Factoring Quadratic Trinomials

Student Probe

Factor $x^2 - 3x - 10$.

Answer: $(x-5)(x+2)$

Lesson Description

This lesson uses the area model of multiplication to factor quadratic trinomials. Part 1 of the lesson consists of "circle puzzles" as a preparation for the actual factoring lesson in Part 2. While the goal of the lesson is for students to be able to factor quadratic trinomials without devices such as circle puzzles or rectangles, it may take some time before they develop fluency when factoring. Part 3 of the lesson uses the graphing calculator to help students relate the factors of a quadratic trinomial to its *x*-intercepts.

Rationale

Factoring is a valuable tool which helps students become aware of the connections between the roots of a quadratic function and its intercepts. While many, in fact most, polynomial functions are prime over the rational numbers, students can gain valuable insight into function behavior by analyzing functions in this manner.

When students are required to solve quadratic equations, factoring will be one of the methods used.

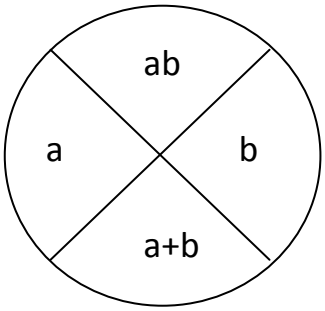
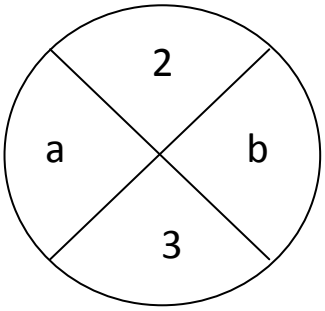
At a Glance

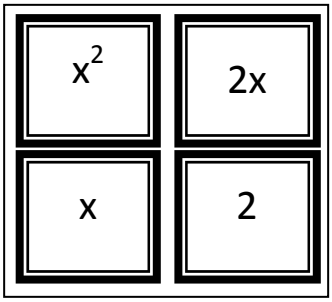
What: Factor quadratic trinomials
Common Core State Standard: CC.9-12.A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.
Matched Arkansas Standard. CC.9-12.A.SSE.3a. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
(a) Factor a quadratic expression to reveal the zeros of the function it defines.
Mathematical Practices:
Look for and make use of structure.
Look for and express regularity in repeated reasoning.
Who: Students who cannot factor quadratic trinomials.
Grade Level: Algebra 1
Prerequisite Vocabulary: factor, quadratic, trinomial
Prerequisite Skills: polynomial arithmetic, factoring whole numbers, distributive property
Delivery Format: individual, small group, whole group
Lesson Length: 30 minutes per day, over several days
Materials, Resources, Technology: graphing calculator
Student Worksheets: none

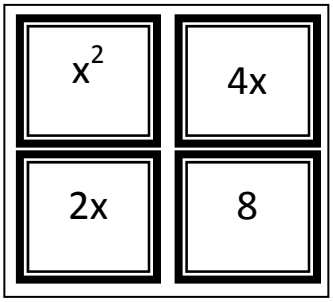
Preparation

For Part 1 of the lesson, prepare several circle puzzles for students to solve.
For Part 2, prepare several rectangles for students to use to write the areas as products.
Examples of each of these appear at the end of the lesson.
For Part 3, provide a graphing calculator for each student.

Lesson

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
Part 1		
1. We are going to solve a "circle puzzle". Circle puzzles have this format:  Notice that the product of two numbers, a and b, is in the top section of the puzzle and the sum of the two numbers is in the lower section. If any two sections of the puzzle are filled in, it is possible to determine the remaining two sections.		What does product mean? What does sum mean?
2. Complete this circle puzzle.  	a = 2 and b = 1 or a = 1 and b= 2	What two numbers have a product of 2? Which of those number pairs have a sum of 3?
3. Repeat circle puzzles with a variety of number combinations. (See Circle Puzzles at the end of the lesson.)		

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
Part 2		
4. Write the area of the rectangle as a sum.  	$x^2+2x+x+2$	What is the area of each part of the rectangle?
5. Can we combine any like terms? What do we get?	x^2+3x+2	Refer to Addition and Subtraction of Polynomials.
6. What are the dimensions of the large rectangle?	$(x+1)$ and $(x+2)$	Refer to Multiplying Polynomial Expressions.
7. What is its area as a product?	$(x+1)(x+2)$	How do we find the area of a rectangle?
8. Since the area of the rectangle is the same regardless of how we write it, we can say $x^2+3x+2=(x+1)(x+2)$. Multiply $(x+1)(x+2)$ to verify our results.	$(x+1)(x+2)=x^2+x+2x+2$ $=x^2+3x+2$	Refer to Multiplying Polynomial Expressions.
9. Writing an expression as a product is called factoring.		
10. Repeat Steps 4-8 with a variety of rectangles. (See Rectangles at the end of the lesson.)		

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
11. Factor this polynomial by writing the sum as a product: $x^2+4x+2x+8$ You may use the rectangle to help you.	 $(x+4)(x+2)$	Model for students.
12. Since 4x and 2x are like terms, this polynomial can be written as x^2+6x+8. How do we know?	$4x+2x=6x$	Refer to Addition and Subtraction of Polynomials.
13. When we did the Circle Puzzles, we looked for numbers with a certain product and a certain sum. What numbers have a product of 8 and a sum of 6?	4 and 2	What numbers have a product of 8? (1 and 8 or 2 and 4) Which of these have a sum of 6?
14. So we can say $x^2+6x+8=(x+4)(x+2)$.		
15. Repeat Steps 11-14 with a variety of quadratic trinomials, moving from writing the trinomial with 4 terms to writing it with 3 terms.		
Part 3		
16. Now that we know how to factor a quadratic trinomial, let's see what the graph can tell us.		

