Chapter 6
Continuous Distributions

LEARNING OBJECTIVES

The primary learning objective of Chapter 6 is to help you understand continuous distributions, thereby enabling you to:

1. Solve for probabilities in a continuous uniform distribution
2. Solve for probabilities in a normal distribution using *z* scores and for the mean, the standard deviation, or a value of *x* in a normal distribution when given information about the area under the normal curve
3. Solve problems from the discrete binomial distribution using the continuous normal distribution and correcting for continuity
4. Solve for probabilities in an exponential distribution and contrast the exponential distribution to the discrete Poisson distribution

CHAPTER OUTLINE

6.1 The Uniform Distribution
- Determining Probabilities in a Uniform Distribution
- Using the Computer to Solve for Uniform Distribution Probabilities

6.2 Normal Distribution
- History of the Normal Distribution
- Probability Density Function of the Normal Distribution
- Standardized Normal Distribution
- Solving Normal Curve Problems
- Using the Computer to Solve for Normal Distribution Probabilities

6.3 Using the Normal Curve to Approximate Binomial Distribution Problems
- Correcting for Continuity

6.4 Exponential Distribution
- Probabilities of the Exponential Distribution
- Using the Computer to Determine Exponential Distribution Probabilities

KEY TERMS

Correction for Continuity	Standardized Normal Distribution
Exponential Distribution	Uniform Distribution
Normal Distribution	*z* Distribution
Rectangular Distribution	*z* Score

STUDY QUESTIONS

1. The uniform distribution is sometimes referred to as the ______________________ distribution.

2. Suppose a set of data are uniformly distributed from $x = 5$ to $x = 13$. The height of the distribution is _____________________. The mean of this distribution is _____________________. The standard deviation of this distribution is ______________________.

3. Suppose a set of data are uniformly distributed from $x = 27$ to $x = 44$. The height of this distribution is ______________________. The mean of this distribution is ______________________. The standard deviation of this distribution is _________________________.

4. A set of values is uniformly distributed from 84 to 98. The probability of a value occurring between 89 and 93 is ______________________. The probability of a value occurring between 80 and 90 is ___________________. The probability of a value occurring that is greater than 75 is ______________________.

5. Probably the most widely known and used of all distributions is the ________________ distribution.

6. Many human characteristics can be described by the ________________ distribution.

7. The area under the curve of a normal distribution is ________.

8. In working normal curve problems using the raw values of x, the mean, and the standard deviation, a problem can be converted to ________ scores.

9. A z score value is the number of ________________ ________________ a value is from the mean.

10. Within a range of z scores of $\pm 1\sigma$ from the mean, fall _________% of the values of a normal distribution.

11. Suppose a population of values is normally distributed with a mean of 155 and a standard deviation of 12. The z score for $x = 170$ is ________.

12. Suppose a population of values is normally distributed with a mean of 76 and a standard deviation of 5.2. The z score for $x = 73$ is ________.

13. Suppose a population of values is normally distributed with a mean of 250 and a variance of 225. The z score for $x = 286$ is ________.

14. Suppose a population of values is normally distributed with a mean of 9.8 and a standard deviation of 2.5. The probability that a value is greater than 11 in the distribution is ________.

15. A population is normally distributed with a mean of 80 and a variance of 400. The probability that x lies between 50 and 100 is ________.

16. A population is normally distributed with a mean of 115 and a standard deviation of 13. The probability that a value is less than 85 is ________.

17. A population is normally distributed with a mean of 64. The probability that a value from this population is more than 70 is .0485. The standard deviation is __________.

18. A population is normally distributed with a mean of 90. 85.99% of the values in this population are greater than 75. The standard deviation of this population is ________.

19. A population is normally distributed with a standard deviation of 18.5. 69.85% of the values in this population are greater than 93. The mean of the population is ________.

20. A population is normally distributed with a variance of 50. 98.17% of the values of the population are less than 27. The mean of the population is ________.

21. A population is normally distributed with a mean of 340 and a standard deviation of 55. 10.93% of values in the population are less than ________.

22. In working a binomial distribution problem by using the normal distribution, the interval, ________, should lie between 0 and n.

23. A binomial distribution problem has an n of 10 and a p of .20. This problem _______________ be worked by the normal distribution because of the size of n and p.

24. A binomial distribution problem has an n of 15 and a p of .60. This problem _______________ be worked by the normal distribution because of the size of n and p.

25. A binomial distribution problem has an n of 30 and a p of .35. A researcher wants to determine the probability of x being greater than 13 and to use the normal distribution to work the problem. After correcting for continuity, the value of x that he/she will be solving for is ________.

26. A binomial distribution problem has an n of 48 and a p of .80. A researcher wants to determine the probability of x being less than or equal to 35 and wants to work the problem using the normal distribution. After correcting for continuity, the value of x that he/she will be solving for is _______________.

27. A binomial distribution problem has an n of 60 and a p value of .72. A researcher wants to determine the probability of x being exactly 45 and use the normal distribution to work the problem. After correcting for continuity, he/she will be solving for the area between ________ and ________.

28. A binomial distribution problem has an n of 27 and a p of .53. If this problem were converted to a normal distribution problem, the mean of the distribution would be ________. The standard deviation of the distribution would be ________.

29. A binomial distribution problem has an n of 113 and a p of .29. If this problem were converted to a normal distribution problem, the mean of the distribution would be ________. The standard deviation of the distribution would be ________.

