COMPUTERS & STRUCTURES, INC.

STRUCTURAL AND EARTHQUAKE ENGINEERING SOFTWARE

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Integrated Building Design Software

Post-Tensioned Slab Design Manual

Post-Tensioned Concrete Slab Design Manual

For ETABS® 2016

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Chapter 1
Introduction

1.1 Overview

Part I of this manual describes the methodology and design algorithms performed by ETABS for the analysis and design of post-tensioned structural slabs. It presents the methods used by ETABS to model tendon objects, prestress losses, post-tensioning loads, and the automation of tendon layouts.

There are two possible ways to apply prestressing to concrete, namely, post-tensioning and pre-tensioning. ETABS considers only the post-tensioning of slabs. The post-tensioning tendons may be bonded or unbonded.

1.2 Post-Tensioning System in ETABS

In ETABS, tendon elements are used to provide the post-tensioning. Tendons can be placed anywhere and in any plan direction (see Chapter 5). Each tendon consists of a specific number of strands. Figure 1-1 provides a schematic of the aspects involved in including post-tensioning, from material definition through to detailed output.

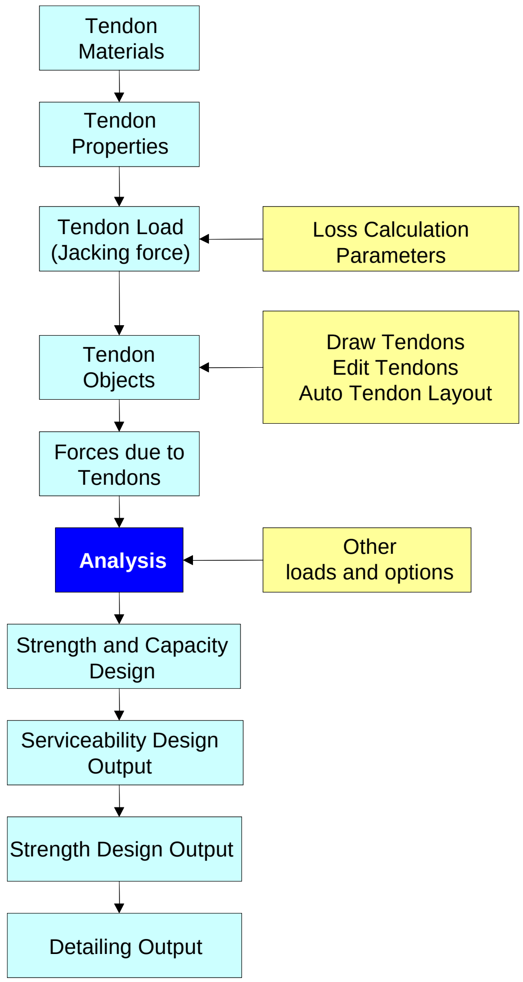


Figure 1-1 Schematic of post-tensioning system and process

Specific analysis and design procedures used in ETABS are intended to comply with the relevant design codes, as presented in Part II of this manual.

1.3 Definition of Terms

Terms used in this manual, within the context of prestressed concrete, are as follows:

Prestressed Concrete - This term refers to concrete that has been pre-compressed, often before application of other loads, and in this manual refers to post-tensioning only.

Post-Tensioning - A procedure in which the steel tendons are tensioned after the concrete has been cast.

Tendon Object - Consists of a number of high-strength steel wires or strands enveloped by a duct, placed anywhere in the slab.

Post-Tensioning Loads - The forces that the tendon exerts on the structure. This includes both the vertical loads due to tendon profile and end forces due to anchorage of the tendon. The force due to friction loss is uniformly distributed along the length of the tendon.

Self Weight - Weight of the structure due to gravity, computed automatically by ETABS from object dimensions and specified density of materials.

1.4 Analysis and Design Procedure

After a ETABS model has been completed and all of the material property and section property definitions, model geometry (including tendon layouts, profiles, and jacking force assignments), member assignments, and loading criteria have been specified, an analysis is ready to be performed.

During the analysis phase, ETABS will calculate reactions, member displacements, slab forces, and slab stresses for all specified load patterns and combinations. ETABS then performs a design in accordance with the specified design code and calculates the required amount of mild steel reinforcement and carries out the appropriate punching shear checks.

ETABS automates several slab and mat design tasks. Specifically, it integrates slab design moments across design strips and designs the required reinforcement, and it checks slab punching shear around column supports and concentrated loads. The actual design algorithms vary based on the specific design code chosen by the user. Part II of this manual describes the algorithms used for the various codes.

It should be noted that the design of post-tensioned reinforced concrete slabs is a complex subject and the design codes cover many aspects of this process. ETABS is a tool to help the user in this process. Only the aspects of design documented in this manual are automated by ETABS design capabilities. The user must check the results produced and address other aspects not covered by ETABS.

Chapter 2
The Tendon Object in ETABS

2.1 Overview

Tendons are a special type of object that can be embedded in concrete elements to represent the effect of post-tensioning. These tendon objects pass through slab objects, attach to them, and impose loads upon them. The tendons are modeled as independent elements.

Any number of tendons may be defined. Each tendon is drawn or defined as a type of line object between two joints, **i** and **j**. The two joints must not share the same location in space. The two ends of the tendon are denoted end I and end J, respectively. The tendon may have an arbitrary curved or segmented shape in three dimensions between those points.

2.2 Tendon Geometry

The vertical profile of a tendon can be defined or modified using the form shown in Figure 2-1.

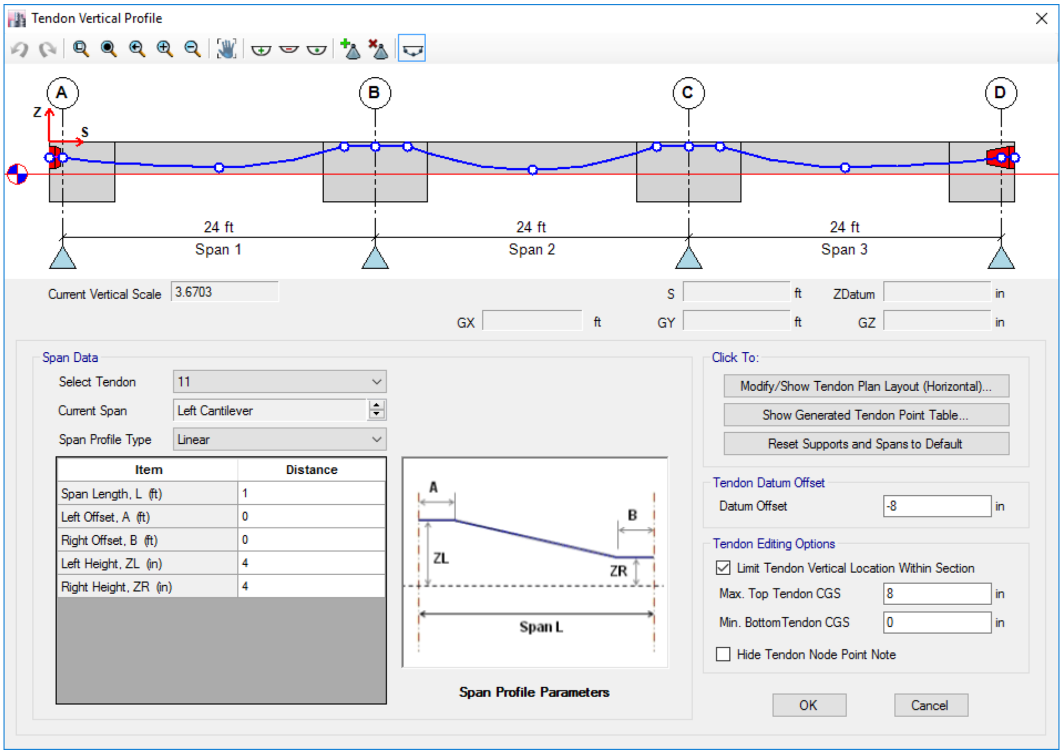


Figure 2-1 Tendon Vertical Profile form, use to define or modify the tendon profile

If a vertical profile is not specified, ETABS will provide a default profile using the maximum drapes allowed by the clearance conditions specified for the slab top and bottom. The automated tendon layout capabilities also automate the tendon profile, as described in Chapter 5.

2.3 Tendon Discretization

A tendon may be a long object with complicated geometry, but internally, it will be discretized automatically into shorter segments for the purposes of analysis. The maximum length of these discretization segments is specified as the maximum mesh size using the **Analyze menu** > **Automatic Mesh Settings for Floors** command. These lengths can affect how the tendons load the structure and the accuracy of the analysis results. It is recommended that shorter lengths be used for tendons with highly curved geometry or for tendons that pass through parts of the structure with complicated geometry or changes in properties. If unsure what value to use, try several different lengths to evaluate the effect on the results.

2.4 Tendon Material Property

The material properties for tendons are defined in terms of the weight density, modulus of elasticity (E), minimum yield stress (f_y), and minimum tensile stress (f_u). Use the **Define menu** > **Materials** command, **Add New Material** button, and the form shown in Figure 2-2 to specify the tendon material properties. Multiple properties can be specified if necessary.

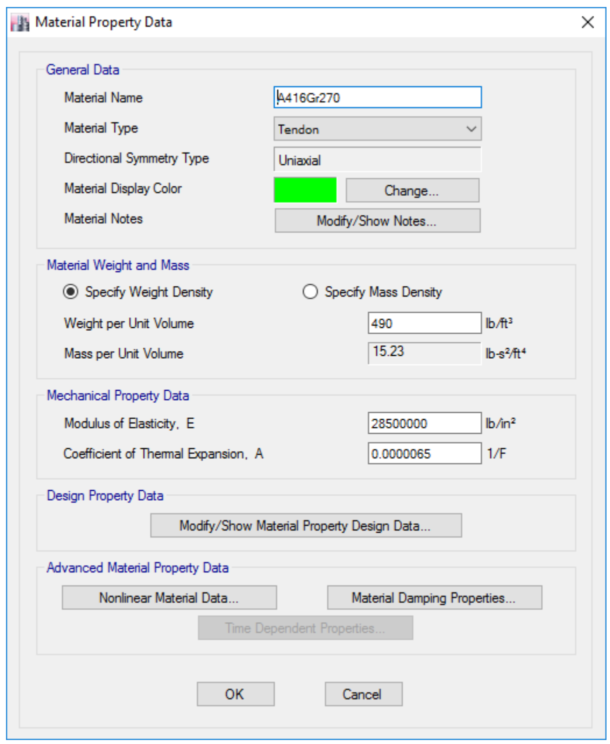


Figure 2-2 Material Property Data form

2.5 Tendon Property

The tendon property contains the strand area and tendon material type. Since tendons can represent single or multiple strands, the area of only a single strand should be specified in the *Tendon Property Data* form, shown in Figure 2-3, which is accessed using the **Define menu > Tendon Properties** command and the **Add Property** button. The number of strands is specified when assigning tendon properties or editing a tendon (refer to the respective Assign or Edit menu command).

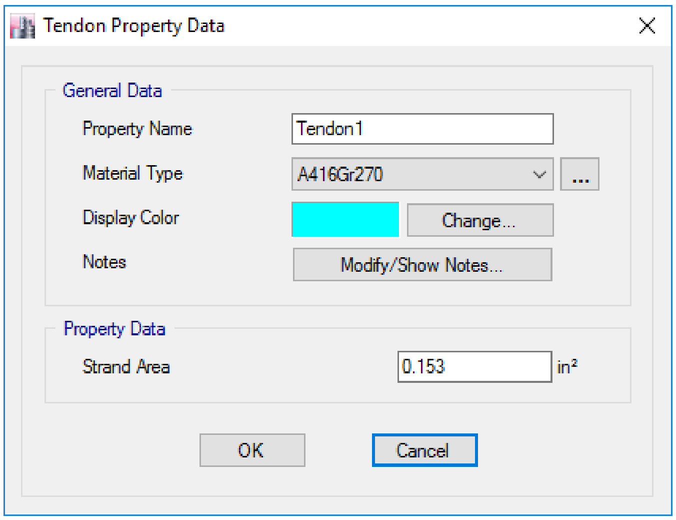


Figure 2-3 Tendon Property Data form

2.6 Tendon Loads

After the tendons have been added to the ETABS model, tendon loads can be specified. Loads can be assigned to a single tendon or multiple tendons by first selecting the tendons to be loaded, selecting the **Assign menu > Tendon Loads > Tendon Loads** command, and then modifying the data in the form shown in Figure 2-4.

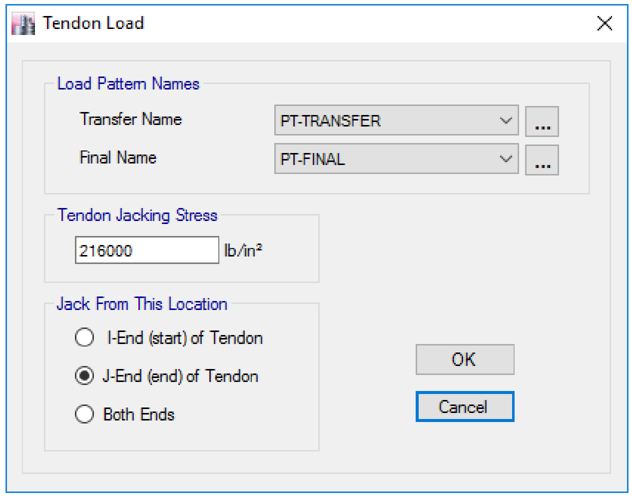


Figure 2-4 Tendon Load form

The load pattern names, jacking locations, and tendon jacking stress are defined in this form. The tendon load (jacking stress) is the total load applied to one or both ends of the tendon. The actual tendon force will vary along the length of the tendon as governed by the frictional and other loss parameters.

Tendon losses can be assigned to a single tendon or multiple tendons by first selecting the tendons, selecting the **Assign menu** > **Tendon Loads** > **Tendon Losses** command and then modifying the data in the form shown in Figure 2-5.

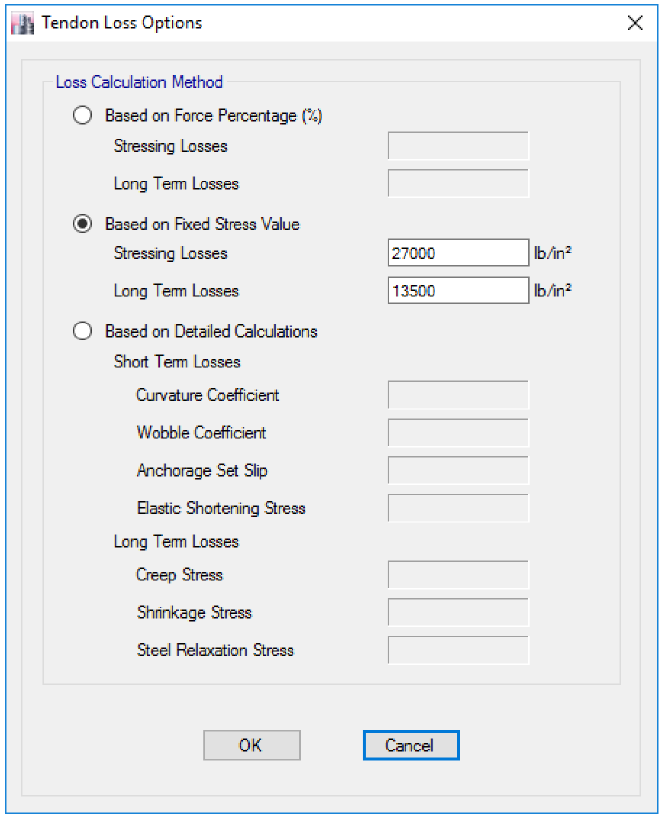


Figure 2-5 Tendon Losses form

Chapter 3
Computing Prestress Losses

3.1 Overview

The tendon load for a given load case refers to the user-defined jacking force. The actual load that is applied to slabs will be less than the jacking force because of prestress losses. The prestress losses are categorized in ETABS into short-term losses and long-term losses, as follows:

Short-term or Stressing losses - These are losses that occur during and immediately after the post-tensioning operations and are caused by friction between the tendons and the duct, elastic shortening, and seating of anchors.

Long-term losses - These types of losses happen over time and also may be referred to as time-dependent losses and include creep, shrinkage, and steel relaxation.

Using the **Assign menu** > **Tendon Loads** > **Tendon Losses** command displays the form shown in Figure 3-1 and allows the prestress losses to be specified using one of three methods.

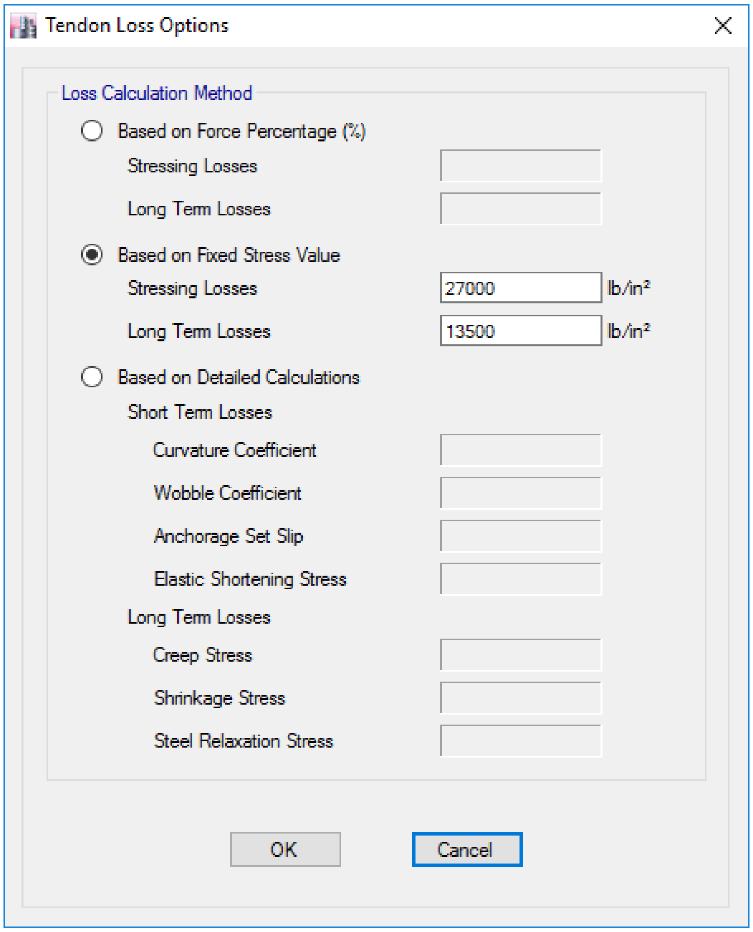


Figure 3-1 Tendon Load form

The first two Loss Calculation Methods on the form can be used to specify the prestress losses as a force percentage or fixed stress value for the Stressing Losses and Long-Term Losses. The third option allows a more detailed calculation of the prestress losses based on a number of input values for both Short-Term and Long-Term Losses. Frictional losses are computed internally and explicitly by ETABS based on the specified wobble and curvature coefficients. All other losses are directly input on this form.

Other factors, such as changes in temperature and flexing of the structure under loading, do not significantly lower the prestress level and are not considered explicitly.

Understanding the stress distribution along the length of a member with respect to short-term or long-term effects is important for correctly analyzing the model and interpreting the results. The prestress losses are evident in terms of the stress distribution along the length, as shown in Figure 3-2. The actual variation in stress varies exponentially in accordance with Eqn 3.1 in the following section.

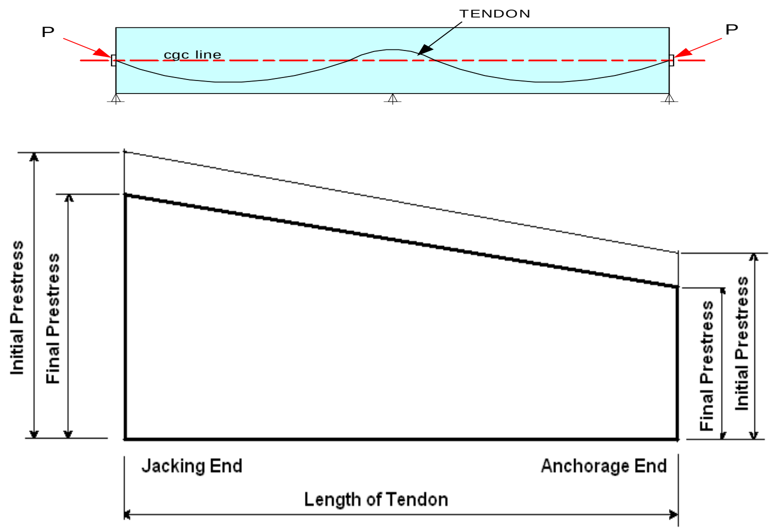


Figure 3-2 Prestress load variation along tendon length

The jacking stress is commonly specified as $0.80f_{pu}$, where f_{pu} is the specified ultimate strength of the strand. Figure 3-2 shows a representation of the tendon force variation with the tendon jacked from the left end. If the tendon were to be jacked from the right end, Figure 3-2 would be reversed. If the tendon were jacked from both ends, the maximum initial prestress force (jacking force) would exist at each end and would vary to a minimum value midway along the length of the tendon. The initial prestress forces are reduced to the final prestress forces in accordance with the long-term losses specified and shown diagrammatically as the Final Prestress in Figure 3-2.

3.2 Computation of Short-Term Losses

3.2.1 Stress Loss Due to Friction (Curvature and Wobble)

When "Based on Detailed Calculations" is the Loss Calculation Method selected, the frictional losses are calculated using the curvature and wobble coefficients specified by the user. The frictional loss due to curvature is calculated in ETABS as:

$$P_{(X)} = P_0 e^{-(\mu\alpha + Kx)}, \text{ where} \qquad \text{(Eqn. 3.1)}$$

μ = curvature friction coefficient

α = sum of the tendon angular change from the tendon jacking end to a distance x

K = wobble friction coefficient (*rad/unit length*^{2})

$P_{(X)}$ = Post-tensioning force at a distance x

P_0 = Post-tensioning force at stressing

The post-tensioning losses due to friction result in a force distribution along the length of the tendon that is exponentially decreasing from the jacking point.

In the empirical coefficient, K is the cumulative effect of the rigidity of the sheathing, the diameter of the sheathing, the spacing of the sheath supports (Figure 3-3), the tendon type, and the sheath type, including the form of construction.

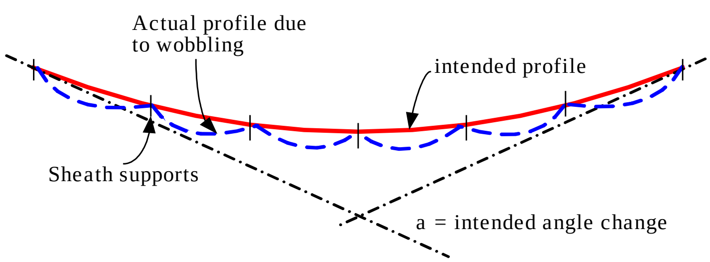


Figure 3-3 Wobble friction loss

3.2.2 Anchorage Set Slip Losses

At the last stage of the stressing operation, the tendons usually are anchored with two-piece conical wedges. Anchoring operations normally result in an additional prestress loss due to seating of the wedges, considering that the strand retracts when it is released and pulls the wedges into the anchoring device.

Calculation of the stress losses is typically performed in an iterative manner. As shown in Figure 3-4, the distance "c" refers to the extent of influence of an anchor set. Procedurally, anchor set is chosen first (usually about 0.25 to 0.375 in or 6 to 8 mm), then the distance "c" is set, and finally the corresponding stress is computed, with the assumption that the stresses vary linearly from the jacking point.

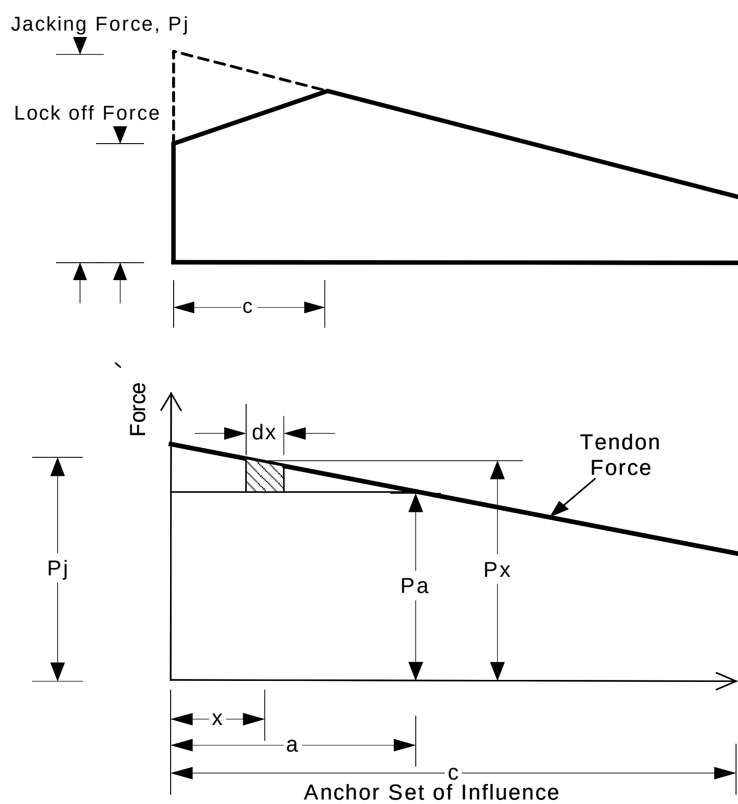


Figure 3-4 Anchor set influence distance diagram

The seating loss is then calculated using the following equation:

$$SL \approx a = \frac{\int (\sigma_j - \sigma_x) dx}{E_s} \quad \text{(Eqn. 3.2)}$$

The iteration process stops when the calculated seating loss is almost equal to the anchor set "*a*"; then the maximum stress is calculated, as follows:

$$\sigma_{\max} = \sigma_j - (\sigma_j - \sigma_x) \quad \text{(Eqn. 3.3)}$$

Further, the elongation shall be calculated as follows:

$$\Delta_a = \frac{\int (P_x - P_a) dx}{AE_s} \quad \text{(Eqn. 3.4)}$$

where Δ_a is the elongation associated with the assumed anchor set distance "*a*"; P_x is the tendon force at a distance *x* from the jacking point; P_a is the force in the tendon under jacking stress at the assumed anchor set distance "*a*"; *dx* is the length of the elements along the tendon; *A* is the cross-sectional area of the tendon; and E_s is the modulus of elasticity of the tendon material.

3.2.3 Elastic Shortening of Concrete

Elastic shortening refers to the shortening of the concrete as the post-tensioning force is applied. As the concrete shortens, the tendon length also shortens, resulting in a loss of prestress. If sequential jacking steps are used, the first tendon jacked and locked off will suffer the maximum amount of loss from elastic shortening. Conversely, there will be no loss because of elastic shortening for the last tendon in a sequence or in a single tendon because the elastic shortening will take place before the tendon is locked into the anchoring device. The user-specified amount of prestress loss from elastic shortening is applied uniformly over the entire length of the tendon.

3.3 Computation of Long-Term Losses

The long-term prestress losses of a member include creep, shrinkage, and steel relaxation effects.

Several methods can be used to determine the long-term stress losses; however, ETABS relies on the user-defined values input in the *Tendon Losses* form shown in Figure 3-1. Lump sum values input into ETABS should reflect the appropriate conditions that exist for the structure being modeled. Creep, shrinkage, and steel relaxation effects are governed by material properties and, in some cases, other environmental conditions that need to be accounted for when specifying the long-term loss values. Each stress loss is treated separately and then summed up, as follows:

$$TL = CR + SH + RE \qquad \text{(Eqn. 3.7)}$$

where TL is the total loss of stress; CR is the stress loss due to creep of the concrete; SH is the stress loss due to shrinkage of the concrete; and RE is the stress loss due to relaxation in the tendon steel. The sum of these losses is applied to the initial (jacking) load of the tendon, as represented in Figure 3-2. All of the long-term losses are uniformly applied over the length of the tendon.

Chapter 4
Loads Due to Post-Tensioning

4.1 Overview

ETABS does not rely on an approximate ‘equivalent loading’ method for calculating member responses subjected to post-tensioning loads. Instead, ETABS uses a finite element method that includes the tendon effects as a load. When a parabolic drape is specified for the tendon, ETABS performs a numerical integration across the finite element using the actual parabolic shape function that defines the tendon’s geometry. This approach is considered to be more accurate, especially when deeper members are being considered.

One of the consequences of applying a post-tensioning load to a member is the introduction of secondary (hyperstatic) forces. These effects and load cases are discussed in this chapter.

ETABS uses the dead load balancing method as the primary procedure for the determination of tendon profiles when they are requested to be automated (see Chapter 5). This chapter also provides information regarding the approach used to perform a load balanced design.

4.2 Dead Load-Balancing

The dead load balancing method is used in ETABS to determine an initial tendon layout (including the profile, number of strands, and the jacking force) when the automated tendon layout feature is used. The basic concept of dead load balancing is that the prestress bending stresses, $f = Pe/S$, are equal but opposite to the applied dead load bending stresses, $f = Mc/I$. When the Self Load Balancing Ratio and the Precompression Level in the *Quick Tendon Layout* form, shown in Figure 4-1, are specified, ETABS iterates the position of the tendon as necessary to find the eccentricity, *e*, that balances the specified dead load stresses.

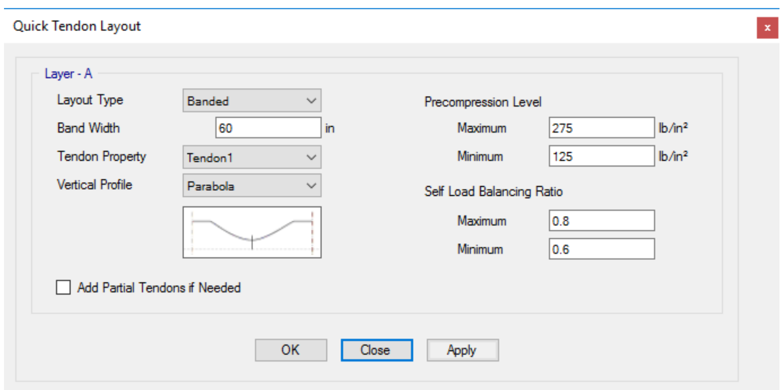


Figure 4-1 Quick Tendon Layout form

The stress diagrams in Figure 4-2 illustrate the dead load balancing concept. The specified precompression limit stress is applied first, (a). Then the dead load stresses are computed, (b), followed by iterating the tendon location to balance the dead load stresses, (c), that finally results in the precompression state shown in (d).

The final stress distribution is the result of this precompression stress combined with the stresses resulting from the application of all remaining loads and design combinations. If the final stress distribution contains tension stresses that exceed the design allowable limit, ETABS calculates the required amount of mild steel reinforcement. Chapter 5 details the steps used by ETABS in the automation of the tendon layout.

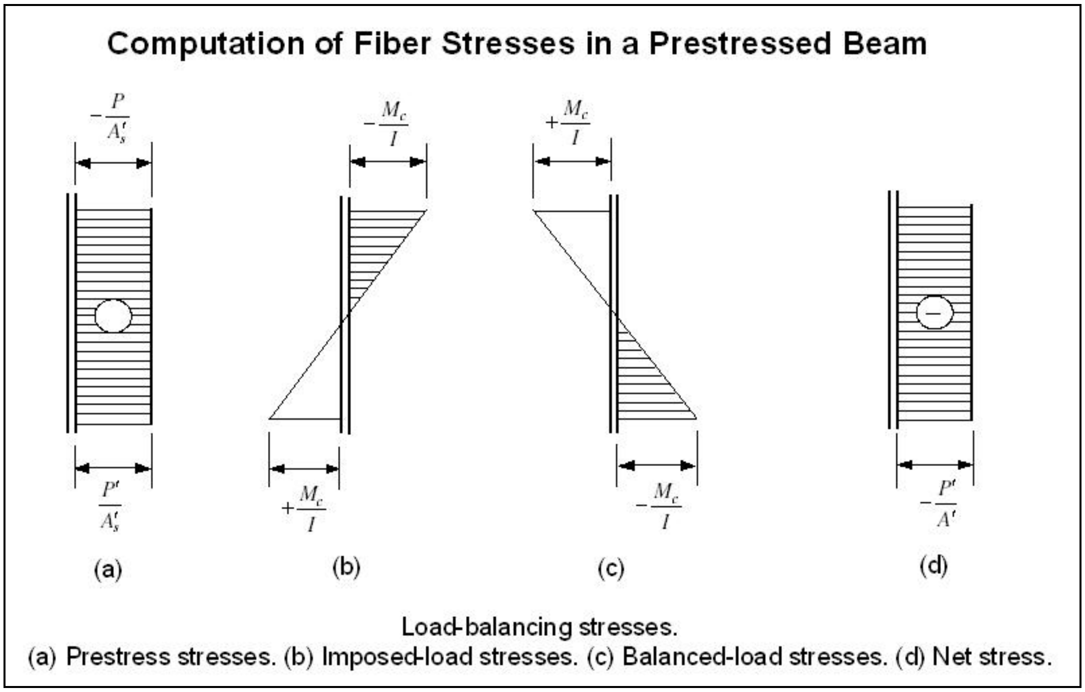


Figure 4-2 Precompression and Load Balancing Stresses

4.3 Primary Moments

If a section cut is made of a uniformly loaded beam, the actions at the cut sections will include the concentric force P_x, a primary moment M_P, and a shear V_x. The primary moment at this section is necessary to maintain equilibrium of the loading, which can be expressed as:

$$M_p = \int (wdx)x + P_L a \qquad \text{(Eqn. 4.1)}$$

where, w, is the intensity of loading at a distance "x," P_L is the vertical component of tendon force at the left anchorage, and a is the distance to the cut section measured from the left anchorage.

Similarly, a free-body diagram of the tendon would show the concentric force P_x and a shear V_x at the cut sections, along with the loading "w." In the same manner, the force P_x taking moments about the CGC line from an eccentricity *e'* or the distance from the tendon's centroid to the neutral axis of the member yields:

$$P_x e' = \int (wdx)x + P_L a \qquad \text{(Eqn. 4.2)}$$

The right-hand sides of Eqns. 4.1 and 4.2 are identical, therefore the primary moment can be defined as:

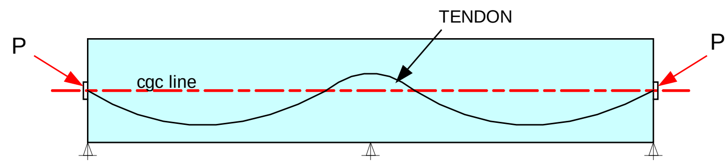
$$M_p = P_x e' \qquad \text{(Eqn. 4.3)}$$

4.4 Secondary (Hyperstatic) Moments

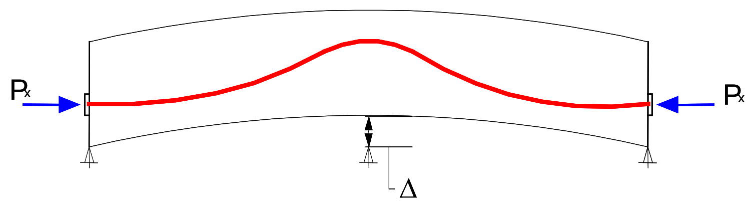
The reactions caused by the post-tensioning forces in continuous slabs or beams are often referred to as secondary (hyperstatic) reactions. The two-span beam shown in Figure 4-3 illustrates the reactions and moments because of the eccentric post-tensioning forces.

If the center support is eliminated for the two-span beam shown in Figure 4-3, the application of the post-tensioning would result in a beam upward displacement of Δ. The application of the force necessary to displace the beam by the amount, $-\Delta$, can be represented as, R_i. From Figure 4-3 (d) and (e), the hyperstatic reactions in the amount $R_i/2$ are produced at each end of the beam and the hyperstatic moment M is produced over the center support. At any section along the beam, the hyperstatic reactions induce a hyperstatic moment M_{hyp} and a hyperstatic shear V_{hyp}.

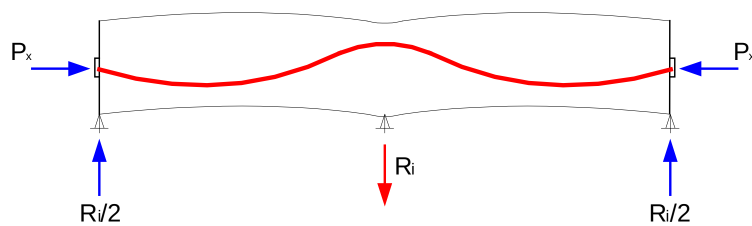
Hyperstatic analysis results can be reviewed by defining a hyperstatic load case using the **Define menu > Load Cases** command to add a new load case with a hyperstatic Load Case Type, as shown in Figure 4-4.



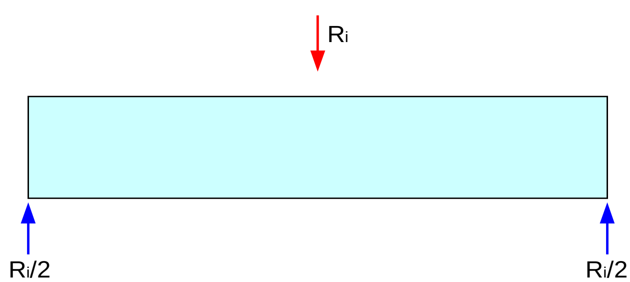
(a) Two-span post-tensioned beam



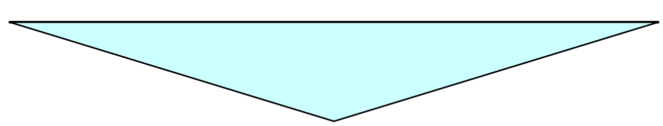
(b) Tendon forces cause the beam to lift off the center support with a deflection Δ upward



(c) Additional hyperstatic reactions develop at the ends due to application of the force, R_i, which is needed to prevent the beam from lifting off the support



(d) Secondary (hyperstatic) reaction R_i in a theoretical, simply supported beam



(e) Secondary (hyperstatic) moment diagram due to R_i

Figure 4-3 Secondary (hyperstatic) actions due to post-tensioning

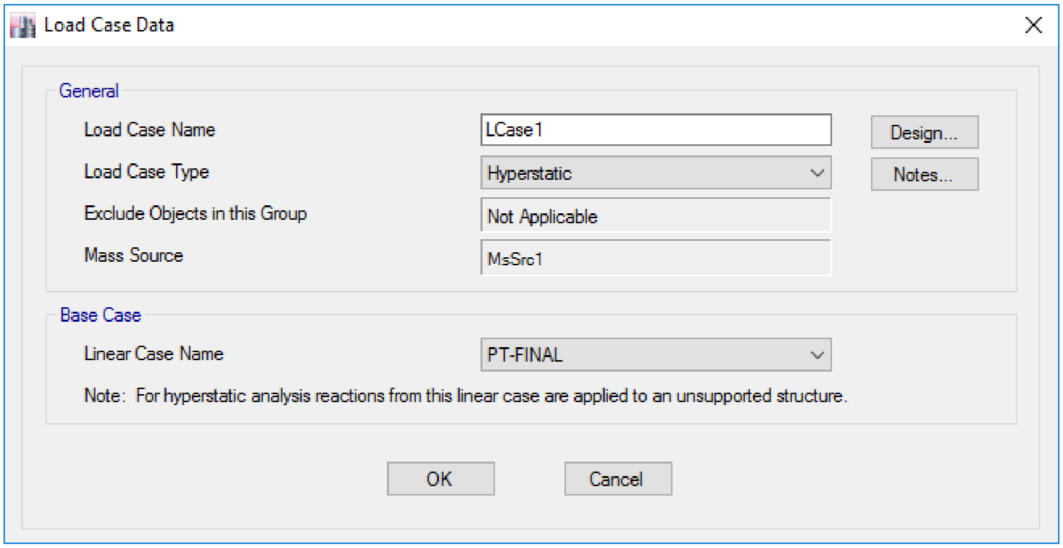


Figure 4-4 Hyperstatic Load Case Data form

In the design process, the secondary moment is assumed to be resisted through a compression block and a tensile force such that:

$$C = T \qquad \text{(Eqn. 4.4)}$$

$$M_{\text{sec}} = Tz = Cz \qquad \text{(Eqn. 4.7)}$$

where C is the total compression force, T is the combined tension force due to post-tensioning tendons and mild reinforcement, and Z is the lever arm of the section, as illustrated in Figure 4-5.

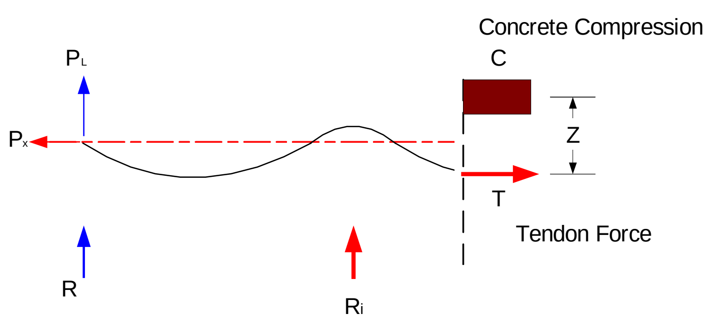


Figure 4-5 Section actions due to post-tensioning and internal distribution of hyperstatic forces

Thus, the combination of forces stipulated in most design codes for gravity conditions simply considers the addition of the hyperstatic effect to the combinations used for non-prestressed concrete.

Chapter 5
Automated Tendon Layout

5.1 Overview

In the past, the analysis and design of post-tensioned floor slabs has been difficult because of the high degree of indeterminacy of the structure, large number of design requirements, and the need to provide an economical design. Some analysis programs rely on simplified approximations in the analysis and the design. ETABS eliminates the need for engineers to oversimplify an analysis model and provides the tools to automate the tendon layout, profile, and jacking forces.

This chapter describes the various methods for adding tendons to a ETABS model and the methodology used to automate the tendon input data. Not all of the methods used to add tendons to a ETABS model are suited for the automation as explained herein.

The automation of tendon layout, profiles, and jacking forces serves as a starting point in the analysis and design process. If it is necessary to make further adjustments to the tendon layout, profiles, or jacking forces, these adjustments should be made manually. ETABS **does not** perform any revision to the initial tendon automations. The parameters related to the tendons can be modified easily, followed by re-analyzing and re-designing the structure as necessary.

5.2 Adding Tendons to a ETABS Model

Four methods are available for adding tendons to a ETABS model:

Template modeling – If a ETABS model is initialized using the **File menu > New Model** command and the appropriate initial model is selected along with toggling the *Add P/T* option, post-tensioning data can be defined. The *Quick Tendon Layout* form shown in Figure 5-1 allows specification of the tendon layout for the Layer A and B directions, as well as the precompression levels and self-load balancing ratios. Tendons with the defined layout parameters are then included in the template model. This can be a quick and easy method to place a large number of tendons into a ETABS model. The tendon profiles satisfy the specified clearances.

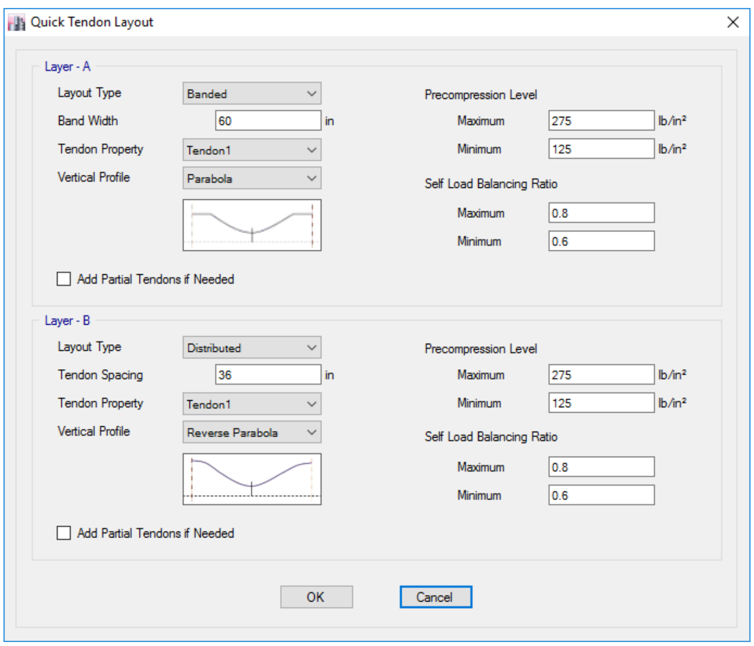


Figure 5-1 Quick Tendon Layout form

Figure 5-2 shows two of several tendon layout options using banded and uniform tendon layout types.

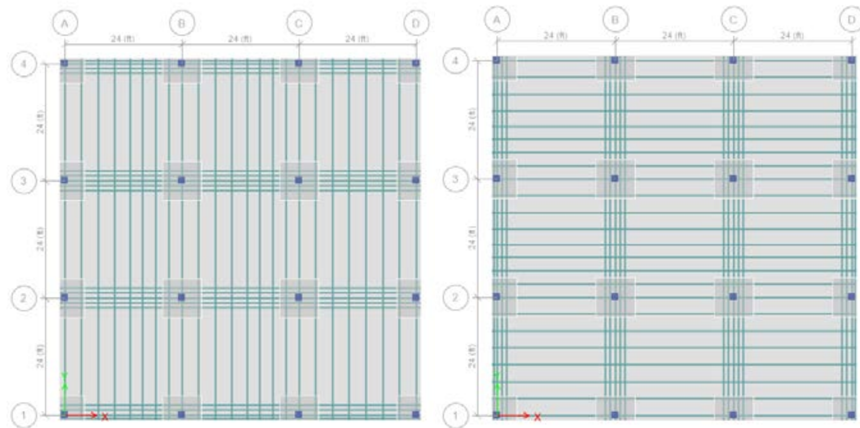


Figure 5-2 Template models with tendon layout options

Tendon Draw commands – Using the **Draw menu > Draw Tendons** command, any number of points can be input to place tendons into a ETABS model. Default tendon profile data is provided; however, it is expected that it will be edited to provide the proper tendon profile and other tendon data as required to satisfy the design requirements. Multiple tendons with the same layout can be generated easily using the **Edit menu > Replicate** command. When this option is used, ETABS replicates the tendon profile of the source tendon.

Note: No automation of the tendon layout, profile, number of strands, or jacking force is performed by ETABS when the **Draw menu > Draw Tendons** command is used to place tendons in a model.

Add Tendons in Strips – The **Edit menu > Add/Edit Tendons > Add Tendons in Strips** command can be used to add tendons to an existing ETABS model. The tendon layouts, profiles, number of strands, and jacking forces are all automated when tendons are added in this manner, based on the input in the *Quick Tendon Layout* form shown in Figure 5-3. The ETABS model can be further modified by adding additional tendons as necessary.

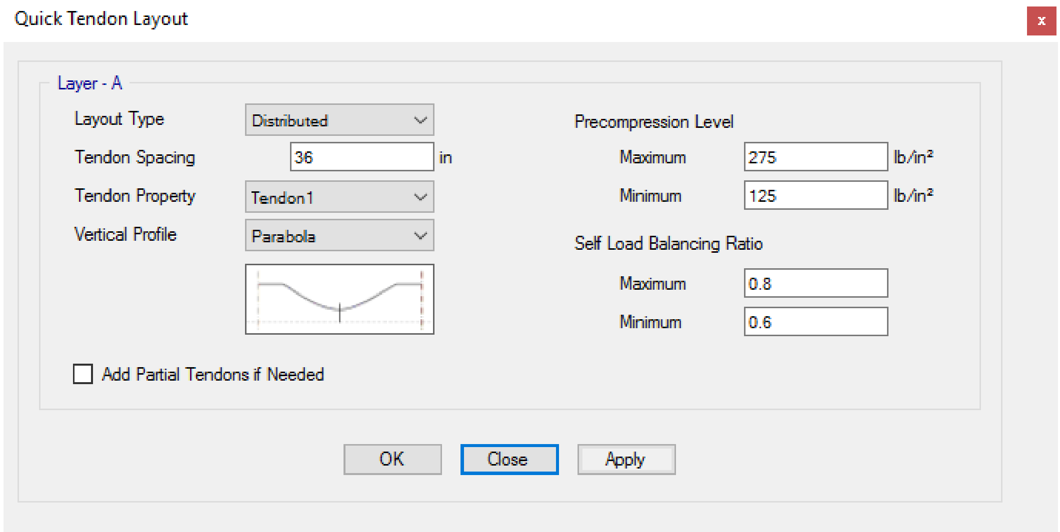


Figure 5-3 Quick Tendon Layout form

5.3 Procedures Used in Automated Tendon Layout

The automated tendon layouts (including profiles, number of strands, and jacking forces) are generated based on the design strip definitions. Automated tendon layouts are developed only on tendons that have been added to design strips. Each strip is modeled as an equivalent continuous beam with the cross-section derived from the slab objects lying within the strip width. The self weight loads are calculated to obtain the load to be used in the load balancing calculation. Only the loads that are applied within the boundary area of a particular strip are included in the determination of the automated tendon layout. As an example, if a column strip is defined as 60 inches wide, only a tributary width of 60 inches is used to determine the load for use in the self load balancing calculation to determine the tendon layout.

A representative tendon is placed in the equivalent beam, centered on the design strip. The supports of the strips are derived from the intersection with perpendicular design strips and by any column supports within the strip width.

Note: ETABS does not automatically consider the intersections of strips and beams to be points of supports for the strips. If it is desired to consider a particular beam as a support point for a strip, then a strip should be defined at the beam location.

The support locations are used to determine the spans. For each span, the tendon profile is automated based on the profile type specified for the tendon (parabola or reverse parabola). An iterative procedure is then used to determine the effective jacking force necessary to satisfy the range of dead load to be balanced and the average precompression stress required. The jacking force is initially calculated to satisfy the minimum required self load balancing ratio and minimum precompression level for the longest span in the strip. The tendon profiles in other spans are then adjusted so as not to exceed the maximum dead load balancing ratios.

A value of 60 to 80 percent is generally used as the self load balancing ratios. Typically precompression levels generally range between 0.125 to 0.275 ksi.

Note: It is important to note that it is possible that an automated tendon layout cannot satisfy the specified dead load balancing ratios and precompression levels. In such cases, ETABS generates a warning so that necessary manual adjustments to the tendon layout and profile can be made, or other modifications to the ETABS model can be applied where required.

Note: If the addition of partial tendons is active, ETABS may add additional tendons in long spans or in exterior spans to satisfy the self load balancing and precompression constraints.

After the total jacking force and profile have been determined for the equivalent tendon, the actual number and spacing of tendons is determined based on the following criteria:

For a banded tendon layout, the number of tendons is initially determined based on the specified Tendon Property (material property and strand area), Precompression Level, and Dead Load Balancing Ratios. The prestress losses are estimated using the Fixed Stress Values from the Tendon Load assignments. If the number of tendons is too large to fit within the band width with a minimum spacing of 12 *in* (300 *mm)*, a larger tendon size is automatically selected by increasing the number of strands. Similarly, if the spacing of the tendons is too large (greater than 60 *in* or 1.5 *m*) or 16 times the slab thickness, a smaller tendon is selected, with fewer strands.

For a uniform tendon layout, a similar procedure as outlined above for the banded tendon layout is used.

Chapter 6
Design for ACI 318-08

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the American code ACI 318-08 [ACI 2008]. Various notations used in this chapter are listed in Table 6-1. For referencing to the pertinent sections of the ACI code in this chapter, a prefix "ACI" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on inch-pound-second units. For simplicity, all equations and descriptions presented in this chapter correspond to inch-pound-second units unless otherwise noted.

6.1 Notations

The following table identifies the various notations used in this chapter.

Table 6-1 List of Symbols Used in the ACI 318-08 Code

A_{cp}	Area enclosed by the outside perimeter of the section, in^2
A_g	Gross area of concrete, in^2
A_l	Total area of longitudinal reinforcement to resist torsion, in^2
A_o	Area enclosed by the shear flow path, sq-in
A_{oh}	Area enclosed by the centerline of the outermost closed transverse torsional reinforcement, sq-in
A_{ps}	Area of prestressing steel in flexural tension zone, in^2
A_s	Area of tension reinforcement, in^2
A'_s	Area of compression reinforcement, in^2
$A_{s(required)}$	Area of steel required for tension reinforcement, in^2
A_t /s	Area of closed shear reinforcement per unit length of member for torsion, sq-in/in
A_v	Area of shear reinforcement, in^2
A_v /s	Area of shear reinforcement per unit length of member, in^2/in
a	Depth of compression block, in
a_b	Depth of compression block at balanced condition, in
$a_{\max}$	Maximum allowed depth of compression block, in
b	Width of member, in
b_f	Effective width of flange (T-beam section), in
b_w	Width of web (T-beam section), in
b_0	Perimeter of the punching critical section, in
b_1	Width of the punching critical section in the direction of bending, in
b_2	Width of the punching critical section perpendicular to the direction of bending, in
c	Depth to neutral axis, in
c_b	Depth to neutral axis at balanced conditions, in

Table 6-1 List of Symbols Used in the ACI 318-08 Code

d	Distance from compression face to tension reinforcement, in
d'	Concrete cover to center of reinforcing, in
d_e	Effective depth from compression face to centroid of tension reinforcement, in
d_s	Thickness of slab (T-beam section), in
d_p	Distance from extreme compression fiber to centroid of prestressing steel, in
E_c	Modulus of elasticity of concrete, psi
E_s	Modulus of elasticity of reinforcement, assumed as 29,000,000 psi (ACI 8.5.2)
f'_c	Specified compressive strength of concrete, psi
f'_{ci}	Specified compressive strength of concrete at time of initial prestress, psi
f_{pe}	Compressive stress in concrete due to effective prestress forces only (after allowance of all prestress losses), psi
f_{ps}	Stress in prestressing steel at nominal flexural strength, psi
f_{pu}	Specified tensile strength of prestressing steel, psi
f_{py}	Specified yield strength of prestressing steel, psi
f_t	Extreme fiber stress in tension in the precompressed tensile zone using gross section properties, psi
f_y	Specified yield strength of flexural reinforcement, psi
f_{ys}	Specified yield strength of shear reinforcement, psi
h	Overall depth of a section, in
h_f	Height of the flange, in
ϕM_n^0	Design moment resistance of a section with tendons only, N-mm

Table 6-1 List of Symbols Used in the ACI 318-08 Code

ϕM_n^{bal}	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
M_u	Factored moment at section, lb-in
N_c	Tension force in concrete due to unfactored dead load plus live load, lb
P_u	Factored axial load at section, lb
s	Spacing of the shear reinforcement along the length of the beam, in
T_u	Factored torsional moment at section, lb-in
V_c	Shear force resisted by concrete, lb
$V_{\max}$	Maximum permitted total factored shear force at a section, lb
V_u	Factored shear force at a section, lb
V_s	Shear force resisted by steel, lb
β_1	Factor for obtaining depth of compression block in concrete
β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
$\varepsilon_{c,\max}$	Maximum usable compression strain allowed in extreme concrete fiber (0.003 in/in)
ε_{ps}	Strain in prestressing steel
ε_s	Strain in reinforcing steel
$\varepsilon_{s,\min}$	Minimum tensile strain allowed in steel reinforcement at nominal strength for tension controlled behavior (0.005 in/in)
φ	Strength reduction factor
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear

Table 6-1 List of Symbols Used in the ACI 318-08 Code

λ	Shear strength reduction factor for light-weight concrete
θ	Angle of compression diagonals, degrees

6.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For ACI 318-08, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the load combinations in the following sections may need to be considered (ACI 9.2.1).

For post-tensioned concrete design, the user can specify the prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning are defined in the following sections.

6.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress forces, in accordance with ACI 318-08 clause 18.4.1. The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0D + 1.0PT (ACI 18.4.1)

6.2.2 Service Load Combination

The following load combinations are used for checking the requirements of prestress for serviceability in accordance with ACI 318-08 clauses 18.3.3, 18.4.2(b), and 18.9.3.2. It is assumed that all long-term losses have already occurred at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT (ACI 18.4.2(b))

6.2.3 Long-Term Service Load Combination

The following load combinations are used for checking the requirements of pre-stress in accordance with ACI 318-08 clause 18.4.2(a). The permanent load for this load combination is taken as 50 percent of the live load. It is assumed that all long-term losses have already occurred at the service stage.

1.0D + 1.0PT
1.0D + 0.5L + 1.0PT (ACI 18.4.2(b))

6.2.4 Strength Design Load Combination

The following load combinations are used for checking the requirements of pre-stress for strength in accordance with ACI 318-08, Chapters 9 and 18.

The strength design combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are automatically determined by ETABS by subtracting out the primary PT moments when the flexural design is carried out.

1.4D + 1.0PT*	(ACI 9.2.1)
1.2D + 1.6L + 1.0PT*	(ACI 9.2.1)
1.2D + 1.6(0.75 PL) + 1.0PT*	(ACI 9.2.1, 13.7.6.3)
0.9D ± 1.6W +1.0PT* 1.2D + 1.0L ± 1.6W + 1.0PT*	(ACI 9.2.1)
0.9D ± 1.0E + 1.0PT* 1.2D + 1.0L ± 1.0E + 1.0PT*	(ACI 9.2.1)
1.2D + 1.6L + 0.5S + 1.0PT* 1.2D + 1.0L + 1.6S + 1.0PT*	(ACI 9.2.1)
1.2D + 1.6S ± 0.8W + 1.0PT* 1.2D + 1.0L + 0.5S ± 1.6W + 1.0PT*	(ACI 9.2.1)
1.2D + 1.0L + 0.2S ± 1.0E + 1.0PT*	(ACI 9.2.1)

* — Replace PT by H for flexural design only

The IBC 2006 basic load combinations (Section 1605.2.1) are the same. These also are the default design load combinations in ETABS whenever the ACI 318-08 code is used. The user should use other appropriate load combinations if roof live load is treated separately, or if other types of loads are present.

6.3 Limits on Material Strength

The concrete compressive strength, f'_c, should not be less than 2500 psi (ACI 5.1.1). The upper limit of the reinforcement yield strength, f_y, is taken as 80 ksi (ACI 9.4) and the upper limit of the reinforcement shear strength, f_{yt}, is taken as 60 ksi (ACI 11.5.2).

ETABS enforces the upper material strength limits for flexure and shear design of slabs. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

6.4 Strength Reduction Factors

The strength reduction factors, ϕ, are applied on the specified strength to obtain the design strength provided by a member. The ϕ factors for flexure, shear, and torsion are as follows:

ϕ_t = 0.90 for flexure (tension controlled) (ACI 9.3.2.1)

ϕ_c = 0.65 for flexure (compression controlled) (ACI 9.3.2.2(b))

ϕ = 0.75 for shear and torsion. (ACI 9.3.2.3)

The value of ϕ varies from compression-controlled to tension-controlled based on the maximum tensile strain in the reinforcement at the extreme edge, ε_t (ACI 9.3.2.2).

Sections are considered compression-controlled when the tensile strain in the extreme tension reinforcement is equal to or less than the compression-controlled strain limit at the time the concrete in compression reaches its assumed strain limit of $\varepsilon_{c.\max}$, which is 0.003. The compression-controlled strain limit is

the tensile strain in the reinforcement at the balanced strain condition, which is taken as the yield strain of the reinforcement, (f_y/E) (ACI 10.3.3).

Sections are tension-controlled when the tensile strain in the extreme tension reinforcement is equal to or greater than 0.005, just as the concrete in compression reaches its assumed strain limit of 0.003 (ACI 10.3.4).

Sections with ε_t between the two limits are considered to be in a transition region between compression-controlled and tension-controlled sections (ACI 10.3.4).

When the section is tension-controlled, ϕ_t is used. When the section is compression-controlled, ϕ_c is used. When the section is in the transition region, ϕ is linearly interpolated between the two values (ACI 9.3.2).

The user is allowed to overwrite these values. However, caution is advised.

6.5 Design Assumptions for Prestressed Concrete

Strength design of prestressed members for flexure and axial loads shall be based on assumptions given in ACI 10.2.

- The strain in the reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis (ACI 10.2.2).
- The maximum usable strain at the extreme concrete compression fiber shall be assumed equal to 0.003 (ACI 10.2.3).
- The tensile strength of the concrete shall be neglected in axial and flexural calculations (ACI 10.2.5).
- The relationship between the concrete compressive stress distribution and the concrete strain shall be assumed to be rectangular by an equivalent rectangular concrete stress distribution (ACI 10.2.7).
- The concrete stress of $0.85f'_c$ shall be assumed uniformly distributed over an equivalent-compression zone bounded by edges of the cross-section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain (ACI 10.2.7.1).

- The distance from the fiber of maximum strain to the neutral axis, c shall be measured in a direction perpendicular to the neutral axis (ACI 10.2.7.2).

Elastic theory shall be used with the following two assumptions:

- The strains shall vary linearly with depth through the entire load range (ACI 18.3.2.1).
- At cracked sections, the concrete resists no tension (ACI 18.3.2.1).

Prestressed concrete members are investigated at the following three stages (ACI 18.3.2):

- At transfer of prestress force
- At service loading
- At nominal strength

The prestressed flexural members are classified as Class U (uncracked), Class T (transition), and Class C (cracked) based on f_t, the computed extreme fiber stress in tension in the precompressed tensile zone at service loads (ACI 18.3.3).

The precompressed tensile zone is that portion of a prestressed member where flexural tension, calculated using gross section properties, would occur under unfactored dead and live loads if the prestress force was not present. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress.

For Class U and Class T flexural members, stresses at service load are determined using uncracked section properties, while for Class C flexural members, stresses at service load are calculated based on the cracked section (ACI 18.3.4).

A prestressed two-way slab system is designed as Class U only with $f_t \leq 6\sqrt{f'_c}$ (ACI R18.3.3); otherwise, an over-stressed (O/S) condition is reported.

The following table provides a summary of the conditions considered for the various section classes.

	Prestressed	

	Class U	Class T	Class C	Nonprestressed
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 18.3.4	Gross section 18.3.4	Cracked section 18.3.4	No requirement
Allowable stress at transfer	18.4.1	18.4.1	18.4.1	No requirement
Allowable compressive stress based on uncracked section properties	18.4.2	18.4.2	No requirement	No requirement
Tensile stress at service loads 18.3.3	$\leq 7.5\sqrt{f'_c}$	$7.5\sqrt{f'_c} < f_t \leq 12\sqrt{f'_c}$	No requirement	No requirement

6.6 Serviceability Requirements of Flexural Members

6.6.1 Serviceability Check at Initial Service Load

The stresses in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits:

- Extreme fiber stress in compression: $0.60 f'_{ci}$ (ACI 18.4.1(a))
- Extreme fiber stress in tension: $3\sqrt{f'_{ci}}$ (ACI 18.4.1(b))
- Extreme fiber stress in tension at ends of simply supported members: $6\sqrt{f'_{ci}}$ (ACI 18.4.1(c))

The extreme fiber stress in tension at the ends of simply supported members is currently **NOT** checked by ETABS.

6.6.2 Serviceability Checks at Service Load

The stresses in the concrete for Class U and Class T prestressed flexural members at service loads, and after all prestress losses occur, are checked against the following limits:

- Extreme fiber stress in compression due to prestress plus total load: $0.60 f'_c$ (ACI 18.4.2(b))
- Extreme fiber stress in tension in the precompressed tensile zone at service loads:

– Class U beams and one-way slabs: $f_t \le 7.5\sqrt{f'_c}$ (ACI 18.3.3)

– Class U two-way slabs: $f_t \le 6\sqrt{f'_c}$ (ACI 18.3.3)

– Class T beams: $7.5\sqrt{f'_c} < f_t \le 12\sqrt{f'_c}$ (ACI 18.3.3)

– Class C beams: $f_t \ge 12\sqrt{f'_c}$ (ACI 18.3.3)

For Class C prestressed flexural members, checks at service loads are not required by the code. However, for Class C prestressed flexural members not subject to fatigue or to aggressive exposure, the spacing of bonded reinforcement nearest the extreme tension face shall not exceed that given by ACI 10.6.4 (ACI 18.4.4). It is assumed that the user has checked the requirements of ACI 10.6.4 and ACI 18.4.4.1 to 18.4.4 independently, as these sections are not checked by the program.

6.6.3 Serviceability Checks at Long-Term Service Load

The stresses in the concrete for Class U and Class T prestressed flexural members at long-term service loads, and after all prestress losses occur, are checked against the same limits as for the normal service load, except for the following:

- Extreme fiber stress in compression due to prestress plus total load:

$$0.45 f'_c \quad \text{(ACI 18.4.2(a))}$$

6.6.4 Serviceability Checks of Prestressing Steel

The program also performs checks on the tensile stresses in the prestressing steel (ACI 18.5.1). The permissible tensile stress checks, in all types of prestressing steel, in terms of the specified minimum tensile stress f_{pu}, and the minimum yield stress, f_y, are summarized as follows:

- Due to tendon jacking force: min(0.94f_{py}, 0.80f_{pu}) (ACI 18.5.1(a))
- Immediately after force transfer: min(0.82f_{py}, 0.74f_{pu}) (ACI 18.5.1(b))

- At anchors and couplers after force transfer: $0.70f_{pu}$ (ACI 18.5.1(c))

6.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

6.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

6.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of prestressed concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or a flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases the beam may be designed as a rectangular or inverted flanged beam.

6.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 6-1 (ACI 10.2). Furthermore, it is assumed that the net tensile strain in the reinforcement shall not be less than 0.005 (tension controlled) (ACI 10.3.4). When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression reinforcement and additional tension reinforcement.

The design procedure used by ETABS, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow. It is assumed that the design ultimate axial force does not exceed $\phi(0.1f'_cA_g)$ (ACI 10.3.5); hence all beams are designed for major direction flexure, shear, and torsion only.

6.7.1.2.1 Design of Rectangular Beams

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, the moment capacity ϕM_n^0 is determined as follows:

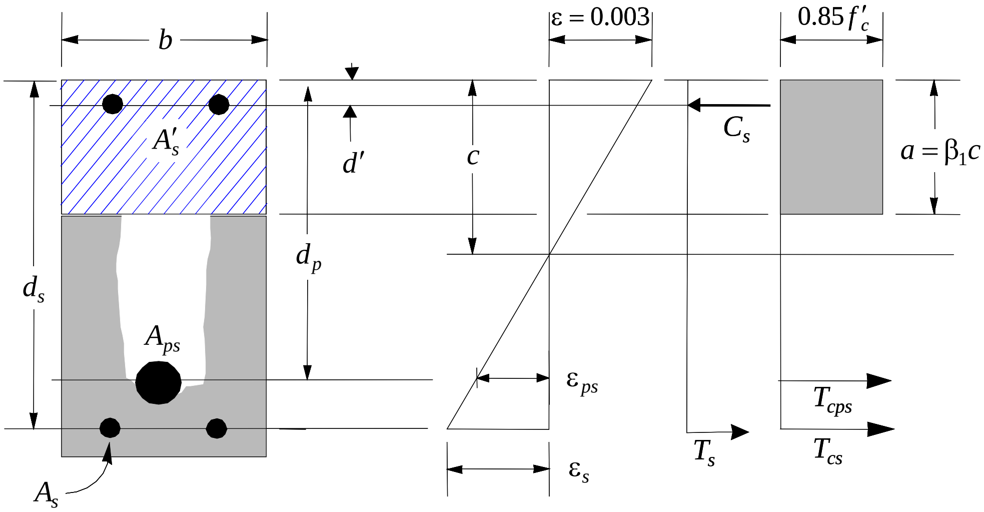


Figure 6-1 Rectangular Beam Design

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tension reinforcement strain shall not be less than ε_{smin}, which is equal to 0.005 for tension-controlled behavior (ACI 10.3.4):

$$c_{max} = \left(\frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}}\right)d \qquad \text{(ACI 10.2.2)}$$

where,

$$\varepsilon_{cmax} = 0.003 \qquad \text{(ACI 10.2.3)}$$

$$\varepsilon_{smin} = 0.005 \qquad \text{(ACI 10.3.4)}$$

Therefore, the limit $c \le c_{max}$ is set for tension-controlled sections.

The maximum allowable depth of the rectangular compression block, a_{max}, is given by:

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 10.2.7.1)}$$

where β_1 is calculated as:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right), \quad 0.65 \le \beta_1 \le 0.85 \qquad \text{(ACI 10.2.7.3)}$$

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{ps}, is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel.

Based on the calculated f_{ps}, the depth of the neutral axis is recalculated, and f_{ps} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \beta_1 c$$

- If $c \le c_{max}$ (ACI 10.3.4), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$\phi M_n^0 = \phi A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)$$

- If $c > c_{max}$ (ACI 10.3.4), a failure condition is declared.

If $M_u > \phi M_n^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of the neutral axis, c is equal to c_{max}. The stress in the post-tensioning steel, f_{ps} is then calculated and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = 0.85 f'_c a_{max} b$$

$$T = A_{ps} f_{ps}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{0.85 f'_c a_{\max} b - A_{ps} f_{ps}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$\phi M_n^{bal} = \phi A_{ps} f_{ps}^{bal} \left(d_p - \frac{a_{\max}}{2} \right) + \phi A_s^{bal} f_s^{bal} \left(d_s - \frac{a_{\max}}{2} \right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, c.

6.7.1.2.1.1 Case 1: Post-tensioning steel is adequate

When $M_u < \phi M_n^0$, the amount of post-tensioning steel is adequate to resist the design moment M_u. Minimum reinforcement is provided to satisfy ductility requirements (ACI 18.9.3.2 and 18.9.3.3), i.e., $M_u < \phi M_n^0$.

6.7.1.2.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M_u, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{max}$.

When $\phi M_n^0 < M_u < \phi M_n^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M_u and reports this required area of tension reinforcement. Since M_u is bounded by ϕM_n^0 at the lower end and ϕM_n^{bal} at the upper end, and ϕM_n^0 is associated with $A_s = 0$ and ϕM_n^{bal} is associated with $A_s = A_s^{bal}$, the required area will fall within the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M_u is positive, or at the top if M_u is negative.

6.7.1.2.1.3 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M_u > \phi M_n^{bal}$, compression reinforcement is required (ACI 10.3.5). In this case ETABS assumes that the depth of the neutral axis, c, is equal to c_{max}. The values of f_{ps} and f_s reach their respective balanced condition values, f_{ps}^{bal} and f_s^{bal}. The area of compression reinforcement, $A_s^{'}$, is then determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M_u - \phi M_n^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(f'_s - 0.85 f'_c\right)\left(d_e - d'\right)\phi}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c\max}\left[\frac{c_{\max} - d'}{c_{\max}}\right] \le f_y \qquad \text{(ACI 10.2.2, 10.2.3, 10.2.4)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{f_y\left(d_s - d'\right)\phi}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and vice versa if M_u is negative.

6.7.1.2.2 Design of Flanged Beams

6.7.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_u (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as above, i.e., no flanged beam data is used.

6.7.1.2.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that A_s = 0. In that case, the moment capacity ϕM_n^0 is determined as follows:

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tension reinforcement strain shall not be less than ε_{smin}, which is equal to 0.005 for tension-controlled behavior (ACI 10.3.4):

$$c_{max} = \left(\frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}}\right) d \qquad \text{(ACI 10.2.2)}$$

where,

$$\varepsilon_{cmax} = 0.003 \qquad \text{(ACI 10.2.3)}$$

$$\varepsilon_{smin} = 0.005 \qquad \text{(ACI 10.3.4)}$$

Therefore, the limit $c \le c_{max}$ is set for tension-controlled section:

The maximum allowable depth of the rectangular compression block, a_{max}, is given by:

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 10.2.7.1)}$$

where β_1 is calculated as:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right), \qquad 0.65 \le \beta_1 \le 0.85 \qquad \text{(ACI 10.2.7.3)}$$

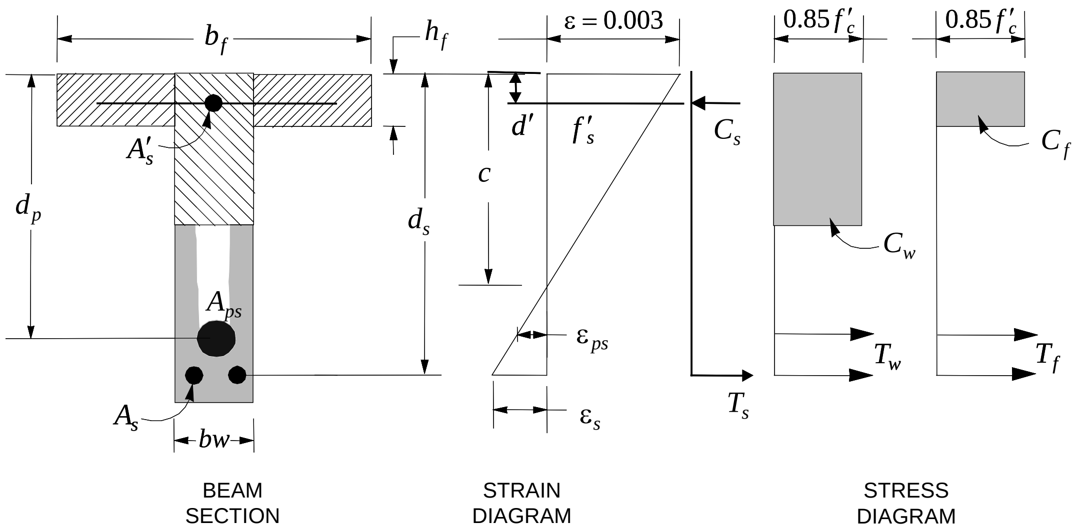


Figure 6-2 T-Beam Design

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{ps} is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel. Based on the calculated f_{ps}, the depth of the neutral axis is recalculated, and f_{ps} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \beta_1 c$$

- If $c \le c_{\max}$ (ACI 10.3.4), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$\phi M_n^0 = \phi A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right)$$

- If $c > c_{\max}$ (ACI 10.3.4), a failure condition is declared.

If $M_u > \phi M_n^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension-controlled case. In that case, it is assumed that the depth of the neutral axis c is equal to c_{max}. The stress in the post-tensioning steel, f_{ps}, is then calculated and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in that case the width of the beam is taken as b_f. Compression reinforcement is required if $a > a_{max}$.

- If $a > h_f$, the calculation for A_s is given by:

$$C = 0.85 f'_c A_c^{comp}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$A_c^{com} = b_f h_f + b_w \left(a_{\max} - h_f\right)$$

$$T = A_{ps} f_{ps}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{0.85 f'_c A_c^{com} - A_{ps} f_{ps}^{bal}}{f_s^{bal}}$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, c.

Case 1: Post-tensioning steel is adequate

When $M_u < \phi M_n^0$ the amount of post-tensioning steel is adequate to resist the design moment M_u. Minimum reinforcement is provided to satisfy ductility requirements (ACI 18.9.3.2 and 18.9.3.3), i.e., $M_u < \phi M_n^0$.

Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M_u, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{max}$.

When $\phi M_n^0 < M_u < \phi M_n^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M_u and reports this required area of tension reinforcement. Since M_u is bounded by ϕM_n^0 at the lower end and ϕM_n^{bal} at the upper end, and ϕM_n^0 is associated with $A_s = 0$ and ϕM_n^{bal} is associated with $A_s = A_s^{bal}$, the required area will fall within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if M_u is positive, or at the top if M_u is negative.

Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M_u > \phi M_n^{bal}$, compression reinforcement is required (ACI 10.3.5). In that case, ETABS assumes that the depth of the neutral axis, c, is equal to c_{max}. The value of f_{ps} and f_s reach their respective balanced condition values, f_{ps}^{bal} and f_s^{bal}. The area of compression reinforcement, $A_s^{'}$, is then determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M_u - \phi M_n^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(f'_s - 0.85 f'_c\right)\left(d_s - d'\right)\phi}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c\max}\left[\frac{c_{\max} - d'}{c_{\max}}\right] \le f_y \qquad \text{(ACI 10.2.2, 10.2.3, and 10.2.4)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{f_y\left(d_s - d'\right)\phi}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and vice versa if M_u is negative.

6.7.1.2.3 Ductility Requirements

ETABS also checks the following condition by considering the post-tensioning steel and tension reinforcement to avoid abrupt failure.

$$\phi M_n \geq 1.2 M_{cr} \qquad \text{(ACI 18.8.2)}$$

The preceding condition is permitted to be waived for the following:

(a) Two-way, unbonded post-tensioned slabs

(b) Flexural members with shear and flexural strength at least twice that required by ACI 9.2.

These exceptions currently are **NOT** handled by ETABS.

6.7.1.2.4 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following limit:

$$A_s \geq 0.004\, A_{ct} \qquad \text{(ACI 18.9.2)}$$

where, A_{ct} is the area of the cross-section between the flexural tension face and the center of gravity of the gross section.

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.4bd & \text{Rectangular beam} \\ 0.4b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

6.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular loading combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V_u.
- Determine the shear force, V_c that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

6.7.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

6.7.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as:

$$V_c = \min(V_{ci}, V_{cw}) \qquad \text{(ACI 11.3.3)}$$

where,

$$V_{ci} = 0.6\lambda\sqrt{f'_c}\,b_w d_p + V_d + \frac{V_i M_{cre}}{M_{\max}} \geq 1.7\lambda\sqrt{f'_c}\,b_w d \qquad \text{(ACI 11.3.3.1)}$$

$$V_{cw} = \left(3.5\lambda\sqrt{f'_c} + 0.3 f_{pc}\right) b_w d_p + V_p \qquad \text{(ACI 11.3.3.2)}$$

$$d_p \geq 0.80h \qquad \text{(ACI 11.3.3.1)}$$

$$M_{cre} = \left(\frac{I}{y_t}\right)\left(6\lambda\sqrt{f'_c} + f_{pe} - f_d\right) \qquad \text{(ACI 11.3.3.1)}$$

where,

f_d = stress due to unfactored dead load, at the extreme fiber of the section where tensile stress is caused by externally applied loads, psi

f_{pe} = compress stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at the extreme fiber of the section where tensile stress is caused by externally applied loads, psi

V_d = shear force at the section due to unfactored dead load, lbs

V_p = vertical component of effective prestress force at the section, lbs

V_{ci} = nominal shear strength provided by the concrete when diagonal cracking results from combined shear and moment

M_{cre} = moment causing flexural cracking at the section because of externally applied loads

$M_{\max}$ = maximum factored moment at section because of externally applied loads

V_i = factored shear force at the section because of externally applied loads occurring simultaneously with $M_{\max}$

V_{cw} = nominal shear strength provided by the concrete when diagonal cracking results from high principal tensile stress in the web

For light-weight concrete, the $\sqrt{f'_c}$ term is multiplied by the shear strength reduction factor λ.

6.7.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{max} = V_c + \left(8\sqrt{f'_c}\right)b_w d \qquad \text{(ACI 11.4.7.9)}$$

Given V_u, V_c, and V_{max}, the required shear reinforcement is calculated as follows where, ϕ, the strength reduction factor, is 0.75 (ACI 9.3.2.3).

- If $V_u \le 0.5\phi V_c$

$$\frac{A_v}{s} = 0 \qquad \text{(ACI 11.4.6.1)}$$

- If $0.5\phi V_c < V_u \le \phi V_{max}$

$$\frac{A_v}{s} = \frac{\left(V_u - \phi V_c\right)}{\phi\, f_{yt} d} \qquad \text{(ACI 11.4.7.1, 11.4.7.2)}$$

$$\frac{A_v}{s} \ge \max\left(\frac{0.75\lambda\sqrt{f'_c}}{f_{yt}}b_w, \quad \frac{50b_w}{f_{yt}}\right) \qquad \text{(ACI 11.4.6.3)}$$

- If $V_u > \phi V_{max}$, a failure condition is declared (ACI 11.4.7.9).

For members with an effective prestress force not less than 40 percent of the tensile strength of the flexural reinforcement, the required shear reinforcement is computed as follows (ACI 11.5.6.3, 11.5.6.4):

$$\frac{A_v}{s} \ge \min\begin{cases} \max\left(\dfrac{0.75\lambda\sqrt{f'_c}}{f_y}b_w, \dfrac{50}{f_y}b_w\right) \\ \dfrac{A_{ps} f_{pu}}{80 f_{yt} d}\sqrt{\dfrac{d}{b_w}} \end{cases}$$

- If V_u exceeds the maximum permitted value of ϕV_{max}, the concrete section should be increased in size (ACI 11.5.7.9).

Note that if torsion design is considered and torsion reinforcement is needed, the equation given in ACI 11.5.6.3 does not need to be satisfied independently. See the next section *Design of Beam Torsion Reinforcement* for details.

If the beam depth h is less than the minimum of 10 in, $2.5h_f$, and $0.5b_w$, the minimum shear reinforcement given by ACI 11.5.6.3 is not enforced (ACI 11.5.6.1(c)).

The maximum of all of the calculated A_v/s values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

6.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_u.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

6.7.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors (ACI 11.6.2).

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T_u is permitted to be reduced in accordance with the code (ACI 11.6.2.2). However, the program does not automatically redistribute the internal forces and reduce T_u. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

6.7.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_{cp}, A_{oh}, A_o, p_{cp}, and p_h are calculated. These properties are described in the following (ACI 2.1).

A_{cp} = Area enclosed by outside perimeter of concrete cross-section

A_{oh} = Area enclosed by centerline of the outermost closed transverse torsional reinforcement

A_o = Gross area enclosed by shear flow path

p_{cp} = Outside perimeter of concrete cross-section

p_h = Perimeter of centerline of outermost closed transverse torsional reinforcement

In calculating the section properties involving reinforcement, such as A_{oh}, A_o, and p_h, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 1.75 inches. This is equivalent to 1.5 inches clear cover and a #4 stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a rectangular beam section are given as:

$$A_{cp} = bh \qquad \text{(ACI 11.6.1, 2.1)}$$

$$A_{oh} = (b - 2c)(h - 2c) \qquad \text{(ACI 11.6.3.1, 2.1, R11.6.3.6(b))}$$

$$A_o = 0.85\,A_{oh} \qquad \text{(ACI 11.6.3.6, 2.1)}$$

p_{cp} = $2b + 2h$ (ACI 11.6.1, 2.1)

p_h = $2(b - 2c) + 2(h - 2c)$ (ACI 11.6.3.1, 2.1)

where, the section dimensions b, h, and c are shown in Figure 6-3. Similarly, the special section properties for a flanged beam section are given as:

A_{cp} = $b_w h + (b_f - b_w)h_f$ (ACI 11.6.1, 2.1)

A_{oh} = $(b_w - 2c)(h - 2c)$ (ACI 11.6.3.1, 2.1, R11.6.3.6(b))

A_o = $0.85\ A_{oh}$ (ACI 11.6.3.6, 2.1)

p_{cp} = $2b_f + 2h$ (ACI11.6.1, 2.1)

p_h = $2(h - 2c) + 2(b_w - 2c)$ (ACI 11.6.3.1, 2.1)

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 6-3. Note that the flange width on either side of the beam web is limited to the smaller of $4h_f$ or $(h - h_f)$ (ACI 13.2.4).

6.7.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, T_{cr}, for which the torsion in the section can be ignored is calculated as:

$$T_{cr} = \phi\lambda\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\sqrt{f'_c}}} \qquad \text{(ACI 11.6.1(b))}$$

where A_{cp} and p_{cp} are the area and perimeter of the concrete cross-section as described in detail in the previous section; f_{pc} is the concrete compressive stress at the centroid of the section; ϕ is the strength reduction factor for torsion, which is equal to 0.75 by default (ACI 9.3.2.3); and f'_c is the specified concrete compressive strength.

6.7.3.4 Determine Torsion Reinforcement

If the factored torsion T_u is less than the threshold limit, T_{cr}, torsion can be safely ignored (ACI 11.6.1). In that case, the program reports that no torsion reinforcement is required. However, if T_u exceeds the threshold limit, T_{cr}, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonal (ACI R11.6.3.6).

If $T_u > T_{cr}$ the required closed stirrup area per unit spacing, A_t /s, is calculated as:

$$\frac{A_t}{s} = \frac{T_u \tan\theta}{\phi 2 A_o f_{yt}} \qquad \text{(ACI 11.6.3.6)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{T_u \, p_h}{\phi 2 A_o f_y \tan\theta} \qquad \text{(ACI 11.6.3.7, 11.6.3.6)}$$

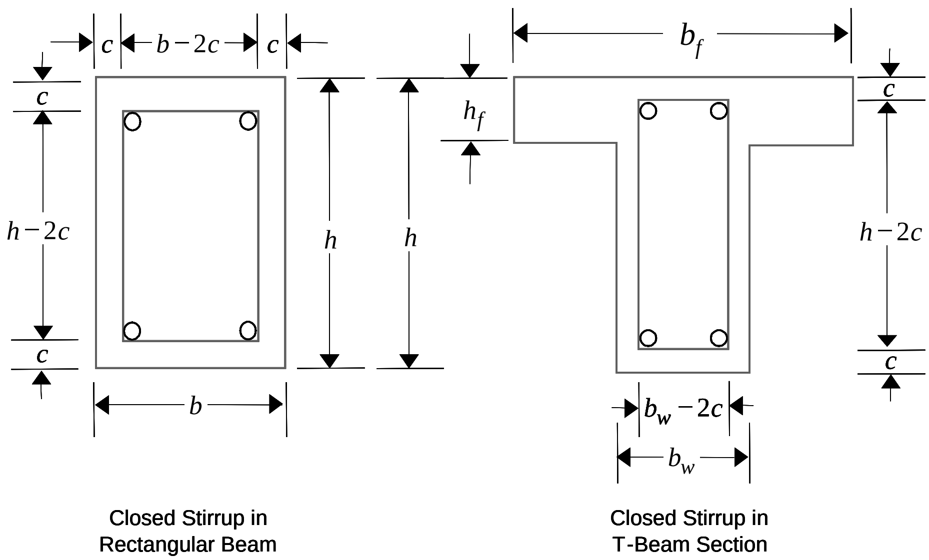


Figure 6-3 Closed stirrup and section dimensions for torsion design

where, the minimum value of A_t /s is taken as:

$$\frac{A_t}{s} = \frac{25}{f_{yt}} b_w \qquad \text{(ACI 11.6.5.3)}$$

and the minimum value of A_l is taken as follows:

$$A_l = \frac{5\lambda\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right)p_h\left(\frac{f_{yt}}{f_y}\right) \qquad \text{(ACI 11.6.5.3)}$$

In the preceding expressions, θ is taken as 45 degrees for prestressed members with an effective prestress force less than 40 percent of the tensile strength of the longitudinal reinforcement; otherwise θ is taken as 37.5 degrees.

An upper limit of the combination of V_u and T_u that can be carried by the section is also checked using the equation:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \qquad \text{(ACI 11.6.3.1)}$$

For rectangular sections, b_w is replaced with b. If the combination of V_u and T_u exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_u > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

$$\left(\frac{A_v}{s} + 2\frac{A_t}{s}\right) \ge \max\left\{0.75\lambda\frac{\sqrt{f'_c}}{f_{yt}}b_w, \frac{50b_w}{f_y}\right\} \qquad \text{(ACI 11.6.5.2)}$$

If this equation is not satisfied with the originally calculated A_v/s and A_t/s, A_v/s is increased to satisfy this condition. In that case, A_v/s does not need to satisfy the ACI Section 11.5.6.3 independently.

The maximum of all of the calculated A_l and A_t/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements considered by the program are based purely on strength considerations. Any minimum stirrup requirements and longitudinal reinforcement requirements to satisfy spacing considerations must be investigated independently of the program by the user.

6.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is completed using the ultimate strength design method (ACI 318-08) for prestressed reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "ETABS Design Features" in the *Key Features and Terminology* manual.

6.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is completed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Determine the capacity of post-tensioned sections.
- Design flexural reinforcement for the strip.

These three steps are described in the subsection that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

6.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

6.8.1.2 Determine Capacity of Post-Tensioned Sections

Calculation of the post-tensioned section capacity is identical to that described earlier for rectangular beam sections.

6.8.1.3 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

6.8.1.3.1 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (ACI 7.12.2):

$$A_{s,\min} = 0.0020\, bh \text{ for } f_y = 40 \text{ ksi or } 50 \text{ ksi} \qquad \text{(ACI 7.12.2.1(a))}$$

$$A_{s,\min} = 0.0018\, bh \text{ for } f_y = 60 \text{ ksi} \qquad \text{(ACI 7.12.2.1(b))}$$

$$A_{s,\min} = \frac{0.0018 \times 60000}{f_y} bh \text{ for } f_y > 60 \text{ ksi} \qquad \text{(ACI 7.12.2.1(c))}$$

Reinforcement is not required in positive moment areas where f_t, the extreme fiber stress in tension in the precompressed tensile zone at service loads (after all prestress losses occurs) does not exceed $2\sqrt{f'_c}$ (ACI 18.9.3.1).

In positive moment areas where the computed tensile stress in the concrete at service loads exceeds $2\sqrt{f_c'}$, the minimum area of bonded reinforcement is computed as:

$$A_{s,\min} = \frac{N_c}{0.5 f_y}\text{, where } f_y \leq 60 \text{ ksi} \qquad \text{(ACI 18.9.3.2)}$$

In negative moment areas at column supports, the minimum area of bonded reinforcement in the top of slab in each direction is computed as:

$$A_{s,\min} = 0.00075 A_{cf} \qquad \text{(ACI 18.9.3.3)}$$

where A_{cf} is the larger gross cross-sectional area of the slab-beam strip in the two orthogonal equivalent frames intersecting a column in a two-way slab system.

When spacing of tendons exceed 54 inches, additional bonded shrinkage and temperature reinforcement (as computed above, ACI 7.12.2.1) is required between the tendons at slab edges, extending from the slab edge for a distance equal to the tendon spacing (ACI 7.12.3.3)

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area. Note that the requirements when f_y > 60 ksi currently are not handled.

6.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code specific items are described in the following sections.

6.8.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (ACI 11.11.1.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (ACI 11.11.1.3). Figure 6-4 shows the auto punching perimeters considered by ETABS for the various column shapes.

The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

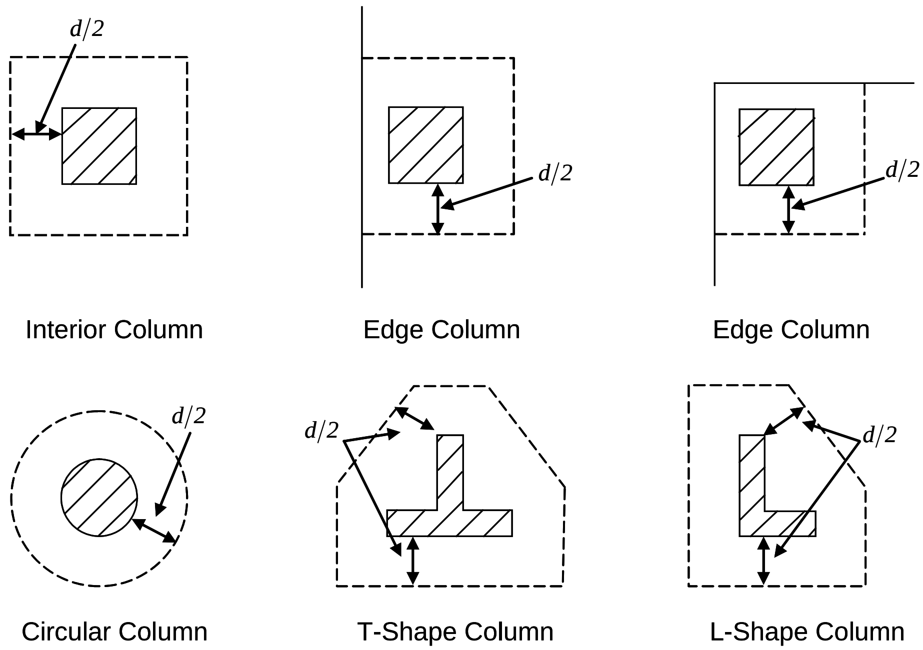


Figure 6-4 Punching Shear Perimeters

6.8.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$.

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} \qquad \text{(ACI 13.5.3.2)}$$

$$\gamma_v = 1 - \gamma_f \qquad \text{(ACI 13.5.3.1)}$$

For flat plates, γ_v is determined from the following equations taken from ACI 421.2R-07 [ACI 2007] *Seismic Design of Punching Shear Reinforcement in Flat Plates*.

For interior columns,

$$\gamma_{vx} = 1 - \frac{1}{1 + (2/3)\sqrt{l_y / l_x}} \qquad \text{(ACI 421.2 C-11)}$$

$$\gamma_{vy} = 1 - \frac{1}{1 + (2/3)\sqrt{l_x / l_y}} \qquad \text{(ACI 421.2 C-12)}$$

For edge columns,

γ_{vx} = same as for interior columns (ACI 421.2 C-13)

$$\gamma_{vy} = 1 - \frac{1}{1 + (2/3)\sqrt{l_x / l_y - 0.2}} \qquad \text{(ACI 421.2 C-14)}$$

$\gamma_{vy} = 0$ when $l_x/l_y \leq 0.2$

For corner columns,

$\gamma_{vx} = 0.4$ (ACI 421.2 C-15)

γ_{vy} = same as for edge columns (ACI 421.2 C-16)

NOTE: Program uses ACI 421.2-12 and ACI 421.2-15 equations in lieu of ACI 421.2 C-14 and ACI 421.2 C-16 which are currently NOT enforced.

where b_1 is the width of the critical section measured in the direction of the span and b_2 is the width of the critical section measured in the direction perpendicular to the span. The values l_x and l_y are the projections of the shear-critical section onto its principal axes, x and y, respectively.

6.8.2.3 Determine Concrete Capacity

The concrete punching shear stress capacity of a two-way prestressed section is taken as:

$$v_c = \phi\left(\beta_p \sqrt{f'_c} + 0.3 f_{pc}\right) + v_p \qquad \text{(ACI 11.11.2.2)}$$

$$\beta_p = \min\left(3.5, \left(\frac{\alpha_s d}{b_o} + 1.5\right)\right) \qquad \text{(ACI 11.11.2.2)}$$

where, β_p is the factor used to compute v_c in prestressed slab; b_o is the perimeter of the critical section; f_{pc} is the average value of f_{pc} in the two directions; v_p is the vertical component of all effective prestress stresses crossing the critical section; and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 40 & \text{for interior columns,} \\ 30 & \text{for edge columns, and} \\ 20 & \text{for corner columns.} \end{cases} \qquad \text{(ACI 11.11.2.1)}$$

The concrete capacity v_c computed from ACI 11.12.2.2 is permitted only when the following conditions are satisfied:

- The column is farther than four times the slab thickness away from any discontinuous slab edges.
- The value of $\sqrt{f'_c}$ is taken no greater than 70 psi.
- In each direction, the value of f_{pc} is within the range:

  $125 \le f_{pc} \le 500$ psi

In thin slabs, the slope of the tendon profile is hard to control and special care should be exercised in computing v_p. In case of uncertainty between the design and as-built profile, a reduced or zero value for v_p should be used.

If the preceding three conditions are not satisfied, the concrete punching shear stress capacity of a two-way prestressed section is taken as the minimum of the following three limits:

$$v_c = \min \begin{cases} \phi\left(2 + \dfrac{4}{\beta_c}\right)\lambda\sqrt{f'_c} \\ \phi\left(2 + \dfrac{\alpha_s d}{b_c}\right)\lambda\sqrt{f'_c} \\ \phi 4\lambda\sqrt{f'_c} \end{cases} \qquad \text{(ACI 11.11.2.1)}$$

where, β_c is the ratio of the maximum to the minimum dimensions of the critical section, b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section (ACI 11.12.2.1).

A limit is imposed on the value of $\sqrt{f'_c}$ as:

$$\sqrt{f'_c} \leq 100 \qquad \text{(ACI 11.1.2)}$$

6.8.2.4 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS.

6.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 6 inches, and not less than 16 times the shear reinforcement bar diameter (ACI 11.11.3). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

6.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a two-way prestressed section with punching shear reinforcement is as previously determined, but limited to:

$$v_c \leq \phi 2\lambda\sqrt{f'_c} \text{ for shear links} \qquad \text{(ACI 11.11.3.1)}$$

$$v_c \leq \phi 3\lambda\sqrt{f'_c} \text{ for shear studs} \qquad \text{(ACI 11.11.5.1)}$$

6.8.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{max} = 6\sqrt{f'_c}\, b_o d \text{ for shear links} \quad \text{(ACI 11.11.3.2)}$$

$$V_{max} = 8\sqrt{f'_c}\, b_o d \text{ for shear studs} \quad \text{(ACI 11.11.5.1)}$$

Given V_u, V_c, and V_{max}, the required shear reinforcement is calculated as follows, where, ϕ, the strength reduction factor, is 0.75 (ACI 9.3.2.3).

$$\frac{A_v}{s} = \frac{\left(V_u - \phi V_c\right)}{\phi f_{ys} d} \quad \text{(ACI 11.4.7.1, 11.4.7.2)}$$

$$\frac{A_v}{s} \geq 2\frac{\sqrt{f'_c}}{f_y} b_o \quad \text{for shear studs}$$

- If $V_u > \phi V_{max}$, a failure condition is declared. (ACI 11.11.3.2)
- If V_u exceeds the maximum permitted value of ϕV_{max}, the concrete section should be increased in size.

6.8.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 6-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed d/2 (ACI R11.3.3, 11.11.5.2). The spacing between adjacent shear reinforcement in the first line of shear reinforcement shall not exceed 2*d* measured in a direction parallel to the column face (ACI 11.11.3.3).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

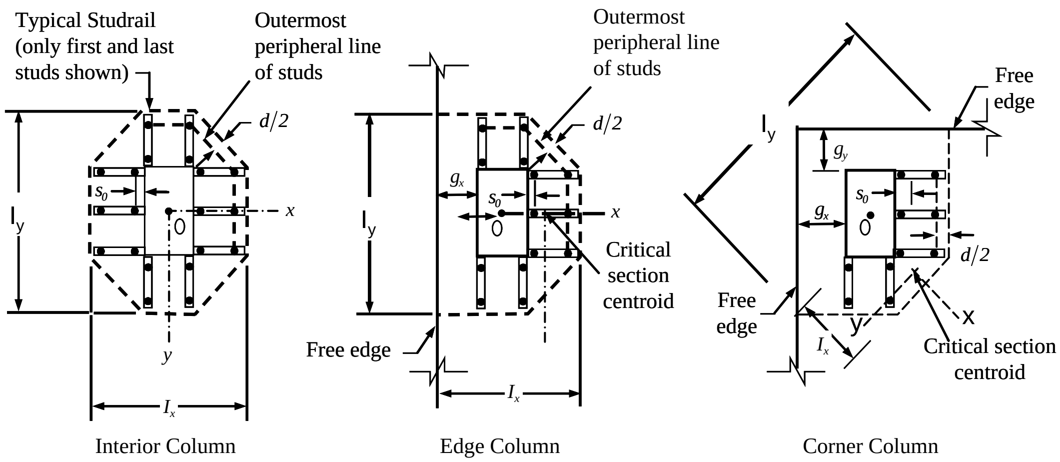


Figure 6-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

6.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in ACI 7.7 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 3/8-, 1/2-, 5/8-, and 3/4-inch diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.35d. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(ACI 11.11.5.2)}$$

$$s \leq \begin{cases} 0.75d & \text{for} \quad v_u \leq 6\phi\lambda\sqrt{f'_c} \\ 0.50d & \text{for} \quad v_u > 6\phi\lambda\sqrt{f'_c} \end{cases} \qquad \text{(ACI 11.11.5.2)}$$

$$g \leq 2d \qquad \text{(ACI 11.11.5.3)}$$

The limits of s_o and the spacing, s, between the links are specified as:

$$s_o \leq 0.5d \qquad \text{(ACI 11.11.3)}$$

$$s \leq 0.50d \qquad \text{(ACI 11.11.3)}$$

Chapter 7
Design for AS 3600-01

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the Australian code AS 3600-2001 [AS 2001], which also incorporates Amendment Nos. 1 (May 2002), and 2 (October 2004). Various notations used in this chapter are listed in Table 7-1. For referencing to the pertinent sections of the AS code in this chapter, a prefix "AS" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

7.1 Notations

The following table identifies the various notations used in this chapter.

