

CSA 23.3-04 PT-SL EXAMPLE 001
Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm as shown in shown in Figure 1.

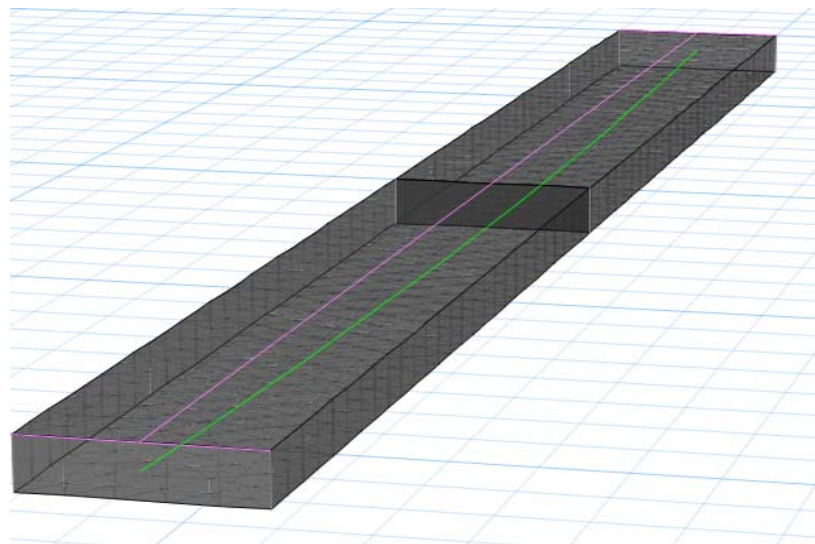
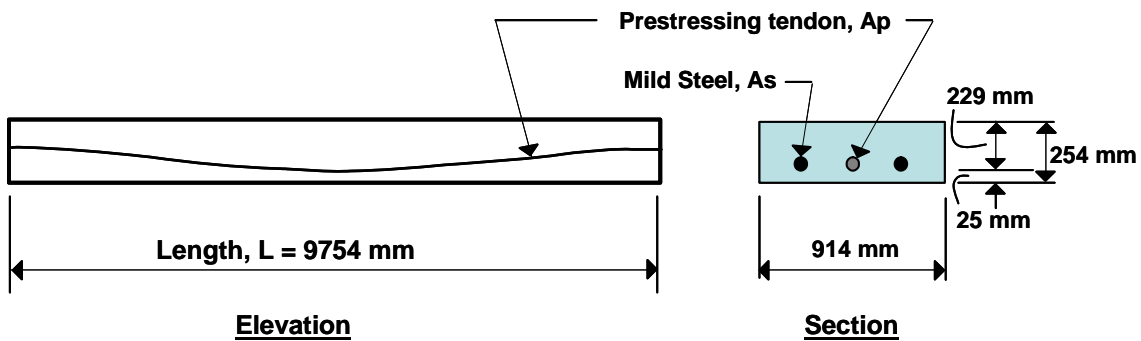


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span

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perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2 , has been added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 KN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the midspan of the slab. Independent hand calculations have been compared with the ETABS results and summarized for verification and validation of the ETABS results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	$T, h =$	254	mm
Effective depth	$d =$	229	mm
Clear span	$L =$	9754	mm
Concrete strength	$f'_c =$	30	MPa
Yield strength of steel	$f_y =$	400	MPa
Prestressing, ultimate	$f_{pu} =$	1862	MPa
Prestressing, effective	$f_e =$	1210	MPa
Area of Prestress (single strand)	$A_p =$	198	mm^2
Concrete unit weight	$w_c =$	23.56	KN/m^3
Modulus of elasticity	$E_c =$	25000	N/mm^2
Modulus of elasticity	$E_s =$	200,000	N/mm^2
Poisson's ratio	$\nu =$	0	
Dead load	$w_d =$	self	KN/m^2
Live load	$w_l =$	4.788	KN/m^2

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

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RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	159.4	159.4	0.00%
Area of Mild Steel req'd, As (sq-cm)	16.25	16.33	0.49%
Transfer Conc. Stress, top (D+PT _I), MPa	-5.058	-5.057	-0.02%
Transfer Conc. Stress, bot (D+PT _I), MPa	2.839	2.839	0.00%
Normal Conc. Stress, top (D+L+PT _F), MPa	-10.460	-10.467	0.07%
Normal Conc. Stress, bot (D+L+PT _F), MPa	8.402	8.409	0.08%
Long-Term Conc. Stress, top (D+0.5L+PT _{F(L)}), MPa	-7.817	-7.818	0.01%
Long-Term Conc. Stress, bot (D+0.5L+PT _{F(L)}), MPa	5.759	5.760	0.02%

COMPUTER FILE: CSA A23.3-04 PT-SL Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

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HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing

