Magnetics Design for Switching Power Supplies

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Section 1 Introduction and Basic Magnetics

Introduction

Experienced SwitchMode Power Supply designers know that SMPS success or failure depends heavily on the proper design and implementation of the magnetic components. Parasitic elements inherent in high frequency transformers or inductors cause a variety of circuit problems including: high losses, high voltage spikes necessitating snubbers or clamps, poor cross regulation between multiple outputs, noise coupling to input or output, restricted duty cycle range, etc. Figure 1 represents a simplified equivalent circuit of a two-output forward converter power transformer, showing leakage inductances, core characteristics including mutual inductance, dc hysteresis and saturation, core eddy current loss resistance, and winding distributed capacitance, all of which affect SMPS performance.

With rare exception, schools of engineering provide very little instruction in practical magnetics relevant to switching power supply applications. As a result, magnetic component design is usually delegated to a self-taught expert in this "black art". There are many aspects in the design of practical, manufacturable, low cost magnetic devices that unquestionably benefit from years of experience in this field. However, the magnetics expert is unlikely to be sufficiently aware of the SMPS circuit problems caused by the various parasitic elements and the impact of the specific circuit locations of these elements. This often results in poor decisions in the magnetic component design.

This collection of topics on magnetics is intended to give the SMPS designer the confidence and the ability to: (1) Develop a reasonably accurate electrical circuit model of any magnetic device, to enable prediction of circuit performance, (2) Relate the electrical circuit model to the magnetic device structure, thus providing the insight needed to achieve an optimized design, (3) Collaborate effectively with experts in magnetics design, and possibly (4) Become a "magnetics expert" in his own right.

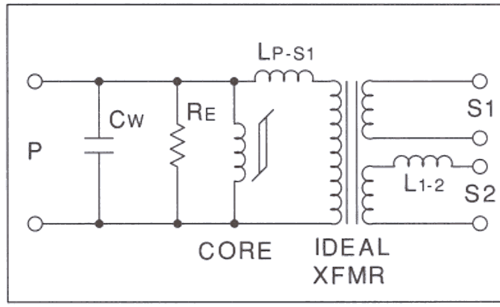

Figure 1-1 Transformer Equivalent Circuit

Obstacles to learning magnetics design

In addition to the lack of instruction in practical magnetics mentioned above, there are several other problems that make it difficult for the SMPS designer to feel "at home" in the magnetics realm:

- *Archaic concepts and practices.* Our great-grandparents probably had a better understanding of practical magnetics than we do today. Unfortunately, in an era when computation was difficult, our ancestors developed concepts intended to minimize computation (such as "circular mils") and other shortcuts, still in use, which make the subject of magnetics more complex and confusing to us today. Ancient design equations intended for sinusoidal waveforms (and not clearly defined as such) are often incorrectly applied to rectangular SMPS waveforms.
- *The CGS system of units.* Magnetics design relationships expressed in the modern SI system of units (rationalized MKS) are much simpler than in the archaic CGS system. SI equations are much easier to understand and remember. Unfortunately, in the U.S., core and magnet wire data is

usually published in the old-fashioned CGS system, with dimensions often in inches, requiring data conversion to apply to the SI system.

- *Energy/time vs. power.* Circuit designers are comfortable in the electrical realm of Volts and Amperes. On a Volt/Ampere plot, area represents power. Time is not directly involved.

  But on a plot of magnetic flux density, B, vs. field intensity, H, area represents *energy*. Time is always required to change flux density, because an energy change must take place. A change in flux density, ΔB, within a winding equates to the *integral* volt-seconds per turn across the winding (Faraday's Law). Time is definitely involved. This concept takes some getting used to.

An appeal to suppliers of core materials and wire: Old-time magnetics designers in the U.S. are acclimated to the CGS system, and may prefer magnetics data expressed in Gauss and Oersteds. But newcomers to magnetics design, as well as experienced designers outside the U.S. prefer the internationally accepted SI system – Tesla and Ampere-Turns. Also, cores and wire dimensional data should *definitely* be provided in metric units.

Core losses are usually characterized as a function of frequency using sinusoidal current-driven waveforms. This is erroneous and misleading for high frequency SMPS applications. High frequency core losses are primarily caused by eddy currents, which depend on rate of flux change, *not* on frequency per se. (At a fixed switching frequency, higher V_{IN} with short duty cycle results in higher loss.) It would be most helpful if materials intended primarily for SMPS applications were characterized using rectangular voltage-driven waveforms, with examples shown of core loss and minor hysteresis loops under these conditions.