30. A binomial distribution problem is to determine the probability that x is less than 22 when the sample size is 40 and the value of p is .50. Using the normal distribution to work this problem produces a probability of ________.

31. A binomial distribution problem is to determine the probability that x is exactly 14 when the sample size is 20 and the value of p is .60. Using the normal distribution to work this problem produces a probability of ________.

32. A binomial distribution problem is to determine the probability that x is greater than or equal to 18 when the sample size is 30 and the value of p is .55. Using the normal distribution to work this problem produces a probability of ________.

33. A binomial distribution problem is to determine the probability that x is greater than 10 when the sample size is 20 and the value of p is .60. Using the normal distribution to work this problem produces a probability of ________. If this problem had been worked using the binomial tables, the obtained probability would have been ________. The difference in answers using these two techniques is ________.

34. The exponential distribution is a________________ distribution.

35. The exponential distribution is closely related to the ________________ distribution.

36. The exponential distribution is skewed to the ________.

37. Suppose random arrivals occur at a rate of 5 per minute. Assuming that random arrivals are Poisson distributed, the probability of there being at least 30 seconds between arrivals is ________.

38. Suppose random arrivals occur at a rate of 1 per hour. Assuming that random arrivals are Poisson distributed, the probability of there being less than 2 hours between arrivals is ________.

39. Suppose random arrivals occur at a rate of 1.6 every five minutes. Assuming that random arrivals are Poisson distributed, the probability of there being between three minutes and six minutes between arrivals is ________.

40. Suppose that the mean time between arrivals is 40 seconds and that random arrivals are Poisson distributed. The probability that at least one minute passes between two arrivals is ________. The probability that at least two minutes pass between two arrivals is ________.

41. Suppose that the mean time between arrivals is ten minutes and that random arrivals are Poisson distributed. The probability that no more than seven minutes pass between two arrivals is ________.

42. The mean of an exponential distribution equals ________.

43. Suppose that random arrivals are Poisson distributed with an average arrival of 2.4 per five minutes. The associated exponential distribution would have a mean of ________________ and a standard deviation of ________________.

44. An exponential distribution has an average interarrival time of 25 minutes. The standard deviation of this distribution is ________________.

ANSWERS TO STUDY QUESTIONS

1. Rectangular
2. 1/8, 9, 2.3094
3. 1/17, 35.5, 4.9075
4. .2857, .7143, 1.000
5. Normal
6. Normal
7. 1
8. *z*
9. Standard deviations
10. 68%
11. 1.25
12. -0.58
13. 2.40
14. .3156
15. .7745
16. .0104
17. 3.614
18. 13.89
19. 102.62
20. 12.22
21. 272.35
22. $\mu \pm 3\sigma$
23. Cannot
24. Can
25. 13.5
26. 35.5
27. 44.5, 45.5
28. 14.31, 2.59
29. 32.77, 4.82
30. .6808
31. .1212
32. .3557
33. .7517, .7550, .0033
34. Continuous
35. Poisson
36. Right
37. .0821
38. .8647
39. .2363
40. .2231, .0498
41. .5034
42. $1/\lambda$
43. 2.08 Minutes, 2.08 Minutes
44. 25 Minutes

SOLUTIONS TO THE ODD-NUMBERED PROBLEMS IN CHAPTER 6

6.1 $a = 200 \quad b = 240$

a) $f(x) = \frac{1}{b-a} = \frac{1}{240-200} = \frac{1}{40}$ = **.025**

b) $\mu = \frac{a+b}{2} = \frac{200+240}{2}$ = **220**

$\sigma = \frac{b-a}{\sqrt{12}} = \frac{240-200}{\sqrt{12}} = \frac{40}{\sqrt{12}}$ = **11.547**

c) $P(x > 230) = \frac{240-230}{240-200} = \frac{10}{40}$ = **.250**

d) $P(205 \le x \le 220) = \frac{220-205}{240-200} = \frac{15}{40}$ = **.375**

e) $P(x \le 225) = \frac{225-200}{240-200} = \frac{25}{40}$ = **.625**

6.3 $a = 2.80 \quad b = 3.14$

$\mu = \frac{a+b}{2} = \frac{2.80+3.14}{2}$ = **2.97**

$\sigma = \frac{b-a}{\sqrt{12}} = \frac{3.14-2.80}{\sqrt{12}}$ = **0.098**

$P(3.00 < x < 3.10) = \frac{3.10-3.00}{3.14-2.80}$ = **0.2941**

6.5 $\mu = 639$ $a = 253$ $b = 1025$

$$\sigma = \frac{b-a}{\sqrt{12}} = \frac{1025-253}{\sqrt{12}} = \mathbf{222.857}$$

Height =

$$\frac{1}{b-a} = \frac{1}{1025-253} = \mathbf{.0013}$$

$P(x > 850) = \frac{1025-850}{1025-253} = \mathbf{.2267}$

$P(x > 1200)$ = **.0000** since 1200 is above the upper limit of the data.

