Chapter 5
Discrete Distributions

LEARNING OBJECTIVES

The overall learning objective of Chapter 5 is to help you understand a category of probability distributions that produces only discrete outcomes, thereby enabling you to:

1. Define a random variable in order to differentiate between a discrete distribution and a continuous distribution
2. Determine the mean, variance, and standard deviation of a discrete distribution
3. Solve problems involving the binomial distribution using the binomial formula and the binomial table
4. Solve problems involving the Poisson distribution using the Poisson formula and the Poisson table
5. Solve problems involving the hypergeometric distribution using the hypergeometric formula

CHAPTER OUTLINE

5.1 Discrete Versus Continuous Distributions

5.2 Describing a Discrete Distribution
- Mean, Variance, and Standard Deviation of Discrete Distributions
  - Mean or Expected Value
  - Variance and Standard Deviation of a Discrete Distribution

5.3 Binomial Distribution
- Solving a Binomial Problem
- Using the Binomial Table
- Using the Computer to Produce a Binomial Distribution
- Mean and Standard Deviation of the Binomial Distribution
- Graphing Binomial Distributions

5.4 Poisson Distribution
- Working Poisson Problems by Formula
- Using the Poisson Tables
- Mean and Standard Deviation of a Poisson Distribution
- Graphing Poisson Distributions
- Using the Computer to Generate Poisson Distributions
- Approximating Binomial Problems by the Poisson Distribution

5.5 Hypergeometric Distribution
- Using the Computer to Solve for Hypergeometric Distribution Probabilities

KEY TERMS

Binomial Distribution
Continuous Distributions
Continuous Random Variables
Discrete Distributions
Discrete Random Variables
Hypergeometric Distribution
Lambda (λ)
Mean, or Expected Value
Poisson Distribution
Random Variable

STUDY QUESTIONS

1. Variables that take on values at every point over a given interval are called ________________ ________________ variables.

2. If the set of all possible values of a variable is at most finite or a countably infinite number of possible values, then the variable is called a ________________ ________________ variable.

3. An experiment in which a die is rolled six times will likely produce values of a ________________ random variable.

4. An experiment in which a researcher counts the number of customers arriving at a supermarket checkout counter every two minutes produces values of a ________________ random variable.

5. An experiment in which the time it takes to assemble a product is measured is likely to produce values of a ________________ random variable.

6. A binomial distribution is an example of a ________________ distribution.

7. The normal distribution is an example of a ________________ distribution.

8. The long-run average of a discrete distribution is called the ___________________ or ____.

Use the following discrete distribution to answer 9 and 10

x	$P(x)$
1	.435
2	.241
3	.216
4	.108

9. The mean of the discrete distribution above is _____________.

10. The variance of the discrete distribution above is ________________.

11. On any one trial of a binomial experiment, there can be only ________________ possible outcomes.

12. Suppose the probability that a given part is defective is .10. If four such parts are randomly drawn from a large population, the probability that exactly two parts are defective is _________.

13. Suppose the probability that a given part is defective is .04. If thirteen such parts are randomly drawn from a large population, the expected value or mean of the binomial distribution that describes this experiment is _________.

14. Suppose a binomial experiment is conducted by randomly selecting 20 items where p = .30. The standard deviation of the binomial distribution is ________________.

15. Suppose forty-seven percent of the workers in a large corporation are under thirty-five years of age. If fifteen workers are randomly selected from this corporation, the probability of selecting exactly ten who are under thirty-five years of age is _______________.

16. Suppose that twenty-three percent of all adult Americans fly at least once a year. If twelve adult Americans are randomly selected, the probability that exactly four have flown at least once last year is _______________.

17. Suppose that sixty percent of all voters support the President of the United States at this time. If twenty voters are randomly selected, the probability that at least eleven support the President is _______________.

18. The Poisson distribution was named after the French mathematician _______________.

19. The Poisson distribution focuses on the number of discrete occurrences per _______________.

20. The Poisson distribution tends to describe _______________ occurrences.

21. The long-run average or mean of a Poisson distribution is _______________.

22. The variance of a Poisson distribution is equal to _______________.

23. If Lambda is 2.6 occurrences over an interval of five minutes, the probability of getting six occurrences over one five minute interval is _______________.

24. Suppose that in the long-run a company determines that there are 1.2 flaws per every twenty pages of typing paper produced. If ten pages of typing paper are randomly selected, the probability that more than two flaws are found is _______________.

25. If Lambda is 1.8 for a four minute interval, an adjusted new Lambda of _______ would be used to analyze the number of occurrences for a twelve minute interval.

26. Suppose a binomial distribution problem has an $n = 200$ and a $p = .03$. If this problem is worked using the Poisson distribution, the value of Lambda is ________.

27. The hypergeometric distribution should be used when a binomial type experiment is being conducted without replacement and the sample size is greater than or equal to ________% of the population.