Magnetic Field Relationships

Understanding the rules that govern the magnetic field is extremely valuable in many aspects of switching power supply design, especially in minimizing parasitics in circuit wiring, as well as in magnetic device design.

Figure 1-2 shows the field surrounding two parallel conductors, each carrying the same current but in opposite directions, i.e. a pair of wires connecting an electrical source to a load.

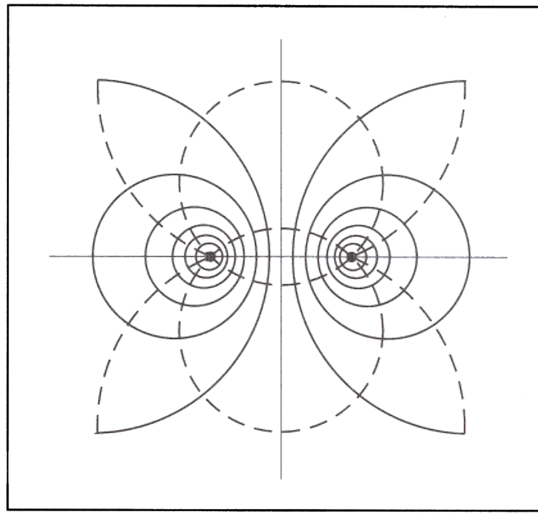

Figure. 1-2 Field Around Conductor Pair

The solid lines represent magnetic *flux*, while the dash lines represent an edge view of the *magnetic field equipotential surfaces*. Each wire has an individual field, symmetrical and radial, with field intensity diminishing in inverse proportion to the distance from the conductor. These two fields are of equal magnitude but opposite in polarity because the currents that generate the fields are in opposite directions. As shown in Fig. 1-2, the two fields, by superposition, reinforce each other in the region between the two wires, but elsewhere they tend to cancel, especially at a distance from the wires where the opposing field intensities become nearly equal.

Figure 1-3 shows the field associated with a simple air cored winding. The individual fields from each wire combine to produce a highly concentrated and fairly linear field within the winding. Outside the winding, the field diverges and weakens. Although the stored energy density is high within the winding, considerable energy is stored in the weaker field outside the winding because the volume extends to infinity.

A magnetic field cannot be blocked by "insulating" it from its surroundings – magnetic "insulation" does not exist. However, the magnetic field *can* be short-circuited – by placing the coil of Fig 1-3 inside

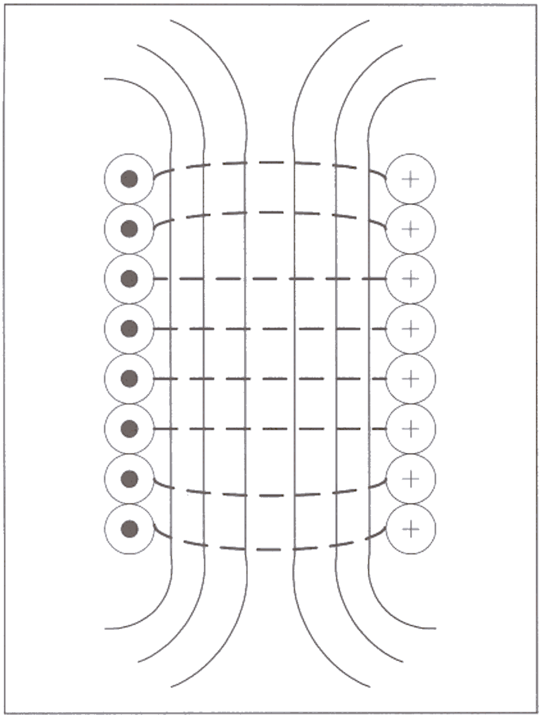

Figure. 1-3 Air-Core Solenoid

a box of high permeability magnetic material, which provides an easy path for the return flux, shielding the coil from the environment external to the box