Table 7-1 List of Symbols Used in the AS 3600-01 Code

A_g	Gross area of concrete, mm^2

Table 7-1 List of Symbols Used in the AS 3600-01 Code

A_l	Area of longitudinal reinforcement for torsion, mm^2
A_p	Area of prestressing steel in flexural tension zone, sq-mm
A_s	Area of tension reinforcement, mm^2
A_{sc}	Area of compression reinforcement, mm^2
A_{st}	Area of tension reinforcement, mm^2
$A_{s(\text{required})}$	Area of required tension reinforcement, mm^2
A_{sv}	Area of shear reinforcement, mm^2
$A_{sv,\min}$	Minimum area of shear reinforcement, mm^2
A_{sv}/s	Area of shear reinforcement per unit length, mm^2 /mm
A_{sw}/s	Area of shear reinforcement per unit length consisting of closed ties, mm^2/mm
A_t	Area of a polygon with vertices at the center of longitudinal bars at the corners of a section, mm^2
s	Spacing of shear reinforcement along the length, mm
a	Depth of compression block, mm
a_b	Depth of compression block at balanced condition, mm
$a_{\max}$	Maximum allowed depth of compression block, mm
b	Width of member, mm
b_{ef}	Effective width of flange (flanged section), mm
b_w	Width of web (flanged section), mm
c	Depth to neutral axis, mm
d	Distance from compression face to tension reinforcement, mm
d'	Concrete cover to compression reinforcement, mm
d_o	Distance from the extreme compression fiber to the centroid of the outermost tension reinforcement, mm
d_{om}	Mean value of d_o, averaged around the critical shear perimeter, mm
D	Overall depth of a section, mm

Table 7-1 List of Symbols Used in the AS 3600-01 Code

D_s	Thickness of slab (flanged section), mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, MPa
f'_c	Specified compressive strength of concrete, MPa
f'_{ci}	Specified compressive strength of concrete at time of initial prestress, MPa
f_{pe}	Compressive stress in concrete due to effective prestress forces only (after allowance of all prestress losses), MPa
f_p	Stress in prestressing steel at nominal flexural strength, MPa
f_{pu}	Specified tensile strength of prestressing steel, MPa
f_{py}	Specified yield strength of prestressing steel, MPa
f_{ct}	Characteristic principal tensile strength of concrete, MPa
f'_{cf}	Characteristic flexural tensile strength of concrete, MPa
f_{cv}	Concrete shear strength, MPa
f_{sy}	Specified yield strength of flexural reinforcement, MPa
$f_{sy,f}$	Specified yield strength of shear reinforcement, MPa
f'_s	Stress in the compression reinforcement, MPa
D	Overall depth of a section, mm
J_t	Torsional modulus, mm^3
k_u	Ratio of the depth to the neutral axis from the compression face, to the effective depth, d
M_u^0	Design moment resistance of a section with tendons only, N-mm
M_u^{bal}	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
M_{ud}	Reduced ultimate strength in bending without axial force, N-mm
M^*	Factored moment at section, N-mm

Table 7-1 List of Symbols Used in the AS 3600-01 Code

N^*	Factored axial load at section, N
s	Spacing of shear reinforcement along the beam, mm
T_{uc}	Torsional strength of section without torsional reinforcement, N-mm
$T_{u.\max}$	Maximum permitted total factored torsion at a section, N-mm
T_{us}	Torsion strength of section with torsion reinforcement, N-mm
T^*	Factored torsional moment at a section, N-mm
u_t	Perimeter of the polygon defined by A_t, mm
V^*	Factored shear force at a section, N
$V_{u.\max}$	Maximum permitted total factored shear force at a section, N
$V_{u.\min}$	Shear strength provided by minimum shear reinforcement, N
V_{uc}	Shear force resisted by concrete, N
V_{us}	Shear force resisted by reinforcement, N
γ_1	Factor for obtaining depth of compression block in concrete
β_h	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
$\varepsilon_{c,\ \max}$	Maximum usable compression strain allowed in extreme concrete fiber, (0.003 mm/mm)
ε_s	Strain in reinforcement
ϕ	Strength reduction factor
θ_t	Angle of compression strut for torsion, degrees
θ_v	Angle of compression strut for shear, degrees

7.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For AS 3600-01, if a structure is subjected to dead load (D), live load (L), pattern live load (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the load combinations in the following sections may need to be considered (AS 3.3.1, 3.4 and 7.6.7).

For post-tensioned concrete design, the user can specify the prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning are defined in the following sections.

7.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress forces in accordance with AS 3600-01 clauses 3.4 and 7.6.7. The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0D + 1.0PT (AS 3.4)
1.15D + 1.15PT (AS 3.4)
0.9D + 1.15PT (AS 3.4)

7.2.2 Service Load Combination

The following load combinations are used for checking the requirements of prestress for serviceability in accordance with AS3.4 and 7.6.7. It is assumed that all long-term losses have occurred already at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT

7.2.3 Ultimate Limit State Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with AS 3.3.1, 3.4 and 7.6.7.

The ultimate limit state combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are determined automatically by ETABS by subtracting out the primary PT moments when the flexural design is carried out.

1.35D + 1.0PT*	(AS/NZS 1170.0-02, 4.2.2(a))
1.2D + 1.5L + 1.0PT*	(AS/NZS 1170.0-02, 4.2.2(b))
1.2D + 1.5(0.75PL) + 1.0PT*	(AS/NZS 1170.0-02, 4.2.2(b))
1.2D + 0.4L + 1.0S + 1.0PT*	(AS/NZS 1170.0-02, 4.2.2(g))
0.9D ± 1.0W +1.0PT*	(AS/NZS 1170.0-02, 4.2.2(e))
1.2D ± 1.0W + 1.0PT*	(AS/NZS 1170.0-02, 4.2.2(d))
1.2D + 0.4L ± 1.0W + 1.0PT*	(AS/NZS 1170.0-02, 4.2.2(d))
1.0D ± 1.0E + 1.0PT* 1.0D + 0.4L ± 1.0E + 1.0PT*	(AS/NZS 1170.0-02, 4.2.2(f))

* — Replace PT by H for flexural design only

Note that the 0.4 factor on the live load in three of the combinations is not valid for live load representing storage areas. These are also the default design load combinations in ETABS whenever the AS 3600-2001 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

7.3 Limits on Material Strength

The upper and lower limits of f'_c are 65 MPa and 20 MPa, respectively, for all framing type (AS 6.1.1.1(b)).

$$f'_c \leq 65\,\text{MPa} \qquad \text{(AS 6.1.1.1)}$$

$$f'_c \geq 20\,\text{MPa} \qquad \text{(AS 6.1.1.1)}$$

The upper limit of f_{sy} is 500 MPa (AS 6.2.1, Table 6.2.1).

The code allows use of f'_c and f_{sy} beyond the given limits, provided special care is taken regarding the detailing and ductility (AS 6.1.1, 6.2.1, 19.2.1.1).

ETABS enforces the upper material strength limits for flexure and shear design of slabs. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

7.4 Strength Reduction Factors

The strength reduction factor, ϕ, is defined as given in the following table (AS 2.3(c), Table 2.3):

Type of action effect	Strength reduction factor (ϕ)
(a) Axial force without bending —	
(i) Tension	0.8
(ii) Compression	0.6
(b) Bending without axial tension or compression where:	
(i) $k_u \leq 0.4$	0.8
(ii) $k_u > 0.4$	$0.8\ M_{ud}/M_{uo} \geq 0.6$
(c) Bending with axial tension	ϕ +[(0.8 – ϕ)(N_u/N_{uot})] ϕ is obtained from (b)
(d) Bending with axial compression where:	
(i) $N_u \geq N_{ub}$	0.6
(ii) $N_u < N_{ub}$	0.6+ [(ϕ- 0.6)(1 – N_u/N_{ub})] ϕ is obtained from (b)
(e) Shear	0.7
(f) Torsion	0.7

The value M_{ud} is the reduced ultimate strength of the cross-section in bending where k_u = 0.4 and tensile force has been reduced to balance the reduced compressive forces (AS 8.1.3).

These values can be overwritten; however, caution is advised.

7.5 Design Assumptions for Prestressed Concrete Structures

Ultimate limit state of prestressed members for flexure and axial loads shall be based on assumptions given in AS 8.1.

- The strain distribution in the concrete in compression is derived from the assumption that the plane section remains plane (AS 8.1.2.1(a)).
- Tensile strength of the concrete is ignored (AS 8.1.2.1 (b)).
- The design stresses in the concrete in compression are taken as 0.85 f_c'. The maximum strain at the extreme concrete compression fiber shall be assumed equal to 0.003 (AS 8.1.2.1 (c), 8.1.2.2).
- The strain in bonded prestressing tendons and in any additional reinforcement (compression or tension) is derived from the assumption that plane section remains plane (AS 8.1.2.1(a)).

Prestressed concrete members are investigated at the following three stages:

- At transfer of prestress force
- At service loading
- At nominal strength

The prestressed flexural members are classified as uncracked and cracked based on tensile strength f_t, the computed extreme fiber stress in tension in the precompressed tensile zone at service loads.

The precompressed tensile zone is that portion of a prestressed member where flexural tension, calculated using gross section properties, would occur under unfactored dead and live loads if the prestress force was not present. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress.

7.6 Serviceability Requirements of Flexural Members

7.6.1 Serviceability Check at Initial Service Load

The stresses in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits (AS 8.1.4.2):

- Extreme fiber stress in compression: $0.50\, f_{cp}$

The extreme fiber stress in tension should not exceed the cracking stress; otherwise the section should be designed as a cracked section.

7.6.2 Serviceability Check at Service Load

Flexural cracking in a prestressed beam shall be deemed to be controlled if under short-term service loads the resulting maximum tensile stress in concrete does not exceed $0.25\sqrt{f_c'}$; in that case, no further checks are needed (AS 8.6.2). However, if this limit is exceeded, flexural cracking shall be deemed to be controlled by providing reinforcement or bonded tendons, or both, near the tensile face and achieving either of the following (AS 8.6.2, 9.4.2):

(a) limiting the calculated maximum flexural tensile stress under short-term service loads to $0.6\sqrt{f_c'}$; or

(b) limiting both of the following

(i) the increment in steel stress near the tension face to 200 MPa, as the load increases from its value when the extreme concrete tensile fiber is at zero stress to the short-term service load value; and

(ii) the center-to-center spacing of reinforcement, including bonded tendons, to 200 mm. (This sub clause is a detailing requirement not checked by the program.)

The program checks the stresses in the concrete prestressed flexural members at service loads and after all prestress losses against the following limit (AS 8.6.2):

- Extreme fiber stress in tension in the precompressed tensile zone at service loads:
  - Extreme fiber stresses in tension for cracked section: $0.5\sqrt{f_c'}$

Thus, although cracking is allowed, it is assumed that the user is limiting the tensile stress at the service stage as presented in AS 8.6.2.

7.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

7.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

7.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of prestressed concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or a flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

7.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 7-1 (AS 8.1.2.2).

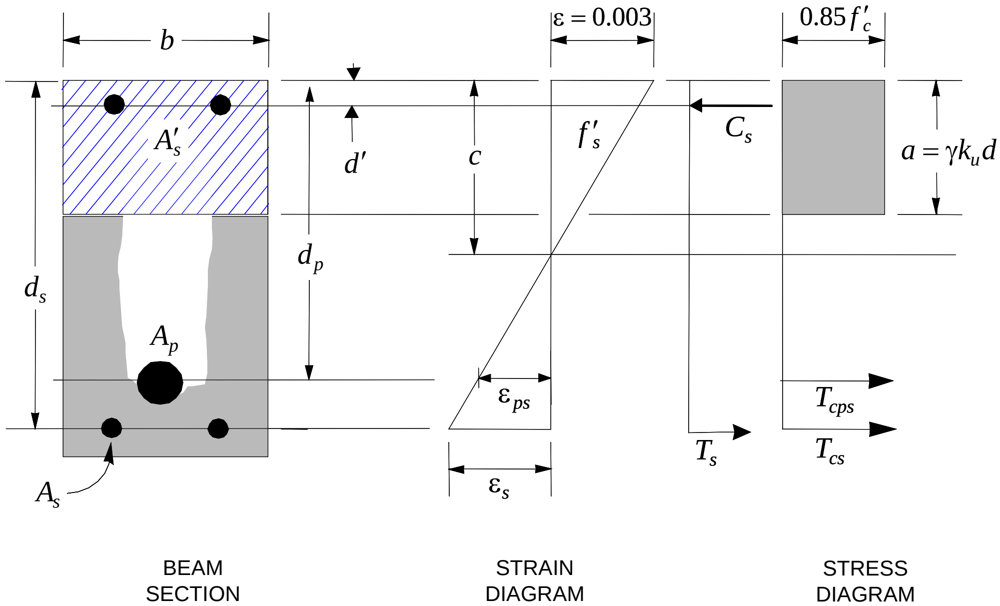


Figure 7-1 Rectangular Beam Design

The design procedure used by ETABS for both rectangular and flanged sections (L- and T-beams) is summarized in the following subsections. It is assumed that the design ultimate axial force does not exceed ($A_{sc}f_{sy} > 0.15N^{*}$) (AS 10.7.1a); hence all beams are designed for major direction flexure, shear, and torsion only.

7.7.1.2.1 Design of Rectangular Beams

The amount of post-tensioning steel adequate to resist the design moment M and minimum reinforcement are provided to satisfy the flexural cracking requirements (AS 8.1.4.1).

ETABS determines the depth of the neutral axis, a, by imposing force equilibrium, i.e., C = T, and performs an iteration to compute the depth of the neutral axis, which is based on stress-strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel. After the depth of the neutral axis has been found, the stress in the post-tensioning reinforcement f_{pb} is computed based on strain compatibility.

The following assumptions are used for the stress block used to compute the flexural bending capacity of rectangular sections (AS 8.1.2.2).

- The maximum strain in the extreme compression fiber is taken as 0.003.
- A uniform compressive stress of $0.85f'_c$ acts on an area bounded by:
  - The edges of the cross-sections.
  - A line parallel to the neutral axis at the strength limit under the loading concerned, and located at a distance $\gamma k_u d$ from the extreme compression fiber.

The maximum allowable depth of the rectangular compression block, a_{max}, is given by

$$a_{\max} = \gamma k_u d \quad \text{where,} \qquad \text{(AS 8.1.3)}$$

$$\gamma = \left[0.85 - 0.007\left(f'_c - 28\right)\right]$$

$$0.65 \le \gamma \le 0.85 \qquad \text{(AS 8.1.2.2)}$$

$$k_u = 0.4$$

- If $a \le a_{\max}$ (AS 8.1.3), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_u^0 = A_p f_p \left(d_p - \frac{a}{2}\right)$$

- If $a > a_{\max}$ (AS 8.1.3), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of neutral axis c is equal to c_{max}. The stress in the post-tensioning steel, f_p is then calculated based on strain compatibility and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = 0.85 f'_c b a_{\max} \qquad \text{(AS 8.1.2.2)}$$

$$T = A_p f_p^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{0.85f'_c - A_p f_p^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_u^{bal} = A_p f_p^{bal}\left(d_p - \frac{a_{\max}}{2}\right) + A_s^{bal} f_s^{bal}\left(d_s - \frac{a_{\max}}{2}\right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of neutral axis, c.

7.7.1.2.1.1 Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment M. Minimum reinforcement is provided to satisfy the ductility requirements, i.e., $M < M_u^0$.

7.7.1.2.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports this required area of tension reinforcement. Since M is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end, and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will fall between the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if *M* is positive, or at the top if *M* is negative.

7.7.1.2.1.3 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required (AS 8.1.3). In that case, ETABS assumes that the depth of neutral axis, *c*, is equal to $c_{\max}$. The values of f_p and f_s reach their respective balanced condition values, f_p^{bal} and f_s^{bal}. The area of compression reinforcement, $A_s^{'}$, is determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A_{sc} = \frac{M_{us}}{\left(f'_s - 0.85 f'_c\right)\left(d - d'\right)\phi}, \text{ where}$$

$$f'_s = 0.003 E_s \left[\frac{c - d'}{c}\right] \leq f_{sy} \qquad \text{(AS 8.1.2.1, 6.2.2)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{\phi f_y \left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if *M* is positive, and vice versa if *M* is negative.

7.7.1.2.2 Design of Flanged Beams

7.7.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

7.7.1.2.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, moment capacity M_u^0 is determined as follows:

ETABS determines the depth of the neutral axis, a, by imposing force equilibrium, i.e., C = T, and performs an iteration to compute the depth of the neutral axis, which is based on stress-strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pb} is computed based on strain compatibility.

The maximum allowable depth of the rectangular compression block, a_{max}, is given by

$$a_{\max} = \gamma k_u d \text{ where, } k_u = 0.4 \qquad \text{(AS 8.1.3)}$$

- If $a \le a_{\max}$ (AS 8.1.3), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_u^0 = A_p f_p \left(d_p - \frac{a}{2} \right)$$

- If $a > a_{\max}$ (AS 8.1.3), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case it is assumed that the depth of neutral axis c is equal to c_{max}. The stress in the post-tensioning steel, f_p is then calculated based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \le D_s$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as b_f. Compression reinforcement is required when $a_{\max} = \gamma k_u d$ where, $k_u = 0.4$.

- If $a > D_s$, the calculation for A_s is given by

$$C = 0.85 f_c' a_{\max} A_c^{com}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$A_c^{com} = b_f D_s + b_w \left(a_{\max} - D_s \right)$$

$$T = A_p f_p^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{0.85 f_c' a_{max} A_c^{com} - A_p f_p^{bal}}{f_s^{bal}}$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of neutral axis, c.

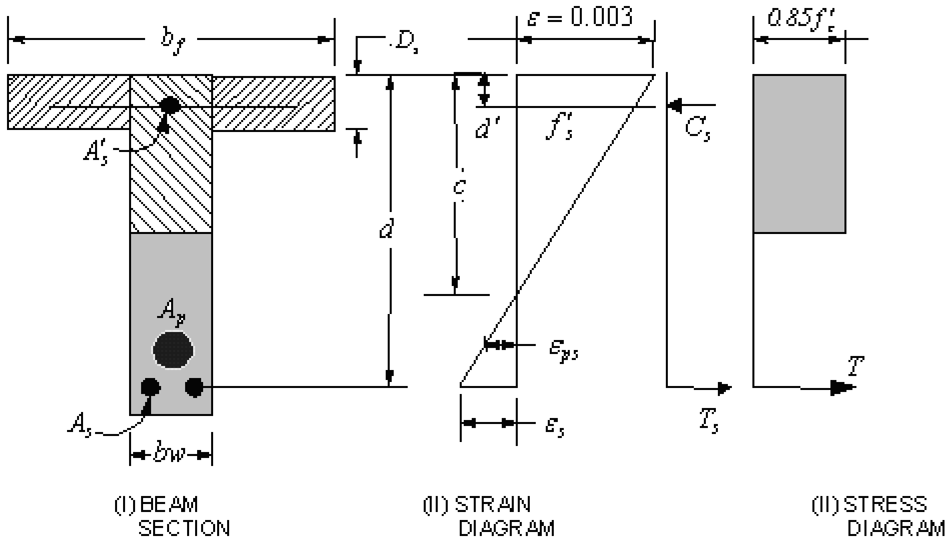


Figure 7-2 T-Beam Design

7.7.1.2.2.3 Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment *M*. Minimum reinforcement is provided to satisfy ductility requirements, i.e., $M < M_u^0$.

7.7.1.2.2.4 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist *M*, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{max}$.

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment *M* and reports the required area of tension reinforcement. Since *M* is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end, and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if *M* is positive, or at the top if *M* is negative.

7.7.1.2.2.5 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required. In that case, ETABS assumes that the depth of the neutral axis, *c*, is equal to $c_{\max}$. The values of f_p and f_s reach their respective balanced condition values, f_p^{bal} and f_s^{bal}. The area of compression reinforcement, $A_s^{'}$, is then determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A_s' = \frac{M_{us}}{\phi\left(f_s' - 0.85 f_c'\right)\left(d - d'\right)}, \text{ where}$$

$$f_s' = 0.003 E_s \left[\frac{c_{\max} - d'}{c_{\max}}\right] \le f_{sy} \qquad \text{(AS 8.1.2.1, 6.2.2)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{\phi f_y \left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom, and A'_s is to be placed at the top if *M* is positive and vice versa if *M* is negative.

7.7.1.3 Minimum and Maximum Reinforcement

Reinforcement in post-tensioned concrete beams is computed to increase the strength of sections as documented for the flexural design of post-tensioned beams or to comply with the shear link requirements. The minimum flexural tension reinforcement required for a beam section to comply with the cracking requirements needs to be separately investigated by the user.

The ultimate strength in bending (M_{uo}), at critical sections shall not be less than $\left(M_{uo}\right)_{\min}$ given by:

$$\left(M_{uo}\right)_{\min} = 1.2\left[Z\left(f'_{cf} + P/A_g\right) + Pe\right] \qquad \text{(AS 8.1.4.1)}$$

where

Z = the section modulus of the uncracked section, referred to the extreme fiber at which flexural cracking occurs

f'_{cf} = the characteristic flexural tensile strength of the concrete

e = the eccentricity of the prestressing force (P), measured from the centroidal axis of the uncracked section

The minimum flexural tension reinforcement required in a beam section is given by the following limit:

$$A_{st.\min} = 0.22\left(\frac{D}{d}\right)^2 \frac{f'_{cf}}{f_{sy}} bd, \text{ where} \qquad \text{(AS 8.1.4.1)}$$

$$f'_{cf} = 0.6\sqrt{f'_c} \qquad \text{(AS 6.1.1.2)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_{st} \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

$$A_{sc} \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

7.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination in the major direction of the beam. In designing the shear reinforcement for a particular beam for a particular load combination, the following steps are involved

- Determine the factored shear force, V^*.
- Determine the shear force, V_{uc}, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three subsections describe in detail the algorithms associated with these steps.

7.7.2.1 Determine Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

7.7.2.2 Determine Concrete Shear Capacity

The ultimate shear strength (V_{uc}) of a prestressed beam, excluding the contribution of shear reinforcement, is the lesser of the values obtained from the following, unless the cross-section under consideration is cracked in flexure, in which case only Flexural-Shear Cracking, Item (a), applies:

(a) Flexural-Shear Cracking

$$V_{uc} = \beta_1 \beta_2 \beta_3 b_v d_o \left[\frac{\left(A_{st} + A_{pt} \right) f'_c}{b_w d_o} \right]^{1/3} + V_o + P_v \qquad \text{(AS 8.2.7.2(a))}$$

where,

$$\beta_1 = 1.1\left(1.6 - \frac{d_o}{1000}\right) \geq 1.1 \qquad \text{(AS 8.2.7.1)}$$

β_2 = 1, or (AS 8.2.7.1)

$$= 1 - \left(\frac{N^*}{3.5A_g} \right) \geq 0$$ for members subject to significant axial tension, or

$$= 1 + \left(\frac{N^*}{14A_g} \right)$$ for members subject to significant axial compression.

β_3 = 1

V_o = the shear force that would occur at the section when the bending moment at that section was equal to the decompression moment (M_o) given by:

$$M_o = Z\sigma_{cp.f}$$

where

$\sigma_{cp.f}$ = the compressive stress because of prestress, at the extreme fiber where cracking occurs

b) Web-shear cracking

$$V_{uc} = V_t + P_v$$ (AS 8.2.7.2(b))

where

Vt = the shear force, which, in combination with the prestressing force and other action effects at the section, would produce a principal tensile stress of $0.33\sqrt{f'_c}$ at either the centroidal axis or the intersection of flange and web, whichever is the more critical.

Where significant reversal of loads may occur, causing cracking in a zone usually in compression, the value of V_{uc} obtained from Clause 8.2.7.1 may not apply.

7.7.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{u,\min} = V_{uc} + 0.6 b_v d_o \qquad \text{(AS 8.2.9)}$$

$$V_{u,\max} = 0.2 f'_c\, b d_o + P_v \qquad \text{(AS 8.2.6)}$$

Given V^*, V_{uc}, and $V_{u.\max}$, the required shear reinforcement is calculated as follows, where, ϕ, the strength reduction factor, is 0.6 by default (AS 2.3).

- If $V^* \le \phi V_{uc} / 2$,

  $\frac{A_{sv}}{s} = 0$, if $D \le 750$ mm, otherwise $A_{sv.\min}$ shall be provided (AS 8.2.5).

- If $\left(\phi V_{uc} / 2\right) < V^* \le \phi V_{u.\min}$,

  $\frac{A_{sv}}{s} = 0$, if $D < b_w/2$ or 250 mm, whichever is greater (AS 8.2.5(c)(i)), otherwise $A_{sv.\min}$ shall be provided.

- If $\phi V_{u.\min} < V^* \le \phi V_{u.\max}$,

$$\frac{A_{sv}}{s} = \frac{\left(V^* - \phi V_{uc}\right)}{f_{sy.f}\, d_o \cot \theta_v}, \qquad \text{(AS 8.2.10)}$$

  and greater than $A_{sv.\min}$, defined as:

$$\frac{A_{sv.\min}}{s} = \left(0.35 \frac{b_w}{f_{sy.f}}\right) \qquad \text{(AS 8.2.8)}$$

  θ_v = the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when $V^* = \phi V_{u.\min}$ to 45 degrees when $V^* = \phi\, V_{u,\max}$.

- If $V^* > \phi V_{\max}$, a failure condition is declared. (AS 8.2.6)

- If V^* exceeds its maximum permitted value $\phi V_{\max}$, the concrete section size should be increased (AS 8.2.6).

Note that if torsion design is considered and torsion reinforcement is required, the calculated shear reinforcement is ignored. Closed stirrups are designed for combined shear and torsion in accordance with AS 8.3.4(b).

The maximum of all of the calculated A_{sv}/s values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

7.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T^*.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

7.7.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T^* is permitted to be reduced in accordance with the code (AS 8.3.2). However, the program does not automatically redistribute the internal forces and reduce T^*. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

7.7.3.2 Determine Special Section Properties

For torsion design, special section properties such as A_t, J_t, and u_t are calculated. These properties are described in the following (AS 8.3).

A_t = Area of a polygon with vertices at the center of longitudinal bars at the corners of the cross-section

u_t = Perimeter of the polygon defined by A_t

J_t = Torsional modulus

In calculating the section properties involving reinforcement, such as A_{sw}/s and A_l, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 mm. This is equivalent to 38-mm clear cover and a 12-mm-diameter stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{uc} calculation. With this assumption, the special properties for a rectangular beam section are given as:

$$A_t = \left(b-2c\right)\left(h-2c\right), \quad \text{(AS 8.3.5)}$$

$$u_t = 2\left(b-2c\right)+2\left(h-2c\right), \quad \text{(AS 8.3.6)}$$

$$J_t = 0.4x^2y \quad \text{(AS 8.3.3)}$$

where, the section dimensions b, h and, c are as shown in Figure 7-3. Similarly, the special section properties for a flanged beam section are given as:

$$A_t = \left(b_w-2c\right)\left(h-2c\right), \quad \text{(AS 8.3.5)}$$

$$u_t = 2\left(h-2_c\right)+2\left(b_w-2_c\right), \quad \text{(AS 8.3.6)}$$

$$J_t = 0.4\Sigma x^2y \quad \text{(AS 8.3.3)}$$

where the section dimensions b_w, h, and c for a flanged beam are as shown in Figure 7-3. The values x and y refer to the smaller and larger dimensions of a component rectangle, respectively.

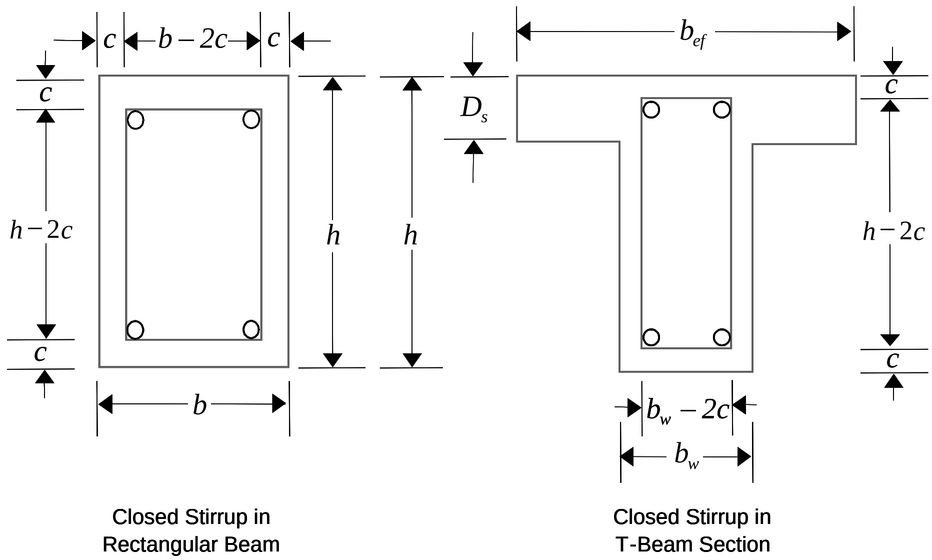


Figure 7-3 Closed stirrup and section dimensions for torsion design

7.7.3.3 Determine Torsion Reinforcement

The torsional strength of the section without torsion reinforcement, T_{uc}, is calculated as:

$$T_{uc} = 0.3 J_t \sqrt{f'_c} \qquad \text{(AS 8.3.5)}$$

where J_t is the torsion modulus of the concrete cross-section as described in detail in the previous section

Torsion reinforcement also can be ignored if any of the following is satisfied:

$$T^* \leq 0.25\phi T_{uc} \qquad \text{(AS 8.3.4(a)(i))}$$

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 0.5 \qquad \text{(AS 8.3.4(a)(ii))}$$

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 1 \text{ and } D \leq \max(250\text{mm}, b/2) \qquad \text{(AS 8.3.4(a)(iii))}$$

If the factored torsion T^* alone or in combination with V^* does not satisfy any of the preceding three conditions, torsion reinforcement is needed. It is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (AS 8.3).

- If $T^* > T_{cr}$, the required closed stirrup area per unit spacing, A_{sw}/s, is calculated as:

$$\frac{A_{sw}}{s} = \frac{T^* \tan\theta_t}{\phi 2 f_{sy.f} A_t} \qquad \text{(AS 8.3.5(b))}$$

where, the minimum value of A_{sw}/s is taken as follows:

$$\frac{A_{sw.\min}}{s} = \frac{0.35 b_w}{f_{sy.f}} \qquad \text{(AS 8.2.8)}$$

The value θ_t is the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when $T^* = \phi T_{uc}$ to 45 degrees when $T^* = \phi T_{u,\max}$.

The following equation shall also be satisfied for combined shear and torsion by adding additional shear stirrups.

$$\frac{T^*}{\phi T_{us}} + \frac{V^*}{\phi V_{us}} \leq 1.0 \qquad \text{(AS 8.3.4(b))}$$

where,

$$T_{us} = f_{sy.f}\left(\frac{A_{sw}}{s}\right) 2A_t \cot\theta_t \qquad \text{(AS 8.3.5(b))}$$

$$V_{us} = \left(A_{sv} f_{sy.f} d_o / s\right)\cot\theta_v \qquad \text{(AS 8.2.10(a))}$$

The required longitudinal rebar area is calculated as:

$$A_l = \frac{0.5 f_{sy.f} \left(\frac{A_{sw}}{s} \right) u_t \cot^2 \theta_t}{f_{sy}} \qquad \text{(AS 8.3.6(a))}$$

An upper limit of the combination of V^* and T^* that can be carried by the section also is checked using the equation:

$$\frac{T^*}{\phi T_{u.\max}} + \frac{V^*}{\phi V_{u.\max}} \leq 1.0 \qquad \text{(AS 8.3.3)}$$

where,

$$V_{u.\max} = 0.2 f'_c b_w d_o \qquad \text{(AS 8.2.6)}$$

$$T_{u.\max} = 0.2 f'_c J_t \qquad \text{(AS 8.3.5(a))}$$

For rectangular sections, b_w is replaced with b. If the combination of V^* and T^* exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T^* > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups satisfy the following limit.

$$\left(\frac{A_{sv}}{s} + 2\frac{A_{sw}}{s} \right) \geq \frac{0.35b}{f_{sy.f}} \qquad \text{(AS 8.3.7, 8.2.8)}$$

If this equation is not satisfied with the originally calculated A_{sv}/s and A_{sw}/s, A_{sv}/s is increased to satisfy this condition. In that case, A_{sv}/s does not need to satisfy AS Section 8.2.8 independently.

The maximum of all of the calculated A_l and A_{sv}/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements and longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

7.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips typically are governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (AS 3600-01) for prestressed reinforced concrete as described in the following subsections. To learn more about the design strips, refer to the section entitled "ETABS Design Techniques" in the *Key Features and Terminology* manual.

7.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Determine the capacity of post-tensioned sections
- Design flexural reinforcement for the strip

These three steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

7.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

7.8.1.2 Determine Capacity of Post-Tensioned Sections

The calculation of the post-tensioned section capacity is identical to that described earlier for rectangular beam sections.

7.8.1.3 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). Where the slab properties (depth and so forth) vary over the width of the strip, the program automatically designs slab widths of each property separately for the bending moment to which they are subjected before summing up the reinforcement for the full width. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

7.8.1.4 Minimum and Maximum Slab Reinforcement

The minimum requirements for untensioned reinforcement in one-way bonded slabs is the same as for beams (AS 9.1.1). Flexural cracking in prestressed slabs shall be deemed controlled if under short-term service loads the resulting stress is less than $0.25\sqrt{f_c'}$; in that case, no further checks are needed (AS 9.4.2). However, if that limit is exceeded, flexural cracking shall be deemed under control by providing reinforcement or bonded tendons, or both, near the tensile face and accomplishing either of the following (AS 9.4.2):

(a) limiting the calculated maximum flexural tensile stress under short-term service loads to $0.5\sqrt{f_c'}$; or

(b) limiting both of the following:

(i) the increment in steel stress near the tension face to 150 MPa, as the load increases from its value when the extreme concrete tensile fiber is at zero stress to the short-term service load value; and

(ii) the center-to-center spacing of reinforcement, including bonded tendons, to 500 mm. (This sub clause is a detailing requirement that is not checked by the program.)

The program checks the stresses in the concrete prestressed flexural members at service loads and after all prestress losses have occurred against the following limit (AS 9.4.2):

- Extreme fiber stress in tension in the precompressed tensile zone at service loads:
  - Extreme fiber stresses in tension for cracked section: $0.5\sqrt{f_c^{'}}$

Thus, although cracking is allowed, it is assumed that the user is limiting the tensile stress at the service stage as presented in AS 9.4.2.

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

7.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Key Features and Terminology* manual. Only the code specific items are described in the following sections.

7.8.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d_{om}/2$ from the face of the support (AS 9.2.1.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (AS 9.2.1.3). Figure 7-4 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

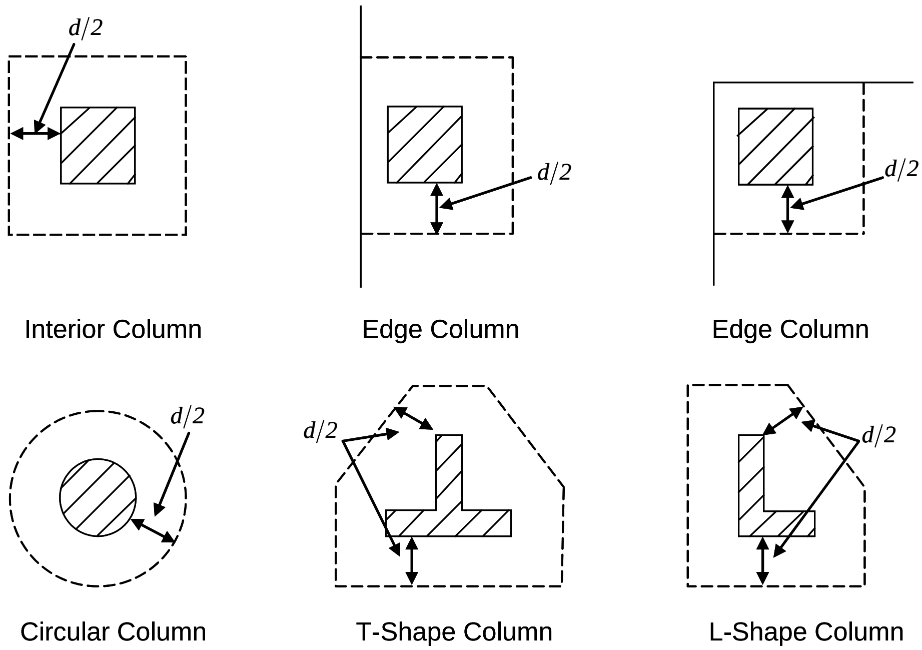


Figure 7-4 Punching Shear Perimeters

7.8.2.2 Determine Concrete Capacity

(i) The ultimate shear strength of a slab where M_v^* is zero, V_{uo}, is given as (AS 9.2.3(a)):

a. when no shear link/stud is present

$$V_{uo} = ud_{om}\left(f_{cv} + 0.3\sigma_{cp}\right) \qquad \text{(AS 9.2.3(a))}$$

b. when shear link/stud is present

$$V_{uo} = ud_{om}\left(0.5\sqrt{f_c'} + 0.3\sigma_{cp}\right) \leq 0.2\sqrt{f_c'}ud_{om} \qquad \text{(AS 9.2.3(b))}$$

where f_{cv} is taken as the minimum of the following two limits:

$$f_{cv} = min\begin{cases} 0.17\left(1+\dfrac{2}{\beta_h}\right)\sqrt{f'_c} \\ 0.34\sqrt{f'_c} \end{cases} \qquad \text{(AS 9.2.3(a))}$$

where, β_h is the ratio of the longest to the minimum dimensions of the critical section.

(ii) The ultimate shear strength of a slab where M_v^* is not zero and no shear reinforcement is provided, V_u, is given as (AS 9.2.4(a)):

$$V_u = V_{uo} / \left[1.0 + \left(uM_v / 8V^* ad_{om}\right)\right] \qquad \text{(AS 9.2.4(a))}$$

7.8.2.3 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by ETABS.

7.8.3 Design Punching Shear Reinforcement

The design guidelines for shear links or shear studs are not available in AS 3600-2001. ETABS uses the NZS 3101-06 guidelines to design shear studs or shear links.

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 150 mm, and not less than 16 times the shear reinforcement bar diameter (NZS 12.7.4.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

7.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

7.8.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{\max} = 3\ V_{u,\min} = 3 \times V_u \qquad \text{(AS 92.2.4(a), (d))}$$

Where V_u is computed from AS 9.2.3 or 9.2.4.Given V^*, V_u, and $V_{u,\max}$, the required shear reinforcement is calculated as follows, where, ϕ, is the strength reduction factor.

$$\frac{A_{sv}}{s} = \frac{\left(V^* - \phi V_u\right)}{f_{sy} d_{om}}, \qquad \text{(AS 8.2.10)}$$

Minimum punching shear reinforcement should be provided such that:

$$V_s \geq \frac{1}{16}\sqrt{f'_c}\,u d_{om} \qquad \text{(NZS 12.7.4.3)}$$

- If $V^* > \phi V_{\max}$, a failure condition is declared. (NZS 12.7.3.4)
- If V^* exceeds the maximum permitted value of $\phi V_{\max}$, the concrete section should be increased in size.

7.8.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 7-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed $2d$ measured in a direction parallel to the column face (NZS 12.7.4.4).

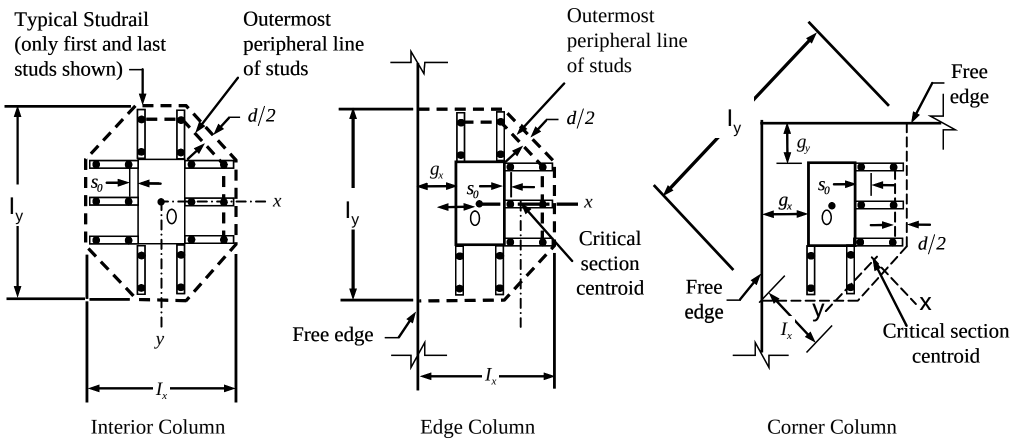


Figure 7-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

7.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in NZS 3.11 plus half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5*d*. The spacing between adjacent shear studs, *g*, at the first peripheral line of studs shall not exceed 2*d* and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of s_o and the spacing, *s*, between the peripheral lines are specified as:

$s_o \leq 0.5d$ (NZS 12.7.4.4)

$s \leq 0.5d$ (NZS 12.7.4.4)

$g \leq 2d$ (NZS 12.7.4.4)

Chapter 8
Design for BS 8110-97

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when British code BS 8110-97 [BSI 1997] is selected. For light-weight concrete and torsion, reference is made to BS 8110-2:1985 [BSI 1985]. Various notations used in this chapter are listed in Table 8-1. For referencing to the pertinent sections of the BS code in this chapter, a prefix "BS" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

8.1 Notations

The following table identifies the various notations used in this chapter.

Table 8-1 List of Symbols Used in the BS 8110-97 Code

A_{cv}	Area of section for shear resistance, mm^2

Table 8-1 List of Symbols Used in the BS 8110-97 Code

A_g	Gross area of cross-section, mm^2
A_s	Area of tension reinforcement, mm^2
A_{ps}	Area of prestress steel, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sv}	Total cross-sectional area of links at the neutral axis, mm^2
A_{sv} / s_v	Area of shear reinforcement per unit length of the member, mm^2/mm
a	Depth of compression block, mm
$a_{\max}$	Maximum depth of the compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
d or d_e	Effective depth of tension reinforcement, mm
d'	Depth to center of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f_{ci}	Concrete strength at transfer, MPa
f_{cu}	Characteristic cube strength at 28 days, MPa
f_{pu}	Characteristic strength of a prestressing tendon, MPa
f_{pb}	Design tensile stress in tendon, MPa
$f_s^{'}$	Compressive stress in a beam compression steel, MPa
f_y	Characteristic strength reinforcement, MPa
f_{yv}	Characteristic strength of link reinforcement, MPa (< 500 MPa)
h	Overall depth of a section in the plane of bending, mm

Table 8-1 List of Symbols Used in the BS 8110-97 Code

h_f	Flange thickness, mm
k_1	Shear strength enhancement factor for support compression
k_2	Concrete shear strength factor, $\left[f_{cu}/25\right]^{1/3}$
M	Design moment at a section, N-mm
M_u	Design moment resistance of a section, N-mm
M_u^0	Design moment resistance of a section with tendons only, N-mm
M_u^{bal}	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
s_v	Spacing of the links along the length of the beam, mm
s	Spacing of shear rails, mm
T	Tension force, N
V	Design shear force at ultimate design load, N
u	Perimeter of the punching critical section, mm
v	Design shear stress at a beam cross-section or at a punch critical section, MPa
v_c	Design ultimate shear stress resistance of a concrete beam, MPa
v_{co}	Ultimate shear stress resistance of an uncracked concrete section, MPa
v_{cr}	Ultimate shear stress resistance of a cracked concrete section, MPa
$v_{\max}$	Maximum permitted design factored shear stress at a beam section or at the punch critical section, MPa
v_t	Torsional shear stress, MPa
x	Neutral axis depth, mm
x_{bal}	Depth of neutral axis in a balanced section, mm

Table 8-1 List of Symbols Used in the BS 8110-97 Code

z	Lever arm, mm
β	Torsional stiffness constant
β_b	Moment redistribution factor in a member
γ_f	Partial safety factor for load
γ_m	Partial safety factor for material strength
ε_c	Maximum concrete strain, 0.0035
ε_{ps}	Strain in prestressing steel
ε_s	Strain in tension steel
ε'_s	Strain in compression steel

8.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. The design load combinations are obtained by multiplying the characteristic loads by appropriate partial factors of safety, γ_f (BS 2.4.1.3). For BS 8110-97, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), and wind (W) loads, and considering that wind forces are reversible, the load combinations in the following sections may need to be considered (BS 2.4.3, 4.1.7.1, 4.3.4 and 4.3.5).

For post-tensioned concrete design, the user can specify the prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning are defined in the following sections.

8.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress forces in accordance with BS 8110-97 clause 4.3.5. The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0D + 1.0PT

8.2.2 Service Load Combination

The following load combinations are used for checking the requirements of prestress for serviceability in accordance with BS 4.3.4. It is assumed that all long-term losses have already occurred at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT

8.2.3 Ultimate Limit State Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with BS 2.4.3.1.1, Table 2.1.

The strength design combinations required for punching shear require the full PT forces (primary and secondary). Flexural design only requires the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are automatically determined by ETABS by subtracting out the primary PT moments when the flexural design is carried out.

1.4D + 1.0PT*	(BS 2.4.3)
1.4D + 1.6L + 1.0PT*	(BS 2.4.3)
1.4D + 1.6(0.75PL) + 1.0PT*	(BS 2.4.3)
1.0D ± 1.4W +1.0PT* 1.4D ± 1.4W + 1.0PT* 1.2D + 1.2L ± 1.2W + 1.0PT*	(BS 2.4.3)
1.4D + 1.6L + 1.6S + 1.0PT* 1.2D + 1.2S ± 1.2W + 1.0PT* 1.2D + 1.2L + 1.2S ± 1.2W + 1.0PT*	(BS 2.4.3)

* — Replace PT with H for flexural design only

Other appropriate loading combinations should be used if roof live load is separately treated, or other types of loads are present.

8.3 Limits on Material Strength

Grade C28/C35 and C32/C40 are the minimum recommended for post-tensioning and pre-tensioning respectively. In both cases the concrete strength at transfer should not be less than 25 MPa (BS 4.1.8.1).

The specified characteristic strength of untensioned reinforcement is given as follows (BS 4.1.8.2, 3.1.7.4):

Hot rolled mild reinforcement -	250 MPa	(BS 3.1.7.4, Table 3.1)
High yield reinforcement -	500 MPa	(BS 3.1.7.4, Table 3.1)

The specified characteristic strength of prestressing steel should conform to BS 448 and BS 5896.

ETABS also checks the tensile strength in the prestressing steel (BS 4.7.1). The permissible tensile stresses in all types of prestressing steel, in terms of the specified minimum tensile strength f_{pu}, are summarized as follows:

a. Due to tendon jacking force: $0.75\ f_{pu}$

b. Immediately after prestress transfer: $0.70\ f_{pu}$

In any circumstances, the initial prestressing forces shall not exceed $0.75\ f_{pu}$.

8.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength of the material by a partial safety factor, γ_m. The values of γ_m used in the program are listed in the table that follows, as taken from BS Table 2.2 (BS 2.4.4.1):

Values of γ_m for the ultimate limit state	
Reinforcement, γ_{ms}	1.15
Prestressing steel, γ_{mp}	1.15
Concrete in flexure and axial load, γ_{mc}	1.50
Shear strength without shear reinforcement, γ_{mv}	1.25

These factors are already incorporated in the design equations and tables in the code. Note that for reinforcement, the default factor of 1.15 is for Grade 500 reinforcement. If other grades are used, this value should be overwritten as necessary. Changes to the partial safety factors are carried through the design equations where necessary, typically affecting the material strength portions of the equations.

8.5 Design Assumptions for Prestressed Concrete Structures

The ultimate limit state of prestressed members for flexure and axial loads shall be based on assumptions given in BS 4.3.7.1.

- The strain distribution in the concrete in compression is derived from the assumption that a plane section remains plane (BS 4.3.7.1(a)).
- The design stresses in the concrete in compression are taken as 0.45 f_{cu}. The maximum strain at the extreme concrete compression fiber shall be assumed equal to 0.0035 (BS 4.3.7.1(b)).
- Tensile strength of the concrete is ignored (BS 4.3.7.1(c)).
- The strain in bonded prestressing tendons and in any additional reinforcement (compression or tension) is derived from the assumption that the plane section remains plane (BS 4.3.7.1(d)).

The serviceability limit state of prestressed members uses the following assumptions given in BS 4.3.4.1.

- Plane sections remain plane, i.e., strain varies linearly with depth through the entire load range (BS 4.3.4.1(a)).
- Elastic behavior exists by limiting the concrete stresses to the values given in BS 4.3.4.2, 4.3.4.3, and 4.3.5 (BS 4.3.4.1(b)).
- In general, it is only necessary to calculate design stresses resulting from the load arrangements immediately after the transfer of prestress and after all losses or prestress have occurred; in both cases the effects of dead and imposed loads on the strain and force in the tendons may be ignored (BS 4.3.4.1(c)).

Prestressed concrete members are investigated at the following three stages (BS 4.3.4.2 and 4.3.4.3):

- At transfer of prestress force
- At service loading
- At nominal strength

The prestressed flexural members are classified as Class 1 (uncracked), Class 2 (cracked but no visible cracking), and Class 3 (cracked) based on tensile strength f_t, the computed extreme fiber stress in tension in the precompressed tensile zone at service loads (BS 4.1.3).

The precompressed tensile zone is that portion of a prestressed member where flexural tension, calculated using gross section properties, would occur under unfactored dead and live loads if the prestress force was not present. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress.

Class 1: No flexural tensile stresses

Class 2: Flexural tensile stresses but no visible cracking

Class 3: Flexural tensile stresses but surface width of cracks are as follows:

- Crack width ≤ 0.1 mm for members in exposure class XS2, XS3, XD2, XD3, XF3 and XF4 (Table A.1 of BS 8500-1)
- Crack width ≤ 0.2 mm for all other members

8.6 Serviceability Requirements of Flexural Members

8.6.1 Serviceability Check at Initial Service Load

The stresses in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits (BS 4.3.5.1 and 4.3.5.2):

- Extreme fiber stress in compression: $0.50 f_{ci}$
- Extreme fiber stress in tension for Class 1: ≤ 1.0 MPa
- Extreme fiber stress in tension for Class 2:

  pre-tensioned member $0.45\sqrt{f_{ci}}$

  post-tensioned member $0.36\sqrt{f_{ci}}$

The extreme fiber stress in tension for Class 3 should not exceed the appropriate value for a Class 2 member; otherwise the section should be designed as a cracked section.

8.6.2 Serviceability Check at Service Load

The stresses in the concrete for Class 1 and Class 2 prestressed flexural members at service loads, and after all prestress losses have occurred, are checked against the following limits (BS 4.3.4.2, 4.3.4.3):

- Extreme fiber stress in compression due to prestress plus total load: $0.33 f_{cu}$
- Extreme fiber stress in compression due to prestress plus total load for continuous beams and other statically indeterminate structures: $0.4 f_{cu}$
- Extreme fiber stress in tension in the precompressed tensile zone at service loads:
  - Extreme fiber stresses in tension for Class 1: No tensile stress
  - Extreme fiber stresses in tension for Class 2:

    pre-tensioned member $0.45\sqrt{f_{cu}}$

    post-tensioned member $0.36\sqrt{f_{cu}}$

Although cracking is allowed for Class 3, it is assumed that the concrete section is uncracked and the user is limiting the tensile stress at service stage as presented in Table 4.2, modified by the coefficients in Table 4.3 of BS 8110-1997. The user needs to provide the tension limits for Class 3 elements at service loads in the design preferences BS 4.3.4.3(c)).

8.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

8.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

8.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of prestressed concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or a flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases the beam may be designed as a rectangular or inverted flanged beam.

8.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 8-1 (BS 3.4.4.4, 4.3.7.1). Furthermore, it is assumed that moment redistribution in the member does not exceed 10 percent (i.e., $\beta_b \geq 0.9$) (BS 3.4.4.4). The code also places a limitation on the neutral axis depth, $x/d \leq 0.5$, to safeguard against non-ductile failures (BS 3.4.4.4). In addition, the area of compression reinforcement is calculated on the assumption that the neutral axis depth remains at the maximum permitted value.

The design procedure used by ETABS, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow. It is assumed that the design ultimate axial force does not exceed 0.1 $f_{cu} A_g$ (BS 3.4.4.1); hence all beams are designed for major direction flexure, shear, and torsion only.

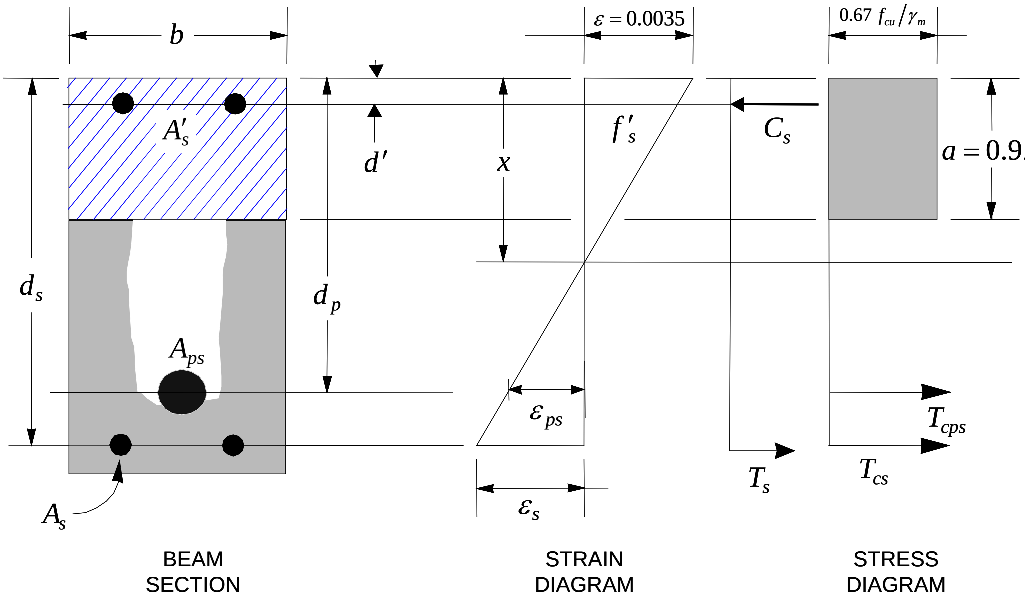


Figure 8-1 Rectangular Beam Design

8.7.1.2.1 Design of Rectangular Beams

The amount of post-tensioning steel adequate to resist the design moment *M* and minimum reinforcement are provided to satisfy the flexural cracking requirements (BS 4.12.6).

ETABS determines the depth of the neutral axis, *x*, by imposing force equilibrium, i.e., C = T, and performs an iteration to compute the depth of the neutral axis, which is based on stress-strain compatibility. After the depth of the neutral axis has been found, the stress in the post-tensioning reinforcement f_{pb} is computed based on strain compatibility.

The ductility of a section is controlled by limiting the *x/d* ratio (BS 3.4.4.4):

$$x/d = 0.5 \qquad \text{(BS 3.4.4.4)}$$

The maximum depth of the compression block is given by:

$$a = 0.9x \qquad \text{(BS 3.4.4.1(b), 4.3.7.3)}$$

The lever arm of the section must not be greater than 0.95 times the effective depth (BS 3.4.4.1).

$$z = d - 0.45x \le 0.95d_e \qquad \text{(BS 3.4.4.1(e))}$$

- If $a \le a_{\max}$ (BS 3.4.4.4), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_u^0 = A_{ps} f_{pb} \left(d_p - 0.45x \right) \qquad \text{(BS 4.3.7.3)}$$

- If $a > a_{\max}$ (BS 3.4.4.4), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of neutral axis x is equal to $c_{\max}$. The stress in the post-tensioning steel, f_{pb} is then calculated based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = \frac{0.67 f_{cu}}{\gamma_m} a_{\max} b$$

$$T = A_{ps} f_{pb}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\dfrac{0.67 f_{cu}}{\gamma_m} a_{\max} b - A_{ps} f_{pb}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_u^{bal} = A_{ps} f_{pb}^{bal} \left(d_p - \frac{a_{\max}}{2} \right) + A_s^{bal} f_s^{bal} \left(d_s - \frac{a_{\max}}{2} \right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of neutral axis, x.

8.7.1.2.1.1 Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment M.. Minimum reinforcement is provided to satisfy the ductility requirements, i.e., $M < M_u^0$.

8.7.1.2.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports this required area of tension reinforcement. Since M is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end, and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will fall between the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

8.7.1.2.1.3 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required (BS 3.4.4.4). In that case, ETABS assumes that the depth of neutral axis, x, is equal to $x_{\max}$. The values of f_{pb} and f_s reach their respective balanced condition values, f_{pb}^{bal} and f_s^{bal}. The area of compression reinforcement, $A_s^{'}$, is determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(f'_s - \frac{0.67 f_{cu}}{\gamma_c}\right)(d - d')}, \text{ where} \qquad \text{(BS 3.4.4.4)}$$

$$f'_s = E_s \varepsilon_c \left[\frac{a_{max} - d'}{a_{max}}\right] \leq 0.87 f_y$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{0.87 f_y (d_s - d')}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M is positive, and vice versa if M is negative.

8.7.1.2.2 Design of Flanged Beams

8.7.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

8.7.1.2.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, moment capacity M_u^0 is determined as follows:

ETABS determines the depth of the neutral axis, x, by imposing force equilibrium, i.e., C = T, and performs an iteration to compute the depth of neutral axis, which is based on stress-strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pb} is computed based on strain compatibility.

The ductility of a section is controlled by limiting the x/d ratio (BS 3.4.4.4):

$$x/d = 0.5 \qquad \text{(BS 3.4.4.4)}$$

The maximum depth of the compression block is given by:

$$a = 0.9x \qquad \text{(BS 3.4.4.1(b), 4.3.7.3)}$$

The lever arm of the section must not be greater than 0.95 times its effective depth (BS 3.4.4.1):

$$z = d - 0.45x \leq 0.95d_e \qquad \text{(BS 3.4.4.1(e))}$$

- If $a \leq a_{\max}$ (BS 3.4.4.4), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_u^0 = A_{ps} f_{pb} \left(d_p - 0.45x\right)$$

- If $a > a_{\max}$ (BS 3.4.4.4), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case it is assumed that the depth of neutral axis x is equal to $x_{\max}$. The stress in the post-tensioning steel, f_{pb} is then calculated based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \leq h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as b_f. Compression reinforcement is required when $x/d > 0.5$.

- If $a > h_f$, the calculation for A_s is given by

$$C = \frac{0.67\, f_{cu}}{\gamma_c} a_{\max} A_c^{com}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$A_c^{com} = b_f h_f + b_w \left(a_{\max} - h_f \right)$$

$$T = A_{ps} f_{pb}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\frac{0.67\, f_{cu}}{\gamma_m} a_{\max} A_c^{com} - A_{ps} f_{pb}^{bal}}{f_s^{bal}}$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of neutral axis, x.

8.7.1.2.2.3 Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment M. Minimum reinforcement is provided to satisfy ductility requirements.

8.7.1.2.2.4 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

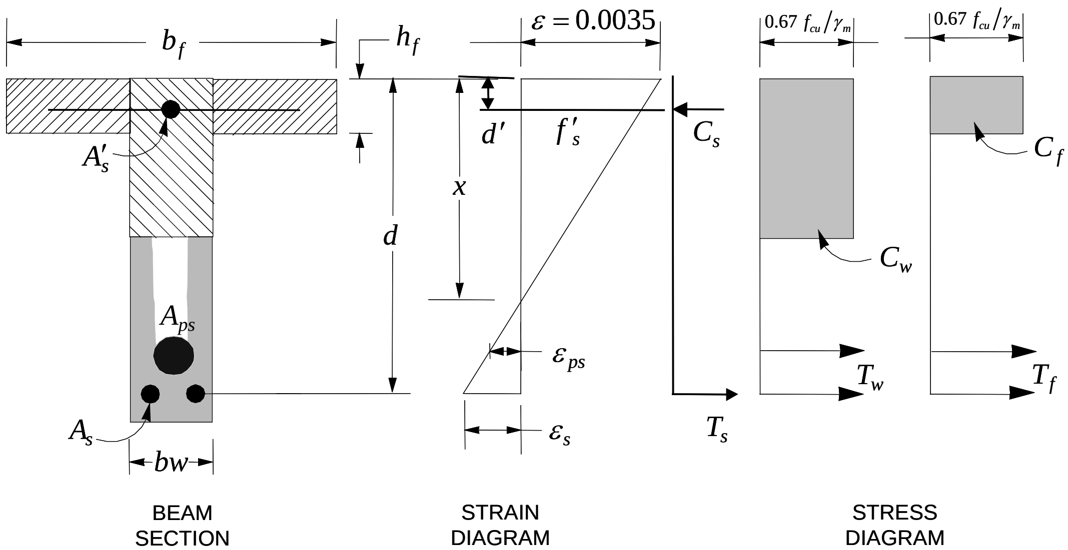


Figure 8-2 T-Beam Design

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports the required area of tension reinforcement. Since M is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end, and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

8.7.1.2.2.5 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required (BS 3.4.4.4). In that case ETABS assumes that the depth of the neutral axis, x, is equal to x_{max}. The values of f_{pb} and f_s reach their respective balanced condition values, f_{pb}^{bal} and f_s^{bal}. The area of compression reinforcement, A'_s, is then determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(f'_s - \frac{0.67 f_{cu}}{\gamma_c}\right)(d - d')}, \text{ where} \qquad \text{(BS 3.4.4.4)}$$

$$f'_s = E_s \varepsilon_c \left[\frac{a_{\max} - d'}{a_{\max}}\right] \leq 0.87 f_y.$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{0.87 f_y (d_s - d')}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom, and A'_s is to be placed at the top if M is positive and vice versa if M is negative.

8.7.1.3 Minimum and Maximum Reinforcement

Reinforcement in post-tensioned concrete beams is computed to increase the strength of sections as documented for the flexural design of post-tensioned beams or to comply with the shear link requirements. The minimum flexural tension reinforcement required for a beam section to comply with the cracking requirements needs to be separately investigated by the user.

For bonded tendons, there is no minimum un-tensioned reinforcement required.

For unbounded tendons, the minimum flexural reinforcement provided in a rectangular or flanged beam section is given by the following table, which is taken from BS Table 3.25 (BS 3.12.5.3) with interpolation for reinforcement of intermediate strength:

Section	Situation	Definition of percentage	Minimum percentage f_y = 250 MPa	Minimum percentage f_y = 460 MPa
Rectangular	—	$100 \frac{A_s}{bh}$	0.24	0.13
T- or L-Beam with web in tension	$\frac{b_w}{b_f} < 0.4$	$100 \frac{A_s}{b_w h}$	0.32	0.18
	$\frac{b_w}{b_f} \geq 0.4$	$100 \frac{A_s}{b_w h}$	0.24	0.13
T-Beam with web in compression	—	$100 \frac{A_s}{b_w h}$	0.48	0.26
L-Beam with web in compression	—	$100 \frac{A_s}{b_w h}$	0.36	0.20

The minimum flexural compression reinforcement, if it is required at all, is given by the following table, which is taken from BS Table 3.25 (BS 3.12.5.3), with interpolation for reinforcement of intermediate strength:

Section	Situation	Definition of percentage	Minimum percentage
Rectangular	—	$100 \frac{A'_s}{bh}$	0.20
T or L-Beam	Web in tension	$100 \frac{A'_s}{b_f h_f}$	0.40
	Web in compression	$100 \frac{A'_s}{b_w h}$	0.20

In addition, an upper limit on both the tension reinforcement and compression reinforcement is imposed to be 0.04 times the gross cross-sectional area (BS 3.12.6.1).

8.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination in the major direction of the beam. In designing the shear reinforcement for a particular beam for a particular load combination, the following steps are involved (BS 3.4.5):

- Determine the shear stress, v.
- Determine the shear stress, v_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three subsections describe in detail the algorithms associated with these steps.

8.7.2.1 Determine Shear Stress

In the design of the beam shear reinforcement, the shear forces for a particular load combination at a particular beam section are obtained by factoring the associated shear forces for different load cases with the corresponding load combination factors.

$$v = \frac{V}{b_w d} \qquad \text{(BS 3.4.5.2)}$$

The maximum allowable shear stress, $v_{\max}$ is defined as:

$$v_{\max} = \min(0.8\sqrt{f_{cu}},\ 5 \text{ MPa}) \qquad \text{(BS 3.4.5.2)}$$

For light-weight concrete, $v_{\max}$ is defined as:

$$v_{\max} = \min(0.63\sqrt{f_{cu}},\ 4 \text{ MPa}) \qquad \text{(BS 8110-2:1985 5.4)}$$

8.7.2.2 Determine Concrete Shear Capacity

The design ultimate shear resistance of the concrete alone, V_c should be considered at sections that are as follows:

Uncracked sections in flexure ($M < M_o$) (BS 4.3.8.3)

Cracked sections in flexural ($M \geq M_o$) (BS 4.3.8.3)

where,

M is the design bending moment at the section

M_o is the moment necessary to produce zero stress in the concrete at the extreme tension fiber; in this calculation, only 0.8 of the stress due to post-tensioning should be taken into account.

8.7.2.2.1.1 Case 1: Uncracked section in flexure

The ultimate shear resistance of the section, V_{co}, is computed as follows:

$$V_{co} = 0.67 b_v h \sqrt{\left(f_t^{\,2} + 0.8 f_{cp} f_t \right)}, \qquad \text{(BS 4.3.8.4)}$$

where,

f_t is the maximum design principal stress (BS 4.3.8.4)

$$f_t = 0.24 \sqrt{f_{cu}} \qquad \text{(BS 4.3.8.4)}$$

f_{cp} = design compressive stress at the centroidal axis due to post-tensioning, taken as positive. (BS 4.3.8.4)

$$V_c = V_{co} + P \sin \beta \qquad \text{(BS 4.3.8.4)}$$

8.7.2.2.1.2 Case 2: Cracked section in flexure

The ultimate shear resistance of the section, V_{cr}, is computed as follows:

$$V_{cr} = \left(1 - 0.55 \frac{f_{pe}}{f_{pu}} \right) v_c b_v d + M_o \frac{V}{M}, \text{ and} \qquad \text{(BS 4.3.8.5)}$$

$$V_{cr} \geq 0.1 b_v d \sqrt{f_{cu}} \qquad \text{(BS 4.3.8.5)}$$

$$V_c = \min\left(V_{co}, V_{cr} \right) + P \sin \beta \qquad \text{(BS 4.3.8.5)}$$

8.7.2.3 Determine Required Shear Reinforcement

Given v, v_c and v_{max} , the required shear reinforcement is calculated as follows (BS 4.3.8.7):

- If $v \leq v_c + 0.4$,

$$\frac{A_{sv}}{s_v} = \frac{0.4\, b_w}{0.87\ f_{yv}} \qquad \text{(BS 4.3.8.7)}$$

- If $(v_c + 0.4) < v \leq v_{max}$,

$$\frac{A_{sv}}{s_v} = \frac{\left(v - v_c\right) b_v}{0.87\ f_{yv}} \qquad \text{(BS 4.3.8.7)}$$

- If $v > v_{max}$, a failure condition is declared. (BS 3.4.5.2)

In the preceding expressions, a limit is imposed on f_{yv} as:

$f_{yv} \leq 500$ MPa. (BS 3.4.5.1, 4.3.8.1)

The maximum of all of the calculated A_{sv}/s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

8.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the torsional shear stress, v_t
- Determine special section properties

- Determine critical torsion stress
- Determine the torsion reinforcement required

Note that references in this section are to BS 8110-2:1985 [BSI 1985].

8.7.3.1 Determine Torsional Shear Stress

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In typical framed construction, specific consideration of torsion is not usually required where torsional cracking is adequately controlled by shear reinforcement. If the design relies on the torsional resistance of a beam, further consideration should be given using the following algorithms (BS 8110-2:85 3.4.5.13).

The torsional shear stress, v_t, for a rectangular section is computed as:

$$v_t = \frac{2T}{h_{\min}^2 \left(h_{\max} - h_{\min}/3\right)} \qquad \text{(BS 8110-2:85 2.4.4.1)}$$

For flanged sections, the section is considered as a series of rectangular segments and the torsional shear stress is computed for each rectangular component using the preceding equation, but considering a torsional moment attributed to that segment, calculated as:

$$T_{\text{seg}} = T\left(\frac{h_{\min}^3 h_{\max}}{\sum\left(h_{\min}^3 h_{\max}\right)}\right) \qquad \text{(BS 8110-2:85 2.4.4.2)}$$

h_{mzx} = Larger dimension of a rectangular section

$h_{\min}$ = Smaller dimension of a rectangular section

If the computed torsional shear stress, v_t, exceeds the following limit for sections with the larger center-to-center dimension of the closed link less than 550 mm, a failure condition is generated if the torsional shear stress does not satisfy:

$$v_t \leq \min(0.8\sqrt{f_{cu}}, 5\text{N/mm}^2) \times \frac{y_1}{550} \qquad \text{(BS 8110-2:85 2.4.5)}$$

8.7.3.2 Determine Critical Torsion Stress

The critical torsion stress, $v_{t,\min}$, for which the torsion in the section can be ignored is calculated as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.4\text{N/mm}^2\right) \qquad \text{(BS 8110-2:85 2.4.6)}$$

where f_{cu} is the specified concrete compressive strength.

For light-weight concrete, $v_{t,\min}$ is defined as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.4\text{N/mm}^2\right) \times 0.8 \qquad \text{(BS 8110-2:85 5.5)}$$

8.7.3.3 Determine Torsion Reinforcement

If the factored torsional shear stress, v_t is less than the threshold limit, $v_{t,\min}$, torsion can be safely ignored (BS 8110-2:85 2.4.6). In that case, the program reports that no torsion reinforcement is required. However, if v_t exceeds the threshold limit, $v_{t,\min}$, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (BS 8110-2:85 2.4.6).

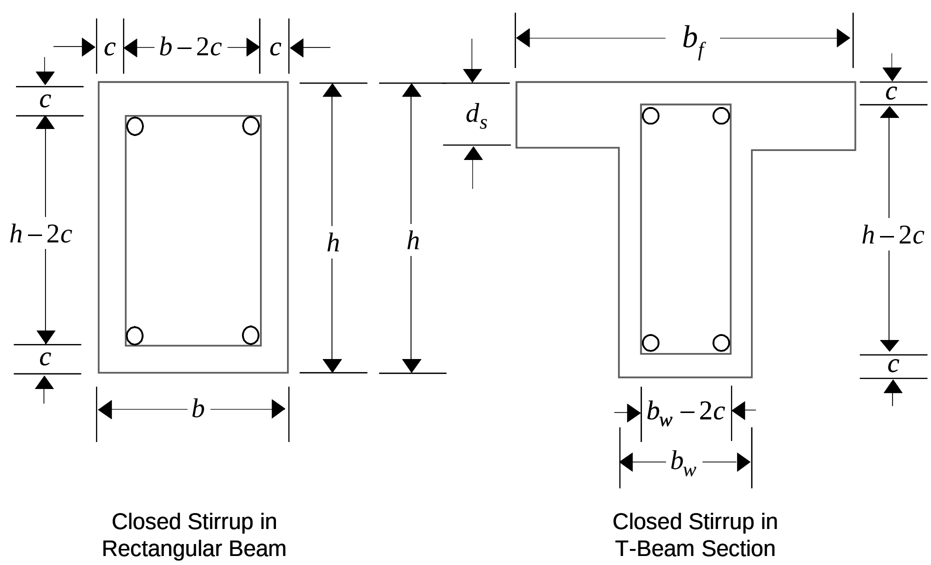


Figure 8-3 Closed stirrup and section dimensions for torsion design

- If $v_t > v_{t,\min}$, the required closed stirrup area per unit spacing, $A_{sv,t}/s_v$, is calculated as:

$$\frac{A_{sv,t}}{s_v} = \frac{T}{0.8x_1y_1\left(0.87 f_{yv}\right)} \quad \text{(BS 8110-2:85 2.4.7)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{A_{sv,t} f_{yv}\left(x_1 + y_1\right)}{s_v f_y} \quad \text{(BS 8110-2:85 2.4.7)}$$

In the preceding expressions, x_1 is the smaller center-to-center dimension of the closed link and y_1 is the larger center-to-center dimension of the closed link.

An upper limit of the combination of v and v_t that can be carried by the section also is checked using the equation:

$$v + v_t \le \min\left(0.8\sqrt{f_{cu}}\,,\ 5\text{N/mm}^2\right) \quad \text{(BS 8110-2:85 2.4.5)}$$

For light-weight concrete, $v_{\max}$ is defined as:

$$v_{\max} = \min(0.63\sqrt{f_{cu}}\,,\ 4\text{ MPa}) \quad \text{(BS 8110-2:85 5.4)}$$

If the combination of shear stress, v and torsional shear stress, v_t exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated A_l and $A_{sv,t}/s_v$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

8.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips typically are governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (BS 8110-97) for prestressed reinforced concrete as described in the following subsections. To learn more about the design strips, refer to the section entitled "ETABS Design Techniques" in the *Key Features and Terminology* manual.

8.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Determine the capacity of post-tensioned sections
- Design flexural reinforcement for the strip

These three steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

8.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

8.8.1.2 Determine Capacity of Post-Tensioned Sections

The calculation of the post-tensioned section capacity is identical to that described earlier for rectangular beam sections.

8.8.1.3 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

8.8.1.4 Minimum and Maximum Slab Reinforcement

There are no minimum requirements for untensioned reinforcement in one-way bonded slabs. One-way spanning floors with unbounded tendons should have minimum reinforcement requirements in accordance with BS Table 3.25 (BS 3.12.5.3)

In flat slabs, reinforcement is added at the top over supports to be 0.00075 times the gross cross-sectional area. This reinforcement extends 1.5 times the slab depth on each side of the column. The length of the reinforcement should be at least 0.2*L* where *L* is the span of the slab.

There are no minimum requirements for span zone. However, additional untensioned reinforcement shall be designed to accommodate the full tension force generated by assumed flexural tensile stresses in the concrete for the following situations (Concrete Society, Technical Report 43):

- all locations in one-way spanning floors using unbonded tendons
- all locations in one-way spanning floors where transfer tensile stress exceeds $0.36\sqrt{f_{ci}}$
- support zones in all flat slabs
- span zones in flat slabs using unbonded tendons where the tensile stress exceeds $0.15\sqrt{f_{cu}}$.

The reinforcement should be designed to act at a stress of $5/8f_y$ as follows:

$$A_s = \frac{F_t}{(5/8)\, f_y}$$

where

$$F_t = -\frac{f_{ct}(h-x)b}{2}$$

The value of f_{ct} will be negative in tension.

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (BS 3.12.6.1).

8.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Key Features and Terminology* manual. Only the code specific items are described in the following sections.

8.8.2.1 Critical Section for Punching Shear

The punching shear is checked at the face of the column (BS 3.7.6.4) and at a critical section at a distance of 1.5*d* from the face of the support (BS 3.7.7.6).

For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (BS 3.7.7.1). Figure 8-4 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

8.8.2.2 Determine Concrete Capacity

The design ultimate shear resistance of the concrete alone V_c should be considered at sections that are as follows:

Uncracked sections in flexure ($M < M_o$) (BS 4.3.8.3)

Cracked sections in flexural ($M \geq M_o$) (BS 4.3.8.3)

where,

M is the design bending moment at the section

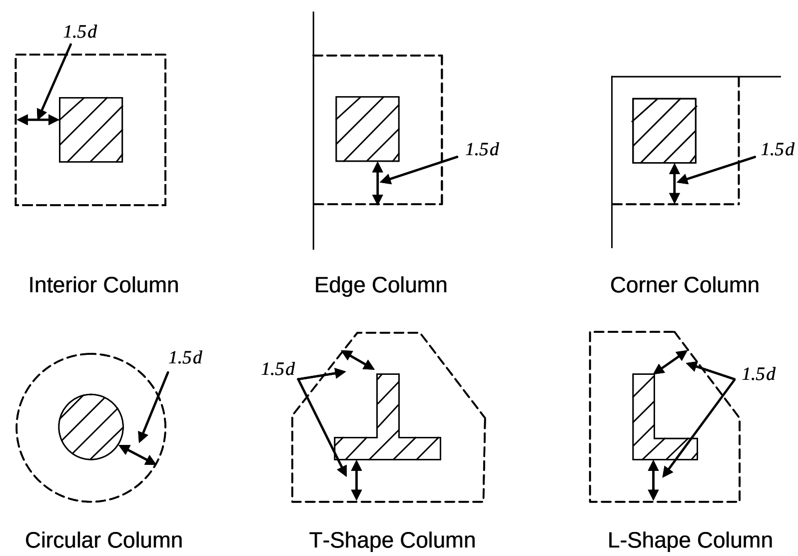


Figure 8-4 Punching Shear Perimeters

M_o is the moment necessary to produce zero stress in the concrete at the extreme tension fiber; in this calculation only 0.8 of the stress due to post-tensioning should be taken into account.

8.8.2.2.1.1 Case 1: Uncracked section in flexure

The ultimate shear resistance of the section, V_{co}, is computed as follows:

$$V_{co} = 0.67 b_v h \sqrt{\left(f_t^2 + 0.8 f_{cp} f_t\right)}, \qquad \text{(BS 4.3.8.4)}$$

where,

f_t is the maximum design principal stress (BS 4.3.8.4)

$$f_t = 0.24\sqrt{f_{cu}} \qquad \text{(BS 4.3.8.4)}$$

f_{cp} = design compressive stress at the centoidal axis due to prestress, taken as positive. (BS 4.3.8.4)

$$V_c = V_{co} + P\text{Sin}\beta \qquad \text{(BS 4.3.8.4)}$$

8.8.2.2.1.2 Case 2: Cracked section in flexure

The ultimate shear resistance of the section, V_{cr}, is computed as follows:

$$V_{cr} = \left(1 - 0.55\frac{f_{pe}}{f_{pu}}\right) v_c b_v d + M_o \frac{V}{M}, \text{ and} \qquad \text{(BS 4.3.8.5)}$$

$$V_{cr} \geq 0.1 b_v d \sqrt{f_{cu}} \qquad \text{(BS 4.3.8.5)}$$

$$V_c = \min\left(V_{co}, V_{cr}\right) + P\sin\beta \qquad \text{(BS 4.3.8.5)}$$

8.8.2.3 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the nominal design shear stress, v, is calculated as:

$$v = \frac{V_{eff}}{u\,d}, \text{ where} \qquad \text{(BS 3.7.7.3)}$$

$$V_{eff} = V \left\{ f + 1.5\frac{M_y}{V\,x} + 1.5\frac{M_x}{V\,y} \right\}, \qquad \text{(BS 3.7.6.2, 3.7.6.3)}$$

u is the perimeter of the critical section

x and y are the length of the side of the critical section parallel to the axis of bending

M_x and M_y are the design moments transmitted from the slab to the column at the connection

V is the total punching shear force

f is a factor to consider the eccentricity of punching shear force and is taken as:

$$f = \begin{cases} 1.00 & \text{for interior columns} \\ 1.25 & \text{for edge columns, and} \\ 1.25 & \text{for corner columns} \end{cases} \qquad \text{(BS 3.7.6.2 and BS 3.7.6.3)}$$

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS.

8.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (BS 3.7.7.5). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed, and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* as described in the earlier sections remains unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

8.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

8.8.3.2 Determine Required Shear Reinforcement

The shear stress is limited to a maximum limit of

$$v_{max} = 2\, v_c \qquad \text{(BS 3.7.7.5)}$$

Given v, v_c and v_{max}, the required shear reinforcement is calculated as follows (BS 3.7.7.5).

- If $v \le 1.6 v_c$

$$\frac{A_v}{s} = \frac{(v - v_c)ud}{0.87\, f_{yv}} \ge \frac{0.4ud}{0.87\, f_{yv}}, \qquad \text{(BS 3.7.7.5)}$$

- If $1.6 v_c \le v < 2.0 v_c$

$$\frac{A_v}{s} = \frac{5(0.7v - v_c)ud}{0.87\, f_{yv}} \ge \frac{0.4ud}{0.87\, f_{yv}}, \qquad \text{(BS 3.7.7.5)}$$

- If $v > v_{max}$, a failure condition is declared. (BS 3.7.7.5)

If v exceeds the maximum permitted value of v_{max}, the concrete section should be increased in size.

8.8.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 8-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner columns.

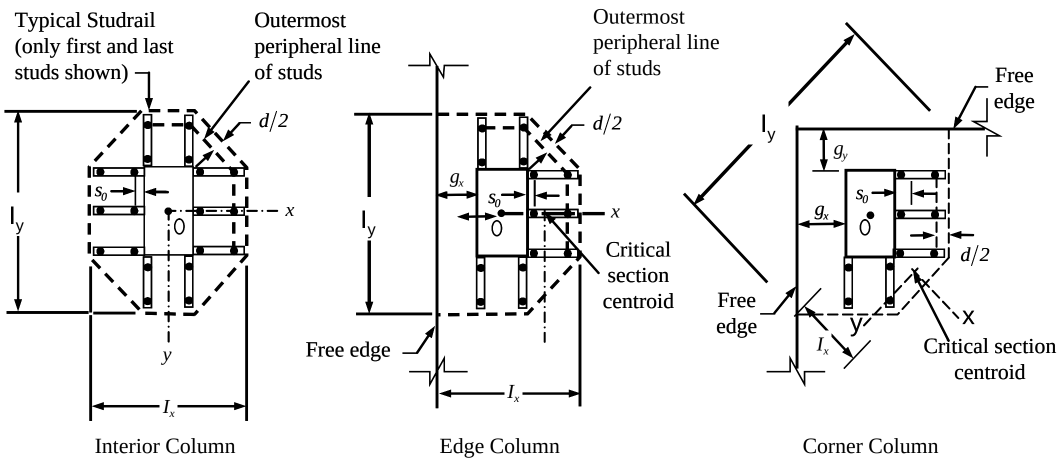


Figure 8-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between column face and the first line of shear reinforcement shall not exceed *d*/2. The spacing between adjacent shear reinforcement in the first line of shear reinforcement shall not exceed 1.5*d* measured in a direction parallel to the column face (BS 11.12.3.3).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8 for corner, edge, and interior columns respectively.

8.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in BS 3.3 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5*d*. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed 1.5d. The limit of s_o and the spacing, s, between the peripheral lines are specified as:

$s_o \leq 0.5d$ (BS 3.7.7.6)

$s \leq 0.75d$ (BS 3.7.7.6)

$g \leq 1.5d$ (BS 3.7.7.6)

Chapter 9
Design for CSA A23.3-04

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the Canadian code CSA A23.3-04 [CSA 2004]. Various notations used in this chapter are listed in Table 9-1. For referencing to the pertinent sections of the CSA code in this chapter, a prefix "CSA" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

9.1 Notations

The following table identifies the various notations used in this chapter.

Table 9-1 List of Symbols Used in the CSA A23.3-04 Code

A_p	Area of tension prestressing tendons, mm^2

Table 9-1 List of Symbols Used in the CSA A23.3-04 Code

A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2
$A_{s(required)}$	Area of steel required for tension reinforcement, mm^2
A_v	Area of shear reinforcement, mm^2
A_v / s	Area of shear reinforcement per unit length of the member, mm^2/mm
A_{vs}	Area of headed shear reinforcement, mm^2
A_{vs} / s	Area of headed shear reinforcement per unit length of the member, mm^2/mm
a	Depth of compression block, mm
b	Width of member, mm
b_f	Effective width of flange (T-beam section), mm
b_w	Width of web (T-beam section), mm
b_0	Perimeter of the punching critical section, mm
b_1	Width of the punching critical section in the direction of bending, mm
b_2	Width of the punching critical section perpendicular to the direction of bending, mm
c	Depth to neutral axis, mm
d	Distance from compression face to tension reinforcement, mm
d'	Concrete cover to center of reinforcing, mm
d_p	Distance from compression face to prestressing tendons, mm
d_s	Thickness of slab, mm

Table 9-1 List of Symbols Used in the CSA A23.3-04 Code

d_v	Effective shear depth, mm
E_c	Modulus of elasticity of concrete, MPa
E_p	Modulus of elasticity of prestressing tendons, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 2×10^5 MPa
f'_{ci}	Specified compressive strength of concrete at time of prestress transfer, MPa
f'_c	Specified compressive strength of concrete, MPa
f_y	Specified yield strength of flexural reinforcement, MPa
f_{yh}	Specified yield strength of shear reinforcement, MPa
f_{yv}	Specified yield strength of headed shear reinforcement, MPa
h	Overall depth of a section, mm
I_g	Moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement.
M_f	Factored moment at section, N-mm
ϕM_r^0	Design moment resistance of a section with tendons only, N-mm
ϕM_r^{bal}	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
s	Spacing of the shear reinforcement along the length of the beam, mm
V_c	Shear resisted by concrete, N
$V_{r,max}$	Maximum permitted total factored shear force at a section, N

Table 9-1 List of Symbols Used in the CSA A23.3-04 Code

V_f	Factored shear force at a section, N
V_s	Shear force at a section resisted by steel, N
α_1	Ratio of average stress in rectangular stress block to the specified concrete strength
β_1	Factor for obtaining depth of compression block in concrete
β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
ε_{cu}	Maximum strain in concrete at ultimate
ε_p	Strain in prestressing tendons
ε_s	Strain in reinforcing steel
φ_c	Resistance factor for concrete
φ_p	Resistance factor for prestressing tendons
φ_s	Resistance factor for steel
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear
λ	Shear strength factor

9.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For CSA A23.3-04, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the load combinations in the following sections may need to be considered (CSA 8.3.2, Table C.1).

For post-tensioned concrete design, the user also can specify the prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning are defined in the following sections.

9.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress forces, in accordance with CSA 18.3.1. The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0D + 1.0PT

9.2.2 Service Load Combinations

The following load combinations are used for checking the requirements of prestress for serviceability in accordance with CSA 18.3.2. It is assumed that long-term losses have already occurred at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT

9.2.3 Long-Term Service Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with CSA 18.3.2(a). The permanent load for this load combination is taken as 50 percent of the live load. It is assumed that all long term losses have already occurred at the service stage.

1.0D + 1.0PT
1.0D + 0.5L + 1.0PT

9.2.4 Strength Design Load Combination

The following load combinations are used for checking the requirements of prestress for strength in accordance with CSA A23.3-04, Chapters 8 and 18.

The strength design combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are determined automatically by ETABS by subtracting the primary PT moments when the flexural design is carried out.

1.4D + 1.0PT*	(CSA 8.3.2,Table C.1, Case 1)
1.25D + 1.5L + 1.0PT* 1.25D + 1.5L + 1.0PT* ± 0.4W 1.25D + 1.5L + 1.0PT* + 0.5S 0.9D + 1.5L + 1.0PT* 0.9D + 1.5L + 1.0PT* ± 0.4W 0.9D + 1.5L + 1.0PT* + 0.5S	(CSA 8.3.2,Table C.1, Case 2)
1.25D + 1.5(0.75 PL) + 1.0PT*	(CSA 13.8.4.3)
1.25D + 1.5S + 1.0PT* 1.25D + 1.5S + 1.0PT* + 0.5L 1.25D + 1.5S + 1.0PT* ± 0.4W 0.9D + 1.5S + 1.0PT* 0.9D + 1.5S + 1.0PT* + 0.5L 0.9D + 1.5S + 1.0PT* ± 0.4W	(CSA 8.3.2,Table C.1, Case 3)
1.25D ± 1.4W + 1.0PT* 1.25D ± 1.4W + 1.0PT* + 0.5L 1.25D ± 1.4W + 1.0PT* + 0.5S 0.9D ± 1.4W + 1.0PT* 0.9D ± 1.4W + 1.0PT* + 0.5L 0.9D ± 1.4W + 1.0PT* + 0.5S	(CSA 8.3.2,Table C.1, Case 4)
1.0D ± 1.0E + 1.0PT* 1.0D ± 1.0E + 0.5L + 1.0PT* 1.0D ± 1.0E + 0.25S + 1.0PT* 1.0D + 0.5L + 0.25S ± 1.0E + 1.0PT*	(CSA 8.3.2,Table C.1, Case 5)

* — Replace PT by H for flexural design only

These are also the default design combinations in ETABS whenever the CSA A23.3-04 code is used. The user should use other appropriate load combinations if roof live load is treated separately, or if other types of loads are present.

9.3 Limits on Material Strength

The upper and lower limits of f'_c are 80 MPa and 20 MPa respectively. The upper limit of f_y is 500 MPa for non-prestressed reinforcement (CSA 8.6.1.1).

For compression reinforcement with f_y exceeding 400 MPa, the value of f_y assumed in design calculations shall not exceed the stress corresponding to a strain of 0.0035 (CSA 8.5.21).

ETABS enforces the upper material strength limits for flexure and shear design of slabs. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

ETABS also checks the following tensile strength limits in prestressing steel (CSA 18.4). The permissible tensile stresses in all types of prestressing steel, in terms of the specified minimum tensile strength f_{pu}, are summarized as follows:

- Due to tendon jacking force for post-tensioning tendons:

  $0.85\, f_{pu} \le 0.94\, f_{py}$

- Due to tendon jacking force for pretensioning tendons:

  $0.80\, f_{pu}$

- Immediately after prestress transfer:

  $0.82\, f_{py} \le 0.74\, f_{pu}$

- Post-tensioning tendons, at anchorages and couplers, immediately after tendon anchorage:

  $0.70\, f_{pu}$

The specified yield strength of prestressing tendons is based on the requirements specified in ASTM A 416/A 416 M, ASTM A 421/A421 M, and ASTM A 722/A 722 m, which specify the following minimum values for f_{py}:

- low-relaxation wire and strands $f_{py} = 0.90\ f_{pu}$
- stress-relieved wire and strands, and plain bars $f_{py} = 0.85\ f_{pu}$
- deformed bar $f_{py} = 0.80\ f_{pu}$

9.4 Strength Reduction Factors

The strength reduction factors, ϕ, are material dependent and defined as:

$\phi_c =$ 0.65 for concrete (CSA 8.4.2)

$\phi_s =$ 0.85 for reinforcement (CSA 8.4.3a)

$\phi_p =$ 0.90 for post-tensioning tendons (CSA 8.4.3a)

The preceding values for ϕ_c, ϕ_s, and ϕ_p are the default values. These values can be modified in the design preferences. For structural concrete manufactured in prequalified manufacturing plants, ϕ_c can be taken as 0.7 (CSA 8.4.2, 16.1.3).

9.5 Design Assumptions for Prestressed Concrete

Strength design of prestressed members for flexure and axial loads shall be based on assumptions given in CSA 10.1.

- The strain in the reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis, except for unbonded tendons (CSA 10.1.2).
- The maximum usable strain at the extreme concrete compression fiber shall be assumed equal to 0.0035 (CSA 10.1.3).
- The balanced strain condition shall exist at a cross-section when tension reinforcement reaches its yield strain just as the concrete in compression reaches its maximum strain of 0.0035 (CSA 10.1.4).

- The tensile strength of concrete shall be neglected in the calculation of the factored flexural resistance of prestressed concrete members (CSA 10.1.5).
- The relationship between the concrete compressive stress distribution and the concrete strain shall be assumed to be rectangular by an equivalent rectangular concrete stress distribution (CSA 10.1.7).
- The concrete stress of $\alpha_c \varphi_c f_c^{'}$ shall be assumed uniformly distributed over an equivalent-compression zone bounded by edges of the cross-section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain (CSA 10.1.7(a)).
- The distance from the fiber of maximum strain to the neutral axis, *c*, shall be measured in a direction perpendicular to the neutral axis (CSA 10.1.7.(b)).
- The factors α_1 and β_1 shall be taken as follows (CSA 10.1.7.(c)).
  - $\alpha_1 = 0.85 - 0.0015 f_c^{'} \geq 0.67$
  - $\beta_1 = 0.97 - 0.0025 f_c^{'} \geq 0.67$

Prestressed concrete members are investigated at the following three stages (CSA 18.3):

- At transfer of prestress force
- At service loading
- At nominal strength

9.6 Serviceability Requirements of Flexural Members

9.6.1 Serviceability Check at Initial Service Load

The stress in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits (CSA 18.3.1.1(a), 18.3.1.1(b) and 18.3.1.1(c)):

- Extreme fiber stress in compression:

$$0.60 f'_{ci}$$

- Extreme fiber stress in tension, except as permitted in the subsequent item:

$$0.25\lambda\sqrt{f'_{ci}}$$

- Extreme fiber stress in tension at ends of simply supported members:

$$0.5\lambda\sqrt{f'_{ci}}$$

The extreme fiber stress in tension at the ends of simply supported members is currently **NOT** checked by ETABS.

9.6.2 Serviceability Check at Service Load

The stresses in prestressed concrete flexural members at service loads, and after all prestress losses occur, are checked against the following limits (CSA 18.3.2):

- Extreme fiber stress in compression due to prestress plus total load:

$$0.60 f'_{c}$$

- Extreme fiber stress in tension in the precompressed tensile zone at service loads:

$$0.5\lambda\sqrt{f'_{c}}$$

- Extreme fiber stress in tension in the precompressed tensile zone at service loads, exposed to corrosive environment:

$$0.25\lambda\sqrt{f'_{c}}$$

9.6.3 Serviceability Check at Long-Term Service Load

The stresses in prestressed concrete flexural members at long-term service loads, and after all prestress losses have occurred, are checked against the same limits as for the normal service load, except for the following (CSA 18.3.2):

Extreme fiber stress in compression due to prestress plus sustained load:

$$0.45 f_c'$$

9.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

9.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement if required

9.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of post-tensioned beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases the beam may be designed as a rectangular or inverted flanged beam.

9.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 9-1 (CSA 10.1.7). Furthermore it is assumed that the compression carried by the concrete is less than or equal to that which can be carried at the balanced condition (CSA 10.1.4). When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by ETABS, for both rectangular and flanged sections (L- and T-beams) is summarized in the subsections that follow. It is assumed that the design ultimate axial force in a beam is negligible; hence all the beams are designed for major direction flexure, shear, and torsion only.

9.7.1.2.1 Design of Rectangular Beams

ETABS first determines whether the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, moment capacity ϕM_n^0 is determined as follows:

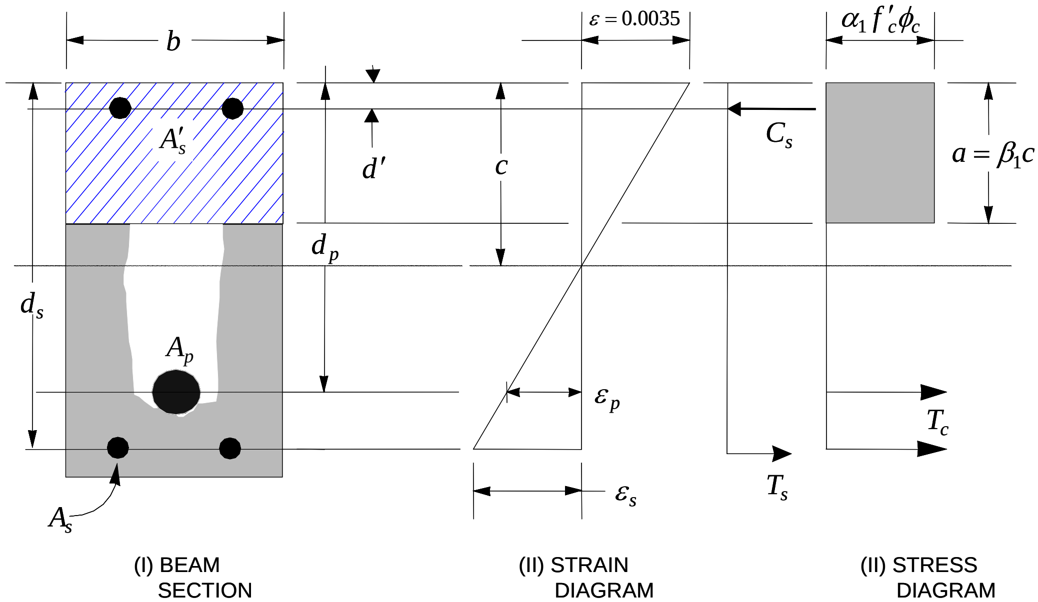


Figure 9-1 Rectangular Beam Design

The maximum depth of the compression zone, c_{max}, is calculated based on strain-stress compatibility (CSA 18.6.1):

$$c_{max} = \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_p}\right) E_p d_p \qquad \text{(CSA 18.6.1)}$$

where,

$$\varepsilon_{cu} = 0.0035 \qquad \text{(CSA 10.1.4)}$$

Therefore, the limits $c \leq c_{max}$ is set for tension-controlled sections.

The ductility of a section is ensured by limiting the *c/d* ratio and strength reduction factor ϕ. The minimum ductility required by the CSA code is limited as $c/d_p \leq 0.5$ (CSA 18.6.2).

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by:

$$a_{\max} = \beta_1 c_{\max} \qquad \text{(CSA 10.1.7(a))}$$

where β_1 is calculated as:

$$\beta_1 = 0.97 - 0.0025\ f'_c \geq 0.67 \qquad \text{(CSA 10.1.7)}$$

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pr} is computed based on strain compatibility. On the basis of the calculated f_{pr}, the depth of the neutral axis is recalculated, and f_{pr} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \beta_1 c$$

- If $c \leq c_{\max}$ (CSA 18.6.2), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$\phi M_r^0 = \phi A_{ps} f_{pr}\left(d_p - \frac{a}{2}\right)$$

- If $c > c_{\max}$ (CSA 18.6.2), a failure condition is declared.

- If $M_f > \phi M_r^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of the neutral axis c is equal to $c_{\max}$. The stress in the post-tensioning steel, f_{pr} is then calculated based on strain compatibility and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = \alpha_1 f'_c \phi_c a_{\max} b$$

$$T = A_p f_{pr}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\alpha_1 f_c^{'} \phi_c a_{\max} b - A_p f_{pr}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$\phi M_r^{bal} = \phi A_p f_{pr}^{bal} \left(d_p - \frac{a_{\max}}{2} \right) + \phi A_s^{bal} f_s^{bal} \left(d_s - \frac{a_{\max}}{2} \right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, *c*.

9.7.1.2.1.1 Case 1: Post-tensioning steel is adequate

When $\left(M_f < \phi M_r^0 \right)$, the amount of post-tensioning steel is adequate to resist the design moment M_f. Minimum reinforcement is provided to satisfy the ductility requirements (CSA 18.3.13, 18.7 and 18.8), i.e., $\left(M_f < \phi M_r^0 \right)$.

9.7.1.2.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist M_f, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $\phi M_r^0 < M_f < \phi M_r^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M_f and reports the required area of tension reinforcement. Since M_f is bounded by ϕM_r^0 at the lower end and ϕM_r^{bal} at the upper end, and ϕM_r^0 is associated with $A_s = 0$ and ϕM_r^{bal}

is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if M_f is positive or at the top if M_f is negative.

9.7.1.2.1.3 Case 3: Post-tensioning steel and tension reinforcement is not adequate

When $\left(M_f > \phi M_r^{bal}\right)$, compression reinforcement is required (CSA 18.6.2). In that case, ETABS assumes that the depth of the neutral axis, c, is equal to $c_{\max}$. The values of f_{pr} and f_s reach their respective balanced condition values, f_{pr}^{bal} and f_s^{bal}. The area of compression reinforcement, A_s', is determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{rs} = M_f - \phi M_r^{bal}$$

The required compression reinforcement is given by:

$$A_s' = \frac{M_{rs}}{\left(\phi_s f'_s - \phi_c \alpha_1 f'_c\right)\left(d_s - d'\right)}, \text{ where}$$

$$f'_s = 0.0035 E_s \left[\frac{c - d'}{c}\right] \le f_y \qquad \text{(CSA 10.1.2, 10.1.3)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{rs}}{f_y\left(d - d'\right)\phi_s}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_f is positive, and vice versa if M_f is negative.