28. Suppose a population contains sixteen items of which seven are X and nine are Y. If a random sample of five of these population items is selected, the probability that exactly three of the five are X is ________.

29. Suppose a population contains twenty people of which eight are members of the Catholic church. If a sample of four of the population is taken, the probability that at least three of the four are members of the Catholic church is ________.

30. Suppose a lot of fifteen personal computer printers contains two defective printers. If three of the fifteen printers are randomly selected for testing, the probability that no defective printers are selected is _______________.

ANSWERS TO STUDY QUESTIONS

1. Continuous Random
2. Discrete Random
3. Discrete
4. Discrete
5. Continuous
6. Discrete
7. Continuous
8. Mean, Expected Value
9. 1.997
10. 1.083
11. Two
12. .0486
13. 0.52
14. 2.049
15. .0661
16. .1712
17. .755
18. Poisson
19. Interval
20. Rare
21. Lambda
22. Lambda
23. .0319
24. .0232
25. 5.4
26. 6.0
27. 5
28. .2885
29. .1531
30. .6286

SOLUTIONS TO THE ODD-NUMBERED PROBLEMS IN CHAPTER 5

5.1

x	$P(x)$	$x \cdot P(x)$	$(x-\mu)^2$	$(x-\mu)^2 \cdot P(x)$
1	.238	.238	2.775556	0.6605823
2	.290	.580	0.443556	0.1286312
3	.177	.531	0.111556	0.0197454
4	.158	.632	1.779556	0.2811700
5	.137	.685	5.447556	0.7463152

$\mu = \sum[x \cdot P(x)] =$ **2.666** $\sigma^2 = \sum[(x-\mu)^2 \cdot P(x)] =$ **1.836444**

$\sigma = \sqrt{1.836444} =$ **1.355155**

5.3

x	$P(x)$	$x \cdot P(x)$	$(x-\mu)^2$	$(x-\mu)^2 \cdot P(x)$
0	.461	.000	0.913936	0.421324
1	.285	.285	0.001936	0.000552
2	.129	.258	1.089936	0.140602
3	.087	.261	4.177936	0.363480
4	.038	.152	9.265936	0.352106

$E(x) = \mu = \sum[x \cdot P(x)] =$ **0.956** $\sigma^2 = \sum[(x-\mu)^2 \cdot P(x)] =$ **1.278064**

$\sigma = \sqrt{1.278064} =$ **1.1305**

5.5 a) $n = 4$ $p = .10$ $q = .90$

$P(x=3) = {}_4C_3(.10)^3(.90)^1 = 4(.001)(.90) =$ **.0036**

b) $n = 7$ $p = .80$ $q = .20$

$P(x=4) = {}_7C_4(.80)^4(.20)^3 = 35(.4096)(.008) =$ **.1147**

c) $n = 10$ $p = .60$ $q = .40$

$P(x \geq 7) = P(x=7) + P(x=8) + P(x=9) + P(x=10) =$

${}_{10}C_7(.60)^7(.40)^3 + {}_{10}C_8(.60)^8(.40)^2 + {}_{10}C_9(.60)^9(.40)^1 + {}_{10}C_{10}(.60)^{10}(.40)^0 =$

$120(.0280)(.064) + 45(.0168)(.16) + 10(.0101)(.40) + 1(.0060)(1) =$

$.2150 + .1209 + .0403 + .0060 =$ **.3822**

d) $n = 12$ $p = .45$ $q = .55$

$P(5 \leq x \leq 7) = P(x=5) + P(x=6) + P(x=7) =$

${}_{12}C_5(.45)^5(.55)^7 + {}_{12}C_6(.45)^6(.55)^6 + {}_{12}C_7(.45)^7(.55)^5 =$

$792(.0185)(.0152) + 924(.0083)(.0277) + 792(.0037)(.0503) =$

$.2225 + .2124 + .1489 =$ **.5838**

5.7 a) $n = 20$ $p = .70$ $q = .30$

$\mu = n \cdot p = 20(.70) =$ **14**

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{20(.70)(.30)} = \sqrt{4.2} =$ **2.05**

b) $n = 70$ $p = .35$ $q = .65$

$\mu = n \cdot p = 70(.35) =$ **24.5**

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{70(.35)(.65)} = \sqrt{15.925} =$ **3.99**

c) $n = 100$ $p = .50$ $q = .50$

$\mu = n \cdot p = 100(.50) =$ **50**

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{100(.50)(.50)} = \sqrt{25} =$ **5**

5.9 Looking at the graph, the highest probability appears to be at $x = 1$ followed by $x = 2$. It is most likely that the mean falls between $x = 1$ and $x = 2$ but because of the size of the drop from $x = 1$ to $x = 2$, the mean is much closer to $x = 1$. Thus, the mean is something like 1.2. Since the expected value $\mu = n \cdot p$, if $n = 6$ and μ is 1.2, solving for p results in $p = .20$. So, p is near to .20 and the mean or expected value is around 1.2.