Important Magnetic Field Principles

- The total magnetic field integrated around any closed path equals the total ampere-turns enclosed by that path (Ampere's Law)
- Magnetic field equipotentials are *surfaces*, not lines. (Alternatively, field intensity can be represented as a vector normal to the surface.)
- Magnetic field equipotential surfaces are *bounded* and *terminated* by the current which generates the field. They are not closed surfaces, as with electric field equipotentials.
- Flux is in the form of *lines*, not surfaces. (Flux can also be represented as a vector.)
- Flux lines are *always closed loops* – they never begin or end. In any arbitrary volume, the number of flux lines entering must equal the number leaving, regardless of the contents of that volume.
- Flux lines are always normal to the magnetic field equipotential surfaces.
- At any location, flux density is always proportional to field intensity: $B = \mu H$

Conservation of energy: At any moment of time, the magnetic field and the current flow distribute themselves so as to minimize the energy taken from the source. If alternative current paths exist, current will initially flow in the path(s) resulting in minimum stored energy, but in the long term, the current flow redistributes so as to minimize I^2R loss. Reference (R2) has more on this subject.

Transformation of Axes

SMPS circuit designers are obviously interested in the electrical characteristics of the magnetic device as seen at the device terminals.

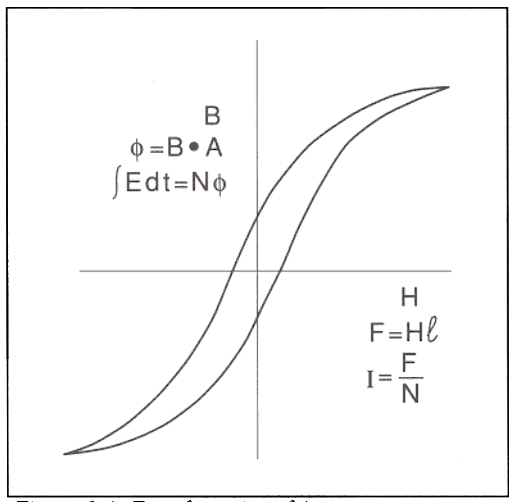

Figure. 1-4 Transformation of Axes

Figure 1-4 shows how the horizontal and vertical axes scale factors can be altered so that the *B-H* characteristic (defining a core *material*) is translated into a ϕ vs. $\mathcal{F}$ (mmf) characteristic (defining a *specific core* with magnetic area A_e and path length ℓ_e). Transforming the axes once again using Faraday's Law and Ampere's Law, the same curve now represents the equivalent electrical characteristics of that core when wound with N turns, $\int Edt$ vs. I.

Note that the slope of the *B-H* characteristic is *permeability*, the slope of the ϕ vs. $\mathcal{F}$ characteristic is *permeance*, while the slope of the $\int Edt$ vs. I characteristic is *inductance.*

Systems of Units

The internationally accepted SI system of units (Système International d'Unités) is a rationalized system, in which permeability, $\mu = \mu_0 \bullet \mu_r$ (μ_0 is the absolute permeability of free space or nonmagnetic material = $4\pi \bullet 10^{-7}$; μ_r is the relative permeability of a magnetic material). In the unrationalized CGS system, $\mu_0 = 1$, therefore μ_0 is omitted from CGS equations so that $\mu = \mu_r$. But the rationalization constant μ_0 doesn't just disappear in the CGS system – instead, portions of this constant show up in all the CGS equations, complicating them and making them more difficult to intuitively grasp. In the SI system, all of the "garbage" is gathered into μ_0, thereby simplifying the SI equations.

The equations below are given in both systems – SI and CGS. It is suggested that beginners in magnetics design stick to the SI equations and ignore the CGS system until completely comfortable with the principles involved. Then, it may be helpful to use the CGS system when working with magnetics data expressed in CGS units, rather than convert the units.

Table I
Magnetic Parameters and Conversion Factors

		SI	CGS	CGS to SI
FLUX DENSITY	B	Tesla	Gauss	10^{-4}
FIELD INTENSITY	H	A-T/m	Oersted	$1000/4\pi$
PERMEABILITY (space)	μ_0	$4\pi \bullet 10^{-7}$	1	$4\pi \bullet 10^{-7}$
PERMEABILITY (relative)	μ_r			
AREA (Core Window)	A_e, A_w	m	cm^2	10^{-4}
LENGTH (Core, Gap)	ℓ_e, ℓ_g	m	cm	10^{-2}
TOTAL FLUX = $\int BdA$	ϕ	Weber	Maxwell	10^{-8}
TOTAL FIELD = $\oint Hd\ell$	$\mathcal{F}$,mmf	A-T	Gilbert	$10/4\pi$
RELUCTANCE = $\mathcal{F}/\phi$	$\mathcal{R}$	A-T/Wb	Gb/Mx	$10^9/4\pi$
PERMEANCE = $1/\mathcal{R}$	$\mathcal{P}$			$4\pi \bullet 10^{-9}$
INDUCTANCE = $\mathcal{P} \bullet N^2$ (SI)	L	Henry	(Henry)	
ENERGY	W	Joule	Erg	10^{-7}

Ampere's Law and Faraday's Law jointly govern the important relationship between the magnetic elements and the equivalent electrical circuit as seen across the windings.