$$f_{cu} = 30 \text{ MPa}$$

$$f_y = 400 \text{ MPa}$$

Post-Tensioning

$$f_{pu} = 1862 \text{ MPa}$$

$$f_{py} = 1675 \text{ MPa}$$

$$\text{Stressing Loss} = 186 \text{ MPa}$$

$$\text{Long-Term Loss} = 94 \text{ MPa}$$

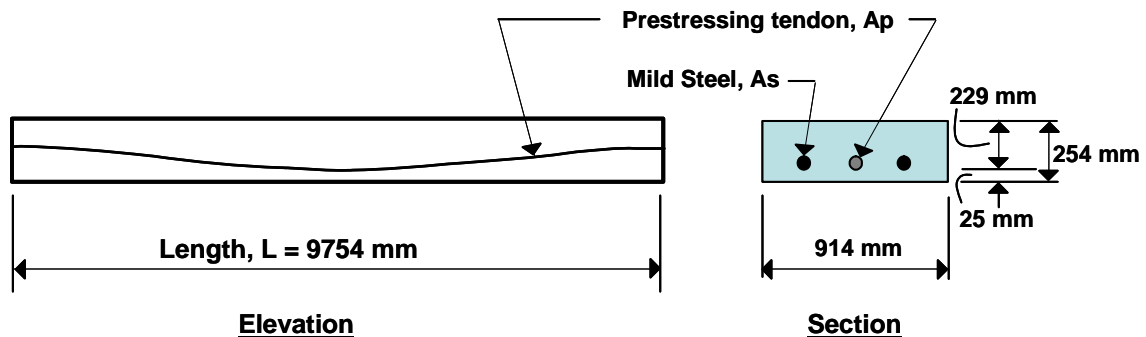
$$f_i = 1490 \text{ MPa}$$

$$f_e = 1210 \text{ MPa}$$

$$\phi_c = 0.65, \quad \phi_s = 0.85$$

$$\alpha_l = 0.85 - 0.0015f'_c \geq 0.67 = 0.805$$

$$\beta_l = 0.97 - 0.0025f'_c \geq 0.67 = 0.895$$



Loads:

$$\text{Dead, self-wt} = 0.254 \text{ m} \times 23.56 \text{ kN/m}^3 = 5.984 \text{ kN/m}^2 \text{ (D)} \times 1.25 = 7.480 \text{ kN/m}^2 \text{ (D}_u\text{)}$$

$$\text{Live,} = 4.788 \text{ kN/m}^2 \text{ (L)} \times 1.50 = 7.182 \text{ kN/m}^2 \text{ (L}_u\text{)}$$

$$\text{Total} = 10.772 \text{ kN/m}^2 \text{ (D+L)} = 14.662 \text{ kN/m}^2 \text{ (D+L)}_{\text{ult}}$$

$$\omega = 10.772 \text{ kN/m}^2 \times 0.914 \text{ m} = 9.846 \text{ kN/m}, \quad \omega_u = 16.039 \text{ kN/m}^2 \times 0.914 \text{ m} = 13.401 \text{ kN/m}$$

$$\text{Ultimate Moment, } M_U = \frac{w l_1^2}{8} = 13.401 \times (9.754)^2 / 8 = 159.42 \text{ kN-m}$$

Ultimate Stress in strand, $f_{pb} = f_{pe} + \frac{8000}{l_o}(d_p - c_y)$

$$c_y = \frac{\phi_p A_p f_{pr} + \phi_s A_s f_y}{\alpha_1 \phi_c f'_c \beta_1 b} = \frac{0.9(197)(1347) + 0.85(1625)(400)}{0.805(0.65)(30.0)(0.895)(914)} = 61.66 \text{ mm}$$

$$f_{pb} = 1210 + \frac{8000}{9754}(229 - 61.66) = 1347 \text{ MPa}$$

Depth of the compression block, a , is given as:

$$\begin{aligned} \text{Stress block depth, } a &= d - \sqrt{d^2 - \frac{2M^*}{\alpha_1 f'_c \phi_c b}} \\ &= 0.229 - \sqrt{0.229^2 - \frac{2(159.42)}{0.805(30000)(0.65)(0.914)}} = 55.18 \end{aligned}$$

Ultimate force in PT, $F_{ult,PT} = A_p(f_{ps}) = 197(1347)/1000 = 265.9 \text{ kN}$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT} \left(d - \frac{a}{2} \right) \phi = 265.9 \left(0.229 - \frac{55.18}{2} \right) (0.85) = 45.52 \text{ kN-m}$$

Net Moment to be resisted by A_s , $M_{NET} = M_U - M_{PT}$
 $= 159.42 - 45.52 = 113.90 \text{ kN-m}$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_{NET}}{0.87 f_y z} = \frac{113.90}{0.87(400) \left(229 - \frac{55.18}{2} \right)} (1e6) = 1625 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (D+PT_i) = 1.0D+0.0L+1.0PT₁

Tendon stress at transfer = jacking stress – stressing losses = 1490 – 186 = 1304 MPa

The force in the tendon at transfer, = 1304(197.4)/1000 = 257.4 kN

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Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \frac{F_{PTI}}{A} \pm \frac{M_D - M_{PT}}{S} = \frac{-257.4}{0.254(0.914)} \pm \frac{65.04 - 26.23}{0.00983}$
 where $S = 0.00983\text{m}^3$

$$f = -1.109 \pm 3.948 \text{ MPa}$$

$$f = -5.058(\text{Comp}) \text{ max}, 2.839(\text{Tension}) \text{ max}$$

Normal Condition, load combinations: $(D+L+PT_F) = 1.0D+1.0L+1.0PT_F$

Tendon stress at normal = jacking – stressing – long-term = $1490 - 186 - 94 = 1210$ MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for $(D+L+PT_F)$,

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$

$$f = -1.029 \pm 9.431$$

$$f = -10.460(\text{Comp}) \text{ max}, 8.402(\text{Tension}) \text{ max}$$

Long-Term Condition, load combinations: $(D+0.5L+PT_{F(L)}) = 1.0D+0.5L+1.0PT_F$

Tendon stress at normal = jacking – stressing – long-term = $1490 - 186 - 94 = 1210$ MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for $(D+0.5L+PT_{F(L)})$,

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$

$$f = -1.029 \pm 6.788$$

$$f = -7.817(\text{Comp}) \text{ max}, 5.759(\text{Tension}) \text{ max}$$