

COMPUTERS & STRUCTURES, INC.

STRUCTURAL AND EARTHQUAKE ENGINEERING SOFTWARE

ETABS® 2016
Integrated Building Design Software

Concrete Frame Design Manual

ACI 318-11





Concrete Frame Design Manual

ACI 318-11

For ETABS® 2016

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Chapter 1

Introduction

The design of concrete frames is seamlessly integrated within the program. Initiation of the design process, along with control of various design parameters, is accomplished using the Design menu.

Automated design at the object level is available for any one of a number of user-selected design codes, as long as the structures have first been modeled and analyzed by the program. Model and analysis data, such as material properties and member forces, are recovered directly from the model database, and no additional user input is required if the design defaults are acceptable.

The design is based on a set of user-specified loading combinations. However, the program provides default load combinations for each design code supported. If the default load combinations are acceptable, no definition of additional load combinations is required.

In the design of columns, the program calculates the required longitudinal and shear reinforcement. However, the user may specify the longitudinal steel, in which case a column capacity ratio is reported. The column capacity ratio gives an indication of the stress condition with respect to the capacity of the column.

The biaxial column capacity check is based on the generation of consistent three-dimensional interaction surfaces. It does not use any empirical formulations that extrapolate uniaxial interaction curves to approximate biaxial action.

Interaction surfaces are generated for user-specified column reinforcing configurations. The column configurations may be rectangular, square or circular, with similar reinforcing patterns. The calculation of moment magnification factors, unsupported lengths, and strength reduction factors is automated in the algorithm.

Every beam member is designed for flexure, shear, and torsion at output stations along the beam span.

All beam-column joints are investigated for existing shear conditions.

For special moment resisting frames (ductile frames), the shear design of the columns, beams, and joints is based on the probable moment capacities of the members. Also, the program will produce ratios of the beam moment capacities with respect to the column moment capacities, to investigate weak beam/strong column aspects, including the effects of axial force.

Output data can be presented graphically on the model, in tables for both input and output data, or on the calculation sheet prepared for each member. For each presentation method, the output is in a format that allows the engineer to quickly study the stress conditions that exist in the structure and, in the event the member reinforcing is not adequate, aids the engineer in taking appropriate remedial measures, including altering the design member without rerunning the entire analysis.

1.1 Organization

This manual is designed to help you quickly become productive with the concrete frame design options of ACI 318-11. Chapter 2 provides detailed descriptions of the Design Prerequisites used for ACI 318-11. Chapter 3 provides detailed descriptions of the code-specific process used for ACI 318-11. The appendices provide details on certain topics referenced in this manual.

1.2 Recommended Reading/Practice

It is strongly recommended that you read this manual and review any applicable “Watch & Learn” Series™ tutorials, which are found on our web site, <http://www.csiamerica.com>, before attempting to design a concrete frame. Additional information can be found in the on-line Help facility available from within the program’s main menu.

Chapter 2 Design Prerequisites

This chapter provides an overview of the basic assumptions, design preconditions, and some of the design parameters that affect the design of concrete frames.

In writing this manual it has been assumed that the user has an engineering background in the general area of structural reinforced concrete design and familiarity with the ACI 318-11 code.

2.1 Design Load Combinations

The design load combinations are used for determining the various combinations of the load cases for which the structure needs to be designed/checked. The load combination factors to be used vary with the selected design code. The load combination factors are applied to the forces and moments obtained from the associated load cases and are then summed to obtain the factored design forces and moments for the load combination.

For multi-valued load combinations involving response spectrum, time history, moving loads and multi-valued combinations (of type enveloping, square-root of the sum of the squares or absolute) where any correspondence between interacting quantities is lost, the program automatically produces multiple sub

combinations using maxima/minima permutations of interacting quantities. Separate combinations with negative factors for response spectrum cases are not required because the program automatically takes the minima to be the negative of the maxima for response spectrum cases and the permutations just described generate the required sub combinations.

When a design combination involves only a single multi-valued case of time history or moving load, further options are available. The program has an option to request that time history combinations produce sub combinations for each time step of the time history. Also an option is available to request that moving load combinations produce sub combinations using maxima and minima of each design quantity but with corresponding values of interacting quantities.

For normal loading conditions involving static dead load, live load, snow load, wind load, and earthquake load, or dynamic response spectrum earthquake load, the program has built-in default loading combinations for each design code. These are based on the code recommendations and are documented for each code in the corresponding manuals.

For other loading conditions involving moving load, time history, pattern live loads, separate consideration of roof live load, snow load, and so on, the user must define design loading combinations either in lieu of or in addition to the default design loading combinations.

The default load combinations assume all load cases declared as dead load to be additive. Similarly, all cases declared as live load are assumed additive. However, each load case declared as wind or earthquake, or response spectrum cases, is assumed to be non additive with each other and produces multiple lateral load combinations. Also wind and static earthquake cases produce separate loading combinations with the sense (positive or negative) reversed. If these conditions are not correct, the user must provide the appropriate design combinations.

The default load combinations are included in design if the user requests them to be included or if no other user-defined combination is available for concrete design. If any default combination is included in design, all default combinations will automatically be updated by the program any time the design code is changed or if static or response spectrum load cases are modified.

Live load reduction factors can be applied to the member forces of the live load case on an element-by-element basis to reduce the contribution of the live load to the factored loading.

The user is cautioned that if moving load or time history results are not requested to be recovered in the analysis for some or all of the frame members, the effects of those loads will be assumed to be zero in any combination that includes them.

2.2 Seismic Load Effects

IBC 2012 requires that all structural element design resists earthquake motions in accordance with ASCE 7-10. The software allows users to activate Special seismic load effects using appropriate commands on the Define menu. The special seismic loads are computed in accordance with ASCE 7-10 sections 12.3.4 and 12.4.

The reliability factor, ρ , and DL multiplier are automatically applied to all program default design combinations when the ACI 318-11 code is selected. The DL multiplier represents the $0.2S_{DS}$ factor in Equation 12.4-4 of ASCE 7-10. When seismic load E is combined with the effects of other loads, the following load combination shall be used in lieu of the seismic load combinations in section 9.2.1 of the code.

$$\begin{aligned}(0.9 - 0.2S_{DS}) D \pm \rho E \\(1.2 + 0.2S_{DS}) D + 1.0L \pm \rho E \\(1.2 + 0.2S_{DS}) D + 1.0L + 0.2S \pm \rho E\end{aligned}$$

2.3 Design and Check Stations

For each load combination, each element is designed or checked at a number of locations along the length of the element. The locations are based on equally spaced segments along the clear length of the element. The number of segments in an element is requested by the user before the analysis is performed. The user can refine the design along the length of an element by requesting more segments.

When using the ACI 318-11 design code, requirements for joint design at the beam-to-column connections are evaluated at the top most station of each

column. The program also performs a joint shear analysis at the same station to determine if special considerations are required in any of the joint panel zones. The ratio of the beam flexural capacities with respect to the column flexural capacities considering axial force effect associated with the weak-beam/strong-column aspect of any beam/column intersection are reported.

2.4 Identifying Beams and Columns

In the program, all beams and columns are represented as frame elements, but design of beams and columns requires separate treatment. Identification for a concrete element is accomplished by specifying the frame section assigned to the element to be of type beam or column. If any brace element exists in the frame, the brace element also would be identified as a beam or a column element, depending on the section assigned to the brace element.

2.5 Design of Beams

In the design of concrete beams, in general, the program calculates and reports the required areas of steel for flexure and shear based on the beam moments, shears, load combination factors, and other criteria, which are described in detail in the code-specific manuals. The reinforcement requirements are calculated at a user-defined number of stations along the beam span.

All beams are designed for major direction flexure, shear, and torsion only. Effects caused by any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

In designing the flexural reinforcement for the major moment at a particular section of a particular beam, the steps involve the determination of the maximum factored moments and the determination of the reinforcing steel. The beam section is designed for the maximum positive and maximum negative factored moment envelopes obtained from all of the load combinations. Negative beam moments produce top steel. In such cases, the beam is always designed as a Rectangular section. Positive beam moments produce bottom steel. In such cases, the beam may be designed as a Rectangular beam or a T-beam. For the design of flexural reinforcement, the beam is first designed as a singly reinforced beam. If the beam section is not adequate, the required compression reinforcement is calculated.

In designing the shear reinforcement for a particular beam for a particular set of loading combinations at a particular station associated with beam major shear, the steps involve the determination of the factored shear force, the determination of the shear force that can be resisted by concrete, and the determination of the reinforcement steel required to carry the balance.

As noted previously, special considerations for seismic design are incorporated into the program for the ACI 318-11 code.

2.6 Design of Columns

In the design of the columns, the program calculates the required longitudinal steel, or if the longitudinal steel is specified, the column stress condition is reported in terms of a column capacity ratio, which is a factor that gives an indication of the stress condition of the column with respect to the capacity of the column. The design procedure for the reinforced concrete columns of the structure involves the following steps:

- Generate axial force-biaxial moment interaction surfaces for all of the different concrete section types in the model.
- Check the capacity of each column for the factored axial force and bending moments obtained from each loading combination at each end of the column. This step is also used to calculate the required reinforcement (if none was specified) that will produce a capacity ratio of 1.0.

The generation of the interaction surface is based on the assumed strain and stress distributions and some other simplifying assumptions. These stress and strain distributions and the assumptions are documented in Chapter 3.

The shear reinforcement design procedure for columns is very similar to that for beams, except that the effect of the axial force on the concrete shear capacity must be considered.

For certain special seismic cases, the design of columns for shear is based on the capacity shear. The capacity shear force in a particular direction is calculated from the moment capacities of the column associated with the factored axial force acting on the column. For each load combination, the factored axial load is calculated using the load cases and the corresponding load combination factors. Then, the moment capacity of the column in a particular direction under the in-

fluence of the axial force is calculated, using the uniaxial interaction diagram in the corresponding direction, as documented in Chapter 3.

2.7 Design of Joints

To ensure that the beam-column joint of special moment resisting frames possesses adequate shear strength, the program performs a rational analysis of the beam-column panel zone to determine the shear forces that are generated in the joint. The program then checks this against design shear strength.

Only joints that have a column below the joint are designed. The material properties of the joint are assumed to be the same as those of the column below the joint. The joint analysis is performed in the major and the minor directions of the column. The joint design procedure involves the following steps:

- Determine the panel zone design shear force
- Determine the effective area of the joint
- Check panel zone shear stress

The joint design details are documented in Chapter 3.

2.8 P-Delta Effects

The program design process requires that the analysis results include P-delta effects. The P-delta effects are considered differently for “braced” or “non-sway” and “unbraced” or “sway” components of moments in columns or frames. For the braced moments in columns, the effect of P-delta is limited to “individual member stability.” For unbraced components, “lateral drift effects” should be considered in addition to individual member stability effect. The program assumes that “braced” or “nonsway” moments are contributed from the “dead” or “live” loads, whereas, “unbraced” or “sway” moments are contributed from all other types of loads.

For the individual member stability effects, the moments are magnified with moment magnification factors, as documented in Chapter 3 of this manual.

For lateral drift effects, the program assumes that the P-delta analysis is performed and that the amplification is already included in the results. The mo-

ments and forces obtained from P-delta analysis are further amplified for individual column stability effect if required by the governing code, as in the ACI 318-11/IBC 2012 codes.

Users of the program should be aware that the default analysis option is turned OFF for P-delta effect. The user can turn the P-delta analysis ON and set the maximum number of iterations for the analysis. The default number of iteration for P-delta analysis is 1. Further details about P-delta analysis are provided in Appendix A of this design manual.

2.9 Element Unsupported Lengths

To account for column slenderness effects, the column unsupported lengths are required. The two unsupported lengths are l_{33} and l_{22} . These are the lengths between support points of the element in the corresponding directions. The length l_{33} corresponds to instability about the 3-3 axis (major axis), and l_{22} corresponds to instability about the 2-2 axis (minor axis).

Normally, the unsupported element length is equal to the length of the element, i.e., the distance between END-I and END-J of the element. The program, however, allows users to assign several elements to be treated as a single member for design. This can be accomplished differently for major and minor bending, as documented in Appendix B of this design manual.

The user has options to specify the unsupported lengths of the elements on an element-by-element basis.

2.10 Choice of Input Units

English as well as SI and MKS metric units can be used for input. The codes are based on a specific system of units. All equations and descriptions presented in the subsequent chapters correspond to that specific system of units unless otherwise noted. For example, the ACI code is published in inch-pound-second units. By default, all equations and descriptions presented in the “Design Process” chapter correspond to inch-pound-second units. However, any system of units can be used to define and design a structure in the program.

Chapter 3 Design Process

This chapter provides a detailed description of the code-specific algorithms used in the design of concrete frames when the ACI 318-11 code have been selected. For simplicity, all equations and descriptions presented in this chapter correspond to inch-lbs-second units unless otherwise noted.

