Similarly:

$$\overline{(\alpha\beta)}_{i.} = A + B\alpha_i + E\alpha_i^2 + F\sum\beta_j^2/b \quad (3)$$

because $\sum\beta_j = 0$ and $\sum_j(\alpha\beta)_{ij} = 0$. From (2) and (3) we obtain:

$$C\beta_j + F\beta_j^2 = -A - E\sum\alpha_i^2/a \quad (4)$$

$$B\alpha_i + E\alpha_i^2 = -A - F\sum\beta_j^2/b \quad (5)$$

Substituting (4) and (5) in (1) yields:

$$(\alpha\beta)_{ij} = -A - E\sum\alpha_i^2/a - F\sum\beta_j^2/b + D\alpha_i\beta_j \quad (6)$$

Averaging (6) over j yields:

$$\overline{(\alpha\beta)}_{i.} = -A - E\sum\alpha_i^2/a - F\sum\beta_j^2/b = 0 \quad (7)$$

Using (7) in (6) yields:

$$(\alpha\beta)_{ij} = D\alpha_i\beta_j \quad (8)$$

Chapter 21

RANDOMIZED COMPLETE BLOCK DESIGNS

21.5. b. e_{ij}:

i	$j=1$	$j=2$	$j=3$
1	-2.50000	1.50000	1.00000
2	1.50000	$-.50000$	-1.00000
3	2.16667	$-.83333$	-1.33333
4	$.16667$	$-.83333$	$.66667$
5	4.16667	-4.83333	$.66667$
6	1.50000	$-.50000$	-1.00000
7	-1.50000	-1.50000	3.00000
8	-2.83333	3.16667	$-.33333$
9	-1.50000	2.50000	-1.00000
10	-1.16667	1.83333	$-.66667$

$r = .984$

d. H_0: $D = 0$, H_a: $D \neq 0$. $SSBL.TR^* = .13$, $SSRem^* = 112.20$,

$F^* = (.13/1) \div (112.20/17) = .02$, $F(.99; 1, 17) = 8.40$. If $F^* \leq 8.40$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .89$

21.6. a.

Source	SS	df	MS
Blocks	433.36667	9	48.15185
Training methods	1,295.00000	2	647.50000
Error	112.33333	18	6.24074
Total	1,840.70000	29	

b. $\bar{Y}_{.1} = 70.6$, $\bar{Y}_{.2} = 74.6$, $\bar{Y}_{.3} = 86.1$

c. H_0: all τ_j equal zero ($j = 1, 2, 3$), H_a: not all τ_j equal zero.

$F^* = 647.50000/6.24074 = 103.754$, $F(.95; 2, 18) = 3.55$. If $F^* \leq 3.55$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

d. $\hat{D}_1 = \bar{Y}_{.1} - \bar{Y}_{.2} = -4.0$, $\hat{D}_2 = \bar{Y}_{.1} - \bar{Y}_{.3} = -15.5$, $\hat{D}_3 = \bar{Y}_{.2} - \bar{Y}_{.3} = -11.5$, $s\{\hat{D}_i\} = 1.1172$ ($i = 1, 2, 3$), $q(.90; 3, 18) = 3.10$, $T = 2.192$

$$\begin{array}{ll} -4.0 \pm 2.192(1.1172) & -6.45 \leq D_1 \leq -1.55 \\ -15.5 \pm 2.192(1.1172) & -17.95 \leq D_2 \leq -13.05 \\ -11.5 \pm 2.192(1.1172) & -13.95 \leq D_3 \leq -9.05 \end{array}$$

e. H_0: all ρ_i equal zero ($i = 1, ..., 10$), H_a: not all ρ_i equal zero.

$F^* = 48.15185/6.24074 = 7.716$, $F(.95; 9, 18) = 2.46$.

If $F^* \leq 2.46$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .0001

21.7. b. e_{ij}:

i	$j=1$	$j=2$	$j=3$
1	−.05267	.00533	.04733
2	−.01267	−.00467	.01733
3	.00400	−.00800	.00400
4	−.02267	.01533	.00733
5	.08400	−.00800	−.07600

$r = .956$

d. H_0: $D = 0$, H_a: $D \neq 0$. $SSBL.TR^* = .0093$, $SSRem^* = .01002$,

$F^* = (.0093/1) \div (.01002/7) = 6.50$, $F(.99; 1, 7) = 12.2$.

If $F^* \leq 12.2$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .038

21.8. a.

Source	SS	df	MS
Blocks	1.41896	4	.35474
Fat content	1.32028	2	.66014
Error	.01932	8	.002415
Total	2.75856	14	

b. $\bar{Y}_{.1} = 1.110$, $\bar{Y}_{.2} = .992$, $\bar{Y}_{.3} = .430$

c. H_0: all τ_j equal zero ($j = 1, 2, 3$), H_a: not all τ_j equal zero.

$F^* = .66014/.002415 = 273.35$, $F(.95; 2, 8) = 4.46$.

If $F^* \leq 4.46$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $\hat{D}_1 = .118$, $\hat{D}_2 = .562$, $s\{\hat{D}_i\} = .03108$ ($i = 1, 2$), $B = t(.9875; 8) = 2.7515$

$$\begin{array}{ll} .118 \pm 2.7515(.03108) & .032 \leq D_1 \leq .204 \\ .562 \pm 2.7515(.03108) & .476 \leq D_2 \leq .648 \end{array}$$

e. H_0: all ρ_i equal zero ($i = 1, ..., 5$), H_a: not all ρ_i equal zero.

$F^* = .35474/.002415 = 146.89$, $F(.95; 4, 8) = 3.84$.

If $F^* \leq 3.84$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

21.9. c. e_{jik}:

	$k=1$			$k=2$	
i	$j=1$	$j=2$	i	$j=1$	$j=2$
1	−.01875	.01875	1	.00625	−.00625
2	.13125	−.03125	2	−.04375	−.05625
3	.05625	−.00625	3	−.11875	.06875
4	.08125	−.08125	4	.00625	−.00625
5	−.09375	−.15625	5	.23125	.01875
6	−.09375	−.05625	6	.03125	.11875
7	−.09375	.24375	7	.03125	−.18125
8	.03125	.06875	8	−.14375	.04375

$r = .984$

e. H_0: $D = 0$, H_a: $D \neq 0$. $SSBL.TR^* = .00503$, $SSRem^* = .29872$,

$F^* = (.00503/1) \div (.29872/20) = .337$, $F(.99; 1, 20) = 8.10$.

If $F^* \leq 8.10$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .57

21.10. a.

Source	SS	df	MS
Blocks	5.59875	7	.79982
A	2.31125	1	2.31125
B	3.38000	1	3.38000
AB interactions	.04500	1	.04500
Error	.30375	21	.01446
Total	11.63875	31	

b. H_0: all $(\alpha\beta)_{jk}$ equal zero, H_a: not all $(\alpha\beta)_{jk}$ equal zero.

$F^* = .04500/.01446 = 3.112$, $F(.99; 1, 21) = 8.017$.

If $F^* \leq 8.017$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .092

c. $\bar{Y}_{.1.} = .88750$, $\bar{Y}_{.2.} = 1.42500$, $\bar{Y}_{..1} = .83125$, $\bar{Y}_{..2} = 1.42500$

d. H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_1 and α_2 equal zero.

$F^* = 2.31125/.01446 = 159.84$, $F(.99; 1, 21) = 8.017$.

If $F^* \leq 8.017$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = 3.38000/.01446 = 233.75$, $F(.99; 1, 21) = 8.017$.

If $F^* \leq 8.017$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. $\hat{L}_1 = \bar{Y}_{.1.} - \bar{Y}_{.2.} = -.53750$, $\hat{L}_2 = \bar{Y}_{..1} - \bar{Y}_{..2} = -.65000$, $s\{\hat{L}_1\} = s\{\hat{L}_2\} = .0425$, $B = t(.9875; 21) = 2.414$

$$-.53750 \pm 2.414(.0425) \qquad -.640 \leq L_1 \leq -.435$$
$$-.65000 \pm 2.414(.0425) \qquad -.753 \leq L_2 \leq -.547$$

f. H_0: all ρ_i equal zero ($i = 1, ..., 8$), H_a: not all ρ_i equal zero.

$F^* = .79982/.01446 = 55.31$, $F(.99; 7, 21) = 3.64$.

If $F^* \leq 3.64$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

21.12. b. $\bar{Y}_{.1..} = 7.25$, $\bar{Y}_{.2..} = 12.75$, $\hat{L} = \bar{Y}_{.1..} - \bar{Y}_{.2..} = -5.50$, $s\{\hat{L}\} = 1.25$,

$t(.995; 8) = 3.355$, $-5.50 \pm 3.355(1.25)$, $-9.69 \leq L \leq -1.31$

21.13. a. $Y_{ijk} = \mu_{..} + \rho_i + \tau_j + (\rho\tau)_{ij} + \epsilon_{ijk}$

b.

Source	SS	df	MS
Blocks	523.20000	4	130.80000
Treatments	1,796.46667	2	898.23333
$BL.TR$ interactions	87.20000	8	10.90000
Error	207.00000	15	13.80000
Total	2,613.86667	29	

c. H_0: all τ_j equal zero ($j = 1, 2, 3$), H_a: not all τ_j equal zero.

$F^* = 898.23333/13.80000 = 65.089$, $F(.99; 2, 15) = 6.36$.

If $F^* \leq 6.36$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $\bar{Y}_{.1.} = 68.9$, $\bar{Y}_{.2.} = 77.1$, $\bar{Y}_{.3.} = 87.8$, $\hat{L}_1 = \bar{Y}_{.1.} - \bar{Y}_{.2.} = -8.2$, $\hat{L}_2 = \bar{Y}_{.1.} - \bar{Y}_{.3.} = -18.9$, $\hat{L}_3 = \bar{Y}_{.2.} - \bar{Y}_{.3.} = -10.7$, $s\{\hat{L}_i\} = 1.6613$ ($i = 1, 2, 3$), $q(.95; 3, 15) = 3.67$, $T = 2.595$

$-8.2 \pm 2.595(1.6613)$ $\quad -12.51 \leq L_1 \leq -3.89$

$-18.9 \pm 2.595(1.6613)$ $\quad -23.21 \leq L_2 \leq -14.59$

$-10.7 \pm 2.595(1.6613)$ $\quad -15.01 \leq L_3 \leq -6.39$

e. e_{ijk}:

	$j = 1$		$j = 2$		$j = 3$	
i	$k = 1$	$k = 2$	$k = 1$	$k = 2$	$k = 1$	$k = 2$
1	1.5	−1.5	3.0	−3.0	−2.0	2.0
2	2.0	−2.0	−4.0	4.0	−2.5	2.5
3	4.0	−4.0	3.5	−3.5	1.5	−1.5
4	−2.5	2.5	−2.5	2.5	−1.5	1.5
5	1.5	−1.5	−1.5	1.5	3.5	−3.5

$r = .956$

f. H_0: all $(\rho\tau)_{ij}$ equal zero, H_a: not all $(\rho\tau)_{ij}$ equal zero.

$F^* = 10.90000/13.80000 = .7899$, $F(.99; 8, 15) = 4.00$.

If $F^* \leq 4.00$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .62

21.14. $\phi = \frac{1}{2.5}\sqrt{\frac{10(18)}{3}} = 3.098$, $\nu_1 = 2$, $\nu_2 = 27$, $1 - \beta > .99$

21.15. $\phi = \frac{1}{.04}\sqrt{\frac{5(.02)}{3}} = 4.564$, $\nu_1 = 2$, $\nu_2 = 12$, $1 - \beta > .99$

21.16. $n_b = 49$ blocks

21.17. a. $n_b = 21$ blocks

b. $n_b = 7$ blocks

21.18. $\hat{E} = 3.084$

21.19. $\hat{E}' = 40.295$

21.20. $\hat{E} = 13.264$

21.21. $L = \prod_{i=1}^{3}\prod_{j=1}^{2} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(Y_{ij} - \mu_{..} - \rho_i - \tau_j)^2\right]$

$$\log_e L = -3 \log_e 2\pi - 3 \log_e \sigma^2 - \frac{1}{2\sigma^2}\sum\sum(Y_{ij} - \mu_{..} - \rho_i - \tau_j)^2$$

$$\frac{\partial(\log_e L)}{\partial \mu_{..}} = -\frac{2}{2\sigma^2}\sum\sum(Y_{ij} - \mu_{..} - \rho_i - \tau_j)(-1)$$

$$\frac{\partial(\log_e L)}{\partial \rho_i} = -\frac{2}{2\sigma^2}\sum_j(Y_{ij} - \mu_{..} - \rho_i - \tau_j)(-1)$$

$$\frac{\partial(\log_e L)}{\partial \tau_j} = -\frac{2}{2\sigma^2}\sum_i(Y_{ij} - \mu_{..} - \rho_i - \tau_j)(-1)$$

Setting each partial derivative equal to zero, utilizing the constraints $\sum \rho_i = \sum \tau_j = 0$, simplifying, and substituting the maximum likelihood estimators yields:

$$\sum\sum Y_{ij} = \sum\sum \hat{\mu}_{..} \quad \text{or} \quad \bar{Y}_{..} = \hat{\mu}_{..}$$

$$\sum_j Y_{ij} = \sum_j(\hat{\mu}_{..} + \hat{\rho}_i) \quad \text{or} \quad \bar{Y}_{i.} - \hat{\mu}_{..} = \hat{\rho}_i$$

$$\sum_i Y_{ij} = \sum_i(\hat{\mu}_{..} + \hat{\tau}_j) \quad \text{or} \quad \bar{Y}_{.j} - \hat{\mu}_{..} = \hat{\tau}_j$$

21.22. $E\{MSTR\} = E\left\{\frac{n_b\sum(\bar{Y}_{.j} - \bar{Y}_{..})^2}{r-1}\right\}$

$$= \frac{n_b}{r-1}E\{\sum(\bar{Y}_{.j} - \bar{Y}_{..})^2\}$$

Since:

$$(\bar{Y}_{.j} - \bar{Y}_{..}) = (\mu_{..} + \tau_j + \bar{\epsilon}_{.j}) - (\mu_{..} + \bar{\epsilon}_{..}) = \tau_j + (\bar{\epsilon}_{.j} - \bar{\epsilon}_{..})$$

and:

$$\sum(\bar{Y}_{.j} - \bar{Y}_{..})^2 = \sum\tau_j^2 + \sum(\bar{\epsilon}_{.j} - \bar{\epsilon}_{..})^2 + 2\sum\tau_j(\bar{\epsilon}_{.j} - \bar{\epsilon}_{..})$$

we find:

$$E\{\sum\tau_j^2\} = \sum\tau_j^2$$

$$E\left\{\sum(\bar{\epsilon}_{.j} - \bar{\epsilon}_{..})^2\right\} = (r-1)\left(\frac{\sigma^2}{n_b}\right)$$

$$E\left\{2\sum\tau_j(\bar{\epsilon}_{.j} - \bar{\epsilon}_{..})\right\} = 0$$

Hence:

$$E\{MSTR\} = \frac{n_b}{r-1}\left[\sum\tau_j^2 + \frac{r-1}{n_b}\sigma^2\right] = \frac{n_b}{r-1}\sum\tau_j^2 + \sigma^2$$

21.23. From (A.69):

$$(t^*)^2 = \left[\frac{\bar{W}}{s\{\bar{W}\}}\right]^2 = \frac{n_b(n_b-1)(\bar{Y}_{.1}-\bar{Y}_{.2})^2}{\Sigma[(Y_{i1}-Y_{i2})-(\bar{Y}_{.1}-\bar{Y}_{.2})]^2}$$

From (27.6b):

$$MSTR = n_b\left(\left[\bar{Y}_{.1} - \left(\frac{\bar{Y}_{.1}+\bar{Y}_{.2}}{2}\right)\right]^2 + \left[\bar{Y}_{.2} - \left(\frac{\bar{Y}_{.1}+\bar{Y}_{.2}}{2}\right)\right]^2\right)$$

$$= \frac{n_b}{2}(\bar{Y}_{.1}-\bar{Y}_{.2})^2$$

From (27.6c):

$$MSBL.TR = \frac{\sum\limits_i \left[\left(Y_{i1}-\bar{Y}_{i.}-\bar{Y}_{.1}+\bar{Y}_{..}\right)^2 + (Y_{i2}-\bar{Y}_{i.}-\bar{Y}_{.2}+\bar{Y}_{..})^2\right]}{(n_b-1)(2-1)}$$

Using:

$$\bar{Y}_{i.} = \frac{Y_{i1}+Y_{i2}}{2} \qquad \bar{Y}_{..} = \frac{\bar{Y}_{.1}+\bar{Y}_{.2}}{2}$$

we obtain:

$$MSBL.TR = \frac{1}{n_b-1}\sum_i \frac{1}{4}\left[(Y_{i1}-Y_{i2}-\bar{Y}_{.1}+\bar{Y}_{.2})^2 + (Y_{i2}-Y_{i1}-\bar{Y}_{.2}+\bar{Y}_{.1})^2\right]$$

$$= \frac{1}{2(n_b-1)}\sum_i \left[(Y_{i1}-Y_{i2})-(\bar{Y}_{.1}-\bar{Y}_{.2})\right]^2$$

Therefore:

$$F^* = \frac{n_b(n_b-1)(\bar{Y}_{.1}-\bar{Y}_{.2})^2}{\Sigma[(Y_{i1}-Y_{i2})-(\bar{Y}_{.1}-\bar{Y}_{.2})]^2} = (t^*)^2$$

21.24. When there are no ties:

$$R_{..}^2 = [n_b r(r+1)/2]^2 \qquad \sum R_{i.}^2 = n_b\left[\frac{r(r+1)}{2}\right]^2$$

$$\sum\sum R_{ij}^2 = n_b[r(r+1)(2r+1)/6]$$

Then:

$$\frac{n_b(r-1)SSTR}{SSTR+SSBL.TR}$$

$$= \frac{(r-1)\left(\Sigma R_{.j}^2 - \frac{R_{..}^2}{r}\right)}{\left(\frac{\Sigma R_{.j}^2}{n_b} - \frac{R_{..}^2}{rn_b}\right) + \left(\Sigma\Sigma R_{ij}^2 - \frac{\Sigma R_{i.}^2}{r} - \frac{\Sigma R_{.j}^2}{n_b} + \frac{R_{..}^2}{rn_b}\right)}$$

$$= \frac{(r-1)\left(\sum R_{.j}^2 - \frac{R_{..}^2}{r}\right)}{\sum\sum R_{ij}^2 - \frac{\Sigma R_{i.}^2}{r}}$$

$$= (r-1)\left[\sum R_{.j}^2 - \frac{n_b^2(r+1)^2 r}{4}\right] \div \left[\frac{n_b r(r+1)(2r+1)}{6} - \frac{n_b r(r+1)^2}{4}\right]$$

$$= \frac{12\sum R_{.j}^2}{n_b r(r+1)} - 3n_b(r+1)$$

Chapter 22

ANALYSIS OF COVARIANCE

22.5. a. $B = t(.9917; 11) = 2.820$

22.6. $Y_{ij} = \mu_. + \tau_i + \gamma_1(X_{ij1} - \bar{X}_{..1}) + \gamma_2(X_{ij2} - \bar{X}_{..2}) + \gamma_3(X_{ij1} - \bar{X}_{..1})^2$
$+\gamma_4(X_{ij2} - \bar{X}_{..2})^2 + \epsilon_{ij}$, $i = 1, ..., 4$

22.7. a. e_{ij} :

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	$-.5281$	.4061	.0089	.4573	$-.1140$	$-.1911$
2	$-.2635$	$-.2005$	.3196	.2995	$-.1662$	.0680
3	$-.1615$	.2586	$-.0099$	$-.3044$	.0472	.1700

i	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$
1	.0660	$-.0939$	$-.0112$			
2	$-.0690$	$-.1776$	$-.0005$	.0653	.0251	.0995

b. $r = .988$

c. $Y_{ij} = \mu_. + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \beta_1 I_{ij1} x_{ij} + \beta_2 I_{ij2} x_{ij} + \varepsilon_{ij}$

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$SSE(F) = .9572$, $SSE(R) = 1.3175$,

$F^* = (.3603/2) \div (.9572/21) = 3.95$, $F(.99; 2, 21) = 5.78$.

If $F^* \leq 5.78$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .035

d. Yes, 5

22.8. b. Full model: $Y_{ij} = \mu_. + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \varepsilon_{ij}$, ($\bar{X}_{..} = 9.4$).

Reduced model: $Y_{ij} = \mu_. + \gamma x_{ij} + \varepsilon_{ij}$.

c. Full model: $\hat{Y} = 7.80627 + 1.65885 I_1 - .17431 I_2 + 1.11417x$, $SSE(F) = 1.3175$

Reduced model: $\hat{Y} = 7.95185 + .54124x$, $SSE(R) = 5.5134$

H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (4.1959/2) \div (1.3175/23) = 36.625$, $F(.95; 2, 23) = 3.42$.

If $F^* \leq 3.42$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $MSE(F) = .0573$, $MSE = .6401$

e. $\hat{Y} = \hat{\mu}_. + \hat{\tau}_2 - .4\hat{\gamma} = 7.18629$, $s^2\{\hat{\mu}_.\} = .00258$, $s^2\{\hat{\tau}_2\} = .00412$, $s^2\{\hat{\gamma}\} = .00506$, $s\{\hat{\mu}_., \hat{\tau}_2\} = -.00045$, $s\{\hat{\tau}_2, \hat{\gamma}\} = -.00108$, $s\{\hat{\mu}_., \hat{\gamma}\} = -.00120$, $s\{\hat{Y}\} = .09183$, $t(.975; 23) = 2.069$, $7.18629 + 2.069(.09183)$, $6.996 \leq \mu_. + \tau_2 - .4\gamma \leq 7.376$

f. $\hat{D}_1 = \hat{\tau}_1 - \hat{\tau}_2 = 1.83316$, $\hat{D}_2 = \hat{\tau}_1 - \hat{\tau}_3 = 2\hat{\tau}_1 + \hat{\tau}_2 = 3.14339$, $\hat{D}_3 = \hat{\tau}_2 - \hat{\tau}_3 = 2\hat{\tau}_2 + \hat{\tau}_1 = 1.31023$, $s^2\{\hat{\tau}_1\} = .03759$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.00418$, $s\{\hat{D}_1\} = .22376$, $s\{\hat{D}_2\} = .37116$, $s\{\hat{D}_3\} = .19326$, $F(.90; 2, 23) = 2.55$, $S = 2.258$

$$
\begin{array}{ll}
1.83316 \pm 2.258(.22376) & 1.328 \leq D_1 \leq 2.338 \\
3.14339 \pm 2.258(.37116) & 2.305 \leq D_2 \leq 3.981 \\
1.31023 \pm 2.258(.19326) & .874 \leq D_3 \leq 1.747
\end{array}
$$

22.9. a. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	$-.5474$	$-.1325$	.2465	$-.0567$	.4901
2	$-.2747$	.1215	.2655	.0346	$-.1468$
3	$-.4225$	$-.0128$	.1090	.5290	$-.2027$

b. $r = .994$

c. $Y_{ij} = \mu_. + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \beta_1 I_{ij1} x_{ij} + \beta_2 I_{ij2} x_{ij} + \varepsilon_{ij}$.

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$SSE(F) = .7682$, $SSE(R) = 1.3162$,

$F^* = (.5480/2) \div (.7682/9) = 3.21$, $F(.995; 2, 9) = 10.1$.

If $F^* \leq 10.1$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .089

d. No

22.10. b. Full model: $Y_{ij} = \mu_. + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \varepsilon_{ij}$, $(\bar{X}_{..} = 280)$.

Reduced model: $Y_{ij} = \mu_. + \gamma x_{ij} + \varepsilon_{ij}$.

c. Full model: $\hat{Y} = 29.00000 + .14361 I_1 + 1.48842 I_2 - .02981x$, $SSE(F) = 1.3162$

Reduced model: $\hat{Y} = 29.00000 - .02697x$, $SSE(R) = 24.7081$

H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (23.3919/2) \div (1.3162/11) = 97.748$, $F(.90; 2, 11) = 2.86$.

If $F^* \leq 2.86$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $MSE(F) = .1197$, $MSE = 9.70$

e. $\hat{Y} = \hat{\mu}_. + \hat{\tau}_1 = 29.14361$, $s^2\{\hat{\mu}_.\} = .00798$, $s^2\{\hat{\tau}_1\} = .01602$, $s\{\hat{\mu}_., \hat{\tau}_1\} = 0$, $s\{\hat{Y}\} = .15492$, $t(.95; 11) = 1.796$, $29.14361 \pm 1.796(.15492)$, $28.865 \leq \mu_. + \tau_1 \leq 29.422$

f. $\hat{D}_1 = \hat{\tau}_1 - \hat{\tau}_2 = -1.34481$, $\hat{D}_2 = \hat{\tau}_1 - \hat{\tau}_3 = 2\hat{\tau}_1 + \hat{\tau}_2 = 1.77564$, $\hat{D}_3 = \hat{\tau}_2 - \hat{\tau}_3 = 2\hat{\tau}_2 + \hat{\tau}_1 = 3.12045$, $s^2\{\hat{\tau}_2\} = .01678$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.00822$, $s\{\hat{D}_1\} = .2219$, $s\{\hat{D}_2\} = .2190$, $s\{\hat{D}_3\} = .2242$, $F(.90; 2, 11) = 2.86$, $S = 2.392$

$$
\begin{array}{ll}
-1.34481 \pm 2.392(.2219) & -1.876 \leq D_1 \leq -.814 \\
1.77564 \pm 2.392(.2190) & 1.252 \leq D_2 \leq 2.299 \\
3.12045 \pm 2.392(.2242) & 2.584 \leq D_3 \leq 3.657
\end{array}
$$

22.11. a. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	.2070	−.4503	−.3648	−.2324	.8999
2	.1361	.0001	−.6691	−.9300	.3190
3	.7938	−.2099	.2295	.2801	−1.0389

i	$j=6$	$j=7$	$j=8$	$j=9$	$j=10$
1	.1178	−.5440	.3668		
2	−.3813	.4201	.5837	−.1635	.6848
3	−.0545				

b. $r = .995$

c. $Y_{ij} = \mu_. + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \beta_1 I_{ij1} x_{ij} + \beta_2 I_{ij2} x_{ij} + \varepsilon_{ij}$

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$SSE(F) = 5.94391$, $SSE(R) = 6.16575$,

$F^* = (.221834/2) \div (5.94391/18) = .336$, $F(.95; 2, 18) = 3.55$.

If $F^* \leq 3.55$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .72

d. No

22.12. b. Full model: $Y_{ij} = \mu_. + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \varepsilon_{ij}$, ($\bar{X}_{..} = 23.575$).

Reduced model: $Y_{ij} = \mu_. + \gamma x_{ij} + \varepsilon_{ij}$.

c. Full model: $\hat{Y} = 31.42704 + 3.52342 I_1 + 1.67605 I_2 + 1.16729x$, $SSE(F) = 6.16575$

Reduced model: $\hat{Y} = 32.00000 + 1.47113x$, $SSE(R) = 252.24945$

H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (246.08370/2) \div (6.16575/20) = 399.114$, $F(.99; 2, 20) = 5.85$.

If $F^* \leq 5.85$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $MSE(F) = .30829$, $MSE = 19.8095$

e. $\hat{Y} = \hat{\mu}_. + \hat{\tau}_2 + .425\hat{\gamma} = 33.59919$, $s^2\{\hat{\mu}_.\} = .013423$, $s^2\{\hat{\tau}_2\} = .024459$, $s^2\{\hat{\gamma}\} = .001025$, $s\{\hat{\mu}_., \hat{\tau}_2\} = -.003069$, $s\{\hat{\mu}_., \hat{\gamma}\} = .000082$, $s\{\hat{\tau}_2, \hat{\gamma}\} = .000886$, $s\{\hat{Y}\} = .180975$, $t(.995; 20) = 2.845$, $33.59919 \pm 2.845(.180975)$, $33.0843 \leq \mu_. + \tau_2 + .425\gamma \leq 34.1141$

f. $\hat{D}_1 = \hat{\tau}_1 - \hat{\tau}_2 = 1.84738$, $\hat{D}_2 = \hat{\tau}_1 - \hat{\tau}_3 = 2\hat{\tau}_1 + \hat{\tau}_2 = 8.72289$, $\hat{D}_3 = \hat{\tau}_2 - \hat{\tau}_3 = 2\hat{\tau}_2 + \hat{\tau}_1 = 6.87551$, $s^2\{\hat{\tau}_1\} = .0336934$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.0120919$, $s\{\hat{D}_1\} = .28705$, $s\{\hat{D}_2\} = .33296$, $s\{\hat{D}_3\} = .28838$, $B = t(.99167; 20) = 2.613$

$$\begin{array}{ll} 1.84738 \pm 2.613(.28705) & 1.097 \leq D_1 \leq 2.597 \\ 8.72289 \pm 2.613(.33296) & 7.853 \leq D_2 \leq 9.529 \\ 6.87551 \pm 2.613(.28838) & 6.122 \leq D_3 \leq 7.629 \end{array}$$

22.13. a. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	−1.7973	−6.7636	2.2280	.5922	5.7406
2	−3.4017	−4.9059	1.4415	3.9373	2.9288
3	−3.0314	−2.5019	.7781	2.6297	2.1255

b. $r = .983$

c. $Y_{ij} = \mu_{.} + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \beta_1 I_{ij1} x_{ij} + \beta_2 I_{ij2} x_{ij} + \varepsilon_{ij}$

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$SSE(F) = 145.2007$, $SSE(R) = 176.5300$,

$F^* = (31.3293/2) \div (145.2007/9) = .971$, $F(.95; 2, 9) = 4.26$.

If $F^* \leq 4.26$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .415

d. No

22.14. b. Full model: $Y_{ij} = \mu_{.} + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \varepsilon_{ij}$, ($\bar{X}_{..} = 70.46667$).

Reduced model: $Y_{ij} = \mu_{.} + \gamma x_{ij} + \varepsilon_{ij}$.

c. Full model: $\hat{Y} = 66.40000 - 13.57740 I_1 + 5.54806 I_2 + .83474 x$, $SSE(F) = 176.5300$

Reduced model: $\hat{Y} = 66.40000 + .81587x$, $SSE(R) = 1,573.8109$

H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (1,397.2809/2) \div (176.5300/11) = 43.53$, $F(.95; 2, 11) = 3.98$.

If $F^* \leq 3.98$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $MSE(F) = 16.0482$, $MSE = 113.9333$

e. $\hat{Y} = \hat{\mu}_{.} + \hat{\tau}_2 + 4.5333\hat{\gamma} = 75.7322$, $s^2\{\hat{\mu}_{.}\} = 1.06988$, $s^2\{\hat{\tau}_2\} = 2.40689$, $s^2\{\hat{\gamma}\} = .00939$, $s\{\hat{\mu}_{.}, \hat{\tau}_2\} = s\{\hat{\mu}_{.}, \hat{\gamma}\} = 0$, $s\{\hat{\tau}_2, \hat{\gamma}\} = -.05009$, $s\{\hat{Y}\} = 1.7932$, $t(.975; 11) = 2.201$, $75.7322 \pm 2.201(1.7932)$, $71.785 \leq \mu_{.} + \tau_2 + 4.5333\gamma \leq 79.679$

f. $\hat{D}_1 = \hat{\tau}_1 - \hat{\tau}_2 = -19.12546$, $\hat{D}_2 = \hat{\tau}_1 - \hat{\tau}_3 = 2\hat{\tau}_1 + \hat{\tau}_2 = -21.60674$, $\hat{D}_3 = \hat{\tau}_2 - \hat{\tau}_3 = 2\hat{\tau}_2 + \hat{\tau}_1 = -2.48128$, $s^2\{\hat{\tau}_1\} = 2.14043$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -1.08324$, $s\{\hat{D}_1\} = 2.5911$, $s\{\hat{D}_2\} = 2.5760$, $s\{\hat{D}_3\} = 2.7267$, $F(.90; 2, 11) = 2.86$, $S = 2.392$

$$\begin{array}{rr}
-19.12546 \pm 2.392(2.5911) & -25.323 \leq D_1 \leq -12.928 \\
-21.60674 \pm 2.392(2.5760) & -27.769 \leq D_2 \leq -15.445 \\
-2.48128 \pm 2.392(2.7267) & -9.004 \leq D_3 \leq 4.041
\end{array}$$

22.15. a. e_{ijk}:

i	$j=1$	$j=2$	i	$j=1$	$j=2$	i	$j=1$	$j=2$
1	−.1184	−.3510	2	−.6809	.2082	3	.9687	.6606
	−.3469	−.0939		.8660	−.1877		−.0150	.0565
	.0041	.0286		−.1177	.2531		.8789	−.1109
	−.6041	.0735		.2905	−.2327		−1.1211	−.0660
	1.2000	−.0163		−.3912	−.2776		.0912	−.4293
	−.1347	.3592		.0333	.2367		−.8027	−.1109

b. $r = .974$

c. $$\begin{aligned}Y_{ijk} = {} & \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + \beta_1 I_{ijk3} + (\alpha\beta)_{11} I_{ijk1} I_{ijk3} \\ & + (\alpha\beta)_{21} I_{ijk2} I_{ijk3} + \gamma x_{ijk} + \delta_1 I_{ijk1} x_{ijk} + \delta_2 I_{ijk2} x_{ijk} \\ & + \delta_3 I_{ijk3} x_{ijk} + \delta_4 I_{ijk1} I_{ijk3} x_{ijk} + \delta_5 I_{ijk2} I_{ijk3} x_{ijk} + \epsilon_{ijk}\end{aligned}$$

H_0: all δ_i equal zero ($i = 1, ..., 5$), H_a: not all δ_i equal zero.

$SSE(R) = 8.2941$, $SSE(F) = 6.1765$,

$F^* = (2.1176/5) \div (6.1765/24) = 1.646$, $F(.99; 5, 24) = 3.90$.

If $F^* \leq 3.90$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .19

22.16. a. $Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + \beta_1 I_{ijk3} + (\alpha\beta)_{11} I_{ijk1} I_{ijk3}$
$+ (\alpha\beta)_{21} I_{ijk2} I_{ijk3} + \gamma x_{ijk} + \epsilon_{ijk}$

$$I_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk2} = \begin{cases} 1 & \text{if case from level 2 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk3} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 2 for factor } B \end{cases}$$

$x_{ijk} = X_{ijk} - \bar{X}_{...} \qquad (\bar{X}_{...} = 3.4083)$

$\hat{Y} = 23.55556 - 2.15283 I_1 + 3.68152 I_2 + .20907 I_3 - .06009 I_1 I_3 - .04615 I_2 I_3 + 1.06122 x$

$SSE(F) = 8.2941$

b. Interactions:

$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + \beta_1 I_{ijk3} + \gamma x_{ijk} + \epsilon_{ijk}$

$\hat{Y} = 23.55556 - 2.15400 I_1 + 3.67538 I_2 + .20692 I_3 + 1.07393 x$

$SSE(R) = 8.4889$

Factor A:

$Y_{ijk} = \mu_{..} + \beta_1 I_{ijk3} + (\alpha\beta)_{11} I_{ijk1} I_{ijk3} + (\alpha\beta)_{21} I_{ijk2} I_{ijk3} + \gamma x_{ijk} + \epsilon_{ijk}$

$\hat{Y} = 23.55556 + .12982 I_3 + .01136 I_1 I_3 + .06818 I_2 I_3 + 1.52893 x$

$SSE(R) = 240.7835$

Factor B:

$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + (\alpha\beta)_{11} I_{ijk1} I_{ijk3}$
$+ (\alpha\beta)_{21} I_{ijk2} I_{ijk3} + \gamma x_{ijk} + \epsilon_{ijk}$

$\hat{Y} = 23.55556 - 2.15487 I_1 + 3.67076 I_2 - .05669 I_1 I_3 - .04071 I_2 I_3 + 1.08348 x$

$SSE(R) = 9.8393$

c. H_0: $(\alpha\beta)_{11} = (\alpha\beta)_{21} = 0$, H_a: not both $(\alpha\beta)_{11}$ and $(\alpha\beta)_{21}$ equal zero.

$F^* = (.1948/2) \div (8.2941/29) = .341$, $F(.95; 2, 29) = 3.33$.

If $F^* \leq 3.33$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .714

d. H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_1 and α_2 equal zero.

$F^* = (232.4894/2) \div (8.2941/29) = 406.445$, $F(.95; 2, 29) = 3.33$.

If $F^* \leq 3.33$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$.

$F^* = (1.5452/1) \div (8.2941/29) = 5.403$, $F(.95; 1, 29) = 4.18$.

If $F^* \leq 4.18$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .027

f. $\hat{D}_1 = \hat{\alpha}_1 - \hat{\alpha}_2 = -5.83435$, $\hat{D}_2 = \hat{\alpha}_1 - \hat{\alpha}_3 = 2\hat{\alpha}_1 + \hat{\alpha}_2 = -.62414$,

$\hat{D}_3 = \hat{\alpha}_2 - \hat{\alpha}_3 = 2\hat{\alpha}_2 + \hat{\alpha}_1 = 5.21021$, $\hat{D}_4 = \hat{\beta}_1 - \hat{\beta}_2 = 2\hat{\beta}_1 = .41814$,

$s^2\{\hat{\alpha}_1\} = .01593$, $s^2\{\hat{\alpha}_2\} = .01708$, $s\{\hat{\alpha}_1, \hat{\alpha}_2\} = -.00772$, $s^2\{\hat{\beta}_1\} = .00809$,

$s\{\hat{D}_1\} = .22011$, $s\{\hat{D}_2\} = .22343$, $s\{\hat{D}_3\} = .23102$, $s\{\hat{D}_4\} = .17989$,

$B = t(.9875; 29) = 2.364$

$$\begin{array}{rr}
-5.83435 \pm 2.364(.22011) & -6.355 \leq D_1 \leq -5.314 \\
-.62414 \pm 2.364(.22343) & -1.152 \leq D_2 \leq -.096 \\
5.21021 \pm 2.364(.23102) & 4.664 \leq D_3 \leq 5.756 \\
.41814 \pm 2.364(.17989) & -.007 \leq D_4 \leq .843
\end{array}$$

22.17. a. e_{ijk}:

i	$j=1$	$j=2$	i	$j=1$	$j=2$
1	.1707	−.8159	2	.3035	.2069
	.0810	1.3979		1.0448	1.5776
	−.4586	−.8383		−.7190	−2.0965
	−1.2448	−.5796		−.9776	.6672
	1.4517	.8359		.3483	−.3552

b. $r = .988$

c. $Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \beta_1 I_{ijk2} + (\alpha\beta)_{11} I_{ijk1} I_{ijk2} + \gamma x_{ijk}$
$+ \delta_1 I_{ijk1} x_{ijk} + \delta_2 I_{ijk2} x_{ijk} + \delta_3 I_{ijk1} I_{ijk2} x_{ijk} + \epsilon_{ijk}$

H_0: all δ_i equal zero ($i = 1, 2, 3$), H_a: not all δ_i equal zero.

$SSE(F) = 16.8817$, $SSE(R) = 18.5364$,

$F^* = (1.6547/3) \div (16.8817/12) = .392$, $F(.995; 3, 12) = 7.23$.

If $F^* \leq 7.23$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .76

22.18. a. $Y_{ijk} = \mu_{.} + \alpha_1 I_{ijk1} + \beta_1 I_{ijk2} + (\alpha\beta)_{11} I_{ijk1} I_{ijk2} + \gamma x_{ijk} + \varepsilon_{ijk}$, ($\bar{X}_{...} = 44.55$).

$\hat{Y} = 13.05000 - .36284 I_1 - 1.11905 I_2 + .09216 I_1 I_2 + .32586 x$

$SSE(F) = 18.5364$

b. Interactions:

$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \beta_1 I_{ijk2} + \gamma x_{ijk} + \epsilon_{ijk}$

$\hat{Y} = 13.05000 - .37286 I_1 - 1.12552 I_2 + .32333 x$

$SSE(R) = 18.7014$

Factor A:

$Y_{ijk} = \mu_{..} + \beta_1 I_{ijk2} + (\alpha\beta)_{11} I_{ijk1} I_{ijk2} + \gamma x_{ijk} + \epsilon_{ijk}$

$\hat{Y} = 13.05000 - 1.04962 I_2 + .12074 I_1 I_2 + .35309 x$

$SSE(R) = 20.3891$

<u>Factor B</u>:

$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + (\alpha\beta)_{11} I_{ijk1} I_{ijk2} + \gamma x_{ijk} + \epsilon_{ijk}$

$\hat{Y} = 13.05000 - .10397 I_1 + .16097 I_1 I_2 + .39140 x$

$SSE(R) = 39.8416$

c. H_0: $(\alpha\beta)_{11} = 0$, H_a: $(\alpha\beta)_{11} \neq 0$.

$F^* = (.1650/1) \div (18.5364/15) = .1335$, $F(.99; 1, 15) = 8.68$.

If $F^* \leq 8.68$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .72$

d. H_0: $\alpha_1 = 0$, H_a: $\alpha_1 \neq 0$.

$F^* = (1.8527/1) \div (18.5364/15) = 1.499$, $F(.99; 1, 15) = 8.68$.

If $F^* \leq 8.68$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .24$

e. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$.

$F^* = (21.3052/1) \div (18.5364/15) = 17.241$, $F(.99; 1, 15) = 8.68$.

If $F^* \leq 8.68$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

f. $\hat{D} = \hat{\beta}_1 - \hat{\beta}_2 = 2\hat{\beta}_1 = -2.2381$, $s\{\hat{D}\} = .539$, $t(.995; 15) = 2.947$,

$-2.2381 \pm 2.947(.539)$, $-3.8265 \leq D \leq -.6497$

g. $\hat{Y} = \hat{\mu}_{..} + \hat{\alpha}_1 - \hat{\beta}_1 - \widehat{(\alpha\beta)}_{11} - 4.55\hat{\gamma} = 12.2314$, $s^2\{\hat{\mu}_{..}\} = .06179$, $s^2\{\hat{\alpha}_1\} = .08782$, $s^2\{\hat{\beta}_1\} = .07264$, $s^2\{\hat{\gamma}\} = .00167$, $s^2\{\widehat{(\alpha\beta)}_{11}\} = .06363$, $s\{\hat{\alpha}_1, \hat{\beta}_1\} = .01680$, $s\{\hat{\alpha}_1, \widehat{(\alpha\beta)}_{11}\} = .00692$, $s\{\hat{\beta}_1, \widehat{(\alpha\beta)}_{11}\} = .00447$, $s\{\hat{\alpha}_1, \hat{\gamma}\} = .00659$, $s\{\hat{\beta}_1, \hat{\gamma}\} = .00425$, $s\{\hat{\gamma}, \widehat{(\alpha\beta)}_{11}\} = .00175$, $s\{\hat{\mu}_{..}, \hat{\alpha}_1\} = s\{\hat{\mu}_{..}, \hat{\beta}_1\} = s\{\hat{\mu}_{..}, \widehat{(\alpha\beta)}_{11}\} = s\{\hat{\mu}_{..}, \hat{\gamma}\} = 0$, $s\{\hat{Y}\} = .5259$, $t(.995; 15) = 2.947$,

$12.2314 \pm 2.947(.5259)$, $10.682 \leq \mu_{..} + \alpha_1 - \beta_1 - (\alpha\beta)_{11} - 4.55\gamma \leq 13.781$

22.19. b. $Y_{ij} = \mu_{..} + \rho_1 I_{ij1} + \rho_2 I_{ij2} + \rho_3 I_{ij3} + \rho_4 I_{ij4} + \rho_5 I_{ij5} + \rho_6 I_{ij6}$
$+ \rho_7 I_{ij7} + \rho_8 I_{ij8} + \rho_9 I_{ij9} + \tau_1 I_{ij10} + \tau_2 I_{ij11} + \gamma x_{ij} + \epsilon_{ij}$

$$I_{ij1} = \begin{cases} 1 & \text{if experimental unit from block 1} \\ -1 & \text{if experimental unit from block 10} \\ 0 & \text{otherwise} \end{cases}$$

$I_{ij2}, \ldots, I_{ij9}$ are defined similarly

$$I_{ij10} = \begin{cases} 1 & \text{if experimental unit received treatment 1} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ij11} = \begin{cases} 1 & \text{if experimental unit received treatment 2} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

$x_{ij} = X_{ij} - \bar{X}_{..}$ $\quad$ $(\bar{X}_{..} = 80.033333)$

c. $\hat{Y} = 77.10000 + 4.87199 I_1 + 3.87266 I_2 + 2.21201 I_3 + 3.22003 I_4$
$+ 1.23474 I_5 + .90876 I_6 - 1.09124 I_7 - 3.74253 I_8 - 4.08322 I_9$

$-6.50033I_{10} - 2.49993I_{11} + .00201x$

$SSE(F) = 112.3327$

d. $Y_{ij} = \mu_{..} + \rho_1 I_{ij1} + \rho_2 I_{ij2} + \rho_3 I_{ij3} + \rho_4 I_{ij4} + \rho_5 I_{ij5} + \rho_6 I_{ij6}$

$+\rho_7 I_{ij7} + \rho_8 I_{ij8} + \rho_9 I_{ij9} + \gamma x_{ij} + \epsilon_{ij}$

$\hat{Y} = 77.10000 + 6.71567I_1 + 5.67233I_2 + 3.61567I_3 + 4.09567I_4$

$+1.14233I_5 + .33233I_6 - 1.66767I_7 - 5.33100I_8 - 5.18767I_9 - .13000x$

$SSE(R) = 1,404.5167$

e. H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (1,292.18/2) \div (112.3327/17) = 97.777$, $F(.95; 2, 17) = 3.59$.

If $F^* \leq 3.59$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

f. $\hat{\tau}_1 = -6.50033$, $\hat{\tau}_2 = -2.49993$, $\hat{L} = -4.0004$, $L^2\{\hat{\tau}_1\} = .44162$, $s^2\{\hat{\tau}_2\} = .44056$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.22048$, $s\{\hat{L}\} = 1.1503$, $t(.975; 17) = 2.11$,

$-4.0004 \pm 2.11(1.1503)$, $-6.43 \leq L \leq -1.57$

22.20. a. $Y_{ij} = \mu_{..} + \rho_1 I_{ij1} + \rho_2 I_{ij2} + \rho_3 I_{ij3} + \rho_4 I_{ij4} + \tau_1 I_{ij5} + \tau_2 I_{ij6} + \gamma x_{ij} + \epsilon_{ij}$

$$I_{ij1} = \begin{cases} 1 & \text{if experimental unit from block 1} \\ -1 & \text{if experimental unit from block 5} \\ 0 & \text{otherwise} \end{cases}$$

$I_{ij2}, \ldots, I_{ij4}$ are defined similarly

$$I_{ij5} = \begin{cases} 1 & \text{if experimental unit received treatment 1} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ij6} = \begin{cases} 1 & \text{if experimental unit received treatment 2} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

$x_{ij} = X_{ij} - \bar{X}_{..} \qquad (\bar{X}_{..} = 104.46667)$

b. $\hat{Y} = .84400 - .25726I_1 - .18916I_2 - .16649I_3 + .27012I_4 + .26663I_5$

$+.15238I_6 + .009385x$

$SSE(F) = .007389$

c. $Y_{ij} = \mu_{..} + \rho_1 I_{ij1} + \rho_2 I_{ij2} + \rho_3 I_{ij3} + \rho_4 I_{ij4} + \gamma x_{ij} + \epsilon_{ij}$

$\hat{Y} = .84400 - .34176I_1 - .24725I_2 - .17555I_3 + .32143I_4 - .00193x$

$SSE(R) = 1.339085$

d. H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (1.331696/2) \div (.007389/7) = 630.79$, $F(.95; 2, 7) = 4.737$.

If $F^* \leq 4.737$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. $\hat{\tau}_1 = .26663$, $\hat{\tau}_2 = .15238$, $\hat{L}_1 = \hat{\tau}_1 - \hat{\tau}_2 = .11425$, $\hat{L}_2 = \hat{\tau}_1 + 2\hat{\tau}_2 = .57139$, $s^2\{\hat{\tau}_1\} = .0001408$, $s^2\{\hat{\tau}_2\} = .0001424$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.0000701$, $s\{\hat{L}_1\} = .02058$, $s\{\hat{L}_2\} = .02074$, $B = t(.9875; 7) = 2.841$

$.11425 \pm 2.841(.02058) \qquad .0558 \le L_1 \le .1727$
$.57139 \pm 2.841(.02074) \qquad .5125 \le L_2 \le .6303$

22.21. a.