Ampere's Law

SI:

$$F = \oint Hd\ell = NI \approx H\ell \quad \text{A-T}$$

$$H \approx NI/\ell \qquad \text{A-T/m (1)}$$

$$F = \oint Hd\ell = .4\pi NI \approx H\ell \qquad \text{Gilberts}$$

$$H \approx .4\pi NI/\ell \qquad \text{Oersteds (1a)}$$

Amperes Law states that the total magnetic force, $\mathcal{F}$, along a closed path is proportional to the ampere-turns in a winding or windings linked to that path, i.e., that the path passes through. In the SI system, the units of magnetic force are expressed in ampere-turns. When the field *intensity* H varies along the path, H must be integrated along the path length. Fortunately, the simplified form shown in Eq. 1 and 1a can be used in most situations.

Faraday's Law

SI:

$$\frac{d\phi}{dt} = -\frac{E}{N}; \quad \Delta\phi = \frac{}{N}\int Edt \qquad \text{Weber (2)}$$

$$\frac{d\phi}{dt} = -\frac{E}{N} \times 10^8 \quad \Delta\phi = \frac{10^8}{N}\int Edt \quad \text{Maxwell (2a)}$$

Faraday's Law equates the flux *rate of change* through a winding to the volts/turn applied to the winding. Thus, the flux change is proportional to the integral volt-seconds per turn (directly equal in the SI system). Faraday's Law operates bilaterally – that is, if 2.5 volts/turn is applied to winding A, the flux through A will change by 2.5 Webers/second. If a second winding, B, is linked to all of the flux produced by winding A, then 2.5 Volts/turn will be *induced* in B.

Faraday's Law makes it clear that flux cannot change instantaneously. Any flux change requires time and usually a change in energy. Time is required to move along the ϕ or B axis, which is more obvious considering the electrical equivalent scale dimensions are Volt-seconds.

Note that all flux lines follow a closed loop path. Flux lines have no beginning or end.

Energy

SI:

$$W/m^3 = \int HdB \approx \frac{1}{2}BH$$

$$W = \int Vol \cdot HdB = \int I \cdot Edt \qquad \text{Joules} \quad (3)$$

CGS:

$$W = \int Vol \cdot HdB = \int I \cdot Edt \cdot 10^{-7} \qquad \text{Ergs} \quad (3a)$$

Energy put into and removed from the magnetic system can be determined by integrating the area between the characteristic and the *vertical* axis (B, ϕ, $\int Edt$) on the energy plane. Energy must be integrated over time, which is a factor on the vertical axis, not the horizontal.

It is much easier to understand this process by using the electrical equivalent axes, Volt-seconds and Amperes. Referring to Fig. 1-5, from point A to B, energy from the external circuit is put into the magnetic system, as shown by the shaded area between A-B and the vertical axis. From B to C, magnetically stored energy is returned to the electrical circuit. The difference between the energy put in and taken out is hysteresis loss, the area between the two curves. At C, the magnetically stored energy is zero.

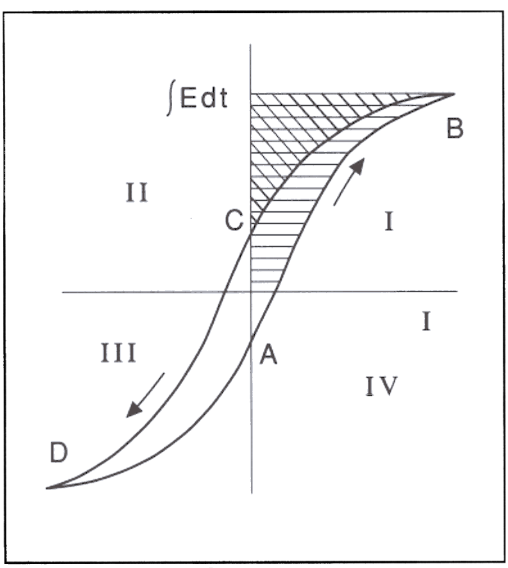

Figure 1-5 Energy Plane

From C to D, energy is put into the system. From D back to A energy is returned to the electrical circuit. The area between the curves is loss. At A, the remaining stored energy is zero.