9.7.1.2.2 Design of Flanged Beams

9.7.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_f (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

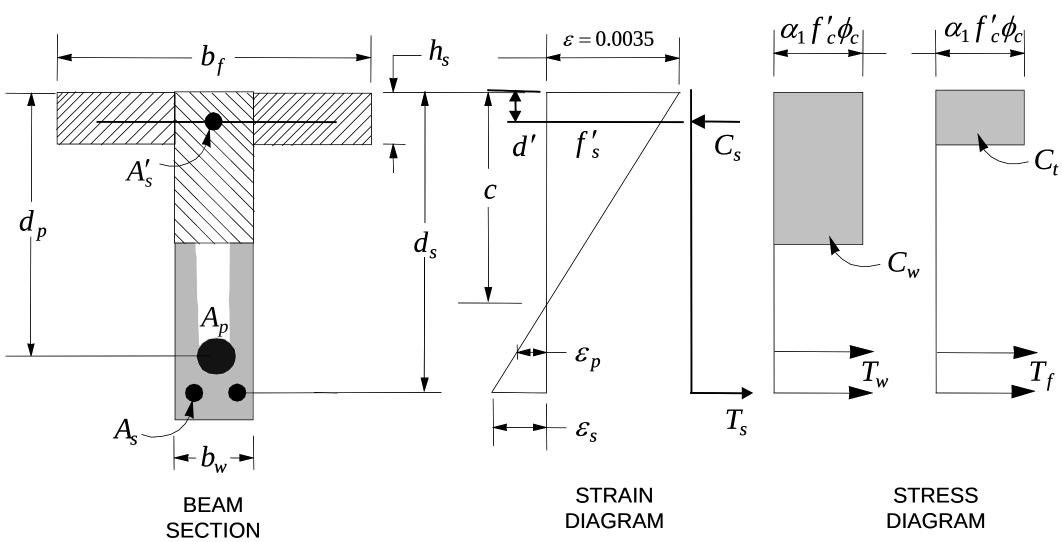


Figure 9-2 T-Beam Design

9.7.1.2.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, the moment capacity ϕM_n^0 is determined as follows:

The maximum depth of the compression zone, c_{max}, is calculated based on strain-stress compatibility (CSA 18.6.1):

$$c_{\max} = \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_p}\right) E_p d_p \qquad \text{(CSA 18.6.1)}$$

where,

$\varepsilon_{cu} = 0.0035$ (CSA 10.1.4)

Therefore, the limits $c \le c_{\max}$ is set for tension-controlled sections.

The ductility of a section is ensured by limiting the *c/d* ratio and strength reduction factor ϕ. The minimum ductility required by the CSA code is limited to $c/d_p \le 0.5$ (CSA 18.6.2).

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by:

$$a_{\max} = \beta_1 c_{\max} \quad \text{(CSA 10.1.7(a))}$$

where β_1 is calculated as:

$$\beta_1 = 0.97 - 0.0025\ f'_c \ge 0.67 \quad \text{(CSA 10.1.7)}$$

ETABS determines the depth of the neutral axis, *c*, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pr} is computed based on strain compatibility. Based on the calculated f_{pr}, the depth of the neutral axis is recalculated, and f_{pr} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \beta_1 c$$

- If $c \le c_{\max}$ (CSA 18.6.2), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$\phi M_r^0 = \phi A_{ps} f_{pr}\left(d_p - \frac{a}{2}\right)$$

- If $c > c_{\max}$ (CSA 18.6.2), a failure condition is declared.

- If $M_f > \phi M_r^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of neutral axis *c* is equal to $c_{\max}$. The stress in the post-tensioning steel, f_{pr} is then calculated based on strain compatibility and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \le h_s$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as b_f. Compression reinforcement is required when $a > a_{\max}$.

- If $a > h_s$, the calculation for A_s is given by:

$$C = \alpha_1 f'_c \phi_c a_{\max} A_c^{com}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$A_c^{com} = b_f d_s + b_w \left(a_{\max} - d_s \right)$$

$$T = A_p f_{pr}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\alpha_1 f'_c \phi_c a_{\max} A_c^{com} - A_p f_{pr}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$\phi M_r^{bal} = \phi A_p f_{pr}^{bal} \left(d_p - \frac{a_{\max}}{2} \right) + \phi A_s^{bal} f_s^{bal} \left(d_s - \frac{a_{\max}}{2} \right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcing steel, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of neutral axis, c.

9.7.1.2.2.3 Case 1: Post-tensioning steel is adequate

When $\left(M_f < \phi M_r^0\right)$ the amount of post-tensioning steel is adequate to resist the design moment M_f. Minimum reinforcement is provided to satisfy ductility requirements (CSA 18.3.13, 18.7 and 18.8), i.e., $\left(M_f < \phi M_r^0\right)$.

9.7.1.2.2.4 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist M_f, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $\phi M_r^0 < M_f < \phi M_r^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M_f and reports this required area of tension reinforcement. Since M_f is bounded by ϕM_r^0 at the lower end and ϕM_r^{bal} at the upper end, and ϕM_r^0 is associated with $A_s = 0$ and ϕM_r^{bal} is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if M_f is positive, or at the top if M_f is negative.

9.7.1.2.2.5 Case 3: Post-tensioning steel and tension reinforcement is not adequate

When $\left(M_f > \phi M_r^{bal}\right)$, compression reinforcement is required (CSA 18.6.2). In that case, ETABS assumes that the depth of the neutral axis, c, is equal to $c_{\max}$. The values of f_{pr} and f_s reach their respective balanced condition values, f_{pr}^{bal} and f_s^{bal}. Then the area of compression reinforcement, A_s', is determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{rs} = M_f - \phi M_r^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{rs}}{\left(\phi_s f'_s - \phi_c \alpha_1 f'_c\right)\left(d_s - d'\right)}, \text{ where}$$

$$f'_s = 0.0035 E_s \left[\frac{c - d'}{c}\right] \leq f_y. \qquad \text{(CSA 10.1.2, 10.1.3)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{rs}}{f_y \left(d - d'\right) \phi_s}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_f is positive, and vice versa if M_f is negative.

9.7.1.2.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the limits specified in CSA 18.8.2, Table 18.1.

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

9.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored forces acting on the section, M_f and v_f. Note that M_f is needed for the calculation of v_c.
- Determine the shear stress, v_c that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

9.7.2.1 Determine Shear Force

In the design of the beam shear reinforcement of a concrete beam, the shear forces for a particular load combination at a particular beam section are obtained by factoring the associated shear forces and moments with the cor-responding load combination factors.

9.7.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as:

$$V_c = \phi_c \lambda \beta \sqrt{f'_c} b_w d_v \qquad \text{(CSA 11.3.4)}$$

where,

$$\sqrt{f'_c} \leq 8 \text{ MPa} \qquad \text{(CSA 11.3.4)}$$

ϕ_c is the resistance factor for concrete. By default it is taken as 0.65 (CSA 8.4.2). For concrete produced in a pre-qualified manufacturing plant, the value can be taken as 0.70 (CSA 16.1.3). This value can be overwritten in the design preferences.

λ is the strength reduction factor to account for low density concrete (CSA 2.2). For normal density concrete, its value is 1 (CSA 8.6.5), which is taken by the program as the default value. For concrete using lower density aggregate, the user can change the value of λ in the material property data. The recommended value for λ is as follows (CSA 8.6.5).

$$\lambda = \begin{cases} 1.00, & \text{for normal density concrete,} \\ 0.85, & \text{for semi-low-density concrete} \\ & \text{in which all of the fine aggregate is natural sand,} \\ 0.75, & \text{for semi-low-density concrete} \\ & \text{in which none of the fine aggregate is natural sand.} \end{cases}$$

β is the factor for accounting for the shear resistance of cracked concrete (CSA 2.2). Its value is normally between 0.1 and 0.4. It is determined according to CSA 11.3.6 and described further in the following sections.

b_w is the effective web width. For rectangular beams, it is the width of the beam. For flanged beams, it is the width of the web of the beam.

d_v is the effective shear depth. It is taken as the greater of 0.9*d* or 0.72*h* (CSA 2.3), where *d* is the distance from the extreme compression fiber to the centroid of tension reinforcement, and *h* is the overall depth of the cross-section in the direction of the shear force.

The value of β is preferably taken as the special value (CSA 11.3.6.2), or it is determined using the simplified method (CSA 11.3.6.3), if applicable. When the conditions of the special value or simplified method do not apply, the general method is used (CSA 11.3.6.4).

If the overall beam depth, *h*, is less than 250 mm or if the beam depth of a flanged beam below the slab is not greater than one-half of the width of the web or 350 mm, β is taken as 0.21 (CSA 11.3.6.2).

$$\beta = 0.21 \qquad \text{(CSA 11.3.6.2)}$$

When the specified yield strength of the longitudinal reinforcing f_y does not exceed 400 MPa, and the specified concrete strength f'_c does not exceed 60 MPa, β is determined in accordance with the simplified method, as follows (CSA 11.6.3.3):

- When the section contains at least the minimum transverse reinforcement, β is taken as 0.18 (CSA 11.3.6.3a).

$$\beta = 0.18 \qquad \text{(CSA 11.3.6.3.a)}$$

When the section contains no transverse reinforcement, β is determined based on the specified maximum nominal size of coarse aggregate, a_g.

For maximum size of coarse aggregate not less than 20 mm, β is taken as:

$$\beta = \frac{230}{1000 + d_v} \quad \text{(CSA 11.3.6.3 b)}$$

where d_v is the effective shear depth expressed in millimeters, which is described in preceding sections.

For a maximum size of coarse aggregate less than 20 mm, β is taken as:

$$\beta = \frac{230}{1000 + s_{ze}} \quad \text{(CSA 11.3.6.3 c)}$$

where, $S_{ze} = \frac{35}{15 + a_g} S_z \geq 0.85 S_z$ (CSA 11.3.6.3.c)

In the preceding expression, the crack spacing parameter, s_{ze}, shall be taken as the minimum of d_v and the maximum distance between layers of distributed longitudinal reinforcement. However, s_{ze} is conservatively taken as equal to d_v.

In summary, for simplified cases, β can be expressed as follows:

$$\beta = \begin{cases} 0.18, & \text{if minimum transverse reinforcement is provided,} \\ \dfrac{230}{1000 + d_v}, & \text{if no transverse reinforcement is provided, and } a_g \geq 20\text{mm,} \\ \dfrac{230}{1000 + S_{ze}}, & \text{if no transverse reinforcement is provided, and } a_g < 20\text{mm.} \end{cases}$$

- When the specified yield strength of the longitudinal reinforcing f_y is greater than 400 MPa, the specified concrete strength f'_c is greater than 60 MPa, or tension is not negligible, β is determined in accordance with the general method as follows (CSA 11.3.6.1, 11.3.6.4):

$$\beta = \frac{0.40}{(1+1500\varepsilon_x)} \bullet \frac{1300}{(1000+S_{ze})} \qquad \text{(CSA 11.3.6.4)}$$

In the preceding expression, the equivalent crack spacing parameter, s_{ze} is taken equal to 300 mm if minimum transverse reinforcement is provided (CSA 11.3.6.4). Otherwise it is determined as stated in the simplified method.

$$S_{ze} = \begin{cases} 300 & \text{if minimum transverse reinforcement is provided,} \\ \dfrac{35}{15+a_g} S_z \geq 0.85 S_z & \text{otherwise.} \end{cases}$$

(CSA 11.3.6.3, 11.3.6.4)

The value of a_g in the preceding equations is taken as the maximum aggregate size for f'_c of 60 MPa, is taken as zero for f'_c of 70 MPa, and is linearly interpolated between these values (CSA 11.3.6.4).

The longitudinal strain, ε_x at mid-depth of the cross-section is computed from the following equation:

$$\varepsilon_x = \frac{M_f/d_v + V_f + 0.5N_f}{2(E_s A_s)} \qquad \text{(CSA 11.3.6.4)}$$

In evaluating ε_x the following conditions apply:

- ε_x is positive for tensile action.
- V_f and M_f are taken as positive quantities. (CSA 11.3.6.4(a))
- M_f is taken as a minimum of $V_f d_v$. (CSA 11.3.6.4(a))
- N_f is taken as positive for tension. (CSA 2.3)

A_s is taken as the total area of longitudinal reinforcement in the beam. It is taken as the envelope of the reinforcement required for all design load combinations. The actual provided reinforcement might be slightly higher than this quantity. The reinforcement should be developed to achieve full strength (CSA 11.3.6.3(b)).

If the value of ε_x is negative, it is recalculated with the following equation, in which A_{ct} is the area of concrete in the flexural tensile side of the beam, taken as half of the total area.

$$\varepsilon_x = \frac{M_f / d_v + V_f + 0.5 N_f}{2(E_s A_s + E_c A_{ct})} \qquad \text{(CSA 11.3.6.4(c))}$$

$$E_s = 200{,}000 \text{ MPa} \qquad \text{(CSA 8.5.4.1)}$$

$$E_c = 4500\sqrt{f_c^{'}} \text{ MPa} \qquad \text{(CSA 8.6.2.3)}$$

If the axial tension is large enough to induce tensile stress in the section, the value of ε_x is doubled (CSA 11.3.6.4(e)).

For sections closer than d_v from the face of the support, ε_x is calculated based on M_f and V_f of a section at a distance d_v from the face of the support (CSA 11.3.6.4(d)). This condition currently is not checked by ETABS.

An upper limit on ε_x is imposed as:

$$\varepsilon_x \leq 0.003 \qquad \text{(CSA 11.3.6.4(f))}$$

In both the simplified and general methods, the shear strength of the section due to concrete, v_c, depends on whether the minimum transverse reinforcement is provided. To check this condition, the program performs the design in two passes. In the first pass, it is assumes that no transverse shear reinforcement is needed. When the program determines that shear reinforcement is needed, the program performs the second pass assuming that at least minimum shear reinforcement is provided.

9.7.2.3 Determine Required Shear Reinforcement

The shear force is limited to $V_{r,\max}$ where:

$$V_{r,\max} = 0.25 \phi_c f'_c b_w d \qquad \text{(CSA 11.3.3)}$$

Given V_f, V_c, and $V_{r,\max}$, the required shear reinforcement is calculated as follows:

- If $V_f \le V_c$

$$\frac{A_v}{s} = 0 \qquad \text{(CSA 11.3.5.1)}$$

- If $V_c < V_f \le V_{r,\max}$

$$\frac{A_v}{s} = \frac{\left(V_f - V_c\right)\tan\theta}{\phi_s f_{yt} d_v} \qquad \text{(CSA 11.3.3, 11.3.5.1)}$$

- If $V_f > V_{r,\max}$, (CSA 11.3.3)

  a failure condition is declared.

A minimum area of shear reinforcement is provided in the following regions (CSA 11.2.8.1):

(a) in regions of flexural members where the factored shear force V_f exceeds V_c

(b) in regions of beams with an overall depth greater than 750 mm

(c) in regions of beams where the factored torsion T_f exceeds $0.25T_{cr}$

Where the minimum shear reinforcement is required by CSA 11.2.8.1, or by calculations, the minimum area of shear reinforcement per unit spacing is taken as:

$$\frac{A_v}{s} \ge 0.06\frac{\sqrt{f_c'}}{f_y} b_w \qquad \text{(CSA 11.2.8.2)}$$

In the preceding equations, the term θ is used where θ is the angle of inclination of the diagonal compressive stresses with respect to the longitudinal axis of the member. The θ value is normally between 22 and 44 degrees. It is determined according to CSA 11.3.6.

Similar to the β factor, which was described previously, the value of θ is preferably taken as the special value (CSA 11.3.6.2) or it is determined using the simplified method (CSA 11.3.6.3), whenever applicable. The program uses the general method when conditions for the simplified method are not satisfied (CSA 11.3.6.4).

- If the overall beam depth, h, is less than 250 mm or if the depth of the flanged beam below the slab is not greater than one-half of the width of the web or 350 mm, θ is taken as 42 degrees (CSA 11.3.6.2).
- If the specified yield strength of the longitudinal reinforcing f_y does not exceed 400 MPa, or the specified concrete strength f'_c does not exceed 60 MPa, θ is taken to be 35 degree (CSA 11.3.6.3).

$$\theta = 35^{\circ} \text{ for } P_f \le 0 \text{ or } f_y \le 400 \text{ MPa or } f'_c \le 60 \text{ MPa} \qquad \text{(CSA11.3.6.3)}$$

- If the axial force is tensile, the specified yield strength of the longitudinal reinforcing f_y > 400 MPa, and the specified concrete strength f'_c > 60 MPa, θ is determined using the general method as follows (CSA 11.3.6.4),

$$\theta = 29 + 7000\varepsilon_x \text{ for } P_f < 0,\ f_y > 400 \text{ MPa, } f'_c \le 60 \text{ MPa} \qquad \text{(CSA11.3.6.4)}$$

where ε_x is the longitudinal strain at the mid-depth of the cross-section for the factored load. The calculation procedure has been described in preceding sections.

The maximum of all of the calculated A_v/s values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements reported by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric requirements must be investigated independently of the program by the user.

9.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_f
- Determine special section properties
- Determine critical torsion capacity
- Determine the torsion reinforcement required

9.7.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur because of redistribution of internal forces upon cracking, the design T_f is permitted to be reduced in accordance with the code (CSA 11.2.9.2). However, the program does not automatically redistribute the internal forces and reduce T_f. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

9.7.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_c, A_{oh}, A_o, p_c, and p_h are calculated. These properties are described in the following (CSA 2.3).

A_c = Area enclosed by outside perimeter of concrete cross-section

A_{oh} = Area enclosed by centerline of the outermost closed transverse torsional reinforcement

A_o = Gross area enclosed by shear flow path

p_c = Outside perimeter of concrete cross-section

p_h = Perimeter of centerline of outermost closed transverse torsional reinforcement

In calculating the section properties involving reinforcement, such as A_{oh}, A_o, and p_h, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 millimeters. This is equivalent to a 38-mm clear cover and a 12-mm stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a rectangular beam section are given as follows:

$$A_c = bh \qquad \text{(CSA 11.2.9.1)}$$

$$A_{oh} = (b - 2c)(h - 2c) \qquad \text{(CSA 11.3.10.3)}$$

$$A_o = 0.85\, A_{oh} \qquad \text{(CSA 11.3.10.3)}$$

$$p_c = 2b + 2h \qquad \text{(CSA 11.2.9.1)}$$

$$p_h = 2(b - 2c) + 2(h - 2c) \qquad \text{(CSA 11.3.10.4)}$$

where, the section dimensions b, h, and c are shown in Figure 9-3. Similarly, the special section properties for a flanged beam section are given as follows:

$$A_c = b_w h + (b_f - b_w) h_s \qquad \text{(CSA 11.2.9.1)}$$

$$A_{oh} = (b_w - 2c)(h - 2c) \qquad \text{(CSA 11.3.10.3)}$$

$$A_o = 0.85\, A_{oh} \qquad \text{(CSA 11.3.10.3)}$$

$$p_c = 2b_f + 2h \qquad \text{(CSA 11.2.9.1)}$$

$$p_h = 2(h - 2c) + 2(b_w - 2c) \qquad \text{(CSA 11.3.10.4)}$$

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 9-3. Note that the flange width on either side of the beam web is limited to the smaller of $6h_s$ or 1/12 the span length (CSA 10.3.4).

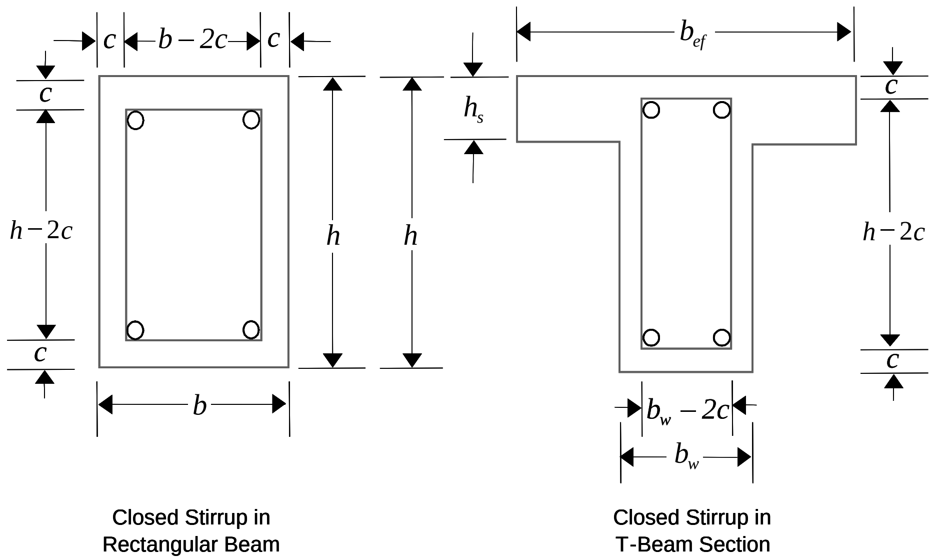


Figure 9-3 Closed stirrup and section dimensions for torsion design

9.7.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, T_{cr}, for which the torsion in the section can be ignored is calculated as:

$$T_{cr} = \frac{0.38\lambda\phi_c\sqrt{f'_c}\left(\frac{A_c^2}{p_c}\right)}{4} \qquad \text{(CSA 11.2.9.1)}$$

where A_{cp} and p_c are the area and perimeter of the concrete cross-section as described in the previous section; λ is a factor to account for low-density concrete; ϕ_c is the strength reduction factor for concrete, which is equal to 0.65; and f'_c is the specified concrete compressive strength.

9.7.3.4 Determine Torsion Reinforcement

If the factored torsion T_f is less than the threshold limit, T_{cr}, torsion can be safely ignored (CSA 11.2.9.1). In that case, the program reports that no torsion reinforcement is required. However, if T_f exceeds the threshold limit, T_{cr}, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CSA 11.3).

- If $T_f > T_{cr}$, the required closed stirrup area per unit spacing, A_t /s, is calculated as:

$$\frac{A_t}{s} = \frac{T_f \tan\theta}{\phi_s 2A_o f_{yt}} \qquad \text{(CSA 11.3.10.3)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{\dfrac{M_f}{d_v} + 0.5N_f + \sqrt{\left(V_f - 0.5V_s\right)^2 + \left(\dfrac{0.45 p_h T_f}{2A_o}\right)^2} \cot\theta}{\phi_s f_y}$$

(CSA 11.3.10.6, 11.3.9)

In the preceding expressions, θ is computed as previously described for shear, except that if the general method is being used, the value ε_x is calculated as specified in CSA 11.3.6.4 is replaced by:

$$\varepsilon_x = \frac{\dfrac{M_f}{d_v} + \sqrt{V_f^{\,2} + \left(\dfrac{0.9 p_h T_f}{2A_o}\right)^2} + 0.5N_f}{2\left(E_s A_s\right)} \qquad \text{(CSA 11.3.10.5)}$$

An upper limit of the combination of V_u and T_u that can be carried by the section also is checked using the equation:

$$\sqrt{\left(\frac{V_f}{b_w d_v}\right)^2 + \left(\frac{T_f\, p_h}{1.7A_{oh}^2}\right)^2} \le 0.25\phi_c f'_c \qquad \text{(CSA 11.3.10.4(b))}$$

For rectangular sections, b_w is replaced with b. If the combination of V_f and T_f exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_f > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

$$\left(\frac{A_v}{s}+2\frac{A_t}{s}\right)\geq 0.06\sqrt{f'_c}\frac{b_w}{f_{yt}} \qquad \text{(CSA 11.2.8.2)}$$

If this equation is not satisfied with the originally calculated A_v/s and A_t/s, A_v/s is increased to satisfy this condition.

The maximum of all of the calculated A_l and A_t/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

9.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method (CSA A 23.3-04) for prestressed reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "ETABS Design Techniques" in the *Key Features and Terminology* manual.

9.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element

boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

9.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

9.8.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

9.8.1.3 Minimum and Maximum Slab Reinforcement

If the computed tensile stress in the concrete immediately after prestress transfer exceeds $0.25\lambda\sqrt{f'_{ci}}$ (CSA 18.3.1.1), the bonded reinforcement with a minimum area of A_s is provided in the tensile zone to resist the total tensile force, N_c, in the concrete computed on the basis of an uncracked section (CSA 18.3.1.3).

$$A_s = N_c / (0.5 f_y) \qquad \text{(CSA 18.3.1.3)}$$

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limit (CSA 18.8.1, 18.8.2):

	Concrete stress (see Clause 18.3.2(c))			
	Tensile stress $\leq 0.5\lambda\sqrt{f'_c}$		Tensile stress $> 0.5\lambda\sqrt{f'_c}$	
	Type of tendon		Type of tendon	
Type of member	Bonded	Unbonded	Bonded	Unbonded
Beams	0	0.004*A*	0.003*A*	0.005*A*
One-way slabs	0	0.003*A*	0.002*A*	0.004*A*
Two-way slabs				
Negative moment regions	0	$0.0006hl_n$	$0.00045hl_n$	$0.00075hl_n$
Positive moment regions, concrete stress $> 0.2\lambda\sqrt{f'_c}$	0	0.004*A*	0.003*A*	0.005*A*
Positive moment regions, concrete tensile stress $\leq 0.2\lambda\sqrt{f'_c}$	0	0	--	--

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

9.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code specific items are described in the following sections.

9.8.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of *d*/*2* from the face of the support (CSA 13.3.3.1 and CSA 13.3.3.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CSA 13.3.3.3). Figure 9-4 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

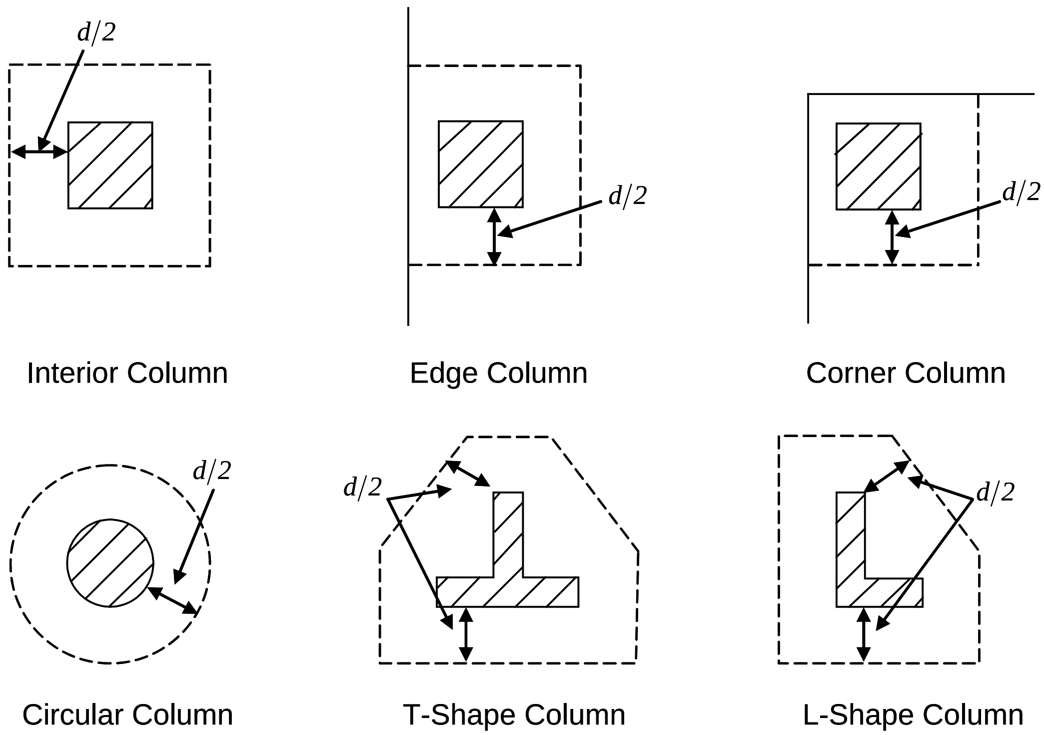


Figure 9-4 Punching Shear Perimeters

9.8.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$, where

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \text{ and} \qquad \text{(CSA 13.10.2)}$$

$$\gamma_v = 1-\frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \qquad \text{(CSA 13.3.5.3)}$$

where b_1 is the width of the critical section measured in the direction of the span and b_2 is the width of the critical section measured in the direction perpendicular to the span.

9.8.2.3 Determine Concrete Capacity

The concrete punching shear factored strength is taken as the minimum of the following three limits:

$$v_c = \min \begin{cases} \varphi_c \left(1 + \dfrac{2}{\beta_c}\right) 0.19 \lambda \sqrt{f'_c} \\ \varphi_c \left(0.19 + \dfrac{\alpha_s d}{b_0}\right) \lambda \sqrt{f'_c} \\ \varphi_c 0.38 \lambda \sqrt{f'_c} \end{cases} \qquad \text{(CSA 13.3.4.1)}$$

where, β_c is the ratio of the minimum to the maximum dimensions of the critical section, b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 4 & \text{for interior columns,} \\ 3 & \text{for edge columns, and} \\ 2 & \text{for corner columns} \end{cases} \qquad \text{(CSA 13.3.4.1(b))}$$

The value of $\sqrt{f'_c}$ is limited to 8 MPa for the calculation of the concrete shear capacity (CSA 13.3.4.2)

If the effective depth, *d*, exceeds 300 mm, the value of v_c is reduced by a factor equal to 1300/(1000 + *d*) (CSA 13.3.4.3).

9.8.2.4 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS.

9.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 120 mm (CSA 13.2.1).

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as follows.

9.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a two-way prestressed section with punching shear reinforcement is:

$$v_c = 0.28\lambda\phi_c\sqrt{f'_c} \quad \text{for shear studs} \qquad \text{(CSA 13.3.8.3)}$$

$$v_c = 0.19\lambda\phi_c\sqrt{f'_c} \quad \text{for shear stirrups} \qquad \text{(CSA 13.3.9.3)}$$

9.8.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of $v_{r,\max}$, where

$$v_{r,\max} = 0.75\lambda\phi_c\sqrt{f'_c} \quad \text{for shear studs} \qquad \text{(CSA 13.3.8.2)}$$

$$v_{r,\max} = 0.55\lambda\phi_c\sqrt{f'_c} \quad \text{for shear stirrups} \qquad \text{(CSA 13.3.9.2)}$$

Given v_f, v_c, and $v_{f,,\max}$, the required shear reinforcement is calculated as follows, where, ϕ_s, is the strength reduction factor.

- If $v_f > v_{r,\max}$,

$$\frac{A_v}{s} = \frac{\left(v_f - v_c\right)}{\phi_s f_{yv}} b_o \qquad \text{(CSA 13.3.8.5, 13.3.9.4)}$$

- If $v_f > v_{r,\max}$, (CSA 13.3.8.2)

a failure condition is declared.

- If V_f exceeds the maximum permitted value of $V_{r,max}$, the concrete section should be increased in size.

9.8.3.3 Determine the Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 9-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner columns.

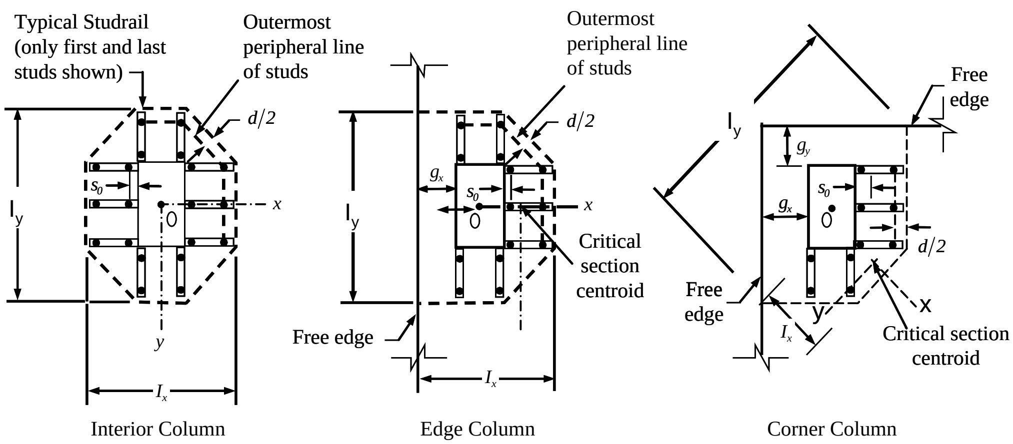


Figure 9-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed 0.4*d*. The spacing between adjacent shear reinforcement in the first line of shear reinforcement shall not exceed 2*d* measured in a direction parallel to the column face.

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

9.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CSA 7.9 plus one half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 9.5-, 12.7-, 15.9-, and 19.1-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.4d. The limits of the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.4d \quad \text{(CSA 13.3.8.6)}$$

$$s \leq \begin{cases} 0.75d & v_f \leq 0.56\lambda\phi_c\sqrt{f'_c} \\ 0.50d & v_f > 0.56\lambda\phi_c\sqrt{f'_c} \end{cases} \quad \text{(CSA 13.3.8.6)}$$

For shear stirrups,

$$s_o \leq 0.25d \quad \text{(CSA 13.3.9.5)}$$

$$s \leq 0.25d \quad \text{(CSA 13.3.9.5)}$$

The minimum depth for reinforcement should be limited to 300 mm (CSA 13.3.9.1).

Chapter 10
Design for Eurocode 2-2004

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the European code Eurocode 2-2004 [EN 1992-1-1:2004]. For the load combinations reference also is made to Eurocode 0 [EN 1990], which is identified with the prefix “EC0.” Various notations used in this chapter are listed in Table 10-1. For referencing to the pertinent sections of the EC code in this chapter, a prefix “EC2” followed by the section number is used. It also should be noted that this section describes the implementation of the CEN Default version of Eurocode 2-2004, without a country specific National Annex. Where Nationally Determined Parameters [NDPs] are to be considered, this is highlighted in the respective section by the notation [*NDP*].

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

10.1 Notations

The following table identifies the various notations used in this chapter.

Table 10-1 List of Symbols Used in the Eurocode 2-2004 Code

A_c	Area of concrete section, mm^2
A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sw}	Total cross-sectional area of links at the neutral axis, mm^2
A_{sw}/s_v	Area of shear reinforcement per unit length of the member, mm^2
a	Depth of compression block, mm
$a_{\max}$	Maximum depth of the compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
d	Effective depth of tension reinforcement, mm
d'	Effective depth of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f_{cd}	Design concrete strength = $\alpha_{cc} f_{ck} / \gamma_c$, MPa
f_{ck}	Characteristic compressive concrete cylinder strength at 28 days, MPa

Table 10-1 List of Symbols Used in the Eurocode 2-2004 Code

f_{cwd}	Design concrete compressive strength for shear design = $\alpha_{cc} \, f_{cwk}/\gamma_c$, MPa
f_{cwk}	Characteristic compressive cylinder strength for shear design, MPa
f_{yd}	Design yield strength of reinforcing steel = f_{yk}/γ_s, MPa
f_{yk}	Characteristic strength of shear reinforcement, MPa
f'_s	Compressive stress in beam compression steel, MPa
f_{ywd}	Design strength of shear reinforcement = f_{ywk}/γ_s, MPa
f_{ywk}	Characteristic strength of shear reinforcement, MPa
h	Overall thickness of slab, mm
h_f	Flange thickness, mm
M	Design moment at a section, N-mm
m	Normalized design moment, $M/bd^2\eta f_{cd}$
m_{lim}	Limiting normalized moment capacity as a singly reinforced beam
$M_{ED}{}^{0}$	Design moment resistance of a section with tendons only, N-mm
$M_{ED}{}^{bal}$	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
s_v	Spacing of the shear reinforcement along the length of the beam, mm
u	Perimeter of the punch critical section, mm

Table 10-1 List of Symbols Used in the Eurocode 2-2004 Code

V_{Rdc}	Design shear resistance from concrete alone, N
$V_{Rd,\max}$	Design limiting shear resistance of a cross-section, N
V_{Ed}	Shear force at ultimate design load, N
x	Depth of neutral axis, mm
$x_{\lim}$	Limiting depth of neutral axis, mm
η	Concrete strength reduction factor for sustained loading and stress-block
β	Enhancement factor of shear resistance for concentrated load; also the coefficient that takes account of the eccentricity of loading in determining punching shear stress; factor for the depth of compressive stress block
γ_f	Partial safety factor for load
γ_c	Partial safety factor for concrete strength
γ_s	Partial safety factor for steel strength
δ	Redistribution factor
ε_c	Concrete strain
ε_s	Strain in tension steel
ε'_s	Strain in compression steel.
ν	Effectiveness factor for shear resistance without concrete crushing
ρ	Tension reinforcement ratio
ω	Normalized tensile steel ratio, $A_s f_{yd} / \eta f_{cd} bd$
ω'	Normalized compression steel ratio, $A'_s f_{yd} \gamma_s / \alpha f'_s bd$

Table 10-1 List of Symbols Used in the Eurocode 2-2004 Code

ω_{lim}	Normalized limiting tensile steel ratio

10.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be checked. Eurocode 0-2002 allows load combinations to be defined based on EC0 Eqation 6.10 or the less favorable of EC0 Equations 6.10a and 6.10b [*NDP*].

$$\sum_{j\geq1}\gamma_{G,j}G_{k,j}+\gamma_P P+\gamma_{Q,1}Q_{k,1}+\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \qquad \text{(EC0 Eqn. 6.10)}$$

$$\sum_{j\geq1}\gamma_{G,j}G_{k,j}+\gamma_P P+\gamma_{Q,1}\psi_{0,1}Q_{k,1}+\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \qquad \text{(EC0 Eqn. 6.10a)}$$

$$\sum_{j\geq1}\xi_j\gamma_{G,j}G_{k,j}+\gamma_P P+\gamma_{Q,1}Q_{k,1}+\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \qquad \text{(EC0 Eqn. 6.10b)}$$

Load combinations considering seismic loading are generated automatically based on EC0 Eqation 6.12b.

$$\sum_{j\geq1}G_{k,j}+P+A_{Ed}+\sum_{i>1}\psi_{2,i}Q_{k,i} \qquad \text{(EC0 Eqn. 6.12b)}$$

For this code, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the load combinations in the following sections may need to be considered (Eurocode 0-2002, Eqation 6.10 or the less favorable of Equations 6.10a and 6.10b).

For post-tensioned concrete design, the user can specify the prestressing load (PT) by providing the tendon profile or by using load balancing options in the program. The default load combinations for post-tensioning are defined in the following sections.

10.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress force in accordance with EC0 6.5.3 and Table A1.1. The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0D + 1.0PT

10.2.2 Service Load Combination

The following characteristic load combinations are used for checking the requirements of prestress for serviceability in accordance with EC0 6.5.3 and Table A1.1. It is assumed that all long-term losses have occurred already at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT

10.2.3 Ultimate Limit State Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with EC2-2004.

The combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are determined automatically by ETABS by subtracting out the primary PT moments when the flexural design is carried out.

The following load combinations need to be considered if Equation 6.10 is specified for generation of the load combinations (EC0 6.4.3) [*NDP*].

$\gamma_{Gj,sup}\,D + 1.0PT^*$ (EC0 Eqn. 6.10)

$\gamma_{Gj,sup}D + \gamma_{Q,1}\,L + 1.0PT^*$ (EC0 Eqn. 6.10)

$\gamma_{Gj,sup}D + (0.75)\gamma_{Q,1}\,PL + 1.0PT^*$ (EC0 Eqn. 6.10)

$\gamma_{Gj,inf}\,D \pm \gamma_{Q,1}\,W + 1.0PT^*$
$\gamma_{Gj,sup}\,D \pm \gamma_{Q,1}\,W + 1.0PT^*$ (EC0 Eqn. 6.10)

$$\gamma_{Gj,sup} D + \gamma_{Q,1} L \pm \gamma_{Q,i} \psi_{0,i} W + 1.0PT^*$$
$$\gamma_{Gj,sup} D + \gamma_{Q,1} L + \gamma_{Q,i} \psi_{0,i} S + 1.0PT^*$$
$$\gamma_{Gj,sup} D \pm \gamma_{Q,1} W + \gamma_{Q,i} \psi_{0,i} L + 1.0PT^* \qquad \text{(EC0 Eqn. 6.10)}$$
$$\gamma_{Gj,sup} D \pm \gamma_{Q,1} W + \gamma_{Q,i} \psi_{0,i} S + 1.0PT^*$$
$$\gamma_{Gj,sup} D + \gamma_{Q,1} S \pm \gamma_{Q,i} \psi_{0,i} W + 1.0PT^*$$
$$\gamma_{Gj,sup} D + \gamma_{Q,1} S + \gamma_{Q,i} \psi_{0,i} L + 1.0PT^*$$

$$\gamma_{Gj,sup} D + \gamma_{Q,1} L + \gamma_{Q,i} \psi_{0,i} S \pm \gamma_{Q,i} \psi_{0,i} W + 1.0PT^*$$
$$\gamma_{Gj,sup} D \pm \gamma_{Q,1} W + \gamma_{Q,i} \psi_{0,i} L + \gamma_{Q,i} \psi_{0,i} S + 1.0PT^* \qquad \text{(EC0 Eqn. 6.10)}$$
$$\gamma_{Gj,sup} D + \gamma_{Q,1} S \pm \gamma_{Q,i} \psi_{0,i} W + \gamma_{Q,i} \psi_{0,i} L + 1.0PT^*$$

$$D \pm 1.0E + 1.0PT^*$$
$$D \pm 1.0E + \psi_{2,i} L + 1.0PT^* \qquad \text{(EC0 Eqn. 6.12b)}$$
$$D \pm 1.0E + \psi_{2,i} L + \psi_{2,i} S + 1.0PT^*$$

* — Replace PT with H for flexural design only

If the load combinations are specified to be generated from the max of EC0 Equations. 6.10a and 6.10b, the following load combinations from both equations are considered in the program.

$$\gamma_{Gj,sup} D + 1.0PT^* \qquad \text{(EC0 Eqn. 6.10a)}$$
$$\xi\, \gamma_{Gj,sup} D + 1.0PT^* \qquad \text{(EC0 Eqn. 6.10b)}$$

$$\gamma_{Gj,sup} D + \gamma_{Q,1} \psi_{0,1} L + 1.0PT^* \qquad \text{(EC0 Eqn. 6.10a)}$$
$$\xi\, \gamma_{Gj,sup} D + \gamma_{Q,1} L + 1.0PT^* \qquad \text{(EC0 Eqn. 6.10b)}$$

$$\gamma_{Gj,sup} D + (0.75)\gamma_{Q,1} \psi_{0,1} PL + 1.0PT^* \qquad \text{(EC0 Eqn. 6.10a)}$$
$$\xi\, \gamma_{Gj,sup} D + (0.75)\gamma_{Q,1} PL + 1.0PT^* \qquad \text{(EC0 Eqn. 6.10b)}$$

$$\gamma_{Gj,inf} D \pm \gamma_{Q,1} \psi_{0,1} W + 1.0PT^* \qquad \text{(EC0 Eqn. 6.10a)}$$
$$\gamma_{Gj,sup} D \pm \gamma_{Q,1} \psi_{0,1} W + 1.0PT^* \qquad \text{(EC0 Eqn. 6.10a)}$$
$$\gamma_{Gj,inf} D \pm \gamma_{Q,1} W + 1.0PT^* \qquad \text{(EC0 Eqn. 6.10b)}$$
$$\xi\, \gamma_{Gj,sup} D \pm \gamma_{Q,1} W + 1.0PT^* \qquad \text{(EC0 Eqn. 6.10b)}$$

$$\gamma_{Gj,sup} D + \gamma_{Q,1} \psi_{0,1} L \pm \gamma_{Q,i} \psi_{0,i} W + 1.0PT^*$$
$$\gamma_{Gj,sup} D + \gamma_{Q,1} \psi_{0,1} L + \gamma_{Q,i} \psi_{0,i} S + 1.0PT^*$$
$$\gamma_{Gj,sup} D \pm \gamma_{Q,1} \psi_{0,1} W + \gamma_{Q,i} \psi_{0,i} L + 1.0PT^* \qquad \text{(EC0 Eqn. 6.10a)}$$
$$\gamma_{Gj,sup} D \pm \gamma_{Q,1} \psi_{0,1} W + \gamma_{Q,i} \psi_{0,i} S + 1.0PT^*$$
$$\gamma_{Gj,sup} D + \gamma_{Q,1} \psi_{0,1} S + \gamma_{Q,i} \psi_{0,i} L + 1.0PT^*$$
$$\gamma_{Gj,sup} D + \gamma_{Q,1} \psi_{0,1} S \pm \gamma_{Q,i} \psi_{0,i} W + 1.0PT^*$$

$\xi\ \gamma_{Gj,sup}\,D + \gamma_{Q,1}\,L \pm \gamma_{Q,i}\,\psi_{0,i}\,W + 1.0PT^*$
$\xi\ \gamma_{Gj,sup}\,D + \gamma_{Q,1}\,L + \gamma_{Q,i}\,\psi_{0,i}\,S + 1.0PT^*$
$\xi\ \gamma_{Gj,sup}\,D + \gamma_{Q,1}\,S \pm \gamma_{Q,i}\,\psi_{0,i}\,W + 1.0PT^*$ (EC0 Eqn. 6.10b)
$\xi\ \gamma_{Gj,sup}\,D + \gamma_{Q,1}\,S + \gamma_{Q,i}\,\psi_{0,i}\,L + 1.0PT^*$
$\gamma_{Gj,inf}\,D \pm \gamma_{Q,1}\,W + \gamma_{Q,i}\,\psi_{0,i}\,L + 1.0PT^*$
$\gamma_{Gj,inf}\,D \pm \gamma_{Q,1}\,W + \gamma_{Q,i}\,\psi_{0,i}\,S + 1.0PT^*$

$D \pm 1.0E + 1.0PT^*$
$D \pm 1.0E + \psi_{2,i}\,L + 1.0PT^*$ (EC0 Eqn. 6.12b)
$D \pm 1.0E + \psi_{2,i}\,L + \psi_{2,i}\,S + 1.0PT^*$

* — Replace PT with H for flexural design only

For both sets of load combinations, the variable values for the CEN Default version of the load combinations are defined in the list that follows [*NDP*].

$\gamma_{Gj,sup}$ = 1.35 (EC0 Table A1.2(B))

$\gamma_{Gj,inf}$ = 1.00 (EC0 Table A1.2(B))

$\gamma_{Q,1}$ = 1.5 (EC0 Table A1.2(B))

$\gamma_{Q,i}$ = 1.5 (EC0 Table A1.2(B))

$\psi_{0,i}$ = 0.7 (live load, assumed not to be storage) (EC0 Table A1.1)

$\psi_{0,i}$ = 0.6 (wind load) (EC0 Table A1.1)

$\psi_{0,i}$ = 0.5 (snow load, assumed $H \leq 1000$ m) (EC0 Table A1.1)

ξ = 0.85 (EC0 Table A1.2(B))

$\psi_{2,i}$ = 0.3 (live, assumed office/residential space) (EC0 Table A1.1)

$\psi_{2,i}$ = 0 (snow, assumed $H \leq 1000$ m) (EC0 Table A1.1)

These are also the default design load combinations in ETABS whenever the Eurocode 2-2004 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

10.3 Limits on Material Strength

The characteristic strengths of concrete are provided in EC2 Table 3.1 where characteristic strengths of concrete range between 12 and 90 MPa.

$$12\,\text{MPa} \leq f_{ck} \leq 90\,\text{MPa} \qquad \text{(EC2 Table 3.1)}$$

Grades C28/C35 and C32/C40 are the minimum recommended for post-tensioning and pre-tensioning respectively. In both cases, the concrete strength at transfer should not be less than 25 MPa.

The specified characteristic strength of reinforcement is given as follows (EC2 3.2.2(3)):

$$400\,\text{MPa} \leq f_{yk} \leq 600\,\text{MPa} \qquad \text{(EC2 3.2.2(3))}$$

The specified characteristic strength of prestressed steel should conform to EN 10138, Part 2 to 4 or European Technical Approval (EC2 3.3.2).

The program also checks the following tensile strength in prestressing steel (EC2 5.10.2.1). The maximum stresses applied to the tendon, $\sigma_{p,\max}$, in all types of prestressing steel, in terms of the specified minimum tensile strength f_{pk}, are summarized as follows:

$$\sigma_{p,\max} = \min\left\{k_1 f_{pk}, k_2 f_{p0.1k}\right\} \qquad \text{(EC2 5.10.2.1)}$$

The recommended value for k_1 and k_2 are 0.8 and 0.9 where, $\left(f_{p0.1k}\right)$ is defined as the characteristic value of 0.1% proof load and $\left(f_{pk}\right)$ is the characteristic maximum load in axial tension (EC2 3.3.3, Figure 3.9).

The stress in tendons immediately after tensioning or after prestress transfer is also limited to the following:

$$\sigma_{pm0} = \min\left\{k_7 f_{pk}, k_8 f_{p0.1k}\right\} \qquad \text{(EC2 5.10.3)}$$

The recommended values for k_7 and k_8 are 0.75 and 0.85.

10.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength, f_{ck}, f_{pk} and $f_{p0.1k}$ of the material by a partial factor of safety, γ_s and γ_c, as follows (EC2 3.1.6, 3.2.7, 3.3.6(6)) [*NDP*].

$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c \quad \text{(EC2 3.1.6 (1))}$$

$$f_{cwd} = \alpha_{cc} f_{cwk} / \gamma_c \quad \text{(EC2 3.1.6 (1))}$$

$$f_{yd} = f_{yk} / \gamma_s \quad \text{(EC2 3.2.7 (2))}$$

$$f_{ywd} = f_{ywk} / \gamma_s \quad \text{(EC2 3.2.7 (2))}$$

$$f_{pd} = f_{p0.1k} / \gamma_s \quad \text{(EC2 3.3.6 (6))}$$

The value α_{cc} is the coefficient that accounts for long-term effects on the compressive strength; α_{cc} is taken as 1.0 by default and can be overwritten by the user (EC2 3.1.6(1)).

The values of partial safety factors, γ_s and γ_c, for the materials and the design strengths of concrete and reinforcement used in the program are listed in the following table (EC2 2.4.2.4 (1), Table 2.1N):

Values of γ_m for the ultimate limit state [*NDP*]	
Reinforcement, $\boldsymbol{\gamma_s}$	1.15
Prestressing steel, $\boldsymbol{\gamma_p}$	1.15
Concrete in flexure and axial load, $\boldsymbol{\gamma_c}$	1.50

These values are recommended by the code to give an acceptable level of safety for normal structures under typical design situations (EC2 3.1.6(1)). For accidental and earthquake situations, the recommended values are less than the tabulated value. The user should consider those cases separately.

These factors are already incorporated into the design equations and tables in the code. The user is allowed to overwrite these values; however, caution is advised.

10.5 Design Assumptions for Prestressed Concrete Structures

Ultimate limit state design of prestressed members for flexure and axial loads shall be based on assumptions given in EC2 6.1(2).

- The strain distribution in the concrete in compression is derived from the assumption that plane sections remain plane.
- The design stresses in the concrete in compression are taken as ηf_{cd}. Maximum strain at the extreme concrete compression fiber shall be assumed equal to ε_{cu3}.
- The tensile strength of the concrete is ignored.
- The strains in bonded post-tensioning tendons and in any additional reinforcement (compression or tension) are the same as that in the surrounding concrete.

The serviceability limit state of prestressed members uses the following assumptions given in EC2 7.2.

- Plane sections remain plane, i.e., strain varies linearly with depth through the entire load range.
- Elastic behavior exists by limiting the concrete stresses to the values given in EC2 7.2(3).
- In general, it is only necessary to calculate design stresses due to the load arrangements immediately after the transfer of prestress and after all losses or prestress have occurred; in both cases the effects of dead and imposed loads on the strain and force in the tendons may be ignored.

Prestressed concrete members are investigated at three stages:

- At transfer of prestress force
- At service loading
- At nominal strength

10.6 Serviceability Requirements of Flexural Members

10.6.1 Serviceability Check at Initial Service Load

The stresses in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits (EC2 5.10.2.2 and 7.1):

- Extreme fiber stresses in compression:

$$0.60\, f_{ck}(\mathrm{t}) \qquad \text{(EC2 5.10.2.2(5))}$$

Unless reinforcing steel has been added, the stress limits will normally be "without bonded reinforcement" values, as any bonded tendons normally will be at the compression face at transfer.

- Extreme fiber stresses in tension (EC2 7.1)

$$\leq f_{ctm}(\mathrm{t}) \text{ where,} \qquad \text{(EC2 7.1(2))}$$

$$f_{ctm} = 0.30\, f_{ck}^{(2/3)} \qquad \text{for } f_{ck} \leq \text{C50/C60} \qquad \text{(EC2 Table 3.1)}$$

$$f_{ctm} = 2.12 ln\left(1 + f_{cm} / 10\right) \qquad \text{for } f_{ck} > \text{C50/C60} \qquad \text{(EC2 Table 3.1)}$$

$$f_{cm} = f_{ck} + 8\text{MPa} \qquad \text{(EC2 Table 3.1)}$$

- Extreme fiber stresses in tension should not exceed f_{ctm}; otherwise, the section should be designed as a cracked section (EC2 7.1).

10.6.2 Serviceability Check at Service Load

The stresses in the concrete for prestressed concrete flexural members at service loads, and after all prestress losses have occurred, are checked against the following limits (EC2 7.2(2)):

- Extreme fiber stress in compression due to prestress plus total load:

$$0.6\, f_{ck} \qquad \text{(EC2 7.2(2))}$$

- Extreme fiber stresses in tension in the precompressed tensile zone at characteristic service loads are defined as follows (EC2 7.2(5)):
  - Extreme fiber stresses in tension for reinforcement:

$$0.8\, f_{yk} \qquad \text{(EC2 7.2(5))}$$

  - Extreme fiber stresses in tension for prestressing tendons:

$$0.75\, f_{pk} \qquad \text{(EC2 7.2(5))}$$

Although cracking is permitted for Exposure Classes X0, XC1, XC2, XC3, and XC4, it may be assumed that the design hypothetical tensile stresses exist at the limiting crack widths given in Eurocode 2, Table 7.1N. Limits to the design hypothetical tensile stresses under Frequent Load combinations are given in the following table (TR43, Second Edition):

Group	Limiting crack width(mm)	Design stress
Bonded Tendons	0.1	1.35 f_{ctm}
	0.2	1.65 f_{ctm}
Unbonded tendons	-	1.35 f_{ctm}

10.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

10.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

10.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of prestressed concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or a flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases the beam may be designed as a rectangular or inverted flanged beam.

10.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 10-1 (EC2 3.1.7(3)).

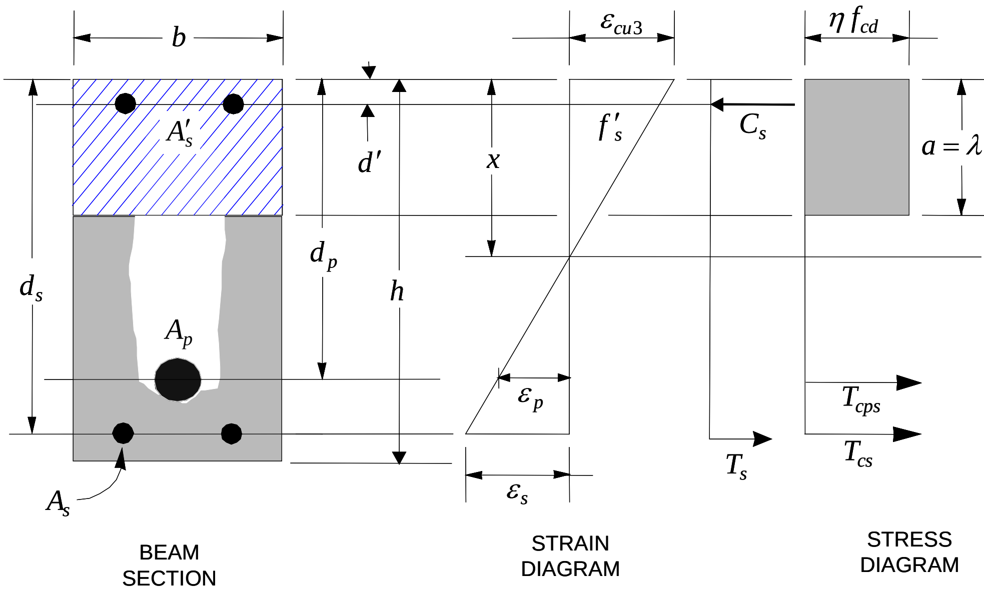


Figure 10-1 Rectangular Beam Design

The area of the stress block and the depth of the center of the compressive force from the most compressed fiber are taken as:

$$F_c = \eta f_{cd}\, a\, b$$

$$a = \lambda\, x$$

where x is the depth of the neutral axis; the factor λ defines the effective height of the compression zone; and the factor η defines the effective strength, as follows:

$\lambda = 0.8$	for $f_{ck} \leq 50$ MPa	(EC2 3.1.7(3))
$\lambda = 0.8\left[\frac{f_{ck} - 50}{400}\right]$	for $50 \leq f_{ck} \leq 90$MPa	(EC2 3.1.7(3))
$\eta = 1.0$	for $f_{ck} \leq 50$ MPa and	(EC2 3.1.7(3))

$$\eta = 1.0 - \left(\frac{f_{ck} - 50}{200} \right) \quad \text{for } 50 \le f_{ck} \ge 90\,\text{MPa} \qquad \text{(EC2 3.1.7(3))}$$

Furthermore, it is assumed that moment redistribution in the beam does not exceed the code specified limiting value. The code also places a limitation on the neutral axis depth, to safeguard against non-ductile failures (EC2 5.5(4)). When the applied moment exceeds the limiting moment capacity as a singly reinforced beam, the area of compression reinforcement is calculated on the assumption that the neutral axis depth remains at the maximum permitted value.

The design procedure used by ETABS, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow.

10.7.1.2.1 Design of Rectangular Beams

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that A_s = 0. In that case, the moment capacity M_{ED}^{0} is determined as follows:

The maximum depth of the compression zone, $x_{\max}$, is calculated based on the limitation that the tension reinforcement strain shall not be less than $\varepsilon_{s\min}$:

$$c_{\max} = \left(\frac{\varepsilon_{cu3}}{\varepsilon_{cu3} + \varepsilon_{s\min}} \right) d_p$$

where,

$$\varepsilon_{cu3} = 0.0035$$

Therefore, the limit $x \le x_{\max}$ is set for tension-controlled sections.

The maximum allowable depth of the compression block is given by:

$$a_{\max} = \lambda x_{\max} \qquad \text{(EC2 3.1.7(3))}$$

where,

$$\lambda = 0.8 \qquad \text{if} \quad f_{ck} < 50\,\text{MPa} \qquad \text{(EC2 3.1.7)}$$

$$\lambda = 0.8 - \left(\frac{f_{ck} - 50}{400}\right) \quad \text{if} \quad f_{ck} > 50\,\text{MPa} \qquad \text{(EC2 3.1.7)}$$

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pk} is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel.

Based on the calculated f_{pk}, the depth of the neutral axis is recalculated, and f_{pk} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \lambda x$$

- If $a \le a_{\max}$ (EC2 3.1.7(3)), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_{ED}^{0} = f_{pk} A_p \left(d_p - \frac{a}{2}\right)$$

- If $a > a_{\max}$ (EC2 3.1.7(3)), a failure condition is declared.

- If $M > M_{ED}^{0}$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension-controlled case. In that case, it is assumed that the depth of the neutral axis x is equal to $x_{\max}$. The stress in the post-tensioning steel, f_{pk}, is then calculated based on strain compatibility and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = \eta f_{cd} a_{\max} b$$

$$T = A_p f_{pk}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\eta f_{cd} a_{\max} b - A_p f_{pk}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_{ED}^{bal} = A_p f_{pk}^{bal}\left(d_p - \frac{a_{\max}}{2}\right) + A_s^{bal} f_s^{bal}\left(d_s - \frac{a_{\max}}{2}\right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, x.

10.7.1.2.1.1 Case 1: Post-tensioning steel is adequate

When $M < M_{ED}^0$, the amount of post-tensioning steel is adequate to resist the design moment M. A minimum reinforcement is provided to satisfy the flexural cracking requirements (EC2 7.3.2).

10.7.1.2.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $\left(M_{ED}^0 < M < M_{ED}^{bal}\right)$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports this required area of tension reinforcement. Since M is bound by M_{ED}^0 at the lower end and M_{ED}^{bal} at the upper end, and M_{ED}^0 is associated with $A_s = 0$ and M_{ED}^{bal} is associated with $A_s = A_s^{bal}$, the required area will fall within the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

10.7.1.2.1.3 Case 3: Post-tensioning steel and tension reinforcement is not adequate

When $\left(M > M_{ED}^{bal}\right)$, compression reinforcement is required (EC2 5.5 (4)). In that case, ETABS assumes that the depth of the neutral axis, x, is equal to $x_{\max}$. The values of f_{pk} and f_s reach their respective balanced condition values, f_{pk}^{bal} and f_s^{bal}. Then the area of compression reinforcement, A_s', is determined as follows:

- The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{ED,s} = M - M_{ED}^{bal}$$

The required compression reinforcement is given by:

$$A_s' = \frac{M_{ED,s}}{\left(0.87 f_s' - \eta f_{cd}\right)\left(d - d'\right)}, \text{ where}$$

$$f_s' = \varepsilon_{cu3} E_s \left[\frac{a_{\max} - d'}{a_{\max}}\right] \leq 0.87 f_y.$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{ED,s}}{0.87 f_y \left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M is positive, and vice versa if M is negative.

10.7.1.2.2 Design of Flanged Beams

10.7.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

10.7.1.2.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, the moment capacity M_{ED}^{0} is determined as follows:

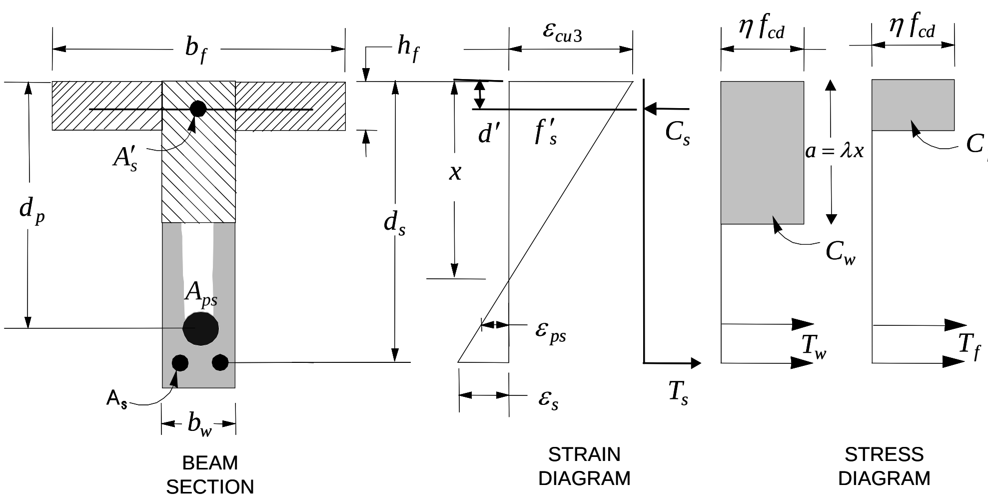


Figure 10-2 T-Beam Design

The maximum depth of the compression zone, x_{max}, is calculated based on the limitation that the tension reinforcement strain shall not be less than ε_{smin}:

$$c_{max} = \left(\frac{\varepsilon_{cu3}}{\varepsilon_{cu3} + \varepsilon_{s\min}} \right) d_p$$

where,

$\varepsilon_{cu3} = 0.0035$

Therefore, the program limit for the depth of the neutral axis is $x \leq x_{\max}$ for tension-controlled sections.

The maximum depth of the compression block is given by:

$$a_{\max} = \lambda x_{\max} \qquad \text{(EC2 3.1.7(3))}$$

where,

$$\lambda = 0.8 \qquad \text{if} \quad f_{ck} < 50\,\text{MPa} \qquad \text{(EC2 3.1.7)}$$

$$\lambda = 0.8 - \left(\frac{f_{ck} - 50}{400}\right) \qquad \text{if} \quad f_{ck} > 50\,\text{MPa} \qquad \text{(EC2 3.1.7)}$$

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pk}, is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps}, in the post-tensioning steel. Based on the calculated f_{pk}, the depth of the neutral axis is recalculated, and f_{pk} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \lambda x$$

- If $a \leq a_{\max}$ (EC2 3.1.7(3)), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_{ED}^{0} = f_{pk} A_p \left(d_p - \frac{a}{2} \right)$$

- If $a > a_{\max}$ (EC2 3.1.7(3)), a failure condition is declared.

- If $M > M_{ED}^{0}$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension- controlled case. In that case it is assumed that the depth of the neutral axis x is equal to $x_{\max}$. The stress in the post-tensioning steel, f_{pk}, is then calculated based on strain compatibility, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \leq h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case, the width of the beam is taken as b_f. Compression reinforcement is required when $a > a_{max}$.

- If $a > h_f$, the calculation for A_s is given by:

$$C = \eta f_{cd} a_{\max} A_c^{com}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$T = A_p f_{pk}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\eta f_{cd} a_{\max} A_c^{com} - A_p f_{pk}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_{ED}^{bal} = A_p f_{pk}^{bal} \left(d_p - \frac{a_{\max}}{2} \right) + A_s^{bal} f_s^{bal} \left(d_s - \frac{a_{\max}}{2} \right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcing steel, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, x.

10.7.1.2.2.2.1 Case 1: Post-tensioning steel is adequate

When $M < M_{ED}^0$, the amount of post-tensioning steel is adequate to resist the design moment M. Minimum reinforcement is provided to satisfy the flexural cracking requirements (EC2 7.3.2).

10.7.1.2.2.2.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{max}$.

When $\left(M_{ED}^{0} < M < M_{ED}^{bal}\right)$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports the required area of tension reinforcement. Since M is bounded by M_{ED}^{0} at the lower end and M_{ED}^{bal} at the upper end, and M_{ED}^{0} is associated with $A_s = 0$ and M_{ED}^{bal} is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

10.7.1.2.2.2.3 Case 3: Post-tensioning steel and tension reinforcement is not adequate

When $\left(M > M_{ED}^{bal}\right)$, compression reinforcement is required (EC2 5.5 (4)). In that case ETABS assumes that the depth of the neutral axis, x, is equal to $x_{\max}$. The values of f_{pk} and f_s reach their respective balanced condition values, f_{pk}^{bal} and f_s^{bal}. Then the area of compression reinforcement, A'_s, is determined as follows:

- The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{ED,s} = M - M_{ED}^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{ED,s}}{\left(0.87 f'_s - \eta f_{cd}\right)\left(d - d'\right)}, \text{ where}$$

$$f'_s = \varepsilon_{cu3} E_s \left[\frac{a_{\max} - d'}{a_{\max}}\right] \leq 0.87 f_y.$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{ED,s}}{0.87 f_y \left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M is positive, and vice versa if M is negative.

10.7.1.2.3 Minimum and Maximum Reinforcement

Reinforcement in prestressed concrete beams is computed to increase the strength of sections as required in the flexural design of prestressed beam or to comply with shear link requirements. The minimum flexural tension reinforcement required for a beam section to comply with the cracking requirements must be separately investigated by the user.

For bonded tendons, there is no minimum un-tensioned reinforcement requirements.

For unbonded tendons, the minimum flexural reinforcement provided in a rectangular or flanged beam section is given as [*NDP*]:

$$A_{s,\min} = 0.26 \frac{f_{ctm}}{f_{yk}} bd \geq 0.0013bd \qquad \text{(EC2 9.2.1.1)}$$

where f_{ctm} is the mean value of axial tensile strength of the concrete and is computed as:

$$f_{ctm} = 0.30 f_{ck}^{(2/3)} \qquad \text{for } f_{ck} \leq 50 \text{ MPa} \qquad \text{(EC2 3.12, Table 3.1)}$$

$$f_{ctm} = 2.12 ln\left(1 + f_{cm}/10\right) \qquad \text{for } f_{ck} > 50 \text{ MPa} \qquad \text{(EC2 3.12, Table 3.1)}$$

$$f_{cm} = f_{ck} + 8 \text{ MPa} \qquad \text{(EC2 3.12, Table 3.1)}$$

The minimum flexural tension reinforcement required for control of cracking should be investigated independently by the user.

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (EC2 9.2.1.1(3)).

10.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each loading combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular loading combination, at a particular station due to the beam major shear, the following steps are involved (EC2 6.2):

- Determine the factored shear force, V
- Determine the shear force, $V_{Rd,c}$, that can be resisted by the concrete
- Determine the shear reinforcement required to carry the balance

The following three sections describe in detail the algorithms associated with these steps.

10.7.2.1 Determine Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the cor-responding shear forces for different load cases with the corresponding load combination factors.

10.7.2.2 Determine Concrete Shear Capacity

The design shear resistance of the beam without shear reinforcement, $V_{Rd,c}$ is calculated as:

$$V_{Rd,c} = [C_{Rd,c}\, k(100\, \rho_1 f_{ck})^{1/3} + k_1\, \sigma_{cp}\,]\,(b_w d) \qquad \text{(EC2 6.2.2(1))}$$

$$V_{Rd,c} \geq [v_{\min} + k_1\, \sigma_{cp}\,]\,(b_w d), \qquad \text{(EC2 6.2.2(1))}$$

where f_{ck} is in MPa

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \text{ with } d \text{ in mm} \qquad \text{(EC2 6.2.2(1))}$$

$$\rho_1 = \text{ tension reinforcement ratio} = \frac{\left(A_{s1} + A_{ps}\right)}{b_w d} \leq 0.02 \qquad \text{(EC2 6.2.2(1))}$$

A_{s1} = area of mild-tension reinforcement (EC2 6.2.2(1))

A_{ps} = area of prestress-tension reinforcement (EC2 6.2.2(1))

σ_{cp} = average stress in concrete due to axial force N_{Ed}/A_c (EC2 6.2.2(1))

$$\sigma_{cp} = N_{Ed}/A_c < 0.2\, f_{cd} z \text{ MPa} \quad \text{(EC2 6.2.2(1))}$$

A_c = the total gross area of concrete section

The value of $C_{Rd,c}$, $v_{\min}$, and k_1 for use in a country may be found in its National Annex. The program default values for $C_{Rd,c}$ [*NDP*], $v_{\min}$ [*NDP*], and k_1 [*NDP*] are given as follows (EC2 6.2.2(1)):

$C_{Rd,c} = 0.18/\gamma_{c,}$

$v_{\min} = 0.035\, k^{3/2} f_{ck}^{1/2}$

$k_1 = 0.15.$

If light-weight concrete:

$C_{Rd,c} = 0.18/\gamma_c$ (EC2 11.6.1(1))

$v_{\min} = 0.03\, k^{3/2} f_{ck}^{1/2}$ (EC2 11.6.1(1))

$k_1 = 0.15.$ (EC2 11.6.1(1))

10.7.2.3 Determine Required Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. The assumptions in designing the shear reinforcement are as follows:

- The beam sections are assumed to be prismatic. The effect of any variation of width in the beam section on the concrete shear capacity is neglected.
- The effect on the concrete shear capacity of any concentrated or distributed load in the span of the beam between two columns is ignored. Also, the effect of the direct support on the beams provided by the columns is ignored.

- All shear reinforcement is assumed to be perpendicular to the longitudinal reinforcement.

In designing the shear reinforcement for a particular beam, for a particular load combination, the following steps of the standard method are involved (EC2 6.2).

- Obtain the design value of the applied shear force *V* from the ETABS analysis results (EC2 6.2.3(3)).

The shear force is limited to a maximum of:

$$V_{Rd,\max} = \frac{\alpha_{cw} b_w z v_1 f_{cd}}{\cot\theta + \tan\theta},\ \text{where} \qquad \text{(EC2 6.2.3(3))}$$

α_{cw} [*NDP*] is conservatively taken as 1 (EC2 6.2.3(3))

The strength reduction factor for concrete cracked in shear, v_1 [*NDP*] is defined as:

$$v_1 = 0.6\left(1 - \frac{f_{ck}}{250}\right) \qquad \text{(EC2 6.2.2(6))}$$

$$z = 0.9d \qquad \text{(EC2 6.2.3(1))}$$

θ is optimized by program and is set to 45° for combinations including seismic loading (EC2 6.2.3(2)).

- Given V_{Ed}, V_{Rdc}, $V_{Rd,\max}$, the required shear reinforcement in area/unit length is calculated as follows:

- If $V_{Ed} \le V_{Rdc}$,

$$\frac{A_{sw}}{s_v} = \frac{A_{sw,\min}}{s}$$

- If $V_{R,dc} < V_{Ed} \le V_{Rd,\max}$

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{z f_{ywd} \cot\theta} \ge \frac{A_{sw,\min}}{s} \qquad \text{(EC2 6.2.3(3))}$$

- If $V_{Ed} > V_{Rd,\max}$

a failure condition is declared. (EC2 6.2.3(3))

The maximum of all the calculated A_{sw}/s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

The minimum shear reinforcement is defined as:

$$\frac{A_{sw,\min}}{s} = \frac{0.08\sqrt{f_{ck}}}{f_{yk}} b_w \qquad \text{(EC2 9.2.2(5))}$$

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

10.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_{Ed}
- Determine special section properties
- Determine critical torsion capacity
- Determine the torsion reinforcement required

10.7.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T_{Ed} is permitted to be reduced in accordance with the code (EC2 6.3.1(2)).

However, the program does not automatically redistribute the internal forces and reduce T_{Ed}. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

10.7.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_k, t_{ef}, u, u_k, and z_i are calculated. These properties are described in the following (EC2 6.3.2).

A = Area enclosed by the outside perimeter of the cross-section

A_k = Area enclosed by centerlines of the connecting walls, where the centerline is located a distance of $t_{ef}/2$ from the outer surface

t_{ef} = Effective wall thickness, A/u. It is taken as at least twice the distance between the edge and center of the longitudinal rebar.

u = Outer perimeter of the cross-section

u_k = Perimeter of the area A_k

z_i = Side length of wall i, defined as the distance between the intersection points of the wall centerlines

In calculating the section properties involving reinforcement, such as A_k, and u_k, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 mm. This is equivalent to 38-mm clear cover and a 12-mm stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during calculation of torsion section properties. With this assumption, the special properties for a rectangular beam section are given as:

$$A = bh \qquad \text{(EC2 6.3.2(1))}$$

$$A_k = (b - t_{ef})(h - t_{ef}) \qquad \text{(EC2 6.3.2(1))}$$

$$u = 2b + 2h \qquad \text{(EC2 6.3.2(1))}$$

$$u_k = 2(b - t_{ef}) + 2(h - t_{ef}) \qquad \text{(EC2 6.3.2(3))}$$

where, the section dimensions b, h, and c are shown in Figure 10-3. Similarly, the special section properties for a flanged beam section are given as:

$$A = b_w h \qquad \text{(EC2 6.3.2(1))}$$

$$A_k = (b_w - t_{ef})(h - t_{ef}) \qquad \text{(EC2 6.3.2(1))}$$

$$u = 2b_w + 2h \qquad \text{(EC2 6.3.2(1))}$$

$$u_k = 2(h - t_{ef}) + 2(b_w - t_{ef}) \qquad \text{(EC2 6.3.2(3))}$$

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 10-3.

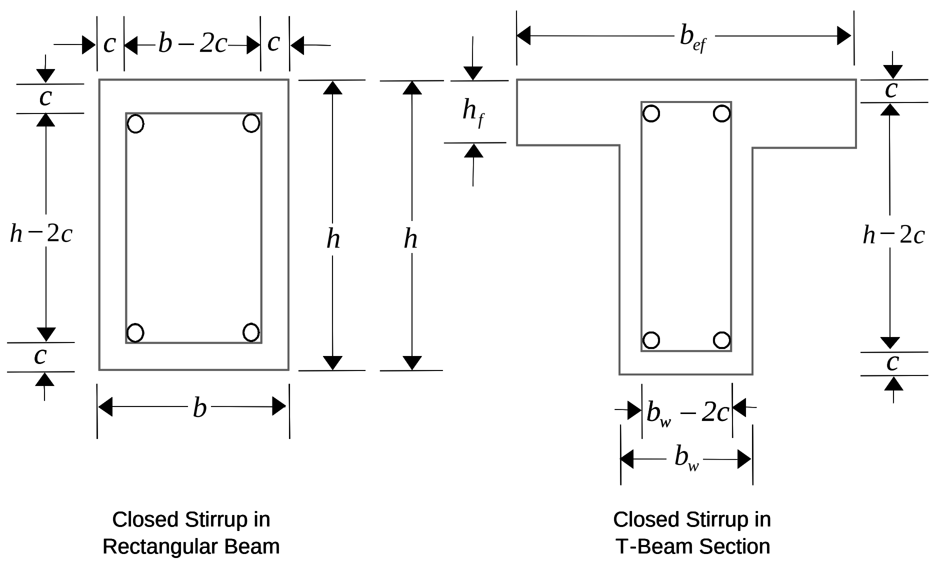


Figure 10-3 Closed stirrup and section dimensions for torsion design

10.7.3.3 Determine Critical Torsion Capacity

The torsion in the section can be ignored with only minimum shear reinforcement (EC2 9.2.1.1) required if the following condition is satisfied:

$$\frac{T_{Ed}}{T_{Rd,c}} + \frac{V_{Ed}}{V_{Rd,c}} \leq 1.0 \qquad \text{(EC2 6.3.2(5))}$$

where $V_{Rd,c}$ is as defined in the previous section and $T_{Rd,c}$ is the torsional cracking moment, calculated as:

$$T_{Rd,c} = f_{ctd} t_{ef} \, 2A_k \qquad \text{(EC2 6.3.2(1), 6.3.2(5))}$$

where t_{ef}, and f_{ctd}, the design tensile strength, are defined as:

$$t_{ef} = A/u \qquad \text{(EC2 6.3.2(1))}$$

$$f_{ctd} = \alpha_{ct} f_{ctk0.05}/\gamma_c \qquad \text{(EC2 Eq. 3.16)}$$

where A is the gross cross-section area, u is the outer circumference of the cross-section, α_{ct} [*NDP*] is a coefficient, taken as 1.0, taking account of long- term effects on the tensile strength, and $f_{ctk0.05}$ is defined as:

$$f_{ctk0.05} = 0.7 f_{ctm} \qquad \text{(EC2 Table 3.1)}$$

10.7.3.4 Determine Torsion Reinforcement

If the expression in the previous subsection is satisfied, torsion can be safely ignored (EC2 6.3.2(5)) with only minimum shear reinforcement required. In that case, the program reports that no torsion reinforcement is required. However, if the equation is not satisfied, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals.

If torsion reinforcement in the form of closed stirrups is required, the shear due to this torsion, V_t, is first calculated, followed by the required stirrup area, as:

$$\frac{A_t}{s} = \frac{V_t}{z f_{ywd} \cot\theta} \qquad \text{(EC2 6.2.3(3))}$$

$$V_t = \left(h - t_{ef}\right)\frac{T_{Ed} - T_{con}}{2A_k} \qquad \text{(EC2 6.3.2(1))}$$

The required longitudinal reinforcement for torsion is defined as:

$$T_{con} = \left(1 - \frac{V_{Ed}}{V_{Rd,c}}\right)T_{Rd,c} \qquad \text{(EC2 6.3.2(5))}$$

$$A_{sl} = \frac{T_{Ed}}{2A_k}\cot\theta\frac{u_k}{f_{yd}} \qquad \text{(EC2 6.3.2(3))}$$

where θ is the angle of the compression struts, as previously defined for beam shear. In the preceding expressions, θ is taken as 45 degrees. The code allows any value between 21.8 and 45 degrees (EC2 6.2.3(2)), while the program assumes the conservative value of 45 degrees.

When torsional reinforcement is required, an upper limit on the combination of V_{Ed} and T_{Ed} that can be carried by the section without exceeding the capacity of the concrete struts also is checked using:

$$\frac{T_{Ed}}{T_{Rd,\max}} + \frac{V_{Ed}}{V_{Rd,\max}} \leq 1.0 \qquad \text{(EC2 6.3.2(4))}$$

where $T_{Rd,\max}$, the design torsional resistance moment is defined as:

$$T_{Rd,\max} = 2\nu\alpha_{cw} f_{cd} A_k t_{ef} \sin\theta\cos\theta \qquad \text{(EC2 6.3.2(4))}$$

If this equation is not satisfied, a failure condition is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated A_{sl} and A_t/s values obtained from each load combination is reported, along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

10.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips usually are governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method for prestressed reinforced concrete (EC2-2004) as described in the following sections. To learn more about the design strips, refer to the section entitled "ETABS Design Techniques" in the *Key Features and Terminology* manual.

10.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.

- Design flexural reinforcement for the strip.

These two steps are described in the subsection that follows and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

10.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

10.8.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

10.8.1.2.1 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (EC2 9.3.1.1, 9.2.1.1, UK, NA Table NA.1) [*NDP*]:

$$A_{s,\min} = 0.26\frac{f_{ctm}}{f_{yk}}bd \qquad \text{(EC2 9.2.1.1(1))}$$

$$A_{s,\min} = 0.0013bd \qquad \text{(EC2 9.2.1.1(1))}$$

where f_{ctm} is the mean value of axial tensile strength of the concrete and is computed as:

$$f_{ctm} = 0.30 f_{ck}^{(2/3)} \quad \text{for } f_{ck} \leq 50 \text{ MPa} \qquad \text{(EC2 Table 3.1)}$$

$$f_{ctm} = 2.12ln\left(1 + f_{cm}/10\right) \quad \text{for } f_{ck} > 50 \text{ MPa} \qquad \text{(EC2 Table 3.1)}$$

$$f_{cm} = f_{ck} + 8 \text{ MPa} \qquad \text{(EC2 Table 3.1)}$$

The minimum flexural tension reinforcement required for control of cracking should be investigated independently by the user.

An upper limit on the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (EC2 9.2.1.1(3)).

10.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code specific items are described in the following.

10.8.2.1 Critical Section for Punching Shear

The punching shear is checked at the face of the column (EC2 6.4.1(4)) and at a critical section at a distance of 2.0*d* from the face of the support (EC2 6.4.2(1)). The perimeter of the critical section should be constructed such that its length is minimized. Figure 10-4 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

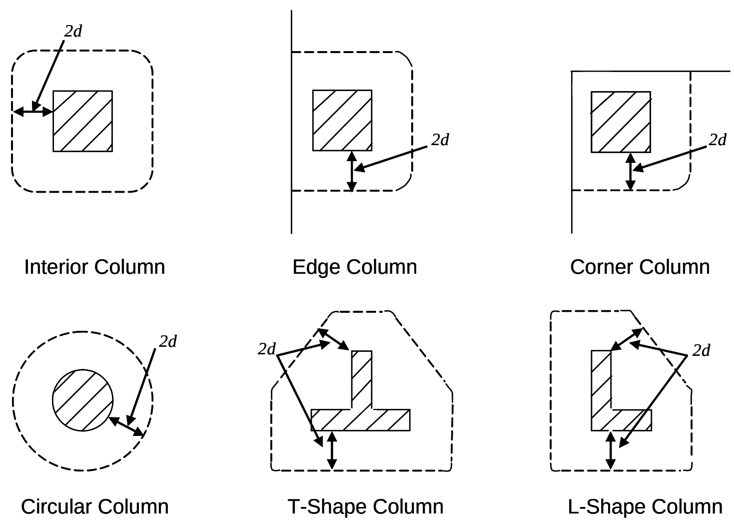


Figure 10-4 Punching Shear Perimeters

10.8.2.2 Determine Concrete Capacity

The concrete punching shear stress capacity is taken as:

$$V_{Rd,c} = \left[C_{Rd,c} k \left(100 \rho_1 f_{ck} \right)^{1/3} + k_1 \sigma_{cp} \right] \qquad \text{(EC2 6.4.4(1))}$$

with a minimum of:

$$V_{Rd,c} = \left(v_{\min} + k_1 \sigma_{cp}\right) \qquad \text{(EC2 6.4.4(1))}$$

where f_{ck} is in MPa and

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \text{ with } d \text{ in mm} \qquad \text{(EC2 6.4.4(1))}$$

$$\rho_1 = \sqrt{\rho_{1x}\rho_{1y}} \leq 0.02 \qquad \text{(EC2 6.4.4(1))}$$

where ρ_{1x} and ρ_{1y} are the reinforcement ratios in the x and y directions respectively, which is taken as the average tension reinforcement ratios of design strips in Layer A and layer B where Layer A and Layer design strips are in orthogonal directions. When design strips are not present in both orthogonal directions then tension reinforcement ratio is taken as zero in the current implementation, and

$$\sigma_{cp} = (\sigma_{cx} + \sigma_{cy})/2 \qquad \text{(EC2 6.4.4(1))}$$

where σ_{cx} and σ_{cy} are the normal concrete stresses in the critical section in the x and y directions respectively, conservatively taken as zeros.

$$C_{Rd,c} = 0.18/\gamma_c \ [NDP] \qquad \text{(EC2 6.4.4(1))}$$

$$v_{\min} = 0.035 k^{3/2} f_{ck}^{\ 1/2} \ [NDP] \qquad \text{(EC2 6.4.4(1))}$$

$$k_1 = 0.15 \ [NDP] \qquad \text{(EC2 6.4.4(1))}$$

10.8.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear, the nominal design shear stress, v_{Ed}, is calculated as:

$$v_{Ed} = \frac{V_{Ed}}{ud}\left[1 + k\frac{M_{Ed,2}u_1}{V_{Ed}W_{1,2}} + k\frac{M_{Ed,3}u_1}{V_{Ed}W_{1,3}}\right], \text{ where} \qquad \text{(EC2 6.4.4(2))}$$

k is the function of the aspect ratio of the loaded area in Table 6.1 of EN 1992-1-1

u_1 is the effective perimeter of the critical section

d is the mean effective depth of the slab

M_{Ed} is the design moment transmitted from the slab to the column at the connection along bending axis 2 and 3

V_{Ed} is the total punching shear force

W_1 accounts for the distribution of shear based on the control perimeter along bending axis 2 and 3.

10.8.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

10.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted provided that the effective depth of the slab is greater than or equal to 200 mm.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* as described in the earlier section remain unchanged. The design of punching shear reinforcement is as described in the following subsections.

10.8.3.1 Determine Required Shear Reinforcement

The shear stress is limited to a maximum limit of

$$\mathrm{V}_{Rd,\max} = \frac{\alpha_{cw} v_1 f_{cd}}{\left(\cot\theta + \tan\theta\right)} b_w z \quad \text{where} \qquad \text{(EC2 6.2.3(3))}$$

α_{cw} is conservatively taken as 1

$$v_1 = 0.6\left(1 - \frac{f_{ck}}{250}\right)$$

$z = 0.9d$ (EC2 6.2.3(1))

$1 \le \cot\theta \le 2.5$, program default value is 1, which can be overwritten by the user (EC2 6.2.3(2))

Given v_{Ed}, $v_{Rd,c}$ and $v_{Rd,\max}$, the required shear reinforcement is calculated as follows (EC2 6.4.5):

- If $v_{Ed} < v_{Rd,\max}$,

$$A_w = \frac{\left(v_{Ed} - 0.75 v_{Rd,c}\right)u}{1.5 f_{ywd}} s_r \quad \text{(EC2 6.4.5)}$$

- If $v_{Ed} > v_{Rd,\max}$, a failure condition is declared. (EC2 6.4.3)
- If v_{Ed} exceeds the maximum permitted value of $v_{RD,\max}$, the concrete section should be increased in size.

10.8.3.2 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of a rectangular columns should be arranged on peripheral lines, i.e., a line running parallel to and at constant distances from the sides of the column. Figure 10-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed 2*d*. The spacing between adjacent shear reinforcement in the first line of shear reinforcement shall not exceed 2*d* measured in a direction parallel to the column face (EC2 6.4.5(4)).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

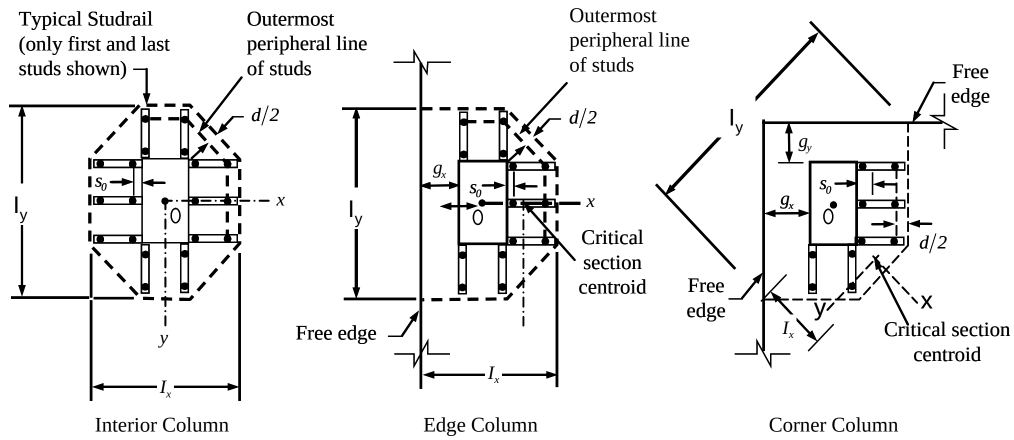


Figure 10-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

10.8.3.3 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in EC2 4.4.1 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.3d. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed 1.5d and should not exceed 2d at additional perimeters. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$0.3d \le s_o \le 2d$ (EC2 9.4.3(1))

$s \le 0.75d$ (EC2 9.4.3(1))

$g \le 1.5d$ (first perimeter) (EC2 9.4.3(1))

$g \le 2d$ (additional perimeters) (EC2 9.4.3(1))

10.9 Nationally Determined Parameters (NDPs)

The Comité Européen de Normalisation (CEN) version of Eurocode 2-2004 specifies a set of clauses in the design code, for which Nationally Determined Parameters (NDPs) are permitted to be adjusted by each member country within their National Annex. Variations in these parameters between countries are considered in the program by choosing the desired country from the **Options menu > Preferences > Concrete Frame Design** command. This appendix lists the NDPs as adopted in the program for the CEN Default version of the design code. Additional tables are provided that list the NDPs that differ from the CEN Default values for each country supported in the program.

Table 10-2 CEN Default NDPs

NDP	Clause	Value
γ_c	2.4.2.4(1)	1.5
γ_s	2.4.2.4(1)	1.15
α_{cc}	3.1.6(1)	1.0
α_{ct}	3.1.6(2)	1.0
max f_{yk}	3.2.2(3)	600MPa
Load Combinations	5.1.3(1)	Combinations from Eq. 6.10
θ_0	5.2(5)	0.005
k_1	5.5(4)	0.44
k_2	5.5(4)	$1.25(0.6 + 0.0014/\varepsilon_{cu2})$
k_3	5.5(4)	0.54
k_4	5.5(4)	$1.25(0.6 + 0.0014/\varepsilon_{cu2})$
λ_{lim}	5.8.3.1(1)	$20 \cdot A \cdot B \cdot C / \sqrt{n}$

Table 10-2 CEN Default NDPs

NDP	Clause	Value
$C_{Rd,c}$	6.2.2(1)	$0.18/\gamma_c$
$v_{\min}$	6.2.2(1)	$0.035k^{3/2}f_{ck}^{1/2}$
k_1	6.2.2(1)	0.15
θ	6.2.3(2)	45 degrees
ν_1	6.2.3(3)	$0.6\left[1-\frac{f_{ck}}{250}\right]$
α_{cw}	6.2.3(3)	1.0
Beam $A_{s,\min}$	9.2.1.1(1)	$0.26\frac{f_{ctm}}{f_{yk}}b_t d \geq 0.0013 b_t d$
Beam $A_{s,\max}$	9.2.1.1(3)	$0.04A_c$
Beam $\rho_{w,\min}$	9.2.2(5)	$\left(0.08\sqrt{f_{ck}}\right)/f_{yk}$
α_{lcc}	11.3.5(1)	0.85
α_{lct}	11.3.5(2)	0.85
$C_{lRd,c}$	11.6.1(1)	$0.15/\gamma_c$
$v_{l,\min}$	11.6.1(1)	$0.30k^{3/2}f_{lck}^{1/2}$
k_1	11.6.1(1)	0.15
ν_1	11.6.2(1)	$0.5\eta_1(1 - f_{lck}/250)$

Table 10-3 Denmark NDPs

NDP	Clause	Value
γ_c	2.4.2.4(1)	1.45
γ_s	2.4.2.4(1)	1.20
Max f_{yk}	3.2.2(3)	650MPa
Load Combinations	5.1.3(1)	Combinations from Eq. 6.10a/b
λ_{lim}	5.8.3.1(1)	$20 \cdot \sqrt{\frac{A_c f_{cd}}{N_{Ed}}}$
Beam $\rho_{w,\text{min}}$	9.2.2(5)	$\left(0.063\sqrt{f_{ck}}\right) / f_{yk}$
α_{lcc}	11.3.5(1)	1.0
α_{lct}	11.3.5(2)	1.0
$v_{l,\text{min}}$	11.6.1(1)	$0.03k^{2/3}f_{lck}^{1/2}$

Table10-4 Finland NDPs

NDP	Clause	Value
α_{cc}	3.1.6(1)	0.85
Max f_{yk}	3.2.2(3)	700MPa
Load Combinations	5.1.3(1)	Combinations from Eq. 6.10a/b
k_2	5.5(4)	1.10
Beam $A_{s,\text{max}}$	9.2.1.1(3)	Unlimited

Table 10-5 Norway NDPs

NDP	Clause	Value
α_{cc}	3.1.6(1)	0.85
α_{ct}	3.1.6(2)	0.85
λ_{lim}	5.8.3.1(1)	$13(2 - r_m)A_f$
k_1	6.2.2(1)	0.15 for compression 0.3 for tension
v_{min}	6.2.2(1)	$0.035k^{3/2}f_{ck}^{1/2}$
Beam $\rho_{w,\text{min}}$	9.2.2(5)	$\left(0.1\sqrt{f_{ck}}\right)/\ f_{yk}$
$v_{\text{l,min}}$	11.6.1(1)	$0.03\text{k}^{2/3}f_{lck}^{1/2}$
k_1	11.6.1(1)	0.15 for compression 0.3 for tension
ν_1	11.6.2(1)	$0.5(1 - f_{lck}/250)$

Table 10-6 Singapore NDPs

NDP	Clause	Value
α_{cc}	3.1.6(1)	0.85
k_1	5.5(4)	0.4
k_2	5.5(4)	$0.6 + 0.0014/\varepsilon_{cu2}$
k_3	5.5(4)	0.54
k_4	5.5(4)	$0.6 + 0.0014/\varepsilon_{cu2}$
v_{lim}	5.8.3.1(1)	$0.30\text{k}^{3/2}f_{lck}^{1/2}$

Table 10-7 Slovenia NDPs

NDP	Clause	Value
Same As CEN Default		

Table 10-8 Sweden NDPs

NDP	Clause	Value
Beam $A_{s,max}$	9.2.1.1(3)	Unlimited
α_{lcc}	11.3.5(1)	1.0
α_{lct}	11.3.5(2)	1.0

Table 10-9 United Kingdom NDPs

NDP	Clause	Value
$\psi_{0,i}$ (wind load)	EC0 Combos	0.5
α_{cc}	3.1.6(1)	0.85
k_1	5.5(4)	0.4
k_2	5.5(4)	$0.6 + 0.0014/\varepsilon_{cu2}$
k_3	5.5(4)	0.4
k_4	5.5(4)	$0.6 + 0.0014/\varepsilon_{cu2}$
$v_{l,min}$	11.6.1(1)	$0.30k^{3/2}f_{lck}^{1/2}$

Chapter 11
Design for Hong Kong CP 04

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the Hong Kong limit state code CP-04 [CP 04], which also incorporates Amendment 1 published in June 2007. Various notations used in this chapter are listed in Table 11-1. For referencing to the pertinent sections of the Hong Kong CP code in this chapter, a prefix "CP" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

11.1 Notations

The following table identifies the various notations used in this chapter.

Table 11-1 List of Symbols Used in the Hong Kong CP 04 Code

A_{cv}	Area of section for shear resistance, mm^2

Table 11-1 List of Symbols Used in the Hong Kong CP 04 Code

A_g	Gross area of cross-section, mm^2
A_s	Area of tension reinforcement, mm^2
A_{ps}	Area of prestress steel, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sv}	Total cross-sectional area of links at the neutral axis, mm^2
A_{sv} / s_v	Area of shear reinforcement per unit length of the member, mm^2/mm
a	Depth of compression block, mm
a_{max}	Maximum depth of the compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
d or d_e	Effective depth of tension reinforcement, mm
d'	Depth to center of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f_{ci}	Concrete strength at transfer, MPa
f_{cu}	Characteristic cube strength, MPa
f_{pu}	Characteristic strength of a prestressing tendon, MPa
f_{pb}	Design tensile stress in tendon, MPa
f'_s	Compressive stress in a beam compression steel, MPa
f_y	Characteristic strength reinforcement, MPa
f_{yv}	Characteristic strength of link reinforcement, MPa
h	Overall depth of a section in the plane of bending, mm
h_f	Flange thickness, mm

Table 11-1 List of Symbols Used in the Hong Kong CP 04 Code

k_1	Shear strength enhancement factor for support compression
k_2	Concrete shear strength factor, $\left[f_{cu}/25\right]^{1/3}$
M	Design moment at a section, MPa
M_u	Design moment resistance of a section, MPa
M_u^0	Design moment resistance of a section with tendons only, N-mm
M_u^{bal}	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
s_v	Spacing of the links along the length of the beam, mm
s	Spacing of shear rails, mm
T	Tension force, N
V	Design shear force at ultimate design load, N
u	Perimeter of the punching critical section, mm
v	Design shear stress at a beam cross-section or at a punch critical section, MPa
v_c	Design ultimate shear stress resistance of a concrete beam, MPa
v_{co}	Ultimate shear stress resistance of an uncracked concrete section, MPa
v_{cr}	Ultimate shear stress resistance of a cracked concrete section, MPa
$v_{\max}$	Maximum permitted design factored shear stress at a beam section or at the punch critical section, MPa
v_t	Torsional shear stress, MPa
x	Neutral axis depth, mm
x_{bal}	Depth of neutral axis in a balanced section, mm
z	Lever arm, mm

Table 11-1 List of Symbols Used in the Hong Kong CP 04 Code

β	Torsional stiffness constant
β_b	Moment redistribution factor in a member
γ_f	Partial safety factor for load
γ_m	Partial safety factor for material strength
ε_c	Maximum concrete strain
ε_{ps}	Strain in prestressing steel
ε_s	Strain in tension steel
ε'_s	Strain in compression steel

11.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. The design load combinations are obtained by multiplying the characteristic loads by appropriate partial factors of safety, γ_f (CP 2.3.2.1, Table 2.1). For Hong Kong CP 04, if a structure is subjected to dead (D), live (L), pattern live (PL), and wind (W) loads, and considering that wind forces are reversible, the load combinations in the following sections may need to be considered (CP 2.3.2.1, 12.3.4.2, 12.3.4.3 and 12.3.5.1).

For post-tensioned concrete design, the user can specify the prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning are defined in the following sections.

11.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress forces in accordance with Hong Kong CP 04 clause 12.3.5. The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0D + 1.0PT

11.2.2 Service Load Combination

The following load combinations are used for checking the requirements of prestress for serviceability in accordance with CP 12.3.4. It is assumed that all long-term losses have occurred already at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT

11.2.3 Ultimate Limit State Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with CP 2.3.2.1, Table 2.1.

The combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are determined automatically by ETABS by subtracting the primary PT moments when the flexural design is completed.

1.4D + 1.0PT*

1.4D + 1.6L + 1.0PT*

1.4D + 1.6(0.75PL) + 1.0PT*

1.0D ± 1.4W +1.0PT*
1.4D ± 1.4W + 1.0PT*
1.2D + 1.2L ± 1.2W + 1.0PT*

* — Replace PT with H for flexural design only

Other appropriate loading combinations should be used if roof live load is separately treated, or other types of loads are present.

11.3 Limits on Material Strength

Grade C28/C35 and C32/C40 are the minimum recommended for post-tensioning and pre-tensioning respectively. In both cases the concrete strength at transfer should not be less than 25 MPa (CP 12.1.8.1).

The specified characteristic strength of un-tensioned reinforcement is given as follows (CP 3.2.3, Table 3.3):

Hot rolled mild reinforcement -	250 MPa	(CP 3.2.3, Table 3.3)
High yield reinforcement -	460 MPa	(CP 3.2.3, Table 3.3)

The specified characteristic strength of prestressing steel should conform to CP 04 clause 3.3.