Source	SS	df	MS
Between treatments	25.5824	2	12.7912
Error	1.4650	24	.0610
Total	27.0474	26	

b. Covariance: $MSE = .0573$, $\hat{\gamma} = 1.11417$

22.22. a.

Source	SS	df	MS
Between treatments	1,417.7333	2	708.8667
Error	223.2000	12	18.6000
Total	1,640.9333	14	

b. Covariance: $MSE = 16.048$, $\hat{\gamma} = .83474$

22.23. $Y_{ij} = \mu_. + \tau_i + \gamma(X_{ij} - \bar{X}_{..}) + \epsilon_{ij} = \Delta_i + \gamma(X_{ij} - \bar{X}_{..}) + \epsilon_{ij}$

$Q = \sum\sum[Y_{ij} - \Delta_i - \gamma(X_{ij} - \bar{X}_{..})]^2$

$$\frac{\partial Q}{\partial \Delta_i} = 2\sum_j [Y_{ij} - \Delta_i - \gamma(X_{ij} - \bar{X}_{..})](-1)$$

$$\frac{\partial Q}{\partial \gamma} = 2\sum\sum[Y_{ij} - \Delta_i - \gamma(X_{ij} - \bar{X}_{..})][-(X_{ij} - \bar{X}_{..})]$$

Setting the partial derivatives equal to zero, simplifying, and substituting the least squares estimators yields:

$$\sum_j Y_{ij} - \hat{\gamma}\sum_j (X_{ij} - \bar{X}_{..}) = n_i \hat{\Delta}_i$$

or:

$$\hat{\Delta}_i = \bar{Y}_{i.} - \hat{\gamma}(\bar{X}_{i.} - \bar{X}_{..})$$

and:

$$\sum\sum[Y_{ij} - \hat{\Delta}_i - \hat{\gamma}(X_{ij} - \bar{X}_{..})](X_{ij} - \bar{X}_{..}) = 0$$

or:

$$\sum\sum Y_{ij}(X_{ij} - \bar{X}_{..}) - \sum\sum[\bar{Y}_{i.} - \hat{\gamma}(\bar{X}_{i.} - \bar{X}_{..})](X_{ij} - \bar{X}_{..}) = \hat{\gamma}\sum\sum(X_{ij} - \bar{X}_{..})^2$$

or:

$$\hat{\gamma} = \frac{\sum\sum(Y_{ij} - \bar{Y}_{i.})(X_{ij} - \bar{X}_{i.})}{\sum\sum(X_{ij} - \bar{X}_{i.})^2}$$

It needs to be recognized in the development that:

$\sum\sum(Y_{ij} - \bar{Y}_{i.})(X_{ij} - \bar{X}_{..}) = \sum\sum(Y_{ij} - \bar{Y}_{i.})(X_{ij} - \bar{X}_{i.})$

$\sum\sum(X_{ij} - \bar{X}_{..})(X_{ij} - \bar{X}_{i.}) = \sum\sum(X_{ij} - \bar{X}_{i.})^2$

22.24. b. $r = .907$

c. $Y_{ij} = \mu_. + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \tau_3 I_{ij3} + \gamma x_{ij} + \beta_1 I_{ij1} x_{ij} + \beta_2 I_{ij2} x_{ij} + \beta_3 I_{ij3} x_{ij} + \epsilon_{ij}$

H_0: $\beta_1 = \beta_2 = \beta_3 = 0$, H_a: not all β_i equal zero.

$SSE(F) = 147.8129$, $SSE(R) = 151.3719$,

$F^* = (3.5590/3) \div (147.8129/56) = .449$, $F(.995; 3, 56) = 4.76$.

If $F^* \leq 4.76$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .72$

22.25. b. Full model: $Y_{ij} = \mu_{.} + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \tau_3 I_{ij3} + \gamma x_{ij} + \epsilon_{ij}$

$$I_{ij1} = \begin{cases} 1 & \text{if case from region NE} \\ -1 & \text{if case from region W} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ij2} = \begin{cases} 1 & \text{if case from region NC} \\ -1 & \text{if case from region W} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ij3} = \begin{cases} 1 & \text{if case from region S} \\ -1 & \text{if case from region W} \\ 0 & \text{otherwise} \end{cases}$$

$x_{ij} = X_{ij} - \bar{X}_{..}$ $\qquad (\bar{X}_{..} = 42.75625)$

Reduced model: $Y_{ij} = \mu_{.} + \gamma x_{ij} + \epsilon_{ij}$

c. <u>Full model</u>: $\hat{Y} = 9.58406 + 1.60061 I_1 + .05250 I_2 - .26776 I_3 + .02579x$,

$SSE(F) = 151.3719$

<u>Reduced model</u>: $\hat{Y} = 9.58406 + .04013x$,

$SSE(R) = 221.2543$

H_0: all τ_i equal zero $(i = 1, 2, 3)$, H_a: not all τ_i equal zero.

$F^* = (69.8824/3) \div (151.3719/59) = 9.079$, $F(.95; 3, 59) = 2.76$.

If $F^* \leq 2.76$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

d. $\hat{D}_1 = \hat{\tau}_1 - \hat{\tau}_2 = 1.54811$, $\hat{D}_2 = \hat{\tau}_1 - \hat{\tau}_3 = 1.86837$, $\hat{D}_3 = \hat{\tau}_1 - \hat{\tau}_4 = 2\hat{\tau}_1 + \hat{\tau}_2 + \hat{\tau}_3 = 2.98596$, $\hat{D}_4 = \hat{\tau}_2 - \hat{\tau}_3 = .32026$, $\hat{D}_5 = \hat{\tau}_2 - \hat{\tau}_4 = 2\hat{\tau}_2 + \hat{\tau}_1 + \hat{\tau}_3 = 1.43785$, $\hat{D}_6 = \hat{\tau}_3 - \hat{\tau}_4 = 2\hat{\tau}_3 + \hat{\tau}_1 + \hat{\tau}_2 = 1.11759$, $s^2\{\hat{\tau}_1\} = .12412$, $s^2\{\hat{\tau}_2\} = .12188$, $s^2\{\hat{\tau}_3\} = .12355$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.03759$, $s\{\hat{\tau}_1, \hat{\tau}_3\} = -.04365$, $s\{\hat{\tau}_2, \hat{\tau}_3\} = -.04240$, $s\{\hat{D}_1\} = .56673$, $s\{\hat{D}_2\} = .57877$, $s\{\hat{D}_3\} = .57632$, $s\{\hat{D}_4\} = .57466$, $s\{\hat{D}_5\} = .57265$, $s\{\hat{D}_6\} = .56641$, $B = t(.99167; 59) = 2.464$

$$\begin{array}{rr}
1.54811 \pm 2.464(.56673) & .1517 \leq D_1 \leq 2.9445 \\
1.86837 \pm 2.464(.57877) & .4423 \leq D_2 \leq 3.2945 \\
2.98596 \pm 2.464(.57632) & 1.5659 \leq D_3 \leq 4.4060 \\
.32026 \pm 2.464(.57466) & -1.0957 \leq D_4 \leq 1.7362 \\
1.43785 \pm 2.464(.57265) & .0268 \leq D_5 \leq 2.8489 \\
1.11759 \pm 2.464(.56641) & -.2780 \leq D_6 \leq 2.5132
\end{array}$$

22.26. b. $r = .9914$

c. $Y_{ij} = \mu_{.} + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \tau_3 I_{ij3} + \gamma x_{ij} + \beta_1 I_{ij1} x_{ij} + \beta_2 I_{ij2} x_{ij} + \beta_3 I_{ij3} x_{ij} + \epsilon_{ij}$

H_0: $\beta_1 = \beta_2 = \beta_3 = 0$, H_a: not all β_i equal zero.

$SSE(F) = .6521$, $SSE(R) = .6778$,

$F^* = (.0257/3) \div (.6521/28) = .37$, $F(.95; 3, 28) = 2.95$.

If $F^* \leq 2.95$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .78$

22.27. b. Full model: $Y_{ij} = \mu_{.} + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \tau_3 I_{ij3} + \gamma x_{ij} + \epsilon_{ij}$

$$I_{ij1} = \begin{cases} 1 & \text{if case from (Var5 Var6)=(0,0)} \\ -1 & \text{if case from (Var5 Var6)=(1,1)} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ij2} = \begin{cases} 1 & \text{if case from (Var5 Var6)=(1,0)} \\ -1 & \text{if case from (Var5 Var6)=(1,1)} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ij3} = \begin{cases} 1 & \text{if case from (Var5 Var6)=(0,1)} \\ -1 & \text{if case from (Var5 Var6)=(1,1)} \\ 0 & \text{otherwise} \end{cases}$$

$x_{ij} = X_{ij} - \bar{X}_{..}$ $\qquad (\bar{X}_{..} = 2.3244)$

Reduced model: $Y_{ij} = \mu_{.} + \gamma x_{ij} + \epsilon_{ij}$

c. Full model: $\hat{Y} = 2.619 - .217I_1 + .109I_2 - .178I_3 - .344x$, $SSE(F) = .6778$

Reduced model: $\hat{Y} = 2.664 - .306x$, $SSE(R) = 2.3593$

H_0: $\tau_1 = \tau_2 = \tau_3 = 0$, H_a: not all τ_i equal zero.

$F^* = (1.6815/3) \div (.6778/31) = 25.64$, $F(.99; 3, 31) = 4.51$.

If $F^* \leq 4.51$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

d. In Project 16.45, $SSE = .7850$.

No, almost none.

e. (2.5005, 2.8353)

f. $\hat{D}_1 = \hat{\tau}_1 - \hat{\tau}_2 = -.326$, $\hat{D}_2 = \hat{\tau}_1 - \hat{\tau}_3 = -.039$,

$\hat{D}_3 = \hat{\tau}_1 - \hat{\tau}_4 = 2\hat{\tau}_1 + \hat{\tau}_2 + \hat{\tau}_3 = -.503$, $\hat{D}_4 = \hat{\tau}_2 - \hat{\tau}_3 = .287$,

$\hat{D}_5 = \hat{\tau}_2 - \hat{\tau}_4 = 2\hat{\tau}_2 + \hat{\tau}_1 + \hat{\tau}_3 = -.177$, $\hat{D}_6 = \hat{\tau}_3 - \hat{\tau}_4 = 2\hat{\tau}_3 + \hat{\tau}_1 + \hat{\tau}_2 = -.464$,

$s^2\{\hat{\tau}_1\} = .002028$, $s^2\{\hat{\tau}_2\} = .002024$, $s^2\{\hat{\tau}_3\} = .002239$,

$s\{\hat{\tau}_1, \hat{\tau}_2\} = -.00071$, $s\{\hat{\tau}_1, \hat{\tau}_3\} = -.00085$, $s\{\hat{\tau}_2, \hat{\tau}_3\} = -.00085$,

$s\{\hat{D}_1\} = .073927$, $s\{\hat{D}_2\} = .077232$, $s\{\hat{D}_3\} = .066790$,

$s\{\hat{D}_4\} = .077165$, $s\{\hat{D}_5\} = .066743$, $s\{\hat{D}_6\} = .069400$,

$B = t(.99583; 31) = 2.818$ ($S^2 = 3F(.95; 3, 31) = 3(2.9113)$, $S = 2.955$)

$$\begin{array}{rr} -.326 \pm 2.818(.073927) & -.534 \leq D_1 \leq -.118 \\ -.039 \pm 2.818(.077232) & -.257 \leq D_2 \leq .179 \\ -.503 \pm 2.818(.066790) & -.691 \leq D_3 \leq -.315 \\ .287 \pm 2.818(.077165) & .070 \leq D_4 \leq .504 \\ -.177 \pm 2.818(.066743) & -.365 \leq D_5 \leq .011 \\ -.464 \pm 2.818(.069400) & -.660 \leq D_6 \leq -.268 \end{array}$$

22.28. b. $r = .991$

c. $$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + \alpha_3 I_{ijk3} + \beta_1 I_{ijk4}$$
$$+(\alpha\beta)_{11} I_{ijk1} I_{ijk4} + (\alpha\beta)_{21} I_{ijk2} I_{ijk4} + (\alpha\beta)_{31} I_{ijk3} I_{ijk4}$$
$$+\gamma x_{ijk} + \delta_1 I_{ijk1} x_{ijk} + \delta_2 I_{ijk2} x_{ijk} + \delta_3 I_{ijk3} x_{ijk}$$
$$+\delta_4 I_{ijk4} x_{ijk} + \delta_5 I_{ijk1} I_{ijk4} x_{ijk} + \delta_6 I_{ijk2} I_{ijk4} x_{ijk} + \delta_7 I_{ijk3} I_{ijk4} x_{ijk} + \epsilon_{ijk}$$

H_0: all δ_i equal zero ($i = 1, ..., 7$), H_a: not all δ_i equal zero.

$SSE(F) = .0093126$, $SSE(R) = .0108089$,

$F^* = (.0014963/7) \div (.0093126/40) = .92$, $F(.999; 7, 40) = 4.436$.

If $F^* \leq 4.436$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .46$

22.29. a. $$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + \alpha_3 I_{ijk3} + \beta_1 I_{ijk4} + (\alpha\beta)_{11} I_{ijk1} I_{ijk4}$$
$$+(\alpha\beta)_{21} I_{ijk2} I_{ijk4} + (\alpha\beta)_{31} I_{ijk3} I_{ijk4} + \gamma x_{ijk} + \epsilon_{ijk}$$

$$I_{ijk1} = \begin{cases} 1 & \text{if case from region NE} \\ -1 & \text{if case from region W} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk2} = \begin{cases} 1 & \text{if case from region NC} \\ -1 & \text{if case from region W} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk3} = \begin{cases} 1 & \text{if case from region S} \\ -1 & \text{if case from region W} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk4} = \begin{cases} 1 & \text{if percent of poverty less than 8.0 percent} \\ -1 & \text{if percent of poverty 8.0 percent or more} \end{cases}$$

$x_{ijk} = X_{ijk} - \bar{X}_{...} \qquad (\bar{X}_{...} = 12.521)$

$$\hat{Y} = .0632 - .0239 I_1 - .0115 I_2 + .0254 I_3 - .00548 I_4$$
$$+.00149 I_1 I_4 + .00643 I_2 I_4 - .00904 I_3 I_4 + .000627 x$$

$SSE(F) = .0108089$

b. Interactions:

$$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + \alpha_3 I_{ijk3} + \beta_1 I_{ijk4} + \gamma x_{ijk} + \epsilon_{ijk}$$

$$\hat{Y} = .0632 - .0224 I_1 - .0117 I_2 + .0255 I_3 - .00557 I_4 - .000061 x$$

$SSE(R) = .0122362$

Factor A:

$$Y_{ijk} = \mu_{..} + \beta_1 I_{ijk4} + (\alpha\beta)_{11} I_{ijk1} I_{ijk4} + (\alpha\beta)_{21} I_{ijk2} I_{ijk4}$$
$$+(\alpha\beta)_{31} I_{ijk3} I_{ijk4} + \gamma x_{ijk} + \epsilon_{ijk}$$

$$\hat{Y} = .0632 - .00565 I_4 + .00001 I_1 I_4 + .00463 I_2 I_4 - .00532 I_3 I_4 - .000719 x$$

$SSE(R) = .0298356$

Factor B:

$$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + \alpha_3 I_{ijk3} + (\alpha\beta)_{11} I_{ijk1} I_{ijk4}$$
$$+(\alpha\beta)_{21} I_{ijk2} I_{ijk4} + (\alpha\beta)_{31} I_{ijk3} I_{ijk4} + \gamma x_{ijk} + \epsilon_{ijk}$$
$$\hat{Y} = .0632 - .0241 I_1 - .0115 I_2 + .0254 I_3 + .00157 I_1 I_4$$
$$+.00652 I_2 I_4 - .00923 I_3 I_4 + .000695 x$$
$SSE(R) = .012488$

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero.
$F^* = (.0014273/3) \div (.0108089/47) = 2.069$, $F(.99; 3, 47) = 4.23$.
If $F^* \leq 4.23$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .12

d. H_0: $\alpha_1 = \alpha_2 = \alpha_3 = 0$, H_a: not all α_i equal zero.
$F^* = (.0190267/3) \div (.0108089/47) = 27.57$, $F(.99; 3, 47) = 4.23$.
If $F^* \leq 4.23$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$.
$F^* = (.0016791/1) \div (.0108089/47) = 7.30$, $F(.99; 1, 47) = 7.21$.
If $F^* \leq 7.21$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .0096

22.30. b. $r = .983$

c. $$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \beta_1 I_{ijk2} + (\alpha\beta)_{11} I_{ijk1} I_{ijk4} + \gamma x_{ijk}$$
$$+\delta_1 I_{ijk1} x_{ijk} + \delta_2 I_{ijk2} x_{ijk} + \delta_3 I_{ijk1} I_{ijk2} x_{ijk} + \epsilon_{ijk}$$
H_0: $\delta_1 = \delta_2 = \delta_3 = 0$, H_a: not all δ_i equal zero.
$SSE(F) = .48044$, $SSE(R) = .51032$,
$F^* = (.02988/3) \div (.48044/20) = .41$, $F(.95; 3, 20) = 3.10$.
If $F^* \leq 3.10$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .75

22.31. a. $$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \beta_1 I_{ijk2} + (\alpha\beta)_{11} I_{ijk1} I_{ijk2} + \epsilon_{ijk}$$
$$I_{ijk1} = \begin{cases} 1 & \text{no discount price} \\ -1 & \text{discount price} \end{cases}$$
$$I_{ijk2} = \begin{cases} 1 & \text{no package promotion} \\ -1 & \text{package promotion} \end{cases}$$
$$x_{ijk} = X_{ijk} - \bar{X}_{...} \qquad (\bar{X}_{...} = 2.2716)$$
$$\hat{Y} = 2.644 - .197 I_1 - .0605 I_2 + .0533 I_1 I_2 - .276 x$$
$SSE(F) = .51032$

b. Interactions:
$$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \beta_1 I_{ijk2} + \gamma x_{ijk} + \epsilon_{ijk}$$
$$\hat{Y} = 2.644 - .189 I_1 - .0608 I_2 - .451 x$$
$SSE(R) = .57864$

Factor A:

$Y_{ijk} = \mu_{..} + \beta_1 I_{ijk2} + (\alpha\beta)_{11} I_{ijk1} I_{ijk2} + \gamma x_{ijk} + \epsilon_{ijk}$

$\hat{Y} = 2.644 - .0616 I_2 + .0241 I_1 I_2 - .962x$

$SSE(R) = 1.42545$

Factor B:

$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + (\alpha\beta)_{11} I_{ijk1} I_{ijk2} + \gamma x_{ijk} + \epsilon_{ijk}$

$\hat{Y} = 2.644 - .198 I_1 + .0536 I_1 I_2 - .267x$

$SSE(R) = .61269$

c. H_0: $(\alpha\beta)_{11} = 0$, H_a: $(\alpha\beta)_{11} \neq 0$

$F^* = (.06832/1) \div (.51032/23) = 3.08$, $F(.99; 1, 23) = 7.88$.

If $F^* \leq 7.88$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .09

d. H_0: $\alpha_1 = 0$, H_a: $\alpha_1 \neq 0$

$F^* = (.91513/1) \div (.51032/23) = 41.24$, $F(.99; 1, 23) = 7.88$.

If $F^* \leq 7.88$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$.

$F^* = (.10237/1) \div (.51032/23) = 4.61$, $F(.99; 1, 23) = 7.88$.

If $F^* \leq 7.88$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .04

Chapter 23

TWO-FACTOR STUDIES WITH UNEQUAL SAMPLE SIZES

23.3. a. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \beta_1 X_{ijk2} + (\alpha\beta)_{11} X_{ijk1} X_{ijk2} + \epsilon_{ijk}$

$$X_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 2 for factor } A \end{cases}$$

$$X_{ijk2} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 2 for factor } B \end{cases}$$

b. $\mathbf{Y}$ entries: in order Y_{111}, ..., Y_{115}, Y_{121}, ..., Y_{125}, Y_{211}, ...

$\boldsymbol{\beta}$ entries: $\mu_{..}$, α_1, β_1, $(\alpha\beta)_{11}$

$\mathbf{X}$ entries:

A	B	Freq.		X_1	X_2	X_1X_2
1	1	5	1	1	1	1
1	2	5	1	1	-1	-1
2	1	5	1	-1	1	-1
2	2	5	1	-1	-1	1

c. $\mathbf{X}\boldsymbol{\beta}$ entries:

A	B	
1	1	$\mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11}$
1	2	$\mu_{..} + \alpha_1 - \beta_1 - (\alpha\beta)_{11} = \mu_{..} + \alpha_1 + \beta_2 + (\alpha\beta)_{12}$
2	1	$\mu_{..} - \alpha_1 + \beta_1 - (\alpha\beta)_{11} = \mu_{..} + \alpha_2 + \beta_1 + (\alpha\beta)_{21}$
2	2	$\mu_{..} - \alpha_1 - \beta_1 + (\alpha\beta)_{11} = \mu_{..} + \alpha_2 + \beta_2 + (\alpha\beta)_{22}$

d. $\hat{Y} = 13.05 - 1.65X_1 - 1.95X_2 - .25X_1X_2$

$\mu_{..}$

e.

Source	SS	df
Regression	131.75	3
X_1	54.45	1] A
$X_2 \vert X_1$	76.05	1] B
$X_1X_2 \vert X_1, X_2$	1.25	1] AB
Error	97.20	16
Total	228.95	19

Yes.

f. See Problem 19.13c and d.

23.4. a. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + \beta_2 X_{ijk4} + (\alpha\beta)_{11} X_{ijk1} X_{ijk3}$
$+(\alpha\beta)_{12} X_{ijk1} X_{ijk4} + (\alpha\beta)_{21} X_{ijk2} X_{ijk3} + (\alpha\beta)_{22} X_{ijk2} X_{ijk4} + \epsilon_{ijk}$

$$X_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk2} = \begin{cases} 1 & \text{if case from level 2 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk3} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk4} = \begin{cases} 1 & \text{if case from level 2 for factor } B \\ -1 & \text{if case from level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

b. **Y** entries: in order Y_{111}, ..., Y_{114}, Y_{121}, ..., Y_{124}, Y_{131}, ..., Y_{134},Y_{211}, ...

$\boldsymbol{\beta}$ entries: $\mu_{..}$, α_1, α_2, β_1, β_2, $(\alpha\beta)_{11}$, $(\alpha\beta)_{12}$, $(\alpha\beta)_{21}$, $(\alpha\beta)_{22}$

X entries:

A	B	Freq.		X_1	X_2	X_3	X_4	X_1X_3	X_1X_4	X_2X_3	X_2X_4
1	1	4	1	1	0	1	0	1	0	0	0
1	2	4	1	1	0	0	1	0	1	0	0
1	3	4	1	1	0	−1	−1	−1	−1	0	0
2	1	4	1	0	1	1	0	0	0	1	0
2	2	4	1	0	1	0	1	0	0	0	1
2	3	4	1	0	1	−1	−1	0	0	−1	−1
3	1	4	1	−1	−1	1	0	−1	0	−1	0
3	2	4	1	−1	−1	0	1	0	−1	0	−1
3	3	4	1	−1	−1	−1	−1	1	1	1	1

c. $\mathbf{X}\boldsymbol{\beta}$ entries:

A	B	
1	1	$\mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11}$
1	2	$\mu_{..} + \alpha_1 + \beta_2 + (\alpha\beta)_{12}$
1	3	$\mu_{..} + \alpha_1 - \beta_1 - \beta_2 - (\alpha\beta)_{11} - (\alpha\beta)_{12} = \mu_{..} + \alpha_1 + \beta_3 + (\alpha\beta)_{13}$
2	1	$\mu_{..} + \alpha_2 + \beta_1 + (\alpha\beta)_{21}$
2	2	$\mu_{..} + \alpha_2 + \beta_2 + (\alpha\beta)_{22}$
2	3	$\mu_{..} + \alpha_2 - \beta_1 - \beta_2 - (\alpha\beta)_{21} - (\alpha\beta)_{22} = \mu_{..} + \alpha_2 + \beta_3 + (\alpha\beta)_{23}$
3	1	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_1 - (\alpha\beta)_{11} - (\alpha\beta)_{21} = \mu_{..} + \alpha_3 + \beta_1 + (\alpha\beta)_{31}$
3	2	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_2 - (\alpha\beta)_{12} - (\alpha\beta)_{22} = \mu_{..} + \alpha_3 + \beta_2 + (\alpha\beta)_{32}$
3	3	$\mu_{..} - \alpha_1 - \alpha_2 - \beta_1 - \beta_2 + (\alpha\beta)_{11} + (\alpha\beta)_{12} + (\alpha\beta)_{21} + (\alpha\beta)_{22}$ $= \mu_{..} + \alpha_3 + \beta_3 + (\alpha\beta)_{33}$

d. $\hat{Y} = 7.18333 - 3.30000X_1 + .65000X_2 - 2.55000X_3 + .75000X_4$
$+1.14167X_1X_3 - .03333X_1X_4 + .16667X_2X_3 + .34167X_2X_4$

$\alpha_1 = \mu_{1.} - \mu_{..}$

e.

Source	SS	*df*
Regression	373.125	8
X_1	212.415	1] A
$X_2 \mid X_1$	7.605	1] A
$X_3 \mid X_1, X_2$	113.535	1] B
$X_4 \mid X_1, X_2, X_3$	10.125	1] B
$X_1X_3 \mid X_1, X_2, X_3, X_4$	26.7806	1] AB
$X_1X_4 \mid X_1, X_2, X_3, X_4, X_1X_3$	.2269	1] AB
$X_2X_3 \mid X_1, X_2, X_3, X_4, X_1X_3, X_1X_4$	1.3669	1] AB
$X_2X_4 \mid X_1, X_2, X_3, X_4, X_1X_3, X_1X_4, X_2X_3$	1.0506	1] AB
Error	1.625	27
Total	374.730	35

Yes.

f. See Problem 19.15c and d.

23.5. a. See Problem 23.4a.

b. $\hat{Y} = 55.82222 - .48889X_1 - .55556X_2 + .31111X_3 + .77778X_4$
$+4.15556X_1X_3 - 8.31111X_1X_4 - 7.17778X_2X_3 + 5.15556X_2X_4$

$\beta_1 = \mu_{.1} - \mu_{..}$

c.

Source	SS	*df*
Regression	1,268.17778	8
X_1	17.63333	1] A
$X_2 \mid X_1$	6.94445	1] A
$X_3 \mid X_1, X_2$	14.70000	1] B
$X_4 \mid X_1, X_2, X_3$	13.61111	1] B
$X_1X_3 \mid X_1, X_2, X_3, X_4$	105.80000	1] AB
$X_1X_4 \mid X_1, X_2, X_3, X_4, X_1X_3$	493.06667	1] AB
$X_2X_3 \mid X_1, X_2, X_3, X_4, X_1X_3, X_1X_4$	317.40000	1] AB
$X_2X_4 \mid X_1, X_2, X_3, X_4, X_1X_3, X_1X_4, X_2X_3$	299.02222	1] AB
Error	1,872.40000	36
Total	3,140.57778	44

Yes.

d. See Problem 19.17c and d.

23.6. a. $Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

$Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + (\alpha\beta)_{11} X_{ijk1} X_{ijk3}$
$+(\alpha\beta)_{21} X_{ijk2} X_{ijk3} + \epsilon_{ijk}$

$$X_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk2} = \begin{cases} 1 & \text{if case from level 2 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk3} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 2 for factor } B \end{cases}$$

b. $\boldsymbol{\beta}$ entries: $\mu_{..}$, α_1, α_2, β_1, $(\alpha\beta)_{11}$, $(\alpha\beta)_{21}$

X entries:

A	B	Freq.		X_1	X_2	X_3	X_1X_3	X_2X_3
1	1	6	1	1	0	1	1	0
1	2	6	1	1	0	-1	-1	0
2	1	5	1	0	1	1	0	1
2	2	6	1	0	1	-1	0	-1
3	1	6	1	-1	-1	1	-1	-1
3	2	5	1	-1	-1	-1	1	1

c. $\mathbf{X}\boldsymbol{\beta}$ entries:

A	B	
1	1	$\mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11}$
1	2	$\mu_{..} + \alpha_1 - \beta_1 - (\alpha\beta)_{11} = \mu_{..} + \alpha_1 + \beta_2 + (\alpha\beta)_{12}$
2	1	$\mu_{..} + \alpha_2 + \beta_1 + (\alpha\beta)_{21}$
2	2	$\mu_{..} + \alpha_2 - \beta_1 - (\alpha\beta)_{21} = \mu_{..} + \alpha_2 + \beta_2 + (\alpha\beta)_{22}$
3	1	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_1 - (\alpha\beta)_{11} - (\alpha\beta)_{21} = \mu_{..} + \alpha_3 + \beta_1 + (\alpha\beta)_{31}$
3	2	$\mu_{..} - \alpha_1 - \alpha_2 - \beta_1 + (\alpha\beta)_{11} + (\alpha\beta)_{21} = \mu_{..} + \alpha_3 + \beta_2 + (\alpha\beta)_{32}$

d. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + \epsilon_{ijk}$

e. <u>Full model</u>:

$\hat{Y} = 23.56667 - 2.06667X_1 + 4.16667X_2 + .36667X_3 - .20000X_1X_3 - .30000X_2X_3$,

$SSE(F) = 71.3333$

<u>Reduced model</u>:

$\hat{Y} = 23.59091 - 2.09091X_1 + 4.16911X_2 + .36022X_3$,

$SSE(R) = 75.5210$

H_0: $(\alpha\beta)_{11} = (\alpha\beta)_{21} = 0$, H_a: not both $(\alpha\beta)_{11}$ and $(\alpha\beta)_{21}$ equal zero.

$F^* = (4.1877/2) \div (71.3333/28) = .82$, $F(.95; 2, 28) = 3.34$.

If $F^* \leq 3.34$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .45$

f. <u>A effects</u>:

$\hat{Y} = 23.50000 + .17677X_3 - .01010X_1X_3 - .49495X_2X_3$,

$SSE(R) = 359.9394$

H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_1 and α_2 equal zero.

$F^* = (288.6061/2) \div (71.3333/28) = 56.64,\ F(.95; 2, 28) = 3.34.$

If $F^* \leq 3.34$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

$\underline{B \text{ effects}}$:

$\hat{Y} = 23.56667 - 2.06667X_1 + 4.13229X_2 - .17708X_1X_3 - .31146X_2X_3,$

$SSE(R) = 75.8708$

H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$.

$F^* = (4.5375/1) \div (71.3333/28) = 1.78,\ F(.95; 1, 28) = 4.20.$

If $F^* \leq 4.20$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .19

g. $\hat{D}_1 = \hat{\alpha}_1 - \hat{\alpha}_2 = -6.23334$, $\hat{D}_2 = \hat{\alpha}_1 - \hat{\alpha}_3 = 2\hat{\alpha}_1 + \hat{\alpha}_2 = .03333$, $\hat{D}_3 = \hat{\alpha}_2 - \hat{\alpha}_3 = 2\hat{\alpha}_2 + \hat{\alpha}_1 = 6.26667$, $s^2\{\hat{\alpha}_1\} = .14625$, $s^2\{\hat{\alpha}_2\} = .15333$, $s\{\hat{\alpha}_1, \hat{\alpha}_2\} = -.07313$, $s\{\hat{D}_1\} = .6677$, $s\{\hat{D}_2\} = .6677$, $s\{\hat{D}_3\} = .6834$, $q(.90; 3, 28) = 3.026$, $T = 2.140$

$$\begin{array}{rc}
-6.23334 \pm 2.140(.6677) & -7.662 \leq D_1 \leq -4.804 \\
.03333 \pm 2.140(.6677) & -1.396 \leq D_2 \leq 1.462 \\
6.26667 \pm 2.140(.6834) & 4.804 \leq D_3 \leq 7.729
\end{array}$$

h. $\hat{L} = .3\bar{Y}_{12.} + .6\bar{Y}_{22.} + .1\bar{Y}_{32.} = .3(21.33333) + .6(27.66667) + .1(20.60000) = 25.06000$, $s\{\hat{L}\} = .4429$, $t(.975; 28) = 2.048$, $25.06000 \pm 2.048(.4429)$, $24.153 \leq L \leq 25.967$

23.7. a. $Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

$$\begin{aligned}
Y_{ijk} = \mu_{..} &+ \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + \beta_2 X_{ijk4} \\
&+ (\alpha\beta)_{11} X_{ijk1} X_{ijk3} + (\alpha\beta)_{12} X_{ijk1} X_{ijk4} + (\alpha\beta)_{21} X_{ijk2} X_{ijk3} \\
&+ (\alpha\beta)_{22} X_{ijk2} X_{ijk4} + \epsilon_{ijk}
\end{aligned}$$

$$X_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk2} = \begin{cases} 1 & \text{if case from level 2 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk3} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk4} = \begin{cases} 1 & \text{if case from level 2 for factor } B \\ -1 & \text{if case from level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

b. $\boldsymbol{\beta}$ entries: $\mu_{..}$, α_1, α_2, β_1, β_2, $(\alpha\beta)_{11}$, $(\alpha\beta)_{12}$, $(\alpha\beta)_{21}$, $(\alpha\beta)_{22}$

$\mathbf{X}$ entries:

A	B	Freq.		X_1	X_2	X_3	X_4	X_1X_3	X_1X_4	X_2X_3	X_2X_4
1	1	3	1	1	0	1	0	1	0	0	0
1	2	4	1	1	0	0	1	0	1	0	0
1	3	4	1	1	0	-1	-1	-1	-1	0	0
2	1	4	1	0	1	1	0	0	0	1	0
2	2	2	1	0	1	0	1	0	0	0	1
2	3	4	1	0	1	-1	-1	0	0	-1	-1
3	1	4	1	-1	-1	1	0	-1	0	-1	0
3	2	4	1	-1	-1	0	1	0	-1	0	-1
3	3	4	1	-1	-1	1	-1	1	1	1	1

c. $\mathbf{X}\boldsymbol{\beta}$ entries:

A	B	
1	1	$\mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11}$
1	2	$\mu_{..} + \alpha_1 + \beta_2 + (\alpha\beta)_{12}$
1	3	$\mu_{..} + \alpha_1 - \beta_1 - \beta_2 - (\alpha\beta)_{11} - (\alpha\beta)_{12} = \mu_{..} + \alpha_1 + \beta_3 + (\alpha\beta)_{13}$
2	1	$\mu_{..} + \alpha_2 + \beta_1 + (\alpha\beta)_{21}$
2	2	$\mu_{..} + \alpha_2 + \beta_2 + (\alpha\beta)_{22}$
2	3	$\mu_{..} + \alpha_2 - \beta_1 - \beta_2 - (\alpha\beta)_{21} - (\alpha\beta)_{22} = \mu_{..} + \alpha_2 + \beta_3 + (\alpha\beta)_{23}$
3	1	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_1 - (\alpha\beta)_{11} - (\alpha\beta)_{21} = \mu_{..} + \alpha_3 + \beta_1 + (\alpha\beta)_{31}$
3	2	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_2 - (\alpha\beta)_{12} - (\alpha\beta)_{22} = \mu_{..} + \alpha_3 + \beta_2 + (\alpha\beta)_{32}$
3	3	$\mu_{..} - \alpha_1 - \alpha_2 - \beta_1 - \beta_2 + (\alpha\beta)_{11} + (\alpha\beta)_{12} + (\alpha\beta)_{21} + (\alpha\beta)_{22}$ $= \mu_{..} + \alpha_3 + \beta_3 + (\alpha\beta)_{33}$

d. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + \beta_2 X_{ijk4} + \epsilon_{ijk}$

e. Full model:

$$\hat{Y} = 7.18704 - 3.28426X_1 + .63796X_2 - 2.53426X_3 + .73796X_4$$
$$+1.16481X_1X_3 - .04074X_1X_4 + .15926X_2X_3 + .33704X_2X_4,$$

$SSE(F) = 1.5767$

Reduced model:

$$\hat{Y} = 7.12711 - 3.33483X_1 + .62861X_2 - 2.58483X_3 + .72861X_4,$$

$SSE(R) = 29.6474$

H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero.

$F^* = (28.0707/4) \div (1.5767/24) = 106.82$, $F(.95; 4, 24) = 2.78$.

If $F^* \leq 2.78$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

f. $\bar{Y}_{11.} = 2.5333$, $\bar{Y}_{12.} = 4.6000$, $\bar{Y}_{13.} = 4.57500$, $\bar{Y}_{21.} = 5.45000$, $\bar{Y}_{22.} = 8.90000$, $\bar{Y}_{23.} = 9.12500$, $\bar{Y}_{31.} = 5.97500$, $\bar{Y}_{32.} = 10.27500$, $\bar{Y}_{33.} = 13.25000$, $\hat{L}_1 = 2.0542$, $\hat{L}_2 = 3.5625$, $\hat{L}_3 = 5.7875$, $\hat{L}_4 = 1.5083$, $\hat{L}_5 = 3.7333$, $\hat{L}_6 = 2.2250$, $s\{\hat{L}_1\} = .1613$, $s\{\hat{L}_2\} = .1695$, $s\{\hat{L}_3\} = .1570$, $s\{\hat{L}_4\} = .2340$, $s\{\hat{L}_5\} = .2251$, $s\{\hat{L}_6\} = .2310$, $F(.90; 8, 24) = 1.94$, $S = 3.9395$

$2.0542 \pm 3.9395(.1613)$	$1.419 \leq L_1 \leq 2.690$
$3.5625 \pm 3.9395(.1695)$	$2.895 \leq L_2 \leq 4.230$
$5.7875 \pm 3.9395(.1570)$	$5.169 \leq L_3 \leq 6.406$
$1.5083 \pm 3.9395(.2340)$	$.586 \leq L_4 \leq 2.430$
$3.7333 \pm 3.9395(.2251)$	$2.846 \leq L_5 \leq 4.620$
$2.2250 \pm 3.9395(.2310)$	$1.315 \leq L_6 \leq 3.135$

23.8. a. $Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

Regression model: see (22.4).

b. $\boldsymbol{\beta}$ entries: $\mu_{..}$, α_1, β_1, β_2, $(\alpha\beta)_{11}$, $(\alpha\beta)_{12}$

$\mathbf{X}$ entries:

A	B	Freq.		X_1	X_2	X_3	X_1X_2	X_1X_3
1	1	10	1	1	1	0	1	0
1	2	9	1	1	0	1	0	1
1	3	10	1	1	−1	−1	−1	−1
2	1	9	1	−1	1	0	−1	0
2	2	10	1	−1	0	1	0	−1
2	3	9	1	−1	−1	−1	1	1

c. $\mathbf{X}\boldsymbol{\beta}$ entries:

A	B	
1	1	$\mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11}$
1	2	$\mu_{..} + \alpha_1 + \beta_2 + (\alpha\beta)_{12}$
1	3	$\mu_{..} + \alpha_1 - \beta_1 - \beta_2 - (\alpha\beta)_{11} - (\alpha\beta)_{12} = \mu_{..} + \alpha_1 + \beta_3 + (\alpha\beta)_{13}$
2	1	$\mu_{..} - \alpha_1 + \beta_1 - (\alpha\beta)_{11} = \mu_{..} + \alpha_2 + \beta_1 + (\alpha\beta)_{21}$
2	2	$\mu_{..} - \alpha_1 + \beta_2 - (\alpha\beta)_{12} = \mu_{..} + \alpha_2 + \beta_2 + (\alpha\beta)_{22}$
2	3	$\mu_{..} - \alpha_1 - \beta_1 - \beta_2 + (\alpha\beta)_{11} + (\alpha\beta)_{12} = \mu_{..} + \alpha_2 + \beta_3 + (\alpha\beta)_{23}$

d. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \beta_1 X_{ijk2} + \beta_2 X_{ijk3} + \epsilon_{ijk}$

e. Full model:

$\hat{Y} = .69139 + .08407X_1 - .27492X_2 - .01281X_3 - .05706X_1X_2 + .01355X_1X_3$,

$SSE(F) = 5.3383$

Reduced model:

$\hat{Y} = .69092 + .08407X_1 - .27745X_2 - .01305X_3$,

$SSE(R) = 5.4393$

H_0: $(\alpha\beta)_{11} = (\alpha\beta)_{12} = 0$, H_a: not both $(\alpha\beta)_{11}$ and $(\alpha\beta)_{12}$ equal zero.

$F^* = (.1010/2) \div (5.3383/51) = .48$, $F(.95; 2, 51) = 3.179$.

If $F^* \leq 3.179$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .62$

f. Duration:

$Y_{ijk} = \mu_{..} + \beta_1 X_{ijk2} + \beta_2 X_{ijk3} + (\alpha\beta)_{11} X_{ijk1} X_{ijk2} + (\alpha\beta)_{12} X_{ijk1} X_{ijk3} + \epsilon_{ijk}$

$\hat{Y} = .69287 - .27197X_2 - .01871X_3 - .05706X_1X_2 + .01355X_1X_3$,

$SSE(R) = 5.7400$

H_0: $\alpha_1 = 0$, H_a: $\alpha_1 \neq 0$.

$F^* = (.4017/1) \div (5.3383/51) = 3.84$, $F(.95; 1, 51) = 4.03$.

If $F^* \leq 4.03$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .06

Weight gain:

$Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + (\alpha\beta)_{11} X_{ijk1} X_{ijk2} + (\alpha\beta)_{12} X_{ijk1} X_{ijk3} + \epsilon_{ijk}$

$\hat{Y} = .69139 + .08452 X_1 - .07198 X_1 X_2 + .01377 X_1 X_3$,

$SSE(R) = 8.3421$

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = (3.0038/2) \div (5.3383/51) = 14.35$, $F(.95; 2, 51) = 3.179$.

If $F^* \leq 3.179$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

g. H_0: $\mu_{.1} \leq .5$, H_a: $\mu_{.1} > .5$. $\hat{\mu}_{.1} = (\bar{Y}_{11.} + \bar{Y}_{21.})/2 = (.44348 + .38946)/2 = .41647$, $s\{\hat{\mu}_{.1}\} = .0743$, $t^* = -.08353/.0743 = -1.12$, $t(.95; 51) = 1.675$. If $t^* \leq 1.675$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .87

h. $\bar{Y}_{11.} = .44348$, $\bar{Y}_{12.} = .77619$, $\bar{Y}_{13.} = 1.10670$, $\bar{Y}_{21.} = .38946$, $\bar{Y}_{22.} = .58096$, $\bar{Y}_{23.} = .85155$, $\hat{D}_1 = .16813$, $\hat{D}_2 = .26211$, $\hat{D}_3 = .56266$, $\hat{D}_4 = .30055$, $s\{\hat{D}_1\} = .08582$, $s\{\hat{D}_i\} = .10511$ $(i = 2, 3, 4)$, $B = t(.9875; 51) = 2.3096$

$$\begin{array}{lr} .16813 \pm 2.3096(.08582) & -.0301 \leq D_1 \leq .3663 \\ .26211 \pm 2.3096(.10511) & .0193 \leq D_2 \leq .5049 \\ .56266 \pm 2.3096(.10511) & .3199 \leq D_3 \leq .8054 \\ .30055 \pm 2.3096(.10511) & 0578 \leq D_4 \leq .5433 \end{array}$$

23.9. a. $Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

$$\begin{aligned} Y_{ijk} = \mu_{..} &+ \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} \\ &+ (\alpha\beta)_{11} X_{ijk1} X_{ijk4} + (\alpha\beta)_{12} X_{ijk1} X_{ijk5} + (\alpha\beta)_{21} X_{ijk2} X_{ijk4} \\ &+ (\alpha\beta)_{22} X_{ijk2} X_{ijk5} + (\alpha\beta)_{31} X_{ijk3} X_{ijk4} + (\alpha\beta)_{32} X_{ijk3} X_{ijk5} + \epsilon_{ijk} \end{aligned}$$

a. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + \beta_2 X_{ijk4} + (\alpha\beta)_{11} X_{ijk1} X_{ijk3}$
$+ (\alpha\beta)_{12} X_{ijk1} X_{ijk4} + (\alpha\beta)_{21} X_{ijk2} X_{ijk3} + (\alpha\beta)_{22} X_{ijk2} X_{ijk4} + \epsilon_{ijk}$

$$X_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 4 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

X_{ijk2} and X_{ijk3} are defined similarly

$$X_{ijk4} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk5} = \begin{cases} 1 & \text{if case from level 2 for factor } B \\ -1 & \text{if case from level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

b. $\boldsymbol{\beta}$ entries: $\mu_{..}$, α_1, α_2, α_3 , β_1, β_2, $(\alpha\beta)_{11}$, $(\alpha\beta)_{12}$, $(\alpha\beta)_{21}$,
$(\alpha\beta)_{22}$, $(\alpha\beta)_{31}$, $(\alpha\beta)_{32}$

$\mathbf{X}$ entries:

A	B	Freq.		X_1	X_2	X_3	X_4	X_5	X_1X_4
1	1	2	1	1	0	0	1	0	1
1	2	2	1	1	0	0	0	1	0
1	3	8	1	1	0	0	−1	−1	−1
2	1	4	1	0	1	0	1	0	0
2	2	5	1	0	1	0	0	1	0
2	3	4	1	0	1	0	−1	−1	0
3	1	2	1	0	0	1	1	0	0
3	2	4	1	0	0	1	0	1	0
3	3	5	1	0	0	1	−1	−1	0
4	1	2	1	−1	−1	−1	1	0	−1
4	2	2	1	−1	−1	−1	0	1	0
4	3	5	1	−1	−1	−1	−1	−1	1

A	B	X_1X_5	X_2X_4	X_2X_5	X_3X_4	X_3X_5
1	1	0	0	0	0	0
1	2	1	0	0	0	0
1	3	−1	0	0	0	0
2	1	0	1	0	0	0
2	2	0	0	1	0	0
2	3	0	−1	−1	0	0
3	1	0	0	0	1	0
3	2	0	0	0	0	1
3	3	0	0	0	−1	−1
4	1	0	−1	0	−1	0
4	2	−1	0	−1	0	−1
4	3	1	1	1	1	1

c. $\mathbf{X}\boldsymbol{\beta}$ entries:

A	B	
1	1	$\mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11}$
1	2	$\mu_{..} + \alpha_1 + \beta_2 + (\alpha\beta)_{12}$
1	3	$\mu_{..} + \alpha_1 - \beta_1 - \beta_2 - (\alpha\beta)_{11} - (\alpha\beta)_{12} = \mu_{..} + \alpha_1 + \beta_3 + (\alpha\beta)_{13}$
2	1	$\mu_{..} + \alpha_2 + \beta_1 + (\alpha\beta)_{21}$
2	2	$\mu_{..} + \alpha_2 + \beta_2 + (\alpha\beta)_{22}$
2	3	$\mu_{..} + \alpha_2 - \beta_1 - \beta_2 - (\alpha\beta)_{21} - (\alpha\beta)_{22} = \mu_{..} + \alpha_2 + \beta_3 + (\alpha\beta)_{23}$
3	1	$\mu_{..} + \alpha_3 + \beta_1 + (\alpha\beta)_{31}$
3	2	$\mu_{..} + \alpha_3 + \beta_2 + (\alpha\beta)_{32}$
3	3	$\mu_{..} + \alpha_3 - \beta_1 - \beta_2 - (\alpha\beta)_{31} - (\alpha\beta)_{32} = \mu_{..} + \alpha_3 + \beta_3 + (\alpha\beta)_{33}$
4	1	$\mu_{..} - \alpha_1 - \alpha_2 - \alpha_3 + \beta_1 - (\alpha\beta)_{11} - (\alpha\beta)_{21} - (\alpha\beta)_{31}$ $= \mu_{..} + \alpha_4 + \beta_1 + (\alpha\beta)_{41}$
4	2	$\mu_{..} - \alpha_1 - \alpha_2 - \alpha_3 + \beta_2 - (\alpha\beta)_{12} - (\alpha\beta)_{22} - (\alpha\beta)_{32}$ $= \mu_{..} + \alpha_4 + \beta_2 + (\alpha\beta)_{42}$
4	3	$\mu_{..} - \alpha_1 - \alpha_2 - \alpha_3 - \beta_1 - \beta_2 + (\alpha\beta)_{11} + (\alpha\beta)_{12} + (\alpha\beta)_{21} + (\alpha\beta)_{22}$ $+(\alpha\beta)_{31} + (\alpha\beta)_{32} = \mu_{..} + \alpha_4 + \beta_3 + (\alpha\beta)_{43}$

d. See Problem 23.10c for fitted model.

e_{ijk}:

$i = 1$

k	$j=1$	$j=2$	$j=3$
1	$-.10$	$-.15$	$-.20$
2	.10	.15	.00
3			.20
4			$-.20$
5			$-.10$
6			.10
7			.00
8			.20

$i = 2$

k	$j=1$	$j=2$	$j=3$
1	.05	.18	.05
2	$-.15$	$-.12$	$-.15$
3	.15	.08	.15
4	.05	$-.12$	$-.05$
5		$-.02$	

$i = 3$

k	$j=1$	$j=2$	$j=3$
1	$-.05$	.05	$-.04$
2	.05	.15	$-.14$
3		$-.05$	$-.04$
4		$-.15$	.06
5			.16

$i = 4$

k	$j=1$	$j=2$	$j=3$
1	$-.05$	$-.25$	$-.12$
2	.05	.25	$-.02$
3			$-.12$
4			.08
5			.18

e. $r = .986$

23.10. a. $\bar{Y}_{11.} = 1.80$, $\bar{Y}_{12.} = 1.95$, $\bar{Y}_{13.} = 2.70$, $\bar{Y}_{21.} = 2.45$, $\bar{Y}_{22.} = 2.52$, $\bar{Y}_{23.} = 3.45$, $\bar{Y}_{31.} = 2.75$, $\bar{Y}_{32.} = 2.85$, $\bar{Y}_{33.} = 3.74$, $\bar{Y}_{41.} = 2.55$, $\bar{Y}_{42.} = 2.55$, $\bar{Y}_{43.} = 3.42$

b. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + \epsilon_{ijk}$

c. Full model:

$$\hat{Y} = 2.72750 - .57750X_1 + .07917X_2 - .38583X_3 - .34000X_4 - .26000X_5$$
$$-.01000X_1X_4 + .06000X_1X_5 - .01667X_2X_4 - .02667X_2X_5$$
$$-.02333X_3X_4 - .00333X_3X_5,$$

$SSE(F) = .7180$

Reduced Model:

$$\hat{Y} = 2.72074 - .59611X_1 + .08412X_2 + .39964X_3 - .33756X_4 - .26317X_5,$$

$SSE(R) = .7624$

H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero.