3.1 Notation

The various notations used in this chapter are described herein:

A_{cp}	Area enclosed by outside perimeter of concrete cross-section, in ²
A_{cv}	Area of concrete used to determine shear stress, in ²
A_g	Gross area of concrete, in ²
A_o	Gross area enclosed by shear flow path, in ²
A_{oh}	Area enclosed by centerline of the outermost closed transverse torsional reinforcement, in ²
A_s	Area of tension reinforcement, in ²
A'_s	Area of compression reinforcement, in ²

A_t	Area of longitudinal torsion reinforcement, in ²
A_t/s	Area of transverse torsion reinforcement (closed stirrups) per unit length of the member, in ² /in
$A_{s(\text{required})}$	Area of steel required for tension reinforcement, in ²
A_{st}	Total area of column longitudinal reinforcement, in ²
A_v	Area of shear reinforcement, in ²
A_v/s	Area of shear reinforcement per unit length of the member, in ² /in
C_m	Coefficient, dependent upon column curvature, used to calculate moment magnification factor
E_c	Modulus of elasticity of concrete, psi
E_s	Modulus of elasticity of reinforcement, assumed as 29×10^6 psi
I_g	Moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in ⁴
I_{se}	Moment of inertia of reinforcement about centroidal axis of member cross-section, in ⁴
L	Clear unsupported length, in
M_a	Smaller factored end moment in a column, lb-in
M_b	Larger factored end moment in a column, lb-in
M_c	Factored moment to be used in design, lb-in
M_{ns}	Non-sway component of factored end moment, lb-in
M_s	Sway component of factored end moment, lb-in
M_u	Factored moment at a section, lb-in
M_{u2}	Factored moment at a section about 2-axis, lb-in
M_{u3}	Factored moment at a section about 3-axis, lb-in
P_b	Axial load capacity at balanced strain conditions, lb
P_c	Critical buckling strength of column, lb

P_{\max}	Maximum axial load strength allowed, lb
P_0	Axial load capacity at zero eccentricity, lb
P_u	Factored axial load at a section, lb
V_c	Shear force resisted by concrete, lb
V_E	Shear force caused by earthquake loads, lb
V_{D+L}	Shear force from span loading, lb
V_{\max}	Maximum permitted total factored shear force at a section, lb
V_p	Shear force computed from probable moment capacity, lb
V_s	Shear force resisted by steel, lb
V_u	Factored shear force at a section, lb
a	Depth of compression block, in
a_b	Depth of compression block at balanced condition, in
a_{\max}	Maximum allowed depth of compression block, in
b	Width of member, in
b_f	Effective width of flange (T-Beam section), in
b_w	Width of web (T-Beam section), in
c	Depth to neutral axis, in
c_b	Depth to neutral axis at balanced conditions, in
d	Distance from compression face to tension reinforcement, in
d'	Concrete cover to center of reinforcing, in
d_s	Thickness of slab (T-Beam section), in
f'_c	Specified compressive strength of concrete, psi
f_y	Specified yield strength of flexural reinforcement, psi.
f_{yt}	Specified yield strength of shear reinforcement, psi.
h	Overall depth of a column section, in

k	Effective length factor
p_{cp}	Outside perimeter of the concrete cross-section, in
p_h	Perimeter of centerline of outermost closed transverse torsional reinforcement, in
r	Radius of gyration of column section, in
α	Reinforcing steel overstrength factor
λ	Modification factor reflecting the reduced mechanical properties of light-weight concrete, all relative to normal weight concrete of the same compressive strength
β_1	Factor for obtaining depth of compression block in concrete
β_{dns}	Absolute value of ratio of maximum factored axial dead load to maximum factored axial total load
δ_s	Moment magnification factor for sway moments
δ_{ns}	Moment magnification factor for non-sway moments
ϵ_c	Strain in concrete
$\epsilon_{c, \max}$	Maximum usable compression strain allowed in extreme concrete fiber (0.003 in/in)
ϵ_s	Strain in reinforcing steel
$\epsilon_{s, \min}$	Minimum tensile strain allowed in steel rebar at nominal strength for tension controlled behavior (0.005 in/in)
ϕ	Strength reduction factor

3.2 Design Load Combinations

The design load combinations are the various combinations of the prescribed response cases for which the structure is to be checked. The program creates a number of default design load combinations for a concrete frame design. Users can add their own design load combinations as well as modify or delete the program default design load combinations. An unlimited number of design load combinations can be specified.

To define a design load combination, simply specify one or more response cases, each with its own scale factor. The scale factors are applied to the forces and moments from the load cases to form the factored design forces and moments for each design load combination. There is one exception to the preceding. For spectral analysis modal combinations, any correspondence between the signs of the moments and axial loads is lost. The program uses eight design load combinations for each such loading combination specified, reversing the sign of axial loads and moments in major and minor directions.

As an example, if a structure is subjected to dead load, D, and live load, L, only, the ACI 318-11 design check may need one design load combination only, namely, $1.2D + 1.6L$. However, if the structure is subjected to wind, earthquake, or other loads, numerous additional design load combinations may be required.

The program allows live load reduction factors to be applied to the member forces of the reducible live load case on a member-by-member basis to reduce the contribution of the live load to the factored responses.

The design load combinations are the various combinations of the load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead (D), live (L), pattern live (PL), wind (W), earthquake (E), and snow (S) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be defined (ACI 9.2.1):

$1.4D$	(ACI 9-1)
$1.2D + 1.6L + 0.5L_r$	(ACI 9-2)
$1.2D + 1.0L + 1.6L_r$	(ACI 9-3)
$1.2D + 1.6(0.75 PL) + 0.5L_r$	(ACI 9-2, 13.7.6.3)
$1.2D + 1.6L + 0.5S$	(ACI 9-2)
$1.2D + 1.0L + 1.6S$	(ACI 9-3)
$0.9D \pm 1.0W$	(ACI 9-6)
$1.2D + 1.0L + 0.5L_r \pm 1.0W$	(ACI 9-4)
$1.2D + 1.6L_r \pm 0.5W$	(ACI 9-3)
$1.2D + 1.6S \pm 0.5W$	(ACI 9-3)
$1.2D + 1.0L + 0.5S \pm 1.0W$	(ACI 9-4)

$$0.9D \pm 1.0E \quad (\text{ACI 9-7})$$

$$1.2D + 1.0L + 0.2S \pm 1.0E \quad (\text{ACI 9-5})$$

These are also the default design load combinations in the program whenever the ACI 318-11 code is used. Also, refer to Section 2.2 *Seismic Load Effects* when special seismic load effects are included that modify the scale factors for Dead and Earthquake loads. The user should use other appropriate design load combinations if other types of loads are present. PL is the live load multiplied by the Pattern Live Load Factor. The Pattern Live Load Factor can be specified in the Preferences.

Live load reduction factors can be applied to the member forces of the live load analysis on a member-by-member basis to reduce the contribution of the live load to the factored loading.

When using this code, the program assumes that a P-Delta analysis has been performed.

3.3 Limits on Material Strength

The concrete compressive strength, f'_c , should not be less than 2500 psi (ACI 5.1.1). The upper limit of the reinforcement yield strength, f_y , is taken as 80 ksi (ACI 9.4) and the upper limit of the reinforcement shear strength, f_{yt} , is taken as 60 ksi (ACI 11.4.2).

ETABS enforces the upper material strength limits for flexure and shear design of beams, columns and slabs or for torsion design of beams. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

3.4 Column Design

The program can be used to check column capacity or to design columns. If the geometry of the reinforcing bar configuration of each concrete column section has been defined, the program will check the column capacity. Alternatively, the program can calculate the amount of reinforcing required to design the column

based on provided reinforcing bar configuration. The reinforcement requirements are calculated or checked at a user-defined number of check/design stations along the column span. The design procedure for the reinforced concrete columns of the structure involves the following steps:

- Generate axial force-biaxial moment interaction surfaces for all of the different concrete section types of the model. A typical biaxial interacting diagram is shown in Figure 3-1. For reinforcement to be designed, the program generates the interaction surfaces for the range of allowable reinforcement: 1 to 8 percent for Ordinary and Intermediate Moment Resisting Frames (ACI 10.9.1) and 1 to 6 percent for Special Moment Resisting Frames (ACI 21.6.3.1).
- Calculate the capacity ratio or the required reinforcing area for the factored axial force and biaxial (or uniaxial) bending moments obtained from each loading combination at each station of the column. The target capacity ratio is taken as the Utilization Factor Limit when calculating the required reinforcing area.
- Design the column shear reinforcement.

The following three sections describe in detail the algorithms associated with this process.

3.4.1 Generation of Biaxial Interaction Surfaces

The column capacity interaction volume is numerically described by a series of discrete points that are generated on the three-dimensional interaction failure surface. In addition to axial compression and biaxial bending, the formulation allows for axial tension and biaxial bending considerations. A typical interaction surface is shown in Figure 3-1.

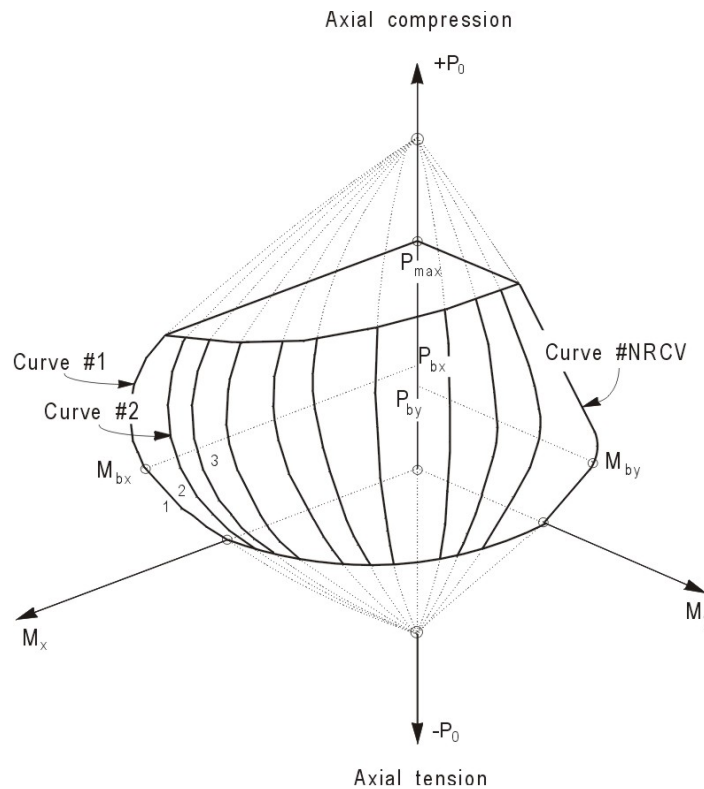


Figure 3-1 A typical column interaction surface

The coordinates of these points are determined by rotating a plane of linear strain in three dimensions on the section of the column, as shown in Figure 3-2. The linear strain diagram limits the maximum concrete strain, ϵ_c , at the extremity of the section, to 0.003 (ACI 10.2.3). The formulation is based consistently on the general principles of ultimate strength design (ACI 10.3).

The stress in the steel is given by the product of the steel strain and the steel modulus of elasticity, $\epsilon_s E_s$, and is limited to the yield stress of the steel, f_y (ACI 10.2.4). The area associated with each reinforcing bar is assumed to be placed at the actual location of the center of the bar, and the algorithm does not assume any further simplifications with respect to distributing the area of steel over the cross-section of the column, as shown in Figure 3-2.

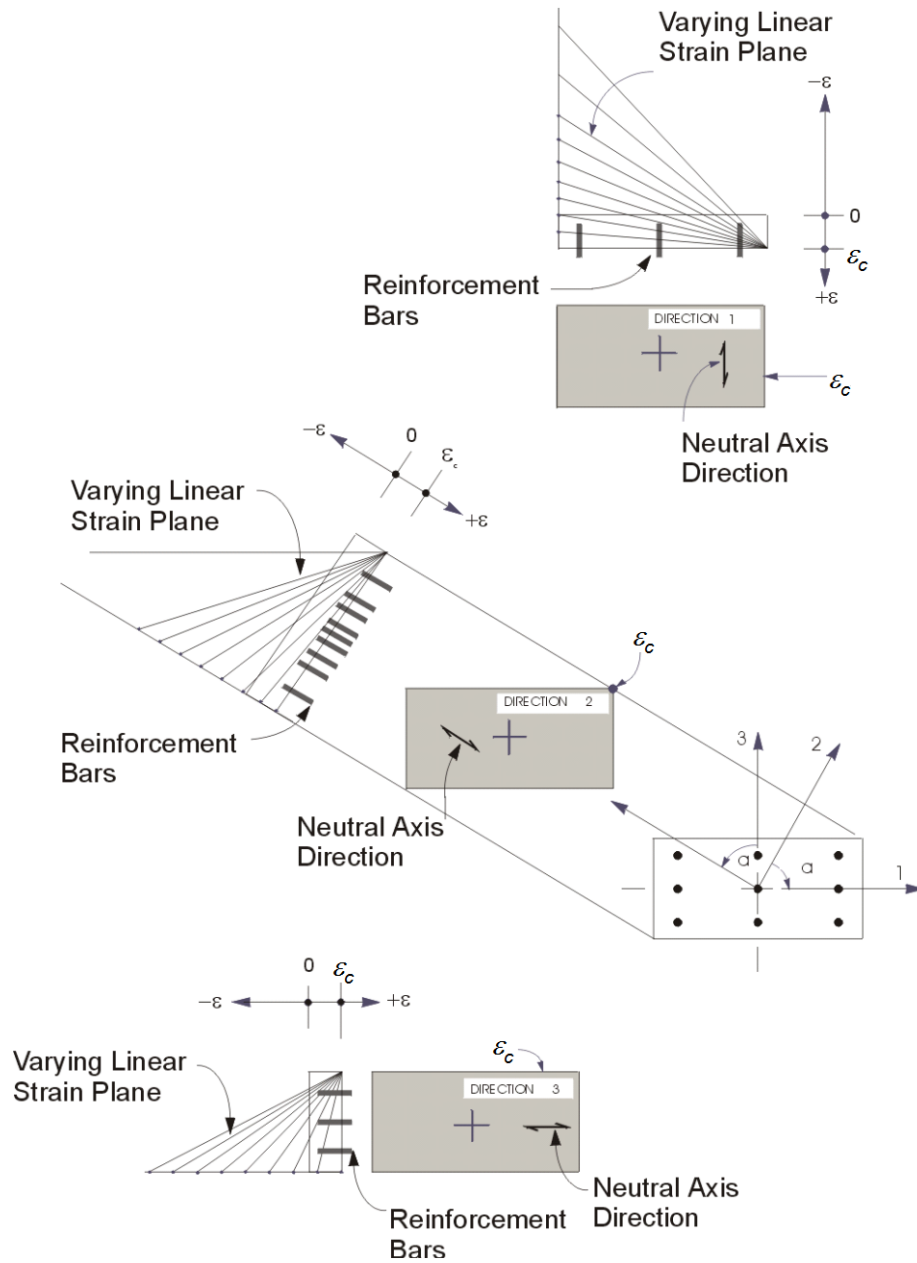


Figure 3-2 Idealized strain distribution for generation of interaction surface

The concrete compression stress block is assumed to be rectangular, with a stress value of $0.85f'_c$ (ACI 10.2.7.1), as shown in Figure 3-3.

