Solutions Manual to Accompany

Fundamentals of Microelectronics, 1st Edition

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2.1 (a)

$$k = 8.617 \times 10^{-5} \text{ eV/K}$$

$$n_i(T = 300 \text{ K}) = 1.66 \times 10^{15} (300 \text{ K})^{3/2} \exp\left[-\frac{0.66 \text{ eV}}{2\,(8.617 \times 10^{-5} \text{ eV/K})\,(300 \text{ K})}\right] \text{ cm}^{-3}$$

$$= \boxed{2.465 \times 10^{13} \text{ cm}^{-3}}$$

$$n_i(T = 600 \text{ K}) = 1.66 \times 10^{15} (600 \text{ K})^{3/2} \exp\left[-\frac{0.66 \text{ eV}}{2\,(8.617 \times 10^{-5} \text{ eV/K})\,(600 \text{ K})}\right] \text{ cm}^{-3}$$

$$= \boxed{4.124 \times 10^{16} \text{ cm}^{-3}}$$

Compared to the values obtained in Example 2.1, we can see that the intrinsic carrier concentration in Ge at $T = 300$ K is $\frac{2.465\times10^{13}}{1.08\times10^{10}} = 2282$ times higher than the intrinsic carrier concentration in Si at $T = 300$ K. Similarly, at $T = 600$ K, the intrinsic carrier concentration in Ge is $\frac{4.124\times10^{16}}{1.54\times10^{15}} =$ 26.8 times higher than that in Si.

(b) Since phosphorus is a Group V element, it is a donor, meaning $N_D = 5 \times 10^{16}$ cm^{-3}. For an n-type material, we have:

$$n = N_D = \boxed{5 \times 10^{16} \text{ cm}^{-3}}$$

$$p(T = 300 \text{ K}) = \frac{[n_i(T = 300 \text{ K})]^2}{n} = \boxed{1.215 \times 10^{10} \text{ cm}^{-3}}$$

$$p(T = 600 \text{ K}) = \frac{[n_i(T = 600 \text{ K})]^2}{n} = \boxed{3.401 \times 10^{16} \text{ cm}^{-3}}$$

2. (a) Mobility of electrons in Si = 1350 $cm^2/V\text{-}s$
Mobility of holes in Si = 480 $cm^2/V\text{-}s$

$\Rightarrow$ velocity of electrons $= \mu_n E = \left(1350 \frac{cm^2}{V\text{-}s}\right)\left(0.1 \frac{V}{\mu m}\right)$

$= 1.35 \cdot 10^4$ m/s

velocity of holes $= \mu_p E = \left(480 \frac{cm^2}{V\text{-}s}\right)\left(0.1 \frac{V}{\mu m}\right)$

$= 4.8 \cdot 10^3$ m/s

(b) Given $E = 0.1\ V/\mu m$ hole current negligible
$\mu_n = 1350\ cm^2/V\text{-}s$ $\mu_p = 480\ cm^2/V\text{-}s$

$$J_{tot} = 1\ mA/\mu m^2 = q[\mu_n n E + \mu_p p E] \approx q \mu_n n E$$

$$\therefore n = \frac{J_{tot}}{q \mu_n E} = \frac{1\ mA/\mu m^2}{(1.6 \cdot 10^{-19} C)(1350\ cm^2/V\text{-}s)(0.1\ V/\mu m)}$$

$$= 4.6 \cdot 10^{17}\ cm^{-3}$$

2.3 (a) Since the doping is uniform, we have no diffusion current. Thus, the total current is due only to the drift component.

$$\begin{aligned} I_{tot} &= I_{drift} \\ &= q(n\mu_n + p\mu_p)AE \\ n &= 10^{17}\ \text{cm}^{-3} \\ p &= n_i^2/n = (1.08 \times 10^{10})^2/10^{17} = 1.17 \times 10^3\ \text{cm}^{-3} \\ \mu_n &= 1350\ \text{cm}^2/\text{V} \cdot \text{s} \\ \mu_p &= 480\ \text{cm}^2/\text{V} \cdot \text{s} \\ E &= V/d = \frac{1\ \text{V}}{0.1\ \mu\text{m}} \\ &= 10^5\ \text{V/cm} \\ A &= 0.05\ \mu\text{m} \times 0.05\ \mu\text{m} \\ &= 2.5 \times 10^{-11}\ \text{cm}^2 \end{aligned}$$

Since $n\mu_n \gg p\mu_p$, we can write

$$\begin{aligned} I_{tot} &\approx qn\mu_n AE \\ &= \boxed{54.1\ \mu\text{A}} \end{aligned}$$

(b) All of the parameters are the same except n_i, which means we must re-calculate p.

$$\begin{aligned} n_i(T = 400\ \text{K}) &= 3.657 \times 10^{12}\ \text{cm}^{-3} \\ p = n_i^2/n &= 1.337 \times 10^8\ \text{cm}^{-3} \end{aligned}$$

Since $n\mu_n \gg p\mu_p$ still holds (note that n is 9 orders of magnitude larger than p), the hole concentration once again drops out of the equation and we have

$$\begin{aligned} I_{tot} &\approx qn\mu_n AE \\ &= \boxed{54.1\ \mu\text{A}} \end{aligned}$$

2.4 (a) From Problem 1, we can calculate n_i for Ge.

$$\begin{aligned}
n_i(T = 300\text{ K}) &= 2.465 \times 10^{13}\text{ cm}^{-3} \\
I_{tot} &= q(n\mu_n + p\mu_p)AE \\
n &= 10^{17}\text{ cm}^{-3} \\
p &= n_i^2/n = 6.076 \times 10^{9}\text{ cm}^{-3} \\
\mu_n &= 3900\text{ cm}^2/\text{V}\cdot\text{s} \\
\mu_p &= 1900\text{ cm}^2/\text{V}\cdot\text{s} \\
E &= V/d = \frac{1\text{ V}}{0.1\ \mu\text{m}} \\
&= 10^5\text{ V/cm} \\
A &= 0.05\ \mu\text{m} \times 0.05\ \mu\text{m} \\
&= 2.5 \times 10^{-11}\text{ cm}^2
\end{aligned}$$

Since $n\mu_n \gg p\mu_p$, we can write

$$\begin{aligned}
I_{tot} &\approx qn\mu_n AE \\
&= \boxed{156\ \mu\text{A}}
\end{aligned}$$

(b) All of the parameters are the same except n_i, which means we must re-calculate p.

$$\begin{aligned}
n_i(T = 400\text{ K}) &= 9.230 \times 10^{14}\text{ cm}^{-3} \\
p &= n_i^2/n = 8.520 \times 10^{12}\text{ cm}^{-3}
\end{aligned}$$

Since $n\mu_n \gg p\mu_p$ still holds (note that n is 5 orders of magnitude larger than p), the hole concentration once again drops out of the equation and we have

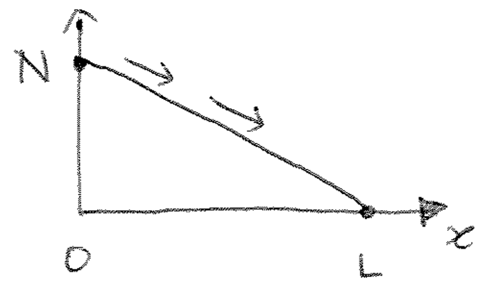
$$\begin{aligned}
I_{tot} &\approx qn\mu_n AE \\
&= \boxed{156\ \mu\text{A}}
\end{aligned}$$

2.5 Since there's no electric field, the current is due entirely to diffusion. If we define the current as positive when flowing in the positive x direction, we can write

$$
\begin{aligned}
I_{tot} &= I_{diff} = AJ_{diff} = Aq\left[D_n \frac{dn}{dx} - D_p \frac{dp}{dx}\right] \\
A &= 1\ \mu\text{m} \times 1\ \mu\text{m} = 10^{-8}\ \text{cm}^2 \\
D_n &= 34\ \text{cm}^2/\text{s} \\
D_p &= 12\ \text{cm}^2/\text{s} \\
\frac{dn}{dx} &= -\frac{5 \times 10^{16}\ \text{cm}^{-3}}{2 \times 10^{-4}\ \text{cm}} = -2.5 \times 10^{20}\ \text{cm}^{-4} \\
\frac{dp}{dx} &= \frac{2 \times 10^{16}\ \text{cm}^{-3}}{2 \times 10^{-4}\ \text{cm}} = 10^{20}\ \text{cm}^{-4} \\
I_{tot} &= \left(10^{-8}\ \text{cm}^2\right)\left(1.602 \times 10^{-19}\ \text{C}\right)\left[\left(34\ \text{cm}^2/\text{s}\right)\left(-2.5 \times 10^{20}\ \text{cm}^{-4}\right) - \left(12\ \text{cm}^2/\text{s}\right)\left(10^{20}\ \text{cm}^{-4}\right)\right] \\
&= \boxed{-15.54\ \mu\text{A}}
\end{aligned}
$$

6. Given Area = a

find total e ectrons stored.



$$n(x) = -\frac{N}{L}x + N$$

∴ total electrons stored

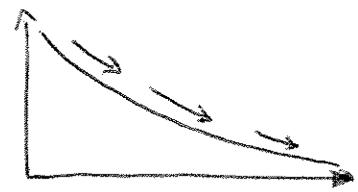
$$= \int a \cdot n(x)\, dx = \int_0^L a\left(-\frac{N}{L}x + N\right)dx$$

$$= aN\left(-\frac{x^2}{2L} + x\right)\Big|_0^L = \frac{aNL}{2}$$

7. Given Area = a

find total electrons stored.

$n(x) = N \cdot \exp\left(\frac{-x}{L_d}\right)$



$\therefore$ total electrons stored

$$= \int a\, n(x)\, dx = \int_0^\infty a \cdot N \cdot \exp\left(\frac{-x}{L_d}\right) dx$$

$$= aN \left(-L_d \cdot \exp \frac{-x}{L_d}\right)\Bigg|_0^\infty = aNL_d.$$

For the linear profile, the result depends on the length, L.

For the exponential profile, the result is constant (since L_d is constant.)

2.8 Assume the diffusion lengths L_n and L_p are associated with the electrons and holes, respectively, in this material and that $L_n, L_p \ll 2$ µm. We can express the electron and hole concentrations as functions of x as follows:

$$
\begin{aligned}
n(x) &= Ne^{-x/L_n} \\
p(x) &= Pe^{(x-2)/L_p} \\
\text{\# of electrons} &= \int_0^2 an(x)dx \\
&= \int_0^2 aNe^{-x/L_n}dx \\
&= -aNL_n \left(e^{-x/L_n}\right)\Big|_0^2 \\
&= -aNL_n \left(e^{-2/L_n} - 1\right) \\
\text{\# of holes} &= \int_0^2 ap(x)dx \\
&= \int_0^2 aPe^{(x-2)/L_p}dx \\
&= aPL_p \left(e^{(x-2)/L_p}\right)\Big|_0^2 \\
&= aPL_p \left(1 - e^{-2/L_p}\right)
\end{aligned}
$$

Due to our assumption that $L_n, L_p \ll 2$ µm, we can write

$$
\begin{aligned}
e^{-2/L_n} &\approx 0 \\
e^{-2/L_p} &\approx 0 \\
\text{\# of electrons} &\approx \boxed{aNL_n} \\
\text{\# of holes} &\approx \boxed{aPL_p}
\end{aligned}
$$

9. Drift is analogous to water flow in a river.

Water flows from top of mountain to bottom because of gravitational field; electron flows from one terminal to the other because of electric field.

DRIFT		WATER FLOW
electrons	$\longleftrightarrow$	water
electric field	$\longleftrightarrow$	gravitational field.
drift / current	$\longleftrightarrow$	water flow

2.10 (a)

$$n_n = N_D = \boxed{5 \times 10^{17}\ \text{cm}^{-3}}$$
$$p_n = n_i^2/n_n = \boxed{233\ \text{cm}^{-3}}$$
$$p_p = N_A = \boxed{4 \times 10^{16}\ \text{cm}^{-3}}$$
$$n_p = n_i^2/p_p = \boxed{2916\ \text{cm}^{-3}}$$

(b) We can express the formula for V_0 in its full form, showing its temperature dependence:

$$V_0(T) = \frac{kT}{q} \ln\left[\frac{N_A N_D}{\left(5.2 \times 10^{15}\right)^2 T^3 e^{-E_g/kT}}\right]$$
$$V_0(T = 250\ \text{K}) = \boxed{906\ \text{mV}}$$
$$V_0(T = 300\ \text{K}) = \boxed{849\ \text{mV}}$$
$$V_0(T = 350\ \text{K}) = \boxed{789\ \text{mV}}$$

Looking at the expression for $V_0(T)$, we can expand it as follows:

$$V_0(T) = \frac{kT}{q}\left[\ln(N_A) + \ln(N_D) - 2\ln\left(5.2 \times 10^{15}\right) - 3\ln(T) + E_g/kT\right]$$

Let's take the derivative of this expression to get a better idea of how V_0 varies with temperature.

$$\frac{dV_0(T)}{dT} = \frac{k}{q}\left[\ln(N_A) + \ln(N_D) - 2\ln\left(5.2 \times 10^{15}\right) - 3\ln(T) - 3\right]$$

From this expression, we can see that if $\ln(N_A) + \ln(N_D) < 2\ln\left(5.2 \times 10^{15}\right) + 3\ln(T) + 3$, or equivalently, if $\ln(N_A N_D) < \ln\left[\left(5.2 \times 10^{15}\right)^2 T^3\right] - 3$, then V_0 will decrease with temperature, which we observe in this case. In order for this not to be true (i.e., in order for V_0 to increase with temperature), we must have either very high doping concentrations or very low temperatures.

2.11 Since the p-type side of the junction is undoped, its electron and hole concentrations are equal to the intrinsic carrier concentration.

$$n_n = N_D = 3 \times 10^{16}\ \text{cm}^{-3}$$
$$p_p = n_i = 1.08 \times 10^{10}\ \text{cm}^{-3}$$
$$V_0 = V_T \ln\left(\frac{N_D n_i}{n_i^2}\right)$$
$$= (26\ \text{mV}) \ln\left(\frac{N_D}{n_i}\right)$$
$$= \boxed{386\ \text{mV}}$$

2.12 (a)

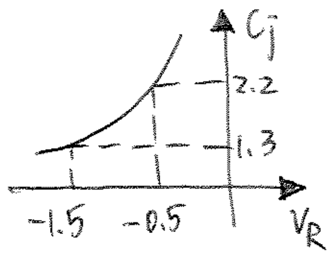
$$\begin{aligned}
C_{j0} &= \sqrt{\frac{q\epsilon_{\text{Si}}}{2}\frac{N_A N_D}{N_A+N_D}\frac{1}{V_0}} \\
C_j &= \frac{C_{j0}}{\sqrt{1-V_R/V_0}} \\
N_A &= 2\times 10^{15}\ \text{cm}^{-3} \\
N_D &= 3\times 10^{16}\ \text{cm}^{-3} \\
V_R &= -1.6\ \text{V} \\
V_0 &= V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 701\ \text{mV} \\
C_{j0} &= 14.9\ \text{nF/cm}^2 \\
C_j &= 8.22\ \text{nF/cm}^2 \\
&= \boxed{0.082\ \text{fF/cm}^2}
\end{aligned}$$

(b) Let's write an equation for C_j' in terms of C_j assuming that C_j' has an acceptor doping of N_A'.

$$\begin{aligned}
C_j' &= 2C_j \\
\sqrt{\frac{q\epsilon_{\text{Si}}}{2}\frac{N_A' N_D}{N_A'+N_D}\frac{1}{V_T\ln(N_A'N_D/n_i^2)-V_R}} &= 2C_j \\
\frac{q\epsilon_{\text{Si}}}{2}\frac{N_A' N_D}{N_A'+N_D}\frac{1}{V_T\ln(N_A'N_D/n_i^2)-V_R} &= 4C_j^2 \\
q\epsilon_{\text{Si}}N_A'N_D &= 8C_j^2(N_A'+N_D)(V_T\ln(N_A'N_D/n_i^2)-V_R) \\
N_A'\left[q\epsilon_{\text{Si}}N_D - 8C_j^2(V_T\ln(N_A'N_D/n_i^2)-V_R)\right] &= 8C_j^2 N_D(V_T\ln(N_A'N_D/n_i^2)-V_R) \\
N_A' &= \frac{8C_j^2 N_D(V_T\ln(N_A'N_D/n_i^2)-V_R)}{q\epsilon_{\text{Si}}N_D - 8C_j^2(V_T\ln(N_A'N_D/n_i^2)-V_R)}
\end{aligned}$$

We can solve this by iteration (you could use a numerical solver if you have one available). Starting with an initial guess of $N_A' = 2\times 10^{15}$ cm^{-3}, we plug this into the right hand side and solve to find a new value of $N_A' = 9.9976\times 10^{15}$ cm^{-3}. Iterating twice more, the solution converges to $N_A' = 1.025\times 10^{16}$ cm^{-3}. Thus, we must increase the N_A by a factor of $N_A'/N_A = 5.125 \approx \boxed{5}$.

13.



$$\frac{C_{j0}}{\sqrt{1+\frac{0.5}{V_0}}} = 2.2 \quad \text{——} \quad ①$$

$$\frac{C_{j0}}{\sqrt{1+\frac{1.5}{V_0}}} = 1.3 \quad \text{——} \quad ②$$

① ÷ ② : $\frac{1+1.5/V_0}{1+0.5/V_0} = \left(\frac{2.2}{1.3}\right)^2 \Rightarrow V_0 = 0.0365\ V$

Substitute V_0 into ①:

$$C_{j0} = 2.2\sqrt{1+\frac{0.5}{V_0}} \approx 8.43\ fF/\mu m^2$$

$$\Rightarrow \frac{N_A N_D}{N_A+N_D} = (C_{j0})^2 \cdot V_0 \cdot \frac{2}{\varepsilon_{si} q}$$

$$= \left(8.43\ \frac{fF}{\mu m^2}\right)^2 \times (0.0365V) \cdot \frac{2}{\varepsilon_{si} q} \approx 3.13 \cdot 10^{17}\ cm^{-3}$$

Fix a value for $N_A > \frac{N_A N_D}{N_A + N_D} \hat{=} y$

$$N_A = 2 \cdot 10^{18}\,cm^{-3} \Rightarrow N_D = \frac{y N_A}{N_A - y}$$

$$= \frac{(3.13 \cdot 10^{17}\,cm^{-3})(2 \cdot 10^{18}\,cm^{-3})}{(2 \cdot 10^{18} - 3.13 \cdot 10^{17})\,cm^{-3}}$$

$$\approx 3.71 \cdot 10^{17}\,cm^{-3}$$

14 (a) In forward bias, $I_D = 1mA$, $V_D = 750mV$

$$\therefore I_S \approx I_D e^{-\frac{V_D}{V_T}} = (1mA)\exp[-750mV/26mV]$$

$$= 2.97 \cdot 10^{-16} A$$

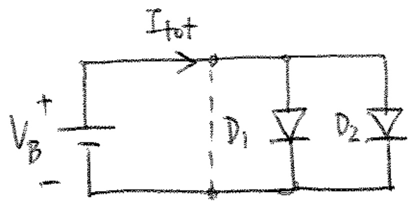
(b) Since $I_S \propto$ Area, doubling area implies doubling I_S. From (a),

$$I_D = 1\ mA = 2 \times I_S e^{V_D/V_T}$$

$$\therefore V_D = V_T \ln\left(\frac{I_D}{2I_S}\right) = (26mV)\ln\left(\frac{1mA}{2 \cdot 2.97 \times 10^{-16} A}\right)$$

$$= 0.732\ V$$

15 (a)



$$I_{tot} = I_{D_1} + I_{D_2} = I_{S_1}(e^{V_B/V_T}-1) + I_{S_2}(e^{V_B/V_T}-1)$$

$$= (I_{S_1} + I_{S_2})(e^{V_B/V_T}-1)$$

Therefore, the parallel combination operates as an exponential device, with an equivalent saturation current of $I_{S_1} + I_{S_2}$.

(b) By KVL, $V_{D_1} = V_{D_2}$

$$\Rightarrow V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{D_2}}{I_{S_2}}\right)$$

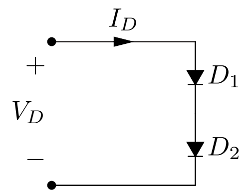
Also, $I_{tot} = I_{D_1} + I_{D_2} \Rightarrow I_{D_2} = I_{tot} - I_{D_1}$

$$\therefore V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{tot} - I_{D_1}}{I_{S_2}}\right)$$

$$\Rightarrow I_{D_1} = I_{tot}\left(\frac{I_{S_1}}{I_{S_1} + I_{S_2}}\right)$$

$$\Rightarrow I_{D_2} = I_{tot}\left(\frac{I_{S_2}}{I_{S_1} + I_{S_2}}\right)$$

2.16 (a) The following figure shows the series diodes.



Let V_{D1} be the voltage drop across D_1 and V_{D2} be the voltage drop across D_2. Let $I_{S1} = I_{S2} = I_S$, since the diodes are identical.

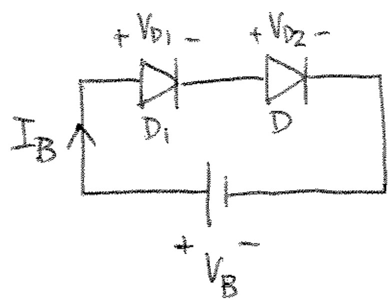
$$\begin{aligned} V_D &= V_{D1} + V_{D2} \\ &= V_T \ln\left(\frac{I_D}{I_S}\right) + V_T \ln\left(\frac{I_D}{I_S}\right) \\ &= 2V_T \ln\left(\frac{I_D}{I_S}\right) \\ I_D &= I_S e^{V_D/2V_T} \end{aligned}$$

Thus, the diodes in series act like a single device with an exponential characteristic described by $I_D = I_S e^{V_D/2V_T}$.

(b) Let V_D be the amount of voltage required to get a current I_D and V_D' the amount of voltage required to get a current $10I_D$.

$$\begin{aligned} V_D &= 2V_T \ln\left(\frac{I_D}{I_S}\right) \\ V_D' &= 2V_T \ln\left(\frac{10I_D}{I_S}\right) \\ V_D' - V_D &= 2V_T \left[\ln\left(\frac{10I_D}{I_S}\right) - \ln\left(\frac{I_D}{I_S}\right)\right] \\ &= 2V_T \ln(10) \\ &= \boxed{120 \text{ mV}} \end{aligned}$$

17.



Find I_B, V_{D_1}, V_{D2} in terms of V_B, $I_{,}$, I_{S2}

By KVL, $V_B = V_{D_1} + V_{D2} = V_T \ln\left(\frac{I_B}{I_{S_1}}\right) + V_T \ln\left(\frac{I_B}{I_{S2}}\right)$

$$\Rightarrow \quad V_B = V_T \ln\left(\frac{I_B^2}{I_{S_1} I_{S2}}\right)$$

$$\therefore \; I_B = \sqrt{I_{S_1} I_{S2} \cdot \exp V_B/V_T} = \sqrt{I_{S_1} I_{S2}} \cdot \exp\left(\frac{V_B}{2V_T}\right).$$

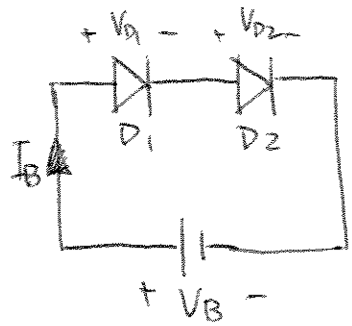
$$V_{D_1} = V_T \ln\left(\frac{I_B}{I_{S_1}}\right) = V_T \ln\left(\frac{\sqrt{I_{S_1} I_{S2}}}{I_{S_1}} \cdot \exp \frac{V_B}{2V_T}\right)$$

$$= V_T \ln\sqrt{\frac{I_{S2}}{I_{S_1}}} + \frac{V_B}{2}$$

$$V_{D2} = V_T \ln\left(\frac{I_B}{I_{S2}}\right) = V_T \ln\left(\frac{\sqrt{I_{S_1} I_{S2}}}{I_{S2}} \cdot \exp \frac{V_B}{2V_T}\right)$$

$$= V_T \ln\sqrt{\frac{I_{S1}}{I_{S2}}} + \frac{V_B}{2}$$

18.



$$V_B = V_T \ln \frac{I_B}{I_{S1}} + V_T \ln \frac{I_B}{I_{S2}} = V_T \ln\left(\frac{I_B^2}{I_{S1} I_{S2}}\right)$$

$$\Rightarrow I_B = \sqrt{I_{S1} I_{S2}} \cdot \exp \frac{V_B}{2V_T}$$

Increase I_B by 10 times:

$$I_{B,new} = 10 I_B$$

$$\Rightarrow V_{B,new} = V_T \ln\left(\frac{I_{B,new}^2}{I_{S1} I_{S2}}\right) = V_T \ln\left[\frac{(10 I_B)^2}{I_{S1} I_{S2}}\right]$$

$$= V_T \ln\left(\frac{I_B^2}{I_{S1} I_{S2}}\right) + V_T \ln 100$$

$$= V_B + V_T \ln 100 \approx V_B + 0.120\,V$$

$\therefore$ V_B increases by 0.120 V.

2.19

$$
\begin{aligned}
V_X &= I_X R_1 + V_{D1} \\
&= I_X R_1 + V_T \ln\left(\frac{I_X}{I_S}\right) \\
I_X &= \frac{V_X}{R_1} - \frac{V_T}{R_1} \ln\left(\frac{I_X}{I_S}\right)
\end{aligned}
$$

For each value of V_X, we can solve this equation for I_X by iteration. Doing so, we find

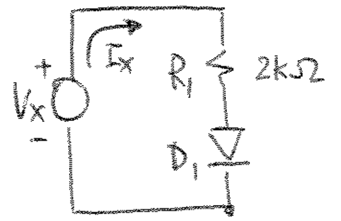
$$
\begin{aligned}
I_X(V_X = 0.5\ \text{V}) &= 0.435\ \mu\text{A} \\
I_X(V_X = 0.8\ \text{V}) &= 82.3\ \mu\text{A} \\
I_X(V_X = 1\ \text{V}) &= 173\ \mu\text{A} \\
I_X(V_X = 1.2\ \text{V}) &= 267\ \mu\text{A}
\end{aligned}
$$

Once we have I_X, we can compute V_D via the equation $V_D = V_T \ln(I_X/I_S)$. Doing so, we find

$$
\begin{aligned}
V_D(V_X = 0.5\ \text{V}) &= \boxed{499\ \text{mV}} \\
V_D(V_X = 0.8\ \text{V}) &= \boxed{635\ \text{mV}} \\
V_D(V_X = 1\ \text{V}) &= \boxed{655\ \text{mV}} \\
V_D(V_X = 1.2\ \text{V}) &= \boxed{666\ \text{mV}}
\end{aligned}
$$

As expected, V_D varies very little despite rather large changes in I_D (in particular, as I_D experiences an increase by a factor of over 3, V_D changes by about 5 %). This is due to the exponential behavior of the diode. As a result, a diode can allow very large currents to flow once it turns on, up until it begins to overheat.

20.



Since $I_{S_1} \propto$ Area, I_{D_1} becomes:

$$I_{D_1} = \underbrace{10 \times (2 \cdot 10^{-15} A)}_{I_{S_1}'} \left(e^{\frac{V_{D1}}{V_T}} - 1\right)$$

$V_X = 0.8\ V$ Suppose D_1 is on. Assume $V_{D_1} = 0.7V$

$$V_{D_1} = 0.7\ V \Rightarrow I_X = \frac{V_X - V_{D_1}}{R_1} = \frac{0.1\ V}{2\ k\Omega} = 0.05\ mA$$

$$\Rightarrow V_{D_1} = V_T \ln(I_X / I_{S_1}') = (0.026 V) \ln\left(\frac{0.05 mA}{20 \cdot 10^{-15} A}\right)$$

$$= 0.563\ V$$

$$V_{D_1} = 0.563V \Rightarrow I_X = \frac{(0.8 - 0.563)\ V}{2\ k\Omega} = 0.12\ mA$$

$$\Rightarrow V_{D_1} = (0.026\ V) \ln\left(\frac{0.12\ mA}{20 \cdot 10^{-15} A}\right) \approx 0.585\ V$$

$$V_{D_1} = 0.585 V \Rightarrow I_X = \frac{(0.8 - 0.585) V}{2\ k\Omega} = 0.11\ mA$$

$$\Rightarrow V_{D_1} = (0.026\ V) \ln\left(\frac{0.11 mA}{20 \cdot 10^{-15} A}\right) \approx 0.583 V$$

$$V_{D_1} = 0.583\ V \Rightarrow I_X = \frac{(0.8 - 0.583) V}{2\ k\Omega} = 0.11\ mA$$

$$\Rightarrow V_{D_1} = 0.583\ V$$

$\therefore$ $V_{D_1} \approx 0.583\ V$

$I_X \approx 0.11\ mA.$

$\boxed{V_X = 1.2\,V}$ Suppose D_1 is on. Use results from previous calculations as starting point.

$$V_{D_1} = 0.583\,V \Rightarrow I_X = \frac{(1.2 - 0.583)V}{2\,k\Omega} = 0.31\,mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.31mA}{20\times10^{-15}A}\right) \approx 0.610\,V$$

$$V_{D_1} = 0.610\,V \Rightarrow I_X = \frac{(1.2 - 0.610)V}{2\,k\Omega} = 0.30\,mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.30\,mA}{20\cdot10^{-15}A}\right) \approx 0.609\,V$$

$$V_{D_1} = 0.609\,V \Rightarrow I_X = \frac{(1.2 - 0.609)V}{2\,k\Omega} = 0.30\,mA$$

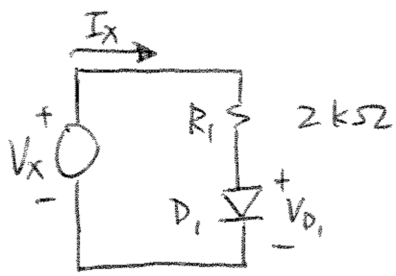
$$\Rightarrow V_{D_1} = 0.609\,V$$

$\therefore$ $V_{D_1} \approx 0.609\,V$

$I_X \approx 0.30\,mA.$

By increasing the cross-section area of D_1, intuitively this means D_1 can conduct same amount of current with less V_{D_1}. The results have shown that in this problem, V_{D_1} is less and I_X is more.

21.



Given: @ $V_x = 2V$, $V_{D_1} = 850mV$

$$\Rightarrow I_x = \frac{V_x - V_{D_1}}{R_1} = 0.58\ mA$$

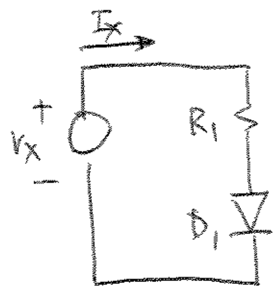
$$\therefore I_s = \frac{I_x}{(e^{V_D/V_T} - 1)} \approx I_x \exp[-V_{D}/V_T]$$

$$= (0.58\ mA) \exp[-0.85/0.026] \approx 3.64 \cdot 10^{-18} A$$

2.22

$$
\begin{aligned}
V_X/2 &= I_X R_1 = V_{D1} = V_T \ln(I_X/I_S) \\
I_X &= \frac{V_T}{R_1} \ln(I_X/I_S) \\
I_X &= 367\ \mu\text{A (using iteration)} \\
V_X &= 2 I_X R_1 \\
&= \boxed{1.47\ \text{V}}
\end{aligned}
$$

23.



Given $V_X = 1V \Rightarrow I_X = 0.2mA$

$V_X = 2V \Rightarrow I_X = 0.5mA$

Find R_1 and I_S.

By KVL, $V_{D_1} = V_X - I_X R_1 = V_T \ln\left(\frac{I_X}{I_S}\right)$

$$\Rightarrow 1 - (0.2mA)R_1 = (0.026V)\ln\left(\frac{0.2mA}{I_S}\right) \quad \text{—— ①}$$

$$2 - (0.5mA)R_1 = (0.026V)\ln\left(\frac{0.5mA}{I_S}\right) \quad \text{—— ②}$$

② − ① : $1 - (0.3mA)R_1 = (0.026V)\ln\left(\frac{0.5}{0.2}\right)$

$$\Rightarrow R_1 = \frac{1-(0.026)\ \ V}{0.3m\ \ A} = 3.25\ k\Omega$$

Substitute R_1 into ①:

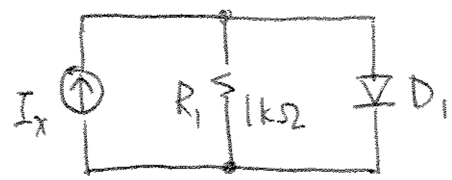
$$I_S = I_X \cdot \exp\left[-\frac{V_X - I_X R_1}{V_T}\right]$$

$$= (0.2mA)\exp\left[-\frac{1-(0.2m)(3.25k)}{0.026}\right] \approx 2.94 \cdot 10^{-10} A$$

$\therefore R_1 \approx 3.25\ k\Omega$

$I_S \approx 2.94 \cdot 10^{-10} A.$

24.



Given $I_S = 3\cdot10^{-16}$ A, find V_{D_1}.

By KCL, $I_X = \frac{V_{D_1}}{R_1} + I_{D_1} = \frac{V_T}{R_1}\ln\left(\frac{I_{D_1}}{I_S}\right) + I_{D_1}$

Since I_X, V_T, R_1 and I_S are known, this can be solved directly with special programs or graphing calculators. However, this can be also solved by iterations: Assume a V_{D_1}, calculate I_{D_1}, and re-iterate on V_{D_1}.

Assume $V_{D_1} = 0.7$ V as starting point.

$I_X = 1$ mA

$V_{D_1} = 0.7$ V $\Rightarrow I_{D_1} = I_X - V_{D_1}/R_1 = 1\text{mA} - \frac{0.7\text{V}}{1\text{k}\Omega} = 0.3\text{mA}$

$\Rightarrow V_{D_1} = V_T \ln\left(I_X/I_S\right)$

$= (0.026\text{V})\ln\left(\frac{0.3\text{mA}}{3\cdot10^{-16}\text{A}}\right) \approx 0.718$ V

$V_{D_1} = 0.718$ V $\Rightarrow I_{D_1} = 1\text{ mA} - \frac{0.718\text{ V}}{1\text{ k}\Omega} = 0.28\text{ mA}$

$\Rightarrow V_{D_1} = (0.026\text{V})\ln\left(\frac{0.28\text{mA}}{3\cdot10^{-16}\text{A}}\right) \approx 0.717$ V

$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 1mA - \dfrac{0.717V}{1\,k\Omega} = 0.28\,mA$

$\Rightarrow V_{D_1} = 0.717\,V$

$\therefore V_{D_1} \approx 0.717\,V.$

$\boxed{I_X = 2mA}$ Assume $V_{D_1} = 0.717\,V$ from previous result.

$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 2mA - \dfrac{0.717V}{1\,k\Omega} = 1.28\,mA$

$\Rightarrow V_{D_1} = (0.026V)\ln\left(\dfrac{1.28\,mA}{3\cdot 10^{-16}A}\right) \approx 0.756V$

$V_{D_1} = 0.756\,V \Rightarrow I_{D_1} = 2mA - \dfrac{0.756V}{1\,k\Omega} = 1.24\,mA$

$\Rightarrow V_{D_1} = (0.026\,V)\ln\left(\dfrac{1.24mA}{3\cdot 10^{-16}A}\right) \approx 0.755V$

$V_{D_1} = 0.755V \Rightarrow I_{D_1} = 2mA - \dfrac{0.755\,V}{1\,k\Omega} = 1.24\,mA$

$\Rightarrow V_{D_1} = 0.755V$

$\therefore V_{D_1} = 0.755\,V$

$\boxed{I_X = 4\ mA}$ Assume $V_{D_1} = 0.755V$ from previous result.

$$V_{D_1} = 0.755V \Rightarrow I_{D_1} = 4mA - \frac{0.755V}{1k\Omega} = 3.25\ mA$$

$$\Rightarrow V_{D_1} = (0.026)V \ln\left(\frac{3.25mA}{3 \cdot 10^{-16}A}\right) \approx 0.780V$$

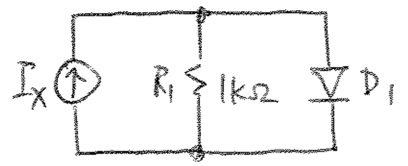
$$V_{D_1} = 0.780\ V \Rightarrow I_{D_1} = 4mA - \frac{0.780V}{1k\Omega} = 3.22\ mA$$

$$\Rightarrow V_{D_1} = (0.026\ V)\ln\left(\frac{3.22mA}{3 \cdot 10^{-16}A}\right) \approx 0.780V$$

$\therefore\ V_{D_1} \approx 0.780\ V.$

Note: As I_X increases, I_{D_1} increases, while (V_{D1}/R_1) stays relatively the same. Because of the exponential characteristic, the diode, once on, will <u>absorb</u> as much current as necessary to satisfy KCL.

25.



Given $I_{D_1} = 0.5mA$ when $I_X = 1.3mA$, find I_S.

This means $V_{D_1} = (I_X - I_{D_1})R_1$

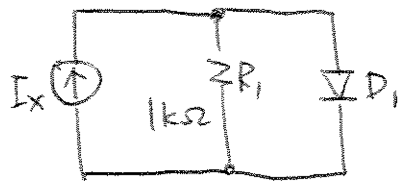
$= (0.8mA)1k\Omega = 0.8V$

$\Rightarrow I_S = I_{D_1} \cdot exp[-V_{D_1}/V_T]$

$= (0.5mA)\ exp[-0.8V/0.026V]$

$\approx 2.17 \cdot 10^{-17} A$

2b



Given $I_{R_1} = I_x/2$

$I_S = 3\cdot 10^{-16}\,A$

find I_x.

$$V_{D_1} = \frac{I_x}{2}\cdot R_1 = V_T \ln\left(\frac{I_x/2}{I_S}\right)$$

This can be solved directly with special programs or graphing calculators. Alternatively, one can solve this iteratively by hand.

Assume $V_D = 0.8\ V$.

$$V_D = 0.8\,V \Rightarrow (I_x/2) = \frac{V_D}{R_1} = \frac{0.8\,V}{1\,k\Omega} = 0.8\,mA$$

$$\Rightarrow V_D = V_T \ln\left(\frac{I_x/2}{I_S}\right) = (0.026V)\ln\left(\frac{0.8\,mA}{3\cdot 10^{-16}A}\right)$$

$$\approx 0.744\ V$$

$$V_D = 0.744\,V \Rightarrow I_x/2 = \frac{0.744V}{1k\Omega} = 0.744\ mA$$

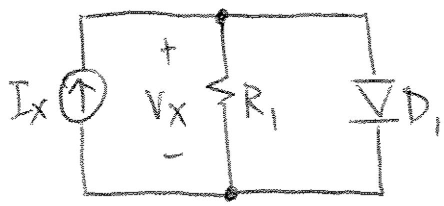
$$\Rightarrow V_D = (0.026V)\ln\left(\frac{0.744\,mA}{3\cdot 10^{-16}A}\right) \approx 0.742\ V$$

$$V_D = 0.742\text{V} \Rightarrow I_X/2 = \frac{0.742\text{V}}{1\text{k}\Omega} = 0.742 \text{ mA}$$

$$\Rightarrow V_D = (0.026\text{V}) \ln\left(\frac{0.742\text{mA}}{3 \cdot 10^{-16}\text{A}}\right) \approx 0.742\text{V}$$

$$\therefore I_X = 2(0.742\text{mA}) = 1.48 \text{ mA}$$

27.



Given $I_X = 1mA \rightarrow V_X = 1.2V$

$I_X = 2mA \rightarrow V_X = 1.8V$

find R_1 and I_S.

$$I_{D_1} = I_X - V_X/R_1 \quad \text{(KCL)}$$

By KVL, $V_X = V_T \ln\left(\frac{I_{D_1}}{I_S}\right) = V_T \ln\left(\frac{I_X - V_X/R_1}{I_S}\right)$

$$\Rightarrow (1.2\ V) = (0.026\ V)\ln\left[\frac{(1mA) - (1.2V)/R_1}{I_S}\right] \quad \text{——— ①}$$

$$(1.8\ V) = (0.026\ V)\ln\left[\frac{(2mA) - (1.8\ V)/R_1}{I_S}\right] \quad \text{——— ②}$$

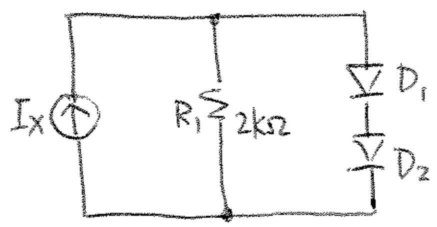
$$② - ①:\quad 0.6V = (0.026V)\ln\left(\frac{2mA - 1.8V/R_1}{1mA - 1.2V/R_1}\right)$$

$$\Rightarrow R_1 = \frac{1.2 \cdot \exp\left[0.6/0.026\right] - 1.8}{1\,mA \cdot \exp\left[0.6/0.026\right] - 2\,mA} \approx 1.2\ k\Omega$$

$$I_S = I_D \exp\left[-V_X/V_T\right] = \left(2mA - \frac{1.8V}{1.2k\Omega}\right)\exp\left[\frac{-1.8V}{0.026V}\right]$$

$$\approx 4.29 \cdot 10^{-34}\ A.$$

28.



Given $D_1 = D_2$ with $I_S = 5 \cdot 10^{-16} A$

Find V_{R_1} for $I_X = 2mA$.

Current through the diodes $= I_D$
$= I_X - \frac{V_{R_1}}{R_1}$ where V_{R_1} = voltage across R_1

$$\Rightarrow V_{R_1} = 2 \cdot V_T \ln\left(\frac{I_D}{I_S}\right) = 2\left[V_T \ln\left(\frac{I_X}{I_S} - \frac{V_{R_1}}{I_S R_1}\right)\right]$$

This can be solved directly with special programs or graphing calculators or by hand iteratively.

Assume a V_{R_1}, calculate I_D, and re-iterate on new $V_{R_1} = (2 \times V_{D_1})$. From experience, most diodes conduct at $V_D \approx 0.7\,V$. Assume $V_{R_1} = 1.4\,V$.

$$V_{R_1} = 1.4\,V \Rightarrow I_D = I_X - \frac{V_{R_1}}{R_1} = 2mA - \frac{1.4V}{2k\Omega} = 1.3mA$$

$$\Rightarrow V_{R_1} = 2 V_T \ln\left(\frac{I_D}{I_S}\right)$$

$$= 2(0.026V)\ln\left(\frac{1.3mA}{5 \times 10^{-16}A}\right) \approx 1.49\,V$$

$$V_{R_1} = 1.49\,V \Rightarrow I_D = 2\,mA - \frac{1.49}{2\,k\Omega} = 1.26\,mA$$

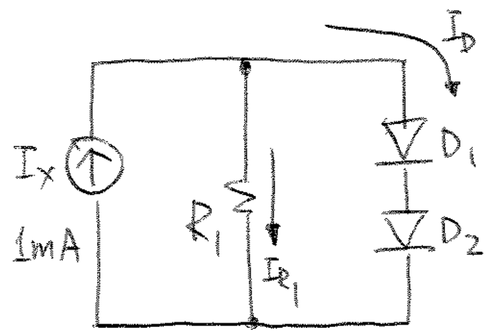
$$\Rightarrow V_{R_1} = 2\,(0.026\,V)\ln\left(\frac{1.26\,mA}{5\cdot 10^{-16}\,A}\right) \approx 1.48\,V$$

$$V_{R_1} = 1.48\,V \Rightarrow I_D = 2\,mA - \frac{1.48\,V}{2\,k\Omega} = 1.26\,mA$$

$$\Rightarrow V_{R_1} = 1.48\,V$$

$\therefore$ voltage across $R_1 = 1.48\,V$

29.



Given $I_{R_1} = 0.5\,mA$, $I_S = 5 \cdot 10^{-16}\,A$ for each diode.

Find R_1.

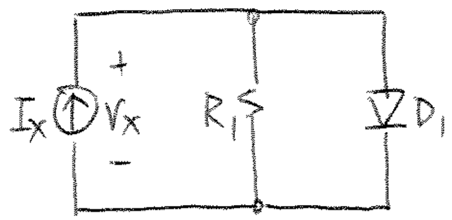
By KCL, $I_D = I_X - I_{R_1} = 0.5\,mA$

$$\Rightarrow V_{D_1} = V_{D_2} = V_T \ln\left(\frac{I_D}{I_S}\right) = 0.026 \ln\left(\frac{0.5mA}{5 \cdot 10^{-16}A}\right)$$

$$\approx 0.718\ V$$

$$\therefore R_1 = \frac{V_{R_1}}{I_{R_1}} = \frac{2V_{D_1}}{I_{R_1}} = \frac{2(0.718V)}{0.5mA} = 2.87\,k\Omega$$

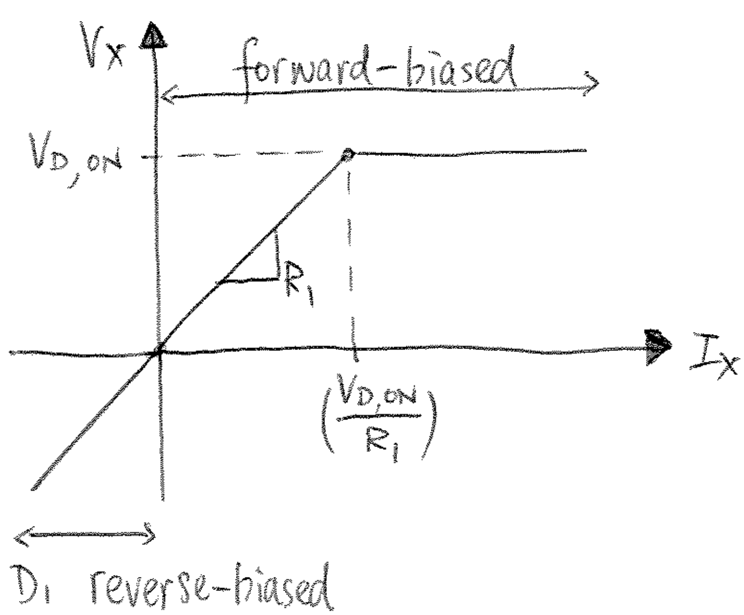
30.



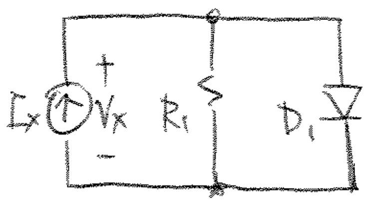
(a) Constant-voltage model:
Consider, first, the extreme cases: when D_1 is off, we have the following:

This implies V_X is linearly proportional to I_X

When D_1 is on, V_X is <u>fixed</u> (by KVL) by D_1 ($= V_{D,ON}$). This implies that any additional current from I_X cannot flow through R_1, which means D_1 will absorb all the currents to satisfy KVL.



(b) exponential model:



Assume I_S negligible.