$F^* = (.0444/6) \div (.7180/33) = .34$, $F(.99; 6, 33) = 3.41$.

If $F^* \leq 3.41$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .91$

d. Subject matter:

$$Y_{ijk} = \mu_{..} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + (\alpha\beta)_{11} X_{ijk1} X_{ijk4} + (\alpha\beta)_{12} X_{ijk1} X_{ijk5}$$
$$+(\alpha\beta)_{21} X_{ijk2} X_{ijk4} + (\alpha\beta)_{22} X_{ijk2} X_{ijk5} + (\alpha\beta)_{31} X_{ijk3} X_{ijk4}$$
$$+(\alpha\beta)_{32} X_{ijk3} X_{ijk5} + \epsilon_{ijk}$$

$\hat{Y} = 2.75121 - .34885X_4 - .19441X_5 + .19925X_1X_4 + .19481X_1X_5$
$\quad - .01178X_2X_4 - .08433X_2X_5 - .22413X_3X_4 + .00731X_3X_5,$

$SSE(R) = 4.9506$

H_0: all α_i equal zero ($i = 1, 2, 3$), H_a: not all α_i equal zero.

$F^* = (4.2326/3) \div (.7180/33) = 64.845$, $F(.99; 3, 33) = 4.437$.

If $F^* \leq 4.437$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

Degree:

$Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + (\alpha\beta)_{11} X_{ijk1} X_{ijk4}$
$\quad + (\alpha\beta)_{12} X_{ijk1} X_{ijk5} + (\alpha\beta)_{21} X_{ijk2} X_{ijk4} + (\alpha\beta)_{22} X_{ijk2} X_{ijk5}$
$\quad + (\alpha\beta)_{31} X_{ijk3} X_{ijk4} + (\alpha\beta)_{32} X_{ijk3} X_{ijk5} + \epsilon_{ijk}$

$\hat{Y} = 2.88451 - .44871X_1 - .09702X_2 + .36160X_3 - .06779X_1X_4 + .08939X_1X_5$
$\quad - .05349X_2X_4 - .03742X_2X_5 + .07190X_3X_4 - .10851X_3X_5,$

$SSE(R) = 8.9467$

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = (8.2287/2) \div (.7180/33) = 189.10$, $F(.99; 2, 33) = 5.321$.

If $F^* \leq 5.321$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. $\hat{D}_1 = \hat{\mu}_{1.} - \hat{\mu}_{2.} = 2.1500 - 2.8067 = -.6567$, $\hat{D}_2 = \hat{\mu}_{1.} - \hat{\mu}_{3.} = 2.1500 - 3.1133 = -.9633$, $\hat{D}_3 = \hat{\mu}_{1.} - \hat{\mu}_{4.} = 2.1500 - 2.8400 = -.6900$, $\hat{D}_4 = \hat{\mu}_{2.} - \hat{\mu}_{3.} = -.3066$, $\hat{D}_5 = \hat{\mu}_{2.} - \hat{\mu}_{4.} = -.0333$, $\hat{D}_6 = \hat{\mu}_{3.} - \hat{\mu}_{4.} = .2733$, $s\{\hat{D}_1\} = .06642$, $s\{\hat{D}_2\} = .07083$, $s\{\hat{D}_3\} = .07497$, $s\{\hat{D}_4\} = .06316$, $s\{\hat{D}_5\} = .06777$, $s\{\hat{D}_6\} = .07209$, $q(.95; 4, 33) = 3.825$, $T = 2.705$

$$\begin{array}{rr}
-.6567 \pm 2.705(.06642) & -.836 \leq D_1 \leq -.477 \\
-.9633 \pm 2.705(.07083) & -1.155 \leq D_2 \leq -.772 \\
-.6900 \pm 2.705(.07497) & -.893 \leq D_3 \leq -.487 \\
-.3066 \pm 2.705(.06316) & -.477 \leq D_4 \leq -.136 \\
-.0333 \pm 2.705(.06777) & -.217 \leq D_5 \leq .150 \\
.2733 \pm 2.705(.07209) & .078 \leq D_6 \leq .468
\end{array}$$

f. $\hat{D}_1 = \hat{\mu}_{.1} - \hat{\mu}_{.2} = 2.3875 - 2.4675 = -.0800$, $\hat{D}_2 = \hat{\mu}_{.1} - \hat{\mu}_{.3} = 2.3875 - 3.3350 = -.9475$, $\hat{D}_3 = \hat{\mu}_{.2} - \hat{\mu}_{.3} = -.8675$, $s\{\hat{D}_1\} = .06597$, $s\{\hat{D}_2\} = .05860$, $s\{\hat{D}_3\} = .05501$, $q(.95; 3, 33) = 3.470$, $T = 2.4537$

$$\begin{array}{rr}
-.0800 \pm 2.4537(.06597) & -.242 \leq D_1 \leq .082 \\
-.9475 \pm 2.4537(.05860) & -1.091 \leq D_2 \leq -.804 \\
-.8675 \pm 2.4537(.05501) & -1.002 \leq D_3 \leq -.733
\end{array}$$

23.11. a. Full model:

$Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + \epsilon_{ijk}$

X_{ijk1}, $X_{ijk2} X_{ijk3}$, X_{ijk4}, X_{ijk5} defined same as in Problem 23.9a

Reduced models:

Factor A: $Y_{ijk} = \mu_{..} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + \epsilon_{ijk}$

Factor B: $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \epsilon_{ijk}$

b. Full model:

$\hat{Y} = 2.72074 - .59611X_1 + .08412X_2 + .33964X_3 - .33756X_4 - .26317X_5$,

$SSE(F) = .762425$, $df_F = 39$

Reduced models:

Factor A:

$\hat{Y} = 2.72494 - .32494X_4 - .18648X_5$, $SSE(R) = 6.741678$, $df_R = 42$

H_0: $\alpha_1 = \alpha_2 = \alpha_3 = 0$, H_a: not all α_i equal zero.

$F^* = (5.979253/3) \div (.762425/39) = 101.95$, $F(.95; 3, 39) = 2.845$.

If $F^* \leq 2.845$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

Factor B:

$\hat{Y} = 2.86983 - .44483X_1 - .08521X_2 + .36654X_3$, $SSE(R) = 9.144878$, $df_R = 41$

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = (8.382453/2) \div (.762425/39) = 214.39$, $F(.95; 2, 39) = 3.238$.

If $F^* \leq 3.238$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

23.12. a. See Problem 19.14a. $\hat{D}_1 = \bar{Y}_{13.} - \bar{Y}_{11.} = 2.100$, $\hat{D}_2 = \bar{Y}_{23.} - \bar{Y}_{21.} = 3.675$, $\hat{D}_3 = \bar{Y}_{33.} - \bar{Y}_{31.} = 7.275$, $\hat{L}_1 = \hat{D}_1 - \hat{D}_2 = -1.575$, $\hat{L}_2 = \hat{D}_1 - \hat{D}_3 = -5.175$, $MSE = .06406$, $s\{\hat{D}_i\} = .1790$ $(i = 1, 2, 3)$, $s\{\hat{L}_i\} = .2531$ $(i = 1, 2)$, $B = t(.99; 24) = 2.492$

$2.100 \pm 2.492(.1790)$	$1.654 \leq D_1 \leq 2.546$
$3.675 \pm 2.492(.1790)$	$3.229 \leq D_2 \leq 4.121$
$7.275 \pm 2.492(.1790)$	$6.829 \leq D_3 \leq 7.721$
$-1.575 \pm 2.492(.2531)$	$-2.206 \leq L_1 \leq -.944$
$-5.175 \pm 2.492(.2531)$	$-5.806 \leq L_2 \leq -4.544$

b. H_0: $\mu_{12} - \mu_{13} = 0$, H_a: $\mu_{12} - \mu_{13} \neq 0$. $\hat{D} = \bar{Y}_{12.} - \bar{Y}_{13.} = .025$, $s\{\hat{D}\} = .1790$, $t^* = .025/.1790 = .14$, $t(.99; 24) = 2.492$. If $|t^*| \leq 2.492$ conclude H_0, otherwise H_a. Conclude H_0.

H_0: $\mu_{32} - \mu_{33} = 0$, H_a: $\mu_{32} - \mu_{33} \neq 0$. $\hat{D} = \bar{Y}_{32.} - \bar{Y}_{33.} = -2.975$, $s\{\hat{D}\} = .1790$, $t^* = -2.975/.1790 = -16.62$, $t(.99; 24) = 2.492$. If $|t^*| \leq 2.492$ conclude H_0, otherwise H_a. Conclude H_a. $\alpha \leq .04$

23.13. a. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \beta_1 X_{ijk2} + \beta_2 X_{ijk3} + \epsilon_{ijk}$

$$X_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 2 for factor } A \end{cases}$$

$$X_{ijk2} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk3} = \begin{cases} 1 & \text{if case from level 2 for factor } B \\ -1 & \text{if case from level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$\hat{Y} = .66939 + .11733X_1 - .34323X_2 + .02608X_3$, $SSE(F) = 4.4898$

Factor A:

$Y_{ijk} = \mu_{..} + \beta_1 X_{ijk2} + \beta_2 X_{ijk3} + \epsilon_{ijk}$

$\hat{Y} = .70850 - .26502X_2 - .01303X_3$, $SSE(R) = 5.0404$

Factor B:

$Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \epsilon_{ijk}$

$\hat{Y} = .77520 + .03152X_1$, $SSE(R) = 7.1043$

b. H_0: $\alpha_1 = 0$, H_a: $\alpha_1 \neq 0$.

$F^* = (.5506/1) \div (4.4898/46) = 5.641$, $F(.95; 1, 46) = 4.05$.

If $F^* \leq 4.05$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .022

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = (2.6145/2) \div (4.4898/46) = 13.393$, $F(.95; 2, 46) = 3.20$.

If $F^* \leq 3.20$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

23.14. a. See Problem 19.20a. $\hat{D}_1 = \bar{Y}_{12.} - \bar{Y}_{13.} = 46.0$, $\hat{D}_2 = \bar{Y}_{22.} - \bar{Y}_{23.} = 6.0$, $\hat{L}_1 = \hat{D}_1 - \hat{D}_2 = 40.0$, $MSE = 88.50$, $s\{\hat{D}_1\} = s\{\hat{D}_2\} = 6.652$, $s\{\hat{L}_1\} = 9.407$, $B = t(.99167; 15) = 2.694$

$$\begin{array}{rr}
46.0 \pm 2.694(6.652) & 28.080 \leq D_1 \leq 63.920 \\
6.0 \pm 2.694(6.652) & -11.920 \leq D_2 \leq 23.920 \\
40.0 \pm 2.694(9.407) & 14.658 \leq L_1 \leq 65.342
\end{array}$$

b. H_0: $\mu_{22} - \mu_{23} \leq 0$, H_a: $\mu_{22} - \mu_{23} > 0$. $\hat{D} = \bar{Y}_{22.} - \bar{Y}_{23.} = 6.0$, $s\{\hat{D}\} = 6.652$, $t^* = 6.0/6.652 = .90$, $t(.95; 15) = 1.753$. If $t^* \leq 1.753$ conclude H_0 otherwise H_a. Conclude H_0. P-value = .19

23.15. a. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + \epsilon_{ijk}$

$$X_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 4 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

X_{ijk2} and X_{ijk3} are defined similarly

$$X_{ijk4} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk5} = \begin{cases} 1 & \text{if case from level 2 for factor } B \\ -1 & \text{if case from level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$\hat{Y} = 2.71932 - .59897X_1 + .08546X_2 + .40036X_3 - .34043X_4 - .26218X_5$,

$SSE(F) = .7419$

Factor A:

$Y_{ijk} = \mu_{..} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + \epsilon_{ijk}$

$\hat{Y} = 2.77494 - .22494X_4 - .23648X_5$, $SSE(R) = 5.8217$

Factor B:

$Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \epsilon_{ijk}$

$\hat{Y} = 2.90108 - .35108X_1 - .11646X_2 + .33529X_3$, $SSE(R) = 8.1874$

b. H_0: $\alpha_1 = \alpha_2 = \alpha_3 = 0$, H_a: not all α_i equal zero.

$F^* = (5.0798/3) \div (.7419/37) = 84.45$, $F(.99; 3, 37) = 4.360$.

If $F^* \leq 4.360$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = (7.4455/2) \div (.7419/37) = 185.66$, $F(.99; 2, 37) = 5.229$.

If $F^* \leq 5.229$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

23.16. a. $Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + \rho_3 X_{ij3} + \rho_4 X_{ij4} + \rho_5 X_{ij5} + \rho_6 X_{ij6}$
$+ \rho_7 X_{ij7} + \rho_8 X_{ij8} + \rho_9 X_{ij9} + \tau_1 X_{ij10} + \tau_2 X_{ij11} + \epsilon_{ij}$

$$I_{ij1} = \begin{cases} 1 & \text{if experimental unit from block 1} \\ -1 & \text{if experimental unit from block 10} \\ 0 & \text{otherwise} \end{cases}$$

$I_{ij2}, \ldots, I_{ij9}$ are defined similarly

$$I_{ij10} = \begin{cases} 1 & \text{if experimental unit received treatment 1} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ij11} = \begin{cases} 1 & \text{if experimental unit received treatment 2} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

b. $\hat{Y} = 77.10000 + 4.90000X_1 + 3.90000X_2 + 2.23333X_3 + 3.23333X_4 + 1.23333X_5$
$+ .90000X_6 - 1.10000X_7 - 3.76667X_8 - 4.10000X_9 - 6.50000X_{10} - 2.50000X_{11}$

c.

Source	SS	df	MS
Regression	1,728.3667	1	157.1242
$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9$	433.3667	9	48.1519
$X_{10}, X_{11} \mid X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9$	1,295.0000	2	647.5000
Error	112.3333	18	6.2407
Total	1,840.7000	29	

d. H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (1,295.0000/2 \div (112.3333/18) = 103.754$, $F(.95; 2, 18) = 3.55$.

If $F^* \leq 3.55$ conclude H_0, otherwise H_a. Conclude H_a.

23.17. a. $Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + \rho_3 X_{ij3} + \rho_4 X_{ij4} + \tau_1 X_{ij5} + \tau_2 X_{ij6} + \epsilon_{ij}$

$$I_{ij1} = \begin{cases} 1 & \text{if experimental unit from block 1} \\ -1 & \text{if experimental unit from block 5} \\ 0 & \text{otherwise} \end{cases}$$

$I_{ij2}, \ldots, I_{ij4}$ are defined similarly

$$I_{ij5} = \begin{cases} 1 & \text{if experimental unit received treatment 1} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ij6} = \begin{cases} 1 & \text{if experimental unit received treatment 2} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

b. $\hat{Y} = .84400 - .32733X_1 - .23733X_2 - .17400X_3 + .31267X_4 + .26600X_5 + .14800X_6$

c.

Source	SS	df	MS
Regression	2.7392	6	.4565
X_1, X_2, X_3, X_4	1.4190	4	.35475
$X_5, X_6, \mid X_1, X_2, X_3, X_4$	1.3203	2	.6602
Error	.0193	8	.0024
Total	2.7585	14	

d. H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (1.3203/2) \div (.0193/8) = 273.637$, $F(.95; 2, 8) = 4.46$.

If $F^* \leq 4.46$ conclude H_0, otherwise H_a. Conclude H_a.

23.18. a. $Y_{ij} = \mu_{..} + \rho_i + \tau_j + \epsilon_{ij}$

$$Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + \rho_3 X_{ij3} + \rho_4 X_{ij4} + \rho_5 X_{ij5} + \rho_6 X_{ij6}$$
$$+\rho_7 X_{ij7} + \rho_8 X_{ij8} + \rho_9 X_{ij9} + \tau_1 X_{ij10} + \tau_2 X_{ij11} + \epsilon_{ij}$$

$$I_{ij1} = \begin{cases} 1 & \text{if experimental unit from block 1} \\ -1 & \text{if experimental unit from block 10} \\ 0 & \text{otherwise} \end{cases}$$

$I_{ij2}, \ldots, I_{ij9}$ are defined similarly

$$I_{ij10} = \begin{cases} 1 & \text{if experimental unit received treatment 1} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ij11} = \begin{cases} 1 & \text{if experimental unit received treatment 2} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

b. $Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + +\rho_3 X_{ij3} + \rho_4 X_{ij4} + \rho_5 X_{ij5} + \rho_6 X_{ij6}$

$+\rho_7 X_{ij7} + \rho_8 X_{ij8} + \rho_9 X_{ij9} + \epsilon_{ij}$

c. <u>Full model</u>: $\hat{Y} = 77.15556 + 4.84444X_1 + 4.40000X_2 + 2.17778X_3$

$+3.17778X_4 + 1.17778X_5 + .84444X_6 - 1.15556X_7$

$-3.82222X_8 - 4.15556X_9 - 6.55556X_{10} - 2.55556X_{11}$

$SSE(F) = 110.6667$

Reduced model: $\hat{Y} = 76.70000 + 5.30000X_1 + .30000X_2 + 2.63333X_3$
$+3.63333X_4 + 1.63333X_5 + 1.30000X_6 - .70000X_7$
$-3.36667X_8 - 3.70000X_9$

$SSE(R) = 1,311.3333$

H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (1,200.6666/2) \div (110.6667/17) = 92.22$, $F(.95; 2, 17) = 3.59$.

If $F^* \leq 3.59$ conclude H_0, otherwise H_a. Conclude H_a.

d. $\hat{L} = \hat{\tau}_2 - \hat{\tau}_3 = 2\hat{\tau}_2 + \hat{\tau}_1 = -11.66667$, $s^2\{\hat{\tau}_i\} = .44604$ $(i = 1, 2)$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.20494$, $s\{\hat{L}\} = 1.1876$, $t(.975; 17) = 2.11$,

$-11.66667 \pm 2.11(1.1876)$, $-14.17 \leq L \leq -9.16$

23.19. a. $Y_{ij} = \mu_{..} + \rho_i + \tau_j + \epsilon_{ij}$

$Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + \rho_3 X_{ij3} + \rho_4 X_{ij4} + \tau_1 X_{ij5} + \tau_2 X_{ij6} + \epsilon_{ij}$

$$I_{ij1} = \begin{cases} 1 & \text{if experimental unit from block 1} \\ -1 & \text{if experimental unit from block 5} \\ 0 & \text{otherwise} \end{cases}$$

$I_{ij2}, \ldots, I_{ij4}$ are defined similarly

$$I_{ij5} = \begin{cases} 1 & \text{if experimental unit received treatment 1} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ij6} = \begin{cases} 1 & \text{if experimental unit received treatment 2} \\ -1 & \text{if experimental unit received treatment 3} \\ 0 & \text{otherwise} \end{cases}$$

b. $Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + \rho_3 X_{ij3} + \rho_4 X_{ij4} + \epsilon_{ij}$

c. Full model: $\hat{Y} = .82941 - .33613X_1 - .22274X_2 - .15941X_3 + .32726X_4$
$+.25085X_5 + .16259X_6$

$SSE(F) = .0035$

Reduced model: $\hat{Y} = .84567 - 14567X_1 - .23900X_2 - .17567X_3 + .31100X_4$

$SSE(R) = .9542$

H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (.9507/2) \div (.0035/6) = 814.89$, $F(.95; 2, 6) = 5.14$.

If $F^* \leq 5.14$ conclude H_0, otherwise H_a. Conclude H_a.

d. $\hat{L} = \hat{\tau}_1 - \hat{\tau}_3 = 2\hat{\tau}_1 + \hat{\tau}_2 = .66429$, $s^2\{\hat{\tau}_1\} = .000105$, $s^2\{\hat{\tau}_2\} = .000087$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.000043$, $s\{\hat{L}\} = .0183$, $t(.99; 6) = 3.143$,

$.66429 \pm 3.143(.0183)$, $.607 \leq L \leq .722$

23.20. See Problem 19.10a. $L_1 = .3\mu_{11} + .6\mu_{21} + .1\mu_{31}$, $L_2 = .3\mu_{12} + .6\mu_{22} + .1\mu_{32}$.

H_0: $L_1 = L_2$, H_a: $L_1 \neq L_2$.

$\hat{L}_1 - \hat{L}_2 = 25.43332 - 25.05001 = .38331$, $MSE = 2.3889$, $s\{\hat{L}_1 - \hat{L}_2\} = .6052$,

$t^* = .38331/.6052 = .63$, $t(.975; 30) = 2.042$.

If $|t^*| \leq 2.042$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .53

23.21. a.

$$H_0: \begin{array}{l} L_1 = \dfrac{3\mu_{11} + \mu_{21}}{4} - \dfrac{3\mu_{12} + \mu_{22}}{4} = 0 \\ L_2 = \dfrac{3\mu_{11} + \mu_{21}}{4} - \dfrac{3\mu_{13} + \mu_{23}}{4} = 0 \end{array}$$

H_a: not both L_1 and L_2 equal zero

b. Regression model equivalent to (19.15) using 1, 0 indicator variables:

Full model:

$$Y_{ijk} = \mu_{11}X_{ijk1} + \mu_{12}X_{ijk2} + \mu_{13}X_{ijk3} + \mu_{21}X_{ijk4} + \mu_{22}X_{ijk5} + \mu_{23}X_{ijk6} + \epsilon_{ijk}$$

$$X_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \text{ and level 1 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk2} = \begin{cases} 1 & \text{if case from level 1 for factor } A \text{ and level 2 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk3} = \begin{cases} 1 & \text{if case from level 1 for factor } A \text{ and level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk4} = \begin{cases} 1 & \text{if case from level 2 for factor } A \text{ and level 1 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk5} = \begin{cases} 1 & \text{if case from level 2 for factor } A \text{ and level 2 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk6} = \begin{cases} 1 & \text{if case from level 2 for factor } A \text{ and level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

c. Reduced model:

$\mu_{11} = \mu_{12} + \mu_{22}/3 - \mu_{21}/3$

$\mu_{13} = \mu_{12} + \mu_{22}/3 - \mu_{23}/3$

$$Y_{ijk} = (\mu_{12} + \mu_{22}/3 - \mu_{21}/3)X_{ijk1} + \mu_{12}X_{ijk2} + (\mu_{12} + \mu_{22}/3 - \mu_{23}/3)X_{ijk3} + \mu_{21}X_{ijk4} + \mu_{22}X_{ijk5} + \mu_{23}X_{ijk6} + \epsilon_{ijk}$$

or

$$Y_{ijk} = \mu_{12}Z_{ijk1} + \mu_{21}Z_{ijk2} + \mu_{22}Z_{ijk3} + \mu_{23}Z_{ijk4} + \epsilon_{ijk}$$

where:

$$\begin{aligned} Z_{ijk1} &= X_{ijk1} + X_{ijk2} + X_{ijk3} \\ Z_{ijk2} &= -X_{ijk1}/3 + X_{ijk4} \\ Z_{ijk3} &= (X_{ijk1} + X_{ijk3})/3 + X_{ijk5} \\ Z_{ijk4} &= (-X_{ijk3}/3) + X_{ijk6} \end{aligned}$$

d. $SSE(F) = 5.468$, $df_F = 54$, $SSE(R) = 8.490$, $df_R = 56$,

$F^* = [(8.490 - 5.468)/2] \div (5.468/54) = 14.92$, $F(.95; 2, 54) = 3.17$.

If $F^* \leq 3.17$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

e. See Problem 19.18a. $\hat{L}_2 = (.75\bar{Y}_{11.} + .25\bar{Y}_{21.}) - (.75\bar{Y}_{13.} + .25\bar{Y}_{23.}) = -.61446$, $s\{\hat{L}_2\} = .1125$, $t(.975; 54) = 2.005$, $-.61446 \pm 2.005(.1125)$, $-.840 \leq L_2 \leq -.389$

23.22. a.

$$H_0\text{:}\quad \begin{aligned} L_1 &= \frac{\mu_{11} + 2\mu_{12} + 7\mu_{13}}{10} - \frac{\mu_{21} + 2\mu_{22} + 7\mu_{23}}{10} = 0 \\ L_2 &= \frac{\mu_{11} + 2\mu_{12} + 7\mu_{13}}{10} - \frac{\mu_{31} + 2\mu_{32} + 7\mu_{33}}{10} = 0 \\ L_3 &= \frac{\mu_{11} + 2\mu_{12} + 7\mu_{13}}{10} - \frac{\mu_{41} + 2\mu_{42} + 7\mu_{43}}{10} = 0 \end{aligned}$$

H_a: not all L_i equal zero ($i = 1, 2, 3$)

b. $\boldsymbol{\beta}$ entries: μ_{11}, μ_{12}, μ_{13}, μ_{21}, μ_{22}, μ_{23}, μ_{31}, μ_{32}, μ_{33}, μ_{41}, μ_{42}, μ_{43}

$\mathbf{X}$ entries:

A	B	Freq.												
1	1	2	1	0	0	0	0	0	0	0	0	0	0	0
1	2	2	0	1	0	0	0	0	0	0	0	0	0	0
1	3	8	0	0	1	0	0	0	0	0	0	0	0	0
2	1	4	0	0	0	1	0	0	0	0	0	0	0	0
2	2	5	0	0	0	0	1	0	0	0	0	0	0	0
2	3	4	0	0	0	0	0	1	0	0	0	0	0	0
3	1	2	0	0	0	0	0	0	1	0	0	0	0	0
3	2	4	0	0	0	0	0	0	0	1	0	0	0	0
3	3	5	0	0	0	0	0	0	0	0	1	0	0	0
4	1	2	0	0	0	0	0	0	0	0	0	1	0	0
4	2	2	0	0	0	0	0	0	0	0	0	0	1	0
4	3	5	0	0	0	0	0	0	0	0	0	0	0	1

c.

$$\mathbf{C} = \begin{bmatrix} .1 & .2 & .7 & -.1 & -.2 & -.7 & 0 & 0 & 0 & 0 & 0 & 0 \\ .1 & .2 & .7 & 0 & 0 & 0 & -.1 & -.2 & -.7 & 0 & 0 & 0 \\ .1 & .2 & .7 & 0 & 0 & 0 & 0 & 0 & 0 & -.1 & -.2 & -.7 \end{bmatrix} \quad \mathbf{h} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

d. $SSE(R) - SSE(F) = 5.6821$

e. $SSE(F) = .7180$, $F^* = (5.6821/3) \div (.7180/33) = 87.05$, $F(.99; 3, 33) = 4.437$.

If $F^* \leq 4.437$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

f. See Problem 23.10a. $\hat{L}_2 = (.1\bar{Y}_{11.} + .2\bar{Y}_{12.} + .7\bar{Y}_{13.}) - (.1\bar{Y}_{31.} + .2\bar{Y}_{32.} + .7\bar{Y}_{33.}) = -1.003$, $s\{\hat{L}_2\} = .0658$, $t(.995; 33) = 2.733$,

$-1.003 \pm 2.733(.0658)$, $-1.183 \leq L_2 \leq -.823$

23.23. H_0: $\frac{4\mu_{11}+4\mu_{12}+2\mu_{13}}{10} = \frac{4\mu_{21}+4\mu_{22}+3\mu_{23}}{11}$, H_a: equality does not hold.

$\bar{Y}_{...} = 93.714$, $\bar{Y}_{1..} = 143$, $\bar{Y}_{2..} = 48.91$

$SSA = 10(143 - 93.714)^2 + 11(48.91 - 93.714)^2 = 46,372$

$F^* = (46,372/1) \div (1,423.1667/15) = 488.8$, $F(.99; 1, 15) = 8.68$.

If $F^* \leq 8.68$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

23.24. H_0: $\frac{2}{12}\mu_{11} + \frac{2}{12}\mu_{12} + \frac{8}{12}\mu_{13} = \frac{4}{13}\mu_{21} + \frac{5}{13}\mu_{22} + \frac{4}{13}\mu_{23} = \frac{2}{11}\mu_{31} + \frac{4}{11}\mu_{32} + \frac{5}{11}\mu_{33} = \frac{2}{9}\mu_{41} + \frac{2}{9}\mu_{42} + \frac{5}{9}\mu_{43}$,

H_a: not all equalities hold.

$\bar{Y}_{...} = 2.849$, $\bar{Y}_{1..} = 2.425$, $\bar{Y}_{2..} = 2.785$, $\bar{Y}_{3..} = 3.236$, $\bar{Y}_{4..} = 3.033$

$SSA = 12(2.425 - 2.849)^2 + 13(2.785 - 2.849)^2 + 11(3.236 - 2.849)^2 + 9(3.033 - 2.849)^2 = 4.163$

$F^* = (4.163/3) \div (.718/33) = 63.78$, $F(.95; 3, 33) = 2.89$.

If $F^* \leq 2.89$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

23.25.
$$\sigma^2\{\hat{L}\} = \sigma^2\{\sum c_i\hat{\mu}_{i.}\} = \sum c_i^2\sigma^2\{\hat{\mu}_{i.}\} = \sum_i c_i^2\sigma^2\left\{\frac{\sum_j \bar{Y}_{ij.}}{b}\right\}$$
$$= \frac{1}{b^2}\sum_i c_i^2 \sum_j \frac{\sigma^2}{n_{ij}} = \frac{\sigma^2}{b^2}\sum_i c_i^2 \sum_j \frac{1}{n_{ij}}$$

because of independence of $\bar{Y}_{ij.}$.

23.26.
$$E\left\{s^2\{\hat{L}\}\right\} = E\left\{MSE \sum\sum \frac{c_{ij}^2}{n_{ij}}\right\} = \sum\sum \frac{c_{ij}^2}{n_{ij}} E\{MSE\} = \sigma^2 \sum\sum \frac{c_{ij}^2}{n_{ij}} = \sigma^2\{\hat{L}\}$$

23.27. a.

$$\mathbf{X} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b.

$$\mathbf{X}\boldsymbol{\beta} = \begin{bmatrix} \beta_0 + \beta_1 + \beta_3 \\ \beta_0 + \beta_1 + \beta_4 \\ \beta_0 + \beta_1 \\ \beta_0 + \beta_2 + \beta_3 \\ \beta_0 + \beta_2 + \beta_4 \\ \beta_0 + \beta_2 \\ \beta_0 + \beta_3 \\ \beta_0 + \beta_4 \\ \beta_0 \end{bmatrix} = \begin{bmatrix} \mu_{..} + \rho_1 + \tau_1 \\ \mu_{..} + \rho_1 + \tau_2 \\ \mu_{..} + \rho_1 + \tau_3 \\ \mu_{..} + \rho_2 + \tau_1 \\ \mu_{..} + \rho_2 + \tau_2 \\ \mu_{..} + \rho_2 + \tau_3 \\ \mu_{..} + \rho_3 + \tau_1 \\ \mu_{..} + \rho_3 + \tau_2 \\ \mu_{..} + \rho_3 + \tau_3 \end{bmatrix}$$

$\beta_0 = \mu_{..} + \rho_3 + \tau_3$ $\quad$ $\beta_2 = \rho_2 - \rho_3$

$\beta_4 = \tau_2 - \tau_3$ $\quad$ $\beta_1 = \rho_1 - \rho_3$

$\beta_3 = \tau_1 - \tau_3$

23.28.

$$\mathbf{Y} = \begin{bmatrix} Y_{111} \\ Y_{112} \\ Y_{121} \\ Y_{122} \\ Y_{211} \\ Y_{212} \\ Y_{221} \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{\beta} = \begin{bmatrix} \mu_{11} \\ \mu_{12} \\ \mu_{21} \\ \mu_{22} \end{bmatrix} \quad \mathbf{b}_F = \begin{bmatrix} \bar{Y}_{11.} \\ \bar{Y}_{12.} \\ \bar{Y}_{21.} \\ \bar{Y}_{22.} \end{bmatrix}$$

$$\mathbf{C}\boldsymbol{\beta} = \begin{bmatrix} 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} \mu_{11} \\ \mu_{12} \\ \mu_{21} \\ \mu_{22} \end{bmatrix} = \mu_{11} - \mu_{12} - \mu_{21} + \mu_{22} = 0$$

From (23.46):

$$\mathbf{b}_R = \begin{bmatrix} \bar{Y}_{11.} \\ \bar{Y}_{12.} \\ \bar{Y}_{21.} \\ \bar{Y}_{22.} \end{bmatrix} - \begin{bmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \left(\frac{5}{2}\right)^{-1} \left(\bar{Y}_{11.} - \bar{Y}_{12.} - \bar{Y}_{21.} + \bar{Y}_{22.} - 0 \right)$$

$$= \begin{bmatrix} \bar{Y}_{11.} \\ \bar{Y}_{12.} \\ \bar{Y}_{21.} \\ \bar{Y}_{22.} \end{bmatrix} - \frac{2}{5} \left(\bar{Y}_{11.} - \bar{Y}_{12.} - \bar{Y}_{21.} + \bar{Y}_{22.} \right) \begin{bmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1 \end{bmatrix}$$

$$\hat{\mu}_{22} = \bar{Y}_{22.} - \frac{2}{5}(\bar{Y}_{11.} - \bar{Y}_{12.} - \bar{Y}_{21.} + \bar{Y}_{22.})$$

23.29. a. $\boldsymbol{\beta}$ entries: μ_{11}, μ_{12}, μ_{13}, μ_{22}

X entries:

A	B	Freq.				
1	1	10	1	0	0	0
1	2	10	0	1	0	0
1	3	10	0	0	1	0
2	2	10	0	0	0	1
2	3	10	0	-1	1	1

b. $\mathbf{C} = [0 \quad 1 \quad 0 \ -1] \qquad \mathbf{h} = 0$

23.30. a. $Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

$$\begin{aligned} Y_{ijk} = \mu_{..} &+ \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} \\ &+(\alpha\beta)_{11} X_{ijk1} X_{ijk4} + (\alpha\beta)_{12} X_{ijk1} X_{ijk5} + (\alpha\beta)_{21} X_{ijk2} X_{ijk4} \\ &+(\alpha\beta)_{22} X_{ijk2} X_{ijk5} + (\alpha\beta)_{31} X_{ijk3} X_{ijk4} + (\alpha\beta)_{32} X_{ijk3} X_{ijk4} + \epsilon_{ijk} \end{aligned}$$

$$I_{ijk1} = \begin{cases} 1 & \text{if case from NE} \\ -1 & \text{if case from W} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk2} = \begin{cases} 1 & \text{if case from NC} \\ -1 & \text{if case from W} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk3} = \begin{cases} 1 & \text{if case from S} \\ -1 & \text{if case from W} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk4} = \begin{cases} 1 & \text{if average age under 52.0} \\ -1 & \text{if average age 55.0 or more} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk5} = \begin{cases} 1 & \text{if average age 52.0 - under 55.0} \\ -1 & \text{if average age 55.0 or more} \\ 0 & \text{otherwise} \end{cases}$$

b. $\hat{Y} = 9.40661 + 1.45009X_1 + .23601X_2 - .24406X_3 - .38373X_4$
$+.19446X_5 - .76296X_1X_4 - .57198X_1X_5 + .44674X_2X_4$
$+.17515X_2X_5 + .35707X_3X_4 - .38986X_3X_5$, $SSE(F) = 261.2341$

c. $r = .959$

23.31. a. $\bar{Y}_{11.} = 9.71000$, $\bar{Y}_{12.} = 10.47917$, $\bar{Y}_{13.} = 12.38091$, $\bar{Y}_{21.} = 9.70563$,
$\bar{Y}_{22.} = 10.01222$, $\bar{Y}_{23.} = 9.21000$, $\bar{Y}_{31.} = 9.13588$, $\bar{Y}_{32.} = 8.96714$,
$\bar{Y}_{33.} = 9.38462$, $\bar{Y}_{41.} = 7.54000$, $\bar{Y}_{42.} = 8.94571$, $\bar{Y}_{43.} = 7.40800$

b. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_1 X_{ijk2} + \alpha_3 X_{ijk3} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + \epsilon_{ijk}$

c. $\hat{Y} = 9.52688 + 1.50561X_1 + .23119X_2 - .30781X_3 - .26423X_4 - .00518X_5$,
$SSE(R) = 300.4100$

H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero.

$F^* = (39.1759/6) \div (261.2341/101) = 2.524$, $F(.99; 6, 101) = 2.99$.

If $F^* \leq 2.99$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .026

d. $Y_{ijk} = \mu_{..} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + (\alpha\beta)_{11} X_{ijk1} X_{ijk4}$
$+ (\alpha\beta)_{12} X_{ijk1} X_{ijk5} + (\alpha\beta)_{21} X_{ijk2} X_{ijk4} + (\alpha\beta)_{22} X_{ijk2} X_{ijk5}$
$+ (\alpha\beta)_{31} X_{ijk3} X_{ijk4} + (\alpha\beta)_{32} X_{ijk3} X_{ijk5} + \epsilon_{ijk}$

$\hat{Y} = 9.52473 - .50240X_4 + .24409X_5 - 1.44778X_1X_4 - .17942X_1X_5$
$+.61345X_2X_4 + .11924X_2X_5 + .34296X_3X_4 - .24453X_3X_5$,
$SSE(R) = 345.4833$

H_0: all α_i equal zero $(i = 1, 2, 3)$, H_a: not all α_i equal zero.

$F^* = (84.2492/3) \div (261.2341/101) = 10.858$, $F(.99; 3, 101) = 3.98$.

If $F^* \leq 3.98$ conclude H_0, otherwise H_a. Conclude H_a. P-value 0+

e. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + (\alpha\beta)_{11} X_{ijk1} X_{ijk4}$
$+ (\alpha\beta)_{12} X_{ijk1} X_{ijk5} + (\alpha\beta)_{21} X_{ijk2} X_{ijk4} + (\alpha\beta)_{22} X_{ijk2} X_{ijk5}$
$+ (\alpha\beta)_{31} X_{ijk3} X_{ijk4} + (\alpha\beta)_{32} X_{ijk3} X_{ijk5} + \epsilon_{ijk}$

$\hat{Y} = 9.42456 + 1.53811X_1 + .15829X_2 - .31129X_3 - .69021X_1X_4$
$-.60603X_1X_5 + .29855X_2X_4 + .26600X_2X_5 + .18805X_3X_4 - .35618X_3X_5$,

$SSE(R) = 267.7103$

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = (6.4762/2) \div (261.2341/101) = 1.252$, $F(.99; 2, 101) = 4.82$.

If $F^* \leq 4.82$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .29

f. $n_{11} = 5$, $n_{12} = 12$, $n_{13} = 11$, $n_{21} = 16$, $n_{22} = 9$, $n_{23} = 7$,

$n_{31} = 17$, $n_{32} = 7$, $n_{33} = 13$, $n_{41} = 4$, $n_{42} = 7$, $n_{43} = 5$,

$\hat{D}_1 = \hat{\mu}_{1.} - \hat{\mu}_{2.} = 10.85669 - 9.64262 = 1.21407$,

$\hat{D}_2 = \hat{\mu}_{1.} - \hat{\mu}_{3.} = 10.85669 - 9.16255 = 1.69414$,

$\hat{D}_3 = \hat{\mu}_{1.} - \hat{\mu}_{4.} = 10.85669 - 7.96457 = 2.89212$,

$\hat{D}_4 = \hat{\mu}_{2.} - \hat{\mu}_{3.} = .48007$, $\hat{D}_5 = \hat{\mu}_{2.} - \hat{\mu}_{4.} = 1.67805$,

$\hat{D}_6 = \hat{\mu}_{3.} - \hat{\mu}_{4.} = 1.19798$, $MSE = 2.5865$, $s\{\hat{D}_1\} = .4455$, $s\{\hat{D}_2\} = .4332$,

$s\{\hat{D}_3\} = .5272$, $s\{\hat{D}_4\} = .4135$, $s\{\hat{D}_5\} = .5112$, $s\{\hat{D}_6\} = .5004$,

$q(.95; 4, 101) = 3.694$, $T = 2.612$

$$\begin{array}{rr}
1.21407 \pm 2.612(.4455) & .050 \leq D_1 \leq 2.378 \\
1.69414 \pm 2.612(.4332) & .563 \leq D_2 \leq 2.826 \\
2.89212 \pm 2.612(.5272) & 1.515 \leq D_3 \leq 4.269 \\
.48007 \pm 2.612(.4135) & -.600 \leq D_4 \leq 1.560 \\
1.67805 \pm 2.612(.5112) & .343 \leq D_5 \leq 3.013 \\
1.19798 \pm 2.612(.5004) & -.109 \leq D_6 \leq 2.505
\end{array}$$

23.32. a. ANOVA model: $Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

Regression: $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5}$
$+ (\alpha\beta)_{11} X_{ijk1} X_{ijk4} + (\alpha\beta)_{12} X_{ijk1} X_{ijk5} + (\alpha\beta)_{21} X_{ijk2} X_{ijk4}$
$+ (\alpha\beta)_{22} X_{ijk2} X_{ijk5} + (\alpha\beta)_{31} X_{ijk3} X_{ijk4} + (\alpha\beta)_{32} X_{ijk3} X_{ijk5} + \epsilon_{ijk}$

$$I_{ijk1} = \begin{cases} 1 & \text{if case from NE} \\ -1 & \text{if case from W} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk2} = \begin{cases} 1 & \text{if case from NC} \\ -1 & \text{if case from W} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk3} = \begin{cases} 1 & \text{if case from S} \\ -1 & \text{if case from W} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk4} = \begin{cases} 1 & \text{if poverty level below 6.0 percent} \\ -1 & \text{if poverty level is 10 percent or more} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk5} = \begin{cases} 1 & \text{if poverty level between 6.0 and under 10.0 percent} \\ -1 & \text{if poverty level is 10 percent or more} \\ 0 & \text{otherwise} \end{cases}$$

b. $\hat{Y} = .0568 - .00852X_1 - .00475X_2 + .00983X_3 - .0114X_4 - .00173X_5 - .00206X_1X_4$
$-.00629X_1X_5 - .00106X_2X_4 + .00069X_2X_5 - .00102X_3X_4 + .00133X_3X_5,$
$SSE(F) = .23111$

c. $r = .932$

23.33. a. $\bar{Y}_{11.} = .0348$, $\bar{Y}_{12.} = .0402$, $\bar{Y}_{13.} = .0697$, $\bar{Y}_{21.} = .0396$, $\bar{Y}_{22.} = .0510$, $\bar{Y}_{23.} = .0655$,
$\bar{Y}_{31.} = .0542$, $\bar{Y}_{32.} = .0662$, $\bar{Y}_{33.} = .0794$, $\bar{Y}_{41.} = .0530$, $\bar{Y}_{42.} = .0627$, $\bar{Y}_{43.} = .0649$

b. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + \epsilon_{ijk}$

c. $\hat{Y} = .0563 - .0105X_1 - .0043X_2 + .0106X_3 - .0111X_4 - .0013X_5,$
$SSE(R) = .23589$
H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero.
$F^* = (.00478/6) \div (.23111/428) = 1.476$. $F(.995; 6, 428) = 3.14$.
If $F^* \leq 3.14$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .18

d. $$Y_{ijk} = \mu_{..} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + (\alpha\beta)_{11} X_{ijk1} X_{ijk4} + (\alpha\beta)_{12} X_{ijk1} X_{ijk5} + (\alpha\beta)_{21} X_{ijk2} X_{ijk4} + (\alpha\beta)_{22} X_{ijk2} X_{ijk5} + (\alpha\beta)_{31} X_{ijk3} X_{ijk4} + (\alpha\beta)_{32} X_{ijk3} X_{ijk5} + \epsilon_{ijk}$$
$\hat{Y} = .0578 - .0141X_4 - .0024X_5 - .0043X_1X_4 - .0090X_1X_5 + .0020X_2X_4$
$-.0005X_2X_5 - .0040X_3X_4 + .0039X_3X_5,$
$SSE(R) = .25118$
H_0: $\alpha_1 = \alpha_2 = \alpha_3$, H_a: not all α_i equal zero.
$F^* = (.02007/3) \div (.23111/428) = 12.39$, $F(.995; 3, 428) = 4.34$.
If $F^* \leq 4.34$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. $$Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + (\alpha\beta)_{11} X_{ijk1} X_{ijk4} + (\alpha\beta)_{12} X_{ijk1} X_{ijk5} + (\alpha\beta)_{21} X_{ijk2} X_{ijk4} + (\alpha\beta)_{22} X_{ijk2} X_{ijk5} + (\alpha\beta)_{31} X_{ijk3} X_{ijk4} + (\alpha\beta)_{32} X_{ijk3} X_{ijk5} + \epsilon_{ijk}$$
$\hat{Y} = .0558 - .0138X_1 - .0047X_2 + .0135X_3 - .0015X_1X_4 - .0009X_1X_5$
$+3.15459X_2X_4 - .23458X_2X_5 - .0032X_3X_4 - .0025X_3X_5,$
$SSE(R) = .26209$
H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.
$F^* = (.03098/2) \div (.23111/428) = 28.69$, $F(.995; 2, 428) = 5.36$.
If $F^* \leq 5.36$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

f. $n_{11} = 52$, $n_{12} = 38$, $n_{13} = 13$, $n_{21} = 32$, $n_{22} = 50$, $n_{23} = 26$,
$n_{31} = 25$, $n_{32} = 52$, $n_{33} = 75$, $n_{41} = 20$, $n_{42} = 33$, $n_{43} = 24$,
$\hat{D}_1 = \hat{\mu}_{1.} - \hat{\mu}_{2.} = -.0038$, $\hat{D}_2 = \hat{\mu}_{1.} - \hat{\mu}_{3.} = -.0184$, $\hat{D}_3 = \hat{\mu}_{1.} - \hat{\mu}_{4.} = -.0120$,
$\hat{D}_4 = \hat{\mu}_{2.} - \hat{\mu}_{3.} = -.0146$, $\hat{D}_5 = \hat{\mu}_{2.} - \hat{\mu}_{4.} = -.0082$, $\hat{D}_6 = \hat{\mu}_{3.} - \hat{\mu}_{4.} = .0064$,

$MSE = .00054$, $q(.95; 4, 428) = 3.63$, $T = 2.567$

$s\{\hat{D}_1\} = .00357$, $s\{\hat{D}_2\} = .00342$, $s\{\hat{D}_3\} = .00383$,

$s\{\hat{D}_4\} = .00312$, $s\{\hat{D}_5\} = .00356$, $s\{\hat{D}_6\} = .00342$,

$$\begin{array}{rr} -.0038 \pm 2.567(.00357) & -.0130 \leq D_1 \leq .0054 \\ -.0184 \pm 2.567(.00342) & -.0271 \leq D_2 \leq -.0096 \\ -.0120 \pm 2.567(.00383) & -.0218 \leq D_3 \leq -.0021 \\ -.0146 \pm 2.567(.00312) & -.0226 \leq D_4 \leq -.0066 \\ -.0082 \pm 2.567(.00356) & -.0173 \leq D_5 \leq .0010 \\ .0064 \pm 2.567(.00342) & -.0024 \leq D_6 \leq .0152 \end{array}$$

23.34. a. ANOVA model: $Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

Regression: $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \beta_1 X_{ijk2} + (\alpha\beta)_{11} X_{ijk1} X_{ijk2} + \epsilon_{ijk}$

$$X_{ijk1} = \begin{cases} 1 & \text{if no discount price (level 0 of variable 5)} \\ -1 & \text{if discount price (level 1 of variable 5)} \end{cases}$$

$$X_{ijk2} = \begin{cases} 1 & \text{if no package promotion (level 0 of variable 6)} \\ -1 & \text{if package promotion (level 1 of variable 6)} \end{cases}$$

b. $\hat{Y} = 2.620 - .199X_1 - .0446X_2 + .0366X_1X_2$

$SSE(F) = .7850$

c. $r = .990$

23.35. a. $\bar{Y}_{00.} = 2.4125$, $\bar{Y}_{01.} = 2.4286$, $\bar{Y}_{10.} = 2.7375$, $\bar{Y}_{11.} = 2.9000$,

b. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \beta_1 X_{ijk2} + \epsilon_{ijk}$

c. $\hat{Y} = 2.625 - .201X_1 - .0498X_2$, $SSE(R) = .8306$

H_0: $(\alpha\beta)_{11} = 0$, H_a: $(\alpha\beta)_{11} \neq 0$.