5.11 a) $n = 20$ $p = .27$ $x = 8$

${}_{20}C_8\,(.27)^8(.73)^{12} = 125{,}970(.000028243)(.022902) =$ **.0815**

b) $n = 20$ $p = .30$ $x = 0$

${}_{20}C_0\,(.30)^0(.70)^{20} = (1)(1)(.0007979) =$ **.0007979**

c) $n = 20$ $p = .30$ $x > 7$

Use table A.2:

$P(x{=}8) + P(x{=}9) + \ldots + P(x{=}14) =$

$.114 + .065 + .031 + .012 + .004 + .001 + .000 + \ldots =$ **.227**

5.13 $n = 25$ $p = .60$

a) $x \geq 15$

$P(x \geq 15) = P(x = 15) + P(x = 16) + \cdots + P(x = 25)$

Using Table A.2 $n = 25$, $p = .60$

x	Prob
15	.161
16	.151
17	.120
18	.080
19	.044
20	.020
21	.007
22	.002
	.585

b) $x > 20$

$P(x > 20) = P(x = 21) + P(x = 22) + P(x = 23) + P(x = 24) + P(x = 25) =$

Using Table A.2 $n = 25$, $p = .60$

.007 + .002 + .000 + .000 + .000 = **.009**

c) $P(x < 10)$

Using Table A.2 $n = 25$, $p = .60$ and $x = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

x	Prob.
9	.009
8	.003
7	.001
≤ 6	.000
	.013

5.15 $n = 15$ $p = .20$

a) $P(x = 5) = {}_{15}C_5(.20)^5(.80)^{10} = 3003(.00032)(.1073742) =$ **.1032**

b) $P(x > 9)$: Using Table A.2

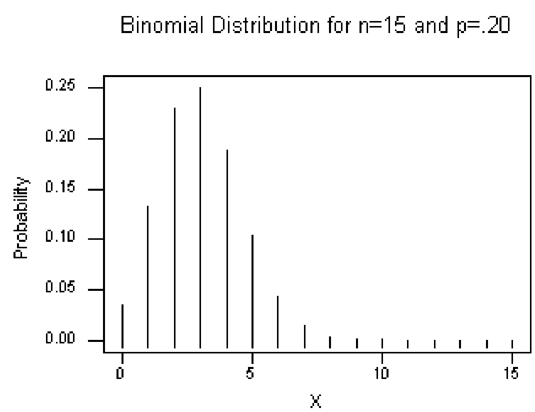
$P(x = 10) + P(x = 11) + \ldots + P(x = 15) = .000 + .000 + \ldots + .000 =$ **.000**

c) $P(x = 0) = {}_{15}C_0(.20)^0(.80)^{15} = (1)(1)(.035184) =$ **.0352**

d) $P(4 \le x \le 7)$: Using Table A.2

$P(x = 4) + P(x = 5) + P(x = 6) + P(x = 7) = .188 + .103 + .043 + .014 =$ **.348**

e) Graph



5.17 a) $P(x=5 \mid \lambda = 2.3) = \dfrac{2.3^5 \cdot e^{-2.3}}{5!} = \dfrac{(64.36343)(.100259)}{120} =$ **.0538**

b) $P(x=2 \mid \lambda = 3.9) = \dfrac{3.9^2 \cdot e^{-3.9}}{2!} = \dfrac{(15.21)(.020242)}{2} =$ **.1539**

c) $P(x \leq 3 \mid \lambda = 4.1) = P(x=3) + P(x=2) + P(x=1) + P(x=0) =$

$$\frac{4.1^3 \cdot e^{-4.1}}{3!} = \frac{(68.921)(.016573)}{6} = .1904$$

$$\frac{4.1^2 \cdot e^{-4.1}}{2!} = \frac{(16.81)(.016573)}{2} = .1393$$

$$\frac{4.1^1 \cdot e^{-4.1}}{1!} = \frac{(4.1)(.016573)}{1} = .0679$$

$$\frac{4.1^0 \cdot e^{-4.1}}{0!} = \frac{(1)(.016573)}{1} = .0166$$

.1904 + .1393 + .0679 + .0166 = **.4142**

d) $P(x=0 \mid \lambda = 2.7) =$

$\dfrac{2.7^0 \cdot e^{-2.7}}{0!} = \dfrac{(1)(.06721)}{1} =$ **.0672**

e) $P(x=1 \mid \lambda = 5.4)=$

$\dfrac{5.4^1 \cdot e^{-5.4}}{1!} = \dfrac{(5.4)(.0045166)}{1} =$ **.0244**

f) $P(4 < x < 8 \mid \lambda = 4.4)$: $P(x=5 \mid \lambda = 4.4) + P(x=6 \mid \lambda = 4.4) + P(x=7 \mid \lambda = 4.4)=$

