EXAMPLE CSA A23.3-14 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8 m spans in each direction, as shown in Figure 1.

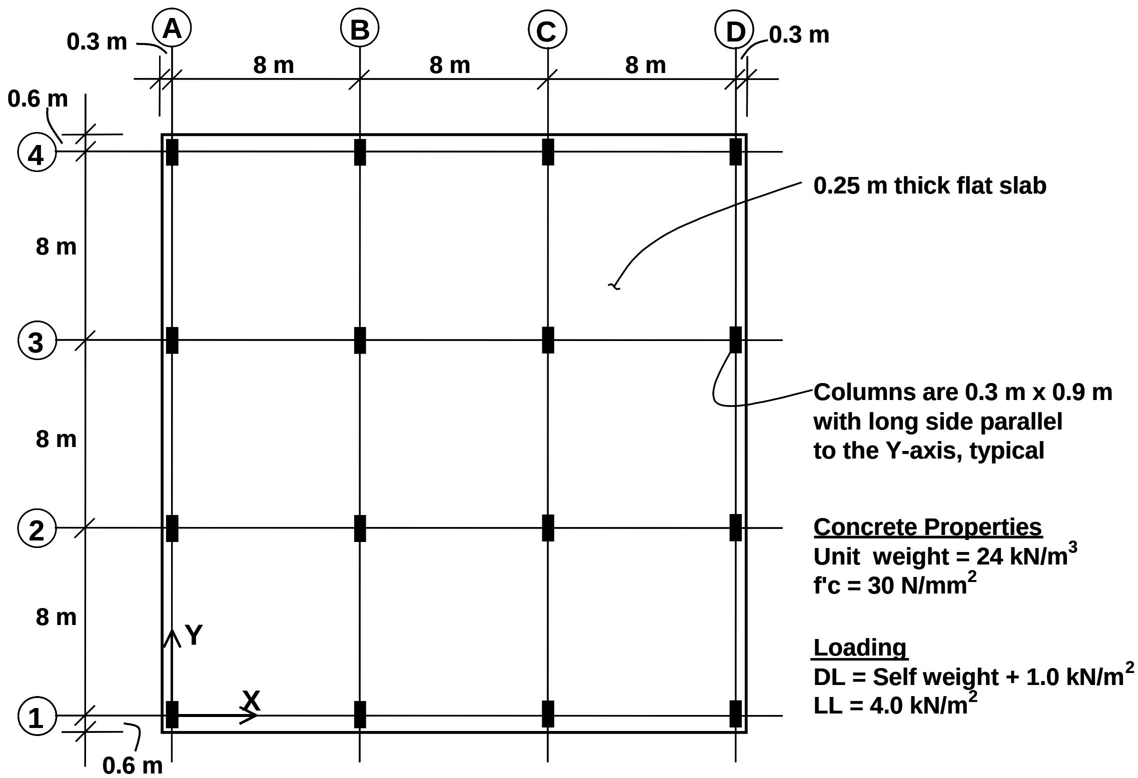

Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f'_c of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
SAFE	1.792	1.127	1.59
Calculated	1.792	1.127	1.59

COMPUTER FILE: CSA A23.3-14 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using SAFE Method

$$d = [(250 - 26) + (250 - 38)]/2 = 218 \text{ mm}$$

Refer to Figure 2.