$F^* = (.0456/1) \div (.7850/32) = 1.86$. $F(.95; 1, 32) = 4.15$.

If $F^* \leq 4.15$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .18$

d. $Y_{ijk} = \mu_{..} + \beta_1 X_{ijk2} + (\alpha\beta)_{11} X_{ijk1} X_{ijk2} + \epsilon_{ijk}$

$\hat{Y} = 2.648 - .0726X_2 + .0494X_1X_2$, $SSE(R) = 2.1352$

H_0: $\alpha_1 = 0$, H_a: $\alpha_1 \neq 0$.

$F^* = (1.3502/1) \div (.7850/32) = 55.04$, $F(.95; 1, 32) = 4.15$.

If $F^* \leq 4.15$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

e. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + (\alpha\beta)_{11} X_{ijk1} X_{ijk2} + \epsilon_{ijk}$

$\hat{Y} = 2.623 - .205X_1 + .0429X_1X_2$, $SSE(R) = .8529$

H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$

$F^* = (.0679/1) \div (.7850/32) = 2.77$, $F(.95; 1, 32) = 4.15$.

If $F^* \leq 4.15$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .11$

23.36. a. H_0: $\frac{5\mu_{11}+12\mu_{12}+11\mu_{13}}{28} = \frac{16\mu_{21}+9\mu_{22}+7\mu_{23}}{32} = \frac{17\mu_{31}+7\mu_{32}+13\mu_{33}}{37} = \frac{4\mu_{41}+7\mu_{42}+5\mu_{43}}{16}$,

H_a: not all equalities hold.

$F^* = (103.55418/3) \div (261.23406/101) = 13.346$, $F(.99; 3, 101) = 3.98$.

If $F^* \leq 3.98$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

b. H_0: $\frac{5\mu_{11}+16\mu_{21}+17\mu_{31}+4\mu_{41}}{42} = \frac{12\mu_{12}+9\mu_{22}+7\mu_{32}+7\mu_{42}}{35} = \frac{11\mu_{13}+7\mu_{23}+13\mu_{33}+5\mu_{43}}{36}$,

H_a: not all equalities hold.

$F^* = (10.63980/2) \div (261.23406/101) = 2.057$, $F(.99; 2, 101) = 4.82$.

If $F^* \leq 4.82$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .13

23.37. a. H_0: $\frac{52\mu_{11}+38\mu_{12}+13\mu_{13}}{103} = \frac{32\mu_{21}+50\mu_{22}+26\mu_{23}}{108} = \frac{25\mu_{31}+52\mu_{32}+75\mu_{33}}{152} = \frac{20\mu_{41}+33\mu_{42}+24\mu_{43}}{77}$,

H_a: not all equalities hold.

$\bar{Y}_{...} = .05729$, $\bar{Y}_{1..} = .04123$, $\bar{Y}_{2..} = .05111$, $\bar{Y}_{3..} = .07074$, $\bar{Y}_{4..} = .06088$

$F^* = (.0592/3) \div (.23111/428) = 36.54$, $F(.995; 3, 428) = 4.34$.

If $F^* \leq 4.34$ conclude H_0, otherwise H_a . Conclude H_a. P-value = 0+

b. H_0: $\frac{52\mu_{11}+32\mu_{21}+25\mu_{31}+20\mu_{41}}{129} = \frac{38\mu_{12}+50\mu_{22}+52\mu_{32}+33\mu_{42}}{173} = \frac{13\mu_{13}+26\mu_{23}+75\mu_{33}+24\mu_{43}}{138}$,

H_a: not all equalities hold.

$\bar{Y}_{...} = .05729$, $\bar{Y}_{.1.} = .04259$, $\bar{Y}_{.2.} = .05544$, $\bar{Y}_{.3.} = .07334$,

$F^* = (.0640/2) \div (.23111/428) = 59.26$, $F(.995; 2, 428) = 5.36$.

If $F^* \leq 5.36$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

Chapter 24

MULTIFACTOR STUDIES

24.1. a. $\beta_1 = \mu_{.1.} - \mu_{...} = -2$, $\beta_2 = \mu_{.2.} - \mu_{...} = -.5$, $\beta_3 = \mu_{.3.} - \mu_{...} = 2.5$

b. $(\beta\gamma)_{12} = \mu_{.12} - \mu_{.1.} - \mu_{..2} + \mu_{...} = 1$

c. $(\alpha\beta\gamma)_{212} = \mu_{212} - \mu_{21.} - \mu_{.12} - \mu_{2.2} + \mu_{2..} + \mu_{.1.} + \mu_{..2} - \mu_{...} = -.5$

24.4. a. $\alpha_1 = \mu_{1..} - \mu_{...} = 138 - 131.5 = 6.5$, $\alpha_2 = \mu_{2..} - \mu_{...} = 131.5 - 131.5 = 0$

$\alpha_3 = \mu_{3..} - \mu_{...} = 125 - 131.5 = -6.5$

b. $\beta_2 = \mu_{.2.} - \mu_{...} = 134 - 131.5 = 2.5$, $\gamma_1 = \mu_{..1} - \mu_{...} = 128.5 - 131.5 = -3$

c. $(\alpha\beta)_{12} = \mu_{12.} - \mu_{1..} - \mu_{.2.} + \mu_{...} = 141 - 138 - 134 + 131.5 = .5$

$(\alpha\gamma)_{21} = \mu_{2.1} - \mu_{2..} - \mu_{..1} + \mu_{...} = 128 - 131.5 - 128.5 + 131.5 = -.5$

$(\beta\gamma)_{12} = \mu_{.12} - \mu_{.1.} - \mu_{..2} + \mu_{...} = 132 - 129 - 134.5 + 131.5 = 0$

d. $(\alpha\beta\gamma)_{111} = \mu_{111} - \mu_{.11} - \mu_{1.1} - \mu_{11.} + \mu_{1..} + \mu_{.1.} + \mu_{..1} - \mu_{...}$

$= 130 - 126 - 134 - 135 + 138 + 129 + 128.5 - 131.5 = -1$

$(\alpha\beta\gamma)_{322} = \mu_{322} - \mu_{32.} - \mu_{3.2} - \mu_{.22} + \mu_{3..} + \mu_{.2.} + \mu_{..2} - \mu_{...}$

$= 131 - 128 - 126.5 - 137 + 125 + 134 + 134.5 - 131.5 = 1.5$

24.6. a. e_{ijkm}:

	$k=1$			$k=2$	
i	$j=1$	$j=2$	i	$j=1$	$j=2$
1	3.7667	1.1667	1	$-.5000$	-1.0333
	-3.9333	-1.6333		.4000	1.3667
	.1667	.4667		.1000	$-.3333$
2	-1.7000	$-.8333$	2	1.1333	$-.5667$
	1.1000	1.3667		-1.6667	1.7333
	.6000	$-.5333$		.5333	-1.1667

b. $r = .973$

24.7. a. $\bar{Y}_{111.} = 36.1333$, $\bar{Y}_{112.} = 56.5000$, $\bar{Y}_{121.} = 52.3333$, $\bar{Y}_{122.} = 71.9333$,

$\bar{Y}_{211.} = 46.9000$, $\bar{Y}_{212.} = 68.2667$, $\bar{Y}_{221.} = 64.1333$, $\bar{Y}_{222.} = 83.4667$

b.

Source	SS	df	MS
Between treatments	4,772.25835	7	681.75119
A (chemical)	788.90667	1	788.90667
B (temperature)	1,539.20167	1	1,539.20167
C (time)	2,440.16667	1	2,440.16667
AB interactions	.24000	1	.24000
AC interactions	.20167	1	.20167
BC interactions	2.94000	1	2.94000
ABC interactions	.60167	1	.60167
Error	53.74000	16	3.35875
Total	4,825.99835	23	

c. H_0: all $(\alpha\beta\gamma)_{ijk}$ equal zero, H_a: not all $(\alpha\beta\gamma)_{ijk}$ equal zero. $F^* = .60167/3.35875 = .18$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .68$

d. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = .24000/3.35875 = .07$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .79$

H_0: all $(\alpha\gamma)_{ik}$ equal zero, H_a: not all $(\alpha\gamma)_{ik}$ equal zero. $F^* = .20167/3.35875 = .06$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .81$

H_0: all $(\beta\gamma)_{jk}$ equal zero, H_a: not all $(\beta\gamma)_{jk}$ equal zero. $F^* = 2.94000/3.35875 = .875$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .36$

e. H_0: all α_i equal zero $(i = 1, 2)$, H_a: not all α_i equal zero. $F^* = 788.90667/3.35875 = 234.88$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

H_0: all β_j equal zero $(j = 1, 2)$, H_a: not all β_j equal zero. $F^* = 1,539.20167/3.35875 = 458.27$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

H_0: all γ_k equal zero $(k = 1, 2)$, H_a: not all γ_k equal zero. $F^* = 2,440.1667/3.35875 = 726.51$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

f. $\alpha \leq .1624$

24.8. a. $\hat{D}_1 = 65.69167 - 54.22500 = 11.46667$, $\hat{D}_2 = 67.96667 - 51.95000 = 16.01667$,

$\hat{D}_3 = 70.04167 - 49.87500 = 20.16667$, $MSE = 3.35875$,

$s\{\hat{D}_i\} = .7482$ $(i = 1, 2, 3)$, $B = t(.99167; 16) = 2.673$

$$11.46667 \pm 2.673(.7482) \qquad 9.467 \leq D_1 \leq 13.467$$
$$16.01667 \pm 2.673(.7482) \qquad 14.017 \leq D_2 \leq 18.017$$
$$20.16667 \pm 2.673(.7482) \qquad 18.167 \leq D_3 \leq 22.167$$

b. $\bar{Y}_{222.} = 83.46667$, $s\{\bar{Y}_{222.}\} = 1.0581$, $t(.975;\ 16) = 2.120$,

$83.46667 \pm 2.120(1.0581)$, $81.2235 \leq \mu_{222} \leq 85.7098$

24.9. a. e_{ijkm}:

$k=1$			$k=2$		
i	$j=1$	$j=2$	i	$j=1$	$j=2$
1	2.250	−1.825	1	1.450	−4.475
	−1.450	2.975		−1.050	3.325
	−1.350	−1.525		2.250	3.725
	.550	.375		−2.650	−2.575
2	−1.925	.950	2	2.625	2.100
	−2.325	−1.850		−1.875	.500
	4.075	−2.850		−2.075	.100
	.175	3.750		1.325	−2.700
3	−.850	4.375	3	−1.350	−2.450
	3.550	−2.525		.650	2.450
	−2.950	.975		3.550	−1.250
	.250	−2.825		−2.850	1.250

b. $r = .974$

24.10. a. $\bar{Y}_{111.} = 122.050$, $\bar{Y}_{112.} = 111.250$, $\bar{Y}_{121.} = 116.925$, $\bar{Y}_{122.} = 92.675$,

$\bar{Y}_{211.} = 121.225$, $\bar{Y}_{212.} = 110.975$, $\bar{Y}_{221.} = 116.250$, $\bar{Y}_{222.} = 90.600$,

$\bar{Y}_{311.} = 91.750$, $\bar{Y}_{312.} = 79.950$, $\bar{Y}_{321.} = 85.525$, $\bar{Y}_{322.} = 61.050$

b. $\bar{Y}_{1...} = 110.7250$, $\bar{Y}_{2...} = 109.7625$, $\bar{Y}_{3...} = 79.5688$

c.

Source	SS	df	MS
Between treatments	16,291.75564	11	1,481.06870
A (fee)	10,044.27125	2	5,022.13563
B (scope)	1,833.97688	1	1,833.97688
C (supervision)	3,832.40021	1	3,832.40021
AB interactions	1.60125	2	.80062
AC interactions	.78792	2	.39396
BC interactions	574.77521	1	574.77521
ABC interactions	3.94292	2	1.97146
Error	266.13750	36	7.39271
Total	16,557.89314	47	

d. H_0: all $(\alpha\beta\gamma)_{ijk}$ equal zero, H_a: not all $(\alpha\beta\gamma)_{ijk}$ equal zero. $F^* = 1.97146/7.39271 = .27$, $F(.99; 2, 36) = 5.25$. If $F^* \leq 5.25$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .77$

e. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = .80062/7.39271 = .11$, $F(.99; 2, 36) = 5.25$. If $F^* \leq 5.25$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .90$

H_0: all $(\alpha\gamma)_{ik}$ equal zero, H_a: not all $(\alpha\gamma)_{ik}$ equal zero. $F^* = .39396/7.39271 = .05$, $F(.99; 2, 36) = 5.25$. If $F^* \leq 5.25$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .95

H_0: all $(\beta\gamma)_{jk}$ equal zero, H_a: not all $(\beta\gamma)_{jk}$ equal zero. $F^* = 574.77521/7.39271 = 77.75$, $F(.99; 1, 36) = 7.40$. If $F^* \leq 7.40$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

f. H_0: all α_i equal zero ($i = 1, 2, 3$), H_a: not all α_i equal zero. $F^* = 5,022.13563/7.39271 = 679.34$, $F(.99; 2, 36) = 5.25$. If $F^* \leq 5.25$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

g. $\alpha \leq .049$

24.11. a. $\hat{D}_1 = .9625$, $\hat{D}_2 = 30.1937$, $\hat{D}_3 = 31.1562$, $\hat{D}_4 = 111.6750 - 100.7250 = 10.9500$

$\hat{D}_5 = 106.2333 - 81.4417 = 24.7916$, $\hat{L}_1 = -13.8416$,

$s\{\hat{D}_i\} = .9613$ ($i = 1, 2, 3$), $s\{\hat{D}_4\} = s\{\hat{D}_5\} = 1.1100$, $s\{\hat{L}_1\} = 1.5698$,

$B = t(.99167;\ 36) = 2.511$

$$\begin{array}{rr}
.9625 \pm 2.511(.9613) & -1.451 \leq D_1 \leq 3.376 \\
30.1937 \pm 2.511(.9613) & 27.780 \leq D_2 \leq 32.608 \\
31.1562 \pm 2.511(.9613) & 28.742 \leq D_3 \leq 33.570 \\
10.9500 \pm 2.511(1.1100) & 8.163 \leq D_4 \leq 13.737 \\
24.7916 \pm 2.511(1.1100) & 22.004 \leq D_5 \leq 27.579 \\
-13.8416 \pm 2.511(1.5698) & -17.783 \leq L_1 \leq -9.900
\end{array}$$

b. $\hat{D} = 116.925 - 116.250 = .675$, $s\{\hat{D}\} = 1.9226$, $t(.975; 36) = 2.028$,

$.675 \pm 2.028(1.9226)$, $-3.224 \leq D \leq 4.574$

c. $s\{\hat{D}\} = 1.9226$, $q(.90;\ 12,\ 36) = 4.52$, $T = 3.196$, $Ts\{\hat{D}\} = 6.14$,

$\bar{Y}_{111.} = 122.050$, $\bar{Y}_{211.} = 121.225$, $\bar{Y}_{121.} = 116.925$, $\bar{Y}_{221.} = 116.250$

24.12. a. e_{ijkm}:

$k=1$				$k=2$			
i	$j=1$	$j=2$	$j=3$	i	$j=1$	$j=2$	$j=3$
1	31.4	44.8	−1.2	1	−30.0	−3.4	−18.2
	−43.6	−23.2	−28.2		48.0	−12.4	15.8
	17.4	−33.2	−17.2		18.0	.6	5.8
	20.4	20.8	13.8		−55.0	−25.4	25.8
	−25.6	−9.2	32.8		19.0	40.6	−29.2

$k=1$				$k=2$			
i	$j=1$	$j=2$	$j=3$	i	$j=1$	$j=2$	$j=3$
2	29.6	27.6	.6	2	−6.6	−4.6	−19.4
	39.6	−34.4	−.4		−22.6	12.4	4.6
	−32.4	−26.4	14.6		10.4	25.4	−43.4
	−34.4	50.6	−20.4		21.4	−34.6	50.6
	−2.4	−17.4	5.6		−2.6	1.4	7.6

b. $r = .992$

24.13. a. $\bar{Y}_{111.} = 1,218.6$, $\bar{Y}_{112.} = 1,051.0$, $\bar{Y}_{121.} = 1,274.2$, $\bar{Y}_{122.} = 1,122.4$,
$\bar{Y}_{131.} = 1,218.2$, $\bar{Y}_{132.} = 1,051.2$, $\bar{Y}_{211.} = 1,036.4$, $\bar{Y}_{212.} = 870.6$,
$\bar{Y}_{221.} = 1,077.4$, $\bar{Y}_{222.} = 931.6$, $\bar{Y}_{231.} = 1,020.4$, $\bar{Y}_{232.} = 860.4$

b.

Source	SS	df	MS
Between treatments	973,645.933	11	88,513.267
A (gender)	540,360.600	1	540,360.600
B (sequence)	49,319.633	2	24,659.817
C (experience)	382,401.667	1	382,401.667
AB interactions	542.500	2	271.250
AC interactions	91.267	1	91.267
BC interactions	911.233	2	455.617
ABC interactions	19.033	2	9.517
Error	41,186.000	48	858.042
Total	1,014,831.933	59	

c. H_0: all $(\alpha\beta\gamma)_{ijk}$ equal zero, H_a: not all $(\alpha\beta\gamma)_{ijk}$ equal zero. $F^* = 9.517/858.042 = .01$, $F(.95; 2, 48) = 3.19$. If $F^* \leq 3.19$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .99

d. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = 271.250/858.042 = .32$, $F(.95; 2, 48) = 3.19$. If $F^* \leq 3.19$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .73

H_0: all $(\alpha\gamma)_{ik}$ equal zero, H_a: not all $(\alpha\gamma)_{ik}$ equal zero. $F^* = 91.267/858.042 = .11$, $F(.95; 1, 48) = 4.04$. If $F^* \leq 4.04$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .75

H_0: all $(\beta\gamma)_{jk}$ equal zero, H_a: not all $(\beta\gamma)_{jk}$ equal zero. $F^* = 455.617/858.042 = .53$, $F(.95; 2, 48) = 3.19$. If $F^* \leq 3.19$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .59

e. H_0: all α_i equal zero ($i = 1, 2$), H_a: not all α_i equal zero. $F^* = 540,360.600/858.042 = 629.76$, $F(.95; 1, 48) = 4.04$. If $F^* \leq 4.04$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: all β_j equal zero ($j = 1, 2, 3$), H_a: not all β_j equal zero. $F^* = 24,659.817/858.042 = 28.74$, $F(.95; 2, 48) = 3.19$. If $F^* \leq 3.19$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: all γ_k equal zero ($k = 1, 2$), H_a: not all γ_k equal zero. $F^* = 382,401.667/858.042 = 445.67$, $F(.95; 1, 48) = 4.04$. If $F^* \leq 4.04$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

f. $\alpha \leq .302$

24.14. a. $\bar{Y}_{1...} = 1,155.933$, $\bar{Y}_{2...} = 966.133$, $\bar{Y}_{.1..} = 1,044.150$, $\bar{Y}_{.2..} = 1,101.400$,
$\bar{Y}_{.3..} = 1,037.550$, $\bar{Y}_{..1.} = 1,140.867$, $\bar{Y}_{..2.} = 981.200$
$\hat{D}_1 = 189.800$, $\hat{D}_2 = -57.250$, $\hat{D}_3 = 6.600$, $\hat{D}_4 = 63.850$, $\hat{D}_5 = 159.667$,
$MSE = 858.042$, $s\{\hat{D}_1\} = 7.5633$, $s\{\hat{D}_i\} = 9.2631$ $(i = 2, 3, 4)$,

$s\{\hat{D}_5\} = 7.5633$, $B = t(.99;\, 48) = 2.406$

$$\begin{array}{rr}
189.800 \pm 2.406(7.5633) & 171.603 \leq D_1 \leq 207.997 \\
-57.250 \pm 2.406(9.2631) & -79.537 \leq D_2 \leq -34.963 \\
6.600 \pm 2.406(9.2631) & -15.687 \leq D_3 \leq 28.887 \\
63.850 \pm 2.406(9.2631) & 41.563 \leq D_4 \leq 86.137 \\
159.667 \pm 2.406(7.5633) & 141.470 \leq D_5 \leq 177.864
\end{array}$$

b. $\bar{Y}_{231.} = 1{,}020.4$, $s\{\bar{Y}_{231.}\} = 13.0999$, $t(.975;\, 48) = 2.011$,

$1{,}020.4 \pm 2.011(13.0999)$, $994.056 \leq \mu_{231} \leq 1{,}046.744$

24.15. a. $Y_{ijkm} = \mu_{...} + \alpha_1 X_{ijkm1} + \beta_1 X_{ijkm2} + \gamma_1 X_{ijkm3} + (\alpha\beta)_{11} X_{ijkm1} X_{ijkm2} + (\alpha\gamma)_{11} X_{ijkm1} X_{ijkm3}$
$+ (\beta\gamma)_{11} X_{ijkm2} X_{ijkm3} + (\alpha\beta\gamma)_{111} X_{ijkm1} X_{ijkm2} X_{ijkm3} + \epsilon_{ijkm}$

$$X_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 2 for factor } A \end{cases}$$

$$X_{ijk2} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 2 for factor } B \end{cases}$$

$$X_{ijk3} = \begin{cases} 1 & \text{if case from level 1 for factor } C \\ -1 & \text{if case from level 2 for factor } C \end{cases}$$

b. $Y_{ijkm} = \mu_{...} + \beta_1 X_{ijkm2} + \gamma_1 X_{ijkm3} + (\alpha\beta)_{11} X_{ijkm1} X_{ijkm2} + (\alpha\gamma)_{11} X_{ijkm1} X_{ijkm3}$
$+ (\beta\gamma)_{11} X_{ijkm2} X_{ijkm3} + (\alpha\beta\gamma)_{111} X_{ijkm1} X_{ijkm2} X_{ijkm3} + \epsilon_{ijkm}$

c. Full model:

$\hat{Y} = 60.01667 - 5.67500X_1 - 8.06667X_2 - 10.02500X_3 + .04167X_1X_2$
$+ .15000X_1X_3 - .40833X_2X_3 + .10000X_1X_2X_3$,

$SSE(F) = 49.4933$

Reduced model:

$\hat{Y} = 61.15167 - 9.20167X_2 - 8.89000X_3 - 1.09333X_1X_2 + 1.28500X_1X_3$
$-1.54333X_2X_3 - 1.03500X_1X_2X_3$,

$SSE(R) = 667.8413$

H_0: $\alpha_1 = 0$, H_a: $\alpha_1 \neq 0$.

$F^* = (618.348/1) \div (49.4933/14) = 174.91$, $F(.975; 1, 14) = 6.298$.

If $F^* \leq 6.298$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $\hat{D} = \hat{\mu}_{2..} - \hat{\mu}_{1..} = \hat{\alpha}_2 - \hat{\alpha}_1 = -2\hat{\alpha}_1 = 11.35000$, $s^2\{\hat{\alpha}_1\} = .18413$, $s\{\hat{D}\} = .8582$, $t(.975;\, 14) = 2.145$,

$11.35000 \pm 2.145(.8582)$, $9.509 \leq D \leq 13.191$

24.16. a. $Y_{ijkm} = \mu_{...} + \alpha_1 X_{ijkm1} + \beta_1 X_{ijkm2} + \beta_2 X_{ijkm3} + \gamma_1 X_{ijkm4}$
$+ (\alpha\beta)_{11} X_{ijkm1} X_{ijkm2} + (\alpha\beta)_{12} X_{ijkm1} X_{ijkm3} + (\alpha\gamma)_{11} X_{ijkm1} X_{ijkm4}$
$+ (\beta\gamma)_{11} X_{ijkm2} X_{ijkm4} + (\beta\gamma)_{21} X_{ijkm3} X_{ijkm4}$
$+ (\alpha\beta\gamma)_{111} X_{ijkm1} X_{ijkm2} X_{ijkm4} + (\alpha\beta\gamma)_{121} X_{ijkm1} X_{ijkm3} X_{ijkm4} + \epsilon_{ijkm}$

$$X_{ijkm1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 2 for factor } A \end{cases}$$

$$X_{ijkm2} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijkm3} = \begin{cases} 1 & \text{if case from level 2 for factor } B \\ -1 & \text{if case from level 3 for factor } B \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijkm4} = \begin{cases} 1 & \text{if case from level 1 for factor } C \\ -1 & \text{if case from level 2 for factor } C \end{cases}$$

b. $$\begin{aligned} Y_{ijkm} = \mu_{...} + \alpha_1 X_{ijkm1} + \beta_1 X_{ijkm2} + \beta_2 X_{ijkm3} + (\alpha\beta)_{11} X_{ijkm1} X_{ijkm2} \\ +(\alpha\beta)_{12} X_{ijkm1} X_{ijkm3} + (\alpha\gamma)_{11} X_{ijkm1} X_{ijkm4} + (\beta\gamma)_{11} X_{ijkm2} X_{ijkm4} \\ +(\beta\gamma)_{21} X_{ijkm3} X_{ijkm4} + (\alpha\beta\gamma)_{111} X_{ijkm1} X_{ijkm2} X_{ijkm4} \\ +(\alpha\beta\gamma)_{121} X_{ijkm1} X_{ijkm3} X_{ijkm4} + \epsilon_{ijkm} \end{aligned}$$

c. Full model:

$$\begin{aligned} \hat{Y} = 1,062.16667 + 94.82500 X_1 - 17.85417 X_2 + 42.47083 X_3 + 79.80000 X_4 \\ -4.33750 X_1 X_2 + 2.01250 X_1 X_3 + .20833 X_1 X_4 + 3.38750 X_2 X_4 \\ -5.33750 X_3 X_4 + .40417 X_1 X_2 X_4 - 1.94583 X_1 X_3 X_4, \end{aligned}$$

$SSE(F) = 39,499.9000$

Reduced model:

$$\begin{aligned} \hat{Y} = 1,063.73137 + 96.38971 X_1 - 14.72475 X_2 + 40.90613 X_3 - 10.59632 X_1 X_2 \\ +9.83603 X_1 X_3 + 1.77304 X_1 X_4 + 3.38750 X_2 X_4 - 10.03162 X_3 X_4 \\ +3.53358 X_1 X_2 X_4 - 3.51054 X_1 X_3 X_4, \end{aligned}$$

$SSE(R) = 399,106.8647$

H_0: $\gamma_1 = 0$, H_a: $\gamma_1 \neq 0$.

$F^* = (359,606.9647/1) \div (39,499.9000/45) = 409.68$, $F(.95; 1, 45) = 4.06$.

If $F^* \leq 4.06$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

d. $\hat{D} = \hat{\mu}_{..1} - \hat{\mu}_{..2} = \hat{\gamma}_1 - \hat{\gamma}_2 = 2\hat{\gamma}_1 = 159.60000$, $s^2\{\hat{\gamma}_1\} = 15.54394$, $s\{\hat{D}\} = 7.8852$, $t(.975; 45) = 2.014$,

$159.60000 \pm 2.014(7.8852)$, $143.719 \leq D \leq 175.481$

24.17. $\dfrac{2\sqrt{n}}{1.8} = 4.1475$, $n = 14$

24.18. $t[.99; 12(n-1)]\sqrt{(29)^2/2n} = \pm 20$, $n = 6$

24.19. $$\begin{aligned} \sum_i (\alpha\beta\gamma)_{ijk} = \sum_i (\mu_{ijk} - \mu_{ij.} - \mu_{i.k} - \mu_{.jk} + \mu_{i..} + \mu_{.j.} + \mu_{..k} - \mu_{...}) \\ = a\mu_{.jk} - a\mu_{.j.} - a\mu_{..k} - a\mu_{.jk} + a\mu_{...} + a\mu_{.j.} + a\mu_{..k} - a\mu_{...} = 0 \end{aligned}$$

24.20. $Y_{ijk} = \mu_{...} + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + \epsilon_{ijk}$

Source	SS	df	MS
A	SSA	$a-1$	MSA
B	SSB	$b-1$	MSB
C	SSC	$c-1$	MSC
AB	$SSAB$	$(a-1)(b-1)$	$MSAB$
AC	$SSAC$	$(a-1)(c-1)$	$MSAC$
BC	$SSBC$	$(b-1)(c-1)$	$MSBC$
Error	SSE	$(a-1)(b-1)(c-1)$	MSE
Total	$SSTO$	$abc-1$	

24.21. $\sigma^2\{\hat{L}\} = \sigma^2\{\sum\sum c_{ij}\bar{Y}_{ij..}\} = \sum\sum c_{ij}^2\sigma^2\{\bar{Y}_{ij..}\}$ (because of independence)

$$= \sum\sum c_{ij}^2 \frac{\sigma^2}{cn} = \frac{\sigma^2}{cn}\sum\sum c_{ij}^2$$

24.22. c. $r = .992$

24.23. a. $\bar{Y}_{111.} = 8.80000$, $\bar{Y}_{112.} = 9.68667$, $\bar{Y}_{113.} = 8.33000$, $\bar{Y}_{114.} = 7.50333$,
$\bar{Y}_{121.} = 10.07667$, $\bar{Y}_{122.} = 9.56333$, $\bar{Y}_{123.} = 10.02667$, $\bar{Y}_{124.} = 8.16000$,
$\bar{Y}_{211.} = 10.55333$, $\bar{Y}_{212.} = 8.79000$, $\bar{Y}_{213.} = 8.77333$, $\bar{Y}_{214.} = 8.00667$,
$\bar{Y}_{221.} = 12.48000$, $\bar{Y}_{222.} = 10.01667$, $\bar{Y}_{223.} = 10.20000$, $\bar{Y}_{224.} = 8.33000$

b.

Source	SS	df	MS
Between treatments	69.63346	15	4.64223
A (age)	4.69375	1	4.69375
B (facilities)	13.26152	1	13.26152
C (region)	37.43491	3	12.47830
AB interactions	.36575	1	.36575
AC interactions	9.03731	3	3.01244
BC interactions	3.38421	3	1.12807
ABC interactions	1.45601	3	.48534
Error	34.18440	32	1.06826
Total	103.81786	47	

c. H_0: all $(\alpha\beta\gamma)_{ijk}$ equal zero, H_a: not all $(\alpha\beta\gamma)_{ijk}$ equal zero. $F^* = .48534/1.06826 = .45$, $F(.99; 3, 32) = 4.46$. If $F^* \leq 4.46$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .72$

d. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = .36575/1.06826 = .34$, $F(.99; 1, 32) = 7.50$. If $F^* \leq 7.50$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .56$

H_0: all $(\alpha\gamma)_{ik}$ equal zero, H_a: not all $(\alpha\gamma)_{ik}$ equal zero. $F^* = 3.01244/1.06826 = 2.82$, $F(.99; 3, 32) = 4.46$. If $F^* \leq 4.46$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .055$

H_0: all $(\beta\gamma)_{jk}$ equal zero, H_a: not all $(\beta\gamma)_{jk}$ equal zero. $F^* = 1.12807/1.06826 = 1.06$, $F(.99; 3, 32) = 4.46$. If $F^* \leq 4.46$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .38$

e. H_0: all α_i equal zero ($i = 1, 2$), H_a: not all α_i equal zero. $F^* = 4.69375/1.06826 = 4.39$, $F(.99; 1, 32) = 7.50$. If $F^* \leq 7.50$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .044$

H_0: all β_j equal zero ($j = 1, 2$), H_a: not all β_j equal zero. $F^* = 13.26152/1.06826 = 12.41$, $F(.99; 1, 32) = 7.50$. If $F^* \leq 7.50$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= .0013$

H_0: all γ_k equal zero ($k = 1, ..., 4$), H_a: not all γ_k equal zero. $F^* = 12.47830/1.06826 = 11.68$, $F(.99; 3, 32) = 4.46$. If $F^* \leq 4.46$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

f. $\bar{Y}_{1...} = 9.01833$, $\bar{Y}_{2...} = 9.64375$, $\bar{Y}_{.1..} = 8.80542$, $\bar{Y}_{.2..} = 9.85667$,

$\bar{Y}_{..1.} = 10.47750$, $\bar{Y}_{..2.} = 9.51417$, $\bar{Y}_{..3.} = 9.33250$, $\bar{Y}_{..4.} = 8.00000$

$\hat{D}_1 = \bar{Y}_{.1..} - \bar{Y}_{.2..} = -1.05125$, $\hat{D}_2 = \bar{Y}_{..1.} - \bar{Y}_{..2.} = .96333$, $\hat{D}_3 = \bar{Y}_{..1.} - \bar{Y}_{..3.} = 1.14500$,

$\hat{D}_4 = \bar{Y}_{..1.} - \bar{Y}_{..4.} = 2.47750$, $\hat{D}_5 = \bar{Y}_{..2.} - \bar{Y}_{..3.} = .18167$, $\hat{D}_6 = \bar{Y}_{..2.} - \bar{Y}_{..4.} = 1.51417$,

$\hat{D}_7 = \bar{Y}_{..3.} - \bar{Y}_{..4.} = 1.33250$, $MSE = 1.06826$,

$s\{\hat{D}_1\} = .29836$, $s\{\hat{D}_i\} = .42195$ ($i = 2, ..., 7$), $B = t(.99286; 32) = 2.5915$

$$\begin{array}{rr}
-1.05125 \pm 2.5915(.29836) & -1.824 \leq D_1 \leq -.278 \\
.96333 \pm 2.5915(.42195) & -.130 \leq D_2 \leq 2.057 \\
1.14500 \pm 2.5915(.42195) & .052 \leq D_3 \leq 2.238 \\
2.47750 \pm 2.5915(.42195) & 1.384 \leq D_4 \leq 3.571 \\
.18167 \pm 2.5915(.42195) & -.912 \leq D_5 \leq 1.275 \\
1.51417 \pm 2.5915(.42195) & .421 \leq D_6 \leq 2.608 \\
1.33250 \pm 2.5915(.42195) & .239 \leq D_7 \leq 2.426
\end{array}$$

24.24. c. $r = .920$

24.25. a. $\bar{Y}_{111.} = .03303$, $\bar{Y}_{112.} = .03886$, $\bar{Y}_{121.} = .03553$, $\bar{Y}_{122.} = .05415$,

$\bar{Y}_{211.} = .04076$, $\bar{Y}_{212.} = .05128$, $\bar{Y}_{221.} = .05516$, $\bar{Y}_{222.} = .06056$,

$\bar{Y}_{311.} = .05841$, $\bar{Y}_{312.} = .05997$, $\bar{Y}_{321.} = .07738$, $\bar{Y}_{322.} = .07915$,

$\bar{Y}_{411.} = .05655$, $\bar{Y}_{412.} = .04688$, $\bar{Y}_{421.} = .06755$, $\bar{Y}_{422.} = .06442$

b. $$\begin{aligned}
Y_{ijkm} = \mu_{...} &+ \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \beta_1 X_{ijk4} + \gamma_1 X_{ijk5} \\
&+ (\alpha\beta)_{11} X_{ijk1} X_{ijk4} + (\alpha\beta)_{21} X_{ijk2} X_{ijk4} + (\alpha\beta)_{31} X_{ijk3} X_{ijk4} \\
&+ (\alpha\gamma)_{11} X_{ijk1} X_{ijk5} + (\alpha\gamma)_{21} X_{ijk2} X_{ijk5} + (\alpha\gamma)_{31} X_{ijk3} X_{ijk5} \\
&+ (\beta\gamma)_{11} X_{ijk4} X_{ijk5} + (\alpha\beta\gamma)_{111} X_{ijk1} X_{ijk4} X_{ijk5} \\
&+ (\alpha\beta\gamma)_{211} X_{ijk2} X_{ijk4} X_{ijk5} + (\alpha\beta\gamma)_{311} X_{ijk3} X_{ijk4} X_{ijk5} + \epsilon_{ijkm}
\end{aligned}$$

$$X_{ijk1} = \begin{cases} 1 & \text{if case from NE} \\ -1 & \text{if case from W} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk2} = \begin{cases} 1 & \text{if case from NC} \\ -1 & \text{if case from W} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk3} = \begin{cases} 1 & \text{if case from S} \\ -1 & \text{if case from W} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk4} = \begin{cases} 1 & \text{if poverty level below 8 percent} \\ -1 & \text{if poverty level 8 percent or higher} \end{cases}$$

$$X_{ijk5} = \begin{cases} 1 & \text{if percent of population 65 or older} < 12.0\% \\ -1 & \text{if percent of population 65 or older} \geq 12.0\% \end{cases}$$

$$\begin{aligned}\hat{Y} = {} & .05498 - .0146X_1 - .00303X_2 + .0137X_3 - .00676X_4 \\ & -.00193X_5 + .00231X_1X_4 + .00084X_2X_4 - .00278X_3X_4 - .00418X_1X_5 \\ & -.00205X_2X_5 + .00110X_3X_5 + .00090X_4X_5 + .00230X_1X_4X_5 \\ & -.00218X_2X_4X_5 - .00085X_3X_4X_5,\end{aligned}$$

$SSE(F) = .23779$

c. $\underline{ABC}$ interactions;

$$\begin{aligned}\hat{Y} = {} & .0552 - .0133X_1 - .00412X_2 + .0135X_3 - .00712X_4 \\ & -.00157X_5 + .00085X_1X_4 + .00128X_2X_4 - .00254X_3X_4 \\ & -.00278X_1X_5 - .00251X_2X_5 + .00083X_3X_5 + .00046X_4X_5,\end{aligned}$$

$SSE(R) = .23849$

H_0: all $(\alpha\beta\gamma)_{ijk}$ equal zero, H_a: not all $(\alpha\beta\gamma)_{ijk}$ equal zero.

$F^* = (.0007/3) \div (.23779/424) = .42$, $F(.975; 3, 424) = 3.147$.

If $F^* \leq 3.147$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .74

$\underline{AB}$ interactions:

$$\begin{aligned}\hat{Y} = {} & 0.0556 - 0.0133X_1 - 0.00355X_2 + 0.0136X_3 - 0.00743X_4 \\ & -0.00135X_5 - 0.00315X_1X_5 - 0.00190X_2X_5 + 0.00040X_3X_5 + 0.00026X_4X_5 \\ & +0.00095X_1X_4X_5 - 0.00161X_2X_4X_5 - 0.00081X_3X_4X_5,\end{aligned}$$

$SSE(R) = .23897$

H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero.

$F^* = (.00118/3) \div (.23779/424) = .70$, $F(.975; 3, 424) = 3.147$.

If $F^* \leq 3.147$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .55

$\underline{AC}$ interactions:

$$\begin{aligned}\hat{Y} = {} & 0.0562 - 0.0129X_1 - 0.00419X_2 + 0.0127X_3 - 0.00727X_4 \\ & -0.00161X_5 + 0.00038X_1X_4 + 0.00021X_2X_4 - 0.00222X_3X_4 \\ & -0.00023X_4X_5 + 0.00052X_1X_4X_5 - 0.00119X_2X_4X_5 + 0.00011X_3X_4X_5,\end{aligned}$$

$SSE(R) = .24070$

H_0: all $(\alpha\gamma)_{ik}$ equal zero, H_a: not all $(\alpha\gamma)_{ik}$ equal zero.

$F^* = (.00291/3) \div (.23779/424) = 1.73$, $F(.975; 3, 424) = 3.147$.

If $F^* \leq 3.147$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .16

$\underline{BC \text{ interactions}}$:

$\hat{Y} = 0.0553 - 0.0142X_1 - 0.00303X_2 + 0.0134X_3 - 0.00687X_4 - 0.00179X_5$
$+0.00152X_1X_4 + 0.00092X_2X_4 - 0.00253X_3X_4 - 0.00344X_1X_5 - 0.00214X_2X_5$
$+0.00085X_3X_5 + 0.00183X_1X_4X_5 - 0.00204X_2X_4X_5 - 0.00042X_3X_4X_5,$

$SSE(R) = .23801$

H_0: all $(\beta\gamma)_{jk}$ equal zero, H_a: not all $(\beta\gamma)_{jk}$ equal zero.

$F^* = (.00022/1) \div (.23779/424) = .39$, $F(.975; 1, 424) = 5.06$.

If $F^* \leq 5.06$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .53

d. $\underline{A \text{ effects}}$:

$\hat{Y} = 0.0585 - 0.0104X_4 + 0.00191X_5 - 0.00575X_1X_4 + 0.00409X_2X_4$
$-0.00150X_3X_4 + 0.00401X_1X_5 - 0.00558X_2X_5 + 0.00016X_3X_5$
$-0.00218X_4X_5 - 0.00445X_1X_4X_5 - 0.00227X_2X_4X_5 + 0.00280X_3X_4X_5,$

$SSE(R) = .27011$

H_0: all α_i equal zero $(i = 1, ..., 4)$, H_a: not all α_i equal zero.

$F^* = (.03232/3) \div (.23779/424) = 19.21$, $F(.975; 3, 424) = 3.147$.

If $F^* \leq 3.147$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

$\underline{B \text{ effects}}$:

$\hat{Y} = 0.0539 - 0.0202X_1 - 0.00235X_2 + 0.0156X_3 - 0.00465X_5$
$+0.00583X_1X_4 - 0.00024X_2X_4 - 0.00598X_3X_4 - 0.00705X_1X_5$
$-0.00208X_2X_5 + 0.00362X_3X_5 + 0.00174X_4X_5 + 0.00828X_1X_4X_5$
$-0.00275X_2X_4X_5 - 0.00270X_3X_4X_5,$

$SSE(R) = .25047$

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = (.01268/1) \div (.23779/424) = 22.61$, $F(.975; 1, 424) = 5.06$.

If $F^* \leq 5.06$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

$\underline{C \text{ effects}}$:

$\hat{Y} = 0.0552 - 0.0129X_1 - 0.00320X_2 + 0.0132X_3 - 0.00754X_4$
$+0.00150X_1X_4 + 0.00083X_2X_4 - 0.00206X_3X_4 - 0.00318X_1X_5$
$-0.00236X_2X_5 + 0.00018X_3X_5 + 0.00061X_4X_5 + 0.00069X_1X_4X_5$
$-0.00198X_2X_4X_5 - 0.00032X_3X_4X_5,$

$SSE(R) = .23882$

H_0: $\gamma_1 = \gamma_2 = 0$, H_a: not both γ_1 and γ_2 equal zero.

$F^* = (.00103/1) \div (.23779/424) = 1.84$, $F(.975; 1, 424) = 5.06$.

If $F^* \leq 5.06$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .175

e. $\hat{D}_1 = \hat{\mu}_{1..} - \hat{\mu}_{2..} = \hat{\alpha}_1 - \hat{\alpha}_2 = -.01155$, $\hat{D}_2 = \hat{\mu}_{1..} - \hat{\mu}_{3..} = \hat{\alpha}_1 - \hat{\alpha}_3 = -.02834$,

$\hat{D}_3 = \hat{\mu}_{1..} - \hat{\mu}_{4..} = -.01846$, $\hat{D}_4 = \hat{\mu}_{2..} - \hat{\mu}_{3..} = -.016784$,

$\hat{D}_5 = \hat{\mu}_{2..} - \hat{\mu}_{4..} = -.006907$, $\hat{D}_6 = \hat{\mu}_{3..} - \hat{\mu}_{4..} = .009877$,

$$\begin{aligned}
-.02276 \le D_1 \le & \quad -.00034 \\
-.03878 \le D_2 \le & \quad -.01790 \\
-.03030 \le D_3 \le & \quad -.00662 \\
-.02531 \le D_4 \le & \quad -.00826 \\
-.01711 \le D_5 \le & \quad .00329 \\
.00053 \le D_6 \le & \quad .01922
\end{aligned}$$

Chapter 25

RANDOM AND MIXED EFFECTS MODELS

25.3. (1) I, (2) II, (3) I, (4) II

25.5. b. H_0: $\sigma_\mu^2 = 0$, H_a: $\sigma_\mu^2 > 0$. $F^* = .45787/.03097 = 14.78$, $F(.95; 5, 114) = 2.29$.
If $F^* \leq 2.29$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $\bar{Y}_{..} = .22767$, $n_T = 120$, $s\{\bar{Y}_{..}\} = .06177$, $t(.975; 5) = 2.571$,
$.22767 \pm 2.571(.06177)$, $.0689 \leq \mu_. \leq .3865$

25.6. a. $F(.025; 5, 114) = .1646$, $F(.975; 5, 114) = 2.680$, $L = .22583$, $U = 4.44098$

$$.1842 \leq \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma^2} \leq .8162$$

b. $\chi^2(.025; 114) = 90.351$, $\chi^2(.975; 114) = 145.441$, $.02427 \leq \sigma^2 \leq .03908$

c. $s_\mu^2 = .02135$

d. Satterthwaite:

$$df = (ns_\mu^2)^2 \div [(MSTR)^2/(r-1) + (MSE)^2/r(n-1)]$$
$$= [20(.02135)]^2 \div [(.45787)^2/5 + (.03907)^2/6(19)] = 4.35,$$

$\chi^2(.025; 4) = .484$, $\chi^2(.975; 4) = 11.143$

$$.0083 = \frac{4.35(.02135)}{11.143} \leq \sigma_\mu^2 \leq \frac{4.35(.02135)}{.484} = .192$$

MLS: $c_1 = .05$, $c_2 = -.05$, $MS_1 = .45787$, $MS_2 = .03907$, $df_1 = 5$, $df_2 = 114$, $F_1 = F(.975; 5, \infty) = 2.57$, $F_2 = F(.975; 114, \infty) = 1.28$, $F_3 = F(.975; \infty, 5) = 6.02$, $F_4 = F(.975; \infty, 114) = 1.32$, $F_5 = F(.975; 5, 114) = 2.68$, $F_6 = F(.975; 114, 5) = 6.07$, $G_1 = .6109$, $G_2 = .2188$, $G_3 = .0147$, $G_4 = -.2076$, $H_L = .014$, $H_U = .115$, $.02135 - .014$, $.02135 + .115$, $.0074 \leq \sigma_\mu^2 \leq .1364$

25.7. a.

Source	SS	df	MS
Between brands	854.52917	5	170.90583
Error	30.07000	42	.71595
Total	884.59917	47	

H_0: $\sigma_\mu^2 = 0$, H_a: $\sigma_\mu^2 > 0$. $F^* = 170.90583/.71595 = 238.71$, $F(.99; 5, 42) = 3.49$.