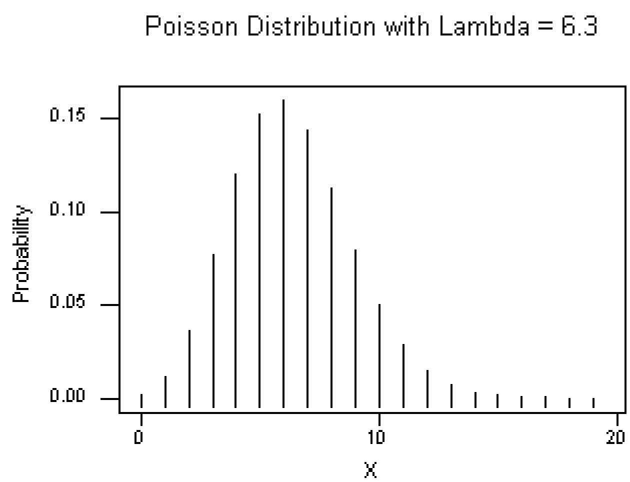
$$\frac{4.4^5 \cdot e^{-4.4}}{5!} + \frac{4.4^6 \cdot e^{-4.4}}{6!} + \frac{4.4^7 \cdot e^{-4.4}}{7!} =$$

$$\frac{(1649.1622)(.01227734)}{120} + \frac{(7256.3139)(.01227734)}{720} + \frac{(31{,}927.781)(.01227734)}{5040}$$

= .1687 + .1237 + .0778 = **.3702**

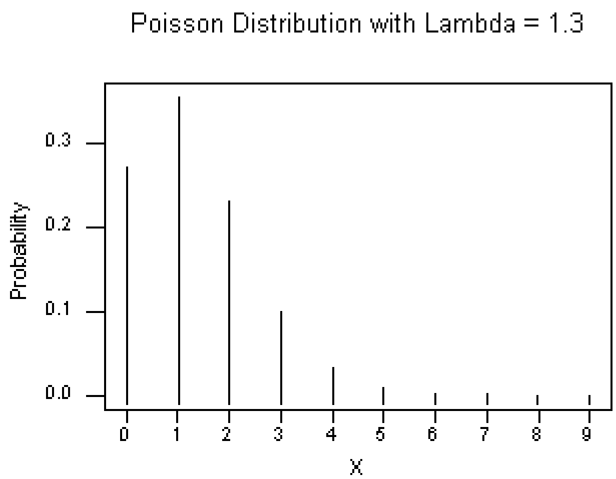
5.19 a) $\lambda = 6.3$ mean = **6.3** Standard deviation = $\sqrt{6.3}$ = **2.51**

x	Prob
0	.0018
1	.0116
2	.0364
3	.0765
4	.1205
5	.1519
6	.1595
7	.1435
8	.1130
9	.0791
10	.0498
11	.0285
12	.0150
13	.0073
14	.0033
15	.0014
16	.0005
17	.0002
18	.0001
19	.0000



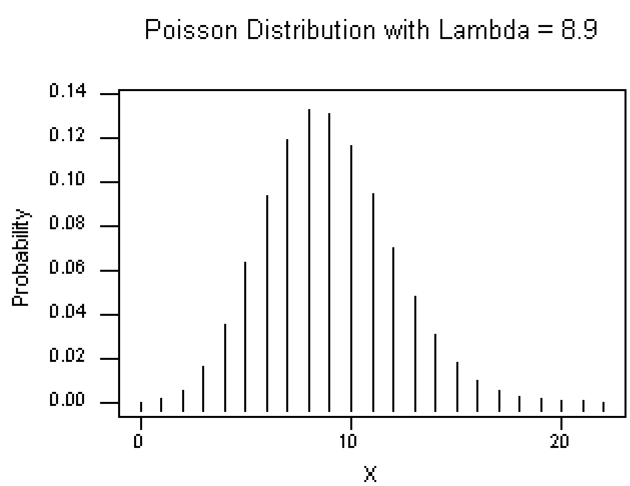
b) λ = 1.3 mean = **1.3** standard deviation = $\sqrt{1.3}$ = **1.14**

x	Prob
0	.2725
1	.3542
2	.2303
3	.0998
4	.0324
5	.0084
6	.0018
7	.0003
8	.0001
9	.0000