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
17. Using your graphing calculator, graph $y = x^2 + 6x + 8 = (x+4)(x+2)$. What do you notice about the graph? (See Teacher Notes.)	Answers will vary, but listen for "the *x*-intercepts are -4 and -2 and the *y*-intercept is 8".	Model for students. It may take several examples before students see the relationship.
18. Repeat Step 17 with additional trinomials until students make the connection between the factors and the x-intercepts.		
19. Can someone summarize what we have discovered?	The *x*-intercepts are the opposites of the numbers in the factors. The *y*-intercept is the constant term in the trinomial.	
20. Use your graphing calculator to graph $y = x^2 + 2x + 2$. What do you notice?	Answers may vary, but listen for, "there are no *x*-intercepts".	Where does the graph cross the *x*-axis?
21. What do you think this means about its factors?	There are not any factors.	Not all trinomials have real factors.

Teacher Notes:

1. It is suggested that this lesson be taught over a number of days. Students seldom develop fluency in factoring quickly.
2. It is recommended that teachers use the "circle puzzles" in part one as warm-up activities for several days prior to the actual factoring lesson. These puzzles will help students think about numbers in ways that will help them in the actual factoring lesson.
3. Use a variety of circle puzzles, including positive and negative numbers.
4. Help students remember the format of circle puzzles by placing the diagram in Step 1 where students can refer to it.
5. When solving circle puzzles, it may be helpful for students to list all of the factor pairs for a number.
6. Once students can solve circle puzzles, move to writing areas as sums and as products in Part 2.
7. Remind students that the word factoring means to write an expression as a product.
8. It may take several examples of graphing quadratic trinomials before students see the relationship between the factors and the x-intercepts. Be patient and let the students "discover" this relationship.

Variations

Algebra tiles may be used in place of, or in addition to, the rectangles.

Formative Assessment

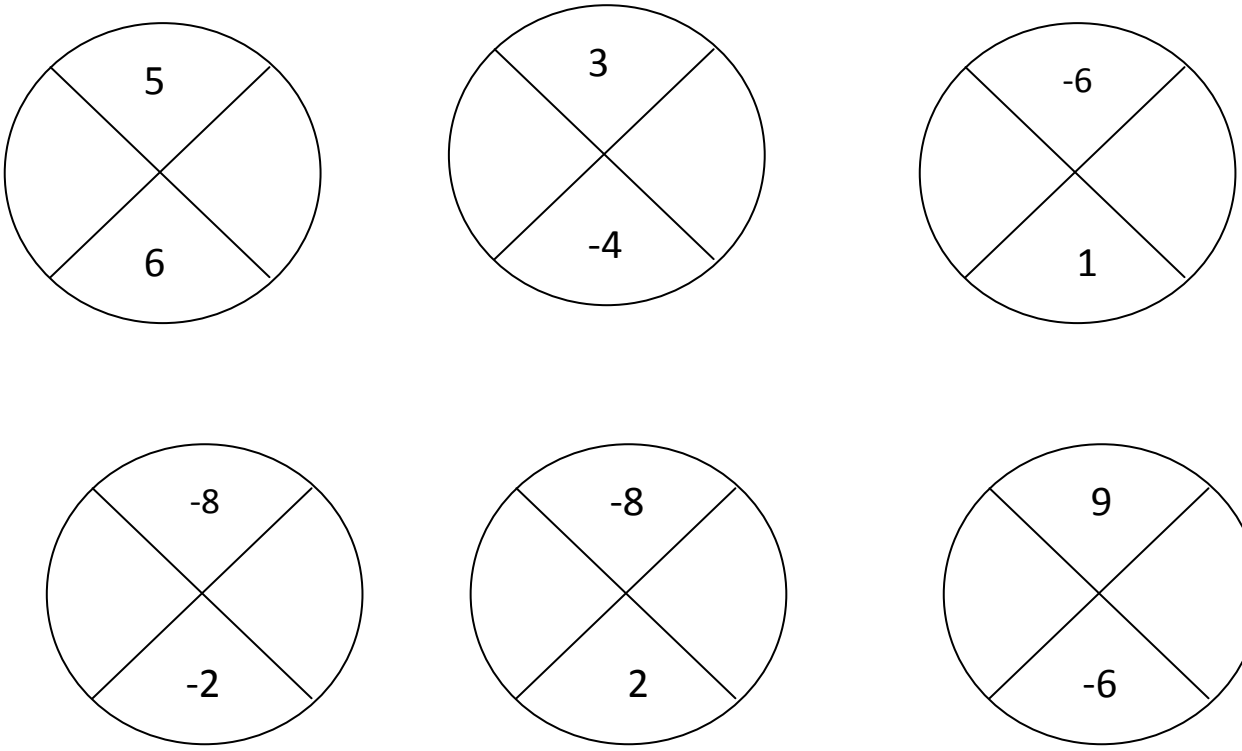
Factor $x^2 - 5x - 36$.

Answer: $(x-9)(x+4)$

References

Russell Gersten, P. (n.d.). *RTI and Mathematics IES Practice Guide - Response to Intervention in Mathematics*. Retrieved 2 25, 2011, from rti4sucess.

Circle Puzzles



Rectangles