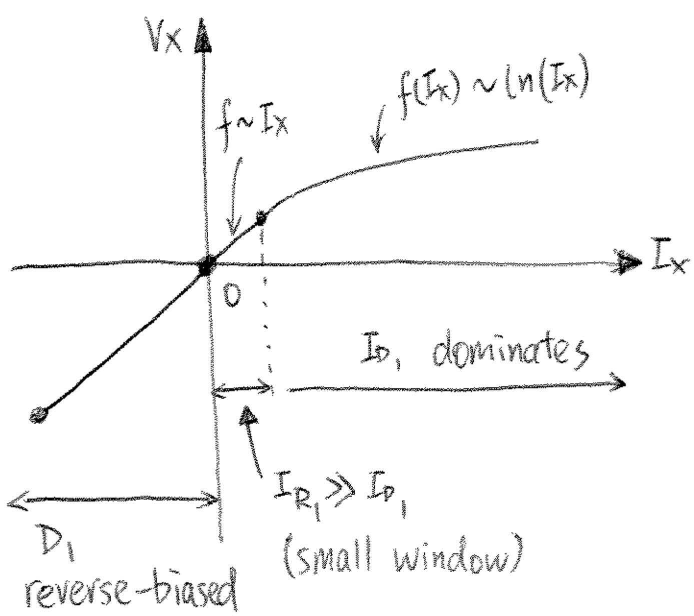
When D_1 is off, most of I_X flows through R_1. When D_1 is on, V_{D_1} ($= V_X$) follows this relationship:

$$V_{D_1} = V_X = V_T \ln\left(\frac{I_{D_1}}{I_S}\right) = V_T \ln\left(\frac{I_X}{I_S} - \frac{V_X}{I_X R_1}\right)$$

$$\Rightarrow I_X = I_S \exp(V_X/V_T) + V_X/R_1$$

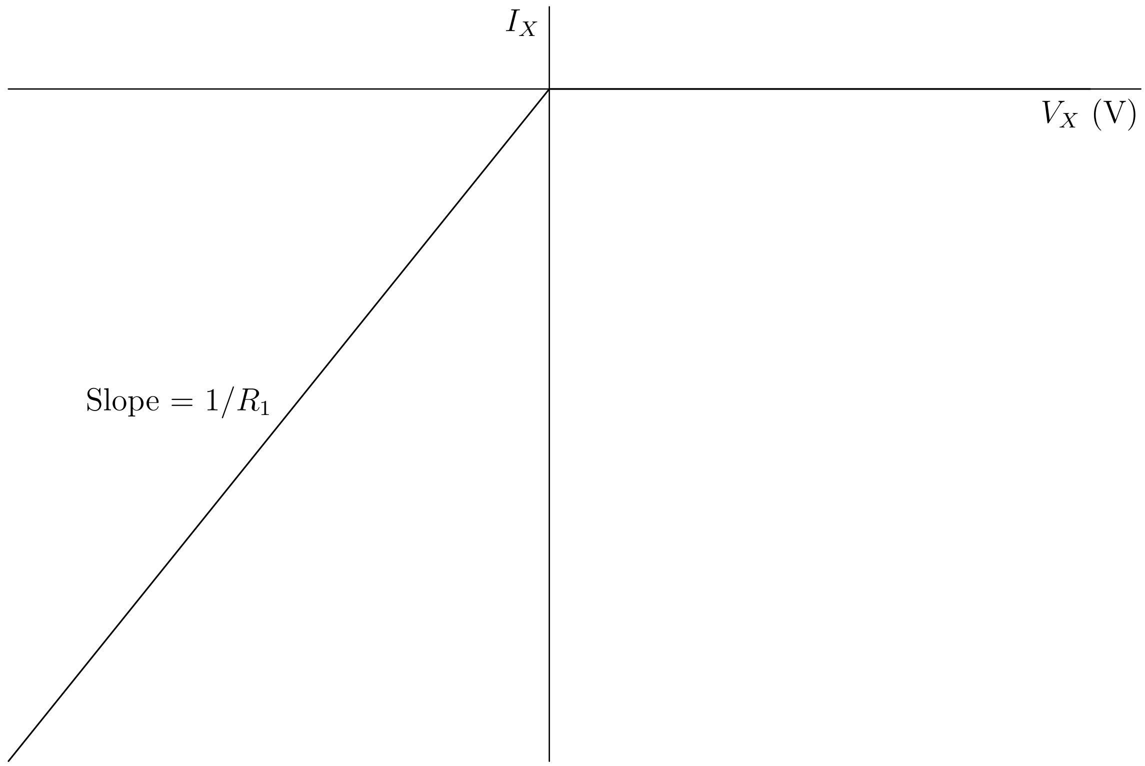
$\approx I_S \exp(V_X/V_T)$ when D_1 is forward-biased ($V_X > V_T$)

i.e. $V_X \approx V_T \ln(I_X/I_S)$



3.1 (a)

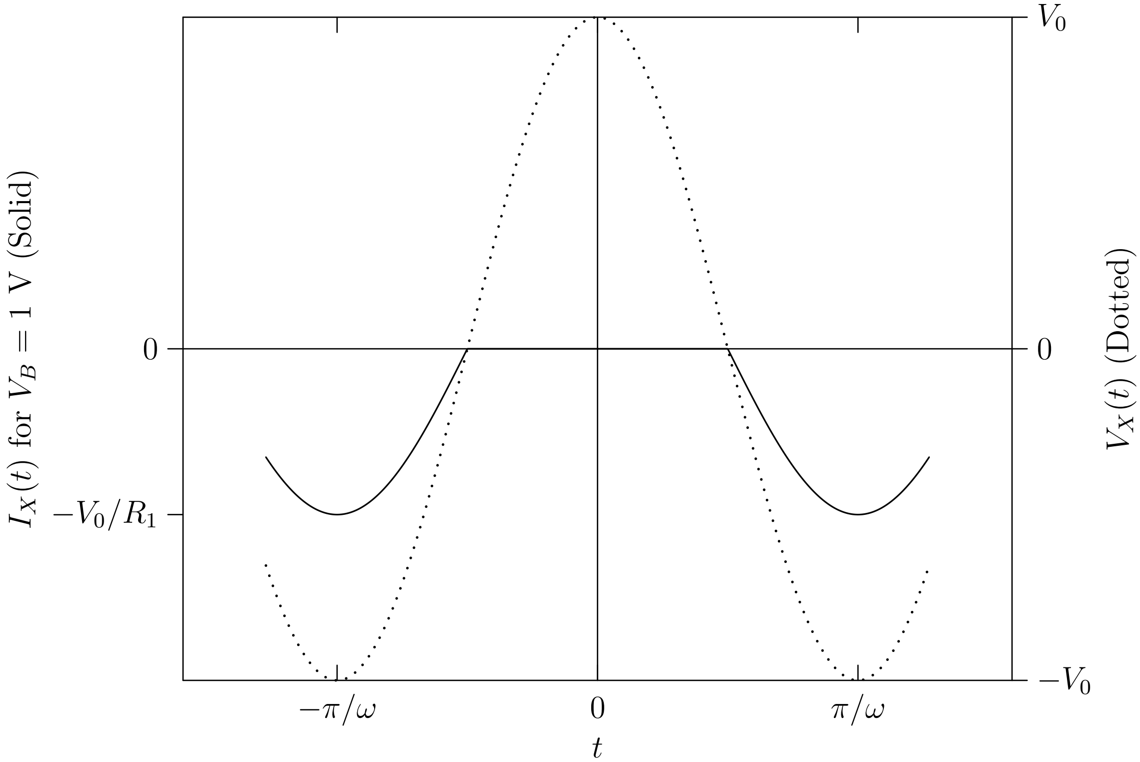
$$I_X = \begin{cases} \frac{V_X}{R_1} & V_X < 0 \\ 0 & V_X > 0 \end{cases}$$



3.2

$$I_X = \begin{cases} \frac{V_X}{R_1} & V_X < 0 \\ 0 & V_X > 0 \end{cases}$$

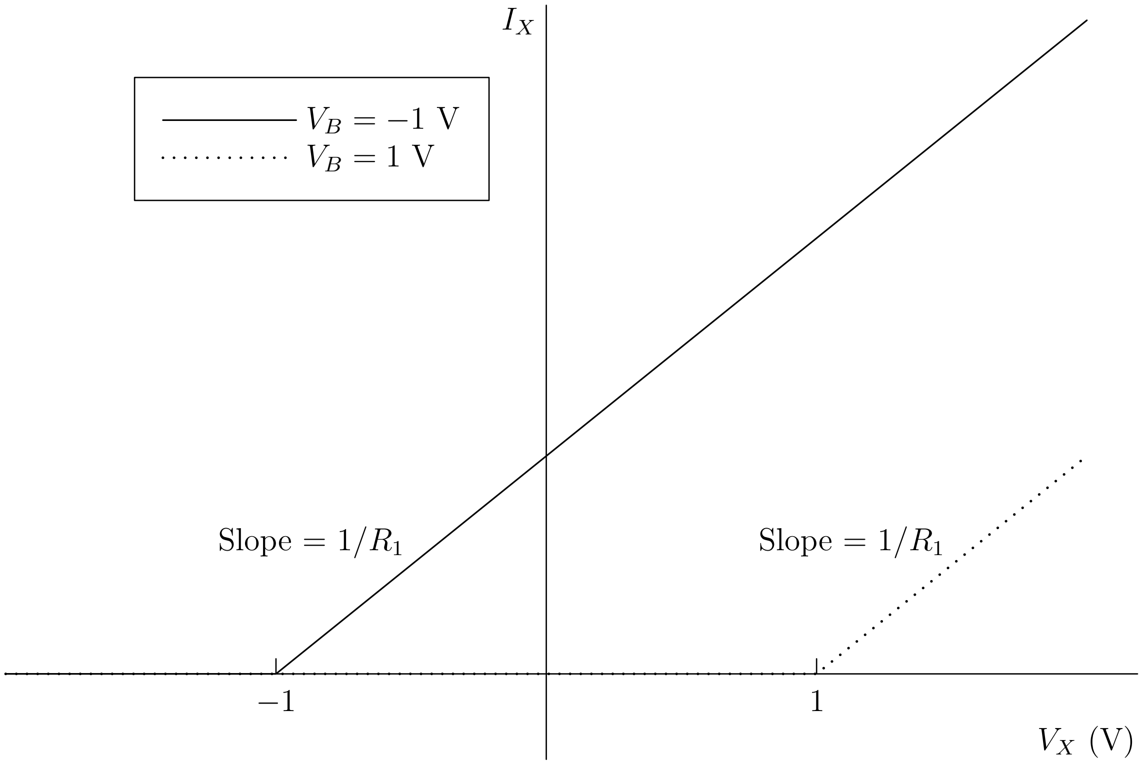
Plotting $I_X(t)$, we have



3.3

$$I_X = \begin{cases} 0 & V_X < V_B \\ \frac{V_X - V_B}{R_1} & V_X > V_B \end{cases}$$

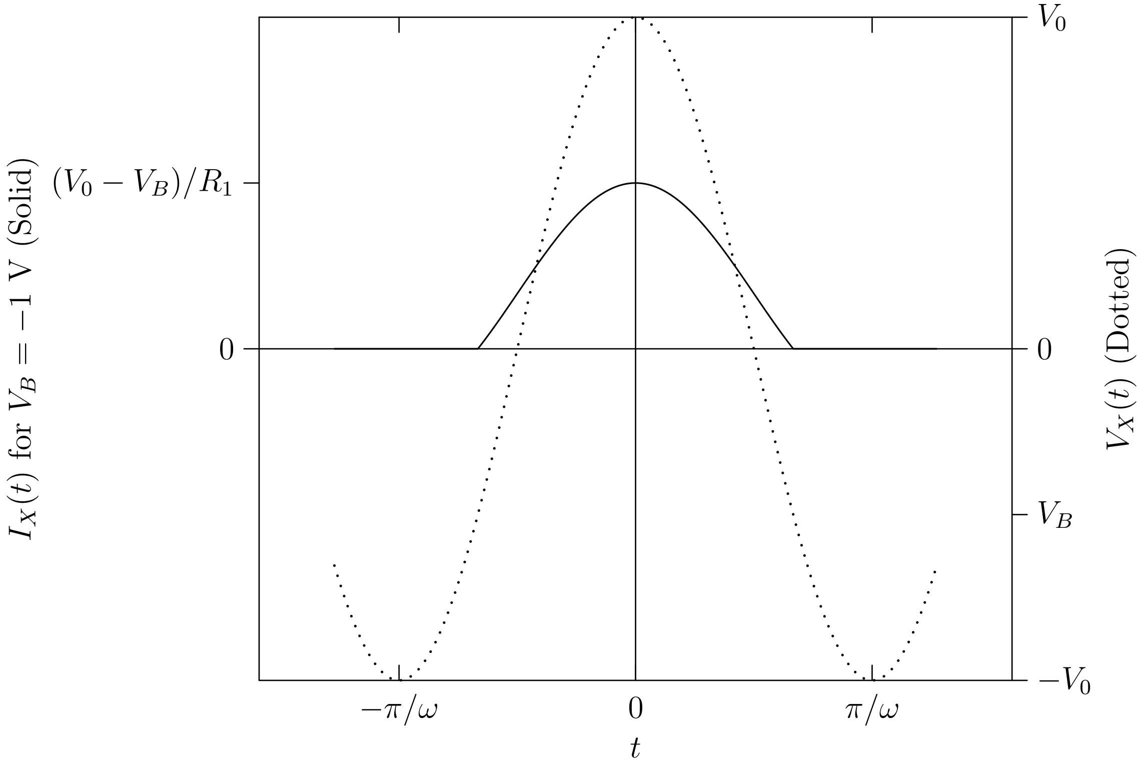
Plotting I_X vs. V_X for $V_B = -1$ V and $V_B = 1$ V, we get:



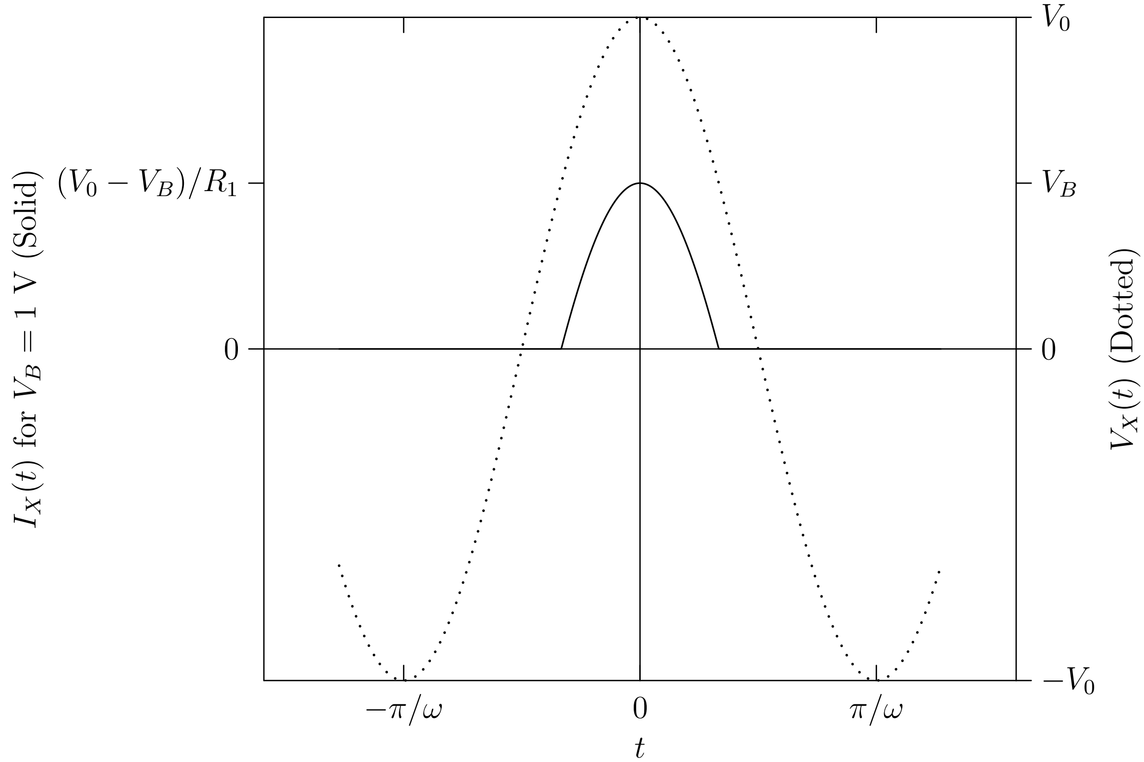
3.4

$$I_X = \begin{cases} 0 & V_X < V_B \\ \frac{V_X - V_B}{R_1} & V_X > V_B \end{cases}$$

Let's assume $V_0 > 1$ V. Plotting $I_X(t)$ for $V_B = -1$ V, we get



Plotting $I_X(t)$ for $V_B = 1$ V, we get

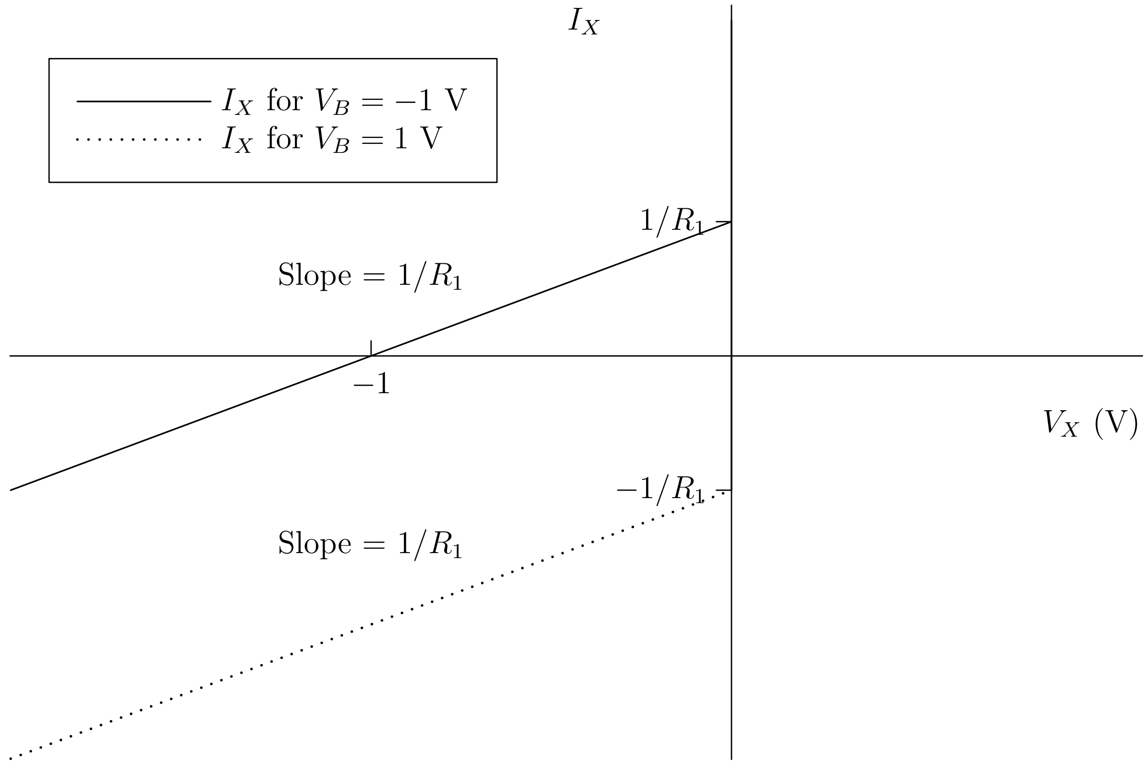


$I_X(t)$ for $V_B = 1$ V (Solid)
$(V_0 - V_B)/R_1$
0
V_0
V_B
0
$-V_0$
$V_X(t)$ (Dotted)
$-\pi/\omega$
0
π/ω
t

3.5

$$I_X = \begin{cases} \frac{V_X - V_B}{R_1} & V_X < 0 \\ \infty & V_X > 0 \end{cases}$$

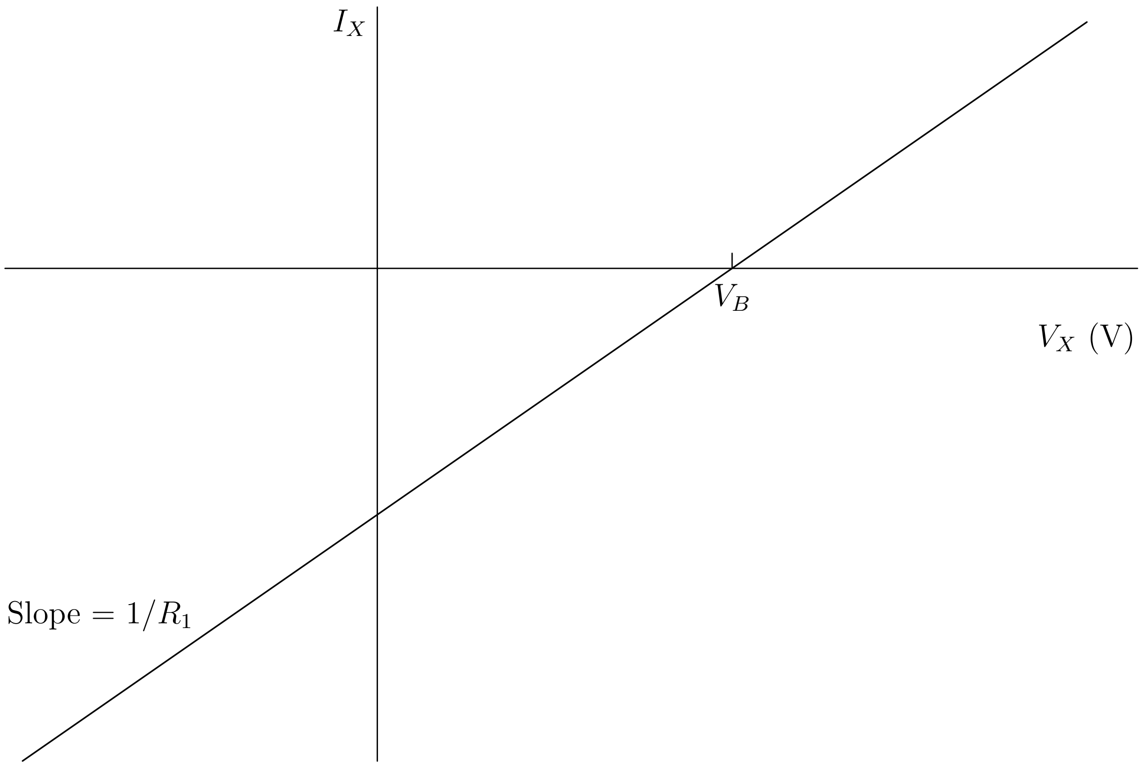
Plotting I_X vs. V_X for $V_B = -1$ V and $V_B = 1$ V, we get:



3.6 First, note that $I_{D1} = 0$ always, since D_1 is reverse biased by V_B (due to the assumption that $V_B > 0$). We can write I_X as

$$I_X = (V_X - V_B)/R_1$$

Plotting this, we get:

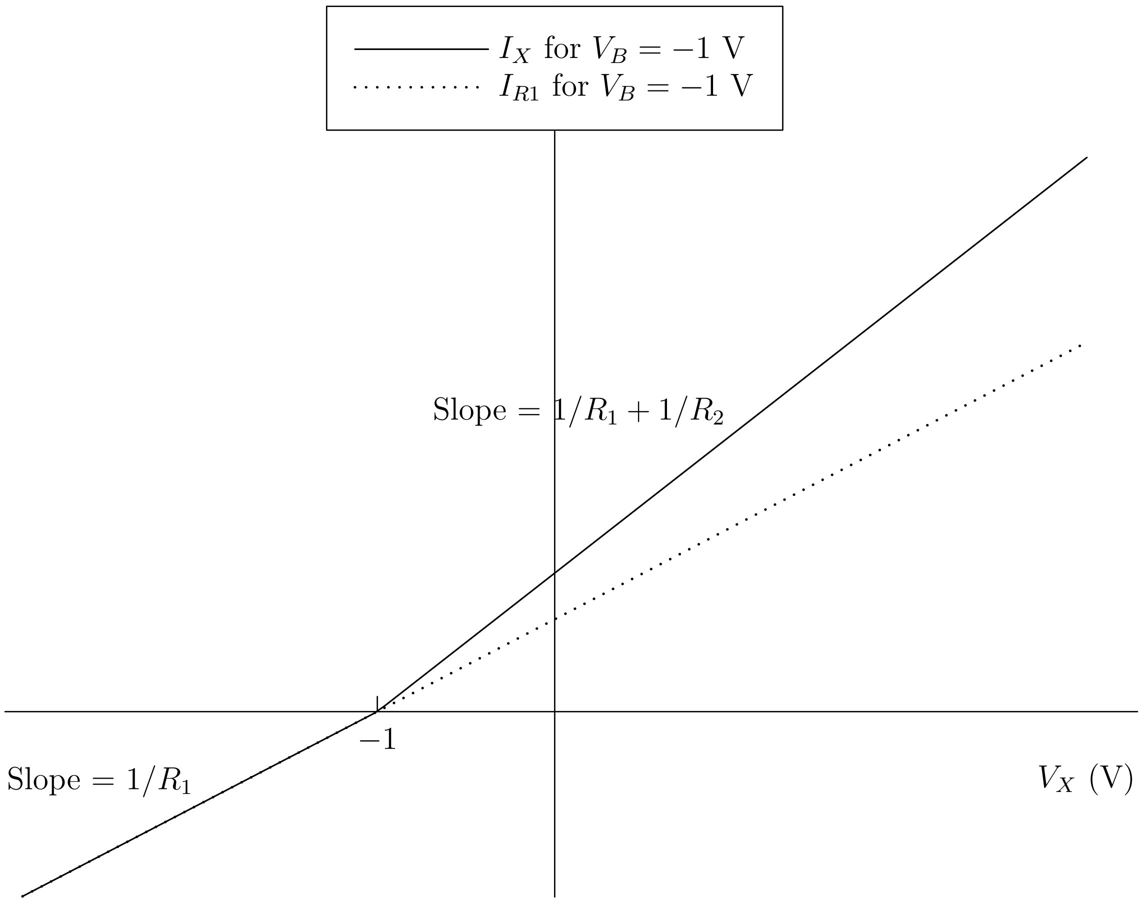


3.7

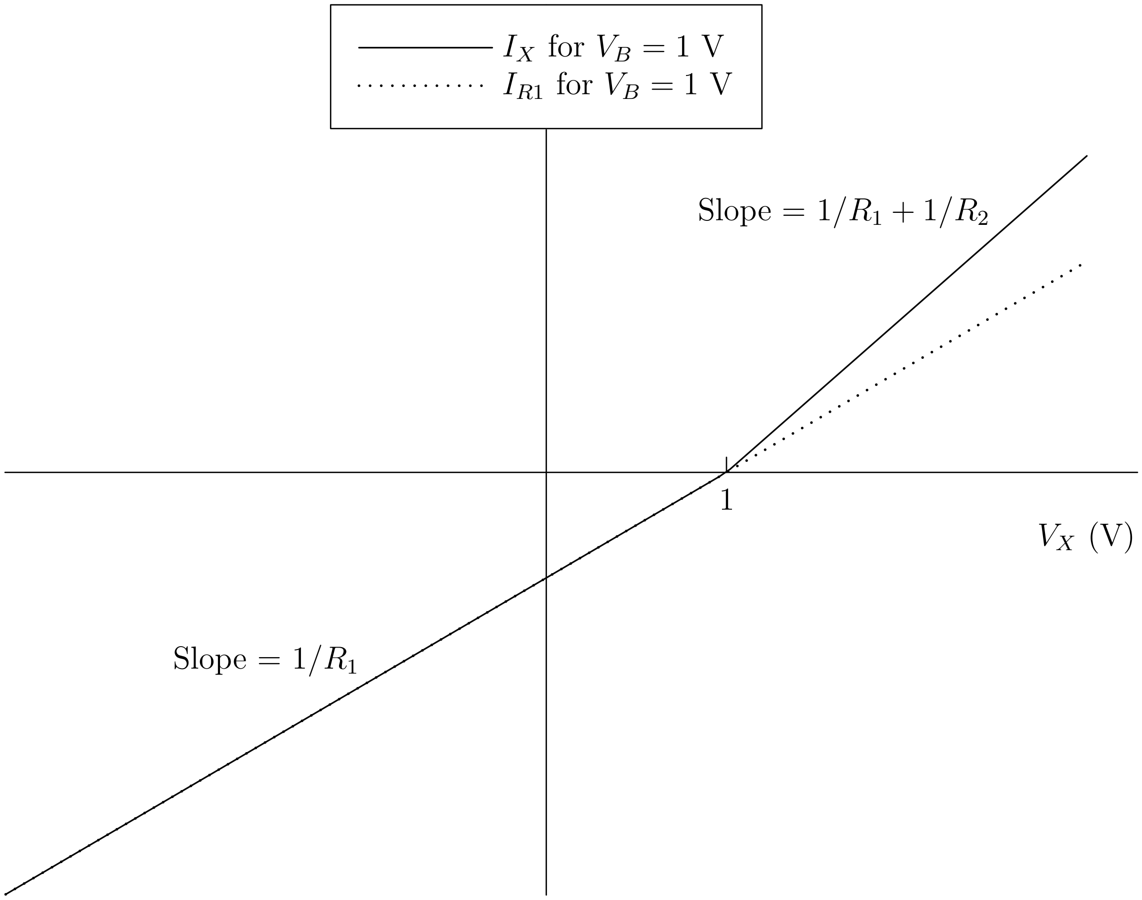
$$I_X = \begin{cases} \frac{V_X - V_B}{R_1} & V_X < V_B \\ \frac{V_X - V_B}{R_1 \| R_2} & V_X > V_B \end{cases}$$

$$I_{R1} = \frac{V_X - V_B}{R_1}$$

Plotting I_X and I_{R1} for $V_B = -1$ V, we get:



Plotting I_X and I_{R1} for $V_B = 1$ V, we get:



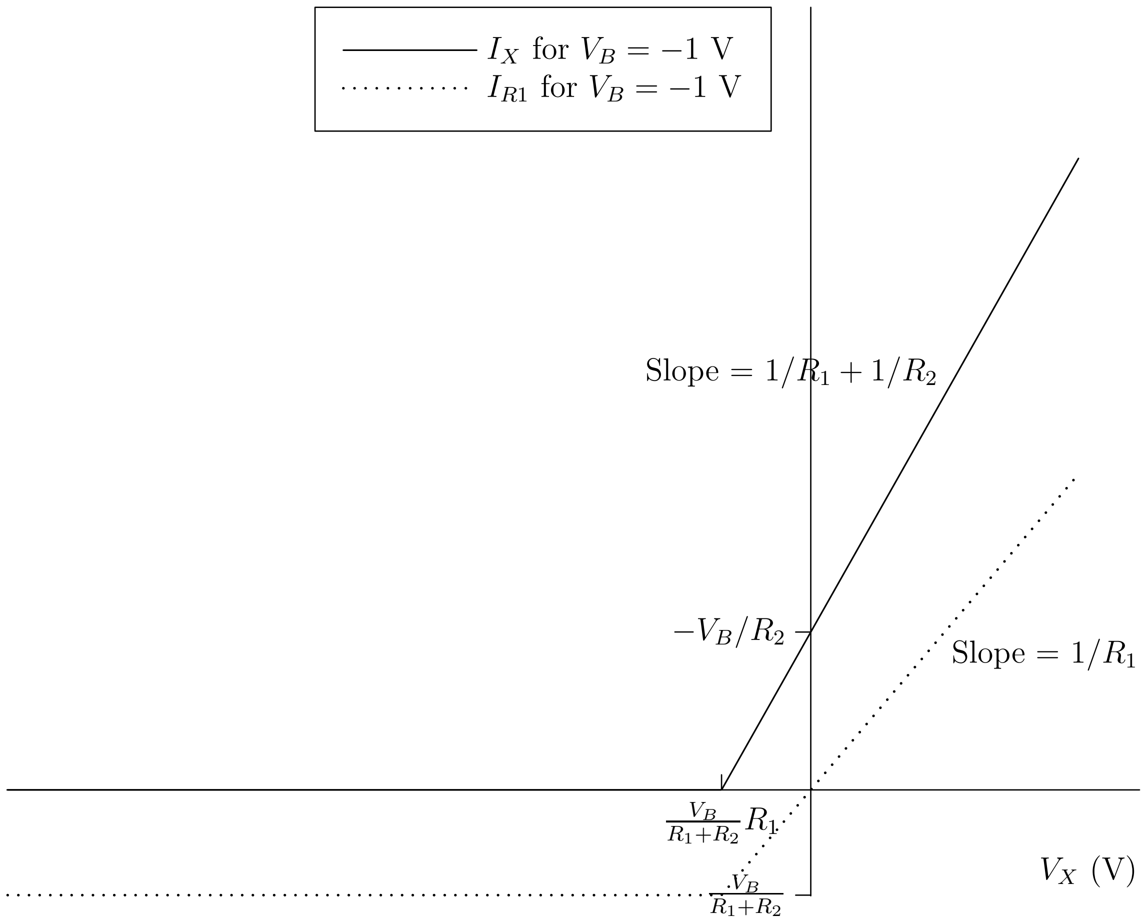
I_X for $V_B = 1$ V
I_{R1} for $V_B = 1$ V
Slope $= 1/R_1 + 1/R_2$
1
V_X (V)
Slope $= 1/R_1$

3.8

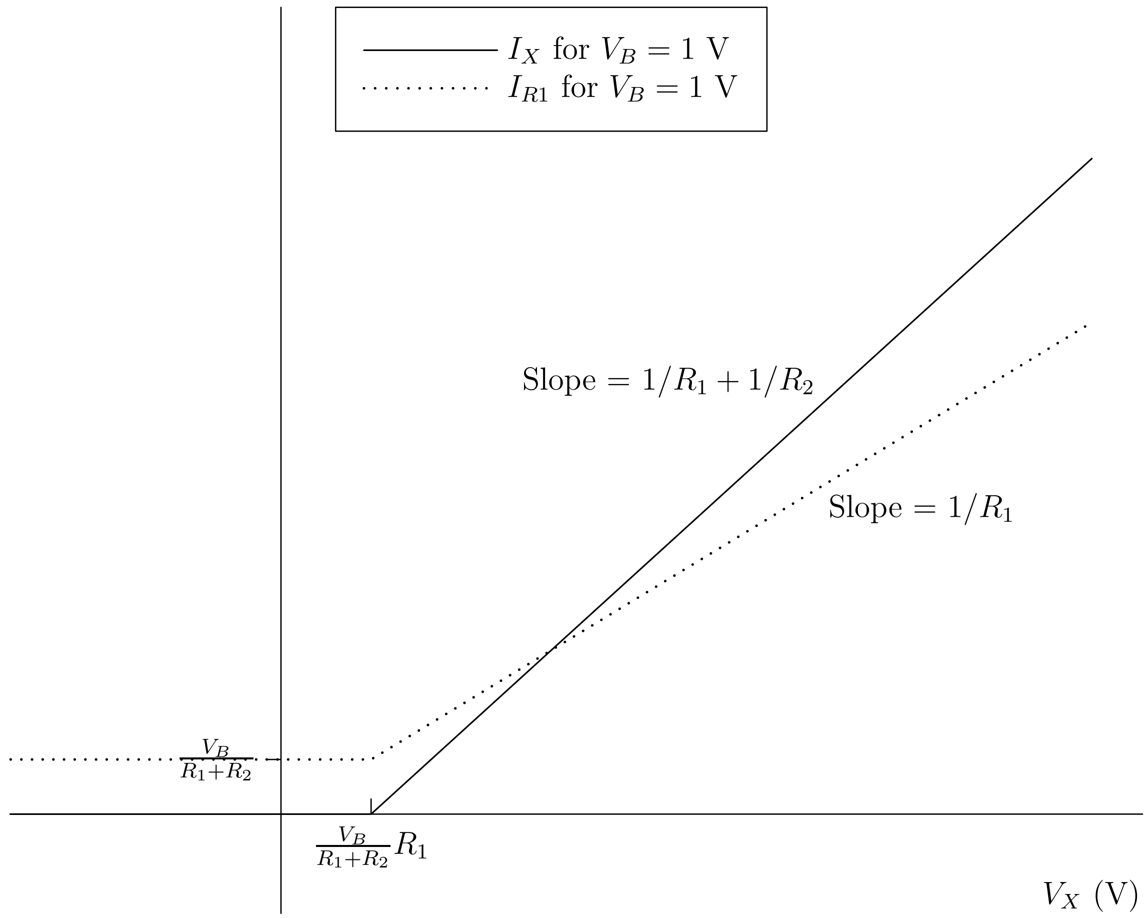
$$I_X = \begin{cases} 0 & V_X < \frac{V_B}{R_1+R_2}R_1 \\ \frac{V_X}{R_1} + \frac{V_X - V_B}{R_2} & V_X > \frac{V_B}{R_1+R_2}R_1 \end{cases}$$

$$I_{R1} = \begin{cases} \frac{V_B}{R_1+R_2} & V_X < \frac{V_B}{R_1+R_2}R_1 \\ \frac{V_X}{R_1} & V_X > \frac{V_B}{R_1+R_2}R_1 \end{cases}$$

Plotting I_X and I_{R1} for $V_B = -1$ V, we get:



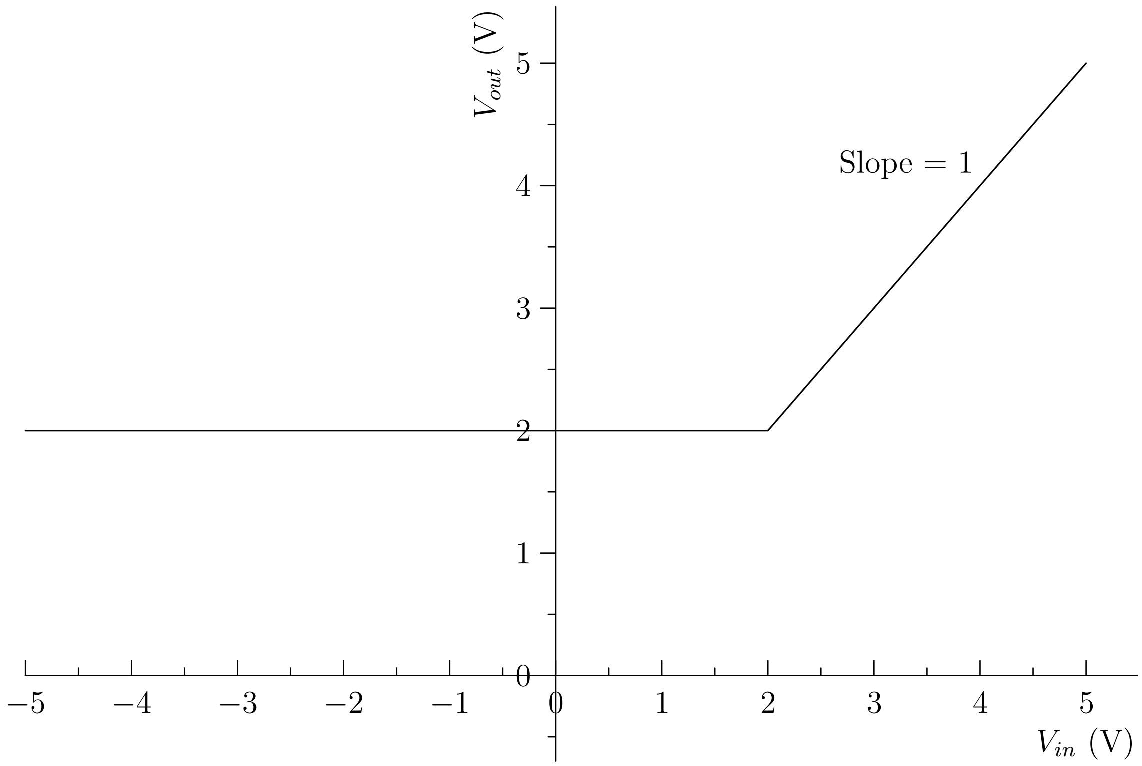
Plotting I_X and I_{R1} for $V_B = 1$ V, we get:



I_X for $V_B = 1$ V
I_{R1} for $V_B = 1$ V
Slope $= 1/R_1 + 1/R_2$
Slope $= 1/R_1$
$\frac{V_B}{R_1+R_2}$
$\frac{V_B}{R_1+R_2}R_1$
V_X (V)

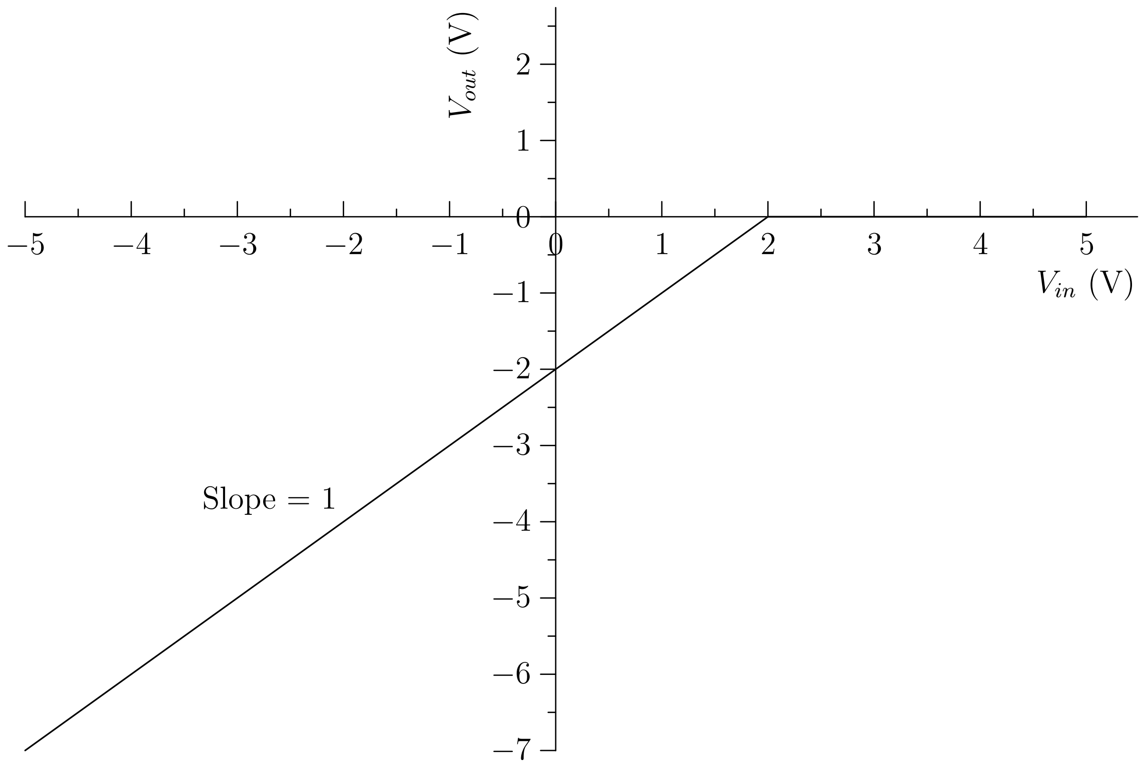
3.9 (a)

$$V_{out} = \begin{cases} V_B & V_{in} < V_B \\ V_{in} & V_{in} > V_B \end{cases}$$



(b)

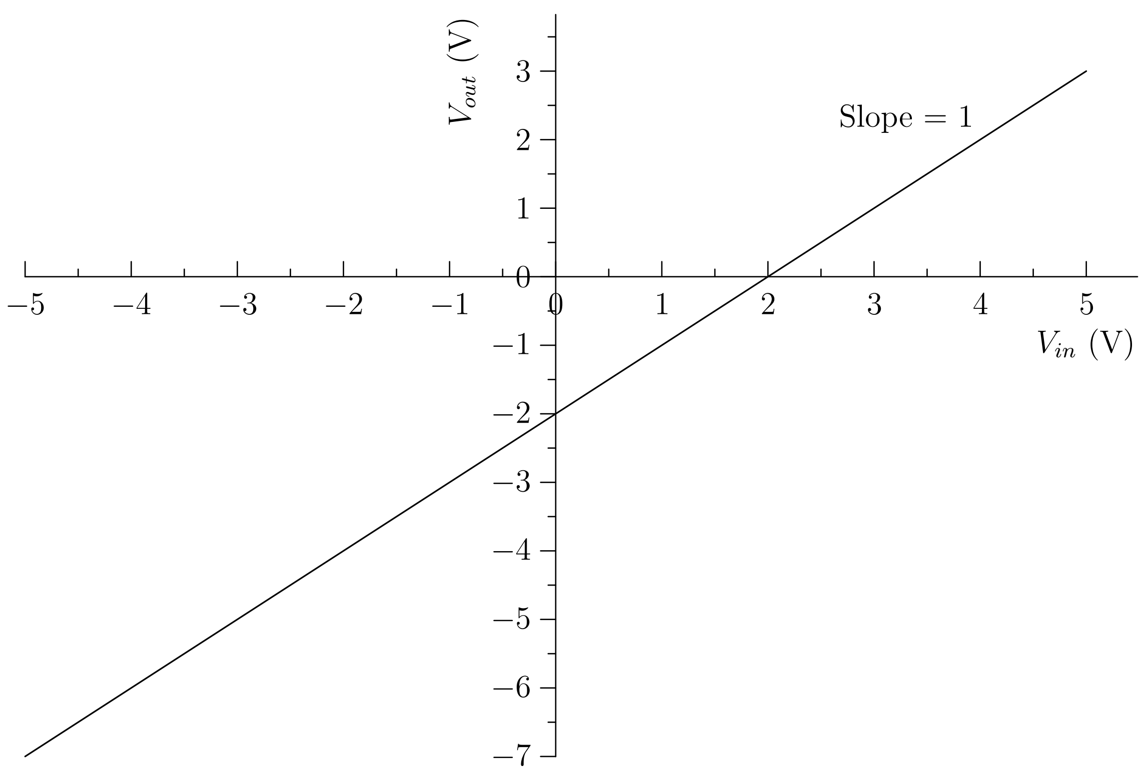
$$V_{out} = \begin{cases} V_{in} - V_B & V_{in} < V_B \\ 0 & V_{in} > V_B \end{cases}$$



V_{out} (V)
2
1
0
−1
−2
−3
−4
−5
−6
−7
−5
−4
−3
−2
−1
0
1
2
3
4
5
V_{in} (V)
Slope = 1

(c)

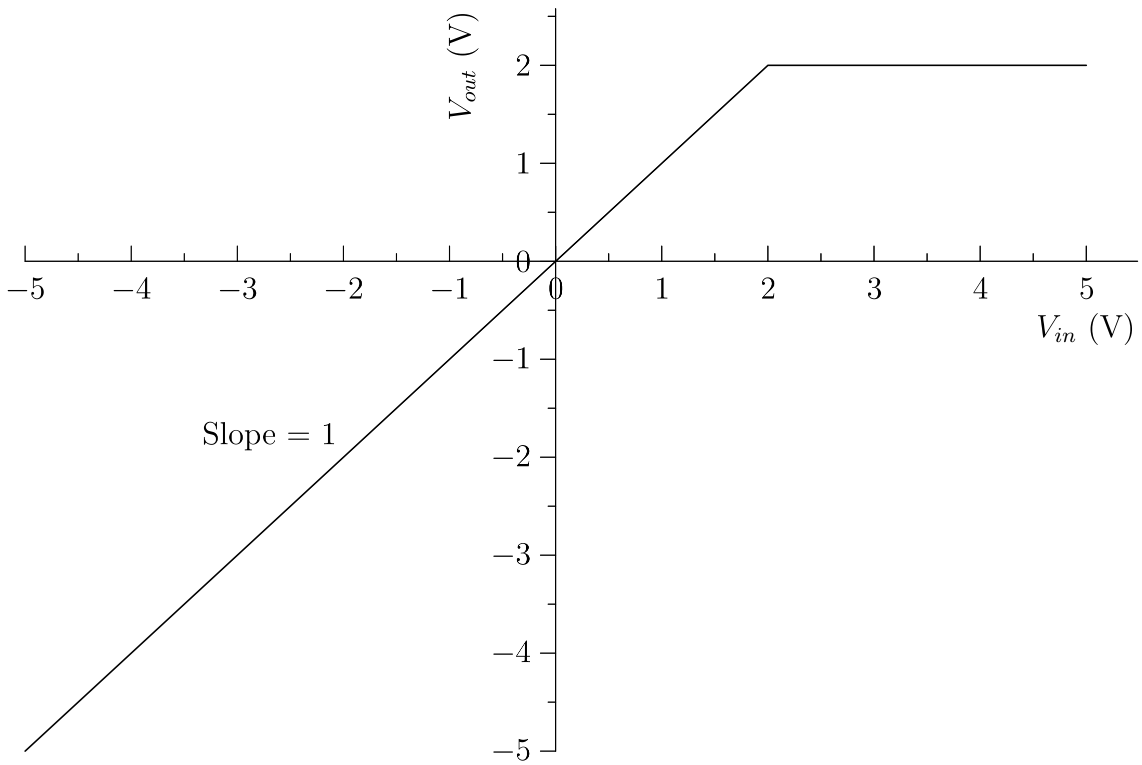
$V_{out} = V_{in} - V_B$



V_{out} (V)
3
2
1
0
−1
−2
−3
−4
−5
−6
−7
−5
−4
−3
−2
−1
0
1
2
3
4
5
V_{in} (V)
Slope = 1

