Introduction

The numbering system we use on a daily basis is *base 10* or the **decimal** system. It turns out that while the numbering system we use is a natural choice for humans, when computers are concerned, a **binary** numbering system is more suitable.

Base 10 or Decimal Numbers

Let's revisit how we were taught to read and write **decimal** numbers. This will help us understand the structure of **binary** numbers.

Digits: There are 10 digits 0 1 2 3 4 5 6 7 8 9

Place Value: The position of each **digit** determines what that value is. Recall that from right to left the first 5 place values are: *ones, tens, hundreds, thousands, ten thousands.*

ten thousands	thousands	hundreds	tens	ones
10,000	1,000	100	10	1

Base 10: Each place value can be represented in terms of 10 by using exponents. 10 is the base and each place value is a **power of 10** The table below shows the place values represented in terms of 10.

10^4	10^3	10^2	10^1	10^0
10,000	1,000	100	10	1

Expanded Form: Consider the decimal number **123**. Recall that the **digit** and **place value** determine what that number is. We can add the quantities in each place value to get the number 123.

100s place	10s place	1s place
1	**2**	**3**
(1×100)	(2×10)	(3×1)
100	20	3

$$\mathbf{123} = (1 \times 100) + (2 \times 10) + (3 \times 1)$$
$$\mathbf{123} = 100 + 20 + 3$$

Quick Practice: Write the following numbers in their expanded form like the example above.

432 =

76 =

8632 =

154 =

90 =

5120 =

801 =

43 =

2922 =

Base 2 or Binary Numbers

A **binary** number system follows the same structure, but instead of powers of 10, place values are **powers of 2** as shown below:

Binary

2^3	2^2	2^1	2^0
8	4	2	1

Decimal

10^3	10^2	10^1	10^0
1,000	100	10	1

Digits: There are 2 digits 0 and 1

Expanded Form: Consider the binary number **1110**. Recall that the **digit** and **place value** determine what that number is. We can add the quantities in each place value to rewrite 1110.

8	4	2	1
1	**1**	**1**	**0**
(1×8)	(1×4)	(1×2)	(0×1)
8	4	2	0

$$\mathbf{1110} = (1 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1)$$
$$\mathbf{1110} = 8 + 4 + 2 + 0$$
$$\mathbf{1110} = 14$$

Shortcut: Since there are only two digits, 0 and 1, you can simply add every place value with a digit of 1.

8	4	2	1
1	1	1	1

$8 + 4 + 2 + 1 = 15$

8	4	2	1
1	0	0	1

$8 + 1 = 9$

Quick Practice: Convert the following binary numbers to decimal numbers as shown above. You may use a table with binary **place values**.

2^3	2^2	2^1	2^0
8	4	2	1

0 0 1 0 = 0 1 1 0 = 0 0 1 1 = 1 0 0 1 =

1 1 1 1 = 0 1 0 0 = 0 0 0 1 = 0 0 1 1 =

0 1 1 0 = 1 0 0 0 = 0 1 1 1 = 1 1 0 1 =