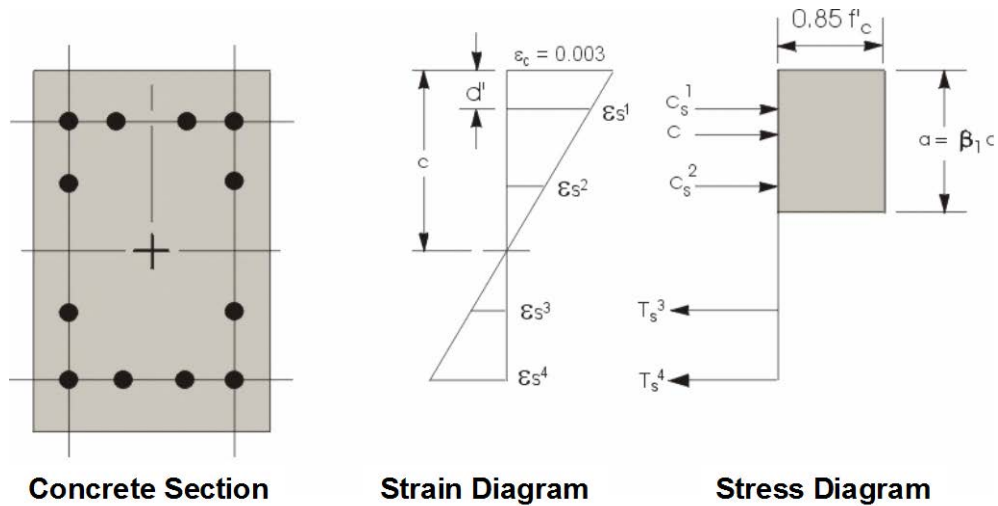


Figure 3-3 Idealization of stress and strain distribution in a column section

The interaction algorithm provides correction to account for the concrete area that is displaced by the reinforcement in the compression zone. The depth of the equivalent rectangular block, a , is taken as:

$$a = \beta_1 c \quad (\text{ACI 10.2.7.1})$$

where c is the depth of the stress block in compression strain and,

$$\beta_1 = 0.85 - 0.05 \left(\frac{f'_c - 4000}{1000} \right), \quad 0.65 \leq \beta_1 \leq 0.85. \quad (\text{ACI 10.2.7.3})$$

The effect of the strength reduction factor, ϕ , is included in the generation of the interaction surface. The value of ϕ used in the interaction diagram varies from compression controlled ϕ to tension controlled ϕ based on the maximum tensile strain in the reinforcing at the extreme edge, ϵ_t (ACI 9.3.2).

Sections are considered compression controlled when the tensile strain in the extreme tension steel is equal to or less than the compression controlled strain limit at the time the concrete in compression reaches its assumed strain limit of $\epsilon_{c,\max}$, which is 0.003. The compression controlled strain limit is the tensile strain

in the reinforcement at balanced strain condition, which is taken as the yield strain of the steel reinforcing, $\frac{f_y}{E}$ (ACI 10.3.3).

Sections are tension controlled when the tensile strain in the extreme tension steel is equal to or greater than 0.005, just as the concrete in compression reaches its assumed strain limit of 0.003 (ACI 10.3.4).

Sections with ε_t between the two limits are considered to be in a transition region between compression controlled and tension controlled sections (ACI 10.3.4).

When the section is tension controlled, a ϕ factor for tension control is used. When the section is compression controlled, a ϕ factor for compression control is used. When the section is within the transition region, ϕ is linearly interpolated between the two values (ACI 9.3.2), as shown in the following:

$$\phi = \begin{cases} \phi_c & \text{if } \varepsilon_t \leq \varepsilon_y \\ \phi_t - (\phi_t - \phi_c) \left(\frac{0.005 - \varepsilon_t}{0.005 - \varepsilon_y} \right) & \text{if } \varepsilon_y < \varepsilon_t \leq 0.005, \\ \phi_t & \text{if } \varepsilon_t \geq 0.005, \text{ where} \end{cases} \quad (\text{ACI 9.3.2})$$

$$\phi_t = \phi \text{ for tension controlled sections,} \\ \text{which is 0.90 by default} \quad (\text{ACI 9.3.2.1})$$

$$\phi_c = \phi \text{ for compression controlled sections} \\ = 0.75 \text{ (by default) for column sections} \\ \text{with spiral reinforcement} \quad (\text{ACI 9.3.2.2a})$$

$$= 0.65 \text{ (by default) for column sections} \\ \text{with tied reinforcement} \quad (\text{ACI 9.3.2.2b})$$

Default values for ϕ_c and ϕ_t are provided by the program but can be overwritten using the Preferences.

The maximum compressive axial load is limited to $\phi P_{n(\max)}$, where

$$\phi P_{n(\max)} = 0.85\phi [0.85f'_c (A_g - A_{st}) + f_y A_{st}], \text{ spiral} \quad (\text{ACI 10.3.6.1})$$

$$\phi P_{n(\max)} = 0.80\phi [0.85f'_c (A_g - A_{st}) + f_y A_{st}], \text{ tied} \quad (\text{ACI 10.3.6.2})$$

In calculating the $\phi P_{n(\max)}$, the ϕ for a compression controlled case is used. A limit of 80,000 psi on f_y has been imposed (ACI 9.4).

$$f_y \leq 80,000 \text{ psi} \quad (\text{ACI 9.4})$$

If input f_y is larger than the preceding limit, f_y is set equal to the limiting value during calculations.

3.4.2 Calculate Column Capacity Ratio

The column capacity ratio is calculated for each design load combination at each output station of each column. The following steps are involved in calculating the capacity ratio of a particular column for a particular design load combination at a particular location:

- Determine the factored moments and forces from the load cases and the specified load combination factors to give P_u , M_{u2} , and M_{u3} .
- Determine the moment magnification factors for the column moments.
- Apply the moment magnification factors to the factored moments. Determine if the point, defined by the resulting axial load and biaxial moment set, lies within the interaction volume.

The factored moments and corresponding magnification factors depend on the identification of the individual column as either “sway” or “non-sway.”

The following three sections describe in detail the algorithms associated with that process.

3.4.2.1 Determine Factored Moments and Forces

The loads for a particular design load combination are obtained by applying the corresponding factors to all of the load cases, giving P_u , M_{u2} , and M_{u3} . The factored moments are further increased, if required, to obtain minimum eccentricities of $(0.6 + 0.03h)$ inches, where h is the dimension of the column in the corresponding direction (ACI 10.10.6.5). The minimum eccentricity is applied in only one direction at a time. The minimum eccentricity moments are amplified for second order effects (ACI 10.10, 10.10.6.5).

3.4.2.2 Determine Moment Magnification Factors

The moment magnification factors are calculated separately for sway (overall stability effect), δ_s , and for non-sway (individual column stability effect), δ_{ns} . Also, the moment magnification factors in the major and minor directions are, in general, different (ACI R10.10.5, R10.10.5).

The moment obtained from analysis is separated into two components: the sway M_s and the non-sway M_{ns} components. The sway components are identified by the “s” subscript. The sway moments are primarily caused by lateral loads and are related to the cause of sidesway. The non-sway components, which are identified by “ns” subscripts, are primarily caused by gravity load.

For individual columns or column-members, the magnified moments about two axes at any station of a column can be obtained as

$$M = M_{ns} + \delta_s M_s \quad (\text{ACI 10.10.7, Eqn. 10-18, 10-19})$$

The factor δ_s is the moment magnification factor for moments causing sidesway. The program takes this factor to be 1 because the component moments M_s and M_{ns} are assumed to be obtained from a second order elastic (P- Δ) analysis (ACI R10.10, 10.10.4, R10.10.4). For more information about P- Δ analysis, refer to Appendix A.

For the P- Δ analysis, the analysis combination should correspond to a load of 1.2 (dead load) + 1.6 (live load) (ACI 9.2.1). See also White and Hajjar (1991). The user should use reduction factors for the moments of inertia in the program as specified in ACI 10.10.4.1. The moment of inertia reduction for sustained lateral load involves a factor β_{ds} (ACI 10.10.4.2). This β_{ds} for sway frames in second-order analysis is different from the one that is defined later for nonsway moment magnification (ACI 2.1, R10.10.4.2, 10.10.6.2). The default moment of inertia factor in this program is 1.

The computed moments are further amplified for individual column stability effect (ACI 10.10.6, 10.10) by the nonsway moment magnification factor, δ_{ns} , as follows:

$$M_c = \delta_{ns} M \quad (\text{ACI Eqn. 10-11})$$

M_c is the factored moment to be used in design.

The nonsway moment magnification factor, δ_{ns} , associated with the major or minor direction of the column is given by

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0 \text{ where} \quad (\text{ACI 10.10.6, Eqn. 10-12})$$

$$C_m = \begin{cases} 0.6 + 0.4 \frac{M_a}{M_b}, & \text{with no transverse load,} \\ 1.0, & \text{with transverse load,} \end{cases} \quad (\text{ACI 10.10.6.4, Eqn. 10-16})$$

where M_a and M_b are the moments at the ends of the column, and M_b is numerically larger than M_a . M_a/M_b is positive for single curvature bending and negative for double curvature bending. The preceding expression of C_m is valid if there is no transverse load applied between the supports. If transverse load is present on the span, or the length is overwritten, $C_m = 1$. The user can overwrite C_m on an object-by-object basis.

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \quad (\text{ACI 10.10.6, Eqn. 10-13})$$

k is conservatively taken as 1; however, the program allows the user to overwrite this value (ACI 10.10.1). l_u is the unsupported length of the column for the direction of bending considered. The two unsupported lengths are l_{22} and l_{33} , corresponding to instability in the minor and major directions of the object, respectively, as shown in Figure B-1 in Appendix B. These are the lengths between the support points of the object in the corresponding directions.

Refer to Appendix B for more information about how the program automatically determines the unsupported lengths. The program allows users to overwrite the unsupported length ratios, which are the ratios of the unsupported lengths for the major and minor axes bending to the overall member length.

EI is associated with a particular column direction:

$$EI = \frac{0.4E_c I_g}{1 + \beta_{dns}} \quad (\text{ACI 10.10.6.1, Eqn. 10-15})$$

$$\beta_{dns} = \frac{\text{maximum factored axial sustained (dead) load}}{\text{maximum factored axial total load}} \leq 1.0 \text{ (ACI 10.10.6.2)}$$

The magnification factor, δ_{ns} , must be a positive number and greater than one. Therefore, P_u must be less than $0.75P_c$. If P_u is found to be greater than or equal to $0.75P_c$, a failure condition is declared.

The preceding calculations are performed for major and minor directions separately. That means that δ_n , δ_{ns} , C_m , k , l_u , EI , and P_c assume different values for major and minor directions of bending.

If the program assumptions are not satisfactory for a particular member, the user can explicitly specify values of δ_n and δ_{ns} .

3.4.2.3 Determine Capacity Ratio

As a measure of the stress condition of the column, a capacity ratio is calculated. The capacity ratio is basically a factor that gives an indication of the stress condition of the column with respect to the capacity of the column.

Before entering the interaction diagram to check the column capacity, the moment magnification factors are applied to the factored loads to obtain P_u , M_{u2} , and M_{u3} . The point (P_u, M_{u2}, M_{u3}) is then placed in the interaction space shown as point L in Figure 3-4. If the point lies within the interaction volume, the column capacity is adequate. However, if the point lies outside the interaction volume, the column is overstressed.

This capacity ratio is achieved by plotting the point L and determining the location of point C . Point C is defined as the point where the line OL (if extended outwards) will intersect the failure surface. This point is determined by three-dimensional linear interpolation between the points that define the failure surface, as shown in Figure 3-4. The capacity ratio, CR , is given by the ratio OL/OC .

- If $OL = OC$ (or $CR = 1$), the point lies on the interaction surface and the column is stressed to capacity.
- If $OL < OC$ (or $CR < 1$), the point lies within the interaction volume and the column capacity is adequate.

- If $OL > OC$ (or $CR > 1$), the point lies outside the interaction volume and the column is overstressed.

The maximum of all values of CR calculated from each design load combination is reported for each check station of the column along with the controlling P_u , M_{u2} , and M_{u3} set and associated design load combination name.

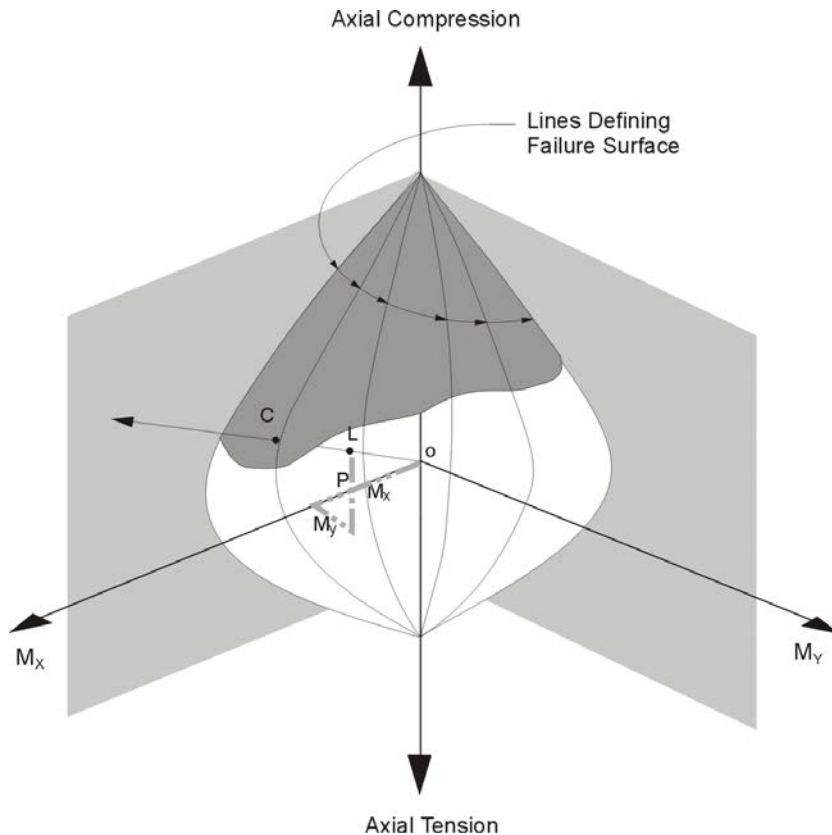


Figure 3-4 Geometric representation of column capacity ratio

3.4.3 Required Reinforcing Area

If the reinforcing area is not defined, the program computes the reinforcement that will give a column capacity ratio equal to the Utilization Factor Limit, which is set to 0.95 by default.

3.4.4 Design Column Shear Reinforcement

The shear reinforcement is designed for each design combination in the major and minor directions of the column. The following steps are involved in designing the shear reinforcing for a particular column for a particular design load combination resulting from shear forces in a particular direction:

- Determine the factored forces acting on the section, P_u and V_u . Note that P_u is needed for the calculation of V_c .
- Determine the shear force, V_c , which can be resisted by concrete alone.
- Calculate the reinforcement steel required to carry the balance.

For Intermediate Moment Frames (Ductile frames), the shear design of the columns is based on the smaller of the following two conditions:

- a) The shear associated with the development of nominal moment strengths of the columns at each restrained end of the unsupported length (ACI 21.3.3.2(a)),
- b) The maximum shear obtained from design load combinations that include earthquake load (E), with E increased by Ω_o (IBC 1901.2, ACI 21.3.3.2(b)).

For Special Moment Frames (Ductile frames), the shear design of the columns is based on the maximum probable strength at the end of each member or the maximum shear obtained from design load combinations that include earthquake load (E) (IBC 1901.2, ACI 21.6.5.1).

Columns of Ordinary Moment Frames that have a clear-height-to-plan-dimension ratio of 5 or less and that are assigned a Seismic Design Category B or higher are designed for capacity shear force in accordance with ACI 21.3.3.2 (Intermediate moment frame) in addition to the factored shear force (IBC 1901.2, ACI 11.9.2, 21.2.3). Effects of the axial forces on the column moment capacities are included in the formulation for all three cases stated here.

