## Lesson 4.1• Triangle Sum Conjecture

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-9, determine the angle measures.

1. $p=$ $\qquad$ , $q=$ $\qquad$

2. $x=$ $\qquad$ , $y=$ $\qquad$
3. $a=$ $\qquad$ $b=$ $\qquad$

4. $r=$ $\qquad$ , $s=$ $\qquad$
$t=$ $\qquad$

5. $x=$ $\qquad$ , $y=$ $\qquad$
6. $y=$ $\qquad$

7. $s=$

$\qquad$
8. $m=$ $\qquad$

9. $m \angle P=$ $\qquad$

10. Find the measure of $\angle Q P T$.

11. Use the diagram to explain why $\angle A$ and $\angle B$ are complementary.

12. Find the sum of the measures of the marked angles.

13. Use the diagram to explain why $m \angle A+m \angle B=m \angle C+m \angle D$.


## Lesson 4.2•Properties of Isosceles Triangles

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, find the angle measures.

## 1. $m \angle T=$ <br> $\qquad$


2. $m \angle G=$ $\qquad$


In Exercises 4-6, find the measures.
4. $m \angle A=$ $\qquad$ , perimeter of $\triangle A B C=$ $\qquad$
5. The perimeter of $\triangle L M O$
is $536 \mathrm{~m} . L M=$ $\qquad$ , $m \angle M=$
$\qquad$

3. $x=$ $\qquad$

6. The perimeter of $\triangle Q R S$ is
$344 \mathrm{~cm} . m \angle Q=$ $\qquad$ —, QR = $\qquad$

7. a. Name the angle(s) congruent to $\angle D A B$.
b. Name the angle(s) congruent to $\angle A D B$.

c. What can you conclude about $\overline{A D}$ and $\overline{B C}$ ? Why?
8. $x=$ $\qquad$ , $y=$ $\qquad$

9. $P R=Q R$ and $Q S=R S$.

If $m \angle R S Q=120^{\circ}$, what is $m \angle Q P R$ ?

10. Use the diagram to explain why $\triangle P Q R$ is isosceles.


## Lesson 4.3•Triangle Inequalities

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1 and 2, determine whether it is possible to draw a triangle with sides of the given measures. If it is possible, write yes. If it is not possible, write no and make a sketch demonstrating why it is not possible.

1. $16 \mathrm{~cm}, 30 \mathrm{~cm}, 45 \mathrm{~cm}$
2. $9 \mathrm{~km}, 17 \mathrm{~km}, 28 \mathrm{~km}$
3. If 17 and 36 are the lengths of two sides of a triangle, what is the range of possible values for the length of the third side?

In Exercises 4-6, arrange the unknown measures in order from greatest to least.
4.

5.

6.

7. $x=$ $\qquad$

8. $x=$ $\qquad$
9. What's wrong with this picture?

10. Explain why $\triangle P Q S$ is isosceles.


In Exercises 11 and 12, use a compass and straightedge to construct a triangle with the given sides. If it is not possible, explain why not.
11.

12.


## Lesson 4.4•Are There Congruence Shortcuts?

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, name the conjecture that leads to each congruence.

1. $\triangle P A T \cong \triangle I M T$

2. $\triangle S I D \cong \triangle J A N$

3. $\overline{T S}$ bisects $\overline{M A}, \overline{M T} \cong \overline{A T}$, and $\triangle M S T \cong \triangle A S T$


In Exercises 4-9, name a triangle congruent to the given triangle and state the congruence conjecture. If you cannot show any triangles to be congruent from the information given, write "cannot be determined" and redraw the triangles so that they are clearly not congruent.
4. $M$ is the midpoint of $\overline{A B}$ and $\overline{P Q}$.
$\triangle A P M \cong \triangle$ $\qquad$

5. KITE is a kite with $K I=T I$.
$\triangle K I E \cong \triangle$

6. $\triangle A B C \cong$ $\qquad$
7. $\triangle M O N \cong$ $\qquad$
8. $\triangle S Q R \cong$

9. $\triangle T O P \cong$ $\qquad$



In Exercises 10-12, use a compass and a straightedge or patty paper and a straightedge to construct a triangle with the given parts. Then, if possible, construct a different (noncongruent) triangle with the same parts. If it is not possible, explain why not.

12.