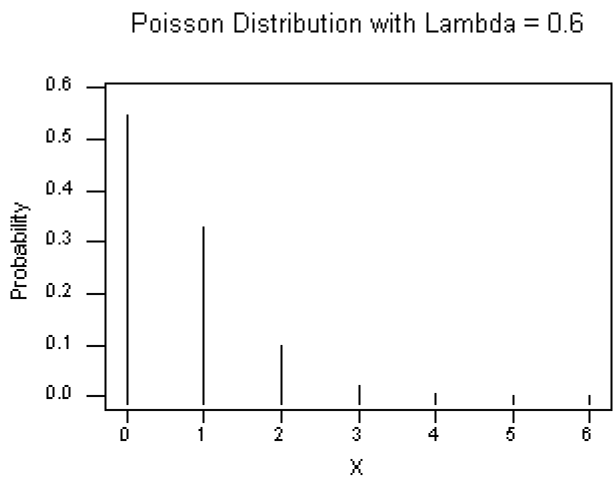
c) λ = 8.9 mean = **8.9** standard deviation = $\sqrt{8.9}$ = **2.98**

x	Prob
0	.0001
1	.0012
2	.0054
3	.0160
4	.0357
5	.0635
6	.0941
7	.1197
8	.1332
9	.1317
10	.1172
11	.0948
12	.0703
13	.0481
14	.0306
15	.0182
16	.0101
17	.0053
18	.0026
19	.0012
20	.0005
21	.0002
22	.0001



d) λ = 0.6 mean = **0.6** standard deviation = $\sqrt{0.6}$ = **.775**

x	Prob
0	.5488
1	.3293
2	.0988
3	.0198
4	.0030
5	.0004
6	.0000



5.21 $\lambda = \Sigma x/n = 126/36 =$ **3.5**

Using Table A.3

a) $P(x = 0) =$ **.0302**

b) $P(x \geq 6) = P(x = 6) + P(x = 7) + \ldots =$

.0771 + .0385 + .0169 + .0066 + .0023 +

.0007 + .0002 + .0001 = **.1424**

c) $P(x < 4 \mid 10 \text{ minutes})$

Double Lambda to $\lambda = 7.0 \mid 10$ minutes

$P(x < 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) =$

.0009 + .0064 + .0223 + .0521 = **.0817**

d) $P(3 \leq x \leq 6 \mid 10 \text{ minutes})$

$\lambda = 7.0 \mid 10$ minutes

$P(3 \leq x \leq 6) = P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6)$

= .0521 + .0912 + .1277 + .1490 = **.42**

e) $P(x = 8 \mid 15 \text{ minutes})$

Change Lambda for a 15 minute interval by multiplying the original Lambda by 3.

$\lambda = 10.5 \mid 15$ minutes

$$P(x = 8 \mid 15 \text{ minutes}) = \frac{\lambda^x \cdot e^{-\lambda}}{x!} = \frac{(10.5^8)(e^{-10.5})}{8!} = \mathbf{.1009}$$

5.23 λ = 1.2 collisions | 4 months

a) $P(x=0 \mid \lambda = 1.2)$:

from Table A.3 = **.3012**

b) $P(x=2 \mid 2 \text{ months})$:

The interval has been decreased (by ½)

New Lambda = λ = 0.6 collisions | 2 months

$P(x=2 \mid \lambda = 0.6)$:

from Table A.3 = **.0988**

c) $P(x \leq 1 \text{ collision} \mid 6 \text{ months})$:

The interval length has been increased (by 1.5)

New Lambda = λ = 1.8 collisions | 6 months

$P(x \leq 1 \mid \lambda = 1.8)$:

from Table A.3

x	Prob.
0	.1653
1	.2975
	.4628

The result is likely to happen almost half the time (46.26%). Ship channel and weather conditions are about normal for this period. Safety awareness is about normal for this period. There is no compelling reason to reject the lambda value of 0.6 collisions per 4 months based on an outcome of 0 or 1 collisions per 6 months.

5.25 $n = 100{,}000$ $p = .00004$

a) $P(x \geq 7 \mid n = 100{,}000 \;\; p = .00004)$:

$\lambda = \mu = n \cdot p = 100{,}000(.00004) = 4.0$

Since $n > 20$ and $n \cdot p \leq 7$, the Poisson approximation to this binomial problem is close enough.

$P(x \geq 7 \mid \lambda = 4)$:

Using Table A.3

x	Prob.
7	.0595
8	.0298
9	.0132
10	.0053
11	.0019
12	.0006
13	.0002
14	.0001
	.1106

b) $P(x > 10 \mid \lambda = 4)$:

Using Table A.3

x	Prob.
11	.0019
12	.0006
13	.0002
14	.0001
	.0028

c) Since getting more than 10 is a rare occurrence, this particular geographic region appears to have a higher average rate than other regions. An investigation of particular characteristics of this region might be warranted.