A positive energy sign indicates energy put in; a negative sign indicates energy returning to the external circuit. From A to B, voltage and current are both positive, so the energy sign is positive. Although the integrated Volt-seconds are negative at A, *upward movement indicates positive voltage*. From B to C, current is positive but voltage is negative (downward movement). Therefore the energy sign is negative. From C to D, current and voltage are both negative, hence positive energy. From D to A, negative current with positive voltage indicates returning energy.

Permeability

SI:

$$\mu = \mu_0 \mu_r = B/H \qquad \text{Tesla/A-T/m}$$

CGS:

$$\mu = \mu_r = B/H \qquad \text{Gauss/Oersted}$$

Permeability is a measure of a magnetic *material* – the amount of flux which a magnetic field can push through a unit volume of the material. Permeability is roughly analogous to conductivity in the electrical realm.

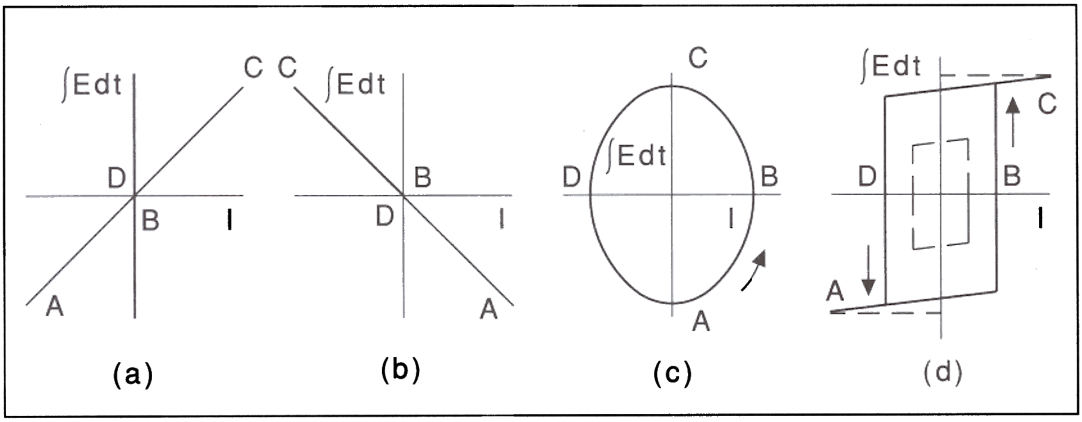

Figure 1-6 – Energy Plane, Sinusoidal Voltage Drive Examples

Permeance, Reluctance

SI:

$$\mathcal{P} = 1/\mathcal{R} = \phi/\mathcal{F} = BA/H\ell$$

Webers/A-T

$$\mathcal{P} = 1/\mathcal{R} = \mu_0\mu_r \bullet A/\ell$$

CGS:

$$\mathcal{P} = 1/\mathcal{R} = \phi/\mathcal{F} = BA/H\ell$$

Maxwells/Gilbert

$$\mathcal{P} = 1/\mathcal{R} = \mu_r \bullet A/\ell$$

When the material characteristic, permeability, is applied to a magnetic element of specific area and length, the result is *permeance*. In the SI system, permeance is equal to the inductance of a single turn.

Reluctance, the reciprocal of permeance, is analogous to resistance in an electrical circuit. (Don't push this analogy too far – reluctance is an energy storage element, whereas resistance is a dissipative element.) Reluctance and permeance can be defined for the entire magnetic device as seen from the electrical terminals, but it is most useful to define the reluctance of specific elements/regions within the device. This enables the construction of a reluctance model – a *magnetic* circuit diagram – which sheds considerable light on the performance of the device and how to improve it. From the reluctance model, using a duality process, a magnetic device can be translated into its equivalent electrical circuit, including parasitic inductances, such as shown in Fig. 1-1.

This will be discussed in a later section.