If $F^* \leq 3.49$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

b. $\bar{Y}_{..} = 17.62917$, $n_T = 48$, $s\{\bar{Y}_{..}\} = 1.8869$, $t(.995; 5) = 4.032$,

$17.62917 \pm 4.032(1.8869)$, $10.021 \leq \mu_. \leq 25.237$

25.8. a. $F(.005; 5, 42) = .0795$, $F(.995; 5, 42) = 3.95$, $L = 7.4292$, $U = 375.20828$

$$.8814 \leq \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma^2} \leq .9973$$

b. $MSE = .71595$, $s_\mu^2 = 21.27374$

c. $\chi^2(.005;\ 42) = 22.138$, $\chi^2(.995;\ 42) = 69.336$, $.4337 \leq \sigma^2 \leq 1.3583$

d. H_0: $\sigma_\mu^2 \leq 2\sigma^2$, H_a: $\sigma_\mu^2 > 2\sigma^2$. $F^* = [MSTR/(2n+1)] \div MSE = 14.042$, $F(.99; 5, 42) = 3.49$. If $F^* \leq 3.49$ conclude H_0, otherwise H_a. Conclude H_a.

e. $c_1 = .125$, $c_2 = -.125$, $df_1 = 5$, $df_2 = 42$, $F_1 = F(.995; 5, \infty) = 3.35$, $F_2 = F(.995; 42, \infty) = 1.66$, $F_3 = F(.995; \infty, 5) = 12.1$, $F_4 = F(.995; \infty, 42) = 1.91$, $F_5 = F(.995; 5, 42) = 3.95$, $F_6 = F(.995; 42, 5) = 12.51$, $G_1 = .7015$, $G_2 = .3976$, $G_3 = .0497$, $G_4 = -1.2371$, $H_L = 14.990$, $H_U = 237.127$, $21.2737 - 14.990$, $21.2737 + 237.127$, $6.284 \leq \sigma_\mu^2 \leq 258.401$

25.9. a.

Source	SS	df	MS
Between machines	602.5000	3	200.8333
Error	257.4000	36	7.1500
Total	859.9000	39	

H_0: $\sigma_\mu^2 = 0$, H_a: $\sigma_\mu^2 > 0$. $F^* = 200.8333/7.1500 = 28.09$, $F(.90; 3, 36) = 2.25$.

If $F^* \leq 2.25$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

b. $\bar{Y}_{..} = 205.05$, $n_T = 40$, $s\{\bar{Y}_{..}\} = 2.2407$, $t(.95; 3) = 2.353$,

$205.05 \pm 2.353(2.2407)$, $199.778 \leq \mu_. \leq 210.322$

25.10. a. $F(.05; 3, 36) = .117$, $F(.95; 3, 36) = 2.87$, $L = .8787$, $U = 23.9073$

$$.4677 \leq \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma^2} \leq .9599$$

b. H_0: $\sigma_\mu^2 = \sigma^2$, H_a: $\sigma_\mu^2 \neq \sigma^2$.

$F^* = [MSTR/(n+1)] \div MSE = 2.554$, $F(.05; 3, 36) = .117$, $F(.95; 3, 36) = 2.87$.

If $.117 \leq F^* \leq 2.87$ conclude H_0, otherwise H_a. Conclude H_0.

c. $\chi^2(.05; 36) = 23.269$, $\chi^2(.95; 36) = 50.998$, $5.047 \leq \sigma^2 \leq 11.062$

d. $s_\mu^2 = 19.3683$

e. Satterthwaite:

$df = [10(19.3683)]^2 \div [(200.8333)^2/3 + (7.1500)^2/36] = 2.79$,

$\chi^2(.05; 3) = .352$, $\chi^2(.95; 3) = 7.815$,

$$6.915 = \frac{2.79(19.3683)}{7.815} \leq \sigma_\mu^2 \leq \frac{2.79(19.3683)}{.352} = 153.516$$

MLS: $c_1 = .10$, $c_2 = -.10$, $df_1 = 3$, $df_2 = 36$, $F_1 = F(.95; 3, \infty) = 2.60$, $F_2 = F(.95; 36, \infty) = 1.42$, $F_3 = F(.95; \infty, 3) = 8.53$, $F_4 = F(.95; \infty, 36) = 1.55$, $F_5 = F(.95; 3, 36) = 2.87$, $F_6 = F(.95; 36, 3) = 8.60$, $G_1 = .6154$, $G_2 = .2958$, $G_3 = .0261$, $G_4 = -.6286$, $H_L = 12.381$, $H_U = 151.198$, $19.3683 - 12.381$, $19.3683 + 151.198$, $6.987 \leq \sigma_\mu^2 \leq 170.566$

25.13. a. $E\{MSA\} = 115$, $E\{MSB\} = 185$, $E\{MSAB\} = 35$

b. $E\{MSA\} = 85$, $E\{MSB\} = 155$, $E\{MSAB\} = 5$

25.15. a.

Source	SS	df	MS
Factor A (driver)	280.28475	3	93.42825
Factor B (car)	94.71350	4	23.67838
AB interactions	2.44650	12	.20388
Error	3.51500	20	.17575
Total	380.95975	39	

H_0: $\sigma_{\alpha\beta}^2 = 0$, H_a: $\sigma_{\alpha\beta}^2 > 0$. $F^* = .20388/.17575 = 1.16$, $F(.95; 12, 20) = 2.28$.

If $F^* \leq 2.28$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .37

b. H_0: $\sigma_\alpha^2 = 0$, H_a: $\sigma_\alpha^2 > 0$. $F^* = 93.42825/.20388 = 458.25$, $F(.95; 3, 12) = 3.49$.

If $F^* \leq 3.49$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: $\sigma_\beta^2 = 0$, H_a: $\sigma_\beta^2 > 0$. $F^* = 23.67838/.20388 = 116.14$, $F(.95; 4, 12) = 3.26$.

If $F^* \leq 3.26$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $s_\alpha^2 = 9.3224$, $s_\beta^2 = 2.9343$

d. $c_1 = .10$, $c_2 = -.10$, $MS_1 = 93.42825$, $MS_2 = .203875$, $df_1 = 3$, $df_2 = 12$, $F_1 = F(.975; 3, \infty) = 3.12$, $F_2 = F(.975; 12, \infty) = 1.94$, $F_3 = F(.975; \infty, 3) = 13.9$, $F_4 = F(.975; \infty, 12) = 2.72$, $F_5 = F(.975; 3, 12) = 4.47$, $F_6 = F(.975; 12, 3) = 14.3$, $G_1 = .6795$, $G_2 = .4845$, $G_3 = -.0320$, $G_4 = -2.6241$, $H_L = 6.348$, $H_U = 120.525$, $9.3244 - 6.348$, $9.3224 + 120.525$, $2.974 \leq \sigma_\alpha^2 \leq 129.847$

e. $df = [8(2.9343)]^2 \div [(23.678375)^2/4 + (.203875)^2/12] = 3.93$

$\chi^2(.025; 4) = .484$, $\chi^2(.975; 4) = 11.143$,

$$1.03 = \frac{3.93(2.9343)}{11.143} \leq \sigma_\beta^2 \leq \frac{3.93(2.9343)}{.484} = 23.83$$

25.16. a. H_0: $\sigma_{\alpha\beta}^2 = 0$, H_a: $\sigma_{\alpha\beta}^2 > 0$. $F^* = 303.822/52.011 = 5.84$, $F(.99; 4, 36) = 3.89$.

If $F^* \leq 3.89$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .001

b. $s_{\alpha\beta}^2 = 50.362$

c. H_0: $\sigma_\alpha^2 = 0$, H_a: $\sigma_\alpha^2 > 0$. $F^* = 12.289/52.011 = .24$, $F(.99; 2, 36) = 5.25$.

If $F^* \leq 5.25$ conclude H_0, otherwise H_a. Conclude H_0.

d. H_0: all β_j equal zero ($j = 1, 2, 3$), H_a: not all β_j equal zero.

$F^* = 14.156/303.822 = .047,\ F(.99; 2, 4) = 18.0.$

If $F^* \leq 18.0$ conclude H_0, otherwise H_a. Conclude H_0.

e. $\bar{Y}_{.1.} = 56.133,\ \bar{Y}_{.2.} = 56.600,\ \bar{Y}_{.3.} = 54.733,\ \hat{D}_1 = \bar{Y}_{.1.} - \bar{Y}_{.2.} = -.467,\ \hat{D}_2 = \bar{Y}_{.1.} - \bar{Y}_{.3.} = -1.400,\ \hat{D}_3 = \bar{Y}_{.2.} - \bar{Y}_{.3.} = 1.867,\ s\{\hat{D}_i\} = 6.3647\ (i = 1, 2, 3),$ $q(.95; 3, 4) = 5.04,\ T = 3.5638$

$$\begin{array}{rl} -.467 \pm 3.5638(6.3647) & -23.150 \leq D_1 \leq 22.216 \\ -1.400 \pm 3.5638(6.3647) & -24.083 \leq D_2 \leq 21.283 \\ 1.867 \pm 3.5638(6.3647) & -20.816 \leq D_3 \leq 24.550 \end{array}$$

f. $\hat{\mu}_{.1} = 56.1333,\ MSA = 12.28889,\ MSAB = 303.82222,\ s^2\{\hat{\mu}_{.1}\} = (2/45)(303.82222)+ (1/45)(12.28889) = 13.7763,\ s\{\hat{u}_{.1}\} = 3.712,\ df = (13.7763)^2 \div \{[(2/45)(303.82222)]^2/4+ [(1/45)(12.28889)]^2/2\} = 4.16,\ t(.995; 4) = 4.60,$

$56.1333 \pm 4.60(3.712),\ 39.06 \leq \mu_{.1} \leq 73.21$

g. $MSA = 12.28889,\ MSE = 52.01111,\ s^2_\alpha = (MSA - MSE)/nb = -2.648,$ $c_1 = 1/15,\ c_2 = -1/15,\ df_1 = 2,\ df_2 = 36,\ F_1 = F(.995; 2, \infty) = 5.30,\ F_2 = F(.995; 36, \infty) = 1.71,\ F_3 = F(.995; \infty, 2) = 200,\ F_4 = F(.995; \infty, 36) = 2.01,$ $F_5 = F(.995; 2, 36) = 6.16,\ F_6 = F(.995; 36, 2) = 199.5,\ G_1 = .8113,\ G_2 = .4152,$ $G_3 = .1022,\ G_4 = -35.3895,\ H_L = 3.605,\ H_U = 162.730, -2.648 - 3.605,$ $-2.648 + 162.730,\ -6.253 \leq \sigma^2_\alpha \leq 160.082$

25.17. a.

Source	SS	df	MS
Factor A (coats)	150.3879	2	75.1940
Factor B (batch)	152.8517	3	50.9506
AB interactions	1.8521	6	.3087
Error	173.6250	36	4.8229
Total	478.7167	47	

H_0: $\sigma^2_{\alpha\beta} = 0$, H_a: $\sigma^2_{\alpha\beta} > 0$. $F^* = .3087/4.8229 = .06,\ F(.95; 6, 36) = 2.36.$

If $F^* \leq 2.36$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .999

b. H_0: all α_i equal zero $(i = 1, 2, 3)$, H_a: not all α_i equal zero. $F^* = 75.1940/.3087 = 243.58,\ F(.95; 2, 6) = 5.14$. If $F^* \leq 5.14$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: $\sigma^2_\beta = 0$, H_a: $\sigma^2_\beta > 0$. $F^* = 50.9506/4.8229 = 10.56,\ F(.95; 3, 36) = 2.87$. If $F^* \leq 2.87$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $\bar{Y}_{1..} = 73.10625,\ \bar{Y}_{2..} = 76.79375,\ \bar{Y}_{3..} = 76.92500,\ \hat{D}_1 = \bar{Y}_{2..} - \bar{Y}_{1..} = 3.68750,$ $\hat{D}_2 = \bar{Y}_{3..} - \bar{Y}_{2..} = .13125,\ s\{\hat{D}_i\} = .1964\ (i = 1, 2),\ B = t(.975; 6) = 2.447$

$$\begin{array}{rl} 3.68750 \pm 2.447(.1964) & 3.2069 \leq D_1 \leq 4.1681 \\ .13125 \pm 2.447(.1964) & -.3493 \leq D_2 \leq .6118 \end{array}$$

d. $\hat{\mu}_{2.} = 76.79375,\ s^2\{\hat{\mu}_{2.}\} = (2/48)(.30868) + (1/48)(50.95056) = 1.0743,\ s\{\hat{\mu}_{2.}\} = 1.0365,\ df = (1.0743)^2 \div \{[(2/48)(.30868)]^2/6 + [(1/48)(50.95056)]^2/3\} = 3.07,$ $t(.975; 3) = 3.182,\ 76.79375 \pm 3.182(1.0365),\ 73.496 \leq \mu_{2.} \leq 80.092$

e. $s^2_\beta = (MSB - MSE)/na = 3.844,\ c_1 = 1/12,\ c_2 = -1/12,\ df_1 = 3,\ df_2 = 36,$ $F_1 = F(.95; 3, \infty) = 2.60,\ F_2 = F(.95; 36, \infty) = 1.42,\ F_3 = F(.95; \infty, 3) = 8.53,$

$F_4 = F(.95; \infty, 36) = 1.55$, $F_5 = F(.95; 3, 36) = 2.87$, $F_6 = F(.95; 36, 3) = 8.60$, $G_1 = .6154$, $G_2 = .2958$, $G_3 = .0261$, $G_4 = -.6286$, $H_L = 2.631$, $H_U = 31.989$, $3.844 - 2.631$, $3.844 + 31.989$, $1.213 \leq \sigma_\beta^2 \leq 35.833$

25.18. a. H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = 47.0450/.1150 = 409.09$, $F(.95; 1, 3) = 10.1$.

If $F^* \leq 10.1$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .0003

25.19. a. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	−.175	−1.300	.325	−2.050	3.200
2	.025	4.900	−3.475	1.150	−2.600
3	−.575	−1.700	.925	−1.450	2.800
4	.025	1.900	−1.475	2.150	−2.600
5	.025	−1.100	.525	−.850	1.400
6	.025	−1.100	2.525	−.850	−.600
7	−.175	−.300	−.675	2.950	−1.800
8	.825	−1.300	1.325	−1.050	.200

$r = .985$

c. H_0: $D = 0$, H_a: $D \neq 0$. $SSBL.TR^* = 27.729$, $SSRem^* = 94.521$,

$F^* = (27.729/1) \div (94.521/27) = 7.921$, $F(.995; 1, 27) = 9.34$.

If $F^* \leq 9.34$ conclude H_0, otherwise H_a. Conclude H_0.

25.20. a.

Source	SS	df	MS
Blocks	4,826.375	7	689.48214
Paint type	531.350	4	132.83750
Error	122.250	28	4.36607
Total	5,479.975	39	

b. H_0: all τ_j equal zero ($j = 1, ..., 5$), H_a: not all τ_j equal zero.

$F^* = 132.83750/4.36607 = 30.425$, $F(.95; 4, 28) = 2.71$.

If $F^* \leq 2.71$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $\bar{Y}_{.1} = 20.500$, $\bar{Y}_{.2} = 23.625$, $\bar{Y}_{.3} = 19.000$, $\bar{Y}_{.4} = 29.375$, $\bar{Y}_{.5} = 21.125$, $\hat{L}_1 = \bar{Y}_{.1} - \bar{Y}_{.2} = -3.125$, $\hat{L}_2 = \bar{Y}_{.1} - \bar{Y}_{.3} = 1.500$, $\hat{L}_3 = \bar{Y}_{.1} - \bar{Y}_{.4} = -8.875$, $\hat{L}_4 = \bar{Y}_{.1} - \bar{Y}_{.5} = -.625$, $s\{\hat{L}_i\} = 1.0448$ ($i = 1, ..., 4$), $B = t(.9875; 28) = 2.369$

$$-3.125 \pm 2.369(1.0448) \qquad -5.60 \leq L_1 \leq -.65$$
$$1.500 \pm 2.369(1.0448) \qquad -.98 \leq L_2 \leq 3.98$$
$$-8.875 \pm 2.369(1.0448) \qquad -11.35 \leq L_3 \leq -6.40$$
$$-.625 \pm 2.369(1.0448) \qquad -3.10 \leq L_4 \leq 1.85$$

d. $\hat{L} = \frac{1}{3}(\bar{Y}_{.1} + \bar{Y}_{.3} + \bar{Y}_{.5}) - \frac{1}{2}(\bar{Y}_{.2} + \bar{Y}_{.4}) = -6.29167$, $s\{\hat{L}\} = .6744$, $t(.975; 28) = 2.048$, $-6.29167 \pm 2.048(.6744)$, $-7.67 \leq L \leq -4.91$

25.21. a. e_{ij}:

i	$j=1$	$j=2$	$j=3$
1	$-.1333$	$.4667$	$-.3333$
2	$-.1333$	$-.5333$	$.6667$
3	$-.4667$	1.1333	$-.6667$
4	$.8667$	$-.5333$	$-.3333$
5	$-.4667$	1.1333	$-.6667$
6	1.2000	-1.2000	$.0000$
7	$-.1333$	1.4667	-1.3333
8	$.8667$	-1.5333	$.6667$
9	-2.1333	$-.5333$	2.6667
10	$.5333$	$.1333$	$-.6667$

$r = .985$

c. H_0: $D = 0$, H_a: $D \neq 0$. $SSBL.TR^* = 4.5365$, $SSRem^* = 24.6635$,

$F^* = (4.5365/1) \div (24.6635/17) = 3.127$, $F(.975; 1, 17) = 6.042$.

If $F^* \leq 6.042$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .095

25.22. a.

Source	SS	df	MS
Blocks	$1,195.5000$	9	132.8333
Reagents	123.4667	2	61.7333
Error	29.2000	18	1.6222
Total	$1,348.1667$	29	

b. H_0: all τ_j equal zero ($j = 1, 2, 3$), H_a: not all τ_j equal zero.

$F^* = 61.7333/1.6222 = 38.055$, $F(.975; 2, 18) = 4.56$.

If $F^* \leq 4.56$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $\bar{Y}_{.1} = 15.3$, $\bar{Y}_{.2} = 19.7$, $\bar{Y}_{.3} = 19.5$, $B = t(.9875;\ 18) = 2.445$, $\hat{L}_1 = .2$, $\hat{L}_2 = 4.3$, $s\{\hat{L}_1\} = .5696$, $s\{\hat{L}_2\} = .4933$

$.2 \pm 2.445(.5696) \qquad -1.193 \leq L_1 \leq 1.593$

$4.3 \pm 2.445(.4933) \qquad 3.094 \leq L_2 \leq 5.506$

d. H_0: $\sigma^2_\rho = 0$, H_a: $\sigma^2_\rho > 0$. $F^* = 132.8333/1.6222 = 81.885$, $F(.975; 9, 18) = 2.929$. If $F^* \leq 2.929$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

25.23. a. H_0: $\sigma^2_{\alpha\beta\gamma} = 0$, H_a: $\sigma^2_{\alpha\beta\gamma} > 0$. $F^* = MSABC/MSE = 1.49/2.30 = .648$, $F(.975; 8, 60) = 2.41$. If $F^* \leq 2.41$ conclude H_0, otherwise H_a. Conclude H_0. P-value=.27.

b. H_0: $\sigma^2_{\alpha\beta} = 0$, H_a: $\sigma^2_{\alpha\beta} > 0$. $F^* = MSAB/MSABC = 2.40/1.49 = 1.611$,

$F(.99; 2, 8) = 8.65$. If $F^* \leq 8.65$ conclude H_0, otherwise H_a. Conclude H_0.

c. H_0: $\sigma^2_\beta = 0$, H_a: $\sigma^2_\beta > 0$. $F^{**} = MSB/(MSAB + MSBC - MSABC) = 4.20/(2.40 + 3.13 - 1.49) = 1.04$, $df = 16.32161/5.6067 = 2.91$, $F(.99; 1, 3) = 34.1$. If $F^{**} \leq 34.1$ conclude H_0, otherwise H_a. Conclude H_0.

d. $s^2_\alpha = (MSA - MSAB - MSAC + MSABC)/nbc = .126$,

$df = [(8.650/30) - (2.40/30) - (3.96/30) + (1.49/30)]^2$

$$\div \left[\frac{(8.65/30)^2}{2} + \frac{(2.40/30)^2}{2} + \frac{(3.96/30)^2}{8} + \frac{(1.49/30)^2}{8} \right] = .336$$

$\chi^2(.025; 1) = .001$, $\chi^2(.975; 1) = 5.02$

$$.008 = \frac{.336(.126)}{5.02} \leq \sigma^2_\alpha \leq \frac{.336(.126)}{.001} = 42.336$$

25.24. a. $F^* = MSAC/MSABC$, $F^* = MSB/MSE$

b. H_0: all $(\alpha\gamma)_{ik}$ equal zero, H_a: not all $(\alpha\gamma)_{ik}$ equal zero. $F^* = 91.267/9.517 = 9.59$, $F(.95; 1, 2) = 18.5$. If $F^* \leq 18.5$ conclude H_0, otherwise H_a. Conclude H_0.

c. H_0: $\sigma^2_\beta = 0$, H_a: $\sigma^2_\beta > 0$. $F^* = 24,659.817/858.042 = 28.74, F\ (.95; 2, 48) = 3.19$. If $F^* \leq 3.19$ conclude H_0, otherwise H_a. Conclude H_a.

d. $s^2_\beta = (MSB - MSE)/acn = (24,659.817 - 858.042)/20 = 1,190.09$, $c_1 = .05$, $c_2 = -.05$, $df_1 = 2$, $df_2 = 48$, $F_1 = F(.975; 2, \infty) = 3.69$, $F_2 = F(.975; 48, \infty) = 1.44$, $F_3 = F(.975; \infty, 2) = 39.5$, $F_4 = F(.975; \infty, 48) = 1.56$, $F_5 = F(.975; 2, 48) = 3.99$, $F_6 = F(.975; 48, 2) = 39.5$, $G_1 = .7290$, $G_2 = .3056$, $G_3 = .0416$, $G_4 = -3.6890$, $H_L = 900.39$, $H_U = 47,468.09$, 1,190.09 – 900.39, $1,190.09 + 47,468.09$, $289.70 \leq \sigma^2_\beta \leq 48,658.18$

25.25. $F^{**} = MSA/(MSAB + MSAC - MSABC)$

$$df = (MSAB + MSAC - MSABC)^2 \div \left(\frac{(MSAB)^2}{df_{AB}} + \frac{(MSAC)^2}{df_{AC}} + \frac{(MSABC)^2}{df_{ABC}} \right)$$

25.26. a. $\hat{\mu}_{..} = 55.593$, $\hat{\beta}_1 = .641$, $\hat{\beta}_2 = .218$, $\hat{\sigma}^2_\alpha = 5.222$, $\hat{\sigma}^2_{\alpha\beta} = 15.666, \hat{\sigma}^2 = 55.265$, no

(Note: Unrestricted estimators are same except that variance component for random effect A is zero.)

b. Estimates remain the same.

c. H_0: $\sigma^2_{\alpha\beta} = 0$, H_a: $\sigma^2_{\alpha\beta} > 0$. $z(.99) = 2.326$, $s\{\hat{\sigma}^2_{\alpha\beta}\} = 13.333$, $z^* = 15.666/13.333 = 1.175$. If $z^* \leq 2.326$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .12.

d. H_0: $\beta_1 = \beta_2 = \beta_3 = 0$, H_a: not all $\beta_j = 0$ $(j = 1, 2, 3)$. $-2\log_e L(R) = 295.385$, $-2\log_e L(F) = 295.253$, $X^2 = 295.385 - 295.253 = .132$, $\chi^2(.99; 2) = 9.21$. If $X^2 \leq 9.21$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .94.

e. $z(.995) = 2.576$, $15.666 \pm 2.576(13.333)$, $-18.680 \leq \alpha^2_{\alpha\beta} \leq 50.012$

25.27. a. $\hat{\mu}_{..} = 75.817$, $\hat{\alpha}_1 = -2.398$, $\hat{\alpha}_2 = .977$, $\hat{\sigma}^2_\beta = 2.994$, $\hat{\sigma}^2_{\alpha\beta} = 0$, $\hat{\sigma}^2 = 3.103$, yes

b. Estimates remain the same.

c. H_0: $\sigma^2_\beta = 0$, H_a: $\sigma^2_\beta > 0$. $-2\log_e L(R) = 214.034$, $-2\log_e L(F) = 192.599$, $X^2 = 214.034 - 192.599 = 21.435$, $\chi^2(.95; 1) = 3.84$. If $X^2 \leq 3.84$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. H_0: $\alpha_1 = \alpha_2 = \alpha_3 = 0$, H_a: not all $\alpha_i = 0$ $(i = 1, 2, 3)$. $-2\log_e L(R) = 221.722$, $-2\log_e L(F) = 192.599$, $X^2 = 221.722 - 192.599 = 29.123$, $\chi^2(.95; 2) = 5.99$. If $X^2 \leq 5.99$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. $s\{\hat{\sigma}^2_\beta\} = 2.309$, $z(.975) = 1.96$, $2.994 \pm 1.96(2.309)$, $-1.532 \leq \sigma^2_\beta \leq 7.520$

(Note: Answers to parts (c) and (e) are not consistent; may be due to large-sample approximation not being appropriate here.)

25.28. $n' = \frac{1}{r-1}\left[(\sum n_i) - \frac{\sum n_i^2}{\sum n_i}\right] = \frac{1}{r-1}(rn - rn^2/rn) = \frac{1}{r-1}(rn-n) = n$

25.29. From (25.12), $\sigma^2\{\bar{Y}_{..}\} = (\sigma_\mu^2/r) + (\sigma^2/n_T)$. When n_T is fixed, $\sigma^2\{\bar{Y}_{..}\}$ is minimized by making r as large as possible, *i.e.*, $r = n_T$.
In that case, $n = 1$ since $rn = n_T$.

25.30. $L \leq \frac{\sigma_\mu^2}{\sigma^2} \leq U$ or $\frac{1}{L} \geq \frac{\sigma^2}{\sigma_\mu^2} \geq \frac{1}{U}$ or $\frac{1}{L} + 1 \geq \frac{\sigma^2}{\sigma_\mu^2} + 1 \geq \frac{1}{U} + 1$ or

$$\frac{1+L}{L} \geq \frac{\sigma^2 + \sigma_\mu^2}{\sigma_\mu^2} \geq \frac{1+U}{U} \text{ or } L^* = \frac{L}{1+L} \leq \frac{\sigma_\mu^2}{\sigma^2 + \sigma_\mu^2} \leq \frac{U}{1+U} = U^*$$

25.31. $\sigma^2\{\bar{Y}_{i..}\} = \sigma^2\{\mu_{..} + \alpha_i + \bar{\beta}_{.} + (\overline{\alpha\beta})_{i.} + \bar{\epsilon}_{i..}\}$

$$= \sigma_\alpha^2 + \frac{\sigma_\beta^2}{b} + \frac{\sigma_{\sigma\beta}^2}{b} + \frac{\sigma^2}{bn} \quad \text{because of independence.}$$

25.32. $\sigma^2\{Y_{ij}\} = \sigma^2\{\mu_{..} + \rho_i + \tau_j + \epsilon_{ij}\} = \sigma_\tau^2 + \sigma^2$

$$\sigma^2\{\bar{Y}_{.j}\} = \sigma^2\{\mu_{..} + \frac{\sum \rho_i}{n_b} + \tau_j + \bar{\epsilon}_{.j}\} = \sigma_\tau^2 + \frac{\sigma^2}{n_b}$$

25.33. a. $Y_{ijk} = \mu_{...} + \rho_i + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$

b. $F^* = MSAB/MSBL.TR$, $F^* = MSA/MSBL.TR$, $F^* = MSB/MSBL.TR$

25.34. $\sigma\{Y_{ij}, Y_{ij'}\} = E\{(Y_{ij} - E\{Y_{ij}\})(Y_{ij'} - E\{Y_{ij'}\})\}$

$$= E\{[(\mu_{..} + \rho_i + \tau_j + \epsilon_{ij}) - (\mu_{..} + \tau_j)][(\mu_{..} + \rho_i + \tau_{j'} + \epsilon_{ij'}) - (\mu_{..} + \tau_{j'})]\}$$
$$= E\{(\rho_i + \epsilon_{ij})(\rho_i + \epsilon_{ij'})\}$$
$$= E\{\rho_i^2\} + E\{\rho_i\epsilon_{ij}\} + E\{\rho_i\epsilon_{ij'}\} + E\{\epsilon_{ij}\epsilon_{ij'}\} = \sigma_\rho^2$$

since ρ_i, ϵ_{ij}, and $\epsilon_{ij'}$ are pairwise independent and have expectations equal to zero.

25.35. $\sigma^2\{\bar{Y}_{i...}\} = \sigma^2\{\mu_{...} + \alpha_i + \bar{\beta}_{.} + \bar{\gamma}_{.} + (\overline{\alpha\beta})_{i.} + (\overline{\alpha\gamma})_{i.} + (\overline{\beta\gamma})_{..} + (\overline{\alpha\beta\gamma})_{i..} + \bar{\epsilon}_{i...}\}$

$$= \sigma_\alpha^2 + \frac{\sigma_\beta^2}{b} + \frac{\sigma_\gamma^2}{c} + \frac{\sigma_{\alpha\beta}^2}{b} + \frac{\sigma_{\alpha\gamma}^2}{c} + \frac{\sigma_{\beta\gamma}^2}{bc} + \frac{\sigma_{\alpha\beta\gamma}^2}{bc} + \frac{\sigma^2}{nbc}$$

25.36. e. $E\{MSA\} = 248.5$, $E\{MSAB\} = 8.5$

25.37. a.

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} 137.4286 & 145.3571 & 131.2857 & 156.6429 & 117.2143 \\ & 158.5536 & 135.4286 & 167.1607 & 121.6250 \\ & & 128.5714 & 148.4286 & 113.8571 \\ & & & 179.4107 & 133.2321 \\ & & & & 102.9821 \end{bmatrix}$$

25.38. a.

$$\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 37.5667 & 42.9889 & 40.8333 \\ & 51.1222 & 47.3889 \\ & & 47.3889 \end{bmatrix}$$

25.39. a. $\hat{\mu}_{..} = 30.051$, $\hat{\sigma}^2_\alpha = 7.439$, $\hat{\sigma}^2_\beta = 2.757$, $\hat{\sigma}^2_{\alpha\beta} = .011$, $\hat{\sigma}^2 = .183$, $s\{\hat{\sigma}^2_\alpha\} = 5.570$, $s\{\hat{\sigma}^2_\beta\} = 1.958$, $s\{\hat{\sigma}^2_{\alpha\beta}\} = .053$, $s\{\hat{\sigma}^2\} = .059$.

Chapter 26

NESTED DESIGNS, SUBSAMPLING, AND PARTIALLY NESTED DESIGNS

26.4. a. e_{ijk} :

	$i = 1$			
k	$j = 1$	$j = 2$	$j = 3$	$j = 4$
1	3.2	.2	−6.6	−7.6
2	−3.8	−5.8	2.4	3.4
3	1.2	7.2	−4.6	1.4
4	−4.8	−3.8	7.4	−4.6
5	4.2	2.2	1.4	7.4

	$i = 2$			
k	$j = 1$	$j = 2$	$j = 3$	$j = 4$
1	−1.8	−6.2	−3.8	−4.0
2	5.2	.8	.2	1.0
3	.2	4.8	6.2	6.0
4	4.2	2.8	2.2	−2.0
5	−7.8	−2.2	−4.8	−1.0

	$i = 3$			
k	$j = 1$	$j = 2$	$j = 3$	$j = 4$
1	−7.8	−6.6	6.6	−6.4
2	6.2	.4	−2.4	5.6
3	−2.8	2.4	−1.4	−.4
4	1.2	−1.6	1.6	3.6
5	3.2	5.4	−4.4	−2.4

$r = .986$

26.5. a. No

b. $\bar{Y}_{ij.}$:

i	$j = 1$	$j = 2$	$j = 3$	$j = 4$
1	61.8	67.8	62.6	52.6
2	75.8	75.2	55.8	77.0
3	76.8	69.6	74.4	73.4

c.

Source	SS	df	MS
Machines (A)	$1,695.63$	2	847.817
Operators, within mahcines [$B(A)$]	$2,272.30$	9	252.478
Error (E)	$1,132.80$	48	23.600
Total	$5,100.73$	59	

d. H_0: all α_i equal zero ($i = 1,2,3$), H_a: not all α_i equal zero.

$F^* = 847.817/23.600 = 35.924$, $F(.99; 2, 48) = 5.075$.

If $F^* \leq 5.075$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. H_0: all $\beta_{j(i)}$ equal zero, H_a: not all $\beta_{j(i)}$ equal zero.

$F^* = 252.478/23.600 = 10.698$, $F(.99; 9, 48) = 2.802$.

If $F^* \leq 2.802$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

f.

i	$SSB(A_i)$
1	599.20
2	$1,538.55$
3	134.55

H_0: all $\beta_{j(1)}$ equal zero, H_a: not all $\beta_{j(1)}$ equal zero.

$F^* = (599.20/3) \div 23.600 = 8.46$, $F(.99; 3, 48) = 4.22$.

If $F^* \leq 4.22$ conclude H_0, otherwise H_a. Conclude H_a.

H_0: all $\beta_{j(2)}$ equal zero, H_a: not all $\beta_{j(2)}$ equal zero.

$F^* = (1,538.55/3) \div 23.600 = 21.73$, $F(.99; 3, 48) = 4.22$.

If $F^* \leq 4.22$ conclude H_0, otherwise H_a. Conclude H_a.

H_0: all $\beta_{j(3)}$ equal zero, H_a: not all $\beta_{j(3)}$ equal zero.

$F^* = (134.55/3) \div 23.600 = 1.90$, $F(.99; 3, 48) = 4.22$.

If $F^* \leq 4.22$ conclude H_0, otherwise H_a. Conclude H_0.

g. $\alpha \leq .05$

26.6. a. $\bar{Y}_{1..} = 61.20$, $\bar{Y}_{2..} = 70.95$, $\bar{Y}_{3..} = 73.55$, $\hat{L}_1 = \bar{Y}_{1..} - \bar{Y}_{2..} = -9.75$, $\hat{L}_2 = \bar{Y}_{1..} - \bar{Y}_{3..} = -12.35$, $\hat{L}_3 = \bar{Y}_{2..} - \bar{Y}_{3..} = -2.60$, $s\{\hat{L}_i\} = 1.536$ ($i = 1,2,3$), $q(.95; 3, 48) = 3.42$, $T = 2.418$

$$\begin{array}{rc} -9.75 \pm 2.418(1.536) & -13.46 \leq L_1 \leq -6.04 \\ -12.35 \pm 2.418(1.536) & -16.06 \leq L_2 \leq -8.64 \\ -2.60 \pm 2.418(1.536) & -6.31 \leq L_3 \leq 1.11 \end{array}$$

b. $\bar{Y}_{11.} = 61.8$, $\bar{Y}_{12.} = 67.8$, $\bar{Y}_{13.} = 62.6$, $\bar{Y}_{14.} = 52.6$, $\hat{L}_1 = \bar{Y}_{11.} - \bar{Y}_{12.} = -6.0$, $\hat{L}_2 = \bar{Y}_{11.} - \bar{Y}_{13.} = -.8$, $\hat{L}_3 = \bar{Y}_{11.} - \bar{Y}_{14.} = 9.2$, $\hat{L}_4 = \bar{Y}_{12.} - \bar{Y}_{13.} = 5.2$, $\hat{L}_5 = \bar{Y}_{12.} - \bar{Y}_{14.} = 15.2$, $\hat{L}_6 = \bar{Y}_{13.} - \bar{Y}_{14.} = 10.0$, $s\{\hat{L}_i\} = 3.0725$ ($i = 1, ..., 6$), $B = t(.99583; 48) = 2.753$

$-6.0 \pm 3.0725(2.753)$	$-14.46 \leq L_1 \leq 2.46$
$-.8 \pm 3.0725(2.753)$	$-9.26 \leq L_2 \leq 7.66$
$9.2 \pm 3.0725(2.753)$	$.74 \leq L_3 \leq 17.66$
$5.2 \pm 3.0725(2.753)$	$-3.26 \leq L_4 \leq 13.66$
$15.2 \pm 3.0725(2.753)$	$6.74 \leq L_5 \leq 23.66$
$10.0 \pm 3.0725(2.753)$	$1.54 \leq L_6 \leq 18.46$

c. $\hat{L} = 11.467$, $s\{\hat{L}\} = 2.5087$, $t(.995; 48) = 2.682$,

$11.467 \pm 2.682(2.5087)$, $4.74 \leq L \leq 18.20$

26.7. a. $\beta_{j(i)}$ are independent $N(0, \sigma_\beta^2)$; $\beta_{j(i)}$ are independent of $\epsilon_{k(ij)}$.

b. $\hat{\sigma}_\beta^2 = 45.7756$

c. H_0: $\sigma_\beta^2 = 0$, H_a: $\sigma_\beta^2 > 0$. $F^* = 252.478/23.600 = 10.698$, $F(.90; 9, 48) = 1.765$.

If $F^* \leq 1.765$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $c_1 = .2$, $c_2 = -.2$, $MS_1 = 252.478$, $MS_2 = 23.600$, $df_1 = 9$, $df_2 = 48$,

$F_1 = F(.95; 9, \infty) = 1.88$, $F_2 = F(.95; 48, \infty) = 1.36$, $F_3 = F(.95; \infty, 9) = 2.71$,

$F_4 = F(.95; \infty, 48) = 1.45$, $F_5 = F(.95; 9, 48) = 2.08$, $F_6 = F(.95; 48, 9) = 2.81$,

$G_1 = .4681$, $G_2 = .2647$, $G_3 = .00765$, $G_4 = -.07162$, $H_L = 23.771$, $H_U = 86.258$,

$45.7756 - 23.771$, $45.7756 + 86.258$, $22.005 \leq \sigma_\beta^2 \leq 132.034$

e. H_0: all α_i equal zero ($i = 1, 2, 3$), H_a: not all α_i equal zero.

$F^* = 847.817/252.478 = 3.358$, $F(.90; 2, 9) = 3.01$.

If $F^* \leq 3.01$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .081

f. See Problem 26.6a. $s\{\hat{L}_i\} = 5.025$ ($i = 1, 2, 3$), $q(.90; 3, 9) = 3.32$, $T = 2.348$

$-9.75 \pm 2.348(5.025)$	$-21.55 \leq L_1 \leq 2.05$
$-12.35 \pm 2.348(5.025)$	$-21.15 \leq L_2 \leq -.55$
$-2.60 \pm 2.348(5.025)$	$-14.40 \leq L_3 \leq 9.20$

g. H_0: all $\sigma^2\{\beta_{j(i)}\}$ are equal ($i = 1, 2, 3$), H_a: not all $\sigma^2\{\beta_{j(i)}\}$ are equal. $\widetilde{Y}_1 = 62.2$, $\widetilde{Y}_2 = 75.5$, $\widetilde{Y}_3 = 73.9$, $MSTR = 11.6433$, $MSE = 38.0156$, $F_{BF}^* = 11.6433/38.0156 = .31$, $F(.99; 2, 9) = 8.02$. If $F_{BF}^* \leq 8.02$ conclude H_0, otherwise H_a. Conclude H_0.

26.8. a. α_i are independent $N(0, \sigma_\alpha^2)$; $\beta_{j(i)}$ are independent $N(0, \sigma_\beta^2)$;

α_i, $\beta_{j(i)}$, and $\epsilon_{k(ij)}$ are independent.

b. $\hat{\sigma}_\beta^2 = 45.7756$, $\hat{\sigma}_\alpha^2 = 29.7669$

c. H_0: $\sigma_\alpha^2 = 0$, H_a: $\sigma_\alpha^2 > 0$. $F^* = 847.817/252.478 = 3.358$, $F(.95; 2, 9) = 4.26$.

If $F^* \leq 4.26$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .081

d. $c_1 = .2$, $c_2 = -.2$, $MS_1 = 252.478$, $MS_2 = 23.600$, $df_1 = 9$, $df_2 = 48$,

$F_1 = F(.975; 9, \infty) = 2.11$, $F_2 = F(.975; 48, \infty) = 1.44$, $F_3 = (.975; \infty, 9) = 3.33$,

$F_4 = F(.975; \infty, 48) = 1.56$, $F_5 = F(.975; 9, 48) = 2.39$, $F_6 = F(.975; 48, 9) = 3.48$,

$G_1 = .5261$, $G_2 = .3056$, $G_3 = .01577$, $G_4 = -.1176$, $H_L = 26.766$, $H_U = 117.544$,

$45.7756 - 26.766$, $45.7756 + 117.544$, $19.01 \leq \sigma_\beta^2 \leq 163.32$

e. $\bar{Y}_{...} = 68.56667$, $s\{\bar{Y}_{...}\} = 3.759$, $t(.975; 2) = 4.303$, $68.56667 \pm 4.303(3.759)$, $52.392 \leq \mu_{..} \leq 84.742$

26.9. a. e_{ijk}:

	$i=1$		
k	$j=1$	$j=2$	$j=3$
1	1.8	−12.8	−9.6
2	15.8	−.8	7.4
3	−5.2	3.2	16.4
4	−.2	−3.8	−14.6
5	−12.2	14.2	.4

	$i=2$		
k	$j=1$	$j=2$	$j=3$
1	−7.2	−2.6	8.8
2	3.8	−15.6	−8.2
3	−15.2	6.4	−10.2
4	7.8	11.4	11.8
5	10.8	.4	−2.2

	$i=3$		
k	$j=1$	$j=2$	$j=3$
1	−5.8	−9.8	−12.0
2	11.2	12.2	0.0
3	−.8	−.8	17.0
4	−12.8	3.2	2.0
5	8.2	−4.8	−7.0

$r = .987$

26.10. a.

Source	*SS*	*df*	*MS*
States (A)	6,976.84	2	3,488.422
Cities within states [$B(A)$]	167.60	6	27.933
Error (E)	3,893.20	36	108.144
Total	11,037.64	44	

b. H_0: all α_i equal zero ($i = 1, 2, 3$), H_a: not all α_i equal zero. $F^* = 3,488.422/108.144 = 32.257$, $F(.95; 2, 36) = 3.26$. If $F^* \leq 3.26$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. H_0: all $\beta_{j(i)}$ equal zero, H_a: not all $\beta_{j(i)}$ equal zero. $F^* = 27.933/108.144 = .258$, $F(.95; 6, 36) = 2.36$. If $F^* \leq 2.36$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .95

d. $\alpha \leq .10$

26.11. a. $\bar{Y}_{11.} = 40.2$, $s\{\bar{Y}_{11.}\} = 4.6507$, $t(.975; 36) = 2.0281$,

$40.2 \pm 2.0281(4.6507)$, $30.77 \leq \mu_{11} \leq 49.63$

b. $\bar{Y}_{1..} = 40.8667$, $\bar{Y}_{2..} = 57.3333$, $\bar{Y}_{3..} = 26.8667$, $s\{\bar{Y}_{i..}\} = 2.6851$ ($i = 1, 2, 3$), $t(.995; 36) = 2.7195$

$40.8667 \pm 2.7195(2.6851)$ $\quad 33.565 \leq \mu_{1.} \leq 48.169$
$57.3333 \pm 2.7195(2.6851)$ $\quad 50.031 \leq \mu_{2.} \leq 64.635$
$26.8667 \pm 2.7195(2.6851)$ $\quad 19.565 \leq \mu_{3.} \leq 34.169$

c. $\hat{L}_1 = \bar{Y}_{1..} - \bar{Y}_{2..} = -16.4666$, $\hat{L}_2 = \bar{Y}_{1..} - \bar{Y}_{3..} = 14.0000$, $\hat{L}_3 = \bar{Y}_{2..} - \bar{Y}_{3..} = 30.4666$, $s\{\hat{L}_i\} = 3.7973$ $(i = 1, 2, 3)$, $q(.90; 3, 36) = 2.998$, $T = 2.120$

$$\begin{array}{ll} -16.4666 \pm 2.120(3.7973) & -24.52 \leq L_1 \leq -8.42 \\ 14.0000 \pm 2.120(3.7973) & 5.95 \leq L_2 \leq 22.05 \\ 30.4666 \pm 2.120(3.7973) & 22.42 \leq L_3 \leq 38.52 \end{array}$$

d. $\hat{L} = 12.4$, $s\{\hat{L}\} = 6.5771$, $t(.975; 36) = 2.0281$, $12.4 \pm 2.0281(6.5771)$, $-.94 \leq L \leq 25.74$

26.12. a. $\beta_{j(i)}$ are independent $N(0, \sigma_\beta^2)$; $\beta_{j(i)}$ are independent of $\epsilon_{k(j)}$.

b. $\hat{\sigma}_\beta^2 = 0$, yes.

c. H_0: $\sigma_\beta^2 = 0$, H_a: $\sigma_\beta^2 > 0$. $F^* = 27.933/108.144 = .258$, $F(.90; 6, 36) = 1.94$.

If $F^* \leq 1.94$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .95

d. H_0: all α_i equal zero $(i = 1, 2, 3)$, H_a: not all α_i equal zero. $F^* = 3,488.422/27.933 = 124.885$, $F(.90; 2, 6) = 3.46$. If $F^* \leq 3.46$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. See Problem 26.11c. $s\{\hat{L}_i\} = 1.9299$ $(i = 1, 2, 3)$, $q(.90; 3, 6) = 3.56$, $T = 2.5173$

$$\begin{array}{ll} -16.4666 \pm 2.5173(1.9299) & -21.32 \leq L_1 \leq -11.61 \\ 14.0000 \pm 2.5173(1.9299) & 9.14 \leq L_2 \leq 18.86 \\ 30.4666 \pm 2.5173(1.9299) & 25.61 \leq L_3 \leq 35.32 \end{array}$$

f. H_0: all $\sigma^2\{\beta_{j(i)}\}$ are equal $(i = 1, 2, 3)$, H_a: not all $\sigma^2\{\beta_{j(i)}\}$ are equal.

$H^* = 37.27/16.07 = 2.32$, $H(.95; 3, 2) = 87.5$.

If $H^* \leq 87.5$ conclude H_0, otherwise H_a. Conclude H_0.

26.13. a. α_i are independent $N(0, \sigma_\alpha^2)$; $\beta_{j(i)}$ are independent $N(0, \sigma_\beta^2)$;

α_i, $\beta_{j(i)}$, and $\epsilon_{k(ij)}$ are independent.

b. $\hat{\sigma}_\beta^2 = 0$, $\hat{\sigma}_\alpha^2 = 230.699$

c. H_0: $\sigma_\alpha^2 = 0$, H_a: $\sigma_\alpha^2 > 0$. $F^* = 3,488.422/27.933 = 124.885$, $F(.99; 2, 6) = 10.9$.