The following three sections describe in detail the algorithms associated with this process.

3.4.4.1 Determine Section Forces

- In the design of the column shear reinforcement of an Ordinary Moment Resisting concrete frame, the forces for a particular design load combination, namely, the column axial force, P_u , and the column shear force, V_u , in a particular direction are obtained by factoring the load cases with the corresponding design load combination factors.
- For Ordinary Moment Frames that are assigned a Seismic Design Category B or higher and columns for which the clear-span-to-maximum-plan-dimension ratio is 5 or less, the shear design of those columns is similar to that of an Intermediate Moment Resisting Frame (IBC 1901.2, ACI 1.1.9.2, 21.2.3, 21.3.3.2)
- In the shear design of Special Moment Frames (i.e., seismic design), the shear capacity of the column is checked for capacity shear in addition to the requirement for the Ordinary Moment Resisting Frames. The capacity shear force in the column, V_u , is determined from consideration of the maximum forces that can be generated at the column. Two different capacity shears are calculated for each direction (major and minor). The first is based on the maximum probable moment strength of the column, while the second is computed from the maximum probable moment strengths of the beams framing into the column. The design strength is taken as the minimum of these two values, but never less than the factored shear obtained from the design load combination.

$$V_u = \min \{ V_e^c, V_e^b \} \geq V_{u,\text{factored}} \quad (\text{ACI 21.6.5.1, IBC 1901.2})$$

where

V_e^c = Capacity shear force of the column based on the maximum probable maximum flexural strengths of the two ends of the column.

V_e^b = Capacity shear force of the column based on the maximum probable moment strengths of the beams framing into the column.

In calculating the capacity shear of the column, V_e^c , the maximum probable flexural strength at the two ends of the column is calculated for the existing factored axial load. Clockwise rotation of the joint at one end and the associ-

ated counter-clockwise rotation of the other joint produces one shear force. The reverse situation produces another capacity shear force, and both of these situations are checked, with the maximum of these two values taken as the V_e^c .

For each design load combination, the factored axial load, P_u , is calculated. Then, the maximum probable positive and negative moment strengths, M_{pr}^+ and M_{pr}^- , of the column in a particular direction under the influence of the axial force P_u is calculated using the uniaxial interaction diagram in the corresponding direction. Then the capacity shear force is obtained by applying the calculated maximum probable ultimate moment strengths at the two ends of the column acting in two opposite directions. Therefore, V_e^c is the maximum of V_{e1}^c and V_{e2}^c ,

$$V_e^c = \max \{V_{e1}^c, V_{e2}^c\} \quad (\text{ACI 21.6.5.1, R21.6.5.1, Fig. R21.5.4})$$

where,

$$V_{e1}^c = \frac{M_I^- + M_J^+}{L}, \quad (\text{ACI 21.6.5.1, Fig. R21.5.4})$$

$$V_{e2}^c = \frac{M_I^+ + M_J^-}{L}, \quad (\text{ACI 21.6.5.1, Fig. R21.5.4})$$

M_I^+, M_I^- = Positive and negative probable maximum moment strengths (M_{pr}^+, M_{pr}^-) at end I of the column using a steel yield stress value of αf_y and no reduction factor ($\phi = 1.0$),

M_J^+, M_J^- = Positive and negative probable maximum moment capacities (M_{pr}^+, M_{pr}^-) at end J of the column using a steel yield stress value of αf_y and no reduction factor ($\phi = 1.0$), and

L = Clear span of the column.

The maximum probable moment strengths are determined using a strength reduction factor of 1.0 and the reinforcing steel stress equal to αf_y , where α is set equal to 1.25 (ACI 2.1, 21.6.5.1, Fig. R21.5.4, R21.6.5.1). If the column

section was identified as a section to be checked, the user-specified reinforcing is used for the interaction curve. If the column section was identified as a section to be designed, the reinforcing area envelope is calculated after completing the flexural (P - M - M) design of the column. This envelope of reinforcing area is used for the interaction curve.

If the column section is a variable (non-prismatic) section, the cross-sections at the two ends are used, along with the user-specified reinforcing or the envelope of reinforcing for check or design sections, as appropriate. If the user overwrites the length factor, the full span length is used. However, if the length factor is not overwritten by the user, the clear span length will be used. In the latter case, the maximum of the negative and positive moment capacities will be used for both the positive and negative moment capacities in determining the capacity shear.

In calculating the capacity shear of the column based on the flexural strength of the beams framing into it, V_e^b , the program calculates the maximum probable positive and negative moment strengths of each beam framing into the top joint of the column. Then the sum of the beam moments is calculated as a resistance to joint rotation. Both clockwise and counter-clockwise rotations are considered separately, as well as the rotation of the joint in both the major and minor axis directions of the column. The shear force in the column is determined assuming that the point of inflection occurs at mid-span of the columns above and below the joint. The effects of load reversals are investigated and the design is based on the maximum of the joint shears obtained from the two cases.

$$V_e^b = \max \{V_{e1}^b, V_{e2}^b\} \quad (\text{ACI 21.6.5.1})$$

where,

V_{e1}^b = Column capacity shear based on the maximum probable flexural strengths of the beams for clockwise joint rotation,

V_{e2}^b = Column capacity shear based on the maximum probable flexural strengths of the beams for counter-clockwise joint rotation,

$$V_{e1}^b = \frac{M_{r1}}{H},$$

$$V_{e2}^b = \frac{M_{r2}}{H},$$

M_{r1} = Sum of beam moment resistances with clockwise joint rotations,

M_{r2} = Sum of beam moment resistances with counter-clockwise joint rotations, and

H = Distance between the inflection points, which is equal to the mean height of the columns above and below the joint. If there is no column at the top of the joint, the distance is taken as one-half of the height of the column at the bottom of the joint.

For the case shown in Figure 3-5, V_{e1} can be calculated as follows:

$$V_{e1}^b = \frac{M_u^L + M_u^R}{H}$$

It should be noted that the points of inflection shown in Figure 3-5 are taken at midway between actual lateral support points for the columns, and H is taken as the mean of the two column heights. If no column is present at the top of the joint, H is taken to be equal to one-half the height of the column below the joint.

The expression V_e^b is applicable for determining both the major and minor direction shear forces. The calculated shear force is used for the design of the column below the joint. When beams are not oriented along the major and minor axes of the column, the appropriate components of the flexural capacities are used. If the beam is oriented at an angle θ with the column major axis, the appropriate component— $M_{pr} \cos\theta$ or $M_{pr} \sin\theta$ —of the beam flexural strength is used in calculating M_{r1} and M_{r2} . Also the positive and negative moment capacities are used appropriately based on the orientation of the beam with respect to the column local axis.

- For Intermediate Moment Frames, the shear capacity of the column also is checked for the capacity shear based on the nominal moment capacities at the

ends and the factored gravity loads, in addition to the check required for Ordinary Moment Resisting Frames. The design shear force is taken to be the minimum of that based on the nominal ($\phi = 1.0$) moment capacity and modified factored shear force.

$$V_u = \min \{ V_e, V_{ef} \} \geq V_{u, \text{factored}} \quad (\text{ACI 21.3.3.2, R21.3, IBC 1901.2})$$

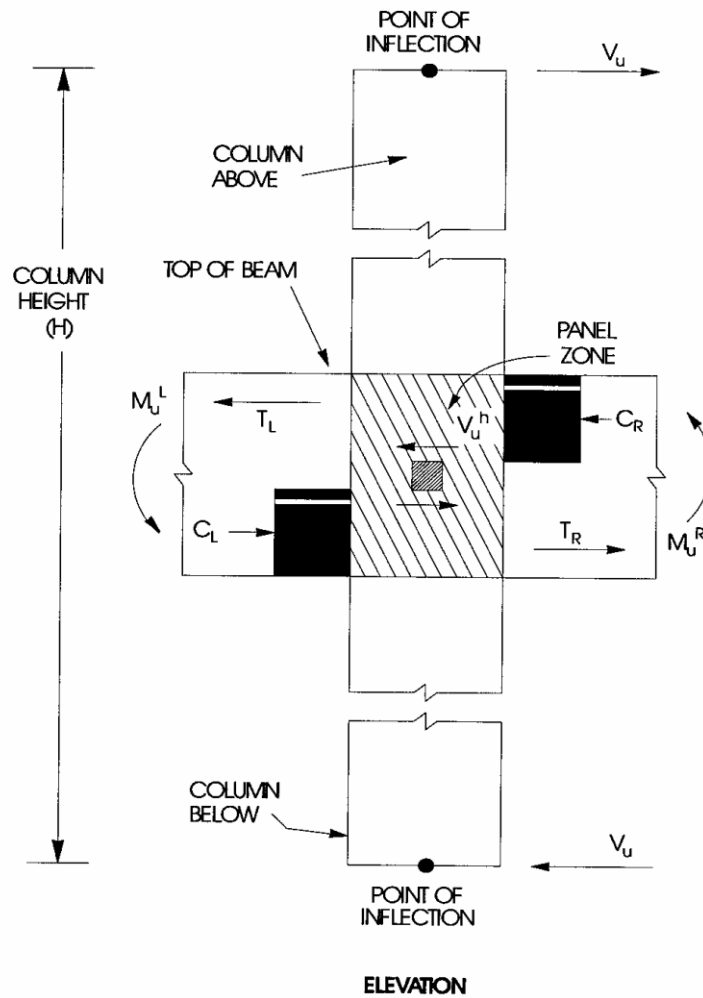


Figure 3-5 Column shear force V_u

where, V_e is the capacity shear force in the column determined from the nominal moment capacities of the column and the beams framing into it.

$$V_e = \min \left\{ V_e^c, V_e^b \right\} \quad (\text{ACI 21.3.3a, Fig. R21.3.3})$$

where, V_e^c is the capacity shear force of the column based on the nominal flexural strength of the column ends alone. V_e^b is the capacity shear force of the column based on the nominal flexural strengths of the beams framing into it. The calculation of V_e^c and V_e^b is the same as that described for Special Moment Resisting Frames, except that in determining the flexural strengths of the column and the beams, the nominal capacities are used. In that case, ϕ is taken as 1.0 as before, but α is taken as 1.0 rather than 1.25 (ACI 21.3.3, Fig. R21.3.3).

V_{ef} is the shear force in the column obtained from the modified design load combinations. In that case, the factored design forces (P_u , V_u , M_u) are based on the specified design load factors, except that the earthquake load factors are increased by Ω_o (ACI 21.3.3b). When designing for this modified shear force, the modified P_u and M_u are used for calculating concrete shear strength. However, the modified P_u and M_u are not used for the P - M - M interaction.

In designing for V_e , the factored P_u and M_u are used for calculating concrete shear strength. In no case is the column designed for a shear force less than the original factored shear force.

- For Ordinary Moment Frames that are assigned a Seismic Design Category B or higher and columns for which the clear-height-to-maximum-plan-dimension ratio is 5 or less, the shear capacity for those columns is checked based on the nominal moment capacities at the ends and the factored gravity loads, in addition to the check required for other Ordinary Moment Resisting Frames (IBC 1901.2, ACI 1.1.9.2, 21.2.3, 21.3.3). This special case is similar to the Intermediate Moment Frames. The design shear force is taken to be the minimum of that based on the nominal ($\phi = 1.0$) moment capacity and modified factored shear force.

$$V_u = \min \left\{ V_e, V_{ef} \right\} \geq V_{u,\text{factored}} \quad (\text{ACI 21.2.3, 21.3.3, R21.3})$$

V_e , V_{ef} , and $V_{u,\text{factored}}$ are calculated exactly in the same way as they are calculated for a column in an Intermediate Moment Resisting Frame.

3.4.4.2 Determine Concrete Shear Capacity

Given the design force set P_u and V_u , the shear force carried by the concrete, V_c , is calculated as follows:

- If the column is subjected to axial compression, i.e., P_u is positive,

$$V_c = 2\lambda\sqrt{f'_c} \left(1 + \frac{P_u}{2,000A_g} \right) A_{cv}, \text{ where} \quad (\text{ACI 11.2.1.2, Eqn. 11-4})$$

$$V_c \leq 3.5\lambda\sqrt{f'_c} \sqrt{\left(1 + \frac{P_u}{500A_g} \right)} A_{cv}. \quad (\text{ACI 11.2.2.2, Eqn. 11-7})$$

The term $\frac{P_u}{A_g}$ must have *psi* units. A_{cv} is the effective shear area, which is shown shaded in Figure 3-6. For circular columns, A_{cv} is taken to be equal to the gross area of the section (ACI 11.2.3, R11.2.3).

If the column is subjected to axial tension, P_u is negative

$$V_c = 2\lambda\sqrt{f'_c} \left(1 + \frac{P_u}{500A_g} \right) A_{cv} \geq 0 \quad (\text{ACI 11.2.2.3, Eqn. 11-8})$$

- For Special Moment Frame design, if the factored axial compressive force, P_u , including the earthquake effect, is small ($P_u < f'_c A_g / 20$), if the shear force contribution from earthquake, V_E , is more than half of the total factored maximum shear force V_u ($V_E \geq 0.5V_u$) over the length of the member, and if the station is within a distance l_o from the face of the joint, then the concrete capacity V_c is taken as zero (ACI 21.6.5.2). Note that for capacity shear design, V_e is considered to be contributed solely by earthquakes, so the second condition is automatically satisfied. The length l_o is taken as the section width, one-sixth the clear span of the column, or 18 *in*, whichever is larger (ACI 21.6.4.1; 21.3.5.2).