b_0 = 518+ 1118 + 1118 + 518 = 3272 mm

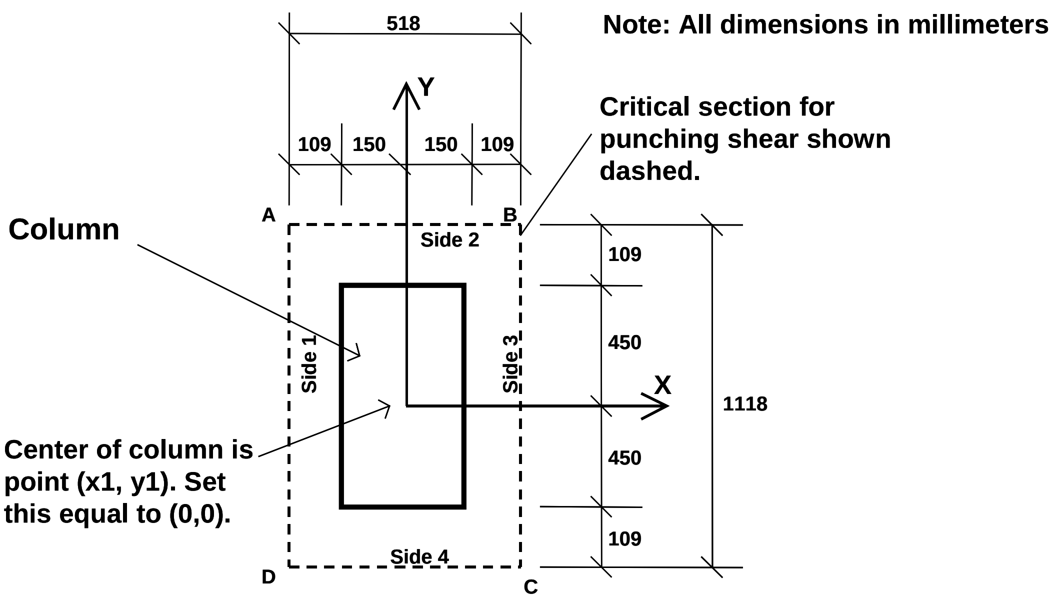

Figure 2: Interior Column, Grid B-2 in SAFE Model

$$\gamma_{V2} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{1118}{518}}} = 0.495$$

$$\gamma_{V3} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{518}{1118}}} = 0.312$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–259	0	259	0	N.A.
y_2	0	559	0	–559	N.A.
L	1118	518	1118	518	b_0 = 3272
d	218	218	218	218	N.A.
Ld	243724	112924	243724	112924	713296
Ldx_2	–63124516	0	63124516	0	0
Ldy_2	0	63124516	0	–63124516	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{713296} = 0\,mm$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{713296} = 0\,mm$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	1118	518	1118	518	N.A.
d	218	218	218	218	N.A.
$x_2 - x_3$	–259	0	259	0	N.A.
$y_2 - y_3$	0	559	0	–559	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	2.64E+10	3.53E+10	2.64E+10	3.53E+10	1.23E+11
I_{YY}	1.63E+10	2.97E+09	1.63E+10	2.97E+09	3.86E+10
I_{XY}	0	0	0	0	0

From the SAFE output at Grid B-2:

V_f = 1126.498 kN

$\gamma_{V2} M_{f,2}$ = –25.725 kN-m

$\gamma_{V3} M_{f,3}$ = 14.272 kN-m

At the point labeled A in Figure 2, x_4 = –259 and y_4 = 559, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6[3.86 \bullet 10^{10}(559-0)-(0)(-259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6[1.23 \bullet 10^{11}(-259-0)-(0)(559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_f = 1.5793 – 0.1169 – 0.0958 = **1.3666 N/mm²** at point A

At the point labeled B in Figure 2, x_4 = 259 and y_4 = 559, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6[3.86 \bullet 10^{10}(559-0)-(0)(259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6[1.23 \bullet 10^{11}(259-0)-(0)(559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_f = 1.5793 – 0.1169 + 0.0958 =**1.5582 N/mm²** at point B

At the point labeled C in Figure 2, x_4 = 259 and y_4 = –559, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6[3.86 \bullet 10^{10}(-559-0)-(0)(259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6[1.23 \bullet 10^{11}(259-0)-(0)(-559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_f = 1.5793 + 0.1169 + 0.0958 = **1.792 N/mm²** at point C

At the point labeled D in Figure 2, x_4 = –259 and y_4 = –559, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6[3.86 \bullet 10^{10}(-559-0)-(0)(-259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6[1.23 \bullet 10^{11}(-259-0)-(0)(-559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_f = 1.5793 + 0.1169 – 0.0958 = **1.6004 N/mm²** at point D

Point C has the largest absolute value of v_u, thus v_{max} = **1.792 N/mm²**

The shear capacity is calculated based on the minimum of the following three limits:

$$v_v = \min\begin{cases} \phi_c\left(1+\dfrac{2}{\beta_c}\right)0.19\lambda\sqrt{f'_c} \\ \phi_c\left(0.19+\dfrac{\alpha_s d}{b_0}\right)\lambda\sqrt{f'_c} \\ \phi_c 0.38\lambda\sqrt{f'_c} \end{cases} \qquad 1.127 \text{ N/mm}^2 \text{ in accordance with CSA 13.3.4.1}$$

CSA 13.3.4.1 yields the smallest value of $\boxed{v_v = 1.127 \text{ N/mm}^2}$, and thus this is the shear capacity.

$$\boxed{\text{Shear Ratio} = \frac{v_U}{\varphi v_v} = \frac{1.792}{1.127} = 1.59}$$