5.27 a) $P(x = 3 \mid N = 11, A = 8, n = 4)$

$$\frac{{}_8C_3 \cdot {}_3C_1}{{}_{11}C_4} = \frac{(56)(3)}{330} = \mathbf{.5091}$$

b) $P(x < 2) \mid N = 15, A = 5, n = 6)$

$P(x = 1) + P(x = 0) =$

$$\frac{{}_5C_1 \cdot {}_{10}C_5}{{}_{15}C_6} + \frac{{}_5C_0 \cdot {}_{10}C_6}{{}_{15}C_6} = \frac{(5)(252)}{5005} + \frac{(1)(210)}{5005}$$

.2517 + .0420 = **.2937**

c) $P(x=0 \mid N = 9, A = 2, n = 3)$

$$\frac{{}_2C_0 \cdot {}_7C_3}{{}_9C_3} = \frac{(1)(35)}{84} = \mathbf{.4167}$$

d) $P(x > 4 \mid N = 20, A = 5, n = 7) =$

$P(x = 5) + P(x = 6) + P(x = 7) =$

$$\frac{{}_5C_5 \cdot {}_{15}C_2}{{}_{20}C_7} + \frac{{}_5C_6 \cdot {}_{15}C_1}{{}_{20}C_7} + \frac{{}_5C_7 \cdot {}_{15}C_0}{{}_{20}C_7} =$$

$\frac{(1)(105)}{77520}$ + ${}_5C_6$ (impossible) + ${}_5C_7$(impossible) = **.0014**

5.29 $N = 17$ $A = 8$ $n = 4$

a) $P(x = 0) = \dfrac{{}_8C_0 \cdot {}_9C_4}{{}_{17}C_4} = \dfrac{(1)(126)}{2380} =$ **.0529**

b) $P(x = 4) = \dfrac{{}_8C_4 \cdot {}_9C_0}{{}_{17}C_4} = \dfrac{(70)(1)}{2380} =$ **.0294**

c) $P(x = 2 \text{ non computer}) = \dfrac{{}_9C_2 \cdot {}_8C_2}{{}_{17}C_4} = \dfrac{(36)(28)}{2380} =$ **.4235**

5.31 $N = 10$ $n = 4$

a) $A = 2$ $x = 2$ $P(x = 2)$:

$$\frac{{}_2C_2 \cdot {}_8C_2}{{}_{10}C_4} = \frac{(1)(28)}{210} = .1333$$

b) $A = 5$ $x = 0$ $P(x = 0)$:

$$\frac{{}_5C_0 \cdot {}_5C_4}{{}_{10}C_4} = \frac{(1)(5)}{210} = .0238$$

c) $A = 4$ $x = 3$ $P(x = 3)$:

$$\frac{{}_4C_3 \cdot {}_6C_1}{{}_{10}C_4} = \frac{(4)(6)}{210} = .1143$$

5.33 $N = 18$ $A = 11$ Hispanic $n = 5$

$P(x \leq 1) = P(1) + P(0) =$

$\dfrac{{}_{11}C_1 \cdot {}_7C_4}{{}_{18}C_5} + \dfrac{{}_{11}C_0 \cdot {}_7C_5}{{}_{18}C_5} = \dfrac{(11)(35)}{8568} + \dfrac{(1)(21)}{8568} = .0449 + .0025 =$ **.0474**

It is fairly unlikely that these results occur by chance. A researcher might want to further investigate this result to determine causes. Were officers selected based on leadership, years of service, dedication, prejudice, or some other reason?

5.35 a) $P(x = 14 \mid n = 20 \text{ and } p = .60) =$ **.124**

b) $P(x < 5 \mid n = 10 \text{ and } p = .30) =$

$P(x = 4) + P(x = 3) + P(x = 2) + P(x = 1) + P(x=0) =$

x	Prob.
0	.028
1	.121
2	.233
3	.267
4	.200
	.849

c) $P(x \geq 12 \mid n = 15 \text{ and } p = .60) =$

$P(x = 12) + P(x = 13) + P(x = 14) + P(x = 15)$

x	Prob.
12	.063
13	.022
14	.005
15	.000
	.090

d) $P(x > 20 \mid n = 25 \text{ and } p = .40) = P(x = 21) + P(x = 22) +$

$P(x = 23) + P(x = 24) + P(x=25) =$

x	Prob.
21	.000
22	.000
23	.000
24	.000
25	.000
	.000

5.37 a) $P(x = 3 \mid \lambda = 1.8)$ = **.1607**

b) $P(x < 5 \mid \lambda = 3.3) =$

$P(x = 4) + P(x = 3) + P(x = 2) + P(x = 1) + P(x = 0) =$

x	Prob.
0	.0369
1	.1217
2	.2008
3	.2209
4	.1823
	.7626

c) $P(x \geq 3 \mid \lambda = 2.1) =$

x	Prob.
3	.1890
4	.0992
5	.0417
6	.0146
7	.0044
8	.0011
9	.0003
10	.0001
11	.0000
	.3504

d) $P(2 < x \leq 5 \mid \lambda = 4.2)$:

$P(x=3) + P(x=4) + P(x=5) =$

x	Prob.
3	.1852
4	.1944
5	.1633
	.5429

5.39 $n = 25$ $p = .20$ retired

a) from Table A.2: $P(x = 7) = $ **.1108**

b) $P(x \geq 10)$: $P(x = 10) + P(x = 11) + \ldots + P(x = 25) = .012 + .004 + .001 = $ **.017**

c) Expected Value = $\mu = n \cdot p = 25(.20) = $ **5**

d) $n = 20$ $p = .40$ mutual funds

$P(x = 8) = $ **.1797**

e) $P(x < 6) = P(x = 0) + P(x = 1) + \ldots + P(x = 5) =$

$.000 + .000 + .003 + .012 + .035 + .075 = $ **.125**

f) $P(x = 0) = $ **.000**

g) $P(x \geq 12) = P(x = 12) + P(x = 13) + \ldots + P(x = 20) = .035 + .015 + .005 + .001 = $ **.056**

h) $x = $ **8** Expected Number = $\mu = n \cdot p = 20(.40) = $ **8**

5.41 $N = 32$ A = 10 $n = 12$

a) $P(x = 3) = \dfrac{{}_{10}C_3 \cdot {}_{22}C_9}{{}_{32}C_{12}} = \dfrac{(120)(497,420)}{225,792,840} = $ **.2644**

b) $P(x = 6) = \dfrac{{}_{10}C_6 \cdot {}_{22}C_6}{{}_{32}C_{12}} = \dfrac{(210)(74,613)}{225,792,840} = $ **.0694**

c) $P(x = 0) = \dfrac{{}_{10}C_0 \cdot {}_{22}C_{12}}{{}_{32}C_{12}} = \dfrac{(1)(646,646)}{225,792,840} = $ **.0029**

d) $A = 22$ $P(7 \leq x \leq 9) = \dfrac{{}_{22}C_7 \cdot {}_{10}C_5}{{}_{32}C_{12}} + \dfrac{{}_{22}C_8 \cdot {}_{10}C_4}{{}_{32}C_{12}} + \dfrac{{}_{22}C_9 \cdot {}_{10}C_3}{{}_{32}C_{12}}$

$= \dfrac{(170,544)(252)}{225,792,840} + \dfrac{(319,770)(210)}{225,792,840} + \dfrac{(497,420)(120)}{225,792,840}$

$= .1903 + .2974 + .2644 = $ **.7521**

5.43 a) $n = 20$ and $p = .25$

The expected number = $\mu = n \cdot p = (20)(.25) =$ **5.00**

b) $P(x \leq 1 \mid n = 20 \text{ and } p = .25) =$

$P(x = 1) + P(x = 0) = {}_{20}C_1(.25)^1(.75)^{19} + {}_{20}C_0(.25)^0(.75)^{20}$

$= (20)(.25)(.00423) + (1)(1)(.0032) = .0212 +. 0032 =$ **.0244**

Since the probability is so low, the population of your state may have a lower percentage of chronic heart conditions than those of other states.

5.45 $n = 12$

a.) $P(x = 0$ long hours):

$p = .20$ ${}_{12}C_0(.20)^0(.80)^{12} =$ **.0687**

b.) $P(x \geq 6)$ long hours):

$p = .20$

Using Table A.2: .016 + .003 + .001 = **.020**

c) $P(x = 5$ good financing):

$p = .25$, ${}_{12}C_5(.25)^5(.75)^7 =$ **.1032**

d.) $p = .19$ (good plan), expected number = $\mu = n(p) = 12(.19) =$ **2.28**

5.47 $P(x \le 3) \mid n = 8$ and $p = .60)$: From Table A.2:

x	Prob.
0	.001
1	.008
2	.041
3	.124
	.174

17.4% of the time in a sample of eight, three or fewer customers are walk-ins by chance. Other reasons for such a low number of walk-ins might be that she is retaining more old customers than before or perhaps a new competitor is attracting walk-ins away from her.

5.49 λ = 1.2 hours$\mid$week

a) $P(x = 0 \mid \lambda = 1.2)$ = (from Table A.3) **.3012**

b) $P(x \ge 3 \mid \lambda = 1.2)$ = (from Table A.3)

x	Prob.
3	.0867
4	.0260
5	.0062
6	.0012
7	.0002
8	.0000
	.1203

a) $P(x < 5 \mid$ 3 weeks)

If $\lambda = 1.2$ for 1 week, the $\lambda = 3.6$ for 3 weeks.