ETABS also checks the tensile strength in the prestressing steel (CP 12.7.1). The permissible tensile stresses in all types of prestressing steel, in terms of the specified minimum tensile strength f_{pu}, are summarized as follows:

a. Due to tendon jacking force:	$0.75\ f_{pu}$
b. Immediately after prestress transfer:	$0.70\ f_{pu}$

In any circumstances, the initial prestressing forces shall not exceed $0.75\ f_{pu}$.

11.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength of the material by a partial safety factor, γ_m. The values of γ_m used in the program are listed in the table that follows, as taken from CP Table 2.2 (CP 2.4.3.2):

Values of γ_m for the ultimate limit state	
Reinforcement, γ_{ms}	1.15
Prestressing steel, γ_{mp}	1.15
Concrete in flexure and axial load, γ_{mc}	1.50
Shear strength without shear reinforcement, γ_{mv}	1.25

These factors are already incorporated in the design equations and tables in the code. Note that for reinforcement, the default factor of 1.15 is for Grade 460 reinforcement. If other grades are used, this value should be overwritten as necessary. Changes to the partial safety factors are carried through the design equations where necessary, typically affecting the material strength portions of the equations.

11.5 Design Assumptions for Prestressed Concrete Structures

The ultimate limit state of prestressed members for flexure and axial loads shall be based on assumptions given in CP 12.3.7.1.

- The strain distribution in the concrete in compression is derived from the assumption that a plane section remains plane (CP 12.3.7.1).
- The design stresses in the concrete in compression are taken as 0.45 f_{cu}. The maximum strain at the extreme concrete compression fiber shall be assumed equal to 0.0035 (CP 12.3.7.1).
- Tensile strength of the concrete is ignored (CP 12.3.7.1).
- The strain in bonded prestressing tendons and in any additional reinforcement (compression or tension) is derived from the assumption that plane section remains plane (CP 12.3.7.1).

The serviceability limit state of prestressed members uses the following assumptions given in CP 12.3.4.1.

- Plane sections remain plane, i.e., strain varies linearly with depth through the entire load range (CP 12.3.4.1).
- Elastic behavior exists by limiting the concrete stresses to the values given in CP 12.3.4.2, 12.3.4.3 and 12.3.5 (CP 12.3.4.1).
- In general, it is only necessary to calculate design stresses due to the load arrangements immediately after the transfer of prestress and after all losses or prestress have occurred; in both cases the effects of dead and imposed loads on the strain and force in the tendons may be ignored (CP 12.3.4.1).

Prestressed concrete members are investigated at the following three stages (CP 12.3.4.2 and 12.3.4.3):

- At transfer of prestress force
- At service loading
- At nominal strength

The prestressed flexural members are classified as Group a (uncracked), Group b (cracked but no visible cracking), and Group c (cracked) based on tensile strength f_t, the computed extreme fiber stress in tension in the precompressed tensile zone at service loads (CP 12.1.3).

The precompressed tensile zone is that portion of a prestressed member where flexural tension, calculated using gross section properties, would occur under unfactored dead and live loads if the prestress force was not present. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress.

Group a: No flexural tensile stresses

Group b: Flexural tensile stresses with no visible cracking

Group c: Flexural tensile stresses with surface crack widths as follows:

- ≤ 0.1 mm for members in exposure conditions 3 and 4 (Table 4.1 of CP 04)
- ≤ 0.2 mm for all other members

11.6 Serviceability Requirements of Flexural Members

11.6.1 Serviceability Check at Initial Service Load

The stresses in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits (CP 12.3.5.1 and 12.3.5.2):

- Extreme fiber stress in compression: $0.50 f_{ci}$
- Extreme fiber stress in tension for Group a: ≤ 1.0 MPa
- Extreme fiber stress in tension for Group b:

  pre-tensioned member $0.45\sqrt{f_{ci}}$

post-tensioned member $0.36\sqrt{f_{ci}}$

The extreme fiber stress in tension for Group c should not exceed the appropriate value for a Group b member; otherwise the section should be designed as a cracked section.

11.6.2 Serviceability Check at Service Load

The stresses in the concrete for Group a and Group b prestressed flexural members at service loads, and after all prestress losses have occurred, are checked against the following limits (CP 12.3.4.2 an 12.3.4.3):

- Extreme fiber stress in compression due to prestress plus total load: $0.33 f_{cu}$
- Extreme fiber stress in compression due to prestress plus total load for continuous beams and other statically indeterminate structures: $0.4 f_{cu}$
- Extreme fiber stress in tension in the precompressed tensile zone at service loads:
  - Extreme fiber stresses in tension for Group a: No tensile stress
  - Extreme fiber stresses in tension for Group b:

    pre-tensioned member $0.45\sqrt{f_{cu}}$

    post-tensioned member $0.36\sqrt{f_{cu}}$

Although cracking is allowed for Group c, it is assumed that the concrete section is uncracked and the user is limiting the tensile stress at the service stage as presented in Table 12.2, modified by the coefficients in Table 12.3 of CP 04. The user needs to provide the tension limits for Group c elements at service loads in the design preferences (CP 12.3.4.3).

11.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

11.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

11.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of prestressed concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or a flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases the beam may be designed as a rectangular or inverted flanged beam.

11.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 12-1 (CP 6.1.2.4(a)), where $\varepsilon_{c,\max}$ is defined as:

$$\varepsilon_{c,\max} = \begin{cases} 0.0035 & \text{if} \quad f_{cu} \leq 60 \text{ MPA} \\ 0.0035 - 0.00006\left(f_{cu} - 60\right)^{1/2} & \text{if} \quad f_{cu} > 60 \text{ MPA} \end{cases}$$

Furthermore, it is assumed that moment redistribution in the member does not exceed 10% (i.e., $\beta_b \geq 0.9$) (CP 6.1.2.4(b)). The code also places a limitation on the neutral axis depth,

$$\frac{x}{d} \leq \begin{cases} 0.5 & \text{for} \quad f_{cu} \leq 45 \text{ N/mm}^2 \\ 0.4 & \text{for} \quad 45 < f_{cu} \leq 70 \text{ N/mm}^2 \\ 0.33 & \text{for} \quad 70 < f_{cu} \leq 100 \text{ N/mm}^2 \end{cases}$$

to safeguard against non-ductile failures (CP 6.1.2.4(b)). In addition, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The design procedure used by ETABS, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow. It is assumed that the design ultimate axial force does not exceed 0.1 $f_{cu} A_g$ (CP 6.1.2.4(a)); hence all beams are designed for major direction flexure, shear, and torsion only.

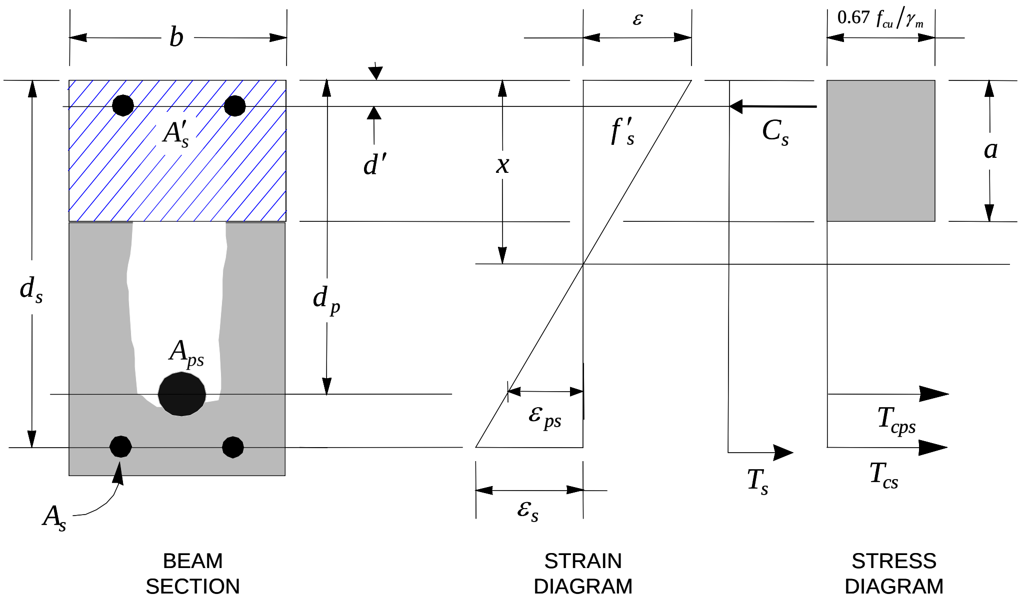


Figure 11-1 Rectangular Beam Design

11.7.1.2.1 Design of Rectangular Beams

The amount of post-tensioning steel adequate to resist the design moment *M* and minimum reinforcement are provided to satisfy the flexural cracking requirements (CP 19.2.1).

ETABS determines the depth of the neutral axis, *x*, by imposing force equilibrium, i.e., C = T, and performs an iteration to compute the depth of the neutral axis, which is based on stress-strain compatibility. After the depth of the neutral axis has been found, the stress in the post-tensioning reinforcement f_{pb} is computed based on strain compatibility.

The ductility of a section is controlled by limiting the *x*/*d* ratio (CP 6.1.2.4(b)):

$$\frac{x}{d} = \begin{cases} 0.5, & \text{for} \quad f_{cu} \leq 45 \text{ N/mm}^2 \\ 0.4, & \text{for} \quad 45 < f_{cu} \leq 70 \text{ N/mm}^2 \\ 0.33, & \text{for} \quad 70 < f_{cu} \leq 100 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(b))}$$

The maximum depth of the compression block is given by:

$$a = \begin{cases} 0.9x & \text{for} \quad f_{cu} \le 45 \text{ N/mm}^2 \\ 0.8x & \text{for} \quad 45 < f_{cu} \le 70 \text{ N/mm}^2 \\ 0.72x & \text{for} \quad 70 < f_{cu} \le 100 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(a))}$$

The lever arm of the section must not be greater than 0.95 times the effective depth (CP 6.1.2.4(c)).

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \le 0.95d, \qquad \text{(CP 6.1.2.4(c))}$$

- If $a \le a_{\max}$ (CP 6.1.2.4), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_u^0 = A_{ps} f_{pb}\left(d_p - \frac{a}{2}\right)$$

- If $a > a_{\max}$ (CP 6.1.2.4), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of neutral axis x is equal to $x_{\max}$. The stress in the post-tensioning steel, f_{pb} is then calculated based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = \frac{0.67 f_{cu}}{\gamma_m} a_{\max} b$$

$$T = A_{ps} f_{pb}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\dfrac{0.67 f_{cu}}{\gamma_m} a_{\max} b - A_{ps} f_{pb}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_u^{bal} = A_{ps} f_{pb}^{bal}\left(d_p - \frac{a_{\max}}{2}\right) + A_s^{bal} f_s^{bal}\left(d_s - \frac{a_{\max}}{2}\right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of neutral axis, x.

11.7.1.2.1.1 Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment M.. Minimum reinforcement is provided to satisfy the ductility requirements, i.e., $M < M_u^0$.

11.7.1.2.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports this required area of tension reinforcement. Since M is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end, and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will be between the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

11.7.1.2.1.3 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required (CP 6.1.2.4). In that case, ETABS assumes that the depth of neutral axis, x, is equal to x_{max}. The values of f_{pb} and f_s reach their respective balanced condition values, f_{pb}^{bal} and f_s^{bal}. The area of compression reinforcement, A'_s, is determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(f'_s - \frac{0.67 f_{cu}}{\gamma_c}\right)(d - d')}, \text{ where} \qquad \text{(CP 6.1.2.4(c))}$$

$$f'_s = E_s \varepsilon_c \left(1 - \frac{d'}{x}\right) \leq 0.87 f_y, \qquad \text{(CP 6.1.2.4(c), 3.2.6, Fig. 3.9)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{0.87 f_y \left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M is positive, and vice versa if M is negative.

11.7.1.2.2 Design of Flanged Beams

11.7.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

11.7.1.2.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_S = 0$. In that case, moment capacity M_u^0 is determined as follows:

ETABS determines the depth of the neutral axis, x, by imposing force equilibrium, i.e., C = T, and performs an iteration to compute the depth of neutral axis, which is based on stress-strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pb} is computed based on strain compatibility.

The ductility of a section is controlled by limiting the x/d ratio (CP 6.1.2.4(b)):

$$\frac{x}{d} = \begin{cases} 0.5, & \text{for} \quad f_{cu} \leq 45 \text{ N/mm}^2 \\ 0.4, & \text{for} \quad 45 < f_{cu} \leq 70 \text{ N/mm}^2 \\ 0.33, & \text{for} \quad 70 < f_{cu} \leq 100 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(b))}$$

The maximum depth of the compression block is given by:

$$a = \begin{cases} 0.9x & \text{for} \quad f_{cu} \leq 45 \text{ N/mm}^2 \\ 0.8x & \text{for} \quad 45 < f_{cu} \leq 70 \text{ N/mm}^2 \\ 0.72x & \text{for} \quad 70 < f_{cu} \leq 100 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(a))}$$

The lever arm of the section must not be greater than 0.95 times the effective depth (CP 6.1.2.4(c)).

$$z = d \left\{ 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \right\} \leq 0.95d, \qquad \text{(CP 6.1.2.4(c))}$$

- If $a \leq a_{\max}$ (CP 6.1.2.4), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_u^0 = A_{ps} f_{pb} \left(d_p - \frac{a}{2} \right)$$

- If $a > a_{\max}$ (CP 6.1.2.4), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case it is assumed that the depth of neutral axis x is equal to c_{max}. The stress in the post-tensioning steel, f_{pb} is then calculated based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as b_f. Compression reinforcement is required when $K > K'$.

- If $a > h_f$, the calculation for A_s is given by

$$C = \frac{0.67 f_{cu}}{\gamma_c} a_{\max} A_c^{com}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$A_c^{com} = b_f h_f + b_w \left(a_{\max} - h_f \right)$$

$$T = A_{ps} f_{pb}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\frac{0.67 f_{cu}}{\gamma_m} a_{\max} A_c^{com} - A_{ps} f_{pb}^{bal}}{f_s^{bal}}$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of neutral axis, c.

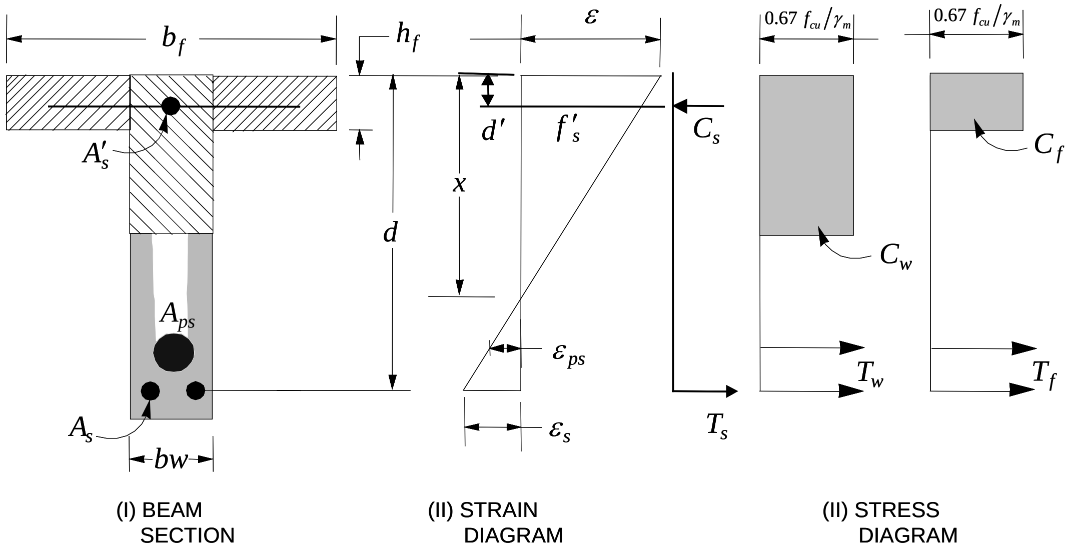


Figure 11-2 T-Beam Design

11.7.1.2.2.3 Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment *M*. Minimum reinforcement is provided to satisfy ductility requirements.

11.7.1.2.2.4 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{PS}, alone is not sufficient to resist *M*, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{max}$.

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_S, iteratively to satisfy the design moment *M* and reports the required area of tension reinforcement. Since *M* is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end, and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

11.7.1.2.2.5 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required (CP 6.1.2.4). In that case ETABS assumes that the depth of the neutral axis, x, is equal to x_{max}. The values of f_{pb} and f_s reach their respective balanced condition values, f_{pb}^{bal} and f_s^{bal}. The area of compression reinforcement, A_s', is then determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(f'_s - \frac{0.67 f_{cu}}{\gamma_c}\right)(d - d')}, \text{ where} \qquad \text{(CP 6.1.2.4(c))}$$

$$f'_s = E_s \varepsilon_c \left(1 - \frac{d'}{x}\right) \leq 0.87 f_y \qquad \text{(CP 6.1.2.4(c))}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{0.87 f_y (d_s - d')}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom, and A'_s is to be placed at the top if M is positive and vice versa if M is negative.

11.7.1.3 Minimum and Maximum Reinforcement

Reinforcement in post-tensioned concrete beams is computed to increase the strength of sections as documented for the flexural design of post-tensioned beams or to comply with the shear link requirements. The minimum flexural tension reinforcement required for a beam section to comply with the cracking requirements needs to be separately investigated by the user.

For bonded tendons, there is no minimum untensioned reinforcement required.

For unbounded tendons, the minimum flexural reinforcement provided in a rectangular or flanged beam section is given by the following table, which is taken from CP Table 9.1(CP 9.2.1.1) with interpolation for reinforcement of intermediate strength:

Section	Situation	Definition of percentage	Minimum percentage $f_y = 250$ MPa	Minimum percentage $f_y = 460$ MPa
Rectangular	—	$100\ \frac{A_s}{bh}$	0.24	0.13
T- or L-Beam with web in tension	$\frac{b_w}{b_f} < 0.4$	$100\ \frac{A_s}{b_w h}$	0.32	0.18
	$\frac{b_w}{b_f} \geq 0.4$	$100\ \frac{A_s}{b_w h}$	0.24	0.13
T-Beam with web in compression	—	$100\ \frac{A_s}{b_w h}$	0.48	0.26
L-Beam with web in compression	—	$100\ \frac{A_s}{b_w h}$	0.36	0.20

The minimum flexural compression reinforcement, if it is required at all, is given by the following table, which is taken from CP Table 9.1(CP 9.2.1.1) with interpolation for reinforcement of intermediate strength:

Section	Situation	Definition of percentage	Minimum percentage
Rectangular	—	$100\ \dfrac{A'_s}{bh}$	0.20
T- or L-Beam	Web in tension	$100\ \dfrac{A'_s}{b_f h_f}$	0.40
	Web in compression	$100\ \dfrac{A'_s}{b_w h}$	0.20

In addition, an upper limit on both the tension reinforcement and compression reinforcement is imposed to be 0.04 times the gross cross-sectional area (CP 3.12.6.1).

11.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination in the major direction of the beam. In designing the shear reinforcement for a particular beam for a particular load combination, the following steps are involved (CP 6.1.2.5):

- Determine the shear stress, v.
- Determine the shear stress, v_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three subsections describe in detail the algorithms associated with these steps.

11.7.2.1 Determine Shear Stress

In the design of the beam shear reinforcement, the shear forces for a particular load combination at a particular beam section are obtained by factoring the associated shear forces for different load cases, with the corresponding load combination factors.

$$v = \frac{V}{b_w d} \qquad \text{(CP 6.1.2.5(a))}$$

The maximum allowable shear stress, v_{max} is defined as:

$$v_{max} = \min(0.8\sqrt{f_{cu}},\ 7\ \text{MPa}) \qquad \text{(CP 6.1.2.5(a), 12.3.8.2)}$$

For light-weight concrete, v_{max} is defined as:

$$v_{max} = \min(0.63\sqrt{f_{cu}},\ 5.6\ \text{MPa}) \qquad \text{(BS 8110-2:1985 5.4)}$$

11.7.2.2 Determine Concrete Shear Capacity

The design ultimate shear resistance of the concrete alone, V_C should be considered at sections that are as follows:

Uncracked sections in flexure ($M < M_o$) (CP 12.3.8.3)

Cracked sections in flexural ($M \geq M_o$) (CP 12.3.8.3)

where,

M is the design bending moment at the section

M_o is the moment necessary to produce zero stress in the concrete at the extreme tension fiber; in this calculation, only 0.8 of the stress due to post-tensioning should be taken into account.

11.7.2.2.1.1 Case 1: Uncracked section in flexure

The ultimate shear resistance of the section, V_{co}, is computed as follows:

$$V_{co} = 0.67 b_v h \sqrt{\left(f_t^2 + 0.8 f_{cp} f_t\right)}, \qquad \text{(CP 12.3.8.4)}$$

where,

f_t is the maximum design principal stress (CP 12.3.8.4)

$$f_t = 0.24\sqrt{f_{cu}} \qquad \text{(CP 12.3.8.4)}$$

f_{cp} = design compressive stress at the centroidal axis due to post-tensioning, taken as positive. (CP 12.3.8.4)

$$V_c = V_{co} + P \sin \beta \qquad \text{(CP 12.3.8.4)}$$

11.7.2.2.1.2 Case 2: Cracked section in flexure

The ultimate shear resistance of the section, V_{cr}, is computed as follows:

$$V_{cr} = \left(1 - 0.55 \frac{f_{pe}}{f_{pu}}\right) v_c b_v d + M_o \frac{V}{M}, \text{ and} \qquad \text{(CP 12.3.8.5)}$$

$$V_{cr} \geq 0.1 b_v d \sqrt{f_{cu}} \qquad \text{(CP 12.3.8.5)}$$

$$V_c = \min\left(V_{co}, V_{cr}\right) + P \sin \beta \qquad \text{(CP 12.3.8.5)}$$

11.7.2.3 Determine Required Shear Reinforcement

Given v, v_c, and $v_{\max}$, the required shear reinforcement is calculated as follows (CP 12.3.8.6):

- Calculate the design average shear stress that can be carried by minimum shear reinforcement, v_r, as:

$$v_r = \begin{cases} 0.4 f_{cu} & \text{if} \quad f_{cu} \leq 40 \text{ N/mm}^2 \\ 0.4\left(f_{cu}/40\right)^{2/3} & \text{if } 40 < f_{cu} \leq 80 \text{ N/mm}^2 \\ 0.4\left(80/40\right)^{2/3} & \text{if} \quad f_{cu} > 80 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 12.3.8.7)}$$

$$f_{cu} \leq 80 \text{ N/mm}^2 \text{ (for calculation purpose only)} \qquad \text{(CP 6.1.2.5(c))}$$

- If $v \leq v_c + v_r$, minimum reinforcement is required:

$$\frac{A_s}{s_v} = \frac{v_r b}{0.87 f_{yv}}, \qquad \text{(CP 12.3.8.7)}$$

- If $v > v_c + v_r$,

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c)b}{0.87 f_{yv}} \quad \text{(CP 12.3.8.8)}$$

- If $v > v_{max}$, a failure condition is declared.

In the preceding expressions, a limit is imposed on f_{yv} as:

$f_{yv} \leq 460$ MPa.

The maximum of all of the calculated A_{sv}/s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

11.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the torsional shear stress, v_t
- Determine special section properties
- Determine critical torsion stress
- Determine the torsion reinforcement required

11.7.3.1 Determine Torsional Shear Stress

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In typical framed construction, specific consideration of torsion is not usually required where torsional cracking is adequately controlled by shear reinforcement. If the design relies on the torsional resistance of a beam, further consideration should be given using the following algorithms (CP 6.3.1).

The torsional shear stress, v_t, for a rectangular section is computed as:

$$v_t = \frac{2T}{h_{\min}^2 \left(h_{\max} - h_{\min}/3\right)} \qquad \text{(CP 6.3.3(a))}$$

For flanged sections, the section is considered as a series of rectangular segments and the torsional shear stress is computed for each rectangular component using the preceding equation, but considering a torsional moment attributed to that segment, calculated as:

$$T_{seg} = T\left(\frac{h_{\min}^3 h_{\max}}{\sum\left(h_{\min}^3 h_{\max}\right)}\right) \qquad \text{(CP 6.3.3(b))}$$

h_{max} = Larger dimension of a rectangular section

h_{min} = Smaller dimension of a rectangular section

If the computed torsional shear stress, v_t, exceeds the following limit for sections with the larger center-to-center dimension of the closed link less than 550 mm, a failure condition is generated if the torsional shear stress does not satisfy:

$$v_t \le \min(0.8\sqrt{f_{cu}}, 7\text{N/mm}^2) \times \frac{y_1}{550} \qquad \text{(CP 6.3.4, Table 17)}$$

11.7.3.2 Determine Critical Torsion Stress

The critical torsion stress, $v_{t,\min}$, for which the torsion in the section can be ignored is calculated as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.6\text{ N/mm}^2\right) \qquad \text{(CP 6.3.4, Table 17)}$$

where f_{cu} is the specified concrete compressive strength.

For light-weight concrete, $v_{t,min}$ is defined as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.4\text{N/mm}^2\right) \times 0.8 \qquad \text{(BS 8110-2:85 5.5)}$$

11.7.3.3 Determine Torsion Reinforcement

If the factored torsional shear stress, v_t is less than the threshold limit, $v_{t,\min}$, torsion can be safely ignored (CP 6.3.5). In that case, the program reports that no torsion reinforcement is required. However, if v_t exceeds the threshold limit, $v_{t,\min}$, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CP 6.3.5).

- If $v_t > v_{t,\min}$, the required closed stirrup area per unit spacing, $A_{sv,t}/s_v$, is calculated as:

$$\frac{A_{sv,t}}{s_v} = \frac{T}{0.8 x_1 y_1 \left(0.87 f_{yv}\right)} \qquad \text{(CP 6.3.6)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{A_{sv,t} f_{yv}\left(x_1 + y_1\right)}{s_v f_y} \qquad \text{(CP 6.3.6)}$$

In the preceding expressions, x_1 is the smaller center-to-center dimension of the closed link, and y_1 is the larger center-to-center dimension of the closed link.

An upper limit of the combination of v and v_t that can be carried by the section is also checked using the equation:

$$v + v_t \le v_{\max} \qquad \text{(CP 6.3.4)}$$

$$v_{\max} \le \min\left(0.8\sqrt{f_{cu}}, 7\ \text{N/mm}^2\right) \qquad \text{(CP 6.3.4)}$$

For light-weight concrete, $v_{\max}$ is defined as:

$$v_{\max} = \min(0.63\sqrt{f_{cu}}, 4\ \text{MPa}) \qquad \text{(BS 8110-2:85 5.4)}$$

If the combination of shear stress, v, and torsional shear stress, v_t, exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated A_l and $A_{sv,t}/s_v$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

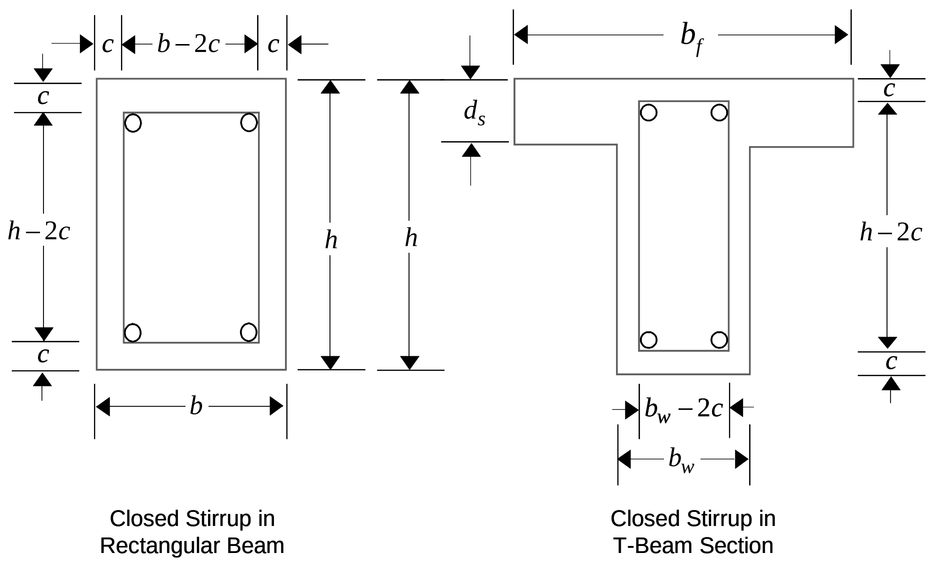


Figure 11-3 Closed stirrup and section dimensions for torsion design

11.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips typically are governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (Hong Kong CP 04) for prestressed reinforced concrete as described in the following subsections. To learn more about the design strips, refer to the section entitled "ETABS Design Techniques" in the *Key Features and Terminology* manual.

11.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Determine the capacity of post-tensioned sections
- Design flexural reinforcement for the strip

These three steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

11.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

11.8.1.2 Determine Capacity of Post-Tensioned Sections

The calculation of the post-tensioned section capacity is identical to that described earlier for rectangular beam sections.

11.8.1.3 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

11.8.1.4 Minimum and Maximum Slab Reinforcement

There are no minimum requirements for untensioned reinforcement in one-way bonded slabs. One-way spanning floors with unbounded tendons should have minimum reinforcement requirements in accordance with CP Table 3.25 (CP 3.12.5.3)

In flat slabs, reinforcement is added at the top over supports to be 0.00075 times the gross cross-sectional area. This reinforcement extends 1.5 times the slab depth on each side of the column. The length of the reinforcement should be at least 0.2*L* where *L* is the span of the slab.

There are no minimum requirements for span zone. However, additional un-tensioned reinforcement shall be designed for the full tension force generated by assumed flexural tensile stresses in the concrete for the following situations (Concrete Society, Technical Report 43):

- all locations in one-way spanning floors using unbonded tendons
- all locations in one-way spanning floors where transfer tensile stress exceeds $0.36\sqrt{f_{ci}}$
- support zones in all flat slabs
- span zones in flat slabs using unbonded tendons where the tensile stress exceeds $0.15\sqrt{f_{cu}}$.

The reinforcement should be designed to act at a stress of 5/8f_y as follows:

$$A_s = \frac{F_t}{(5/8)\, f_y}$$

where

$$F_t = -\frac{f_{ct}(h-x)b}{2}$$

The value of f_{ct} will be negative in tension.

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (CP9.2.1.3).

11.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Key Features and Terminology* manual. Only the code specific items are described in the following sections.

11.8.2.1 Critical Section for Punching Shear

The punching shear is checked at a critical section at a distance of 1.5d from the face of the support (CP 6.1.5.7(f)). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads. Figure 11-4 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

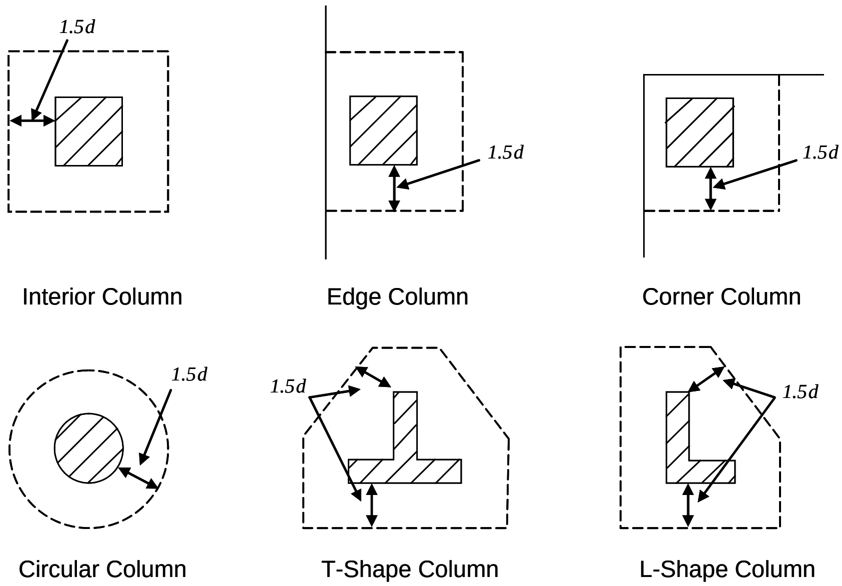


Figure 11-4 Punching Shear Perimeters

11.8.2.2 Determine Concrete Capacity

The design ultimate shear resistance of the concrete alone, V_c, should be considered at sections that are as follows:

Uncracked sections in flexure ($M < M_o$) (CP 12.3.8.3)

Cracked sections in flexural ($M \geq M_o$) (CP 12.3.8.3)

where,

M is the design bending moment at the section

M_o is the moment necessary to produce zero stress in the concrete at the extreme tension fiber; in this calculation, only 0.8 of the stress due to post-tensioning should be taken into account.

11.8.2.2.1.1 Case 1: Uncracked section in flexure

The ultimate shear resistance of the section, V_{co}, is computed as follows:

$$V_{co} = 0.67 b_v h \sqrt{\left(f_t^{\,2} + 0.8 f_{cp} f_t \right)},$$ (CP 12.3.8.4)

where,

f_t is the maximum design principal stress (CP 12.3.8.4)

$$f_t = 0.24 \sqrt{f_{cu}}$$ (CP 12.3.8.4)

f_{cp} = design compressive stress at the centoidal axis due to prestress, taken as positive. (CP 12.3.8.4)

$$V_c = V_{co} + P \sin \beta$$ (CP 12.3.8.4)

11.8.2.2.1.2 Case 2: Cracked section in flexure

The ultimate shear resistance of the section, V_{cr}, is computed as follows:

$$V_{cr} = \left(1 - 0.55 \frac{f_{pe}}{f_{pu}} \right) v_c b_v d + M_o \frac{V}{M}, \text{ and}$$ (CP 12.3.8.5)

$$V_{cr} \geq 0.1 b_v d \sqrt{f_{cu}}$$ (CP 12.3.8.5)

$$V_c = \min \left(V_{co}, V_{cr} \right) + P \sin \beta$$ (CP 12.3.8.5)

11.8.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the bending axis, the nominal design shear stress, $v_{\max}$, is calculated as:

$$V_{eff,x} = V \left(f + \frac{1.5 M_x}{Vy} \right)$$ (CP 6.1.5.6(b), 6.1.5.6(c))

$$V_{eff,y} = V \left(f + \frac{1.5 M_y}{Vx} \right)$$ (CP 6.1.5.6(b), 6.1.5.6(c))

$$v_{\max} = \max\begin{cases} \dfrac{V_{eff,x}}{u\,d} \\ \dfrac{V_{eff,y}}{u\,d} \end{cases} \qquad \text{(CP 6.1.5.7)}$$

where,

u is the perimeter of the critical section,

x and y are the lengths of the sides of the critical section parallel to the axis of bending,

M_x and M_y are the design moments transmitted from the slab to the column at the connection,

V is the total punching shear force, and

f is a factor to consider the eccentricity of punching shear force and is taken as

$$f = \begin{cases} 1.00 & \text{for interior columns} \\ 1.25 & \text{for edge columns} \\ 1.25 & \text{for corner columns} \end{cases} \qquad \text{(CP 6.1.5.6(b), 6.1.5.6(c))}$$

11.8.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

11.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (CP 6.1.5.7(e)). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* as described in the earlier sections remains unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

11.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

11.8.3.2 Determine Required Shear Reinforcement

The shear stress is limited to a maximum limit of

$$v_{max} = \min(0.8\sqrt{f_{cu}},\ 7\text{ MPa}) \quad \text{(CP 6.1.2.5(a))}$$

$$v_r = \begin{cases} 0.4 f_{cu} & \text{if} \quad f_{cu} \leq 40\ \text{N}/\text{mm}^2 \\ 0.4\left(f_{cu}/40\right)^{2/3} & \text{if } 40 < f_{cu} \leq 80\ \text{N}/\text{mm}^2 \\ 0.4\left(80/40\right)^{2/3} & \text{if} \quad f_{cu} > 80\ \text{N}/\text{mm}^2 \end{cases} \quad \text{(CP 12.3.8.7)}$$

$$f_{cu} \leq 80\ \text{N}/\text{mm}^2 \text{ (for calculation purpose only)} \quad \text{(CP 6.1.2.5(c))}$$

- If $v \leq v_c + v_r$, minimum reinforcement is required:

$$\frac{A_s}{s_v} = \frac{v_r b}{0.87 f_{yv}}, \quad \text{(CP 12.3.8.7)}$$

- If $v > v_c + v_r$,

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c)b}{0.87 f_{yv}} \quad \text{(CP 12.3.8.8)}$$

- If $v > v_{max}$, a failure condition is declared.

If v exceeds the maximum permitted value of v_{max}, the concrete section should be increased in size.

11.8.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 11-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner columns.

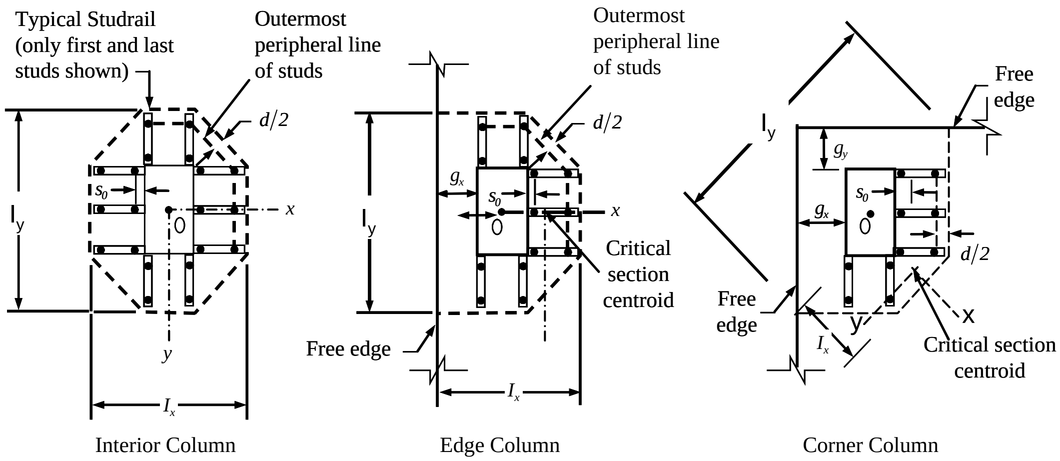


Figure 11-5 Typical arrangement of shear studs and critical sections outside the shear-reinforced zone

The distance between column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line of shear reinforcement shall not exceed $0.75d$ measured in a direction parallel to the column face (CP12.3.8.10). When $V > 1.8V_c$, the maximum spacing is reduced to $0.5\,d$.

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8 for corner, edge, and interior columns respectively.

11.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CP 4.2.4 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

The following information is taken from the BS 8110-1997 code. When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5*d*. The spacing between adjacent shear studs, *g*, at the first peripheral line of studs shall not exceed 1.5*d*. The limit of s_o and the spacing, *s*, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(CP 6.1.5.7(f))}$$

$$s \leq 0.75d \qquad \text{(CP 6.1.5.7(f))}$$

$$g \leq 1.5d \qquad \text{(CP 6.1.5.7(f))}$$

Stirrups are permitted only when slab thickness is greater than 200 mm (CP 6.1.5.7(e)).

Chapter 12
Design for IS 1343-1980

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the Indian code IS: 456-2000. When the aforementioned code is selected in ETABS, program design meets the requirement of the Indian code IS: 1343-1980 [IS: 1980]. Various notations used in this chapter are listed in Table 12-1. For referencing to the pertinent sections of the IS:1343 Code of Practice for Prestressed Concrete in this chapter, a prefix "IS" followed by the section number is used. Additionally, the latter portion of the chapter references Indian Code IS:456-2000, Plain and Reinforced Concrete-Code of Practice. For referencing the pertinent sections of that code, a prefix "IS:456 " followed by the section number is used to distinguish it from IS:1343.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

12.1 Notations

The following table identifies the various notations used in this chapter.

Table 12-1 List of Symbols Used in the IS 1343-1980 Code

A_{cv}	Area of section for shear resistance, mm^2
A_g	Gross area of cross-section, mm^2
A_s	Area of tension reinforcement, mm^2
A_{ps}	Area of prestress steel, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sv}	Total cross-sectional area of links at the neutral axis, mm^2
A_{sv} / s_v	Area of shear reinforcement per unit length of the member, mm^2/mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
d or d_e	Effective depth of tension reinforcement, mm
d_v	Effective depth of beam in shear, mm
d'	Depth to center of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f_{ci}	Concrete strength at transfer, MPa
f_{ck}	Characteristic cube strength at 28 days, MPa

Table 12-1 List of Symbols Used in the IS 1343-1980 Code

f_p	Characteristic strength of a prestressing tendon, MPa
f_{pe}	Maximum prestress in tendon after losses, MPa
f_{pi}	Maximum initial prestress in tendon, MPa
$f_s^{'}$	Compressive stress in a beam compression steel, MPa
f_y	Characteristic strength of non-prestressed reinforcement, MPa
f_{yv}	Characteristic strength of non-prestressed shear stirrup reinforcement, MPa (< 500 MPa)
D	Overall depth of a section in the plane of bending, mm
D_f	Flange thickness, mm
M	Design moment at a section, MPa
M_u	Design moment resistance of a section, MPa
M_u^0	Design moment resistance of a section with tendons only, N-mm
M_u^{bal}	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
s_v	Spacing of the shear stirrups along the length of the beam, mm
s	Spacing of shear rails, mm
T	Tension force, N
V	Design shear force at ultimate design load, N
u	Perimeter of the punching critical section, mm
v	Design shear stress at a beam cross-section or at a punch critical section, MPa

Table 12-1 List of Symbols Used in the IS 1343-1980 Code

v_c	Design ultimate shear stress resistance of a concrete beam, MPa
v_{co}	Ultimate shear stress resistance of an uncracked concrete section, MPa
v_{cr}	Ultimate shear stress resistance of a cracked concrete section, MPa
v_{max}	Maximum permitted design factored shear stress at a beam section or at the punch critical section, MPa
x_u	Neutral axis depth, mm
a_{max}	Depth of neutral axis in a balanced section, mm
z	Lever arm, mm
β_b	Moment redistribution factor in a member
γ_f	Partial safety factor for load
γ_m	Partial safety factor for material strength
ε_c	Maximum concrete strain, 0.0035
ε_{sp}	Strain in prestressing steel
ε_s	Strain in tension steel
ε_s'	Strain in compression steel

12.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For IS 1343-1980, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the load combinations in the following sections may need to be defined (IS 20.4.2 and IS Table 5).

For post-tensioned concrete design, the user can specify prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning are defined in the following subsections.

12.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress force in accordance with IS 19.3 and IS 20.4. The prestressing forces are considered without any long-term losses for the initial service load combination check.

1.0D + 1.0PT

12.2.2 Service Load Combination

The following load combinations are used for checking the requirements of prestress for serviceability in accordance with IS 19.3. It is assumed that all long-term losses have occurred already at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT

12.2.3 Ultimate Limit State Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with IS 20.4.2, Table 5.

The strength design combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are determined automatically by ETABS by subtracting out the primary PT moments when the flexural design is carried out.

1.5D + 1.0PT*

1.5D + 1.5L + 1.0PT*

1.5D + 1.5(0.75 PL) + 1.0PT*

1.5D ± 1.5W +1.0PT*
0.9D ± 1.5W + 1.0PT*
1.2D + 1.2L ± 1.2W + 1.0

1.5D ± 1.5E + 1.0PT*
0.9D ± 1.5E + 1.0PT*
1.2D + 1.2L ± 1.2E + 1.0

* — Replace PT by H for flexural design only

Other appropriate loading combinations should be used if roof live load is treated separately, or if other types of loads are present.

12.3 Limits on Material Strength

Grade M30 and M40 are the minimum recommended for post-tensioning and pre-tensioning respectively. In both cases the concrete strength at transfer should not be less than that specified in IS 15.1.

The specified characteristic strength of nonprestressed reinforcement is given as follows:

▪ Hot rolled mild steel	250 MPa	(IS:432)
▪ Hot rolled deformed steel bars	415 MPa	(IS:1139)
▪ High yield strength deformed steel	415 MPa	(IS:1786)

The specified characteristic strength of prestressing steel should conform to IS 1785 for plain cold drawn wires and to IS 2090 or IS 6006 for high tensile steel bars and strands respectively.

ETABS also checks the following tensile strength in prestressing steel (IS 18.5.1). The permissible tensile stresses in all types of prestressing steel, in terms of the specified minimum tensile strength f_{pu}, are summarized as follows:

a. Initial prestress behind anchorages at transfer: Not more than 0.80 f_{pu}

b. Residual prestress in tendon after losses: Not less than 0.45f_{pu}

12.4 Partial Safety Factors

The design strength for concrete and reinforcement is obtained by dividing the characteristic strength of the material by a partial factor of safety, γ_m. The values of γ_m used in the program and listed in the following table are taken from IS 20.4.1.1:

Values of γ_m for the Ultimate Limit State	
Nonprestressed reinforcement, γ_{ms}	1.15
Prestressing steel, γ_{mp}	1.15
Concrete in flexure and axial load, γ_{mc}	1.50

These factors are already incorporated in the design equations and tables in the code. The user is allowed to overwrite these values. However, caution is advised.

The design load combinations are obtained by multiplying the characteristic loads by the appropriate partial factor of safety, γ_f (IS 20.4.2).

12.5 Design Requirements of Prestressed Concrete Structures

The structural design is based on “Limit State Concepts” (IS 19.1). The limit state of collapse and limit states of serviceability are considered in design (IS 19.1.1), (IS 19.2) and (IS 19.3).

12.5.1 Limit State of Collapse

The resistance of prestressed structural elements to flexure and shear at every section will be designed such that the resistance is always greater than the demand imposed by all types of loads, with the most unfavorable combination using appropriate partial safety factors (IS 19.2). This will include the design for the following limit states:

- Limit state of collapse in flexure
- Limit state of collapse in shear

Limit State of Collapse: Flexure

Limit state of collapse of prestressed members for flexure is based on the following assumptions given in IS 1343 clause 22.1.

- The strain distribution in the concrete in compression is derived from the assumption that plane sections normal to the axis remain plane after bending (IS 22.1.1(a)).
- The design stresses in the concrete in compression are taken as $0.67 f_{cu}$. Maximum strain at the extreme concrete compression fiber shall be assumed equal to 0.0035 (IS 22.1.1(b)) and (IS 22.1.1(c)).
- Tensile strength of the concrete is ignored (IS 22.1.1(d)).
- The stresses in bonded post-tensioning tendons and in any additional reinforcement (compression or tension) are derived from the representative stress-strain curve for the type of steel used given by the manufacturer or typical curves given in Figure 5 of IS:1343 or in Figure 23 of IS:456 for non-prestressed reinforcement. For design purposes, the partial safety factor shall be applied as indicated earlier in this chapter.
- The stress in unbonded post-tensioning tendons will be obtained from a rigorous analysis (IS 22.1.1(f)).

12.5.2 Limit State of Serviceability

The check for limit state of serviceability involves checking for deflection, cracking, and maximum compressive stresses at prestress transfer and under service conditions. For checking limit states of serviceability, prestressed structural elements are classified into the following groups:

- ***Class 1***: No flexural tensile stresses permitted (IS 19.3.2 (a))
- ***Class 2***: Flexural tensile stresses are allowed, but no visible cracking (IS 19.3.2 (b))
- ***Class 3***: Flexural tensile stresses are allowed, but the surface width of cracks should be limited to 0.1 mm for members in aggressive environments and 0.2 mm for other members

By default all prestressed concrete elements are assumed to be of type ***Class-1.*** This setting can be overwritten in the Design Preferences.

ETABS checks the following limit states of serviceability (IS 19.3.1):

12.6 Maximum Compression Check

ETABS checks the maximum compressive stress in flexure as defined in the subsections that follow.

12.6.1 Maximum Compressive Stress at Transfer

Maximum compressive stress due to bending and direct force at the time of transfer of prestress shall not be more than $0.54f_{ci}$ for concrete grade M30 and $0.37f_{ci}$ for concrete grade M60, where f_{ci} is the cube strength of the concrete at transfer, which will not be less than $0.5f_{ck}$. For all other grades of concrete, the limiting stress can be linearly interpolated between these values.

12.6.2 Maximum Compressive Stress Under Service Conditions

Maximum compressive stress due to bending under service conditions for applied prestress and service loads after deduction of the full losses in prestress will be limited to the following values.

12.6.2.1 Case I

When compressive stress is not likely to increase during service, maximum compressive stress shall not be more than $0.41f_{ck}$ for concrete grade M30 and $0.35f_{ck}$ for concrete grade M60, where f_{ck} is the cube strength of the concrete at 28 days. For all other grades of concrete, the limiting stress can be linearly interpolated between these values.

12.6.2.2 Case II

When compressive stress is likely to increase during service, maximum compressive stress shall not be more than $0.34f_{ck}$ for concrete grade M30 and $0.27f_{ck}$ for concrete grade M60, where f_{ck} is the cube strength of the concrete at 28 days. For all other grades of concrete, the limiting stress can be linearly interpolated between these values.

12.7 Beam Design (for Reference Only)

***Important Note:** Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure and shear based on the beam moments, shear forces, load combination factors, and other criteria described in the following subsections. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure and shear only. Effects of minor direction bending that may exist in the beams must be investigated independently by the user.

12.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine design bending moments, shears, and torsion
- Determine required flexural reinforcement

12.7.1.1 Effects of Torsion

IS 1343 clause 22.5.1 states that wherever torsional stiffness of the member is taken into account in the analysis of the structure, the member will be designed for torsion. However, if torsional stiffness of the member has been ignored in the analysis, no specific calculations for torsion will be required. It should be noted that the program will not automatically ignore or redistribute torsion. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

12.7.1.2 Determine Factored Moments, Shears, and Torsional Moments

In the design of flexural reinforcement of post-tensioned concrete beams, the factored moments, shears, and torsional moments for each load combination at a particular beam station are obtained by factoring the corresponding design forces for different load cases with the corresponding load factors.

IS 1343 uses a simplified approach based on skew bending theory, for design of post-tensioned concrete members subjected to bending moment, shear, and torsion. In this method, torsion and bending moment are combined as an equivalent bending moment and the beam is designed for the equivalent moment. Positive equivalent moments produce bottom reinforcement. In such cases, the beam may be designed as a rectangular or a flanged beam. Negative equivalent moments produce top reinforcement. In such cases, the beam may be designed as a rectangular or an inverted flanged beam. Torsion and shear are considered together as detailed in subsequent sections.

12.7.1.2.1 Determine Design Moments when Torsion is Excluded

In the design of flexural reinforcement of post-tensioned concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments produce bottom steel. In such cases, the beam may be designed as a rectangular or a flanged beam. Negative beam moments produce top steel. In such cases, the beam may be designed as a rectangular or an inverted flanged beam.

12.7.1.2.2 Determine Design Moments when Torsion is Included

In the design of flexural reinforcement of post-tensioned concrete beams, the factored moments and torsion for each load combination at a particular beam section are obtained by factoring the corresponding moments and torsion for different load cases with the corresponding load factors.

The equivalent moment at a particular design station is computed as described in the following text. The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all the of the load combinations. Positive beam moments produce bottom steel. In such cases, the beam may be designed as a rectangular or a flanged beam. Negative beam moments produce top steel. In such cases, the beam may be designed as a rectangular or an inverted flanged beam.

The equivalent moment is calculated from the following equation:

$M_{e1} = M_u + M_t$, where (IS 22.5.3.1)

$$M_t = T_u \sqrt{1 + \frac{2D}{b}}$$

and D and b are the overall depth and width of the beam, respectively.

If M_t exceeds M_u, additional reinforcement will be computed for the moment M_{e2} applied in the opposite sense of M_u. Effectively, this will result in additional longitudinal reinforcement on the compression face of the beam due to reversal of the moment sign. The additional moment M_{e2} is computed using the following equation:

$M_{e2} = M_t - M_u$ (IS 22.5.3.2)

In addition to the preceding equation, when M_t exceeds M_u the beam shall be designed to withstand an equivalent transverse moment M_{e3} not acting simultaneously with M_{e1}, given by the following equation:

$$M_{e3} = M_t \left(1 + \frac{x_1}{2e}\right)^2 \left(\frac{1 + \frac{2b}{D}}{1 + \frac{2D}{b}}\right)$$ (IS 22.5.3.3)

where x_1 is the smaller dimension of the closed hoop used as a torsional shear reinforcement and $e = T / V$.

After the design moments have been worked out, the design proceeds with equivalent moments for their respective axes of application, as previously outlined.

12.7.1.3 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the parabolic stress block, as shown in Figure 12-1. (IS 22.1.1(c))

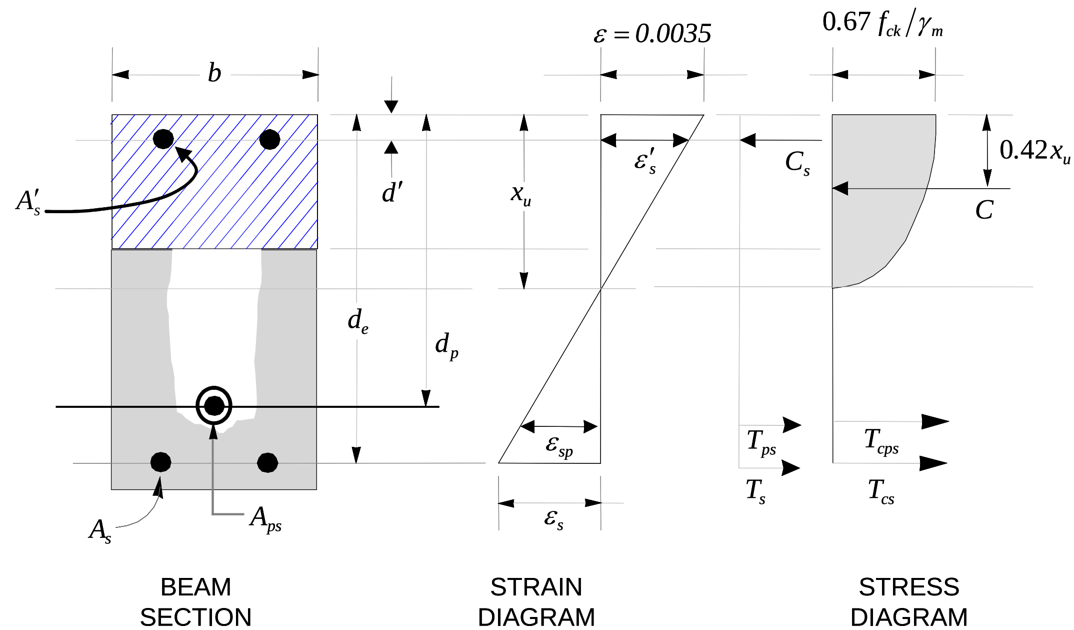


Figure 12-1 Rectangular Beam Design

The design procedure used by ETABS for both rectangular and flanged sections (L- and T-beams) is summarized in the subsections that follow. All beams are designed for major direction flexure and shear only.

12.7.1.3.1 Design of Rectangular Beams

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, the moment capacity M_u^0 is determined as follows:

The maximum depth of the compression zone, $x_{u,\max}$, is calculated based on the limitation that the tension reinforcement strain shall not be less than $\varepsilon_{\min}$.

$$x_{u,\max} = \left(\frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}} \right) d$$

where,

$\varepsilon_{c\max} = 0.0035$

Therefore, the limit $x_u \leq x_{u,\max}$ is set for tension-controlled sections. The ductility of a section is controlled by limiting the x_u/d ratio (IS 21.1.1(d)):

$x_u/d \leq 0.5$ (IS 21.1.1(d))

ETABS determines the depth of the neutral axis, x_u, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pb} is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel. Based on the calculated f_{pb}, the depth of the neutral axis is recalculated, and f_{pb} is further updated. After this iteration process has converged, the depth of compression block, x_u is determined:

- If $x_u \leq x_{u,\max}$ (IS 21.1.1(d)), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_u^0 = A_{ps} f_{pb} \left(d_p - 0.42 x_u \right)$$

- If $x_u > x_{u,\max}$ (IS 21.1.1(d)), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension-controlled case. In that case, it is assumed that the depth of the neutral axis x_u is equal to $x_{u,\max}$. The stress in the post-tensioning steel, f_{pb}, is then calculated based on strain compatibility, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = 0.36 f_{ck} x_u b$$

$$T = A_{ps} f_{pb}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{0.36 f_{ck} x_u b - A_{ps} f_{pb}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_u^{bal} = A_{ps} f_{pb}^{bal} \left(d_p - 0.42 x_u\right) + A_s^{bal} f_s^{bal} \left(d_s - 0.42 x_u\right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. This case does not involve any iteration in determining the depth of neutral axis, x_u.

12.7.1.3.1.1 Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment M_u. Minimum reinforcement is provided to satisfy the flexural cracking requirements (IS 18.6.3.3(a)).

12.7.1.3.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $x_u < x_{u,\max}$.

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports this required area of tension reinforcement. Since M is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

12.7.1.3.1.3 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required. In that case, ETABS assumes that the depth of the neutral axis, x_u, is equal to $x_{u,\max}$. The values of f_{pb} and f_s reach their respective balanced condition values, f_{pb}^{bal} and f_s^{bal}. Then the area of compression reinforcement, A_s', is determined as follows:

- The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(0.87 f'_s - f'_c\right)\left(d_s - d'\right)}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c\max} \left[\frac{x_{u,\max} - d'}{x_{u,\max}}\right] \leq 0.87 f_y$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{0.87 f_y \left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M is positive, and vice versa if M is negative.

12.7.1.3.2 Design of Flanged Beams

12.7.1.3.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

12.7.1.3.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, the moment capacity M_u^0 is determined as follows:

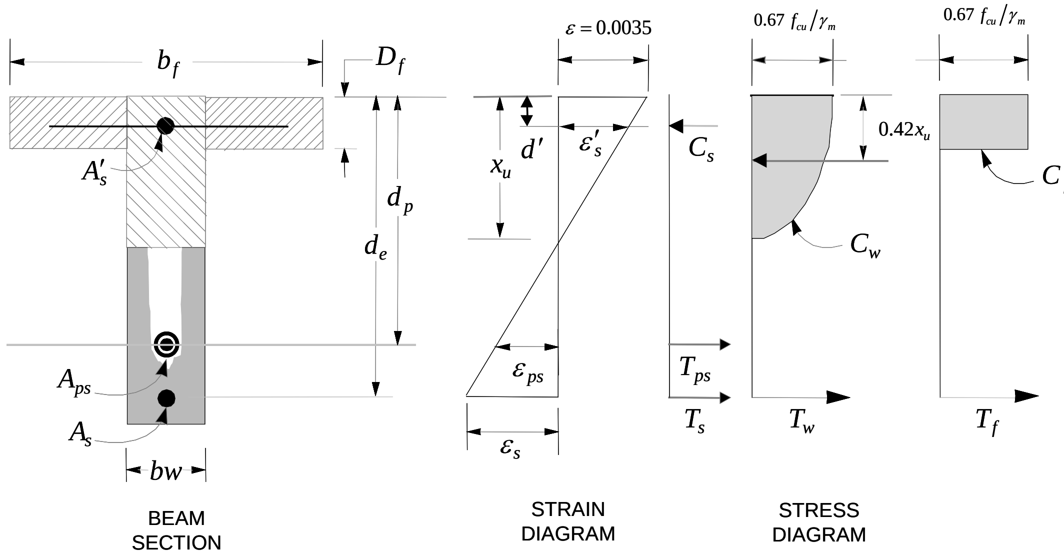


Figure 12-2 T-Beam Design

The maximum depth of the compression zone, $x_{u,\max}$, is calculated based on the limitation that the tension reinforcement strain shall not be less than $\varepsilon_{\text{smin}}$.

$$x_{u,\max} = \left(\frac{\varepsilon_{\text{cmax}}}{\varepsilon_{\text{cmax}} + \varepsilon_{\text{smin}}}\right) d_p$$

where,

$$\varepsilon_{\text{cmax}} = 0.0035$$

Therefore, the limit $x_u \leq x_{u,\max}$ is set for tension-controlled sections. The ductility of a section is controlled by limiting the x_u/d ratio (IS 21.1.1(d)):

$$x_u/d \leq 0.5 \qquad \text{(IS 21.1.1(d))}$$

ETABS determines the depth of the neutral axis, x_u, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pb}, is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps}, in the post-tensioning steel. Based on the calculated f_{pb}, the depth of the neutral axis is recalculated, and f_{pb} is further updated. After the iteration process has converged, the depth of the compression block, x_u, is determined as follows:

- If $x_u \le x_{u,\max}$ (IS 21.1.1(d)), the moment capacity of the section provided by post-tensioning steel only is computed as:

$$M_u^0 = A_{ps} f_{pb}\left(d_p - 0.42 x_u\right)$$

- If $x_u > x_{u,\max}$ (IS 21.1.1(d)), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension-controlled case. In that case, it is assumed that the depth of neutral axis x_u is equal to $x_{u,\max}$. The stress in the post-tensioning steel, f_{pb}, is then calculated based on strain compatibility and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $x_u \le D_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case, the width of the beam is taken as b_f. Compression reinforcement is required when $x_u > x_{u,\max}$.

- If $x_u > D_f$, the calculation for A_s is given by:

$$C = 0.36 f_{ck} x_u A_c^{com}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$A_c^{com} = b_f D_f + b_w\left(x_{u,\max} - D_f\right)$$

$$T = A_{ps} f_{pb}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{0.36 f_{ck} x_u A_c^{com} - A_{ps} f_{pb}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_u^{bal} = A_{ps} f_{pb}^{bal}\left(d_p - 0.42 x_u\right) + A_s^{bal} f_s^{bal}\left(d_s - 0.42 x_u\right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcing steel, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. This case does not involve any iteration in determining the depth of the neutral axis, x_u.

12.7.1.3.2.2.1 Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment *M*. Minimum reinforcement is provided to satisfy the flexural cracking requirements (IS 18.6.3.3(a)).

12.7.1.3.2.2.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist *M*, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $x_u < x_{u,\max}$.

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment *M* and reports this required area of tension reinforcement. Since M is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will fall within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if *M* is positive, or at the top if *M* is negative.

12.7.1.3.2.2.3 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required. In that case, ETABS assumes that the depth of the neutral axis, x_u, is equal to $x_{u,\max}$. The values of f_{pb} and f_s reach their respective balanced condition values, f_{pb}^{bal} and f_s^{bal}. Then the area of compression reinforcement, A_s', is determined as follows:

- The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(0.87 f'_s - f'_c\right)\left(d_s - d'\right)}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c\max} \left[\frac{x_{u,\max} - d'}{x_{u,\max}}\right] \le 0.87 f_y$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{0.87 f_y \left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M is positive, and vice versa if M is negative.

12.7.1.4 Minimum and Maximum Reinforcement

Reinforcement in post-tensioned concrete beams is computed to increase the flexural strength of sections or to comply with the shear link requirements. The minimum flexural tension reinforcement required for a beam section to comply with the cracking requirements must be separately investigated by the user.

For bonded tendons, there are no minimum untensioned reinforcement requirements.

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (IS:456 26.5.1.1, 26.5.1.2).

12.7.2 Design Beam Shear Reinforcement (Torsion Excluded)

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear, the following steps are involved (IS 22.4):

- Determine the factored shear force, V.
- Determine the shear force, v_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three subsections describe in detail the algorithms associated with these steps.

12.7.2.1 Determine Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces and moments for different load cases with the corresponding load combination factors.

12.7.2.2 Determine Concrete Shear Capacity

The design ultimate shear resistance of the concrete alone, V_{co}, should be considered at sections as follows:

- Uncracked sections in flexure ($M < M_o$)
- Cracked sections in flexural ($M \geq M_o$)

where,

M is the design bending moment at the section due to the load combination,

M_o is the moment necessary to produce zero stress in the concrete at the extreme tension fiber; in this calculation only 0.8 times the stress due to post-tensioning should be taken into account.

Case 1: Uncracked section in flexure

The ultimate shear resistance of the section, V_{co}, is computed as follows:

$$V_{co} = 0.67 b_w D \sqrt{\left(f_t^2 + 0.8 f_{cp} f_t \right)} \quad \text{(IS 22.4.1)}$$

where,

f_t is the maximum design principal stress, (IS 22.4.1)

$f_t = 0.24\sqrt{f_{ck}}$, and (IS 22.4.1)

f_{cp} = design compressive stress at the centoidal axis due to prestress, taken as positive. (IS 22.4.1)

$$V_c = V_{co} + P \sin \beta \quad \text{(IS 22.4.1)}$$

Case 2: Cracked section in flexure

The ultimate shear resistance of the section, V_{cr}, is computed as follows:

$$V_{cr} = \left(1 - 0.55 \frac{f_{pe}}{f_{pu}} \right) v_c b_v d + M_o \frac{V}{M} \quad \text{(IS 22.4.2)}$$

In the preceding, the shear stress capacity of the concrete, v_c is taken from IS Table 6 for a provided percentage of post-tensioning steel in the section.

$$V_{cr} \geq 0.1 b_w d \sqrt{f_{ck}} \quad \text{(IS 22.4.2)}$$

$$V_c = \min\left(V_{co}, V_{cr}\right) + P \sin \beta \quad \text{(IS 22.4.2)}$$

12.7.2.3 Determine Required Shear Reinforcement

For the section under consideration, compute the following:

$v = \dfrac{V}{A_{cv}}$, $A_{cv} = b_w d$, where

$v \leq v_{max}$, and (IS 22.4.4)

v_{max} will not exceed the values defined in IS Table 7

Given v, v_c and v_{max}, the required shear reinforcement is calculated as follows (IS 22.4.3.1):

- If $v \le 0.5v_c$ then no shear reinforcement is to be provided
- If $0.5\,v_c < v \le v_c$,

$$\frac{A_{sv}}{s_v} = \frac{0.4\,b_w}{0.87\,f_{yv}} \qquad \text{(IS 22.4.3.1)}$$

- If $v_c < v \le v_{max}$,

$$\frac{A_{sv}}{s_v} = \frac{\left(v - v_c\right) b_v}{0.87\,f_{yv}} \qquad \text{(IS 22.4.3.2)}$$

- If $v > v_{max}$, a failure condition is declared. (IS 22.4.4)

In the preceding expressions, a limit is imposed on f_{yv} as

$$f_{yv} \le 415 \text{ MPa} \qquad \text{(IS 40.4)}$$

The maximum of all of the calculated A_{sv} / s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

12.7.3 Design Beam Shear Reinforcement (Torsion Included)

The transverse reinforcement to resist shear and torsion is designed for each load combination at each station along the length of the beam. In designing the transverse reinforcement for a particular beam, for a particular load combination, at a particular station due to the beam major shear and torsion, the following steps are involved (IS 22.5.4):

- Determine the factored shear force, V, and factored torsion T.
- Determine the shear force, V_{c1}, that can be resisted by the concrete.

- Determine the torsional moment, T_{c1}, that is carried by concrete.
- Determine the shear reinforcement required to carry the balance shear and torsion.

The following three sections describe in detail the algorithms associated with these steps.

12.7.3.1 Determine Shear Force and Torsional Moment

In the design of the beam transverse reinforcement, the shear force and torsional moment for each load combination at a particular beam station are obtained by factoring the corresponding shear force and torsional moment for different load cases with the corresponding load combination factors.

12.7.3.2 Determine Torsional Moment Carried by Concrete

The torsional moment carried by the concrete is computed using the following equation:

$$T_{c1} = T_c\left(\frac{e}{e+e_c}\right), \text{ where} \qquad \text{(IS 22.5.4.1)}$$

$$T_c = \sum 1.5 b^2 D\left(1-\frac{b}{30}\right)\lambda_p\sqrt{f_{ck}}$$

$$e = T/V$$

$$e_c = T_c / V_c$$

$$\lambda_p = \sqrt{\left(1+\frac{12 f_{cp}}{f_{ck}}\right)}$$

$$V_c = \min\left(V_{co}, V_{cr}\right)$$

Please refer to the previous section for the relevant equations for V_{co} and V_{cr}.

12.7.3.3 Determine Shear Force Carried by Concrete

The shear force carried by the concrete is computed using the following equation:

$$V_{c1} = V_c\left(\frac{e}{e+e_c}\right),\ \text{where} \qquad \text{(IS 22.5.4.2)}$$

$V_c = \min\,(V_{co}, V_{cr})$

$e = T/V$

$e_c = T_c\,/V_c$

12.7.3.4 Determine Required Shear Reinforcement

For the section under consideration, compute the area of transverse reinforcement using the following two equations and provide the larger area:

$$\frac{A_{sv}}{s_v} = \frac{M_t}{1.5 b_1 d_1 f_{yv}} \Big/ 2 \qquad \text{(IS22.5.4.3)}$$

$$\frac{A_{sv}}{s_v} = \left(\frac{(V - V_c)}{0.87 f_{yv} d_1}\right) \Big/ 2 + \left(\frac{(T - T_{c1})}{0.87 b_1 d_1 f_{yv}}\right) \qquad \text{(IS22.5.4.3)}$$

In the preceding expressions, a limit is imposed on f_{yv} as

$f_{yv} \leq 415$ MPa. (IS 40.4)

The maximum of all of the calculated A_{sv}/s_v values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam transverse shear and torsional reinforcement requirements considered by the program are based purely on shear and torsional strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

12.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the limit state of collapse for prestressed concrete, as described in IS 1343. In general, provisions of IS:456 will also apply to the design of post-tensioned concrete slabs, as described in the following sections. To learn more about the design strips, refer to the section entitled "ETABS Design Techniques" in the *Key Features and Terminology* manual.

12.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of the element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the text that follows and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

12.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

12.8.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

12.8.1.3 Minimum and Maximum Slab Reinforcement

There are no minimum requirements for un-tensioned reinforcement in one-way bonded slabs.

In flat slabs, reinforcement is added at the top over supports to be 0.00075 times the gross cross-sectional area. This reinforcement extends 1.5 times the slab depth on each side of the column. The length of reinforcement is at least 0.2*L* where *L* is the span of the slab.

There are no minimum requirements for span zones. However, additional untensioned reinforcement shall be designed for the full tension force generated by assumed flexural tensile stresses in the concrete for the following situations (Concrete Society, Technical report 43):

- all locations in one-way spanning floors using unbonded tendons
- all locations in one-way spanning floors where transfer tensile stress exceeds $0.36\sqrt{f_{ci}}$
- support zones in all flat slabs
- span zones in flat slabs using unbonded tendons where the tensile stress exceeds $0.15\sqrt{f_{cu}}$.

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (IS 26.5.1.1).

12.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code specific items are described in the following sections.

12.8.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (IS:456 30.6.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (IS:456 30.6.1). Figure 12-3 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

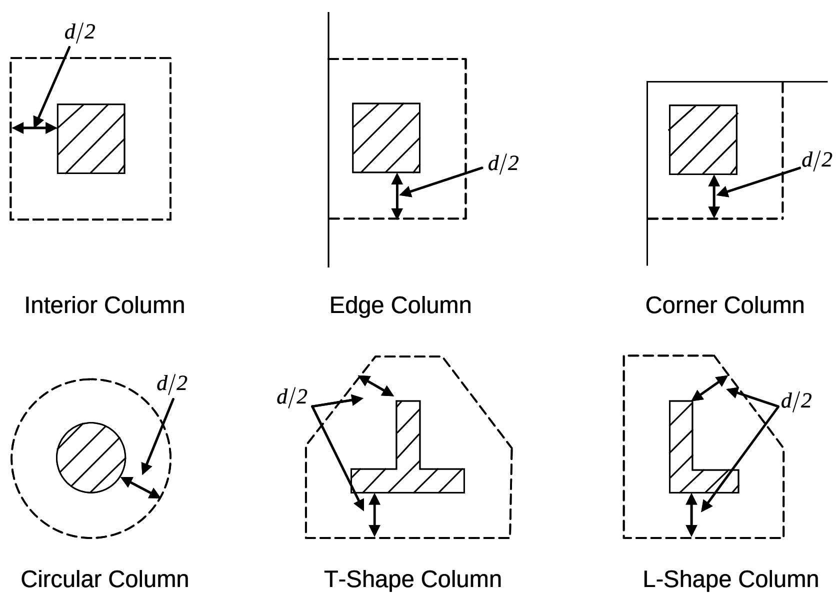


Figure 12-3 Punching Shear Perimeters

12.8.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be αM_u and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $(1 - \alpha)\ M_u$ (IS:456 30.6.2.2), where

$$\alpha = \frac{1}{1+(2/3)\sqrt{a_1/a_2}} \qquad \text{(IS:456 30.3.3)}$$

a_1 is the width of the critical section measured in the direction of the span

a_2 is the width of the critical section measured in the direction perpendicular to the span.

12.8.2.3 Determine Concrete Shear Capacity

The concrete punching shear factored strength is taken as:

$v_c = k_s\ \tau_c$, where (IS:456 30.6.3.1)

$k_s = 0.5 + \beta c \leq 1.0$, (IS:456 30.6.3.1)

$\tau_c = 0.25\ \sqrt{f_{ck}}$, and (IS:456 30.6.3.1)

β_c = ratio of the minimum to the maximum dimensions of the support section

12.8.2.4 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS.

12.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

12.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined, but limited to:

$$v_c \leq 0.5\tau_c \quad \text{(IS 31.6.3.2)}$$

12.8.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{\max} = 1.5\,\tau_c\, b_o d \quad \text{(IS 31.6.3.2)}$$

Given V_u, V_c, and $V_{\max}$, the required shear reinforcement is calculated as follows (IS:456 31.6.3.2).

$$A_v = \frac{\left(V_u - 0.5V_c\right)}{0.87\,f_y} \quad \text{(IS 31.6.3.2, 40.4)}$$

- If $V_u > V_{\max}$, a failure condition is declared. (IS 31.6.3.2)
- If V_u exceeds the maximum permitted value of $V_{\max}$, the concrete section should be increased in size.

12.8.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 12-4 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

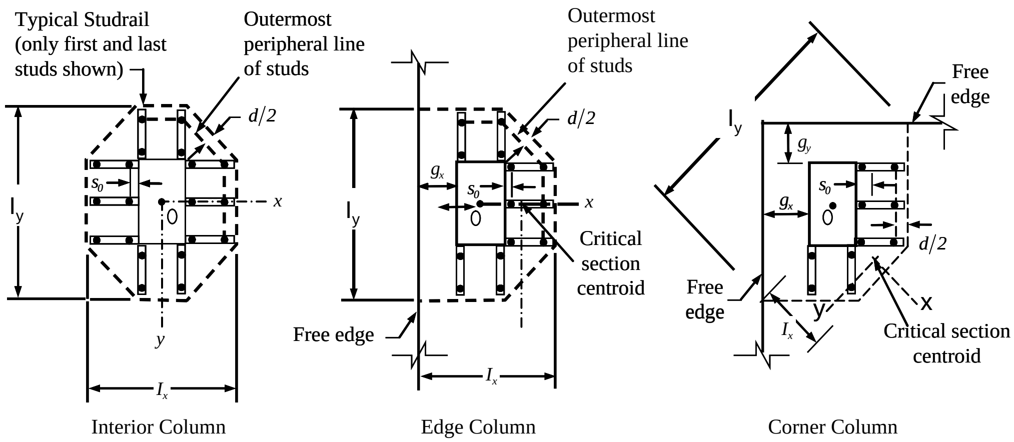


Figure 12-4 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed $2d$ measured in a direction parallel to the column face.

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

12.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in IS:456 26.4 plus half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than $0.5d$. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed $2d$. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.5d$$

$$s \leq 0.5d$$

$$g \leq 2d$$

Chapter 13
Design for NZS 3101:06

This chapter describes in detail the various aspects of the concrete design procedure that is used by ETABS when the New Zealand code NZS 3101:06 [NZS 06] is selected. Various notations used in this chapter are listed in Table 13-1. For referencing to the pertinent sections of the New Zealand code in this chapter, a prefix “NZS” followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

13.1 Notations

The following table identifies the various notations used in this chapter.

Table 13-1 List of Symbols Used in the NZS 3101:06 Code

A_{co}	Area enclosed by perimeter of the section, sq-mm
A_{cv}	Area of concrete used to determine shear stress, sq-mm

Table 13-1 List of Symbols Used in the NZS 3101:06 Code

A_g	Gross area of concrete, sq-mm
A_l	Area of longitudinal reinforcement for torsion, sq-mm
A_{ps}	Area of prestressing steel in flexural tension zone, sq-mm
A_o	Gross area enclosed by shear flow path, sq-mm
A_s	Area of tension reinforcement, sq-mm
A'_s	Area of compression reinforcement, sq-mm
$A_{s(\text{required})}$	Area of steel required for tension reinforcement, sq-mm
A_t /s	Area of closed shear reinforcement per unit length for torsion, sq-mm/mm
A_v	Area of shear reinforcement, sq-mm
A_v /s	Area of shear reinforcement per unit length, sq-mm/mm
a	Depth of compression block, mm
a_b	Depth of compression block at balanced condition, mm
$a_{\max}$	Maximum allowed depth of compression block, mm
b	Width of member, mm
b_f	Effective width of flange (flanged section), mm
b_w	Width of web (flanged section), mm
b_0	Perimeter of the punching critical section, mm
b_1	Width of the punching critical section in the direction of bending, mm
b_2	Width of the punching critical section perpendicular to the direction of bending, mm
c	Distance from extreme compression fiber to the neutral axis, mm
c_b	Distance from extreme compression fiber to neutral axis at balanced condition, mm
d	Distance from extreme compression fiber to tension reinforcement, mm
d'	Distance from extreme compression fiber to compression reinforcement, mm

Table 13-1 List of Symbols Used in the NZS 3101:06 Code

E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f'_c	Specified compressive strength of concrete, MPa
f'_{ci}	Specified compressive strength of concrete at time of initial prestress, MPa
f_{pe}	Compressive stress in concrete due to effective prestress forces only (after allowance of all prestress losses), MPa
f_{ps}	Stress in prestressing steel at nominal flexural strength, MPa
f_{pu}	Specified tensile strength of prestressing steel, MPa
f_{py}	Specified yield strength of prestressing steel, MPa
f_t	Extreme fiber stress in tension in the precompressed tensile zone using gross section properties, MPa
f'_s	Stress in the compression reinforcement, psi
f_y	Specified yield strength of flexural reinforcement, MPa
f_{yt}	Specified yield strength of shear reinforcement, MPa
h	Overall depth of sections, mm
h_f	Thickness of slab or flange, mm
k_a	Factor accounting for influence of aggregate size on shear strength
k_d	Factor accounting for influence of member depth on shear strength
M_u^0	Design moment resistance of a section with tendons only, N-mm
M_u^{bal}	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
M^*	Factored design moment at a section, N-mm
p_c	Outside perimeter of concrete section, mm
p_o	Perimeter of area A_o, mm

Table 13-1 List of Symbols Used in the NZS 3101:06 Code

s	Spacing of shear reinforcement along the length, mm
T^*	Factored design torsion at a section, N-mm
t_c	Assumed wall thickness of an equivalent tube for the gross section, mm
t_o	Assumed wall thickness of an equivalent tube for the area enclosed by the shear flow path, mm
V_c	Shear force resisted by concrete, N
V^*	Factored shear force at a section, N
v^*	Average design shear stress at a section, MPa
v_c	Design shear stress resisted by concrete, MPa
$v_{\max}$	Maximum design shear stress permitted at a section, MPa
v_{tn}	Shear stress due to torsion, MPa
α_s	Punching shear factor accounting for column location
α_1	Concrete strength factor to account for sustained loading and equivalent stress block
β_1	Factor for obtaining depth of compression block in concrete
β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
$\varepsilon_{c,\max}$	Maximum usable compression strain allowed in the extreme concrete fiber, (0.003 in/in)
ε_s	Strain in reinforcement
ϕ_b	Strength reduction factor for bending
ϕ_s	Strength reduction factor for shear and torsion
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear

13.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For NZS 3101:06, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the load combinations in the following subsections may need to be considered (AS/NZS 1170.0, 4.2.2). For post-tensioned concrete design, the user can specify the prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning also are defined in the subsections that follow.

13.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress forces in accordance with NZS 3101:06 clause 19.3.3.5.1(b). The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0D + 1.0PT

13.2.2 Service Load Combination

The following load combinations are used for checking the requirements of prestress for serviceability in accordance with NZS 19.3.3.5.1. It is assumed that all long-term losses have occurred already at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT

13.2.3 Long-Term Service Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with NZS 19.3.3.5.1. The permanent load for this load combination is taken as 50 percent of the live load (taken from ACI 318-08 clause 18.4.2(a)). It is assumed that all long-term losses have occurred already at the service stage.

1.0D + 1.0PT
1.0D + 0.5L + 1.0 PT

13.2.4 Ultimate Limit State Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with AS/NZS 1170.0, 4.2.2.

The design combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are determined automatically by ETABS by subtracting out the primary PT moments when the flexural design is carried out.

1.35D + 1.0PT*	(AS/NZS 1170.0, 4.2.2(a))
1.2D + 1.5L + 1.0PT*	(AS/NZS 1170.0, 4.2.2(b))
1.2D + 1.5(0.75 PL) + 1.0PT*	(AS/NZS 1170.0, 4.2.2(b))
1.2D + 0.4L + 1.0S + 1.0PT*	(AS/NZS 1170.0, 4.2.2(g))
1.2D ± 1.0W + 1.0PT*	(AS/NZS 1170.0, 4.2.2(d))
0.9D ± 1.0W + 1.0PT*	(AS/NZS 1170.0, 4.2.2(e))
1.2D + 0.4L ± 1.0W + 1.0PT*	(AS/NZS 1170.0, 4.2.2(d))
1.0D ± 1.0E + 1.0PT*	(AS/NZS 1170.0, 4.2.2(f))
1.0D + 0.4L ± 1.0E + 1.0PT*	(AS/NZS 1170.0, 4.2.2(f))

* — Replace PT by H for flexural design only

Note that the 0.4 factor on the live load in three of the combinations is not valid for live load representing storage areas. These also are the default design load combinations in ETABS whenever the NZS 3101 code is used. If roof live load is treated separately or if other types of loads are present, other appropriate load combinations should be used.

13.3 Limits on Material Strength

The upper and lower limits of f'_c shall be as follows:

$$25 \leq f'_c \leq 100 \, \text{MPa} \quad \text{(NZS 5.2.1)}$$

The lower characteristic yield strength of longitudinal reinforcement, f_y, should be equal to or less than 500 MPa (NZS 5.3.3). The lower characteristic yield strength of transverse (stirrup) reinforcement, f_{yt}, should not be greater than 500 MPa for shear or 800 MPa for confinement (NZS 5.3.3).

The code allows use of f'_c and f_y beyond the given limits, provided special study is conducted (NZS 5.2.1).

ETABS enforces the upper material strength limits for flexure and shear design of slabs. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

ETABS also checks the tensile strength in the prestressing steel (NZS 19.3.3.6). The permissible tensile stresses in all types of prestressing steel, in terms of the specified minimum tensile strength f_{py}, are summarized as follows:

a. Due to tendon jacking force: $0.94 \, f_{py}$

b. Immediately after prestress transfer: $0.82 \, f_{py}$

In any circumstances, the initial prestressing forces shall not exceed $0.8 \, f_{pu}$.

13.4 Strength Reduction Factors

The strength reduction factors, ϕ, are applied to the specified strength to obtain the design strength provided by a member. The ϕ factors for flexure, shear, and torsion are as follows:

$\phi_b = 0.85$ for flexure (NZS 2.3.2.2)

$\phi_s = 0.75$ for shear and torsion (NZS 2.3.2.2)

These values can be overwritten; however, caution is advised.

13.5 Design Assumptions for Prestressed Concrete Structures

The utimate limit state of prestressed members for flexure and axial loads shall be based on assumptions given in NZS 7.4.2.

- The strain distribution in the concrete in compression is derived from the assumption that a plane section remains plane (NZS 7.4.2.2).
- The design stress in the concrete in compression is taken as 0.45 f_{cu}. The maximum strain at the extreme concrete compression fiber shall be assumed equal to 0.003 (N ZS 7.4.2.3).
- Tensile strength of the concrete is ignored (NZS 7.4.2.5).
- The strain in bonded prestressing tendons and in any additional reinforcement (compression or tension) is derived from the assumption that a plane section remains plane (NZS 7.4.2.2).

The serviceability limit state of prestressed members uses the following assumptions given in NZS 19.3.3.

- Plane sections remain plane, i.e., strain varies linearly with depth through the entire load range (NZS 19.3.3.2).
- Elastic behavior exists by limiting the concrete stresses to the values given in NZS 19.3.3.5.1.
- In general, it is only necessary to calculate design stresses due to the load arrangements immediately after the transfer of prestress and after all losses or prestress have occurred; in both cases the effects of dead and imposed loads on the strain and force in the tendons may be ignored.

Prestressed concrete members are investigated at the following three stages (NZS 19.3.3.5.2):

- At transfer of prestress force
- At service loading
- At nominal strength

The prestressed flexural members are classified as Class U (uncracked), Class T (transition), and Class C (cracked) based on f_t, the computed extreme fiber stress in tension in the precompressed tensile zone at service loads (NZS 19.3.2).

The precompressed tensile zone is that portion of a prestressed member where flexural tension, calculated using gross section properties, would occur under unfactored dead and live loads if the prestress force was not present. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress.

For Class U and Class T flexural members, stresses at service load are determined using uncracked section properties, while for Class C flexural members, stresses at service load are calculated based on the cracked section (NZS 19.3.3).

The following table provides a summary of the conditions considered for the various section classes.

	Prestressed			Nonprestressed
	Class U	Class T	Class C	
Assumed Behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 19.3.3.3	Gross section 19.3.3.3	Cracked section 19.3.3.3	No requirement
Allowable stress at transfer	19.3.3.6.1	19.3.3.6.1	19.3.3.6.1	No requirement
Allowable compressive stress based on uncracked section properties	19.3.3.6.2	19.3.3.6.2	No requirement	No requirement
Tensile stress at service loads 19.3.2	$\leq 0.7\sqrt{f'_c}$	$0.7\sqrt{f'_c} < f_t \leq \sqrt{f'_c}$	No requirement	No requirement

13.6 Serviceability Requirements of Flexural Members

13.6.1 Serviceability Check at Initial Service Load

The stresses in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits (NZS 19.3.3.5.1 and 19.3.3.5.2):

- Extreme fiber stress in tension: $0.25\sqrt{f'_{ci}}$ (NZS 19.3.3.5.2(b))

- Extreme fiber stress in tension at ends of simply supported members: $0.5\sqrt{f'_{ci}}$

The extreme fiber stress in tension at the ends of simply supported members is currently **NOT** checked by ETABS.

13.6.2 Serviceability Check at Service Load

The stresses in the concrete for Class U and Class T prestressed flexural members at service loads, and after all prestress losses occur, are checked against the following limits:

- Extreme fiber stress in compression due to prestress plus total load: $0.60 f'_c$ (NZS 19.3.3.5.1)

- Extreme fiber stress in tension in the precompressed tensile zone at service loads:

  - Class U beams and one-way slabs:

    $$f_t \leq 0.7\sqrt{f'_c} \qquad \text{(NZS 19.3.2, 19.3.3.5.2(a))}$$

  - Class U two-way slabs:

    $$f_t \leq 0.7\sqrt{f'_c} \qquad \text{(NZS 19.3.2, 19.3.3.5.2(a))}$$

  - Class T beams:

    $$0.7\sqrt{f'_c} < f_t \leq \sqrt{f'_c} \qquad \text{(NZS 19.3.2, 19.3.3.5.2(a))}$$

  - Class C beams:

    $$f_t \geq \sqrt{f'_c} \qquad \text{(NZS 19.3.2, 19.3.3.5.2(a))}$$

For Class C prestressed flexural members, checks at service loads are not required by the code. However, for Class C prestressed flexural members not subject to fatigue or to aggressive exposure, the spacing of bonded reinforcement nearest the extreme tension face shall not exceed that given by NZS 19.3.3.5.3.

It is assumed that the user has checked the requirements of NZS 19.3.3.5.3 independently, as these sections are not checked by the program.

13.6.3 Serviceability Checks at Long-Term Service Load

The stresses in the concrete for Class U and Class T prestressed flexural members at long-term service loads and after all prestress losses occur are checked against the same limits as for the normal service load, except for the following:

- Extreme fiber stress in compression due to prestress plus total load:

$$0.45 f'_c \qquad \text{(NZS 19.3.3.5.2(a))}$$

13.6.4 Serviceability Checks of Prestressing Steel

The program also performs checks on the tensile stresses in the prestressing steel (NZS 19.3.3.6.1). The permissible tensile stress checks in all types of prestressing steel in terms of the specified minimum tensile stress, f_{pu}, and the minimum yield stress, f_{py}, are summarized as follows:

- Due to tendon jacking force: $\min(0.94f_{py}, 0.80f_{pu})$ (NZS 19.3.3.6.1 (a))
- Immediately after force transfer: $\min(0.82f_{py}, 0.74f_{pu})$ (NZS 19.3.3.6.1 (b))
- At anchors and couplers after force transfer: $0.70f_{pu}$ (NZS 19.3.3.6.1 (c))

13.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

13.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

13.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of prestressed concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases, the beam may be designed as a rectangular or a flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

13.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 13-1 (NZS 7.4.2.7). Furthermore, it is assumed that the compression carried by the concrete is 0.75 times that which can be carried at the balanced condition (NZS 9.3.8.1). When the applied moment exceeds the moment capacity at the balanced condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression reinforcement and additional tension reinforcement.

The design procedure used by ETABS for both rectangular and flanged sections (L- and T-beams) is summarized in the subsections that follow. The beams are designed for major direction flexure, shear, and torsion only.

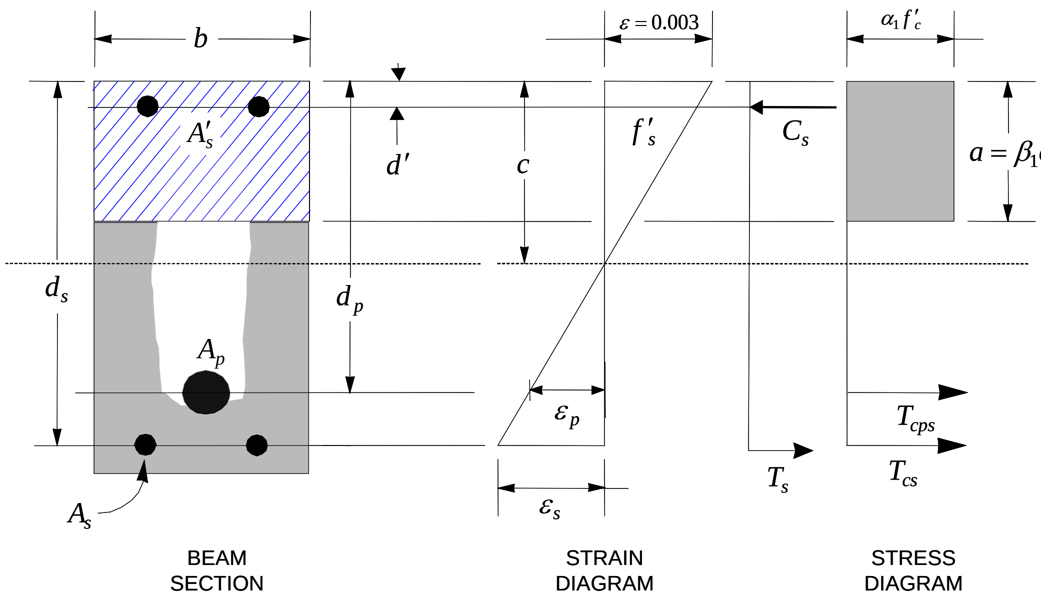


Figure 13-1 Rectangular Beam Design

13.7.1.2.1 Design of Rectangular Beams

The amount of post-tensioning steel adequate to resist the design moment M and minimum reinforcement are provided to satisfy the flexural cracking requirements (NZS 19.3.6.7).

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tension reinforcement strain shall not be less than ε_{smin}, which is equal to 0.0044 for tension-controlled behavior (NZS 7.4.2.8, 19.3.6.6.2):

$$c_b = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_{s,\min}} d \qquad \text{(NZS 7.4.2.8, 19.3.6.6.2)}$$

The maximum allowed depth of the rectangular compression block, a_{max}, is given by:

$$a_{\max} = 0.75\beta_1 c_b \qquad \text{(NZS 7.4.2.7, 9.3.8.1)}$$

where β_1 is calculated as:

$$\beta_1 = 0.85 \quad \text{for } f'_c \leq 30, \qquad \text{(NZS 7.4.2.7)}$$

$$\beta_1 = 0.85 - 0.008(f'_c - 30), \quad 0.65 \leq \beta_1 \leq 0.85 \qquad \text{(NZS 7.4.2.7)}$$

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{ps}, is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel. Based on the calculated f_{ps}, the depth of the neutral axis is recalculated, and f_{ps} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \beta_1 c$$

- If $a \leq a_{max}$ (NZS 9.3.8.1), the moment capacity of the section provided by post-tensioning steel only is computed as:

$$\phi M_n^0 = \phi_b A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right)$$

- If $a > a_{\max}$ (NZS 9.3.8.1), a failure condition is declared.

If $M^* > \phi M_n^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of the neutral axis, c is equal to $c_{\max}$. The stress in the post-tensioning steel, f_{ps} is then calculated based on strain compatibility, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = \alpha_1 f'_c a_{\max} b$$

$$T = A_{ps} f_{ps}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\alpha_1 f'_c a_{\max} b - A_{ps} f_{ps}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$\phi M_n^{bal} = \phi_b A_{ps} f_{ps}^{bal}\left(d_p - \frac{a_{\max}}{2}\right) + \phi_b A_s^{bal} f_s^{bal}\left(d_s - \frac{a_{\max}}{2}\right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. This case does not involve any iteration in determining the depth of the neutral axis, c.

13.7.1.2.1.1 Case 1: Post-tensioning steel is adequate

When $M^* < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment M.. Minimum reinforcement is provided to satisfy the ductility requirements.

13.7.1.2.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M^*, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $M_u^0 < M^* < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports this required area of tension reinforcement. Since M is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will be between the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M^* is positive, or at the top if M^* is negative.

13.7.1.2.1.3 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M^* > M_u^{bal}$, compression reinforcement is required (NZS 9.3.8.1). In that case, ETABS assumes that the depth of neutral axis, c, is equal to $c_{\max}$. The values of f_{pb} and f_s reach their respective balanced condition values, f_{pb}^{bal} and f_s^{bal}. The area of compression reinforcement, A_s', is determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M^* - M_u^{bal}$$

The required compression reinforcement is given by:

$$A_s' = \frac{M_{us}}{\left(f_s' - \alpha_1 f_c'\right)\left(d_s - d'\right)\phi_b}, \text{ where}$$

$$f_s' = \varepsilon_{c,max} E_s \left[\frac{c - d'}{c}\right] \le f_y \qquad \text{(NZS 7.4.2.2, 7.4.2.4)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{\phi_b f_y \left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom, and A'_s is to be placed at the top if M^* is positive, and vice versa if M^* is negative.

13.7.1.2.2 Design of Flanged Beams

13.7.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M^* (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

13.7.1.2.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, moment capacity M_u^0 is determined as follows:

The maximum depth of the compression zone, $c_{\max}$, is calculated based on the limitation that the tension reinforcement strain shall not be less than $\varepsilon_{s\min}$, which is equal to 0.0044 for tension-controlled behavior (NZS 7.4.2.8, 19.3.6.6.2):

$$c_b = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_{s,\min}} d \qquad \text{(NZS 7.4.2.8, 19.3.6.6.2)}$$

The maximum allowed depth of the rectangular compression block, $a_{\max}$, is given by:

$$a_{\max} = 0.75\beta_1 c_b \qquad \text{(NZS 7.4.2.7, 9.3.8.1)}$$

where β_1 is calculated as:

$$\beta_1 = 0.85 \quad \text{for } f'_c \le 30, \qquad \text{(NZS 7.4.2.7)}$$

$$\beta_1 = 0.85 - 0.008(f'_c - 30), \quad 0.65 \le \beta_1 \le 0.85 \qquad \text{(NZS 7.4.2.7)}$$

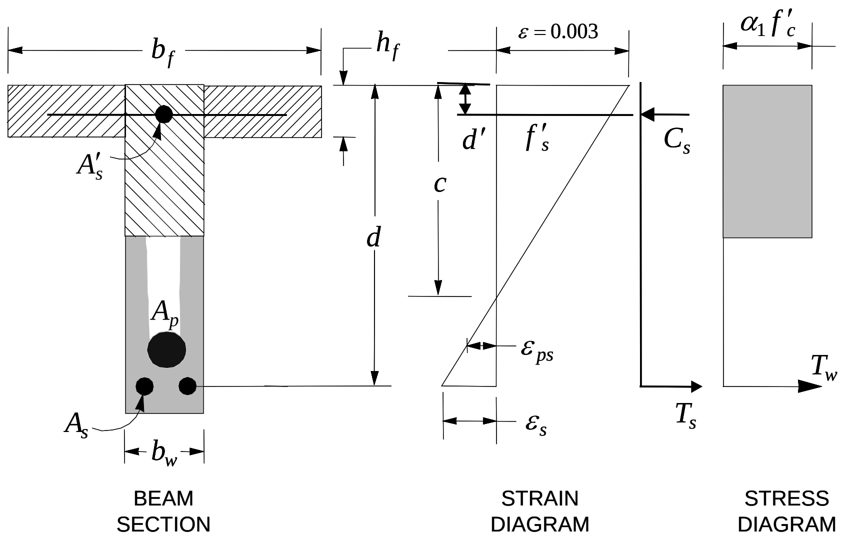


Figure 13-2 T-Beam Design

ETABS determines the depth of the neutral axis, *c*, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{ps}, is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress in the post-tensioning steel. Based on the calculated f_{ps}, the depth of the neutral axis is recalculated, and f_{ps} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \beta_1 c$$

- If $a \le a_{max}$ (NZS 9.3.8.1), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$\phi M_n^0 = \phi_b A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right)$$

- If $a > a_{max}$ (NZS 9.3.8.1), a failure condition is declared.

If $M^* > \phi M_n^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of the neutral axis, *c* is equal to c_{max}. The stress in the post-tensioning steel, f_{ps} is then calculated based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in that case the width of the beam is taken as b_f. Compression reinforcement is required if $a > a_{max}$.

- If $a > h_f$, the calculation for A_s is given by

$$C = \alpha_1 f'_c A_c^{comp}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$A_c^{com} = b_f h_f + b_w \left(a_{\max} - h_f \right)$$

$$T = A_{ps} f_{ps}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\alpha_1 f'_c A_c^{com} - A_{ps} f_{ps}^{bal}}{f_s^{bal}}$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. This case does not involve any iteration in determining the depth of neutral axis, c.

13.7.1.2.2.3 Case 1: Post-tensioning steel is adequate

When $M^* < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment M. Minimum reinforcement is provided to satisfy ductility requirements, i.e., $M^* < M_u^0$.

13.7.1.2.2.4 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M^*, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $M_u^0 < M^* < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports the required area of tension reinforcement. Since M^* is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end, and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if M^* is positive, or at the top if M^* is negative.

13.7.1.2.2.5 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M^* > M_u^{bal}$, compression reinforcement is required (BS 3.4.4.4). In that case ETABS assumes that the depth of the neutral axis, c, is equal to $c_{\max}$. The values of f_{pb} and f_s reach their respective balanced condition values, f_{pb}^{bal} and f_s^{bal}. The area of compression reinforcement, A_s', is then determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M^* - \phi_b M_u^{bal}$$

The required compression reinforcement is given by:

$$A_s' = \frac{M_{us}}{\left(f'_s - \alpha_1 f'_c\right)\left(d_s - d'\right)\phi_b}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c\max}\left[\frac{c_{\max} - d'}{c_{\max}}\right] \le f_y$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{\phi_b f_y \left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom, and A'_s is to be placed at the top if M is positive and vice versa if M is negative.

13.7.1.2.3 Ductility Requirements

ETABS also checks the following condition by considering the post-tensioning steel and tension reinforcement to avoid abrupt failure.

$$\phi M_n \geq 1.2 M_{cr} \qquad \text{(NZS 19.3.6.6.3)}$$

The preceding condition is permitted to be waived for the following:

(a) Two-way, unbonded post-tensioned slabs

(b) Flexural members with shear and flexural strength at least twice that required by AS/NZS 1170 and NZS 1170.5.