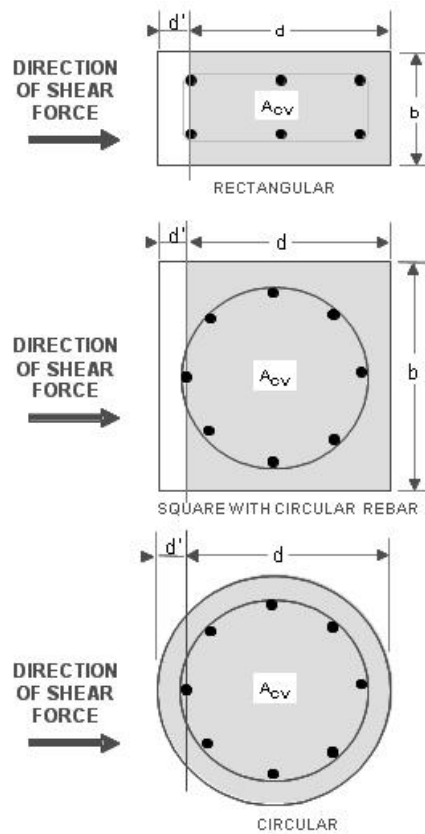


Figure 3-6 Shear stress area, A_{cv}

3.4.4.3 Determine Required Shear Reinforcement

Given V_u and V_c , the required shear reinforcement in the form of stirrups or ties within a spacing, s , is given for rectangular and circular columns by the following:

- The shear force is limited to a maximum of

$$V_{\max} = V_c + (8\sqrt{f'_c})A_{cv} \quad (\text{ACI 11.4.7.9})$$

- The required shear reinforcement per unit spacing, A_v/s , is calculated as follows:

If $V_u \leq (V_c/2)\phi$,

$$\frac{A_v}{s} = 0, \quad (\text{ACI 11.4.6.1})$$

else if $(V_c/2)\phi < V_u \leq \phi V_{\max}$,

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d}, \quad (\text{ACI 11.4.7.1, 11.4.7.2})$$

$$\frac{A_v}{s} \geq \max \left(\frac{0.75\sqrt{f'_c}}{f_y} b_w, \frac{50}{f_y} b_w \right) \quad (\text{ACI 11.4.6.3})$$

else if $V_u > \phi V_{\max}$,

a failure condition is declared. (ACI 11.4.7.9)

In the preceding expressions, for a rectangular section, b_w is the width of the column, d is the effective depth of the column, and A_{cv} is the effective shear area, which is equal to $b_w d$. For a circular section, b_w is replaced with D , which is the external diameter of the column, and d is replaced with $0.8D$ and A_{cv} is replaced with the gross area $\frac{\pi D^2}{4}$ (ACI 11.4.7.3, 11.2.3, R11.2.3).

In the preceding expressions, the strength reduction factor ϕ is taken by default as 0.75 for non-seismic cases (ACI 9.3.2.3), and as 0.60 for seismic cases (ACI 9.3.4.a). However, those values can be overwritten by the user, if so desired.

If V_u exceeds its maximum permitted value ϕV_{\max} , the concrete section size should be increased (ACI 11.4.7.9).

The maximum of all calculated A_v/s values, obtained from each design load combination, is reported for the major and minor directions of the column, along with the controlling combination name.

The column shear reinforcement requirements reported by the program are based purely on shear strength consideration. Any minimum stirrup require-

ments to satisfy spacing considerations or transverse reinforcement volumetric considerations must be investigated independently of the program by the user.

3.5 Beam Design

In the design of concrete beams, the program calculates and reports the required areas of steel for flexure and shear based on the beam moments, shear forces, torsions, design load combination factors, and other criteria described in the text that follows. The reinforcement requirements are calculated at a user-defined number of check/design stations along the beam span.

All beams are designed for major direction flexure, shear and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

3.5.1 Design Beam Flexural Reinforcement

The beam top and bottom flexural steel is designed at check/design stations along the beam span. The following steps are involved in designing the flexural reinforcement for the major moment for a particular beam for a particular section:

- Determine the maximum factored moments
- Determine the reinforcing steel

3.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of Special, Intermediate, or Ordinary Moment concrete frame beams, the factored moments for each design load combination at a particular beam section are obtained by factoring the

corresponding moments for different load cases with the corresponding design load combination factors.

The beam section is then designed for the factored moments obtained from all of the design load combinations. Positive moments produce bottom steel. In such cases, the beam may be designed as a Rectangular or a T-Beam. Negative moments produce top steel. In such cases, the beam is always designed as a rectangular section.

3.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 3-7 (ACI 10.2). Furthermore, it is assumed that the net tensile strain of the reinforcing steel shall not be less than 0.005 (tension controlled) (ACI 10.3.4). When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by the program for both rectangular and flanged sections (T-Beams) is summarized in the following subsections. It is assumed that the design ultimate axial force does not exceed $\phi(0.1f'_cA_g)$ (ACI 10.3.5); hence, all of the beams are designed ignoring axial force.

3.5.1.2.1 Design for Rectangular Beam

In designing for a factored negative or positive moment, M_u (i.e., designing top or bottom steel), the depth of the compression block is given by a (see Figure 3-7), where,

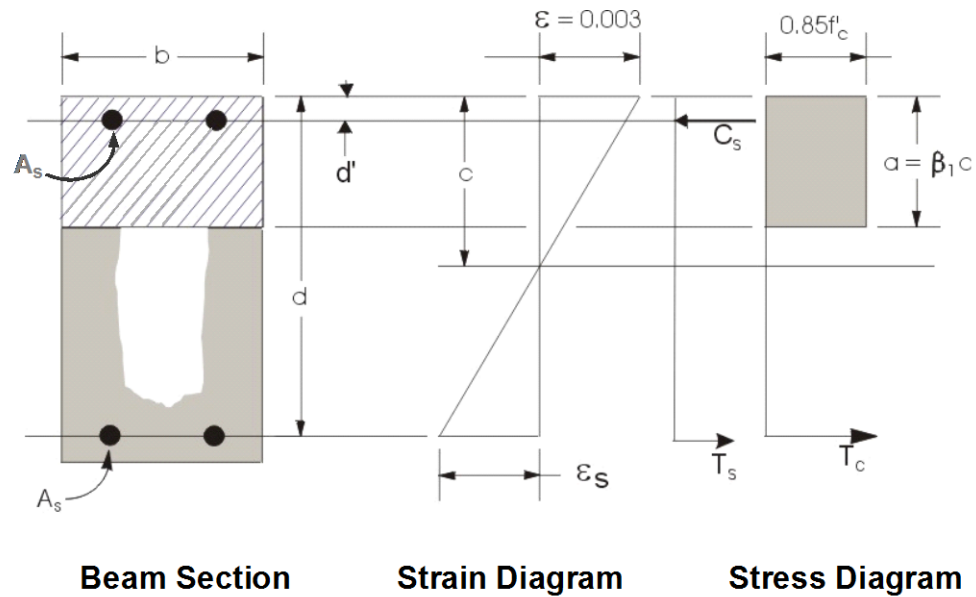


Figure 3-7 Rectangular beam design

$$a = d - \sqrt{d^2 - \frac{2|M_u|}{0.85f'_c\phi b}}, \quad (\text{ACI 10.2})$$

where, the value ϕ is taken as that for a tension controlled section, which is 0.90 by default (ACI 9.3.2.1) in the preceding and the following equations.

The maximum depth of the compression zone, c_{\max} , is calculated based on the limitation that the tensile steel tension shall not be less than $\epsilon_{s,\min}$, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{\max} = \frac{\epsilon_{c,\max}}{\epsilon_{c,\max} + \epsilon_{s,\min}} d \quad \text{where,} \quad (\text{ACI 10.2.2})$$

$$\epsilon_{c,\max} = 0.003 \quad (\text{ACI 10.2.3})$$

$$\epsilon_{s,\min} = 0.005 \quad (\text{ACI 10.3.4})$$

The maximum allowable depth of the rectangular compression block, a_{\max} , is given by

$$a_{\max} = \beta_1 c_{\max} \quad (\text{ACI 10.2.7.1})$$

where β_1 is calculated as follows:

$$\beta_1 = 0.85 - 0.05 \left(\frac{f'_c - 4000}{1000} \right), \quad 0.65 \leq \beta_1 \leq 0.85 \quad (\text{ACI 10.2.7.3})$$

- If $a \leq a_{\max}$ (ACI 10.3.4, 10.3.5), the area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2} \right)}$$

This steel is to be placed at the bottom if M_u is positive, or at the top if M_u is negative.

- If $a > a_{\max}$, compression reinforcement is required (ACI 10.3.4, 10.3.5) and is calculated as follows:

The compressive force developed in concrete alone is given by:

$$C = 0.85 f'_c b a_{\max}, \quad (\text{ACI 10.2.7.1})$$

the moment resisted by concrete compression and tensile steel is:

$$M_{uc} = C \left(d - \frac{a_{\max}}{2} \right) \phi.$$

Therefore, the moment resisted by compression steel and tensile steel is:

$$M_{us} = M_u - M_{uc}.$$

So the required compression steel is given by:

$$A'_s = \frac{M_{us}}{(f'_s - 0.85 f'_c)(d - d') \phi}, \quad \text{where}$$

$$f'_s = E_s \varepsilon_{c \max} \left[\frac{c_{\max} - d'}{c_{\max}} \right] \leq f_y. \quad (\text{ACI 10.2.2, 10.2.3, 10.2.4})$$

The required tensile steel for balancing the compression in concrete is

$$A_{s1} = \frac{M_{uc}}{f_y \left[d - \frac{a_{\max}}{2} \right] \phi}, \text{ and}$$

the tensile steel for balancing the compression in steel is given by

$$A_{s2} = \frac{M_{us}}{f_y (d - d') \phi}.$$

Therefore, the total tensile reinforcement is $A_s = A_{s1} + A_{s2}$, and the total compression reinforcement is A'_s . A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and A'_s is to be placed at the bottom and A_s is to be placed at the top if M_u is negative.

3.5.1.2.2 Design for T-Beam

In designing a T-beam, a simplified stress block, as shown in Figure 3-8, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ignored. In that case, a simplified stress block similar to that shown in Figure 3-8 is assumed in the compression side (ACI 10.2).

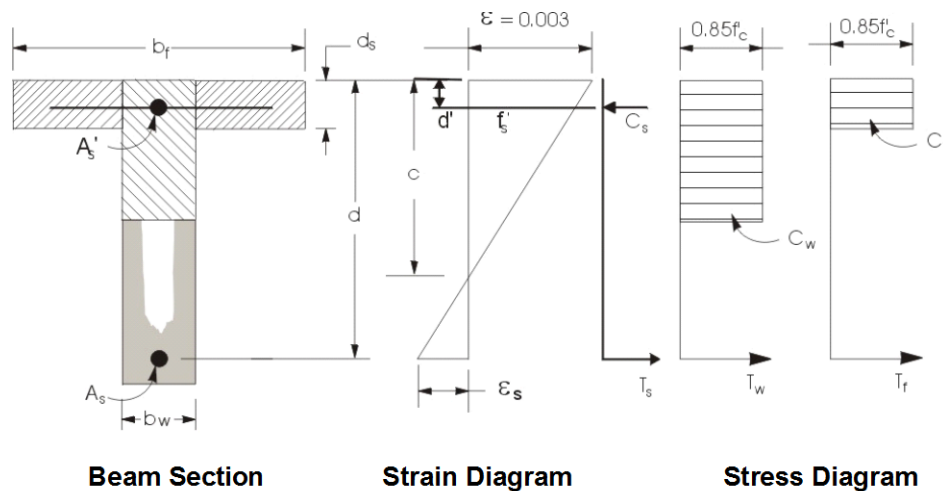


Figure 3-8 T-beam design

Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_u (i.e., designing top steel), the calculation of the steel area is exactly the same as described for a rectangular beam, i.e., no T-Beam data is used.

Flanged Beam Under Positive Moment

If $M_u > 0$, the depth of the compression block is given by

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.85f'_c\phi b_f}}$$

where, the value of ϕ is taken as that for a tension controlled section, which is 0.90 by default (ACI 9.3.2.1) in the preceding and the following equations.

The maximum depth of the compression zone, c_{\max} , is calculated based on the limitation that the tensile steel tension shall not be less than $\epsilon_{s,\min}$, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{\max} = \frac{\epsilon_{c,\max}}{\epsilon_{c,\max} + \epsilon_{s,\min}} d \quad \text{where,} \quad (\text{ACI 10.2.2})$$

$$\epsilon_{c,\max} = 0.003 \quad (\text{ACI 10.2.3})$$

$$\epsilon_{s,\min} = 0.005 \quad (\text{ACI 10.3.4})$$

The maximum allowable depth of the rectangular compression block, a_{\max} , is given by

$$a_{\max} = \beta_1 c_{\max} \quad (\text{ACI 10.2.7.1})$$

where β_1 is calculated as follows:

$$\beta_1 = 0.85 - 0.05 \left(\frac{f'_c - 4000}{1000} \right), \quad 0.65 \leq \beta_1 \leq 0.85 \quad (\text{ACI 10.2.7.3})$$

- If $a \leq d_s$, the subsequent calculations for A_s are exactly the same as previously defined for the Rectangular section design. However, in that case, the width of the beam is taken as b_f , as shown in Figure 3-8. Compression reinforcement is required if $a > a_{\max}$.

- If $a > d_s$, the calculation for A_s has two parts. The first part is for balancing the compressive force from the flange, C_f , and the second part is for balancing the compressive force from the web, C_w , as shown in Figure 3-8. C_f is given by:

$$C_f = 0.85 f'_c (b_f - b_w) * \min(d_s, a_{\max}) \quad (\text{ACI 10.2.7.1})$$

Therefore, $A_{s1} = \frac{C_f}{f_y}$ and the portion of M_u that is resisted by the flange is given by:

$$M_{uf} = C_f \left(d - \frac{\min(d_s, a_{\max})}{2} \right) \phi.$$

Again, the value for ϕ is 0.90 by default. Therefore, the balance of the moment, M_u , to be carried by the web is given by:

$$M_{uw} = M_u - M_{uf}.$$

The web is a rectangular section of dimensions b_w and d , for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f'_c \phi b_w}}. \quad (\text{ACI 10.2})$$

- If $a_1 \leq a_{\max}$ (ACI 10.3.5), the area of tensile steel reinforcement is then given by:

$$A_{s2} = \frac{M_{uw}}{\phi f_y \left(d - \frac{a_1}{2} \right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2}$$

This steel is to be placed at the bottom of the T-beam.

- If $a_1 > a_{\max}$, compression reinforcement is required (ACI 10.3.4, 10.3.5) and is calculated as follows:

The compression force in the web concrete alone is given by:

$$C = 0.85 f'_c b_w a_{\max} \quad (\text{ACI 10.2.7.1})$$

Therefore the moment resisted by the concrete web and tensile steel is:

$$M_{uc} = C \left(d - \frac{a_{\max}}{2} \right) \phi, \text{ and}$$

the moment resisted by compression steel and tensile steel is:

$$M_{us} = M_{uw} - M_{uc}.$$

Therefore, the compression steel is computed as:

$$A'_s = \frac{M_{us}}{(f'_s - 0.85 f'_c) (d - d') \phi}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c \max} \left[\frac{c_{\max} - d'}{c_{\max}} \right] \leq f_y. \quad (\text{ACI 10.2.2, 10.2.3, 10.2.4})$$

The tensile steel for balancing compression in the web concrete is:

$$A_{s2} = \frac{M_{uc}}{f_y \left[d - \frac{a_{\max}}{2} \right] \phi}, \text{ and}$$

the tensile steel for balancing the compression steel is:

$$A_{s3} = \frac{M_{us}}{f_y (d - d') \phi}.$$

The total tensile reinforcement is $A_s = A_{s1} + A_{s2} + A_{s3}$, and the total compression reinforcement is A'_s . A_s is to be placed at the bottom, and A'_s is to be placed at the top.