$P(350 < x < 480) = \frac{480-350}{1025-253} = \mathbf{.1684}$

6.7 $\mu = 22$ $\sigma = 4$

a) $P(x > 17)$:

$$z = \frac{x-\mu}{\sigma} = \frac{17-22}{4} = -1.25$$

area between $x = 17$ and $\mu = 22$ from table A.5 is .3944

$P(x > 17)$ = .3944 + .5000 = **.8944**

b) $P(x < 13)$:

$$z = \frac{x-\mu}{\sigma} = \frac{13-22}{4} = -2.25$$

from table A.5, area = .4878

$P(x < 13)$ = .5000 - .4878 = **.0122**

c) $P(25 \le x \le 31)$:

$$z = \frac{x-\mu}{\sigma} = \frac{31-22}{4} = 2.25$$

from table A.5, area = .4878

$$z = \frac{x-\mu}{\sigma} = \frac{25-22}{4} = 0.75$$

from table A.5, area = .2734

$P(25 \le x \le 31)$ = .4878 - .2734 = **.2144**

6.9 μ = \$1332 σ = \$725

a) $P(x > \$2000)$:

$$z = \frac{x-\mu}{\sigma} = \frac{2000-1332}{725} = 0.92$$

from Table A.5, the z = 0.92 yields: .3212

$P(x > \$2000)$ = .5000 - .3212 = **.1788**

b) $P(\text{owes money}) = P(x < 0)$:

$$z = \frac{x-\mu}{\sigma} = \frac{0-1332}{725} = -1.84$$

from Table A.5, the z = -1.84 yields: .4671

$P(x < 0)$ = .5000 - .4671 = **.0329**

c) $P(\$100 \le x \le \$700)$:

$$z = \frac{x-\mu}{\sigma} = \frac{100-1332}{725} = -1.70$$

from Table A.5, the z = -1.70 yields: .4554

$$z = \frac{x-\mu}{\sigma} = \frac{700-1332}{725} = -0.87$$

from Table A.5, the z = -0.87 yields: .3078

$P(\$100 \le x \le \$700)$ = .4554 - .3078 = **.1476**

6.11 μ = 200, σ = 47 Determine x

a) 60% of the values are greater than x:

Since 50% of the values are greater than the mean, μ = 200, 10% or .1000 lie between x and the mean. From Table A.5, the z value associated with an area of .1000 is z = -0.25. The z value is negative since x is below the mean. Substituting z = -0.25, μ = 200, and σ = 47 into the formula and solving for x:

$$z = \frac{x-\mu}{\sigma}$$

$$-0.25 = \frac{x-200}{47}$$

x = **188.25**

b) x is less than 17% of the values.

Since x is only less than 17% of the values, 33% (.5000- .1700) or .3300 lie between x and the mean. Table A.5 yields a z value of 0.95 for an area of .3300. Using this z = 0.95, μ = 200, and σ = 47, x can be solved for:

$$z = \frac{x-\mu}{\sigma}$$

$$0.95 = \frac{x-200}{47}$$

x = **244.65**

c) 22% of the values are less than x.

Since 22% of the values lie below x, 28% lie between x and the mean (.5000 - .2200). Table A.5 yields a z of -0.77 for an area of .2800. Using the z value of -0.77, μ = 200, and σ = 47, x can be solved for:

$$z = \frac{x - \mu}{\sigma}$$

$$-0.77 = \frac{x - 200}{47}$$

x = **163.81**

d) x is greater than 55% of the values.

Since x is greater than 55% of the values, 5% (.0500) lie between x and the mean. From Table A.5, a z value of 0.13 is associated with an area of .05. Using z = 0.13, μ = 200, and σ = 47, x can be solved for:

$$z = \frac{x - \mu}{\sigma}$$

$$0.13 = \frac{x - 200}{47}$$

x = **206.11**

6.13 μ = 750 σ = ??

Since 29.12% of the values are less than 500 and x = 500 is below the mean, then 20.88% lie between 500 and μ. From table A.5, z = - 0.55.

$$-0.55 = \frac{500 - 750}{\sigma}$$

$$-0.55\sigma = -250$$

$$\sigma = \frac{-250}{-0.55} = \mathbf{454.55}$$

6.15 σ = 6.2. Since 62.5% is greater than 21, x = 21 is in the lower half of the distribution and .1250 (.6250 - .5000) lie between x and the mean. Table A.5 yields a z = -0.32 for an area of .1255 (closest value to .1250)+.

Solving for σ:

$$z = \frac{x-\mu}{\sigma}$$

$$-0.32 = \frac{21-\mu}{6.2}$$

$$-1.984 = 21 - \mu$$

$$\mu = \mathbf{22.984}$$

6.17 a) $P(x \le 16 \mid n = 30 \text{ and } p = .70)$

$\mu = n \cdot p = 30(.70) = 21$

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{30(.70)(.30)} = 2.51$

$\boldsymbol{P(x \le 16.5 \mid \mu = 21 \text{ and } \sigma = 2.51)}$

b) $P(10 < x \le 20 \mid n = 25 \text{ and } p = .50)$

$\mu = n \cdot p = 25(.50) = 12.5$

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{25(.50)(.50)} = 2.5$

$\boldsymbol{P(10.5 \le x \le 20.5 \mid \mu = 12.5 \text{ and } \sigma = 2.5)}$

c) $P(x = 22 \mid n = 40 \text{ and } p = .60)$

$\mu = n \cdot p = 40(.60) = 24$

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{40(.60)(.40)} = 3.10$

$\boldsymbol{P(21.5 \le x \le 22.5 \mid \mu = 24 \text{ and } \sigma = 3.10)}$

d) $P(x > 14\ n = 16 \text{ and } p = .45)$

$\mu = n \cdot p = 16(.45) = 7.2$

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{16(.45)(.55)} = 1.99$

$\boldsymbol{P(x \geq 14.5 \mid \mu = 7.2 \text{ and } \sigma = 1.99)}$

6.19 a) $P(x = 8 \mid n = 25 \text{ and } p = .40)$ $\mu = n \cdot p = 25(.40) = 10$

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{25(.40)(.60)} = 2.449$

$\mu \pm 3\sigma = 10 \pm 3(2.449) = 10 \pm 7.347$

(2.653 to 17.347) lies between 0 and 25.
Approximation by the normal curve is sufficient.