These exceptions currently are **NOT** handled by ETABS.

13.7.1.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section with unbonded tendon is given by the following limit:

$$A_s \geq 0.004\, A \qquad \text{(NZS 19.3.6.7.1)}$$

where A is the area of the cross-section between the flexural tension face and the center of gravity of the gross section.

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \le \begin{cases} 0.4bd & \text{Rectangular beam} \\ 0.4b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

13.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination in the major direction of the beam. In designing the shear reinforcement for a particular beam for a particular load combination, the following steps are involved:

- Determine the factored shear force, V^*.
- Determine the shear force, V_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three subsections describe in detail the algorithms associated with these steps.

13.7.2.1 Determine Shear Force and Moment

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam section are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

13.7.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated using the simplified procedure given in NZS 19.3.11.2.1. It is assumed that the effective prestress force is equal to or greater than 40% of the tensile strength of flexural reinforcement and the member is not subjected to axial tension or self-strain action such as temperature, which can induced significant tensile stresses over part of the element.

The shear strength provided by concrete, V_c, is given by:

$$V_c = \left(\frac{\sqrt{f'_c}}{20} + \frac{5V^* d_c}{M^*} \right) b_w d \qquad \text{(NZS 19.3.11.2.1)}$$

$$\frac{V^* d_c}{M^*} \leq 1.0 \qquad \text{(NZS 19.3.11.2.1)}$$

where V^* and M^* are the design moment and shear force acting simultaneously at the section considered, and d_c is the distance from extreme compression fiber to centroid of the prestressed reinforcement.

The following limit is also enforced:

$$0.14\sqrt{f'_c} b_w d \leq V_c \leq 0.4\sqrt{f'_c} b_w d \qquad \text{(NZS 19.3.11.2.1)}$$

13.7.2.3 Determine Required Shear Reinforcement

The average shear stress is computed for rectangular and flanged sections as:

$$v^* = \frac{V^*}{b_w d} \qquad \text{(NZS 7.5.1)}$$

The average shear stress is limited to a maximum limit of,

$$v_{\max} = \min\left\{0.2 f'_c, 8 \text{ MPa}\right\} \qquad \text{(NZS 7.5.2, 9.3.9.3.3, 19.3.11.1)}$$

The shear reinforcement is computed as follows:

- If $v^* \leq \phi_s \left(v_c / 2\right)$

$$\frac{A_v}{s} = 0 \qquad \text{(NZS 9.3.9.4.13)}$$

- If $\phi_s \left(v_c / 2\right) < v^* \leq \phi_s v_c$,

$$\frac{A_v}{s} = \frac{A_{v,\min}}{s} \qquad \text{(NZS 9.3.9.4.15, 19.3.11.3.4(b))}$$

$$\frac{A_{v,\min}}{s} = \min\begin{cases} \dfrac{1}{16}\sqrt{f'_c}\dfrac{b_w}{f_{yt}} \\ \dfrac{A_{ps} f_{pu}}{80 f_y d}\sqrt{\dfrac{d}{d_w}} \end{cases} \qquad \text{(NZS 9.3.9.4.15, 19.3.11.3.4(b))}$$

- If $\phi_s v_c < v^* \le \phi_s v_{\max}$,

$$\frac{A_v}{s} = \frac{\left(v^* - \phi_s v_c\right)}{\phi_s f_{yt} d} \qquad \text{(NZS 9.3.9.4.2)}$$

$$\frac{A_v}{s} \ge \frac{A_{v,\min}}{s} \qquad \text{(NZS 9.3.9.4.15, 19.3.11.3.4(b))}$$

- If $v^* > v_{\max}$,

  a failure condition is declared. (NZS 7.5.2, 9.3.9.3.3)

If the beam depth h is less than the maximum of 300 mm and $0.5b_w$, no shear reinforcement is required (AS 9.3.9.4.13).

The maximum of all of the calculated A_v/s values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

13.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T^*.
- Determine special section properties.

- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

13.7.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur because of redistribution of internal forces upon cracking, the design T^* is permitted to be reduced in accordance with the code (NZS 7.6.1.3). However, the program does not automatically redistribute the internal forces and reduce T^*. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

13.7.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_{co}, A_o, p_c, t_c, and t_o, are calculated. These properties are described in the following (NZS 7.1).

A_{co} = Area enclosed by outside perimeter of concrete cross-section

A_o = Gross area enclosed by shear flow path

p_c = Outside perimeter of concrete cross-section

p_o = Perimeter of area A_o

t_c = Assumed wall thickness of an equivalent tube for the gross section

t_o = Assumed wall thickness of an equivalent tube for the area enclosed by the shear flow path

In calculating the section properties involving reinforcement, such as A_o, p_o, and t_o, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 mm. This is equivalent to a 38 mm clear cover and a 12 mm stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a rectangular beam section are given as:

$$A_{co} = bh \quad \text{(NZS 7.1)}$$

$$A_o = (b - 2c)(h - 2c) \quad \text{(NZS 7.1)}$$

$$p_c = 2b + 2h \quad \text{(NZS 7.1)}$$

$$p_o = 2(b - 2c) + 2(h - 2c) \quad \text{(NZS 7.1)}$$

$$t_c = 0.75\, A_o / p_o \quad \text{(NZS 7.1)}$$

$$t_o = 0.75\, A_{co} / p_c \quad \text{(NZS 7.1)}$$

where, the section dimensions b, h, and c are shown in Figure 13-3.

Similarly, the special section properties for a flanged beam section are given as:

$$A_{co} = b_w h + (b_f - b_w)h_f \quad \text{(NZS 7.1)}$$

$$A_o = (b_w - 2c)(h - 2c) \quad \text{(NZS 7.1)}$$

$$p_c = 2b_f + 2h \quad \text{(NZS 7.1)}$$

$$p_o = 2(h - 2c) + 2(b_w - 2c) \quad \text{(NZS 7.1)}$$

$$t_c = 0.75\, A_o / p_o \quad \text{(NZS 7.1)}$$

$$t_o = 0.75\, A_{co} / p_c \quad \text{(NZS 7.1)}$$

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 13-3. Note that the flange width on either side of the beam web is limited to the smaller of $3h_f$ (NZS 7.6.1.7).

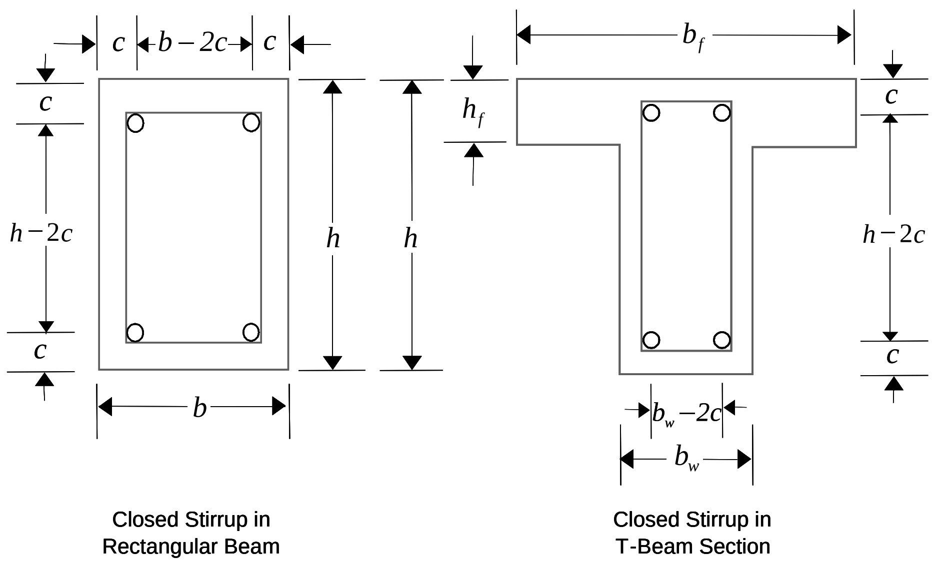


Figure 13-3 Closed stirrup and section dimensions for torsion design

13.7.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, T_{cr}, for which the torsion in the section can be ignored is calculated as:

$$T_{cr} = \phi 0.1 A_{co} t_c \sqrt{f'_c} \qquad \text{(NZS 7.6.1.2)}$$

where A_{co} and t_c are as described in the previous section, and f'_c is the specified concrete compressive strength. The stress due to torsion also should be limited in order to ignore torsion, defined as:

$$\frac{T^*}{\phi 2 A_o t_o} \leq 0.08\sqrt{f'_c} \qquad \text{(NZS 7.6.1.3)}$$

13.7.3.4 Determine Torsion Reinforcement

If the factored torsion, T^*, is less than the threshold limit, T_{cr}, and meets the torsion stress limit, torsion can be safely ignored (NZS 7.6.1). In that case, the program reports that no torsion reinforcement is required. However, if T^* exceeds the threshold limit, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (NZS 7.6.4.1).

- If $T^* > T_{cr}$ or the torsion stress limit is not met, the required closed stirrup area per unit spacing, A_t / s, is calculated as:

$$\frac{A_t}{s} = \frac{v_{tn} t_o}{f_{yt}} \qquad \text{(NZS 7.6.4.2)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{v_{tn} t_o p_o}{f_y} \qquad \text{(NZS 7.6.4.3)}$$

where the torsional shear stress v_{tn} is defined as:

$$v_{tn} = \frac{T^*}{\phi 2 A_o t_o} \qquad \text{(NZS 7.6.1.6, 7.6.1.5)}$$

The minimum closed stirrups and longitudinal reinforcement shall be such that the following is satisfied, where A_t/s can be from any closed stirrups for shear and A_l can include flexure reinforcement, provided it is fully developed.

$$\sqrt{\frac{A_t A_l}{s p_o}} = \frac{1.5 A_o t_c}{f_y A_o} \qquad \text{(NZS 7.6.2)}$$

The term $A_t A_l / p_o$ shall not be taken greater than $7A_t/s$ (NZS 7.6.2.3).

An upper limit of the combination of V^* and T^* that can be carried by the section also is checked using the equation:

$$v_n + v_{tn} < \min(0.2 f'_c, 8 \text{ MPa}) \qquad \text{(NZS 7.6.1.8, 7.5.2)}$$

For rectangular sections, b_w is replaced with b. If the combination of V^* and T^* exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated A_l and A_t/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

13.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips typically are governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (NZS 3101:06) for prestressed reinforced concrete, as described in the following subsections. To learn more about the design strips, refer to the section entitled "ETABS Design Techniques" in the *Key Features and Terminology* manual.

13.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of the element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Determine the capacity of post-tensioned sections
- Design flexural reinforcement for the strip

These three steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

13.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

13.8.1.2 Determine Capacity of Post-Tensioned Sections

The calculation of the post-tensioned section capacity is identical to that described earlier for rectangular beam sections.

13.8.1.3 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

13.8.1.4 Minimum and Maximum Slab Reinforcement

There are no minimum requirements for untensioned reinforcement in one-way bonded slabs. One-way spanning floors with unbounded tendons should have minimum reinforcement requirements in accordance with NZS 19.3.6.7.1.

In flat slabs, reinforcement is added at the top over supports to be 0.00075 times the gross cross-sectional area. This reinforcement extends 1.5 times the slab depth on each side of the column. The length of the reinforcement should be at least $L/6$ where L is the span of the slab.

There are no minimum requirements for the span zone. However, additional untensioned reinforcement shall be designed for the full tension force generated by assumed flexural tensile stresses in the concrete for the following situations:

- all locations in one-way spanning floors using unbonded tendons
- all locations in one-way spanning floors where transfer tensile stress exceeds $0.36\sqrt{f_{ci}}$
- support zones in all flat slabs
- span zones in flat slabs using unbonded tendons where the tensile stress exceeds $0.17\sqrt{f_c^{'}}$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

13.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code specific items are described in the following sections.

13.8.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (NZS 12.7.1(b)). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (NZS 12.7.1(b)). Figure 13-4 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

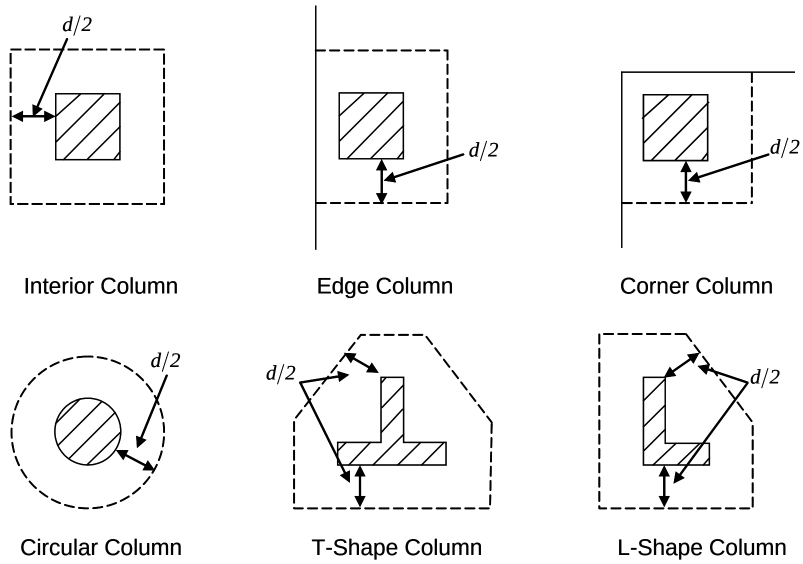


Figure 13-4 Punching Shear Perimeters

13.8.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M^*$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M^*$, where

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}} \qquad \text{(NZS 12.7.7.2)}$$

$$\gamma_v = 1 - \frac{1}{1+(2/3)\sqrt{b_1/b_2}} \qquad \text{(NZS 12.7.7.1)}$$

where b_1 is the width of the critical section measured in the direction of the span, and b_2 is the width of the critical section measured in the direction perpendicular to the span.

13.8.2.3 Determine Concrete Capacity

The concrete punching shear factored strength is taken as the minimum of the following three limits:

$$v_c = \beta_p \sqrt{f'_c} + 0.3 f_{pc} + v_p \qquad \text{(NZS 19.3.11.2.4)}$$

$$\beta_p = \min\left(0.29, \left(\frac{\alpha_s d}{b_o} + 1.5\right) \Big/ 12\right) \qquad \text{(NZS 19.3.11.2.4)}$$

$$v_p = 0$$

where, β_p is the ratio of the maximum to the minimum dimension of the critical section (NZS 12.1, 12.7.3.2(a)), b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 40 & \text{for interior columns,} \\ 30 & \text{for edge columns,} \\ 20 & \text{for corner columns.} \end{cases} \qquad \text{(NZS 19.3.11.2.4)}$$

A limit is imposed on the value of $\sqrt{f'_c}$ as follows:

$$\lambda \sqrt{f'_c} \le \sqrt{100} \qquad \text{(NZS 5.2.1)}$$

13.8.2.4 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS.

13.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 150 mm, and not less than 16 times the shear reinforcement bar diameter (NZS 12.7.4.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed, and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for*

Punching Shear and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

13.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is determined as:

$$v_c = 0.17\sqrt{f'_c} \qquad \text{(NZS 19.3.11.2.4)}$$

13.8.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$v_{\max} = 0.5\sqrt{f'_c} \qquad \text{(NZS 12.7.3.4)}$$

Given v^*, v_c, and $v_{\max}$, the required shear reinforcement is calculated as follows, where, ϕ, is the strength reduction factor.

$$\frac{A_v}{s} = \frac{(v_n - v_c)}{\phi f_{yv} d} \qquad \text{(NZS 12.7.4.2(a))}$$

Minimum punching shear reinforcement should be provided such that:

$$V_s \geq \frac{1}{16}\sqrt{f'_c}\, b_o d \qquad \text{(NZS 12.7.4.3)}$$

- If $v_n > \phi v_{\max}$, a failure condition is declared. (NZS 12.7.3.4)
- If v_n exceeds the maximum permitted value of $\phi v_{\max}$, the concrete section should be increased in size.

13.8.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 13-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

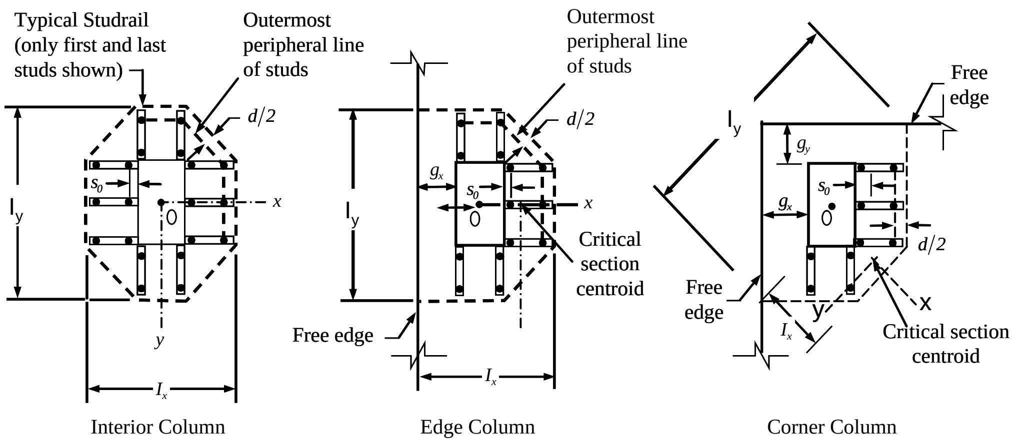


Figure 13-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 2d measured in a direction parallel to the column face (NZS 12.7.4.4).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

13.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in NZS 3.11 plus half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5d. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed $2d$ and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(NZS 12.7.4.4)}$$

$$s \leq 0.5d \qquad \text{(NZS 12.7.4.4)}$$

$$g \leq 2d \qquad \text{(NZS 12.7.4.4)}$$

Chapter 14
Design for Singapore CP 65:99

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the Singapore Standard CP 65 : 99 [CP 1999], which also incorporates Erratum Nos. 1 and BC 2:2008 Design Guide of High Strength Concrete to Singapore Standard CP 65 [BC 2008]. Various notations used in this chapter are listed in Table 14-1. For referencing to the pertinent sections of the CP code in this chapter, a prefix "CP" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

14.1 Notations

The following table identifies the various notations used in this chapter.

Table 14-1 List of Symbols Used in the CP 65:99 Code

A_{cv}	Area of section for shear resistance, mm^2
A_g	Gross area of cross-section, mm^2
A_s	Area of tension reinforcement, mm^2
A_{ps}	Area of prestress steel, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sv}	Total cross-sectional area of links at the neutral axis, mm^2
A_{sv} / s_v	Area of shear reinforcement per unit length of the member, mm^2/mm
a	Depth of compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
d or d_e	Effective depth of tension reinforcement, mm
d'	Depth to center of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f_{ci}	Concrete strength at transfer, MPa
f_{cu}	Characteristic cube strength, MPa
f_{pu}	Characteristic strength of a prestressing tendon, MPa
f_{pb}	Design tensile stress in tendon, MPa
f'_s	Compressive stress in a beam compression steel, MPa
f_y	Characteristic strength reinforcement, MPa
f_{yv}	Characteristic strength of link reinforcement, MPa (< 500 MPa)
h	Overall depth of a section in the plane of bending, mm

Table 14-1 List of Symbols Used in the CP 65:99 Code

h_f	Flange thickness, mm
k_1	Shear strength enhancement factor for support compression
k_2	Concrete shear strength factor, $\left[f_{cu}/30\right]^{1/3}$
M	Design moment at a section, MPa
M_u	Design moment resistance of a section, MPa
M_u^0	Design moment resistance of a section with tendons only, N-mm
M_u^{bal}	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
s_v	Spacing of the links along the length of the beam, mm
s	Spacing of shear rails, mm
T	Tension force, N
V	Design shear force at ultimate design load, N
u	Perimeter of the punching critical section, mm
v	Design shear stress at a beam cross-section or at a punch critical section, MPa
v_c	Design ultimate shear stress resistance of a concrete beam, MPa
v_{co}	Ultimate shear stress resistance of an uncracked concrete section, MPa
v_{cr}	Ultimate shear stress resistance of a cracked concrete section, MPa
$v_{\max}$	Maximum permitted design factored shear stress at a beam section or at the punch critical section, MPa
v_t	Torsional shear stress, MPa
x	Neutral axis depth, mm
x_{bal}	Depth of neutral axis in a balanced section, mm

Table 14-1 List of Symbols Used in the CP 65:99 Code

a_{max}	Depth of neutral axis in a balanced section, mm
z	Lever arm, mm
β	Torsional stiffness constant
β_b	Moment redistribution factor in a member
γ_f	Partial safety factor for load
γ_m	Partial safety factor for material strength
ε_c	Maximum concrete strain, 0.0035
ε_{ps}	Strain in prestressing steel
ε_s	Strain in tension steel
ε'_s	Strain in compression steel

14.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. The design load combinations are obtained by multiplying the characteristic loads by appropriate partial factors of safety, γ_f (CP 2.4.1.3). For CP 65:99, if a structure is subjected to dead (D), live (L), pattern live (PL), and wind (W) loads, and considering that wind forces are reversible, the load combinations in the following subsections may need to be considered (CP 2.4.3, 4.1.7.1, 4.3.4 and 4.3.5).

For post-tensioned concrete design, the user can specify the prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning are defined in the following sections.

14.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress forces in accordance with CP 65:99 clause 4.3.5. The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0D + 1.0PT

14.2.2 Service Load Combination

The following load combinations are used for checking the requirements of prestress for serviceability in accordance with CP 4.3.4. It is assumed that all long-term losses have already occurred at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT

14.2.3 Ultimate Limit State Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with CP 2.4.3.1.1, Table 2.1.

The design combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are determined automatically by ETABS by subtracting the primary PT moments when the flexural design is performed.

1.4D + 1.0PT*

1.4D + 1.6L + 1.0PT*

1.4D + 1.6(0.75PL) + 1.0PT*

1.0D ± 1.4W +1.0PT*
1.4D ± 1.4W + 1.0PT*
1.2D + 1.2L ± 1.2W + 1.0PT*

* — Replace PT by H for flexural design only

Other appropriate loading combinations should be used if roof live load is separately treated, or other types of loads are present. Note that the automatic combination, including pattern live load, is assumed and should be reviewed before using for design.

14.3 Limits on Material Strength

Grade C28/C35 and C32/C40 are the minimum recommended for post-tensioning and pre-tensioning respectively. In both cases the concrete strength at transfer should not be less than 25 MPa (CP 4.1.8.1).

The specified characteristic strength of untensioned reinforcement is given as follows (CP 4.1.8.2, 3.1.7.4):

Hot rolled mild reinforcement -	250 MPa	(CP 3.1.7.4, Table 3.1)
High yield reinforcement -	460 MPa	(CP 3.1.7.4, Table 3.1)

The specified characteristic strength of prestressing steel should conform to SS 2: Part 3: 1987, SS 18: Part 2: 1970, and SS 32 : Part 2: 1988.

ETABS also checks the tensile strength in the prestressing steel (CP 4.7.1). The permissible tensile stresses in all types of prestressing steel, in terms of the specified minimum tensile strength f_{pu}, are summarized as follows:

a. Due to tendon jacking force:	$0.75\ f_{pu}$
b. Immediately after prestress transfer:	$0.70\ f_{pu}$

In any circumstances, the initial prestressing forces shall not exceed $0.75\ f_{pu}$.

14.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength of the material by a partial safety factor, γ_m. The values of γ_m used in the program are listed in the table that follows, as taken from CP Table 2.2 (CP 2.4.4.1):

Values of γ_m for the ultimate limit state	
Reinforcement, γ_{ms}	1.15
Prestressing steel, γ_{mp}	1.15
Concrete in flexure and axial load, γ_{mc}	1.50
Shear strength without shear reinforcement, γ_{mv}	1.25

These factors are already incorporated in the design equations and tables in the code. Note that for reinforcement, the default factor of 1.15 is for Grade 460 reinforcement. If other grades are used, this value should be overwritten as necessary. Changes to the partial safety factors are carried through the design equations where necessary, typically affecting the material strength portions of the equations.

14.5 Design Assumptions for Prestressed Concrete Structures

The ultimate limit state of prestressed members for flexure and axial loads shall be based on assumptions given in CP 4.3.7.1.

- The strain distribution in the concrete in compression is derived from the assumption that a plane section remains plane (CP 4.3.7.1(a)).
- The design stresses in the concrete in compression are taken as 0.45 f_{cu}. The maximum strain at the extreme concrete compression fiber shall be assumed equal to 0.0035 (CP 4.3.7.1(b)).
- Tensile strength of the concrete is ignored (CP 4.3.7.1(c)).
- The strain in bonded prestressing tendons and in any additional reinforcement (compression or tension) is derived from the assumption that a plane section remains plane (CP 4.3.7.1(d)).

The serviceability limit state of prestressed members uses the following assumptions given in CP 4.3.4.1.

- Plane sections remain plane, i.e., strain varies linearly with depth through the entire load range (CP 4.3.4.1(a)).
- Elastic behavior exists by limiting the concrete stresses to the values given in CP 4.3.4.2, 4.3.4.3, and 4.3.5 (CP 4.3.4.1(b)).
- In general, it is only necessary to calculate design stresses due to the load arrangements immediately after the transfer of prestress and after all losses or prestress has occurred; in both cases the effects of dead and imposed loads on the strain and force in the tendons may be ignored (CP 4.3.4.1(c)).

Prestressed concrete members are investigated at the following three stages (CP 4.3.4.2 and 4.3.4.3):

- At transfer of prestress force
- At service loading
- At nominal strength

The prestressed flexural members are classified as Class 1 (uncracked), Class 2 (cracked but no visible cracking), and Class 3 (cracked) based on tensile strength, f_t, the computed extreme fiber stress in tension in the precompressed tensile zone at service loads (CP 4.1.3).

The precompressed tensile zone is that portion of a prestressed member where flexural tension, calculated using gross section properties, would occur under unfactored dead and live loads if the prestress force was not present. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress.

Class 1: No flexural tensile stresses

Class 2: Flexural tensile stresses with no visible cracking

Class 3: Flexural tensile stresses with surface width of cracks as follows:

- Crack width ≤ 0.1 mm for members in very severe environments as specified in CP Table 3.2
- Crack width ≤ 0.2 mm for all other members

14.6 Serviceability Requirements of Flexural Members

14.6.1 Serviceability Check at Initial Service Load

The stresses in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits (CP 4.3.5.1 and 4.3.5.2):

- Extreme fiber stress in compression: $0.50 f_{ci}$
- Extreme fiber stress in tension for Class 1: ≤ 1.0 MPa
- Extreme fiber stress in tension for Class 2:

  pre-tensioned member $0.45\sqrt{f_{ci}}$

  post-tensioned member $0.36\sqrt{f_{ci}}$

The extreme fiber stress in tension for Class 3 should not exceed the appropriate value for a Class 2 member; otherwise the section should be designed as a cracked section.

14.6.2 Serviceability Check at Service Load

The stresses in the concrete for Class 1 and Class 2 prestressed flexural members at service loads, and after all prestress losses occur, are checked against the following limits (CP 4.3.4.2, 4.3.4.3):

- Extreme fiber stress in compression due to prestress plus total load: $0.33 f_{cu}$
- Extreme fiber stress in compression due to prestress plus total load for continuous beams and other statically indeterminate structures: $0.4 f_{cu}$
- Extreme fiber stress in tension in the precompressed tensile zone at service loads:
  - Extreme fiber stresses in tension for Class 1: No tensile stress
  - Extreme fiber stresses in tension for Class 2:

    pre-tensioned member $0.45\sqrt{f_{cu}}$

    post-tensioned member $0.36\sqrt{f_{cu}}$

Although cracking is allowed for Class 3, it is assumed that the concrete section is uncracked and the user is limiting the tensile stress at service stage as presented in Table 4.2, modified by the coefficients in Table 4.3 of CP 65 : 1999. The user needs to provide the tension limits for Class 3 elements at service loads in the Design Preferences (CP 4.3.4.3(c)).

14.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

14.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

14.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of prestressed concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or a flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

14.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 14-1 (CP 3.4.4.4), where $\varepsilon_{c,\max}$ is defined as:

$$\varepsilon_{c,\max} = \begin{cases} 0.0035 & \text{if} \quad f_{cu} \le 60 \text{ MPA} \\ 0.0035 - \dfrac{\left(f_{cu} - 60\right)}{50000} & \text{if} \quad f_{cu} > 60 \text{ MPA} \end{cases}$$

Furthermore, it is assumed that moment redistribution in the member does not exceed 10 percent (i.e., $\beta_b \ge 0.9$; CP 3.4.4.4). The code also places a limitation on the neutral axis depth,

$$\frac{x}{d} \le \begin{cases} 0.5 & \text{for} \quad f_{cu} \le 60 \text{ N/mm}^2 \\ 0.4 & \text{for} \quad 60 < f_{cu} \le 75 \text{ N/mm}^2 \\ 0.33 & \text{for} \quad 75 < f_{cu} \le 105 \text{ N/mm}^2 \end{cases}$$

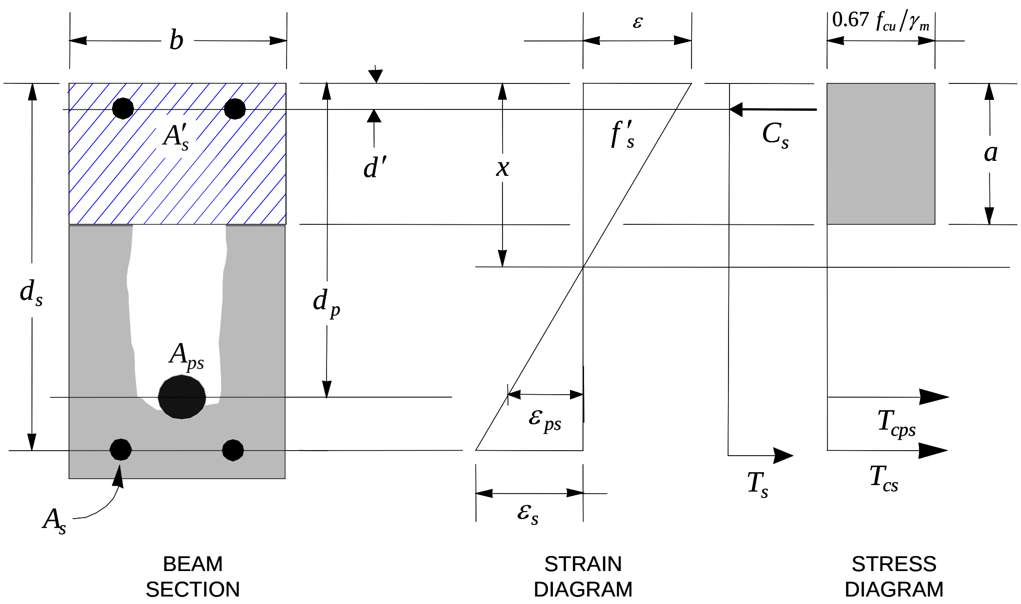


Figure 14-1 Rectangular Beam Design

to safeguard against non-ductile failures (CP 3.4.4.4). In addition, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The design procedure used by ETABS, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow. It is assumed that the design ultimate axial force does not exceed $0.1\, f_{cu} A_g$ (CP 3.4.4.1); hence all beams are designed for major direction flexure, shear, and torsion only.

14.7.1.2.1 Design of Rectangular Beams

The amount of post-tensioning steel adequate to resist the design moment *M* and minimum reinforcement are provided to satisfy the flexural cracking requirements (CP 4.12.6).

ETABS determines the depth of the neutral axis, *x*, by imposing force equilibrium, i.e., C = T, and performs an iteration to compute the depth of the neutral axis, which is based on stress-strain compatibility. After the depth of the neutral axis has been found, the stress in the post-tensioning reinforcement, f_{pb}, is computed based on strain compatibility.

The ductility of a section is controlled by limiting the x/d ratio (CP 3.4.4.4):

$$x = \begin{cases} \dfrac{d-z}{0.45}, & \text{for} \quad f_{cu} \le 60 \text{ N/mm}^2 \\ \dfrac{d-z}{0.40}, & \text{for} \quad 60 < f_{cu} \le 75 \text{ N/mm}^2 \\ \dfrac{d-z}{0.36}, & \text{for} \quad 75 < f_{cu} \le 105 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 3.4.4.4)}$$

The maximum depth of the compression block is given by:

$$a = \begin{cases} 0.9x & \text{for} \quad f_{cu} \le 60 \text{ N/mm}^2 \\ 0.8x & \text{for} \quad 60 < f_{cu} \le 75 \text{ N/mm}^2 \\ 0.72x & \text{for} \quad 75 < f_{cu} \le 105 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 3.4.4.1(b), 4.3.7.3)}$$

The lever arm of the section must not be greater than 0.95 times the effective depth (CP 3.4.4.1).

$$z = d - 0.5a \le 0.95d_e \qquad \text{(CP 3.4.4.1(e))}$$

- If $a \le a_{\max}$ (CP 3.4.4.4), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_u^0 = A_{ps} f_{pb} \left(d_p - \frac{a}{2} \right) \qquad \text{(CP 4.3.7.3)}$$

- If $a > a_{\max}$ (CP 3.4.4.4), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension-controlled case. In that case, it is assumed that the depth of neutral axis x is equal to $c_{\max}$. The stress in the post-tensioning steel, f_{pb}, is then calculated based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps}, in the post-tensioning steel, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = \frac{0.67 f_{cu}}{\gamma_m} a_{\max} b$$

$$T = A_{ps} f_{pb}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\frac{0.67 f_{cu}}{\gamma_m} a_{\max} b - A_{ps} f_{pb}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_u^{bal} = A_{ps} f_{pb}^{bal} \left(d_p - \frac{a_{\max}}{2} \right) + A_s^{bal} f_s^{bal} \left(d_s - \frac{a_{\max}}{2} \right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. This case does not involve any iteration in determining the depth of neutral axis, x.

[redacted]Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment M.. Minimum reinforcement is provided to satisfy the ductility requirements, i.e., $M < M_u^0$.

[redacted]Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment *M* and reports this required area of tension reinforcement. Since *M* is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will be between the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if *M* is positive, or at the top if *M* is negative.

[redacted] Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required (CP 3.4.4.4). In that case, ETABS assumes that the depth of neutral axis, *x*, is equal to $x_{\max}$. The values of f_{pb} and f_s reach their respective balanced condition values, f_{pb}^{bal} and f_s^{bal}. The area of compression reinforcement, A_s', is determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A_s' = \frac{M_{us}}{\left(f_s' - \frac{0.67 f_{cu}}{\gamma_c}\right)(d - d')}, \text{ where} \qquad \text{(CP 3.4.4.4)}$$

$$f_s' = \varepsilon_c E_s \left[\frac{a_{\max} - d'}{a_{\max}}\right] \leq 0.87 f_y$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{0.87 f_y \left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M is positive, and vice versa if M is negative.

14.7.1.2.2 Design of Flanged Beams

[redacted]Flanged Beam Under Negative Moment

In designing for a factored negative moment, M (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

[redacted]Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, moment capacity M_u^0 is determined as follows:

ETABS determines the depth of the neutral axis, x, by imposing force equilibrium, i.e., $C = T$, and performs an iteration to compute the depth of the neutral axis, which is based on stress-strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps}, in the post-tensioning steel. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pb}, is computed based on strain compatibility.

The ductility of a section is controlled by limiting the x/d ratio (CP 3.4.4.4):

$$x = \begin{cases} \dfrac{d-z}{0.45}, & \text{for} \quad f_{cu} \le 60 \text{ N/mm}^2 \\ \dfrac{d-z}{0.40}, & \text{for} \quad 60 < f_{cu} \le 75 \text{ N/mm}^2 \\ \dfrac{d-z}{0.36}, & \text{for} \quad 75 < f_{cu} \le 105 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 3.4.4.4)}$$

The maximum depth of the compression block is given by:

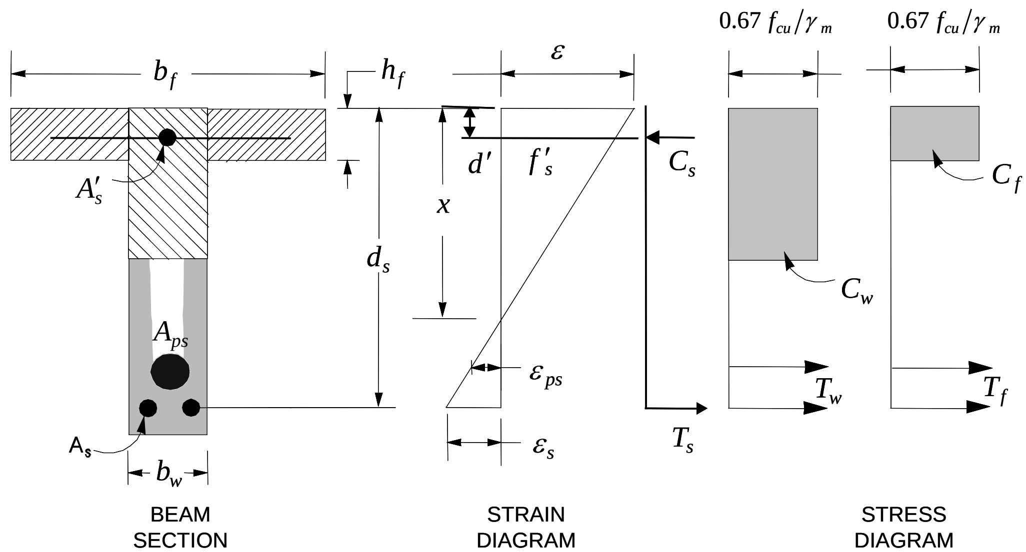


Figure 14-2 T-Beam Design

$$a = \begin{cases} 0.9x & \text{for} \quad f_{cu} \le 60 \text{ N/mm}^2 \\ 0.8x & \text{for} \quad 60 < f_{cu} \le 75 \text{ N/mm}^2 \\ 0.72x & \text{for} \quad 75 < f_{cu} \le 105 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 3.4.4.1(b), 4.3.7.3)}$$

The lever arm of the section must not be greater than 0.95 times its effective depth (CP 3.4.4.1):

$$z = d - 0.5a \le 0.95d_e \qquad \text{(CP 3.4.4.1(e))}$$

- If $a \le a_{\max}$ (CP 3.4.4.4), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_u^0 = A_{ps} f_{pb} \left(d_p - \frac{a}{2} \right) \qquad \text{(CP 4.3.7.3)}$$

- If $a > a_{\max}$ (CP 3.4.4.4), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension-controlled case. In that case, it is assumed that the depth of neutral axis x is equal to $c_{\max}$. The stress in the post-tensioning steel, f_{pb}, is then calculated based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps}, in the post-tensioning steel, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as b_f. Compression reinforcement is required when x/d exceed the limits.

- If $a > h_f$, the calculation for A_s is given by

$$C = \frac{0.67 f_{cu}}{\gamma_c} a_{\max} A_c^{com}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$A_c^{com} = b_f h_f + b_w \left(a_{\max} - h_f \right)$$

$$T = A_{ps} f_{pb}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\frac{0.67 f_{cu}}{\gamma_m} a_{\max} A_c^{com} - A_{ps} f_{pb}^{bal}}{f_s^{bal}}$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. This case does not involve any iteration in determining the depth of neutral axis, x.

[redacted]Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment *M*. Minimum reinforcement is provided to satisfy ductility requirements.

[redacted] Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist *M*, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment *M* and reports the required area of tension reinforcement. Since *M* is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if *M* is positive, or at the top if *M* is negative.

[redacted] Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required (CP 3.4.4.4). In that case, ETABS assumes that the depth of the neutral axis, *x*, is equal to $x_{\max}$. The values of f_{pb} and f_s reach their respective balanced condition values, f_{pb}^{bal} and f_s^{bal}. The area of compression reinforcement, $A_s^{'}$, is then determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(f'_s - \frac{0.67 f_{cu}}{\gamma_c}\right)(d - d')}\text{, where} \qquad \text{(CP 3.4.4.4)}$$

$$f'_s = \varepsilon_c E_s \left[\frac{a_{\max} - d'}{a_{\max}}\right] \leq 0.87 f_y.$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{0.87 f_y (d_s - d')}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom, and A'_s is to be placed at the top if M is positive and vice versa if M is negative.

14.7.1.3 Minimum and Maximum Reinforcement

Reinforcement in post-tensioned concrete beams is computed to increase the strength of sections as documented for the flexural design of post-tensioned beams or to comply with the shear link requirements. The minimum flexural tension reinforcement required for a beam section to comply with the cracking requirements needs to be separately investigated by the user.

For bonded tendons, there is no minimum untensioned reinforcement required.

For unbonded tendons, the minimum flexural reinforcement provided in a rectangular or flanged beam section is given by the following table, which is taken from CP Table 3.27 (CP 3.12.5.3) with interpolation for reinforcement of intermediate strength:

Section	Situation	Definition of percentage	Minimum percentage $f_y = 250$ MPa	Minimum percentage $f_y = 460$ MPa
Rectangular	—	$100 \frac{A_s}{bh}$	0.24	0.13

Section	Situation	Definition of percentage	Minimum percentage $f_y = 250$ MPa	Minimum percentage $f_y = 460$ MPa
T- or L-Beam with web in tension	$\frac{b_w}{b_f} < 0.4$	$100\ \frac{A_s}{b_w h}$	0.32	0.18
	$\frac{b_w}{b_f} \geq 0.4$	$100\ \frac{A_s}{b_w h}$	0.24	0.13
T-Beam with web in compression	—	$100\ \frac{A_s}{b_w h}$	0.48	0.26
L-Beam with web in compression	—	$100\ \frac{A_s}{b_w h}$	0.36	0.20

The minimum flexural compression reinforcement, if it is required at all, is given by the following table, which is taken from CP Table 3.27 (CP 3.12.5.3) with interpolation for reinforcement of intermediate strength:

Section	Situation	Definition of percentage	Minimum percentage
Rectangular	—	$100\ \frac{A'_s}{bh}$	0.20
T- or L-Beam	Web in tension	$100\ \frac{A'_s}{b_f h_f}$	0.40
	Web in compression	$100\ \frac{A'_s}{b_w h}$	0.20

For $f_{cu} > 40$ MPa, the minimum percentage shown in CP Table 3.27 shall be multiplied by a factor of $\left(f_{cu} / 40 \right)^{2/3}$.

In addition, an upper limit on both the tension reinforcement and compression reinforcement is imposed to be 0.04 times the gross cross-sectional area (CP 3.12.6.1).

14.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination in the major direction of the beam. In designing the shear reinforcement for a particular beam for a particular load combination, the following steps are involved (CP 3.4.5):

- Determine the shear stress, v.
- Determine the shear stress, v_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three subsections describe in detail the algorithms associated with these steps.

14.7.2.1 Determine Shear Stress

In the design of the beam shear reinforcement, the shear forces for a particular load combination at a particular beam section are obtained by factoring the associated shear forces for different load cases, with the corresponding load combination factors.

$$v = \frac{V}{b_w d} \qquad \text{(CP 3.4.5.2)}$$

The maximum allowable shear stress, v_{max} is defined as:

$$v_{max} = \min(0.8\sqrt{f_{cu}},\ 7 \text{ MPa}) \qquad \text{(CP 3.4.5.2)}$$

For light-weight concrete, v_{max} is defined as:

$$v_{max} = \min(0.63\sqrt{f_{cu}},\ 5.6 \text{ MPa}) \qquad \text{(CP 65-2 5.4)}$$

14.7.2.2 Determine Concrete Shear Capacity

The design ultimate shear resistance of the concrete alone, V_c, should be considered at sections that are as follows:

Uncracked sections in flexure ($M < M_o$) (CP 4.3.8.3)

Cracked sections in flexural ($M \geq M_o$) (CP 4.3.8.3)

where,

M is the design bending moment at the section

M_o is the moment necessary to produce zero stress in the concrete at the extreme tension fiber; in this calculation, only 0.8 of the stress due to post-tensioning should be taken into account.

[redacted] Case 1: Uncracked section in flexure

The ultimate shear resistance of the section, V_{co}, is computed as follows:

$$V_{co} = 0.67 b_v h \sqrt{\left(f_t^2 + 0.8 f_{cp} f_t\right)},$$ (CP 4.3.8.4)

where,

f_t is the maximum design principal stress (CP 4.3.8.4)

$$f_t = 0.24\sqrt{f_{cu}}$$ (CP 4.3.8.4)

f_{cp} = design compressive stress at the centoidal axis due to post-tensioning, taken as positive. (CP 4.3.8.4)

$$V_c = V_{co} + P \sin \beta$$ (CP 4.3.8.4)

[redacted] Case 2: Cracked section in flexure

The ultimate shear resistance of the section, V_{cr}, is computed as follows:

$$V_{cr} = \left(1 - 0.55 \frac{f_{pe}}{f_{pu}}\right) v_c b_v d + M_o \frac{V}{M}, \text{ and}$$ (CP 4.3.8.5)

$$V_{cr} \geq 0.1 b_v d \sqrt{f_{cu}}$$ (CP 4.3.8.5)

$$V_c = \min\left(V_{co}, V_{cr}\right) + P \sin \beta$$ (CP 4.3.8.5)

14.7.2.3 Determine Required Shear Reinforcement

Given v, v_c and v_{max}, the required shear reinforcement is calculated as follows (CP 4.3.8.7):

- Calculate the design average shear stress that can be carried by minimum shear reinforcement, v_r, as:

$$v_r = \begin{cases} 0.4 & \text{if} \quad f_{cu} \leq 40 \text{ N/mm}^2 \\ 0.4\left(\frac{f_{cu}}{40}\right)^{2/3} & \text{if } 40 < f_{cu} \leq 80 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 3.4.5.3, Table 3.8)}$$

$$f_{cu} \leq 80 \text{ N/mm}^2 \text{ (for calculation purpose only)} \qquad \text{(CP 3.4.5.3, Table 3.8)}$$

- If $v \leq v_c + v_r$,

$$\frac{A_s}{s_v} = \frac{v_r b}{0.87 f_{yv}}, \qquad \text{(CP 3.4.5.3, Table 3.8)}$$

- If $v > v_c + v_r$,

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c)b}{0.87 f_{yv}} \qquad \text{(CP 3.4.5.3, Table 3.8)}$$

- If $v > v_{max}$, a failure condition is declared. (CP 3.4.5.2)

In the preceding expressions, a limit is imposed on f_{yv} as:

$$f_{yv} \leq 460 \text{ MPa.} \qquad \text{(CP 3.4.5.1)}$$

The maximum of all of the calculated A_{sv}/s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

14.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the torsional shear stress, v_t.
- Determine special section properties.
- Determine critical torsion stress.
- Determine the torsion reinforcement required.

Note that references in this section refer to CP 65:Part 2.

14.7.3.1 Determine Torsional Shear Stress

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In typical framed construction, specific consideration of torsion is not usually required where torsional cracking is adequately controlled by shear reinforcement. If the design relies on the torsional resistance of a beam, further consideration should be given using the following algorithms (CP 65-2 3.4.5.13).

The torsional shear stress, v_t, for a rectangular section is computed as:

$$v_t = \frac{2T}{h_{\min}^2\left(h_{\max} - h_{\min}/3\right)} \qquad \text{(CP 65-2 2.4.4.1)}$$

For flanged sections, the section is considered as a series of rectangular segments and the torsional shear stress is computed for each rectangular component using the preceding equation, but considering a torsional moment attributed to that segment, calculated as:

$$T_{seg} = T\left(\frac{h_{\min}^3 h_{\max}}{\sum\left(h_{\min}^3 h_{\max}\right)}\right) \qquad \text{(CP 65-2 2.4.4.2)}$$

h_{mzx} = Larger dimension of a rectangular section

h_{min} = Smaller dimension of a rectangular section

If the computed torsional shear stress, v_t, exceeds the following limit for sections with the larger center-to-center dimension of the closed link less than 550 mm, a failure condition is generated if the torsional shear stress does not satisfy:

$$v_t \leq \min(0.8\sqrt{f_{cu}}, 7\text{N/mm}^2) \times \frac{y_1}{550} \qquad \text{(CP 65-2 2.4.5)}$$

14.7.3.2 Determine Critical Torsion Stress

The critical torsion stress, $v_{t,\min}$, for which the torsion in the section can be ignored is calculated as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.6\text{N/mm}^2\right) \qquad \text{(CP 65-2 2.4.6)}$$

where f_{cu} is the specified concrete compressive strength.

For light-weight concrete, $v_{t,min}$ is defined as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.4\text{N/mm}^2\right) \text{x } 0.8 \qquad \text{(CP 65-2 5.5)}$$

14.7.3.3 Determine Torsion Reinforcement

If the factored torsional shear stress, v_t is less than the threshold limit, $v_{t,\min}$, torsion can be safely ignored (CP 65-2 2.4.6). In that case, the program reports that no torsion reinforcement is required. However, if v_t exceeds the threshold limit, $v_{t,\min}$, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CP 65-2 2.4.6).

- If $v_t > v_{t,\min}$, the required closed stirrup area per unit spacing, $A_{sv,t} / s_v$, is calculated as:

$$\frac{A_{sv,t}}{s_v} = \frac{T}{0.8 x_1 y_1 \left(0.87 f_{yv}\right)} \qquad \text{(CP 65-2 2.4.7)}$$

  and the required longitudinal reinforcement is calculated as:

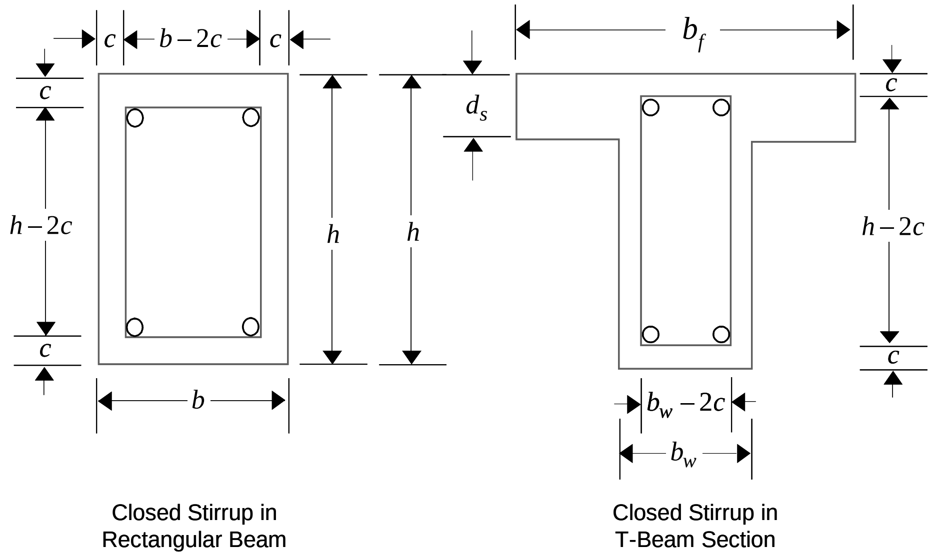


Figure 14-3 Closed stirrup and section dimensions for torsion design

$$A_l = \frac{A_{sv,t} f_{yv} (x_1 + y_1)}{s_v f_y} \qquad \text{(CP 65-2 2.4.7)}$$

In the preceding expressions, x_1 is the smaller center-to-center dimension of the closed link and y_1 is the larger center-to-center dimension of the closed link.

An upper limit of the combination of v and v_t that can be carried by the section also is checked using the equation:

$$v + v_t \le \min\left(0.8\sqrt{f_{cu}},\ 7\text{N/mm}^2\right) \qquad \text{(CP 65-2 2.4.5)}$$

For light-weight concrete, $v_{\max}$ is defined as:

$$v_{\max} = \min(0.63\sqrt{f_{cu}},\ 5.6 \text{ MPa}) \qquad \text{(CP 65-2 5.4)}$$

If the combination of shear stress, v, and torsional shear stress, v_t, exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated A_l and $A_{sv,t} / s_v$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

14.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips typically are governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (CP 65:99) for prestressed reinforced concrete as described in the following subsections. To learn more about the design strips, refer to the section entitled "ETABS Design Techniques" in the *Key Features and Terminology* manual.

14.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Determine the capacity of post-tensioned sections
- Design flexural reinforcement for the strip

These three steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

14.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

14.8.1.2 Determine Capacity of Post-Tensioned Sections

The calculation of the post-tensioned section capacity is identical to that described earlier for rectangular beam sections.

14.8.1.3 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

14.8.1.4 Minimum and Maximum Slab Reinforcement

There are no minimum requirements for untensioned reinforcement in one-way bonded slabs. One-way spanning floors with unbounded tendons should have minimum reinforcement requirements in accordance with CP Table 3.27 (CP 3.12.5.3)

In flat slabs, reinforcement is added at the top, over supports, to be 0.00075 times the gross cross-sectional area. This reinforcement extends 1.5 times the slab depth on each side of the column. The length of the reinforcement should be at least 0.2*L*, where *L* is the span of the slab.

There are no minimum requirements for the span zone. However, additional un-tensioned reinforcement shall be designed for the full tension force generated by assumed flexural tensile stresses in the concrete for the following situations (Concrete Society, Technical Report 43):

- all locations in one-way spanning floors using unbonded tendons
- all locations in one-way spanning floors where transfer tensile stress exceeds $0.36\sqrt{f_{ci}}$
- support zones in all flat slabs
- span zones in flat slabs using unbonded tendons where the tensile stress exceeds $0.15\sqrt{f_{cu}}$.

The reinforcement should be designed to act at a stress of $5/8f_y$ as follows:

$$A_s = \frac{F_t}{(5/8)\, f_y}$$

where

$$F_t = -\frac{f_{ct}(h-x)b}{2}$$

The value of f_{ct} will be negative in tension.

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (CP 3.12.6.1).

14.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code specific items are described in the following sections.

14.8.2.1 Critical Section for Punching Shear

The punching shear is checked at the face of the column (CP 3.7.6.4) and at a critical section at a distance of 1.5*d* from the face of the support (CP 3.7.7.6). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CP 3.7.7.1). Figure 14-4 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

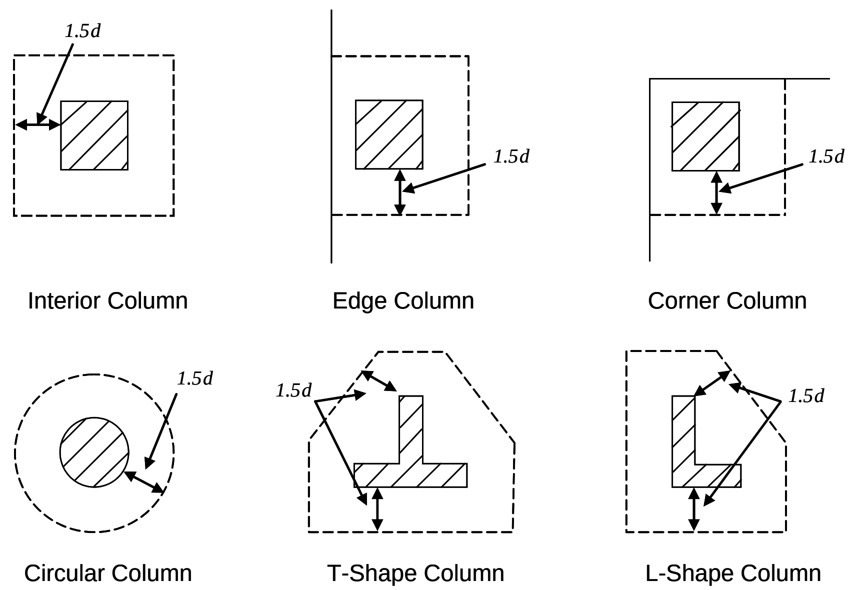


Figure 14-4 Punching Shear Perimeters

14.8.2.2 Determine Concrete Capacity

The design ultimate shear resistance of the concrete alone V_c should be considered at sections that are as follows:

Uncracked sections in flexure ($M < M_o$) (CP 4.3.8.3)

Cracked sections in flexural ($M \geq M_o$) (CP 4.3.8.3)

where,

M is the design bending moment at the section

M_o is the moment necessary to produce zero stress in the concrete at the extreme tension fiber; in this calculation, only 0.8 of the stress due to post-tensioning should be taken into account.

[redacted] Case 1: Uncracked section in flexure

The ultimate shear resistance of the section, V_{co}, is computed as follows:

$$V_{co} = 0.67 b_v h \sqrt{\left(f_t^{\,2} + 0.8 f_{cp} f_t \right)}, \qquad \text{(CP 4.3.8.4)}$$

where,

f_t is the maximum design principal stress (CP 4.3.8.4)

$$f_t = 0.24\sqrt{f_{cu}} \qquad \text{(CP 4.3.8.4)}$$

f_{cp} = design compressive stress at the centoidal axis due to prestress, taken as positive. (CP 4.3.8.4)

$$V_c = V_{co} + P\sin\beta \qquad \text{(CP 4.3.8.4)}$$

[redacted] Case 2: Cracked section in flexure

The ultimate shear resistance of the section, V_{cr}, is computed as follows:

$$V_{cr} = \left(1 - 0.55 \frac{f_{pe}}{f_{pu}} \right) v_c b_v d + M_o \frac{V}{M}, \text{ and} \qquad \text{(CP 4.3.8.5)}$$

$$V_{cr} \geq 0.1 b_v d \sqrt{f_{cu}} \qquad \text{(CP 4.3.8.5)}$$

$$V_c = \min\left(V_{co}, V_{cr} \right) + P\sin\beta \qquad \text{(CP 4.3.8.5)}$$

14.8.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the nominal design shear stress, *v*, is calculated as:

$$V_{eff,x} = V\left(f + \frac{1.5M_x}{Vy} \right) \qquad \text{(CP 3.7.6.2, 3.7.6.3)}$$

$$V_{eff,y} = V\left(f + \frac{1.5M_y}{Vx} \right) \qquad \text{(CP 3.7.6.2, 3.7.6.3)}$$

$$v_{\max} = \max \begin{cases} \dfrac{V_{eff,x}}{u\,d} \\ \dfrac{V_{eff,y}}{u\,d} \end{cases} \qquad \text{(CP 3.7.7.3)}$$

where,

u is the perimeter of the critical section,

x and y are the length of the side of the critical section parallel to the axis of bending

M_x and M_y are the design moments transmitted from the slab to the column at the connection

V is the total punching shear force

f is a factor to consider the eccentricity of punching shear force and is taken as:

$$f = \begin{cases} 1.00 & \text{for interior columns,} \\ 1.25 & \text{for edge columns, and} \\ 1.25 & \text{for corner columns.} \end{cases} \qquad \text{(CP 3.7.6.2, 3.7.6.3)}$$

14.8.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

14.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (CP 3.7.7.5). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* as described in the earlier sections remains unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

14.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

14.8.3.2 Determine Required Shear Reinforcement

The shear stress is limited to a maximum limit of

$$v_{\max} = 2\, v_c \qquad \text{(CP 3.7.7.5)}$$

Given v, v_c and $v_{\max}$, the required shear reinforcement is calculated as follows (CP 3.7.7.5).

- If $v \leq 1.6v_c$,

$$\frac{A_v}{s} = \frac{(v - v_c)ud}{0.87\, f_{yv}} \geq \frac{0.4ud}{0.87\, f_{yv}}, \qquad \text{(CP 3.7.7.5)}$$

- If $1.6v_c \leq v < 2.0v_c$,

$$\frac{A_v}{s} = \frac{5(0.7v - v_c)ud}{0.87\, f_{yv}} \geq \frac{0.4ud}{0.87\, f_{yv}}, \qquad \text{(CP 3.7.7.5)}$$

- If $v > v_{\max}$, a failure condition is declared. (CP 3.7.7.5)

If v exceeds the maximum permitted value of v_{max}, the concrete section should be increased in size.

14.8.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 14-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner columns.

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line of shear reinforcement shall not exceed 1.5d measured in a direction parallel to the column face (CP 11.12.3.3).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8 for corner, edge, and interior columns respectively.

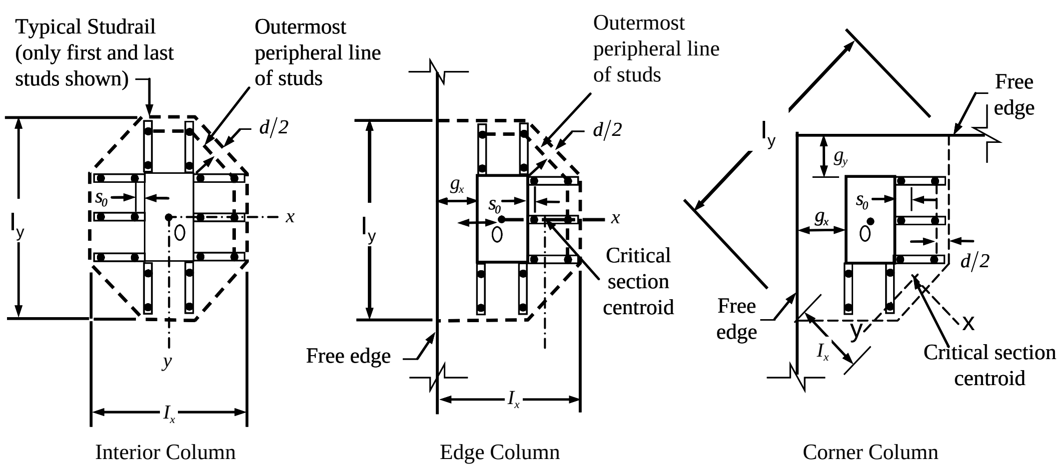


Figure 14-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

14.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CP 3.3 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5*d*. The spacing between adjacent shear studs, *g*, at the first peripheral line of studs shall not exceed 1.5*d*. The limit of s_o and the spacing, *s*, between the peripheral lines are specified as:

$$s_o \le 0.5d \qquad \text{(CP 3.7.7.6)}$$

$$s \le 0.75d \qquad \text{(CP 3.7.7.6)}$$

$$g \le 1.5d \qquad \text{(CP 3.7.7.6)}$$

Chapter 15
Design for AS 3600-09

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the Australian code AS 3600-2009 [AS 2009]. Various notations used in this chapter are listed in Table 15-1. For referencing to the pertinent sections of the AS code in this chapter, a prefix "AS" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

15.1 Notations

The following table identifies the various notations used in this chapter.

Table 15-1 List of Symbols Used in the AS 3600-09 Code

A_g	Gross area of concrete, mm^2
A_l	Area of longitudinal reinforcement for torsion, mm^2

Table 15-1 List of Symbols Used in the AS 3600-09 Code

A_p	Area of prestressing steel in flexural tension zone, sq-mm
A_s	Area of tension reinforcement, mm^2
A_{sc}	Area of compression reinforcement, mm^2
A_{st}	Area of tension reinforcement, mm^2
$A_{s\text{(required)}}$	Area of required tension reinforcement, mm^2
A_{sv}	Area of shear reinforcement, mm^2
$A_{sv,\min}$	Minimum area of shear reinforcement, mm^2
A_{sv}/s	Area of shear reinforcement per unit length, mm^2 /mm
A_{sw}/s	Area of shear reinforcement per unit length consisting of closed ties, mm^2/mm
A_t	Area of a polygon with vertices at the center of longitudinal bars at the corners of a section, mm^2
s	Spacing of shear reinforcement along the length, mm
a	Depth of compression block, mm
a_b	Depth of compression block at balanced condition, mm
$a_{\max}$	Maximum allowed depth of compression block, mm
b	Width of member, mm
b_{ef}	Effective width of flange (flanged section), mm
b_w	Width of web (flanged section), mm
c	Depth to neutral axis, mm
d	Distance from compression face to tension reinforcement, mm
d'	Concrete cover to compression reinforcement, mm
d_o	Distance from the extreme compression fiber to the centroid of the outermost tension reinforcement, mm
d_{om}	Mean value of d_o, averaged around the critical shear perimeter, mm
D	Overall depth of a section, mm
D_s	Thickness of slab (flanged section), mm

Table 15-1 List of Symbols Used in the AS 3600-09 Code

E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, MPa
f'_c	Specified compressive strength of concrete, MPa
f'_{ci}	Specified compressive strength of concrete at time of initial prestress, MPa
f_{pe}	Compressive stress in concrete due to effective prestress forces only (after allowance of all prestress losses), MPa
f_p	Stress in prestressing steel at nominal flexural strength, MPa
f_{pu}	Specified tensile strength of prestressing steel, MPa
f_{py}	Specified yield strength of prestressing steel, MPa
f_{ct}	Characteristic principal tensile strength of concrete, MPa
f'_{cf}	Characteristic flexural tensile strength of concrete, MPa
f_{cv}	Concrete shear strength, MPa
f_{sy}	Specified yield strength of flexural reinforcement, MPa
$f_{sy,f}$	Specified yield strength of shear reinforcement, MPa
f'_s	Stress in the compression reinforcement, MPa
D	Overall depth of a section, mm
J_t	Torsional modulus, mm^3
k_u	Ratio of the depth to the neutral axis from the compression face, to the effective depth, *d*
M_u^0	Design moment resistance of a section with tendons only, N-mm
M_u^{bal}	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
M_{ud}	Reduced ultimate strength in bending without axial force, N-mm
M^*	Factored moment at section, N-mm
N^*	Factored axial load at section, N

Table 15-1 List of Symbols Used in the AS 3600-09 Code

s	Spacing of shear reinforcement along the beam, mm
T_{uc}	Torsional strength of section without torsional reinforcement, N-mm
$T_{u.\max}$	Maximum permitted total factored torsion at a section, N-mm
T_{us}	Torsion strength of section with torsion reinforcement, N-mm
T^*	Factored torsional moment at a section, N-mm
u_t	Perimeter of the polygon defined by A_t, mm
V^*	Factored shear force at a section, N
$V_{u.\max}$	Maximum permitted total factored shear force at a section, N
$V_{u.\min}$	Shear strength provided by minimum shear reinforcement, N
V_{uc}	Shear force resisted by concrete, N
V_{us}	Shear force resisted by reinforcement, N
γ_1	Factor for obtaining depth of compression block in concrete
β_h	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
$\varepsilon_{c,\max}$	Maximum usable compression strain allowed in extreme concrete fiber, (0.003 mm/mm)
ε_s	Strain in reinforcement
ϕ	Strength reduction factor
θ_t	Angle of compression strut for torsion, degrees
θ_v	Angle of compression strut for shear, degrees

15.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For AS 3600-09, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the load combinations in the following sections may need to be considered (AS 2.4.2).

For post-tensioned concrete design, the user can specify the prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning are defined in the following sections.

15.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress forces in accordance with AS 3600-09 clauses 2.4.2. The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0D + 1.0PT (AS 2.4.2)
1.15D + 1.15PT (AS 2.4.2)
0.9D + 1.15PT (AS 2.4.2)

15.2.2 Service Load Combination

The following load combinations are used for checking the requirements of prestress for serviceability in accordance with AS 2.4.2. It is assumed that all long-term losses have occurred already at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT

15.2.3 Ultimate Limit State Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with AS 2.4.2.

The ultimate limit state combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are determined automatically by ETABS by subtracting out the primary PT moments when the flexural design is carried out.

1.35D + 1.0PT* (AS/NZS 1170.0-02, 4.2.2(a))

$1.2D + 1.5L + 1.0PT^*$	(AS/NZS 1170.0-02, 4.2.2(b))
$1.2D + 1.5(0.75PL) + 1.0PT^*$	(AS/NZS 1170.0-02, 4.2.2(b))
$1.2D + 0.4L + 1.0S + 1.0PT^*$	(AS/NZS 1170.0-02, 4.2.2(g))
$0.9D \pm 1.0W +1.0PT^*$	(AS/NZS 1170.0-02, 4.2.2(e))
$1.2D \pm 1.0W + 1.0PT^*$	(AS/NZS 1170.0-02, 4.2.2(d))
$1.2D + 0.4L \pm 1.0W + 1.0PT^*$	(AS/NZS 1170.0-02, 4.2.2(d))
$1.0D \pm 1.0E + 1.0PT^*$ $1.0D + 0.4L \pm 1.0E + 1.0PT^*$	(AS/NZS 1170.0-02, 4.2.2(f))

* — Replace PT with H for flexural design only

Note that the 0.4 factor on the live load in three of the combinations is not valid for live load representing storage areas. These are also the default design load combinations in ETABS whenever the AS 3600-2009 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

15.3 Limits on Material Strength

The upper and lower limits of f'_c are 100 MPa and 20 MPa, respectively, for all framing types (AS 3.1.1.1(b)).

$$f'_c \leq 100\,\text{MPa} \qquad \text{(AS 3.1.1.1)}$$

$$f'_c \geq 20\,\text{MPa} \qquad \text{(AS 3.1.1.1)}$$

The upper limit of f_{sy} is 500 MPa for all frames (AS 3.2.1, Table 3.2.1).

The code allows use of f'_c and f_{sy} beyond the given limits, provided special care is taken regarding the detailing and ductility (AS 3.1.1, 3.2.1, 17.2.1.1).

ETABS enforces the upper material strength limits for flexure and shear design of slabs. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

15.4 Strength Reduction Factors

The strength reduction factor, ϕ, is defined as given in the following table (AS 2.2.2, Table 2.2.2):

Type of action effect	Strength reduction factor (ϕ)
(a) Axial force without bending — (i) Tension (ii) Compression	 0.8 0.6
(b) Bending without axial tension or compression where: (i) *for members with Class N* reinforcement only (ii) *for members with Class L* reinforcement	$0.6 \leq (1.19\text{-}13\ k_{uo}/12) \leq 0.8$ $0.6 \leq (1.19\text{-}13\ k_{uo}/12) \leq 0.64$
(c) Bending with axial tension	$\phi + [(0.8 - \phi)(N_u/N_{uot})]$ ϕ is obtained from (b)
(d) Bending with axial compression where: (i) $N_u \geq N_{ub}$ (ii) $N_u < N_{ub}$	 0.6 $0.6 + [(\phi - 0.6)(1 - N_u/N_{ub})]$ ϕ is obtained from (b)
(e) Shear	0.7
(f) Torsion	0.7

The value M_{ud} is the reduced ultimate strength of the cross-section in bending where k_u = 0.36 and tensile force has been reduced to balance the reduced compressive forces (AS 8.1.5).

These values can be overwritten; however, caution is advised.

15.5 Design Assumptions for Prestressed Concrete Structures

Ultimate limit state of prestressed members for flexure and axial loads shall be based on assumptions given in AS 8.1.

- The strain distribution in the concrete in compression is derived from the assumption that the plane section remains plane (AS 8.1.2.1(a)).

- Tensile strength of the concrete is ignored (AS 8.1.2.1 (b)).

- The design stresses in the concrete in compression are taken as $\alpha_2 f_c'$. The maximum strain at the extreme concrete compression fiber shall be assumed equal to 0.003 (AS 8.1.2.1 (c), 8.1.2.2).
- The strain in bonded prestressing tendons and in any additional reinforcement (compression or tension) is derived from the assumption that plane section remains plane (AS 8.1.2.1(a)).

Prestressed concrete members are investigated at the following three stages:

- At transfer of prestress force
- At service loading
- At nominal strength

The prestressed flexural members are classified as uncracked and cracked based on tensile strength f_t, the computed extreme fiber stress in tension in the precompressed tensile zone at service loads.

The precompressed tensile zone is that portion of a prestressed member where flexural tension, calculated using gross section properties, would occur under unfactored dead and live loads if the prestress force was not present. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress.

15.6 Serviceability Requirements of Flexural Members

15.6.1 Serviceability Check at Initial Service Load

The stresses in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits (AS 8.1.6.2):

- Extreme fiber stress in compression: $0.50 f_{cp}$

The extreme fiber stress in tension should not exceed the cracking stress; otherwise the section should be designed as a cracked section.

15.6.2 Serviceability Check at Service Load

Flexural cracking in a prestressed beam shall be deemed to be controlled if under short-term service loads the resulting maximum tensile stress in concrete does not exceed $0.25\sqrt{f_c'}$; in that case, no further checks are needed (AS 8.6.2). However, if this limit is exceeded, flexural cracking shall be deemed to be controlled by providing reinforcement or bonded tendons, or both, near the tensile face and achieving either of the following (AS 8.6.2, 9.4.2):

(a) limiting the calculated maximum flexural tensile stress under short-term service loads to $0.6\sqrt{f_c'}$; or

(b) limiting both of the following

(i) the increment in steel stress near the tension face to 200 MPa, as the load increases from its value when the extreme concrete tensile fiber is at zero stress to the short-term service load value; and

(ii) the center-to-center spacing of reinforcement, including bonded tendons, to 200 mm. (This sub clause is a detailing requirement not checked by the program.)

The program checks the stresses in the concrete prestressed flexural members at service loads and after all prestress losses against the following limit (AS 8.6.2):

- Extreme fiber stress in tension in the precompressed tensile zone at service loads:
  – Extreme fiber stresses in tension for cracked section: $0.5\sqrt{f_c'}$

Thus, although cracking is allowed, it is assumed that the user is limiting the tensile stress at the service stage as presented in AS 8.6.2.

15.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

15.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments and axial forces
- Determine required flexural reinforcement

15.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of prestressed concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or a flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases, the beam may be designed as a rectangular or inverted flanged beam.

15.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 15-1 (AS 8.1.2.2).

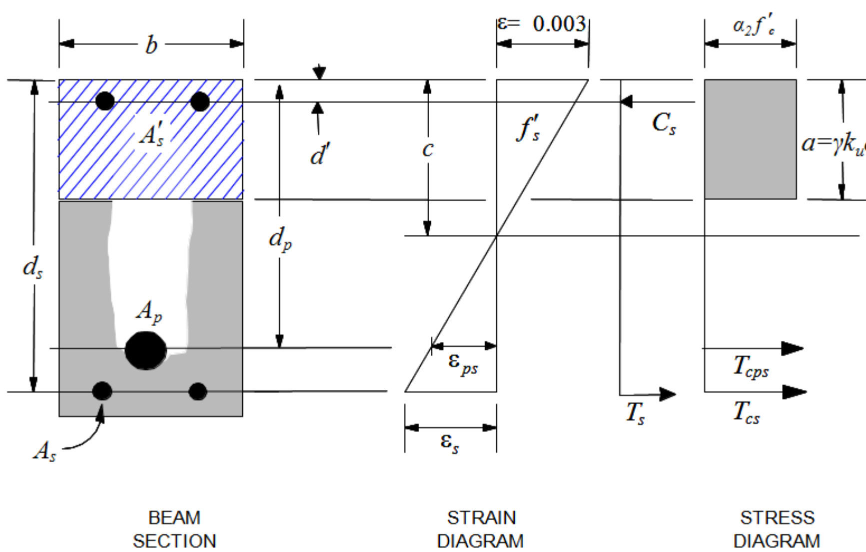


Figure 15-1 Rectangular Beam Design

The design procedure used by ETABS for both rectangular and flanged sections (L- and T-beams) is summarized in the following subsections. It is assumed that the design ultimate axial force does not exceed ($A_{sc}f_{sy} > 0.15N^*$) (AS 10.7.1a); hence all beams are designed for major direction flexure, shear, and torsion only.

15.7.1.2.1 Design of Rectangular Beams

The amount of post-tensioning steel adequate to resist the design moment *M* and minimum reinforcement are provided to satisfy the flexural cracking requirements (AS 8.1.6.1).

ETABS determines the depth of the neutral axis, *a*, by imposing force equilibrium, i.e., C = T, and performs an iteration to compute the depth of the neutral axis, which is based on stress-strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel. After the depth of the neutral axis has been found, the stress in the post-tensioning reinforcement f_{pb} is computed based on strain compatibility.

The following assumptions are applied for the stress block used to compute the flexural bending capacity of rectangular sections (AS 8.1.2).

- The maximum strain in the extreme compression fiber is taken as 0.003(AS 8.1.3(a)).
- A uniform compressive stress of $\alpha_2 f'_c$ acts on an area bounded by (AS 8.1.3(b)):
  - The edges of the cross-sections.
  - A line parallel to the neutral axis at the strength limit under the loading concerned, and located at a distance $\gamma k_u d$ from the extreme compression fiber.

The maximum allowable depth of the rectangular compression block, a_{max}, is given by

$$a_{max} = \gamma k_u d \quad \text{where,} \qquad \text{(AS 8.1.3(b))}$$

$$\alpha_2 = 1.0 - 0.003 f'_c \text{ where, } 0.67 \leq \alpha_2 \leq 0.85 \qquad \text{(AS 8.1.3(1))}$$

$$\gamma = 1.05 - 0.007 f'_c \text{ where, } 0.67 \leq \gamma \leq 0.85 \qquad \text{(AS 8.1.3(2))}$$

$$k_u = 0.36 \qquad \text{(AS 8.1.5)}$$

- If $a \leq a_{max}$ (AS 8.1.5), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_u^0 = A_p f_p \left(d_p - \frac{a}{2} \right)$$

- If $a > a_{\max}$ (AS 8.1.5), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of neutral axis c is equal to $c_{\max}$. The stress in the post-tensioning steel, f_p is then calculated based on strain compatibility and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = \alpha_2 f'_c b a_{\max} \qquad \text{(AS 8.1.3)}$$

$$T = A_p f_p^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\alpha_2 f'_c \text{ - } A_p f_p^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_u^{bal} = A_p f_p^{bal} \left(d_p - \frac{a_{\max}}{2} \right) + A_s^{bal} f_s^{bal} \left(d_s - \frac{a_{\max}}{2} \right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of neutral axis, c.

[redacted] Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment M.. Minimum reinforcement is provided to satisfy the ductility requirements, i.e., $M < M_u^0$.

Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports this required area of tension reinforcement. Since M is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end, and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will fall between the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required (AS 8.1.5). In that case, ETABS assumes that the depth of neutral axis, c, is equal to $c_{\max}$. The values of f_p and f_s reach their respective balanced condition values, f_p^{bal} and f_s^{bal}. The area of compression reinforcement, $A_s^{'}$, is determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A_{sc} = \frac{M_{us}}{\left(f'_s - \alpha_2 f'_c\right)\left(d - d'\right)\phi}, \text{ where}$$

$$f'_s = 0.003 E_s \left[\frac{c - d'}{c}\right] \leq f_{sy} \qquad \text{(AS 8.1.2.1, 3.2.2)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{\phi f_y (d_s - d')}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if *M* is positive, and vice versa if *M* is negative.

15.7.1.2.2 Design of Flanged Beams

[redacted] Flanged Beam Under Negative Moment

In designing for a factored negative moment, *M* (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

[redacted] Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, moment capacity M_u^0 is determined as follows:

ETABS determines the depth of the neutral axis, *a*, by imposing force equilibrium, i.e., C = T, and performs an iteration to compute the depth of the neutral axis, which is based on stress-strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps}, in the post-tensioning steel. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pb}, is computed based on strain compatibility.

The maximum allowable depth of the rectangular compression block, a_{max}, is given by

$$a_{max} = \gamma k_u d \text{ where, } k_u = 0.36 \qquad \text{(AS 8.1.5)}$$

- If $a \le a_{max}$ (AS 8.1.5), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_u^0 = A_p f_p \left(d_p - \frac{a}{2} \right)$$

- If $a > a_{\max}$ (AS 8.1.5), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case it is assumed that the depth of neutral axis c is equal to $c_{\max}$. The stress in the post-tensioning steel, f_p, is then calculated based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \leq D_s$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as b_f. Compression reinforcement is required when $a_{\max} = \gamma k_u d$ where, $k_u = 0.36$.

- If $a > D_s$, the calculation for A_s is given by

$$C = \alpha_2 f'_c a_{\max} A_c^{com}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$A_c^{com} = b_f D_s + b_w \left(a_{\max} - D_s \right)$$

$$T = A_p f_p^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\alpha_2 f'_c a_{max} A_c^{com} - A_p f_p^{bal}}{f_s^{bal}}$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of neutral axis, c.

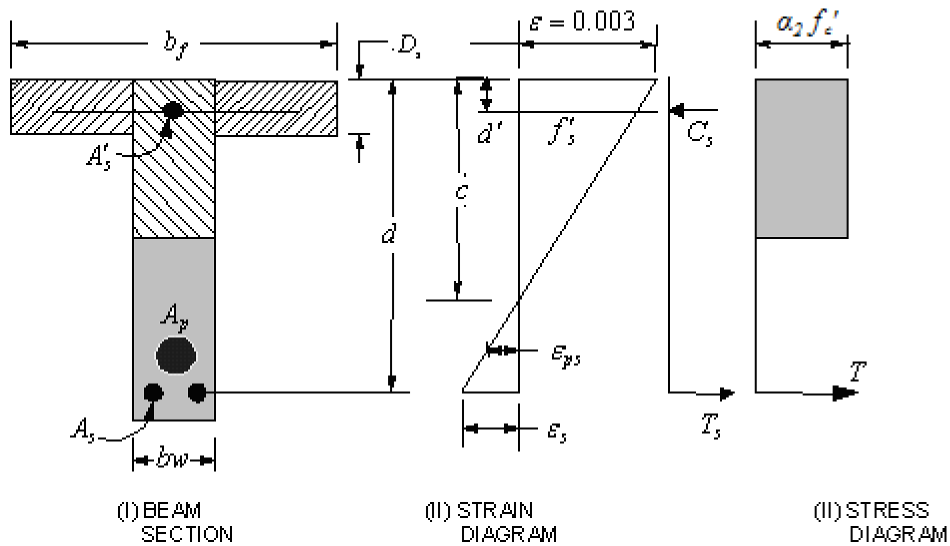


Figure 15-2 T-Beam Design

[redacted] Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment *M*. Minimum reinforcement is provided to satisfy ductility requirements, i.e., $M < M_u^0$.

[redacted] Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist *M*, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{max}$.

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment *M* and reports the required area of tension reinforcement. Since *M* is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end, and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

[redacted] Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required. In that case, ETABS assumes that the depth of the neutral axis, c, is equal to $c_{\max}$. The values of f_p and f_s reach their respective balanced condition values, f_p^{bal} and f_s^{bal}. The area of compression reinforcement, $A_s^{'}$, is then determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A_s' = \frac{M_{us}}{\phi\left(f_s' - \alpha_2 f_c'\right)\left(d - d'\right)},\ \text{where}$$

$$f'_s = 0.003E_s\left[\frac{c_{\max} - d'}{c_{\max}}\right] \le f_{sy} \qquad \text{(AS 8.1.2.1, 3.2.2)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{\phi f_y\left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom, and A'_s is to be placed at the top if M is positive and vice versa if M is negative.

15.7.1.3 Minimum and Maximum Reinforcement

Reinforcement in post-tensioned concrete beams is computed to increase the strength of sections as documented for the flexural design of post-tensioned beams or to comply with the shear link requirements. The minimum flexural tension reinforcement required for a beam section to comply with the cracking requirements needs to be separately investigated by the user.

The ultimate strength in bending (M_{uo}) at critical sections shall not be less than $\left(M_{uo}\right)_{\min}$ given by:

$$\left(M_{uo}\right)_{\min} = 1.2\left[Z\left(f'_{cf} + P/A_g\right) + Pe\right] \qquad \text{(AS 8.1.6.1)}$$

where

Z = the section modulus of the uncracked section, referred to the extreme fiber at which flexural cracking occurs

f'_{cf} = the characteristic flexural tensile strength of the concrete

e = the eccentricity of the prestressing force (P), measured from the centroidal axis of the uncracked section

The minimum flexural tension reinforcement required in a beam section is given by the following limit (AS 8.1.6.1):

$$A_{st.\min} = \alpha_b \left(\frac{D}{d}\right)^2 \frac{f'_{ct,f}}{f_{sy}} bd, \text{ where} \qquad \text{(AS 8.1.6.1(2))}$$

$$\alpha_b = 20, \quad \text{for Rectangular Section} \qquad \text{(AS8.1.6.1(2))}$$

for L- and T-Sections with the web in tension:

$$\alpha_b = 0.20 + \left(\frac{b_f}{b_w} - 1\right)\left(0.4\frac{D_s}{D} - 0.18\right) \geq 0.20\left(\frac{b_f}{b_w}\right)^{1/4}, \qquad \text{(AS8.1.6.1(2))}$$

for L- and T-Sections with the flange in tension:

$$\alpha_b = 0.20 + \left(\frac{b_f}{b_w} - 1\right)\left(0.25\frac{D_s}{D} - 0.08\right) \geq 0.20\left(\frac{b_f}{b_w}\right)^{2/3}, \qquad \text{(AS8.1.6.1(2))}$$

$$f'_{ct,f} = 0.6\sqrt{f'_c} \qquad \text{(AS 3.1.1.3(b))}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_{st} \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

$$A_{sc} \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

15.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination in the major direction of the beam. In designing the shear reinforcement for a particular beam for a particular load combination, the following steps are involved.

- Determine the factored shear force, V^*
- Determine the shear force, V_{uc}, that can be resisted by the concrete
- Determine the shear reinforcement required to carry the balance

The following three subsections describe in detail the algorithms associated with these steps.

15.7.2.1 Determine Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.

15.7.2.2 Determine Concrete Shear Capacity

The ultimate shear strength (V_{uc}) of a prestressed beam, excluding the contribution of shear reinforcement, is the lesser of the values obtained from the following, unless the cross-section under consideration is cracked in flexure, in which case only Flexural-Shear Cracking, Item (a), applies:

(a) Flexural-Shear Cracking

$$V_{uc} = \beta_1 \beta_2 \beta_3 b_v d_o f'_{cv} \left[\frac{\left(A_{st} + A_{pt}\right)}{b_w d_o} \right]^{1/3} + V_o + P_v \qquad \text{(AS 8.2.7.2(a))}$$

where,

$$f'_{cv} = \left(f'_c\right)^{1/3} \leq 4\text{MPa} \qquad \text{(AS 8.2.7.1)}$$

$$\beta_1 = 1.1\left(1.6 - \frac{d_o}{1000}\right) \geq 1.1 \qquad \text{(AS 8.2.7.1)}$$

β_2 = 1, or (AS 8.2.7.1)

= $1 - \left(\frac{N^*}{3.5A_g}\right) \geq 0$ for members subject to significant axial tension, or

= $1 + \left(\frac{N^*}{14A_g}\right)$ for members subject to significant axial compression.

β_3 = 1

$V_o = \frac{M_o}{\left|M^* / V^*\right|}$ = the shear force that would occur at the section when the bending moment at that section was equal to the decompression moment (M_o) given by:

$$M_o = Z\sigma_{cp.f}$$

where

$\sigma_{cp.f}$ = the compressive stress because of prestress, at the extreme fiber where cracking occurs

b) Web-shear cracking

$$V_{uc} = V_t + P_v \qquad \text{(AS 8.2.7.2(b))}$$

where

V_t = the shear force, which, in combination with the prestressing force and other action effects at the section would produce a principal tensile stress of $0.33\sqrt{f'_c}$ at either the centroidal axis or the intersection of flange and web, whichever is the more critical.

Where significant reversal of loads may occur, causing cracking in a zone usually in compression, the value of V_{uc} obtained from Clause 8.2.7.1 may not apply.

15.7.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{u,\min} = V_{uc} + 0.6 b_v d_o \qquad \text{(AS 8.2.9)}$$

$$V_{u,\max} = 0.2 f'_c \, b d_o + P_v \qquad \text{(AS 8.2.6)}$$

Given V^*, V_{uc}, and $V_{u.\max}$, the required shear reinforcement is calculated as follows, where, ϕ, the strength reduction factor, is 0.75 by default (AS 2.2.2).

- If $V^* \leq \phi V_{uc} / 2$,

  $\frac{A_{sv}}{s} = 0$, if $D \leq 750$ mm, otherwise $A_{sv.\min}$ shall be provided. (AS 8.2.5).

- If $\left(\phi V_{uc} / 2\right) < V^* \leq \phi V_{u.\min}$,

  $\frac{A_{sv}}{s} = 0$, if $D < b_w/2$ or 250 mm, whichever is greater (AS 8.2.5(c)(i)), otherwise $A_{sv.\min}$ shall be provided.

- If $\phi V_{u.\min} < V^* \leq \phi V_{u.\max}$,

$$\frac{A_{sv}}{s} = \frac{\left(V^* - \phi V_{uc}\right)}{f_{sy.f}\, d_o \cot \theta_v}, \qquad \text{(AS 8.2.10)}$$

and greater than $A_{sv.\min}$, defined as:

$$\frac{A_{sv.\min}}{s} = \left(0.35 \frac{b_w}{f_{sy.f}}\right) \qquad \text{(AS 8.2.8)}$$

θ_v = the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when $V^* = \phi V_{u.\min}$ to 45 degrees when $V^* = \phi\, V_{u,\max}$.

- If $V^* > \phi V_{\max}$, a failure condition is declared. (AS 8.2.6)
- If V^* exceeds its maximum permitted value $\phi V_{\max}$, the concrete section size should be increased (AS 8.2.6).

Note that if torsion design is considered and torsion reinforcement is required, the calculated shear reinforcement is ignored. Closed stirrups are designed for combined shear and torsion in accordance with AS 8.3.4(b).

The maximum of all of the calculated A_{sv}/s values obtained from each load combination is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

15.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T^*.

- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

15.7.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T^* is permitted to be reduced in accordance with the code (AS 8.3.2). However, the program does not automatically redistribute the internal forces and reduce T^*. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

15.7.3.2 Determine Special Section Properties

For torsion design, special section properties such as A_t, J_t, and u_t are calculated. These properties are described in the following (AS 8.3).

A_t = Area of a polygon with vertices at the center of longitudinal bars at the corners of the cross-section

u_t = Perimeter of the polygon defined by A_t

J_t = Torsional modulus

In calculating the section properties involving reinforcement, such as A_{sw}/s and A_l, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 mm. This is equivalent to 38-mm clear cover and a 12-mm-diameter stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{uc} calculation. With this assumption, the special properties for a rectangular beam section are given as:

$$A_t = (b - 2c)(h - 2c), \qquad \text{(AS 8.3.5)}$$

$$u_t = 2(b - 2c) + 2(h - 2c), \qquad \text{(AS 8.3.6)}$$

$$J_t = 0.33x^2y \qquad \text{(AS 8.3.3)}$$

where, the section dimensions b, h and, c are shown in Figure 15-3. Similarly, the special section properties for a flanged beam section are given as:

$$A_t = (b_w - 2c)(h - 2c), \qquad \text{(AS 8.3.5)}$$

$$u_t = 2(h - 2_c) + 2(b_w - 2_c), \qquad \text{(AS 8.3.6)}$$

$$J_t = 0.33\Sigma x^2y \qquad \text{(AS 8.3.3)}$$

where the section dimensions b_w, h, and c for a flanged beam are shown in Figure 15-3. The values x and y refer to the smaller and larger dimensions of a component rectangle, respectively.

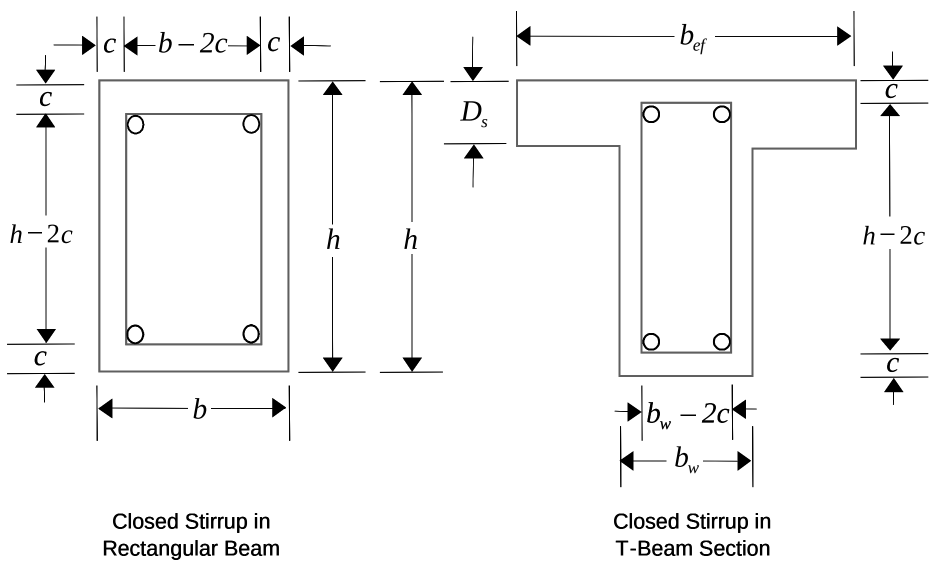


Figure 15-3 Closed stirrup and section dimensions for torsion design

15.7.3.3 Determine Torsion Reinforcement

The torsional strength of the section without torsion reinforcement, T_{uc}, is calculated as:

$$T_{uc} = 0.3\, J_t \sqrt{f'_c} \qquad \text{(AS 8.3.5)}$$

where J_t is the torsion modulus of the concrete cross-section as described in detail in the previous section

Torsion reinforcement also can be ignored if any one of the following is satisfied:

$$T^* \leq 0.25\phi T_{uc} \qquad \text{(AS 8.3.4(a)(i))}$$

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 0.5 \qquad \text{(AS 8.3.4(a)(ii))}$$

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 1 \text{ and } D \leq \max(250\text{mm}, b/2) \qquad \text{(AS 8.3.4(a)(iii))}$$

If the factored torsion T^* alone or in combination with V^* does not satisfy any of the preceding three conditions, torsion reinforcement is needed. It is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (AS 8.3).

- If $T^* > T_{cr}$, the required closed stirrup area per unit spacing, A_{sw}/s, is calculated as:

$$\frac{A_{sw}}{s} = \frac{T^* \tan\theta_t}{\phi 2 f_{sy.f} A_t} \qquad \text{(AS 8.3.5(b))}$$

where, the minimum value of A_{sw}/s is taken as follows:

$$\frac{A_{sw.\min}}{s} = \frac{0.35 b_w}{f_{sy.f}} \qquad \text{(AS 8.2.8)}$$

The value θ_t is the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when $T^* = \phi T_{uc}$ to 45 degrees when $T^* = \phi T_{u,\max}$.

The following equation shall also be satisfied for combined shear and torsion by adding additional shear stirrups.

$$\frac{T^*}{\phi T_{us}} + \frac{V^*}{\phi V_{us}} \leq 1.0 \qquad \text{(AS 8.3.4(b))}$$

where,

$$T_{us} = f_{sy.f}\left(\frac{A_{sw}}{s}\right)2A_t \cot\theta_t \qquad \text{(AS 8.3.5(b))}$$

$$V_{us} = \left(A_{sv} f_{sy.f} d_o / s\right)\cot\theta_v \qquad \text{(AS 8.2.10(a))}$$

The required longitudinal rebar area is calculated as:

$$A_l = \frac{0.5 f_{sy.f}\left(\frac{A_{sw}}{s}\right)u_t \cot^2\theta_t}{f_{sy}} \qquad \text{(AS 8.3.6(a))}$$

An upper limit of the combination of V^* and T^* that can be carried by the section also is checked using the equation:

$$\frac{T^*}{\phi T_{u.\max}} + \frac{V^*}{\phi V_{u.\max}} \leq 1.0 \qquad \text{(AS 8.3.3)}$$

where,

$$V_{u.\max} = 0.2 f'_c b_w d_o \qquad \text{(AS 8.2.6)}$$

$$T_{u.\max} = 0.2 f'_c J_t \qquad \text{(AS 8.3.5(a))}$$

For rectangular sections, b_w is replaced with b. If the combination of V^* and T^* exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T^* > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups satisfy the following limit.

$$\left(\frac{A_{sv}}{s}+2\frac{A_{sw}}{s}\right)\geq\frac{0.35b}{f_{sy.f}} \quad \text{(AS 8.3.7, 8.2.8)}$$

If this equation is not satisfied with the originally calculated A_{sv}/s and A_{sw}/s, A_{sv}/s is increased to satisfy this condition. In that case, A_{sv}/s does not need to satisfy AS Section 8.2.8 independently.

The maximum of all of the calculated A_l and A_{sv}/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements and longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

15.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips typically are governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (AS 3600-09) for prestressed reinforced concrete as described in the following subsections. To learn more about the design strips, refer to the section entitled "ETABS Design Techniques" in the *Key Features and Terminology* manual.

15.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element

boundaries. Controlling reinforcement is computed on either side of these element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Determine the capacity of post-tensioned sections
- Design flexural reinforcement for the strip

These three steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

15.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

15.8.1.2 Determine Capacity of Post-Tensioned Sections

The calculation of the post-tensioned section capacity is identical to that described earlier for rectangular beam sections.

15.8.1.3 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). Where the slab properties (depth and so forth) vary over the width of the strip, the program automatically designs slab widths of each property separately for the bending moment to which they are subjected before summing up the reinforcement for the full width. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

15.8.1.4 Minimum and Maximum Slab Reinforcement

The minimum requirements for untensioned reinforcement in one-way bonded slabs is the same as for beams (AS 9.1.1). Flexural cracking in prestressed slabs shall be deemed controlled if under short-term service loads the resulting stress is less than $0.25\sqrt{f_c'}$; in that case, no further checks are needed (AS 9.4.2). However, if that limit is exceeded, flexural cracking shall be deemed under control by providing reinforcement or bonded tendons, or both, near the tensile face and accomplishing either of the following (AS 9.4.2):

(a) limiting the calculated maximum flexural tensile stress under short-term service loads to $0.6\sqrt{f_c'}$; or

(b) limiting both of the following:

(i) the increment in steel stress near the tension face to 200 MPa, as the load increases from its value when the extreme concrete tensile fiber is at zero stress to the short-term service load value; and

(ii) the center-to-center spacing of reinforcement, including bonded tendons, to 300 mm. (This sub clause is a detailing requirement that is not checked by the program.)

The program checks the stresses in the concrete prestressed flexural members at service loads and after all prestress losses have occurred against the following limit (AS 9.4.2):

- Extreme fiber stress in tension in the precompressed tensile zone at service loads:
  - Extreme fiber stresses in tension for cracked section: $0.6\sqrt{f_c'}$

Thus, although cracking is allowed, it is assumed that the user is limiting the tensile stress at the service stage as presented in AS 9.4.2.

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

15.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code specific items are described in the following sections.

15.8.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d_{om}/2$ from the face of the support (AS 9.2.1.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (AS 9.2.1.3). Figure 15-4 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

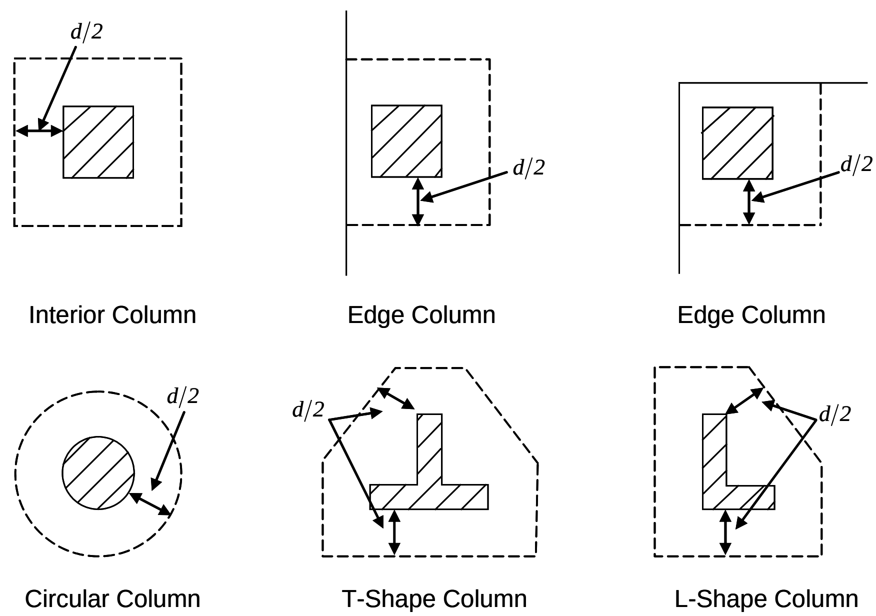


Figure 15-4 Punching Shear Perimeters

15.8.2.2 Determine Concrete Capacity

(i) The ultimate shear strength of a slab where M_v^* is zero, V_{uo}, is given as (AS 9.2.3(a)):

a. when no shear link/stud is present

$$V_{uo} = ud_{om}\left(f_{cv} + 0.3\sigma_{cp}\right) \qquad \text{(AS 9.2.3(a))}$$

b. when shear link/stud is present

$$V_{uo} = ud_{om}\left(0.5\sqrt{f'_c} + 0.3\sigma_{cp}\right) \le 0.2\sqrt{f'_c}\,ud_{om} \qquad \text{(AS 9.2.3(b))}$$

where f_{cv} is taken as the minimum of the following two limits:

$$f_{cv} = \min\begin{cases} 0.17\left(1 + \dfrac{2}{\beta_h}\right)\sqrt{f'_c} \\ 0.34\sqrt{f'_c} \end{cases} \qquad \text{(AS 9.2.3(a))}$$

where, β_h is the ratio of the longest to the minimum dimensions of the critical section.

