Sample data

$X = 7, -5, 6, 4$
$y = 11, -13, 9, 1$

① **Mean.**

$X = 7 + (-5) + 6 + 4 = 12$
$y = 11, + (-13) + 9 + 1 = 8$

$X = \frac{12}{4}$ 3 $\quad Y = \frac{8}{4} = 2$ # divide by no of Sample

$\frac{3+2}{2} = 2.5$ (Mean)

② **Variance.** (Degree of freedom = 1)

- Individual variance (Sample Variance)
- Total variance.

Variance of X

$$\underset{(\text{Sample})}{S} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2}$$

$$= \frac{(7-3)^2 + (-5-3)^2 + (6-3)^2 + (4-3)^2}{4-1}$$

$$= (4)^2 + (-8)^2 + (3)^2 + (1)^2 / 3$$

$$= 16 + 64 + 9 + 1 / 3 = \frac{90}{3} = 30 \quad —①$$

Variance of y

$$\frac{(11-2)^2+(-13-2)^2+(9-2)^2+(1-2)^2}{4-1}$$

$$= \frac{(9)^2+(15)^2+(7)^2+(1)^2}{3}$$

$$= 81+225+49+1/3$$

$$Var_{y_{ddf=1}} = \underline{118.66} \quad \text{—— } (2)$$

Variance of (x, y) (ddf = 1)

$$= (7-2.5)^2+(-5-2.5)^2+(6-2.5)^2+(4-2.5)^2$$
$$+ (11-2.5)^2+(-13-2.5)^2+(9-2.5)^2+(1-2.5)^2$$

$$= (4.5)^2+(-7.5)^2+(3.5)^2+(1.5)^2+(8.5)^2$$
$$+ (-15.5)^2+(6.5)^2+(-1.5)^2$$

$$= 20.25+56.25+12.25+2.25+72.25+240.25+$$
$$42.25+2.25$$

$$= \frac{448}{8-1} = \frac{448}{7} \Rightarrow \underline{64} \quad \text{—— } (2)$$

Now, Variance with (Degree of freedom = 0)

we can use the values calculated in the answers and divide with total no. of sample.

$$\underset{ddf=0}{\text{Variance}(x)} = \frac{90}{4} = 22.5 \quad \text{—} \quad (4)$$

$$\underset{ddf=0}{\text{Variance}(y)} = \frac{356}{4} = 89 \quad \text{—} \quad (5)$$

$$\underset{ddf=0}{\text{Variance}(x,y)} = \frac{448}{8} = 56 \quad \text{—} \quad (6)$$

(3) Sample Covariance. (Degree of freedom = 1)

$$\text{cov}(x,y) = \frac{\text{Sum}(x_i - x_{mean}) * (y_i - y_{mean})}{\text{Samplesize} - 1}$$

$$= (7-3) * (11-2) + (-5-3) * (-13-2) + (6-3) * (9-2) + (4-3) * (1-2) / 3$$

$$= 4*9 + -8*-15 + 3*7 + 1*-1 / 3$$

$$= 176/3 \Rightarrow 58.66$$

(4) Co-variance Matrix.

It can be written as

$$\begin{bmatrix} Var(x) & cov(x,y) \\ Cov(x,y) & Var(y) \end{bmatrix}$$

Since, we already calculated the values. So, we can put the values in the Matrix—

$$\text{Cov Matrix} = \begin{bmatrix} 30, & 58.66 \\ 58.66, & 118.66 \end{bmatrix}$$

(5) X and Y are positively co-related.

⑥ Form Correlation Matrix

$X = 7, -5, 6, 4$

$y = 11, -13, 9, 1$

$X_{mean} = \underline{3}$ $\quad X_{std} = \underline{5.47}$

$Y_{mean} = \underline{2}$ $\quad Y_{std} = \underline{10.89}$

Correlation Matrix $(r) \longrightarrow$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

$$= \frac{1}{3}\left(\left(\frac{7-3}{5.47}\right) * \left(\frac{11-2}{10.89}\right) + \left(\frac{-5-3}{5.47}\right) * \left(\frac{-13-2}{10.89}\right) \right.$$

$$\left. + \left(\frac{6-3}{5.47}\right) * \left(\frac{9-2}{10.89}\right) + \left(\frac{4-3}{5.47}\right) * \left(\frac{1-2}{10.89}\right) \right)$$

$$= \frac{4 \times 9}{5.47 \times 10.89} + \frac{-8 \times -15}{5.47 \times 10.89} + \frac{3 \times 7}{5.47 \times 10.89}$$

$$+ \frac{1 \times -1}{5.47 \times 10.89}$$

$$= \frac{36}{59.56} + \frac{120}{59.56} + \frac{21}{59.56} - \frac{1}{59.56}$$

$$= 2.95/3 = \underline{\underline{0.98}}.$$

$$r = \underline{\underline{0.98}}.$$

So, the Co-relation Matrix will be.

$$\begin{bmatrix} 1 , & 0.98 \\ 0.98 , & 1 \end{bmatrix}$$

(7) Co-relation Coeffient between X and Y is

0.98.

(8) If $y = [-11, 13, -9, -1]$
$x = [7, -5, 6, 4]$

find Covariance & Corelation Matrix

Co-variance (DOf = 1)

$X_{mean} = 3$

$y_{mean} = -2$

$$\underset{(x,y)}{\text{Covariance}} = \text{Sum}(x_i - x_{mean}) * (y_i - y_{mean}) / N-1$$

$$= ((7-3)*(-11--2) + (-5-3)*(13--2) + (6-3)*(-9--2) + (4-3)*(-1--2)) / 3$$

$$= (4*-9 + -8*15 + 3*-7 + 1*1) / 3$$

$$= -36 - 120 - 21 + 1 / 3$$

$$= \frac{-176}{3} = -58.66$$

Co-relation Matrix.

$X = 7, -5, 6, 4$

$y = -11, 13, -9, -1$

$X_{mean} = \underline{3}$ $\quad X_{std} = \underline{5.47}$

$Y_{mean} = \underline{-2}$ $\quad Y_{std} = \underline{10.89}$

Corelation Matrix $(r) \rightarrow$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

$$= \frac{1}{3}\left(\left(\frac{7-3}{5.47}\right) * \left(\frac{-11 - -2}{10.89}\right) + \left(\frac{-5-3}{5.47}\right) * \left(\frac{13 - -2}{10.89}\right) + \left(\frac{6-3}{5.47}\right) * \left(\frac{-9 - -2}{10.89}\right) + \left(\frac{4-3}{5.47}\right) * \left(\frac{-1 - -2}{10.89}\right)\right)$$

$$= \frac{4 \times -9}{5.47 \times 10.89} + \frac{-8 \times 15}{5.47 \times 10.89} + \frac{3 \times -7}{5.47 \times 10.89}$$

$$+ \frac{1 \times 1}{5.47 \times 10.89}$$

$$= \frac{-36}{59.56} - \frac{120}{59.56} - \frac{21}{59.56} + \frac{1}{59.56}$$

$$= -2.95/3$$

$$= -0.98.$$

Corelation matrix $= \begin{bmatrix} 1 , & -0.98 \\ -0.98 , & 1 \end{bmatrix}$