If $F^* \leq 10.9$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $c_1 = 1/15$, $c_2 = -1/15$, $MS_1 = 3488.422$, $MS_2 = 27.933$, $df_1 = 2$, $df_2 = 6$, $F_1 = F(.995; 2, \infty) = 5.30$, $F_2 = F(.995; 6, \infty) = 3.09$, $F_3 = F(.995; \infty, 2) = 200$, $F_4 = F(.995; \infty, 6) = 8.88$, $F_5 = F(.995; 2, 6) = 14.5$, $F_6 = F(.995; 6, 2) = 199$, $G_1 = .8113$, $G_2 = .6764$, $G_3 = -1.2574$, $G_4 = -93.0375$, $H_L = 187.803$, $H_U = 46,279.30$, $230.699 - 187.803$, $230.699 + 46,279.30$, $42.90 \leq \sigma_\alpha^2 \leq 46,510.00$

e. $\bar{Y}_{...} = 41.6889$, $s\{\bar{Y}_{...}\} = 8.8046$, $t(.995; 2) = 9.925$, $41.6889 \pm 9.925(8.8046)$, $-45.70 \leq \mu_{..} \leq 129.07$

26.14. a. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_{1(1)} X_{ijk3} + \beta_{2(1)} X_{ijk4} + \beta_{1(2)} X_{ijk5} + \beta_{1(3)} X_{ijk6} + \epsilon_{ijk}$

$$X_{ijk1} = \begin{cases} 1 & \text{if case from region 1} \\ -1 & \text{if case from region 3} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk2} = \begin{cases} 1 & \text{if case from region 2} \\ -1 & \text{if case from region 3} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk3} = \begin{cases} 1 & \text{if case for team 1 from region 1} \\ -1 & \text{if case for team 3 from region 1} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk4} = \begin{cases} 1 & \text{if case for team 2 from region 1} \\ -1 & \text{if case for team 3 from region 1} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk5} = \begin{cases} 1 & \text{if case for team 1 from region 2} \\ -1 & \text{if case for team 2 from region 2} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk6} = \begin{cases} 1 & \text{if case for team 1 from region 3} \\ -1 & \text{if case for team 2 from region 3} \\ 0 & \text{otherwise} \end{cases}$$

b. Full model: $\hat{Y} = 150.01667 - 9.21667X_1 + 5.28333X_2 + 6.60000X_3$
$+.50000X_4 + 3.70000X_5 - 1.85000X_6$

e_{ijk}:

	$i=1$			$i=2$		$i=3$	
k	$j=1$	$j=2$	$j=3$	$j=1$	$j=2$	$j=1$	$j=2$
1	4.20	1.90	−2.30	4.80	0.00	4.90	4.20
2	−6.20	−1.90	2.30	−4.80		−4.90	−4.20
3	2.00						

$r = .962$

26.15. a. $SSE(F) = 207.2600$

Reduced model: $\hat{Y} = 147.60248 + 4.89938X_3 + 1.35031X_4 + 6.26584X_5 - 1.85000X_6$

$SSE(R) = 838.7766$

H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_1 and α_2 equal zero. $F^* = (631.5166/2) \div (207.2600/7) = 10.664$, $F(.975; 2, 7) = 6.54$. If $F^* \leq 6.54$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .0075

b. Reduced model: $\hat{Y} = 150.74206 - 8.99921X_1 + 5.79127X_2$

$SSE(R) = 483.2338$

H_0: $\beta_{1(1)} = \beta_{2(1)} = \beta_{1(2)} = \beta_{1(3)} = 0$, H_a: not all $\beta_{j(i)}$ equal zero. $F^* = (275.9738/4) \div (207.26/7) = 2.33$, $F(.975; 4, 7) = 5.52$. If $F^* \leq 5.52$ conclude H_0, otherwise H_a. Conclude H_0.

c. $\hat{L} = \hat{\alpha}_1 - \hat{\alpha}_2 = -14.5$, $s^2\{\hat{\alpha}_1\} = 4.0057$, $s^2\{\hat{\alpha}_2\} = 6.2446$, $s\{\hat{\alpha}_1, \hat{\alpha}_2\} = -2.6197$, $s\{\hat{L}\} = 3.9357$, $t(.99; 7) = 2.998$, $-14.5 \pm 2.998(3.9357)$, $-26.30 \leq L \leq -2.70$

26.17. a. e_{ijk}:

	$i=1$				
k	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	-2.0	1.5	1.0	1.5	-1.0
2	2.0	-1.5	-1.0	-1.5	1.0

	$i=2$				
k	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	.5	1.0	1.5	-1.5	2.0
2	$-.5$	-1.0	-1.5	1.5	-2.0

	$i=3$				
k	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	-2.0	-1.5	1.0	2.0	-1.5
2	2.0	1.5	-1.0	-2.0	1.5

$r = .957$

b. H_0: all $\sigma^2\{\epsilon_{j(i)}\}$ are equal $(i = 1, 2, 3)$, H_a: not all $\sigma^2\{\epsilon_{j(i)}\}$ are equal. $\widetilde{Y}_1 = 30$, $\widetilde{Y}_2 = 28$, $\widetilde{Y}_3 = 27$, $MSTR = 2.2167$, $MSE = 6.8750$, $F^*_{BF} = 2.2167/6.8750 = .32$, $F(.99; 2, 12) = 6.93$. If $F^*_{BF} \leq 6.93$ conclude H_0, otherwise H_a. Conclude H_0.

26.18. a.

Source	SS	df	MS
Treatments (colors)	3.2667	2	1.63335
Experimental error	369.4000	12	30.78333
Observational error	67.5000	15	4.50000
Total	440.1667	29	

b. H_0: $\tau_1 = \tau_2 = \tau_3 = 0$, H_a: not all τ_i equal zero.

$F^* = 1.63335/30.78333 = .053$, $F(.95; 2, 12) = 3.89$.

If $F^* \leq 3.89$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .95

c. H_0: $\sigma^2 = 0$, H_a: $\sigma^2 > 0$. $F^* = 30.78333/4.50000 = 6.841$, $F(.95; 12, 15) = 2.48$.

If $F^* \leq 2.48$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .0004

d. $\bar{Y}_{1..} = 29.2$, $s\{\bar{Y}_{1..}\} = 1.7545$, $t(.975; 12) = 2.179$,

$29.2 \pm 2.179(1.7545)$, $25.38 \leq \mu_{1.} \leq 33.02$

e. $\hat{\sigma}^2 = 13.1417$, $\hat{\sigma}^2_\eta = 4.5$

f. For σ^2: $c_1 = .5$, $c_2 = -.5$, $MS_1 = 30.7833$, $MS_2 = 4.5000$, $df_1 = 12$, $df_2 = 15$, $F_1 = F(.975; 12, \infty) = 1.94$, $F_2 = F(.975; 15, \infty) = 1.83$, $F_3 = F(.975; \infty, 12) = 2.72$, $F_4 = F(.975; \infty, 15) = 2.40$, $F_5 = F(.975; 12, 15) = 2.96$, $F_6 = F(.975; 15, 12) = 3.18$, $G_1 = .4845$, $G_2 = .4536$, $G_3 = -.05916$, $G_4 = -.0906$, $H_L = 7.968$, $H_U = 26.434$, $13.1417 - 7.968$, $13.1417 + 26.434$, $5.174 \leq \sigma^2 \leq 39.576$

For σ^2_η: $df = 15$, $\chi^2(.025; 15) = 6.26$, $\chi^2(.975; 15) = 27.49$,

$$2.455 = \frac{15(4.5)}{27.49} \leq \sigma^2_\eta \leq \frac{15(4.5)}{6.26} = 10.783$$

26.19. e_{ijk}:

	$i=1$		
k	$j=1$	$j=2$	$j=3$
1	$-.4000$	.0333	$-.3667$
2	.0000	.3333	.0333
3	.4000	$-.3667$	.3333

	$i=2$		
k	$j=1$	$j=2$	$j=3$
1	.0667	.4333	$-.2000$
2	$-.2333$	.0667	.3000
3	.1667	$-.3667$	$-.1000$

	$i=3$		
k	$j=1$	$j=2$	$j=3$
1	$-.4333$	$-.1333$	$-.3667$
2	.1667	.4667	.3333
3	.2667	$-.3333$	$-.0667$

	$i=4$		
k	$j=1$	$j=2$	$j=3$
1	$-.0667$	$-.3000$	.4000
2	.4333	.2000	.0000
3	$-.3667$	.1000	$-.4000$

$r = .972$

26.20. a.

Source	SS	df	MS
Plants	343.1789	3	114.3930
Leaves, within plants	187.4533	8	23.4317
Observations, within leaves	3.0333	24	.1264
Total	533.6655	35	

b. H_0: $\sigma_\tau^2 = 0$, H_a: $\sigma_\tau^2 > 0$. $F^* = 114.3930/23.4317 = 4.88$, $F(.95; 3, 8) = 4.07$.

If $F^* \leq 4.07$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .03

c. H_0: $\sigma^2 = 0$, H_a: $\sigma^2 > 0$. $F^* = 23.4317/.1264 = 185.38$, $F(.95; 8, 24) = 2.36$.

If $F^* \leq 2.36$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $\bar{Y}_{...} = 14.26111$, $s\{\bar{Y}_{...}\} = 1.7826$, $t(.975; 3) = 3.182$,

$14.26111 \pm 3.182(1.7826)$, $8.59 \leq \mu_{..} \leq 19.93$

e. $\hat{\sigma}_\tau^2 = 10.1068$, $\hat{\sigma}^2 = 7.7684$, $\hat{\sigma}_\eta^2 = .1264$

f. $c_1 = 1/9 = .1111$, $c_2 = -1/9 = -.1111$, $MS_1 = 114.3930$, $MS_2 = 23.4317$, $df_1 = 3$, $df_2 = 8$, $F_1 = F(.95; 3, \infty) = 2.60$, $F_2 = F(.95; 8, \infty) = 1.94$, $F_3 = F(.95; \infty, 3) = 8.53$, $F_4 = F(.95; \infty, 8) = 2.93$, $F_5 = F(.95; 3, 8) = 4.07$, $F_6 = F(.95; 8, 3) = 8.85$, $G_1 = .6154$, $G_2 = .4845$, $G_3 = -.1409$, $G_4 = -1.5134$, $H_L = 9.042$, $H_U = 95.444$, $10.1068-9.042$, $10.1068+95.444$, $1.065 \leq \sigma_\tau^2 \leq 105.551$

26.21. a. e_{ijk}:

	$i=1$			
k	$j=1$	$j=2$	$j=3$	$j=4$
1	.1667	.0667	.0333	$-.0333$
2	$-.0333$	$-.1333$	$-.1667$	.1667
3	$-.1333$	.0667	.1333	$-.1333$

	$i=2$			
k	$j=1$	$j=2$	$j=3$	$j=4$
1	.0333	.0333	$-.0667$	$-.2000$
2	.1333	$-.1667$	$-.0667$	.2000
3	$-.1667$	.1333	.1333	.0000

	$i=3$			
k	$j=1$	$j=2$	$j=3$	$j=4$
1	.0000	.1667	$-.1333$	.0667
2	.1000	.0667	$-.0333$	$-.2333$
3	$-.1000$	$-.2333$	.1667	.1667

	$i=4$			
k	$j=1$	$j=2$	$j=3$	$j=4$
1	$-.1333$	$-.0333$	.1667	$-.0333$
2	.1667	.1667	$-.1333$	.1667
3	$-.0333$	$-.1333$	$-.0333$	$-.1333$

	$i=5$			
k	$j=1$	$j=2$	$j=3$	$j=4$
1	.0333	.1000	.1333	.2000
2	.1333	.1000	$-.0667$	$-.2000$
3	$-.1667$	$-.2000$	$-.0667$	.0000

$r = .981$

b. H_0: all $\sigma^2\{\epsilon_{j(i)}\}$ are equal $(i = 1, ..., 5)$, H_a: not all $\sigma^2\{\epsilon_{j(i)}\}$ are equal.

$H^* = .100833/.014167 = 7.117$, $H(.99; 5, 3) = 151$.

If $H^* \leq 151$ conclude H_0, otherwise H_a. Conclude H_0.

26.22. a.

Source	SS	df	MS
Batches	10.6843	4	2.67108
Barrels, within batches	.6508	15	.04339
Determinations, within barrels	1.0067	40	.02517
Total	12.3418	59	

b. H_0: $\sigma_\tau^2 = 0$, H_a: $\sigma_\tau^2 > 0$. $F^* = 2.67108/.04339 = 61.56$, $F(.99; 4, 15) = 4.89$.

If $F^* \leq 4.89$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. H_0: $\sigma^2 = 0$, H_a: $\sigma^2 > 0$. $F^* = .04339/.02517 = 1.724$, $F(.99; 15, 40) = 2.52$.

If $F^* \leq 2.52$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .085

d. $\bar{Y}_{...} = 2.9117$, $s\{\bar{Y}_{...}\} = .21099$, $t(.995; 4) = 4.604$,

$2.9117 \pm 4.604(.21099)$, $1.94 \leq \mu_{..} \leq 3.88$

e. $\hat{\sigma}_\tau^2 = .2190$, $\hat{\sigma}^2 = .0061$, $\hat{\sigma}_\eta^2 = .0252$

f. $c_1 = .08333$, $c_2 = -.08333$, $MS_1 = 2.67108$, $MS_2 = .04339$, $df_1 = 4$, $df_2 = 15$, $F_1 = F(.975; 4, \infty) = 2.79$, $F_2 = F(.975; 15, \infty) = 1.83$, $F_3 = F(.975; \infty, 4) = 8.26$, $F_4 = F(.975; \infty, 15) = 2.40$, $F_5 = F(.975; 4, 15) = 3.80$, $F_6 = F(.975; 15, 4) = 8.66$, $G_1 = .6416$, $G_2 = .4536$, $G_3 = .1082$, $G_4 = -1.0925$, $H_L = .1432$, $H_U = 1.6157$, $.2190 - .1432$, $.2190 + 1.6157$, $.076 \leq \sigma_\tau^2 \leq 1.835$

26.23.

$$\begin{aligned}\sum\sum\sum(Y_{ijk} - \bar{Y}_{...})^2 &= \sum\sum\sum[(\bar{Y}_{i..} - \bar{Y}_{...}) + (\bar{Y}_{ij.} - \bar{Y}_{i..}) + (Y_{ijk} - \bar{Y}_{ij.})]^2 \\ &= \sum\sum\sum[(\bar{Y}_{i..} - \bar{Y}_{...})^2 + (\bar{Y}_{ij.} - \bar{Y}_{i..})^2 + (Y_{ijk} - \bar{Y}_{ij.})^2 + 2(\bar{Y}_{i..} - \bar{Y}_{...})(\bar{Y}_{ij.} - \bar{Y}_{i..}) \\ &\quad + 2(\bar{Y}_{i..} - \bar{Y}_{...})(Y_{ijk} - \bar{Y}_{ij.}) + 2(\bar{Y}_{ij.} - \bar{Y}_{i..})(Y_{ijk} - \bar{Y}_{ij.})] \\ &= bn\sum(\bar{Y}_{i..} - \bar{Y}_{...})^2 + n\sum\sum(\bar{Y}_{ij.} - \bar{Y}_{i..})^2 + \sum\sum\sum(Y_{ijk} - \bar{Y}_{ij.})^2\end{aligned}$$

All cross products equal zero by arguments similar to that given in Section 16.8.

26.24.

$$\begin{aligned}SSB + SSAB &= \frac{\sum Y_{.j.}^2}{na} - \frac{Y_{...}^2}{nab} + \frac{\sum\sum Y_{ij.}^2}{n} - \frac{\sum Y_{i..}^2}{nb} - \frac{\sum Y_{.j.}^2}{na} + \frac{Y_{...}^2}{nab} \\ &= \frac{\sum\sum Y_{ij.}^2}{n} - \frac{\sum Y_{i..}^2}{nb} = SSB(A)\end{aligned}$$

26.25. a. $\sigma^2\{\bar{Y}_{i..}\} = \sigma^2\{\mu_{..} + \alpha_i + \bar{\beta}_{.(i)} + \bar{\epsilon}_{.(i.)}\} = \dfrac{\sigma_\beta^2}{b} + \dfrac{\sigma^2}{bn}$

$$\sigma^2\{\bar{Y}_{...}\} = \sigma^2\{\mu_{..} + \bar{\beta}_{.(.)} + \bar{\epsilon}_{.(..)}\} = \frac{\sigma_\beta^2}{ab} + \frac{\sigma^2}{abn}$$

b. $[MSB(A) - MSE]/n$

26.26. $\sigma^2\{\bar{Y}_{i..}\} = \sigma^2\{\mu_{..} + \tau_i + \bar{\epsilon}_{.(i)} + \bar{\eta}_{.(i.)}\}$

$$= \frac{\sigma^2}{n} + \frac{\sigma_\eta^2}{mn} = \frac{m\sigma^2 + \sigma_\eta^2}{mn}$$

26.27. $\sigma^2\{\bar{Y}_{...}\} = \sigma^2\{\mu_{..} + \bar{\tau}_{.} + \bar{\epsilon}_{.(.)} + \bar{\eta}_{.(..)}\}$

$$= \frac{\sigma_\tau^2}{r} + \frac{\sigma^2}{rn} + \frac{\sigma_\eta^2}{rnm}$$

$$E\left\{s^2\{\bar{Y}_{...}\}\right\} = E\left\{\frac{MSTR}{rnm}\right\} = \frac{\sigma_\eta^2 + m\sigma^2 + nm\sigma_\tau^2}{rnm} = \sigma^2\{\bar{Y}_{...}\}$$

26.28. $\sigma^2\{\bar{Y}_{1j..} - \bar{Y}_{2j..}\} = \frac{2}{c}\left(\sigma_{\beta\gamma}^2 + \frac{\sigma^2}{n} + \sigma_\gamma^2\right)$

$$df = \frac{[bMSBC(A) + MSC(A) - MSE]^2}{\frac{[bMSBC(A)]^2}{a(b-1)(c-1)} + \frac{[MSC(A)]^2}{a(c-1)} + \frac{(MSE)^2}{abc(n-1)}}$$

26.29. a. $Y_{ijk} = \mu_{..} + \alpha_i + \beta_{j(i)} + \epsilon_{k(ij)}$, $\beta_{j(i)}$ and $\epsilon_{k(ij)}$ random

b. e_{ijk}:

$i = 1$

k	$j=1$	$j=2$	$j=3$	$j=4$
1	−.040	.045	.020	−.035
2	.040	−.045	−.020	.035

$i = 2$

k	$j=1$	$j=2$	$j=3$	$j=4$
1	.035	−.045	−.025	.040
2	−.035	.045	.025	−.040

$i = 3$

k	$j=1$	$j=2$	$j=3$	$j=4$
1	.025	.040	−.035	−.060
2	−.025	−.040	.035	.060

$r = .938$

26.30. a.

Source	SS	df	MS
A (lever press rate)	.89306	2	.44653
$D(A)$ (rats within A)	.12019	9	.01335
Error	.03555	12	.00296
Total	1.04880	23	

b. H_0: $\alpha_1 = \alpha_2 = \alpha_3 = 0$, H_a: not all α_i equal zero.

$F^* = .44653/.01335 = 33.448$, $F(.95; 2, 9) = 4.26$.

If $F^* \leq 4.26$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .0001

c. H_0: $\sigma_\beta^2 = 0$, H_a: $\sigma_\beta^2 > 0$. $F^* = .01335/.00296 = 4.510$, $F(.95; 9, 12) = 2.80$.

If $F^* \leq 2.80$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .009

d. $\bar{Y}_{1..} = .53500$, $\bar{Y}_{2..} = .77375$, $\bar{Y}_{3..} = 1.00750$, $\hat{L}_1 = \bar{Y}_{1..} - \bar{Y}_{2..} = -.23875$, $\hat{L}_2 = \bar{Y}_{1..} - \bar{Y}_{3..} = -.47250$, $\hat{L}_3 = \bar{Y}_{2..} - \bar{Y}_{3..} = -.23375$, $s\{\hat{L}_i\} = .0578$ $(i = 1, 2, 3)$, $q(.90; 3, 9) = 3.32$, $T = 2.3476$

$-.23875 \pm 2.3476(.0578)$	$-.374 \leq L_1 \leq -.103$
$-.47250 \pm 2.3476(.0578)$	$-.608 \leq L_2 \leq -.337$
$-.23375 \pm 2.3476(.0578)$	$-.369 \leq L_3 \leq -.098$

e. $\hat{\sigma}^2_\beta = .005195$, $c_1 = .5$, $c_2 = -.5$, $MS_1 = .013354$, $MS_2 = .002963$, $df_1 = 9$, $df_2 = 12$, $F_1 = F(.95; 9, \infty) = 1.88$, $F_2 = F(.95; 12, \infty) = 1.75$, $F_3 = F(.95; \infty, 9) = 2.71$, $F_4 = F(.95; \infty, 12) = 2.30$, $F_5 = F(.95; 9, 12) = 2.80$, $F_6 = F(.95; 12, 9) = 3.07$, $G_1 = .4681$, $G_2 = .4286$, $G_3 = -.05996$, $G_4 = -.1210$, $H_L = .003589$, $H_U = .01138$, $.005195 - .003589$, $.005195 + .01138$, $.00161 \leq \sigma^2_\beta \leq .0166$

26.31. a. $Y_{ijk} = \mu_{..} + \tau_i + \epsilon_{j(i)} + \eta_{k(ij)}$, $\epsilon_{j(i)}$ and $\eta_{k(ij)}$ random

b. e_{ijk}:

	$i=1$			
k	$j=1$	$j=2$	$j=3$	$j=4$
1	$-.035$	$-.030$	$-.030$	$-.025$
2	.035	.030	.030	.025

	$i=2$			
k	$j=1$	$j=2$	$j=3$	$j=4$
1	.020	.030	$-.035$	$-.020$
2	$-.020$	$-.030$	.035	.020

	$i=3$			
k	$j=1$	$j=2$	$j=3$	$j=4$
1	$-.050$	$-.025$	$-.035$	.045
2	.050	.025	.035	$-.045$

$r = .940$

c. H_0: all $\sigma^2\{\epsilon_{j(i)}\}$ are equal $(i = 1, 2, 3)$, H_a: not all $\sigma^2\{\epsilon_{j(i)}\}$ are equal.

$\widetilde{Y}_1 = 1.9075$, $\widetilde{Y}_2 = 2.2200$, $\widetilde{Y}_3 = 2.4075$, $MSTR = .001431$, $MSE = .004204$,

$F^*_{BF} = .001431/.004204 = .34$, $F(.99; 2, 9) = 8.02$.

If $F^*_{BF} \leq 8.02$ conclude H_0, otherwise H_a. Conclude H_0.

26.32. a.

Source	SS	df	MS
Treatments (lever press rates)	1.013125	2	.50656
Experimental error	.182025	9	.02023
Observational error	.025900	12	.00216
Total	1.221050	23	

b. H_0: all $\tau_1 = \tau_2 = \tau_3 = 0$, H_a: not all τ_i equal zero.

$F^* = .50656/.02023 = 25.040$, $F(.99; 2, 9) = 8.02$.

If $F^* \leq 8.02$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .0002

c. H_0: $\sigma^2 = 0$, H_a: $\sigma^2 > 0$. $F^* = .02023/.00216 = 9.366$, $F(.99; 9, 12) = 4.39$.

If $F^* \leq 4.39$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .0003

d. $\bar{Y}_{1..} = 1.88750$, $\bar{Y}_{2..} = 2.21875$, $\bar{Y}_{3..} = 2.38125$, $\hat{L}_1 = \bar{Y}_{1..} - \bar{Y}_{2..} = -.33125$, $\hat{L}_2 = \bar{Y}_{1..} - \bar{Y}_{3..} = -.49375$, $\hat{L}_3 = \bar{Y}_{2..} - \bar{Y}_{3..} = -.16250$, $s\{\hat{L}_i\} = .071116$ $(i = 1, 2, 3)$, $q(.95; 3, 9) = 3.95$, $T = 2.793$

$-.33125 \pm 2.793(.071116)$ $\quad -.530 \leq L_1 \leq -.133$
$-.49375 \pm 2.793(.071116)$ $\quad -.692 \leq L_2 \leq -.295$
$-.16250 \pm 2.793(.071116)$ $\quad -.361 \leq L_3 \leq .036$

f. <u>For σ^2</u>: $\hat{\sigma}^2 = .00904$, $c_1 = .5$, $c_2 = -.5$, $MS_1 = .020225$, $MS_2 = .0021583$, $df_1 = 9$, $df_2 = 12$, $F_1 = F(.95; 9, \infty) = 1.88$, $F_2 = F(.95; 12, \infty) = 1.75$, $F_3 =$

$F(.95; \infty, 9) = 2.71$, $F_4 = F(.95; \infty, 12) = 2.30$, $F_5 = F(.95; 9, 12) = 2.80$, $F_6 = F(.95; 12, 9) = 3.07$, $G_1 = .4681$, $G_2 = .4286$, $G_3 = -.05996$, $G_4 = -.1210$, $H_L = .00487$, $H_U = .0173$, $.00904 - .00487$, $.00904 + .0173$, $.0042 \leq \sigma^2 \leq .0263$

<u>For σ_η^2:</u> $\hat{\sigma}_\eta^2 = .00216$, $df = 12$, $\chi^2(.05; 12) = 5.23$, $\chi^2(.95; 12) = 21.03$,

$$.0012 = \frac{12(.00216)}{21.03} \leq \sigma_\eta^2 \leq \frac{12(.00216)}{5.23} = .0050$$

Chapter 27

REPEATED MEASURES AND RELATED DESIGNS

27.3. a. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	2.5556	−1.9444	−3.7778	2.4722	−2.7778	3.4722
2	−.1111	.3889	−.4444	2.8056	−3.4444	.8056
3	−2.9444	5.5556	3.7222	−3.0278	−2.2778	−1.0278
4	−2.7778	.7222	−2.1111	1.1389	1.8889	1.1389
5	−2.4444	−3.9444	1.2222	1.4722	1.2222	2.4722
6	−.1111	3.3889	−.4444	−3.9944	.5556	−.1944
7	−.9444	−2.4444	−1.2778	−2.0278	2.7222	3.9722
8	2.3889	−2.1111	4.0556	−1.6944	.0556	−2.6944
9	−.6111	−5.1111	.0556	−1.6944	3.0556	4.3056
10	1.5556	1.0556	−3.7778	3.4722	−.7778	−1.5278
11	1.3889	.8889	.0556	1.3056	1.0556	−4.6944
12	2.0556	3.5556	2.7222	−1.0278	−1.2778	−6.0278

$r = .995$

d. H_0: $D = 0$, H_a: $D \neq 0$. $SSTR.S = 467.3889$, $SSTR.S^* = 8.7643$, $SSRem^* = 458.6246$, $F^* = (8.7643/1) \div (458.6246/54) = 1.032$, $F(.995; 1, 54) = 8.567$. If $F^* \leq 8.567$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .31

27.4. a.

Source	SS	df	MS
Subjects	1,197.4444	11	108.8586
Doses	5,826.2778	5	1,165.2556
Error	467.3889	55	8.4980
Total	7,491.1111	71	

b. H_0: all τ_j equal zero ($j = 1, ..., 6$), H_a: not all τ_j equal zero. $F^* = 1,165.2556/8.4980 = 137.12$, $F(.99; 5, 55) = 3.37$. If $F^* \leq 3.37$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $\bar{Y}_{.1} = 14.6667$, $\bar{Y}_{.2} = 19.1667$, $\bar{Y}_{.3} = 23.0000$, $\bar{Y}_{.4} = 28.7500$, $\bar{Y}_{.5} = 35.0000$, $\bar{Y}_{.6} = 40.7500$, $\hat{L}_1 = \bar{Y}_{.1} - \bar{Y}_{.2} = -4.5000$, $\hat{L}_2 = \bar{Y}_{.2} - \bar{Y}_{.3} = -3.8333$, $\hat{L}_3 = \bar{Y}_{.3} - \bar{Y}_{.4} =$

-5.7500, $\hat{L}_4 = \bar{Y}_{.4} - \bar{Y}_{.5} = -6.2500$, $\hat{L}_5 = \bar{Y}_{.5} - \bar{Y}_{.6} = -5.7500$, $s\{\hat{L}_i\} = 1.1901$ $(i = 1, ..., 5)$, $B = t(.995; 55) = 2.668$

$-4.5000 \pm 2.668(1.1901)$	$-7.6752 \leq L_1 \leq -1.3248$
$-3.8333 \pm 2.668(1.1901)$	$-7.0085 \leq L_2 \leq -.6581$
$-5.7500 \pm 2.668(1.1901)$	$-8.9252 \leq L_3 \leq -2.5748$
$-6.2500 \pm 2.668(1.1901)$	$-9.4252 \leq L_4 \leq -3.0748$
$-5.7500 \pm 2.668(1.1901)$	$-8.9252 \leq L_5 \leq -2.5748$

d. $\hat{E} = 2.83$

27.5. a. $$Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + \rho_3 X_{ij3} + \rho_4 X_{ij4} + \rho_5 X_{ij5} + \rho_6 X_{ij6} + \rho_7 X_{ij7} + \rho_8 X_{ij8} + \rho_9 X_{ij9} + \rho_{10} X_{ij10} + \rho_{11} X_{ij11} + \gamma_1 x_{ij} + \gamma_2 x_{ij}^2 + \gamma_3 x_{ij}^3 + \epsilon_{ij}$$

$$X_{ij1} = \begin{cases} 1 & \text{if experimental unit from block 1} \\ -1 & \text{if experimental unit from block 12} \\ 0 & \text{otherwise} \end{cases}$$

X_{ij2}, ..., X_{ij11} are defined similarly

$$x_{ij} = \begin{cases} -.97 & \text{if experimental unit received treatment 1} \\ -.77 & \text{if experimental unit received treatment 2} \\ -.57 & \text{if experimental unit received treatment 3} \\ -.07 & \text{if experimental unit received treatment 4} \\ .43 & \text{if experimental unit received treatment 5} \\ 1.93 & \text{if experimental unit received treatment 6} \end{cases}$$

b. $$\hat{Y} = 30.3903 + 3.7778X_1 + 4.4444X_2 + .2778X_3 - 2.8889X_4 - 5.2222X_5 + 3.4444X_6 - 4.7222X_7 + 2.9444X_8 + 3.9444X_9 - 8.2222X_{10} + 1.9444X_{11} + 11.5329x - 4.0297x^2 + .4353x^3$$

c. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	2.2076	−1.6998	−3.2045	1.6591	−2.4167	3.4543
2	−.4591	.6335	.1288	1.9925	−3.0834	.7877
3	−3.2924	5.8002	4.2955	−3.8409	−1.9167	−1.0457
4	−3.1257	.9668	−1.5378	.3258	2.2499	1.1210
5	−2.7924	−3.6998	1.7955	.6591	1.5833	2.4543
6	−.4591	3.6335	.1288	−4.0075	.9166	−.2123
7	−1.2934	−2.1998	−.7045	−2.8409	3.0833	3.9543
8	2.0409	−1.8665	4.6288	−2.5075	.4166	−2.7123
9	−.9591	−4.8665	.6288	−2.5075	3.4166	4.2877
10	1.2076	1.3002	−3.2045	2.6591	−.4167	−1.5457
11	1.0409	1.1335	.6288	.4925	1.4166	−4.7123
12	1.7076	3.8002	3.2955	−1.8409	−.9167	−6.0457

d. H_0: $\gamma_3 = 0$, H_a: $\gamma_3 \neq 0$. $SSE(F) = 483.0053$, $SSE(R) = 484.8980$,

$F^* = (1.8927/1) \div (483.0053/57) = .223$, $F(.95; 1, 57) = 4.01$.

If $F^* \leq 4.01$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .64$

27.6. a. e_{ij}:

i	$j=1$	$j=2$	$j=3$
1	-1.2792	$-.2417$	1.5208
2	$-.8458$	.6917	.1542
3	.6208	.0583	$-.6792$
4	.5542	.1917	$-.7458$
5	.5208	$-.3417$	$-.1792$
6	$-.1458$	.3917	$-.2458$
7	.9875	$-.7750$	$-.2125$
8	$-.4125$	.0250	.3875

$r = .992$

d. H_0: $D = 0$, H_a: $D \neq 0$. $SSTR.S = 9.5725$, $SSTR.S^* = 2.9410$, $SSRem^* = 6.6315$, $F^* = (2.9410/1) \div (6.6315/13) = 5.765$, $F(.99; 1, 13) = 9.07$. If $F^* \leq 9.07$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .032

27.7. a.

Source	SS	df	MS
Stores	745.1850	7	106.4550
Prices	67.4808	2	33.7404
Error	9.5725	14	.68375
Total	822.2383	23	

b. H_0: all τ_j equal zero ($j = 1, 2, 3$), H_a: not all τ_j equal zero. $F^* = 33.7404/.68375 = 49.346$, $F(.95; 2, 14) = 3.739$. If $F^* \leq 3.739$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $\bar{Y}_{.1} = 55.4375$, $\bar{Y}_{.2} = 53.6000$, $\bar{Y}_{.3} = 51.3375$, $\hat{L}_1 = \bar{Y}_{.1} - \bar{Y}_{.2} = 1.8375$, $\hat{L}_2 = \bar{Y}_{.1} - \bar{Y}_{.3} = 4.1000$, $\hat{L}_3 = \bar{Y}_{.2} - \bar{Y}_{.3} = 2.2625$, $s\{\hat{L}_i\} = .413446$ ($i = 1, 2, 3$), $q(.95; 3, 14) = 3.70$, $T = 2.616$

$1.8375 \pm 2.616(.413446)$ $\quad .756 \leq L_1 \leq 2.919$
$4.1000 \pm 2.616(.413446)$ $\quad 3.018 \leq L_2 \leq 5.182$
$2.2625 \pm 2.616(.413446)$ $\quad 1.181 \leq L_3 \leq 3.344$

d. $\hat{E} = 48.08$

27.8. H_0: all τ_j equal zero ($j = 1, ..., 6$), H_a: not all τ_j equal zero.

$MSTR = 39.8583$, $MSTR.S = .2883$, $F_R^* = 39.8583/.2883 = 138.24$,

$F(.99; 5, 25) = 3.855$. If $F_R^* \leq 3.855$ conclude H_0, otherwise H_a. Conclude H_a.

27.9. H_0: all τ_j equal zero ($j = 1, 2, 3$), H_a: not all τ_j equal zero. $MSTR = 8$,

$MSTR.S = 0$, $F_R^* = 8/0$. Note: Nonparametric F test results in $SSTR.S = 0$ and therefore should not be used.

27.10. a. H_0: all τ_j equal zero ($j = 1, ..., 5$), H_a: not all τ_j equal zero.

$MSTR = 15.8500$, $MSTR.S = 1.0167$, $F_R^* = 15.8500/1.0167 = 15.59$, $F(.95; 4, 36) = 2.63$. If $F_R^* \leq 2.63$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

b. $\bar{R}_{.1} = 4.0$, $\bar{R}_{.2} = 1.4$, $\bar{R}_{.3} = 2.1$, $\bar{R}_{.4} = 3.1$, $\bar{R}_{.5} = 4.4$, $B = z(.995) = 2.576$, $B[r(r+1)/6n]^{1/2} = 1.82$

Group 1: B, C, D

Group 2: A, D, E

c. $W = .634$

27.11. a. e_{ijk}:

	$j=1$		$j=2$	
i	$k=1$	$k=2$	$k=1$	$k=2$
1	−1.9167	1.9167	−2.3333	2.3333
2	−.4167	.4167	.6667	−.6667
3	1.5833	−1.5833	−.3333	.3333
4	.0833	−.0833	.6667	−.6667
5	1.0833	−1.0833	−.3333	.3333
6	−.4167	.4167	1.6667	−1.6667

$r = .994$

27.12. a.

Source	SS	df	MS
A (incentive stimulus)	975.38	1	975.38
$S(A)$	148.75	10	14.875
B (problem type)	513.37	1	513.37
AB interactions	155.04	1	155.04
$B.S(A)$(Error)	34.08	10	3.408
Total	1826.63	23	

b. $\bar{Y}_{.11} = 12.667$, $\bar{Y}_{.12} = 16.833$, $\bar{Y}_{.21} = 20.333$, $\bar{Y}_{.22} = 34.667$

c. H_0: all $(\alpha\beta)_{jk}$ equal zero, H_a: not all $(\alpha\beta)_{jk}$ equal zero.

$F^* = 155.04/3.408 = 45.49$, $F(.95; 1, 10) = 4.96$.

If $F^* \leq 4.96$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $T = q(.95; 2, 10)/\sqrt{2} = 2.227$, $s^2\{\hat{D}\} = 2(3.408)/6 = 1.136$, $s\{\hat{D}\} = 1.0658$

$-4.17 \pm 2.227(1.0658) \qquad -6.54 \leq L_1 \leq -1.80$
$-14.33 \pm 2.227(1.0658) \qquad -16.70 \leq L_2 \leq -11.96$

e. $df_{adj} = \dfrac{[34.08 + 148.75]^2}{34.08^2/10 + 148.75^2/10} = 14.35$, $T = q(.95; 2, 14)/\sqrt{2} = 2.143$

MS(Within Treatments)=(34.08+148.75)/20=9.1415

$s^2\{\hat{D}\} = 2(9.1415)/6 = 3.0472$, $s\{\hat{D}\} = 1.7456$

$-7.67 \pm 2.143(1.7456) \qquad -11.41 \leq L_1 \leq -3.93$
$-17.83 \pm 2.143(1.7456) \qquad -21.57 \leq L_2 \leq -14.09$

27.13. a. e_{ijk}:

		$k=1$	$k=2$	$k=3$	$k=4$
	$i=1$	9.250	−8.750	1.250	−1.750
	$i=2$	−11.750	−2.750	15.250	−.750
$j=1$	$i=3$	7.750	−5.250	5.750	−8.250
	$i=4$	−5.250	16.750	−22.250	10.750
	$i=1$	3.625	−3.125	−13.875	13.375
$j=2$	$i=2$	15.375	6.625	7.875	−29.875
	$i=3$	−8.375	−3.125	−3.875	15.375
	$i=4$	−10.625	−.375	9.875	1.125

$r = .981$

27.14. a. H_0: $\sigma^2\{\rho_{i(1)}\} = \sigma^2\{\rho_{i(2)}\}$, H_a: $\sigma^2\{\rho_{i(1)}\} \neq \sigma^2\{\rho_{i(2)}\}$.

$SSS(A_1) = 1,478,757.00$, $SSS(A_2) = 1,525,262.25$,

$H^* = (1,525,262.25/3) \div (1,478,757.00/3) = 1.03$, $H(.99; 2, 3) = 47.5$.

If $H^* \leq 47.5$ conclude H_0, otherwise H_a. Conclude H_0.

b. H_0: $\sigma^2\{\epsilon_{1jk}\} = \sigma^2\{\epsilon_{2jk}\}$, H_a: $\sigma^2\{\epsilon_{1jk}\} \neq \sigma^2\{\epsilon_{2jk}\}$.

$SSB.S(A_1) = 1,653.00$, $SSB.S(A_2) = 2,172.25$,

$H^* = (2,172.25/9) \div (1,653.00/9) = 1.31$, $H(.99; 2, 9) = 6.54$.

If $H^* \leq 6.54$ conclude H_0, otherwise H_a. Conclude H_0.

27.15. a.

Source	SS	df	MS
A (type display)	266,085.1250	1	266,085.1250
$S(A)$	3,004,019.2500	6	500,669.8750
B (time)	53,321.6250	3	17,773.8750
AB interactions	690.6250	3	230.2083
Error	3,825.2500	18	212.5139
Total	3,327,941.8750	31	

b. $\bar{Y}_{.11} = 681.500$, $\bar{Y}_{.12} = 696.500$, $\bar{Y}_{.13} = 671.500$, $\bar{Y}_{.14} = 785.500$,

$\bar{Y}_{.21} = 508.500$, $\bar{Y}_{.22} = 512.250$, $\bar{Y}_{.23} = 496.000$, $\bar{Y}_{.24} = 588.750$

c. H_0: all $(\alpha\beta)_{jk}$ equal zero, H_a: not all $(\alpha\beta)_{jk}$ equal zero.

$F^* = 230.2083/212.5139 = 1.08$, $F(.975; 3, 18) = 3.95$.

If $F^* \leq 3.95$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .38

d. H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_j equal zero.

$F^* = 266,085.1250/500,669.8750 = .53$, $F(.975; 1, 6) = 8.81$.

If $F^* \leq 8.81$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .49

H_0: all β_k equal zero ($k = 1, ..., 4$), H_a: not all β_k equal zero.

$F^* = 17,773.8750/212.5139 = 83.636$, $F(.975; 3, 18) = 3.95$.

If $F^* \leq 3.95$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. $\bar{Y}_{.1.} = 708.750$, $\bar{Y}_{.2.} = 526.375$, $\bar{Y}_{..1} = 595.000$, $\bar{Y}_{..2} = 604.375$, $\bar{Y}_{..3} = 583.750$, $\bar{Y}_{..4} = 687.125$, $\hat{L}_1 = 182.375$, $\hat{L}_2 = -9.375$, $\hat{L}_3 = 20.625$, $\hat{L}_4 = -103.375$, $s\{\hat{L}_1\} = 250.1674$, $s\{\hat{L}_i\} = 7.2889$ $(i = 2, 3, 4)$, $B_1 = t(.9875; 6) = 2.969$, $B_i = t(.9875; 18) = 2.445$ $(i = 2, 3, 4)$

$$\begin{array}{rr} 182.375 \pm 2.969(250.1674) & -560.372 \leq L_1 \leq 925.122 \\ -9.375 \pm 2.445(7.2889) & -27.196 \leq L_2 \leq 8.446 \\ 20.625 \pm 2.445(7.2889) & 2.804 \leq L_3 \leq 38.446 \\ -103.375 \pm 2.445(7.2889) & -121.196 \leq L_4 \leq -85.554 \end{array}$$

27.16. a. e_{ijk}:

	$j = 1$		$j = 2$	
i	$k = 1$	$k = 2$	$k = 1$	$k = 2$
1	−.05833	.05833	.05833	−.05833
2	−.05833	.05833	.05833	−.05833
3	−.03333	.03333	.03333	−.03333
4	−.00833	.00833	.00833	−.00833
5	.21667	−.21667	−.21667	.21667
6	−.05833	.05833	.05833	−.05833

$r = .9685$

27.17. a.

Source	SS	df	MS
Subjects	1.0533	5	.2107
A (problem)	16.6667	1	16.6667
B (model)	72.1067	1	72.1067
AB	3.6817	1	3.6817
AS	.5983	5	.1197
BS	.1783	5	.0357
ABS	.2333	5	.0467
Total	94.5183	23	

b. $\bar{Y}_{.11} = 3.367$, $\bar{Y}_{.12} = 7.617$, $\bar{Y}_{.21} = 2.483$, $\bar{Y}_{.22} = 5.167$

c. H_0: all $(\alpha\beta)_{jk}$ equal zero, H_a: not all $(\alpha\beta)_{jk}$ equal zero.

$F^* = 3.6817/.0467 = 78.84$, $F(.99; 1, 5) = 16.3$.

If $F^* \leq 16.3$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $\hat{L}_1 = 4.250$, $\hat{L}_2 = 2.684$, $\hat{L}_3 = -1.566$,

$s\{\hat{L}_i\} = .1248$ $(i = 1, 2)$, $s\{\hat{L}_3\} = .1765$, $B = t(.9917; 5) = 3.538$

$$\begin{array}{rr} 4.250 \pm 3.538(.1248) & 3.808 \leq L_1 \leq 4.692 \\ 2.684 \pm 3.538(.1248) & 2.242 \leq L_2 \leq 3.126 \\ -1.566 \pm 3.538(.1765) & -2.190 \leq L_3 \leq -.942 \end{array}$$

27.18. a. e_{ijk}:

i	$j=1$, $k=1$	$j=1$, $k=2$	$j=2$, $k=1$	$j=2$, $k=2$
1	$-.045$	.045	.045	$-.045$
2	$-.120$	.120	.120	$-.120$
3	.080	$-.080$	$-.080$	.080
4	$-.045$	.045	.045	$-.045$
5	.080	$-.080$	$-.080$	.080
6	.055	$-.055$	$-.055$	.055
7	.030	$-.030$	$-.030$	.030
8	$-.045$	.045	.045	$-.045$
9	.055	$-.055$	$-.055$	.055
10	$-.045$	.045	.045	$-.045$

$r = .973$

27.19. a.

Source	SS	df	MS
Subjects	154.579	9	17.175
A	3.025	1	3.025
B	14.449	1	11.449
AB	.001	1	.001
AS	2.035	9	.226
BS	5.061	9	.562
ABS	.169	9	.019
Total	176.319	39	

b. $\bar{Y}_{.11} = 3.93$, $\bar{Y}_{.12} = 5.01$, $\bar{Y}_{.21} = 4.49$, $\bar{Y}_{.22} = 5.55$

c. H_0: all $(\alpha\beta)_{jk}$ equal zero, H_a: not all $(\alpha\beta)_{jk}$ equal zero.

$F^* = .001/.019 = .05$, $F(.995; 1, 9) = 13.6$.

If $F^* \leq 13.6$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .82

d. H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_j equal zero.

$F^* = 3.025/.226 = 13.38$, $F(.95; 1, 9) = 5.12$.

If $F^* \leq 13.6$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .005

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_k equal zero.

$F^* = 11.449/.562 = 20.36$, $F(.95; 1, 9) = 5.12$.

If $F^* \leq 13.6$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .001

e. $\hat{L}_1 = .56$, $\hat{L}_2 = 1.08$, $\hat{L}_3 = -.52$, $\hat{L}_4 = 1.62$,

$s\{\hat{L}_i\} = .0613$ $(i = 1, ..., 4)$, $B = t(.99375; 9) = 3.11$

$$
\begin{array}{rr}
.56 \pm 3.11(.0613) & .37 \leq L_1 \leq .75 \\
1.08 \pm 3.11(.0613) & .89 \leq L_2 \leq 1.27 \\
-.52 \pm 3.11(.0613) & -.71 \leq L_3 \leq -.33 \\
1.62 \pm 3.11(.0613) & 1.43 \leq L_4 \leq 1.81
\end{array}
$$

27.20. a. e_{ijk}:

i	j	$k=1$	$k=2$
1	1	$-.6$	$.6$
	2	-1.7	1.7
2	1	$.4$	$-.4$
	2	1.3	-1.3
3	1	$-.6$	$.6$
	2	$.3$	$-.3$
4	1	$.4$	$-.4$
	2	$-.2$	$.2$
5	1	$.4$	$-.4$
	2	$.3$	$-.3$

$r = .981$

27.21. a.

Source	SS	df	MS
Whole plots			
Irrigation method (A)	$1,394.45$	1	$1,394.45$
Whole-plot error	837.60	8	104.70
Split plots			
Fertilizer (B)	68.45	1	68.45
AB Interactions	$.05$	1	$.05$
Split-plot error	12.00	8	1.50
Total	$2,312.55$	19	

b. $\bar{Y}_{.11} = 35.4$, $\bar{Y}_{.21} = 52.2$, $\bar{Y}_{.12} = 39.2$, $\bar{Y}_{.22} = 55.8$

c. H_0: all $(\alpha\beta)_{jk}$ equal zero, H_a: not all $(\alpha\beta)_{jk}$ equal zero. $F^* = .05/1.50 = .033$, $F(.95; 1, 8) = 5.32$. If $F^* \leq 5.32$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .86$

d. H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_j equal zero. $F^* = 1,394.45/104.70 = 13.32$, $F(.95; 1, 8) = 5.32$. If $F^* \leq 5.32$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= .006$

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_k equal zero. $F^* = 68.45/1.50 = 45.63$, $F(.95; 1, 8) = 5.32$. If $F^* \leq 5.32$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= .0001$

e. $\bar{Y}_{.1.} = 37.3$, $\bar{Y}_{.2.} = 54.0$, $\bar{Y}_{..1} = 43.8$, $\bar{Y}_{..2} = 47.5$, $\hat{L}_1 = -16.7$, $\hat{L}_2 = -3.7$, $s\{\hat{L}_1\} = 4.5760$, $s\{\hat{L}_2\} = .5477$, $B_1 = t(.975; 8) = 2.306$, $B_2 = t(.975; 8) = 2.306$

$$-16.7 \pm 2.306(4.5760) \qquad -27.252 \leq L_1 \leq -6.148$$
$$-3.7 \pm 2.306(.5477) \qquad -4.963 \leq L_2 \leq -2.437$$

27.22.

$$\begin{aligned}\sum\sum(Y_{ij} - \bar{Y}_{..})^2 &= \sum\sum[(Y_{ij} - \bar{Y}_{i.}) + (\bar{Y}_{i.} - \bar{Y}_{..})]^2 \\ &= \sum\sum(Y_{ij} - \bar{Y}_{i.})^2 + \sum\sum(\bar{Y}_{i.} - \bar{Y}_{..})^2 + 2\sum\sum(Y_{ij} - \bar{Y}_{i.})(\bar{Y}_{i.} - \bar{Y}_{..}) \\ &= \sum\sum(Y_{ij} - \bar{Y}_{i.})^2 + r\sum(\bar{Y}_{i.} - \bar{Y}_{..})^2\end{aligned}$$

Cross-product term equals zero by argument similar to that given in Section 16.5.

27.23.

$$\begin{array}{cc} & j' \\ j & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\left[\begin{array}{cccccc} 27.3333 & 20.8788 & 23.0909 & 19.1818 & 16.7273 & 17.0909 \\ & 29.4242 & 23.4545 & 14.3182 & 12.6364 & 11.0455 \\ & & 30.9091 & 14.7273 & 18.2727 & 16.4545 \\ & & & 18.3864 & 11.1818 & 15.0227 \\ & & & & 17.4545 & 16.8182 \\ & & & & & 27.8409 \end{array}\right] \end{array}$$

27.24.

$$\begin{array}{cc} & j' \\ j & \begin{array}{c} 1 \\ 2 \\ 3 \end{array}\left[\begin{array}{ccc} 29.6084 & 33.0114 & 34.0598 \\ & 37.5886 & 38.7000 \\ & & 40.6255 \end{array}\right] \end{array}$$

27.25. a. $Y_{ij} = \mu_{..} + \rho_i + \tau_j + \epsilon_{(ij)}$

b. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$
1	.02083	−.00917	.00833	−.02000
2	.00083	−.00917	−.00167	.01000
3	.00083	.00083	.00833	−.01000
4	.04083	.02083	−.02167	−.04000
5	−.03167	.04833	−.04417	.02750
6	.01833	−.03167	.05583	−.04250
7	−.00167	−.00167	−.02417	.02750
8	−.00167	.00833	−.01417	.00750
9	−.02417	−.00417	.02333	.00500
10	−.00417	−.04417	.02333	.02500
11	.03333	−.01667	−.03917	.02250
12	−.05167	.03833	.02583	−.01250

$r = .994$

27.26. a.