3.5.1.2.3 Minimum and Maximum Tensile Reinforcement

The minimum flexural tensile steel required in a beam section is given by the minimum of the following two limits:

$$A_s \geq \max \left\{ \frac{3\sqrt{f'_c}}{f_y} b_w d, \frac{200}{f_y} b_w d \right\} \quad (\text{ACI 10.5.1})$$

$$A_s \geq \frac{4}{3} A_{s(\text{required})} \quad (\text{ACI 10.5.3})$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed as follows:

$$A_s \leq \begin{cases} 0.04bd & \text{Rectangular Beam} \\ 0.04b_w d & \text{T-Beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.04bd & \text{Rectangular Beam} \\ 0.04b_w d & \text{T-Beam} \end{cases}$$

- The minimum longitudinal reinforcement shall be provided at both the top and bottom. Any of the top and bottom reinforcement shall not be less than $A_{s(\text{min})}$ (ACI 21.5.2.1).

$$A_{s(\text{min})} \geq \max \left\{ \frac{3\sqrt{f'_c}}{f_y} b_w d, \frac{200}{f_y} b_w d \right\} \quad \text{or} \quad (\text{ACI 21.5.2.1, 10.5.1})$$

$$A_{s(\text{min})} \geq \frac{4}{3} A_{s(\text{required})}. \quad (\text{ACI 21.5.2.1, 10.5.3})$$

- The beam flexural steel is limited to a maximum given by

$$A_s \leq 0.025 b_w d. \quad (\text{ACI 21.5.2.1})$$

- At any end (support) of the beam, the beam positive moment capacity (i.e., associated with the bottom steel) would not be less than 1/2 of the beam negative moment capacity (i.e., associated with the top steel) at that end (ACI 21.5.2.2).
- Neither the negative moment capacity nor the positive moment capacity at any of the sections within the beam would be less than 1/4 of the maximum of positive or negative moment capacities of any of the beam end (support) stations (ACI 21.5.2.2).

For Intermediate Moment Frames (i.e., seismic design), the beam design would satisfy the following conditions:

- At any support of the beam, the beam positive moment capacity would not be less than 1/3 of the beam negative moment capacity at that end (ACI 21.3.4.1).
- Neither the negative moment capacity nor the positive moment capacity at any of the sections within the beam would be less than 1/5 of the maximum of positive or negative moment capacities of any of the beam end (support) stations (ACI 21.3.4.1).

3.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each design load combination at a user-defined number of stations along the beam span. The following steps are involved in designing the shear reinforcement for a particular station because of beam major shear:

- Determine the factored shear force, V_u .
- Determine the shear force, V_c , that can be resisted by the concrete.
- Determine the reinforcement steel required to carry the balance.

For Special and Intermediate Moment frames (ductile frames), the shear design of the beams is also based on the maximum probable moment strengths and the nominal moment strengths of the members, respectively, in addition to the factored design. Effects of axial forces on the beam shear design are neglected.

The following three sections describe in detail the algorithms associated with this process.

3.5.2.1 Determine Shear Force and Moment

- In the design of the beam shear reinforcement of an Ordinary Moment Frame, the shear forces and moments for a particular design load combination at a particular beam section are obtained by factoring the associated shear forces and moments with the corresponding design load combination factors.

- In the design of Special Moment Frames (i.e., seismic design), the shear capacity of the beam is also checked for the capacity shear resulting from the maximum probable moment capacities at the ends along with the factored gravity load. This check is performed in addition to the design check required for Ordinary Moment Frames. The capacity shear force, V_p , is calculated from the maximum probable moment capacities of each end of the beam and the gravity shear forces. The procedure for calculating the design shear force in a beam from the maximum probable moment capacity is the same as that described for a column earlier in this chapter. See Table 3-1 for a summary.

The design shear force is then given by (ACI 21.5.4.1, IBC 1905.1.2):

$$V_u = \max \{ V_{e1}, V_{e2} \} \quad (\text{ACI 21.5.4.1, Fig R21.5.4})$$

$$V_{e1} = V_{p1} + V_{D+L} \quad (\text{ACI 21.5.4.1, Fig R21.5.4})$$

$$V_{e2} = V_{p2} + V_{D+L} \quad (\text{ACI 21.5.4.1, Fig R21.5.4})$$

where V_p is the capacity shear force obtained by applying the calculated maximum probable ultimate moment capacities at the two ends of the beams acting in two opposite directions. Therefore, V_p is the maximum of V_{p1} and V_{p2} , where

$$V_{p1} = \frac{M_I^- + M_J^+}{L}, \text{ and}$$

$$V_{p2} = \frac{M_I^+ + M_J^-}{L}, \text{ where}$$

M_I^- = Moment capacity at end I, with top steel in tension, using a steel yield stress value of αf_y and no reduction factors ($\phi = 1.0$).

M_J^+ = Moment capacity at end J, with bottom steel in tension, using a steel yield stress value of αf_y and no reduction factors ($\phi = 1.0$).

M_I^+ = Moment capacity at end I, with bottom steel in tension, using a steel yield stress value of αf_y and no reduction factors ($\phi = 1.0$).

M_J^- = Moment capacity at end J, with top steel in tension, using a steel yield stress value of αf_y and no reduction factors ($\phi = 1.0$).

L = Clear span of beam.

The moment strengths are determined using a strength reduction factor of 1.0 and the reinforcing steel stress equal to αf_y , where α is equal to 1.25 (ACI 2.1, R21.5.4.1). If the reinforcement area has not been overwritten for ductile beams, the value of the reinforcing area envelope is calculated after completing the flexural design of the beam for all the design load combinations. Then this enveloping reinforcing area is used in calculating the moment capacity of the beam. If the reinforcing area has been overwritten for ductile beams, this area is used in calculating the moment capacity of the beam. If the beam section is a variable cross-section, the cross-sections at the two ends are used along with the user-specified reinforcing or the envelope of reinforcing, as appropriate. If the user overwrites the major direction length factor, the full span length is used. However, if the length factor is not overwritten, the clear length will be used. In the latter case, the maximum of the negative and positive moment capacities will be used for both the negative and positive moment capacities in determining the capacity shear.

V_{D+L} is the contribution of shear force from the in-span distribution of gravity loads with the assumption that the ends are simply supported.

- For Intermediate Moment Frames, the shear capacity of the beam also is checked for the capacity shear based on the nominal moment capacities at the ends along with the factored gravity loads, in addition to the check required for Ordinary moment resisting frames. The design shear force in beams is taken to be the minimum of that based on the nominal moment capacity and modified factored shear force.

$$V_u = \min \{ V_e, V_{ef} \} \geq V_{u, \text{factored}} \quad (\text{ACI 21.3.3, R21.3, IBC 1905.1.2})$$

where, V_e is the capacity shear force in the beam determined from the nominal moment capacities of the beam (ACI 21.3.3a). The calculation of V_e is the same as that described for Special Moment Frames, except that in determining the flexural strength of the beam, nominal moment capacities are used. In that case, ϕ is taken as 1.0 as before, but α is taken as 1.0 rather than 1.25 (ACI 21.3.3.1(a), Fig. R 21.3.3).

V_{ef} is the shear force in the beam obtained from the modified design load combinations. In that case, the factored design forces (P_u , V_u , M_u) are based on the specified design loads, except that the earthquake factors are doubled (ACI 21.3.3b). In no case is the beam designed for a shear force less than the original factored shear force.

The computation of the design shear force in a beam of an Intermediate Moment Frame is the same as described for columns earlier in this chapter. See Table 3-1 for a summary.

3.5.2.2 Determine Concrete Shear Capacity

The allowable concrete shear capacity is given by

$$V_c = 2\lambda\sqrt{f'_c}b_wd, \text{ where} \quad (\text{ACI 11.2.1.1, Eqn. 11-3})$$

for Special Moment Frame design, if the factored axial compressive force, P_u , including the earthquake effect, is less than $f'_c A_g/20$, if the shear force contribution from earthquake, V_E , is more than half of the total maximum shear force over the length of the member V_u (i.e., $V_E \geq 0.5 V_u$), and if the station is within a distance l_o from the face of the joint, the concrete capacity V_c is taken as zero (ACI 21.5.4.2). The length l_o is taken as $2d$ from the face of the support (ACI 21.5.4.2, 21.5.3.1).

3.5.2.3 Determine Required Shear Reinforcement

Given V_u and V_c the required shear reinforcement in area/unit length is calculated as follows:

- The shear force is limited to a maximum of

$$V_{\max} = V_c + (8\sqrt{f'_c})b_wd. \quad (\text{ACI 11.4.7.9})$$

- The required shear reinforcement per unit spacing, A_v/s , is calculated as follows:

$$\text{If } V_u \leq (V_c/2)\phi,$$

$$\frac{A_v}{s} = 0, \quad (\text{ACI 11.4.6.1})$$

else if $(V_c/2)\phi < V_u \leq \phi V_{\max}$,

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d}, \quad (\text{ACI 11.4.7.1, 11.4.7.2})$$

$$\frac{A_v}{s} \geq \max \left(\frac{0.75\sqrt{f'_c}}{f_y} b_w, \frac{50}{f_y} b_w \right) \quad (\text{ACI 11.4.6.3})$$

else if $V_u > \phi V_{\max}$,

a failure condition is declared. (ACI 11.4.7.9)

In the preceding equations, the strength reduction factor ϕ is taken as 0.75 for non-seismic cases (ACI 9.3.2.3), and as 0.6 for seismic cases (ACI 9.3.4.a). However, those values may be overwritten by the user if so desired.

If V_u exceeds the maximum permitted value of ϕV_{\max} , the concrete section should be increased in size (ACI 11.4.7.9).

Note that if torsion design is performed and torsion rebar is needed, the equation given in ACI 11.4.6.3 does not need to be satisfied independently. See the next section *Design Beam Torsion Reinforcement* for details.

The maximum of all of the calculated A_v/s values, obtained from each design load combination, is reported along with the controlling shear force and associated design load combination name.

The beam shear reinforcement requirements reported by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric consideration must be investigated independently of the program by the user.

3.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at a user-defined number of stations along the beam span. The following steps are

involved in designing the shear reinforcement for a particular station because of beam torsion:

- Determine the factored torsion, T_u .
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the reinforcement steel required.

3.5.3.1 Determine Factored Torsion

In the design of torsion reinforcement of any beam, the factored torsions for each design load combination at a particular design station are obtained by factoring the corresponding torsion for different load cases with the corresponding design load combination factors (ACI 11.5.2).

In a statistically indeterminate structure where redistribution of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, the design T_u is permitted to be reduced in accordance with code (ACI 11.5.2.2). However, the program does not try to redistribute the internal forces and to reduce T_u . If redistribution is desired, the user should *release* the torsional DOF in the structural model.

3.5.3.2 Determine Special Section Properties

For torsion design, special section properties such as A_{cp} , A_{oh} , A_o , p_{cp} , and p_n are calculated. These properties are described as follows (ACI 2.1).

A_{cp} = Area enclosed by outside perimeter of concrete cross-section

A_{oh} = Area enclosed by centerline of the outermost closed transverse torsional reinforcement

A_o = Gross area enclosed by shear flow path

p_{cp} = Outside perimeter of concrete cross-section

p_n = Perimeter of centerline of outermost closed transverse torsional

reinforcement

In calculating the section properties involving reinforcement, such as A_{oh} , A_o , and p_n , it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 1.75 inches. This is equivalent to 1.5 inches clear cover and a #4 stirrup placement. For torsion design of T-Beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a Rectangular beam section are given as follows:

$$A_{cp} = bh, \quad (\text{ACI 11.5.1, 2.1})$$

$$A_{oh} = (b - 2c)(h - 2c), \quad (\text{ACI 11.5.3.1, 2.1, R11.5.3.6(b)})$$

$$A_o = 0.85 A_{oh}, \quad (\text{ACI 11.5.3.6, 2.1})$$

$$p_{cp} = 2b + 2h, \text{ and} \quad (\text{ACI 11.5.1, 2.1})$$

$$p_n = 2(b - 2c) + 2(h - 2c), \quad (\text{ACI 11.5.3.1, 2.1})$$

where, the section dimensions b , h and c are shown in Figure 3-9. Similarly, the special section properties for a T-Beam section are given as follows:

$$A_{cp} = b_w h + (b_f - b_w) d_s, \quad (\text{ACI 11.5.1, 2.1})$$

$$A_{oh} = (b_w - 2c)(h - 2c), \quad (\text{ACI 11.5.3.1, 2.1, R11.5.3.6(b)})$$

$$A_o = 0.85 A_{oh}, \quad (\text{ACI 11.5.3.6, 2.1})$$

$$p_{cp} = 2b_f + 2h, \text{ and} \quad (\text{ACI 11.5.1, 2.1})$$

$$p_n = 2(h - 2c) + 2(b_w - 2c), \quad (\text{ACI 11.5.3.1, 2.1})$$

where the section dimensions b_f , b_w , h , d_s and c for a T-beam are shown in Figure 3-9.

3.5.3.3 Determine Critical Torsion Capacity

The critical torsion limits, T_{cr} , for which the torsion in the section can be ignored, is calculated as follows:

$$T_{cr} = \phi 4\lambda\sqrt{f'_c} \left(\frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{P_u}{4A_{cp}\lambda\sqrt{f'_c}}} \quad (\text{ACI 11.5.2.2.c})$$

where A_{cp} and p_{cp} are the area and perimeter of concrete cross-section as described in detail in the previous section, P_u is the factored axial force (compression positive), ϕ is the strength reduction factor for torsion, which is equal to 0.75 by default (ACI 9.3.2.3), and f'_c is the specified concrete strength.

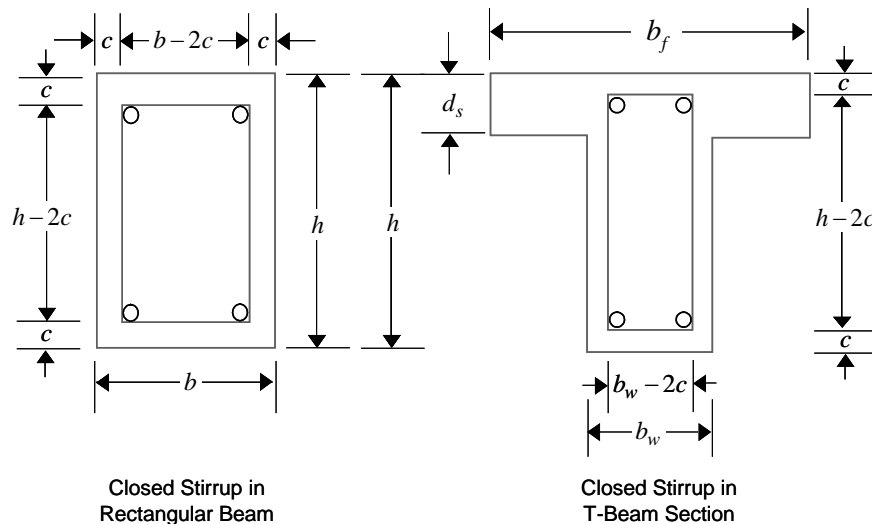


Figure 3-9 Closed stirrup and section dimensions for torsion design

3.5.3.4 Determine Torsion Reinforcement

If the factored torsion T_u is less than the threshold limit, T_{cr} , torsion can be safely ignored (ACI 11.5.1). In that case, the program reports that no torsion is required. However, if T_u exceeds the threshold limit, T_{cr} , it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals (ACI R11.5.3.6).