## Lesson 4.5 • Are There Other Congruence Shortcuts?

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-6, name a triangle congruent to the given triangle and state the congruence conjecture. If you cannot show any triangles to be congruent from the information given, write "cannot be determined" and explain why.

1. $\triangle P I T \cong \triangle$

2. $\triangle X V W \cong \triangle$

3. $\triangle A C N \cong \triangle$ $\qquad$
4. $\triangle E C D \cong \triangle$ $\qquad$

5. $\overline{P S}$ is the angle bisector of $\angle Q P R$.
$\triangle P Q S \cong \triangle$ $\qquad$


6. $E F G H$ is a parallelogram.
$G Q=E Q$.
$\triangle E Q L \cong \triangle$ $\qquad$

7. The perimeter of $\triangle Q R S$ is 350 cm .

Is $\triangle Q R S \cong \triangle M O L$ ? Explain.


8. The perimeter of $\triangle T U V$ is 95 cm .

Is $\triangle T U V \cong \triangle W X V$ ? Explain.


In Exercises 9 and 10, construct a triangle with the given parts. Then, if possible, construct a different (noncongruent) triangle with the same parts.
If it is not possible, explain why not.
9.

10.


## Lesson 4.6•Corresponding Parts of Congruent Triangles

Name $\qquad$ Period $\qquad$ Date $\qquad$

1. Give the shorthand name for each of the four triangle congruence conjectures.

In Exercises 2-5, use the figure at right to explain why each congruence is true. $W X Y Z$ is a parallelogram.
2. $\angle W X Z \cong \angle Y Z X$
3. $\angle W Z X \cong \angle Y X Z$

4. $\triangle W Z X \cong \triangle Y X Z$
5. $\angle W \cong \angle Y$

For Exercises 6 and 7, mark the figures with the given information. To demonstrate whether the segments or the angles indicated are congruent, determine that two triangles are congruent. Then state which conjecture proves them congruent.
6. $M$ is the midpoint of $\overline{W X}$ and
$\overline{Y Z}$. Is $\overline{Y W} \cong \overline{Z X}$ ? Why?

7. $\triangle A B C$ is isosceles and $\overline{C D}$ is the bisector of the vertex angle. Is $\overline{A D} \cong \overline{B D}$ ? Why?


In Exercises 8 and 9, use the figure at right to write a paragraph proof for each statement.
8. $\overline{D E} \cong \overline{C F}$
9. $\overline{E C} \cong \overline{F D}$

10. $T R A P$ is an isosceles trapezoid with $T P=R A$ and $\angle P T R \cong \angle A R T$. Write a paragraph proof explaining why $\overline{T A} \cong \overline{R P}$.


## Lesson 4.7 • Flowchart Thinking

Name $\qquad$ Period $\qquad$ Date $\qquad$

Complete the flowchart for each proof.

1. Given: $\overline{P Q} \| \overline{S R}$ and $\overline{P Q} \cong \overline{S R}$

Show: $\overline{S P} \cong \overline{Q R}$
Flowchart Proof

2. Given: Kite KITE with $\overline{K E} \cong \overline{K I}$

Show: $\overline{K T}$ bisects $\angle E K I$ and $\angle E T I$
Flowchart Proof

3. Given: $A B C D$ is a parallelogram

Show: $\angle A \cong \angle C$


## Flowchart Proof



## Lesson 4.8 • Proving Special Triangle Conjectures

Name $\qquad$
$\qquad$ Date $\qquad$

In Exercises 1-3, use the figure at right.

1. $\overline{C D}$ is a median, perimeter $\triangle A B C=60$, and $A C=22 . A D=$ $\qquad$
2. $\overline{C D}$ is an angle bisector, and $m \angle A=54^{\circ} . m \angle A C D=$ $\qquad$
3. $\overline{C D}$ is an altitude, perimeter $\triangle A B C=42, m \angle A C D=38^{\circ}$, and $A D=8$. $m \angle B=$ $\qquad$ , $C B=$ $\qquad$

4. $\triangle E Q U$ is equilateral.
$m \angle E=$ $\qquad$
5. $\triangle A N G$ is equiangular
and perimeter $\triangle A N G=51$.
$A N=$ $\qquad$
6. $\triangle A B C$ is equilateral, $\triangle A C D$ is isosceles with base $\overline{A C}$, perimeter $\triangle A B C=66$, and perimeter $\triangle A C D=82$. Perimeter $A B C D=$ $\qquad$

7. Complete a flowchart proof for this conjecture: In an isosceles triangle, the altitude from the vertex angle is the median to the base.
Given: Isosceles $\triangle A B C$ with $\overline{A C} \cong \overline{B C}$ and altitude $\overline{C D}$
Show: $\overline{C D}$ is a median
Flowchart Proof

8. Write a flowchart proof for this conjecture: In an isosceles triangle, the median to the base is also the angle bisector of the vertex angle.