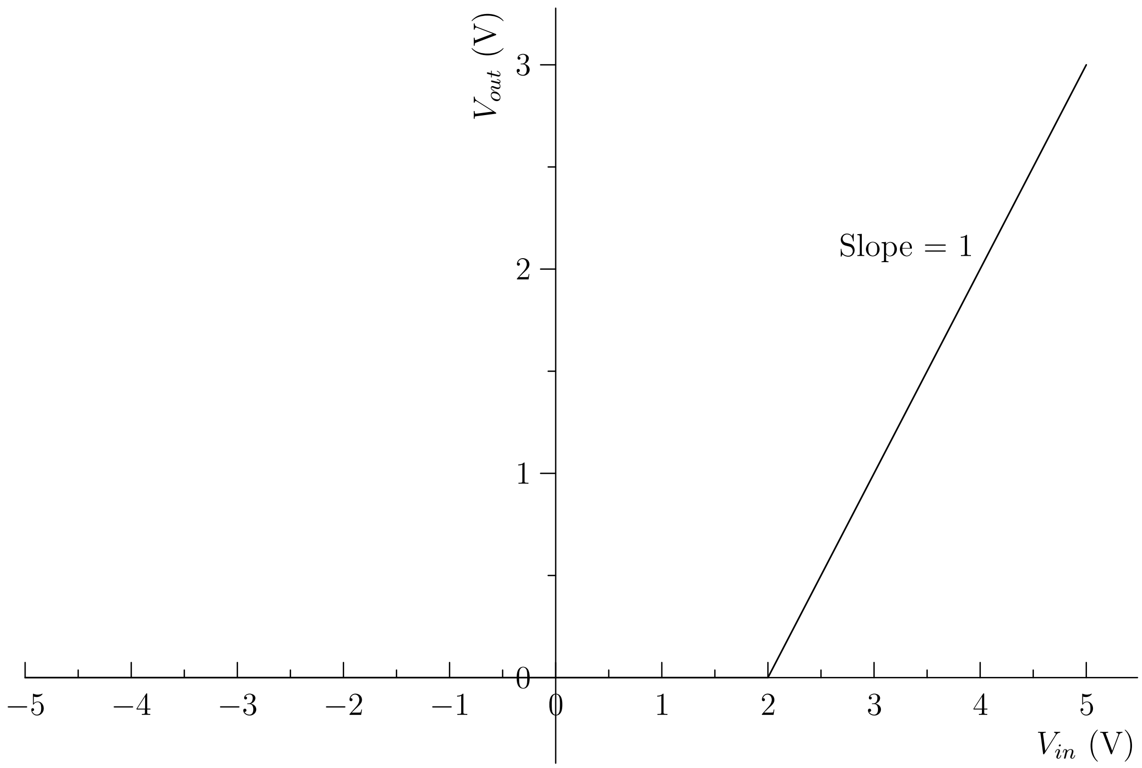
(d)

$$V_{out} = \begin{cases} V_{in} & V_{in} < V_B \\ V_B & V_{in} > V_B \end{cases}$$



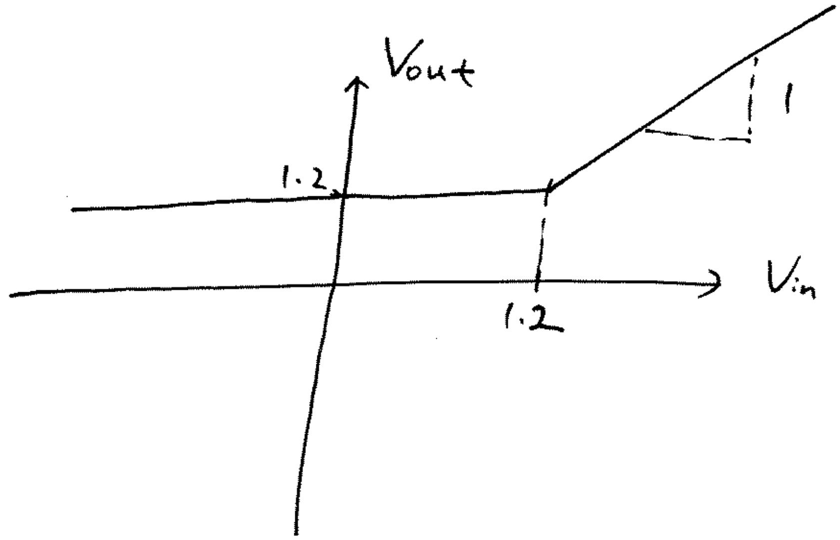
(e)

$$V_{out} = \begin{cases} 0 & V_{in} < V_B \\ V_{in} - V_B & V_{in} > V_B \end{cases}$$

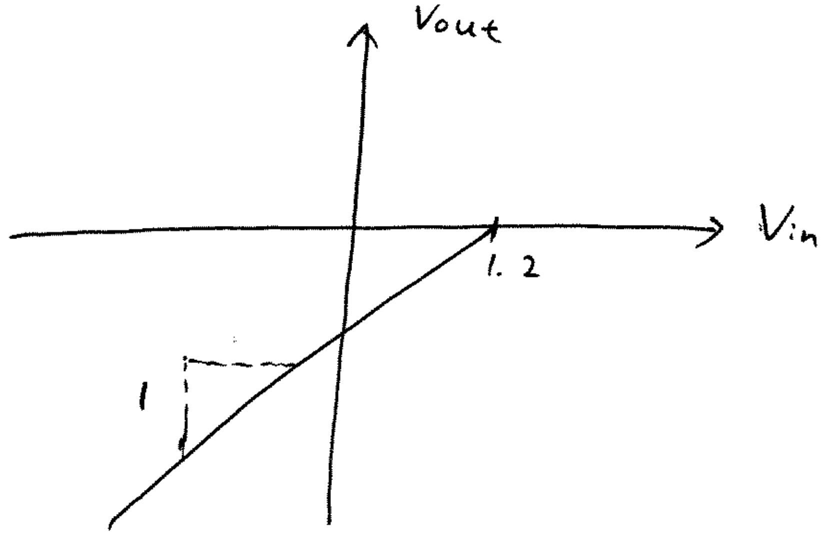


V_{out} (V)
3
2
1
0
Slope = 1
−5
−4
−3
−2
−1
0
1
2
3
4
5
V_{in} (V)

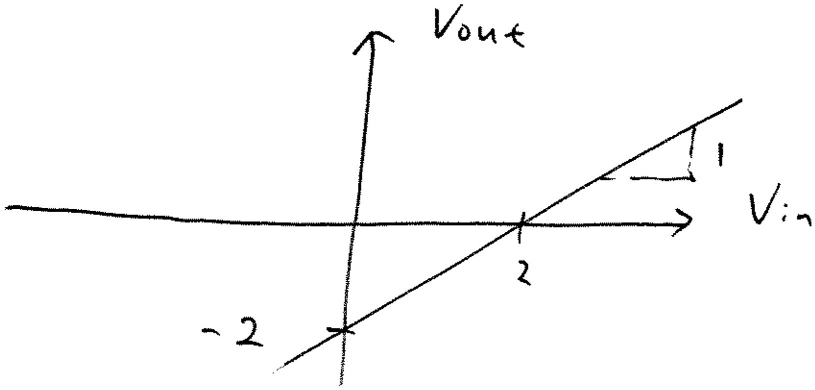
(10) a)



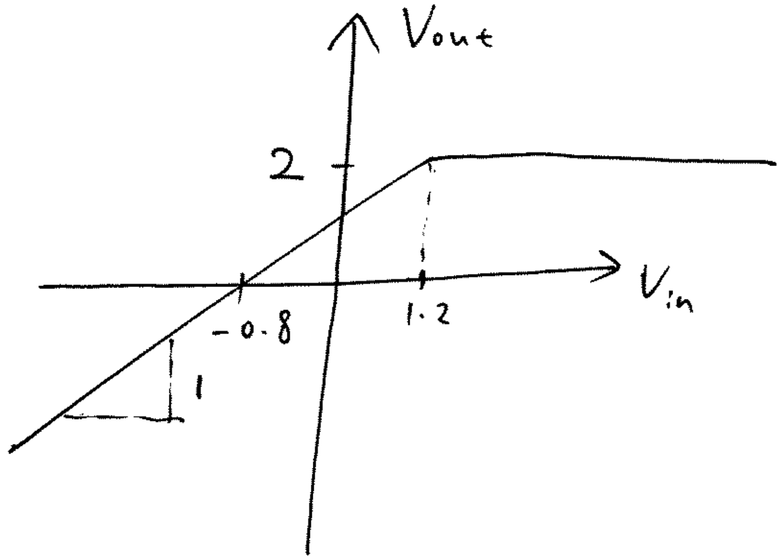
b)



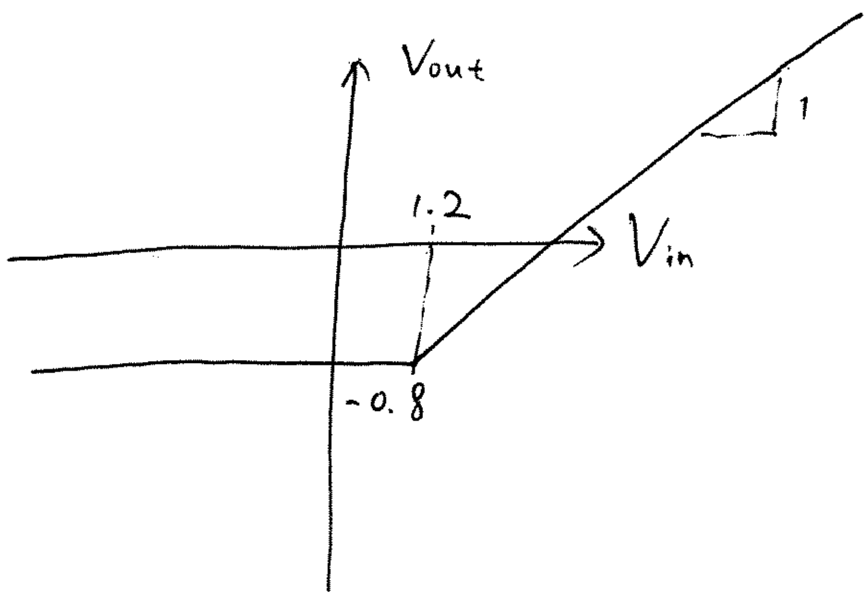
c)



d)



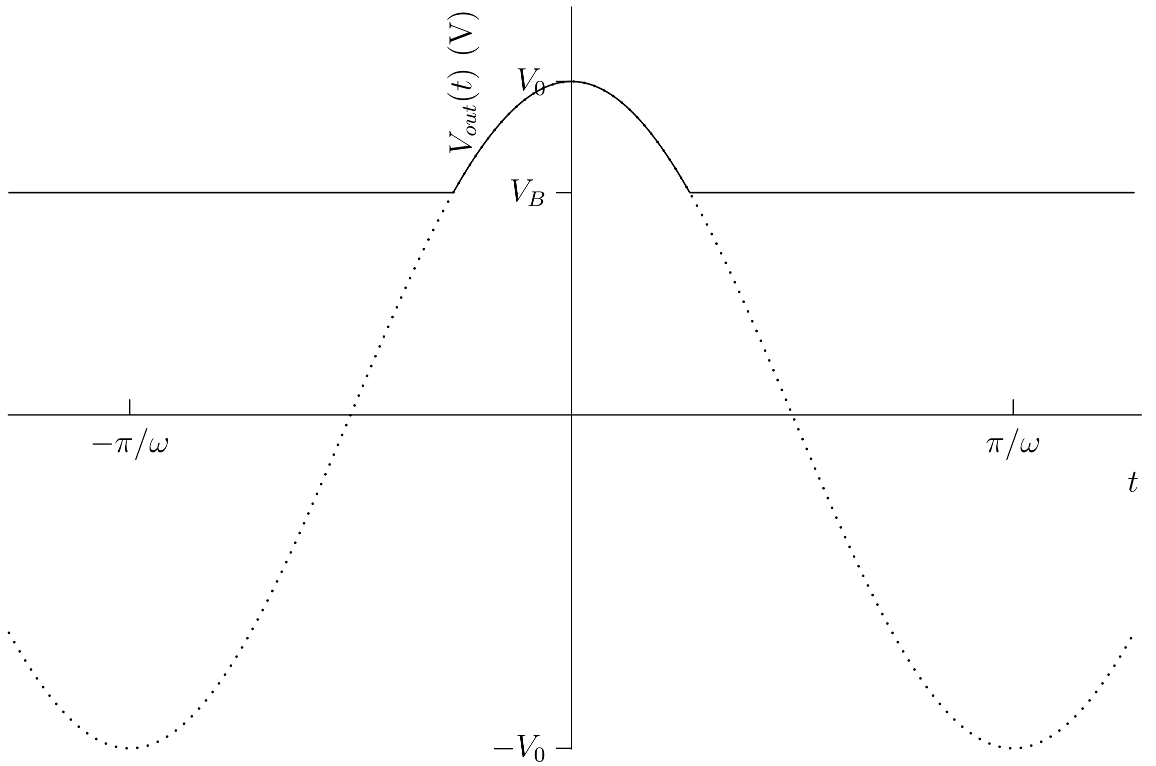
e)



3.11 For each part, the dotted line indicates $V_{in}(t)$, while the solid line indicates $V_{out}(t)$. Assume $V_0 > V_B$.

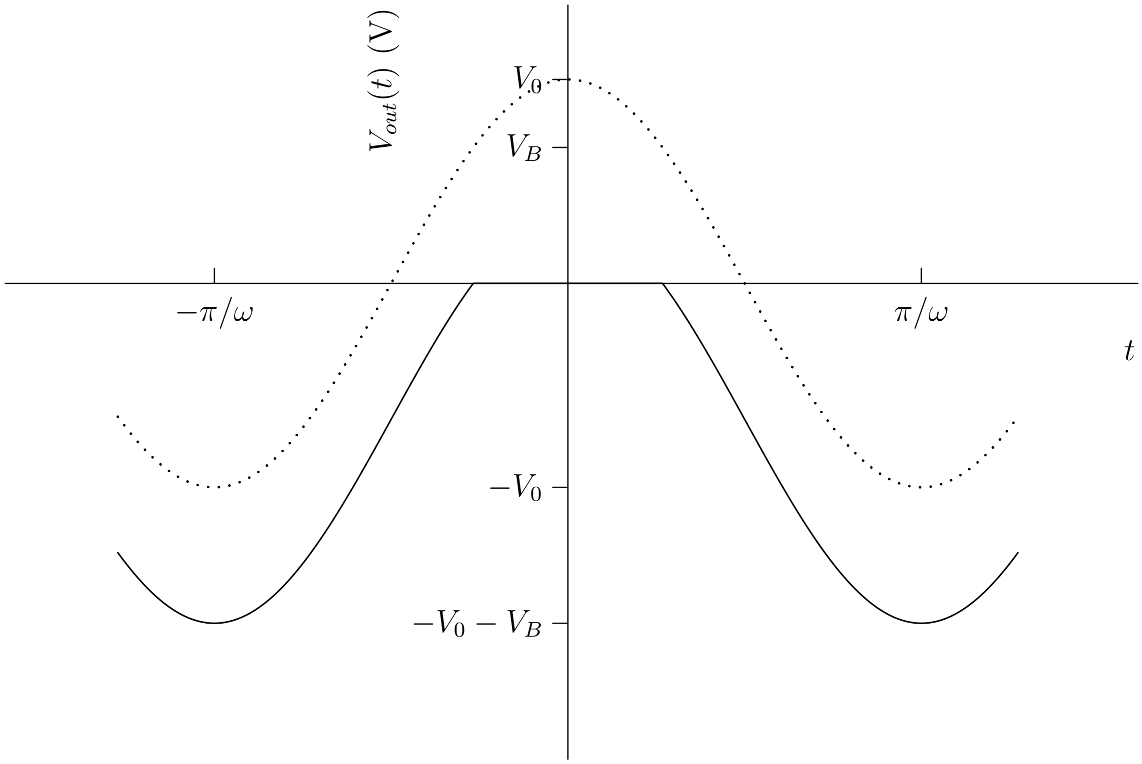
(a)

$$V_{out} = \begin{cases} V_B & V_{in} < V_B \\ V_{in} & V_{in} > V_B \end{cases}$$



(b)

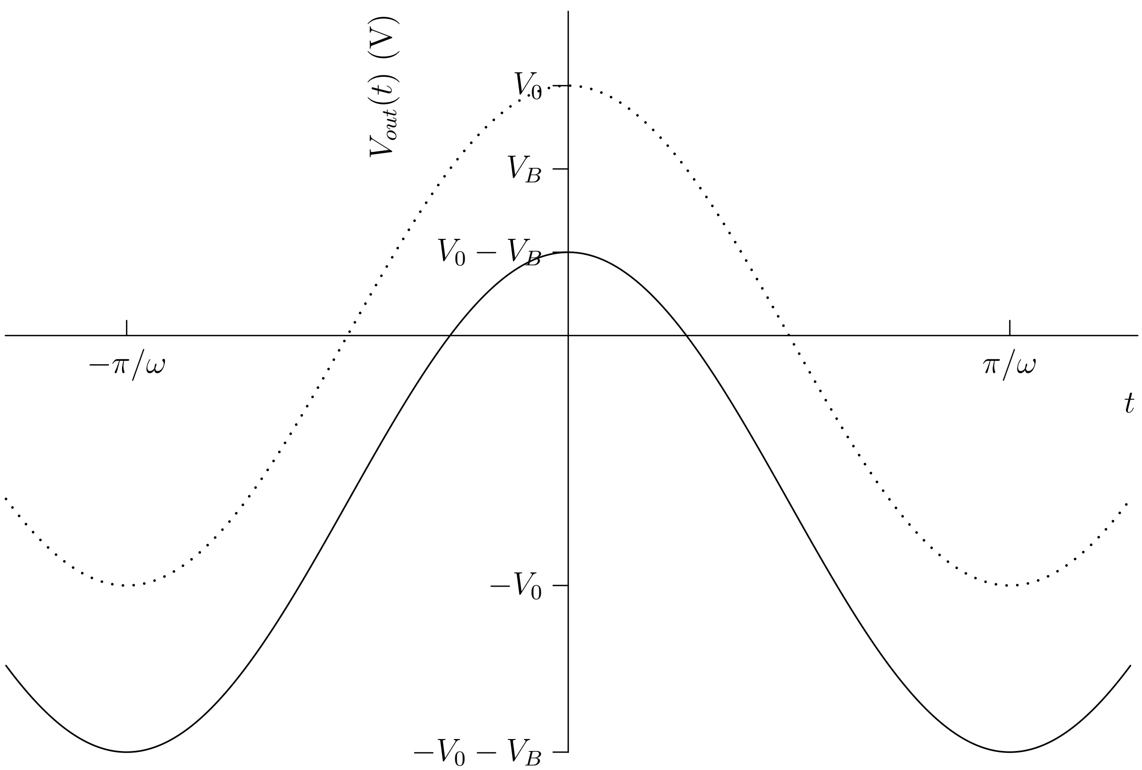
$$V_{out} = \begin{cases} V_{in} - V_B & V_{in} < V_B \\ 0 & V_{in} > V_B \end{cases}$$



$V_{out}(t)$ (V)
V_0
V_B
$-\pi/\omega$
π/ω
t
$-V_0$
$-V_0 - V_B$

(c)

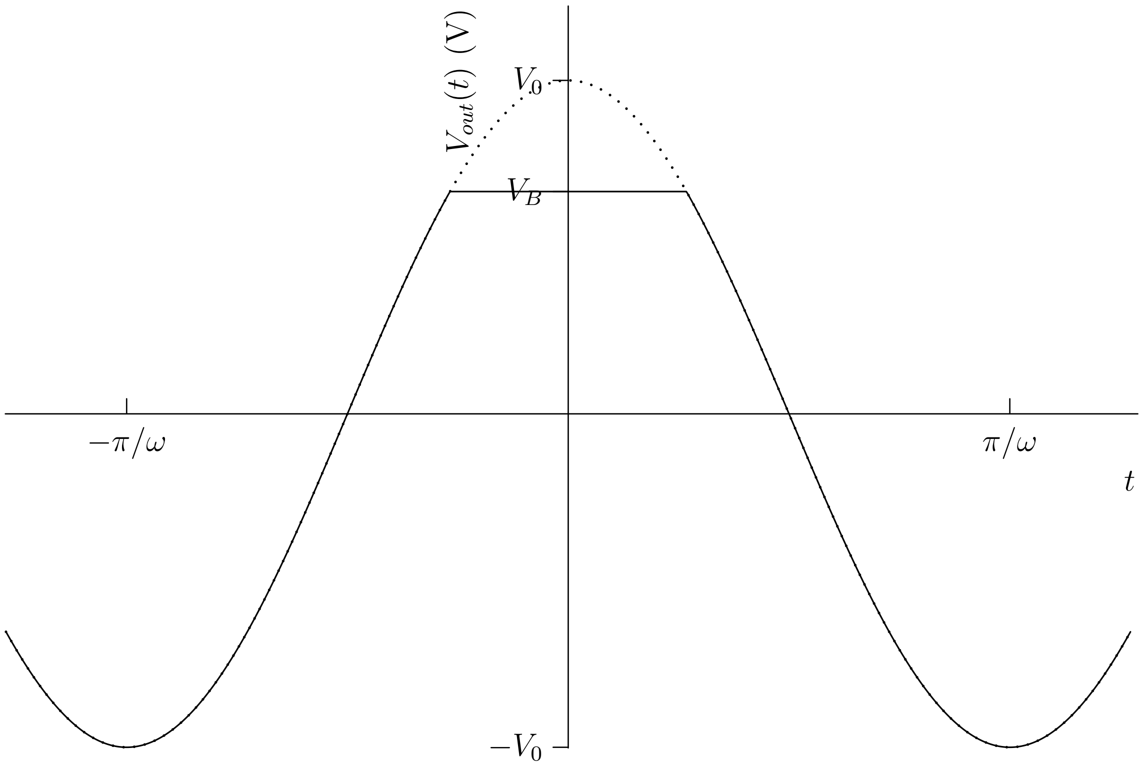
$V_{out} = V_{in} - V_B$



$V_{out}(t)$ (V)
V_0
V_B
$V_0 - V_B$
$-\pi/\omega$
π/ω
t
$-V_0$
$-V_0 - V_B$

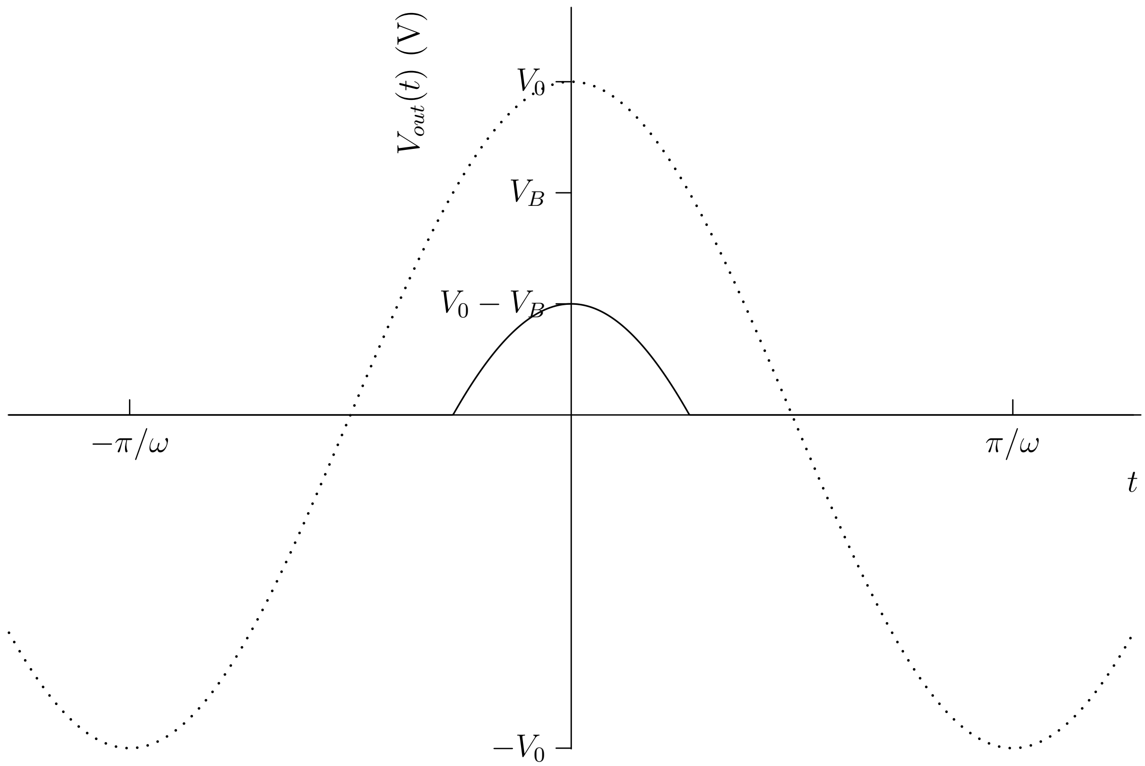
(d)

$$V_{out} = \begin{cases} V_{in} & V_{in} < V_B \\ V_B & V_{in} > V_B \end{cases}$$



(e)

$$V_{out} = \begin{cases} 0 & V_{in} < V_B \\ V_{in} - V_B & V_{in} > V_B \end{cases}$$

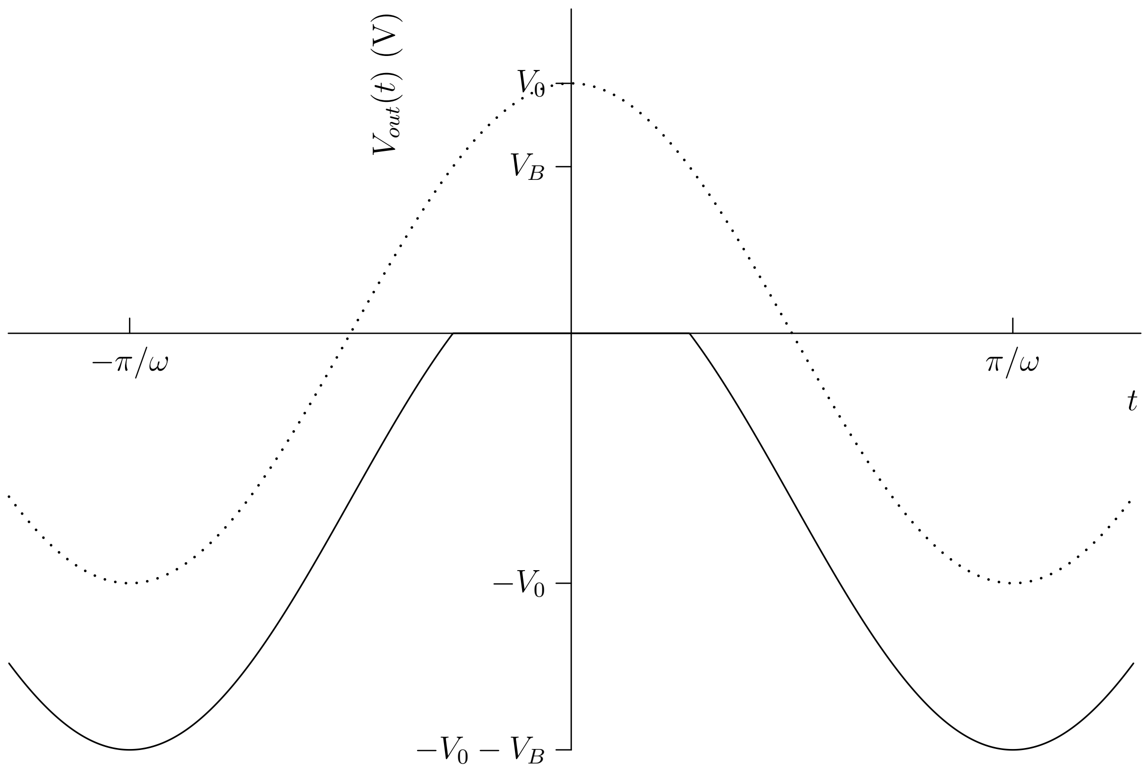


$V_{out}(t)$ (V)
V_0
V_B
$V_0 - V_B$
$-V_0$
$-\pi/\omega$
π/ω
t

3.12 For each part, the dotted line indicates $V_{in}(t)$, while the solid line indicates $V_{out}(t)$. Assume $V_0 > V_B$.

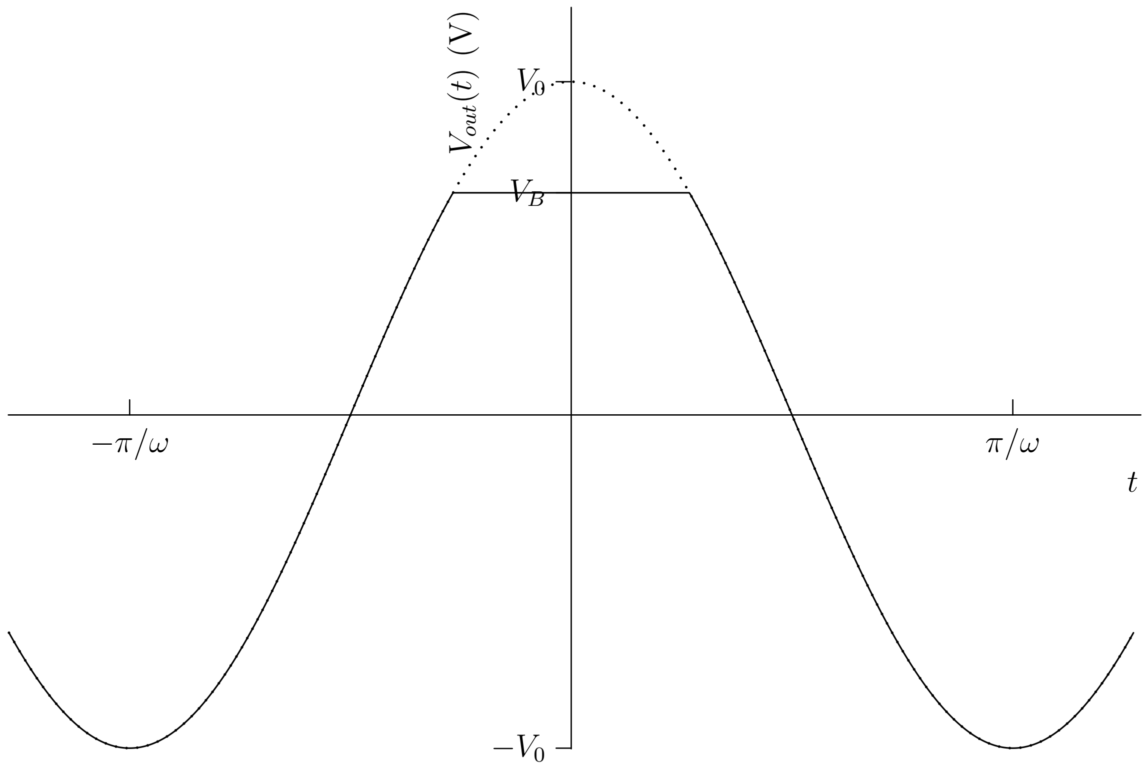
(a)

$$V_{out} = \begin{cases} V_{in} - V_B & V_{in} < V_B \\ 0 & V_{in} > V_B \end{cases}$$



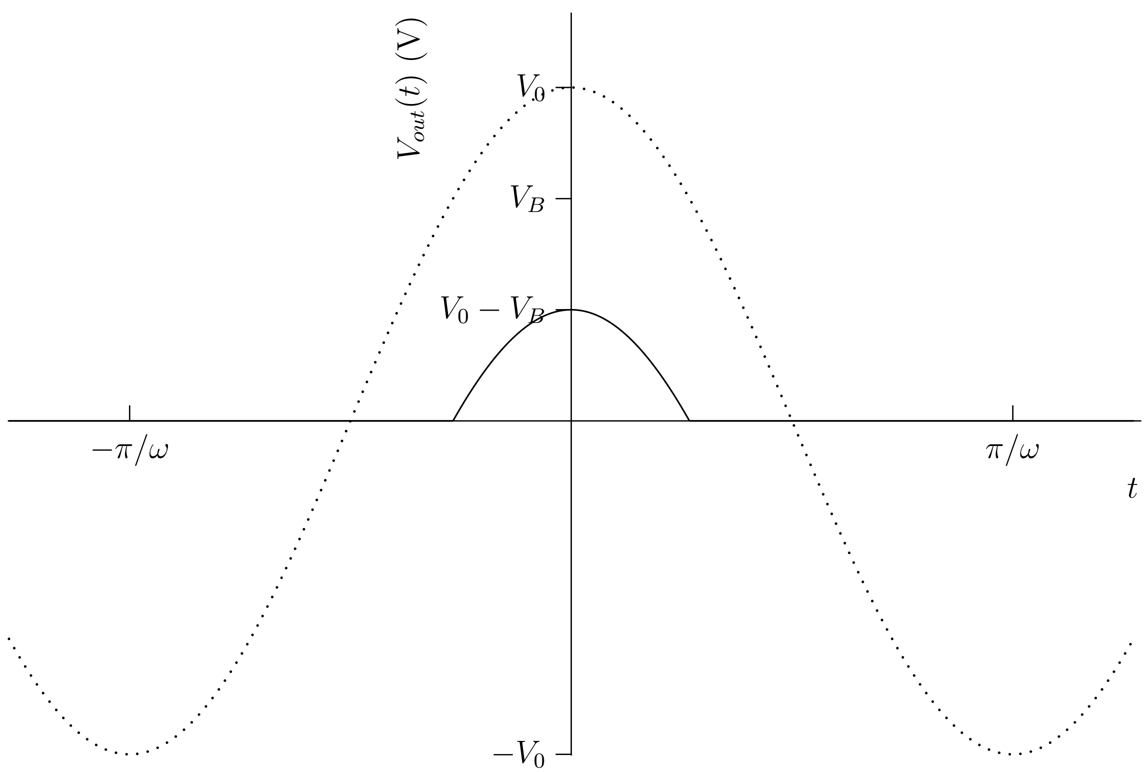
(b)

$$V_{out} = \begin{cases} V_{in} & V_{in} < V_B \\ V_B & V_{in} > V_B \end{cases}$$



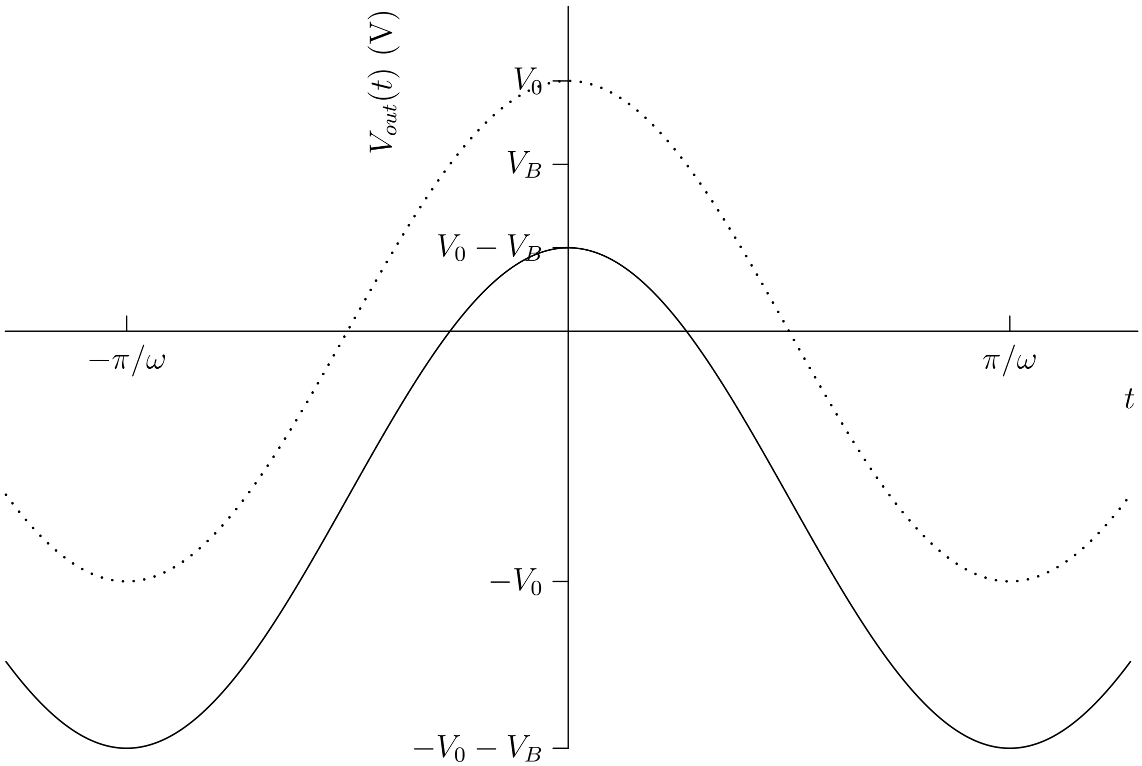
(c)

$$V_{out} = \begin{cases} 0 & V_{in} < V_B \\ V_{in} - V_B & V_{in} > V_B \end{cases}$$



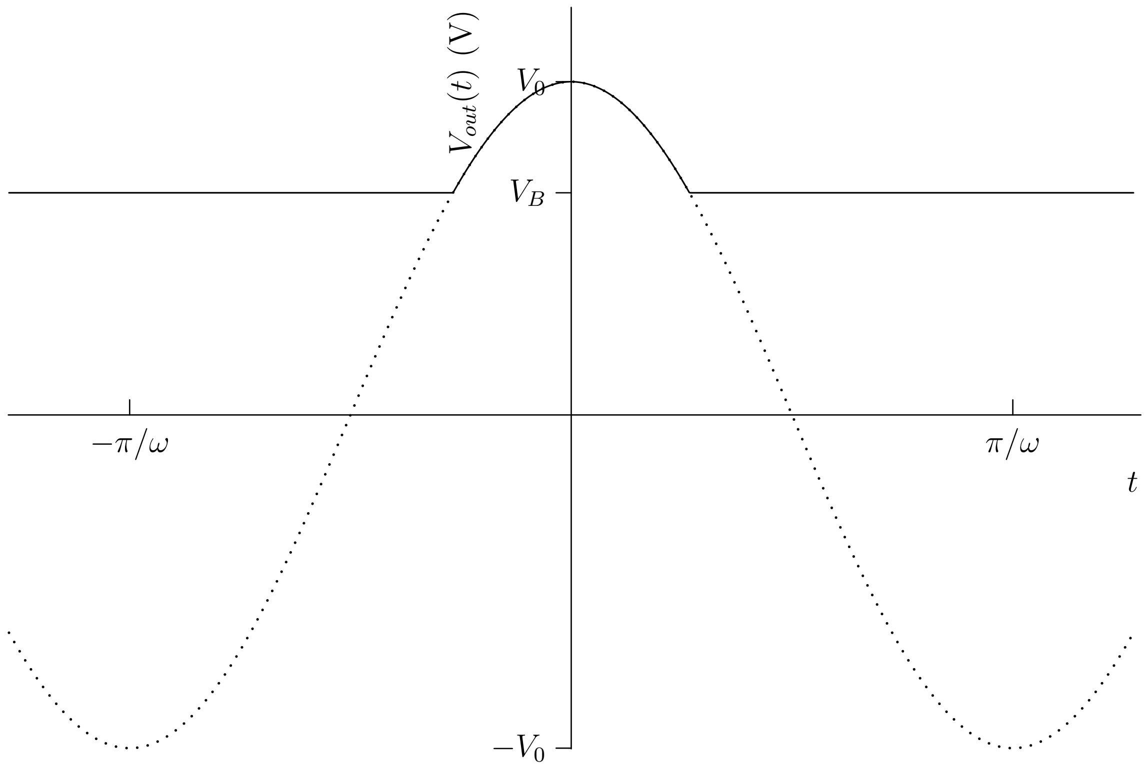
(d)

$$V_{out} = V_{in} - V_B$$



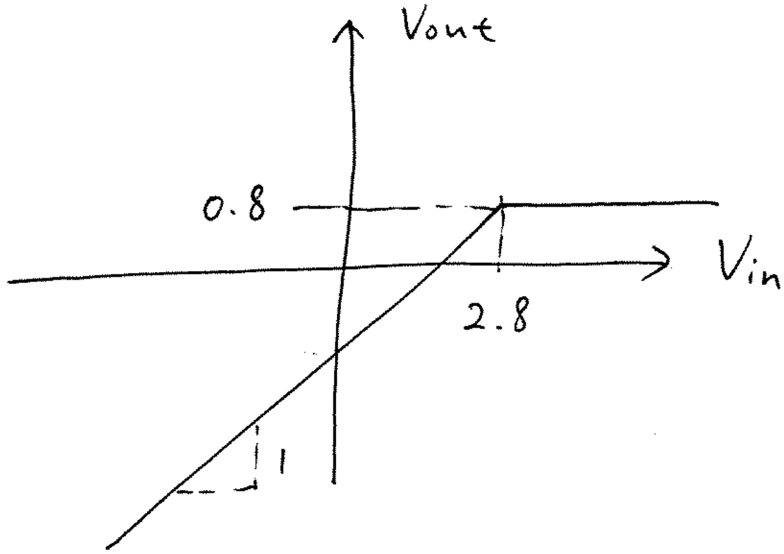
(e)

$$V_{out} = \begin{cases} V_B & V_{in} < V_B \\ V_{in} & V_{in} > V_B \end{cases}$$

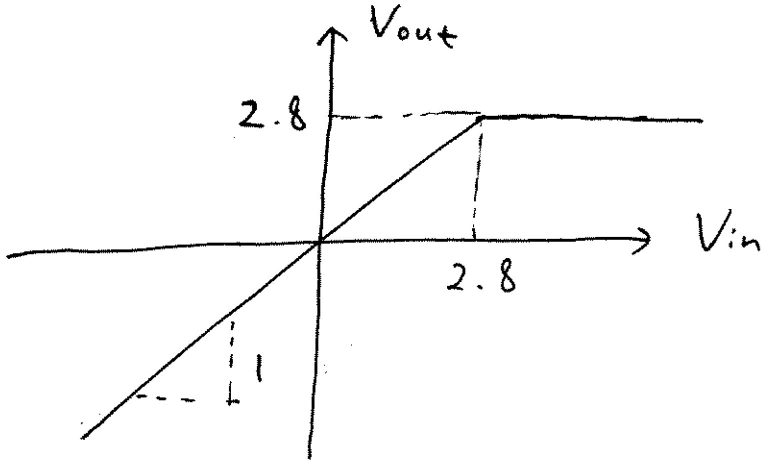


$V_{out}(t)$ (V)
V_0
V_B
$-\pi/\omega$
π/ω
t
$-V_0$

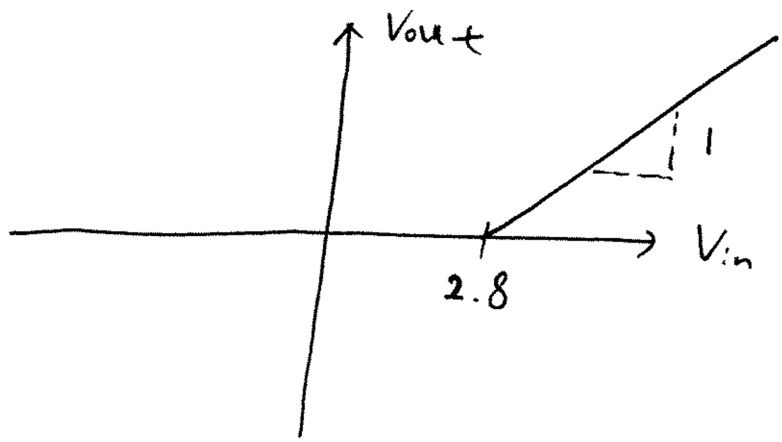
⑬ a)



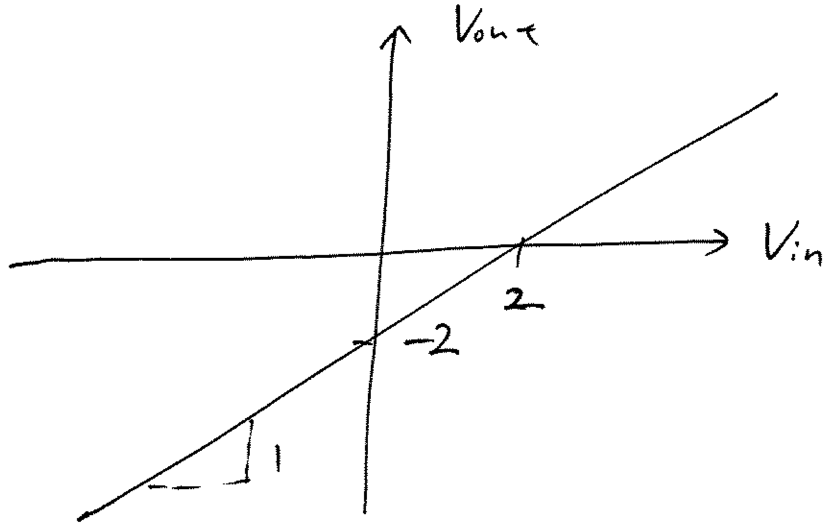
b)



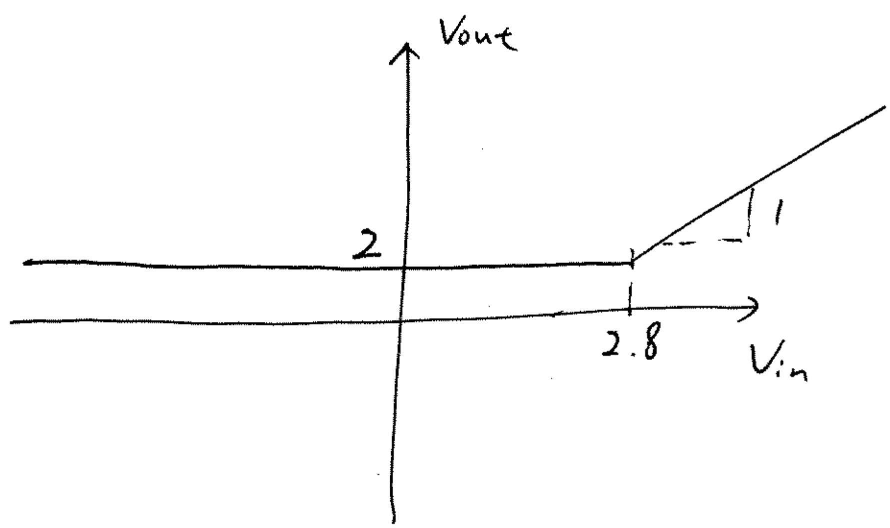
c)



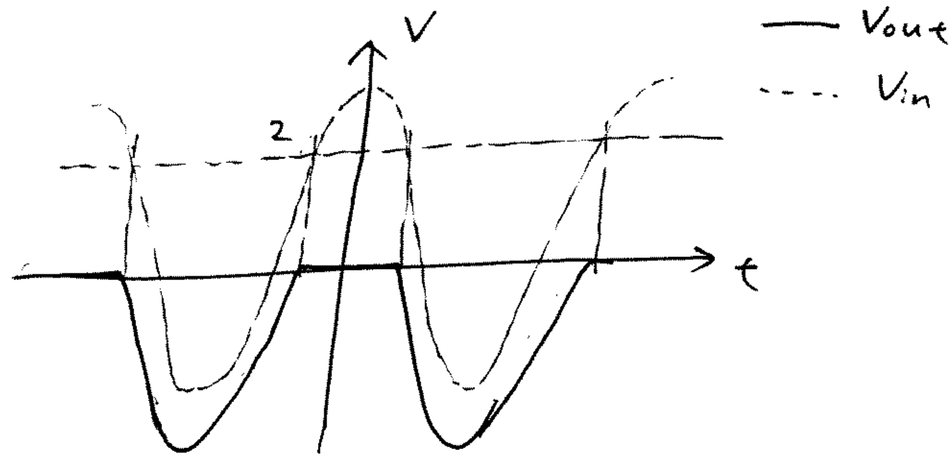
d)



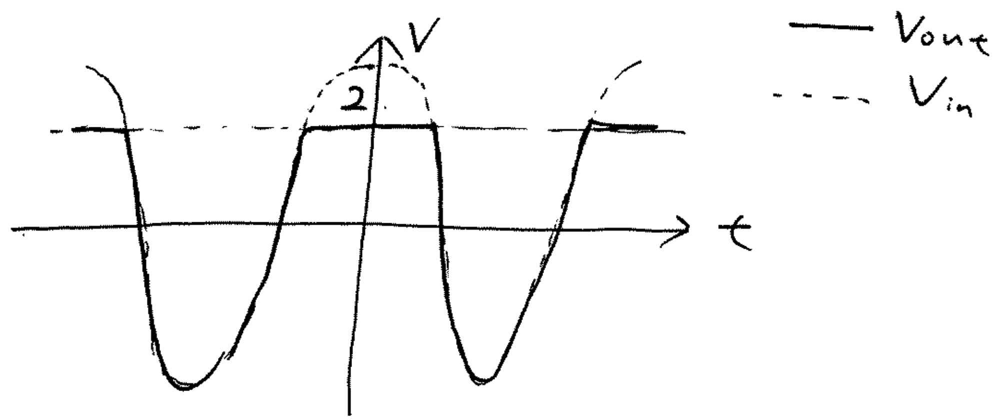
e)



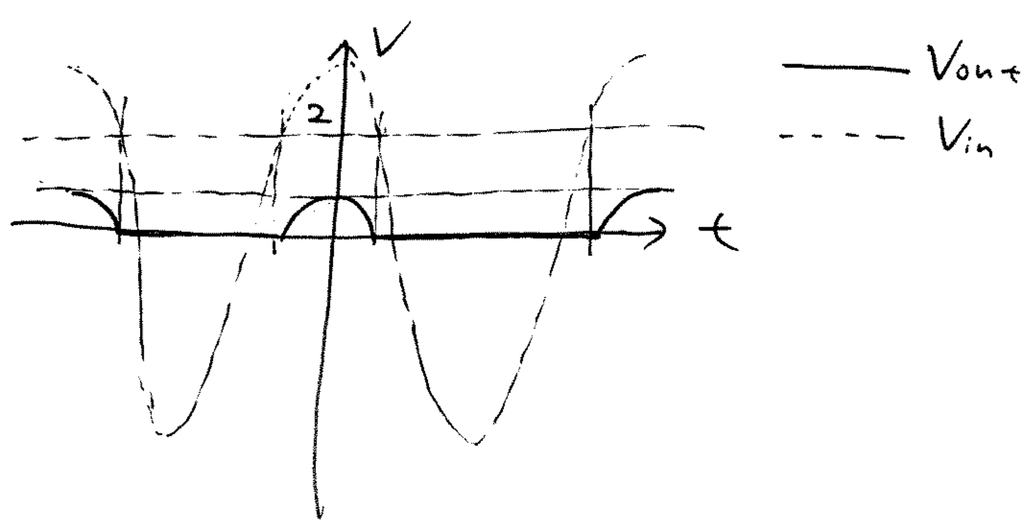
⑭ a)



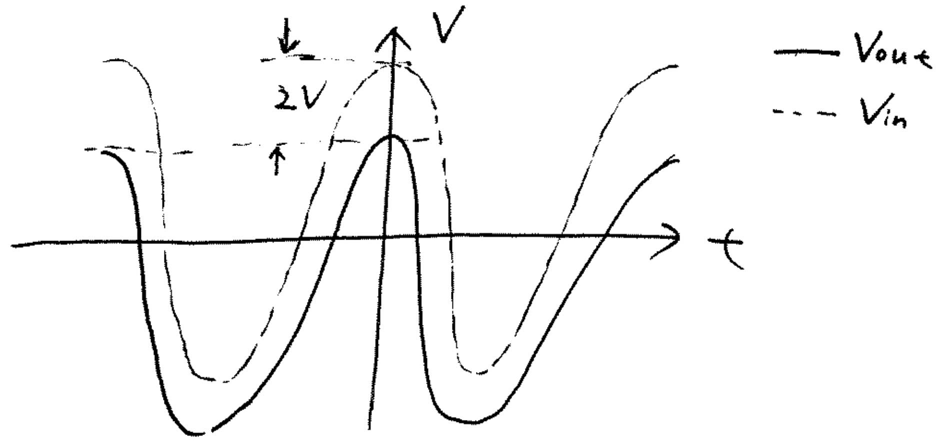
b)



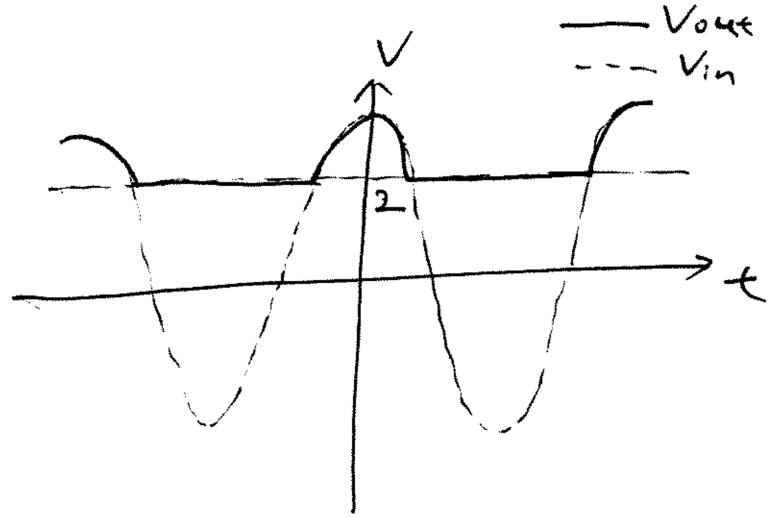
c)



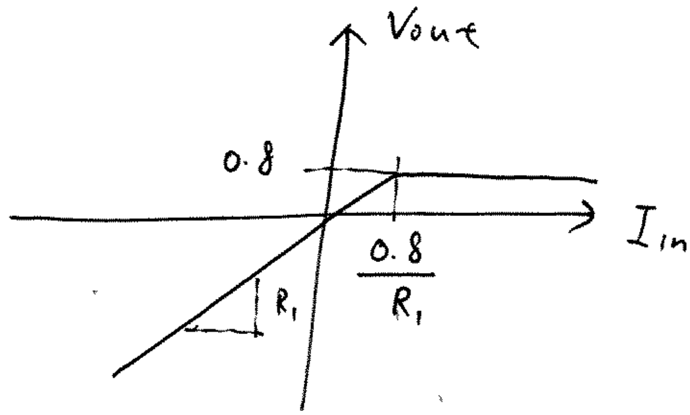
d)



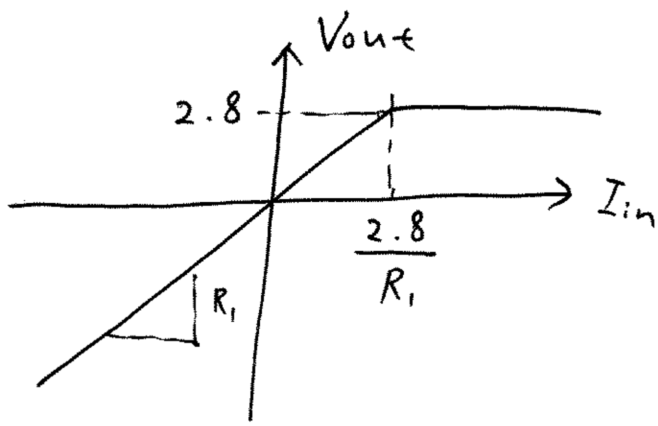
e).