$P(7.5 \leq x \leq 8.5 \mid \mu = 10 \text{ and } \sigma = 2.449)$:

$$z = \frac{7.5 - 10}{2.449} = -1.02$$

From Table A.5, area = .3461

$$z = \frac{8.5 - 10}{2.449} = -0.61$$

From Table A.5, area = .2291

$P(7.5 \leq x \leq 8.5) = .3461 - .2291 =$ **.1170**

From Table A.2 (binomial tables) = **.120**

b) $P(x \geq 13 \mid n = 20 \text{ and } p = .60)$ $\mu = n \cdot p = 20(.60) = 12$

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{20(.60)(.40)} = 2.19$

$\mu \pm 3\sigma = 12 \pm 3(2.19) = 12 \pm 6.57$

(5.43 to 18.57) lies between 0 and 20.
Approximation by the normal curve is sufficient.

$P(x \geq 12.5 \mid \mu = 12 \text{ and } \sigma = 2.19)$:

$$z = \frac{x-\mu}{\sigma} = \frac{12.5-12}{2.19} = 0.23$$

From Table A.5, area = .0910

$P(x \geq 12.5)$ = .5000 -.0910 = **.4090**

From Table A.2 (binomial tables) = **.415**

c) $P(x = 7 \mid n = 15 \text{ and } p = .50)$ $\mu = n \cdot p = 15(.50) = 7.5$

$$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{15(.50)(.50)} = 1.9365$$

$\mu \pm 3\sigma = 7.5 \pm 3(1.9365) = 7.5 \pm 5.81$

(1.69 to 13.31) lies between 0 and 15.
Approximation by the normal curve is sufficient.

$P(6.5 \leq x \leq 7.5 \mid \mu = 7.5 \text{ and } \sigma = 1.9365)$:

$$z = \frac{x-\mu}{\sigma} = \frac{6.5-7.5}{1.9365} = -0.52$$

From Table A.5, area = **.1985**

From Table A.2 (binomial tables) = **.196**

d) $P(x < 3 \mid n = 10 \text{ and } p = .70)$: $\mu = n \cdot p = 10(.70) = 7$

$$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{10(.70)(.30)}$$

$\mu \pm 3\sigma = 7 \pm 3(1.449) = 7 \pm 4.347$

(2.653 to 11.347) does not lie between 0 and 10.
The normal curve is not a good approximation to this problem.

6.21 $n = 70$, $p = .59$ $P(x < 35)$:

Converting to the normal dist.:

$\mu = n(p) = 70(.59) = 41.3$ and $\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{70(.59)(.41)} = 4.115$

Test for normalcy:

$0 \leq \mu \pm 3\sigma \leq n$, $0 \leq 41.3 \pm 3(4.115) \leq 70$

0 < 28.955 to 53.645 < 70, passes the test

correction for continuity, use $x = 34.5$

$$z = \frac{34.5 - 41.3}{4.115} = -1.65$$

from table A.5, area = .4505

$P(x < 35) = .5000 - .4505 =$ **.0495**

6.23 $p = .27 \quad n = 130$

Conversion to normal dist.: $\mu = n(p) = 130(.27) = 35.1$

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{130(.27)(.73)} = 5.062$

a) $P(x > 39)$: Correct for continuity: $x = 39.5$

$$z = \frac{39.5 - 35.1}{5.062} = .87$$

from table A.5, area = .3078

$P(x > 39) = .5000 - .3078 =$ **.1922**

b) $P(28 \le x \le 38)$: Correct for continuity: 27.5 to 38.5

$$z = \frac{27.5 - 35.1}{5.062} = -1.50 \qquad z = \frac{38.5 - 35.1}{5.062} = 0.67$$

from table A.5, area for z = -1.50 is .4322
area for z = 0.67 is .2486

$P(28 \le x \le 38) = .4322 + .2486 =$ **.6808**

c) $P(x < 23)$: correct for continuity: $x = 22.5$

$$z = \frac{22.5 - 35.1}{5.062} = -2.49$$

from table A.5, area for z = -2.49 is .4936

$P(x < 23) = .5000 - .4936 =$ **.0064**

d) $P(x = 33)$: correct for continuity: 32.5 to 33.5

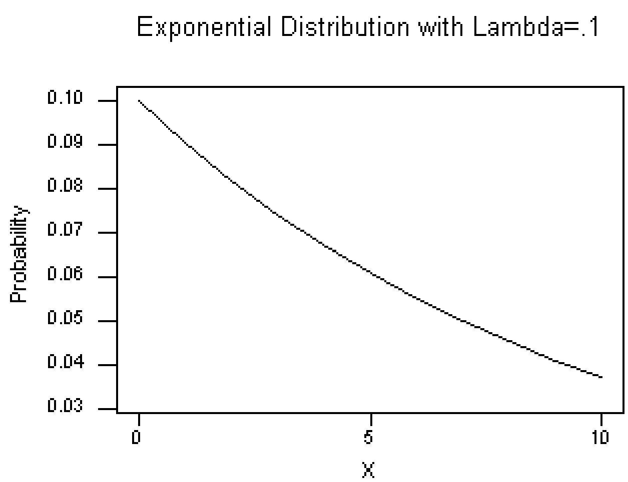
$$z = \frac{32.5 - 35.1}{5.062} = -0.51 \qquad z = \frac{33.5 - 35.1}{5.062} = -0.32$$