If $T_u > T_{cr}$, the required longitudinal rebar area is calculated as:

$$A_t = \frac{T_u p_h}{\phi 2 A_0 f_y \tan \theta} \quad (\text{ACI 11.5.3.7, 11.5.3.6})$$

and the required closed stirrup area per unit spacing, A_t/s , is calculated as:

$$\frac{A_t}{s} = \frac{T_u \tan \theta}{\phi 2 A_0 f_{ys}} \quad (\text{ACI 11.5.3.6})$$

where, the minimum value of A_t/s is taken as:

$$\frac{A_t}{s} \geq \frac{25}{f_{ys}} b_w \quad (\text{ACI 11.5.5.3})$$

and the minimum value of A_t is taken as:

$$A_t \geq \frac{5\sqrt{f'_c} A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{ys}}{f_y}\right). \quad (\text{ACI 11.5.5.3})$$

In the preceding expressions, θ is taken as 45 degrees. The code allows any value between 30 and 60 degrees (ACI 11.5.3.6).

An upper limit of the combination of V_u and T_u that can be carried by the section is also checked using the following equation.

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \leq \phi \left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \quad (\text{ACI 11.5.3.1, 11.4.7.9})$$

For rectangular sections, b_w is replaced with b . If the combination of V_u and T_u exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_u > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups satisfy the following limit.

$$\left(\frac{A_v}{s} + 2\frac{A_t}{s}\right) \geq \max \left\{ 0.75 \frac{\sqrt{f'_c}}{f_{ys}} b_w, \frac{50}{f_y} b_w \right\} \quad (\text{ACI 11.5.5.2, 11.4.6.3})$$

If this equation is not satisfied with the originally calculated A_v/s and A_t/s , A_v/s is increased to satisfy this condition. In that case, A_v/s does not need to satisfy ACI Section 11.4.6.3 independently.

The maximum of all the calculated A_t and A_t/s values obtained from each design load combination is reported along with the controlling combination names.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements and longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

3.6 Joint Design

To ensure that the beam-column joint of Special Moment Frames possesses adequate shear strength, the program performs a rational analysis of the beam-column panel zone to determine the shear forces that are generated in the joint. The program then checks this against design shear strength.

Only joints having a column below the joint are checked. The material properties of the joint are assumed to be the same as those of the column below the joint.

The joint analysis is completed in the major and the minor directions of the column. The joint design procedure involves the following steps:

- Determine the panel zone design shear force, V_u^h
- Determine the effective area of the joint
- Check panel zone shear stress

The algorithms associated with these three steps are described in detail in the following three sections.

3.6.1 Determine the Panel Zone Shear Force

Figure 3-10 illustrates the free body stress condition of a typical beam-column intersection for a column direction, major or minor.

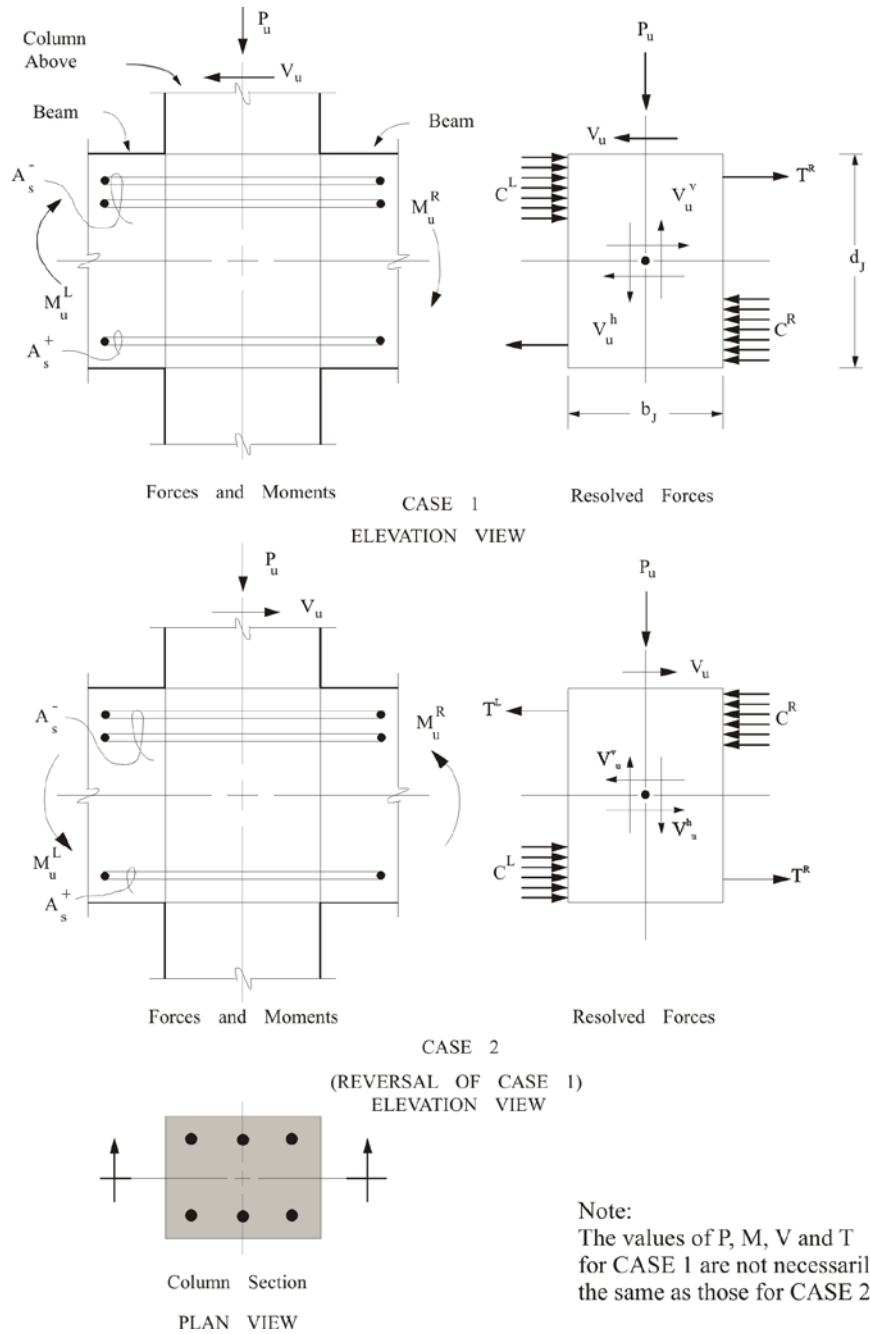


Figure 3-10 Beam-column joint analysis

The force V_u^h is the horizontal panel zone shear force that is to be calculated. The forces that act on the joint are P_u , V_u , M_u^L , and M_u^R . The forces P_u and V_u are axial force and shear force, respectively, from the column framing into the top of the joint. The moments M_u^L and M_u^R are obtained from the beams framing into the joint. The program calculates the joint shear force V_u^h by resolving the moments into C and T forces. Noting that $T_L = C_L$ and $T_R = C_R$,

$$V_u^h = T_L + T_R - V_u$$

The location of C or T forces is determined by the direction of the moment. The magnitude of C or T forces is conservatively determined using basic principles of ultimate strength theory (ACI 10.2).

The moments and the forces from beams that frame into the joint in a direction that is not parallel to the major or minor direction of the column are resolved along the direction that is being investigated, thereby contributing force components to the analysis.

In the design of Special Moment Resisting concrete frames, the evaluation of the design shear force is based on the moment capacities (with reinforcing steel overstrength factor, α , where, $\alpha = 1.25$ and no ϕ factors) of the beams framing into the joint (ACI 21.7.2.1). The C and T force are based on these moment capacities. The program calculates the column shear force V_u from the beam moment capacities, as follows (see Figure 3-5):

$$V_u = \frac{M_u^L + M_u^R}{H}$$

It should be noted that the points of inflection shown on Figure 3-5 are taken as midway between actual lateral support points for the columns. If no column exists at the top of the joint, the shear force from the top of the column is taken as zero.

The effects of load reversals, as illustrated in Case 1 and Case 2 of Figure 3-10, are investigated, and the design is based on the maximum of the joint shears obtained from the two cases.

3.6.2 Determine the Effective Area of Joint

The joint area that resists the shear forces is assumed always to be rectangular in plan view. The dimensions of the rectangle correspond to the major and minor dimensions of the column below the joint, except if the beam framing into the joint is very narrow. The effective width of the joint area to be used in the calculation is limited to the width of the beam plus the depth of the column. The area of the joint is assumed not to exceed the area of the column below. The joint area for joint shear along the major and minor directions is calculated separately (ACI R21.7.4).

It should be noted that if the beam frames into the joint eccentrically, the preceding assumptions may not be conservative and the user should investigate the acceptability of the particular joint.

3.6.3 Check Panel Zone Shear Stress

The panel zone shear stress is evaluated by dividing the shear force by the effective area of the joint and comparing it with the following design shear strengths (ACI 21.5.3).

$$v = \begin{cases} 20\phi\sqrt{f'_c} & \text{for joints confined on all four sides,} \\ 15\phi\sqrt{f'_c} & \text{for joints confined on three faces or} \\ & \text{on two opposite faces,} \\ 12\phi\sqrt{f'_c} & \text{for all other joints,} \end{cases} \quad (\text{ACI 21.7.4.1, 9.3.4(c)})$$

where $\phi = 0.85$ (by default).

A beam that frames into a face of a column at the joint is considered in this program to provide confinement to the joint if at least three-quarters of the face of the joint is covered by the framing member (ACI 21.7.3.2, Fig. R21.7.4).

For light-weight aggregate concrete, the design shear strength of the joint is reduced in the program to at least three-quarters of that of the normal weight concrete by replacing the $\sqrt{f'_c}$ with

$$\min \left\{ f_{cs, \text{factor}} \sqrt{f'_c}, \frac{3}{4} \sqrt{f'_c} \right\} \quad (\text{ACI 21.7.4.2})$$

where the f_{cs} factor is the shear strength reduction factor as defined by the user. For joint design, the program reports the joint shear, the joint shear stress, the allowable joint shear stress, and a capacity ratio.

3.6.4 Beam-Column Flexural Capacity Ratios

The program calculates the ratio of the sum of the beam moment capacities to the sum of the column moment capacities. For Special Moment Frames, at a particular joint for a particular column direction, major or minor (ACI 21.6.2.2):

$$\sum M_{nc} \geq \frac{6}{5} \sum M_{nb} \quad (\text{ACI 21.6.2.2})$$

$\sum M_{nc}$ = Sum of nominal flexural strengths of columns framing into the joint, evaluated at the faces of the joint. Individual column flexural strength is calculated for the associated factored axial force.

$\sum M_{nb}$ = Sum of nominal flexural strengths of the beams framing into the joint, evaluated at the faces of the joint.

The capacities are calculated with no reinforcing overstrength factor α , $\alpha = 1$, and with no ϕ factors ($\phi = 1.0$). The beam capacities are calculated for reversed situations (Cases 1 and 2) as illustrated in Figure 3-10 and the maximum summation obtained is used.

The moment capacities of beams that frame into the joint in a direction that is not parallel to the major or minor direction of the column are resolved along the direction that is being investigated and the resolved components are added to the summation.

The column capacity summation includes the column above and the column below the joint. For each load combination, the axial force, P_u , in each of the columns is calculated from the program design load combinations. For each design load combination, the moment capacity of each column under the influence of the corresponding axial load is then determined separately for the major and minor directions of the column, using the uniaxial column interaction diagram; see Figure 3-11. The moment capacities of the two columns are added to give the capacity summation for the corresponding design load combination.

The maximum capacity summations obtained from all of the design load combinations is used for the beam-column capacity ratio.

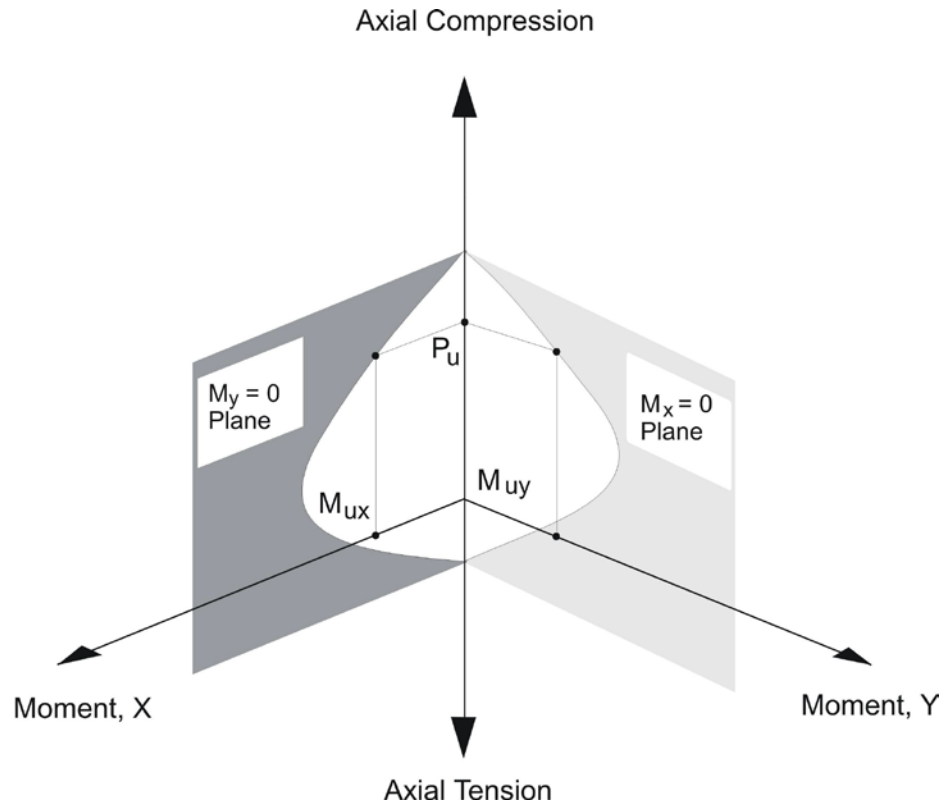


Figure 3-11 Moment capacity M_u at a given axial load P_u

The beam-column capacity ratio is determined for a beam-column joint only when the following conditions are met:

- the frame is a Special Moment Frame
- when a column exists above the beam-column joint, the column is concrete
- all of the beams framing into the column are concrete beams
- the connecting member design results are available

- the load combo involves seismic load

The beam-column flexural capacity ratios ($\sum M_{nb} / \sum M_{nc}$) are reported only for Special Moment Resisting frames involving seismic design load combinations. If this ratio is greater than 5/6, a warning message is printed in the output. The ratio is also reported in the form of $(\frac{6}{5})\sum M_{nb} / M_{nc}$ and $\sum M_{nc} / \sum M_{nb}$.