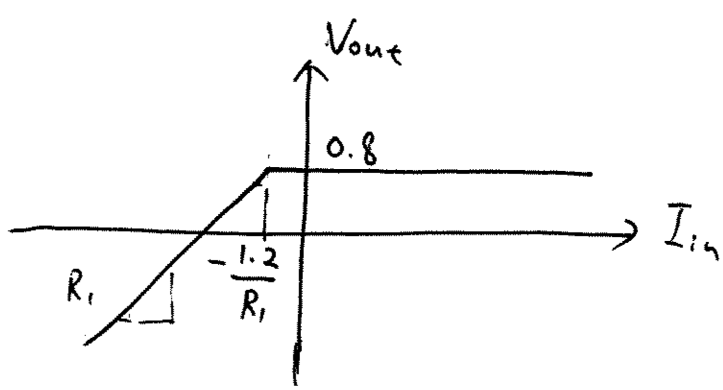
(15) a)


Vout
0.8
Iin
0.8
R1
R1

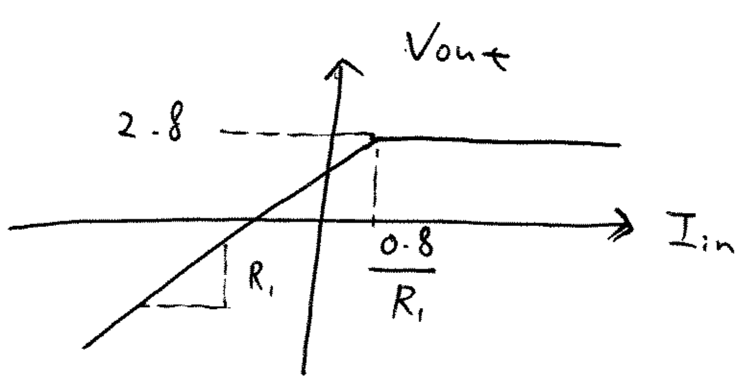
b)


Vout
2.8
Iin
2.8
R1
R1

c)

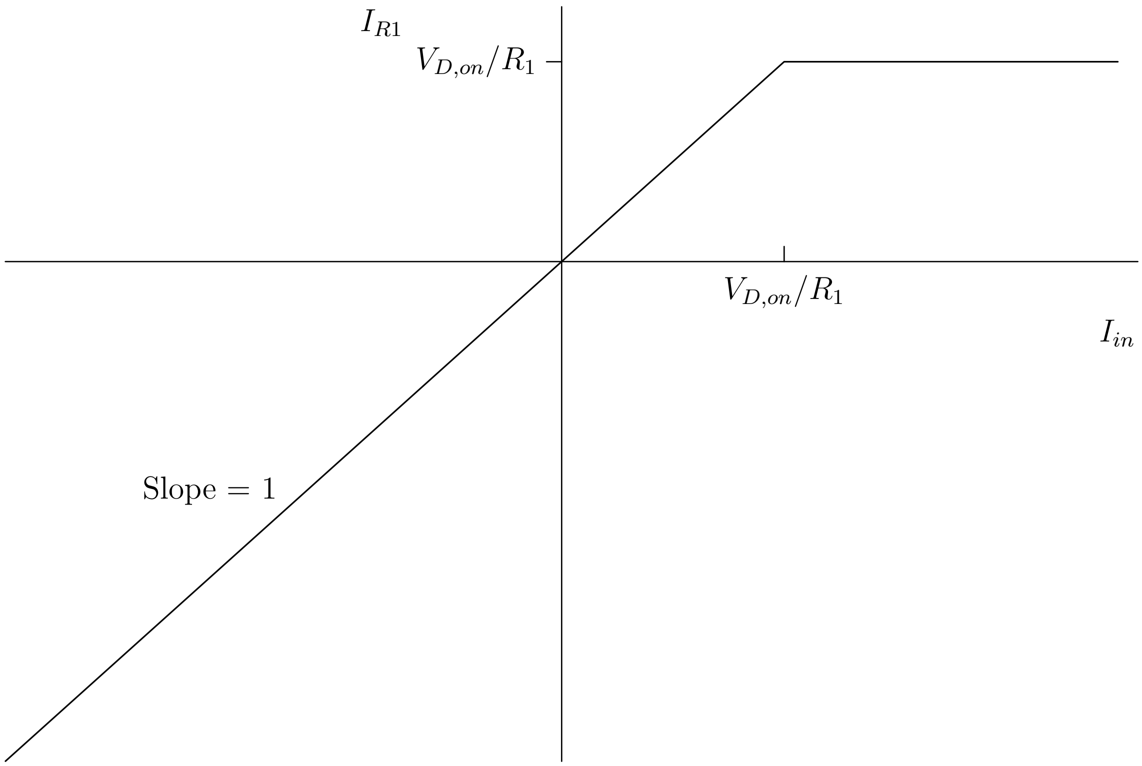

Vout
0.8
Iin
-1.2
R1
R1

d)


Vout
2.8
Iin
0.8
R1
R1

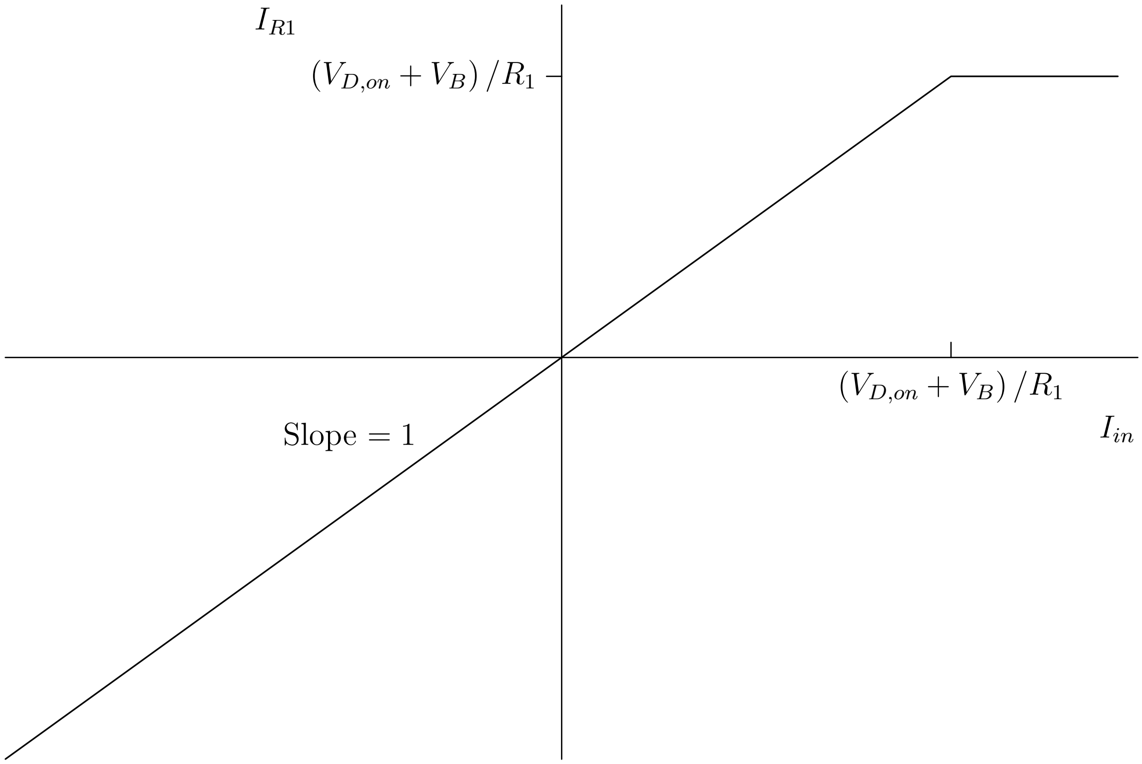
3.16 (a)

$$I_{R1} = \begin{cases} I_{in} & I_{in} < \frac{V_{D,on}}{R_1} \\ \frac{V_{D,on}}{R_1} & I_{in} > \frac{V_{D,on}}{R_1} \end{cases}$$



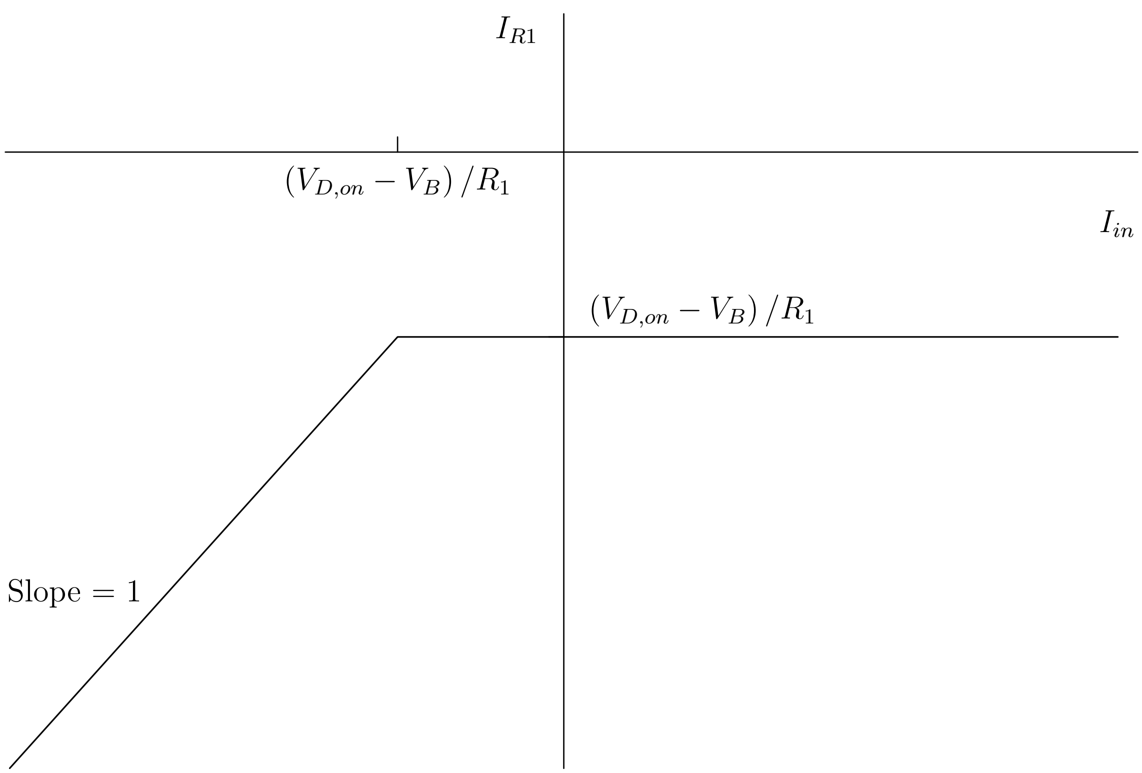
(b)

$$I_{R1} = \begin{cases} I_{in} & I_{in} < \frac{V_{D,on}+V_B}{R_1} \\ \frac{V_{D,on}+V_B}{R_1} & I_{in} > \frac{V_{D,on}+V_B}{R_1} \end{cases}$$



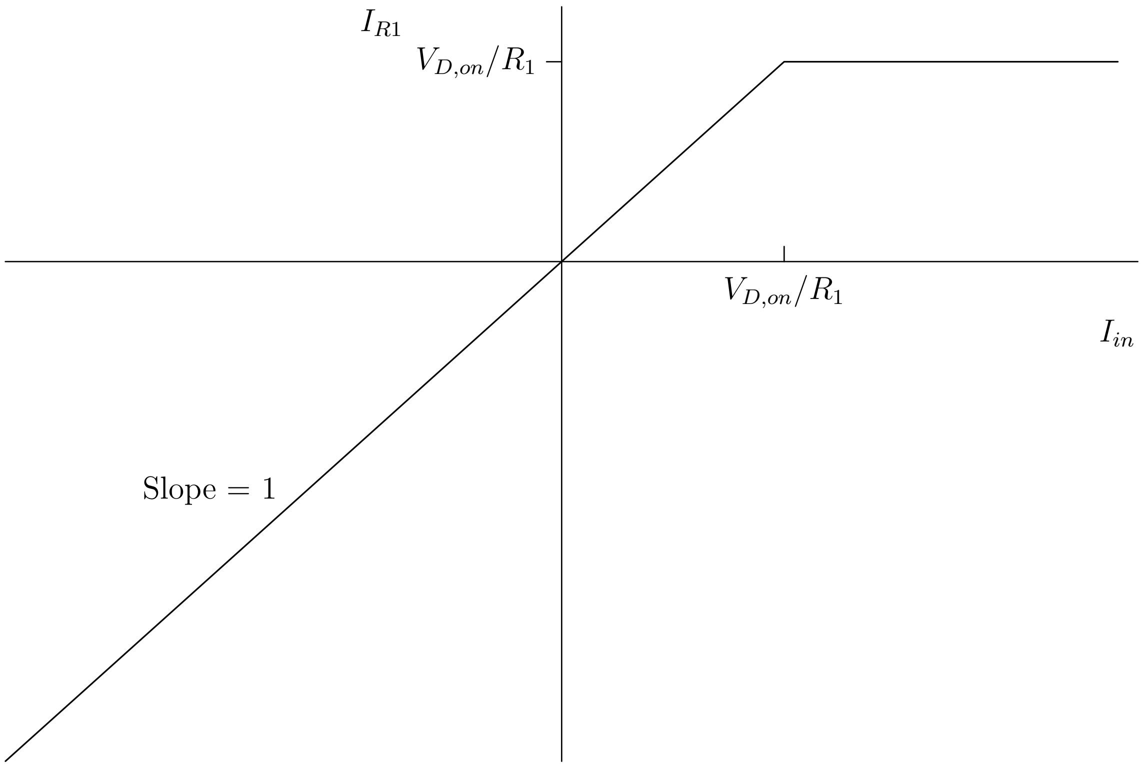
(c)

$$I_{R1} = \begin{cases} I_{in} & I_{in} < \frac{V_{D,on}-V_B}{R_1} \\ \frac{V_{D,on}-V_B}{R_1} & I_{in} > \frac{V_{D,on}-V_B}{R_1} \end{cases}$$



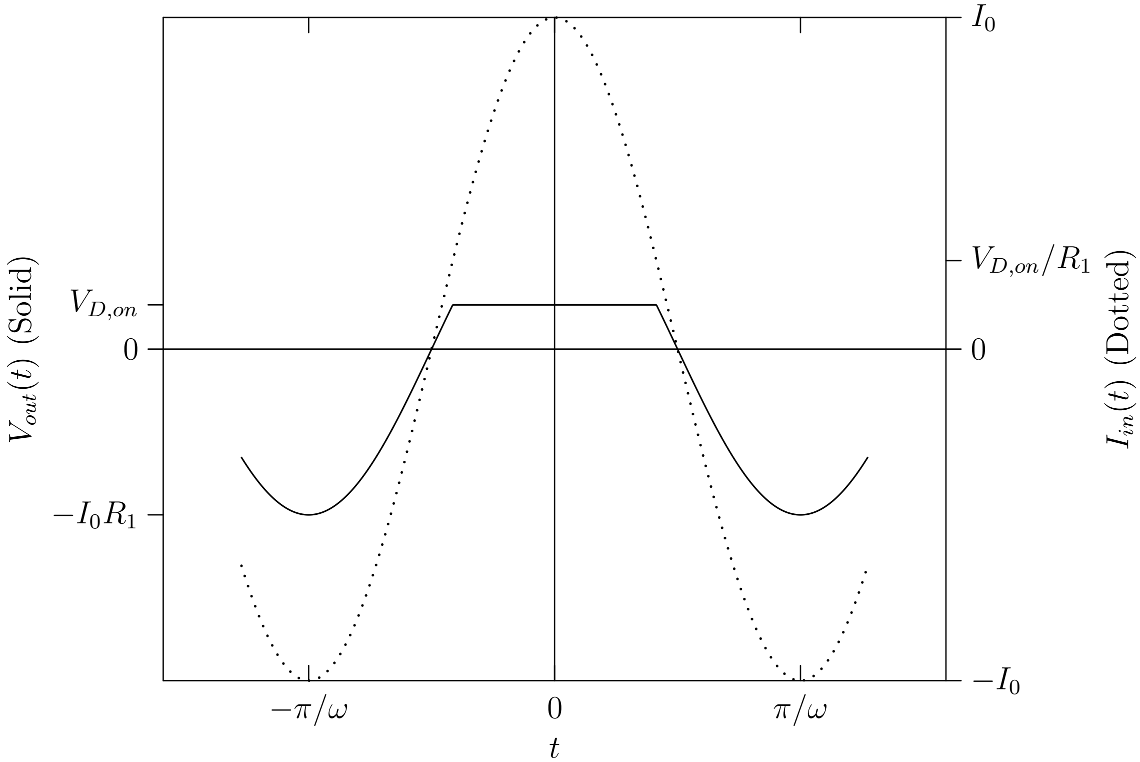
(d)

$$I_{R1} = \begin{cases} I_{in} & I_{in} < \frac{V_{D,on}}{R_1} \\ \frac{V_{D,on}}{R_1} & I_{in} > \frac{V_{D,on}}{R_1} \end{cases}$$



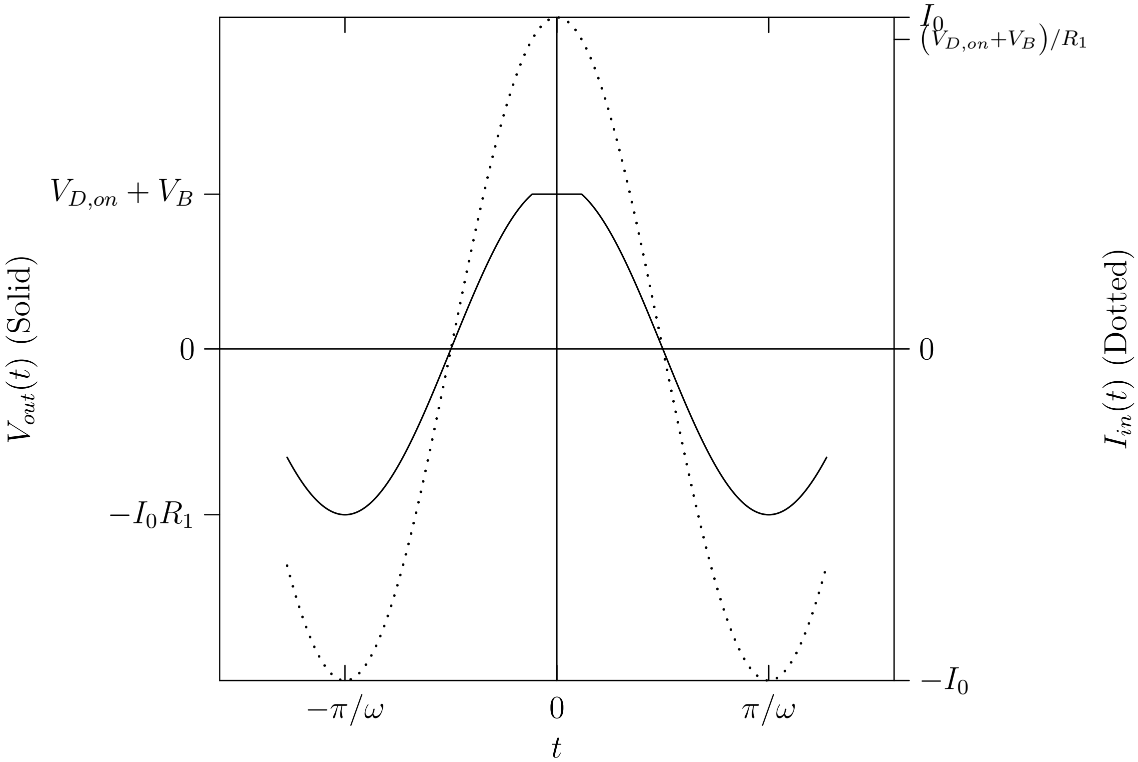
3.17 (a)

$$V_{out} = \begin{cases} I_{in}R_1 & I_{in} < \frac{V_{D,on}}{R_1} \\ V_{D,on} & I_{in} > \frac{V_{D,on}}{R_1} \end{cases}$$



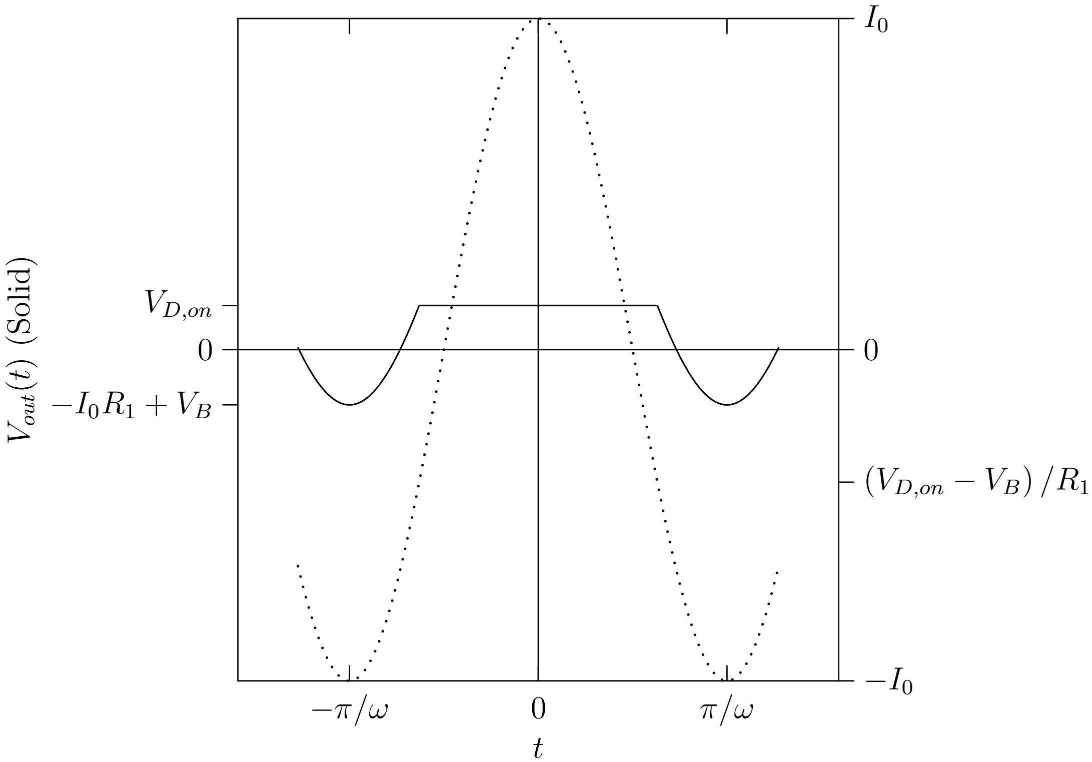
(b)

$$V_{out} = \begin{cases} I_{in}R_1 & I_{in} < \frac{V_{D,on}+V_B}{R_1} \\ V_{D,on}+V_B & I_{in} > \frac{V_{D,on}+V_B}{R_1} \end{cases}$$



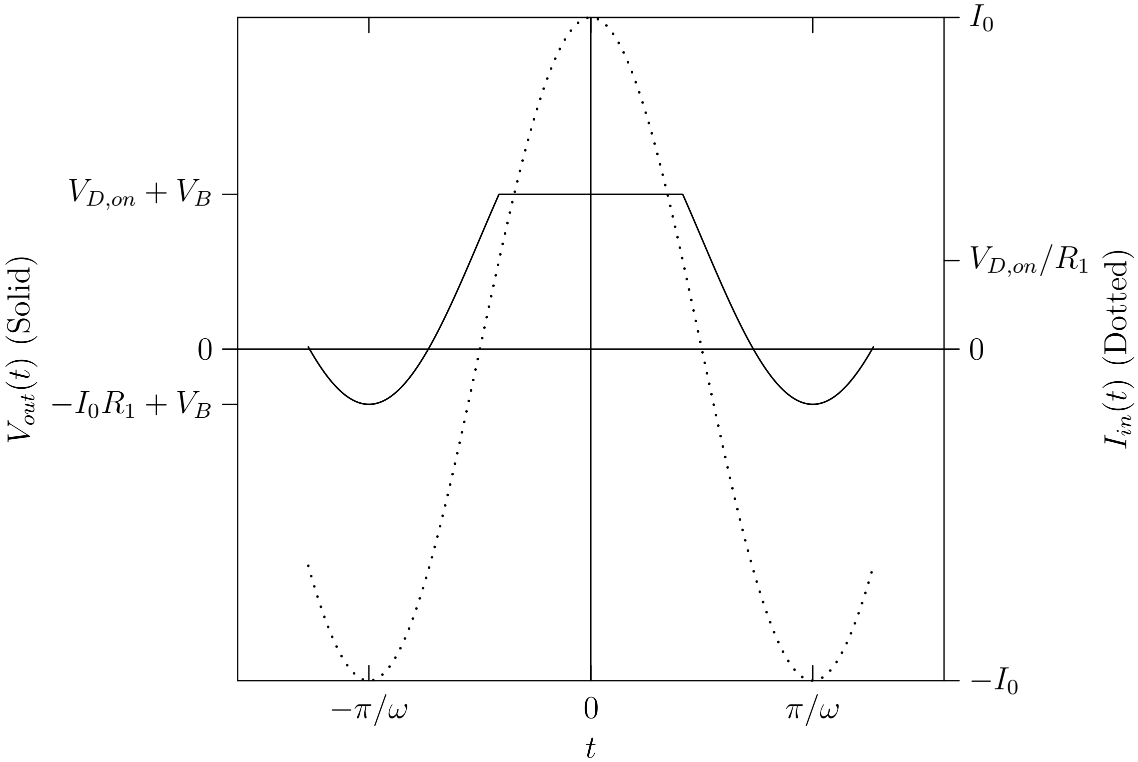
(c)

$$V_{out} = \begin{cases} I_{in}R_1 + V_B & I_{in} < \frac{V_{D,on}-V_B}{R_1} \\ V_{D,on} & I_{in} > \frac{V_{D,on}-V_B}{R_1} \end{cases}$$

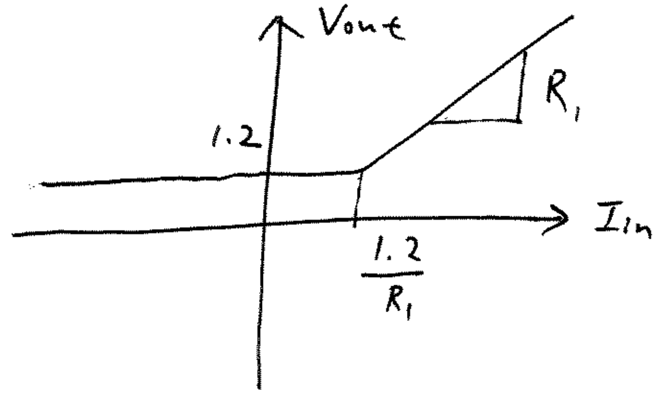


(d)

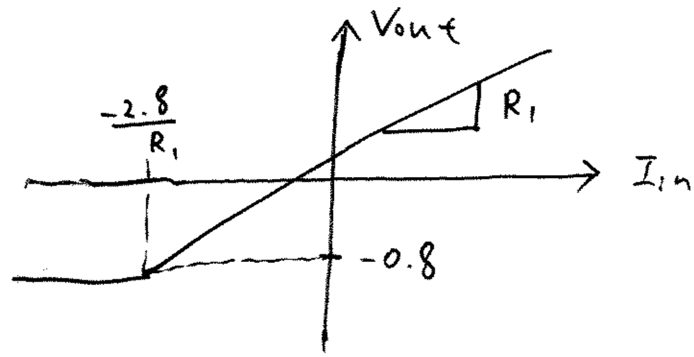
$$V_{out} = \begin{cases} I_{in}R_1 + V_B & I_{in} < \frac{V_{D,on}}{R_1} \\ V_{D,on} + V_B & I_{in} > \frac{V_{D,on}}{R_1} \end{cases}$$



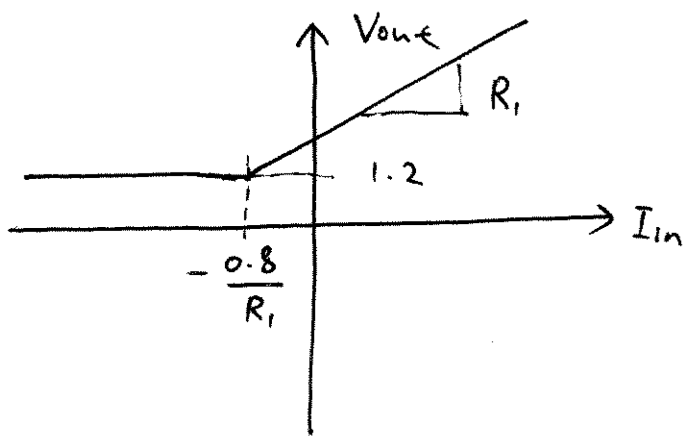
(18) a)



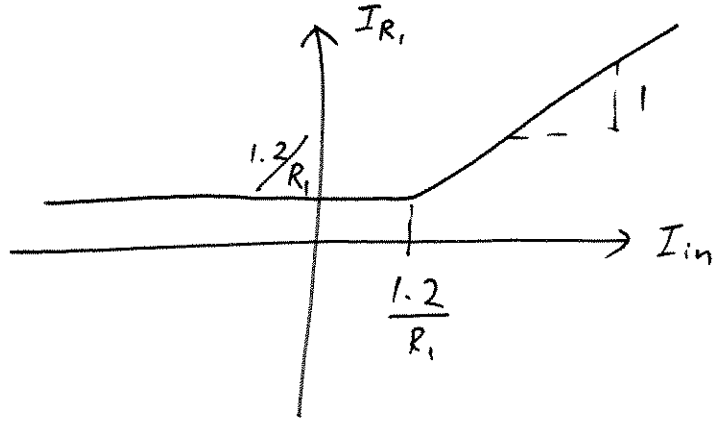
b)



c)

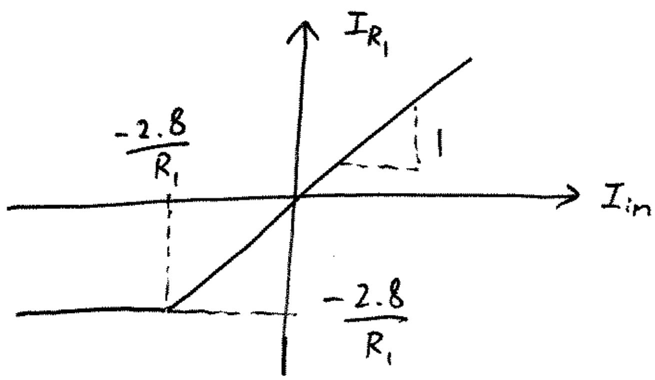


(19) a)



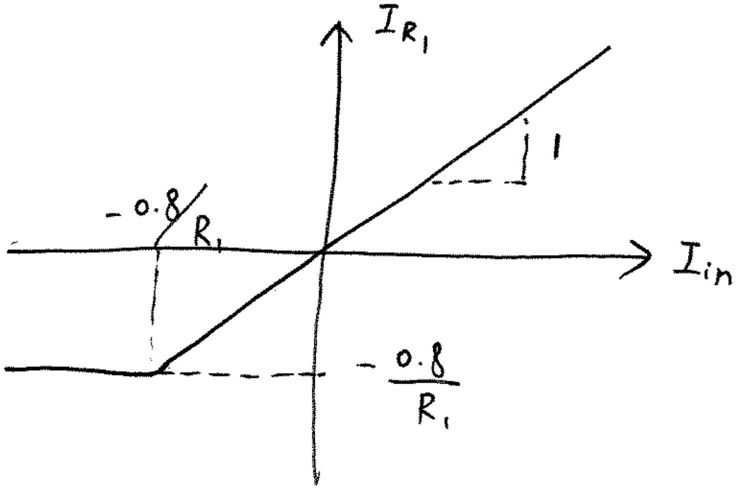
I_{R_1}
$1.2/R_1$
1
I_{in}
$\frac{1.2}{R_1}$

b)



I_{R_1}
$\frac{-2.8}{R_1}$
1
I_{in}
$-\frac{2.8}{R_1}$

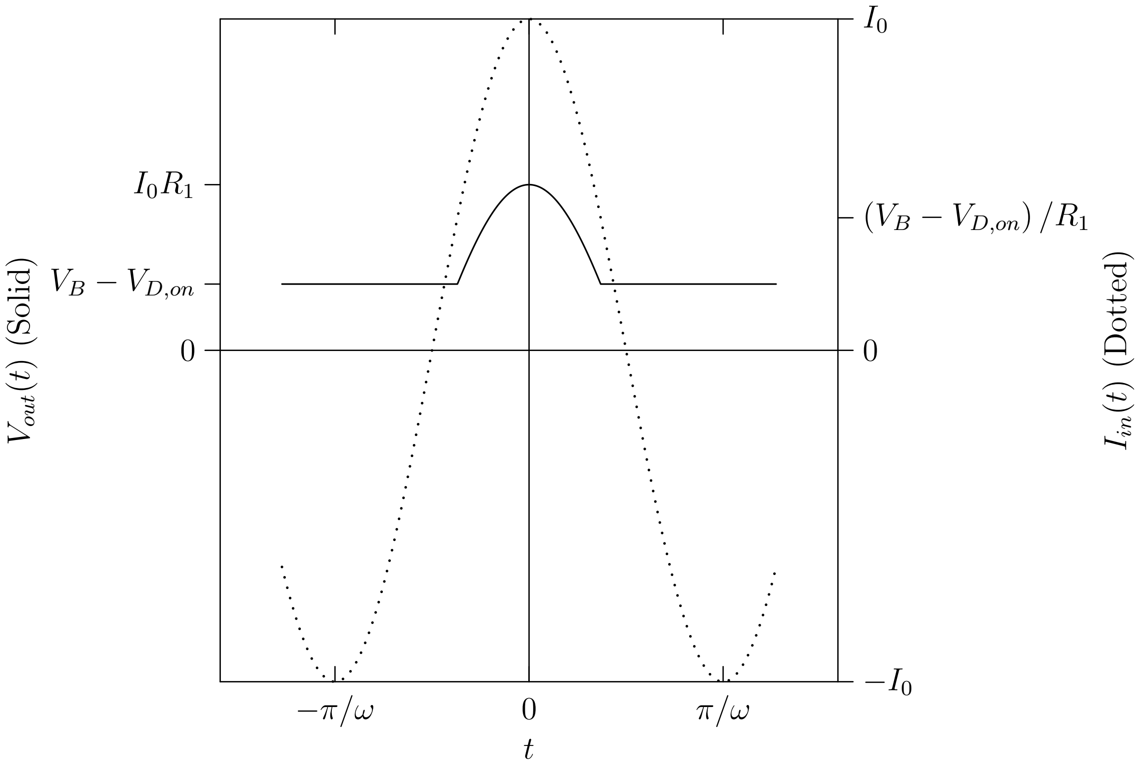
c)



I_{R_1}
$-0.8/R_1$
1
I_{in}
$-\frac{0.8}{R_1}$

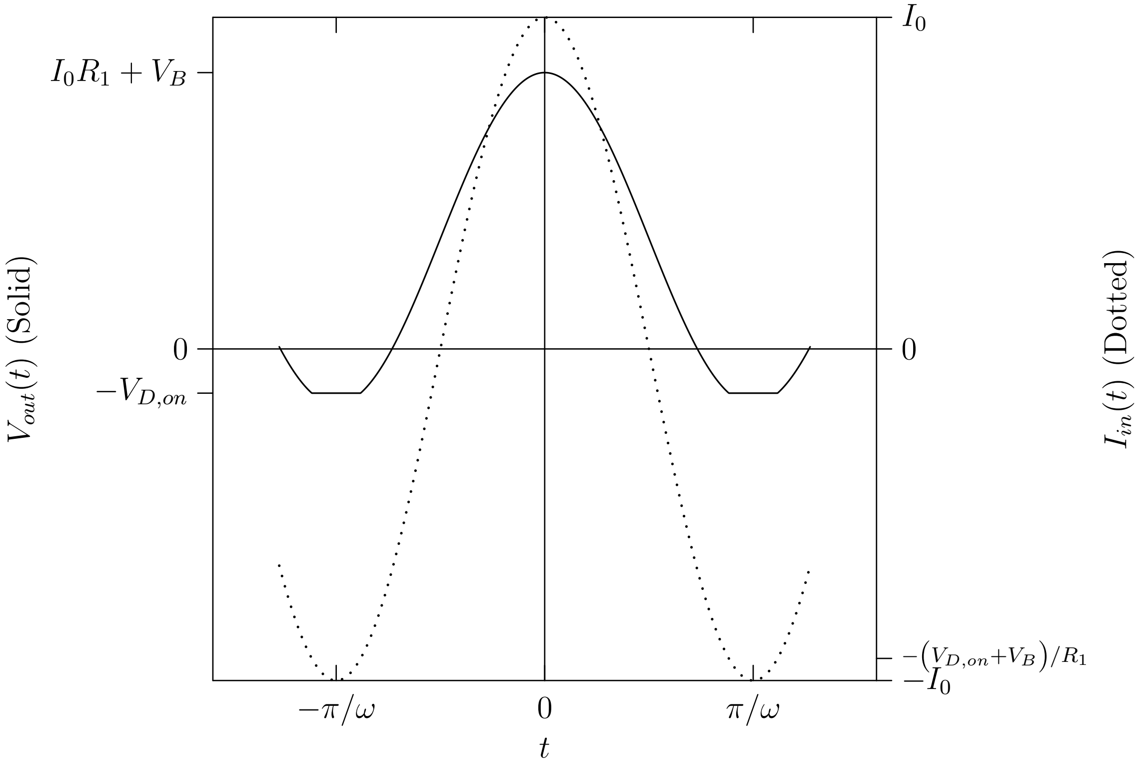
3.20 (a)

$$V_{out} = \begin{cases} I_{in} R_1 & I_{in} > \frac{V_B - V_{D,on}}{R_1} \\ V_B - V_{D,on} & I_{in} < \frac{V_B - V_{D,on}}{R_1} \end{cases}$$



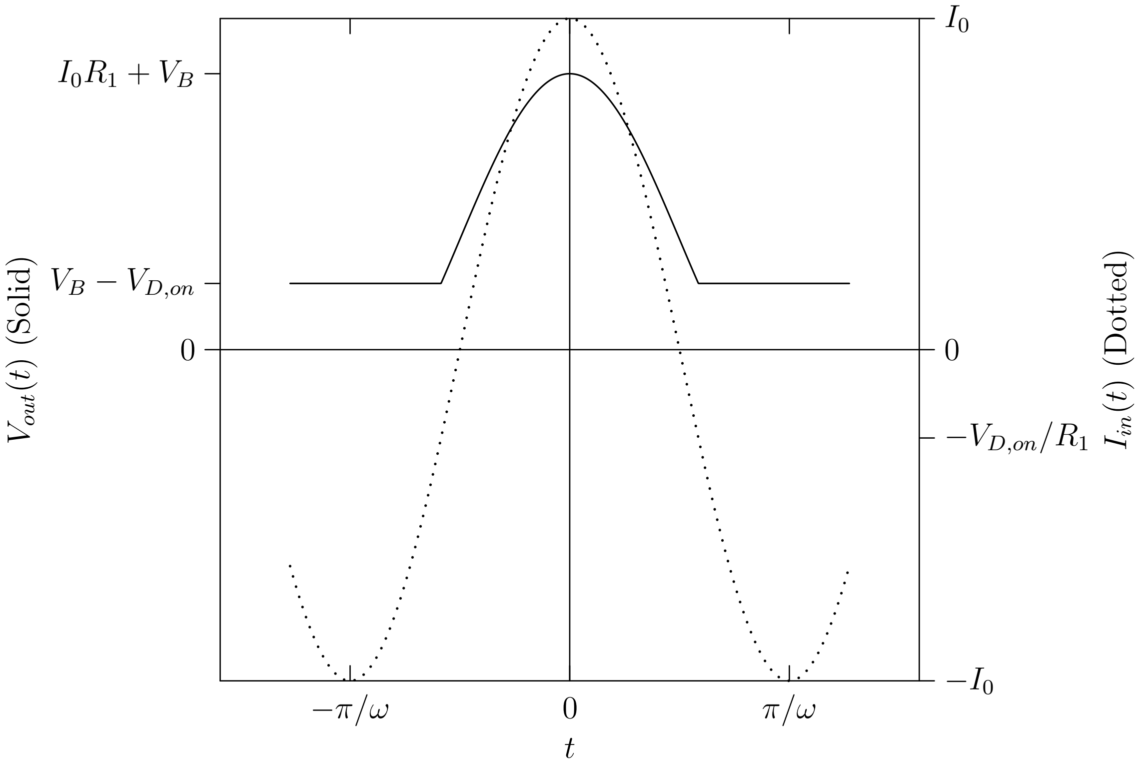
(b)

$$V_{out} = \begin{cases} I_{in} R_1 + V_B & I_{in} > -\frac{V_{D,on} + V_B}{R_1} \\ -V_{D,on} & I_{in} < -\frac{V_{D,on} + V_B}{R_1} \end{cases}$$

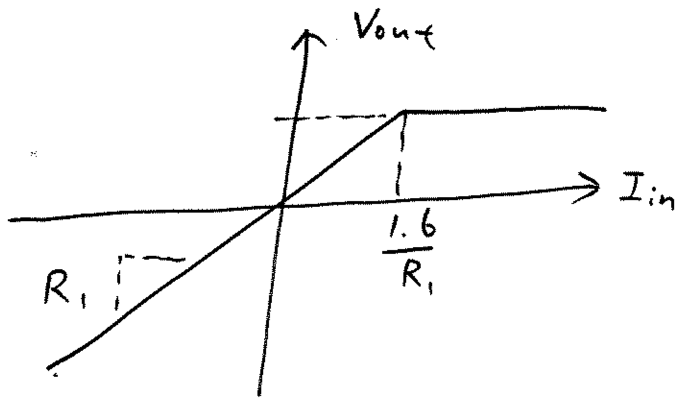


(c)