(ii) The ultimate shear strength of a slab where M_v^* is not zero and no shear reinforcement is provided, V_u, is given as (AS 9.2.4(a)):

$$V_u = V_{uo} \Big/ \left[1.0 + \left(uM_v \big/ 8V^* ad_{om}\right)\right] \qquad \text{(AS 9.2.4(a))}$$

15.8.2.3 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by ETABS.

15.8.3 Design Punching Shear Reinforcement

The design guidelines for shear links or shear studs are not available in AS 3600-2009. ETABS uses the NZS 3101-06 guidelines to design shear studs or shear links.

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 150 mm and not less than 16 times the shear reinforcement bar diameter (NZS 12.7.4.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed, and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

15.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

15.8.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{\max} = 3\ V_{u,\min} = 3 \times V_u \qquad \text{(AS 9.2.4(a), (d))}$$

Where V_u is computed from AS 9.2.3 or 9.2.4. Given V^*, V_u, and $V_{u,\max}$, the required shear reinforcement is calculated as follows, where, ϕ, is the strength reduction factor.

$$\frac{A_{sv}}{s} = \frac{\left(V^* - \phi V_u\right)}{f_{sy} d_{om}}, \qquad \text{(AS 8.2.10)}$$

Minimum punching shear reinforcement should be provided such that:

$$V_s \geq \frac{1}{16}\sqrt{f'_c}\,u d_{om} \qquad \text{(NZS 12.7.4.3)}$$

- If $V^* > \phi V_{\max}$, a failure condition is declared. (NZS 12.7.3.4)
- If V^* exceeds the maximum permitted value of $\phi V_{\max}$, the concrete section should be increased in size.

15.8.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 15-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed $2d$ measured in a direction parallel to the column face (NZS 12.7.4.4).

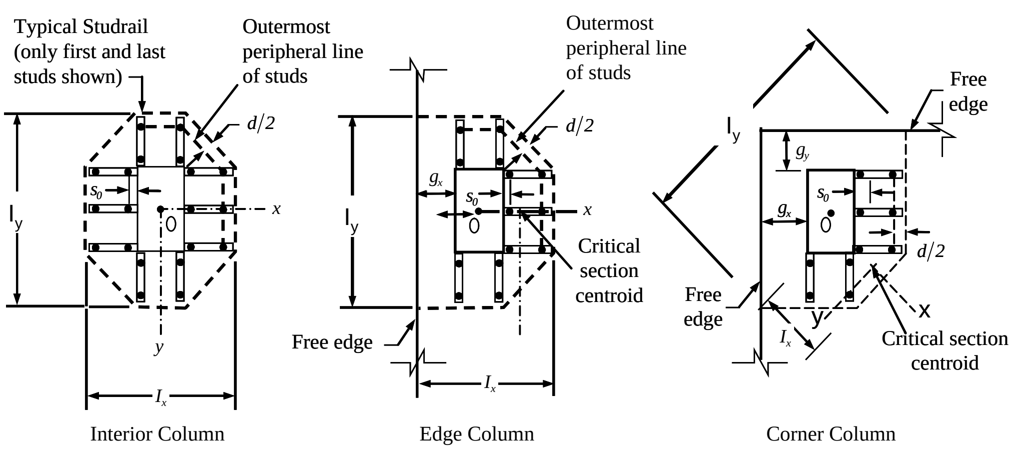


Figure 15-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

15.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in NZS 3.11 plus half of the diameter of the flexural reinforcement.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5d. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed 2d and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.5d \quad \text{(NZS 12.7.4.4)}$$

$$s \leq 0.5d \quad \text{(NZS 12.7.4.4)}$$

$$g \leq 2d \quad \text{(NZS 12.7.4.4)}$$

Chapter 16
Design for ACI 318-11

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the American code ACI 318-11 [ACI 2011]. Various notations used in this chapter are listed in Table 6-1. For referencing to the pertinent sections of the ACI code in this chapter, a prefix “ACI” followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on inch-pound-second units. For simplicity, all equations and descriptions presented in this chapter correspond to inch-pound-second units unless otherwise noted.

16.1 Notations

The following table identifies the various notations used in this chapter.

Table 16-1 List of Symbols Used in the ACI 318-11 Code

A_{cp}	Area enclosed by the outside perimeter of the section, in^2
A_g	Gross area of concrete, in^2
A_l	Total area of longitudinal reinforcement to resist torsion, in^2
A_o	Area enclosed by the shear flow path, sq-in
A_{oh}	Area enclosed by the centerline of the outermost closed transverse torsional reinforcement, sq-in
A_{ps}	Area of prestressing steel in flexural tension zone, in^2
A_s	Area of tension reinforcement, in^2
A'_s	Area of compression reinforcement, in^2
$A_{s(required)}$	Area of steel required for tension reinforcement, in^2
A_t /s	Area of closed shear reinforcement per unit length of member for torsion, sq-in/in
A_v	Area of shear reinforcement, in^2
A_v /s	Area of shear reinforcement per unit length of member, in^2/in
a	Depth of compression block, in
a_b	Depth of compression block at balanced condition, in
$a_{\max}$	Maximum allowed depth of compression block, in
b	Width of member, in
b_f	Effective width of flange (T-beam section), in
b_w	Width of web (T-beam section), in
b_0	Perimeter of the punching critical section, in
b_1	Width of the punching critical section in the direction of bending, in
b_2	Width of the punching critical section perpendicular to the direction of bending, in
c	Depth to neutral axis, in

Table 16-1 List of Symbols Used in the ACI 318-11 Code

c_b	Depth to neutral axis at balanced conditions, in
d	Distance from compression face to tension reinforcement, in
d'	Concrete cover to center of reinforcing, in
d_e	Effective depth from compression face to centroid of tension reinforcement, in
d_s	Thickness of slab (T-beam section), in
d_p	Distance from extreme compression fiber to centroid of prestressing steel, in
E_c	Modulus of elasticity of concrete, psi
E_s	Modulus of elasticity of reinforcement, assumed as 29,000,000 psi (ACI 8.5.2)
f'_c	Specified compressive strength of concrete, psi
f'_{ci}	Specified compressive strength of concrete at time of initial prestress, psi
f_{pe}	Compressive stress in concrete due to effective prestress forces only (after allowance of all prestress losses), psi
f_{ps}	Stress in prestressing steel at nominal flexural strength, psi
f_{pu}	Specified tensile strength of prestressing steel, psi
f_{py}	Specified yield strength of prestressing steel, psi
f_t	Extreme fiber stress in tension in the precompressed tensile zone using gross section properties, psi
f_y	Specified yield strength of flexural reinforcement, psi
f_{ys}	Specified yield strength of shear reinforcement, psi
h	Overall depth of a section, in
h_f	Height of the flange, in
ϕM_n^0	Design moment resistance of a section with tendons only, N-mm

Table 16-1 List of Symbols Used in the ACI 318-11 Code

ϕM_n^{bal}	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
M_u	Factored moment at section, lb-in
N_c	Tension force in concrete due to unfactored dead load plus live load, lb
P_u	Factored axial load at section, lb
s	Spacing of the shear reinforcement along the length of the beam, in
T_u	Factored torsional moment at section, lb-in
V_c	Shear force resisted by concrete, lb
$V_{\max}$	Maximum permitted total factored shear force at a section, lb
V_u	Factored shear force at a section, lb
V_s	Shear force resisted by steel, lb
β_1	Factor for obtaining depth of compression block in concrete
β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
$\varepsilon_{c,\max}$	Maximum usable compression strain allowed in extreme concrete fiber (0.003 in/in)
ε_{ps}	Strain in prestressing steel
ε_s	Strain in reinforcing steel
$\varepsilon_{s,\min}$	Minimum tensile strain allowed in steel reinforcement at nominal strength for tension controlled behavior (0.005 in/in)
φ	Strength reduction factor
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear

Table 16-1 List of Symbols Used in the ACI 318-11 Code

λ	Shear strength reduction factor for light-weight concrete
θ	Angle of compression diagonals, degrees

16.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For ACI 318-11, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the load combinations in the following sections may need to be considered (ACI 9.2.1).

For post-tensioned concrete design, the user can specify the prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning are defined in the following sections.

16.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress forces, in accordance with ACI 318-11 clause 18.4.1. The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0D + 1.0PT (ACI 18.4.1)

16.2.2 Service Load Combination

The following load combinations are used for checking the requirements of prestress for serviceability in accordance with ACI 318-11 clauses 18.3.3, 18.4.2(b), and 18.9.3.2. It is assumed that all long-term losses have already occurred at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT (ACI 18.4.2(b))

16.2.3 Long-Term Service Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with ACI 318-11 clause 18.4.2(a). The permanent load for this load combination is taken as 50 percent of the live load. It is assumed that all long-term losses have already occurred at the service stage.

1.0D + 1.0PT 1.0D + 0.5L + 1.0PT	(ACI 18.4.2(a))

16.2.4 Strength Design Load Combination

The following load combinations are used for checking the requirements of prestress for strength in accordance with ACI 318-11, Chapters 9 and 18.

The strength design combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are automatically determined by ETABS by subtracting out the primary PT moments when the flexural design is carried out.

1.4D + 1.0PT*	(ACI 9.2.1)
1.2D + 1.6L + 1.0PT*	(ACI 9.2.1)
1.2D + 1.6(0.75 PL) + 1.0PT*	(ACI 9.2.1, 13.7.6.3)
0.9D ± 1.0W +1.0PT* 1.2D + 1.0L ± 1.0W + 1.0PT*	(ACI 9.2.1)
0.9D ± 1.0E + 1.0PT* 1.2D + 1.0L ± 1.0E + 1.0PT*	(ACI 9.2.1)
1.2D + 1.6L + 0.5S + 1.0PT* 1.2D + 1.0L + 1.6S + 1.0PT*	(ACI 9.2.1)
1.2D + 1.6S ± 0.5W + 1.0PT* 1.2D + 1.0L + 0.5S ± 1.0W + 1.0PT*	(ACI 9.2.1)
1.2D + 1.0L + 0.2S ± 1.0E + 1.0PT*	(ACI 9.2.1)

* — Replace PT by H for flexural design only

The IBC 2012 basic load combinations (Section 1605.2.1) are the same. These also are the default design load combinations in ETABS whenever the ACI 318-11 code is used. The user should use other appropriate load combinations if roof live load is treated separately, or if other types of loads are present.

16.3 Limits on Material Strength

The concrete compressive strength, f'_c, should not be less than 2500 psi (ACI 5.1.1). The upper limit of the reinforcement yield strength, f_y, is taken as 80 ksi (ACI 9.4) and the upper limit of the reinforcement shear strength, f_{yt}, is taken as 60 ksi (ACI 11.5.2).

ETABS enforces the upper material strength limits for flexure and shear design of slabs. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

16.4 Strength Reduction Factors

The strength reduction factors, ϕ, are applied on the specified strength to obtain the design strength provided by a member. The ϕ factors for flexure, shear, and torsion are as follows:

ϕ_t = 0.90 for flexure (tension controlled) (ACI 9.3.2.1)

ϕ_c = 0.65 for flexure (compression controlled) (ACI 9.3.2.2(b))

ϕ = 0.75 for shear and torsion. (ACI 9.3.2.3)

The value of ϕ varies from compression-controlled to tension-controlled based on the maximum tensile strain in the reinforcement at the extreme edge, ε_t (ACI 9.3.2.2).

Sections are considered compression-controlled when the tensile strain in the extreme tension reinforcement is equal to or less than the compression-controlled strain limit at the time the concrete in compression reaches its assumed strain limit of $\varepsilon_{c.\max}$, which is 0.003. The compression-controlled strain limit is

the tensile strain in the reinforcement at the balanced strain condition, which is taken as the yield strain of the reinforcement, (f_y/E) (ACI 10.3.3).

Sections are tension-controlled when the tensile strain in the extreme tension reinforcement is equal to or greater than 0.005, just as the concrete in compression reaches its assumed strain limit of 0.003 (ACI 10.3.4).

Sections with ε_t between the two limits are considered to be in a transition region between compression-controlled and tension-controlled sections (ACI 10.3.4).

When the section is tension-controlled, ϕ_t is used. When the section is compression-controlled, ϕ_c is used. When the section is in the transition region, ϕ is linearly interpolated between the two values (ACI 9.3.2).

The user is allowed to overwrite these values. However, caution is advised.

16.5 Design Assumptions for Prestressed Concrete

Strength design of prestressed members for flexure and axial loads shall be based on assumptions given in ACI 10.2.

- The strain in the reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis (ACI 10.2.2).
- The maximum usable strain at the extreme concrete compression fiber shall be assumed equal to 0.003 (ACI 10.2.3).
- The tensile strength of the concrete shall be neglected in axial and flexural calculations (ACI 10.2.5).
- The relationship between the concrete compressive stress distribution and the concrete strain shall be assumed to be rectangular by an equivalent rectangular concrete stress distribution (ACI 10.2.7).
- The concrete stress of $0.85f'_c$ shall be assumed uniformly distributed over an equivalent-compression zone bounded by edges of the cross-section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain (ACI 10.2.7.1).

- The distance from the fiber of maximum strain to the neutral axis, c shall be measured in a direction perpendicular to the neutral axis (ACI 10.2.7.2).

Elastic theory shall be used with the following two assumptions:

- The strains shall vary linearly with depth through the entire load range (ACI 18.3.2.1).
- At cracked sections, the concrete resists no tension (ACI 18.3.2.2).

Prestressed concrete members are investigated at the following three stages (ACI 18.3.2):

- At transfer of prestress force
- At service loading
- At nominal strength

The prestressed flexural members are classified as Class U (uncracked), Class T (transition), and Class C (cracked) based on f_t, the computed extreme fiber stress in tension in the precompressed tensile zone at service loads (ACI 18.3.3).

The precompressed tensile zone is that portion of a prestressed member where flexural tension, calculated using gross section properties, would occur under unfactored dead and live loads if the prestress force was not present. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress.

For Class U and Class T flexural members, stresses at service load are determined using uncracked section properties, while for Class C flexural members, stresses at service load are calculated based on the cracked section (ACI 18.3.4).

A prestressed two-way slab system is designed as Class U only with $f_t \leq 6\sqrt{f'_c}$ (ACI R18.3.3); otherwise, an over-stressed (O/S) condition is reported.

The following table provides a summary of the conditions considered for the various section classes.

	Prestressed			Nonprestressed
	Class U	Class T	Class C	
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 18.3.4	Gross section 18.3.4	Cracked section 18.3.4	No requirement
Allowable stress at transfer	18.4.1	18.4.1	18.4.1	No requirement
Allowable compressive stress based on uncracked section properties	18.4.2	18.4.2	No requirement	No requirement
Tensile stress at service loads 18.3.3	$\le 7.5\sqrt{f'_c}$	$7.5\sqrt{f'_c} < f_t \le 12\sqrt{f'_c}$	No requirement	No requirement

16.6 Serviceability Requirements of Flexural Members

16.6.1 Serviceability Check at Initial Service Load

The stresses in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits:

- Extreme fiber stress in compression: $0.60 f'_{ci}$ (ACI 18.4.1(a))
- Extreme fiber stress in tension: $3\sqrt{f'_{ci}}$ (ACI 18.4.1(c))
- Extreme fiber stress in tension at ends of simply supported members: $6\sqrt{f'_{ci}}$ (ACI 18.4.1(c))

The extreme fiber stress in tension at the ends of simply supported members is currently **NOT** checked by ETABS.

16.6.2 Serviceability Checks at Service Load

The stresses in the concrete for Class U and Class T prestressed flexural members at service loads, and after all prestress losses occur, are checked against the following limits:

- Extreme fiber stress in compression due to prestress plus total load: $0.60 f'_c$ (ACI 18.4.2(b))

- Extreme fiber stress in tension in the precompressed tensile zone at service loads:

  – Class U beams and one-way slabs: $f_t \leq 7.5\sqrt{f'_c}$ (ACI 18.3.3)

  – Class U two-way slabs: $f_t \leq 6\sqrt{f'_c}$ (ACI 18.3.3)

  – Class T beams: $7.5\sqrt{f'_c} < f_t \leq 12\sqrt{f'_c}$ (ACI 18.3.3)

  – Class C beams: $f_t \geq 12\sqrt{f'_c}$ (ACI 18.3.3)

For Class C prestressed flexural members, checks at service loads are not required by the code. However, for Class C prestressed flexural members not subject to fatigue or to aggressive exposure, the spacing of bonded reinforcement nearest the extreme tension face shall not exceed that given by ACI 10.6.4 (ACI 18.4.4). It is assumed that the user has checked the requirements of ACI 10.6.4 and ACI 18.4.4.1 to 18.4.4 independently, as these sections are not checked by the program.

16.6.3 Serviceability Checks at Long-Term Service Load

The stresses in the concrete for Class U and Class T prestressed flexural members at long-term service loads, and after all prestress losses occur, are checked against the same limits as for the normal service load, except for the following:

- Extreme fiber stress in compression due to prestress plus total load:

  $0.45 f_c'$ (ACI 18.4.2(a))

16.6.4 Serviceability Checks of Prestressing Steel

The program also performs checks on the tensile stresses in the prestressing steel (ACI 18.5.1). The permissible tensile stress checks, in all types of prestressing steel, in terms of the specified minimum tensile stress f_{pu}, and the minimum yield stress, f_y, are summarized as follows:

- Due to tendon jacking force: $\min(0.94 f_{py}, 0.80 f_{pu})$ (ACI 18.5.1(a))

- At anchors and couplers after force transfer: $0.70f_{pu}$ (ACI 18.5.1(b))

16.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

16.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

16.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of prestressed concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or a flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases the beam may be designed as a rectangular or inverted flanged beam.

16.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 16-1 (ACI 10.2). Furthermore, it is assumed that the net tensile strain in the reinforcement shall not be less than 0.005 (tension controlled) (ACI 10.3.4). When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression reinforcement and additional tension reinforcement.

The design procedure used by ETABS, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow. It is assumed that the design ultimate axial force does not exceed $\phi(0.1f'_cA_g)$ (ACI 10.3.5); hence all beams are designed for major direction flexure, shear, and torsion only.

16.7.1.2.1 Design of Rectangular Beams

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, the moment capacity ϕM_n^0 is determined as follows:

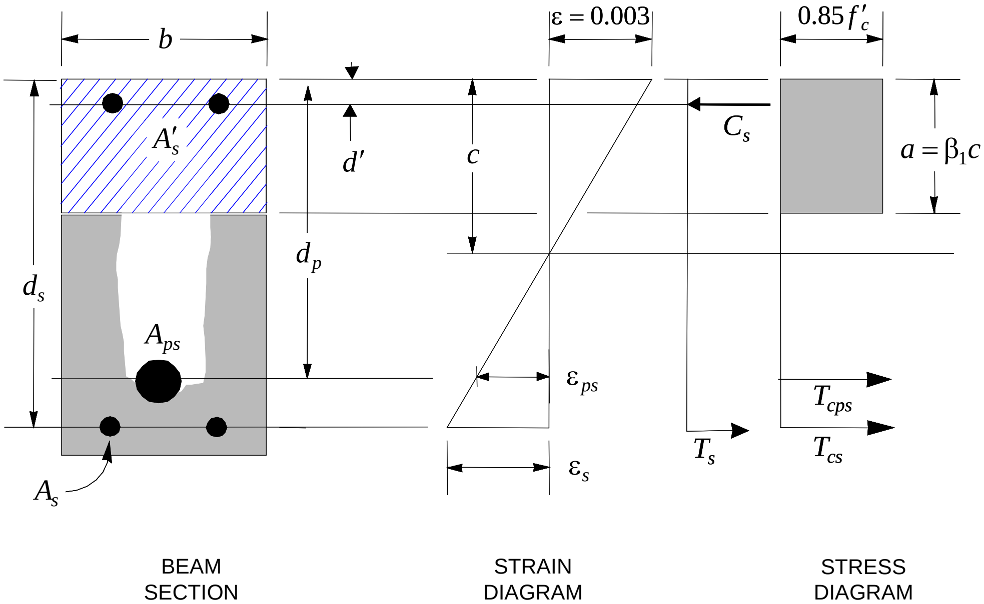


Figure 16-1 Rectangular Beam Design

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tension reinforcement strain shall not be less than ε_{smin}, which is equal to 0.005 for tension-controlled behavior (ACI 10.3.4):

$$c_{max} = \left(\frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}}\right) d \qquad \text{(ACI 10.2.2)}$$

where,

$$\varepsilon_{cmax} = 0.003 \qquad \text{(ACI 10.2.3)}$$

$$\varepsilon_{smin} = 0.005 \qquad \text{(ACI 10.3.4)}$$

Therefore, the limit $c \le c_{max}$ is set for tension-controlled sections.

The maximum allowable depth of the rectangular compression block, a_{max}, is given by:

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 10.2.7.1)}$$

where β_1 is calculated as:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right), \quad 0.65 \le \beta_1 \le 0.85 \qquad \text{(ACI 10.2.7.3)}$$

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{ps}, is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel.

Based on the calculated f_{ps}, the depth of the neutral axis is recalculated, and f_{ps} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \beta_1 c$$

- If $c \le c_{max}$ (ACI 10.3.4), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$\phi M_n^0 = \phi A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)$$

- If $c > c_{max}$ (ACI 10.3.4), a failure condition is declared.

If $M_u > \phi M_n^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of the neutral axis, c is equal to c_{max}. The stress in the post-tensioning steel, f_{ps} is then calculated and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = 0.85 f'_c a_{max} b$$

$$T = A_{ps} f_{ps}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{0.85 f'_c a_{\max} b - A_{ps} f_{ps}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$\phi M_n^{bal} = \phi A_{ps} f_{ps}^{bal} \left(d_p - \frac{a_{\max}}{2} \right) + \phi A_s^{bal} f_s^{bal} \left(d_s - \frac{a_{\max}}{2} \right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, *c*.

16.7.1.2.1.1 Case 1: Post-tensioning steel is adequate

When $M_u < \phi M_n^0$, the amount of post-tensioning steel is adequate to resist the design moment M_u. Minimum reinforcement is provided to satisfy ductility requirements (ACI 18.9.3.2 and 18.9.3.3), i.e., $M_u < \phi M_n^0$.

16.7.1.2.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M_u, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $\phi M_n^0 < M_u < \phi M_n^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M_u and reports this required area of tension reinforcement. Since M_u is bounded by ϕM_n^0 at the lower end and ϕM_n^{bal} at the upper end, and ϕM_n^0 is associated with $A_s = 0$ and ϕM_n^{bal} is associated with $A_s = A_s^{bal}$, the required area will fall within the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M_u is positive, or at the top if M_u is negative.

16.7.1.2.1.3 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M_u > \phi M_n^{bal}$, compression reinforcement is required (ACI 10.3.5). In this case ETABS assumes that the depth of the neutral axis, c, is equal to $c_{\max}$. The values of f_{ps} and f_s reach their respective balanced condition values, f_{ps}^{bal} and f_s^{bal}. The area of compression reinforcement, $A_s^{'}$, is then determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M_u - \phi M_n^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(f'_s - 0.85 f'_c\right)\left(d_e - d'\right)\phi}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c\max}\left[\frac{c_{\max} - d'}{c_{\max}}\right] \le f_y \qquad \text{(ACI 10.2.2, 10.2.3, 10.2.4)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{f_y\left(d_s - d'\right)\phi}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and vice versa if M_u is negative.

16.7.1.2.2 Design of Flanged Beams

16.7.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_u (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as above, i.e., no flanged beam data is used.

16.7.1.2.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that A_s = 0. In that case, the moment capacity ϕM_n^0 is determined as follows:

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tension reinforcement strain shall not be less than ε_{smin}, which is equal to 0.005 for tension-controlled behavior (ACI 10.3.4):

$$c_{max} = \left(\frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}} \right) d \qquad \text{(ACI 10.2.2)}$$

where,

$$\varepsilon_{c\max} = 0.003 \qquad \text{(ACI 10.2.3)}$$

$$\varepsilon_{s\min} = 0.005 \qquad \text{(ACI 10.3.4)}$$

Therefore, the limit $c \le c_{max}$ is set for tension-controlled section:

The maximum allowable depth of the rectangular compression block, a_{max}, is given by:

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 10.2.7.1)}$$

where β_1 is calculated as:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000} \right), \qquad 0.65 \le \beta_1 \le 0.85 \qquad \text{(ACI 10.2.7.3)}$$

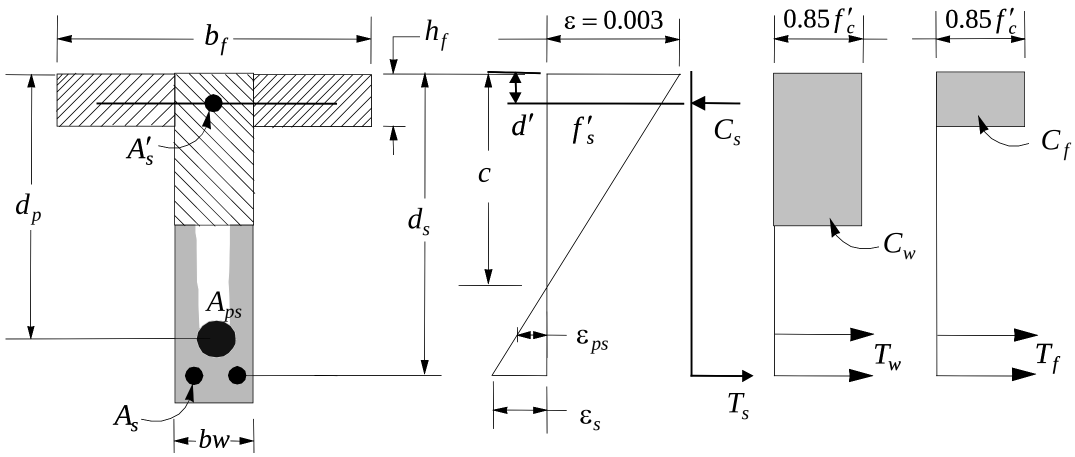


Figure 16-2 T-Beam Design

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{ps} is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel. Based on the calculated f_{ps}, the depth of the neutral axis is recalculated, and f_{ps} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \beta_1 c$$

- If $c \leq c_{\max}$ (ACI 10.3.4), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$\phi M_n^0 = \phi A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right)$$

- If $c > c_{\max}$ (ACI 10.3.4), a failure condition is declared.

If $M_u > \phi M_n^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension-controlled case. In that case, it is assumed that the depth of the neutral axis c is equal to c_{max}. The stress in the post-tensioning steel, f_{ps}, is then calculated and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in that case the width of the beam is taken as b_f. Compression reinforcement is required if $a > a_{max}$.
- If $a > h_f$, the calculation for A_s is given by:

$$C = 0.85 f'_c\, A_c^{comp}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$A_c^{com} = b_f h_f + b_w\left(a_{\max} - h_f\right)$$

$$T = A_{ps} f_{ps}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{0.85 f'_c\, A_c^{com} - A_{ps} f_{ps}^{bal}}{f_s^{bal}}$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, c.

Case 1: Post-tensioning steel is adequate

When $M_u < \phi M_n^0$ the amount of post-tensioning steel is adequate to resist the design moment M_u. Minimum reinforcement is provided to satisfy ductility requirements (ACI 18.9.3.2 and 18.9.3.3), i.e., $M_u < \phi M_n^0$.

Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M_u, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $\phi M_n^0 < M_u < \phi M_n^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M_u and reports this required area of tension reinforcement. Since M_u is bounded by ϕM_n^0 at the lower end and ϕM_n^{bal} at the upper end, and ϕM_n^0 is associated with $A_s = 0$ and ϕM_n^{bal} is associated with $A_s = A_s^{bal}$, the required area will fall within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if M_u is positive, or at the top if M_u is negative.

Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M_u > \phi M_n^{bal}$, compression reinforcement is required (ACI 10.3.5). In that case, ETABS assumes that the depth of the neutral axis, c, is equal to $c_{\max}$. The value of f_{ps} and f_s reach their respective balanced condition values, f_{ps}^{bal} and f_s^{bal}. The area of compression reinforcement, A_s', is then determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M_u - \phi M_n^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(f'_s - 0.85 f'_c\right)\left(d_s - d'\right)\phi}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c\max}\left[\frac{c_{\max} - d'}{c_{\max}}\right] \leq f_y \qquad \text{(ACI 10.2.2, 10.2.3, and 10.2.4)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{f_y\left(d_s - d'\right)\phi}$$

Therefore, the total tension reinforcement, A_s = A_s^{bal} + A_s^{com}, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and vice versa if M_u is negative.

16.7.1.2.3 Ductility Requirements

ETABS also checks the following condition by considering the post-tensioning steel and tension reinforcement to avoid abrupt failure.

$$\phi M_n \geq 1.2 M_{cr} \qquad \text{(ACI 18.8.2)}$$

The preceding condition is permitted to be waived for the following:

(a) Two-way, unbonded post-tensioned slabs

(b) Flexural members with shear and flexural strength at least twice that required by ACI 9.2.

These exceptions currently are **NOT** handled by ETABS.

16.7.1.2.4 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following limit:

$$A_s \geq 0.004\, A_{ct} \qquad \text{(ACI 18.9.2)}$$

where, A_{ct} is the area of the cross-section between the flexural tension face and the center of gravity of the gross section.

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \le \begin{cases} 0.4bd & \text{Rectangular beam} \\ 0.4b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \le \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

16.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular loading combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V_u.
- Determine the shear force, V_c that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

16.7.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

16.7.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as:

$$V_c = \min(V_{ci}, V_{cw}) \qquad \text{(ACI 11.3.3)}$$

where,

$$V_{ci} = 0.6\lambda\sqrt{f'_c}\,b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}} \ge 1.7\lambda\sqrt{f'_c}\,b_w d \qquad \text{(ACI 11.3.3.1)}$$

$$V_{cw} = \left(3.5\lambda\sqrt{f'_c} + 0.3 f_{pc}\right) b_w d_p + V_p \qquad \text{(ACI 11.3.3.2)}$$

$$d_p \geq 0.80h \qquad \text{(ACI 11.3.3.1)}$$

$$M_{cre} = \left(\frac{I}{y_t}\right)\left(6\lambda\sqrt{f'_c} + f_{pe} - f_d\right) \qquad \text{(ACI 11.3.3.1)}$$

where,

f_d = stress due to unfactored dead load, at the extreme fiber of the section where tensile stress is caused by externally applied loads, psi

f_{pe} = compress stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at the extreme fiber of the section where tensile stress is caused by externally applied loads, psi

V_d = shear force at the section due to unfactored dead load, lbs

V_p = vertical component of effective prestress force at the section, lbs

V_{ci} = nominal shear strength provided by the concrete when diagonal cracking results from combined shear and moment

M_{cre} = moment causing flexural cracking at the section because of externally applied loads

$M_{\max}$ = maximum factored moment at section because of externally applied loads

V_i = factored shear force at the section because of externally applied loads occurring simultaneously with $M_{\max}$

V_{cw} = nominal shear strength provided by the concrete when diagonal cracking results from high principal tensile stress in the web

For light-weight concrete, the $\sqrt{f'_c}$ term is multiplied by the shear strength reduction factor λ.

16.7.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{\max} = V_c + \left(8\sqrt{f'_c}\right) b_w d \qquad \text{(ACI 11.4.7.9)}$$

Given V_u, V_c, and $V_{\max}$, the required shear reinforcement is calculated as follows where, ϕ, the strength reduction factor, is 0.75 (ACI 9.3.2.3).

- If $V_u \le 0.5\phi V_c$

$$\frac{A_v}{s} = 0 \qquad \text{(ACI 11.4.6.1)}$$

- If $0.5\phi V_c < V_u \le \phi V_{\max}$

$$\frac{A_v}{s} = \frac{\left(V_u - \phi V_c\right)}{\phi\, f_{yt} d} \qquad \text{(ACI 11.4.7.1, 11.4.7.2)}$$

$$\frac{A_v}{s} \ge \max\left(\frac{0.75\lambda\sqrt{f'_c}}{f_{yt}} b_w, \quad \frac{50 b_w}{f_{yt}}\right) \qquad \text{(ACI 11.4.6.3)}$$

- If $V_u > \phi V_{\max}$, a failure condition is declared (ACI 11.4.7.9).

For members with an effective prestress force not less than 40 percent of the tensile strength of the flexural reinforcement, the required shear reinforcement is computed as follows (ACI 11.4.6.3, 11.4.6.4):

$$\frac{A_v}{s} \ge \min\begin{cases} \max\left(\dfrac{0.75\lambda\sqrt{f'_c}}{f_y} b_w, \dfrac{50}{f_y} b_w\right) \\ \dfrac{A_{ps} f_{pu}}{80 f_{yt} d}\sqrt{\dfrac{d}{b_w}} \end{cases}$$

- If V_u exceeds the maximum permitted value of $\phi V_{\max}$, the concrete section should be increased in size (ACI 11.4.7.9).

Note that if torsion design is considered and torsion reinforcement is needed, the equation given in ACI 11.5.6.3 does not need to be satisfied independently. See the next section *Design of Beam Torsion Reinforcement* for details.

If the beam depth h is less than the minimum of 10 in, $2.5h_f$, and $0.5b_w$, the minimum shear reinforcement given by ACI 11.5.6.3 is not enforced (ACI 11.5.6.1(c)).

The maximum of all of the calculated A_v/s values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

16.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_u.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

16.7.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors (ACI 11.6.2).

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T_u is permitted to be reduced in accordance with the code (ACI 11.6.2.2). However, the program does not automatically redistribute the internal forces and reduce T_u. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

16.7.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_{cp}, A_{oh}, A_o, p_{cp}, and p_h are calculated. These properties are described in the following (ACI 2.1).

A_{cp} = Area enclosed by outside perimeter of concrete cross-section

A_{oh} = Area enclosed by centerline of the outermost closed transverse torsional reinforcement

A_o = Gross area enclosed by shear flow path

p_{cp} = Outside perimeter of concrete cross-section

p_h = Perimeter of centerline of outermost closed transverse torsional reinforcement

In calculating the section properties involving reinforcement, such as A_{oh}, A_o, and p_h, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 1.75 inches. This is equivalent to 1.5 inches clear cover and a #4 stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a rectangular beam section are given as:

$$A_{cp} = bh \qquad \text{(ACI 11.5.1, 2.1)}$$

$$A_{oh} = (b - 2c)(h - 2c) \qquad \text{(ACI 11.5.3.1, 2.1, R11.5.3.6(b))}$$

$$A_o = 0.85\, A_{oh} \qquad \text{(ACI 11.5.3.6, 2.1)}$$

$$p_{cp} = 2b + 2h \qquad \text{(ACI 11.5.1, 2.1)}$$

$$p_h = 2(b - 2c) + 2(h - 2c) \qquad \text{(ACI 11.5.3.1, 2.1)}$$

where, the section dimensions b, h, and c are shown in Figure 16-3. Similarly, the special section properties for a flanged beam section are given as:

$$A_{cp} = b_w h + (b_f - b_w)h_f \qquad \text{(ACI 11.5.1, 2.1)}$$

$$A_{oh} = (b_w - 2c)(h - 2c) \qquad \text{(ACI 11.5.3.1, 2.1, R11.5.3.6(b))}$$

$$A_o = 0.85\, A_{oh} \qquad \text{(ACI 11.5.3.6, 2.1)}$$

$$p_{cp} = 2b_f + 2h \qquad \text{(ACI11.5.1, 2.1)}$$

$$p_h = 2(h - 2c) + 2(b_w - 2c) \qquad \text{(ACI 11.5.3.1, 2.1)}$$

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 16-3. Note that the flange width on either side of the beam web is limited to the smaller of $4h_f$ or $(h - h_f)$ (ACI 13.2.4).

16.7.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, T_{cr}, for which the torsion in the section can be ignored is calculated as:

$$T_{cr} = \varphi\lambda\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\lambda\sqrt{f'_c}}} \qquad \text{(ACI 11.5.1(b))}$$

where A_{cp} and p_{cp} are the area and perimeter of the concrete cross-section as described in detail in the previous section; f_{pc} is the concrete compressive stress at the centroid of the section; ϕ is the strength reduction factor for torsion, which is equal to 0.75 by default (ACI 9.3.2.3); and f'_c is the specified concrete compressive strength.

16.7.3.4 Determine Torsion Reinforcement

If the factored torsion T_u is less than the threshold limit, T_{cr}, torsion can be safely ignored (ACI 11.5.1). In that case, the program reports that no torsion reinforcement is required. However, if T_u exceeds the threshold limit, T_{cr}, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonal (ACI R11.5.3.6).

If $T_u > T_{cr}$ the required closed stirrup area per unit spacing, A_t/s, is calculated as:

$$\frac{A_t}{s} = \frac{T_u \tan\theta}{\phi 2 A_o f_{yt}} \qquad \text{(ACI 11.5.3.6)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{T_u p_h}{\phi 2 A_o f_y \tan\theta} \qquad \text{(ACI 11.5.3.7, 11.5.3.6)}$$

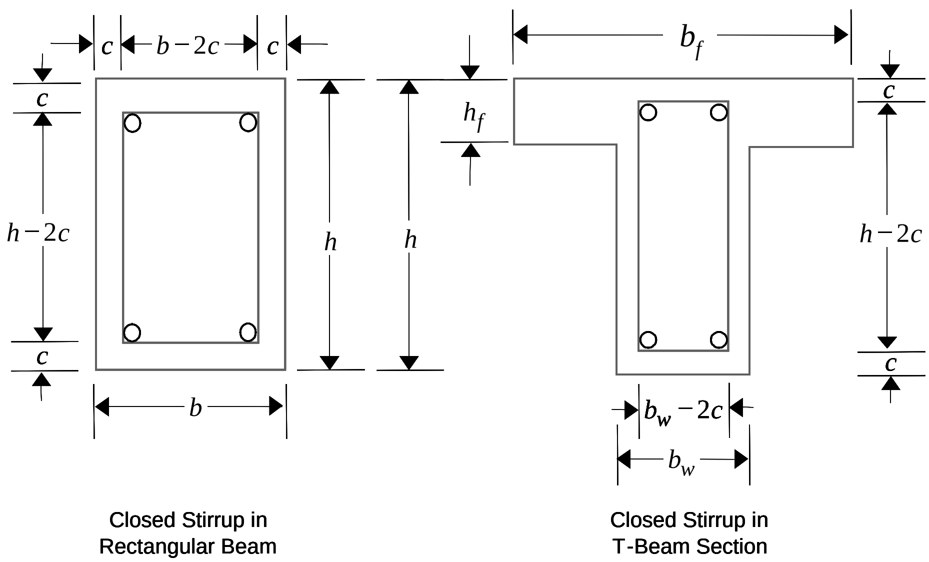


Figure 16-3 Closed stirrup and section dimensions for torsion design

where, the minimum value of A_t/s is taken as:

$$\frac{A_t}{s} = \frac{25}{f_{yt}} b_w \qquad \text{(ACI 11.5.5.3)}$$

and the minimum value of A_l is taken as follows:

$$A_l = \frac{5\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yt}}{f_y}\right) \qquad \text{(ACI 11.5.5.3)}$$

In the preceding expressions, θ is taken as 45 degrees for prestressed members with an effective prestress force less than 40 percent of the tensile strength of the longitudinal reinforcement; otherwise θ is taken as 37.5 degrees.

An upper limit of the combination of V_u and T_u that can be carried by the section is also checked using the equation:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \qquad \text{(ACI 11.5.3.1)}$$

For rectangular sections, b_w is replaced with b. If the combination of V_u and T_u exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_u > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

$$\left(\frac{A_v}{s} + 2\frac{A_t}{s}\right) \ge \max\left\{0.75\lambda\frac{\sqrt{f'_c}}{f_{yt}}b_w, \frac{50b_w}{f_y}\right\} \qquad \text{(ACI 11.5.5.2)}$$

If this equation is not satisfied with the originally calculated A_v/s and A_t/s, A_v/s is increased to satisfy this condition. In that case, A_v/s does not need to satisfy the ACI Section 11.5.6.3 independently.

The maximum of all of the calculated A_l and A_t/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements considered by the program are based purely on strength considerations. Any minimum stirrup requirements and longitudinal reinforcement requirements to satisfy spacing considerations must be investigated independently of the program by the user.

16.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is completed using the ultimate strength design method (ACI 318-11) for prestressed reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "ETABS Design Features" in the *Key Features and Terminology* manual.

16.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is completed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Determine the capacity of post-tensioned sections.
- Design flexural reinforcement for the strip.

These three steps are described in the subsection that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

16.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

16.8.1.2 Determine Capacity of Post-Tensioned Sections

Calculation of the post-tensioned section capacity is identical to that described earlier for rectangular beam sections.

16.8.1.3 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

16.8.1.3.1 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (ACI 7.12.2, 13.3.1):

$$A_{s,\min} = 0.0020\, bh \text{ for } f_y = 40 \text{ ksi or } 50 \text{ ksi} \qquad \text{(ACI 7.12.2.1(a))}$$

$$A_{s,\min} = 0.0018\, bh \text{ for } f_y = 60 \text{ ksi} \qquad \text{(ACI 7.12.2.1(b))}$$

$$A_{s,\min} = \frac{0.0018 \times 60000}{f_y} bh \text{ for } f_y > 60 \text{ ksi} \qquad \text{(ACI 7.12.2.1(c))}$$

Reinforcement is not required in positive moment areas where f_t, the extreme fiber stress in tension in the precompressed tensile zone at service loads (after all prestress losses occurs) does not exceed $2\sqrt{f'_c}$ (ACI 18.9.3.1).

In positive moment areas where the computed tensile stress in the concrete at service loads exceeds $2\sqrt{f_c'}$, the minimum area of bonded reinforcement is computed as:

$$A_{s,\min} = \frac{N_c}{0.5 f_y}, \text{ where } f_y \leq 60 \text{ ksi} \qquad \text{(ACI 18.9.3.2)}$$

In negative moment areas at column supports, the minimum area of bonded reinforcement in the top of slab in each direction is computed as:

$$A_{s,\min} = 0.00075 A_{cf} \qquad \text{(ACI 18.9.3.3)}$$

where A_{cf} is the larger gross cross-sectional area of the slab-beam strip in the two orthogonal equivalent frames intersecting a column in a two-way slab system.

When spacing of tendons exceed 54 inches, additional bonded shrinkage and temperature reinforcement (as computed above, ACI 7.12.2.1) is required between the tendons at slab edges, extending from the slab edge for a distance equal to the tendon spacing (ACI 7.12.3.3)

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area. Note that the requirements when f_y > 60 ksi currently are not handled.

16.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Key Features and Terminology* manual. Only the code specific items are described in the following sections.

16.8.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of *d*/*2* from the face of the support (ACI 11.11.1.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (ACI 11.11.1.3). Figure 16-4 shows the auto punching perimeters considered by ETABS for the various column shapes.

The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

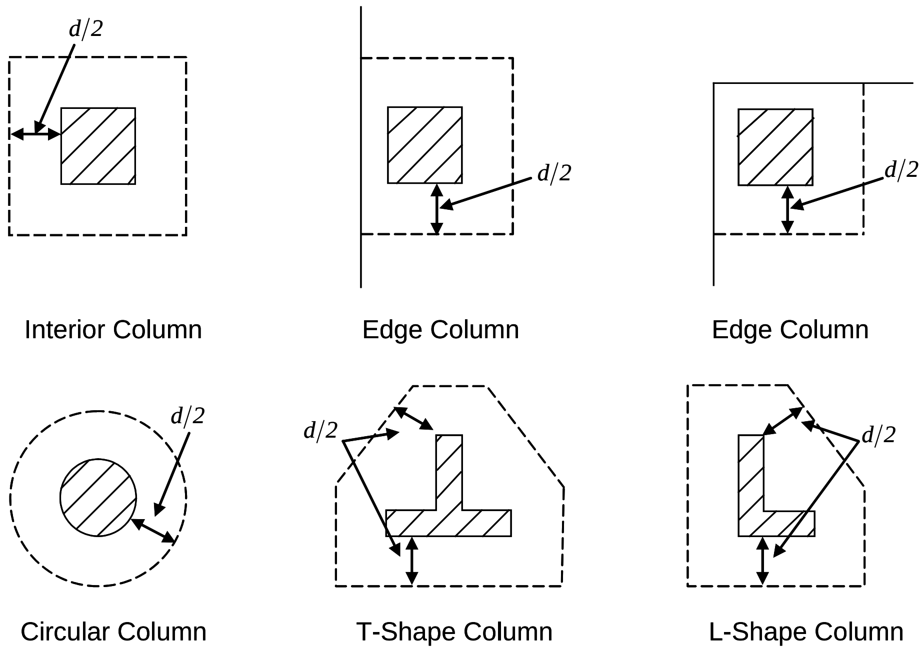


Figure 16-4 Punching Shear Perimeters

16.8.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$.

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}} \qquad \text{(ACI 13.5.3.2)}$$

$$\gamma_v = 1 - \gamma_f \qquad \text{(ACI 13.5.3.1)}$$

For flat plates, γ_v is determined from the following equations taken from ACI 421.2R-07 [ACI 2007] *Seismic Design of Punching Shear Reinforcement in Flat Plates*.

For interior columns,

$$\gamma_{vx} = 1 - \frac{1}{1 + (2/3)\sqrt{l_y/l_x}} \qquad \text{(ACI 421.2 C-11)}$$

$$\gamma_{vy} = 1 - \frac{1}{1 + (2/3)\sqrt{l_x/l_y}} \qquad \text{(ACI 421.2 C-12)}$$

For edge columns,

γ_{vx} = same as for interior columns (ACI 421.2 C-13)

$$\gamma_{vy} = 1 - \frac{1}{1 + (2/3)\sqrt{l_x/l_y - 0.2}} \qquad \text{(ACI 421.2 C-14)}$$

$\gamma_{vy} = 0$ when $l_x/l_y \leq 0.2$

For corner columns,

$\gamma_{vx} = 0.4$ (ACI 421.2 C-15)

γ_{vy} = same as for edge columns (ACI 421.2 C-16)

NOTE: Program uses ACI 421.2-12 and ACI 421.2-15 equations in lieu of ACI 421.2 C-14 and ACI 421.2 C-16 which are currently NOT enforced.

where b_1 is the width of the critical section measured in the direction of the span and b_2 is the width of the critical section measured in the direction perpendicular to the span. The values l_x and l_y are the projections of the shear-critical section onto its principal axes, x and y, respectively.

16.8.2.3 Determine Concrete Capacity

The concrete punching shear stress capacity of a two-way prestressed section is taken as:

$$v_c = \left(\beta_p \lambda \sqrt{f'_c} + 0.3 f_{pc}\right) + v_p \qquad \text{(ACI 11.11.2.2)}$$

$$\beta_p = \min\left(3.5, \left(\frac{\alpha_s d}{b_o} + 1.5\right)\right) \qquad \text{(ACI 11.11.2.2)}$$

where, β_p is the factor used to compute v_c in prestressed slab; b_o is the perimeter of the critical section; f_{pc} is the average value of f_{pc} in the two directions; v_p is the vertical component of all effective prestress stresses crossing the critical section; and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 40 & \text{for interior columns,} \\ 30 & \text{for edge columns, and} \\ 20 & \text{for corner columns.} \end{cases} \qquad \text{(ACI 11.11.2.1)}$$

The concrete capacity v_c computed from ACI 11.12.2.2 is permitted only when the following conditions are satisfied:

- The column is farther than four times the slab thickness away from any discontinuous slab edges.
- The value of $\sqrt{f'_c}$ is taken no greater than 70 psi.
- In each direction, the value of f_{pc} is within the range:

  $125 \le f_{pc} \le 500$ psi

In thin slabs, the slope of the tendon profile is hard to control and special care should be exercised in computing v_p. In case of uncertainty between the design and as-built profile, a reduced or zero value for v_p should be used.

If the preceding three conditions are not satisfied, the concrete punching shear stress capacity of a two-way prestressed section is taken as the minimum of the following three limits:

$$v_c = \min \begin{cases} \left(2 + \dfrac{4}{\beta_c}\right)\lambda\sqrt{f'_c} \\ \left(2 + \dfrac{\alpha_s d}{b_c}\right)\lambda\sqrt{f'_c} \\ 4\lambda\sqrt{f'_c} \end{cases} \qquad \text{(ACI 11.11.2.1)}$$

where, β_c is the ratio of the maximum to the minimum dimensions of the critical section, b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section (ACI 11.12.2.1).

A limit is imposed on the value of $\sqrt{f'_c}$ as:

$$\sqrt{f'_c} \le 100 \qquad \text{(ACI 11.1.2)}$$

16.8.2.4 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS.

16.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 6 inches, and not less than 16 times the shear reinforcement bar diameter (ACI 11.11.3). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

16.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a two-way prestressed section with punching shear reinforcement is as previously determined, but limited to:

$$v_c \le 2\lambda\sqrt{f'_c} \text{ for shear links} \qquad \text{(ACI 11.11.3.1)}$$

$$v_c \le 3\lambda\sqrt{f'_c} \text{ for shear studs} \qquad \text{(ACI 11.11.5.1)}$$

16.8.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$V_{max} = 6\sqrt{f'_c}\, b_o d$ for shear links (ACI 11.11.3.2)

$V_{max} = 8\sqrt{f'_c}\, b_o d$ for shear studs (ACI 11.11.5.1)

Given V_u, V_c, and V_{max}, the required shear reinforcement is calculated as follows, where, ϕ, the strength reduction factor, is 0.75 (ACI 9.3.2.3).

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d} \quad \text{(ACI 11.4.7.1, 11.4.7.2)}$$

$$\frac{A_v}{s} \geq 2\frac{\sqrt{f'_c}}{f_y} b_o \quad \text{for shear studs}$$

- If $V_u > \phi V_{max}$, a failure condition is declared. (ACI 11.11.3.2)
- If V_u exceeds the maximum permitted value of ϕV_{max}, the concrete section should be increased in size.

16.8.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 16-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed d/2 (ACI R11.3.3, 11.11.5.2). The spacing between adjacent shear reinforcement in the first line of shear reinforcement shall not exceed 2*d* measured in a direction parallel to the column face (ACI 11.11.3.3).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

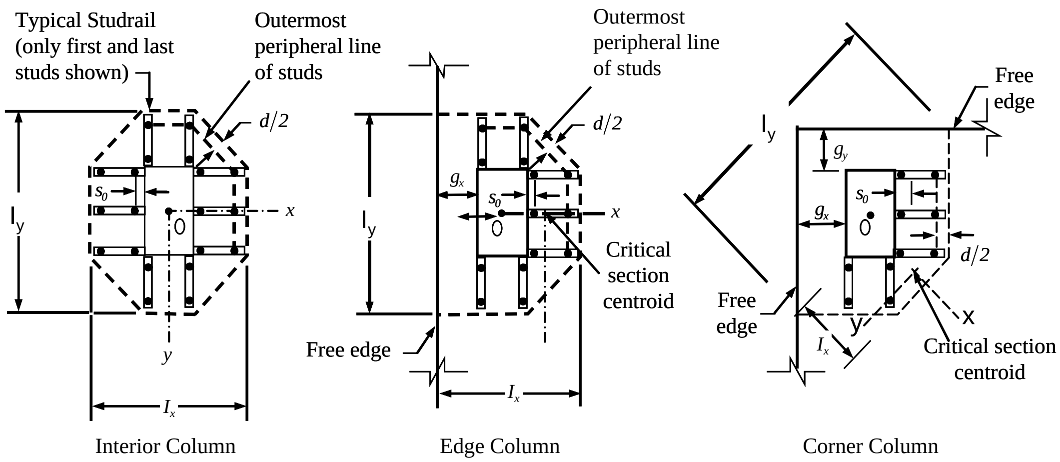


Figure 16-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

16.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in ACI 7.7 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 3/8-, 1/2-, 5/8-, and 3/4-inch diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.35d. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \le 0.5d \qquad \text{(ACI 11.11.5.2)}$$

$$s \le \begin{cases} 0.75d & \text{for} \quad v_u \le 6\phi\lambda\sqrt{f'_c} \\ 0.50d & \text{for} \quad v_u > 6\phi\lambda\sqrt{f'_c} \end{cases} \qquad \text{(ACI 11.11.5.2)}$$

$$g \le 2d \qquad \text{(ACI 11.11.5.3)}$$

The limits of s_o and the spacing, s, between the links are specified as:

$$s_o \leq \ 0.5d \qquad \text{(ACI 11.11.3)}$$

$$s \leq 0.50d \qquad \text{(ACI 11.11.3)}$$

Chapter 17
Design for TS 3233-1979

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the TS 500-2000. When the aforementioned code is selected in ETABS, program design meets the requirement of the TS 3233-1979 [TS 3233]. Various notations used in this chapter are listed in Table 17-1. For referencing to the pertinent sections or equations of the TS code in this chapter, a prefix "TS" followed by the section or equation number is used herein.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

17.1 Notations

The following table identifies the various notations used in this chapter.

Table 17-1 List of Symbols Used in the TS 3233-1979 Code

A_{cp}	Area enclosed by the outside perimeter of the section, mm^2
A_g	Gross area of concrete, mm^2
A_l	Area of longitudinal reinforcement for torsion, mm^2
A_o	Area enclosed by the shear flow path, mm^2
A_{oh}	Area enclosed by the centerline of the outermost closed transverse torsional reinforcement, mm^2
A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{ot}/s	Area of transverse torsion reinforcement (closed stirrups) per unit length of the member, mm^2/mm
A_{ov}/s	Area of transverse shear reinforcement per unit length of the member, mm^2/mm
a	Depth of compression block, mm
A_{sw}	Area of shear reinforcement, mm^2
A_{sw}/s	Area of shear reinforcement per unit length of the member, mm^2/mm
$a_{\max}$	Maximum allowed depth of compression block, mm
b	Width of section, mm
b_f	Effective width of flange (flanged section), mm
b_o	Perimeter of the punching shear critical section, mm
b_w	Width of web (flanged section), mm
b_1	Width of the punching shear critical section in the direction of bending, mm
b_2	Width of the punching shear critical section perpendicular to the direction of bending, mm
c	Depth to neutral axis, mm

Table 17-1 List of Symbols Used in the TS 3233-1979 Code

d	Distance from compression face to tension reinforcement, mm
d'	Distance from compression face to compression reinforcement, in
E_c	Modulus of elasticity of concrete, N/mm^2
E_s	Modulus of elasticity of reinforcement, N/mm^2
f_{cd}	Designed compressive strength of concrete, N/mm^2
f_{ck}	Characteristic compressive strength of concrete, N/mm^2
f_{ctk}	Characteristic tensile strength of concrete, N/mm^2
f_{yd}	Designed yield stress of flexural reinforcement, N/mm^2.
f_{yk}	Characteristic yield stress of flexural reinforcement, N/mm^2.
f_{ywd}	Designed yield stress of transverse reinforcement, N/mm^2.
h	Overall depth of a section, mm
h_f	Height of the flange, mm
M_d	Design moment at a section, N/mm
V_d	Design axial load at a section, N
p_{cp}	Outside perimeter of concrete cross-section, mm
p_h	Perimeter of centerline of outermost closed transverse torsional reinforcement, mm
s	Spacing of shear reinforcement along the beam, mm
T_{cr}	Critical torsion capacity, N/mm
T_d	Design torsional moment at a section, N/mm
V_c	Shear force resisted by concrete, N
V_{max}	Maximum permitted total factored shear force at a section, N
V_s	Shear force resisted by transverse reinforcement, N

Table 17-1 List of Symbols Used in the TS 3233-1979 Code

V_d	Design shear force at a section, N
α_s	Punching shear scale factor based on column location
β_c	Ratio of the maximum to the minimum dimensions of the punching shear critical section
k_1	Factor for obtaining depth of the concrete compression block
ε_c	Strain in the concrete
$\varepsilon_{c\ max}$	Maximum usable compression strain allowed in the extreme concrete fiber, (0.003 mm / mm)
ε_s	Strain in the reinforcement
ε_{cu},	Maximum usable compression strain allowed in extreme concrete fiber (0.003 mm/mm)
ε_s	Strain in reinforcing steel
γ_m	Material factor
γ_{mc}	Material factor for concrete
λ	Shear strength reduction factor for light-weight concrete

17.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For TS 3233-1979, if a structure is subjected to dead (G), live (Q), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the following load combinations may need to be considered (TS 8.4.3).

For post-tensioned concrete design, the user can specify the prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning are defined in the following sections.

These are the default design load combinations in ETABS whenever the TS 3233-1979 code is used. The user should use other appropriate load combinations if roof live load is treated separately, or if other types of loads are present.

17.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress force in accordance with TS 3233 Section 9.1.1. The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0D + 1.0PT

17.2.2 Service Load Combination

The following characteristic load combinations are used for checking the requirements of prestress for serviceability in accordance with TS 3233 Section 9.1.1. It is assumed that all long-term losses have occurred already at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT

17.2.3 Strength Design Load Combination

The following load combinations are used for checking the requirements of prestress for strength in accordance with TS 3233-1979.

The strength design combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are automatically determined by ETABS by subtracting out the primary PT moments when the flexural design is carried out.

1.4G + 1.6Q + 1.0PT*	(TS 8.4.3)
0.9G ± 1.3W + 1.0PT*	(TS 8.4.3)
1.0G + 1.3Q ± 1.3W + 1.0PT*	(TS 8.4.3)
0.9G ± 1.0E + 1.0PT*	(TS 8.4.3)
1.0G + 1.0Q ± 1.0E + 1.0PT*	(TS 8.4.3)

* — Replace PT by hyperstatic (H) for flexural design only

These are also the default design load combinations in ETABS whenever the Turkish TS 3233-1979 code is used. If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

17.3 Limits on Material Strength

The concrete compressive strength, f'_c, should not be less than 25 N/mm^2 (TS 2.3.1).

$$25\,\text{N/mm}^2 \le f_{ck} \le 55\,\text{N/mm}^2 \qquad \text{(TS 2.3.2)}$$

ETABS enforces the upper material strength limits for flexure and shear design of slabs. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

The specified characteristic strength of reinforcement is given as follows:

$$f_{yk} \le 420 \text{ N/mm}^2$$

17.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength, f_{ck}, f_{pk} and f_{yk} of the material by a partial factor of safety, γ_s and γ_c, as follows (TS 8.4.3):

$$f_{cd} = f_{ck} / \gamma_{mc} \qquad \text{(TS 8.4.3)}$$

$$f_{yd} = f_{yk} / \gamma_{ms} \qquad \text{(TS 8.4.3)}$$

$$f_{pd} = f_{pk} / \gamma_{ms} \qquad \text{(TS 8.4.3)}$$

The values of partial safety factors, γ_{ms} and γ_{mc}, for the materials and the design strengths of concrete and reinforcement used in the program are listed in the following table (TS 8.4.3):

Values of γ_m for the Strength Design	
Reinforcement, γ_{ms}	1.15
Prestressing steel, γ_{mp}	1.15
Concrete in flexure and axial load, γ_{mc}	1.50

These values are recommended by the code to give an acceptable level of safety for normal structures under typical design situations.

These factors are already incorporated into the design equations and tables in the code. The user is allowed to overwrite these values; however, caution is advised.

17.5 Design Assumptions for Prestressed Concrete Structures

Strength design of prestressed members for flexure and axial loads shall be based on assumptions given in TS 8.3.

- The strain distribution in the concrete in compression is derived from the assumption that plane sections remain plane.
- The design stresses in the concrete in compression are taken as 0.85 f_{cd}. Maximum strain at the extreme concrete compression fiber shall be assumed equal to ε_{cu}.
- The tensile strength of the concrete is ignored.
- The strains in bonded post-tensioning tendons and in any additional reinforcement (compression or tension) are the same as that in the surrounding concrete.

The serviceability limit state of prestressed members uses the following assumptions given in 9.1, 9.2.

- Plane sections remain plane, i.e., strain varies linearly with depth through the entire load range.

- Elastic behavior exists by limiting the concrete stresses to the values given in TS 8.3
- In general, it is only necessary to calculate design stresses due to the load arrangements immediately after the transfer of prestress and after all losses or prestress have occurred; in both cases the effects of dead and imposed loads on the strain and force in the tendons may be ignored.

Prestressed concrete members are investigated at three stages:

- At transfer of prestress force
- At service loading
- At nominal strength

17.6 Serviceability Requirements of Flexural Members

17.6.1 Serviceability Check at Initial Service Load

The stresses in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits (TS 9.1.1 and 9.1.2):

- Extreme fiber stresses in compression:

$$0.60\, f_{cjk} \text{ in N/mm}^2 \qquad \text{(TS 9.1.1)}$$

Unless reinforcing steel has been added, the stress limits will normally be "without bonded reinforcement" values, as any bonded tendons normally will be at the compression face at transfer.

- Extreme fiber stresses in tension (TS 9.1.2)

$$0.5\sqrt{f_{cjk}} \text{ in N/mm}^2 \qquad \text{(TS 9.1.2)}$$

- Extreme fiber stresses in tension should not exceed $0.5\sqrt{f_{cjk}}$; otherwise, the section should be designed as a cracked section.

17.6.2 Serviceability Check at Service Load

The stresses in the concrete for prestressed concrete flexural members at service loads, and after all prestress losses have occurred, are checked against the following limits (TS 9.2)

- Extreme fiber stress in compression due to prestress plus total load:

$$0.45\, f_{ck} \qquad \text{(TS 9.2.1)}$$

- Extreme fiber stresses in tension in the precompressed tensile zone at characteristic service loads are defined as follows (TS 9.2.2):

  – Extreme fiber stresses in tension for reinforcement:

$$0.5\sqrt{f_{ck}} \qquad \text{(TS 9.2.2)}$$

  – Extreme fiber stresses in tension for prestressing tendons:

$$0.70\, f_{pk} \qquad \text{(TS 9.3)}$$

17.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement

- Design shear reinforcement
- Design torsion reinforcement

17.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

17.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of prestressed concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or a flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases the beam may be designed as a rectangular or inverted flanged beam.

17.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 17-1 (TS 8.3).

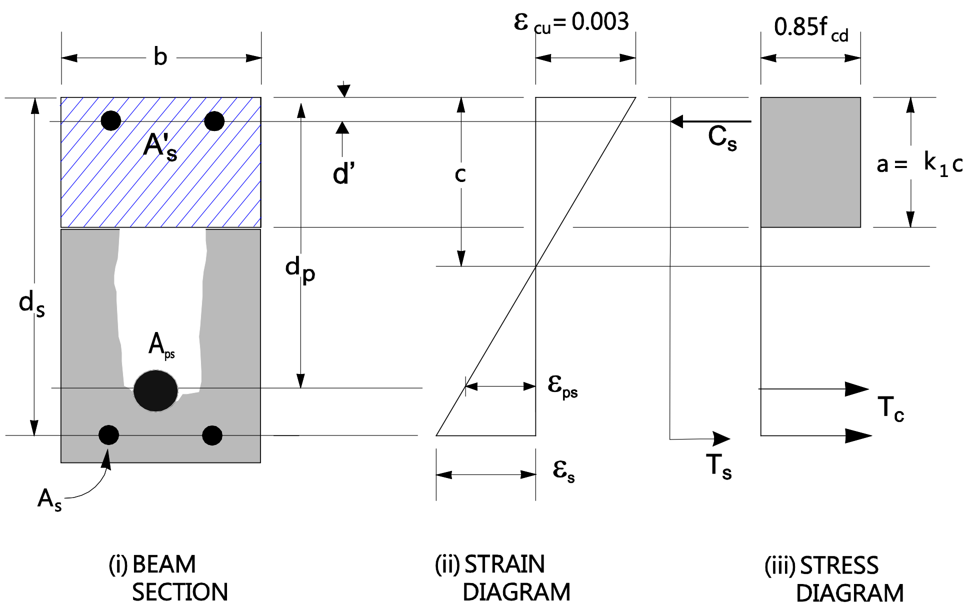


Figure 17-1 Rectangular Beam Design

The maximum depth of the compression zone, c_b, is calculated based on the compressive strength of the concrete and the tensile steel tension using the following equation (TS 8.3):

$$c_b = \frac{\varepsilon_{cu} E_s}{\varepsilon_{cu} E_s + f_{yd}} d \quad \text{(TS 8.3)}$$

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by

$$a_{\max} = 0.85 k_1 c_b \quad \text{(TS 8.3)}$$

where k_1 is calculated as follows:

$$k_1 = 0.85 \text{ for } f_{ck} \leq 30 \text{ N/mm}^2, \quad \text{(TS 8.3)}$$

$$k_1 = 0.85 - 0.0075\left(f_{ck} - 30\right), \quad \text{for } f_{ck} > 30 \text{ N/mm}^2. \quad \text{(TS 8.3)}$$

Furthermore, it is assumed that moment redistribution in the beam does not exceed the code specified limiting value. The code also places a limitation on the neutral axis depth, to safeguard against non-ductile failures. When the applied moment exceeds the limiting moment capacity as a singly reinforced beam, the area of compression reinforcement is calculated on the assumption that the neutral axis depth remains at the maximum permitted value.

The design procedure used by ETABS, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow.

17.7.1.2.1 Design of Rectangular Beams

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, the moment capacity M_{res}^0 is determined as follows:

The maximum depth of the compression zone, $c_{\max}$, is calculated based on the limitation that the tension reinforcement strain shall not be less than $\varepsilon_{s\min}$:

$$c_{\max} = \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{s\min}}\right) d_p$$

where,

$$\varepsilon_{cu} = 0.003$$

Therefore, the limit $c \le c_{\max}$ is set for tension-controlled sections.

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by

$$a_{\max} = 0.85 k_1 c_b \qquad \text{(TS 8.3)}$$

where k_1 is calculated as follows:

$$k_1 = 0.85 \text{ for } f_{ck} \le 30 \text{ N/mm}^2, \qquad \text{(TS 8.3)}$$

$$k_1 = 0.85 - 0.0075\left(f_{ck} - 30\right), \quad \text{for } f_{ck} > 30 \text{ N/mm}^2. \qquad \text{(TS 8.3)}$$

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pk} is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel.

Based on the calculated f_{pk}, the depth of the neutral axis is recalculated, and f_{pk} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = 0.85 k_1 c$$

- If $c \le c_{\max}$, the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_{res}^{0} = A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right)$$

- If $c > c_{\max}$, a failure condition is declared.

- If $M > M_{res}^{0}$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension-controlled case. In that case, it is assumed that the depth of the neutral axis c is equal to $c_{\max}$. The stress in the post-tensioning steel, f_{pk}, is then calculated based on strain compatibility and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = 0.85 f_{cd} a_{\max} b,$$

$$T = A_p f_{pk}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{0.85 f_{cd} a_{\max} b - A_p f_{ps}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_{res}^{bal} = A_p f_{ps}^{bal} \left(d_p - \frac{a_{\max}}{2} \right) + A_s^{bal} f_s^{bal} \left(d_s - \frac{a_{\max}}{2} \right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, c.

17.7.1.2.1.1 Case 1: Post-tensioning steel is adequate

When $M_d < M_{res}^0$, the amount of post-tensioning steel is adequate to resist the design moment M. A minimum reinforcement is provided to satisfy the flexural cracking requirements (TS 10.1.1, 10.2.1).

17.7.1.2.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{max}$.

When $\left(M_{res}^0 < M_d < M_{res}^{bal}\right)$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M_d and reports this required area of tension reinforcement. Since M_d is bound by M_{res}^0 at the lower end and M_{res}^{bal} at the upper end, and M_{res}^0 is associated with $A_s = 0$ and M_{res}^{bal} is associated with $A_s = A_s^{bal}$, the required area will fall within the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

17.7.1.2.1.3 Case 3: Post-tensioning steel and tension reinforcement is not adequate

When $\left(M_d > M_{res}^{bal}\right)$, compression reinforcement is required. In that case, ETABS assumes that the depth of the neutral axis, c, is equal to c_{max}. The values of f_{pk} and f_s reach their respective balanced condition values, f_{pk}^{bal} and f_s^{bal}. Then the area of compression reinforcement, A'_s, is determined as follows:

- The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{d,s} = M_d - M_{res}^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{d,s}}{\left(f'_s - 0.85 f_{cd}\right)\left(d_e - d'\right)}, \text{ where}$$

$$f'_s = \varepsilon_{c3} E_s \left[\frac{c_{\max} - d'}{c_{\max}}\right] \leq f_{yd}.$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{d,s}}{f_{yd}\left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M is positive, and vice versa if M is negative.

17.7.1.2.2 Design of Flanged Beams

17.7.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_d (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

17.7.1.2.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, the moment capacity M^0_{res} is determined as follows:

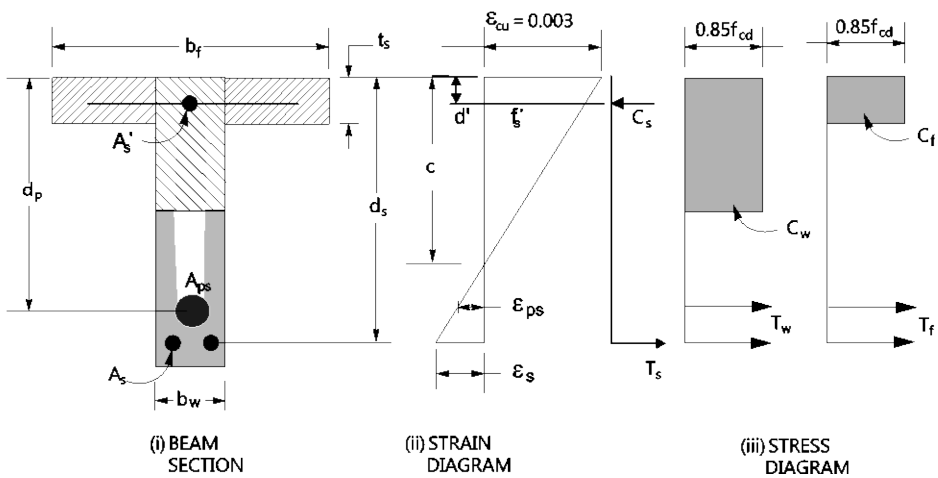


Figure 17-2 T-Beam Design

The maximum allowable depth of the rectangular compression block, a_{max}, is given by

$$a_{max} = 0.85 k_1 c_b \quad \text{(TS 8.3)}$$

where k_1 is calculated as follows:

$$k_1 = 0.85 \text{ for } f_{ck} \leq 30 \text{ N/mm}^2, \quad \text{(TS 8.3)}$$

$$k_1 = 0.85 - 0.0075\left(f_{ck} - 30\right), \quad \text{for } f_{ck} > 30 \text{ N/mm}^2. \quad \text{(TS 8.3)}$$

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pk}, is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps}, in the post-tensioning steel. Based on the calculated f_{pk}, the depth of the neutral axis is recalculated, and f_{pk} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = 0.85 k_1 c$$

- If $c \le c_{\max}$, the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_{res}^{0} = A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right)$$

- If $c > c_{\max}$, a failure condition is declared.

- If $M > M_{res}^{0}$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension- controlled case. In that case it is assumed that the depth of the neutral axis c is equal to $c_{\max}$. The stress in the post-tensioning steel, f_{pk}, is then calculated based on strain compatibility, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case, the width of the beam is taken as b_f. Compression reinforcement is required when $a > a_{\max}$.

- If $a > h_f$, the calculation for A_s is given by:

$$C = 0.85 f_{cd} a_{\max} A_c^{com}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$T = A_p f_{pk}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{0.85 f_{cd} A_c^{com} - A_p f_{pk}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_{res}^{bal} = A_p f_{pk}^{bal} \left(d_p - \frac{a_{\max}}{2} \right) + A_s^{bal} f_s^{bal} \left(d_s - \frac{a_{\max}}{2} \right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcing steel, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, c.

17.7.1.2.2.2.1 Case 1: Post-tensioning steel is adequate

When $M < M_{res}^{0}$, the amount of post-tensioning steel is adequate to resist the design moment M. Minimum reinforcement is provided to satisfy the flexural cracking requirements (TS 10.1.1, 10.2.1).

17.7.1.2.2.2.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $c < c_{\max}$.

When $\left(M_{res}^{0} < M_d < M_{res}^{bal}\right)$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports the required area of tension reinforcement. Since M is bounded by M_{res}^{0} at the lower end and M_{res}^{bal} at the upper end, and M_{res}^{0} is associated with $A_s = 0$ and M_{res}^{bal} is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if M_d is positive, or at the top if M_d is negative.

17.7.1.2.2.2.3 Case 3: Post-tensioning steel and tension reinforcement is not adequate

When $\left(M_d > M_{res}^{bal}\right)$, compression reinforcement is required. In that case ETABS assumes that the depth of the neutral axis, c, is equal to $c_{\max}$. The values of f_{pk} and f_s reach their respective balanced condition values, f_{pk}^{bal} and f_s^{bal}. Then the area of compression reinforcement, A_s', is determined as follows:

- The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{res,s} = M_d - M_{res}^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{res,s}}{\left(f'_s/\gamma_{ms} - 0.85 f_{cd}\right)\left(d - d'\right)}, \text{ where}$$

$$f'_s = \varepsilon_{c3} E_s \left[\frac{c_{\max} - d'}{c_{\max}}\right] \leq f_{yk}/\gamma_{ms} .$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{res,s}}{f_{yd}\left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_d is positive, and vice versa if M_d is negative.

17.7.1.2.3 Minimum and Maximum Tensile Reinforcement

Reinforcement in prestressed concrete beams is computed to increase the strength of sections as required in the flexural design of prestressed beam or to comply with shear link requirements. The minimum flexural tension reinforcement required for a beam section to comply with the cracking requirements must be separately investigated by the user.

For bonded tendons, there is no minimum un-tensioned reinforcement requirements.

For unbonded tendons, the minimum flexural reinforcement provided in a rectangular or flanged beam section is given as:

$$A_{s,min} \geq 0.004\, A_{ct} \qquad \text{(TS 8.6.2)}$$

The minimum flexural tension reinforcement required for control of cracking should be investigated independently by the user.

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

17.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each loading combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular loading combination, at a particular station due to the beam major shear, the following steps are involved (TS 11.1):

- Determine the factored shear force, V_d
- Determine the shear force, V_c, that can be resisted by the concrete
- Determine the shear reinforcement required to carry the balance

The following three sections describe in detail the algorithms associated with these steps.

17.7.2.1 Determine Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the cor-responding shear forces for different load cases with the corresponding load combination factors.

17.7.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as:

$$V_c = \min(V_{cw}, V_{cr}) \quad \text{(TS 11.1.1, 11.1.2)}$$

where,

$$V_{cw} = 0.67 b_w h \left(\sigma_{ct}^2 + 0.8\sigma_{cp}\sigma_{ct}\right)^{0.5} \quad \text{(TS 11.1.1)}$$

$$\sigma_{ct} = 0.25 f_{ck}^{0.25}$$

σ_{cp} = compress stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at the extreme fiber of the section where tensile stress is caused by externally applied loads, N/mm^2

$$V_{cr} = \left(1 - 0.55\frac{\sigma_{pef}}{f_{pk}}\right)\tau_c b_w d + M_o \frac{V_d}{M_d} \geq 0.12\lambda f_{ck} b_w d \qquad \text{(TS 11.1.2)}$$

$$d \geq 0.80h \qquad \text{(TS 11.1.2)}$$

$$M_o = 0.8\sigma_{cpd}\left(\frac{I}{y}\right) \qquad \text{(TS 11.1.2)}$$

where,

σ_{pef} = compress stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at the extreme fiber of the section where tensile stress is caused by externally applied loads, $\sigma_{pef} \leq 0.6 f_{pk}$ N/mm^2

σ_{cpd} = compress stress in concrete due to gravity loading, N/mm^2

f_{pk} = characteristic strength of tendon, N/mm^2

V_d = designed factored shear force at the section, N

M_d = design factored moment at the section, N-mm

M_{cre} = moment causing flexural cracking at the section because of externally applied loads

V_{cw} = shear resistance provided by the concrete when diagonal cracking results from high principal tensile stress in the web

V_{cr} = shear resistance provided by the concrete when diagonal cracking results from combined shear and moment

For light-weight concrete, the f_{ck} term is multiplied by the shear strength reduction factor λ.

17.7.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{\max} = \tau_{\max} b_w d \qquad \text{(TS 11.1)}$$

$$\tau_{\max} = 0.79\, f_{ck}^{0.5}$$

Given V_d, V_c, and $V_{\max}$, the required shear reinforcement is calculated as follows:

- If $V_d \leq 0.5V_c$

$$\frac{A_{sw}}{s} = 0 \qquad \text{(TS 11.2.1)}$$

- If $0.5V_c < V_d \leq V_{\max}$

$$\frac{A_{sw}}{s} = \frac{\left(V_d - V_c\right)}{f_{ywd}\, d} \qquad \text{(TS 11.2.3)}$$

$$\frac{A_{sw}}{s} \geq 0.25 b_w \frac{f_{ctd}}{f_{ywd}} \qquad \text{(TS 11.2.1)}$$

- If $V_d > V_{\max}$, a failure condition is declared (TS 11.1).
- If V_d exceeds the maximum permitted value of $V_{\max}$, the concrete section should be increased in size.

  - Note that if torsion design is performed and torsion rebar is needed, the equation given in TS 500-2000 8.1.5 does not need to be satisfied independently. See the next section *Design of Beam Torsion Reinforcement* for details.

  - The maximum of all of the calculated A_{sw}/s values, obtained from each design load combination, is reported along with the controlling shear force and associated design load combination name.

  - The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

17.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_d.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

17.7.3.1 Determine Factored Torsion

In the design of torsion reinforcement of any beam, the factored torsions for each design load combination at a particular design station are obtained by factoring the corresponding torsion for different analysis cases with the corresponding design load combination factors (TS 8.2).

NOTE: All section listed in torsion design refers to TS 500-2000.

In a statistically indeterminate structure where redistribution of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, the design T_d is permitted to be reduced in accordance with code (TS 8.2.3). However, the program does not try to redistribute the internal forces and to reduce T_d. If redistribution is desired, the user should *release* the torsional DOF in the structural model.

17.7.3.2 Determine Special Section Properties

For torsion design, special section properties such as A_e, S and u_e are calculated. These properties are described as follows (TS 8.2.4).

A_e = Area enclosed by centerline of the outermost closed transverse torsional reinforcement

S = Shape factor for torsion

u_e = Perimeter of area A_e

In calculating the section properties involving reinforcement, such as A_{ov}/s, A_{ot}/s, and u_e, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 30 mm. This is equivalent to 25-mm clear cover and a 10-mm-diameter stirrup placement. For torsion design of T beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a Rectangular beam section are given as follows:

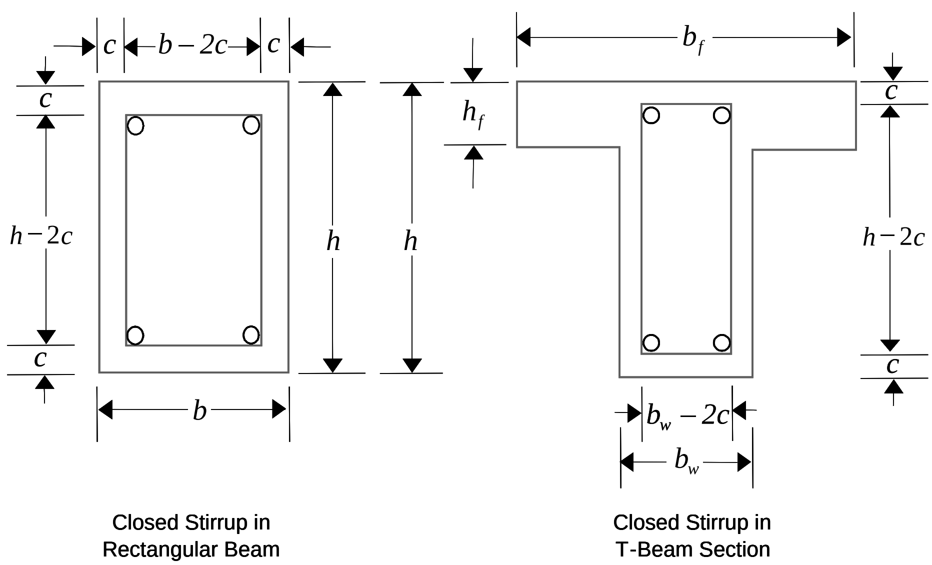


Figure 17-3 Closed stirrup and section dimensions for torsion design

$$A_e = (b-2c)(h-2c), \quad \text{(TS 8.2.4)}$$

$$u_t = 2(b-2c)+2(h-2c), \quad \text{(TS 8.2.4)}$$

$S \quad = \quad x^2y/3$ (TS 8.2.4) (TS 8.2.4) (TS 8.2.4)

where, the section dimensions *b*, *h* and *c* are shown in Figure 3-9. Similarly, the special section properties for a T beam section are given as follows:

$$A_e = (b_w - 2c)(h - 2c), \quad \text{(TS 8.2.4)}$$

$$u_t = 2(h - 2c) + 2(b_w - 2c), \quad \text{(TS 8.2.4)}$$

$$S = \Sigma x^2y/3 \quad \text{(TS 8.2.4)}$$

where the section dimensions b_w, *h* and *c* for a T-beam are shown in Figure 13-3.

17.7.3.3 Determine Critical Torsion Capacity

Design for torsion may be ignored if either of the following is satisfied:

(i) The critical torsion limits, T_{cr}, for which the torsion in the section can be ignored, is calculated as follows:

$$T_d \leq T_{cr} = 0.65 f_{ctd} S \quad \text{(TS 8.2.3, Eqn 8.12)}$$

In that case, the program reports shear reinforcement based on TS 8.1.5, Eqn. 8.6. i.e.,

$$\frac{A_{sw}}{s} \geq 0.3 \frac{f_{ctd}}{f_{ywd}} b_w \quad \text{(TS 8.1.5, Eqn. 8.6)}$$

(ii) When design shear force and torsional moment satisfy the following equation, there is no need to compute torsional stirrups. However, the minimum stirrups and longitudinal reinforcement shown below must be provided:

$$\left(\frac{V_d}{V_{cr}}\right)^2 + \left(\frac{T_d}{T_{cr}}\right)^2 \leq 1 \quad \text{(TS 8.2.2, Eqn 8.10)}$$

where T_{cr} is computed as follows:

$$T_{cr} = 1.35 f_{ctd} S \quad \text{(TS 8.2.2, Eqn 8.11)}$$

The required minimum closed stirrup area per unit spacing, A_o /s, is calculated as:

$$\frac{A_o}{s} = 0.15 \frac{f_{ctd}}{f_{ywd}} \left(1 + \frac{1.3T_d}{V_d b_w} \right) b_w \qquad \text{(TS 8.2.4, Eqn. 8.17)}$$

In Eqn. 8.17, $\frac{T_d}{V_d b_w} \le 1.0$ and for the case of statistically indeterminate structure where redistribution of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, minimum reinforcement will obtained by taking T_d equal to T_{cr}.

and the required minimum longitudinal rebar area, A_{sl}, is calculated as:

$$A_{sl} = \frac{T_d u_e}{2A_e f_{yd}} \qquad \text{(TS 8.2.5, Eqn. 8.18).}$$

17.7.3.4 Determine Torsion Reinforcement

If the factored torsion T_d is less than the threshold limit, T_{cr}, torsion can be safely ignored (TS 8.2.3), when the torsion is not required for equilibrium. In that case, the program reports that no torsion is required. However, if T_d exceeds the threshold limit, T_{cr}, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals (TS 8.2.4 and 8.2.5).

If $T_d > T_{cr}$, the required longitudinal rebar area, A_{sl}, is calculated as:

$$A_{sl} = \frac{T_d u_e}{2A_e f_{yd}} \qquad \text{(TS 8.2.4, Eqn. 8.16)}$$

and the required closed stirrup area per unit spacing, A_{ot} /s, is calculated as:

$$\frac{A_o}{s} = \frac{A_{ov}}{s} + \frac{A_{ot}}{s} \qquad \text{(TS 8.2.4, Eqn. 8.13)}$$

$$\frac{A_{ov}}{s} = \frac{(V_d - V_c)}{d f_{ywd}} \qquad \text{(TS 8.2.4, Eqn. 8.14)}$$

$$\frac{A_{ot}}{s} = \frac{T_d}{2A_e f_{ywd}} \qquad \text{(TS 8.2.4, Eqn. 8.15)}$$

where, the minimum value of A_o/s is taken as:

$$\frac{A_o}{s} = 0.15 \frac{f_{ctd}}{f_{ywd}} \left(1 + \frac{1.3T_d}{V_d b_w}\right) b_w \qquad \text{(TS 8.2.4, Eqn. 8.17)}$$

where, $\frac{1.3T_d}{V_d b_w} \leq 1.0$

An upper limit of the combination of V_d and T_d that can be carried by the section also is checked using the following equation.

$$\frac{T_d}{S} + \frac{V_d}{b_w d} \leq 0.22 f_{cd} \qquad \text{(TS 8.2.5b, Eqn. 8.19)}$$

The maximum of all the calculated A_{sl} and A_o/s values obtained from each design load combination is reported along with the controlling combination names.

The beam torsion reinforcement requirements considered by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal reinforcement requirements to satisfy spacing considerations must be investigated independently of the program by the user.

17.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips usually are governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method for prestressed reinforced concrete (TS 3233-1979) as described in the following sections. To learn more about the design strips, refer to the section entitled "ETABS Design Techniques" in the *Key Features and Terminology* manual.

17.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the subsection that follows and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

17.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

17.8.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

17.8.1.2.1 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits:

For unbonded tendons, the minimum flexural reinforcement provided in a rectangular or flanged beam section is given as:

$$A_{s,min} \geq 0.004\, A_{ct} \qquad \text{(TS 8.6.2)}$$

The minimum flexural tension reinforcement required for control of cracking should be investigated independently by the user.

An upper limit on the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

17.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Key Features and Terminology* manual. Only the code-specific items are described in the following sections.

17.8.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of $d/2$ from the face of the support (TS 8.3.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (TS 8.3.1). Figure 13-4 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

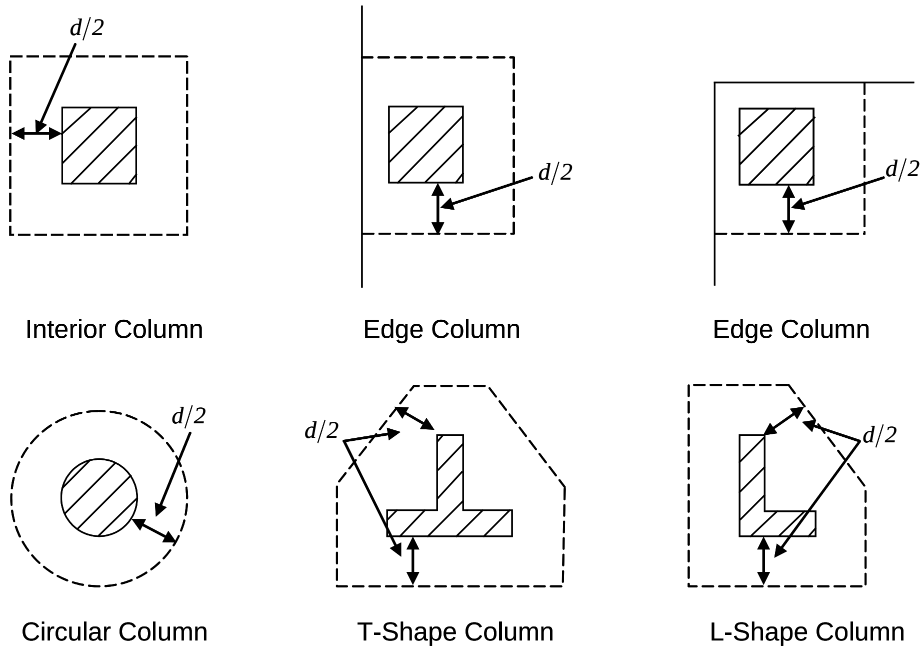


Figure 17-4 Punching Shear Perimeters

17.8.2.2 Determine Concrete Capacity

The concrete punching shear stress capacity is taken as the following limit:

$$v_{pr} = f_{ctd} = 0.35\sqrt{f_{ck}}/\gamma_c \qquad \text{(TS 8.3.1)}$$

17.8.2.3 Computation of Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear, the nominal design shear stress, v_{Ed}, is calculated as:

$$v_{pd} = \frac{V_{pd}}{u_p d}\left[1+\eta\frac{0.4M_{pd,2}u_p d}{V_{pd}W_{m,2}}+\eta\frac{0.4M_{pd,3}u_p d}{V_{pd}W_{m,3}}\right], \text{ where} \qquad \text{(TS 8.3.1)}$$

η factor to be used in punching shear check

$$\eta = \frac{1}{1+\sqrt{b_2 / b_1}} \text{ where } b_2 \geq 0.7 b_1$$

When the aspect ratio of loaded area is greater than 3, the critical perimeter is limited assuming $h = 3b$

u_p is the effective perimeter of the critical section

d is the mean effective depth of the slab

M_{pd} is the design moment transmitted from the slab to the column at the connection along bending axis 2 and 3

V_{pd} is the total punching shear force

W_m section modulus of area within critical punching perimeter (u_p) along bending axis 2 and 3.

17.8.2.4 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

17.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the slab thickness is greater than or equal to 250 mm, a (TS 8.3.2). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is described in the subsections that follow.

17.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is limited to:

$$v_{pr} = f_{ctd} = 0.35\sqrt{f_{ck}} / \gamma_c \qquad \text{(TS 8.3.1)}$$

17.8.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$v_{pr,\max} = 1.5 f_{ctd} = 0.525\sqrt{f_{ck}} / \gamma_c \text{ for shear links/shear studs} \qquad \text{(TS 8.3.1)}$$

Given V_{pd}, V_{pr}, and $V_{\text{pr,max}}$, the required shear reinforcement is calculated as follows,

$$\frac{A_v}{s} = \frac{\left(V_{pd} - V_{pr}\right)}{f_{yd}d} \qquad \text{(TS8.1.4)}$$

- If $V_{pd} > V_{\text{pr,max}}$, a failure condition is declared. (TS 8.3.1)
- If V_{pd} exceeds the maximum permitted value of $V_{\text{pr,max}}$, the concrete section should be increased in size.

17.8.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 13-6 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

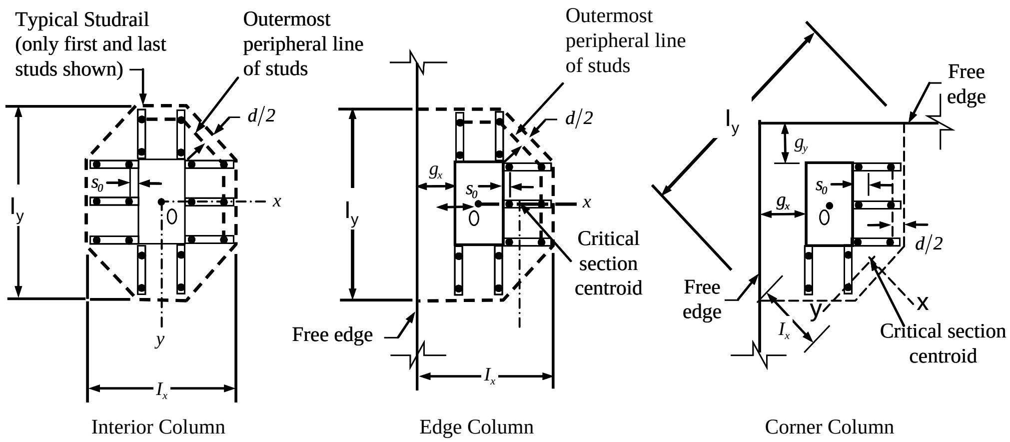


Figure 17-6 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

NOTE: Shear Stud and shear links requirements are computed based on ACI 318-08 code as Turkish TS 500-2000 refers to special literature on this topic.

The distance between the column face and the first line of shear reinforcement shall not exceed $d/2$(ACI R11.3.3, 11.11.5.2. The spacing between adjacent shear reinforcement in the first line (perimeter) of shear reinforcement shall not exceed 2*d* measured in a direction parallel to the column face (ACI 11.11.3.3).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

17.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in ACI 7.7 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5d. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed 2d, and in the case of studs in a radial pattern, the angle between adjacent stud rails shall not exceed 60 degrees. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(ACI 11.11.5.2)}$$

$$s \leq \begin{cases} 0.75d & \text{for} \quad v_u \leq 6\phi\lambda\sqrt{f'_c} \\ 0.50d & \text{for} \quad v_u > 6\phi\lambda\sqrt{f'_c} \end{cases} \qquad \text{(ACI 11.11.5.2)}$$

$$g \leq 2d \qquad \text{(ACI 11.11.5.3)}$$

The limits of s_o and the spacing, s, between for the links are specified as:

$$s_o \leq 0.5d \qquad \text{(ACI 11.11.3)}$$

$$s \leq 0.50d \qquad \text{(ACI 11.11.3)}$$

Chapter 18
Design for Italian NTC 2008

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the Italian code NTC2008 [D.M. 14/01/2008]. For the load combinations reference also is made to NTC2008. Various notations used in this chapter are listed in Table 18-1.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

18.1 Notations

The following table identifies the various notations used in this chapter.

Table 18-1 List of Symbols Used in the Italian NTC 2008 Code

A_c	Area of concrete section, mm^2

Table 18-1 List of Symbols Used in the Italian NTC 2008 Code

A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sw}	Total cross-sectional area of links at the neutral axis, mm^2
A_{sw}/s_v	Area of shear reinforcement per unit length of the member, mm^2
a	Depth of compression block, mm
$a_{\max}$	Maximum depth of the compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
d	Effective depth of tension reinforcement, mm
d'	Effective depth of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f_{cd}	Design concrete strength = $\alpha_{cc} f_{ck} / \gamma_c$, MPa
f_{ck}	Characteristic compressive concrete cylinder strength at 28 days, MPa
f_{cwd}	Design concrete compressive strength for shear design = α_{cc} f_{cwk}/γ_c, MPa

Table 18-1 List of Symbols Used in the Italian NTC 2008 Code

f_{cwk}	Characteristic compressive cylinder strength for shear design, MPa
f_{yd}	Design yield strength of reinforcing steel = f_{yk}/γ_s, MPa
f_{yk}	Characteristic strength of shear reinforcement, MPa
f'_s	Compressive stress in beam compression steel, MPa
f_{ywd}	Design strength of shear reinforcement = f_{ywk}/γ_s, MPa
f_{ywk}	Characteristic strength of shear reinforcement, MPa
h	Overall thickness of slab, mm
h_f	Flange thickness, mm
M	Design moment at a section, N-mm
m	Normalized design moment, $M/bd^2\eta f_{cd}$
$m_{\lim}$	Limiting normalized moment capacity as a singly reinforced beam
$M_{ED}{}^{0}$	Design moment resistance of a section with tendons only, N-mm
$M_{ED}{}^{bal}$	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
s_v	Spacing of the shear reinforcement along the length of the beam, mm
u	Perimeter of the punch critical section, mm
V_{Rdc}	Design shear resistance from concrete alone, N
$V_{Rd,\max}$	Design limiting shear resistance of a cross-section, N
V_{Ed}	Shear force at ultimate design load, N

Table 18-1 List of Symbols Used in the Italian NTC 2008 Code

x	Depth of neutral axis, mm
x_{lim}	Limiting depth of neutral axis, mm
η	Concrete strength reduction factor for sustained loading and stress-block
β	Enhancement factor of shear resistance for concentrated load; also the coefficient that takes account of the eccentricity of loading in determining punching shear stress; factor for the depth of compressive stress block
γ_f	Partial safety factor for load
γ_c	Partial safety factor for concrete strength
γ_s	Partial safety factor for steel strength
δ	Redistribution factor
ε_c	Concrete strain
ε_s	Strain in tension steel
ε'_s	Strain in compression steel.
ν	Effectiveness factor for shear resistance without concrete crushing
ρ	Tension reinforcement ratio
ω	Normalized tensile steel ratio, $A_s f_{yd} / \eta f_{cd} bd$
ω'	Normalized compression steel ratio, $A'_s f_{yd} \gamma_s / \alpha f'_s bd$
ω_{lim}	Normalized limiting tensile steel ratio

18.2 Design Load Combinations

18.2.1 Ultimate Limit State Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with NTC2008.

The combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are determined automatically by ETABS by subtracting out the primary PT moments when the flexural design is carried out.

The design load combinations are the various combinations of the load cases for which the structure needs to be checked. NTC2008 allows load combinations to be defined based on NTC2008 Equation 2.5.1.

$$\sum_{j\geq 1}\gamma_{G1,j}G_{1k,j} + \sum_{l\geq 1}\gamma_{G2k,l}G_{2k,l} + PT + \gamma_{Q,1}Q_{k,1} + \sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i} \qquad \text{(Eq. 2.5.1)}$$

Load combinations considering seismic loading are automatically generated based on NTC2008 Equation 2.5.5.

$$\sum_{j\geq 1}G_{1k,j} + \sum_{l\geq 1}G_{2k,l} + P + E + \sum_{i>1}\psi_{2,i}Q_{k,i} \qquad \text{(Eq. 2.5.5)}$$

For both sets of load combinations, the variable values are defined in the list that follows.

$\gamma_{G1,sup} = 1.30$ (NTC2008 Table 2.6.I)

$\gamma_{G1,inf} = 1.00$ (NTC2008 Table 2.6.I)

$\gamma_{G2,sup} = 1.50$ (NTC2008 Table 2.6.I)

$\gamma_{G2,inf} = 0.00$ (NTC2008 Table 2.6.I)

$\gamma_{Q,1,sup} = 1.5$ (NTC2008 Table 2.6.I)

$\gamma_{Q,1,inf} = 0.0$ (NTC2008 Table 2.6.I)

$\gamma_{Q,I,sup} = 1.5$ (NTC2008 Table 2.6.I)

$\gamma_{Q,I,inf}$ = 0.0 (NTC2008 Table 2.6.I)

$\psi_{0,i}$ = 0.7 (live load, assumed not to be storage) (Table 2.5.I)

$\psi_{0,i}$ = 0.6 (wind load) (Table 2.5.I)

$\psi_{0,i}$ = 0.5 (snow load, assumed H ≤ 1,000 m) (Table 2.5.I)

$\psi_{2,i}$ = 0.3 (live, assumed office/residential space) (Table 2.5.I)

$\psi_{2,i}$ = 0 (snow, assumed H ≤ 1,000 m) (Table 2.5.I)

If roof live load is treated separately or other types of loads are present, other appropriate load combinations should be used.

18.2.2 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress force in accordance with NTC2008 § 4.1.8.1.4. The prestressing forces are considered without any long-term loses for the initial service load combination check.

$$\sum_{j\geq 1} G_{1k,j} + PT$$

18.2.3 Service Load Combination

The following characteristic load combinations are used for checking the requirements of prestress for serviceability in accordance with NTC2008 § 4.1.8.1.3. It is assumed that all long-term losses have occurred already at the service stage.

$$\sum_{j\geq 1} G_{1k,j} + \sum_{l\geq 1} G_{2k,l} + PT + Q_{k,1} + \sum_{i>1} \psi_{0,i} Q_{k,i}$$

18.3 Limits on Material Strength

The concrete compressive strength, f_{ck}, should not be greater than 90 MPa (NTC2008 Tab. 4.1.I). The reinforcement material should be B450C or B450A (NTC2008 §11.3.2).