Inductance

SI:

Henrys (4)

$$L = N^2\mathcal{P} = \mu_0\mu_r N^2 \bullet A/\ell$$

CGS:

Henrys (4a)

$$L = 4\pi\mu_r N^2 \bullet A/\ell \bullet 10^{-9}$$

Inductance has the same value in the SI and CGS systems. In the SI system, inductance is simply the permeance times the number of turns squared.

At Home on the Energy Plane

It is, of course, possible to plot any type of electrical device on the energy plane ∫Edt vs. I. Figure 1-6 shows several different devices – inductors, capacitors, resistors – driven by a sinusoidal voltage waveform. Before looking at Figure 1-7 which shows the waveforms involved, try to identify the devices represented in Fig. 1-6.

The answers:

Fig.1-6 (a) is an air core inductor, ideal and lossless. Current lags the applied voltage, but as shown in Fig. 1-7, the inductor current is in phase with ∫Edt plotted on the vertical axis of the energy plane. As the waveform traverses from B to C, voltage and current are both positive – energy is put into the inductor. from C to D, current is positive but voltage is negative – the same energy previously stored is given back to the circuit.

Fig.1-4 (b) is an ideal, lossless capacitor. Current leads the applied voltage and is therefore 180° out of phase with ∫Edt. From A to B, voltage and current are both positive – energy is put into the capacitor. From B to C, the same energy previously stored is returned to the circuit.

Fig.1-6 (c) is a resistor. Current is in phase with applied voltage, therefore current leads ∫Edt by 90°. Since voltage and current are in phase, their signs are always the same. The energy sign is always positive – energy is always put into the resistor, never returned to the circuit. The entire area within the ellipse represents loss.

Of course, Faraday's Law does not apply to a resistor or capacitor. Therefore the vertical ∫Edt scale for these devices cannot be translated into flux.

Fig.1-6 (d) is an inductor with idealized metal alloy core with low frequency hysteresis, driven into saturation. A tape-wound Permalloy core driven at low frequency (no eddy currents) approaches this characteristic. The area within the characteristic is hysteresis loss. The only energy returned to the circuit is the area of the thin wedges above and below the characteristic. Only when the core saturates, taking on the characteristic of an air core, is any energy stored and returned.

The dashed characteristic shows a minor hysteresis loop occurring with reduced drive, which does not take the core into saturation.