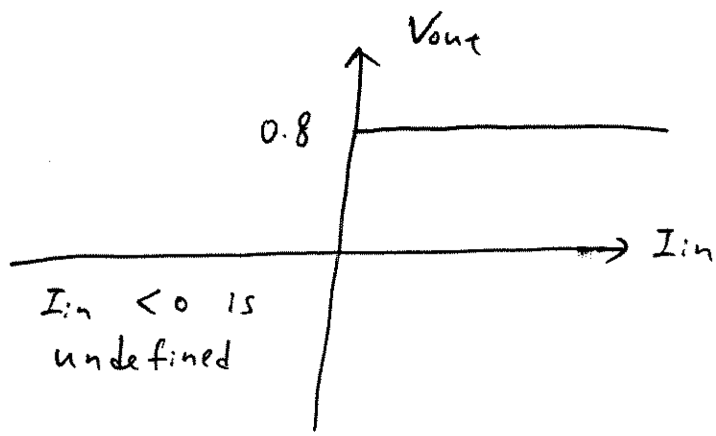
$$V_{out} = \begin{cases} I_{in}R_1 + V_B & I_{in} > -\frac{V_{D,on}}{R_1} \\ V_B - V_{D,on} & I_{in} < -\frac{V_{D,on}}{R_1} \end{cases}$$



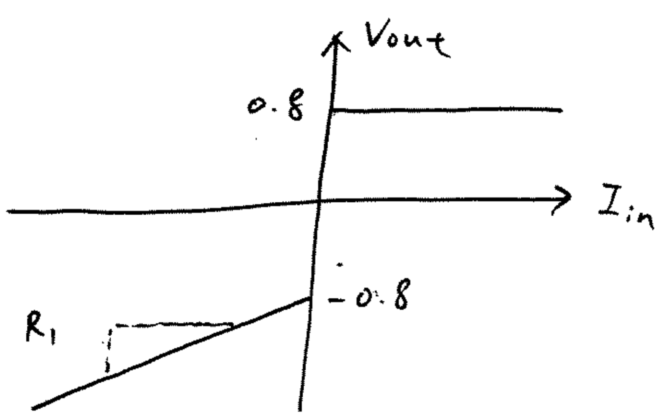
(21) a)


Vout
Iin
1.6/R1
R1

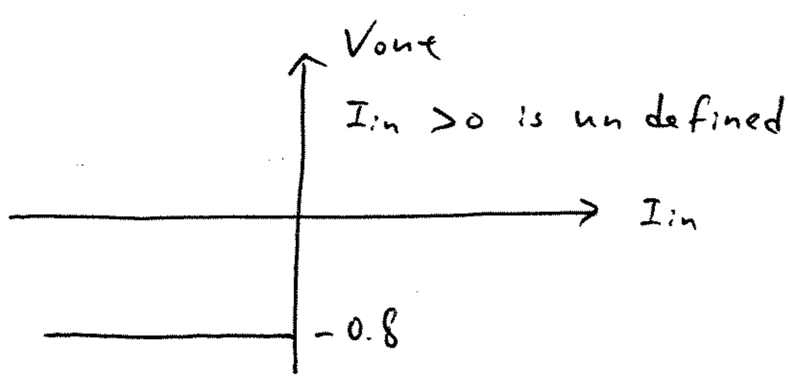
b)


Vout
0.8
Iin
Iin < 0 is undefined

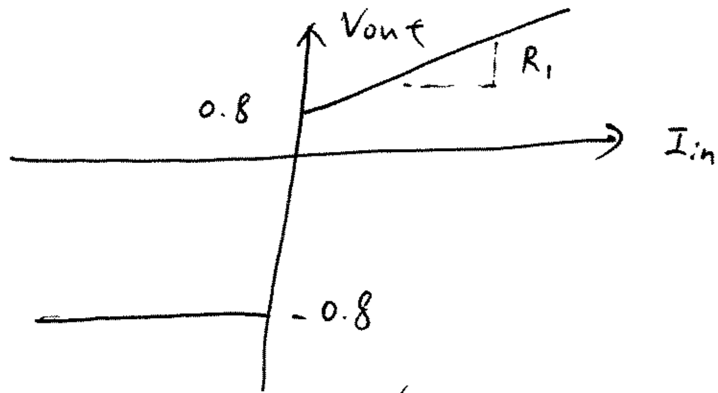
c)


Vout
0.8
Iin
-0.8
R1

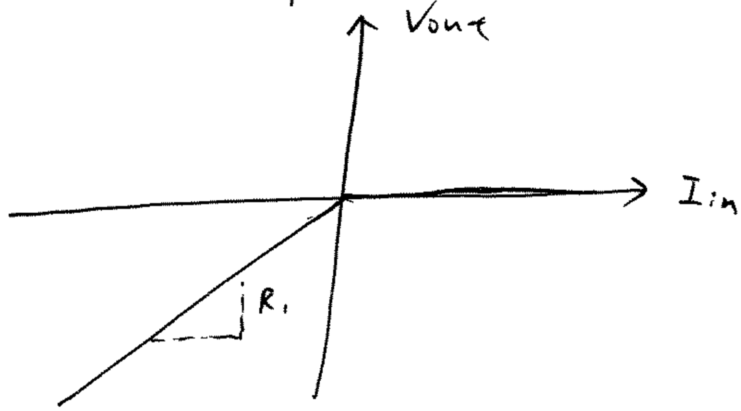
d)


Vout
Iin > 0 is undefined
Iin
-0.8

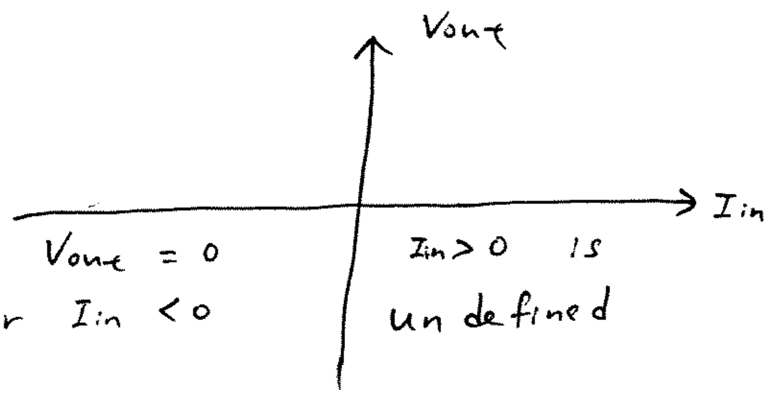
e)


Vout
R1
0.8
Iin
-0.8

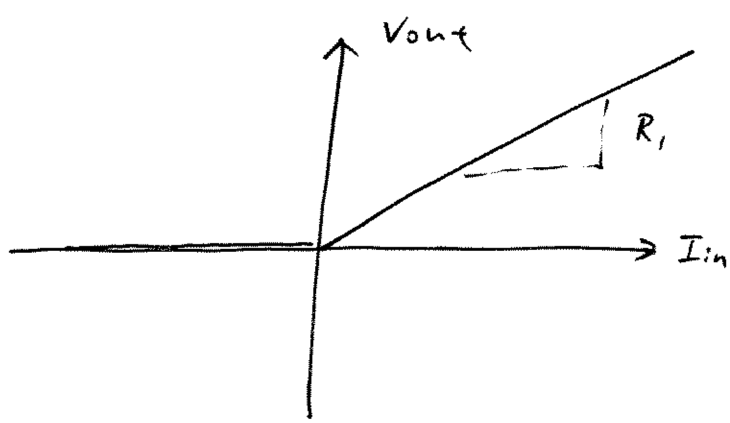
f)


Vout
Iin
R1

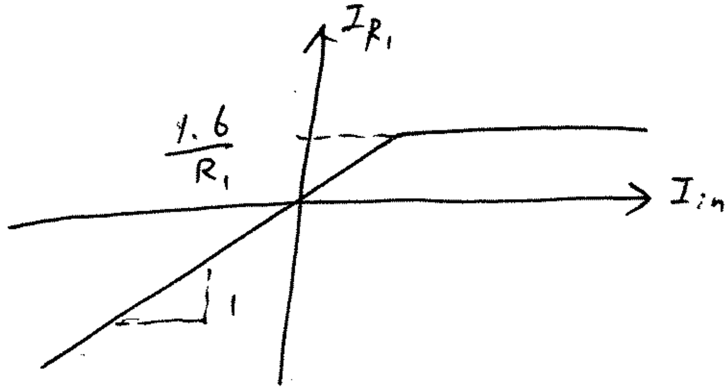
g)


Vout
Iin
Vout = 0
for Iin < 0
Iin > 0 is
undefined

h)

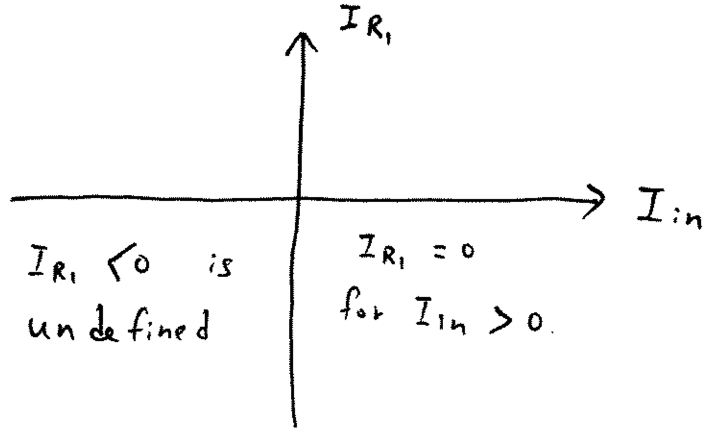

Vout
R1
Iin

(22) a)



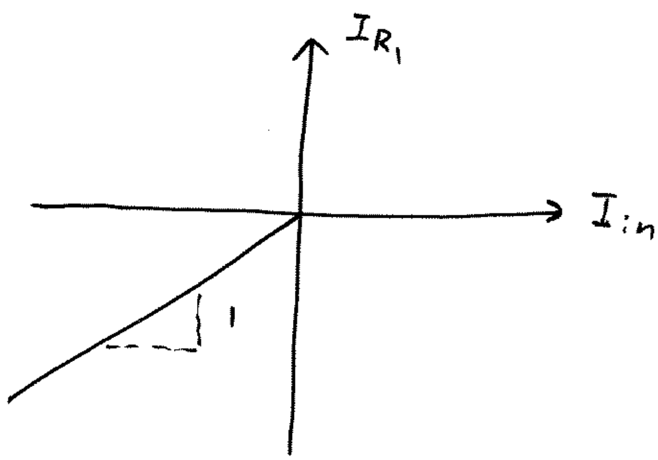
I_{R_1}
$\frac{4.6}{R_1}$
I_{in}
1

b)



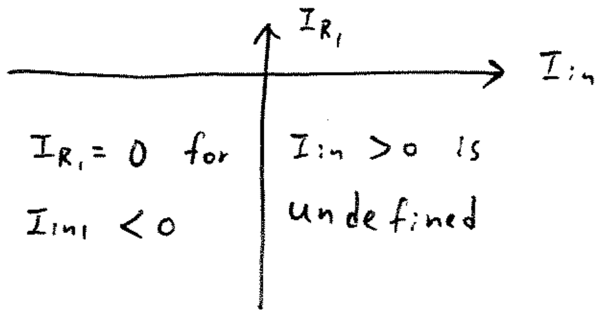
I_{R_1}
I_{in}
$I_{R_1} < 0$ is undefined
$I_{R_1} = 0$ for $I_{in} > 0$

c)



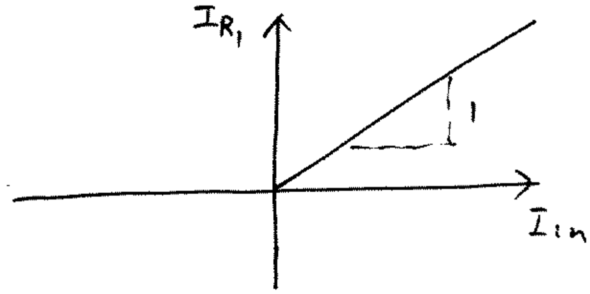
I_{R_1}
I_{in}
1

d)

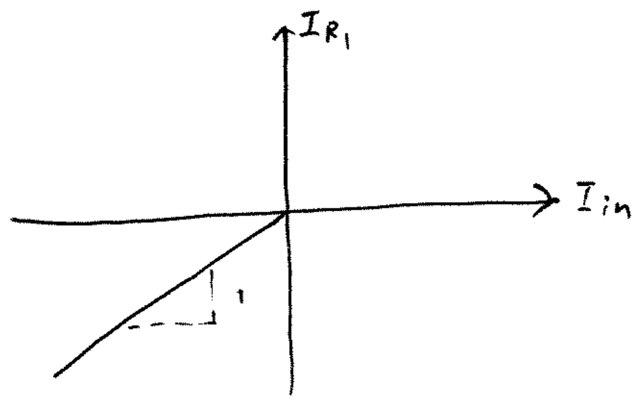


I_{R_1}
I_{in}
$I_{R_1} = 0$ for $I_{in} < 0$
$I_{in} > 0$ is undefined

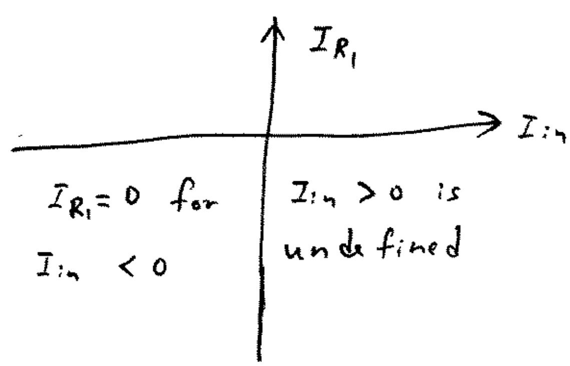
e)


I_{R_1}
1
I_{in}

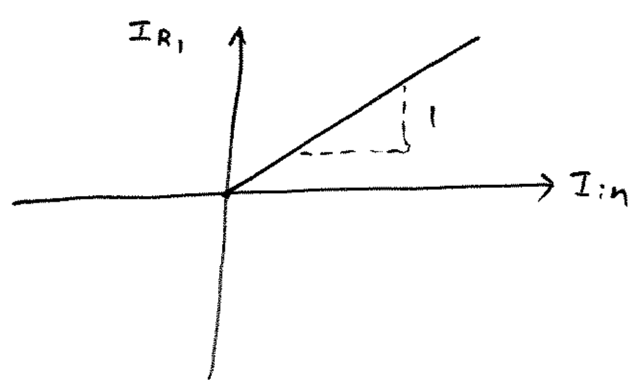
f)


I_{R_1}
I_{in}
1

g)

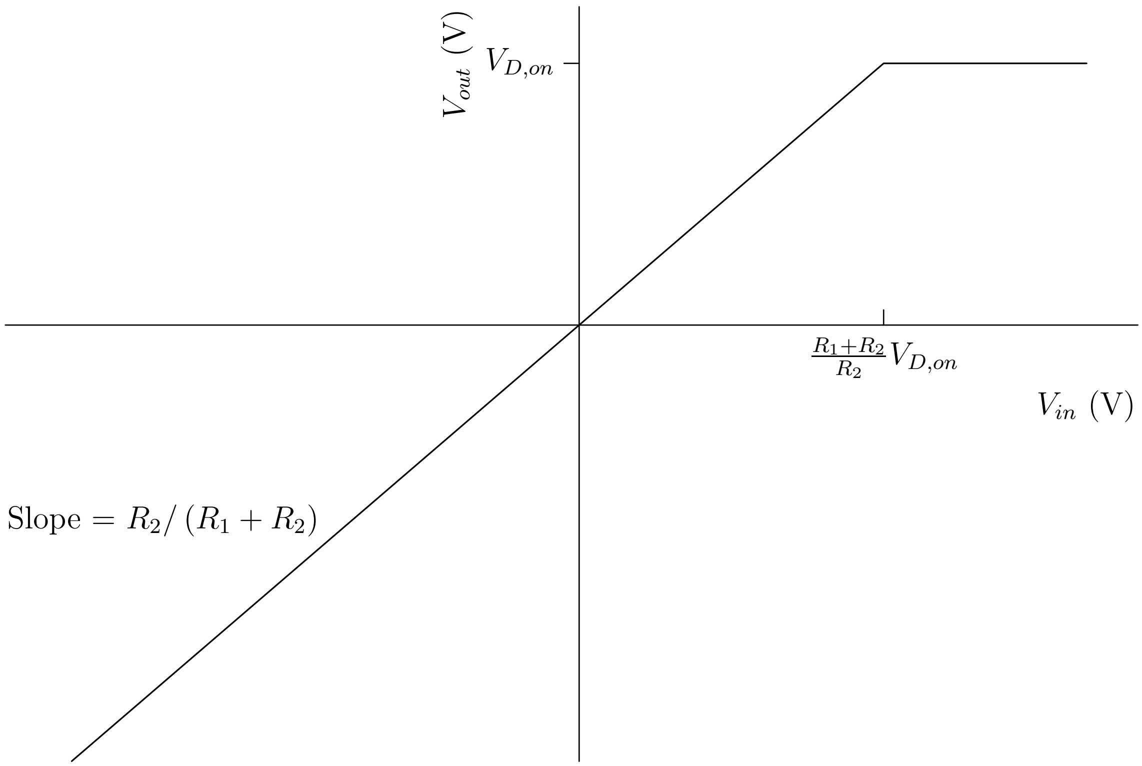

I_{R_1}
I_{in}
$I_{R_1} = 0$ for $I_{in} < 0$
$I_{in} > 0$ is undefined

h)


I_{R_1}
1
I_{in}

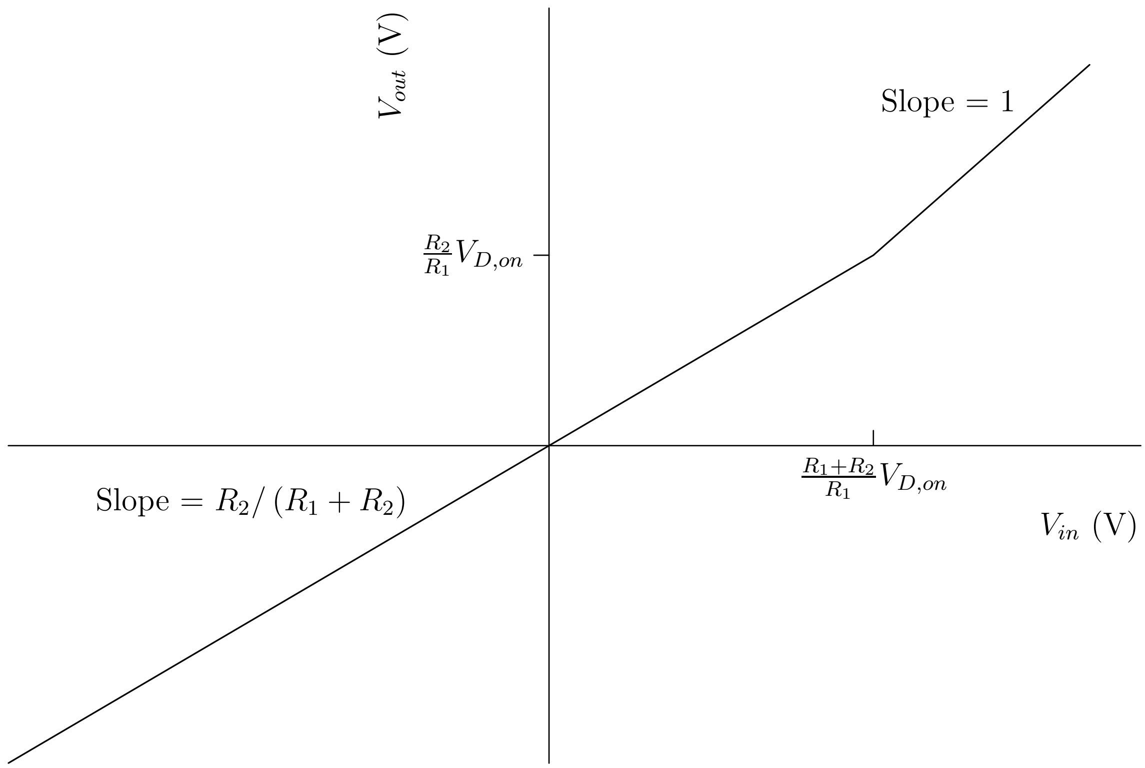
3.23 (a)

$$V_{out} = \begin{cases} \frac{R_2}{R_1+R_2} V_{in} & V_{in} < \frac{R_1+R_2}{R_2} V_{D,on} \\ V_{D,on} & V_{in} > \frac{R_1+R_2}{R_2} V_{D,on} \end{cases}$$



(b)

$$V_{out} = \begin{cases} \frac{R_2}{R_1+R_2} V_{in} & V_{in} < \frac{R_1+R_2}{R_1} V_{D,on} \\ V_{in} - V_{D,on} & V_{in} > \frac{R_1+R_2}{R_1} V_{D,on} \end{cases}$$

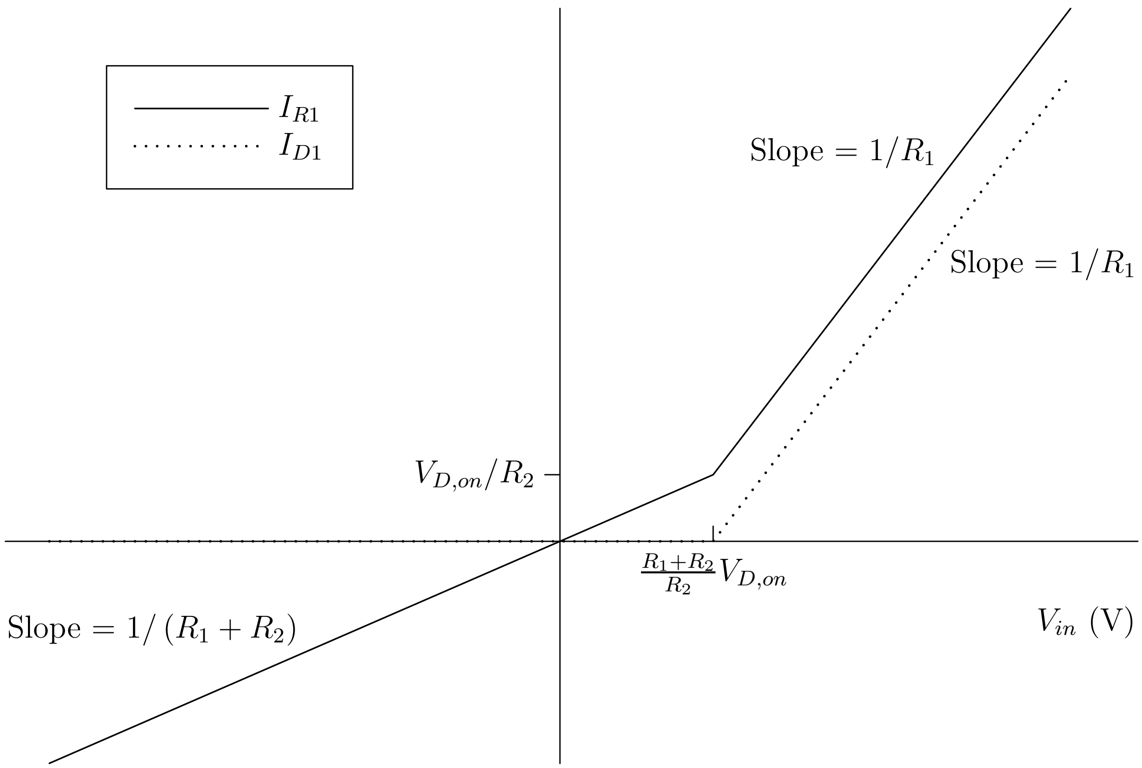


V_{out} (V)
Slope = 1
$\frac{R_2}{R_1}V_{D,on}$
$\frac{R_1+R_2}{R_1}V_{D,on}$
Slope = $R_2/(R_1+R_2)$
V_{in} (V)

3.24 (a)

$$I_{R1} = \begin{cases} \frac{V_{in}}{R_1+R_2} & V_{in} < \frac{R_1+R_2}{R_2} V_{D,on} \\ \frac{V_{in}-V_{D,on}}{R_1} & V_{in} > \frac{R_1+R_2}{R_2} V_{D,on} \end{cases}$$

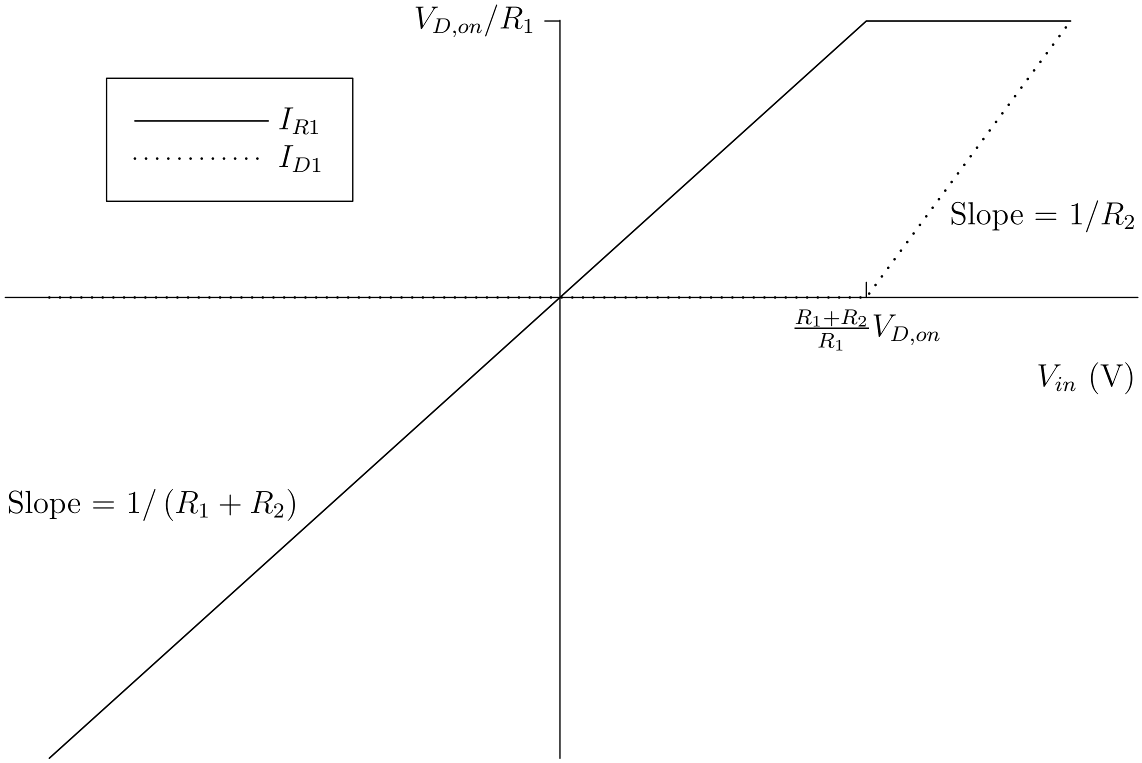
$$I_{D1} = \begin{cases} 0 & V_{in} < \frac{R_1+R_2}{R_2} V_{D,on} \\ \frac{V_{in}-V_{D,on}}{R_1} - \frac{V_{D,on}}{R_2} & V_{in} > \frac{R_1+R_2}{R_2} V_{D,on} \end{cases}$$



(b)

$$I_{R1} = \begin{cases} \frac{V_{in}}{R_1+R_2} & V_{in} < \frac{R_1+R_2}{R_1} V_{D,on} \\ \frac{V_{D,on}}{R_1} & V_{in} > \frac{R_1+R_2}{R_1} V_{D,on} \end{cases}$$

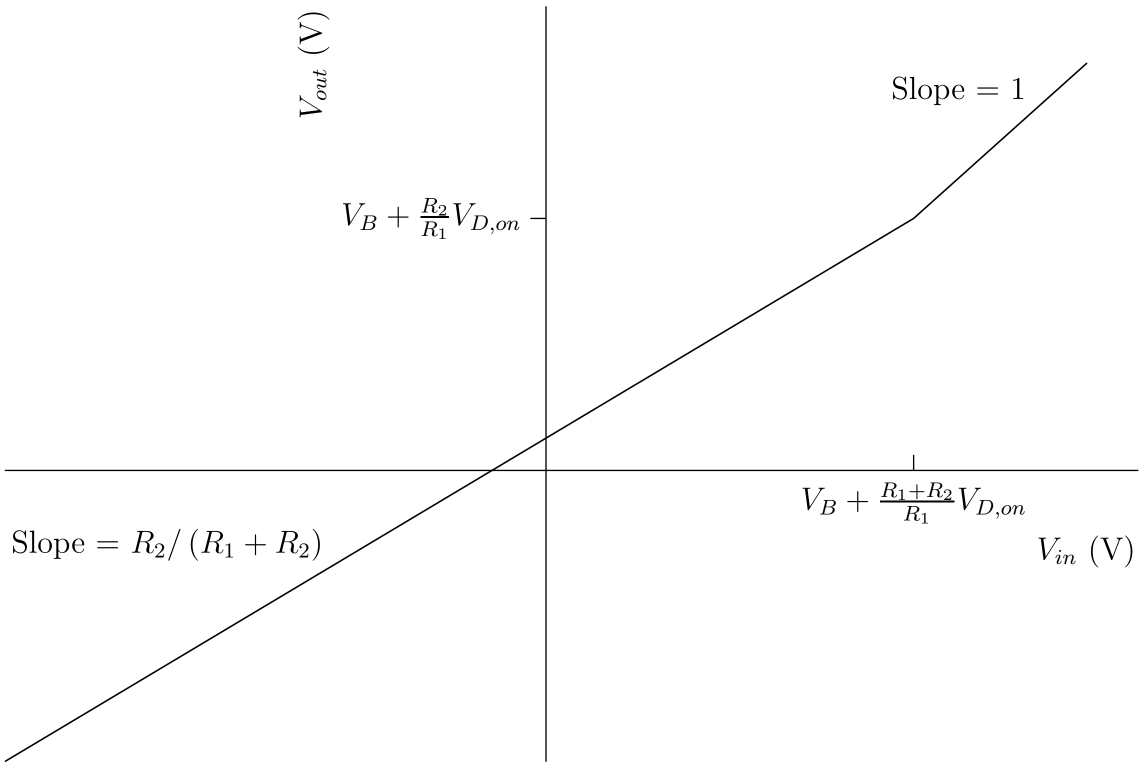
$$I_{D1} = \begin{cases} 0 & V_{in} < \frac{R_1+R_2}{R_1} V_{D,on} \\ \frac{V_{in}-V_{D,on}}{R_2} - \frac{V_{D,on}}{R_1} & V_{in} > \frac{R_1+R_2}{R_1} V_{D,on} \end{cases}$$



$V_{D,on}/R_1$
I_{R1}
I_{D1}
Slope = $1/R_2$
$\frac{R_1+R_2}{R_1}V_{D,on}$
V_{in} (V)
Slope = $1/(R_1+R_2)$

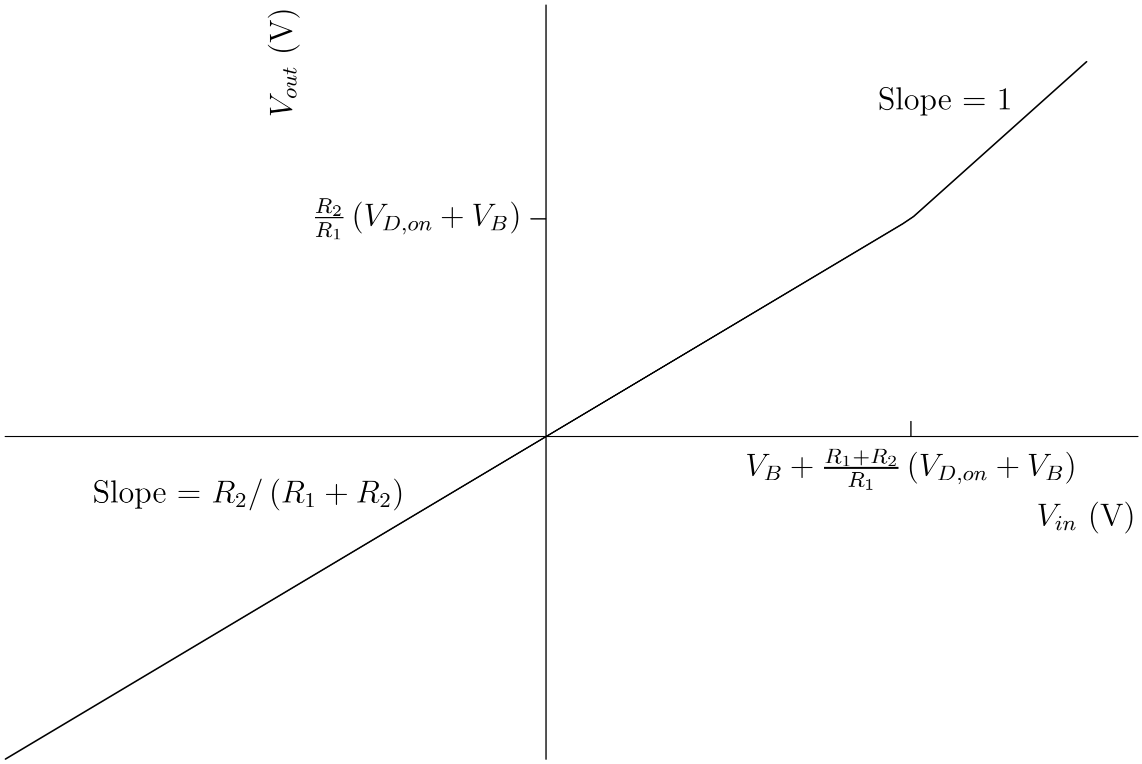
3.25 (a)

$$V_{out} = \begin{cases} V_B + \frac{R_2}{R_1+R_2}\left(V_{in} - V_B\right) & V_{in} < V_B + \frac{R_1+R_2}{R_1}V_{D,on} \\ V_{in} - V_{D,on} & V_{in} > V_B + \frac{R_1+R_2}{R_1}V_{D,on} \end{cases}$$



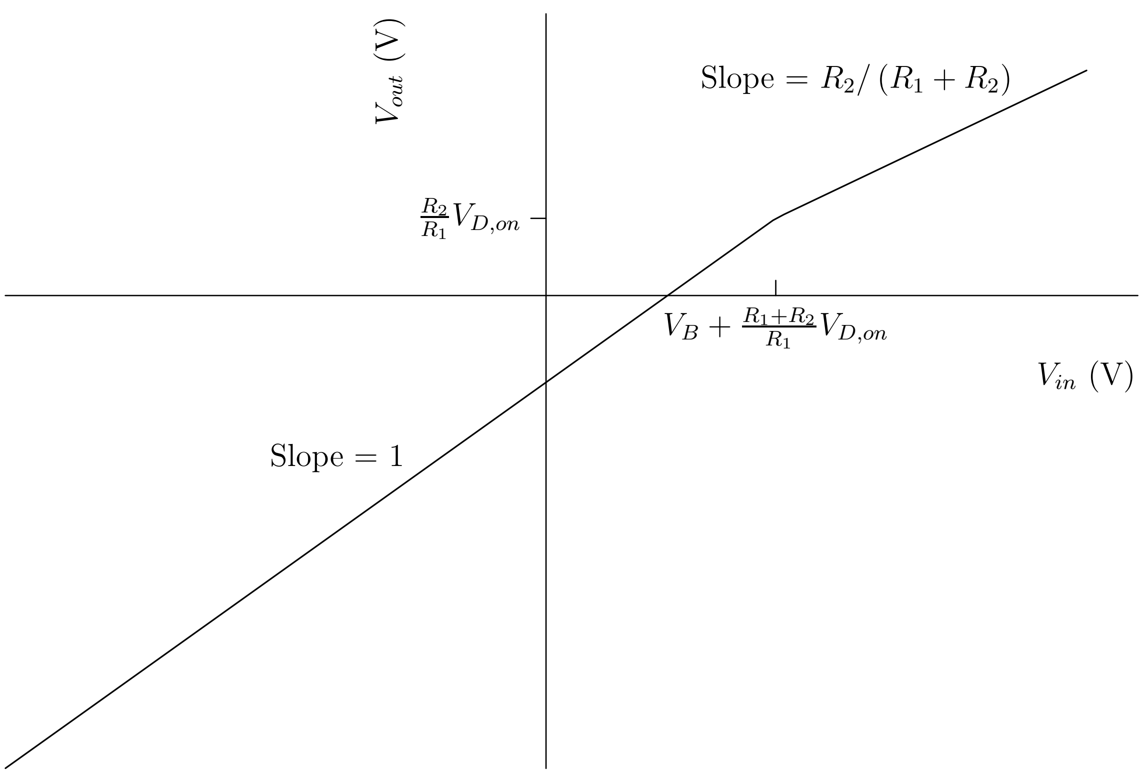
(b)

$$V_{out} = \begin{cases} \frac{R_2}{R_1+R_2}V_{in} & V_{in} < \frac{R_1+R_2}{R_1}\left(V_{D,on} + V_B\right) \\ V_{in} - V_{D,on} - V_B & V_{in} > \frac{R_1+R_2}{R_1}\left(V_{D,on} + V_B\right) \end{cases}$$



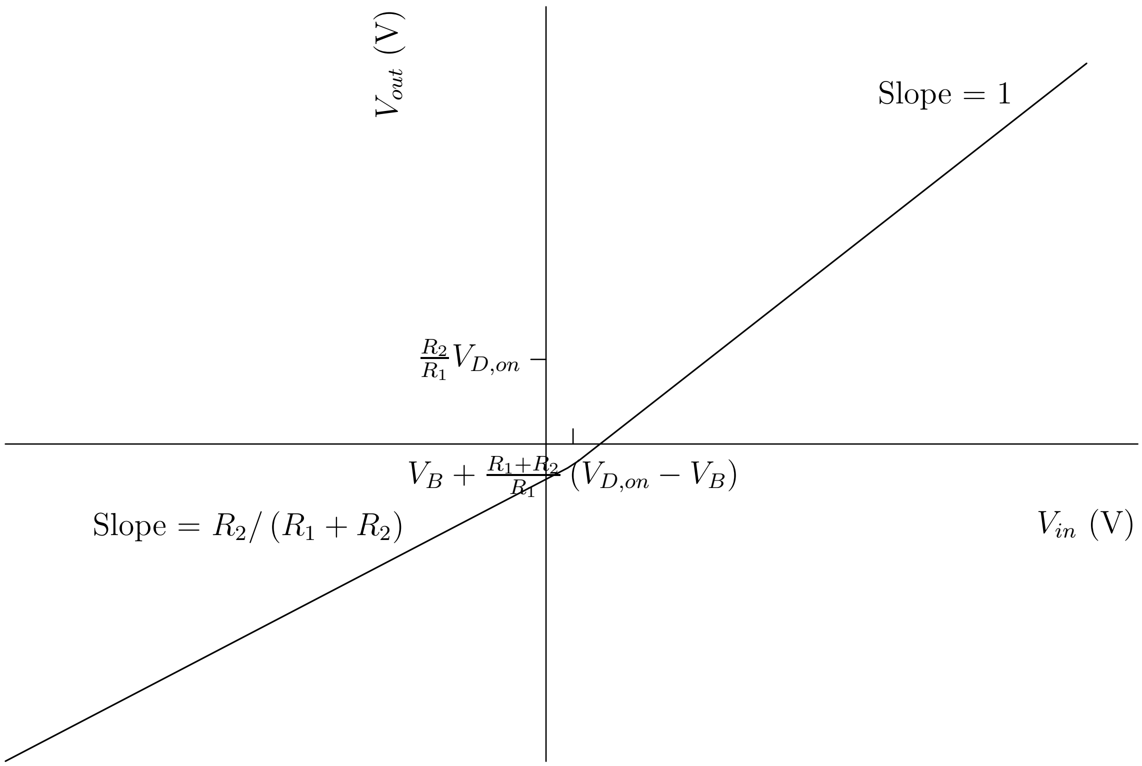
(c)

$$V_{out} = \begin{cases} \frac{R_2}{R_1+R_2}(V_{in}-V_B) & V_{in} > V_B + \frac{R_1+R_2}{R_1}V_{D,on} \\ V_{in}+V_{D,on}-V_B & V_{in} < V_B + \frac{R_1+R_2}{R_1}V_{D,on} \end{cases}$$



(d)

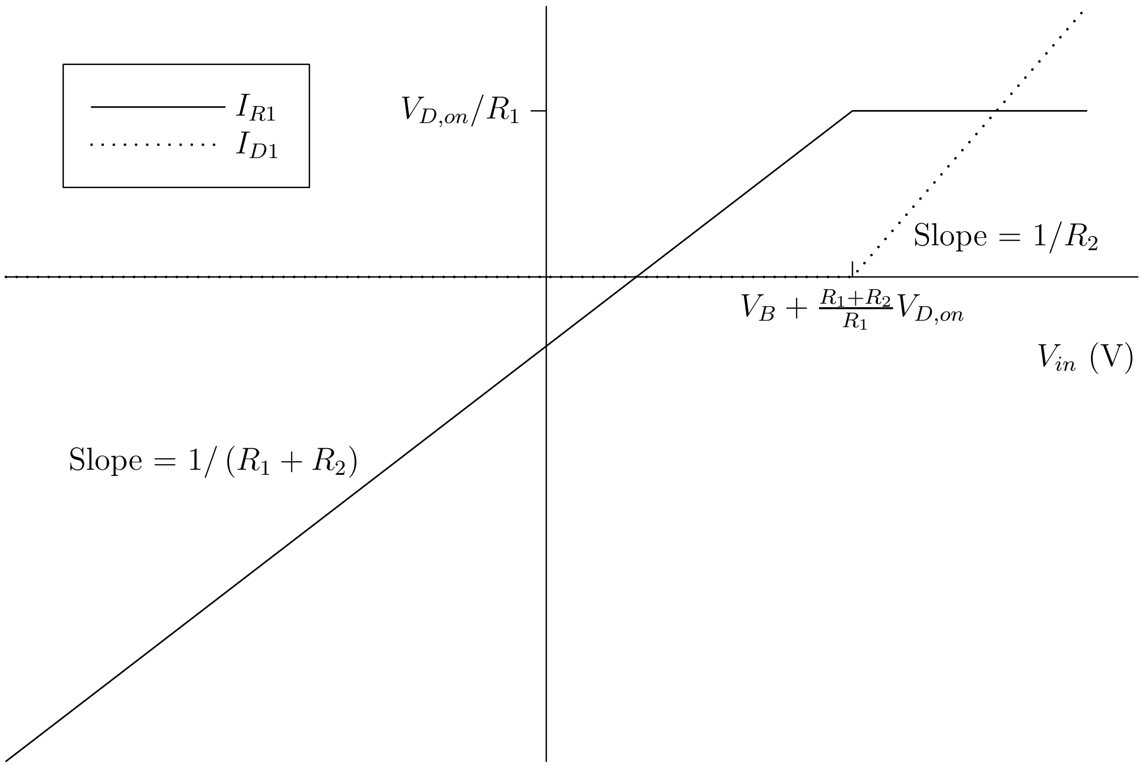
$$V_{out} = \begin{cases} \frac{R_2}{R_1+R_2}\left(V_{in} - V_B\right) & V_{in} < V_B + \frac{R_1+R_2}{R_1}\left(V_{D,on} - V_B\right) \\ V_{in} - V_{D,on} & V_{in} > V_B + \frac{R_1+R_2}{R_1}\left(V_{D,on} - V_B\right) \end{cases}$$



3.26 (a)

$$I_{R1} = \begin{cases} \frac{V_{in}-V_B}{R_1+R_2} & V_{in} < V_B + \frac{R_1+R_2}{R_1}V_{D,on} \\ \frac{V_{D,on}}{R_1} & V_{in} > V_B + \frac{R_1+R_2}{R_1}V_{D,on} \end{cases}$$

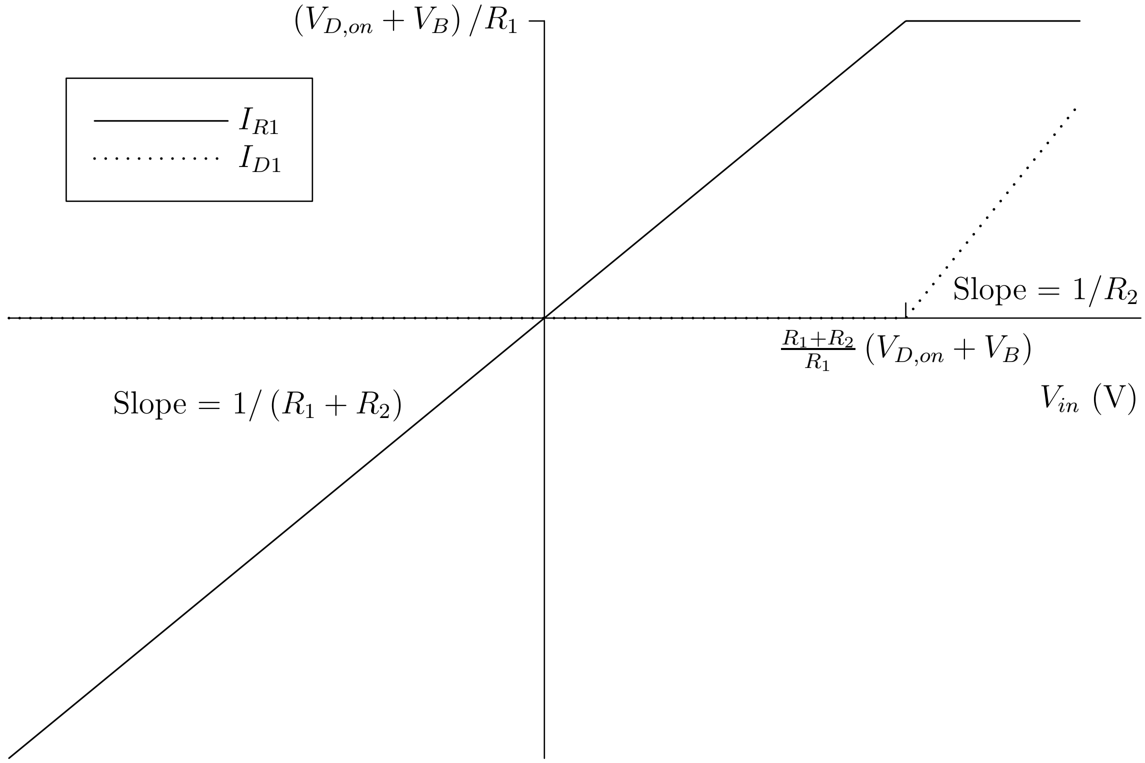
$$I_{D1} = \begin{cases} 0 & V_{in} < V_B + \frac{R_1+R_2}{R_1}V_{D,on} \\ \frac{V_{in}-V_{D,on}-V_B}{R_2} - \frac{V_{D,on}}{R_1} & V_{in} > V_B + \frac{R_1+R_2}{R_1}V_{D,on} \end{cases}$$



(b)

$$I_{R1} = \begin{cases} \frac{V_{in}}{R_1+R_2} & V_{in} < \frac{R_1+R_2}{R_1}(V_{D,on}+V_B) \\ \frac{V_{D,on}+V_B}{R_1} & V_{in} > \frac{R_1+R_2}{R_1}(V_{D,on}+V_B) \end{cases}$$