NTC Table 11.3.Ia:

$f_{y,nom}$	450 N/mm^2
$f_{t,nom}$	540 N/mm^2

NTC Table 11.3.Ib: Material TYPE B450C Properties

Properties	**Prerequisite**	**Fracture %**
Characteristic yield stress, f_{yk}	$\geq f_{y,nom}$	5.0
Characteristic rupture stress, f_{tk}	$\geq f_{y,nom}$	5.0
$(f_t / f_y)_k$	$\geq$ 1.15 < 1.35	 10.0
Elongation at rupture $(f_y / f_{y,nom})_k$ $(A_{gt})_k$	 < 1.25 $\geq$ 7.5 %	 10.0 10.0

NTC Table 11.3.Ic: Material TYPE B450A Properties

Properties	**Prerequisite**	**Fracture %**
Characteristic yield stress, f_{yk}	$\geq f_{y,nom}$	5.0
Characteristic rupture stress, f_{tk}	$\geq f_{y,nom}$	5.0
$(f_t / f_y)_k$	$\geq$ 1.05 < 1.25	 10.0
Elongation at rupture $(f_y / f_{y,nom})_k$ $(A_{gt})_k$	 < 1.25 $\geq$ 2.5 %	 10.0 10.0

The specified characteristic strength of prestressed steel should conform to NTC2008 §11.3.3.

The program also checks the following tensile strength in prestressing steel (EC2 5.10.2.1). The maximum stresses applied to the tendon, $\sigma_{p,\max}$, in all types of prestressing steel, in terms of the specified minimum tensile strength f_{pk}, are summarized as follows:

$$\sigma_{p,\max} = \min\left\{k_1 f_{pk}, k_2 f_{p0.1k}\right\} \qquad \text{(EC2 5.10.2.1)}$$

The recommended value for k_1 and k_2 are 0.8 and 0.9 where, $\left(f_{p0.1k}\right)$ is defined as the characteristic value of 0.1% proof load and $\left(f_{pk}\right)$ is the characteristic maximum load in axial tension (EC2 3.3.3, Figure 3.9).

The stress in tendons immediately after tensioning or after prestress transfer is also limited to the following:

$$\sigma_{pm0} = \min\left\{k_7 f_{pk}, k_8 f_{p0.1k}\right\} \qquad \text{(EC2 5.10.3)}$$

The recommended values for k_7 and k_8 are 0.75 and 0.85.

18.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength, f_{ck}, f_{pk} and $f_{p0.1k}$ of the material by a partial factor of safety, γ_s and γ_c, as follows (EC2 3.1.6, 3.2.7, 3.3.6(6)).

$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c \qquad \text{(NTC Eq. 4.1.4)}$$

$$f_{cwd} = \alpha_{cc} f_{cwk} / \gamma_c \qquad \text{(EC2 3.1.6 (1))}$$

$$f_{yd} = f_{yk} / \gamma_s \qquad \text{(NTC Eq. 4.1.6)}$$

$$f_{ywd} = f_{ywk} / \gamma_s \qquad \text{(NTC Eq. 4.1.6)}$$

$$f_{pd} = f_{p0.1k} / \gamma_p \qquad \text{(EC2 3.3.6 (6))}$$

The value α_{cc} is the coefficient that accounts for long-term effects on the compressive strength; α_{cc} is taken as 0.85 by default and can be overwritten by the user.

The values of partial safety factors, γ_s and γ_c, for the materials and the design strengths of concrete and reinforcement used in the program are listed in the following table:

Values of γ_m for the ultimate limit state	
Reinforcement, γ_s	1.15
Prestressing steel, γ_p	1.15
Concrete in flexure and axial load, γ_c	1.50

These factors are already incorporated into the design equations and tables in the code. The user is allowed to overwrite these values; however, caution is advised.

18.5 Design Assumptions for Prestressed Concrete Structures

For Post-Tensioned elements NTC2008 (§ 4.1.8) refers completely to EC2-2004. So all the particular prescriptions regarding Post-Tensioned elements are checked according to EC2.

Ultimate limit state design of prestressed members for flexure and axial loads shall be based on assumptions given in EC2 6.1(2).

- The strain distribution in the concrete in compression is derived from the assumption that plane sections remain plane.
- The design stresses in the concrete in compression are taken as ηf_{cd}. Maximum strain at the extreme concrete compression fiber shall be assumed equal to ε_{cu3}.
- The tensile strength of the concrete is ignored.
- The strains in bonded post-tensioning tendons and in any additional reinforcement (compression or tension) are the same as that in the surrounding concrete.

The serviceability limit state of prestressed members uses the following assumptions given in EC2 7.2.

- Plane sections remain plane, i.e., strain varies linearly with depth through the entire load range.
- Elastic behavior exists by limiting the concrete stresses to the values given in EC2 7.2(3).
- In general, it is only necessary to calculate design stresses due to the load arrangements immediately after the transfer of prestress and after all losses or prestress have occurred; in both cases the effects of dead and imposed loads on the strain and force in the tendons may be ignored.

Prestressed concrete members are investigated at three stages:

- At transfer of prestress force
- At service loading
- At nominal strength

18.6 Serviceability Requirements of Flexural Members

18.6.1 Serviceability Check at Initial Service Load

The stresses in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits (EC2 5.10.2.2 and 7.1):

- Extreme fiber stresses in compression:

$$0.60\, f_{ck}(\mathrm{t}) \qquad \text{(EC2 5.10.2.2(5))}$$

  Unless reinforcing steel has been added, the stress limits will normally be "without bonded reinforcement" values, as any bonded tendons normally will be at the compression face at transfer.

- Extreme fiber stresses in tension (EC2 7.1)

$\leq f_{ctm}$(t) where, (EC2 7.1(2))

$$f_{ctm} = 0.30 f_{ck}^{(2/3)} \quad \text{for } f_{ck} \leq \text{C50/C60} \quad \text{(EC2 Table 3.1)}$$

$$f_{ctm} = 2.12 ln\left(1 + f_{cm} / 10\right) \quad \text{for } f_{ck} > \text{C50/C60} \quad \text{(EC2 Table 3.1)}$$

$$f_{cm} = f_{ck} + 8\text{MPa} \quad \text{(EC2 Table 3.1)}$$

- Extreme fiber stresses in tension should not exceed f_{ctm}; otherwise, the section should be designed as a cracked section (EC2 7.1).

18.6.2 Serviceability Check at Service Load

The stresses in the concrete for prestressed concrete flexural members at service loads, and after all prestress losses have occurred, are checked against the following limits (EC2 7.2(2)):

- Extreme fiber stress in compression due to prestress plus total load:

$$0.6 f_{ck} \quad \text{(EC2 7.2(2))}$$

- Extreme fiber stresses in tension in the precompressed tensile zone at characteristic service loads are defined as follows (EC2 7.2(5)):
  - Extreme fiber stresses in tension for reinforcement:

$$0.8 f_{yk} \quad \text{(EC2 7.2(5))}$$

  - Extreme fiber stresses in tension for prestressing tendons:

$$0.75 f_{pk} \quad \text{(EC2 7.2(5))}$$

Although cracking is permitted for Exposure Classes X0, XC1, XC2, XC3, and XC4, it may be assumed that the design hypothetical tensile stresses exist at the limiting crack widths given in Eurocode 2, Table 7.1N. Limits to the design hypothetical tensile stresses under Frequent Load combinations are given in the following table (TR43, Second Edition):

Group	Limiting crack width(mm)	Design stress

Bonded Tendons	0.1	1.35 f_{ctm}
	0.2	1.65 f_{ctm}
Unbonded tendons	-	1.35 f_{ctm}

18.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

18.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

18.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of prestressed concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or a flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases the beam may be designed as a rectangular or inverted flanged beam.

18.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 18-1 (EC2 3.1.7(3)).

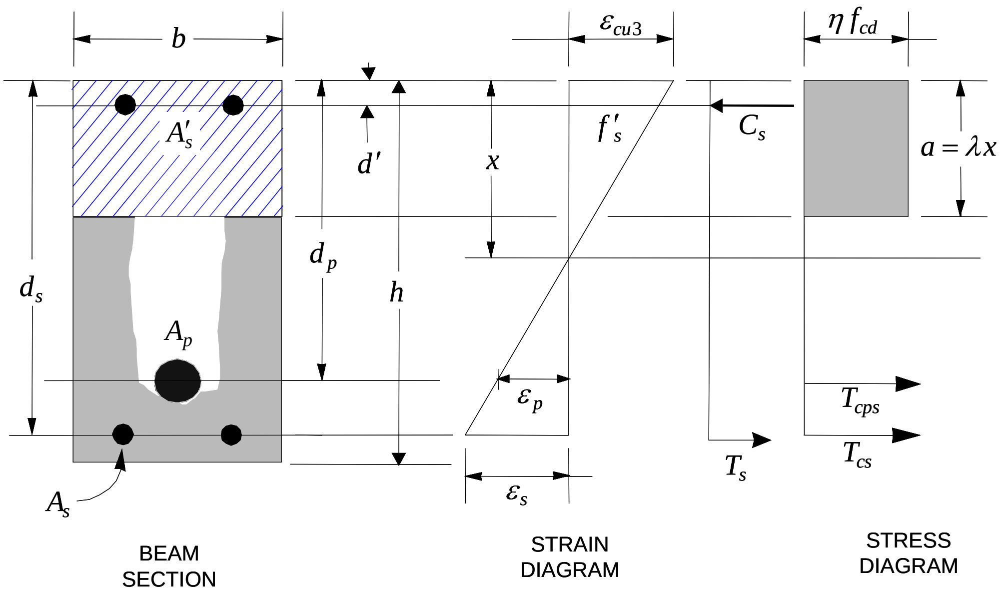


Figure 18-1 Rectangular Beam Design

The area of the stress block and the depth of the center of the compressive force from the most compressed fiber are taken as:

$$F_c = \eta f_{cd}\, a b$$

$$a = \lambda\, x$$

where x is the depth of the neutral axis; the factor λ defines the effective height of the compression zone; and the factor η defines the effective strength, as follows:

$\lambda = 0.8$ for $f_{ck} \leq 50$ MPa (EC2 3.1.7(3))

$\lambda = 0.8\left[\dfrac{f_{ck} - 50}{400}\right]$ for $50 \leq f_{ck} \leq 90$MPa (EC2 3.1.7(3))

$\eta = 1.0$ for $f_{ck} \leq 50$ MPa and (EC2 3.1.7(3))

$\eta = 1.0 - \left(\dfrac{f_{CK} - 50}{200}\right)$ for $50 \leq f_{ck} \geq 90$ MPa (EC2 3.1.7(3))

Furthermore, it is assumed that moment redistribution in the beam does not exceed the code specified limiting value. The code also places a limitation on the neutral axis depth, to safeguard against non-ductile failures (EC2 5.5(4)). When the applied moment exceeds the limiting moment capacity as a singly reinforced beam, the area of compression reinforcement is calculated on the assumption that the neutral axis depth remains at the maximum permitted value.

The design procedure used by ETABS, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow.

18.7.1.2.1 Design of Rectangular Beams

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that A_s = 0. In that case, the moment capacity M_{ED}^{0} is determined as follows:

The maximum depth of the compression zone, x_{max}, is calculated based on the limitation that the tension reinforcement strain shall not be less than ε_{smin}:

$$c_{max} = \left(\frac{\varepsilon_{cu3}}{\varepsilon_{cu3} + \varepsilon_{s\min}} \right) d_p$$

where,

$$\varepsilon_{cu3} = 0.0035$$

Therefore, the limit $x \leq x_{max}$ is set for tension-controlled sections.

The maximum allowable depth of the compression block is given by:

$$a_{max} = \lambda x_{max} \qquad \text{(EC2 3.1.7(3))}$$

where,

$$\lambda = 0.8 \qquad \text{if} \quad f_{ck} < 50\,\text{MPa} \qquad \text{(EC2 3.1.7)}$$

$$\lambda = 0.8 - \left(\frac{f_{ck} - 50}{400} \right) \qquad \text{if} \quad f_{ck} > 50\,\text{MPa} \qquad \text{(EC2 3.1.7)}$$

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pk} is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel.

Based on the calculated f_{pk}, the depth of the neutral axis is recalculated, and f_{pk} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \lambda x$$

- If $a \leq a_{\max}$ (EC2 3.1.7(3)), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_{ED}^{0} = f_{pk} A_p \left(d_p - \frac{a}{2} \right)$$

- If $a > a_{\max}$ (EC2 3.1.7(3)), a failure condition is declared.

- If $M > M_{ED}^{0}$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension-controlled case. In that case, it is assumed that the depth of the neutral axis x is equal to $x_{\max}$. The stress in the post-tensioning steel, f_{pk}, is then calculated based on strain compatibility and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = \eta f_{cd} a_{\max} b$$

$$T = A_p f_{pk}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\eta f_{cd} a_{\max} b - A_p f_{pk}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_{ED}^{bal} = A_p f_{pk}^{bal}\left(d_p - \frac{a_{\max}}{2}\right) + A_s^{bal} f_s^{bal}\left(d_s - \frac{a_{\max}}{2}\right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, x.

18.7.1.2.1.1 Case 1: Post-tensioning steel is adequate

When $M < M_{ED}^{0}$, the amount of post-tensioning steel is adequate to resist the design moment M. A minimum reinforcement is provided to satisfy the flexural cracking requirements (EC2 7.3.2).

18.7.1.2.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $\left(M_{ED}^{0} < M < M_{ED}^{bal}\right)$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports this required area of tension reinforcement. Since M is bound by M_{ED}^{0} at the lower end and M_{ED}^{bal} at the upper end, and M_{ED}^{0} is associated with $A_s = 0$ and M_{ED}^{bal} is associated with $A_s = A_s^{bal}$, the required area will fall within the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

18.7.1.2.1.3 Case 3: Post-tensioning steel and tension reinforcement is not adequate

When $\left(M > M_{ED}^{bal}\right)$, compression reinforcement is required (EC2 5.5 (4)). In that case, ETABS assumes that the depth of the neutral axis, x, is equal to $x_{\max}$. The values of f_{pk} and f_s reach their respective balanced condition values, f_{pk}^{bal} and f_s^{bal}. Then the area of compression reinforcement, A_s', is determined as follows:

- The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{ED,s} = M - M_{ED}^{bal}$$

The required compression reinforcement is given by:

$$A_s' = \frac{M_{ED,s}}{\left(0.87 f_s' - \eta f_{cd}\right)\left(d - d'\right)}, \text{ where}$$

$$f_s' = \varepsilon_{cu3} E_s \left[\frac{a_{\max} - d'}{a_{\max}}\right] \leq 0.87 f_y.$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{ED,s}}{0.87 f_y \left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M is positive, and vice versa if M is negative.

18.7.1.2.2 Design of Flanged Beams

18.7.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

18.7.1.2.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that A_s = 0. In that case, the moment capacity M_{ED}^{0} is determined as follows:

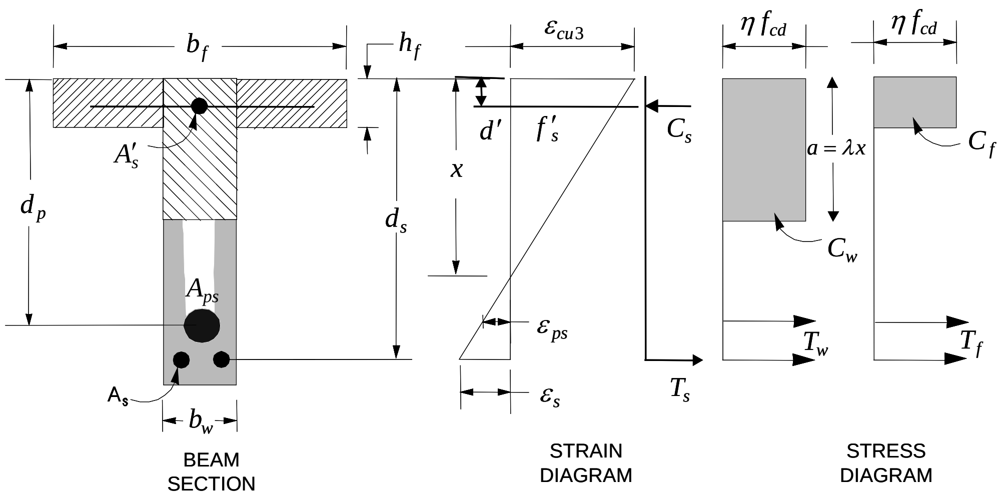


Figure 18-2 T-Beam Design

The maximum depth of the compression zone, x_{max}, is calculated based on the limitation that the tension reinforcement strain shall not be less than ε_{smin}:

$$c_{max} = \left(\frac{\varepsilon_{cu3}}{\varepsilon_{cu3} + \varepsilon_{s\min}}\right) d_p$$

where,

ε_{cu3} = 0.0035

Therefore, the program limit for the depth of the neutral axis is $x \leq x_{max}$ for tension-controlled sections.

The maximum depth of the compression block is given by:

$$a_{max} = \lambda x_{max} \qquad \text{(EC2 3.1.7(3))}$$

where,

$$\lambda = 0.8 \qquad \text{if } f_{ck} < 50\,\text{MPa} \qquad \text{(EC2 3.1.7)}$$

$$\lambda = 0.8 - \left(\frac{f_{ck} - 50}{400}\right) \qquad \text{if } f_{ck} > 50\,\text{MPa} \qquad \text{(EC2 3.1.7)}$$

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pk}, is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps}, in the post-tensioning steel. Based on the calculated f_{pk}, the depth of the neutral axis is recalculated, and f_{pk} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \lambda x$$

- If $a \leq a_{max}$ (EC2 3.1.7(3)), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_{ED}^{0} = f_{pk} A_p \left(d_p - \frac{a}{2}\right)$$

- If $a > a_{max}$ (EC2 3.1.7(3)), a failure condition is declared.

- If $M > M_{ED}^{0}$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension- controlled case. In that case it is assumed that the depth of the neutral axis x is equal to x_{max}. The stress in the post-tensioning steel, f_{pk}, is then calculated based on strain compatibility, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case, the width of the beam is taken as b_f. Compression reinforcement is required when $a > a_{\max}$.

- If $a > h_f$, the calculation for A_s is given by:

$$C = \eta f_{cd} a_{\max} A_c^{com}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$T = A_p f_{pk}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\eta f_{cd} a_{\max} A_c^{com} - A_p f_{pk}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_{ED}^{bal} = A_p f_{pk}^{bal}\left(d_p - \frac{a_{\max}}{2}\right) + A_s^{bal} f_s^{bal}\left(d_s - \frac{a_{\max}}{2}\right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcing steel, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, x.

18.7.1.2.2.2.1 Case 1: Post-tensioning steel is adequate

When $M < M_{ED}^{0}$, the amount of post-tensioning steel is adequate to resist the design moment M. Minimum reinforcement is provided to satisfy the flexural cracking requirements (EC2 7.3.2).

18.7.1.2.2.2.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $\left(M_{ED}^{0} < M < M_{ED}^{bal}\right)$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports the required area of tension reinforcement. Since M is bounded by M_{ED}^{0} at the lower end and M_{ED}^{bal} at the upper end, and M_{ED}^{0} is associated with $A_s = 0$ and M_{ED}^{bal} is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

18.7.1.2.2.2.3 Case 3: Post-tensioning steel and tension reinforcement is not adequate

When $\left(M > M_{ED}^{bal}\right)$, compression reinforcement is required (EC2 5.5 (4)). In that case ETABS assumes that the depth of the neutral axis, x, is equal to $x_{\max}$. The values of f_{pk} and f_s reach their respective balanced condition values, f_{pk}^{bal} and f_s^{bal}. Then the area of compression reinforcement, A_s', is determined as follows:

- The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{ED,s} = M - M_{ED}^{bal}$$

The required compression reinforcement is given by:

$$A_s' = \frac{M_{ED,s}}{\left(0.87 f'_s - \eta f_{cd}\right)\left(d - d'\right)}, \text{ where}$$

$$f'_s = \varepsilon_{cu3} E_s \left[\frac{a_{\max} - d'}{a_{\max}}\right] \leq 0.87 f_y.$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{ED,s}}{0.87 f_y \left(d_s - d'\right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M is positive, and vice versa if M is negative.

18.7.1.2.3 Minimum and Maximum Reinforcement

Reinforcement in prestressed concrete beams is computed to increase the strength of sections as required in the flexural design of prestressed beam or to comply with shear link requirements. The minimum flexural tension reinforcement required for a beam section to comply with the cracking requirements must be separately investigated by the user.

For bonded tendons, there is no minimum un-tensioned reinforcement requirements.

For unbonded tendons, the minimum flexural reinforcement provided in a rectangular or flanged beam section is given as:

$$A_{s,\min} = 0.26 \frac{f_{ctm}}{f_{yk}} bd \geq 0.0013bd \qquad \text{(EC2 9.2.1.1)}$$

where f_{ctm} is the mean value of axial tensile strength of the concrete and is computed as:

$$f_{ctm} = 0.30 f_{ck}^{(2/3)} \quad \text{for } f_{ck} \leq 50 \text{ MPa} \qquad \text{(EC2 3.12, Table 3.1)}$$

$$f_{ctm} = 2.12 ln\left(1 + f_{cm}/10\right) \quad \text{for } f_{ck} > 50 \text{ MPa} \qquad \text{(EC2 3.12, Table 3.1)}$$

$$f_{cm} = f_{ck} + 8 \text{ MPa} \qquad \text{(EC2 3.12, Table 3.1)}$$

The minimum flexural tension reinforcement required for control of cracking should be investigated independently by the user.

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (EC2 9.2.1.1(3)).

18.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each loading combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular loading combination, at a particular station due to the beam major shear, the following steps are involved (EC2 6.2):

- Determine the factored shear force, V
- Determine the shear force, $V_{Rd,c}$, that can be resisted by the concrete
- Determine the shear reinforcement required to carry the balance

The following three sections describe in detail the algorithms associated with these steps.

18.7.2.1 Determine Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors.

18.7.2.2 Determine Concrete Shear Capacity

The design shear resistance of the beam without shear reinforcement, $V_{Rd,c}$ is calculated as:

$$V_{Rd,c} = [\mathrm{C}_{Rd,c}\, k(100\, \rho_1 f_{ck})^{1/3} + k_1\, \sigma_{cp}\,]\,(b_w d) \qquad \text{(EC2 6.2.2(1))}$$

$$V_{Rd,c} \geq [v_{\min} + k_1\, \sigma_{cp}\,]\,(b_w d), \qquad \text{(EC2 6.2.2(1))}$$

where f_{ck} is in MPa

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \text{ with } d \text{ in mm} \qquad \text{(EC2 6.2.2(1))}$$

$$\rho_1 = \text{ tension reinforcement ratio } = \frac{\left(A_{s1} + A_{ps}\right)}{b_w d} \leq 0.02 \qquad \text{(EC2 6.2.2(1))}$$

A_{s1} = area of mild-tension reinforcement (EC2 6.2.2(1))

A_{ps} = area of prestress-tension reinforcement (EC2 6.2.2(1))

σ_{cp} = average stress in concrete due to axial force N_{Ed}/A_c (EC2 6.2.2(1))

$$\sigma_{cp} = N_{Ed}/A_c < 0.2\, f_{cd} z \text{ MPa} \quad \text{(EC2 6.2.2(1))}$$

A_c = the total gross area of concrete section

The value of $C_{Rd,c}$, $v_{\min}$, and k_1 for use in a country may be found in its National Annex. The program default values for $C_{Rd,c}$, $v_{\min}$ and k_1 are given as follows (EC2 6.2.2(1)):

$C_{Rd,c} = 0.18/\gamma_{c,}$

$v_{\min} = 0.035\, k^{3/2} f_{ck}^{1/2}$

$k_1 = 0.15.$

If light-weight concrete:

$C_{Rd,c} = 0.15/\gamma_c$ (EC2 11.6.1(1))

$v_{\min} = 0.03\, k^{3/2} f_{ck}^{1/2}$ (EC2 11.6.1(1))

$k_1 = 0.15.$ (EC2 11.6.1(1))

18.7.2.3 Determine Required Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. The assumptions in designing the shear reinforcement are as follows:

- The beam sections are assumed to be prismatic. The effect of any variation of width in the beam section on the concrete shear capacity is neglected.
- The effect on the concrete shear capacity of any concentrated or distributed load in the span of the beam between two columns is ignored. Also, the effect of the direct support on the beams provided by the columns is ignored.

- All shear reinforcement is assumed to be perpendicular to the longitudinal reinforcement.

In designing the shear reinforcement for a particular beam, for a particular load combination, the following steps of the standard method are involved (EC2 6.2).

- Obtain the design value of the applied shear force *V* from the ETABS analysis results (EC2 6.2.3(3)).

The shear force is limited to a maximum of:

$$V_{Rd,\max} = \frac{\alpha_{cw} b_w z v_1 f_{cd}}{\cot\theta + \tan\theta}, \text{ where} \qquad \text{(EC2 6.2.3(3))}$$

α_{cw} is conservatively taken as 1 (EC2 6.2.3(3))

The strength reduction factor for concrete cracked in shear, v_1 is defined as:

$$v_1 = 0.6\left(1 - \frac{f_{ck}}{250}\right) \qquad \text{(EC2 6.2.2(6))}$$

$z = 0.9d$ (EC2 6.2.3(1))

θ is optimized by program and is set to 45° for combinations including seismic loading (EC2 6.2.3(2)).

- Given V_{Ed}, V_{Rdc}, $V_{Rd,\max}$, the required shear reinforcement in area/unit length is calculated as follows:

- If $V_{Ed} \le V_{Rdc}$,

$$\frac{A_{sw}}{s_v} = \frac{A_{sw,\min}}{s}$$

- If $V_{R,dc} < V_{Ed} \le V_{Rd,\max}$

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{z f_{ywd} \cot\theta} \ge \frac{A_{sw,\min}}{s} \qquad \text{(EC2 6.2.3(3))}$$

- If $V_{Ed} > V_{Rd,\max}$

a failure condition is declared. (EC2 6.2.3(3))

The maximum of all the calculated A_{sw}/s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

The minimum shear reinforcement is defined as:

$$\frac{A_{sw,\min}}{s} = \frac{0.08\sqrt{f_{ck}}}{f_{yk}} b_w \qquad \text{(EC2 9.2.2(5))}$$

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

18.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_{Ed}
- Determine special section properties
- Determine critical torsion capacity
- Determine the torsion reinforcement required

18.7.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T_{Ed} is permitted to be reduced in accordance with the code (EC2 6.3.1(2)).

However, the program does not automatically redistribute the internal forces and reduce T_{Ed}. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

18.7.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_k, t_{ef}, u, u_k, and z_i are calculated. These properties are described in the following (EC2 6.3.2).

A = Area enclosed by the outside perimeter of the cross-section

A_k = Area enclosed by centerlines of the connecting walls, where the centerline is located a distance of $t_{ef}/2$ from the outer surface

t_{ef} = Effective wall thickness, A/u. It is taken as at least twice the distance between the edge and center of the longitudinal rebar.

u = Outer perimeter of the cross-section

u_k = Perimeter of the area A_k

z_i = Side length of wall i, defined as the distance between the intersection points of the wall centerlines

In calculating the section properties involving reinforcement, such as A_k, and u_k, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 mm. This is equivalent to 38-mm clear cover and a 12-mm stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during calculation of torsion section properties. With this assumption, the special properties for a rectangular beam section are given as:

$$A = bh \qquad \text{(EC2 6.3.2(1))}$$

$$A_k = (b - t_{ef})(h - t_{ef}) \qquad \text{(EC2 6.3.2(1))}$$

$$u = 2b + 2h \qquad \text{(EC2 6.3.2(1))}$$

$$u_k = 2(b - t_{ef}) + 2(h - t_{ef}) \qquad \text{(EC2 6.3.2(3))}$$

where, the section dimensions b, h, and c are shown in Figure 18-3. Similarly, the special section properties for a flanged beam section are given as:

$$A = b_w h \qquad \text{(EC2 6.3.2(1))}$$

$$A_k = (b_w - t_{ef})(h - t_{ef}) \qquad \text{(EC2 6.3.2(1))}$$

$$u = 2b_w + 2h \qquad \text{(EC2 6.3.2(1))}$$

$$u_k = 2(h - t_{ef}) + 2(b_w - t_{ef}) \qquad \text{(EC2 6.3.2(3))}$$

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 18-3.

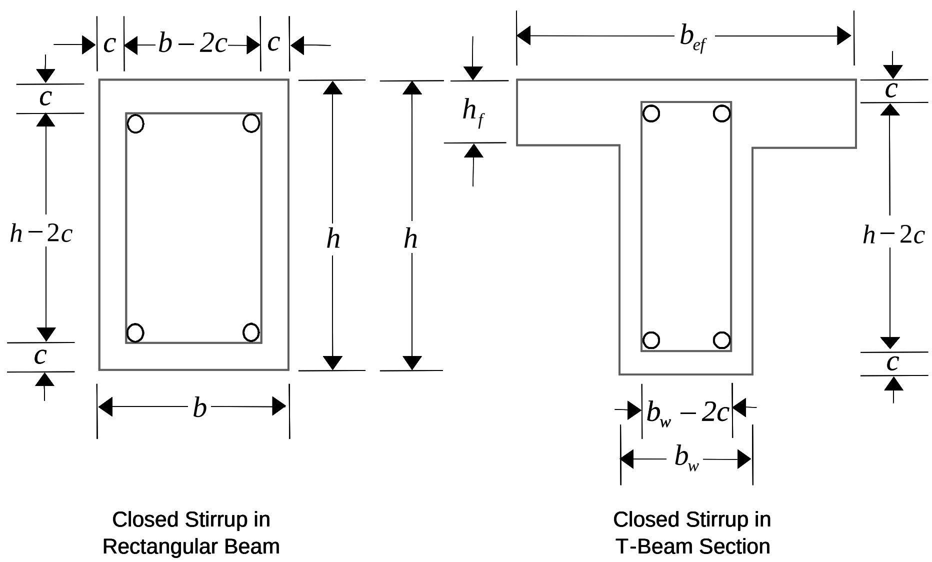


Figure 18-3 Closed stirrup and section dimensions for torsion design

18.7.3.3 Determine Critical Torsion Capacity

The torsion in the section can be ignored with only minimum shear reinforcement (EC2 9.2.1.1) required if the following condition is satisfied:

$$\frac{T_{Ed}}{T_{Rd,c}} + \frac{V_{Ed}}{V_{Rd,c}} \leq 1.0 \qquad \text{(EC2 6.3.2(5))}$$

where $V_{Rd,c}$ is as defined in the previous section and $T_{Rd,c}$ is the torsional cracking moment, calculated as:

$$T_{Rd,c} = f_{ctd} t_{ef} \, 2A_k \qquad \text{(EC2 6.3.2(1), 6.3.2(5))}$$

where t_{ef}, and f_{ctd}, the design tensile strength, are defined as:

$$t_{ef} = A/u \qquad \text{(EC2 6.3.2(1))}$$

$$f_{ctd} = \alpha_{ct} f_{ctk0.05}/\gamma_c \qquad \text{(EC2 Eq. 3.16)}$$

where A is the gross cross-section area, u is the outer circumference of the cross-section, α_{ct} is a coefficient, taken as 1.0, taking account of long- term effects on the tensile strength, and $f_{ctk0.05}$ is defined as:

$$f_{ctk0.05} = 0.7 f_{ctm} \qquad \text{(EC2 Table 3.1)}$$

18.7.3.4 Determine Torsion Reinforcement

If the expression in the previous subsection is satisfied, torsion can be safely ignored (EC2 6.3.2(5)) with only minimum shear reinforcement required. In that case, the program reports that no torsion reinforcement is required. However, if the equation is not satisfied, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals.

If torsion reinforcement in the form of closed stirrups is required, the shear due to this torsion, V_t, is first calculated, followed by the required stirrup area, as:

$$\frac{A_t}{s} = \frac{V_t}{z f_{ywd} \cot\theta} \qquad \text{(EC2 6.2.3(3))}$$

$$V_t = \left(h - t_{ef}\right)\frac{T_{Ed} - T_{con}}{2A_k} \qquad \text{(EC2 6.3.2(1))}$$

The required longitudinal reinforcement for torsion is defined as:

$$T_{con} = \left(1 - \frac{V_{Ed}}{V_{Rd,c}}\right)T_{Rd,c} \qquad \text{(EC2 6.3.2(5))}$$

$$A_{sl} = \frac{T_{Ed}}{2A_k}\cot\theta\frac{u_k}{f_{yd}} \qquad \text{(EC2 6.3.2(3))}$$

where θ is the angle of the compression struts, as previously defined for beam shear. In the preceding expressions, θ is taken as 45 degrees. The code allows any value between 21.8 and 45 degrees (EC2 6.2.3(2)), while the program assumes the conservative value of 45 degrees.

When torsional reinforcement is required, an upper limit on the combination of V_{Ed} and T_{Ed} that can be carried by the section without exceeding the capacity of the concrete struts also is checked using:

$$\frac{T_{Ed}}{T_{Rd,\max}} + \frac{V_{Ed}}{V_{Rd,\max}} \le 1.0 \qquad \text{(EC2 6.3.2(4))}$$

where $T_{Rd,\max}$, the design torsional resistance moment is defined as:

$$T_{Rd,\max} = 2\nu\alpha_{cw} f_{cd} A_k t_{ef} \sin\theta\cos\theta \qquad \text{(EC2 6.3.2(4))}$$

If this equation is not satisfied, a failure condition is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated A_{sl} and A_t/s values obtained from each load combination is reported, along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

18.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips usually are governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method for prestressed reinforced concrete (EC2-2004) as described in the following sections. To learn more about the design strips, refer to the section entitled "ETABS Design Techniques" in the *Key Features and Terminology* manual.

18.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the subsection that follows and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

18.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

18.8.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

18.8.1.2.1 Minimum and Maximum Slab Reinforcement

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limits (EC2 9.3.1.1, 9.2.1.1, UK, NA Table NA.1):

$$A_{s,\min} = 0.26 \frac{f_{ctm}}{f_{yk}} bd \qquad \text{(EC2 9.2.1.1(1))}$$

$$A_{s,\min} = 0.0013 bd \qquad \text{(EC2 9.2.1.1(1))}$$

where f_{ctm} is the mean value of axial tensile strength of the concrete and is computed as:

$$f_{ctm} = 0.30 f_{ck}^{(2/3)} \qquad \text{for } f_{ck} \leq 50 \text{ MPa} \qquad \text{(EC2 Table 3.1)}$$

$$f_{ctm} = 2.12 ln\left(1 + f_{cm}/10\right) \qquad \text{for } f_{ck} > 50 \text{ MPa} \qquad \text{(EC2 Table 3.1)}$$

$$f_{cm} = f_{ck} + 8 \text{ MPa} \qquad \text{(EC2 Table 3.1)}$$

The minimum flexural tension reinforcement required for control of cracking should be investigated independently by the user.

An upper limit on the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (EC2 9.2.1.1(3)).

18.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code specific items are described in the following.

18.8.2.1 Critical Section for Punching Shear

The punching shear is checked at the face of the column (EC2 6.4.1(4)) and at a critical section at a distance of 2.0*d* from the face of the support (EC2 6.4.2(1)). The perimeter of the critical section should be constructed such that its length is minimized. Figure 18-4 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

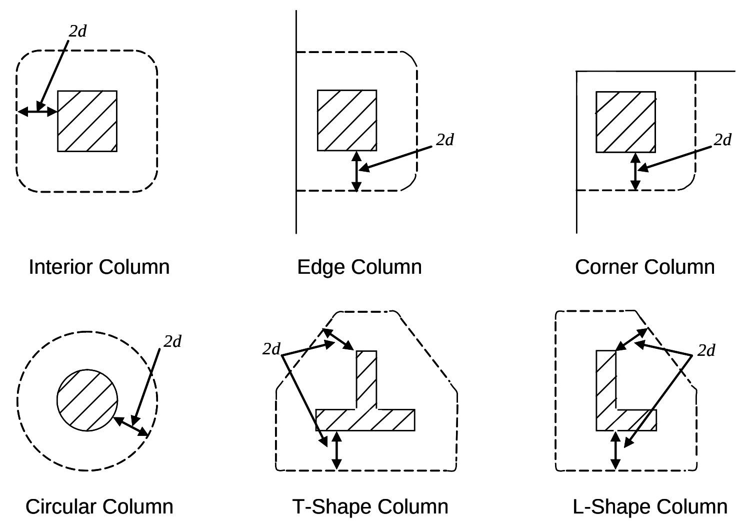


Figure 18-4 Punching Shear Perimeters

18.8.2.2 Determine Concrete Capacity

The concrete punching shear stress capacity is taken as:

$$V_{Rd,c} = \left[C_{Rd,c} k \left(100 \rho_1 f_{ck} \right)^{1/3} + k_1 \sigma_{cp} \right] \qquad \text{(EC2 6.4.4(1))}$$

with a minimum of:

$$V_{Rd,c} = \left(v_{\min} + k_1\sigma_{cp}\right) \qquad \text{(EC2 6.4.4(1))}$$

where f_{ck} is in MPa and

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \text{ with } d \text{ in mm} \qquad \text{(EC2 6.4.4(1))}$$

$$\rho_1 = \sqrt{\rho_{1x}\rho_{1y}} \leq 0.02 \qquad \text{(EC2 6.4.4(1))}$$

where ρ_{1x} and ρ_{1y} are the reinforcement ratios in the x and y directions respectively, conservatively taken as zeros, and

$$\sigma_{cp} = (\sigma_{cx} + \sigma_{cy})/2 \qquad \text{(EC2 6.4.4(1))}$$

where σ_{cx} and σ_{cy} are the normal concrete stresses in the critical section in the x and y directions respectively, conservatively taken as zeros.

$$C_{Rd,c} = 0.18/\gamma_c \qquad \text{(EC2 6.4.4(1))}$$

$$v_{\min} = 0.035k^{3/2} f_{ck}^{\;1/2} \qquad \text{(EC2 6.4.4(1))}$$

$$k_1 = 0.15 \qquad \text{(EC2 6.4.4(1))}$$

18.8.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear, the nominal design shear stress, v_{Ed}, is calculated as:

$$v_{Ed} = \frac{V_{Ed}}{ud}\left[1 + k\frac{M_{Ed,2}u_1}{V_{Ed}W_{1,2}} + k\frac{M_{Ed,3}u_1}{V_{Ed}W_{1,3}}\right], \text{ where} \qquad \text{(EC2 6.4.4(2))}$$

k is the function of the aspect ratio of the loaded area in Table 6.1 of EN 1992-1-1

u_1 is the effective perimeter of the critical section

d is the mean effective depth of the slab

M_{Ed} is the design moment transmitted from the slab to the column at the connection along bending axis 2 and 3

V_{Ed} is the total punching shear force

W_1 accounts for the distribution of shear based on the control perimeter along bending axis 2 and 3.

18.8.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

18.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted provided that the effective depth of the slab is greater than or equal to 200 mm.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* as described in the earlier section remain unchanged. The design of punching shear reinforcement is as described in the following subsections.

18.8.3.1 Determine Required Shear Reinforcement

The shear stress is limited to a maximum limit of

$$V_{Rd,\max} = \frac{\alpha_{cw} v_1 f_{cd}}{\left(\cot\theta + \tan\theta\right)} b_w z \text{ where} \qquad \text{(EC2 6.2.3(3))}$$

α_{cw} is conservatively taken as 1

$$v_1 = 0.6\left(1 - \frac{f_{ck}}{250}\right)$$

$$z = 0.9d \qquad \text{(EC2 6.2.3(1))}$$

$1 \leq \cot\theta \leq 2.5$, program default value is 1, which can be overwritten by the user (EC2 6.2.3(2))

Given v_{Ed}, $v_{Rd,c}$ and $v_{Rd,\max}$, the required shear reinforcement is calculated as follows (EC2 6.4.5):

- If $v_{Ed} < v_{Rd,\max}$,

$$A_w = \frac{\left(v_{Ed} - 0.75 v_{Rd,c}\right)u}{1.5 f_{ywd}} s_r \qquad \text{(EC2 6.4.5)}$$

- If $v_{Ed} > v_{Rd,\max}$, a failure condition is declared. (EC2 6.4.3)
- If v_{Ed} exceeds the maximum permitted value of $v_{RD,\max}$, the concrete section should be increased in size.

18.8.3.2 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of a rectangular columns should be arranged on peripheral lines, i.e., a line running parallel to and at constant distances from the sides of the column. Figure 18-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed 2*d*. The spacing between adjacent shear reinforcement in the first line of shear reinforcement shall not exceed 2*d* measured in a direction parallel to the column face (EC2 6.4.5(4)).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

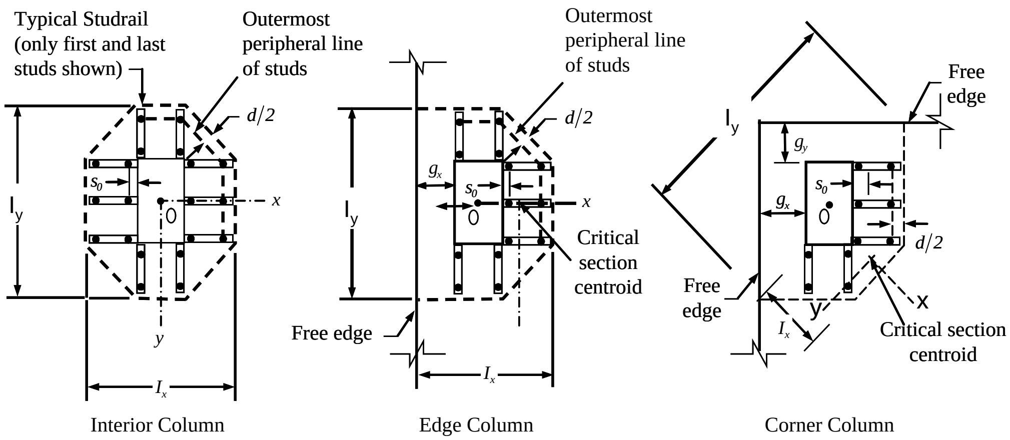


Figure 18-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

18.8.3.3 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in EC2 4.4.1 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.3d. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed 1.5d and should not exceed 2d at additional perimeters. The limits of s_o and the spacing, s, between the peripheral lines are specified as:

$0.3d \leq s_o \leq 2d$ (EC2 9.4.3(1))

$s \leq 0.75d$ (EC2 9.4.3(1))

$g \leq 1.5d$ (first perimeter) (EC2 9.4.3(1))

$g \leq 2d$ (additional perimeters) (EC2 9.4.3(1))

Chapter 19
Design for Hong Kong CP 2013

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the Hong Kong limit state code CP-2013 [CP 2013]. Various notations used in this chapter are listed in Table 19-1. For referencing to the pertinent sections of the Hong Kong CP code in this chapter, a prefix "CP" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

19.1 Notations

The following table identifies the various notations used in this chapter.

Table 19-1 List of Symbols Used in the Hong Kong CP 2013 Code

A_{cv}	Area of section for shear resistance, mm^2

Table 19-1 List of Symbols Used in the Hong Kong CP 2013 Code

A_g	Gross area of cross-section, mm^2
A_s	Area of tension reinforcement, mm^2
A_{ps}	Area of prestress steel, mm^2
A'_s	Area of compression reinforcement, mm^2
A_{sv}	Total cross-sectional area of links at the neutral axis, mm^2
A_{sv} / s_v	Area of shear reinforcement per unit length of the member, mm^2/mm
a	Depth of compression block, mm
$a_{\max}$	Maximum depth of the compression block, mm
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
d or d_e	Effective depth of tension reinforcement, mm
d'	Depth to center of compression reinforcement, mm
E_c	Modulus of elasticity of concrete, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 200,000 MPa
f_{ci}	Concrete strength at transfer, MPa
f_{cu}	Characteristic cube strength, MPa
f_{pu}	Characteristic strength of a prestressing tendon, MPa
f_{pb}	Design tensile stress in tendon, MPa
f'_s	Compressive stress in a beam compression steel, MPa
f_y	Characteristic strength reinforcement, MPa
f_{yv}	Characteristic strength of link reinforcement, MPa
h	Overall depth of a section in the plane of bending, mm
h_f	Flange thickness, mm

Table 19-1 List of Symbols Used in the Hong Kong CP 2013 Code

k_1	Shear strength enhancement factor for support compression
k_2	Concrete shear strength factor, $\left[f_{cu}/25\right]^{1/3}$
M	Design moment at a section, MPa
M_u	Design moment resistance of a section, MPa
M_u^0	Design moment resistance of a section with tendons only, N-mm
M_u^{bal}	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
s_v	Spacing of the links along the length of the beam, mm
s	Spacing of shear rails, mm
T	Tension force, N
V	Design shear force at ultimate design load, N
u	Perimeter of the punching critical section, mm
v	Design shear stress at a beam cross-section or at a punch critical section, MPa
v_c	Design ultimate shear stress resistance of a concrete beam, MPa
v_{co}	Ultimate shear stress resistance of an uncracked concrete section, MPa
v_{cr}	Ultimate shear stress resistance of a cracked concrete section, MPa
$v_{\max}$	Maximum permitted design factored shear stress at a beam section or at the punch critical section, MPa
v_t	Torsional shear stress, MPa
x	Neutral axis depth, mm
x_{bal}	Depth of neutral axis in a balanced section, mm
z	Lever arm, mm

Table 19-1 List of Symbols Used in the Hong Kong CP 2013 Code

β	Torsional stiffness constant
β_b	Moment redistribution factor in a member
γ_f	Partial safety factor for load
γ_m	Partial safety factor for material strength
ε_c	Maximum concrete strain
ε_{ps}	Strain in prestressing steel
ε_s	Strain in tension steel
ε'_s	Strain in compression steel

19.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. The design load combinations are obtained by multiplying the characteristic loads by appropriate partial factors of safety, γ_f (CP 2.3.2.1, Table 2.1). For Hong Kong CP 2013, if a structure is subjected to dead (G), live (Q), pattern live (PQ), and wind (W) loads, and considering that wind forces are reversible, the load combinations in the following sections may need to be considered (CP 2.3.2.1, 12.3.4.2, 12.3.4.3 and 12.3.5.1).

For post-tensioned concrete design, the user can specify the prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning are defined in the following sections.

19.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress forces in accordance with Hong Kong CP 2013 sections 12.3.5. The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0G + 1.0PT

19.2.2 Service Load Combination

The following load combinations are used for checking the requirements of prestress for serviceability in accordance with CP 12.3.4. It is assumed that all long-term losses have occurred already at the service stage.

1.0G + 1.0PT
1.0G + 1.0Q + 1.0PT

19.2.3 Ultimate Limit State Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with CP 2.3.2.1, Table 2.1.

The combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are determined automatically by ETABS by subtracting the primary PT moments when the flexural design is completed.

1.4G + 1.0PT*

1.4G + 1.6Q + 1.0PT*

1.4G + 1.6(0.75PQ) + 1.0PT*

1.0G ± 1.4W +1.0PT*
1.4G ± 1.4W + 1.0PT*
1.2G + 1.2Q ± 1.2W + 1.0PT*

* — Replace PT with H for flexural design only

Other appropriate loading combinations should be used if roof live load is separately treated, or other types of loads are present.

19.3 Limits on Material Strength

Grade C28/C35 and C32/C40 are the minimum recommended for post-tensioning and pre-tensioning respectively. In both cases the concrete strength at transfer should not be less than 25 MPa (CP 12.1.8.1).

The specified characteristic strength of un-tensioned reinforcement is given as follows (CP 3.2.3, Table 3.3):

Hot rolled mild reinforcement -	250 MPa	(CP 3.2.3, Table 3.3)
High yield reinforcement -	500 MPa	(CP 3.2.3, Table 3.3)

The specified characteristic strength of prestressing steel should conform to CP 2013 section 3.3.

ETABS also checks the tensile strength in the prestressing steel (CP 12.7.1). The permissible tensile stresses in all types of prestressing steel, in terms of the specified minimum tensile strength f_{pu}, are summarized as follows:

a. Due to tendon jacking force: $0.75\ f_{pu}$

b. Immediately after prestress transfer: $0.70\ f_{pu}$

In any circumstances, the initial prestressing forces shall not exceed $0.75\ f_{pu}$.

19.4 Partial Safety Factors

The design strengths for concrete and reinforcement are obtained by dividing the characteristic strength of the material by a partial safety factor, γ_m. The values of γ_m used in the program are listed in the table that follows, as taken from CP Table 2.2 (CP 2.4.3.2):

Values of γ_m for the ultimate limit state	
Reinforcement, γ_{ms}	1.15
Prestressing steel, γ_{mp}	1.15
Concrete in flexure and axial load, γ_{mc}	1.50
Shear strength without shear reinforcement, γ_{mv}	1.25

These factors are already incorporated in the design equations and tables in the code. Note that for reinforcement, the default factor of 1.15 is for Grade 460 reinforcement. If other grades are used, this value should be overwritten as necessary. Changes to the partial safety factors are carried through the design equations where necessary, typically affecting the material strength portions of the equations.

19.5 Design Assumptions for Prestressed Concrete Structures

The ultimate limit state of prestressed members for flexure and axial loads shall be based on assumptions given in CP 12.3.7.1.

- The strain distribution in the concrete in compression is derived from the assumption that a plane section remains plane (CP 12.3.7.1).
- The design stresses in the concrete in compression are taken as 0.45 f_{cu}. The maximum strain at the extreme concrete compression fiber shall be assumed equal to 0.0035 (CP 12.3.7.1).
- Tensile strength of the concrete is ignored (CP 12.3.7.1).
- The strain in bonded prestressing tendons and in any additional reinforcement (compression or tension) is derived from the assumption that plane section remains plane (CP 12.3.7.1).

The serviceability limit state of prestressed members uses the following assumptions given in CP 12.3.4.1.

- Plane sections remain plane, i.e., strain varies linearly with depth through the entire load range (CP 12.3.4.1).
- Elastic behavior exists by limiting the concrete stresses to the values given in CP 12.3.4.2, 12.3.4.3 and 12.3.5 (CP 12.3.4.1).
- In general, it is only necessary to calculate design stresses due to the load arrangements immediately after the transfer of prestress and after all losses or prestress have occurred; in both cases the effects of dead and imposed loads on the strain and force in the tendons may be ignored (CP 12.3.4.1).

Prestressed concrete members are investigated at the following three stages (CP 12.3.4.2 and 12.3.4.3):

- At transfer of prestress force
- At service loading
- At nominal strength

The prestressed flexural members are classified as Class 1 (uncracked), Class 2 (cracked but no visible cracking), and Class 3 (cracked) based on tensile strength f_t, the computed extreme fiber stress in tension in the precompressed tensile zone at service loads (CP 12.1.3).

The precompressed tensile zone is that portion of a prestressed member where flexural tension, calculated using gross section properties, would occur under unfactored dead and live loads if the prestress force was not present. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress (CP 12.1.3).

Class 1: No flexural tensile stresses

Class 2: Flexural tensile stresses with no visible cracking

Class 3: Flexural tensile stresses with surface crack widths as follows:

- ≤ 0.1 mm for members in exposure conditions 3 and 4 (Table 4.1 of CP 2013)
- ≤ 0.2 mm for all other members

19.6 Serviceability Requirements of Flexural Members

19.6.1 Serviceability Check at Initial Service Load

The stresses in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits (CP 12.3.5.1 and 12.3.5.2):

- Extreme fiber stress in compression: $0.50 f_{ci}$
- Extreme fiber stress in tension for Class 1: ≤ 1.0 MPa
- Extreme fiber stress in tension for Calss 2:

  pre-tensioned member $0.45\sqrt{f_{ci}}$

post-tensioned member $0.36\sqrt{f_{ci}}$

The extreme fiber stress in tension for Class 3 should not exceed the appropriate value for a Class 2 member; otherwise the section should be designed as a cracked section.

19.6.2 Serviceability Check at Service Load

The stresses in the concrete for Class 1 and Class 2 prestressed flexural members at service loads, and after all prestress losses have occurred, are checked against the following limits (CP 12.3.4.2 an 12.3.4.3):

- Extreme fiber stress in compression due to prestress plus total load: $0.33 f_{cu}$
- Extreme fiber stress in compression due to prestress plus total load for continuous beams and other statically indeterminate structures: $0.4 f_{cu}$
- Extreme fiber stress in tension in the precompressed tensile zone at service loads:
  - Extreme fiber stresses in tension for Class 1: No tensile stress
  - Extreme fiber stresses in tension for Class 2:

    pre-tensioned member $0.45\sqrt{f_{cu}}$

    post-tensioned member $0.36\sqrt{f_{cu}}$

Although cracking is allowed for Class 3, it is assumed that the concrete section is uncracked and the user is limiting the tensile stress at the service stage as presented in Table 12.2, modified by the coefficients in Table 12.3 of CP 2013. The user needs to provide the tension limits for Class 3 elements at service loads in the design preferences (CP 12.3.4.3).

19.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

19.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

19.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of prestressed concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or a flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases the beam may be designed as a rectangular or inverted flanged beam.

19.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block shown in Figure 12-1 (CP 6.1.2.4(a)), where $\varepsilon_{c,\max}$ is defined as:

$$\varepsilon_{c,\max} = \begin{cases} 0.0035 & \text{if} \quad f_{cu} \le 60 \text{ MPA} \\ 0.0035 - 0.00006\left(f_{cu} - 60\right)^{1/2} & \text{if} \quad f_{cu} > 60 \text{ MPA} \end{cases}$$

Furthermore, it is assumed that moment redistribution in the member does not exceed 10% (i.e., $\beta_b \ge 0.9$) (CP 6.1.2.4(b)). The code also places a limitation on the neutral axis depth,

$$\frac{x}{d} \le \begin{cases} 0.5 & \text{for} \quad f_{cu} \le 45 \text{ N/mm}^2 \\ 0.4 & \text{for} \quad 45 < f_{cu} \le 70 \text{ N/mm}^2 \\ 0.33 & \text{for} \quad 70 < f_{cu} \le 100 \text{ N/mm}^2 \end{cases}$$

to safeguard against non-ductile failures (CP 6.1.2.4(b)). In addition, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The design procedure used by ETABS, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow. It is assumed that the design ultimate axial force does not exceed 0.1 $f_{cu} A_g$ (CP 6.1.2.4(a)); hence all beams are designed for major direction flexure, shear, and torsion only.

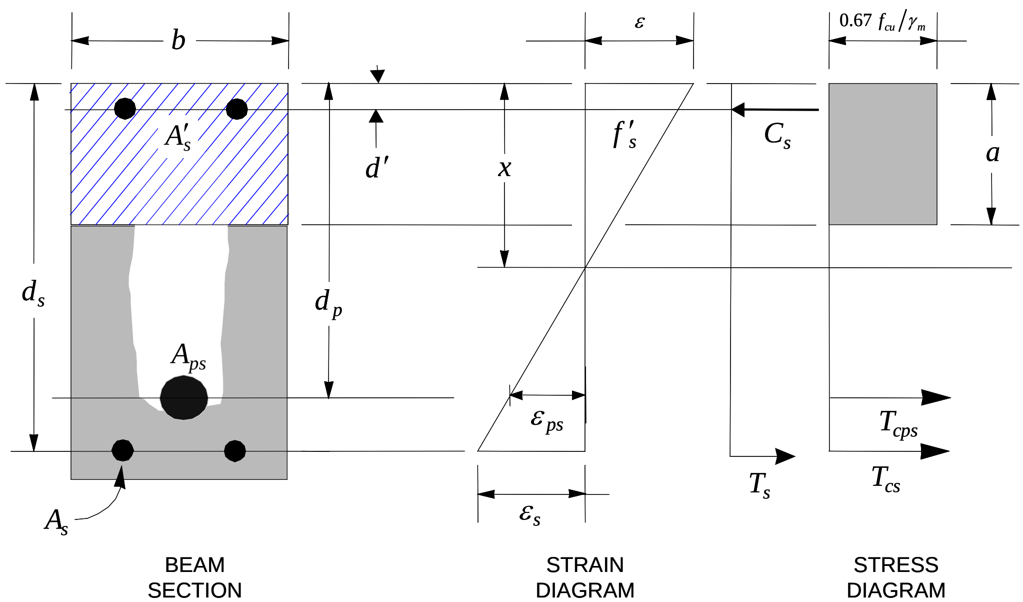


Figure 11-1 Rectangular Beam Design

19.7.1.2.1 Design of Rectangular Beams

The amount of post-tensioning steel adequate to resist the design moment *M* and minimum reinforcement are provided to satisfy the flexural cracking requirements (CP 19.2.1).

ETABS determines the depth of the neutral axis, *x*, by imposing force equilibrium, i.e., C = T, and performs an iteration to compute the depth of the neutral axis, which is based on stress-strain compatibility. After the depth of the neutral axis has been found, the stress in the post-tensioning reinforcement f_{pb} is computed based on strain compatibility.

The ductility of a section is controlled by limiting the *x*/*d* ratio (CP 6.1.2.4(b)):

$$\frac{x}{d} = \begin{cases} 0.5, & \text{for} \quad f_{cu} \le 45 \text{ N/mm}^2 \\ 0.4, & \text{for} \quad 45 < f_{cu} \le 70 \text{ N/mm}^2 \\ 0.33, & \text{for} \quad 70 < f_{cu} \le 100 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(b))}$$

The maximum depth of the compression block is given by:

$$a = \begin{cases} 0.9x & \text{for} \quad f_{cu} \le 45 \text{ N/mm}^2 \\ 0.8x & \text{for} \quad 45 < f_{cu} \le 70 \text{ N/mm}^2 \\ 0.72x & \text{for} \quad 70 < f_{cu} \le 100 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(a))}$$

The lever arm of the section must not be greater than 0.95 times the effective depth (CP 6.1.2.4(c)).

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \le 0.95d, \qquad \text{(CP 6.1.2.4(c))}$$

- If $a \le a_{\max}$ (CP 6.1.2.4), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_u^0 = A_{ps} f_{pb}\left(d_p - \frac{a}{2}\right)$$

- If $a > a_{\max}$ (CP 6.1.2.4), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of neutral axis x is equal to $x_{\max}$. The stress in the post-tensioning steel, f_{pb} is then calculated based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = \frac{0.67 f_{cu}}{\gamma_m} a_{\max} b$$

$$T = A_{ps} f_{pb}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\dfrac{0.67 f_{cu}}{\gamma_m} a_{\max} b - A_{ps} f_{pb}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$M_u^{bal} = A_{ps} f_{pb}^{bal}\left(d_p - \frac{a_{\max}}{2}\right) + A_s^{bal} f_s^{bal}\left(d_s - \frac{a_{\max}}{2}\right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of neutral axis, x.

19.7.1.2.1.1 Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment M.. Minimum reinforcement is provided to satisfy the ductility requirements, i.e., $M < M_u^0$.

19.7.1.2.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M and reports this required area of tension reinforcement. Since M is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end, and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will be between the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

19.7.1.2.1.3 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required (CP 6.1.2.4). In that case, ETABS assumes that the depth of neutral axis, x, is equal to x_{max}. The values of f_{pb} and f_s reach their respective balanced condition values, f_{pb}^{bal} and f_s^{bal}. The area of compression reinforcement, A'_s, is determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(f'_s - \frac{0.67 f_{cu}}{\gamma_c}\right)(d - d')}, \text{ where} \qquad \text{(CP 6.1.2.4(c))}$$

$$f'_s = E_s \varepsilon_c \left(1 - \frac{d'}{x}\right) \le 0.87 f_y, \qquad \text{(CP 6.1.2.4(c), 3.2.6, Fig. 3.9)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{0.87 f_y (d_s - d')}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M is positive, and vice versa if M is negative.

19.7.1.2.2 Design of Flanged Beams

19.7.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

19.7.1.2.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_S = 0$. In that case, moment capacity M_u^0 is determined as follows:

ETABS determines the depth of the neutral axis, x, by imposing force equilibrium, i.e., C = T, and performs an iteration to compute the depth of neutral axis, which is based on stress-strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pb} is computed based on strain compatibility.

The ductility of a section is controlled by limiting the x/d ratio (CP 6.1.2.4(b)):

$$\frac{x}{d} = \begin{cases} 0.5, & \text{for} \quad f_{cu} \le 45 \text{ N/mm}^2 \\ 0.4, & \text{for} \quad 45 < f_{cu} \le 70 \text{ N/mm}^2 \\ 0.33, & \text{for} \quad 70 < f_{cu} \le 100 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(b))}$$

The maximum depth of the compression block is given by:

$$a = \begin{cases} 0.9x & \text{for} \quad f_{cu} \le 45 \text{ N/mm}^2 \\ 0.8x & \text{for} \quad 45 < f_{cu} \le 70 \text{ N/mm}^2 \\ 0.72x & \text{for} \quad 70 < f_{cu} \le 100 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 6.1.2.4(a))}$$

The lever arm of the section must not be greater than 0.95 times the effective depth (CP 6.1.2.4(c)).

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \le 0.95d, \qquad \text{(CP 6.1.2.4(c))}$$

- If $a \le a_{\max}$ (CP 6.1.2.4), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$M_u^0 = A_{ps} f_{pb}\left(d_p - \frac{a}{2}\right)$$

- If $a > a_{\max}$ (CP 6.1.2.4), a failure condition is declared.

If $M > M_u^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case it is assumed that the depth of neutral axis x is equal to $c_{\max}$. The stress in the post-tensioning steel, f_{pb} is then calculated based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel, and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as b_f. Compression reinforcement is required when $K > K'$.

- If $a > h_f$, the calculation for A_s is given by

$$C = \frac{0.67 f_{cu}}{\gamma_c} a_{\max} A_c^{com}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$A_c^{com} = b_f h_f + b_w \left(a_{\max} - h_f \right)$$

$$T = A_{ps} f_{pb}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\frac{0.67 f_{cu}}{\gamma_m} a_{\max} A_c^{com} - A_{ps} f_{pb}^{bal}}{f_s^{bal}}$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of neutral axis, c.

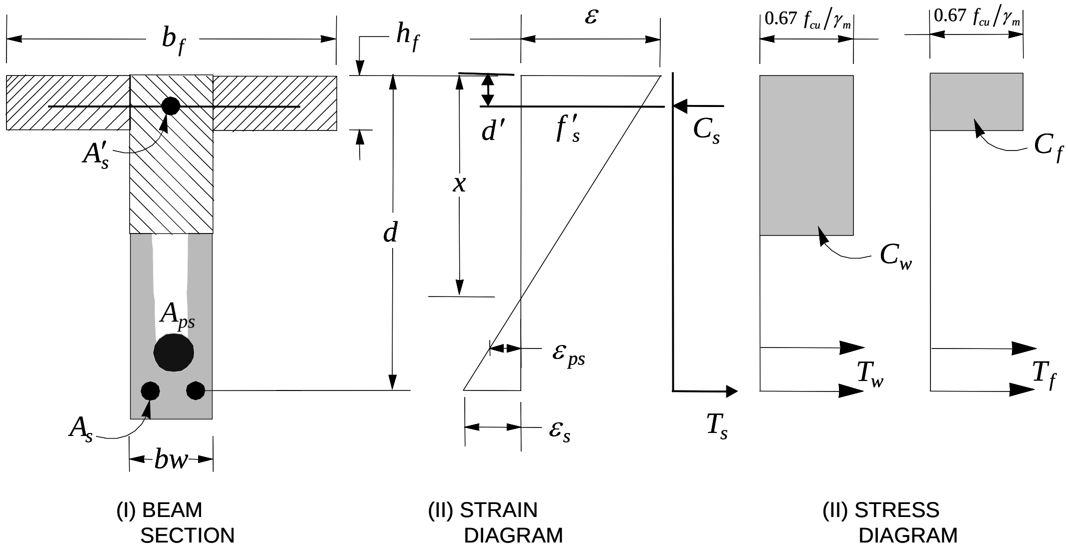


Figure 11-2 T-Beam Design

19.7.1.2.2.3 Case 1: Post-tensioning steel is adequate

When $M < M_u^0$, the amount of post-tensioning steel is adequate to resist the design moment *M*. Minimum reinforcement is provided to satisfy ductility requirements.

19.7.1.2.2.4 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{PS}, alone is not sufficient to resist *M*, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $M_u^0 < M < M_u^{bal}$, ETABS determines the required area of tension reinforcement, A_S, iteratively to satisfy the design moment *M* and reports the required area of tension reinforcement. Since *M* is bounded by M_u^0 at the lower end and M_u^{bal} at the upper end, and M_u^0 is associated with $A_s = 0$ and M_u^{bal} is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M is positive, or at the top if M is negative.

19.7.1.2.2.5 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M > M_u^{bal}$, compression reinforcement is required (CP 6.1.2.4). In that case ETABS assumes that the depth of the neutral axis, x, is equal to $x_{\max}$. The values of f_{pb} and f_s reach their respective balanced condition values, f_{pb}^{bal} and f_s^{bal}. The area of compression reinforcement, A'_s, is then determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M - M_u^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(f'_s - \frac{0.67 f_{cu}}{\gamma_c} \right)(d - d')}, \text{ where} \qquad \text{(CP 6.1.2.4(c))}$$

$$f'_s = E_s \varepsilon_c \left(1 - \frac{d'}{x} \right) \leq 0.87 f_y \qquad \text{(CP 6.1.2.4(c))}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{0.87 f_y \left(d_s - d' \right)}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom, and A'_s is to be placed at the top if M is positive and vice versa if M is negative.

19.7.1.3 Minimum and Maximum Reinforcement

Reinforcement in post-tensioned concrete beams is computed to increase the strength of sections as documented for the flexural design of post-tensioned beams or to comply with the shear link requirements. The minimum flexural tension reinforcement required for a beam section to comply with the cracking requirements needs to be separately investigated by the user.

For bonded tendons, there is no minimum untensioned reinforcement required.

For unbounded tendons, the minimum flexural reinforcement provided in a rectangular or flanged beam section is given by the following table, which is taken from CP Table 9.1(CP 9.2.1.1) with interpolation for reinforcement of intermediate strength:

Section	Situation	Definition of percentage	Minimum percentage	
			f_y = 250 MPa	f_y = 460 MPa
Rectangular	—	$100\ \frac{A_s}{bh}$	0.24	0.13
T- or L-Beam with web in tension	$\frac{b_w}{b_f} < 0.4$	$100\ \frac{A_s}{b_w h}$	0.32	0.18
	$\frac{b_w}{b_f} \geq 0.4$	$100\ \frac{A_s}{b_w h}$	0.24	0.13
T-Beam with web in compression	—	$100\ \frac{A_s}{b_w h}$	0.48	0.26
L-Beam with web in compression	—	$100\ \frac{A_s}{b_w h}$	0.36	0.20

The minimum flexural compression reinforcement, if it is required at all, is given by the following table, which is taken from CP Table 9.1(CP 9.2.1.1) with interpolation for reinforcement of intermediate strength:

Section	Situation	Definition of percentage	Minimum percentage
Rectangular	—	$100\ \dfrac{A'_s}{bh}$	0.20
T- or L-Beam	Web in tension	$100\ \dfrac{A'_s}{b_f h_f}$	0.40
	Web in compression	$100\ \dfrac{A'_s}{b_w h}$	0.20

In addition, an upper limit on both the tension reinforcement and compression reinforcement is imposed to be 0.2013 times the gross cross-sectional area (CP 3.12.6.1).

19.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination in the major direction of the beam. In designing the shear reinforcement for a particular beam for a particular load combination, the following steps are involved (CP 6.1.2.5):

- Determine the shear stress, v.
- Determine the shear stress, v_c, that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three subsections describe in detail the algorithms associated with these steps.

19.7.2.1 Determine Shear Stress

In the design of the beam shear reinforcement, the shear forces for a particular load combination at a particular beam section are obtained by factoring the associated shear forces for different load cases, with the corresponding load combination factors.

$$v = \frac{V}{b_w d} \qquad \text{(CP 6.1.2.5(a))}$$

The maximum allowable shear stress, $v_{\max}$ is defined as:

$$v_{\max} = \min(0.8\sqrt{f_{cu}},\ 7\text{ MPa}) \quad \text{(CP 6.1.2.5(a), 12.3.8.2)}$$

For light-weight concrete, $v_{\max}$ is defined as:

$$v_{\max} = \min(0.63\sqrt{f_{cu}},\ 5.6\text{ MPa}) \quad \text{(BS 8110-2:1985 5.4)}$$

19.7.2.2 Determine Concrete Shear Capacity

The design ultimate shear resistance of the concrete alone, V_C should be considered at sections that are as follows:

Uncracked sections in flexure ($M < M_o$) (CP 12.3.8.3)

Cracked sections in flexural ($M \geq M_o$) (CP 12.3.8.3)

where,

M is the design bending moment at the section

M_o is the moment necessary to produce zero stress in the concrete at the extreme tension fiber; in this calculation, only 0.8 of the stress due to post-tensioning should be taken into account.

19.7.2.2.1.1 Case 1: Uncracked section in flexure

The ultimate shear resistance of the section, V_{co}, is computed as follows:

$$V_{co} = 0.67 b_v h \sqrt{\left(f_t^2 + 0.8 f_{cp} f_t\right)}, \quad \text{(CP 12.3.8.4)}$$

where,

f_t is the maximum design principal stress (CP 12.3.8.4)

$$f_t = 0.24\sqrt{f_{cu}} \quad \text{(CP 12.3.8.4)}$$

f_{cp} = design compressive stress at the centroidal axis due to post-tensioning, taken as positive. (CP 12.3.8.4)

$$V_c = V_{co} + P\sin\beta \qquad \text{(CP 12.3.8.4)}$$

19.7.2.2.1.2 Case 2: Cracked section in flexure

The ultimate shear resistance of the section, V_{cr}, is computed as follows:

$$V_{cr} = \left(1 - 0.55\frac{f_{pe}}{f_{pu}}\right)v_c b_v d + M_o\frac{V}{M}, \text{ and} \qquad \text{(CP 12.3.8.5)}$$

$$V_{cr} \geq 0.1 b_v d\sqrt{f_{cu}} \qquad \text{(CP 12.3.8.5)}$$

$$V_c = \min\left(V_{co}, V_{cr}\right) + P\sin\beta \qquad \text{(CP 12.3.8.5)}$$

19.7.2.3 Determine Required Shear Reinforcement

Given v, v_c, and v_{max}, the required shear reinforcement is calculated as follows (CP 12.3.8.6):

- Calculate the design average shear stress that can be carried by minimum shear reinforcement, v_r, as:

$$v_r = \begin{cases} 0.4 f_{cu} & \text{if } f_{cu} \leq 40\ \text{N/mm}^2 \\ 0.4\left(f_{cu}/40\right)^{2/3} & \text{if } 40 < f_{cu} \leq 80\ \text{N/mm}^2 \\ 0.4\left(80/40\right)^{2/3} & \text{if } f_{cu} > 80\ \text{N/mm}^2 \end{cases} \qquad \text{(CP 12.3.8.7)}$$

$$f_{cu} \leq 80\ \text{N/mm}^2 \text{ (for calculation purpose only)} \qquad \text{(CP 6.1.2.5(c))}$$

- If $v \leq v_c + v_r$, minimum reinforcement is required:

$$\frac{A_s}{s_v} = \frac{v_r b}{0.87 f_{yv}}, \qquad \text{(CP 12.3.8.7)}$$

- If $v > v_c + v_r$,

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c)b}{0.87 f_{yv}} \qquad \text{(CP 12.3.8.8)}$$

- If $v > v_{\max}$, a failure condition is declared.

In the preceding expressions, a limit is imposed on f_{yv} as:

$f_{yv} \leq 460$ MPa.

The maximum of all of the calculated A_{sv}/s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

19.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the torsional shear stress, v_t
- Determine special section properties
- Determine critical torsion stress
- Determine the torsion reinforcement required

19.7.3.1 Determine Torsional Shear Stress

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In typical framed construction, specific consideration of torsion is not usually required where torsional cracking is adequately controlled by shear reinforcement. If the design relies on the torsional resistance of a beam, further consideration should be given using the following algorithms (CP 6.3.1).

The torsional shear stress, v_t, for a rectangular section is computed as:

$$v_t = \frac{2T}{h_{\min}^2 \left(h_{\max} - h_{\min}/3\right)} \qquad \text{(CP 6.3.3(a))}$$

For flanged sections, the section is considered as a series of rectangular segments and the torsional shear stress is computed for each rectangular component using the preceding equation, but considering a torsional moment attributed to that segment, calculated as:

$$T_{seg} = T\left(\frac{h_{\min}^3 h_{\max}}{\sum\left(h_{\min}^3 h_{\max}\right)}\right) \qquad \text{(CP 6.3.3(b))}$$

$h_{\max}$ = Larger dimension of a rectangular section

$h_{\min}$ = Smaller dimension of a rectangular section

If the computed torsional shear stress, v_t, exceeds the following limit for sections with the larger center-to-center dimension of the closed link less than 550 mm, a failure condition is generated if the torsional shear stress does not satisfy:

$$v_t \le \min(0.8\sqrt{f_{cu}}, 7\text{N/mm}^2) \times \frac{y_1}{550} \qquad \text{(CP 6.3.4, Table 17)}$$

19.7.3.2 Determine Critical Torsion Stress

The critical torsion stress, $v_{t,\min}$, for which the torsion in the section can be ignored is calculated as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.6 \text{ N/mm}^2\right) \qquad \text{(CP 6.3.4, Table 17)}$$

where f_{cu} is the specified concrete compressive strength.

For light-weight concrete, $v_{t,\min}$ is defined as:

$$v_{t,\min} = \min\left(0.067\sqrt{f_{cu}}, 0.4\text{N/mm}^2\right) \times 0.8 \qquad \text{(BS 8110-2:85 5.5)}$$

19.7.3.3 Determine Torsion Reinforcement

If the factored torsional shear stress, v_t is less than the threshold limit, $v_{t,\min}$, torsion can be safely ignored (CP 6.3.5). In that case, the program reports that no torsion reinforcement is required. However, if v_t exceeds the threshold limit, $v_{t,\min}$, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CP 6.3.5).

- If $v_t > v_{t,\min}$, the required closed stirrup area per unit spacing, $A_{sv,t}/s_v$, is calculated as:

$$\frac{A_{sv,t}}{s_v} = \frac{T}{0.8 x_1 y_1 \left(0.87 f_{yv}\right)} \qquad \text{(CP 6.3.6)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{A_{sv,t} f_{yv}\left(x_1 + y_1\right)}{s_v f_y} \qquad \text{(CP 6.3.6)}$$

In the preceding expressions, x_1 is the smaller center-to-center dimension of the closed link, and y_1 is the larger center-to-center dimension of the closed link.

An upper limit of the combination of v and v_t that can be carried by the section is also checked using the equation:

$$v + v_t \le v_{\max} \qquad \text{(CP 6.3.4)}$$

$$v_{\max} \le \min\left(0.8\sqrt{f_{cu}}, 7\ \text{N/mm}^2\right) \qquad \text{(CP 6.3.4)}$$

For light-weight concrete, $v_{\max}$ is defined as:

$$v_{\max} = \min(0.63\sqrt{f_{cu}},\ 4\ \text{MPa}) \qquad \text{(BS 8110-2:85 5.4)}$$

If the combination of shear stress, v, and torsional shear stress, v_t, exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

The maximum of all of the calculated A_l and $A_{sv,t}/s_v$ values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

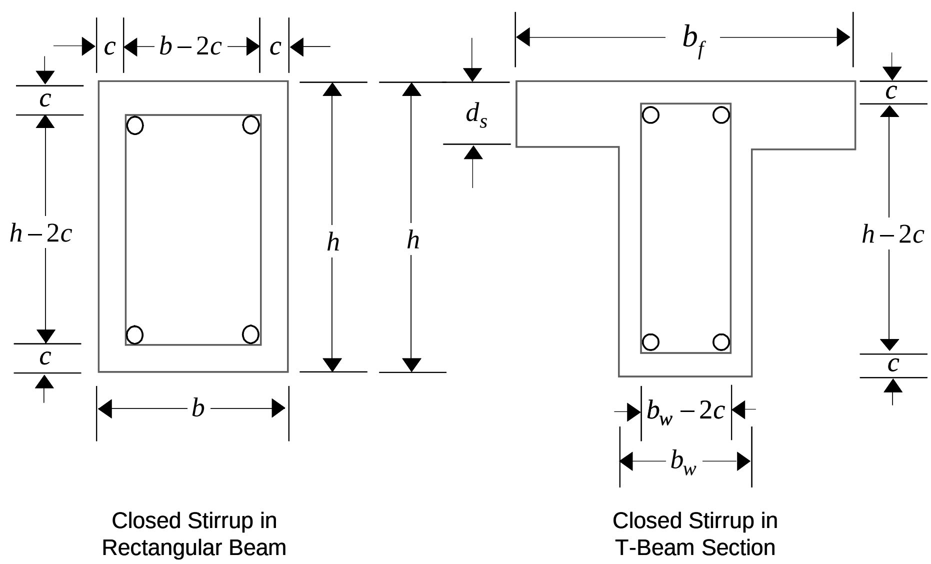


Figure 11-3 Closed stirrup and section dimensions for torsion design

19.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips typically are governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is carried out based on the ultimate strength design method (Hong Kong CP

2013) for prestressed reinforced concrete as described in the following subsections. To learn more about the design strips, refer to the section entitled "ETABS Design Techniques" in the *Key Features and Terminology* manual.

19.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip
- Determine the capacity of post-tensioned sections
- Design flexural reinforcement for the strip

These three steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

19.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

19.8.1.2 Determine Capacity of Post-Tensioned Sections

The calculation of the post-tensioned section capacity is identical to that described earlier for rectangular beam sections.

19.8.1.3 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

19.8.1.4 Minimum and Maximum Slab Reinforcement

There are no minimum requirements for untensioned reinforcement in one-way bonded slabs. One-way spanning floors with unbounded tendons should have minimum reinforcement requirements in accordance with CP Table 3.25 (CP 3.12.5.3)

In flat slabs, reinforcement is added at the top over supports to be 0.00075 times the gross cross-sectional area. This reinforcement extends 1.5 times the slab depth on each side of the column. The length of the reinforcement should be at least 0.2*L* where *L* is the span of the slab.

There are no minimum requirements for span zone. However, additional un-tensioned reinforcement shall be designed for the full tension force generated by assumed flexural tensile stresses in the concrete for the following situations (Concrete Society, Technical Report 43):

- all locations in one-way spanning floors using unbonded tendons
- all locations in one-way spanning floors where transfer tensile stress exceeds $0.36\sqrt{f_{ci}}$
- support zones in all flat slabs
- span zones in flat slabs using unbonded tendons where the tensile stress exceeds $0.15\sqrt{f_{cu}}$.

The reinforcement should be designed to act at a stress of $5/8f_y$ as follows:

$$A_s = \frac{F_t}{(5/8)\, f_y}$$

where

$$F_t = -\frac{f_{ct}(h - x)b}{2}$$

The value of f_{ct} will be negative in tension.

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (CP 9.2.1.3).

19.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code specific items are described in the following sections.

19.8.2.1 Critical Section for Punching Shear

The punching shear is checked at a critical section at a distance of 1.5*d* from the face of the support (CP 6.1.5.7(f)). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads. Figure 11-4 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be over-written using the Punching Check Overwrites.

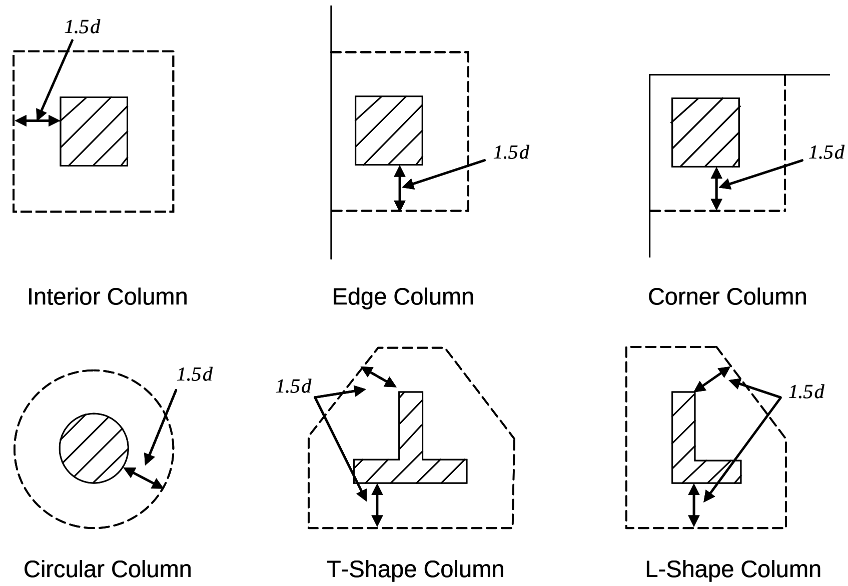


Figure 11-4 Punching Shear Perimeters

19.8.2.2 Determine Concrete Capacity

The design ultimate shear resistance of the concrete alone, V_c, should be considered at sections that are as follows:

Uncracked sections in flexure ($M < M_o$) (CP 12.3.8.3)

Cracked sections in flexural ($M \geq M_o$) (CP 12.3.8.3)

where,

M is the design bending moment at the section

M_o is the moment necessary to produce zero stress in the concrete at the extreme tension fiber; in this calculation, only 0.8 of the stress due to post-tensioning should be taken into account.

19.8.2.2.1.1 Case 1: Uncracked section in flexure

The ultimate shear resistance of the section, V_{co}, is computed as follows:

$$V_{co} = 0.67 b_v h \sqrt{\left(f_t^{\,2} + 0.8 f_{cp} f_t\right)},$$ (CP 12.3.8.4)

where,

f_t is the maximum design principal stress (CP 12.3.8.4)

$$f_t = 0.24\sqrt{f_{cu}}$$ (CP 12.3.8.4)

f_{cp} = design compressive stress at the centoidal axis due to prestress, taken as positive. (CP 12.3.8.4)

$$V_c = V_{co} + P\sin\beta$$ (CP 12.3.8.4)

19.8.2.2.1.2 Case 2: Cracked section in flexure

The ultimate shear resistance of the section, V_{cr}, is computed as follows:

$$V_{cr} = \left(1 - 0.55\frac{f_{pe}}{f_{pu}}\right) v_c b_v d + M_o \frac{V}{M}, \text{ and}$$ (CP 12.3.8.5)

$$V_{cr} \geq 0.1 b_v d\sqrt{f_{cu}}$$ (CP 12.3.8.5)

$$V_c = \min\left(V_{co}, V_{cr}\right) + P\sin\beta$$ (CP 12.3.8.5)

19.8.2.3 Determine Maximum Shear Stress

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the bending axis, the nominal design shear stress, $v_{\max}$, is calculated as:

$$V_{eff,x} = V\left(f + \frac{1.5M_x}{Vy}\right)$$ (CP 6.1.5.6(b), 6.1.5.6(c))

$$V_{eff,y} = V\left(f + \frac{1.5M_y}{Vx}\right)$$ (CP 6.1.5.6(b), 6.1.5.6(c))

$$v_{\max} = \max \begin{cases} \dfrac{V_{eff,x}}{u\,d} \\ \dfrac{V_{eff,y}}{u\,d} \end{cases} \qquad \text{(CP 6.1.5.7)}$$

where,

u is the perimeter of the critical section,

x and y are the lengths of the sides of the critical section parallel to the axis of bending,

M_x and M_y are the design moments transmitted from the slab to the column at the connection,

V is the total punching shear force, and

f is a factor to consider the eccentricity of punching shear force and is taken as

$$f = \begin{cases} 1.00 & \text{for interior columns} \\ 1.25 & \text{for edge columns} \\ 1.25 & \text{for corner columns} \end{cases} \qquad \text{(CP 6.1.5.6(b), 6.1.5.6(c))}$$

19.8.2.4 Determine Capacity Ratio

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS. If this ratio exceeds 1.0, punching shear reinforcement is designed as described in the following section.

19.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 200 mm (CP 6.1.5.7(e)). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* as described in the earlier sections remains unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

19.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a section with punching shear reinforcement is as previously determined for the punching check.

19.8.3.2 Determine Required Shear Reinforcement

The shear stress is limited to a maximum limit of

$$v_{\max} = \min(0.8\sqrt{f_{cu}},\ 7\text{ MPa}) \qquad \text{(CP 6.1.2.5(a))}$$

$$v_r = \begin{cases} 0.4 f_{cu} & \text{if} \quad f_{cu} \le 40 \text{ N/mm}^2 \\ 0.4\left(\dfrac{f_{cu}}{40}\right)^{2/3} & \text{if } 40 < f_{cu} \le 80 \text{ N/mm}^2 \\ 0.4\left(\dfrac{80}{40}\right)^{2/3} & \text{if} \quad f_{cu} > 80 \text{ N/mm}^2 \end{cases} \qquad \text{(CP 12.3.8.7)}$$

$$f_{cu} \le 80 \text{ N/mm}^2 \text{ (for calculation purpose only)} \qquad \text{(CP 6.1.2.5(c))}$$

- If $v \le v_c + v_r$, minimum reinforcement is required:

$$\frac{A_s}{s_v} = \frac{v_r b}{0.87 f_{yv}}, \qquad \text{(CP 12.3.8.7)}$$

- If $v > v_c + v_r$,

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c) b}{0.87 f_{yv}} \qquad \text{(CP 12.3.8.8)}$$

- If $v > v_{\max}$, a failure condition is declared.

If v exceeds the maximum permitted value of v_{max}, the concrete section should be increased in size.

19.8.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 11-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner columns.

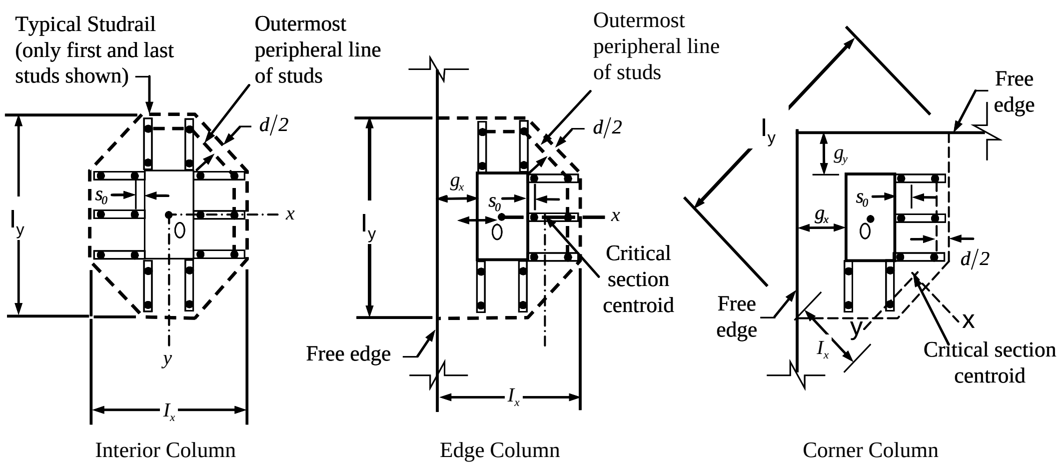


Figure 11-5 Typical arrangement of shear studs and critical sections outside the shear-reinforced zone

The distance between column face and the first line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement in the first line of shear reinforcement shall not exceed $0.75d$ measured in a direction parallel to the column face (CP12.3.8.10). When $V > 1.8V_c$, the maximum spacing is reduced to $0.5\,d$.

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8 for corner, edge, and interior columns respectively.

19.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CP 4.2.4 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 10-, 12-, 14-, 16-, and 20-millimeter diameters.

The following information is taken from the BS 8110-1997 code. When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.5d. The spacing between adjacent shear studs, g, at the first peripheral line of studs shall not exceed 1.5d. The limit of s_o and the spacing, s, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(CP 6.1.5.7(f))}$$

$$s \leq 0.75d \qquad \text{(CP 6.1.5.7(f))}$$

$$g \leq 1.5d \qquad \text{(CP 6.1.5.7(f))}$$

Stirrups are permitted only when slab thickness is greater than 200 mm (CP 6.1.5.7(e)).

Chapter 20
Design for ACI 318-14

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the American code ACI 318-14 [ACI 2014]. Various notations used in this chapter are listed in Table 6-1. For referencing to the pertinent sections of the ACI code in this chapter, a prefix “ACI” followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on inch-pound-second units. For simplicity, all equations and descriptions presented in this chapter correspond to inch-pound-second units unless otherwise noted.

20.1 Notations

The following table identifies the various notations used in this chapter.

Table 20-1 List of Symbols Used in the ACI 318-14 Code

A_{cp}	Area enclosed by the outside perimeter of the section, in^2
A_g	Gross area of concrete, in^2
A_l	Total area of longitudinal reinforcement to resist torsion, in^2
A_o	Area enclosed by the shear flow path, sq-in
A_{oh}	Area enclosed by the centerline of the outermost closed transverse torsional reinforcement, sq-in
A_{ps}	Area of prestressing steel in flexural tension zone, in^2
A_s	Area of tension reinforcement, in^2
A'_s	Area of compression reinforcement, in^2
$A_{s(required)}$	Area of steel required for tension reinforcement, in^2
A_t /s	Area of closed shear reinforcement per unit length of member for torsion, sq-in/in
A_v	Area of shear reinforcement, in^2
A_v /s	Area of shear reinforcement per unit length of member, in^2/in
a	Depth of compression block, in
a_b	Depth of compression block at balanced condition, in
$a_{\max}$	Maximum allowed depth of compression block, in
b	Width of member, in
b_f	Effective width of flange (T-beam section), in
b_w	Width of web (T-beam section), in
b_0	Perimeter of the punching critical section, in
b_1	Width of the punching critical section in the direction of bending, in
b_2	Width of the punching critical section perpendicular to the direction of bending, in
c	Depth to neutral axis, in

Table 20-1 List of Symbols Used in the ACI 318-14 Code

c_b	Depth to neutral axis at balanced conditions, in
d	Distance from compression face to tension reinforcement, in
d'	Concrete cover to center of reinforcing, in
d_e	Effective depth from compression face to centroid of tension reinforcement, in
d_s	Thickness of slab (T-beam section), in
d_p	Distance from extreme compression fiber to centroid of pre-stressing steel, in
E_c	Modulus of elasticity of concrete, psi
E_s	Modulus of elasticity of reinforcement, assumed as 29,000,000 psi (ACI 8.5.2)
f'_c	Specified compressive strength of concrete, psi
f'_{ci}	Specified compressive strength of concrete at time of initial prestress, psi
f_{pe}	Compressive stress in concrete due to effective prestress forces only (after allowance of all prestress losses), psi
f_{ps}	Stress in prestressing steel at nominal flexural strength, psi
f_{pu}	Specified tensile strength of prestressing steel, psi
f_{py}	Specified yield strength of prestressing steel, psi
f_t	Extreme fiber stress in tension in the precompressed tensile zone using gross section properties, psi
f_y	Specified yield strength of flexural reinforcement, psi
f_{ys}	Specified yield strength of shear reinforcement, psi
h	Overall depth of a section, in
h_f	Height of the flange, in
ϕM_n^0	Design moment resistance of a section with tendons only, N-mm

Table 20-1 List of Symbols Used in the ACI 318-14 Code

ϕM_n^{bal}	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
M_u	Factored moment at section, lb-in
N_c	Tension force in concrete due to unfactored dead load plus live load, lb
P_u	Factored axial load at section, lb
s	Spacing of the shear reinforcement along the length of the beam, in
T_u	Factored torsional moment at section, lb-in
V_c	Shear force resisted by concrete, lb
$V_{\max}$	Maximum permitted total factored shear force at a section, lb
V_u	Factored shear force at a section, lb
V_s	Shear force resisted by steel, lb
β_1	Factor for obtaining depth of compression block in concrete
β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
$\varepsilon_{c,\max}$	Maximum usable compression strain allowed in extreme concrete fiber (0.003 in/in)
ε_{ps}	Strain in prestressing steel
ε_s	Strain in reinforcing steel
$\varepsilon_{s,\min}$	Minimum tensile strain allowed in steel reinforcement at nominal strength for tension controlled behavior (0.005 in/in)
φ	Strength reduction factor
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear

Table 20-1 List of Symbols Used in the ACI 318-14 Code

λ	Shear strength reduction factor for light-weight concrete
θ	Angle of compression diagonals, degrees

20.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For ACI 318-14, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the load combinations in the following sections may need to be considered (ACI 5.3.1).

For post-tensioned concrete design, the user can specify the prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning are defined in the following sections.

20.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress forces, in accordance with ACI 318-14 sections 5.3.11 and 24.5.3. The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0D + 1.0PT (ACI 5.3.11, 25.4.3)

20.2.2 Service Load Combination

The following load combinations are used for checking the requirements of prestress for serviceability in accordance with ACI 318-14 sections 5.3.11, 24.5.4. It is assumed that all long-term losses have already occurred at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT (ACI 24.5.4)

20.2.3 Long-Term Service Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with ACI 318-14 sections 5.3.11, 24.5.4.1, and R24.5.4.1. The permanent load for this load combination is taken as 50 percent of the live load. It is assumed that all long-term losses have already occurred at the service stage.

1.0D + 1.0PT 1.0D + 0.5L + 1.0PT	(ACI 5.3.11, 24.5.4.1)

20.2.4 Strength Design Load Combination

The following load combinations are used for checking the requirements of prestress for strength in accordance with ACI 318-14, Chapters 8, 9, and 24.

The strength design combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are automatically determined by ETABS by subtracting out the primary PT moments when the flexural design is carried out.

$1.4D + 1.0PT^{*}$	(ACI Eqn. 5.3.1a)
$1.2D + 1.6L + 0.5L_r + 1.0PT^{*}$	(ACI Eqn.5.3.1b)
$1.2D + 1.0L + 1.6L_r + 1.0PT^{*}$	(ACI Eqn.5.3.1c)
$1.2D + 1.6(0.75\ PL) + 0.5L_r + 1.0PT^{*}$	(ACI Eqn.5.3.1b, 6.4)
$1.2D + 1.6L + 0.5S + 1.0PT^{*}$	(ACI Eqn.5.3.1b)
$1.2D + 1.0L + 1.6S+ 1.0PT^{*}$	(ACI Eqn.5.3.1c)
$0.9D \pm 1.0W + 1.0PT^{*}$	(ACI Eqn.5.3.1f)
$1.2D + 1.0L + 0.5L_r \pm 1.0W + 1.0PT^{*}$	(ACI Eqn.5.3.1d)
$1.2D + 1.6L_r \pm 0.5W + 1.0PT^{*}$	(ACI Eqn.5.3.1c)
$1.2D + 1.6S \pm 0.5W + 1.0PT^{*}$	(ACI Eqn.5.3.1c)
$1.2D + 1.0L + 0.5S \pm 1.0W + 1.0PT^{*}$	(ACI Eqn.5.3.1d)
$0.9D \pm 1.0E + 1.0PT^{*}$	(ACI Eqn.5.3.1g)
$1.2D + 1.0L + 0.2S \pm 1.0E + 1.0PT^{*}$	(ACI Eqn.5.3.1e)

* — Replace PT by H for flexural design only

These also are the default design load combinations in ETABS whenever the ACI 318-14 code is used. The user should use other appropriate load combinations if other types of loads are present.

20.3 Limits on Material Strength

The concrete compressive strength, f'_c, should not be less than 2500 psi (ACI 19.2.1, Table 19.2.1.1). The upper limit of the reinforcement yield strength, f_y, is taken as 80 ksi (ACI 20.2.2.4a, Table 20.2.2.4a) and the upper limit of the reinforcement shear strength, f_{yt}, is taken as 60 ksi (ACI 21.2.2.4a, Table 21.2.2.4a).

ETABS enforces the upper material strength limits for flexure and shear design of slabs. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

20.4 Strength Reduction Factors

The strength reduction factors, ϕ, are applied on the specified strength to obtain the design strength provided by a member. The ϕ factors for flexure, shear, and torsion are as follows:

ϕ_t = 0.90 for flexure (tension controlled) (ACI 21.2.1, Table 21.2.1)

ϕ_c = 0.65 for flexure (compression controlled) (ACI 21.2.1, Table 21.2.1)

ϕ = 0.75 for shear and torsion. (ACI 21.2.1, Table 21.2.1)

The value of ϕ varies from compression-controlled to tension-controlled based on the maximum tensile strain in the reinforcement at the extreme edge, ε_t (ACI 21.2.3).

Sections are considered compression-controlled when the tensile strain in the extreme tension reinforcement is equal to or less than the compression-controlled strain limit at the time the concrete in compression reaches its assumed strain limit of $\varepsilon_{c.\max}$, which is 0.003. The compression-controlled strain limit is

the tensile strain in the reinforcement at the balanced strain condition, which is taken as the yield strain of the reinforcement, (f_y/E) (ACI 21.2.2.1, Table 21.2.2).

Sections are tension-controlled when the tensile strain in the extreme tension reinforcement is equal to or greater than 0.005, just as the concrete in compression reaches its assumed strain limit of 0.003 (Table 21.2.2, Fig R21.2.26).

Sections with ε_t between the two limits are considered to be in a transition region between compression-controlled and tension-controlled sections (ACI 21.2.2, Table 21.2.2).

When the section is tension-controlled, ϕ_t is used. When the section is compression-controlled, ϕ_c is used. When the section is in the transition region, ϕ is linearly interpolated between the two values (ACI 21.2.2, Table 21.2.2).

The user is allowed to overwrite these values. However, caution is advised.

20.5 Design Assumptions for Prestressed Concrete

Strength design of prestressed members for flexure and axial loads shall be based on assumptions given in ACI 22.2.

- The strain in the reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis (ACI 22.2.1.2).
- The maximum usable strain at the extreme concrete compression fiber shall be assumed equal to 0.003 (ACI 22.2.2.1).
- The tensile strength of the concrete shall be neglected in axial and flexural calculations (ACI 22.2.2.2).
- The relationship between the concrete compressive stress distribution and the concrete strain shall be assumed to be rectangular by an equivalent rectangular concrete stress distribution (ACI 22.2.2.3).
- The concrete stress of $0.85f'_c$ shall be assumed uniformly distributed over an equivalent-compression zone bounded by edges of the cross-section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain (ACI 22.2.2.4.1).

- The distance from the fiber of maximum strain to the neutral axis, *c* shall be measured in a direction perpendicular to the neutral axis (ACI 22.2.2.4.2).

Elastic theory shall be used with the following two assumptions:

- The strains shall vary linearly with depth through the entire load range (ACI 24.5.1.2a).
- At cracked sections, the concrete resists no tension (ACI 24.5.1.2b).

Prestressed concrete members are investigated at the following three stages (ACI 24.5.1.2):

- At transfer of prestress force
- At service loading
- At nominal strength

The prestressed flexural members are classified as Class U (uncracked), Class T (transition), and Class C (cracked) based on f_t, the computed extreme fiber stress in tension in the precompressed tensile zone at service loads (ACI 24.5.2.1).

The precompressed tensile zone is that portion of a prestressed member where flexural tension, calculated using gross section properties, would occur under unfactored dead and live loads if the prestress force was not present. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress.

For Class U and Class T flexural members, stresses at service load are determined using uncracked section properties, while for Class C flexural members, stresses at service load are calculated based on the cracked section (ACI 24.5.2.2 and 24.5.2.3).

A prestressed two-way slab system is designed as Class U only with $f_t \leq 6\sqrt{f'_c}$ (ACI R24.5.2.1); otherwise, an over-stressed (O/S) condition is reported.

The following table provides a summary of the conditions considered for the various section classes.

	Prestressed			Nonprestressed
	Class U	Class T	Class C	
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 24.5.2.2	Gross section 24.5.2.2	Cracked section 24.5.2.2	No requirement
Allowable stress at transfer	24.5.3	24.5.3	24.5.3	No requirement
Allowable compressive stress based on uncracked section properties	24.5.4	24.5.4	No requirement	No requirement
Tensile stress at service loads 24.5.2.1	$\le 7.5\sqrt{f'_c}$	$7.5\sqrt{f'_c} < f_t \le 12\sqrt{f'_c}$	No requirement	No requirement

20.6 Serviceability Requirements of Flexural Members

20.6.1 Serviceability Check at Initial Service Load

The stresses in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits:

- Extreme fiber stress in compression: $0.60 f'_{ci}$ (ACI 24.5.3.1)
- Extreme fiber stress in tension: $3\sqrt{f'_{ci}}$ (ACI 24.5.3.2)
- Extreme fiber stress in tension at ends of simply supported members: $6\sqrt{f'_{ci}}$ (ACI 24.5.3.2)

The extreme fiber stress in tension at the ends of simply supported members is currently **NOT** checked by ETABS.