Source	SS	df	MS
Subjects	1.80012	11	.163647
Dosage	.72615	3	.242050
Error	.03220	33	.000976
Total	2.55847	47	

b. H_0: all τ_j equal zero ($j = 1, ..., 4$), H_a: not all τ_j equal zero.

$F^* = .242050/.000976 = 248.0$, $F(.95; 3, 33) = 2.89$.

If $F^* \leq 2.89$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $\bar{Y}_{.1} = 1.03833$, $\bar{Y}_{.2} = 1.05833$, $\bar{Y}_{.3} = 1.06083$, $\bar{Y}_{.4} = .76917$, $\hat{L}_1 = \bar{Y}_{.1} - \bar{Y}_{.2} = -.02000$, $\hat{L}_2 = \bar{Y}_{.2} - \bar{Y}_{.3} = -.00250$, $\hat{L}_3 = \bar{Y}_{.3} - \bar{Y}_{.4} = .29166$, $s\{\hat{L}_i\} = .01275$ $(i = 1, 2, 3)$, $B = t(.983; 33) = 2.22$

$$-.02000 \pm 2.22(.01275) \qquad -.048 \leq L_1 \leq .008$$
$$-.00250 \pm 2.22(.01275) \qquad -.031 \leq L_2 \leq .026$$
$$.29166 \pm 2.22(.01275) \qquad .263 \leq L_3 \leq .320$$

d. $Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + \rho_3 X_{ij3} + \rho_4 X_{ij4} + \rho_5 X_{ij5} + \rho_6 X_{ij6}$

$$+\rho_7 X_{ij7} + \rho_8 X_{ij8} + \rho_9 X_{ij9} + \rho_{10} X_{ij10} + \rho_{11} X_{ij11} + \gamma_1 x_{ij} + \gamma_2 x_{ij}^2 + \epsilon_{ij}$$

$$X_{ij1} = \begin{cases} 1 & \text{if experimental unit from subject 1} \\ -1 & \text{if experimental unit from subject 12} \\ 0 & \text{otherwise} \end{cases}$$

X_{ij2}, ..., X_{ij11} are defined similarly

$$x_{ij} = \begin{cases} -.825 & \text{if experimental unit received treatment 1} \\ -.325 & \text{if experimental unit received treatment 2} \\ .175 & \text{if experimental unit received treatment 3} \\ .975 & \text{if experimental unit received treatment 4} \end{cases}$$

$$\hat{Y} = 1.06647 - .24917X_1 - .26917X_2 - .23917X_3 - .12917X_4 + .02333X_5 - .09667X_6$$
$$-.05667X_7 + .13333X_8 + .18583X_9 + .21583X_{10} + .15833X_{11} - .11341x - .19192x^2$$

e. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$
1	.02976	−.03389	.02842	−.02429
2	.00976	−.03389	.01842	.00571
3	.00976	−.02389	.02842	−.01429
4	.04976	−.00389	−.00158	−.04429
5	−.02274	.02361	−.02408	.02321
6	.02726	−.05639	.07592	−.04679
7	.00726	−.02639	−.00408	.02321
8	.00726	−.01639	.00592	.00321
9	−.01524	−.02889	.04342	.00071
10	.00476	−.06889	.04342	.02071
11	.04226	−.04139	−.01908	.01821
12	−.04274	.01361	.04592	−.01679

f. H_0: $\gamma_2 = 0$, H_a: $\gamma_2 \neq 0$. $SSE(F) = .0456$, $SSE(R) = .2816$,

$F^* = (.2360/1) \div (.0456/34) = 175.96$, $F(.99; 1, 34) = 7.44$.

If $F^* \leq 7.44$ conclude H_0, otherwise H_a. Conclude H_a.

27.27. Note: The subscript for subjects here is l instead of the usual i and the subscripts for factors A, B, and C are i, j, and k, respectively.

a. $Y_{ijklm} = \mu_{....} + \alpha_i + \beta_j + \gamma_k + \rho_{l(ik)} + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \epsilon_{m(ijkl)}$

b. $r = .990$

27.28. a.

Source	SS	df	MS
A (initial lever press rate)	7.99586	2	3.99793
B (dosage level)	25.90210	3	8.63403
C (reinforcement schedule)	59.74172	1	59.74172
AB interactions	.35167	6	.05861
AC interactions	.09465	2	.04733
BC interactions	12.36104	3	4.12035
ABC interactions	.37040	6	.06173
$S(AC)$ (rats, within AC)	1.64179	18	.09121
Error	.36711	150	.00245
Total	108.82634	191	

$$\begin{aligned}
E\{MSA\} &= 64\sum\alpha_i^2/2 + 8\sigma_\rho^2 + \sigma^2\\
E\{MSB\} &= 48\sum\beta_j^2/3 + \sigma^2\\
E\{MSC\} &= 96\sum\gamma_k^2/1 + 8\sigma_\rho^2 + \sigma^2\\
E\{MSS(AC)\} &= 8\sigma_\rho^2 + \sigma^2\\
E\{MSAB\} &= 16\sum\sum(\alpha\beta)_{ij}^2/6 + \sigma^2\\
E\{MSAC\} &= 32\sum\sum(\alpha\gamma)_{ik}^2/2 + 8\sigma_\rho^2 + \sigma^2\\
E\{MSBC\} &= 24\sum\sum(\beta\gamma)_{jk}^2/3 + \sigma^2\\
E\{MSABC\} &= 8\sum\sum\sum(\alpha\beta\gamma)_{ijk}^2/6 + \sigma^2\\
E\{MSE\} &= \sigma^2
\end{aligned}$$

b. H_0: all $(\alpha\beta\gamma)_{ijk}$ equal zero, H_a: not all $(\alpha\beta\gamma)_{ijk}$ equal zero.

$F^* = .06173/.00245 = 25.196$, $F(.99; 6, 150) = 2.92$.

If $F^* \leq 2.92$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

c. $\bar{Y}_{ijk..}$:

k	i	$j=1$	$j=2$	$j=3$	$j=4$
1	1	.81375	.82375	.83625	.53500
	2	1.05375	1.06625	1.05625	.77375
	3	1.25500	1.25625	1.27125	1.00750
2	1	2.15125	2.33625	1.88750	.88125
	2	2.59250	2.58375	2.21875	1.01250
	3	3.04750	2.75125	2.38125	1.29250

27.29. a.

F_R^*	$P(F_R^*)$
0	12/216
.25	90/216
1.00	36/216
1.60	36/216
7.00	36/216
Undefined	6/216

b. $F(.90; 2, 4) = 4.32$, $P(F_R^* \leq 7.00) = .972$, $P(F_R^* \leq 1.60) = .806$

Chapter 28

BALANCED INCOMPLETE BLOCK, LATIN SQUARE, AND RELATED DESIGNS

28.3. One such design, for which $n_b = 3$, $n = 2$, and $n_p = 1$:

1	2
1	3
2	3

28.4. For $r = 7, r_b = 5$, a BIBD exists for $n_b = \dfrac{7!}{5!(7-5)!} = 21$.

Since $n_b r_b = nr$, $n = 21(5)/7 = 15$.

Since $n_p(r-1) = n(r_b - 1)$, $n_p = 15(5-1)/(7-1) = 10$.

28.5. For $r = 8, r_b = 3$, a BIBD exists for $n_b = \dfrac{8!}{3!(8-3)!} = 56$.

Since $n_b r_b = nr$, $n = 56(3)/8 = 21$.

Since $n_p(r-1) = n(r_b - 1)$, $n_p = 21(3-1)/(8-1) = 6$.

28.6. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$	$j=9$
1			$-.704$	.185				.519	
2		.222		$-.111$					$-.111$
3			.556			-1.000			.444
4	$-.481$				$-.259$				.741
5		$-.926$				.296	.630		
6				$-.519$	.481	.037			
7							.815	.259	-1.074
8	.222			.444			$-.667$		
9	$-.111$					.667		$-.556$	
10		$-.222$			.444			$-.222$	
11			1.444		$-.667$		$-.778$		
12	.370	.926	-1.296						

$r = .990$

28.7. a. $\hat{\mu}_{..} = 19.36$, $\hat{\tau}_1 = .33$, $\hat{\tau}_2 = -2.22$, $\hat{\tau}_3 = -6.00$, $\hat{\tau}_4 = -12.89$, $\hat{\tau}_5 = 6.11$, $\hat{\tau}_6 = 3.56$, $\hat{\tau}_7 = 1.22$, $\hat{\tau}_8 = -.22$, $\hat{\tau}_9 = 10.11$.

$\hat{\mu}_{.1} = 19.69$, $\hat{\mu}_{.2} = 17.14$, $\hat{\mu}_{.3} = 13.36$, $\hat{\mu}_{.4} = 6.47$, $\hat{\mu}_{.5} = 25.47$, $\hat{\mu}_{.6} = 22.92$, $\hat{\mu}_{.7} = 20.58$, $\hat{\mu}_{.8} = 19.14$, $\hat{\mu}_{.9} = 29.47$.

b. H_0: all τ_j equal zero ($j = 1, 2, \ldots, 8$), H_a: not all τ_j equal zero. $SSE(F) = 14.519$, $SSE(R) = 1097.33$, $F^* = (1082.811/8) \div (14.519/16) = 149.2$, $F(.95; 8, 16) = 2.59$. If $F^* \leq 2.59$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. H_0: all ρ_i equal zero ($j = 1, 2, \ldots, 11$), H_a: not all ρ_i equal zero. $SSE(F) = 14.519$, $SSE(R) = 25.25$, $F^* = (10.731/11) \div (14.519/16) = 1.08$, $F(.95; 11, 16) = 2.46$. If $F^* \leq 2.46$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .43

d. $\hat{\mu}_{.5} = 25.47$, $s^2(\hat{\mu}_{.5}) = s^2(\hat{\mu}_{..}) + s^2(\hat{\tau}_5) = (.02778 + .29630).907 = .2939$, $B = t(.975; 16) = 2.120$, $25.47 \pm 2.120(.542)$, $24.32 \leq \mu_{.5} \leq 26.62$

e.

95% C.I.	lower	center	upper
$\mu_{.1} - \mu_{.2}$	$-.21$	2.56	5.32
$\mu_{.1} - \mu_{.3}$	3.57	6.33	9.10
$\mu_{.1} - \mu_{.4}$	10.46	13.22	15.99
$\mu_{.1} - \mu_{.5}$	-8.54	-5.78	-3.01
$\mu_{.1} - \mu_{.6}$	-5.99	-3.22	$-.46$
$\mu_{.1} - \mu_{.7}$	-3.66	$-.89$	1.88
$\mu_{.1} - \mu_{.8}$	-2.21	.56	3.32
$\mu_{.1} - \mu_{.9}$	-12.54	-9.78	-7.01
$\mu_{.2} - \mu_{.3}$	1.01	3.78	6.54
$\mu_{.2} - \mu_{.4}$	7.90	10.67	13.43
$\mu_{.2} - \mu_{.5}$	-11.10	-8.33	-5.57
$\mu_{.2} - \mu_{.6}$	-8.54	-5.78	-3.01
$\mu_{.2} - \mu_{.7}$	-6.21	-3.44	$-.68$
$\mu_{.2} - \mu_{.8}$	-4.77	-2.00	.77
$\mu_{.2} - \mu_{.9}$	-15.10	-12.33	-9.57
$\mu_{.3} - \mu_{.4}$	4.12	6.89	9.66
$\mu_{.3} - \mu_{.5}$	-14.88	-12.11	-9.35
$\mu_{.3} - \mu_{.6}$	-12.32	-9.56	-6.79
$\mu_{.3} - \mu_{.7}$	-9.99	-7.22	-4.46
$\mu_{.3} - \mu_{.8}$	-8.54	-5.78	-3.01
$\mu_{.3} - \mu_{.9}$	-18.88	-16.11	-13.35
$\mu_{.4} - \mu_{.5}$	-21.77	-19.00	-16.23
$\mu_{.4} - \mu_{.6}$	-19.21	-16.44	-13.68
$\mu_{.4} - \mu_{.7}$	-16.88	-14.11	-11.35
$\mu_{.4} - \mu_{.8}$	-15.43	-12.67	-9.90
$\mu_{.4} - \mu_{.9}$	-25.77	-23.00	-20.23
$\mu_{.5} - \mu_{.6}$	$-.21$	2.56	5.32
$\mu_{.5} - \mu_{.7}$	2.12	4.89	7.66
$\mu_{.5} - \mu_{.8}$	3.57	6.33	9.10
$\mu_{.5} - \mu_{.9}$	-6.77	-4.00	-1.23
$\mu_{.6} - \mu_{.7}$	$-.43$	2.33	5.10
$\mu_{.6} - \mu_{.8}$	1.01	3.78	6.54
$\mu_{.6} - \mu_{.9}$	-9.32	-6.56	-3.79
$\mu_{.7} - \mu_{.8}$	-1.32	1.44	4.21
$\mu_{.7} - \mu_{.9}$	-11.66	-8.89	-6.12
$\mu_{.8} - \mu_{.9}$	-13.10	-10.33	-7.57

28.8. a. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$
1	13.2083	8.8333	-22.0417	
2	-7.9167	4.7083		3.2083
3	-5.2917		-1.5417	6.8333
4		-13.5417	23.5833	-10.0417

$r = .995$

28.9. a. $\hat{\mu}_{..} = 297.667$, $\hat{\tau}_1 = -45.375$, $\hat{\tau}_2 = -41.000$, $\hat{\tau}_3 = 30.875$, $\hat{\tau}_4 = 55.500$

$\hat{\mu}_{.1} = 252.292$, $\hat{\mu}_{.2} = 256.667$, $\hat{\mu}_{.3} = 328.542$, $\hat{\mu}_{.4} = 353.167$

b. H_0: $\tau_1 = \tau_2 = \tau_3 = 0$, H_a: not all τ_j equal zero. $SSE(F) = 1750.9$, $SSE(R) = 22480$, $F^* = (20729.1/3) \div (1750.9/5) = 19.73$, $F(.95; 3, 5) = 5.41$. If $F^* \leq 5.41$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .003

c. H_0: $\rho_1 = \rho_2 = \rho_3 = 0$, H_a: not all ρ_i equal zero. $SSE(F) = 14.519$, $SSE(R) = 22789$, $F^* = (21038.1/3) \div (1750.9/5) = 20.03$, $F(.95; 3, 5) = 5.41$. If $F^* \leq 5.41$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .003

d. $\hat{\mu}_{.1} = 252.292$, $s^2(\hat{\mu}_{.1}) = s^2(\hat{\mu}_{..}) + s^2(\hat{\tau}_1) = (.08333 + .28125)350.2 = 127.68$, $B = t(.975; 5) = 2.571$, $252.292 \pm 2.571(11.30)$, $223.240 \leq \mu_{.1} \leq 281.344$

e.

95% C.I.	lower	center	upper
$\mu_{.1} - \mu_{.2}$	−64.19	−4.375	55.44
$\mu_{.1} - \mu_{.3}$	−136.07	−76.250	−16.43
$\mu_{.1} - \mu_{.4}$	−160.69	−100.875	−41.06
$\mu_{.2} - \mu_{.3}$	−131.70	−71.87	−12.06
$\mu_{.2} - \mu_{.4}$	−156.30	−96.50	−36.68
$\mu_{.3} - \mu_{.4}$	−84.44	−24.63	35.19

28.10. $r = 4$, and $r_b = 3$, $df_e = 4n - 4 - 4n/3 + 1 = 8n/3 - 3$.

Since $n_p = n(3-1)/(4-1) = 2n/3$, $\sigma^2\{\hat{D}_j\} = 2\sigma^2(3)/(4n_p) = 9\sigma^2/(4n)$

$$T\sigma\{\hat{D}_j\} = \frac{1}{\sqrt{2}}q[.95; 4, 8n/3 - 3]\sqrt{\frac{9\sigma^2}{4n}}$$

For $\sigma^2 = 2.0$ and $T\sigma\{\hat{D}_j\} \leq 1.5$, so we need to iterate to find n so that

$$n \geq q^2[.95; 4, 8n/3 - 3]$$

We iteratively find $n \geq 15$. Since design 2 in Table 28.1 has $n = 3$, we require that design 2 be repeated 5 times. Thus, $n = 15$, and $n_b = 20$.

28.11. $r = 5$, and $r_b = 4$, $df_e = 5n - 5 - 5n/4 + 1 = 15n/4 - 4$.

Since $n_p = n(4-1)/(5-1) = 3n/4$, $\sigma^2\{\hat{D}_j\} = 2\sigma^2(4)/(5n_p) = 32\sigma^2/(15n)$

$$T\sigma\{\hat{D}_j\} = \frac{1}{\sqrt{2}}q[.90; 5, 15n/4 - 4]\sqrt{\frac{32\sigma^2}{15n}}$$

For $\sigma^2 = 1.5$ and $T\sigma\{\hat{D}_j\} \leq 1.25$, so we need to iterate to find n so that

$$n \geq 1.024q^2[.90; 5, 15n/4 - 4]$$

We iteratively find $n \geq 14$. Since design 5 in Table 28.1 has $n = 4$, we require that design 2 be repeated 4 times. Thus, $n = 16$, and $n_b = 20$.

28.14. e_{ijk}:

i	$j=1$	$j=2$	$j=3$	$j=4$
1	−.1375	.0875	−.0125	.0625
2	−.0125	−.0125	.1625	−.1375
3	.1375	−.0875	−.0625	.0125
4	.0125	.0125	−.0875	.0625

$r = .986$

28.15. a. $\bar{Y}_{..1} = 1.725$, $\bar{Y}_{..2} = 1.900$, $\bar{Y}_{..3} = 2.175$, $\bar{Y}_{..4} = 2.425$

b.

Source	SS	df	MS
Rows (sales volumes)	5.98187	3	1.99396
Columns (locations)	.12188	3	.04062
Treatments (prices)	1.13688	3	.37896
Error	.11875	6	.01979
Total	7.35938	15	

H_0: all τ_k equal zero ($k = 1, ..., 4$), H_a: not all τ_k equal zero. $F^* = .37896/.01979 = 19.149$, $F(.95; 3, 6) = 4.76$. If $F^* \leq 4.76$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .002

c. $\hat{L}_1 = \bar{Y}_{..1} - \bar{Y}_{..2} = -.175$, $\hat{L}_2 = \bar{Y}_{..1} - \bar{Y}_{..3} = -.450$, $\hat{L}_3 = \bar{Y}_{..1} - \bar{Y}_{..4} = -.700$, $\hat{L}_4 = \bar{Y}_{..2} - \bar{Y}_{..3} = -.275$, $\hat{L}_5 = \bar{Y}_{..2} - \bar{Y}_{..4} = -.525$, $\hat{L}_6 = \bar{Y}_{..3} - \bar{Y}_{..4} = -.250$, $s\{\hat{L}_i\} = .09947$ ($i = 1, ..., 6$), $q(.90; 4, 6) = 4.07$, $T = 2.8779$

$-.175 \pm 2.8779(.09947)$ $\quad -.461 \leq L_1 \leq .111$
$-.450 \pm 2.8779(.09947)$ $\quad -.736 \leq L_2 \leq -.164$
$-.700 \pm 2.8779(.09947)$ $\quad -.986 \leq L_3 \leq -.414$
$-.275 \pm 2.8779(.09947)$ $\quad -.561 \leq L_4 \leq .011$
$-.525 \pm 2.8779(.09947)$ $\quad -.811 \leq L_5 \leq -.239$
$-.250 \pm 2.8779(.09947)$ $\quad -.536 \leq L_6 \leq .036$

28.16. a. $\hat{E}_1 = 21.1617$, $\hat{E}_2 = 1.2631$, $\hat{E}_3 = 25.9390$

28.17. e_{ijk}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	$-.88$	$-.68$	.92	.32	.32
2	.32	.12	$-.28$	.92	-1.08
3	.52	$-.68$	-1.08	.12	1.12
4	$-.68$	1.92	.52	$-.08$	-1.68
5	.72	$-.68$	$-.08$	-1.28	1.32

$r = .993$

28.18. a. $\bar{Y}_{..1} = 7.0$, $\bar{Y}_{..2} = 7.4$, $\bar{Y}_{..3} = 15.0$, $\bar{Y}_{..4} = 19.0$, $\bar{Y}_{..5} = 13.4$

b.

Source	SS	df	MS
Rows (executives)	220.16	4	55.040
Columns (months)	10.96	4	2.740
Treatments (reports)	527.36	4	131.840
Error	19.28	12	1.607
Total	777.76	24	

H_0: all τ_k equal zero ($k = 1, ..., 5$), H_a: not all τ_k equal zero. $F^* = 131.840/1.607 = 82.04$, $F(.99; 4, 12) = 5.41$. If $F^* \leq 5.41$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $\hat{L}_1 = \bar{Y}_{..1} - \bar{Y}_{..2} = -.4$, $\hat{L}_2 = \bar{Y}_{..1} - \bar{Y}_{..3} = -8.0$, $\hat{L}_3 = \bar{Y}_{..1} - \bar{Y}_{..4} = -12.0$, $\hat{L}_4 = \bar{Y}_{..1} - \bar{Y}_{..5} = -6.4$, $\hat{L}_5 = \bar{Y}_{..2} - \bar{Y}_{..3} = -7.6$, $\hat{L}_6 = \bar{Y}_{..2} - \bar{Y}_{..4} = -11.6$, $\hat{L}_7 = \bar{Y}_{..2} - \bar{Y}_{..5} = -6.0$, $\hat{L}_8 = \bar{Y}_{..3} - \bar{Y}_{..4} = -4.0$, $\hat{L}_9 = \bar{Y}_{..3} - \bar{Y}_{..5} = 1.6$, $\hat{L}_{10} = \bar{Y}_{..4} - \bar{Y}_{..5} = 5.6$, $s\{\hat{L}_i\} = .8017$ $(i = 1, ..., 10)$, $q(.95; 5, 12) = 4.51$, $T = 3.189$

$$\begin{array}{rr}
-.4 \pm 3.189(.8017) & -2.96 \leq L_1 \leq 2.16 \\
-8.0 \pm 3.189(.8017) & -10.56 \leq L_2 \leq -5.44 \\
-12.0 \pm 3.189(.8017) & -14.56 \leq L_3 \leq -9.44 \\
-6.4 \pm 3.189(.8017) & -8.96 \leq L_4 \leq -3.84 \\
-7.6 \pm 3.189(.8017) & -10.16 \leq L_5 \leq -5.04 \\
-11.6 \pm 3.189(.8017) & -14.16 \leq L_6 \leq -9.04 \\
-6.0 \pm 3.189(.8017) & -8.56 \leq L_7 \leq -3.44 \\
-4.0 \pm 3.189(.8017) & -6.56 \leq L_8 \leq -1.44 \\
1.6 \pm 3.189(.8017) & -.96 \leq L_9 \leq 4.16 \\
5.6 \pm 3.189(.8017) & 3.04 \leq L_{10} \leq 8.16
\end{array}$$

28.19. a. $\hat{E}_1 = 6.66$, $\hat{E}_2 = 1.14$, $\hat{E}_3 = 7.65$

28.20. $\phi = 3.399$, $1 - \beta \cong .99$

28.21. $\phi = 2.202$, $1 - \beta \cong .69$

28.22. e_{ijkl}:

i	$j=1$	$j=2$	$j=3$	$j=4$
1	.01625	−.01875	.01625	−.01375
2	.00625	.01875	−.05375	.02875
3	−.00875	−.03125	.03375	.00625
4	−.01375	.03125	.00375	−.02125

$r = .980$

28.23. a. $Y_{ijkl} = \mu_{...} + \rho_i + \kappa_j + \alpha_k + \beta_l + (\alpha\beta)_{kl} + \epsilon_{(ijkl)}$

b.

Source	SS	df	MS
Rows (subjects)	.03462	3	.01154
Columns (periods)	.00592	3	.00197
Treatments	.43333	3	.14444
X	.22801	1	.22801
Y	.19581	1	.19581
XY interactions	.00951	1	.00951
Error	.00904	6	.00151
Total	.48291	15	

H_0: all $(\alpha\beta)_{kl}$ equal zero, H_a: not all $(\alpha\beta)_{kl}$ equal zero. $F^* = .00951/.00151 = 6.298$, $F(.90; 1, 6) = 3.78$. If $F^* \leq 3.78$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= .046$

c. $\bar{Y}_{..1} = .0050$, $\bar{Y}_{..2} = .1950$, $\bar{Y}_{..3} = .1775$, $\bar{Y}_{..4} = .4650$, $\hat{L} = -.0975$, $s\{\hat{L}\} = .03886$, $t(.95; 6) = 1.943$, $-.0975 \pm 1.943(.03886)$, $-.1730 \leq L \leq -.0220$

28.24. a. $Y_{ijk} = \mu_{...} + \rho_1 X_{ijk1} + \rho_2 X_{ijk2} + \rho_3 X_{ijk3} + \kappa_1 X_{ijk4} + \kappa_2 X_{ijk5}$
$+\kappa_3 X_{ijk6} + \tau_1 X_{ijk7} + \tau_2 X_{ijk8} + \tau_3 X_{ijk9} + \epsilon_{(ijk)}$

$$X_{ijk1} = \begin{cases} 1 & \text{if experimental unit from row blocking class 1} \\ -1 & \text{if experimental unit from row blocking class 4} \\ 0 & \text{otherwise} \end{cases}$$

X_{ijk2} and X_{ijk3} are defined similarly

$$X_{ijk4} = \begin{cases} 1 & \text{if experimental unit from column blocking class 1} \\ -1 & \text{if experimental unit from column blocking class 4} \\ 0 & \text{otherwise} \end{cases}$$

X_{ijk5} and X_{ijk6} are defined similarly

$$X_{ijk7} = \begin{cases} 1 & \text{if experimental unit received treatment 1} \\ -1 & \text{if experimental unit received treatment 4} \\ 0 & \text{otherwise} \end{cases}$$

X_{ijk8} and X_{ijk9} are defined similarly

b. Full model:

$\hat{Y} = 2.05625 - .70625X_1 - .45625X_2 + .34375X_3 + .14375X_4$
$-.05625X_5 - .00625X_6 - .33125X_7 - .15625X_8 + .11875X_9$

$SSE(F) = .1188$

Reduced model:

$\hat{Y} = 2.05625 - .70625X_1 - .45625X_2 + .34375X_3 + .14375X_4 - .05625X_5 - .00625X_6$

$SSE(R) = 1.2556$

H_0: all τ_k equal zero ($k = 1, 2, 3$), H_a: not all τ_k equal zero. $F^* = (1.1368/3) \div (.1188/6) = 19.138$, $F(.95; 3, 6) = 4.76$. If $F^* \leq 4.76$ conclude H_0, otherwise H_a. Conclude H_a.

c. $\hat{L} = \hat{\tau}_3 - (-\hat{\tau}_1 - \hat{\tau}_2 - \hat{\tau}_3) = 2\hat{\tau}_3 + \hat{\tau}_1 + \hat{\tau}_2 = -.250$, $s^2\{\hat{\tau}_i\} = .00371$ ($i = 1, 2, 3$), $s\{\hat{\tau}_1, \hat{\tau}_2\} = s\{\hat{\tau}_1, \hat{\tau}_3\} = s\{\hat{\tau}_2, \hat{\tau}_3\} = -.00124$, $s\{\hat{L}\} = .09930$, $t(.975; 6) = 2.447$, $-.250 \pm 2.447(.09930)$, $-.493 \leq L \leq -.007$

d. (i) Full model:

$\hat{Y} = 2.02917 - .67917X_1 - .53750X_2 + .37083X_3 + .17083X_4 - .02917X_5$
$-.08750X_6 - .30417X_7 - .23750X_8 + .14583X_9$

$SSE(F) = .0483$

Reduced model:

$\hat{Y} = 2.05556 - .70556X_1 - .45833X_2 + .34444X_3 + .14444X_4 - .05556X_5 - .00833X_6$

$SSE(R) = 1.2556$

H_0: all τ_k equal zero ($k = 1, 2, 3$), H_a: not all τ_k equal zero. $F^* = (1.2073/3) \div (.0483/5) = 41.66$, $F(.95; 3, 5) = 5.41$. If $F^* \leq 5.41$ conclude H_0, otherwise H_a. Conclude H_a.

(ii) $\hat{L} = \hat{\tau}_1 - \hat{\tau}_2 = -.06667$, $s^2\{\hat{\tau}_1\} = .00191$, $s^2\{\hat{\tau}_2\} = .00272$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.00091$, $s\{\hat{L}\} = .0803$, $t(.975; 5) = 2.571$, $-.06667 \pm 2.571(.0803)$, $-.273 \leq L \leq .140$

28.25. a. Full model:

$$\hat{Y} = 12.54286 + 1.91429X_1 - 3.54286X_2 + 3.25714X_3 - 3.28571X_4 + 1.11429X_5$$
$$-.34286X_6 - .94286X_7 - .74286X_8 - 5.54286X_9$$
$$-5.14286X_{10} + 3.11329X_{11} + 6.71429X_{12}$$

$SSE(F) = 12.6286$

Reduced model:

$$\hat{Y} = 11.96471 + .44706X_1 - 2.96471X_2 + 3.83529X_3 - 3.55294X_4 - .35294X_5$$
$$+.23529X_6 - .36471X_7 - .16471X_8$$

$SSE(R) = 494.2353$

H_0: all τ_k equal zero ($k = 1, ..., 4$), H_a: not all τ_k equal zero. $F^* = (481.6067/4) \div (12.6286/10) = 95.340$, $F(.99; 4, 10) = 5.99$. If $F^* \leq 5.99$ conclude H_0, otherwise H_a. Conclude H_a.

b. $\hat{L} = \hat{\tau}_4 - \hat{\tau}_1 = 12.25715$, $s^2\{\hat{\tau}_1\} = .20927$, $s^2\{\hat{\tau}_4\} = .28144$, $s\{\hat{\tau}_1, \hat{\tau}_4\} = -.06134$, $s\{\hat{L}\} = .7832$, $t(.995; 10) = 3.169$, $12.25715 \pm 3.169(.7832)$, $9.775 \leq L \leq 14.739$

28.26. e_{ijkm}:

i	m	$j=1$	$j=2$	$j=3$	$j=4$
1	1	−1.9375	−1.5625	.6875	1.3125
	2	1.0625	3.4375	−2.3125	−.6875
2	1	−3.6875	6.0625	−1.1875	3.8125
	2	2.3125	.0625	−5.1875	−2.1875
3	1	−4.0625	1.1875	−.6875	4.0625
	2	2.9375	−4.8125	4.3125	−2.9375
4	1	−.3125	.3125	.1875	−2.6875
	2	3.6875	−4.6875	4.1875	−.6875

$r = .990$

28.27. a. $Y_{ijklm} = \mu_{...} + \rho_i + \kappa_j + \alpha_k + \beta_l + (\alpha\beta)_{kl} + \epsilon_{m(ijkl)}$

b.

Source	SS	df	MS
Rows (ages)	658.09375	3	219.36458
Columns (education levels)	18.34375	3	6.11458
Treatments	1,251.34375	3	417.11458
Volumes	399.03125	1	399.03125
Products	850.78125	1	850.78125
Volume-product interactions	1.53125	1	1.53125
Error	285.43750	22	12.97443
Total	2,213.21875	31	

H_0: all $(\alpha\beta)_{kl}$ equal zero, H_a: not all $(\alpha\beta)_{kl}$ equal zero. $F^* = 1.53125/12.97443 = .118$, $F(.99; 1, 22) = 7.95$. If $F^* \leq 7.95$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .73$

c. H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_1 and α_2 equal zero. $F^* = 399.03125/12.97443 = 30.755$, $F(.99; 1, 22) = 7.95$. If $F^* \leq 7.95$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero. $F^* = 850.78125/12.97443 = 65.574$, $F(.99; 1, 22) = 7.95$. If $F^* \leq 7.95$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

d. $\bar{Y}_{..1.} = 61.750$, $\bar{Y}_{..2.} = 69.250$, $\bar{Y}_{..3.} = 72.500$, $\bar{Y}_{..4.} = 79.125$,

$$L_1 = \frac{\mu_{..1} + \mu_{..3}}{2} - \frac{\mu_{..2} + \mu_{..4}}{2}$$

$$L_2 = \frac{\mu_{..1} + \mu_{..2}}{2} - \frac{\mu_{..3} + \mu_{..4}}{2}$$

$\hat{L}_1 = -7.0625$, $\hat{L}_2 = -10.3125$, $s\{\hat{L}_1\} = s\{\hat{L}_2\} = 1.2735$,

$B = t(.9875; 22) = 2.4055$

$$-7.0625 \pm 2.4055(1.2735) \qquad -10.126 \leq L_1 \leq -3.999$$
$$-10.3125 \pm 2.4055(1.2735) \qquad -13.376 \leq L_2 \leq -7.249$$

28.28. e_{ijkm}:

		$j=1$	$j=2$	$j=3$
	$m=1$	4.3704	−2.7407	−1.6296
$i=1$	$m=2$	−3.6296	1.2593	2.3704
	$m=3$	−2.2963	3.5926	−1.2963
	$m=1$	−.9630	−1.1852	2.1481
$i=2$	$m=2$	1.0370	−1.1852	.1481
	$m=3$	2.0370	1.8148	−3.8519
	$m=1$	−3.5185	−.8519	4.3704
$i=3$	$m=2$	.1481	3.8148	−3.9630
	$m=3$	2.8148	−4.5185	1.7037

$r = .986$

28.29. a.

Source	SS	df	MS
Patterns	14.2963	2	7.1481
Order positions	1,803.6296	2	901.8148
Questionnaires	3,472.0741	2	1,736.0370
Subjects (within patterns)	159.5556	6	26.5926
Error	194.9630	14	13.9259
Total	5,644.5185	26	

H_0: all ρ_i equal zero ($i = 1, 2, 3$), H_a: not all ρ_i equal zero. $F^* = 7.1481/26.5926 = .269$, $F(.95; 2, 6) = 5.14$. If $F^* \leq 5.14$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .77$

H_0: all κ_j equal zero ($j = 1, 2, 3$), H_a: not all κ_j equal zero. $F^* = 901.8148/13.9259 = 64.758$, $F(.95; 2, 14) = 3.74$. If $F^* \leq 3.74$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: all τ_k equal zero ($k = 1, 2, 3$), H_a: not all τ_k equal zero. $F^* = 1,736.0370/13.9259 = 126.66$, $F(.95; 2, 14) = 3.74$. If $F^* \leq 3.74$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

b. $\bar{Y}_{..1.} = 22.3333$, $\bar{Y}_{..2.} = 22.4444$, $\bar{Y}_{..3.} = 46.4444$, $\hat{L}_1 = \bar{Y}_{..1.} - \bar{Y}_{..2.} = -.1111$, $\hat{L}_2 = \bar{Y}_{..1.} - \bar{Y}_{..3.} = -24.1111$, $\hat{L}_3 = \bar{Y}_{..2.} - \bar{Y}_{..3.} = -24.0000$, $s\{\hat{L}_i\} = 1.75916$ ($i = 1, 2, 3$), $q(.90; 3, 14) = 3.16$, $T = 2.234$

$$
\begin{array}{rr}
-.1111 \pm 2.234(1.75916) & -4.0411 \leq \mu_{..1} - \mu_{..2} \leq 3.8189 \\
-24.1111 \pm 2.234(1.75916) & -28.0411 \leq \mu_{..1} - \mu_{..3} \leq -20.1811 \\
-24.0000 \pm 2.234(1.75916) & -27.9300 \leq \mu_{..2} - \mu_{..3} \leq -20.0700
\end{array}
$$

Chapter 29

EXPLORATORY EXPERIMENTS – TWO-LEVEL FACTORIAL AND FRACTIONAL FACTORIAL DESIGNS

29.1. $$Y_i = \beta_0 X_{i0} + \beta_1 X_{il} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \beta_{12} X_{i12} + \beta_{13} X_{i13}$$
$$+\beta_{14} X_{i14} + \beta_{23} X_{i23} + \beta_{24} X_{i24} + \beta_{34} X_{i34} + \beta_{123} X_{i123}$$
$$+\beta_{124} X_{i124} + \beta_{134} X_{i134} + \beta_{234} X_{i234} + \beta_{1234} X_{i1234} + \epsilon_i$$

6, 4, 1

29.2. Fractional factorial designs can be used.

29.3. a. Six factors, two levels, 64 trials

b. No

29.4. a. Seven factors, two levels, 8 trials; no

b. Yes, no

29.5.

X_0	X_1	X_2	X_3	X_{12}	X_{13}	X_{23}	X_{123}
1	−1	−1	−1	1	1	1	−1
1	1	−1	−1	−1	−1	1	1
1	−1	1	−1	−1	1	−1	1
1	1	1	−1	1	−1	−1	−1
1	−1	−1	1	1	−1	−1	1
1	1	−1	1	−1	1	−1	−1
1	−1	1	1	−1	−1	1	−1
1	1	1	1	1	1	1	1

$$\mathbf{X}'\mathbf{X} = \begin{bmatrix} 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 \end{bmatrix} = 8\mathbf{I} = n_T\mathbf{I}$$

29.6. a. $\sigma^2\{b_1\} = \sigma^2/n_T = 5^2/64 = .391$. Yes, yes

b. $z(.975) = 1.96$, $n_T = [1.96(5)/(.5)]^2 = 384.16$, $384.16/64 = 6$ replicates

29.7. a. $Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \cdots + \beta_5 X_{i5} + \beta_{12} X_{i12} + \cdots + \beta_{45} X_{i45} + \beta_{123} X_{i123}$

$$+ \cdots + \beta_{345} X_{i345} + \beta_{1234} X_{i1234} + \cdots + \beta_{2345} X_{i2345} + \beta_{12345} X_{i12345} + \epsilon_i$$

Coef.	b_q	Coef.	b_q	Coef.	b_q	Coef.	b_q
b_0	6.853	b_{14}	$-.239$	b_{123}	.070	b_{245}	.076
b_1	1.606	b_{15}	.611	b_{124}	.020	b_{345}	$-.576$
b_2	$-.099$	b_{23}	$-.134$	b_{125}	$-.118$	b_{1234}	.062
b_3	1.258	b_{24}	$-.127$	b_{134}	$-.378$	b_{1235}	.323
b_4	-1.151	b_{25}	$-.045$	b_{135}	$-.138$	b_{1245}	.357
b_5	-1.338	b_{34}	$-.311$	b_{145}	$-.183$	b_{1345}	$-.122$
b_{12}	$-.033$	b_{35}	.912	b_{234}	.233	b_{2345}	$-.292$
b_{13}	.455	b_{45}	$-.198$	b_{235}	.055	b_{12345}	.043

29.8. a. $Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \cdots + \beta_5 X_{i5} + \beta_{12} X_{i12} + \cdots + \beta_{45} X_{i45} + \epsilon_i$

Coef.	b_q	P-value	Coef.	b_q	P-value
b_0	6.853		b_{14}	$-.239$	.340
b_1	1.606	.000	b_{15}	.611	.023
b_2	$-.099$	.689	b_{23}	$-.134$	.589
b_3	1.258	.000	b_{24}	$-.127$	.610
b_4	-1.151	.000	b_{25}	$-.045$	.855
b_5	-1.338	.000	b_{34}	$-.311$	.219
b_{12}	$-.033$	.892	b_{35}	.912	.002
b_{13}	.455	.080	b_{45}	$-.198$	.426

b. H_0: Normal, H_a: not normal. $r = .983$. If $r \geq .9656$ conclude H_0, otherwise H_a. Conclude H_0.

c. H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $s\{b_q\} = .2432$. If P-value $\geq .0034$ conclude H_0, otherwise H_a. Active effects (see part a): β_1, β_3, β_4, β_5, β_{35}

29.9. a. $Y_i = \beta_0 + \beta_1 X_{i1} + \cdots + \beta_4 X_{i4} + \beta_{12} X_{i12} + \cdots + \beta_{34} X_{i34} + \beta_{123} X_{i123}$

$$+ \cdots + \beta_{234} X_{i234} + \beta_{1234} X_{i1234} + \epsilon_i$$

Coef.	b_q	P-value	Coef.	b_q	P-value
b_0	3.7784		b_{23}	$-.0925$	.176
b_1	$-.3113$	.020	b_{24}	.0125	.807
b_2	$-.0062$	.903	b_{34}	$-.2175$	.040
b_3	$-.1463$	.083	b_{123}	$-.0087$	.865
b_4	.0837	.204	b_{124}	.0538	.354
b_{12}	.0050	.922	b_{134}	$-.0363$	.505
b_{13}	.0400	.468	b_{234}	$-.0138$	.788
b_{14}	.0025	.961	b_{1234}	.0050	.922

(Note: P-values based on $MSPE$; see part d.)

d. H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $MSPE = .0324$, $s\{b_q\} = \sqrt{.0324/16} = .0450$. If P-value $\geq .05$ conclude H_0, otherwise H_a. Active effects (see part a): β_1, β_{34}

29.10. a.

Coef.	b_q	P-value
b_0	3.778	
b_1	$-.3112$	.000
b_3	$-.1462$	.006
b_4	.0838	.084
b_{34}	$-.2175$	.000

b. H_0: Normal, H_a: not normal. $r = .970$. If $r \geq .9485$ conclude H_0, otherwise H_a. Conclude H_0.

c. H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $s\{b_q\} = .0449$. If P-value $\geq .01$ conclude H_0, otherwise H_a. Active effects (see part a): β_1, β_3, β_{34}.

d. H_0: No lack of fit, H_a: lack of fit. $SSLF = SSE - SSPE = .45248 - .25617 = .19631$. $F^* = [.19631/4] \div (.25617/10) = 1.92$, $F(.95; 4, 10) = 3.48$. If $F^* \leq 3.48$ conclude H_0, otherwise H_a. Conclude H_0.

e. Set X_1, X_3, X_4 at high levels to minimize failure rate.

29.11. a. $0 = -234$, resolution = III

b. $0 = -234$, $1 = -1234$, $2 = -34$, $3 = -24$, $4 = -23$, $12 = -134$,

$13 = -124$, $14 = -123$

29.12. a.

X_1	X_2	X_3	X_4
-1	-1	-1	-1
1	-1	-1	1
-1	1	-1	1
1	1	-1	-1
-1	-1	1	1
1	-1	1	-1
-1	1	1	-1
1	1	1	1

Resolution = IV

b. For example, dropping X_1 and arranging in standard order:

X_2	X_3	X_4
-1	-1	-1
1	-1	-1
-1	1	-1
1	1	-1
-1	-1	1
1	-1	1
-1	1	1
1	1	1

29.13. No

29.14.

X_1	X_2	X_3	X_4
1	-1	-1	-1
-1	1	-1	-1
-1	-1	1	-1
1	1	1	-1
1	-1	-1	1
-1	1	-1	1
-1	-1	1	1
1	1	1	1

Yes; use $0 = 1234$ for resolution IV.

29.15. Defining relation: $0 = 123 = 245 = 1345$

Confounding scheme:

$$\begin{array}{lllllll}
0 & = & 123 & = & 245 & = & 1345 \\
1 & = & 23 & = & 1245 & = & 345 \\
2 & = & 13 & = & 45 & = & 12345 \\
3 & = & 12 & = & 2345 & = & 145 \\
4 & = & 1234 & = & 25 & = & 135 \\
5 & = & 1235 & = & 24 & = & 134 \\
14 & = & 234 & = & 125 & = & 35 \\
15 & = & 235 & = & 124 & = & 34
\end{array}$$

Resolution = III, no

29.16. Defining relation: $0 = -145 = -234 = 1235$

Confounding scheme:

$$\begin{array}{lllllll}
0 & = & -145 & = & -234 & = & 1235 \\
1 & = & -45 & = & -1234 & = & 235 \\
2 & = & -1245 & = & -34 & = & 135 \\
3 & = & -1345 & = & -24 & = & 125 \\
4 & = & -15 & = & -23 & = & 12345 \\
5 & = & -14 & = & -2345 & = & 123 \\
12 & = & -245 & = & -134 & = & 35 \\
13 & = & -345 & = & -124 & = & 25
\end{array}$$

No

29.17. Defining relation: $0 = 124 = 135 = 2345 = 236 = 1346 = 1256 = 456$

X_1	X_2	X_3	X_4	X_5	X_6
-1	-1	-1	1	1	1
1	-1	-1	-1	-1	1
-1	1	-1	-1	1	-1
1	1	-1	1	-1	-1
-1	-1	1	1	-1	-1
1	-1	1	-1	1	-1
-1	1	1	-1	-1	1
1	1	1	1	1	1

Resolution = III

29.18. a. Defining relation: $0 = 1235 = 2346 = 1247 = 1456 = 3457 = 1367 = 2567$,

resolution = IV, no

b. Omitting four-factor and higher-order interactions:

$$\begin{array}{lllllllll}
1 & = & 235 & = & 247 & = & 367 & = & 456 \\
2 & = & 135 & = & 147 & = & 346 & = & 567 \\
3 & = & 125 & = & 167 & = & 246 & = & 457 \\
4 & = & 127 & = & 156 & = & 236 & = & 357 \\
5 & = & 123 & = & 146 & = & 267 & = & 347 \\
6 & = & 137 & = & 145 & = & 234 & = & 257 \\
7 & = & 124 & = & 136 & = & 256 & = & 345 \\
12 & = & 35 & = & 47 & & & & \\
13 & = & 25 & = & 67 & & & & \\
14 & = & 27 & = & 56 & & & & \\
15 & = & 23 & = & 46 & & & & \\
16 & = & 37 & = & 45 & & & & \\
17 & = & 24 & = & 36 & & & & \\
26 & = & 34 & = & 57 & & & &
\end{array}$$

c. $$Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \cdots + \beta_7 X_{i7} + \beta_{12} X_{i12} + \beta_{13} X_{i13} + \beta_{14} X_{i14}$$
$$+\beta_{15} X_{i15} + \beta_{16} X_{i16} + \beta_{17} X_{i17} + \beta_{26} X_{i26} + \epsilon_i$$

Coef.	b_q	Coef.	b_q	Coef.	b_q
b_0	8.028	b_5	.724	b_{14}	$-.316$
b_1	.127	b_6	$-.467$	b_{15}	.318
b_2	.003	b_7	$-.766$	b_{16}	.117
b_3	.021	b_{12}	.354	b_{17}	.021
b_4	-2.077	b_{13}	$-.066$	b_{26}	$-.182$

e. H_0: $\beta_{12} = \cdots = \beta_{17} = \beta_{26} = 0$, H_a: not all $\beta_q = 0$. $F^* = (6.046/7) \div (.1958/1) = 4.41$, $F(.99; 7, 1) = 5,928$. If $F^* \leq 5,928$ conclude H_0, otherwise H_a. Conclude H_0.

29.19. a.

Coef.	b_q	P-value	Coef.	b_q	P-value
b_0	8.028		b_4	-2.077	.000
b_1	.127	.581	b_5	.724	.011
b_2	.003	.989	b_6	$-.467$	.067
b_3	.021	.928	b_7	$-.766$	.008

b. H_0: Case i not an outlier, H_a: case i an outlier ($i = 3, 14$). $t_3 = 2.70$, $t_{14} = -4.09$, $t(.99844; 7) = 4.41$. If $|\ t_i\ | \leq 4.41$ conclude H_0, otherwise H_a. Conclude H_0 for both cases.

c. H_0: Normal, H_a: not normal. $r = .938$. If $r \geq .929$ conclude H_0, otherwise H_a. Conclude H_0.

d. H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $s\{b_q\} = .2208$. If P-value $\geq .02$ conclude H_0, otherwise H_a. Active effects (see part a): β_4, β_5, β_7

e. Set $X_4 = -1$, $X_5 = 1$, $X_7 = -1$ to maximize extraction.

29.20. a. $Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \cdots + \beta_9 X_{i9} + \epsilon_i$

Coef.	b_q	P-value	Coef.	b_q	P-value
b_0	70.11		b_5	13.49	.060
b_1	13.52	.060	b_6	.12	.984
b_2	$-.99$	.870	b_7	-21.58	.010
b_3	1.32	.829	b_8	-4.07	.512
b_4	2.36	.701	b_9	3.07	.618

d. H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $s\{b_q\} = 5.841$. If P-value $\geq .10$ conclude H_0, otherwise H_a. Active effects (see part a): β_1, β_5, β_7.