3.7 Summary of Special Considerations for Seismic Design

The similarities and differences of special consideration for column design/check, beam design, and joint design checks are reported in Table 3-1.

For Special Moment Frames (seismic design), the beam design satisfies the following additional conditions (see also Table 3-1):

Table 3-1: Design Criteria

Type of Check/Design	Ordinary Moment Frames (Non-Seismic)	Intermediate Moment Frames (Seismic)	Special Moment Frames (Seismic)
Column Check (interaction)	Specified Combinations	Specified Combinations	Specified Combinations
Column Design (interaction)	Specified Combinations $1\% < \rho < 8\%$	Specified Combinations $1\% < \rho < 8\%$	Specified Combinations $1\% < \rho < 6\%$
Column Shears	Specified Combinations (If SDC = B, and $h/B \leq 5$, same as Intermediate)	Modified Combinations (earthquake loads increased by Ω_o) Column Capacity Shear $\phi = 1.0$ and $\alpha = 1.0$	Specified Combinations Column Capacity Shear $\phi = 1.0$ and $\alpha = 1.25$ $V_c = 0$ (conditional)

Table 3-1: Design Criteria

Type of Check/Design	Ordinary Moment Frames (Non-Seismic)	Intermediate Moment Frames (Seismic)	Special Moment Frames (Seismic)
Beam Design Flexure			
	Specified Combinations	Specified Combinations	Specified Combinations
	$\rho \leq 0.04$	$\rho \leq 0.04$	$\rho \leq 0.025$
	$\rho \geq \frac{3\sqrt{f'_c}}{f_y}, \rho \geq \frac{200}{f_y}$	$\rho \geq \frac{3\sqrt{f'_c}}{f_y}, \rho \geq \frac{200}{f_y}$	$\rho \geq \frac{3\sqrt{f'_c}}{f_y}, \rho \geq \frac{200}{f_y}$
	$\rho \geq \frac{3\sqrt{f'_c}}{f_y}, \rho \geq \frac{200}{f_y}$	$\rho \geq \frac{3\sqrt{f'_c}}{f_y}, \rho \geq \frac{200}{f_y}$	$\rho \geq \frac{3\sqrt{f'_c}}{f_y}, \rho \geq \frac{200}{f_y}$
Beam Minimum Moment Override Check			
	No Requirement	$M_{u,end}^+ \geq \frac{1}{3}M_{u,end}^-$	$M_{u,end}^+ \geq \frac{1}{2}M_{u,end}^-$
		$M_{u,span}^+ \geq \frac{1}{5} \max \{M_u^+, M_u^-\}_{end}$	$M_{u,span}^+ \geq \frac{1}{4} \max \{M_u^+, M_u^-\}_{end}$
		$M_{u,span}^- \geq \frac{1}{5} \max \{M_u^+, M_u^-\}_{max}$	$M_{u,span}^- \geq \frac{1}{4} \max \{M_u^+, M_u^-\}_{end}$
Beam Design Shear			
	Specified Combinations	Modified Specified Combinations (earthquake loads doubled)	Specified Combinations
		Beam Capacity Shear (V_e) with $\phi = 1.0$ and $\alpha = 1.0$ plus V_{D+L}	Beam Capacity Shear (V_e) with $\phi = 1.0$ and $\alpha = 1.25$ plus V_{D+L} $V_c = 0$ (conditional)
Joint Design			
	No Requirement	No Requirement	Checked for shear
Beam/Column Capacity Ratio			
	No Requirement	No Requirement	Checked

APPENDICES

Appendix A

Second Order P-Delta Effects

Typically, design codes require that second order P-Delta effects be considered when designing concrete frames. These effects are the global lateral translation of the frame and the local deformation of members within the frame.

Consider the frame object shown in Figure A-1, which is extracted from a story level of a larger structure. The overall global translation of this frame object is indicated by Δ . The local deformation of the member is shown as δ . The total second order P-Delta effects on this frame object are those caused by both Δ and δ .

The program has an option to consider P-Delta effects in the analysis. When P-Delta effects are considered in the analysis, the program does a good job of capturing the effect due to the Δ deformation shown in Figure A-1, but it does not typically capture the effect of the δ deformation (unless, in the model, the frame object is broken into multiple elements over its length).

Consideration of the second order P-Delta effects is generally achieved by computing the flexural design capacity using a formula similar to that shown in the following equation.

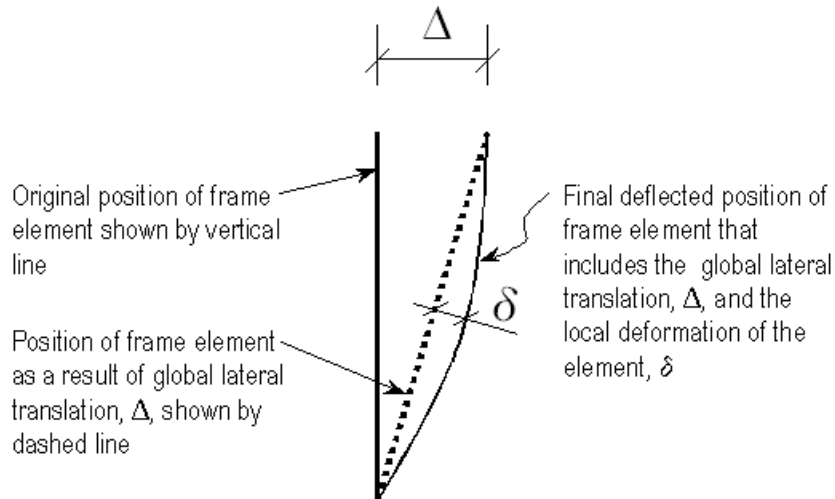


Figure A-1 The total second order P-delta effects on a frame element caused by both Δ and δ

$$M_{CAP} = aM_{nt} + bM_{lt}$$

where,

M_{CAP} = Flexural design capacity required

M_{nt} = Required flexural capacity of the member assuming there is no joint translation of the frame (i.e., associated with the δ deformation in Figure A-1)

M_{lt} = Required flexural capacity of the member as a result of lateral translation of the frame only (i.e., associated with the Δ deformation in Figure A-1)

a = Unitless factor multiplying M_{nt}

b = Unitless factor multiplying M_{lt} (assumed equal to 1 by the program; see the following text)

When the program performs concrete frame design, it assumes that the factor b is equal to 1 and calculates the factor a . That $b = 1$ assumes that P-Delta effects have been considered in the analysis, as previously described. Thus, in general, when performing concrete frame design in this program, **consider P-Delta effects in the analysis before running the program.**

Appendix B

Member Unsupported Lengths and Computation of *K*-Factors

The column unsupported lengths are required to account for column slenderness effects. The program automatically determines the unsupported length ratios, which are specified as a fraction of the frame object length. Those ratios times the frame object length give the unbraced lengths for the members. Those ratios also can be overwritten by the user on a member-by-member basis, if desired, using the overwrite option.

There are two unsupported lengths to consider. They are l_{33} and l_{22} , as shown in Figure B-1. These are the lengths between support points of the member in the corresponding directions. The length l_{33} corresponds to instability about the 3-3 axis (major axis), and l_{22} corresponds to instability about the 2-2 axis (minor axis).

In determining the values for l_{22} and l_{33} of the members, the program recognizes various aspects of the structure that have an effect on those lengths, such as member connectivity, diaphragm constraints and support points. The program automatically locates the member support points and evaluates the corresponding unsupported length.

It is possible for the unsupported length of a frame object to be evaluated by the program as greater than the corresponding member length. For example, assume a column has a beam framing into it in one direction, but not the other, at a floor level. In that case, the column is assumed to be supported in one direction only at that story level, and its unsupported length in the other direction will exceed the story height.

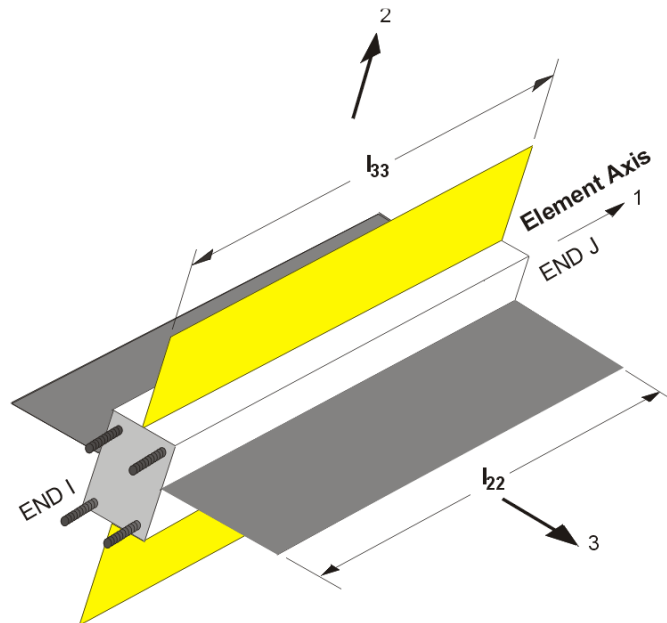


Figure B-1 Axis of bending and unsupported length

Appendix C

Concrete Frame Design Preferences

The concrete frame design preferences are general assignments that are applied to all of the concrete frame members. The design preferences should be reviewed and any changes from the default values made before performing a design. The following table lists the design preferences that are specific to using ACI 318-11; the preferences that are generic to all codes are not included in this table.

Table C-1 Preferences

Item	Possible Values	Default Value	Description
Time History Design	Envelopes, Step-by-Step, Last Step, and so on	Envelopes	Toggle for design load combinations that include a time history designed for the envelope of the time history, or designed step-by-step for the entire time history. If a single design load combination has <i>more than one</i> time history case in it, that design load combination is designed for the envelopes of the time histories, regardless of what is specified here.

Item	Possible Values	Default Value	Description
Number Interaction Curves	Multiple of 4 ≥ 4	24	Number of equally spaced interaction curves used to create a full 360 <i>deg</i> interaction surface (this item should be a multiple of four). We recommend 24 for this item.
Number	Any odd value ≥ 5	11	Number of points used for defining a single curve in a concrete frame; should be odd.
Consider Minimum Eccentricity	No, Yes	Yes	Toggle to specify if minimum eccentricity is considered in design.
Seismic Design Category	A, B, C, D, E, F	D	This item varies with the Seismic Hazard Exposure Group and the effective Peak Velocity Related Acceleration.
Phi (Tension Controlled)	> 0	0.9	Strength reduction factor for tension controlled sections.
Phi (Compression Controlled-Tied)	> 0	0.65	The strength reduction factor for compression controlled sections with spiral reinforcement.
Phi (Compression Controlled-Spiral)	> 0	0.75	The strength reduction factor for compression controlled sections with spiral reinforcement.
Phi (Shear and/ or Torsion)	> 0	0.75	The strength reduction factor for shear and torsion.
Phi (Shear - Seismic)	> 0	0.60	The strength reduction factor for shear in structures that rely on special moment resisting frames or special reinforced concrete structural walls to resist earthquake effects.

Item	Possible Values	Default Value	Description
Phi (Joint Shear)	> 0	0.75	The strength reduction factor for shear and torsion.
Pattern Live Load Factor	≥ 0	0.75	The pattern load factor is used to compute positive live load moment by multiplying Live load with Pattern Load Factor (PLF) and assuming that the beam is simply supported. This option provides a limited pattern loading to frames. Use zero to turn off this option.
Utilization Factor Limit	> 0	0.95	Stress ratios that are less than or equal to this value are considered acceptable.

Appendix D Concrete Frame Overwrites

The concrete frame design overwrites are basic assignments that apply only to those elements to which they are assigned. Table D-1 lists concrete frame design overwrites for ACI 318-11. Default values are provided for all overwrite items. Thus, it is not necessary to specify or change any of the overwrites. However, at least review the default values to ensure they are acceptable. When changes are made to overwrite items, the program applies the changes only to the elements to which they are specifically assigned.

Table D-1 Overwrites

Item	Possible Values	Default Value	Description
Current Design Section	Any defined concrete section	Analysis section	The design section for the selected frame objects.
Element Type	Sway Special, Sway Intermediate, Sway Ordinary, NonSway	From Reference	Frame type in accordance with the moment frame definition given in ACI 21.1. The Framing Type is used for ductility considerations in the design. The program determines its default value based on the Seismic Design Category (SDC) assigned for the structure in the Preferences. If the assigned SDC is A or B, the

Item	Possible Values	Default Value	Description
			Framing Type is set to Ordinary. If the assigned SDC is C, the Framing Type is set to Intermediate. If the assigned SDC is D, E, or F, the Framing Type is set to special (IBC 1908.1.2). These are default values, which the user can overwrite if needed.
Live Load Reduction Factor	≥ 0	Calculated	The reduced live load factor. A reducible live load is multiplied by this factor to obtain the reduced live load for the frame object. Specifying 0 means the value is program determined.
Unbraced Length Ratio (Major)	≥ 0	Calculated	Unbraced length factor for buckling about the frame object major axis. This item is specified as a fraction of the frame object length. Multiplying this factor times the frame object length gives the unbraced length for the object. Specifying 0 means the value is program determined.
Unbraced Length Ratio (Minor)	≥ 0	0.60	Unbraced length factor for buckling about the frame object minor axis. Multiplying this factor times the frame object length gives the unbraced length for the object. Specifying 0 means the value is program determined. This factor is also used in determining the length for lateral-torsional buckling.
Effective Length Factor (K Major)	> 0	Calculated	See ACI 10.10 and Figure R10.10.1.1. Effective length factor for buckling about the frame object major axis. This item is specified as a fraction of the frame object length.

References

- ACI, 2011. Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary, American Concrete Institute, 38800 Country Club Drive, Farmington Hills, Michigan, 48331.
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