from table A.5, area for -0.51 = .1950
area for -0.32 = .1255

$P(x = 33) = .1950 - .1255 =$ **.0695**

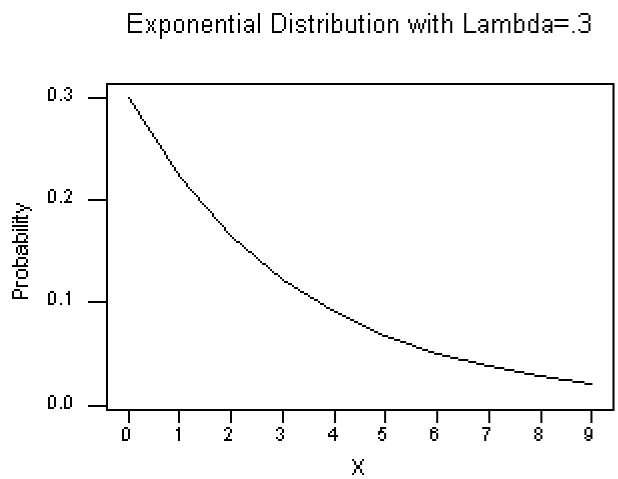
6.25 a) $\lambda = 0.1$

x_0	y
0	.1000
1	.0905
2	.0819
3	.0741
4	.0670
5	.0607
6	.0549
7	.0497
8	.0449
9	.0407
10	.0368



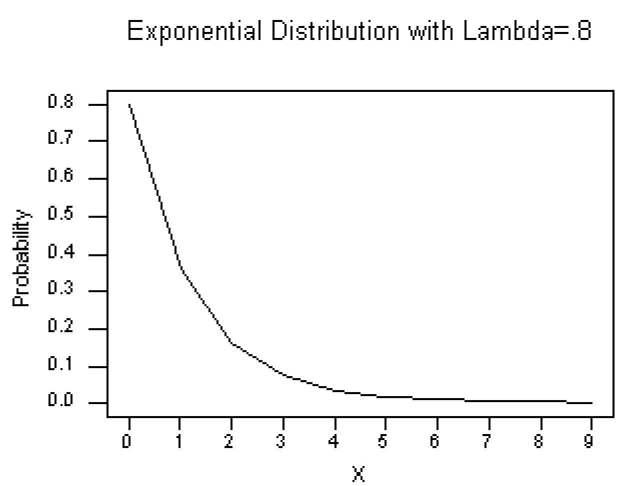
b) $\lambda = 0.3$

x_0	y
0	.3000
1	.2222
2	.1646
3	.1220
4	.0904
5	.0669
6	.0496
7	.0367
8	.0272
9	.0202



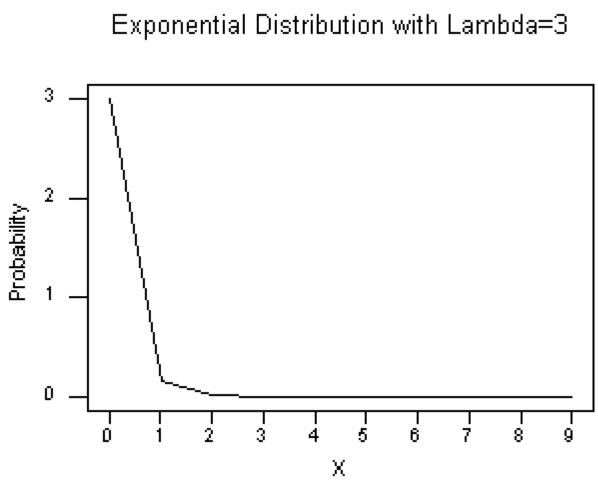
c) $\lambda = 0.8$

x_0	y
0	.8000
1	.3595
2	.1615
3	.0726
4	.0326
5	.0147
6	.0066
7	.0030
8	.0013
9	.0006



d) $\lambda = 3.0$

x_0	y
0	3.0000
1	.1494
2	.0074
3	.0004
4	.0000
5	.0000



6.27 a) $P(x \geq 5 \mid \lambda = 1.35) =$

for $x_0 = 5$: $P(x) = e^{-\lambda x} = e^{-1.35(5)} = e^{-6.75} =$ **.0012**

b) $P(x < 3 \mid \lambda = 0.68) = 1 - P(x \leq 3 \mid \lambda = .68) =$

for $x_0 = 3$: $1 - e^{-\lambda x} = 1 - e^{-0.68(3)} = 1 - e^{-2.04} = 1 - .1300 =$ **.8700**

c) $P(x > 4 \mid \lambda = 1.7) =$

for $x_0 = 4$: $P(x) = e^{-\lambda x} = e^{-1.7(4)} = e^{-6.8} =$ **.0011**

d) $P(x < 6 \mid \lambda = 0.80) = 1 - P(x \geq 6 \mid \lambda = 0.80) =$

for $x_0 = 6$: $P(x) = 1 - e^{-\lambda x} = 1 - e^{-0.80(6)} = 1 - e^{-4.8} = 1 - .0082$