29.21. a. $b_0 = 70.11$, $b_1 = 13.52$, $b_5 = 13.49$, $b_7 = -21.58$

b. H_0: Normal, H_a: not normal. $r = .951$. If $r \geq .941$ conclude H_0, otherwise H_a. Conclude H_0.

c. H_0: No lack of fit, H_a: lack of fit. $SSLF = SSE - SSPE = 3,824 - 1,068 = 2,756$, $F^* = (2,756/4) \div (1,068/8) = 5.16$, $F(.95; 4, 8) = 3.84$. If $F^* \leq 3.84$ conclude H_0, otherwise H_a. Conclude H_a.

29.22. a.
$$Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \beta_5 X_{i5} + \beta_7 X_{i7} + \beta_{15} X_{i15} + \beta_{17} X_{i17}$$
$$+\beta_{57} X_{i57} + \beta_{157} X_{i157} + \epsilon_i$$

Coef.	b_q	P-value	Coef.	b_q	P-value
b_0	70.11		b_{15}	11.68	.004
b_1	13.52	.000	b_{17}	-1.32	.660
b_5	13.49	.000	b_{57}	5.83	.078
b_7	-21.58	.000	b_{157}	.12	.968

H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $s\{b_q\} = 2.889$. If P-value $\geq .01$ conclude H_0, otherwise H_a. Active effects: β_1, β_5, β_7, β_{15}

29.23. a. Defining relation: $0 = 134$

Confounding scheme:

0	=	134	4	=	13
1	=	34	12	=	234
2	=	1234	23	=	124
3	=	14	24	=	123

Yes. Defining relation 0 = 1234 would yield a resolution IV design.

b. $Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \beta_{12} X_{i12}$

$+\beta_{23} X_{i23} + \beta_{24} X_{i24} + \epsilon_i$

Coef.	b_q	Coef.	b_q
b_0	747.50	b_4	88.25
b_1	−207.25	b_{12}	−24.75
b_2	−17.00	b_{23}	−29.00
b_3	108.00	b_{24}	−18.75

29.24. a.

Coef.	b_q	P-value
b_0	747.50	
b_1	−207.25	.003
b_2	−17.00	.538
b_3	108.00	.022
b_4	88.25	.037

H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $s\{b_q\} = 24.53$. If P-value $\geq .05$ conclude H_0, otherwise H_a. Active effects: β_1, β_3, β_4

b. Set $X_1 = -1$, $X_3 = 1$, $X_4 = 1$ to maximize defect-free moldings.

29.25. Confounding scheme for design:

0	=	124	=	135	=	2345			
1	=	24	=	35	=	12345			
2	=	14	=	1235	=	345			
3	=	1234	=	15	=	245			
4	=	12	=	1345	=	235			
5	=	1245	=	13	=	234			
23	=	134	=	125	=	45	=	Block effect
25	=	145	=	123	=	34			

Design:

Block	X_1	X_2	X_3	X_4	X_5
1	−1	1	−1	−1	1
1	1	1	−1	1	−1
1	−1	−1	1	1	−1
1	1	−1	1	−1	1
2	−1	−1	−1	1	1
2	1	−1	−1	−1	−1
2	−1	1	1	−1	−1
2	1	1	1	1	1

29.26. b. The seven block effects are confounded with the following interaction terms: β_{135}, β_{146}, β_{236}, β_{245}, β_{1234}, β_{1256}, β_{3456}

No, no

c. $$Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \cdots + \beta_6 X_{i6} + \beta_{12} X_{i12} + \cdots + \beta_{56} X_{i56} + \beta_{123} X_{i123}$$
$$+ \cdots + \beta_{456} X_{i456} + \beta_{1235} X_{i1235} + \cdots + \beta_{2456} X_{i2456} + \beta_{12345} X_{i12345}$$
$$+ \cdots + \beta_{23456} X_{i23456} + \beta_{123456} X_{i123456} + \alpha_1 Z_{i1} + \cdots + \alpha_7 Z_{i7} + \epsilon_i$$

where α_1, ..., α_7 are the block effects

Coef.	b_q	Coef.	b_q	Coef.	b_q	Coef.	b_q
b_0	63.922	b_{34}	.297	b_{246}	$-.391$	b_{2356}	.766
b_1	2.297	b_{35}	.266	b_{256}	.078	b_{2456}	.203
b_2	5.797	b_{36}	.984	b_{345}	$-.672$	b_{12345}	$-.297$
b_3	2.172	b_{45}	$-.422$	b_{346}	.734	b_{12346}	$-.391$
b_4	2.359	b_{46}	$-.141$	b_{356}	$-.734$	b_{12356}	$-.734$
b_5	2.828	b_{56}	.516	b_{456}	$-.234$	b_{12456}	$-.422$
b_6	2.922	b_{123}	.422	b_{1235}	.578	b_{13456}	$-.109$
b_{12}	.547	b_{124}	.172	b_{1236}	.922	b_{23456}	.203
b_{13}	$-.266$	b_{125}	1.391	b_{1245}	.453	b_{123456}	.016
b_{14}	$-.203$	b_{126}	.984	b_{1246}	.109	Block 1	-4.172
b_{15}	$-.797$	b_{134}	.297	b_{1345}	$-.797$	Block 2	$-.422$
b_{16}	$-.141$	b_{136}	$-.641$	b_{1346}	.547	Block 3	1.203
b_{23}	$-.641$	b_{145}	$-.109$	b_{1356}	-1.109	Block 4	6.703
b_{24}	-1.141	b_{156}	$-.547$	b_{1456}	$-.109$	Block 5	$-.797$
b_{25}	.891	b_{234}	.234	b_{2345}	.328	Block 6	-1.047
b_{26}	.047	b_{235}	.266	b_{2346}	$-.578$	Block 7	-9.547

29.27. a.

Coef.	b_q	P-value	Coef.	b_q	P-value
b_0	63.922		b_{26}	.047	.935
b_1	2.297	.000	b_{34}	.297	.607
b_2	5.797	.000	b_{35}	.266	.645
b_3	2.172	.001	b_{36}	.984	.094
b_4	2.359	.000	b_{45}	$-.422$	.466
b_5	2.828	.000	b_{46}	$-.141$	.807
b_6	2.922	.000	b_{56}	.516	.373
b_{12}	.547	.346	Block 1	-4.172	.009
b_{13}	$-.266$	.645	Block 2	$-.422$	.782
b_{14}	$-.203$	.725	Block 3	1.203	.432
b_{15}	$-.797$	.172	Block 4	6.703	.000
b_{16}	$-.141$	.807	Block 5	$-.797$	.602
b_{23}	$-.641$	.270	Block 6	-1.047	.494
b_{24}	-1.141	.054	Block 7	-9.547	.000
b_{25}	.891	.128			

b. H_0: Normal, H_a: not normal. $r = .989$. If $r \geq .9812$ conclude H_0, otherwise H_a. Conclude H_0.

c. H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $s\{\hat{\alpha}_i\} = 1.513$ for block effects, $s\{b_q\} = .5719$ for factor effects. If P-value $\geq .01$ conclude H_0, otherwise H_a. Active effects (see part a): Block effects 1, 4, 7, all main effects

29.28. a. See Problem 29.27a for estimated factor and block effects. (These do not change with subset model.)

b. Maximum team effectiveness is accomplished by setting each factor at its high level.

c. $\hat{Y}_h = 82.297$, $s\{\text{pred}\} = 4.857$, $t(.975; 50) = 2.009$, $82.297 \pm 2.009(4.857)$, $72.54 \leq Y_{h(new)} \leq 92.05$

29.29. a. Defining relation: $0 = 12345$, resolution = V

b. $Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \cdots + \beta_5 X_{i5} + \beta_{12} X_{i12} + \cdots + \beta_{35} X_{i35} + \alpha_1 Z_{i1} + \epsilon_i$

where α_1 is the block effect

Coef.	b_q	Coef.	b_q
b_0	113.18	b_{14}	-1.44
b_1	26.69	b_{15}	-2.94
b_2	-10.94	b_{23}	1.44
b_3	5.69	b_{24}	5.19
b_4	4.44	b_{25}	2.94
b_5	14.69	b_{34}	-3.44
b_{12}	21.94	b_{35}	$-.94$
b_{13}	.56	Block effect	2.27

d. H_0: $\alpha_1 = 0$, H_a: $\alpha_1 \neq 0$. $s\{\hat{\alpha}_1\} = 3.673$, $t^* = 2.27/3.673 = .62$, $(.975; 6) = 2.447$. If $|\, t^* \,| \leq 2.447$ conclude H_0, otherwise H_a. Conclude H_0.

e.

Coef.	b_q	P-value	Coef.	b_q	P-value
b_0	113.18		b_{14}	-1.44	.625
b_1	26.69	.000	b_{15}	-2.94	.336
b_2	-10.94	.011	b_{23}	1.44	.625
b_3	5.69	.094	b_{24}	5.19	.119
b_4	4.44	.169	b_{25}	2.94	.336
b_5	14.69	.003	b_{34}	-3.44	.268
b_{12}	21.94	.001	b_{35}	$-.94$	.748
b_{13}	.56	.847			

(Note: P-values based on $MSPE$; see part f.)

H_0: No lack of fit, H_a: lack of fit. $SSLF = SSE - SSPE = 1,894.4 - 609.5 = 1,284.9$, $F^* = (1,284.9/2) \div (609.5/5) = 5.270$, $F(.95; 2, 5) = 5.786$. If $F^* \leq 5.786$ conclude H_0, otherwise H_a. Conclude H_0.

f. H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $MSPE = 121.90$, $s\{b_q\} = \sqrt{121.90/16} = 2.760$. If P-value $\geq .025$ conclude H_0, otherwise H_a. Active effects (see part a): β_1, β_2, β_5, β_{12}

29.30. a.

Coef.	b_q
b_0	113.18
b_1	26.69
b_2	-10.94

Coef.	b_q
b_5	14.69
b_{12}	21.94

b. H_0: Normal, H_a: not normal. $r = .961$. If $r \geq .954$ conclude H_0, otherwise H_a. Conclude H_0

c. Set factors 1, 2, 5 at their high levels to maximize whippability.

d. $\hat{Y}_h = 165.56$, $s\{\hat{Y}_h\} = 8.03$, $t(.975;\, 17) = 2.110$, $165.56 \pm 2.110(8.03)$, $148.62 \leq E\{Y_h\} \leq 182.50$

29.31. a.

i	1	2	3	4	5	6	7	8
s_i^2	1.244	1.299	1.103	.992	1.966	1.916	1.589	1.576
$\log_e s_i^2$	.218	.261	.098	$-.008$	.676	.650	.463	.455

i	9	10	11	12	13	14	15	16
s_i^2	2.201	1.818	1.901	1.547	1.681	1.020	1.033	1.151
$\log_e s_i^2$	.789	.598	.642	.437	.520	.020	.033	.141

b. $\widehat{\log_e s_i^2} = .3746 - .0553X_{i1} - .0919X_{i2} - .0048X_{i3} + .0229X_{i4} + .0289X_{i12} + .0021X_{i13} - .0432X_{i14} - .0048X_{i23} + .0077X_{i24} - .2142X_{i34} + .0493X_{i123} + .0453X_{i124} - .0016X_{i134} - .0024X_{i234} + .0284X_{i1234}$

X_{34} appears to be active.

c. $\hat{v}_i = 1.17395$ (for $i = 1, 2, 3, 4, 13, 14, 15, 16$)

$\hat{v}_i = 1.80173$ (for $i = 5, \ldots, 12$)

d. $\hat{Y}_i = 3.7082 - .3754X_{i1}$

e. From location model: $X_1 = +1$; and from location model: $(X_3, X_4) = (-1, -1)$ or $(+1, +1)$

f. From dispersion model: $\hat{s^2} = \exp(.3746 - .2142) = 1.17395$,

and a 95% P.I. is $(\exp(.0453), \exp(.2755))$, or (1.0463, 1.3172).

g. $\widehat{MSE} = 1.17395 + 3.333^2 = 12.284$

29.32. a.

i	1	2	3	4	5	6	7	8
s_i^2	.0164	.0173	.0804	.1100	.0010	.0079	.0953	.1134
$\log_e s_i^2$	-4.109	-4.058	-2.521	-2.207	-6.949	-4.838	-2.351	-2.176

b. $\widehat{\log_e s_i^2} = -3.651 + .331X_{i1} + 1.337X_{i2} - .427X_{i3} - .275X_{i4} - .209X_{i12} + .240X_{i13} + .477X_{i14}$.

X_2 appears to be active.

c. $\hat{v}_i = .00682$ (for $i = 1, 2, 5, 6$)

$\hat{v}_i = .0989$ (for $i = 3, 4, 7, 8$)

d. $\hat{Y}_i = 7.5800 + .0772X_{i1}$

e. From the location model: $X_1 = +1$; from the dispersion model: $X_2 = -1$

f. From dispersion model: $\hat{s^2} = \exp(-3.651 + 1.337(-1)) = .006819$,
and a 95% P.I. is $(\exp(-6.169), \exp(-3.808))$, or (.0021, .0222).

g. $\widehat{MSE} = .00682 + (8 - 7.657)^2 = .124$

29.33. From (2.51), $SSR(X_q) = b_q^2 \sum (X_{iq} - \bar{X}_q)^2$. For coding in (29.2a), $\bar{X}_q = 0$. Then:

$$SSR(X_q) = b_q^2 \sum X_{iq}^2 = b_q^2 \sum_{i=1}^{n_T} (\pm 1)^2 = n_T b_q^2$$

29.34. a.

$$\begin{aligned}
\mathbf{E}\{\hat{\boldsymbol{\beta}}_1\} &= \mathbf{E}\{(\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'\mathbf{Y}\} \\
&= (\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'\mathbf{E}\{\mathbf{Y}\} \\
&= (\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'(\mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2) \\
&= \boldsymbol{\beta}_1 + (\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'\mathbf{X}_2\boldsymbol{\beta}_2 = \boldsymbol{\beta}_1 + \mathbf{A}\boldsymbol{\beta}_2
\end{aligned}$$

b. Let:

$$\boldsymbol{\beta}_1 = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \qquad \boldsymbol{\beta}_2 = \begin{bmatrix} \beta_{12} \\ \beta_{13} \\ \beta_{23} \end{bmatrix}$$

Then:

$$\mathbf{X}_1 = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad \mathbf{X}_2 = \begin{bmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

and:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

The results follow from $\mathbf{E}\{\mathbf{b}_1\} = \boldsymbol{\beta}_1 + \mathbf{A}\boldsymbol{\beta}_2$.

Chapter 30

RESPONSE SURFACE METHODOLOGY

30.2. Second block:

X_1	X_2	X_3	X_4
2	0	0	0
-2	0	0	0
0	2	0	0
0	-2	0	0
0	0	2	0
0	0	-2	0
0	0	0	2
0	0	0	-2

Any number of center points may be added to the second block.

30.7. a. 21

b. 5, 5, 10

c. 21, 27

30.8.

X_1	X_2	X_3	X_4	X_5
-1	-1	-1	-1	-1
1	1	-1	-1	-1
1	-1	1	-1	-1
-1	1	1	-1	-1
1	-1	-1	1	-1
-1	1	-1	1	-1
-1	-1	1	1	-1
1	1	1	1	-1
1	-1	-1	-1	1
-1	1	-1	-1	1
-1	-1	1	-1	1
1	1	1	-1	1
-1	-1	-1	1	1
1	1	-1	1	1
1	-1	1	1	1
-1	1	1	1	1

X_1	X_2	X_3	X_4	X_5
2	0	0	0	0
-2	0	0	0	0
0	2	0	0	0
0	-2	0	0	0
0	0	2	0	0
0	0	-2	0	0
0	0	0	2	0
0	0	0	-2	0
0	0	0	0	2
0	0	0	0	-2
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

30.9. No, base design is resolution III.

30.10. $\alpha = [2^{9-3}(1)/(1)]^{1/4} = 2.828$

30.11. b.

Coef.	b_q	P-value
b_0	1.868	
b_1	.190	.007
b_2	.195	.006
b_3	$-.120$	.039
b_{12}	.162	.020

Coef.	b_q	P-value
b_{13}	$-.038$	.471
b_{23}	$-.062$	.251
b_{11}	.228	.044
b_{22}	$-.047$	.602
b_{33}	.028	.757

d. H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $s\{b_q\} = .0431$ (for linear effects), $s\{b_q\} = .0481$ (for interaction effects), $s\{b_q\} = .0849$ (for quadratic effects). If P-value $\geq .05$ conclude H_0, otherwise H_a. Active effects (see part b): β_1, β_2, β_3, β_{12}, β_{11}

30.12. a.

Coef.	b_q
b_0	1.860
b_1	.190
b_2	.195

Coef.	b_q
b_3	$-.120$
b_{12}	.162
b_{11}	.220

b. H_0: Normal, H_a: not normal. $r = .947$. If $r \geq .938$ conclude H_0, otherwise H_a. Conclude H_0.

30.13. a.

Coef.	b_q
b_0	189.750
b_1	28.247
b_2	$-.772$

Coef.	b_q
b_{12}	13.750
b_{11}	-18.128
b_{22}	-6.875

c. H_0: No lack of fit, H_a: lack of fit. $SSLF = SSE - SSPE = 978.9 - 230.75 = 748.15$, $F^* = (748.15/3) \div (230.75/3) = 3.24$, $F(.99; 3, 3) = 29.5$. If $F^* \leq 29.5$ conclude H_0, otherwise H_a. Conclude H_0.

e. $(1.22, 1.16)$

f. $\hat{Y}_h = 206.54$, $s\{\hat{Y}_h\} = 13.70$, $t(.975; 6) = 2.447$, $206.54 \pm 2.447(13.70)$, $173.0 \leq E\{Y_h\} \leq 240.1$.

30.14. a.

Design Matrix:

X_1	X_2
$-.707$	$-.707$
$.707$	$-.707$
$-.707$	$.707$
$.707$	$.707$
-1	0
1	0
0	-1
0	1
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0

Corner Points:

X_1	X_2
$-.707$	$-.707$
$.707$	$-.707$
$-.707$	$.707$
$.707$	$.707$

b.

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} .125 & 0 & 0 & -.125 & -.125 & 0 \\ 0 & .250 & 0 & 0 & 0 & 0 \\ 0 & 0 & .250 & 0 & 0 & 0 \\ -.125 & 0 & 0 & .5 & 0 & 0 \\ -.125 & 0 & 0 & 0 & .5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

30.15. a.

X_1	X_2
-1	-1
1	-1
-1	1
1	1
-1.414	0
1.414	0
0	-1.414
0	1.414

X_1	X_2
0	0
0	0
0	0
0	0
0	0

$n_0 = 5$

b. Variance function:

$$.20 - .075X_1^2 - .075X_2^2 + .14375X_1^4 + .14375X_2^4 + .2875X_1^2X_2^2$$

30.16. a.

$$\mathbf{b}^* = \begin{bmatrix} -2.077 \\ .724 \end{bmatrix} \qquad s = 2.200$$

b.

t	X_1	X_2
1.5	-1.416	.494
2.5	-2.361	.823
3.5	-3.304	1.152

30.17. a.

$$\mathbf{b}^* = \begin{bmatrix} 13.519 \\ 13.494 \\ -21.581 \end{bmatrix} \qquad s = 28.820$$

b.

t	X_1	X_2	X_3
-1	$-.469$	$-.468$	.749
-2	$-.938$	$-.936$	1.498
-3	-1.407	-1.404	2.246

30.18. a.

Design	Variance Function
1	$.6788 - .5116X + .1710X^2 - .02264X^3 + .001029X^4$
2	$.5266 - .4048X + .1475X^2 - .02012X^3 + .000914X^4$
3	$.6615 - .4504X + .1393X^2 - .01788X^3 + .0008129X^4$

b.

Design	$\bar{V}$
1	.1993
2	.2037
3	.1869

Design 3 preferred

c.

Comparison	E_V
Design 1 relative to design 2	1.02
Design 1 relative to design 3	.94
Design 2 relative to design 3	.92

$1/.94 = 1.06$ times

d.

Design	$\mid (\mathbf{X'X})^{-1} \mid$
1	6.35057 x 10^{-7}
2	4.70419 x 10^{-7}
3	5.57533 x 10^{-7}

Design 2 preferred

e.

Comparison	E_D
Design 1 relative to design 2	.90
Design 1 relative to design 3	.96
Design 2 relative to design 3	1.06

$1/.90 = 1.11$ times

30.19. a. Design 2 is D-optimal

b. Design 3 is V-optimal

30.20. a. Irregular

c.

Design	$\bar{V}$
1	.5235
2	.8962

Design 1 preferred

d. $E_V = 1.712$, $1/1.712 = .584$ times

e.

Design	$\mid (\mathbf{X}'\mathbf{X})^{-1} \mid$
1	.393
2	1.567

Design 1 preferred

f. $E_D = .794$, $1/.794 = 1.26$ times

30.21. a. D-optimal design: $\mid (\mathbf{X}'\mathbf{X})^{-1} \mid = .2998$

X_1	X_2	Number of Replicates
-1	-1	1
$-.5$	-1	1
$-.5$	0	1
.25	0	1
0	1	2
1	1	1
.5	1	2

No

b. V-optimal design: $\bar{V} = .4765$

X_1	X_2	Number of Replicates
-1	-1	1
$-.5$	-1	1
$-.5$	0	1
.25	0	1
0	1	1
.5	1	1
1	1	1
0	.25	2

No

Appendix D: RULES FOR DEVELOPING ANOVA MODELS AND TABLES FOR BALANCED DESIGNS

D.1.

	i	j	k					
	R	R	R		E{MSA}	E{MSB}	E{MSAB}	E{MSE}
	a	b	n	Variance	i	j	ij	$(ij)k$
α_i	1	b	n	σ^2_α	bn	0	0	0
β_j	a	1	n	σ^2_β	0	an	0	0
$(\alpha\beta)_{ij}$	1	1	n	$\sigma^2_{\alpha\beta}$	n	n	n	0
$\epsilon_{k(ij)}$	1	1	1	σ^2	1	1	1	1

D.2. a.

Model Term	Coef-ficient	Symbolic Product	Term to be Squared	Degrees of Freedom
α_i	bcn	$i-1$	$\bar{Y}_{i...} - \bar{Y}_{....}$	$a-1$
β_j	acn	$j-1$	$\bar{Y}_{.j..} - \bar{Y}_{....}$	$b-1$
γ_k	abn	$k-1$	$\bar{Y}_{..k.} - \bar{Y}_{....}$	$c-1$
$(\alpha\beta)_{ij}$	cn	$ij-i-j+1$	$\bar{Y}_{ij..} - \bar{Y}_{i...} - \bar{Y}_{.j..} + \bar{Y}_{....}$	$(a-1)(b-1)$
$(\alpha\gamma)_{ik}$	bn	$ik-i-k+1$	$\bar{Y}_{i.k.} - \bar{Y}_{i...} - \bar{Y}_{..k.} + \bar{Y}_{....}$	$(a-1)(c-1)$
$(\beta\gamma)_{jk}$	an	$jk-j-k+1$	$\bar{Y}_{.jk.} - \bar{Y}_{.j..} - \bar{Y}_{..k.} + \bar{Y}_{....}$	$(b-1)(c-1)$
$(\alpha\beta\gamma)_{ijk}$	n	$ijk-ij-ik$	$\bar{Y}_{ijk.} - \bar{Y}_{ij..} - \bar{Y}_{i.k.} - \bar{Y}_{.jk.}$	$(a-1)(b-1)(c-1)$
		$-jk+i+j+k-1$	$+\bar{Y}_{i...} + \bar{Y}_{.j..} + \bar{Y}_{..k.} - \bar{Y}_{....}$	
$\epsilon_{m(ijk)}$	1	$ijkm-ijk$	$Y_{ijkm} - \bar{Y}_{ijk}$	$abc(n-1)$
Total			$Y_{ijkm} - \bar{Y}_{....}$	$abcn-1$

b.

	i	j	k	m		Expected Mean Square of $--$							
	R	R	R	R	Vari-	A	B	C	AB	AC	BC	ABC	E
	a	b	c	n	ance	i	j	k	ij	ik	jk	ijk	$(ijk)m$
α_i	1	b	c	n	σ^2_α	bcn	0	0	0	0	0	0	0
β_j	a	1	c	n	σ^2_β	0	nac	0	0	0	0	0	0
γ_k	a	b	1	n	σ^2_γ	0	0	abn	0	0	0	0	0
$(\alpha\beta)_{ij}$	1	1	c	n	$\sigma^2_{\alpha\beta}$	nc	nc	0	nc	0	0	0	0
$(\alpha\gamma)_{ik}$	1	b	1	n	$\sigma^2_{\alpha\gamma}$	nb	0	nb	0	nb	0	0	0
$(\beta\gamma)_{jk}$	a	1	1	n	$\sigma^2_{\beta\gamma}$	0	na	na	0	0	na	0	0
$(\alpha\beta\gamma)_{ijk}$	1	1	1	n	$\sigma^2_{\alpha\beta\gamma}$	n	n	n	n	n	n	n	0
$\epsilon_{m(ijk)}$	1	1	1	1	σ^2	1	1	1	1	1	1	1	1

D.3. a. See Problem D.2a.

b.

	i	j	k	m		Expected Mean Square of $--$							
	F	R	R	R	Vari-	A	B	C	AB	AC	BC	ABC	E
	a	b	c	n	ance	i	j	k	ij	ik	jk	ijk	$(ijk)m$
α_i	0	b	c	n	σ^2_α	bcn	0	0	0	0	0	0	0
β_j	a	1	c	n	σ^2_β	0	nac	0	0	0	0	0	0
γ_k	a	b	1	n	σ^2_γ	0	0	abn	0	0	0	0	0
$(\alpha\beta)_{ij}$	1	1	c	n	$\sigma^2_{\alpha\beta}$	nc	0	0	nc	0	0	0	0
$(\alpha\gamma)_{ik}$	1	b	1	n	$\sigma^2_{\alpha\gamma}$	nb	0	0	0	nb	0	0	0
$(\beta\gamma)_{jk}$	a	1	1	n	$\sigma^2_{\beta\gamma}$	0	na	na	0	0	na	0	0
$(\alpha\beta\gamma)_{ijk}$	1	1	1	n	$\sigma^2_{\alpha\beta\gamma}$	n	0	0	n	n	0	n	0
$\epsilon_{m(ijk)}$	1	1	1	1	σ^2	1	1	1	1	1	1	1	1

D.4. a.

Model Term	Symbolic Product	Sum of Squares	Degrees of Freedom
β_j	$j-1$	$an\sum(\bar{Y}_{.j.} - \bar{Y}_{...})^2$	$b-1$
$\alpha_{i(j)}$	$ij-j$	$n\sum\sum(\bar{Y}_{ij.} - \bar{Y}_{.j.})^2$	$b(a-1)$
$\epsilon_{k(ij)}$	$ijk-ij$	$\sum\sum\sum(Y_{ijk} - \bar{Y}_{ij.})^2$	$ab(n-1)$
Total		$\sum\sum\sum(Y_{ijk} - \bar{Y}_{...})^2$	$abn-1$

b.

	j	i	k				
	F	R	R	Vari-	E{MSB}	E{MSA(B)}	E{MSE}
	b	a	n	ance	j	$i(j)$	$(ij)k$
β_j	0	a	n	σ^2_β	an	0	0
$\alpha_{i(j)}$	1	1	n	σ^2_α	n	n	0
$\epsilon_{k(ij)}$	1	1	1	σ^2	1	1	1

$\text{E}\{\text{MSB}\} = \frac{an\sum\beta_j^2}{b-1} + n\sigma_\alpha^2 + \sigma^2$

$\text{E}\{\text{MSE}\} = \sigma^2$

$\text{E}\{\text{MSA(B)}\} = n\sigma_\alpha^2 + \sigma^2$

c. MSA(B)

D.5. a.

Model Term	Coef-ficient	Symbolic Product	Term to be Squared	Degrees of Freedom
ρ_i	r	$i-1$	$\bar{Y}_{i.} - \bar{Y}_{..}$	$n-1$
τ_j	n_b	$j-1$	$\bar{Y}_{.j} - \bar{Y}_{..}$	$r-1$
Error			Remainder = $Y_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..}$	Remainder = $(r-1)(n_b-1)$
Total			$Y_{ij} - \bar{Y}_{..}$	$rn_b - 1$

b.

	i	j				
	F	F	Vari-	E{MSBL}	E{MSTR}	E{MSE}
	n_b	r	ance	i	j	(ij)
ρ_i	0	r	σ_ρ^2	r	0	0
τ_j	n_b	0	σ_τ^2	0	n_b	0
$\epsilon_{(ij)}$	1	1	σ^2	1	1	1

D.6. a. See Problem D.5a.

b.

	i	j				
	F	R	Vari-	E{MSBL}	E{MSTR}	E{MSE}
	n_b	r	ance	i	j	(ij)
ρ_i	0	r	σ_ρ^2	r	0	0
τ_j	n_b	1	σ_τ^2	0	n_b	0
$\epsilon_{(ij)}$	1	1	σ^2	1	1	1

D.7. a. See Problem D.5a.

b.

	i	j				
	R	F	Vari-	E{MSBL}	E{MSTR}	E{MSE}
	n_b	r	ance	i	j	(ij)
ρ_i	1	r	σ_ρ^2	r	0	0
τ_j	n_b	0	σ_τ^2	0	n_b	0
$\epsilon_{(ij)}$	1	1	σ^2	1	1	1

D.8. a.

Model Term	Coef-ficient	Symbolic Product	Term to be Squared	Degrees of Freedom
ρ_i	rm	$i-1$	$\bar{Y}_{i..} - \bar{Y}_{...}$	$n_b - 1$
τ_j	$n_b m$	$j-1$	$\bar{Y}_{.j.} - \bar{Y}_{...}$	$r-1$
$\epsilon_{(ij)}$	m		Remainder = $\bar{Y}_{ij.} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...}$	Remainder = $(n_b-1)(r-1)$
$\eta_{k(ij)}$	1	$ijk - ij$	$Y_{ijk} - \bar{Y}_{ij.}$	$n_b r(m-1)$
Total			$Y_{ijk} - \bar{Y}_{...}$	$n_b rm - 1$

SSBL $= rm \sum(\bar{Y}_{i..} - \bar{Y}_{...})^2$
SSEE $= m \sum\sum(\bar{Y}_{ij.} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...})^2$
SSTR $= n_b m \sum(\bar{Y}_{.j.} - \bar{Y}_{...})^2$
SSOE $= \sum\sum\sum(Y_{ijk} - \bar{Y}_{ij.})^2$

b.

	i	j	k					
	R	F	R	Vari-	E{MSBL}	E{$MSTR$}	E{MSEE}	E{MSOE}
	n_b	r	m	ance	i	j	(ij)	$(ij)k$
ρ_i	1	r	m	σ_ρ^2	rm	0	0	0
τ_j	n_b	0	m	σ_τ^2	0	$n_b m$	0	0
$\epsilon_{(ij)}$	1	1	m	σ^2	m	m	m	0
$\eta_{k(ij)}$	1	1	1	σ_η^2	1	1	1	1

E{MSBL} $= rm\sigma_\rho^2 + m\sigma^2 + \sigma_\eta^2$ E{MSEE} $= m\sigma^2 + \sigma_\eta^2$
E{MSTR} $= \frac{n_b m \sum \tau_j^2}{r-1} + m\sigma^2 + \sigma_\eta^2$ E{MSOE} $= \sigma_\eta^2$

D.9. a.

Model Term	Symbolic Product	Sum of Squares	Degrees of Freedom
α_i	$i-1$	$bcn\sum(\bar{Y}_{i...} - \bar{Y}_{....})^2$	$a-1$
$\beta_{j(k)}$	$jk-k$	$an\sum\sum(\bar{Y}_{.jk.} - \bar{Y}_{..k.})^2$	$c(b-1)$
γ_k	$k-1$	$abn\sum(\bar{Y}_{..k.} - \bar{Y}_{....})^2$	$c-1$
$(\alpha\gamma)_{ik}$	$ik-i-k+1$	$bn\sum\sum(\bar{Y}_{i\cdot k\cdot} - \bar{Y}_{i...} - \bar{Y}_{..k.} + \bar{Y}_{....})^2$	$(a-1)(c-1)$
$(\alpha\beta)_{ij(k)}$	$ijk-ik-jk+k$	$n\sum\sum\sum(\bar{Y}_{ijk\cdot} - \bar{Y}_{i\cdot k\cdot} - \bar{Y}_{.jk.} + \bar{Y}_{..k.})^2$	$(a-1)(b-1)c$
$\epsilon_{m(ijk)}$	$ijkm-ijk$	$\sum\sum\sum\sum(Y_{ijkm} - \bar{Y}_{ijk\cdot})^2$	$abc(n-1)$
Total		$\sum\sum\sum\sum(Y_{ijkm} - \bar{Y}_{....})^2$	$abcn-1$

b.

	i	j	k	m		Expected Mean Square of $--$					
	F	R	R	R	Vari-	A	$B(C)$	C	AC	$AB(C)$	E
	a	b	c	n	ance	i	$j(k)$	k	ik	$ij(k)$	$m(ijk)$
α_i	0	b	c	n	σ^2_α	bcn	0	0	0	0	0
$\beta_{j(k)}$	a	1	1	n	σ^2_β	0	an	an	0	0	0
γ_k	a	b	1	n	σ^2_γ	0	0	abn	0	0	0
$(\alpha\gamma)_{ik}$	0	b	1	n	$\sigma^2_{\alpha\gamma}$	bn	0	0	bn	0	0
$(\alpha\beta)_{ij(k)}$	0	1	1	n	$\sigma^2_{\alpha\beta}$	n	0	0	n	n	0
$\epsilon_{m(ijk)}$	1	1	1	1	σ^2	1	1	1	1	1	1

$E\{MSA\} = \frac{bcn\sum\alpha_i^2}{a-1} + bn\sigma^2_{\alpha\gamma} + n\sigma^2_{\alpha\beta} + \sigma^2$

$E\{MSB(C)\} = an\sigma^2_\beta + \sigma^2$

$E\{MSC\} = an\sigma^2_\beta + abn\sigma^2_\gamma + \sigma$ $\qquad$ $E\{MSAB(C)\} = n\sigma^2_{\alpha\beta} + \sigma^2$

$E\{MSAC\} = bn\sigma^2_{\alpha\gamma} + n\sigma^2_{\alpha\beta} + \sigma^2$ $\qquad$ $E\{MSE\} = \sigma^2$

D.10. e_{ijk} :

		$j=1$	$j=2$	$j=3$
	$k=1$	-2.3333	.3333	-1.6667
$i=1$	$k=2$	1.6667	-1.6667	.3333
	$k=3$	.6667	1.3333	1.3333
	$k=1$	$-.3333$	2.3333	-1.0000
$i=2$	$k=2$	1.6667	$-.6667$	.0000
	$k=3$	-1.3333	-1.6667	1.0000
	$k=1$	-1.6667	-1.6667	-1.3333
$i=3$	$k=2$	1.3333	1.3333	.6667
	$k=3$	.3333	.3333	.6667

$r = .981$

D.11. a.

Source	SS	df	MS
Blocks	520.963	2	260.4815
Treatments	103.185	2	51.5925
Experimental error	5.259	4	1.3148
Observation error	45.333	18	2.5185
Total	674.741	26	

b. H_0 : all τ_j equal zero ($j = 1, 2, 3$), H_a : not all τ_j equal zero. $F^* = 51.5925/1.3148 = 39.24$, $F(.95; 2, 4) = 6.94$. If $F^* \leq 6.94$ conclude H_0, otherwise H_a. Conclude H_a. $P-$value $= .002$.

c. $\bar{Y}_{.1.} = 24.77778$, $\bar{Y}_{.2.} = 20.00000$, $\bar{Y}_{.3.} = 22.66667$, $\hat{L}_1 = \bar{Y}_{.1.} - \bar{Y}_{.2.} = 4.77778$, $\hat{L}_2 = \bar{Y}_{.1.} - \bar{Y}_{.3.} = 2.11111$, $\hat{L}_3 = \bar{Y}_{.2.} - \bar{Y}_{.3.} = -2.66667$, $s\{\hat{L}_i\} = .5405$ ($i = 1, 2, 3$), $q(.90; 3, 4) = 3.98$, $T = 2.8143$

$$4.77778 \pm 2.8143(.5405) \qquad 3.257 \leq L_1 \leq 6.299$$

2.11111	$\pm$	$2.8143(.5405)$	$.590 \le L_2 \le 3.632$
-2.66667	$\pm$	$2.8143(.5405)$	$-4.188 \le L_3 \le -1.146$

d. $\hat{\sigma}^2 = 0$, $\hat{\sigma}^2_\eta = 2.5185$

D.12. a.

Model Term	Coef-ficient	Symbolic Product	Term to Be Squared	Degrees of Freedom
ρ_i	ab	$i-1$	$\bar{Y}_{i..} - \bar{Y}_{...}$	$s-1$
α_j	sb	$j-1$	$\bar{Y}_{.j.} - \bar{Y}_{...}$	$a-1$
β_k	sa	$k-1$	$\bar{Y}_{..k} - \bar{Y}_{...}$	$b-1$
$(\alpha\beta)_{jk}$	s	$jk-j-k+1$	$\bar{Y}_{.jk} - \bar{Y}_{.j.} - \bar{Y}_{..k} + \bar{Y}_{...}$	$(a-1)(b-1)$
Error			Remainder = $Y_{ijk} - \bar{Y}_{i..} - \bar{Y}_{.jk} + \bar{Y}_{...}$	Remainder = $(s-1)(ab-1)$
Total			$Y_{ijk} - \bar{Y}_{...}$	$abs-1$

b.

	i	j	k		Expected Mean Square of $--$				
	R	F	F	Vari-	S	A	B	AB	Rem
	s	a	b	ance	i	j	k	jk	(ijk)
ρ_i	1	a	b	σ^2_ρ	ab	0	0	0	0
α_j	s	0	b	σ^2_α	0	sb	0	0	0
β_k	s	a	0	σ^2_β	0	0	sa	0	0
$(\alpha\beta)_{jk}$	s	0	0	$\sigma^2_{\alpha\beta}$	0	0	0	s	0
$\epsilon_{(ijk)}$	1	1	1	σ^2	1	1	1	1	1

D.13. a.

Model Term	Coef-ficient	Symbolic Product	Term to Be Squared	Degrees of Freedom
α_j	bs	$j-1$	$\bar{Y}_{.j.} - \bar{Y}_{...}$	$a-1$
β_k	as	$k-1$	$\bar{Y}_{..k} - \bar{Y}_{...}$	$b-1$
$(\alpha\beta)_{jk}$	s	$jk-j-k+1$	$\bar{Y}_{.jk} - \bar{Y}_{.j.} - \bar{Y}_{..k} + \bar{Y}_{...}$	$(a-1)(b-1)$
$\rho_{i(j)}$	b	$ij-j$	$\bar{Y}_{ij.} - \bar{Y}_{.j.}$	$a(s-1)$
Error			Remainder = $Y_{ijk} - \bar{Y}_{.jk} - \bar{Y}_{ij.} + \bar{Y}_{.j.}$	Remainder = $a(s-1)(b-1)$
Total			$Y_{ijk} - \bar{Y}_{...}$	$abs-1$

b.

	j	k	i		Expected Mean Square of $--$				
	F	F	R	Vari-	A	B	AB	$S(A)$	Rem
	a	b	s	ance	j	k	jk	$i(j)$	(ijk)
α_j	0	b	s	σ^2_α	bs	0	0	0	0
β_k	a	0	s	σ^2_β	0	as	0	0	0
$(\alpha\beta)_{jk}$	0	0	s	$\sigma^2_{\alpha\beta}$	0	0	s	0	0
$\rho_{i(j)}$	1	b	1	σ^2_ρ	b	0	0	b	0
$\epsilon_{(ijk)}$	1	1	1	σ^2	1	1	1	1	1

D.14. Note: The subscript for subjects here is l instead of the usual i and the subscripts for factor A, B, and C are i, j, and k, respectively.

a.

$$\begin{aligned} Y_{ijklm} &= \mu_{....} + \alpha_i + \beta_j + \gamma_k + \rho_{l(ik)} + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} \\ &\quad +(\alpha\beta\gamma)_{ijk} + \epsilon_{m(ijkl)} \end{aligned}$$

b.

Model Term	Coef- ficient	Symbolic Product	Term to Be Squared	Degrees of Freedom
α_i	$bcrn$	$i-1$	$\bar{Y}_{i....} - \bar{Y}_{.....}$	$a-1$
β_j	$acrn$	$j-1$	$\bar{Y}_{.j...} - \bar{Y}_{.....}$	$b-1$
γ_k	$abrn$	$k-1$	$\bar{Y}_{..k..} - \bar{Y}_{.....}$	$c-1$
$\rho_{l(ik)}$	$abcn$	$ikl-ik$	$\bar{Y}_{i\cdot kl\cdot} - \bar{Y}_{i\cdot k\cdot\cdot}$	$ac(r-1)$
$(\alpha\beta)_{ij}$	crn	$ij-i-j+1$	$\bar{Y}_{ij...} - \bar{Y}_{i....} - \bar{Y}_{.j...} + \bar{Y}_{.....}$	$(a-1)(b-1)$
$(\alpha\gamma)_{ik}$	brn	$ik-i-k+1$	$\bar{Y}_{i\cdot k\cdot\cdot} - \bar{Y}_{i....} - \bar{Y}_{..k..} + \bar{Y}_{.....}$	$(a-1)(c-1)$
$(\beta\gamma)_{jk}$	arn	$jk-j-k+1$	$\bar{Y}_{.jk..} - \bar{Y}_{.j...} - \bar{Y}_{..k..} + \bar{Y}_{.....}$	$(b-1)(c-1)$
$(\alpha\beta\gamma)_{ijk}$	rn	$ijk-ik-jk$ $-ij+i+j+k-1$	$\bar{Y}_{ijk..} - \bar{Y}_{i\cdot k\cdot\cdot} - \bar{Y}_{.jk..} - \bar{Y}_{ij...}$ $+\bar{Y}_{i....} + \bar{Y}_{.j...} + \bar{Y}_{..k..} - \bar{Y}_{.....}$	$(a-1)(b-1)(c-1)$
Error			Remainder $=$ $Y_{ijklm} - \bar{Y}_{i\cdot kl\cdot} - \bar{Y}_{ijk..} + \bar{Y}_{i\cdot k\cdot\cdot}$	Remainder $=$ $abcrn - acr - abc + ac$
Total			$Y_{ijklm} - \bar{Y}_{.....}$	$abcrn-1$

$$\begin{aligned} SSA &= bcrn \sum (\bar{Y}_{i....} - \bar{Y}_{.....})^2 \\ SSB &= acrn \sum (\bar{Y}_{.j...} - \bar{Y}_{.....})^2 \\ SSC &= abrn \sum (\bar{Y}_{..k..} - \bar{Y}_{.....})^2 \\ & etc. \end{aligned}$$

c.

	i	j	k	l	m	
	F	F	F	R	R	
	$a=3$	$b=4$	$c=2$	$r=4$	$n=2$	Variance
α_i	0	b	c	r	n	σ^2_α
β_j	a	0	c	r	n	σ^2_β
γ_k	a	b	0	r	n	σ^2_γ
$\rho_{l(ik)}$	1	b	1	1	n	σ^2_ρ
$(\alpha\beta)_{ij}$	0	0	c	r	n	$\sigma^2_{\alpha\beta}$
$(\alpha\gamma)_{ik}$	0	b	0	r	n	$\sigma^2_{\alpha\gamma}$
$(\beta\gamma)_{jk}$	a	0	0	r	n	$\sigma^2_{\beta\gamma}$
$(\alpha\beta\gamma)_{ijk}$	0	0	0	r	n	$\sigma^2_{\alpha\beta\gamma}$
$\epsilon_{m(ijkl)}$	1	1	1	1	1	σ^2

	Expected Mean Square of $--$								
	A	B	C	$S(AC)$	AB	AC	BC	ABC	E
	i	j	k	$l(ik)$	ij	ik	jk	ijk	$m(ijkl)$
α_i	$bcrn$	0	0	0	0	0	0	0	0
β_j	0	$acrn$	0	0	0	0	0	0	0
γ_k	0	0	$abrn$	0	0	0	0	0	0
$\rho_{l(ik)}$	bn	0	bn	bn	0	bn	0	0	0
$(\alpha\beta)_{ij}$	0	0	0	0	crn	0	0	0	0
$(\alpha\gamma)_{ik}$	0	0	0	0	0	brn	0	0	0
$(\beta\gamma)_{jk}$	0	0	0	0	0	0	arn	0	0
$(\alpha\beta\gamma)_{ijk}$	0	0	0	0	0	0	0	rn	0
$\epsilon_{m(ijkl)}$	1	1	1	1	1	1	1	1	1

$$
\begin{aligned}
\mathrm{E}\{\mathrm{MSA}\} &= 64\sum\frac{\alpha_i^2}{2} + 8\sigma^2_\rho + \sigma^2 \\
\mathrm{E}\{\mathrm{MSB}\} &= 48\sum\frac{\beta_j^2}{3} + \sigma^2 \\
\mathrm{E}\{\mathrm{MSC}\} &= 96\sum\frac{\gamma_k^2}{1} + 8\sigma^2_\rho + \sigma^2 \\
\mathrm{E}\{\mathrm{MSS(AC)}\} &= 8\sigma^2_\rho + \sigma^2 \\
\mathrm{E}\{\mathrm{MSAB}\} &= 16\sum\sum\frac{(\alpha\beta)_{ij}^2}{6} + \sigma^2 \\
\mathrm{E}\{\mathrm{MSAC}\} &= 32\sum\sum\frac{(\alpha\gamma)_{ik}^2}{2} + 8\sigma^2_\rho + \sigma^2 \\
\mathrm{E}\{\mathrm{MSBC}\} &= 24\sum\sum\frac{(\beta\gamma)_{jk}^2}{3} + \sigma^2 \\
\mathrm{E}\{\mathrm{MSABC}\} &= 8\sum\sum\sum\frac{(\alpha\beta\gamma)_{ijk}^2}{6} + \sigma^2 \\
\mathrm{E}\{\mathrm{MSE}\} &= \sigma^2
\end{aligned}
$$

D.15.

	i	j	k					
	F	F	F	Vari-	E{MSROW}	E{MSCOL}	E{MSTR}	E{MSE}
	r	r	r	ance	i	j	k	(ijk)
ρ_i	0	r	r	σ^2_ρ	r	0	0	0
κ_j	r	0	r	σ^2_κ	0	r	0	0
τ_k	r	r	0	σ^2_τ	0	0	r	0
$\epsilon_{(ijk)}$	1	1	1	σ^2	1	1	1	1

D.16.

	i	j	k	m					
	F	F	F	R	Vari-	E{MSROW}	E{MSCOL}	E{MSTR}	E{MSRem}
	r	r	r	n	ance	i	j	k	$m(ijk)$
ρ_i	0	r	r	n	σ^2_ρ	rn	0	0	0
κ_j	r	0	r	n	σ^2_κ	0	rn	0	0
τ_k	r	r	0	n	σ^2_τ	0	0	rn	0
$\epsilon_{m(ijk)}$	1	1	1	1	σ^2	1	1	1	1

$$\mathrm{E}\{\mathrm{MSROW}\} = \sigma^2 + \frac{rn\sum\rho_i^2}{r-1} \qquad \mathrm{E}\{\mathrm{MSTR}\} = \sigma^2 + \frac{rn\sum\tau_k^2}{r-1}$$

$$\mathrm{E}\{\mathrm{MSCOL}\} = \sigma^2 + \frac{rn\sum\kappa_j^2}{r-1} \qquad \mathrm{E}\{\mathrm{MSRem}\} = \sigma^2$$

D.17.

	i	j	k	m						
						Expected Mean Square of −−				
	F	F	F	R	Vari-	P	O	TR	$S(P)$	Rem
	r	r	r	n	ance	i	j	k	$m(i)$	$(ijkm)$
ρ_i	0	r	r	n	σ^2_ρ	rn	0	0	0	0
κ_j	r	0	r	n	σ^2_κ	0	rn	0	0	0
τ_k	r	r	0	n	σ^2_τ	0	0	rn	0	0
$\eta_{m(i)}$	1	r	r	1	σ^2_η	r	0	0	r	0
$\epsilon_{(ijkm)}$	1	1	1	1	σ^2	1	1	1	1	1