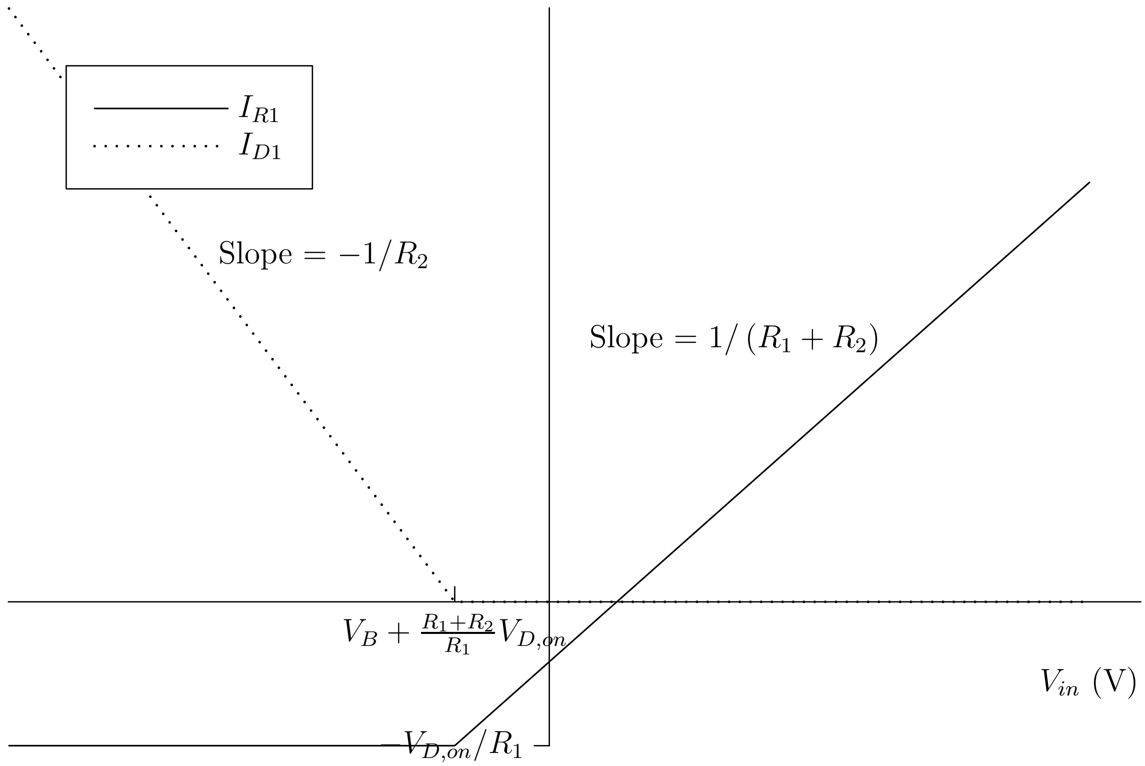
$$I_{D1} = \begin{cases} 0 & V_{in} < \frac{R_1+R_2}{R_1}(V_{D,on}+V_B) \\ \frac{V_{in}-V_{D,on}-V_B}{R_2} - \frac{V_{D,on}+V_B}{R_1} & V_{in} > \frac{R_1+R_2}{R_1}(V_{D,on}+V_B) \end{cases}$$



(c)

$$I_{R1} = \begin{cases} \frac{V_{in}-V_B}{R_1+R_2} & V_{in} > V_B - \frac{R_1+R_2}{R_1}V_{D,on} \\ -\frac{V_{D,on}}{R_1} & V_{in} < V_B - \frac{R_1+R_2}{R_1}V_{D,on} \end{cases}$$

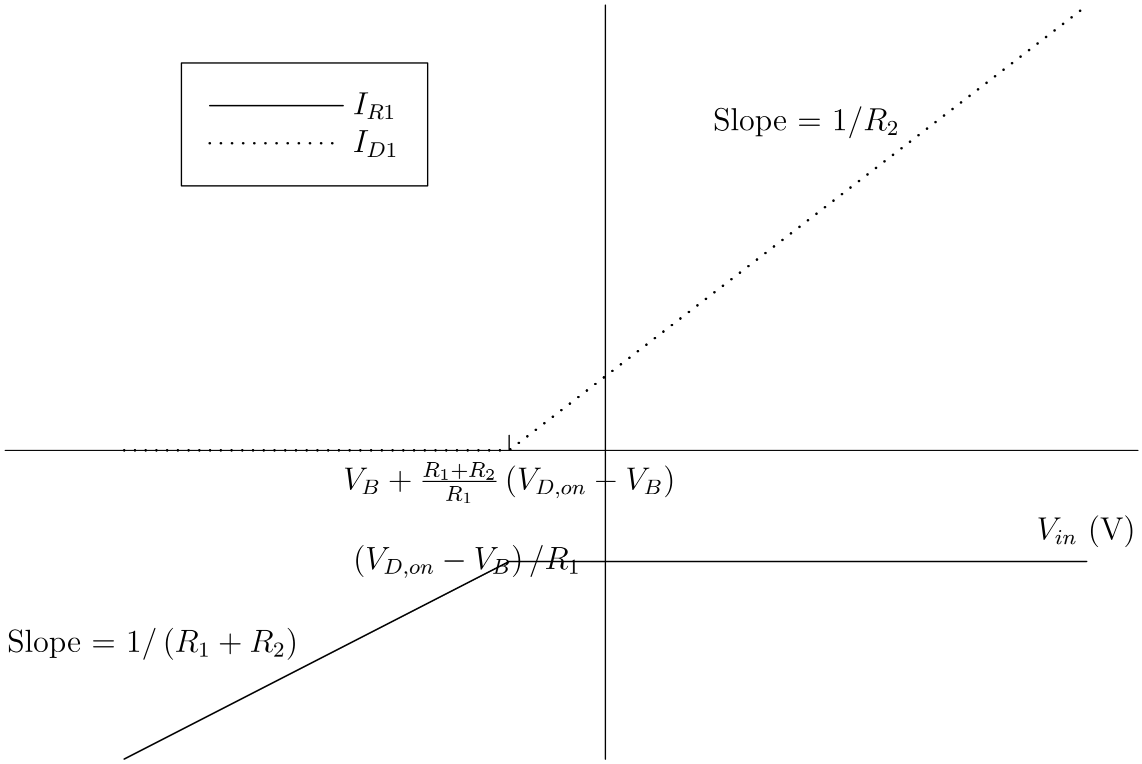
$$I_{D1} = \begin{cases} 0 & V_{in} > V_B - \frac{R_1+R_2}{R_1}V_{D,on} \\ -\frac{V_{in}+V_{D,on}+V_B}{R_2} - \frac{V_{D,on}}{R_1} & V_{in} < V_B - \frac{R_1+R_2}{R_1}V_{D,on} \end{cases}$$



(d)

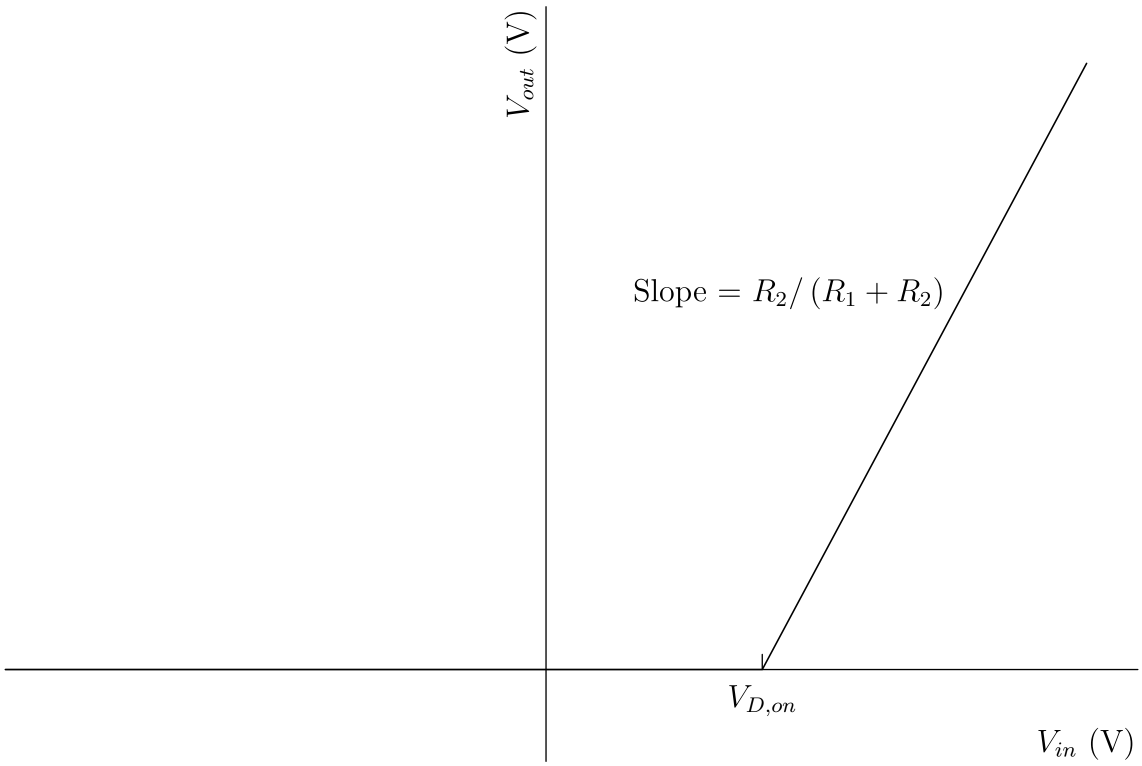
$$I_{R1} = \begin{cases} \frac{V_{in}-V_B}{R_1+R_2} & V_{in} < V_B + \frac{R_1+R_2}{R_1}\left(V_{D,on}-V_B\right) \\ \frac{V_{D,on}-V_B}{R_1} & V_{in} > V_B + \frac{R_1+R_2}{R_1}\left(V_{D,on}-V_B\right) \end{cases}$$

$$I_{D1} = \begin{cases} 0 & V_{in} < V_B + \frac{R_1+R_2}{R_1}\left(V_{D,on}-V_B\right) \\ \frac{V_{in}-V_{D,on}}{R_2} - \frac{V_{D,on}-V_B}{R_1} & V_{in} > V_B + \frac{R_1+R_2}{R_1}\left(V_{D,on}-V_B\right) \end{cases}$$


I_{R1}
I_{D1}
Slope = $1/R_2$
$V_B + \frac{R_1+R_2}{R_1}(V_{D,on} - V_B)$
V_{in} (V)
$(V_{D,on} - V_B)/R_1$
Slope = $1/(R_1 + R_2)$

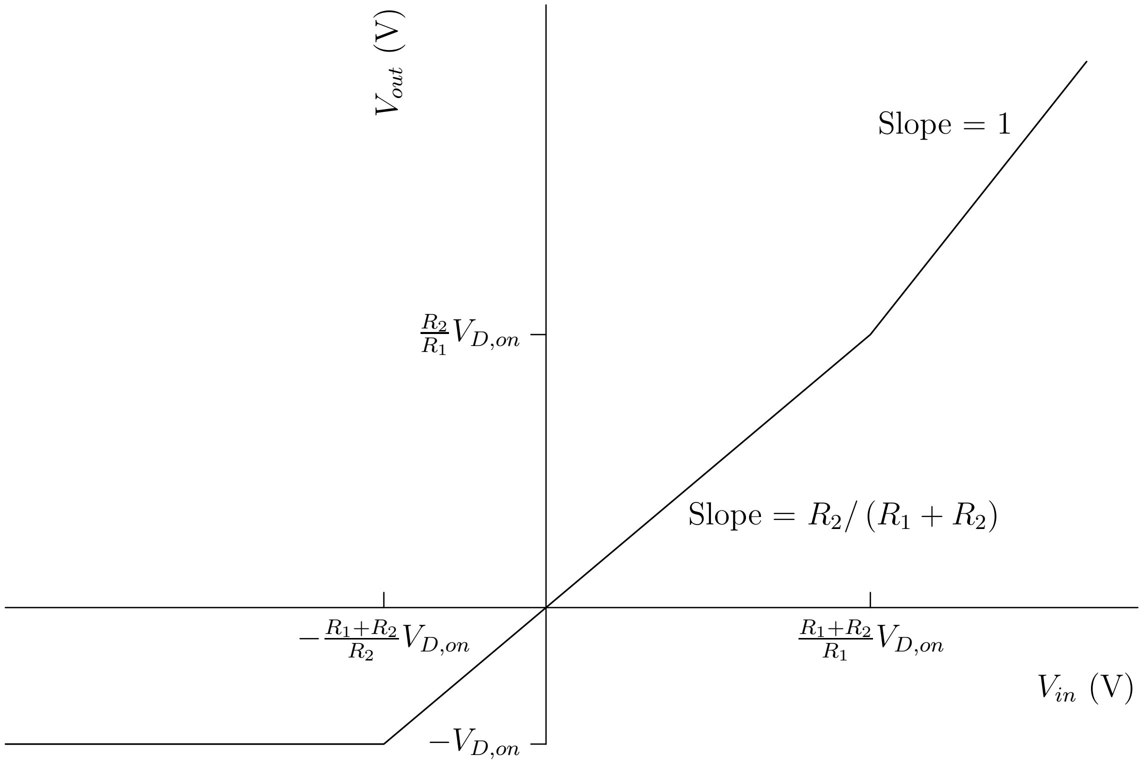
3.27 (a)

$$V_{out} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{R_2}{R_1+R_2}\left(V_{in} - V_{D,on}\right) & V_{in} > V_{D,on} \end{cases}$$



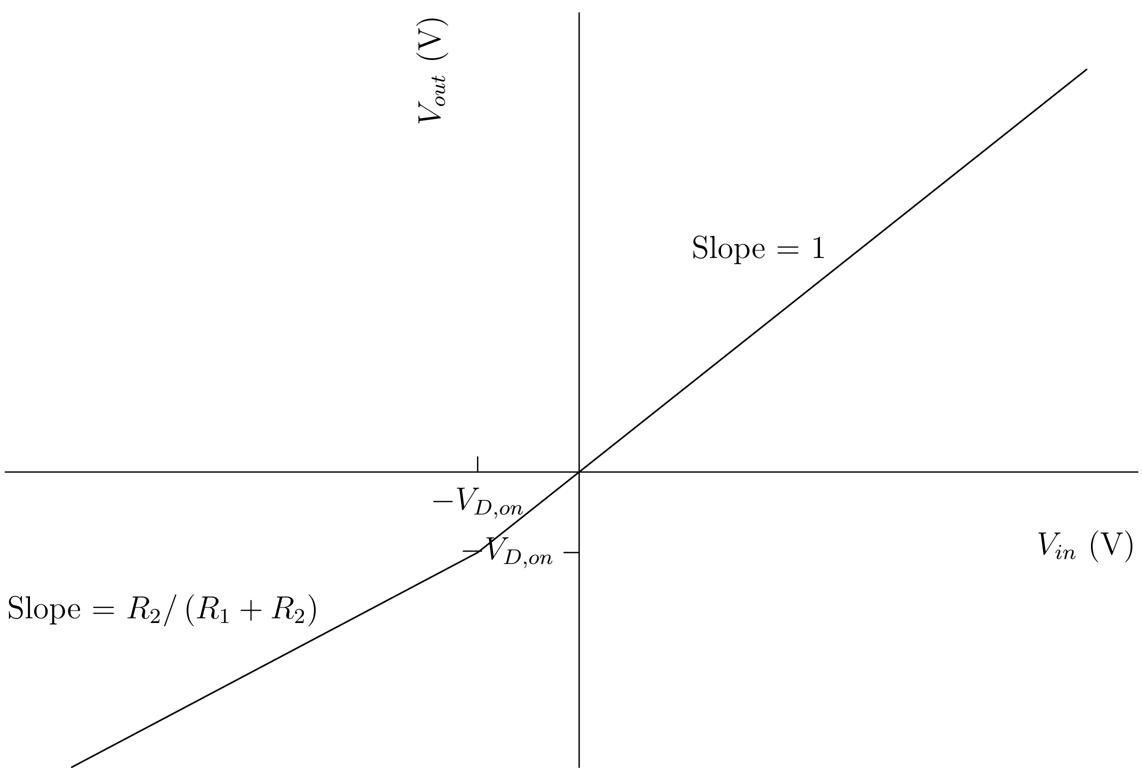
(b)

$$V_{out} = \begin{cases} -V_{D,on} & V_{in} < -\frac{R_1+R_2}{R_2}V_{D,on} \\ \frac{R_2}{R_1+R_2}V_{in} & -\frac{R_1+R_2}{R_2}V_{D,on} < V_{in} < \frac{R_1+R_2}{R_1}V_{D,on} \\ V_{in} - V_{D,on} & V_{in} > \frac{R_1+R_2}{R_1}V_{D,on} \end{cases}$$



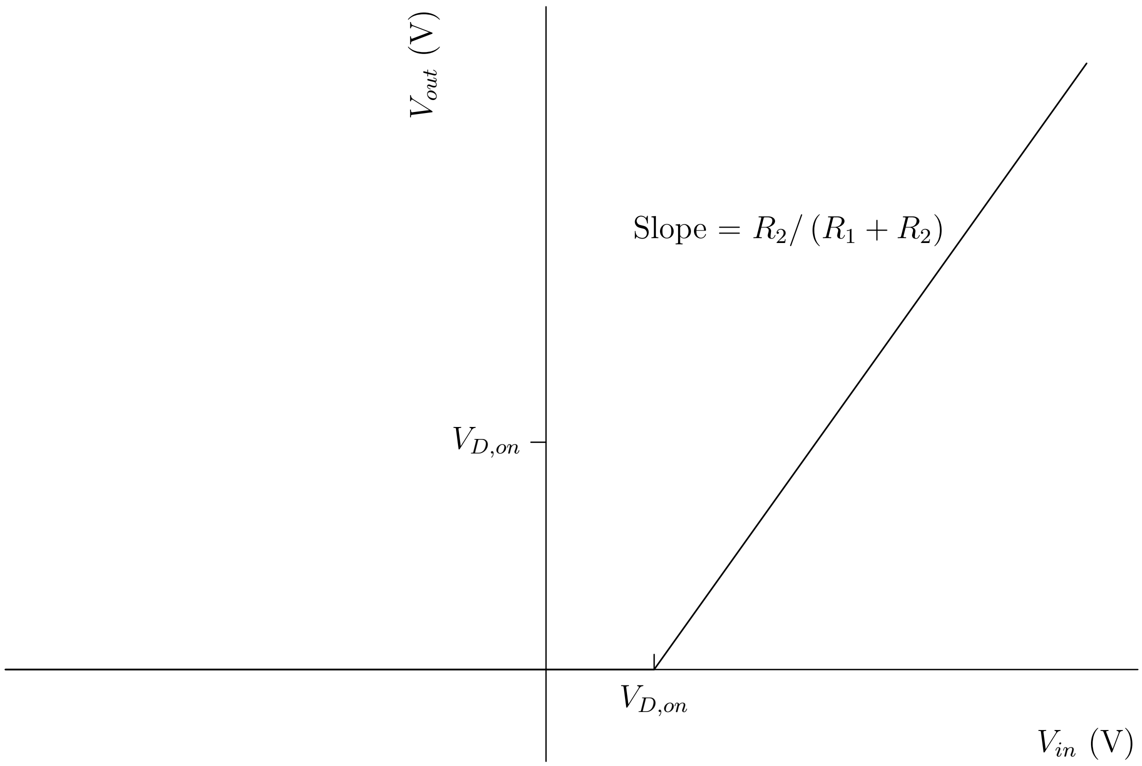
(c)

$$V_{out} = \begin{cases} \frac{R_2}{R_1+R_2}\left(V_{in}+V_{D,on}\right)-V_{D,on} & V_{in} < -V_{D,on} \\ V_{in} & V_{in} > -V_{D,on} \end{cases}$$



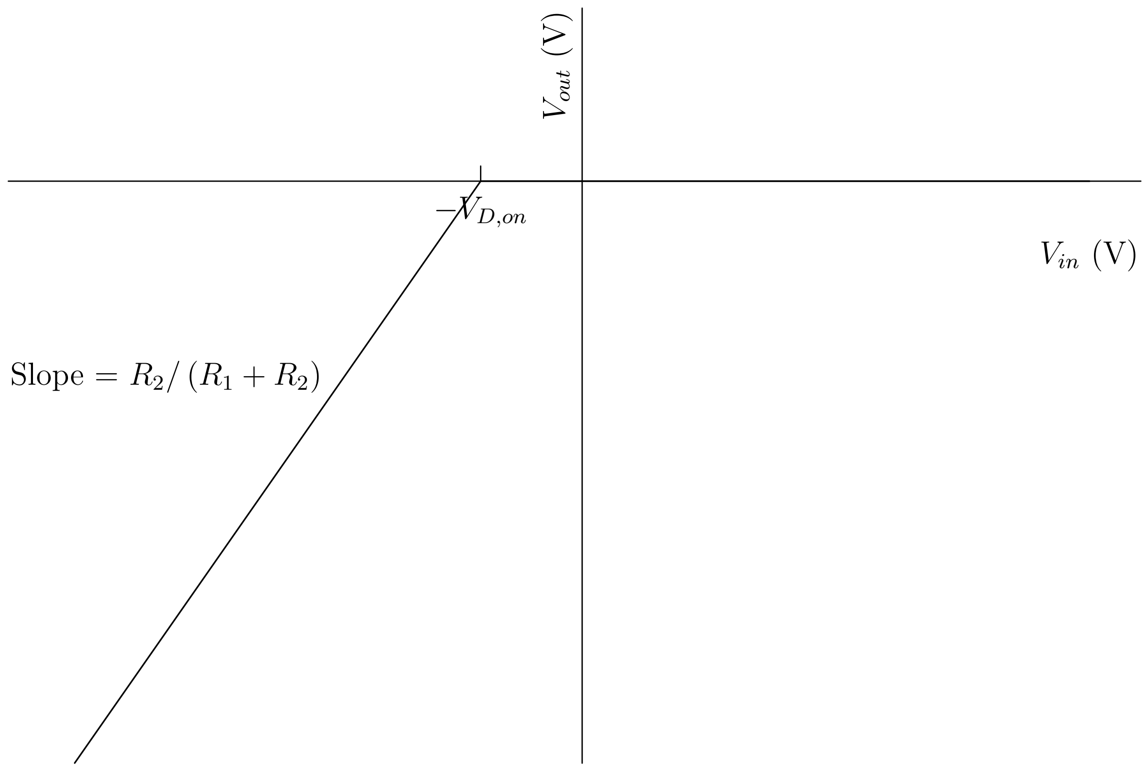
(d)

$$V_{out} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{R_2}{R_1+R_2}\left(V_{in} - V_{D,on}\right) & V_{in} > V_{D,on} \end{cases}$$



(e)

$$V_{out} = \begin{cases} \frac{R_2}{R_1+R_2}\left(V_{in} + V_{D,on}\right) & V_{in} < -V_{D,on} \\ 0 & V_{in} > -V_{D,on} \end{cases}$$

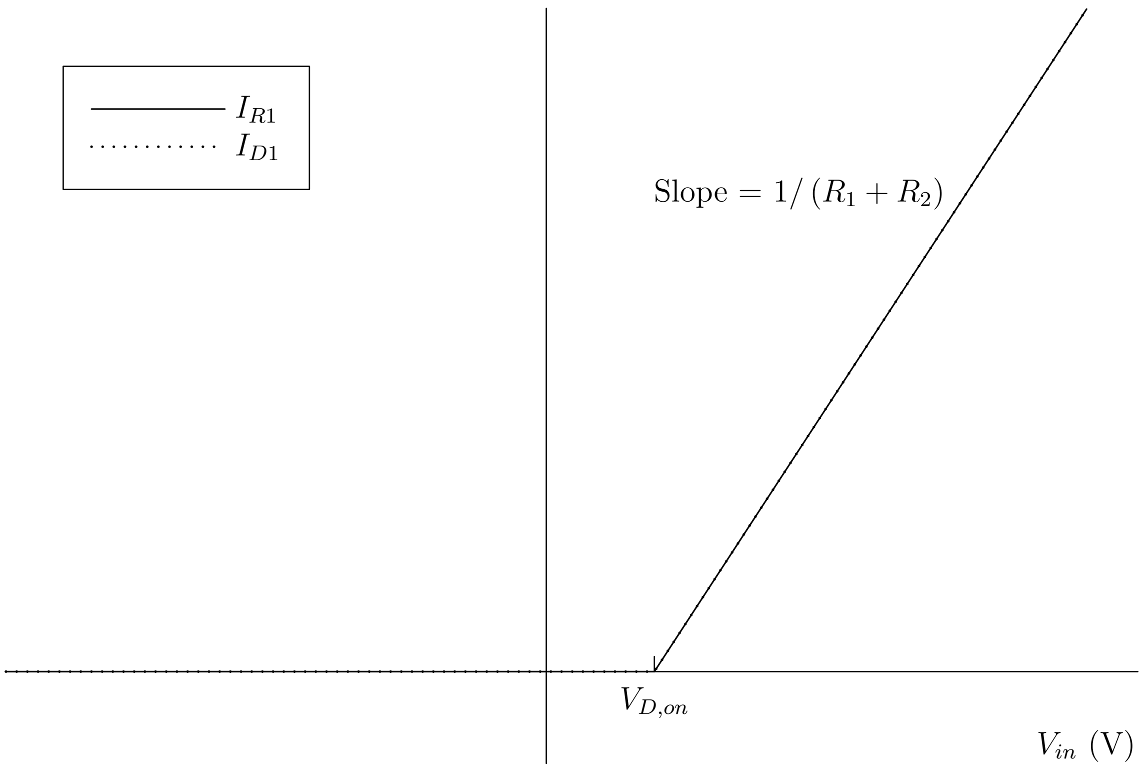


V_{out} (V)
$-V_{D,on}$
V_{in} (V)
Slope = $R_2/(R_1+R_2)$

3.28 (a)

$$I_{R1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_1 + R_2} & V_{in} > V_{D,on} \end{cases}$$

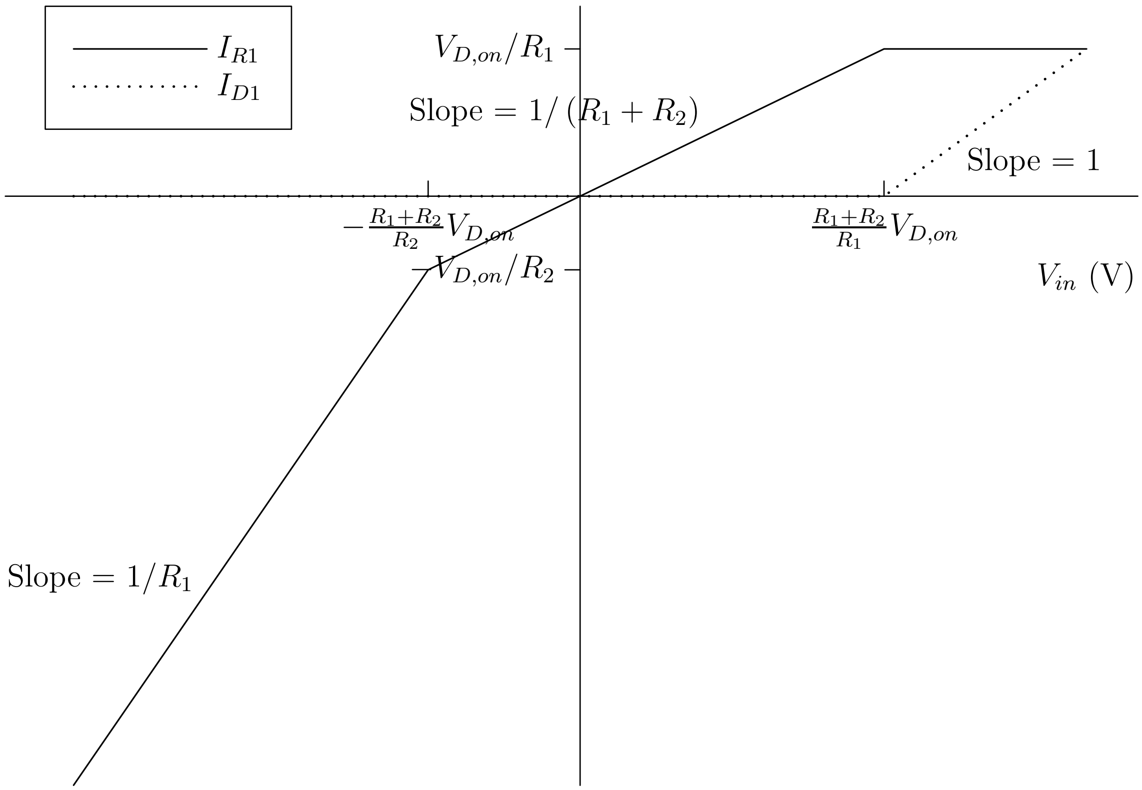
$$I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_1 + R_2} & V_{in} > V_{D,on} \end{cases}$$



(b)

$$I_{R1} = \begin{cases} \frac{V_{in} + V_{D,on}}{R_1} & V_{in} < -\frac{R_1 + R_2}{R_2} V_{D,on} \\ \frac{V_{in}}{R_1 + R_2} & -\frac{R_1 + R_2}{R_2} V_{D,on} < V_{in} < \frac{R_1 + R_2}{R_1} V_{D,on} \\ \frac{V_{D,on}}{R_1} & V_{in} > \frac{R_1 + R_2}{R_1} V_{D,on} \end{cases}$$

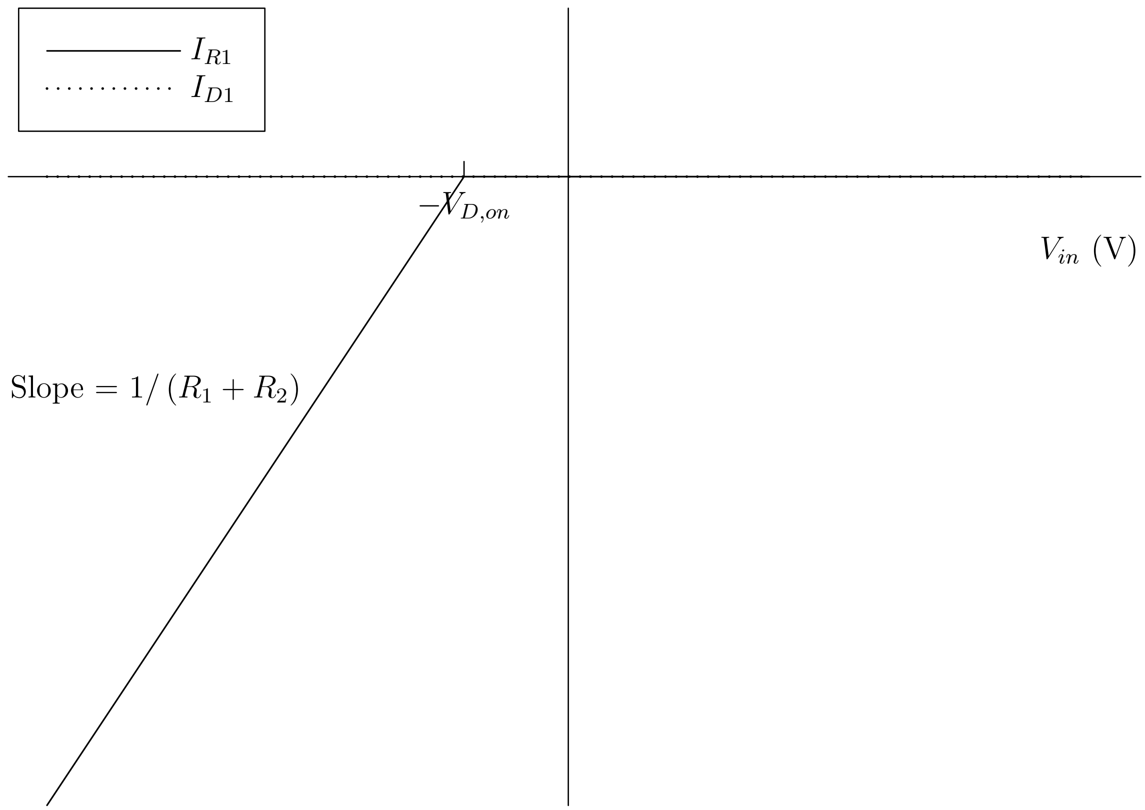
$$I_{D1} = \begin{cases} 0 & V_{in} < -\frac{R_1 + R_2}{R_2} V_{D,on} \\ 0 & -\frac{R_1 + R_2}{R_2} V_{D,on} < V_{in} < \frac{R_1 + R_2}{R_1} V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_2} - \frac{V_{D,on}}{R_1} & V_{in} > \frac{R_1 + R_2}{R_1} V_{D,on} \end{cases}$$



(c)

$$I_{R1} = \begin{cases} \frac{V_{in}+V_{D,on}}{R_1+R_2} & V_{in} < -V_{D,on} \\ 0 & V_{in} > -V_{D,on} \end{cases}$$

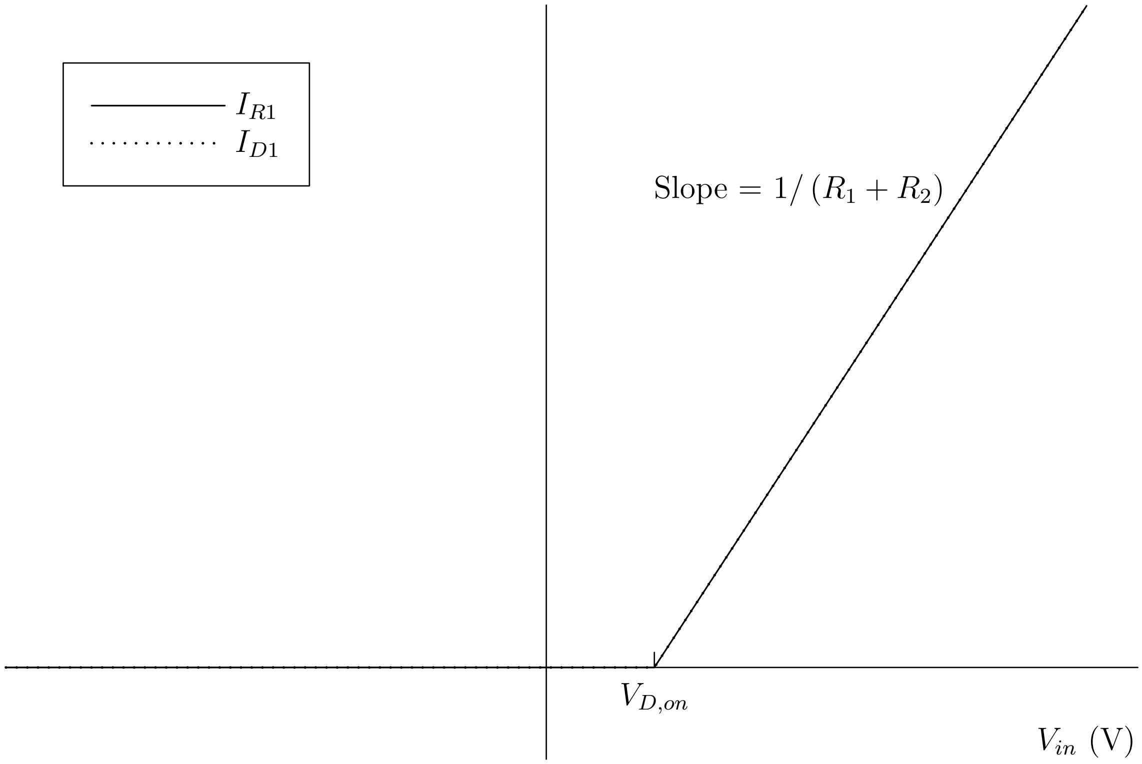
$$I_{D1} = \begin{cases} 0 & V_{in} < -V_{D,on} \\ 0 & V_{in} > -V_{D,on} \end{cases}$$



(d)

$$I_{R1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in}-V_{D,on}}{R_1+R_2} & V_{in} > V_{D,on} \end{cases}$$

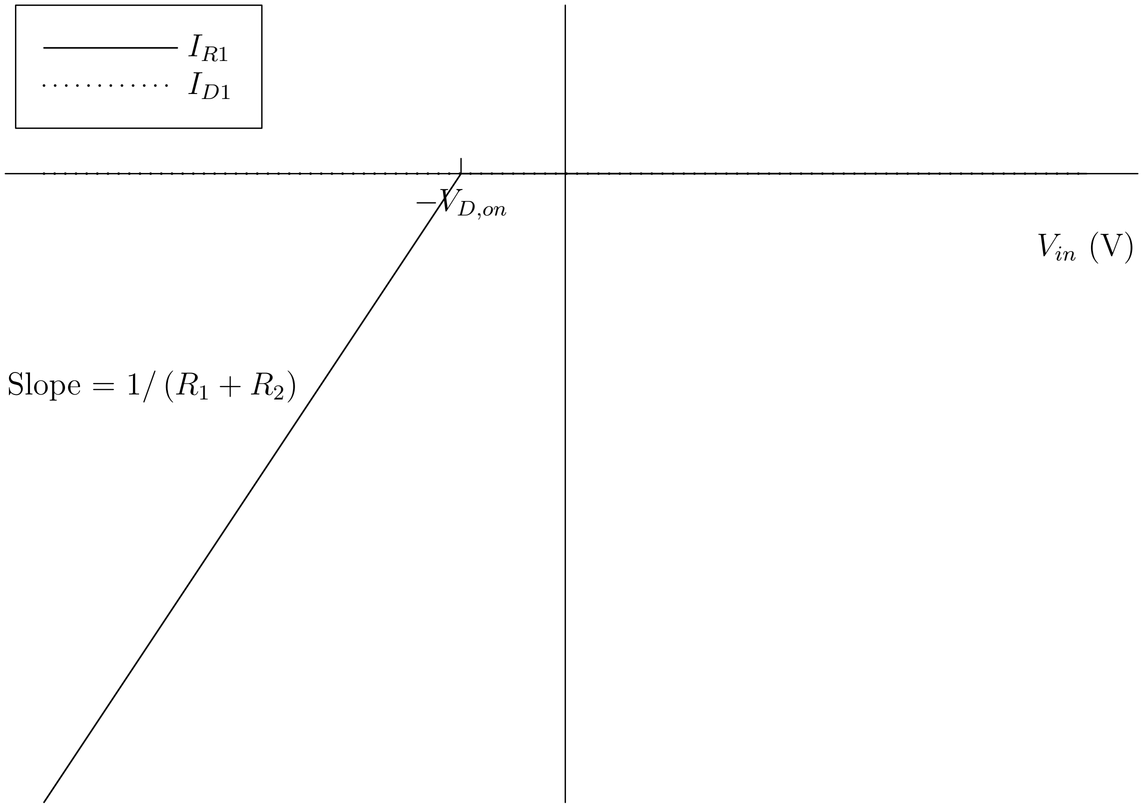
$$I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in}-V_{D,on}}{R_1+R_2} & V_{in} > V_{D,on} \end{cases}$$



(e)

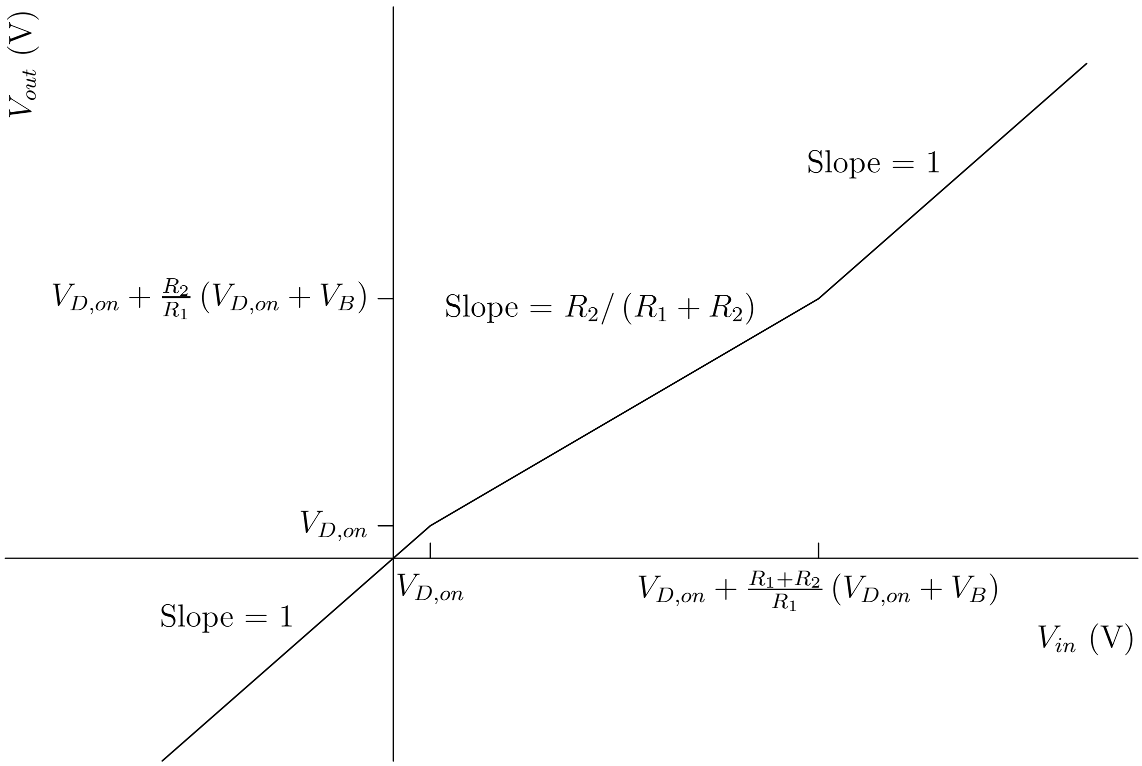
$$I_{R1} = \begin{cases} \frac{V_{in}+V_{D,on}}{R_1+R_2} & V_{in} < -V_{D,on} \\ 0 & V_{in} > -V_{D,on} \end{cases}$$

$$I_{D1} = \begin{cases} 0 & V_{in} < -V_{D,on} \\ 0 & V_{in} > -V_{D,on} \end{cases}$$


I_{R1}
I_{D1}
$-V_{D,on}$
V_{in} (V)
Slope = $1/(R_1 + R_2)$

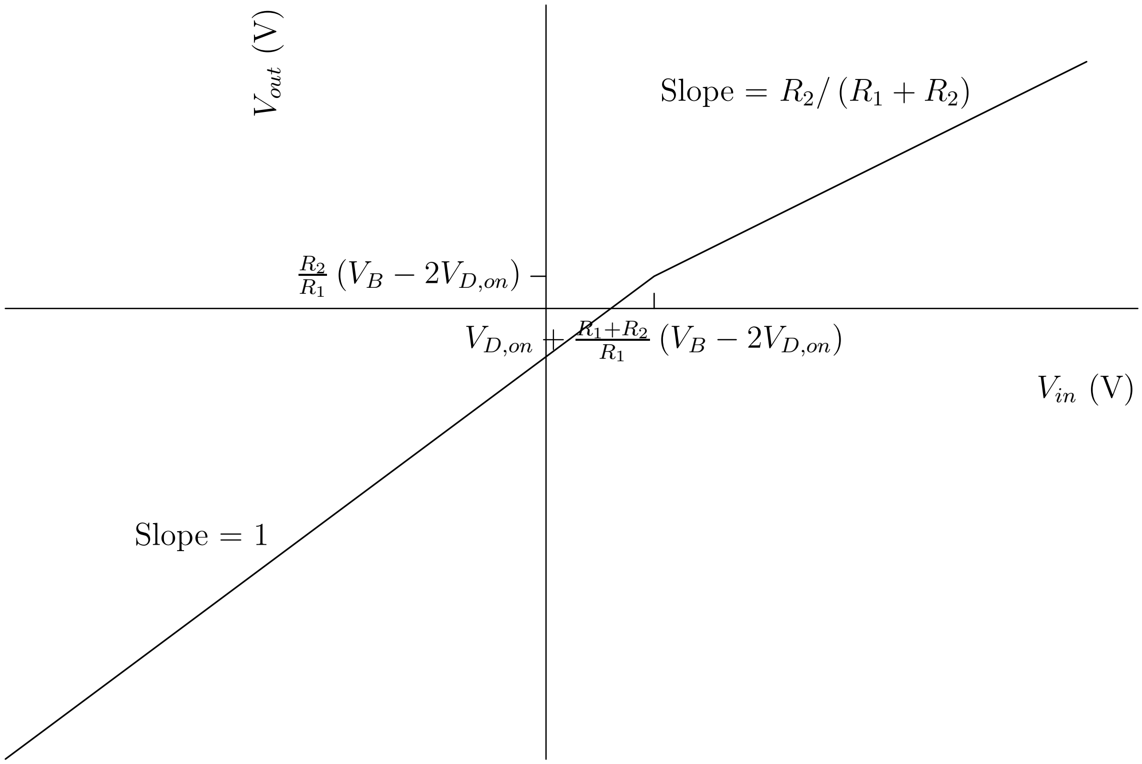
3.29 (a)

$$V_{out} = \begin{cases} V_{in} & V_{in} < V_{D,on} \\ V_{D,on} + \frac{R_2}{R_1+R_2}\left(V_{in} - V_{D,on}\right) & V_{D,on} < V_{in} < V_{D,on} + \frac{R_1+R_2}{R_1}\left(V_{D,on} + V_B\right) \\ V_{in} - V_{D,on} - V_B & V_{in} > V_{D,on} + \frac{R_1+R_2}{R_1}\left(V_{D,on} + V_B\right) \end{cases}$$



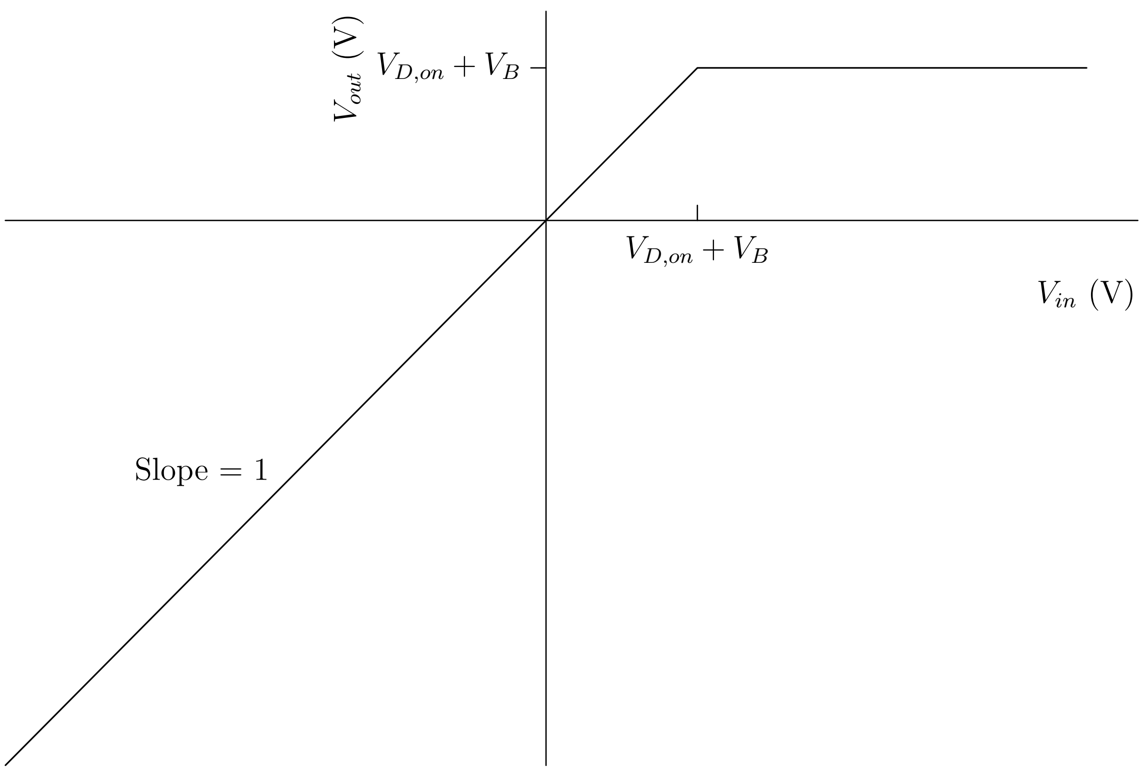
(b)

$$V_{out} = \begin{cases} V_{in} + V_{D,on} - V_B & V_{in} < V_{D,on} + \frac{R_1+R_2}{R_1}\left(V_B - 2V_{D,on}\right) \\ \frac{R_2}{R_1+R_2}\left(V_{in} - V_{D,on}\right) & V_{in} > V_{D,on} + \frac{R_1+R_2}{R_1}\left(V_B - 2V_{D,on}\right) \end{cases}$$