20.6.2 Serviceability Checks at Service Load

The stresses in the concrete for Class U and Class T prestressed flexural members at service loads, and after all prestress losses occur, are checked against the following limits:

- Extreme fiber stress in compression due to prestress plus total load: $0.60 f'_c$ (ACI 18.4.2(b))

- Extreme fiber stress in tension in the precompressed tensile zone at service loads:

  – Class U beams and one-way slabs: $f_t \leq 7.5\sqrt{f'_c}$ (ACI 24.5.2.1)

  – Class U two-way slabs: $f_t \leq 6\sqrt{f'_c}$ (ACI 24.5.2.1)

  – Class T beams: $7.5\sqrt{f'_c} < f_t \leq 12\sqrt{f'_c}$ (ACI 24.5.2.1)

  – Class C beams: $f_t \geq 12\sqrt{f'_c}$ (ACI 24.5.2.1)

For Class C prestressed flexural members, checks at service loads are not required by the code. However, for Class C prestressed flexural members not subject to fatigue or to aggressive exposure, the spacing of bonded reinforcement nearest the extreme tension face shall not exceed that given by ACI 24.3.5 (ACI 24.3.2). It is assumed that the user has checked the requirements of ACI 9.7.2.2 and ACI 24.3.1 to 24.3.4 independently, as these sections are not checked by the program.

20.6.3 Serviceability Checks at Long-Term Service Load

The stresses in the concrete for Class U and Class T prestressed flexural members at long-term service loads, and after all prestress losses occur, are checked against the same limits as for the normal service load, except for the following:

- Extreme fiber stress in compression due to prestress plus total load:

  $0.45 f_c'$ (ACI 24.5.4.1)

20.6.4 Serviceability Checks of Prestressing Steel

The program also performs checks on the tensile stresses in the prestressing steel (ACI 20.3.2.5). The permissible tensile stress checks, in all types of prestressing steel, in terms of the specified minimum tensile stress f_{pu}, and the minimum yield stress, f_y, are summarized as follows:

- Due to tendon jacking force: $\min(0.94f_{py}, 0.80f_{pu})$ (ACI 20.3.2.5.1)

- At anchors and couplers after force transfer: $0.70f_{pu}$ (ACI 20.3.2.5.1)

20.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

20.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

20.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of prestressed concrete beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or a flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases the beam may be designed as a rectangular or inverted flanged beam.

20.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 20-1 (ACI 22.2). Furthermore, it is assumed that the net tensile strain in the reinforcement shall not be less than 0.005 (tension controlled) (ACI 9.3.3). When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression reinforcement and additional tension reinforcement.

The design procedure used by ETABS, for both rectangular and flanged sections (L- and T-beams), is summarized in the subsections that follow. It is assumed that the design ultimate axial force does not exceed $\phi(0.1f'_c A_g)$ (ACI 9.3.3, 9.5.2.1); hence all beams are designed for major direction flexure, shear, and torsion only.

20.7.1.2.1 Design of Rectangular Beams

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, the moment capacity ϕM_n^0 is determined as follows:

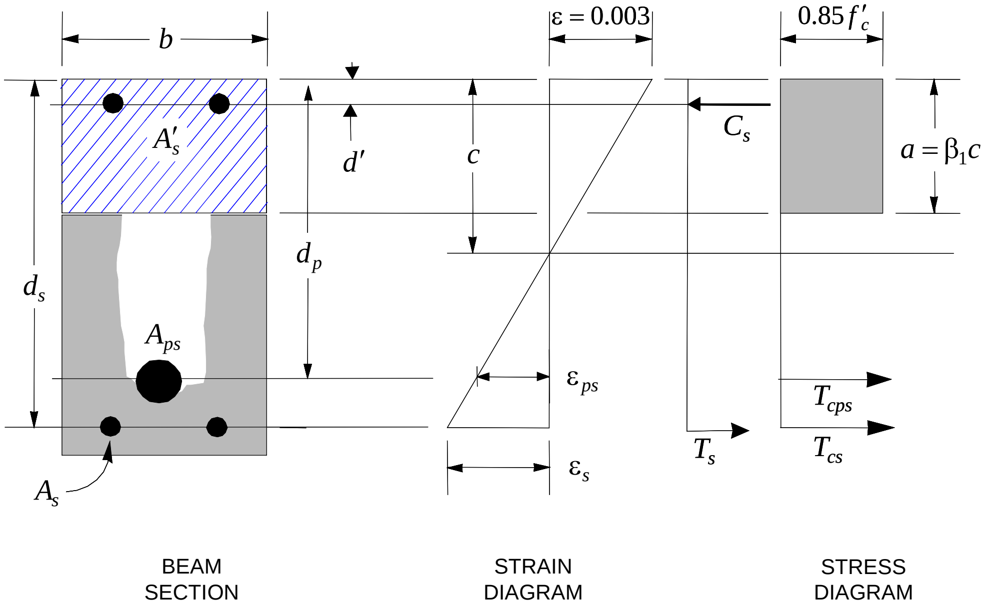


Figure 20-1 Rectangular Beam Design

The maximum depth of the compression zone, $c_{\max}$, is calculated based on the limitation that the tension reinforcement strain shall not be less than $\varepsilon_{s\min}$, which is equal to 0.005 for tension-controlled behavior (ACI 10.3.4):

$$c_{\max} = \left(\frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}} \right) d \qquad \text{(ACI 21.2.2)}$$

where,

$$\varepsilon_{c\max} = 0.003 \qquad \text{(ACI 21.2.2, Fig R21.2)}$$

$$\varepsilon_{s\min} = 0.005 \qquad \text{(ACI 21.2.2, Fig R21.2.26)}$$

Therefore, the limit $c \le c_{\max}$ is set for tension-controlled sections.

The maximum allowable depth of the rectangular compression block, a_{max}, is given by:

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 22.2.2.4.1)}$$

where β_1 is calculated as:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right), \quad 0.65 \le \beta_1 \le 0.85 \qquad \text{(ACI 22.2.2.4.3)}$$

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{ps}, is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel.

Based on the calculated f_{ps}, the depth of the neutral axis is recalculated, and f_{ps} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \beta_1 c$$

- If $c \le c_{max}$ (ACI 9.3.3.1, 21.2.2), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$\phi M_n^0 = \phi A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)$$

- If $c > c_{max}$ (ACI 9.3.3), a failure condition is declared.

If $M_u > \phi M_n^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of the neutral axis, c is equal to c_{max}. The stress in the post-tensioning steel, f_{ps} is then calculated and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = 0.85 f'_c a_{max} b$$

$$T = A_{ps} f_{ps}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{0.85 f'_c a_{\max} b - A_{ps} f_{ps}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$\phi M_n^{bal} = \phi A_{ps} f_{ps}^{bal} \left(d_p - \frac{a_{\max}}{2} \right) + \phi A_s^{bal} f_s^{bal} \left(d_s - \frac{a_{\max}}{2} \right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, *c*.

20.7.1.2.1.1 Case 1: Post-tensioning steel is adequate

When $M_u < \phi M_n^0$, the amount of post-tensioning steel is adequate to resist the design moment M_u. Minimum reinforcement is provided to satisfy ductility requirements (ACI 8.6.2.2.1 and 8.6.2.3), i.e, $M_u < \phi M_n^0$.

20.7.1.2.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M_u, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $\phi M_n^0 < M_u < \phi M_n^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M_u and reports this required area of tension reinforcement. Since M_u is bounded by ϕM_n^0 at the lower end and ϕM_n^{bal} at the upper end, and ϕM_n^0 is associated with $A_s = 0$ and ϕM_n^{bal} is associated with $A_s = A_s^{bal}$, the required area will fall within the range of 0 to A_s^{bal}.

The tension reinforcement is to be placed at the bottom if M_u is positive, or at the top if M_u is negative.

20.7.1.2.1.3 Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M_u > \phi M_n^{bal}$, compression reinforcement is required (ACI 9.3.3.1). In this case ETABS assumes that the depth of the neutral axis, c, is equal to $c_{\max}$. The values of f_{ps} and f_s reach their respective balanced condition values, f_{ps}^{bal} and f_s^{bal}. The area of compression reinforcement, $A_s^{'}$, is then determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M_u - \phi M_n^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(f'_s - 0.85 f'_c\right)\left(d_e - d'\right)\phi}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c\max}\left[\frac{c_{\max} - d'}{c_{\max}}\right] \leq f_y \qquad \text{(ACI 9.2.1.2, 9.5.2.1, 20.2.2, 22.2.1.2)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{f_y\left(d_s - d'\right)\phi}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and vice versa if M_u is negative.

20.7.1.2.2 Design of Flanged Beams

20.7.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_u (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as above, i.e., no flanged beam data is used.

20.7.1.2.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, the moment capacity ϕM_n^0 is determined as follows:

The maximum depth of the compression zone, c_{max}, is calculated based on the limitation that the tension reinforcement strain shall not be less than ε_{smin}, which is equal to 0.005 for tension-controlled behavior (ACI 22.2.2):

$$c_{max} = \left(\frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}}\right) d \qquad \text{(ACI 22.2.1.2)}$$

where,

$$\varepsilon_{cmax} = 0.003 \qquad \text{(ACI 21.2.2, Fig R21.2))}$$

$$\varepsilon_{smin} = 0.005 \qquad \text{(ACI 21.2.2, Fig R21.2.26)}$$

Therefore, the limit $c \leq c_{max}$ is set for tension-controlled section:

The maximum allowable depth of the rectangular compression block, a_{max}, is given by:

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 22.2.2.4.1)}$$

where β_1 is calculated as:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right), \qquad 0.65 \leq \beta_1 \leq 0.85 \quad \text{(ACI 22.2.2.4.3)}$$

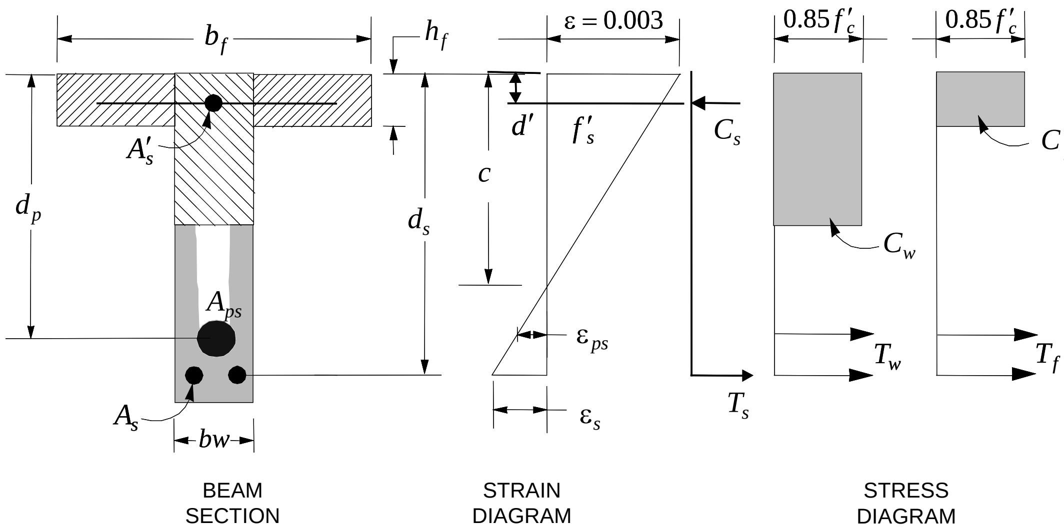


Figure 20-2 T-Beam Design

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{ps} is computed based on strain compatibility for bonded tendons. For unbonded tendons, the code equations are used to compute the stress, f_{ps} in the post-tensioning steel. Based on the calculated f_{ps}, the depth of the neutral axis is recalculated, and f_{ps} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \beta_1 c$$

If $c \le c_{\max}$ (ACI 9.3.3.1, 21.2.2, Fig 21.2.26, 22.2.2.4.1), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$\phi M_n^0 = \phi A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right)$$

- If $c > c_{\max}$ (ACI 9.3.3), a failure condition is declared.

If $M_u > \phi M_n^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension-controlled case. In that case, it is assumed that the depth of the neutral axis c is equal to $c_{\max}$. The stress in the post-tensioning steel, f_{ps}, is then calculated and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \le h_f$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in that case the width of the beam is taken as b_f. Compression reinforcement is required if $a > a_{\max}$.
- If $a > h_f$, the calculation for A_s is given by:

$$C = 0.85 f'_c A_c^{comp}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$A_c^{com} = b_f h_f + b_w \left(a_{\max} - h_f\right)$$

$$T = A_{ps} f_{ps}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{0.85 f'_c A_c^{com} - A_{ps} f_{ps}^{bal}}{f_s^{bal}}$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, c.

Case 1: Post-tensioning steel is adequate

When $M_u < \phi M_n^0$ the amount of post-tensioning steel is adequate to resist the design moment M_u. Minimum reinforcement is provided to satisfy ductility requirements (ACI 8.6.2.2.1 and 8.6.2.3), i.e., $M_u < \phi M_n^0$.

Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_{ps}, alone is not sufficient to resist M_u, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $\phi M_n^0 < M_u < \phi M_n^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M_u and reports this required area of tension reinforcement. Since M_u is bounded by ϕM_n^0 at the lower end and ϕM_n^{bal} at the upper end, and ϕM_n^0 is associated with $A_s = 0$ and ϕM_n^{bal} is associated with $A_s = A_s^{bal}$, the required area will fall within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if M_u is positive, or at the top if M_u is negative.

Case 3: Post-tensioning steel and tension reinforcement are not adequate

When $M_u > \phi M_n^{bal}$, compression reinforcement is required (ACI 9.3.3.1). In that case, ETABS assumes that the depth of the neutral axis, c, is equal to $c_{\max}$. The value of f_{ps} and f_s reach their respective balanced condition values, f_{ps}^{bal} and f_s^{bal}. The area of compression reinforcement, $A_s^{'}$, is then determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{us} = M_u - \phi M_n^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{us}}{\left(f'_s - 0.85 f'_c\right)\left(d_s - d'\right)\phi}, \text{ where}$$

$$f'_s = E_s \varepsilon_{c\max} \left[\frac{c_{\max} - d'}{c_{\max}}\right] \leq f_y \qquad \text{(ACI 9.2.1.2, 9.5.2.1, 20.2.2, 22.2.1.2)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{us}}{f_y\left(d_s - d'\right)\phi}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and vice versa if M_u is negative.

20.7.1.2.3 Ductility Requirements

ETABS also checks the following condition by considering the post-tensioning steel and tension reinforcement to avoid abrupt failure.

$$\phi M_n \geq 1.2 M_{cr} \qquad \text{(ACI 8.6.2.2)}$$

The preceding condition is permitted to be waived for the following:

(a) Two-way, unbonded post-tensioned slabs

(b) Flexural members with shear and flexural strength at least twice that required by ACI 8.6.2.2.1.

These exceptions currently are **NOT** handled by ETABS.

20.7.1.2.4 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following limit:

$$A_s \geq 0.004\, A_{ct} \qquad \text{(ACI 9.6.2.3)}$$

where, A_{ct} is the area of the cross-section between the flexural tension face and the center of gravity of the gross section.

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.4bd & \text{Rectangular beam} \\ 0.4b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

20.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular loading combination, at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored shear force, V_u.
- Determine the shear force, V_c that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

20.7.2.1 Determine Factored Shear Force

In the design of the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases, with the corresponding load combination factors.

20.7.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as:

$$V_c = \min(V_{ci}, V_{cw}) \qquad \text{(ACI 22.5.1.4, 22.5.8.3)}$$

where,

$$V_{ci} = 0.6\lambda\sqrt{f'_c}b_w d_p + V_d + \frac{V_i M_{cre}}{M_{\max}} \geq 1.7\lambda\sqrt{f'_c}b_w d$$

(ACI 22.5.8.3.1a, 22.5.8.3.1b)

$$V_{cw} = \left(3.5\lambda\sqrt{f'_c} + 0.3 f_{pc}\right)b_w d_p + V_p \qquad \text{(ACI 22.5.8.3.2)}$$

$$d_p \geq 0.80h \qquad \text{(ACI 22.5.8.3.2)}$$

$$M_{cre} = \left(\frac{I}{y_t}\right)\left(6\lambda\sqrt{f'_c} + f_{pe} - f_d\right) \qquad \text{(ACI 22.5.8.3.1c)}$$

where,

f_d = stress due to unfactored dead load, at the extreme fiber of the section where tensile stress is caused by externally applied loads, psi

f_{pe} = compress stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at the extreme fiber of the section where tensile stress is caused by externally applied loads, psi

V_d = shear force at the section due to unfactored dead load, lbs

V_p = vertical component of effective prestress force at the section, lbs

V_{ci} = nominal shear strength provided by the concrete when diagonal cracking results from combined shear and moment

M_{cre} = moment causing flexural cracking at the section because of externally applied loads

$M_{\max}$ = maximum factored moment at section because of externally applied loads

V_i = factored shear force at the section because of externally applied loads occurring simultaneously with $M_{\max}$

V_{cw} = nominal shear strength provided by the concrete when diagonal cracking results from high principal tensile stress in the web

For light-weight concrete, the $\sqrt{f'_c}$ term is multiplied by the shear strength reduction factor λ.

20.7.2.3 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$$V_{\max} = V_c + \left(8\sqrt{f'_c}\right)b_w d \qquad \text{(ACI 22.5.1.2)}$$

Given V_u, V_c, and $V_{\max}$, the required shear reinforcement is calculated as follows where, ϕ, the strength reduction factor, is 0.75 (ACI 21.2.2).

- If $V_u \le 0.5\phi V_c$

$$\frac{A_v}{s} = 0 \qquad \text{(ACI 9.6.3.1)}$$

- If $0.5\phi V_c < V_u \le \phi V_{\max}$

$$\frac{A_v}{s} = \frac{\left(V_u - \phi V_c\right)}{\phi\, f_{yt} d} \qquad \text{(ACI 22.5.1.1, 22.5.10.1, 20.5.10.5.3)}$$

$$\frac{A_v}{s} \ge \max\left(\frac{0.75\lambda\sqrt{f'_c}}{f_{yt}} b_w, \quad \frac{50 b_w}{f_{yt}}\right) \qquad \text{(ACI 9.6.3.3, Table 9.6.3.3)}$$

- If $V_u > \phi V_{\max}$, a failure condition is declared (ACI 22.5.1.2).

For members with an effective prestress force not less than 40 percent of the tensile strength of the flexural reinforcement, the required shear reinforcement is computed as follows (ACI 9.6.3.3):

$$\frac{A_v}{s} \ge \min\begin{cases} \max\left(\dfrac{0.75\lambda\sqrt{f'_c}}{f_y} b_w, \dfrac{50}{f_y} b_w\right) \\ \dfrac{A_{ps} f_{pu}}{80 f_{yt} d}\sqrt{\dfrac{d}{b_w}} \end{cases}$$

- If V_u exceeds the maximum permitted value of $\phi V_{\max}$, the concrete section should be increased in size (ACI 22.5.1.2).

Note that if torsion design is considered and torsion reinforcement is needed, the equation given in ACI 9.6.3.3 does not need to be satisfied independently. See the next section *Design of Beam Torsion Reinforcement* for details.

If the beam depth h is less than the minimum of 10 in, $2.5h_f$, and $0.5b_w$, the minimum shear reinforcement given by ACI 9.6.3.3 is not enforced (ACI 9.6.3.1)).

The maximum of all of the calculated A_v/s values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements considered by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

20.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_u.
- Determine special section properties.
- Determine critical torsion capacity.
- Determine the torsion reinforcement required.

20.7.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases with the corresponding load combination factors (ACI 9.4.4.2).

In a statically indeterminate structure where redistribution of the torsion in a member can occur due to redistribution of internal forces upon cracking, the design T_u is permitted to be reduced in accordance with the code (ACI 22.7.3.3). However, the program does not automatically redistribute the internal forces and reduce T_u. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

20.7.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_{cp}, A_{oh}, A_o, p_{cp}, and p_h are calculated. These properties are described in the following (ACI 2.2).

A_{cp} = Area enclosed by outside perimeter of concrete cross-section

A_{oh} = Area enclosed by centerline of the outermost closed transverse torsional reinforcement

A_o = Gross area enclosed by shear flow path

p_{cp} = Outside perimeter of concrete cross-section

p_h = Perimeter of centerline of outermost closed transverse torsional reinforcement

In calculating the section properties involving reinforcement, such as A_{oh}, A_o, and p_h, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 1.75 inches. This is equivalent to 1.5 inches clear cover and a #4 stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a rectangular beam section are given as:

$A_{cp} = bh$ (ACI 2.2, R22.7.5)

$A_{oh} = (b - 2c)(h - 2c)$ (ACI 2.2, R22.7, Fig R22.7.6.1.1))

$A_o = 0.85\ A_{oh}$ (ACI 22.7.6.1.1, Fig R22.7.6.1.1)

$p_{cp} = 2b + 2h$ (ACI 2.2, R22.7.5)

$p_h = 2(b - 2c) + 2(h - 2c)$ (ACI 22.7.6.1.1, Fig R22.7.6.1.1)

where, the section dimensions b, h, and c are shown in Figure 20-3. Similarly, the special section properties for a flanged beam section are given as:

$A_{cp} = b_w h + (b_f - b_w)h_f$ (ACI 2.2, R22.7.5)

$A_{oh} \quad = \quad (b_w - 2c)(h - 2c)$ (ACI 2.2, R22.7, Fig R22.7.6.1.1)

$A_o \quad = \quad 0.85\, A_{oh}$ (ACI 22.7.6.1.1, Fig R22.7.6.1.1)

$p_{cp} \quad = \quad 2b_f + 2h$ (ACI 2.2, R22.7.5)

$p_h \quad = \quad 2(h - 2c) + 2(b_w - 2c)$ (ACI 2.2, R22.7.5)

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 20-3. Note that the flange width on either side of the beam web is limited to the smaller of $4h_f$ or $(h - h_f)$.

20.7.3.3 Determine Critical Torsion Capacity

The threshold torsion limit, T_{th}, and the cracking torsion limits, T_{cr}, for which the torsion in the section can be ignored is calculated as:

$$T_{th} = \lambda \sqrt{f'_c} \left(\frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4\lambda \sqrt{f'_c}}}$$ (ACI 22.7.4.1, Table 22.7.4.1a)

$$T_{th} = 4\lambda \sqrt{f'_c} \left(\frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4\lambda \sqrt{f'_c}}}$$ (ACI 22.7.5.1g, Table 22.7.5.2)

where A_{cp} and p_{cp} are the area and perimeter of the concrete cross-section as described in detail in the previous section; f_{pc} is the concrete compressive stress at the centroid of the section; ϕ is the strength reduction factor for torsion, which is equal to 0.75 by default (ACI 21.2.1g, Table 21.2.1c); and f'_c is the specified concrete compressive strength.

20.7.3.4 Determine Torsion Reinforcement

If the factored torsion T_u is less than the threshold limit, ϕT_{th}, torsion can be safely ignored (ACI 22.7.1.1, 9.6.4.1).). In that case, the program reports that no torsion reinforcement is required. However, if T_u exceeds the cracking torsion limit, ϕT_{cr}, it is assumed that the torsional resistance is provided by closed stirrups, longitudinal bars, and compression diagonals (ACI 22.7.1, 22.7.6.1). If T_u is greater than ϕT_{th} but less than ϕT_{cr}, only minimum tension rebar needs to be provided (ACI 9.6.4.1).

If $T_u > T_{cr}$, the required longitudinal rebar area is calculated as:

$$A_l = \frac{T_u p_h \tan\theta}{\phi 2 A_0 f_y \tan\theta} \qquad \text{(ACI 22.7.6.1)}$$

and the required closed stirrup area per unit spacing, A_t/s, is calculated as:

$$\frac{A_t}{s} = \frac{T_u \tan\theta}{\phi 2 A_0 f_{ys}} \qquad \text{(ACI 22.7.6.1)}$$

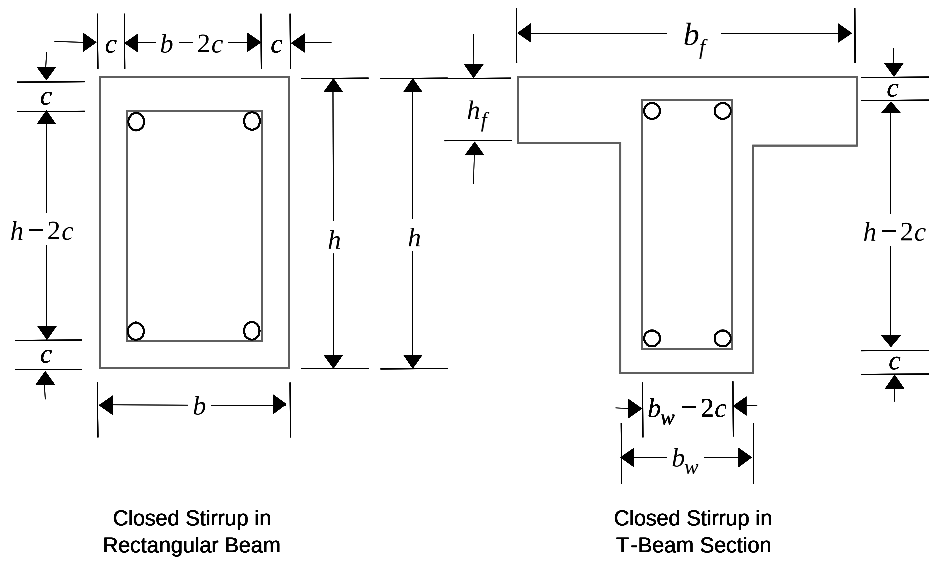


Figure 20-3 Closed stirrup and section dimensions for torsion design

where, the minimum value of A_t/s is taken as:

$$\frac{A_t}{s} = \frac{25}{f_{yt}} b_w \qquad \text{(ACI 11.5.5.3)}$$

and the minimum value of A_l is taken as the least of the following:

$$A_{l,\min} \geq \frac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{ys}}{f_y}\right) \qquad \text{(ACI 9.6.4.3a)}$$

$$A_{l,\min} = \frac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{25}{f_{ys}}b_w\right)p_h\left(\frac{f_{ys}}{f_y}\right) \qquad \text{(ACI 9.6.4.3b)}$$

In the preceding expressions, θ is taken as 45 degrees for prestressed members with an effective prestress force less than 40 percent of the tensile strength of the longitudinal reinforcement; otherwise θ is taken as 37.5 degrees.

An upper limit of the combination of V_u and T_u that can be carried by the section is also checked using the equation:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) \qquad \text{(ACI 22.7.7.1a)}$$

For rectangular sections, b_w is replaced with b. If the combination of V_u and T_u exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_u > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

$$\left(\frac{A_v}{s} + 2\frac{A_t}{s}\right) \ge \max\left\{0.75\lambda\frac{\sqrt{f_c'}}{f_{yt}}b_w, \frac{50b_w}{f_y}\right\} \qquad \text{(ACI 9.6.4.2)}$$

If this equation is not satisfied with the originally calculated A_v/s and A_t/s, A_v/s is increased to satisfy this condition. In that case, A_v/s does not need to satisfy the ACI Section 9.6.6.3 independently.

The maximum of all of the calculated A_l and A_t/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements considered by the program are based purely on strength considerations. Any minimum stirrup requirements and longitudinal reinforcement requirements to satisfy spacing considerations must be investigated independently of the program by the user.

20.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is completed using the ultimate strength design method (ACI 318-14) for prestressed reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "ETABS Design Features" in the *Key Features and Terminology* manual.

20.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is completed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Determine the capacity of post-tensioned sections.
- Design flexural reinforcement for the strip.

These three steps are described in the subsection that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination, is obtained and reported.

20.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

20.8.1.2 Determine Capacity of Post-Tensioned Sections

Calculation of the post-tensioned section capacity is identical to that described earlier for rectangular beam sections.

20.8.1.3 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

20.8.1.3.1 Minimum and Maximum Slab Reinforcement

For one-way slab with bonded prestressed reinforcement, total quantities of A_s and A_{ps} shall be adequate to develop a factored loads at least 1.2 times the cracking load calculated on the basis of modulus of rupture (ACI 7.6.2.1). Currently this check is NOT performed in the program.

For one-way slab with unbounded tendons, the minimum area of bounded deformed longitudinal reinforcement, $A_{s,min}$, has the following limit (ACI 7.6.2.3):

$$A_{s,\min} = 0.004A_{ct} \quad \text{(ACI 7.6.2.3)}$$

where A_{ct} is the area of that part of the cross-section between the flexural tension face and the centroid of the gross-section.

For two-way prestressed slabs, a minimum area of bounded deformed longitudinal reinforcement, $A_{s,min}$, is provided in accordance with ACI section 8.6.2.3, Table 8.6.2.3.

Reinforcement is not required in positive moment areas where f_t, the extreme fiber stress in tension in the precompressed tensile zone at service loads (after all prestress losses occurs) does not exceed $2\sqrt{f_c'}$ (ACI 8.6.2.3).

In positive moment areas where the computed tensile stress in the concrete at service loads exceeds $2\sqrt{f_c'}$, the minimum area of bonded reinforcement is computed as:

$$A_{s,\min} = \frac{N_c}{0.5 f_y}, \text{ where } f_y \leq 60 \text{ ksi} \qquad \text{(ACI 8.6.2.3)}$$

In negative moment areas at column supports, the minimum area of bonded reinforcement in the top of slab in each direction is computed as:

$$A_{s,\min} = 0.00075 A_{cf} \qquad \text{(ACI 8.6.2.3)}$$

where A_{cf} is the larger gross cross-sectional area of the slab-beam strip in the two orthogonal equivalent frames intersecting a column in a two-way slab system.

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area. Note that the requirements when f_y > 60 ksi currently are not handled.

20.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Key Features and Terminology* manual. Only the code specific items are described in the following sections.

20.8.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of *d/2* from the face of the support (ACI 22.6.4.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (ACI 22.6.4.3). Figure 20-4 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

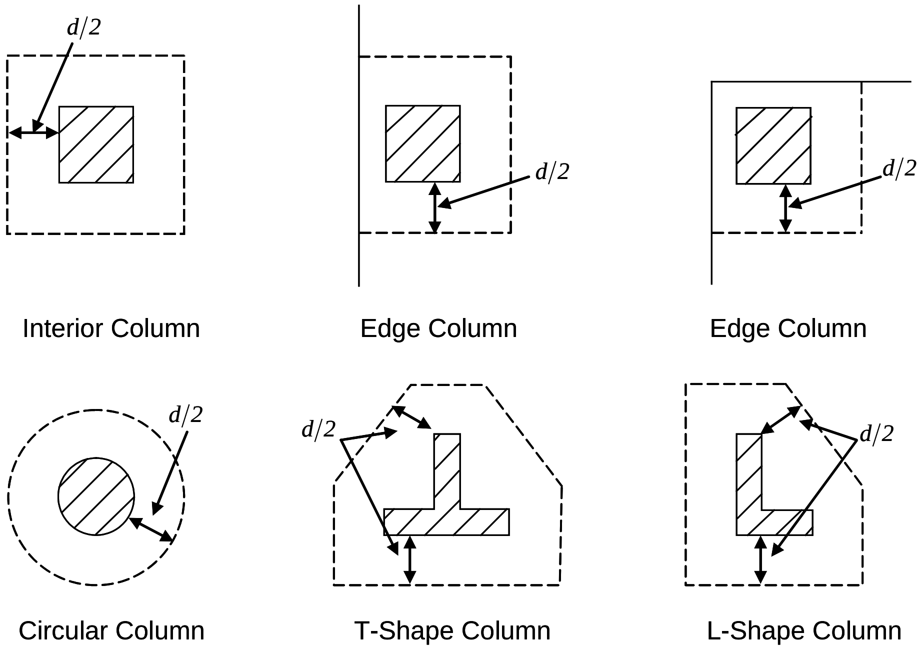


Figure 20-4 Punching Shear Perimeters

20.8.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_{sc}$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_{sc}$.

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}} \qquad \text{(ACI 8.4.2.3)}$$

$$\gamma_v = 1 - \gamma_f \qquad \text{(ACI 8.4.4.2.2)}$$

For reinforced concrete slabs, γ_f is permitted to increase to the maximum modified values provided in ACI Table 8.4.2.3.4 provided that the limitations on v_{ug} and ε_t given in ACI Table 8.4.2.3.4 are satisfied .

Column Location	Span Direction	v_{ug}	ε_t	Maximum modified γ_f
Corner column	Either direction	$\leq 0.5\phi v_c$	≥ 0.004	1.0
Edge column	Perpendicular to the edge	$\leq 0.75\phi v_c$	≥ 0.004	1.0
	Parallel to the edge	$\leq 0.4\phi v_c$	≥ 0.010	$\gamma_f = \dfrac{1.25}{1+(2/3)\sqrt{b_1/b_2}} \leq 1.0$
Interior column	Either direction	$\leq 0.4\phi v_c$	≥ 0.010	$\gamma_f = \dfrac{1.25}{1+(2/3)\sqrt{b_1/b_2}} \leq 1.0$

where b_1 is the width of the critical section measured in the direction of the span and b_2 is the width of the critical section measured in the direction perpendicular to the span.

20.8.2.3 Determine Concrete Capacity

The concrete punching shear stress capacity of a two-way prestressed section is taken as:

$$v_c = \left(\beta_p \lambda \sqrt{f'_c} + 0.3 f_{pc}\right) + v_p \qquad \text{(ACI 22.6.5.5a, 22.6.5.5b)}$$

$$\beta_p = \min\left(3.5, \left(\frac{\alpha_s d}{b_o} + 1.5\right)\right) \qquad \text{(ACI 22.6.5.5a, 22.6.5.5b)}$$

where, β_p is the factor used to compute v_c in prestressed slab; b_o is the perimeter of the critical section; f_{pc} is the average value of f_{pc} in the two directions; v_p is the vertical component of all effective prestress stresses crossing the critical section; and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 40 & \text{for interior columns,} \\ 30 & \text{for edge columns, and} \\ 20 & \text{for corner columns.} \end{cases} \qquad \text{(ACI 22.6.5.3)}$$

The concrete capacity v_c computed from ACI 22.6.5.5 is permitted only when the following conditions are satisfied:

- The column is farther than four times the slab thickness away from any discontinuous slab edges.
- The value of $\sqrt{f'_c}$ is taken no greater than 70 psi.
- In each direction, the value of f_{pc} is within the range:

  $125 \le f_{pc} \le 500$ psi

In thin slabs, the slope of the tendon profile is hard to control and special care should be exercised in computing v_p. In case of uncertainty between the design and as-built profile, a reduced or zero value for v_p should be used.

If the preceding three conditions are not satisfied, the concrete punching shear stress capacity of a two-way prestressed section is taken as the minimum of the following three limits:

$$v_c = \min \begin{cases} \left(2 + \dfrac{4}{\beta_c}\right)\lambda\sqrt{f'_c} \\ 4\lambda\sqrt{f'_c} \end{cases} \qquad \text{(ACI 22.6.5.2)}$$

where, β_c is the ratio of the maximum to the minimum dimensions of the critical section, b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section (ACI 22.6.5.3).

A limit is imposed on the value of $\sqrt{f'_c}$ as:

$$\sqrt{f'_c} \leq 100 \qquad \text{(ACI 22.5.3.1)}$$

20.8.2.4 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS.

20.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 6 inches, and not less than 16 times the shear reinforcement bar diameter (ACI 22.6.7.1). If the slab thickness does not meet these requirements, the punching shear reinforcement is not designed and the slab thickness should be increased by the user.

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as described in the subsections that follow.

20.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a two-way prestressed section with punching shear reinforcement is as previously determined, but limited to:

$$v_c \leq 2\lambda\sqrt{f'_c} \text{ for shear links} \qquad \text{(ACI 22.6.6.1)}$$

$$v_c \leq 3\lambda\sqrt{f'_c} \text{ for shear studs} \qquad \text{(ACI 22.26.6.1)}$$

20.8.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of:

$V_{\max} = 6\sqrt{f'_c}\ b_o d$ for shear links (ACI 22.6.6.2)

$V_{\max} = 8\sqrt{f'_c}\ b_o d$ for shear studs (ACI 22.6.6.2)

Given V_u, V_c, and $V_{\max}$, the required shear reinforcement is calculated as follows, where, ϕ, the strength reduction factor, is 0.75 (ACI 9.3.2.3).

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d}$$ (ACI 22.5.1.1, 22.5.10.1, 20.5.10.5.3)

$$\frac{A_v}{s} \geq 2\frac{\sqrt{f'_c}}{f_y} b_o \quad \text{for shear studs}$$

- If $V_u > \phi V_{\max}$, a failure condition is declared. (ACI 22.5.1.2)
- If V_u exceeds the maximum permitted value of $\phi V_{\max}$, the concrete section should be increased in size.

20.8.3.3 Determine Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 20-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner column.

The distance between the column face and the first line of shear reinforcement shall not exceed d/2 (ACI ACI 8.7.6.3, Table 8.7.6.3). The spacing between adjacent shear reinforcement in the first line of shear reinforcement shall not exceed 2*d* measured in a direction parallel to the column face (ACI 8.7.6.3, Table 8.7.6.3).

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

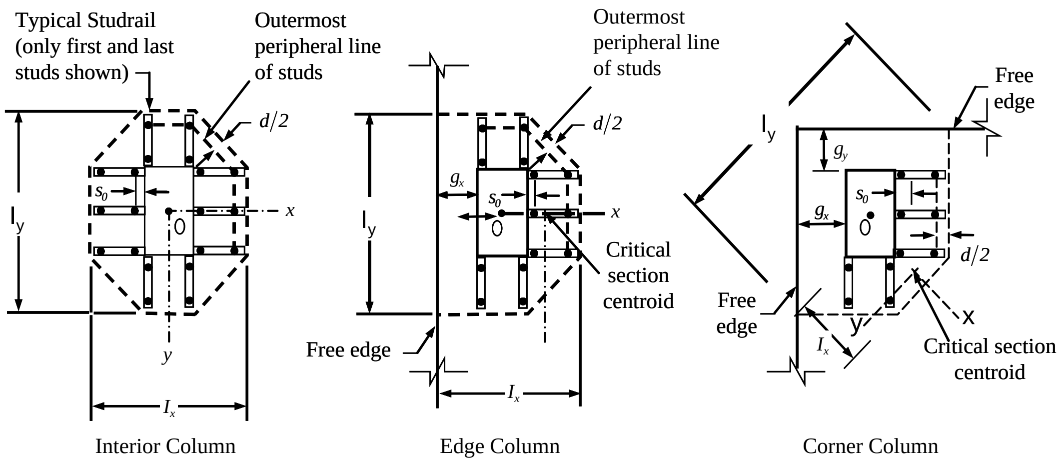


Figure 20-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

20.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in ACI 20.6.1.3 plus half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 3/8-, 1/2-, 5/8-, and 3/4-inch diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.35*d*. The limits of s_o and the spacing, *s*, between the peripheral lines are specified as:

$$s_o \leq 0.5d \qquad \text{(ACI 8.7.7.1.2)}$$

$$s \leq \begin{cases} 0.75d & \text{for} \quad v_u \leq 6\phi\lambda\sqrt{f'_c} \\ 0.50d & \text{for} \quad v_u > 6\phi\lambda\sqrt{f'_c} \end{cases} \qquad \text{(ACI 8.7.7.1.2)}$$

$$g \leq 2d \qquad \text{(ACI 8.7.7.1.2)}$$

The limits of s_o and the spacing, *s*, between the links are specified as:

$$s_o \leq 0.5d \quad \text{(ACI 8.7.6.3)}$$

$$s \leq 0.50d \quad \text{(ACI 8.7.6.3)}$$

Chapter 21
Design for CSA A23.3-14

This chapter describes in detail the various aspects of the post-tensioned concrete design procedure that is used by ETABS when the user selects the Canadian code CSA A23.3-14 [CSA 2014]. Various notations used in this chapter are listed in Table 21-1. For referencing to the pertinent sections of the CSA code in this chapter, a prefix "CSA" followed by the section number is used.

The design is based on user-specified load combinations. The program provides a set of default load combinations that should satisfy the requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-millimeter-second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-millimeter-second units unless otherwise noted.

21.1 Notations

The following table identifies the various notations used in this chapter.

Table 21-1 List of Symbols Used in the CSA A23.3-14 Code

A_p	Area of tension prestressing tendons, mm^2

Table 21-1 List of Symbols Used in the CSA A23.3-14 Code

A_s	Area of tension reinforcement, mm^2
A'_s	Area of compression reinforcement, mm^2
$A_{s(required)}$	Area of steel required for tension reinforcement, mm^2
A_v	Area of shear reinforcement, mm^2
A_v / s	Area of shear reinforcement per unit length of the member, mm^2/mm
A_{vs}	Area of headed shear reinforcement, mm^2
A_{vs} / s	Area of headed shear reinforcement per unit length of the member, mm^2/mm
a	Depth of compression block, mm
b	Width of member, mm
b_f	Effective width of flange (T-beam section), mm
b_w	Width of web (T-beam section), mm
b_0	Perimeter of the punching critical section, mm
b_1	Width of the punching critical section in the direction of bending, mm
b_2	Width of the punching critical section perpendicular to the direction of bending, mm
c	Depth to neutral axis, mm
d	Distance from compression face to tension reinforcement, mm
d'	Concrete cover to center of reinforcing, mm
d_p	Distance from compression face to prestressing tendons, mm
d_s	Thickness of slab, mm

Table 21-1 List of Symbols Used in the CSA A23.3-14 Code

d_v	Effective shear depth, mm
E_c	Modulus of elasticity of concrete, MPa
E_p	Modulus of elasticity of prestressing tendons, MPa
E_s	Modulus of elasticity of reinforcement, assumed as 2×10^5 MPa
f'_{ci}	Specified compressive strength of concrete at time of prestress transfer, MPa
f'_c	Specified compressive strength of concrete, MPa
f_y	Specified yield strength of flexural reinforcement, MPa
f_{yh}	Specified yield strength of shear reinforcement, MPa
f_{yv}	Specified yield strength of headed shear reinforcement, MPa
h	Overall depth of a section, mm
I_g	Moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement.
M_f	Factored moment at section, N-mm
ϕM_r^0	Design moment resistance of a section with tendons only, N-mm
ϕM_r^{bal}	Design moment resistance of a section with tendons and the necessary mild reinforcement to reach the balanced condition, N-mm
s	Spacing of the shear reinforcement along the length of the beam, mm
V_c	Shear resisted by concrete, N
$V_{r,max}$	Maximum permitted total factored shear force at a section, N

Table 21-1 List of Symbols Used in the CSA A23.3-14 Code

V_f	Factored shear force at a section, N
V_s	Shear force at a section resisted by steel, N
α_1	Ratio of average stress in rectangular stress block to the specified concrete strength
β_1	Factor for obtaining depth of compression block in concrete
β_c	Ratio of the maximum to the minimum dimensions of the punching critical section
ε_c	Strain in concrete
ε_{cu}	Maximum strain in concrete at ultimate
ε_p	Strain in prestressing tendons
ε_s	Strain in reinforcing steel
φ_c	Resistance factor for concrete
φ_p	Resistance factor for prestressing tendons
φ_s	Resistance factor for steel
γ_f	Fraction of unbalanced moment transferred by flexure
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear
λ	Shear strength factor

21.2 Design Load Combinations

The design load combinations are the various combinations of the load cases for which the structure needs to be designed. For CSA A23.3-14, if a structure is subjected to dead (D), live (L), pattern live (PL), snow (S), wind (W), and earthquake (E) loads, and considering that wind and earthquake forces are reversible, the load combinations in the following sections may need to be considered (CSA 8.3.2, Table C.1a).

For post-tensioned concrete design, the user also can specify the prestressing load (PT) by providing the tendon profile or by using the load balancing options in the program. The default load combinations for post-tensioning are defined in the following sections.

21.2.1 Initial Service Load Combination

The following load combination is used for checking the requirements at transfer of prestress forces, in accordance with CSA 18.3.1. The prestressing forces are considered without any long-term loses for the initial service load combination check.

1.0D + 1.0PT

21.2.2 Service Load Combinations

The following load combinations are used for checking the requirements of prestress for serviceability in accordance with CSA 18.3.2. It is assumed that long-term losses have already occurred at the service stage.

1.0D + 1.0PT
1.0D + 1.0L + 1.0PT

21.2.3 Long-Term Service Load Combination

The following load combinations are used for checking the requirements of prestress in accordance with CSA 18.3.2(a). The permanent load for this load combination is taken as 50 percent of the live load. It is assumed that all long term losses have already occurred at the service stage.

1.0D + 1.0PT
1.0D + 0.5L + 1.0PT

21.2.4 Strength Design Load Combination

The following load combinations are used for checking the requirements of prestress for strength in accordance with CSA A23.3-14, Chapters 8 and 18.

The strength design combinations required for punching shear require the full PT forces (primary and secondary). Flexural design requires only the hyperstatic (secondary) forces. The hyperstatic (secondary) forces are determined automatically by ETABS by subtracting the primary PT moments when the flexural design is carried out.

$1.4D + 1.0PT^*$ (CSA 8.3.2,Table C.1a, Case 1)

$1.25D + 1.5L + 1.0PT^*$
$1.25D + 1.5L + 1.0PT^* \pm 0.4W$
$1.25D + 1.5L + 1.0PT^* + 0.5S$
$0.9D + 1.5L + 1.0PT^*$
$0.9D + 1.5L + 1.0PT^* \pm 0.4W$
$0.9D + 1.5L + 1.0PT^* + 0.5S$
(CSA 8.3.2,Table C.1a, Case 2)

$1.25D + 1.5(0.75\ PL) + 1.0PT^*$ (CSA 13.8.4.3)

$1.25D + 1.5S + 1.0PT^*$
$1.25D + 1.5S + 1.0PT^* + 0.5L$
$1.25D + 1.5S + 1.0PT^* \pm 0.4W$
$0.9D + 1.5S + 1.0PT^*$
$0.9D + 1.5S + 1.0PT^* + 0.5L$
$0.9D + 1.5S + 1.0PT^* \pm 0.4W$
(CSA 8.3.2,Table C.1a, Case 3)

$1.25D \pm 1.4W + 1.0PT^*$
$1.25D \pm 1.4W + 1.0PT^* + 0.5L$
$1.25D \pm 1.4W + 1.0PT^* + 0.5S$
$0.9D \pm 1.4W + 1.0PT^*$
$0.9D \pm 1.4W + 1.0PT^* + 0.5L$
$0.9D \pm 1.4W + 1.0PT^* + 0.5S$
(CSA 8.3.2,Table C.1a, Case 4)

$1.0D \pm 1.0E + 1.0PT^*$
$1.0D \pm 1.0E + 0.5L + 1.0PT^*$
$1.0D \pm 1.0E + 0.25S + 1.0PT^*$
$1.0D + 0.5L + 0.25S \pm 1.0E + 1.0PT^*$
(CSA 8.3.2,Table C.1a, Case 5)

* — Replace PT by H for flexural design only

These are also the default design combinations in ETABS whenever the CSA A23.3-14 code is used. The user should use other appropriate load combinations if roof live load is treated separately, or if other types of loads are present.

21.3 Limits on Material Strength

The upper and lower limits of f'_c are 80 MPa and 20 MPa respectively. The upper limit of f_y is 500 MPa for non-prestressed reinforcement (CSA 8.6.1.1).

For compression reinforcement with f_y exceeding 400 MPa, the value of f_y assumed in design calculations shall not exceed the stress corresponding to a strain of 0.0035 (CSA 8.5.21).

ETABS enforces the upper material strength limits for flexure and shear design of slabs. The input material strengths are taken as the upper limits if they are defined in the material properties as being greater than the limits. The user is responsible for ensuring that the minimum strength is satisfied.

ETABS also checks the following tensile strength limits in prestressing steel (CSA 18.4). The permissible tensile stresses in all types of prestressing steel, in terms of the specified minimum tensile strength f_{pu}, are summarized as follows:

- Due to tendon jacking force for post-tensioning tendons:

  $0.85\ f_{pu} \leq 0.94\ f_{py}$

- Due to tendon jacking force for pretensioning tendons:

  $0.80\ f_{pu}$

- Immediately after prestress transfer:

  $0.82\ f_{py} \leq 0.74\ f_{pu}$

- Post-tensioning tendons, at anchorages and couplers, immediately after tendon anchorage:

  $0.70\ f_{pu}$

The specified yield strength of prestressing tendons is based on the requirements specified in ASTM A 416/A 416 M, ASTM A 421/A421 M, and ASTM A 722/A 722 m, which specify the following minimum values for f_{py}:

- low-relaxation wire and strands $f_{py} = 0.90\ f_{pu}$
- stress-relieved wire and strands, and plain bars $f_{py} = 0.85\ f_{pu}$
- deformed bar $f_{py} = 0.80\ f_{pu}$

21.4 Strength Reduction Factors

The strength reduction factors, ϕ, are material dependent and defined as:

$\phi_c =$ 0.65 for concrete (CSA 8.4.2)

$\phi_s =$ 0.85 for reinforcement (CSA 8.4.3a)

$\phi_p =$ 0.90 for post-tensioning tendons (CSA 8.4.3a)

The preceding values for ϕ_c, ϕ_s, and ϕ_p are the default values. These values can be modified in the design preferences. For structural concrete manufactured in prequalified manufacturing plants, ϕ_c can be taken as 0.7 (CSA 8.4.2, 16.1.3).

21.5 Design Assumptions for Prestressed Concrete

Strength design of prestressed members for flexure and axial loads shall be based on assumptions given in CSA 10.1.

- The strain in the reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis, except for unbonded tendons (CSA 10.1.2).
- The maximum usable strain at the extreme concrete compression fiber shall be assumed equal to 0.0035 (CSA 10.1.3).
- The balanced strain condition shall exist at a cross-section when tension reinforcement reaches its yield strain just as the concrete in compression reaches its maximum strain of 0.0035 (CSA 10.1.4).

- The tensile strength of concrete shall be neglected in the calculation of the factored flexural resistance of prestressed concrete members (CSA 10.1.5).
- The relationship between the concrete compressive stress distribution and the concrete strain shall be assumed to be rectangular by an equivalent rectangular concrete stress distribution (CSA 10.1.7).
- The concrete stress of $\alpha_c \varphi_c f_c^{'}$ shall be assumed uniformly distributed over an equivalent-compression zone bounded by edges of the cross-section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain (CSA 10.1.7(a)).
- The distance from the fiber of maximum strain to the neutral axis, *c*, shall be measured in a direction perpendicular to the neutral axis (CSA 10.1.7.(b)).
- The factors α_1 and β_1 shall be taken as follows (CSA 10.1.7.(c)).
  - $\alpha_1 = 0.85 - 0.0015\, f_c^{'} \geq 0.67$
  - $\beta_1 = 0.97 - 0.0025\, f_c^{'} \geq 0.67$

Prestressed concrete members are investigated at the following three stages (CSA 18.3):

- At transfer of prestress force
- At service loading
- At nominal strength

21.6 Serviceability Requirements of Flexural Members

21.6.1 Serviceability Check at Initial Service Load

The stress in the concrete immediately after prestress force transfer (before time dependent prestress losses) are checked against the following limits (CSA 18.3.1.1(a), 18.3.1.1(b) and 18.3.1.1(c)):

- Extreme fiber stress in compression:

$$0.60 f'_{ci}$$

- Extreme fiber stress in tension, except as permitted in the subsequent item:

$$0.25\lambda\sqrt{f'_{ci}}$$

- Extreme fiber stress in tension at ends of simply supported members:

$$0.5\lambda\sqrt{f'_{ci}}$$

The extreme fiber stress in tension at the ends of simply supported members is currently **NOT** checked by ETABS.

21.6.2 Serviceability Check at Service Load

The stresses in prestressed concrete flexural members at service loads, and after all prestress losses occur, are checked against the following limits (CSA 18.3.2):

- Extreme fiber stress in compression due to prestress plus total load:

$$0.60 f'_{c}$$

- Extreme fiber stress in tension in the precompressed tensile zone at service loads:

$$0.50\lambda\sqrt{f'_{c}}$$

- Extreme fiber stress in tension in the precompressed tensile zone at service loads, exposed to corrosive environment:

$$0.25\lambda\sqrt{f'_{c}}$$

21.6.3 Serviceability Check at Long-Term Service Load

The stresses in prestressed concrete flexural members at long-term service loads, and after all prestress losses have occurred, are checked against the same limits as for the normal service load, except for the following (CSA 18.3.2):

Extreme fiber stress in compression due to prestress plus sustained load:

$$0.45 f_c^{'}$$

21.7 Beam Design (for Reference Only)

Important Note: *Post-tensioned beam design is not available in the current version of ETABS, but is planned for a future release. This section is provided as reference only for the documentation of post-tensioned slab design.*

In the design of prestressed concrete beams, ETABS calculates and reports the required areas of reinforcement for flexure, shear, and torsion based on the beam moments, shear forces, torsion, load combination factors, and other criteria described in the subsections that follow. The reinforcement requirements are calculated at each station along the length of the beam.

Beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement
- Design torsion reinforcement

21.7.1 Design Flexural Reinforcement

The beam top and bottom flexural reinforcement is designed at each station along the beam. In designing the flexural reinforcement for the major moment of a particular beam, for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement if required

21.7.1.1 Determine Factored Moments

In the design of flexural reinforcement of post-tensioned beams, the factored moments for each load combination at a particular beam station are obtained by factoring the corresponding moments for different load cases, with the corresponding load factors.

The beam is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments can be used to calculate bottom reinforcement. In such cases the beam may be designed as a rectangular or flanged beam. Negative beam moments can be used to calculate top reinforcement. In such cases the beam may be designed as a rectangular or inverted flanged beam.

21.7.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the strength of the concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 21-1 (CSA 10.1.7). Furthermore it is assumed that the compression carried by the concrete is less than or equal to that which can be carried at the balanced condition (CSA 10.1.4). When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by ETABS, for both rectangular and flanged sections (L- and T-beams) is summarized in the subsections that follow. It is assumed that the design ultimate axial force in a beam is negligible; hence all the beams are designed for major direction flexure, shear, and torsion only.

21.7.1.2.1 Design of Rectangular Beams

ETABS first determines whether the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, moment capacity ϕM_n^0 is determined as follows:

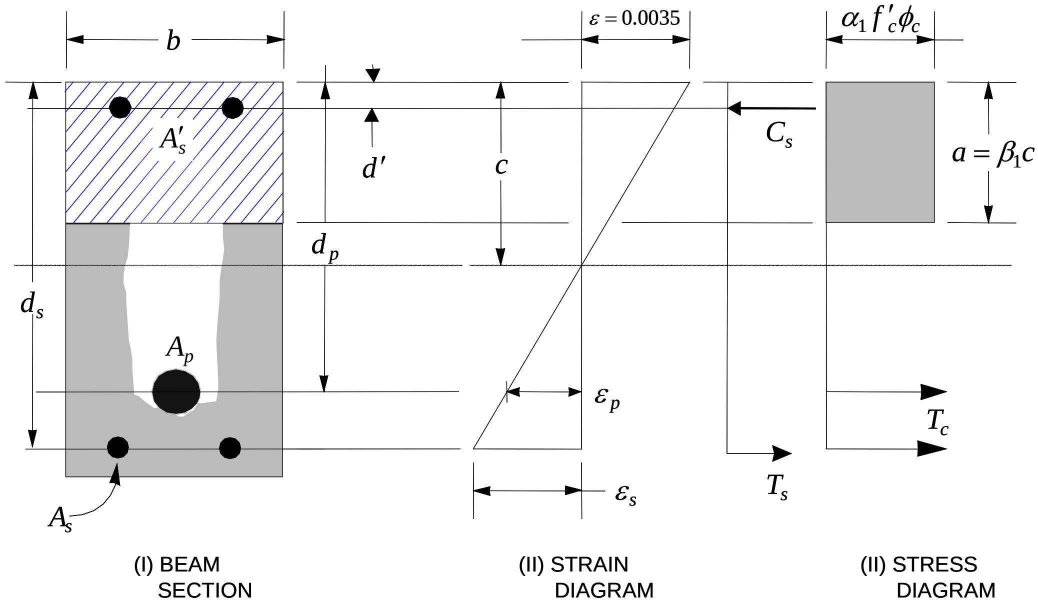


Figure 21-1 Rectangular Beam Design

The maximum depth of the compression zone, c_{max}, is calculated based on strain-stress compatibility (CSA 18.6.1):

$$c_{max} = \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_p}\right) E_p d_p \qquad \text{(CSA 18.6.1)}$$

where,

$$\varepsilon_{cu} = 0.0035 \qquad \text{(CSA 10.1.4)}$$

Therefore, the limits $c \le c_{max}$ is set for tension-controlled sections.

The ductility of a section is ensured by limiting the c/d ratio and strength reduction factor ϕ. The minimum ductility required by the CSA code is limited as $c/d_p \leq 0.5$ (CSA 18.6.2).

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by:

$$a_{\max} = \beta_1 c_{\max} \qquad \text{(CSA 10.1.7(a))}$$

where β_1 is calculated as:

$$\beta_1 = 0.97 - 0.0025\ f_c' \geq 0.67 \qquad \text{(CSA 10.1.7)}$$

ETABS determines the depth of the neutral axis, c, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pr} is computed based on strain compatibility. On the basis of the calculated f_{pr}, the depth of the neutral axis is recalculated, and f_{pr} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \beta_1 c$$

- If $c \leq c_{\max}$ (CSA 18.6.2), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$\phi M_r^0 = \phi A_{ps} f_{pr} \left(d_p - \frac{a}{2} \right)$$

- If $c > c_{\max}$ (CSA 18.6.2), a failure condition is declared.

- If $M_f > \phi M_r^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of the neutral axis c is equal to $c_{\max}$. The stress in the post-tensioning steel, f_{pr} is then calculated based on strain compatibility and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

$$C = \alpha_1 f_c' \phi_c a_{\max} b$$

$$T = A_p f_{pr}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\alpha_1 f_c^{'} \phi_c a_{\max} b - A_p f_{pr}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$\phi M_r^{bal} = \phi A_p f_{pr}^{bal} \left(d_p - \frac{a_{\max}}{2} \right) + \phi A_s^{bal} f_s^{bal} \left(d_s - \frac{a_{\max}}{2} \right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcement, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of the neutral axis, *c*.

21.7.1.2.1.1 Case 1: Post-tensioning steel is adequate

When $\left(M_f < \phi M_r^0 \right)$, the amount of post-tensioning steel is adequate to resist the design moment M_f. Minimum reinforcement is provided to satisfy the ductility requirements (CSA 18.3.13, 18.7 and 18.8), i.e., $\left(M_f < \phi M_r^0 \right)$.

21.7.1.2.1.2 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist M_f, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $\phi M_r^0 < M_f < \phi M_r^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M_f and reports the required area of tension reinforcement. Since M_f is bounded by ϕM_r^0 at the lower end and ϕM_r^{bal} at the upper end, and ϕM_r^0 is associated with $A_s = 0$ and ϕM_r^{bal}

is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if M_f is positive or at the top if M_f is negative.

21.7.1.2.1.3 *Case 3: Post-tensioning steel and tension reinforcement is not adequate*

When $\left(M_f > \phi M_r^{bal}\right)$, compression reinforcement is required (CSA 18.6.2). In that case, ETABS assumes that the depth of the neutral axis, c, is equal to $c_{\max}$. The values of f_{pr} and f_s reach their respective balanced condition values, f_{pr}^{bal} and f_s^{bal}. The area of compression reinforcement, A_s', is determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{rs} = M_f - \phi M_r^{bal}$$

The required compression reinforcement is given by:

$$A_s' = \frac{M_{rs}}{\left(\phi_s f_s' - \phi_c \alpha_1 f_c'\right)\left(d_s - d'\right)}, \text{ where}$$

$$f_s' = 0.0035 E_s \left[\frac{c - d'}{c}\right] \leq f_y \qquad \text{(CSA 10.1.2, 10.1.3)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{rs}}{f_y\left(d - d'\right)\phi_s}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_f is positive, and vice versa if M_f is negative.

21.7.1.2.2 Design of Flanged Beams

21.7.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment, M_f (i.e., designing top reinforcement), the calculation of the reinforcement area is exactly the same as described previously, i.e., no flanged beam data is used.

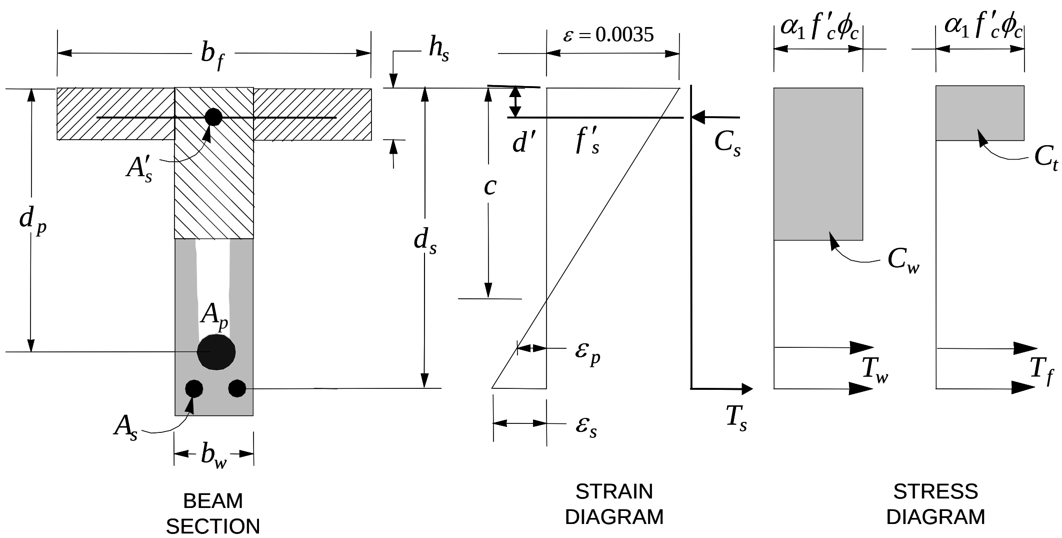


Figure 21-2 T-Beam Design

21.7.1.2.2.2 Flanged Beam Under Positive Moment

ETABS first determines if the moment capacity provided by the post-tensioning tendons alone is enough. In calculating the capacity, it is assumed that $A_s = 0$. In that case, the moment capacity ϕM_n^0 is determined as follows:

The maximum depth of the compression zone, $c_{\max}$, is calculated based on strain-stress compatibility (CSA 18.6.1):

$$c_{\max} = \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_p}\right) E_p d_p \qquad \text{(CSA 18.6.1)}$$

where,

$\varepsilon_{cu} = 0.0035$ (CSA 10.1.4)

Therefore, the limits $c \leq c_{\max}$ is set for tension-controlled sections.

The ductility of a section is ensured by limiting the *c/d* ratio and strength reduction factor ϕ. The minimum ductility required by the CSA code is limited to $c/d_p \leq 0.5$ (CSA 18.6.2).

The maximum allowable depth of the rectangular compression block, $a_{\max}$, is given by:

$$a_{\max} = \beta_1 c_{\max} \quad \text{(CSA 10.1.7(a))}$$

where β_1 is calculated as:

$$\beta_1 = 0.97 - 0.0025 \; f'_c \geq 0.67 \quad \text{(CSA 10.1.7)}$$

ETABS determines the depth of the neutral axis, *c*, by imposing force equilibrium, i.e., $C = T$. After the depth of the neutral axis has been determined, the stress in the post-tensioning steel, f_{pr} is computed based on strain compatibility. Based on the calculated f_{pr}, the depth of the neutral axis is recalculated, and f_{pr} is further updated. After this iteration process has converged, the depth of the rectangular compression block is determined as follows:

$$a = \beta_1 c$$

- If $c \leq c_{\max}$ (CSA 18.6.2), the moment capacity of the section, provided by post-tensioning steel only, is computed as:

$$\phi M_r^0 = \phi A_{ps} f_{pr} \left(d_p - \frac{a}{2} \right)$$

- If $c > c_{\max}$ (CSA 18.6.2), a failure condition is declared.

- If $M_f > \phi M_r^0$, ETABS calculates the moment capacity and the A_s required at the balanced condition. The balanced condition is taken as the marginal tension controlled case. In that case, it is assumed that the depth of neutral axis *c* is equal to $c_{\max}$. The stress in the post-tensioning steel, f_{pr} is then calculated based on strain compatibility and the area of required tension reinforcement, A_s, is determined by imposing force equilibrium, i.e., $C = T$.

- If $a \le h_s$, the subsequent calculations for A_s are exactly the same as previously defined for the rectangular beam design. However, in this case the width of the beam is taken as b_f. Compression reinforcement is required when $a > a_{\max}$.
- If $a > h_s$, the calculation for A_s is given by:

$$C = \alpha_1 f_c' \phi_c a_{\max} A_c^{com}$$

where A_c^{com} is the area of concrete in compression, i.e.,

$$A_c^{com} = b_f d_s + b_w \left(a_{\max} - d_s \right)$$

$$T = A_p f_{pr}^{bal} + A_s^{bal} f_s^{bal}$$

$$A_s^{bal} = \frac{\alpha_1 f_c' \phi_c a_{\max} A_c^{com} - A_p f_{pr}^{bal}}{f_s^{bal}}$$

After the area of tension reinforcement has been determined, the capacity of the section with post-tensioning steel and tension reinforcement is computed as:

$$\phi M_r^{bal} = \phi A_p f_{pr}^{bal} \left(d_p - \frac{a_{\max}}{2} \right) + \phi A_s^{bal} f_s^{bal} \left(d_s - \frac{a_{\max}}{2} \right)$$

In that case, it is assumed that the bonded tension reinforcement will yield, which is true for most cases. In the case that it does not yield, the stress in the reinforcing steel, f_s, is determined from the elastic-perfectly plastic stress-strain relationship. The f_y value of the reinforcement is then replaced with f_s in the preceding four equations. This case does not involve any iteration in determining the depth of neutral axis, c.

21.7.1.2.2.3 Case 1: Post-tensioning steel is adequate

When $\left(M_f < \phi M_r^0\right)$ the amount of post-tensioning steel is adequate to resist the design moment M_f. Minimum reinforcement is provided to satisfy ductility requirements (CSA 18.3.13, 18.7 and 18.8), i.e., $\left(M_f < \phi M_r^0\right)$.

21.7.1.2.2.4 Case 2: Post-tensioning steel plus tension reinforcement

In this case, the amount of post-tensioning steel, A_p, alone is not sufficient to resist M_f, and therefore the required area of tension reinforcement is computed to supplement the post-tensioning steel. The combination of post-tensioning steel and tension reinforcement should result in $a < a_{\max}$.

When $\phi M_r^0 < M_f < \phi M_r^{bal}$, ETABS determines the required area of tension reinforcement, A_s, iteratively to satisfy the design moment M_f and reports this required area of tension reinforcement. Since M_f is bounded by ϕM_r^0 at the lower end and ϕM_r^{bal} at the upper end, and ϕM_r^0 is associated with $A_s = 0$ and ϕM_r^{bal} is associated with $A_s = A_s^{bal}$, the required area will be within the range of 0 to A_s.

The tension reinforcement is to be placed at the bottom if M_f is positive, or at the top if M_f is negative.

21.7.1.2.2.5 Case 3: Post-tensioning steel and tension reinforcement is not adequate

When $\left(M_f > \phi M_r^{bal}\right)$, compression reinforcement is required (CSA 18.6.2). In that case, ETABS assumes that the depth of the neutral axis, c, is equal to $c_{\max}$. The values of f_{pr} and f_s reach their respective balanced condition values, f_{pr}^{bal} and f_s^{bal}. Then the area of compression reinforcement, A_s', is determined as follows:

The moment required to be resisted by compression reinforcement and tension reinforcement is:

$$M_{rs} = M_f - \phi M_r^{bal}$$

The required compression reinforcement is given by:

$$A'_s = \frac{M_{rs}}{\left(\phi_s f'_s - \phi_c \alpha_1 f'_c\right)\left(d_s - d'\right)}, \text{ where}$$

$$f'_s = 0.0035 E_s \left[\frac{c - d'}{c}\right] \leq f_y. \qquad \text{(CSA 10.1.2, 10.1.3)}$$

The tension reinforcement for balancing the compression reinforcement is given by:

$$A_s^{com} = \frac{M_{rs}}{f_y\left(d - d'\right)\phi_s}$$

Therefore, the total tension reinforcement, $A_s = A_s^{bal} + A_s^{com}$, and the total compression reinforcement is A'_s. A_s is to be placed at the bottom and A'_s is to be placed at the top if M_f is positive, and vice versa if M_f is negative.

21.7.1.2.3 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the limits specified in CSA 18.8.2, Table 18.1.

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w d & \text{Flanged beam} \end{cases}$$

21.7.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement for a particular beam, for a particular load combination at a particular station due to the beam major shear, the following steps are involved:

- Determine the factored forces acting on the section, M_f and v_f. Note that M_f is needed for the calculation of v_c.
- Determine the shear stress, v_c that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

The following three sections describe in detail the algorithms associated with these steps.

21.7.2.1 Determine Shear Force

In the design of the beam shear reinforcement of a concrete beam, the shear forces for a particular load combination at a particular beam section are obtained by factoring the associated shear forces and moments with the cor-responding load combination factors.

21.7.2.2 Determine Concrete Shear Capacity

The shear force carried by the concrete, V_c, is calculated as:

$$V_c = \phi_c \lambda \beta \sqrt{f'_c} b_w d_v \qquad \text{(CSA 11.3.4)}$$

where,

$$\sqrt{f'_c} \le 8 \text{ MPa} \qquad \text{(CSA 11.3.4)}$$

ϕ_c is the resistance factor for concrete. By default it is taken as 0.65 (CSA 8.4.2). For concrete produced in a pre-qualified manufacturing plant, the value can be taken as 0.70 (CSA 16.1.3). This value can be overwritten in the design preferences.

λ is the strength reduction factor to account for low density concrete (CSA 2.2). For normal density concrete, its value is 1 (CSA 8.6.5), which is taken by the program as the default value. For concrete using lower density aggregate, the user can change the value of λ in the material property data. The recommended value for λ is as follows (CSA 8.6.5).

$$\lambda = \begin{cases} 1.00, & \text{for normal density concrete,} \\ 0.85, & \text{for semi-low-density concrete} \\ & \text{in which all of the fine aggregate is natural sand,} \\ 0.75, & \text{for semi-low-density concrete} \\ & \text{in which none of the fine aggregate is natural sand.} \end{cases}$$

β is the factor for accounting for the shear resistance of cracked concrete (CSA 2.2) and should be equal to or greater than 0.05. . Its value is normally between 0.1 and 0.4. It is determined according to CSA 11.3.6 and described further in the following sections.

b_w is the effective web width. For rectangular beams, it is the width of the beam. For flanged beams, it is the width of the web of the beam.

d_v is the effective shear depth. It is taken as the greater of 0.9*d* or 0.72*h* (CSA 2.3), where *d* is the distance from the extreme compression fiber to the centroid of tension reinforcement, and *h* is the overall depth of the cross-section in the direction of the shear force.

The value of β is preferably taken as the special value (CSA 11.3.6.2), or it is determined using the simplified method (CSA 11.3.6.3), if applicable. When the conditions of the special value or simplified method do not apply, the general method is used (CSA 11.3.6.4).

If the overall beam depth, *h*, is less than 250 mm or if the beam depth of a flanged beam below the slab is not greater than one-half of the width of the web or 350 mm, β is taken as 0.21 (CSA 11.3.6.2).

$$\beta = 0.21 \qquad \text{(CSA 11.3.6.2)}$$

When the specified yield strength of the longitudinal reinforcing f_y does not exceed 400 MPa, and the specified concrete strength f'_c does not exceed 60 MPa, β is determined in accordance with the simplified method, as follows (CSA 11.6.3.3):

- When the section contains at least the minimum transverse reinforcement, β is taken as 0.18 (CSA 11.3.6.3a).

$\beta = 0.18$ (CSA 11.3.6.3.a)

When the section contains no transverse reinforcement, β is determined based on the specified maximum nominal size of coarse aggregate, a_g.

For maximum size of coarse aggregate not less than 20 mm, β is taken as:

$$\beta = \frac{230}{1000 + d_v} \quad \text{(CSA 11.3.6.3 b)}$$

where d_v is the effective shear depth expressed in millimeters, which is described in preceding sections.

For a maximum size of coarse aggregate less than 20 mm, β is taken as:

$$\beta = \frac{230}{1000 + s_{ze}} \quad \text{(CSA 11.3.6.3 c)}$$

where, $S_{ze} = \dfrac{35}{15 + a_g} S_z \geq 0.85 S_z$ (CSA 11.3.6.3.c)

In the preceding expression, the crack spacing parameter, s_{ze}, shall be taken as the minimum of d_v and the maximum distance between layers of distributed longitudinal reinforcement. However, s_{ze} is conservatively taken as equal to d_v.

In summary, for simplified cases, β can be expressed as follows:

$$\beta = \begin{cases} 0.18, & \text{if minimum transverse reinforcement is provided,} \\ \dfrac{230}{1000 + d_v}, & \text{if no transverse reinforcement is provided, and } a_g \geq 20\text{mm,} \\ \dfrac{230}{1000 + S_{ze}}, & \text{if no transverse reinforcement is provided, and } a_g < 20\text{mm.} \end{cases}$$

- When the specified yield strength of the longitudinal reinforcing f_y is greater than 400 MPa, the specified concrete strength f'_c is greater than 60 MPa, or tension is not negligible, β is determined in accordance with the general method as follows (CSA 11.3.6.1, 11.3.6.4):

$$\beta = \frac{0.40}{(1+1500\varepsilon_x)} \bullet \frac{1300}{(1000+S_{ze})} \qquad \text{(CSA 11.3.6.4)}$$

In the preceding expression, the equivalent crack spacing parameter, s_{ze} is taken equal to 300 mm if minimum transverse reinforcement is provided (CSA 11.3.6.4). Otherwise it is determined as stated in the simplified method.

$$S_{ze} = \begin{cases} 300 & \text{if minimum transverse reinforcement is provided,} \\ \dfrac{35}{15+a_g} S_z \geq 0.85 S_z & \text{otherwise.} \end{cases}$$

(CSA 11.3.6.3, 11.3.6.4)

The value of a_g in the preceding equations is taken as the maximum aggregate size for f'_c of 60 MPa, is taken as zero for f'_c of 70 MPa, and is linearly interpolated between these values (CSA 11.3.6.4).

The longitudinal strain, ε_x at mid-depth of the cross-section is computed from the following equation:

$$\varepsilon_x = \frac{M_f / d_v + V_f + 0.5 N_f}{2(E_s A_s)} \qquad \text{(CSA 11.3.6.4)}$$

In evaluating ε_x the following conditions apply:

- ε_x is positive for tensile action.
- V_f and M_f are taken as positive quantities. (CSA 11.3.6.4(a))
- M_f is taken as a minimum of $V_f d_v$. (CSA 11.3.6.4(a))
- N_f is taken as positive for tension. (CSA 2.3)

A_s is taken as the total area of longitudinal reinforcement in the beam. It is taken as the envelope of the reinforcement required for all design load combinations. The actual provided reinforcement might be slightly higher than this quantity. The reinforcement should be developed to achieve full strength (CSA 11.3.6.3(b)).

If the value of ε_x is negative, it is recalculated with the following equation, in which A_{ct} is the area of concrete in the flexural tensile side of the beam, taken as half of the total area.

$$\varepsilon_x = \frac{M_f / d_v + V_f + 0.5N_f}{2\left(E_s A_s + E_c A_{ct}\right)} \qquad \text{(CSA 11.3.6.4(c))}$$

$$E_s = 200{,}000 \text{ MPa} \qquad \text{(CSA 8.5.4.1)}$$

$$E_c = 4500\sqrt{f_c^{'}} \text{ MPa} \qquad \text{(CSA 8.6.2.3)}$$

If the axial tension is large enough to induce tensile stress in the section, the value of ε_x is doubled (CSA 11.3.6.4(e)).

For sections closer than d_v from the face of the support, ε_x is calculated based on M_f and V_f of a section at a distance d_v from the face of the support (CSA 11.3.6.4(d)). This condition currently is not checked by ETABS.

An upper limit on ε_x is imposed as:

$$\varepsilon_x \le 0.003 \qquad \text{(CSA 11.3.6.4(f))}$$

In both the simplified and general methods, the shear strength of the section due to concrete, v_c, depends on whether the minimum transverse reinforcement is provided. To check this condition, the program performs the design in two passes. In the first pass, it is assumes that no transverse shear reinforcement is needed. When the program determines that shear reinforcement is needed, the program performs the second pass assuming that at least minimum shear reinforcement is provided.

21.7.2.3 Determine Required Shear Reinforcement

The shear force is limited to $V_{r,\max}$ where:

$$V_{r,\max} = 0.25\phi_c f'_c b_w d \qquad \text{(CSA 11.3.3)}$$

Given V_f, V_c, and $V_{r,\max}$, the required shear reinforcement is calculated as follows:

- If $V_f \le V_c$

$$\frac{A_v}{s} = 0 \qquad \text{(CSA 11.3.5.1)}$$

- If $V_c < V_f \le V_{r,\max}$

$$\frac{A_v}{s} = \frac{\left(V_f - V_c\right)\tan\theta}{\phi_s f_{yt} d_v} \qquad \text{(CSA 11.3.3, 11.3.5.1)}$$

- If $V_f > V_{r,\max}$, (CSA 11.3.3)

  a failure condition is declared.

A minimum area of shear reinforcement is provided in the following regions (CSA 11.2.8.1):

(a) in regions of flexural members where the factored shear force V_f exceeds V_c

(b) in regions of beams with an overall depth greater than 750 mm

(c) in regions of beams where the factored torsion T_f exceeds $0.25T_{cr}$

Where the minimum shear reinforcement is required by CSA 11.2.8.1, or by calculations, the minimum area of shear reinforcement per unit spacing is taken as:

$$\frac{A_v}{s} \ge 0.06\frac{\sqrt{f_c'}}{f_y} b_w \qquad \text{(CSA 11.2.8.2)}$$

In the preceding equations, the term θ is used where θ is the angle of inclination of the diagonal compressive stresses with respect to the longitudinal axis of the member. The θ value is normally between 22 and 44 degrees. It is determined according to CSA 11.3.6.

Similar to the β factor, which was described previously, the value of θ is preferably taken as the special value (CSA 11.3.6.2) or it is determined using the simplified method (CSA 11.3.6.3), whenever applicable. The program uses the general method when conditions for the simplified method are not satisfied (CSA 11.3.6.4).

- If the overall beam depth, h, is less than 250 mm or if the depth of the flanged beam below the slab is not greater than one-half of the width of the web or 350 mm, θ is taken as 42 degrees (CSA 11.3.6.2).

- If the specified yield strength of the longitudinal reinforcing f_y does not exceed 400 MPa, or the specified concrete strength f'_c does not exceed 60 MPa, θ is taken to be 35 degree (CSA 11.3.6.3).

$$\theta = 35^{\circ} \text{ for } P_f \leq 0 \text{ or } f_y \leq 400 \text{ MPa or } f'_c \leq 60 \text{ MPa} \qquad \text{(CSA11.3.6.3)}$$

- If the axial force is tensile, the specified yield strength of the longitudinal reinforcing $f_y > 400$ MPa, and the specified concrete strength $f'_c > 60$ MPa, θ is determined using the general method as follows (CSA 11.3.6.4),

$$\theta = 29 + 7000\varepsilon_x \text{ for } P_f < 0,\ f_y > 400 \text{ MPa, } f'_c \leq 60 \text{ MPa} \qquad \text{(CSA11.3.6.4)}$$

where ε_x is the longitudinal strain at the mid-depth of the cross-section for the factored load. The calculation procedure has been described in preceding sections.

The maximum of all of the calculated A_v/s values, obtained from each load combination, is reported along with the controlling shear force and associated load combination.

The beam shear reinforcement requirements reported by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric requirements must be investigated independently of the program by the user.

21.7.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at each station along the length of the beam. The following steps are involved in designing the longitudinal and shear reinforcement for a particular station due to the beam torsion:

- Determine the factored torsion, T_f
- Determine special section properties
- Determine critical torsion capacity
- Determine the torsion reinforcement required

21.7.3.1 Determine Factored Torsion

In the design of beam torsion reinforcement, the torsions for each load combination at a particular beam station are obtained by factoring the corresponding torsions for different load cases, with the corresponding load combination factors.

In a statically indeterminate structure where redistribution of the torsion in a member can occur because of redistribution of internal forces upon cracking, the design T_f is permitted to be reduced in accordance with the code (CSA 11.2.9.2). However, the program does not automatically redistribute the internal forces and reduce T_f. If redistribution is desired, the user should release the torsional degree of freedom (DOF) in the structural model.

21.7.3.2 Determine Special Section Properties

For torsion design, special section properties, such as A_c, A_{oh}, A_o, p_c, and p_h are calculated. These properties are described in the following (CSA 2.3).

A_c = Area enclosed by outside perimeter of concrete cross-section

A_{oh} = Area enclosed by centerline of the outermost closed transverse torsional reinforcement

A_o = Gross area enclosed by shear flow path

p_c = Outside perimeter of concrete cross-section

p_h = Perimeter of centerline of outermost closed transverse torsional reinforcement

In calculating the section properties involving reinforcement, such as A_{oh}, A_o, and p_h, it is assumed that the distance between the centerline of the outermost closed stirrup and the outermost concrete surface is 50 millimeters. This is equivalent to a 38-mm clear cover and a 12-mm stirrup. For torsion design of flanged beam sections, it is assumed that placing torsion reinforcement in the flange area is inefficient. With this assumption, the flange is ignored for torsion reinforcement calculation. However, the flange is considered during T_{cr} calculation. With this assumption, the special properties for a rectangular beam section are given as follows:

$$A_c = bh \qquad \text{(CSA 11.2.9.1)}$$

$$A_{oh} = (b - 2c)(h - 2c) \qquad \text{(CSA 11.3.10.3)}$$

$$A_o = 0.85\, A_{oh} \qquad \text{(CSA 11.3.10.3)}$$

$$p_c = 2b + 2h \qquad \text{(CSA 11.2.9.1)}$$

$$p_h = 2(b - 2c) + 2(h - 2c) \qquad \text{(CSA 11.3.10.4)}$$

where, the section dimensions b, h, and c are shown in Figure 21-3. Similarly, the special section properties for a flanged beam section are given as follows:

$$A_c = b_w h + (b_f - b_w)h_s \qquad \text{(CSA 11.2.9.1)}$$

$$A_{oh} = (b_w - 2c)(h - 2c) \qquad \text{(CSA 11.3.10.3)}$$

$$A_o = 0.85\, A_{oh} \qquad \text{(CSA 11.3.10.3)}$$

$$p_c = 2b_f + 2h \qquad \text{(CSA 11.2.9.1)}$$

$$p_h = 2(h - 2c) + 2(b_w - 2c) \qquad \text{(CSA 11.3.10.4)}$$

where the section dimensions b_f, b_w, h, h_f, and c for a flanged beam are shown in Figure 21-3. Note that the flange width on either side of the beam web is limited to the smaller of $6h_s$ or 1/12 the span length (CSA 10.3.4).

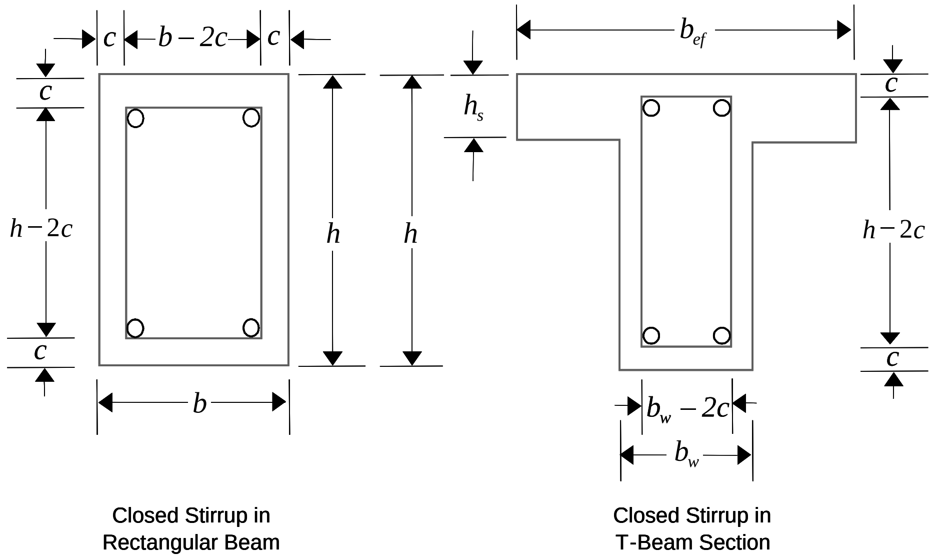


Figure 21-3 Closed stirrup and section dimensions for torsion design

21.7.3.3 Determine Critical Torsion Capacity

The critical torsion capacity, T_{cr}, for which the torsion in the section can be ignored is calculated as:

$$T_{cr} = \frac{0.38\lambda\phi_c\sqrt{f'_c}\left(\dfrac{A_c^2}{p_c}\right)}{4} \qquad \text{(CSA 11.2.9.1)}$$

where A_{cp} and p_c are the area and perimeter of the concrete cross-section as described in the previous section; λ is a factor to account for low-density concrete; ϕ_c is the strength reduction factor for concrete, which is equal to 0.65; and f'_c is the specified concrete compressive strength.

21.7.3.4 Determine Torsion Reinforcement

If the factored torsion T_f is less than the threshold limit, T_{cr}, torsion can be safely ignored (CSA 11.2.9.1). In that case, the program reports that no torsion reinforcement is required. However, if T_f exceeds the threshold limit, T_{cr}, it is assumed that the torsional resistance is provided by closed stirrups and longitudinal bars (CSA 11.3).

- If $T_f > T_{cr}$, the required closed stirrup area per unit spacing, A_t /s, is calculated as:

$$\frac{A_t}{s} = \frac{T_f \tan\theta}{\phi_s 2A_o f_{yt}} \qquad \text{(CSA 11.3.10.3)}$$

and the required longitudinal reinforcement is calculated as:

$$A_l = \frac{\dfrac{M_f}{d_v} + 0.5N_f + \sqrt{\left(V_f - 0.5V_s\right)^2 + \left(\dfrac{0.45 p_h T_f}{2A_o}\right)^2} \cot\theta}{\phi_s f_y}$$

(CSA 11.3.10.6, 11.3.9)

In the preceding expressions, θ is computed as previously described for shear, except that if the general method is being used, the value ε_x is calculated as specified in CSA 11.3.6.4 is replaced by:

$$\varepsilon_x = \frac{\dfrac{M_f}{d_v} + \sqrt{V_f^{\,2} + \left(\dfrac{0.9 p_h T_f}{2A_o}\right)^2} + 0.5N_f}{2\left(E_s A_s\right)} \qquad \text{(CSA 11.3.10.5)}$$

An upper limit of the combination of V_u and T_u that can be carried by the section also is checked using the equation:

$$\sqrt{\left(\frac{V_f}{b_w d_v}\right)^2 + \left(\frac{T_f p_h}{1.7A_{oh}^2}\right)^2} \le 0.25\phi_c f'_c \qquad \text{(CSA 11.3.10.4(b))}$$

For rectangular sections, b_w is replaced with b. If the combination of V_f and T_f exceeds this limit, a failure message is declared. In that case, the concrete section should be increased in size.

When torsional reinforcement is required ($T_f > T_{cr}$), the area of transverse closed stirrups and the area of regular shear stirrups must satisfy the following limit.

$$\left(\frac{A_v}{s}+2\frac{A_t}{s}\right)\geq 0.06\sqrt{f'_c}\,\frac{b_w}{f_{yt}} \qquad \text{(CSA 11.2.8.2)}$$

If this equation is not satisfied with the originally calculated A_v/s and A_t/s, A_v/s is increased to satisfy this condition.

The maximum of all of the calculated A_l and A_t/s values obtained from each load combination is reported along with the controlling combination.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements or longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

21.8 Slab Design

Similar to conventional design, the ETABS slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is carried out based on the ultimate strength design method (CSA A 23.3-04) for prestressed reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "ETABS Design Techniques" in the *Key Features and Terminology* manual.

21.8.1 Design for Flexure

ETABS designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is carried out at specific locations along the length of the strip. These locations correspond to the element

boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps are described in the subsections that follow and are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

21.8.1.1 Determine Factored Moments for the Strip

For each element within the design strip, for each load combination, the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

21.8.1.2 Design Flexural Reinforcement for the Strip

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the flanged beam if the slab is ribbed). In some cases, at a given design section in a design strip, there may be two or more slab properties across the width of the design strip. In that case, the program automatically designs the tributary width associated with each of the slab properties separately using its tributary bending moment. The reinforcement obtained for each of the tributary widths is summed to obtain the total reinforcement for the full width of the design strip at the considered design section. This method is used when drop panels are included. Where openings occur, the slab width is adjusted accordingly.

21.8.1.3 Minimum and Maximum Slab Reinforcement

If the computed tensile stress in the concrete immediately after prestress transfer exceeds $0.25\lambda\sqrt{f'_{ci}}$ (CSA 18.3.1.1), the bonded reinforcement with a minimum area of A_s is provided in the tensile zone to resist the total tensile force, N_c, in the concrete computed on the basis of an uncracked section (CSA 18.3.1.3).

$$A_s = N_c /(0.5 f_y) \qquad \text{(CSA 18.3.1.3)}$$

The minimum flexural tension reinforcement required for each direction of a slab is given by the following limit (CSA 18.8.1, 18.8.2):

	Concrete stress (see Clause 18.3.2(c))			
	Tensile stress $\leq 0.5\lambda\sqrt{f'_c}$		Tensile stress $> 0.5\lambda\sqrt{f'_c}$	
	Type of tendon		Type of tendon	
Type of member	Bonded	Unbonded	Bonded	Unbonded
Beams	0	0.004*A*	0.003*A*	0.005*A*
One-way slabs	0	0.003*A*	0.002*A*	0.004*A*
Two-way slabs				
Negative moment regions	0	$0.0006hl_n$	$0.00045hl_n$	$0.00075hl_n$
Positive moment regions, concrete stress $> 0.2\lambda\sqrt{f'_c}$	0	0.004*A*	0.003*A*	0.005*A*
Positive moment regions, concrete tensile stress $\leq 0.2\lambda\sqrt{f'_c}$	0	0	--	--

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

21.8.2 Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Key Features and Terminology* manual. Only the code specific items are described in the following sections.

21.8.2.1 Critical Section for Punching Shear

The punching shear is checked on a critical section at a distance of *d*/*2* from the face of the support (CSA 13.3.3.1 and CSA 13.3.3.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CSA 13.3.3.3). Figure 21-4 shows the auto punching perimeters considered by ETABS for the various column shapes. The column location (i.e., interior, edge, corner) and the punching perimeter may be overwritten using the Punching Check Overwrites.

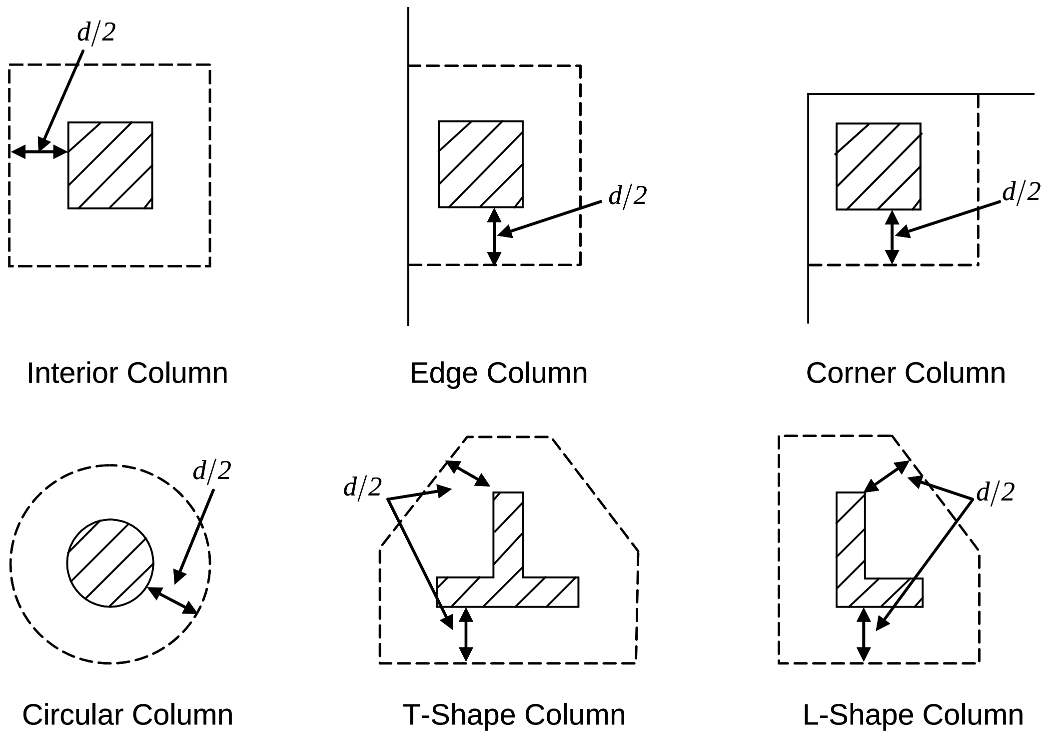


Figure 21-4 Punching Shear Perimeters

21.8.2.2 Transfer of Unbalanced Moment

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$, where

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \text{ and} \tag{CSA 13.10.2}$$

$$\gamma_v = 1 - \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \tag{CSA 13.3.5.3}$$

where b_1 is the width of the critical section measured in the direction of the span and b_2 is the width of the critical section measured in the direction perpendicular to the span.

21.8.2.3 Determine Concrete Capacity

The concrete punching shear factored strength is taken as the minimum of the following three limits:

$$v_c = \min \begin{cases} \phi_c \left(1 + \dfrac{2}{\beta_c}\right) 0.19 \lambda \sqrt{f'_c} \\ \phi_c \left(0.19 + \dfrac{\alpha_s d}{b_0}\right) \lambda \sqrt{f'_c} \\ \phi_c 0.38 \lambda \sqrt{f'_c} \end{cases} \qquad \text{(CSA 13.3.4.1)}$$

where, β_c is the ratio of the minimum to the maximum dimensions of the critical section, b_0 is the perimeter of the critical section, and α_s is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 4 & \text{for interior columns,} \\ 3 & \text{for edge columns, and} \\ 2 & \text{for corner columns} \end{cases} \qquad \text{(CSA 13.3.4.1(b))}$$

The value of $\sqrt{f'_c}$ is limited to 8 MPa for the calculation of the concrete shear capacity (CSA 13.3.4.2)

If the effective depth, d, exceeds 300 mm, the value of v_c is reduced by a factor equal to 1300/(1000 + d) (CSA 13.3.4.3).

21.8.2.4 Determine Capacity Ratio

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported as the punching shear capacity ratio by ETABS.

21.8.3 Design Punching Shear Reinforcement

The use of shear studs as shear reinforcement in slabs is permitted, provided that the effective depth of the slab is greater than or equal to 120 mm (CSA 13.2.1).

The algorithm for designing the required punching shear reinforcement is used when the punching shear capacity ratio exceeds unity. The *Critical Section for Punching Shear* and *Transfer of Unbalanced Moment* as described in the earlier sections remain unchanged. The design of punching shear reinforcement is carried out as follows.

21.8.3.1 Determine Concrete Shear Capacity

The concrete punching shear stress capacity of a two-way prestressed section with punching shear reinforcement is:

$$v_c = 0.28\lambda\phi_c\sqrt{f'_c} \quad \text{for shear studs} \qquad \text{(CSA 13.3.8.3)}$$

$$v_c = 0.19\lambda\phi_c\sqrt{f'_c} \quad \text{for shear stirrups} \qquad \text{(CSA 13.3.9.3)}$$

21.8.3.2 Determine Required Shear Reinforcement

The shear force is limited to a maximum of $v_{r,\max}$, where

$$v_{r,\max} = 0.75\lambda\phi_c\sqrt{f'_c} \quad \text{for shear studs} \qquad \text{(CSA 13.3.8.2)}$$

$$v_{r,\max} = 0.55\lambda\phi_c\sqrt{f'_c} \quad \text{for shear stirrups} \qquad \text{(CSA 13.3.9.2)}$$

Given v_f, v_c, and $v_{f,,\max}$, the required shear reinforcement is calculated as follows, where, ϕ_s, is the strength reduction factor.

- If $v_f > v_{r,\max}$,

$$\frac{A_v}{s} = \frac{\left(v_f - v_c\right)}{\phi_s f_{yv}} b_o \qquad \text{(CSA 13.3.8.5, 13.3.9.4)}$$

- If $v_f > v_{r,\max}$, (CSA 13.3.8.2)

a failure condition is declared.

- If V_f exceeds the maximum permitted value of $V_{r,max}$, the concrete section should be increased in size.

21.8.3.3 Determine the Reinforcement Arrangement

Punching shear reinforcement in the vicinity of rectangular columns should be arranged on peripheral lines, i.e., lines running parallel to and at constant distances from the sides of the column. Figure 21-5 shows a typical arrangement of shear reinforcement in the vicinity of a rectangular interior, edge, and corner columns.

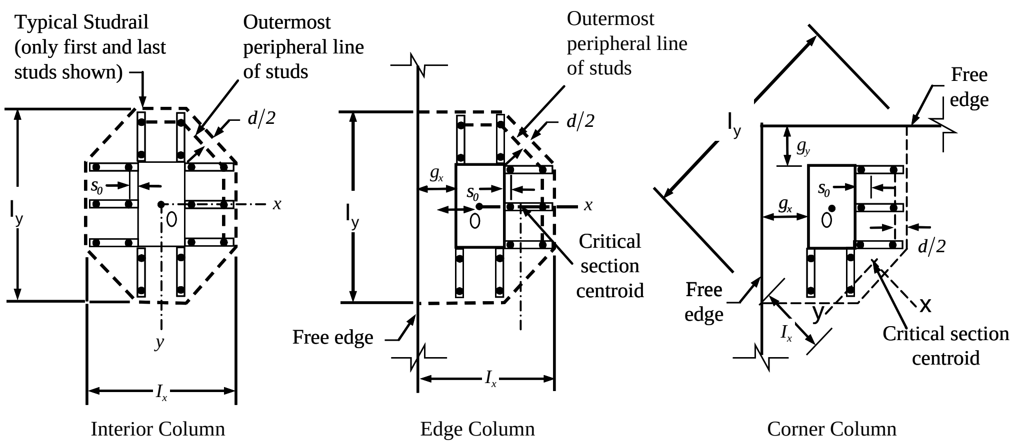


Figure 21-5 Typical arrangement of shear studs and critical sections outside shear-reinforced zone

The distance between the column face and the first line of shear reinforcement shall not exceed 0.4*d*. The spacing between adjacent shear reinforcement in the first line of shear reinforcement shall not exceed 2*d* measured in a direction parallel to the column face.

Punching shear reinforcement is most effective near column corners where there are concentrations of shear stress. Therefore, the minimum number of lines of shear reinforcement is 4, 6, and 8, for corner, edge, and interior columns respectively.

21.8.3.4 Determine Reinforcement Diameter, Height, and Spacing

The punching shear reinforcement is most effective when the anchorage is close to the top and bottom surfaces of the slab. The cover of anchors should not be less than the minimum cover specified in CSA 7.9 plus one half of the diameter of the flexural reinforcement.

Punching shear reinforcement in the form of shear studs is generally available in 9.5-, 12.7-, 15.9-, and 19.1-millimeter diameters.

When specifying shear studs, the distance, s_o, between the column face and the first peripheral line of shear studs should not be smaller than 0.4*d*. The limits of the spacing, *s*, between the peripheral lines are specified as:

$$s_o \leq 0.4d \qquad \text{(CSA 13.3.8.6)}$$

$$s \leq \begin{cases} 0.75d & v_f \leq 0.56\lambda\phi_c\sqrt{f'_c} \\ 0.50d & v_f > 0.56\lambda\phi_c\sqrt{f'_c} \end{cases} \qquad \text{(CSA 13.3.8.6)}$$

For shear stirrups,

$$s_o \leq 0.25d \qquad \text{(CSA 13.3.9.5)}$$

$$s \leq 0.25d \qquad \text{(CSA 13.3.9.5)}$$

The minimum depth for reinforcement should be limited to 300 mm (CSA 13.3.9.1).

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