Given: Isosceles $\triangle A B C$ with $\overline{A C} \cong \overline{B C}$ and median $\overline{C D}$
Show: $\overline{C D}$ bisects $\angle A C B$


## LESSON 3.8•The Centroid

1. 


2.

3. $C P=3.3 \mathrm{~cm}, C Q=5.7 \mathrm{~cm}, C R=4.8 \mathrm{~cm}$

4. $(3,4)$
5. $P C=16, C L=8, Q M=15, C R=14$
6. a. Incenter
b. Centroid
c. Circumcenter
d. Circumcenter
e. Orthocenter
f. Incenter
g. Centroid

## LESSON 4.1•Triangle Sum Conjecture

1. $p=67^{\circ}, q=15^{\circ}$
2. $x=82^{\circ}, y=81^{\circ}$
3. $a=78^{\circ}, b=29^{\circ}$
4. $r=40^{\circ}, s=40^{\circ}, t=100^{\circ}$
5. $x=31^{\circ}, y=64^{\circ}$
6. $y=145^{\circ}$
7. $s=28^{\circ}$
8. $m=72 \frac{1}{2}$ 。
9. $m \angle P=a$
10. $m \angle Q P T=135^{\circ}$
11. $720^{\circ}$
12. The sum of the measures of $\angle A$ and $\angle B$ is $90^{\circ}$ because $m \angle C$ is $90^{\circ}$ and all three angles must be $180^{\circ}$. So, $\angle A$ and $\angle B$ are complementary.
13. $m \angle B E A=m \angle C E D$ because they are vertical angles. Because the measures of all three angles in each triangle add to $180^{\circ}$, if equal measures are subtracted from each, what remains will be equal.
14. $m \angle T=64^{\circ}$
15. $m \angle G=45^{\circ}$
16. $x=125^{\circ}$
17. $m \angle A=39^{\circ}$, perimeter of $\triangle A B C=46 \mathrm{~cm}$
18. $L M=163 \mathrm{~m}, m \angle M=50^{\circ}$
19. $m \angle Q=44^{\circ}, Q R=125$
20. a. $\angle D A B \cong \angle A B D \cong \angle B D C \cong \angle B C D$
b. $\angle A D B \cong \angle C B D$
c. $\overline{A D} \| \overline{B C}$ by the Converse of the AIA Conjecture.
21. $x=21^{\circ}, y=16^{\circ}$
22. $m \angle Q P R=15^{\circ}$
23. $m \angle P R Q=55^{\circ}$ by VA, which makes $m \angle P=55^{\circ}$ by the Triangle Sum Conjecture. So, $\triangle P Q R$ is isosceles by the Converse of the Isosceles Triangle Conjecture.

## LESSON 4.3•Triangle Inequalities

1. Yes
2. No

3. $19<x<53$
4. $b>a>c$
5. $b>c>a$
6. $a>c=d>b$
7. $x=76^{\circ}$
8. $x=79^{\circ}$
9. The interior angle at $A$ is $60^{\circ}$. The interior angle at $B$ is $20^{\circ}$. But now the sum of the measures of the triangle is not $180^{\circ}$.
10. By the Exterior Angles Conjecture, $2 x=x+m \angle P Q S$. So, $m \angle P Q S=x$. So, by the Converse of the Isosceles Triangle Conjecture, $\triangle P Q S$ is isosceles.
11. Not possible. $A B+B C<A C$
12. 



LESSON 4.4•Are There Congruence Shortcuts?

1. SAA or ASA
2. SSS
3. SSS
4. $\triangle B Q M$ (SAS)
5. $\triangle$ TIE (SSS)
6. Cannot be determined, as shown by the figure.