$P(x < 5 \mid \lambda = 3.6)$ = (from Table A.3)

x	Prob.
4	.1912
3	.2125
2	.1771
1	.0984
0	.0273
	.7065

5.51 $N = 24$ $n = 6$ A = 8

a) $P(x = 6) = \dfrac{{}_{8}C_{6} \cdot {}_{16}C_{0}}{{}_{24}C_{6}} = \dfrac{(28)(1)}{134,596} =$ **.0002**

b) $P(x = 0) = \dfrac{{}_{8}C_{0} \cdot {}_{16}C_{6}}{{}_{24}C_{6}} = \dfrac{(1)(8008)}{134,596} =$ **.0595**

d) $A = 16$ East Side

$P(x = 3) = \dfrac{{}_{16}C_{3} \cdot {}_{8}C_{3}}{{}_{24}C_{6}} = \dfrac{(560)(56)}{134,596} =$ **.2330**

5.53 $\lambda = 2.4$ calls$|$1 minute

a) $P(x = 0 | \lambda = 2.4)$ = (from Table A.3) **.0907**

b) Can handle $x \leq 5$ calls Cannot handle $x > 5$ calls

$P(x > 5 | \lambda = 2.4)$ = (from Table A.3)

x	Prob.
6	.0241
7	.0083
8	.0025
9	.0007
10	.0002
11	.0000
	.0358

c) $P(x = 3$ calls$|$2 minutes)

The interval has been increased 2 times.
New Lambda: $\lambda = 4.8$ calls$|$2 minutes.

from Table A.3: **.1517**

d) $P(x \leq 1$ calls$|$15 seconds):

The interval has been decreased by ¼.
New Lambda = $\lambda = 0.6$ calls$|$15 seconds.

$P(x \leq 1 | \lambda = 0.6)$ = (from Table A.3)

$P(x = 1)$ = .3293
$P(x = 0)$ = .5488
.8781

5.55 $p = .005$ $n = 1{,}000$

$\lambda = n \cdot p = (1{,}000)(.005) = 5$

a) $P(x < 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) =$

.0067 + .0337 + .0842 + .1404 = **.265**

b) $P(x > 10) = P(x = 11) + P(x = 12) + \ldots =$

.0082 + .0034 + .0013 + .0005 + .0002 = **.0136**

c) $P(x = 0) =$ **.0067**

5.57 $N = 25$

a) $n = 5$ $x = 3$ $A = 17$

$$\frac{{}_{17}C_3 \cdot {}_8C_2}{{}_{25}C_5} = \frac{(680)(28)}{53{,}130} = .3584$$

b) $n = 8$ $x \le 2$ $A = 5$

$$\frac{{}_5C_0 \cdot {}_{20}C_8}{{}_{25}C_8} + \frac{{}_5C_1 \cdot {}_{20}C_7}{{}_{25}C_8} + \frac{{}_5C_2 \cdot {}_{20}C_6}{{}_{25}C_8} =$$

$$\frac{(1)(125{,}970)}{1{,}081{,}575} + \frac{(5)(77{,}520)}{1{,}081{,}575} + \frac{(10)(38{,}760)}{1{,}081{,}575} =$$

.1165 + .3584 + .3584 = **.8333**

c) $n = 5$ $x = 2$ $A = 3$

${}_5C_2(3/25)^2(22/25)^3 = (10)(.0144)(.681472) =$ **.0981**

5.59 a) λ = 3.05 | 1,000 for U.S.

$$P(x=0)=\frac{3.05^0\cdot e^{-3.05}}{0!}=.0474$$

b) λ = 6.10 | 2,000 for U.S. (just double previous lambda)

$$P(x=6)=\frac{6.10^6\cdot e^{-6.104}}{6!}=\frac{(51{,}520.37)(.002243)}{720}=.1605$$

c) λ = 1.07 | 1,000 and λ = 3.21 | 3,000

from Table A.3:

$P(x < 7) = P(x = 0) + P(x = 1) + \ldots + P(x = 6) =$

$$\frac{(3.21^0)(e^{-3.21})}{0!}+\frac{(3.21^1)(e^{-3.21})}{1!}+\frac{(3.21^2)(e^{-3.21})}{2!}+\frac{(3.21^3)(e^{-3.21})}{3!}+\frac{(3.21^4)(e^{-3.21})}{4!}+\frac{(3.21^5)(e^{-3.21})}{5!}+\frac{(3.21^6)(e^{-3.21})}{6!}$$

.0404 + .1295 + .2079 + .2225 + .1785 + .1146 + .0613 = **.9547**

5.61 This printout contains the probabilities for various values of x from zero to eleven from a Poisson distribution with λ = 2.78. Note that the highest probabilities are at $x = 2$ and $x = 3$ which are near the mean. The probability is slightly higher at $x = 2$ than at $x = 3$ even though $x = 3$ is nearer to the mean because of the "piling up" effect of $x = 0$.

5.63 This is the graph of a Poisson Distribution with λ = 1.784. Note the high probabilities at $x = 1$ and $x = 2$ which are nearest to the mean. Note also that the probabilities for values of $x \geq 8$ are near to zero because they are so far away from the mean or expected value.