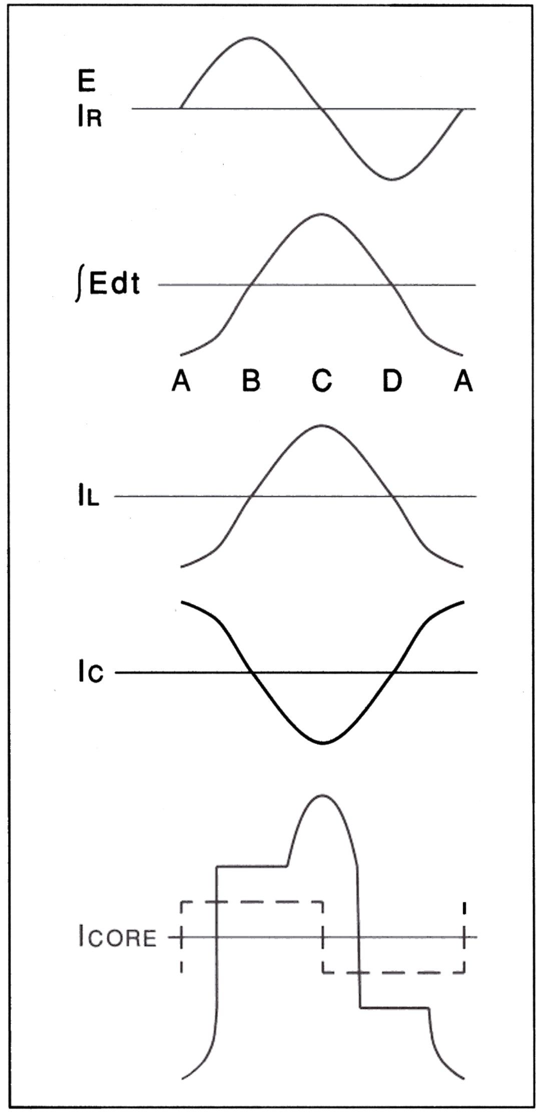

Figure 1-7 Sinusoidal Voltage Drive

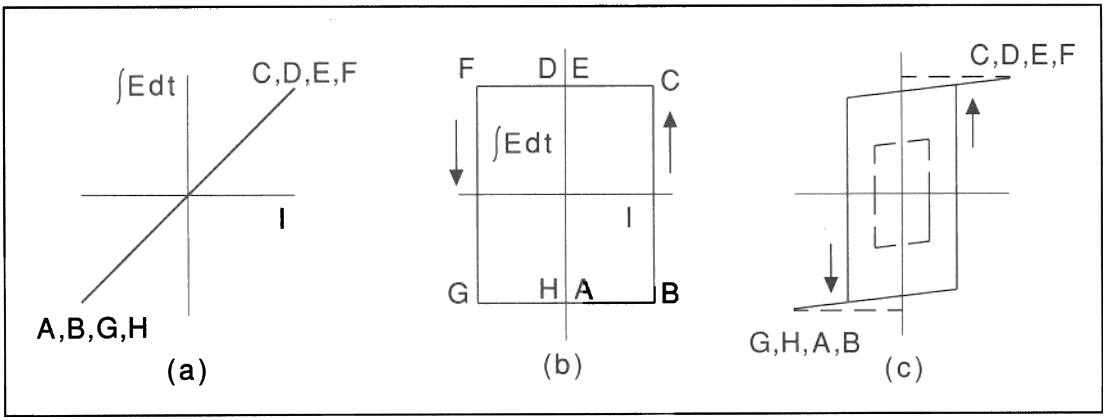

Figure 1-8 – Rectangular Voltage Drive Examples

Rectangular Voltage Drive Waveform

Sinusoidal waveshapes are not relevant in most SMPS applications. Figure 1-8 shows the same devices driven by a symmetrical rectangular voltage waveform (not including the capacitor, which would require an infinite current at the instantaneous voltage transitions). Figure 1-9 shows the corresponding waveforms.

Fig.1-8 (a) is the same lossless air-core inductor as in 1-6 (a). Although the characteristic looks the same as with sinusoidal drive, Fig. 1-9 waveforms reveal that the characteristic dwells at its extremes during times of zero applied voltage.

Fig.1-8 (b) The same resistor as in 1-6 (c) plots as a rectangle rather than an ellipse. The rectangular voltage and current waveforms exist at three distinct levels. From B to C, the voltage and current are both at a constant positive level, while ∫Edt slowly rises. Thus, the current is constant while ∫Edt changes. From C to D, current suddenly collapses to zero, where it dwells until time E, because ∫Edt does not change while the voltage is zero. At time F, the current suddenly changes to its constant negative level, where it remains while ∫Edt slowly drops toward G.

Fig.1-8 (c) The same idealized metal-cored inductor as Fig.1-6 (d) exhibits the same shape on the energy plane although the driving waveshape is quite different.

In fact, with any practical inductor with magnetic core, the low-frequency hysteresis loop (excluding eddy currents) does not change shape radically when frequency *or* voltage *or* the waveshape are changed. But with a resistor, unlike the inductor, the energy plane plot expands in all directions as a function of voltage, the plot changes vertically inversely with frequency, and changes shape as a function of the driving waveshape, as seen in the difference between Fig.1-8 (b) and 1-6 (c).

And no matter how many Volt-seconds are applied to the resistor, it will never saturate like the inductor in Fig.1-8 (c).

Noting the striking similarity between the resistor characteristic of Fig. 1-8 (b) and the dash line unsaturated square-loop inductor characteristic of Fig. 1-8 (c), raises an interesting question: If inductance is defined as the slope on the plot of ∫Edt vs. I, then the resistor in (b) is an inductor – it has infinite inductance along B to C and F to G, just like the unsaturated inductor in 1-8 (c).

But if a resistor is defined as a device that does not store energy, only dissipates energy, then the inductor of Fig.1-8 (c) is a resistor!!

Notes on the SMPS Environment

Transformer Definition: A true transformer is a magnetic device with multiple windings whose purpose is *not* to store energy, but to transfer energy instantaneously from input to output(s). In addition, the windings are often electrically insulated to provide high voltage dc isolation between input and output. The turns ratio can be adjusted to obtain optimum relationship between input and output voltages.