7. $\triangle T N O$ (SAS)
8. Cannot be determined, as shown by the figure.

9. $\triangle D O G$ (SAS)
10. Only one triangle because of SSS.

11. Two possible triangles.

12. Only one triangle because of SAS.


## LESSON 4.5 • Are There Other Congruence Shortcuts?

1. Cannot be determined

2. $\triangle X Z Y$ (SAA)
3. $\triangle A C B$ (ASA or SAA)
4. $\triangle P R S$ (ASA)
5. $\triangle N R A(\mathrm{SAA})$
6. $\triangle G Q K$ (ASA or SAA)
7. Yes, $\triangle Q R S \cong \triangle M O L$ by SSS.
8. No, corresponding sides $\overline{T V}$ and $\overline{W V}$ are not congruent.
9. All triangles will be congruent by ASA. Possible triangle:

10. All triangles will be congruent by SAA. Possible procedure: Use $\angle A$ and $\angle C$ to construct $\angle B$ and then copy $\angle A$ and $\angle B$ at the ends of $\overline{A B}$.


## LESSON 4.6•Corresponding Parts of Congruent Triangles

1. SSS, SAS, ASA, SAA
2. $\overline{Y Z} \| \overline{W X}$, AIA Conjecture
3. $\overline{W Z} \| \overline{X Y}$, AIA Conjecture
4. ASA
5. СРСТС
6. $\triangle Y W M \cong \triangle Z X M$ by SAS. $\overline{Y W} \cong \overline{Z X}$ by CPCTC.
7. $\triangle A C D \cong \triangle B C D$ by SAS. $\overline{A D} \cong \overline{B D}$ by CPCTC.
8. Possible answer: $D E$ and $C F$ are both the distance between $\overleftrightarrow{D C}$ and $\overleftrightarrow{A B}$. Because the lines are parallel, the distances are equal. So, $\overline{D E} \cong \overline{C F}$.
9. Possible answer: $\overline{D E} \cong \overline{C F}$ (see Exercise 8 ). $\angle D E F \cong \angle C F E$ because both are right angles, $\overline{E F} \cong \overline{F E}$ because they are the same segment. So, $\triangle D E F \cong \triangle C F E$ by SAS. $\overline{E C} \cong \overline{F D}$ by CPCTC.
10. Possible answer: It is given that $T P=R A$ and $\angle P T R \cong \angle A R T$, and $\overline{T R} \cong \overline{R T}$ because they are the same segment. So $\triangle P T R \cong \angle A R T$ by SAS and $\overline{T A} \cong \overline{R P}$ by CPCTC.

## LESSON 4.7•Flowchart Thinking

1. (See flowchart proof at bottom of page 101.)
2. (See flowchart proof at bottom of page 101.)
3. (See flowchart proof at bottom of page 101.)

## LESSON 4.8• Proving Special Triangle Conjectures

1. $A D=8$
2. $m \angle A C D=36^{\circ}$
3. $m \angle B=52^{\circ}, C B=13$
4. $m \angle E=60^{\circ}$
5. $A N=17$
6. Perimeter $A B C D=104$
7. (See flowchart proof at bottom of page 102.)
8. Flowchart Proof


## LESSON 5.1• Polygon Sum Conjecture

1. $a=103^{\circ}, b=103^{\circ}, c=97^{\circ}, d=83^{\circ}, e=154^{\circ}$
2. $a=92^{\circ}, b=44^{\circ}, c=51^{\circ}, d=85^{\circ}, e=44^{\circ}, f=136^{\circ}$
3. $170^{\circ}$; 36 sides 4. 15 sides
4. $x=105^{\circ}$
5. $x=18^{\circ}$
6. $m \angle E=150^{\circ}$


## LESSON 5.2•Exterior Angles of a Polygon

1. 12 sides
2. 24 sides
3. 4 sides
4. 6 sides
5. $a=64^{\circ}, b=138 \frac{2}{3}^{\circ}$
6. $a=102^{\circ}, b=9^{\circ}$
7. $a=156^{\circ}, b=132^{\circ}, c=108^{\circ}$
8. $a=135^{\circ}, b=40^{\circ}, c=105^{\circ}, d=135^{\circ}$
9. 



Lesson 4.7, Exercises 1, 2, 3
1.

2.

3.