$=$ **.9918**

6.29 $\lambda = 2.44$/min.

a) $P(x \geq 10 \text{ min} \mid \lambda = 2.44\text{/min}) =$

Let $x_0 = 10$, $e^{-\lambda x} = e^{-2.44(10)} = e^{-24.4} =$ **.0000**

b) $P(x \geq 5 \text{ min} \mid \lambda = 2.44\text{/min}) =$

Let $x_0 = 5$, $e^{-\lambda x} = e^{-2.44(5)} = e^{-12.20} =$ **.0000**

c) $P(x \geq 1 \text{ min} \mid \lambda = 2.44\text{/min}) =$

Let $x_0 = 1$, $e^{-\lambda x} = e^{-2.44(1)} = e^{-2.44} =$ **.0872**

d) Expected time = $\mu = \dfrac{1}{\lambda} = \dfrac{1}{2.44}$ min. = **.41 min = 24.6 sec.**

6.31 λ = 1.31/ 1000 passengers

$$\mu = \frac{1}{\lambda} = \frac{1}{1.31} = .7634$$

(0.7634)(1,000) = **763.4**

a) $P(x > 500)$:

Let x_0 = 500/1,000 passengers = .5

$e^{-\lambda x} = e^{-1.31(.5)} = e^{-.655} =$ **.5194**

b) $P(x < 200)$:

Let x_0 = 200/1,000 passengers = .2

$e^{-\lambda x} = e^{-1.31(.2)} = e^{-.262} = .7695$

$P(x < 200) = 1 - .7695 =$ **.2305**

6.33 λ = 2/month

Average number of time between rain = $\mu = \frac{1}{\lambda} = \frac{1}{2}$ month = **15 days**

$\sigma = \mu =$ **15 days**

$P(x \leq 2 \text{ days} \mid \lambda = 2\text{/month})$:

Change λ to days: $\lambda = \frac{2}{30} = .067\text{/day}$

$P(x \leq 2 \text{ days} \mid \lambda = .067\text{/day}) =$

$1 - P(x > 2 \text{ days} \mid \lambda = .067\text{/day})$

let $x_0 = 2$, $1 - e^{-\lambda x} = 1 - e^{-.067(2)} = 1 - .8746 =$ **.1254**

6.35 a) $P(x < 21 | \mu = 25 \text{ and } \sigma = 4)$:

$$z = \frac{x-\mu}{\sigma} = \frac{21-25}{4} = -1.00$$

From Table A.5, area = .3413

$P(x < 21)$ = .5000 -.3413 = **.1587**

b) $P(x \geq 77 | \mu = 50 \text{ and } \sigma = 9)$:

$$z = \frac{x-\mu}{\sigma} = \frac{77-50}{9} = 3.00$$

From Table A.5, area = .4987

$P(x \geq 77)$ = .5000 -.4987 = **.0013**

c) $P(x > 47 | \mu = 50 \text{ and } \sigma = 6)$:

$$z = \frac{x-\mu}{\sigma} = \frac{47-50}{6} = -0.50$$

From Table A.5, area = .1915

$P(x > 47)$ = .5000 + .1915 = **.6915**

d) $P(13 < x < 29 | \mu = 23 \text{ and } \sigma = 4)$:

$$z = \frac{x-\mu}{\sigma} = \frac{13-23}{4} = -2.50$$

From Table A.5, area = .4938

$$z = \frac{x-\mu}{\sigma} = \frac{29-23}{4} = 1.50$$

From Table A.5, area = .4332

$P(13 < x < 29)$ = .4938 + 4332 = **.9270**

e) $P(x \geq 105 | \mu = 90 \text{ and } \sigma = 2.86)$:

$$z = \frac{x-\mu}{\sigma} = \frac{105-90}{2.86} = 5.24$$

From Table A.5, area = .5000

$P(x \geq 105)$ = .5000 - .5000 = **.0000**

6.37 a) $P(x \geq 3 \mid \lambda = 1.3)$:

let $x_0 = 3$

$P(x \geq 3 \mid \lambda = 1.3) = e^{-\lambda x} = e^{-1.3(3)} = e^{-3.9} =$ **.0202**

b) $P(x < 2 \mid \lambda = 2.0)$:

Let $x_0 = 2$

$P(x < 2 \mid \lambda = 2.0) = 1 - P(x \geq 2 \mid \lambda = 2.0) =$

$1 - e^{-\lambda x} = 1 - e^{-2(2)} = 1 - e^{-4} = 1 - .0183 =$ **.9817**

c) $P(1 \leq x \leq 3 \mid \lambda = 1.65)$:

$P(x \geq 1 \mid \lambda = 1.65)$:

Let $x_0 = 1$

$e^{-\lambda x} = e^{-1.65(1)} = e^{-1.65} = .1920$

$P(x \geq 3 \mid \lambda = 1.65)$:

Let $x_0 = 3$

$e^{-\lambda x} = e^{-1.65(3)} = e^{-4.95} = .0071$

$P(1 \leq x \leq 3) = P(x \geq 1) - P(x \geq 3) = .1920 - .0071 =$ **.1849**

d) $P(x > 2 \mid \lambda = 0.405)$:

Let $x_0 = 2$

$e^{-\lambda x} = e^{-(.405)(2)} = e^{-.81} =$ **.4449**

6.39 $p = 1/5 = .20$ $n = 150$

$P(x > 50)$:

$\mu = 150(.20) = 30$

$\sigma = \sqrt{150(.20)(.80)} = 4.899$

$z = \frac{50.5 - 30}{4.899} = 4.18$

Area associated with $z = 4.18$ is .5000

$P(x > 50) = .5000 - .5000 =$ **.0000**

6.41 $\mu = 237$ $\sigma = 54$

a) $P(x < 150)$:

$$z = \frac{150 - 237}{54} = -1.61$$

from Table A.5, area for $z = -1.61$ is .4463

$P(x < 80) = .5000 - .4463 =$ **.0537**

b) $P(x > 400)$:

$$z = \frac{400 - 237}{54} = 3.02$$

from Table A.5, area for $z = 3.02$ is .4987

$P(x > 400) = .5000 - .4987 =$ **.0013**

c) $P(120 < x < 185)$:

$$z = \frac{120 - 237}{54} = -2.17$$

$$z = \frac{185 - 237}{54} = -0.96$$

from Table A.5, area for z = -2.17 is .4850
area for z = -0.96 is .3315

$P(120 < x < 185)$ = .4850 - .3315 = **.1535**

6.43 $a = 18$ $b = 65$

$$P(25 < x < 50) = \frac{50-25}{65-18} = \frac{25}{47} = \mathbf{.5319}$$

$$\mu = \frac{a+b}{2} = \frac{65+18}{2} = \mathbf{41.5}$$

$$f(x) = \frac{1}{b-a} = \frac{1}{65-18} = \frac{1}{47} = \mathbf{.0213}$$

6.45 $\mu = 951$ $\sigma = 96$

a) $P(x \geq 1000)$:

$$z = \frac{x-\mu}{\sigma} = \frac{1000-951}{96} = 0.51$$

from Table A.5, the area for z = 0.51 is .1950

$P(x \geq 1000)$ = .5000 - .1950 = .**3050**

b) $P(900 < x < 1100)$:

$$z = \frac{x-\mu}{\sigma} = \frac{900-951}{96} = -0.53$$

$$z = \frac{x-\mu}{\sigma} = \frac{1100-951}{96} = 1.55$$

from Table A.5, the area for z = -0.53 is .2019
the area for z = 1.55 is .4394

$P(900 < x < 1100)$ = .2019 + .4394 = **.6413**

c) $P(825 < x < 925)$:

$$z = \frac{x-\mu}{\sigma} = \frac{825-951}{96} = -1.31$$

$$z = \frac{x-\mu}{\sigma} = \frac{925-951}{96} = -0.27$$

from Table A.5, the area for z = -1.31 is .4049
the area for z = -0.27 is .1064

$P(825 < x < 925)$ = .4049 - .1064 = **.2985**

d) $P(x < 700)$:

$$z = \frac{x-\mu}{\sigma} = \frac{700-951}{96} = -2.61$$

from Table A.5, the area for z = -2.61 is .4955

$P(x < 700)$ = .5000 - .4955 = **.0045**

6.47 μ = 50,542 σ = 4,246

a) $P(x > 60{,}000)$:

$$z = \frac{x-\mu}{\sigma} = \frac{60{,}000 - 50{,}542}{4246} = 2.23$$

from Table A.5, the area for z = 2.23 is .4871

$P(x > 50{,}000)$ = .5000 - .4871 = **.0129**

b) $P(x < 45{,}000)$:

$$z = \frac{x-\mu}{\sigma} = \frac{45{,}000 - 50{,}542}{4246} = -1.31$$

from Table A.5, the area for z = -1.31 is .4049

$P(x < 40{,}000)$ = .5000 - .4049 = **.0951**

c) $P(x > 40{,}000)$:

$$z = \frac{x-\mu}{\sigma} = \frac{40{,}000 - 50{,}542}{4246} = -2.48$$

from Table A.5, the area for z = -2.48 is .4934

$P(x > 35{,}000)$ = .5000 + .4934 = **.9934**

d) $P(44{,}000 < x < 52{,}000)$:

$$z = \frac{x-\mu}{\sigma} = \frac{44{,}000 - 50{,}542}{4246} = -1.54$$

$$z = \frac{x-\mu}{\sigma} = \frac{52{,}000 - 50{,}542}{4246} = 0.34$$

from Table A.5, the area for z = -1.54 is .4382
the area for z = 0.34 is .1331

$P(44{,}000 < x < 52{,}000)$ = .4382 + .1331 = **.5713**

6.49 $\mu = 88$ $\sigma = 6.4$

a) $P(x < 70)$:

$$z = \frac{x-\mu}{\sigma} = \frac{70-88}{6.4} = -2.81$$

From Table A.5, area = .4975

$P(x < 70)$ = .5000 - .4975 = **.0025**

b) $P(x > 80)$:

$$z = \frac{x-\mu}{\sigma} = \frac{80-88}{6.4} = -1.25$$

From Table A.5, area = .3944

$P(x > 80)$ = .5000 + .3944 = **.8944**

c) $P(90 \le x \le 100)$:

$$z = \frac{x-\mu}{\sigma} = \frac{100-88}{6.4} = 1.88$$

From Table A.5, area = .4699

$$z = \frac{x-\mu}{\sigma} = \frac{90-88}{6.4} = 0.31$$

From Table A.5, area = .1217

$P(90 \le x \le 100)$ = .4699 - .1217 = **.3482**

6.51 $n = 150$ $p = .75$

$\mu = n \cdot p = 150(.75) = 112.5$

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{150(.75)(.25)} = 5.3033$

a) $P(x < 105)$:

correcting for continuity: $x = 104.5$

$$z = \frac{x-\mu}{\sigma} = \frac{104.5-112.5}{5.3033} = -1.51$$

from Table A.5, the area for $z = -1.51$ is .4345

$P(x < 105) = .5000 - .4345 =$ **.0655**

b) $P(110 \le x \le 120)$:

correcting for continuity: $x = 109.5$, $x = 120.5$

$$z = \frac{109.5-112.5}{5.3033} = -0.57$$

$$z = \frac{120.5-112.5}{5.3033} = 1.51$$

from Table A.5, the area for $z = -0.57$ is .2157
the area for $z = 1.51$ is .4345

$P(110 \le x \le 120) = .2157 + .4345 =$ **.6502**

c) $P(x > 95)$:

correcting for continuity: $x = 95.5$

$$z = \frac{95.5-112.5}{5.3033} = -3.21$$

from Table A.5, the area for -3.21 is .4993

$P(x > 95) = .5000 + .4993 =$ **.9993**

6.53 μ = 85,200

60% are between 75,600 and 94,800

94,800 –85,200 = 9,600

75,600 – 85,200 = 9,600

The 60% can be split into 30% and 30% because the two x values are equal distance from the mean.

The z value associated with .3000 area is 0.84

$$z = \frac{x-\mu}{\sigma}$$

$$.84 = \frac{94{,}800 - 85{,}200}{\sigma}$$

σ = **11,428.57**

6.55 λ = 3 hurricanes | 5 months

a) $P(x \geq 1 \text{ month} \mid \lambda = 3 \text{ hurricanes per 5 months})$:

x = 1 month is 1/5 of the 5 month interval associated with λ. Thus, x_0 = 1/5

$P(x_0 \geq 1) = e^{-\lambda x_0} = e^{-3(1/5)} = e^{-0.6} =$ **.5488**

b) $P(x \leq 2 \text{ weeks})$: 2 weeks = 0.5 month = 1/10 of the 5 month interval

$P(x \leq 2 \text{ weeks} \mid \lambda = 0.6 \text{ per month}) =$

$1 - P(x > 2 \text{ weeks} \mid \lambda = 0.6 \text{ per month})$

But $P(x > 2 \text{ weeks} \mid \lambda = 0.6 \text{ per month})$:

Let $x_0 = 0.1$

$P(x > 2 \text{ weeks}) = e^{-\lambda x_0} = e^{-3(1/10)} = e^{-0.3} = .7408$

$P(x \leq 2 \text{ weeks}) = 1 - P(x > 2 \text{ weeks}) = 1 - .7408 =$ **.2592**

c) Average time = Expected time = $\mu = 1/\lambda$ = **1.67 months**

6.57 μ = 2087 σ = 175

If 20% are less, then 30% lie between x and μ.

$z_{.30}$ = -.84

$$z = \frac{x-\mu}{\sigma}$$

$$-.84 = \frac{x-2087}{175}$$

x = **1940**

If 65% are more, then 15% lie between x and μ

$z_{.15}$ = -0.39

$$z = \frac{x-\mu}{\sigma}$$

$$-.39 = \frac{x-2087}{175}$$

x = **2018.75**

If x is more than 85%, then 35% lie between x and μ.

$z_{.35}$ = 1.04

$$z = \frac{x-\mu}{\sigma}$$

$$1.04 = \frac{x-2087}{175}$$

x = **2269**

6.59 μ = 1,717,000 σ = 50,940

$P(x > 1{,}800{,}000)$:

$$z = \frac{x-\mu}{\sigma} = \frac{1{,}800{,}000 - 1{,}717{,}000}{50{,}940} = 1.63$$

from table A.5 the area for z = 1.63 is .4484

$P(x > 1{,}800{,}000)$ = .5000 - .4484 = **.0516**

$P(x < 1{,}600{,}000)$:

$$z = \frac{x-\mu}{\sigma} = \frac{1{,}600{,}000 - 1{,}717{,}000}{50{,}940} = -2.30$$

from table A.5 the area for z = -2.30 is .4893

$P(x < 1{,}600{,}000)$ = .5000 - .4893 = **.0107**

1.07% of the time.

6.61 This is a uniform distribution with a = 11 and b = 32.

The mean is (11 + 32)/2 = 21.5 and the standard deviation is

$(32 - 11)/\sqrt{12}$ = 6.06. Almost 81% of the time there are less than or equal to 28 sales associates working. One hundred percent of the time there are less than or equal to 34 sales associates working and never more than 34. About 23.8% of the time there are 16 or fewer sales associates working. There are 21 or fewer sales associates working about 48% of the time.

6.63 The lengths of cell phone calls are normally distributed with a mean of 2.35 minutes and a standard deviation of .11 minutes. Almost 99% of the calls are less than or equal to 2.60 minutes, almost 82% are less than or equal to 2.45 minutes, over 32% are less than 2.3 minutes, and almost none are less than 2 minutes.