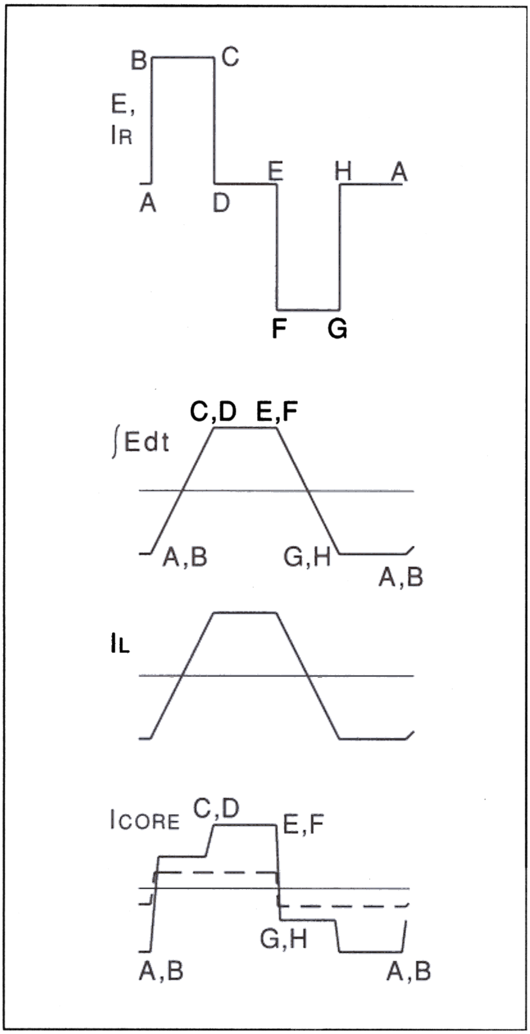

Fig. 1-9 Rectangular Voltage Drive

A practical transformer does store some energy in mutual (magnetizing) inductance and leakage inductances, which degrade circuit performance in several important respects. These inductances are normally considered undesirable parasitics, whose minimization is one of the important goals of transformer design.

Inductor Definition: An inductor is a device whose purpose is to store and release energy. A filter inductor uses this capability to smooth the current through it. A flyback transformer is actually an inductor with multiple windings. It stores energy taken from the input in its mutual inductance during one portion of the switching period, then delivers energy to the output during a subsequent interval.

Since the magnetic core material itself is incapable of storing significant energy, energy storage is accomplished in a non-magnetic gap(s) in series with the core.

Although mutual inductance is an essential element in a flyback transformer, leakage inductances remain undesired parasitic elements.

Core Material Limitations: In dc applications, inductors are thought of as current operated devices. Even the smallest dc voltage will ultimately saturate the magnetic core, unless offset by the IR drop in the winding.

In high frequency SMPS applications, the major core material limitations are saturation and core losses, both of which depend upon flux swing. In these applications, transformer and inductor windings are usually driven with rectangular voltage waveforms derived from low impedance sources. Since the voltage, pulse width, and number of turns are quite accurately known, it is easy to apply Faraday's Law to determine the flux swing and appropriately limit it. In a ferrite core transformer, magnetizing current is difficult to determine accurately. It depends entirely on the core material characteristic which varies widely with temperature and with the flatness of the mating surfaces of the core halves. Fortunately, the magnetizing current in a transformer is small enough to be of less concern than the flux swing.

In an inductor or flyback transformer, the magnetizing current is vitally important, because it represents the energy storage required by the application. In this case, the magnetizing current can be calculated quite accurately using Ampere's Law, because it depends on the very predicable characteristics of the gap in series with the core, and the uncertain core contribution to energy storage is negligible.

Points to Remember:

- Magnetic field equipotentials are surfaces, bounded by the current generating the field.
- All flux lines form complete loops that never begin or end, normal to field equipotentials.
- Flux change cannot occur instantaneously – time is required – an energy change occurs.

- Energy added and removed is quantified by integrating the area between the characteristic and the vertical axis.
- On the energy plane, upward movement in Quadrants I and IV or downward movement in QII and QIII *add* energy to the device. Moving downward in QI and QIV, or upward in QII and QIII returns energy to the circuit.
- The purpose of an inductor is to store energy. In a transformer, energy storage represents an undesired parasitic element.

References

"R-numbered" references are reprinted in the Reference Section at the back of this Manual.

(1) T.G. Wilson, Sr., "Fundamentals of Magnetic Materials," *APEC Tutorial Seminar*, 1987

(R2) "Eddy Current Losses in Transformer Windings and Circuit Wiring," *Unitrode Seminar Manual SEM400*, 1985

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