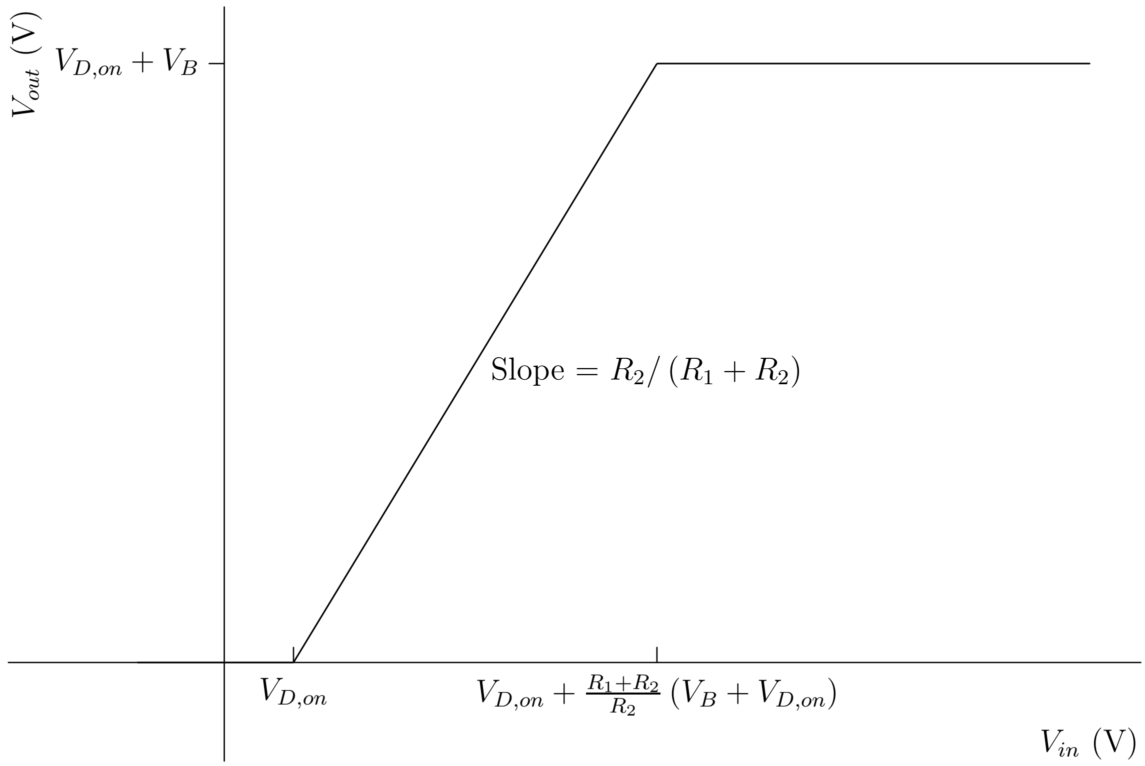
(c)

$$V_{out} = \begin{cases} V_{in} & V_{in} < V_{D,on} + V_B \\ V_{D,on} + V_B & V_{in} > V_{D,on} + V_B \end{cases}$$



(d)

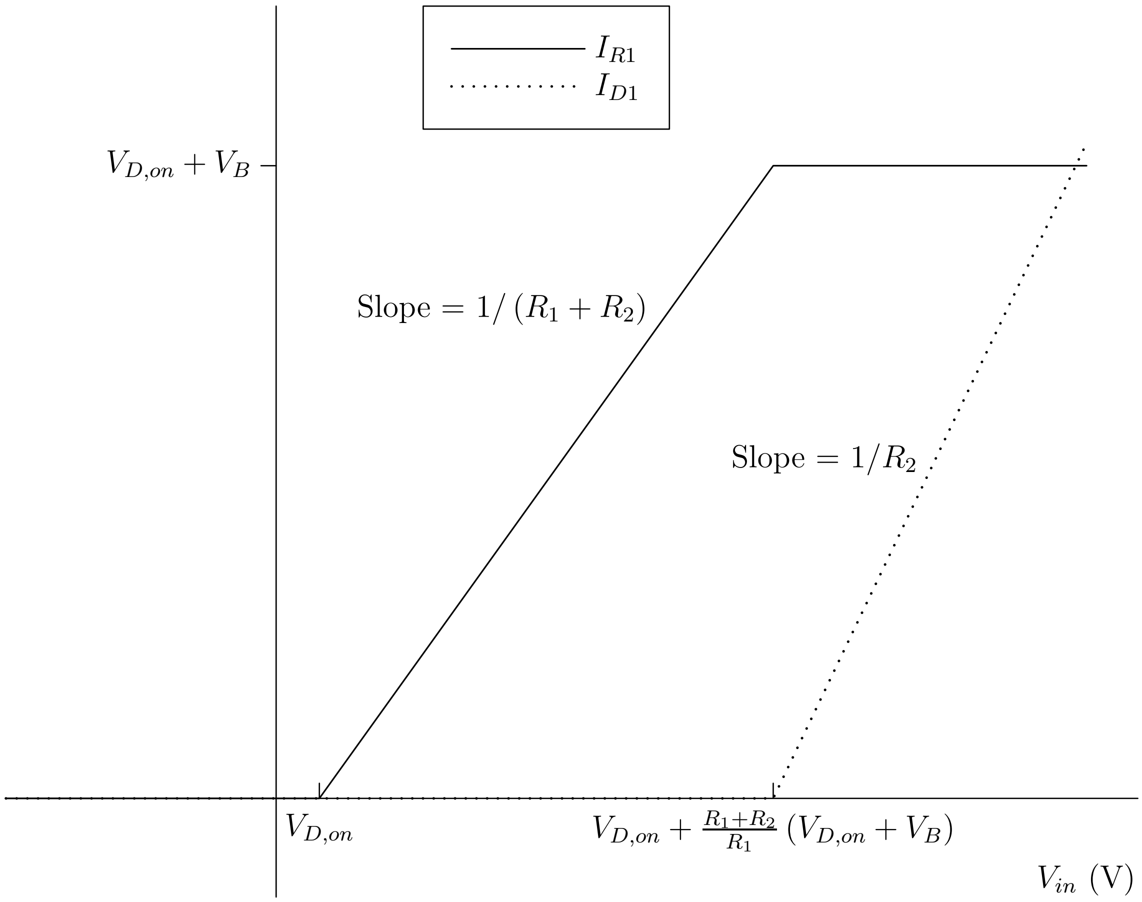
$$V_{out} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{R_2}{R_1+R_2}\left(V_{in} - V_{D,on}\right) & V_{D,on} < V_{in} < V_{D,on} + \frac{R_1+R_2}{R_2}\left(V_B + V_{D,on}\right) \\ V_{D,on} + V_B & V_{in} > V_{D,on} + \frac{R_1+R_2}{R_2}\left(V_B + V_{D,on}\right) \end{cases}$$



3.30 (a)

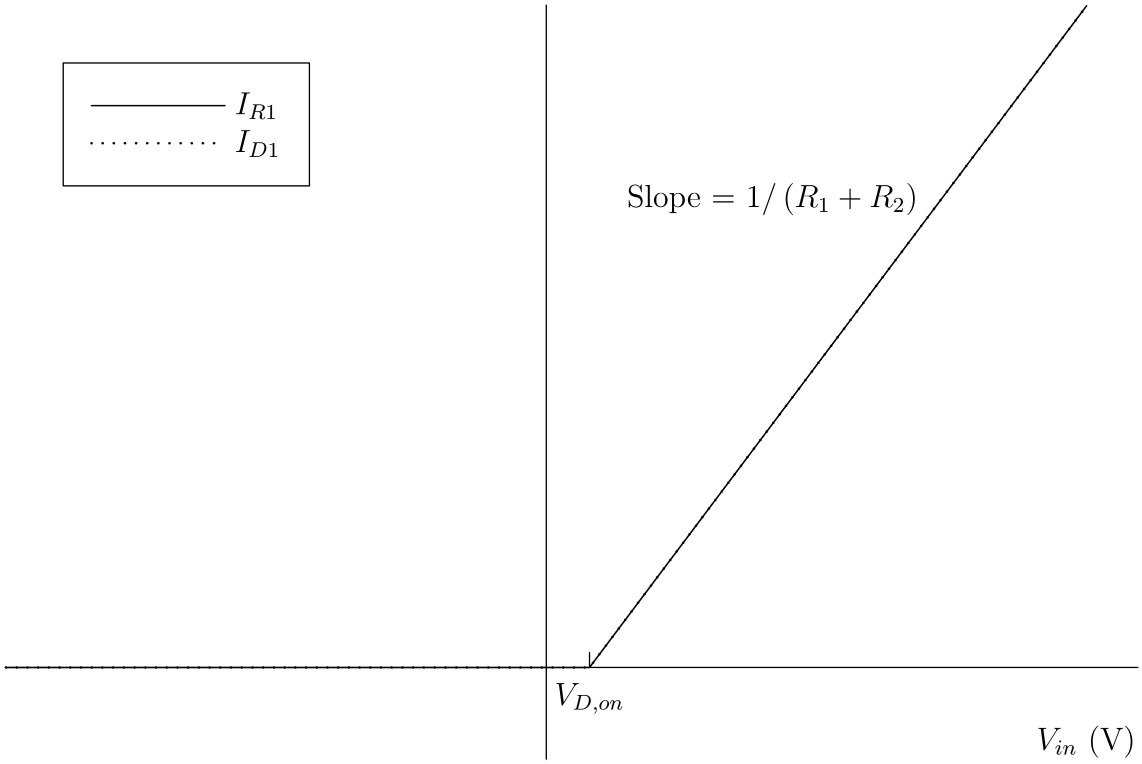
$$I_{R1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_1 + R_2} & V_{D,on} < V_{in} < V_{D,on} + \frac{R_1 + R_2}{R_1}(V_{D,on} + V_B) \\ \frac{V_{D,on} + V_B}{R_1} & V_{in} > V_{D,on} + \frac{R_1 + R_2}{R_1}(V_{D,on} + V_B) \end{cases}$$

$$I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} + \frac{R_1 + R_2}{R_1}(V_{D,on} + V_B) \\ \frac{V_{in} - 2V_{D,on} - V_B}{R_2} - \frac{V_{D,on} + V_B}{R_1} & V_{in} > V_{D,on} + \frac{R_1 + R_2}{R_1}(V_{D,on} + V_B) \end{cases}$$



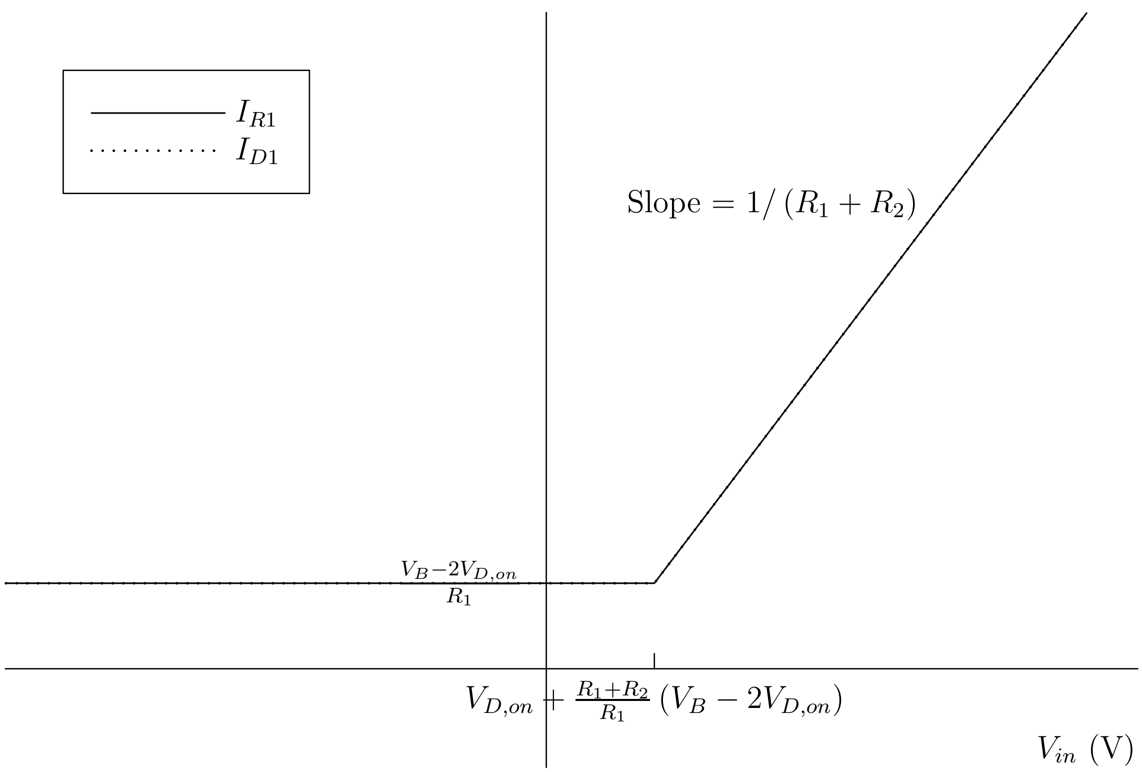
(b) If $V_B < 2V_{D,on}$:

$$I_{R1} = I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_1 + R_2} & V_{in} > V_{D,on} \end{cases}$$



If $V_B > 2V_{D,on}$:

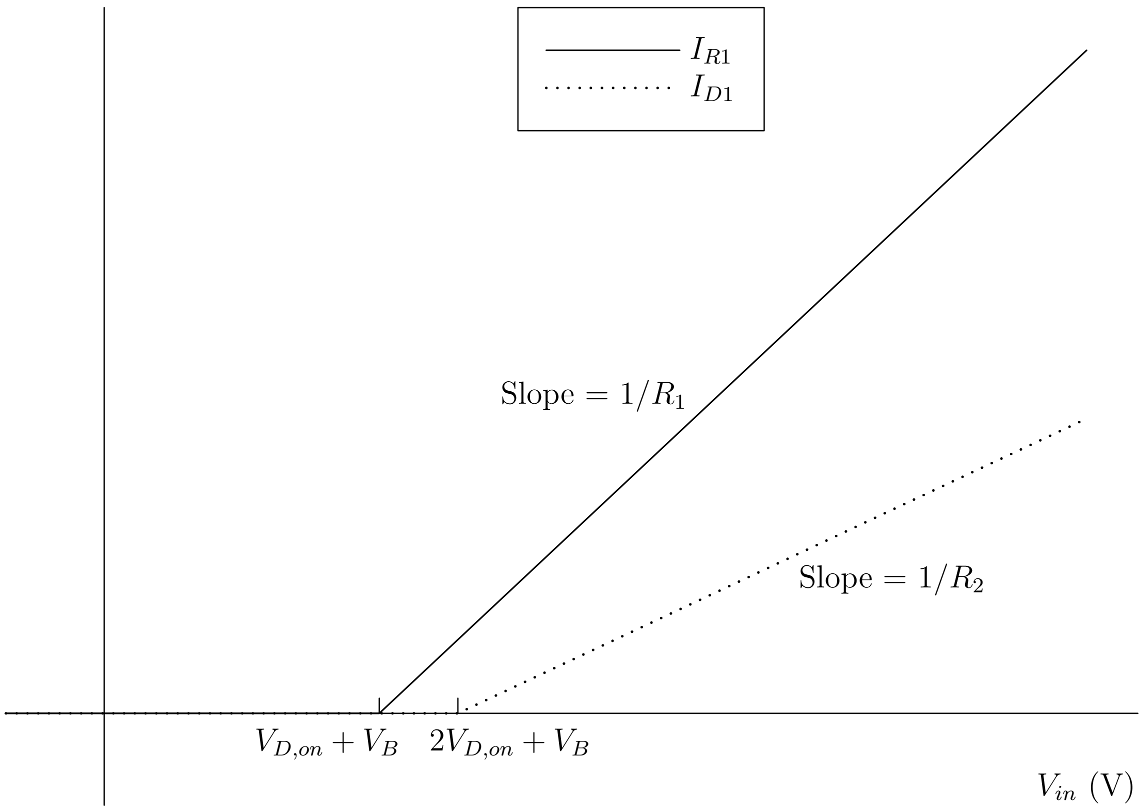
$$I_{R1} = I_{D1} = \begin{cases} \frac{V_B - 2V_{D,on}}{R_1} & V_{in} < V_{D,on} + \frac{R_1+R_2}{R_1}\left(V_B - 2V_{D,on}\right) \\ \frac{V_{in} - V_{D,on}}{R_1+R_2} & V_{in} > V_{D,on} + \frac{R_1+R_2}{R_1}\left(V_B - 2V_{D,on}\right) \end{cases}$$



(c)

$$I_{R1} = \begin{cases} 0 & V_{in} < V_{D,on} + V_B \\ \frac{V_{in} - V_{D,on} - V_B}{R_1} & V_{in} > V_{D,on} + V_B \end{cases}$$

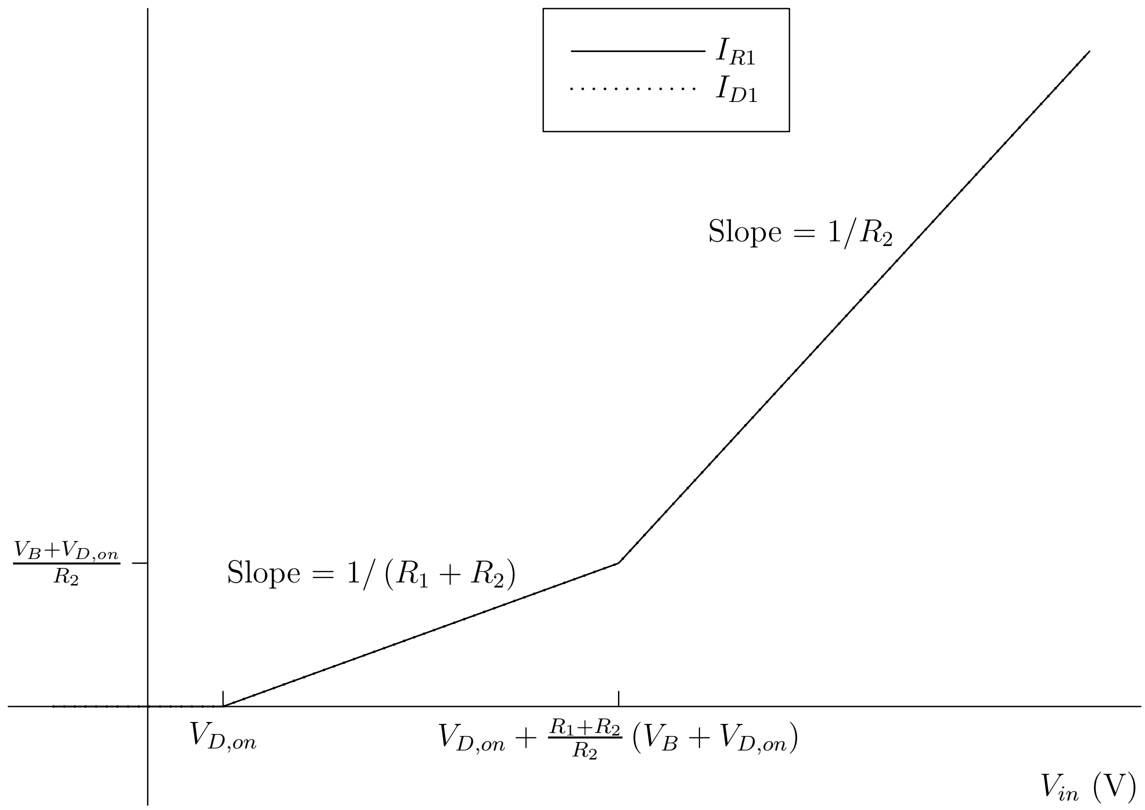
$$I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} + V_B \\ 0 & V_{D,on} + V_B < V_{in} < 2V_{D,on} + V_B \\ \frac{V_{in} - 2V_{D,on} - V_B}{R_2} & V_{in} > 2V_{D,on} + V_B \end{cases}$$



(d)

$$I_{R1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_1 + R_2} & V_{D,on} < V_{in} < V_{D,on} + \frac{R_1 + R_2}{R_2}(V_B + V_{D,on}) \\ \frac{V_{in} - 2V_{D,on} - V_B}{R_1} & V_{in} > V_{D,on} + \frac{R_1 + R_2}{R_2}(V_B + V_{D,on}) \end{cases}$$

$$I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_1 + R_2} & V_{D,on} < V_{in} < V_{D,on} + \frac{R_1 + R_2}{R_2}(V_B + V_{D,on}) \\ \frac{V_{in} - 2V_{D,on} - V_B}{R_1} & V_{in} > V_{D,on} + \frac{R_1 + R_2}{R_2}(V_B + V_{D,on}) \end{cases}$$



I_{R1}
I_{D1}
Slope = $1/R_2$
Slope = $1/(R_1 + R_2)$
$\frac{V_B + V_{D,on}}{R_2}$
$V_{D,on}$
$V_{D,on} + \frac{R_1+R_2}{R_2}(V_B + V_{D,on})$
V_{in} (V)

3.31 (a)

$$I_{D1} = \frac{V_{in} - V_{D,on}}{R_1} = 1.6 \text{ mA}$$

$$r_{d1} = \frac{V_T}{I_{D1}} = 16.25\ \Omega$$

$$\Delta V_{out} = \frac{R_1}{r_d + R_1} \Delta V_{in} = \boxed{98.40 \text{ mV}}$$

(b)

$$I_{D1} = I_{D2} = \frac{V_{in} - 2V_{D,on}}{R_1} = 0.8 \text{ mA}$$

$$r_{d1} = r_{d2} = \frac{V_T}{I_{D1}} = 32.5\ \Omega$$

$$\Delta V_{out} = \frac{R_1 + r_{d2}}{R_1 + r_{d1} + r_{d2}} \Delta V_{in} = \boxed{96.95 \text{ mV}}$$

(c)

$$I_{D1} = I_{D2} = \frac{V_{in} - 2V_{D,on}}{R_1} = 0.8 \text{ mA}$$

$$r_{d1} = r_{d2} = \frac{V_T}{I_{D1}} = 32.5\ \Omega$$

$$\Delta V_{out} = \frac{r_{d2}}{r_{d1} + R_1 + r_{d2}} \Delta V_{in} = \boxed{3.05 \text{ mV}}$$

(d)

$$I_{D2} = \frac{V_{in} - V_{D,on}}{R_1} - \frac{V_{D,on}}{R_2} = 1.2 \text{ mA}$$

$$r_{d2} = \frac{V_T}{I_{D2}} = 21.67\ \Omega$$

$$\Delta V_{out} = \frac{R_2 \parallel r_{d2}}{R_1 + R_2 \parallel r_{d2}} \Delta V_{in} = \boxed{2.10 \text{ mV}}$$

3.32 (a)

$$\Delta V_{out} = \Delta I_{in} R_1 = \boxed{100 \text{ mV}}$$

(b)

$$I_{D1} = I_{D2} = I_{in} = 3 \text{ mA}$$
$$r_{d1} = r_{d2} = \frac{V_T}{I_{D1}} = 8.67\ \Omega$$
$$\Delta V_{out} = \Delta I_{in} (R_1 + r_{d2}) = \boxed{100.867 \text{ mV}}$$

(c)

$$I_{D1} = I_{D2} = I_{in} = 3 \text{ mA}$$
$$r_{d1} = r_{d2} = \frac{V_T}{I_{D1}} = 8.67\ \Omega$$
$$\Delta V_{out} = \Delta I_{in} r_{d2} = \boxed{0.867 \text{ mV}}$$

(d)

$$I_{D2} = I_{in} - \frac{V_{D,on}}{R_2} = 2.6 \text{ mA}$$
$$r_{d2} = \frac{V_T}{I_{D2}} = 10\ \Omega$$
$$\Delta V_{out} = \Delta I_{in} (R_2 \parallel r_{d2}) = \boxed{0.995 \text{ mV}}$$

(33) a) $i_1 = i_{in}$

$= 0.1\ mA$

b) $i_{r_1} = i_{in}$

$= 0.1\ mA$

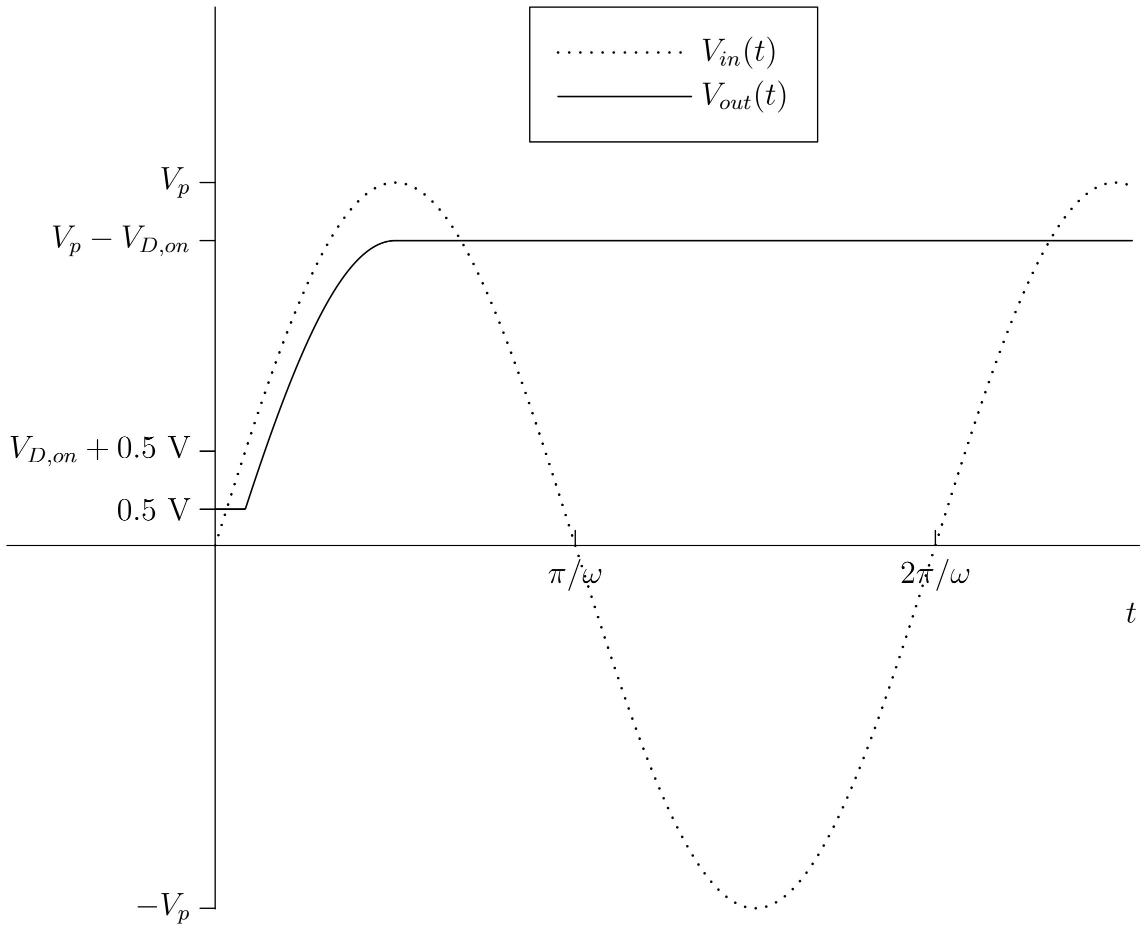
c) $i_{r_1} = i_{in}$

$= 0.1\ mA$

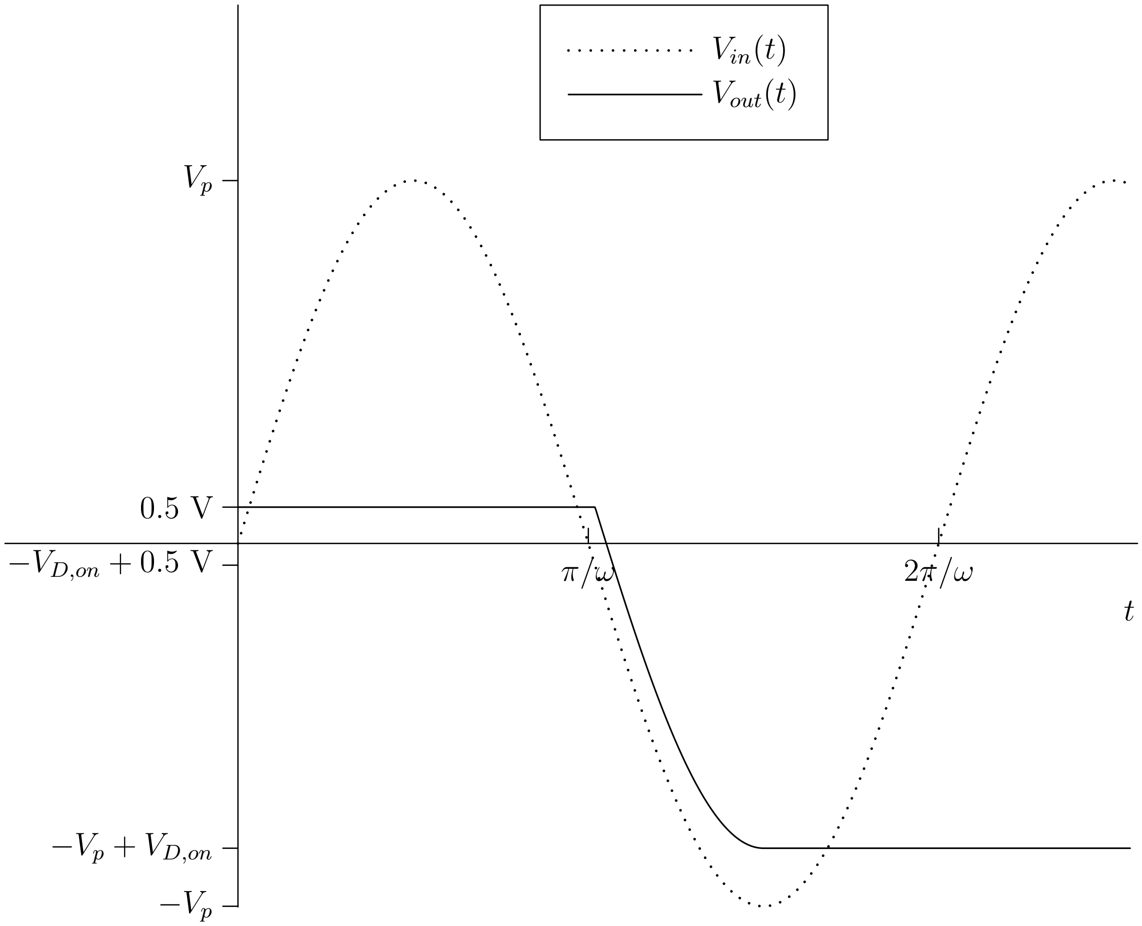
d) $i_{r_1} = i_{in}$

$= 0.1\ mA.$

3.34



3.35



3.36

$$V_R \approx \frac{V_p - V_{D,on}}{R_L C_1 f_{in}}$$

$$V_p = 3.5 \text{ V}$$

$$R_L = 100\ \Omega$$

$$C_1 = 1000\ \mu\text{F}$$

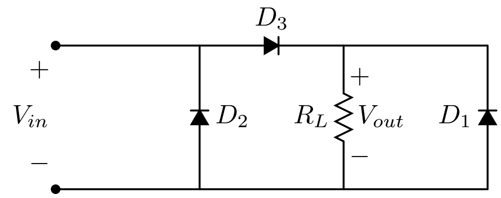
$$f_{in} = 60 \text{ Hz}$$

$$V_R = \boxed{0.45 \text{ V}}$$

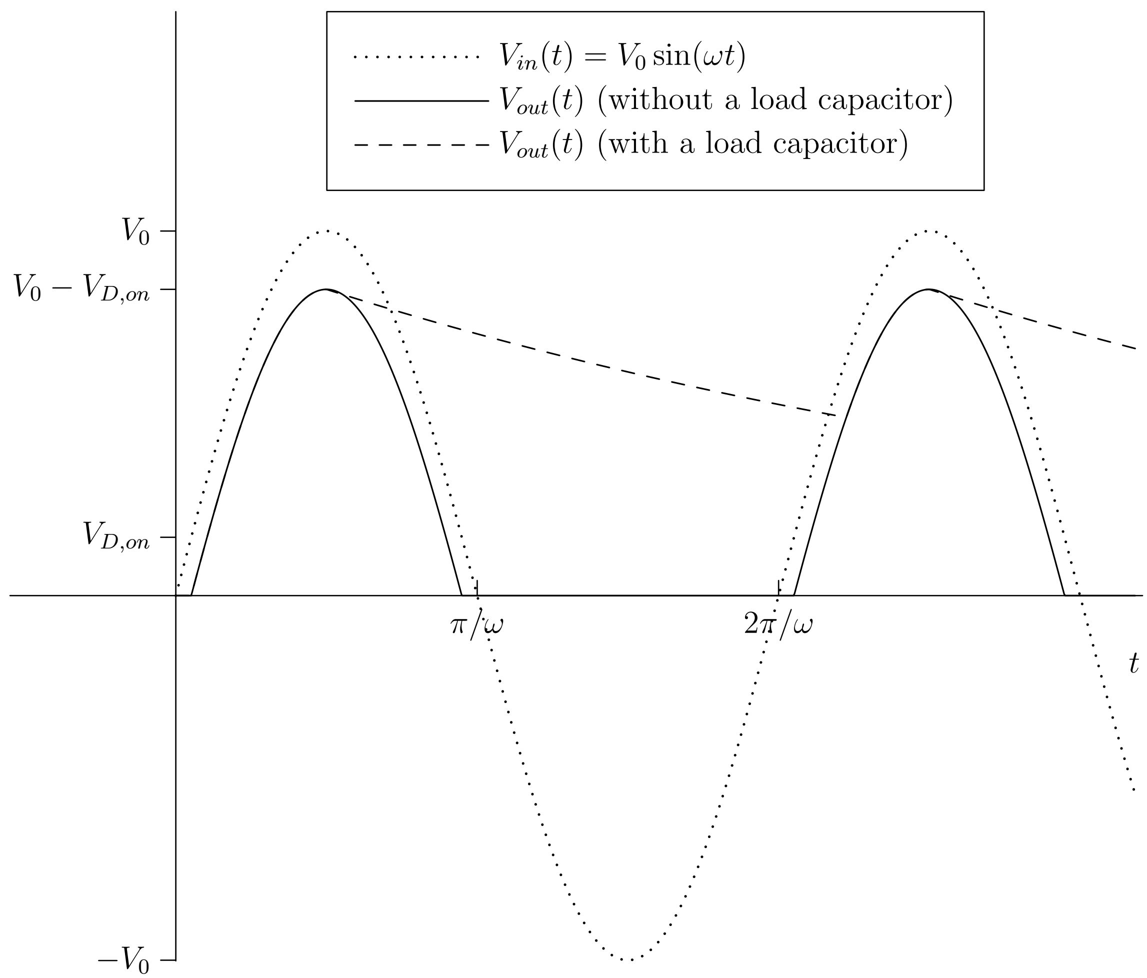
3.37

$$V_R = \frac{I_L}{C_1 f_{in}} \leq 300 \text{ mV}$$
$$f_{in} = 60 \text{ Hz}$$
$$I_L = 0.5 \text{ A}$$
$$C_1 \geq \frac{I_L}{(300 \text{ mV}) f_{in}} = \boxed{27.78 \text{ mF}}$$

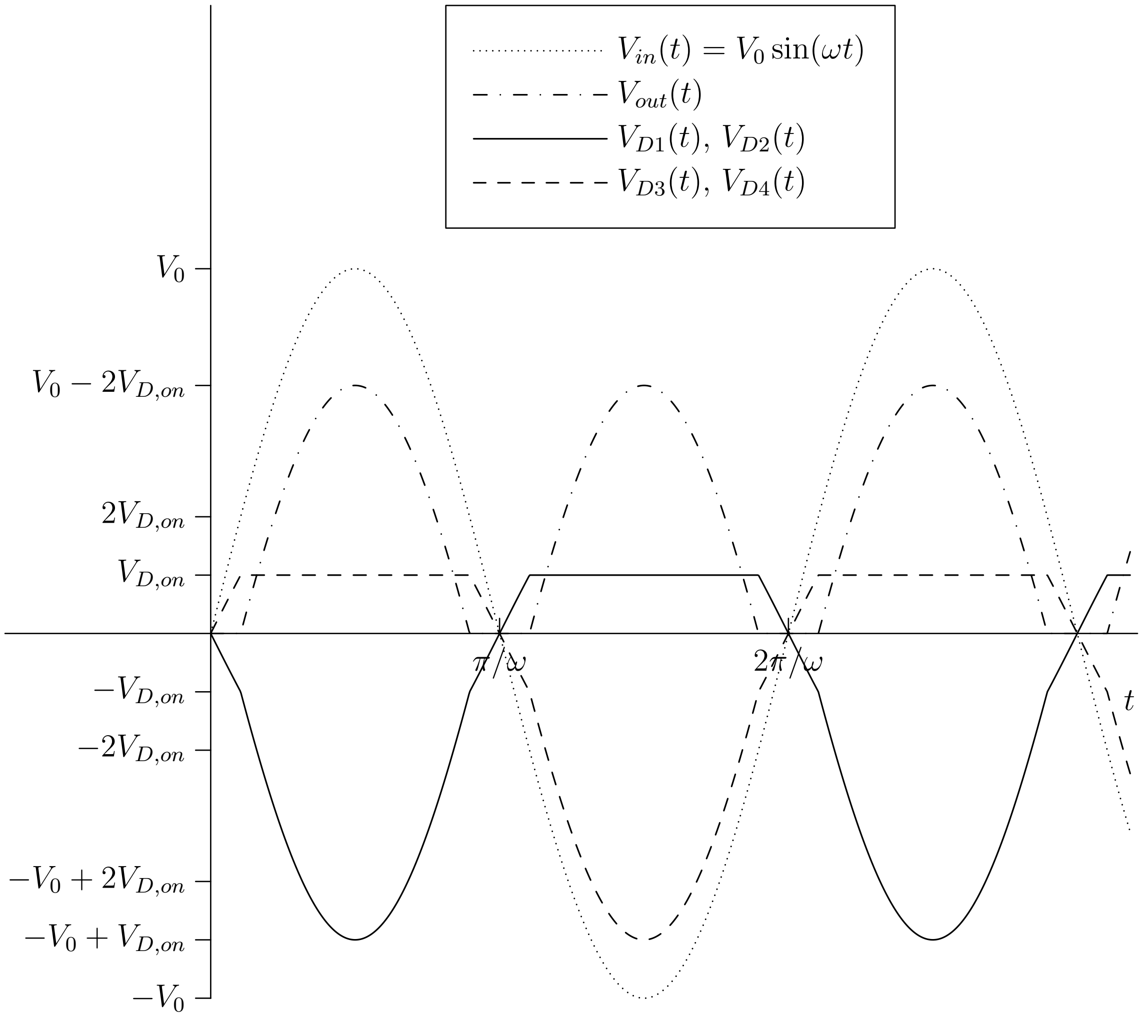
3.38 Shorting the input and output grounds of a full-wave rectifier shorts out the diode D_4 from Fig. 3.38(b). Redrawing the modified circuit, we have:



On the positive half-cycle, D_3 turns on and forms a half-wave rectifier along with R_L (and C_L, if included). On the negative half-cycle, D_2 shorts the input (which could cause a dangerously large current to flow) and the output remains at zero. Thus, the circuit behaves like a half-wave recifier. The plots of $V_{out}(t)$ are shown below.



3.39 Note that the waveforms for V_{D1} and V_{D2} are identical, as are the waveforms for V_{D3} and V_{D4}.



3.40 During the positive half-cycle, D_2 and D_3 will remain reverse-biased, causing V_{out} to be zero as no current will flow through R_L. During the negative half-cycle, D_1 and D_3 will short the input (potentially causing damage to the devices), and once again, no current will flow through R_L (even though D_2 will turn on, there will be no voltage drop across R_L). Thus, V_{out} always remains at zero, and the circuit fails to act as a rectifier.

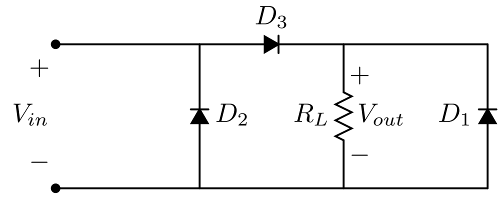
(41) Using Eq. (3.94),

$$V_R \approx \frac{1}{2} \cdot \frac{V_P - 2V_{P,ON}}{R_L C_1 f_{in}}$$

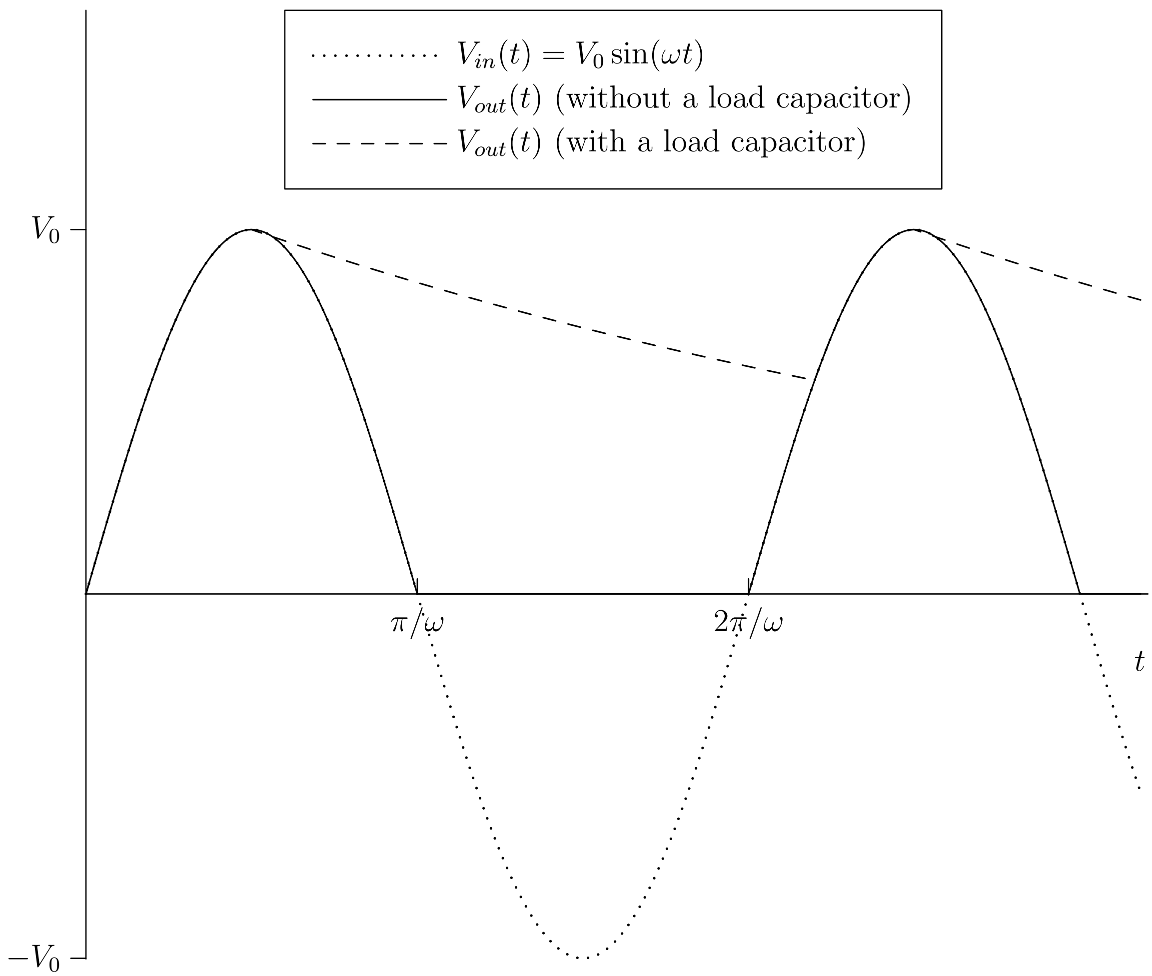
$$= \frac{1}{2} \cdot \frac{3 - 2 \times 0.8}{30 \times 1000 \times 10^{-6} \times 60}$$

$$= 0.389V$$

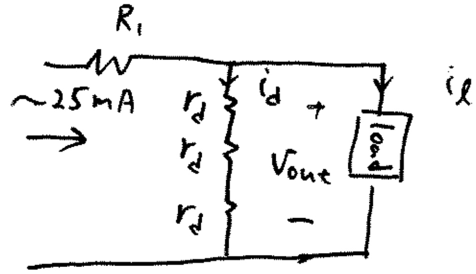
3.42 Shorting the negative terminals of V_{in} and V_{out} of a full-wave rectifier shorts out the diode D_4 from Fig. 3.38(b). Redrawing the modified circuit, we have:



On the positive half-cycle, D_3 turns on and forms a half-wave rectifier along with R_L (and C_L, if included). On the negative half-cycle, D_2 shorts the input (which could cause a dangerously large current to flow) and the output remains at zero. Thus, the circuit behaves like a half-wave recifier. The plots of $V_{out}(t)$ are shown below.



(43) The circuit can be simplified as:



First, find r_d:

$$r_d = \frac{V_T}{I_D} \quad \text{(from eq. 3.60)}$$

$$= \frac{26\,mV}{5\,mA}$$

$$= 5.2\,\Omega$$

since $i_\ell = +1\,mA$.

$i_d = -1\,mA$.

∴ change in V_{out},

ie. $V_{out} = (-1\,mA)(3 \times 5.2)$

$$= -15.6\,mV$$

3.44 (a) We know that when a capacitor is discharged by a constant current at a certain frequency, the ripple voltage is given by $\frac{I}{Cf_{in}}$, where I is the constant current. In this case, we can calculate the current as approximately $\frac{V_p - 5V_{D,on}}{R_1}$ (since $V_p - 5V_{D,on}$ is the voltage drop across R_1, assuming R_1 carries a constant current). This gives us the following:

$$V_R \approx \frac{1}{2}\frac{V_p - 5V_{D,on}}{R_L C_1 f_{in}}$$
$$V_p = 5 \text{ V}$$
$$R_L = 1 \text{ k}\Omega$$
$$C_1 = 100 \text{ µF}$$
$$f_{in} = 60 \text{ Hz}$$
$$V_R = \boxed{166.67 \text{ mV}}$$

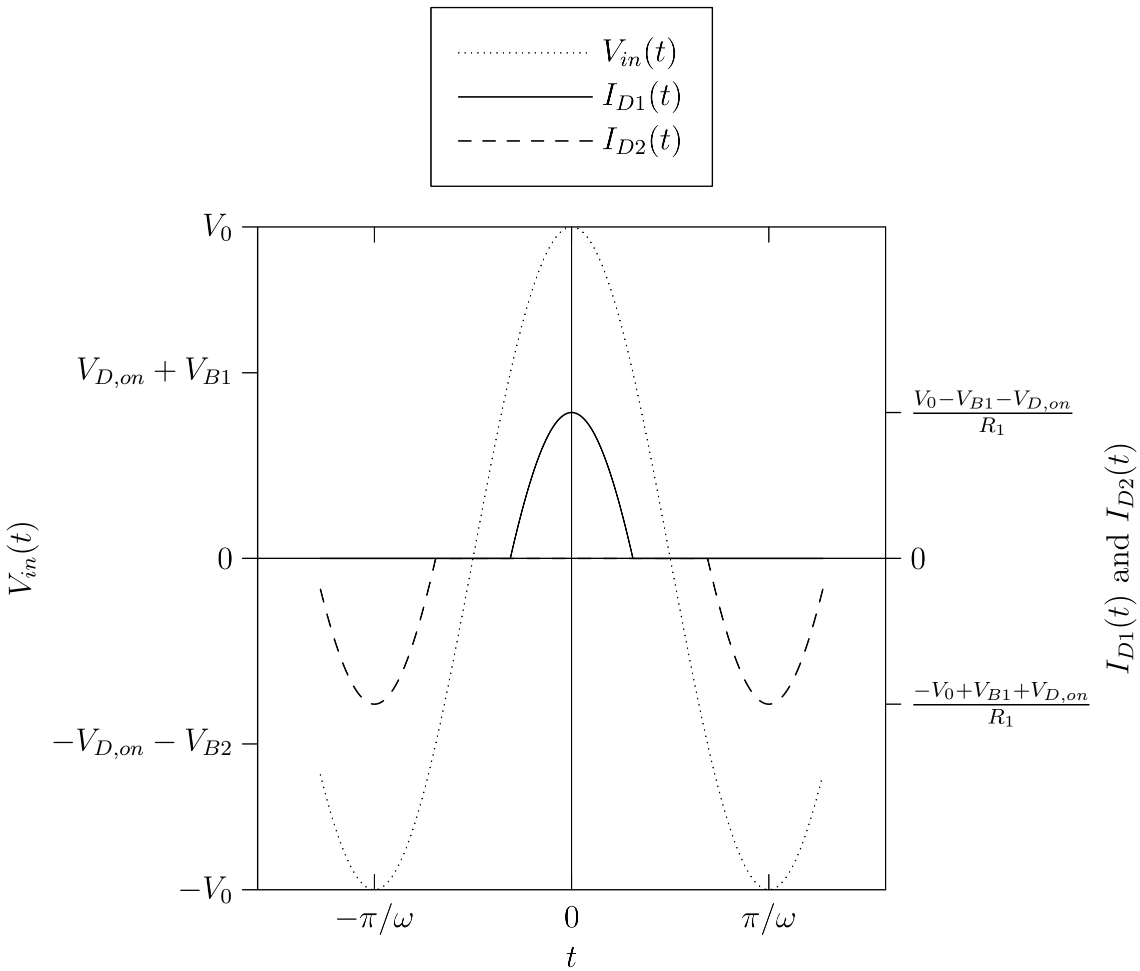
(b) The bias current through the diodes is the same as the bias current through R_1, which is $\frac{V_p - 5V_{D,on}}{R_1} = 1$ mA. Thus, we have:

$$r_d = \frac{V_T}{I_D} = 26\ \Omega$$
$$V_{R,load} = \frac{3r_d}{R_1 + 3r_d} V_R = \boxed{12.06 \text{ mV}}$$

3.45

$$I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} + V_{B1} \\ \frac{V_{in} - V_{D,on} - V_{B1}}{R_1} & V_{in} > V_{D,on} + V_{B1} \end{cases}$$

$$I_{D2} = \begin{cases} \frac{V_{in} + V_{D,on} + V_{B2}}{R_1} & V_{in} < -V_{D,on} - V_{B2} \\ 0 & V_{in} > -V_{D,on} - V_{B2} \end{cases}$$



(46) With positive theshold = + 2.2V,

$$V_{B1} = 2.2 - 0.8$$

$$= +1.4V$$

with negative thres hold = −1.9V,

$$-V_{B2} = -1.9 + 0.8$$

$$= -1.1V.$$

$$V_{B2} = 1.1V$$

To meet the maximum current criterion,

Since $I_{R_1} = I_{D1}$ or I_{D2},

I_{D1} or I_{D2} is at max when

I_{R_1} is at max.

I_{R_1} is at max when $|V_R|$ is max,

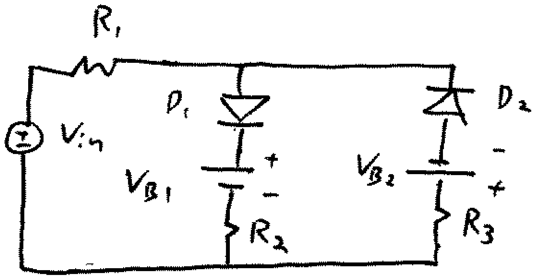
ie. $|V_R| = 5 - 1.9$

$$= 3.1V.$$

Since $I_{R_1} \leq 2$ mA.

$R_1 \geq \frac{3.1}{2mA}$, ie. $R_1 \geq 1550\Omega$

(47) The required circuit is:



Similar to Example 3.34,

$$V_{B1} = V_{B2} = (2 - 0.8)\,V = 1.2\,V$$

To find R_2,

For $V_{in} > 2V$, $\frac{V_{out}}{V_{in}}$ has a slope of 0.5.

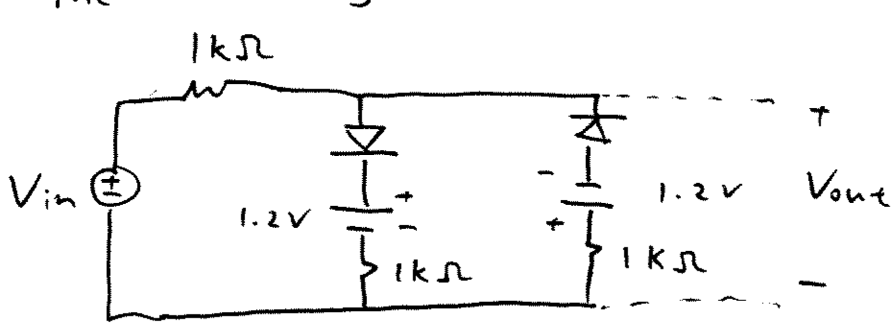
This implies $R_2 = R_1$

(R_1 and R_2 forms a volt. divider).

Similarly, $R_3 = R_1$

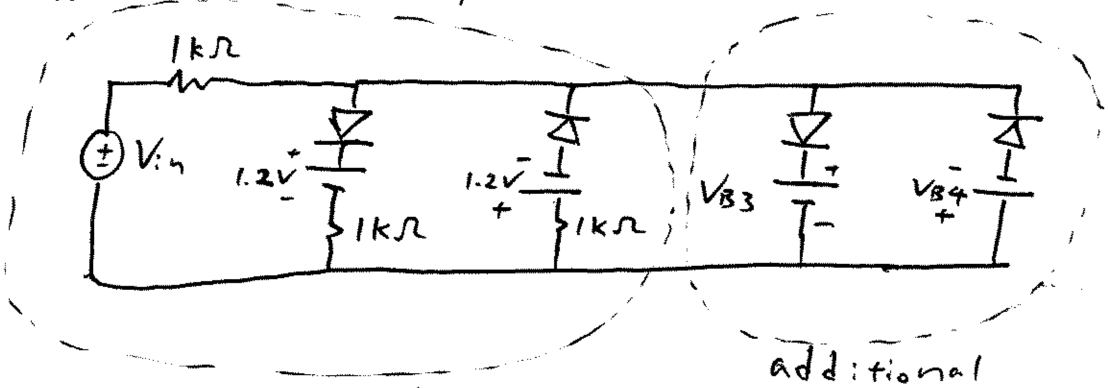
Thus, set $R_1 = R_2 = R_3 = 1\,k\Omega$.

The resulting circuit is:



(48) For $|V_{in}| < 4V$, the $V_{out} - V_{in}$ characteristic is similar to prob.(47).

To get voltage limiting characteristic for $V_{in} > 4V$, and $V_{in} < -4V$, we can shunt the circuit used in prob(47) with two antiparallel diodes as below:



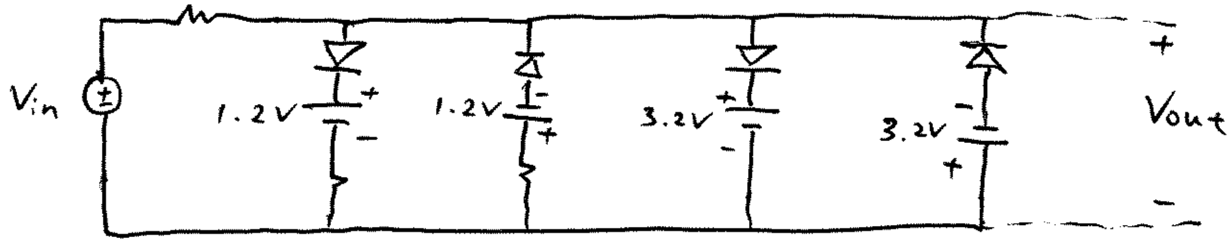
ckt used in prob. 47

additional antiparallel diodes

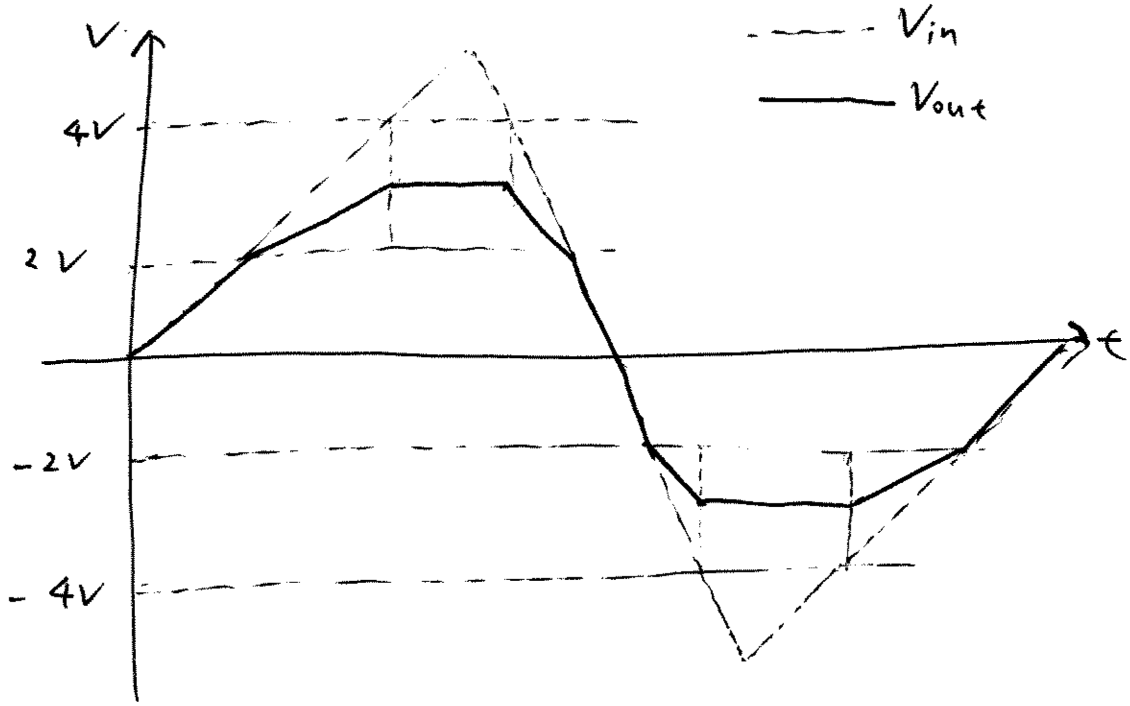
$$V_{B3} = V_{B4} = 4 - 0.8$$

$$= 3.2V$$

Resulting circuit is:

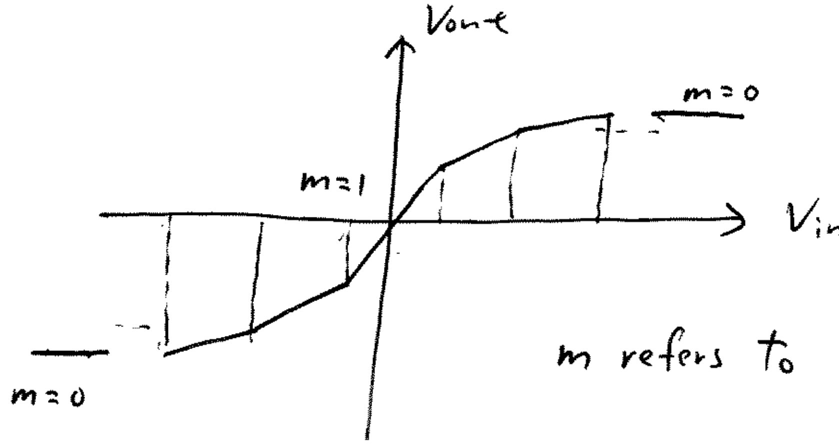


(49)



To get a better approximate of a sinusoid, the slope of the input-output characteristic should decrease <u>more gradually</u> from 1 to 0 through more sections.

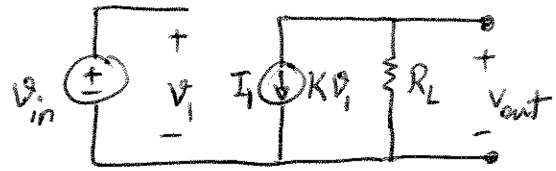
eg:



m refers to slope

Chapter 4

(4.1)



$K = 20\,mA/V$

$\left|\frac{V_{out}}{V_{in}}\right| = 15 \qquad V_{in} = V_1$

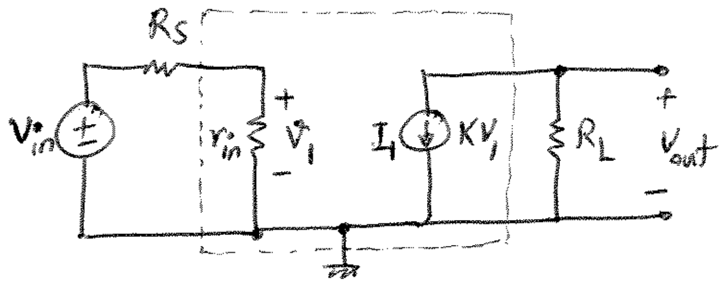
$$V_{out} = -I_1 R_L = -K R_L V_{in}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = -K R_L \Rightarrow \left|\frac{V_{out}}{V_{in}}\right| = K R_L$$

$$\Rightarrow K R_L = 15 \Rightarrow R_L = \frac{15}{20\,mA/V} = 750\,\Omega$$

$$\boxed{R_L = 750\,\Omega}$$

4.2



$$\frac{V_{out}}{V_{in}} = ?$$

$$V_1 = \frac{r_{in}}{r_{in}+R_S} V_{in}$$

$$I_1 = KV_1$$

$$V_{out} = -R_L I_1$$

$$\Rightarrow V_{out} = -KR_L V_1$$

$$\Rightarrow V_{out} = -KR_L \frac{r_{in}}{r_{in}+R_S} V_{in}$$

$$\Rightarrow \boxed{A_V = \frac{V_{out}}{V_{in}} = -KR_L \frac{r_{in}}{r_{in}+R_S}}$$

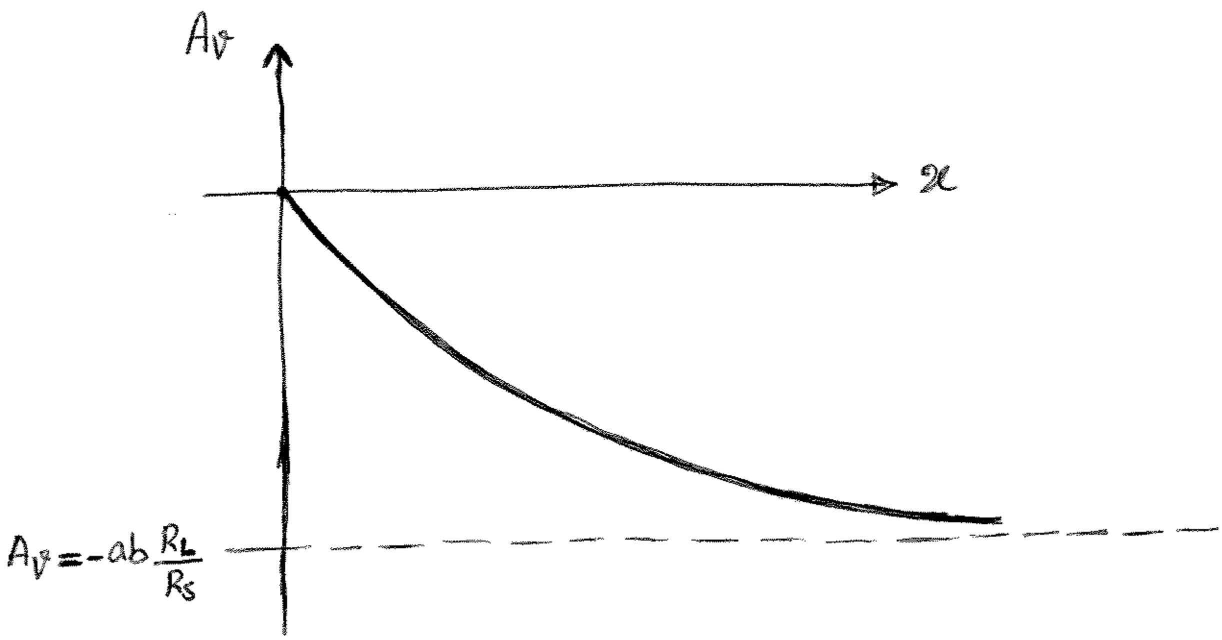
(4.3) From solution for problem 4.2,

$a > 0$
$b > 0$
$x \geq 0$

$$A_v = -K R_L \frac{r_{in}}{r_{in} + R_S}$$

$$\xrightarrow[K = bx]{r_{in} = a/x} A_v = -bx R_L \frac{a/x}{a/x + R_S} = -b R_L \frac{a}{\frac{a}{x} + R_S}$$

$$\Longrightarrow \boxed{A_v = -b R_L \left(\frac{x}{1 + \frac{R_S}{a} x} \right)}$$

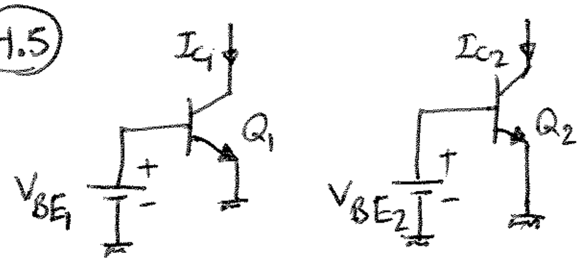


4.4 According to Equation (4.8), we have

$$\begin{aligned} I_C &= \frac{A_E q D_n n_i^2}{N_B W_B} \left(e^{V_{BE}/V_T} - 1 \right) \\ &\propto \frac{1}{W_B} \end{aligned}$$

We can see that if W_B increases by a factor of two, then $\boxed{I_C \text{ decreases by a factor of two}}$.

(4.5)



$V_T = 26\,mV$

$I_{C_1} = I_{C_2}$

$V_{BE_1} - V_{BE_2} = 20\,mV$

$$I_C = \frac{A_E q D_n n_i^2}{N_E W_B}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$$ equation (4.8) page 136

$$\Rightarrow I_C \simeq \frac{A_E q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE}}{V_T}}$$ $A_E \equiv$ Cross Section

if $I_{C_1} = I_{C_2}$

$$\Rightarrow A_{E_1} \frac{q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE_1}}{V_T}} = A_{E_2} \frac{q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE_2}}{V_T}}$$

$$\Rightarrow \frac{A_{E_2}}{A_{E_1}} = \frac{e^{V_{BE_1}/V_T}}{e^{V_{BE_2}/V_T}}$$

$$\Rightarrow \frac{A_{E_2}}{A_{E_1}} = e^{(V_{BE_1} - V_{BE_2})/V_T} = e^{20\,mV/26\,mV}$$

$$\Rightarrow \boxed{\frac{A_{E_2}}{A_{E_1}} = e^{\frac{20}{26}} \simeq 2.16}$$

(6a) $I_X = 1^{mA} \Rightarrow I_{Q_1} = I_{Q_2} = 0.5^{mA}$

$$I_{Q_1} = I_{S_1} e^{\frac{V_{BE_1}}{V_T}} \Rightarrow 5\times10^{-4} = 3\times10^{-16} e^{\frac{V_B}{26^{mV}}}$$

$$\Rightarrow V_B = 26^{mV} \ln\left(\tfrac{5}{3}\times10^{12}\right) \Rightarrow \boxed{V_B \simeq 731.7^{mV}}$$

(6b) $I_Y = I_{S_3} e^{\frac{V_B}{V_T}}$

$$\Rightarrow I_{S_3} = I_Y e^{-\frac{V_B}{V_T}} = 2.5\times10^{-3} \times e^{-\frac{V_B}{26^{mV}}} = 2.5\times10^{-3} \times \frac{1}{\frac{5}{3}\times10^{12}}$$

$$\Rightarrow \boxed{I_{S_3} = 1.5\times10^{-15} A}$$

7a) $I_X = I_1 + I_2$

$$\Rightarrow I_X = I_{S_1} e^{\frac{V_B}{V_T}} + I_{S_2} e^{\frac{V_B}{V_T}} \Rightarrow I_X = (I_{S_1} + I_{S_2}) e^{\frac{V_B}{V_T}}$$

$$\Rightarrow V_B = V_T \ln\left(\frac{I_X}{I_{S_1} + I_{S_2}}\right) \xrightarrow{I_{S_1} = 2I_{S_2}} \boxed{V_B = V_T \ln\left(\frac{I_X}{\frac{3}{2} I_{S_1}}\right)}$$

$$V_B = 26\times10^{-3} \ln\left(\frac{1.2\times10^{-3}}{\frac{3}{2}\times5\times10^{-16}}\right) \Rightarrow \boxed{V_B \simeq 730.6\ mV}$$

7b) Transistors at the edge of the active mode $\Rightarrow V_C = V_B$

applying KVL, we have:

$$V_{CC} = R_C I_X + V_B \Rightarrow \boxed{R_C = \frac{V_{CC} - V_B}{I_X}}$$

$$\Rightarrow R_C = \frac{2.5 - 0.73}{1.2\times10^{-3}}$$

$$\Rightarrow \boxed{R_C \simeq 1475\ \Omega}$$

(8a) Same as 7a,

$$\boxed{V_B \simeq 730.6\,mV}$$

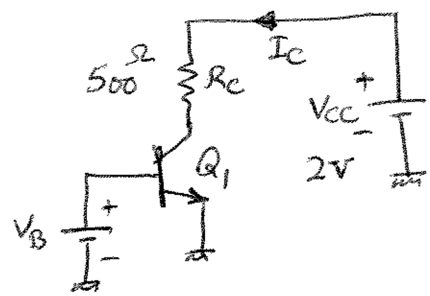
(8b) According to 7b,

$$R_C = \frac{V_{CC} - V_B}{I_X} = \frac{1.5 - 0.73}{1.2 \times 10^{-3}}$$

$$\Rightarrow \boxed{R_C \simeq 642\ \Omega}$$

(9) Q_1 is at the edge of the active region $\Rightarrow V_C = V_B$

applying KVL, we have:



$$V_{CC} = R_C I_C + V_C$$

$$\xRightarrow{V_C = V_B} V_{CC} = R_C I_C + V_B$$

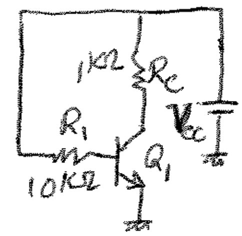
$$\Rightarrow V_{CC} = R_C I_S e^{V_B/V_T} + V_B$$

$$\Rightarrow 500\Omega \times 5\times10^{-16}\, e^{\frac{V_B}{26mV}} + V_B = 2V$$

Using numerical methods or simply, trial & error:

$$\boxed{V_B \simeq 760\ mV}$$

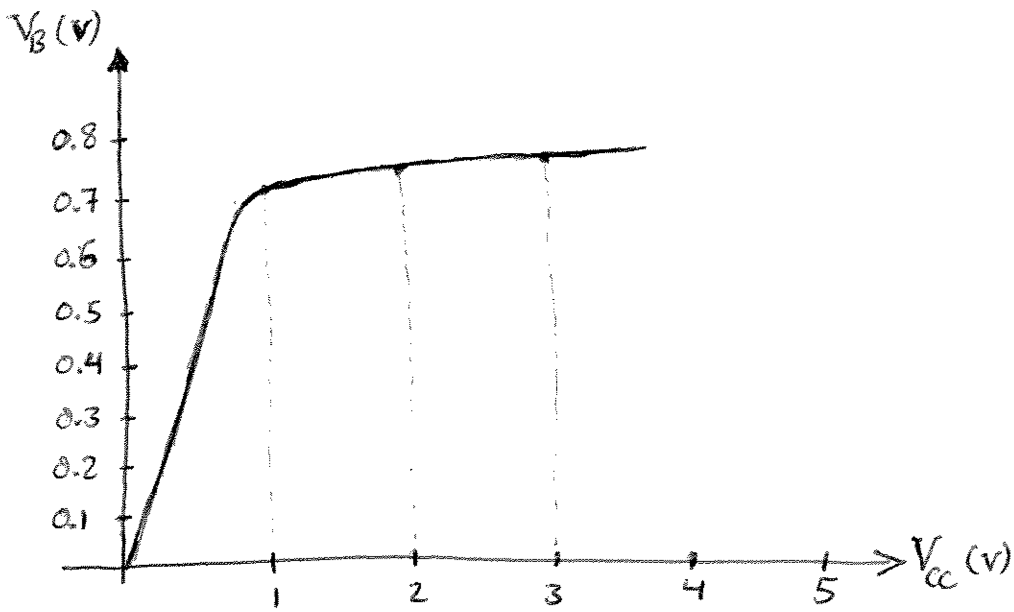
(10) Q_1 at the edge of saturation $\Rightarrow V_C = V_B$



Hence: $V_{CC} = R_C I_C + V_B$

$$\Rightarrow V_{CC} = R_C I_S e^{V_B/V_T} + V_B$$

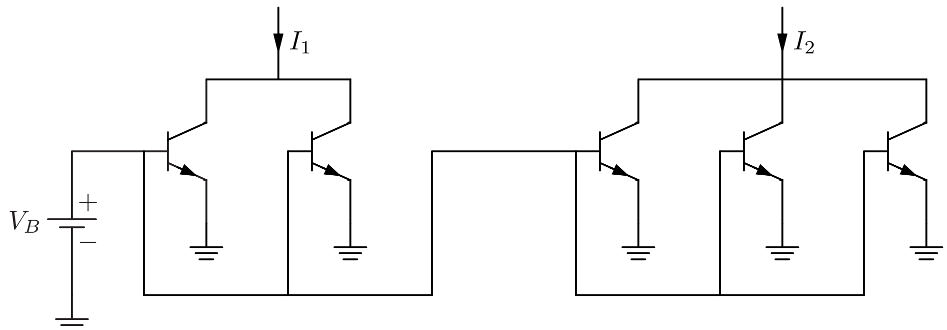
$$\xrightarrow{I_S = 3\times10^{-16}A} V_{CC} = 3\times10^{-13} e^{V_B/V_T} + V_B \xrightarrow[V_{CC}=2^V]{\text{with}} \boxed{V_B \simeq 755^{mV}}$$



4.11

$$
\begin{aligned}
V_{BE} &= 1.5\ \text{V} - I_E(1\ \text{k}\Omega) \\
&\approx 1.5\ \text{V} - I_C(1\ \text{k}\Omega)\ (\text{assuming } \beta \gg 1) \\
&= V_T \ln\left(\frac{I_C}{I_S}\right) \\
I_C &= 775\ \mu\text{A} \\
V_X &\approx I_C(1\ \text{k}\Omega) \\
&= \boxed{775\ \text{mV}}
\end{aligned}
$$

4.12 Since we have only integer multiples of a unit transistor, we need to find the largest number that divides both I_1 and I_2 evenly (i.e., we need to find the largest x such that I_1/x and I_2/x are integers). This will ensure that we use the fewest transistors possible. In this case, it's easy to see that we should pick $x = 0.5$ mA, meaning each transistor should have 0.5 mA flowing through it. Therefore, I_1 should be made up of 1 mA/0.5 mA = 2 parallel transistors, and I_2 should be made up of 1.5 mA/0.5 mA = 3 parallel transistors. This is shown in the following circuit diagram.



Now we have to pick V_B so that $I_C = 0.5$ mA for each transistor.

$$\begin{aligned} V_B &= V_T \ln\left(\frac{I_C}{I_S}\right) \\ &= (26\text{ mV}) \ln\left(\frac{5\times 10^{-4}\text{ A}}{3\times 10^{-16}\text{ A}}\right) \\ &= \boxed{732\text{ mV}} \end{aligned}$$

(13) Using the same technique as in problem 12, we have:

$$\frac{n_1}{I_1} = \frac{n_2}{I_2} = \frac{n_3}{I_3}$$

$$\Rightarrow \frac{n_1}{0.2} = \frac{n_2}{0.3} = \frac{n_3}{0.45} \Rightarrow \boxed{\frac{n_1}{4} = \frac{n_2}{6} = \frac{n_3}{9}}$$

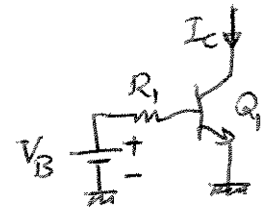
So lets choose $\boxed{\begin{cases} n_1 = 4 \\ n_2 = 6 \\ n_3 = 9 \end{cases}}$

Hence,

$$I_1 = n_1 I_S e^{V_B/V_T} \Rightarrow 0.2 \times 10^{-3} = 4 \times 3 \times 10^{-16} e^{\frac{V_B}{26mV}}$$

$$\Rightarrow \boxed{V_B \simeq 672 \text{ mV}}$$

(14) From KVL,



$$V_B = R_1 I_B + V_{BE_{Q_1}}$$

$$I_B = \frac{I_C}{\beta} = \frac{1^{mA}}{100} \Rightarrow \boxed{I_B = 10^{-5} A}$$

$$V_{BE_{Q_1}} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26 \times 10^{-3} \ln\left(\frac{10^{-3}}{7 \times 10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{BE_{Q_1}} \simeq 727.7 \, mV}$$

Therefore,

$$V_B = R_1 I_B + V_{BE_{Q_1}}$$

$$\simeq 10^{K\Omega} \times 10^{-5} A + 728 \times 10^{-3}$$

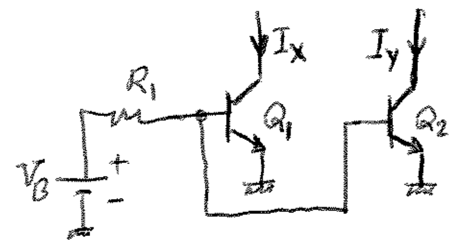
$$\Rightarrow V_B \simeq 0.1 + 0.728 \Rightarrow \boxed{V_B \simeq 0.828 \text{ V}}$$

4.15

$$\begin{aligned} \frac{V_B - V_{BE}}{R_1} &= I_B \\ &= \frac{I_C}{\beta} \\ I_C &= \frac{\beta}{R_1}\left[V_B - V_T \ln(I_C/I_S)\right] \\ I_C &= \boxed{786\ \mu\text{A}} \end{aligned}$$

(16)

$$\begin{cases} I_X = I_{S_1} \exp\left(\frac{V_{BE_1}}{V_T}\right) \\ I_Y = I_{S_2} \exp\left(\frac{V_{BE_2}}{V_T}\right) \\ V_{BE_1} = V_{BE_2} = V_{BE} \end{cases}$$



$$\Rightarrow \frac{I_X}{I_Y} = \frac{I_{S_1}}{I_{S_2}} = \frac{2 I_{S_2}}{I_{S_2}} \Rightarrow \boxed{\frac{I_X}{I_Y} = 2}$$

$$\begin{cases} I_X = \beta_1 I_{B_1} \\ I_Y = \beta_2 I_{B_2} \\ \beta_1 = \beta_2 \end{cases}$$

$$\Rightarrow \boxed{\frac{I_{B_1}}{I_{B_2}} = \frac{I_X}{I_Y} = 2}$$

Applying KVL:

$$V_B = R_1 (I_{B_1} + I_{B_2}) + V_{BE}$$

$$V_{BE} = V_{BE_1} = V_T \ln\left(\frac{I_X}{I_{S_1}}\right) = 26^{mV} \ln\left(\frac{1^{mA}}{4\times10^{-16}}\right) \simeq 742\, mV$$

$$I_{B_1} = \frac{I_X}{\beta} \xrightarrow{\beta=100} I_{B_1} = \frac{1^{mA}}{100} = 10\mu A$$

$$\frac{I_{B_1}}{I_{B_2}} = 2 \longrightarrow I_{B_2} = \frac{I_{B_1}}{2} = \frac{10^{\mu A}}{2} \Rightarrow I_{B_2} = 5\mu A$$

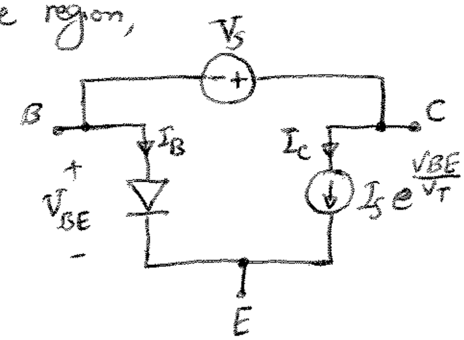
Hence: $V_B = 5\times10^3 \Omega \times (10\mu A + 5\mu A) + 0.742^V$

$$= 0.075 + 0.742 \Rightarrow \boxed{V_B \simeq 0.817^V}$$

4.17 First, note that $V_{BE1} = V_{BE2} = V_{BE}$.

$$\begin{aligned}
V_B &= (I_{B1} + I_{B2})R_1 + V_{BE} \\
&= \frac{R_1}{\beta}(I_X + I_Y) + V_T \ln(I_X/I_{S1}) \\
I_{S2} &= \frac{5}{3} I_{S1} \\
\Rightarrow I_Y &= \frac{5}{3} I_X \\
V_B &= \frac{8R_1}{3\beta} I_X + V_T \ln(I_X/I_{S1}) \\
I_X &= \boxed{509\ \mu\text{A}} \\
I_Y &= \boxed{848\ \mu\text{A}}
\end{aligned}$$

(18) Since Transistor is in Forward active region,



No change across V_{BE}

$\Downarrow$

No change in I_B

$\Downarrow$

No change in I_C

(19) $g_m = \frac{I_C}{V_T}$

$$\Rightarrow g_m = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_T} \Rightarrow \boxed{V_{BE} = V_T \ln\left(\frac{g_m V_T}{I_S}\right)}$$

$$\xrightarrow[g_m = \frac{1}{13\Omega}]{I_S = 6\times 10^{-16} A} V_{BE} = 26^{mV} \cdot \ln\left(\frac{\frac{1}{13\Omega} \times 26\times 10^{-3}}{6\times 10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{BE} \simeq 750 \text{ mV}}$$

(20) $g_m = \frac{I_C}{V_T}$

$$\Delta g_m = \frac{\Delta I_C}{V_T} = \frac{1}{V_T}\Delta\left(I_S e^{V_{BE}/V_T}\right) \simeq \frac{I_S}{V_T^2} e^{\frac{V_{BE}}{V_T}} \Delta V_{BE}$$

$$\Rightarrow \boxed{\Delta g_m \simeq \frac{I_C}{V_T^2} \Delta V_{BE}}$$

$$\Rightarrow \Delta g_m \simeq \frac{g_m}{V_T} \Delta V_{BE}$$

$$\Rightarrow \boxed{\frac{\Delta g_m}{g_m} \simeq \frac{1}{V_T} \Delta V_{BE}}$$

$$\left.\frac{\Delta g_m}{g_m}\right|_{I_C = 1mA} \overset{max}{=\!=} 0.1 \quad \Rightarrow \Delta V_{BE_{max}} = 0.1\, V_T$$

$$\Rightarrow \boxed{\Delta V_{BE} \leqslant 2.6 \text{ mV}}$$

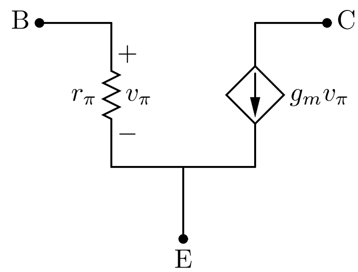
4.21 (a)

$$V_{BE} = \boxed{0.8 \text{ V}}$$
$$I_C = I_S e^{V_{BE}/V_T}$$
$$= \boxed{18.5 \text{ mA}}$$
$$V_{CE} = V_{CC} - I_C R_C$$
$$= \boxed{1.58 \text{ V}}$$

Q_1 is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{710 \text{ mS}}$$
$$r_\pi = \beta/g_m = \boxed{141\ \Omega}$$
$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



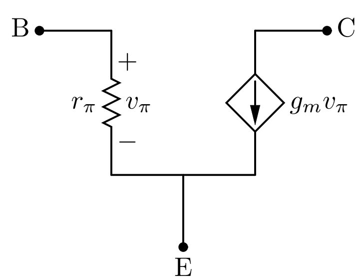
(b)

$$I_B = 10\ \mu\text{A}$$
$$I_C = \beta I_B = \boxed{1 \text{ mA}}$$
$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{724 \text{ mV}}$$
$$V_{CE} = V_{CC} - I_C R_C$$
$$= \boxed{1.5 \text{ V}}$$

Q_1 is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{38.5 \text{ mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.6 \text{ k}\Omega}$$
$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



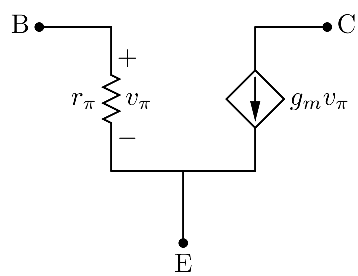
(c)

$$I_E = \frac{V_{CC} - V_{BE}}{R_C} = \frac{1+\beta}{\beta} I_C$$
$$I_C = \frac{\beta}{1+\beta} \frac{V_{CC} - V_T \ln(I_C/I_S)}{R_C}$$
$$I_C = \boxed{1.74 \text{ mA}}$$
$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{739 \text{ mV}}$$
$$V_{CE} = V_{BE} = \boxed{739 \text{ mV}}$$

Q_1 is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{38.5 \text{ mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.6 \text{ k}\Omega}$$
$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



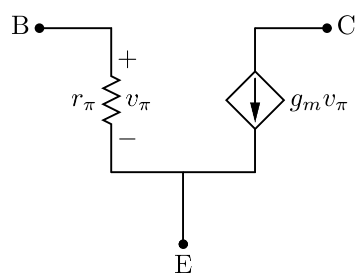
4.22 (a)

$$I_B = 10\ \mu\text{A}$$
$$I_C = \beta I_B = \boxed{1\ \text{mA}}$$
$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{739\ \text{mV}}$$
$$\begin{aligned} V_{CE} &= V_{CC} - I_E(1\ \text{k}\Omega) \\ &= V_{CC} - \frac{1+\beta}{\beta}(1\ \text{k}\Omega) \\ &= \boxed{0.99\ \text{V}} \end{aligned}$$

Q_1 is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{38.5\ \text{mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.6\ \text{k}\Omega}$$
$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



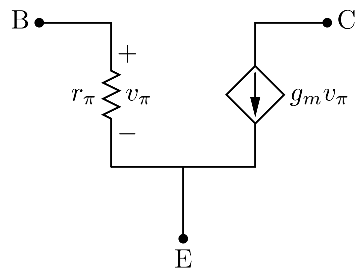
(b)

$$I_E = \frac{V_{CC} - V_{BE}}{1\ \text{k}\Omega} = \frac{1+\beta}{\beta} I_C$$
$$I_C = \frac{\beta}{1+\beta} \frac{V_{CC} - V_T \ln(I_C/I_S)}{1\ \text{k}\Omega}$$
$$I_C = \boxed{1.26\ \text{mA}}$$
$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{730\ \text{mV}}$$
$$V_{CE} = V_{BE} = \boxed{730\ \text{mV}}$$

Q_1 is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{48.3\ \text{mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.07\ \text{k}\Omega}$$
$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



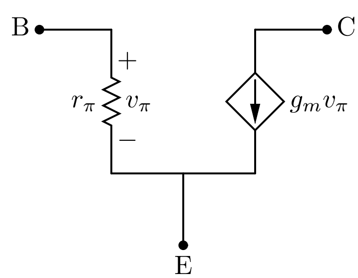
(c)

$$I_E = 1 \text{ mA}$$
$$I_C = \frac{\beta}{1+\beta} I_E = \boxed{0.99 \text{ mA}}$$
$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{724 \text{ mV}}$$
$$V_{CE} = V_{BE} = \boxed{724 \text{ mV}}$$

Q_1 is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{38.1 \text{ mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.63 \text{ k}\Omega}$$
$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



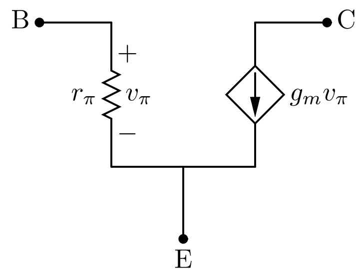
(d)

$$I_E = 1 \text{ mA}$$
$$I_C = \frac{\beta}{1+\beta} I_E = \boxed{0.99 \text{ mA}}$$
$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{724 \text{ mV}}$$
$$V_{CE} = V_{BE} = \boxed{724 \text{ mV}}$$

Q_1 is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{38.1 \text{ mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.63 \text{ k}\Omega}$$
$$r_o = \boxed{\infty}$$

The small-signal model is shown below.

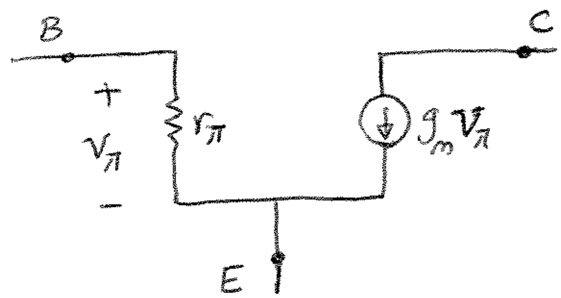


B
C
+
r_π
v_π
−
$g_m v_\pi$
E

(23) $I_C = I_S \exp\left(\frac{V_{BE}}{nV_T}\right)$ $\qquad I_C = \beta I_B$

$$g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{1}{nV_T} I_S \exp\left(\frac{V_{BE}}{nV_T}\right) = \frac{I_C}{nV_T}$$

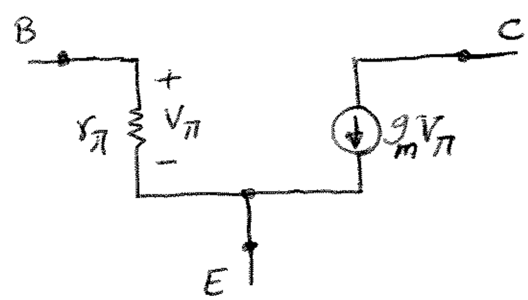
$$r_\pi = \frac{\partial V_{BE}}{\partial I_B} = \frac{\partial V_{BE}}{\frac{1}{\beta}\partial I_C} = \frac{\beta}{g_m} = \frac{n\beta V_T}{I_C}$$



(24) $I_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$, $\qquad I_C = \alpha I_B^2 \Rightarrow \frac{\partial I_B}{\partial I_C} = \frac{1}{2\sqrt{\alpha I_C}}$

$$g_m = \frac{\partial I_C}{\partial v_{BE}} = \frac{I_S}{V_T}\exp\left(\frac{v_{BE}}{V_T}\right) = \boxed{\frac{I_C}{V_T}}$$

$$r_\pi = \frac{\partial V_{BE}}{\partial I_B} = \frac{\partial V_{BE}}{\frac{1}{2\sqrt{\alpha I_C}}\partial I_C} = \frac{2\sqrt{\alpha I_C}}{g_m} = \frac{2\sqrt{\alpha I_C}}{I_C/V_T} = \boxed{2V_T\sqrt{\frac{\alpha}{I_C}}}$$



(25) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$ $\qquad V_{BE}$ is Constant

$$\Delta I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A}\,\Delta V_{CE}$$

$$\Rightarrow \frac{\Delta I_C}{I_C} = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A}\cdot \Delta V_{CE}}{I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]} = \frac{\Delta V_{CE}}{V_A + V_{CE}}$$

$$\frac{\Delta I_C}{I_{C_{min}}} < 0.05 \Rightarrow \frac{\Delta V_{CE}}{V_A + V_{CE_{min}}} < 0.05$$

$$\Rightarrow 20\,\Delta V_{CE} < V_A + V_{CE_{min}}$$

$$\left.\begin{array}{l}\Delta V_{CE} = 2^V \\ V_{CE_{min}} = 1^V\end{array}\right\} \Rightarrow 40 < V_A + 1 \Rightarrow \boxed{V_A > 39^V}$$

(26)

a) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) = 5\times10^{-17} \exp\left(\frac{800^{mV}}{26^{mV}}\right) \simeq 1.15 mA$

$V_X = V_{CC} - R_C I_C = 2.5^{V} - 1^{k\Omega} \times 1.15^{mA}$

$V_X = 1.35 V$ Transistor is in Forward Active Region

b) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$

$\Rightarrow I_C = 5\times10^{-17} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5^{V}}\right]$ equation 1

Also we know: $V_X = V_{CC} - R_C I_C \Rightarrow I_C = \frac{V_{CC} - V_X}{R_C}$ equation 2

equations 1, 2 $\Rightarrow \frac{V_{CC} - V_X}{R_C} = 5\times10^{-17} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5}\right]$

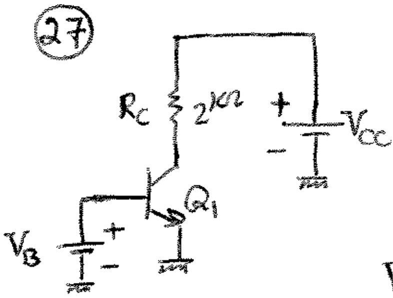
$\Rightarrow V_X + 5\times10^{-14} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5}\right] = 2.5$

$\Rightarrow 1.2306\, V_X \simeq 1.347$

$\Rightarrow V_X \simeq 1.095\ V \xrightarrow{\text{equation 1}} I_C \simeq 1.406 mA$

Transistor is in Forward Active Region

(27)



$I_S =$ [redacted] $\times 10^{-17} A \qquad V_A = 5^V$

Applying KVL:

$$V_{CC} = R_C I_C + V_{CE}$$

$$\Rightarrow V_{CC} = R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right] + V_{CE}$$

$$\xrightarrow{V_{BE}\text{ Constant}} \boxed{\Delta V_{CC} = \left[R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A} + 1\right] \cdot \Delta V_{CE}} \quad \text{equation 1}$$

$$I_C = I_S e^{\frac{V_{BE}}{V_T}}\left[1 + \frac{V_{CE}}{V_A}\right] \Rightarrow \Delta I_C = I_S e^{V_{BE}/V_T} \times \frac{1}{V_A}\,\Delta V_{CE}$$

$$\Rightarrow \boxed{\Delta V_{CE} = \frac{1}{I_S e^{V_{BE}/V_T} \times \frac{1}{V_A}} \cdot \Delta I_C} \quad \text{equation 2}$$

equations 1, 2

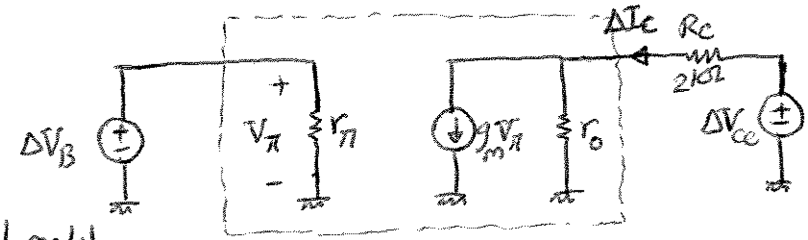
$$\Rightarrow \Delta I_C = \frac{I_S e^{\frac{V_{BE}}{V_T}} \times \frac{1}{V_A}}{1 + R_C I_S e^{\frac{V_{BE}}{V_T}} \times \frac{1}{V_A}} \cdot \Delta V_{CC}$$

$$\Rightarrow \boxed{\Delta I_C = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_A + R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)} \cdot \Delta V_{CC}} = \frac{1}{r_o + R_C} \cdot \Delta V_{CC}$$

(Could also be obtained using small signal model)

$$\Rightarrow \Delta I_C = \frac{2.31 \times 10^{-4}}{5 + 0.4613} \times 0.5 \Rightarrow \Delta I_C \simeq 0.021\,mA$$

(28)



We use small signal model,

Assuming that the required ΔV_B is small enough.

Applying superposition,

$$\Delta I_C = \left(\frac{1}{r_o + R_C}\right)\Delta V_{CC} + \left(\frac{g_m r_o}{r_o + R_C}\right)\Delta V_B$$

$$\Delta I_C = 0 \implies \boxed{\Delta V_B = -\frac{1}{g_m r_o}\Delta V_{CC}}$$

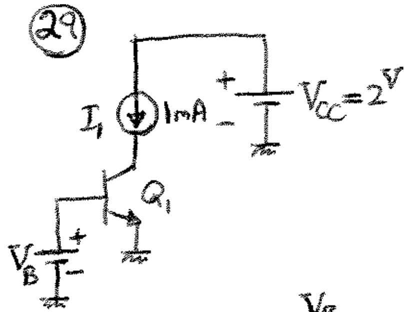
$$\Delta V_B = -\frac{1}{\frac{I_C}{V_T}\cdot\frac{V_A}{I_C}}\Delta V_{CC} \implies \Delta V_B = -\frac{V_T}{V_A}\Delta V_{CC}$$

$$\implies \Delta V_B = -\frac{26\times10^{-3}}{5}\times(3-2.5)$$

$$\implies \boxed{\Delta V_B = -2.6\ \text{mV}}$$

which is small enough for small signal model

(29)



$I_S = 3\times10^{-17} A$

a) $I_C = I_S e^{\frac{V_B}{V_T}} \Longrightarrow V_B = V_T \ln\left(\frac{I_C}{I_S}\right) = 26^{mV} \ln\left(\frac{10^{-3}}{3\times10^{-17}}\right)$

$$\Longrightarrow \boxed{V_B \simeq 809.6 \text{ mV}}$$

b) $I_C = I_S e^{\frac{V_B}{V_T}}\left(1 + \frac{V_{CE}}{V_A}\right)$

$10^{-3} = 3\times10^{-17} e^{V_B/V_T}\left(1 + \frac{1.5}{5}\right) \Rightarrow e^{V_B/V_T} = \frac{10^{14}}{3.9}$

$\Longrightarrow V_B = 26^{mV} \ln\left(\frac{10^{14}}{3.9}\right) \Longrightarrow \boxed{V_B \simeq 802.8 \text{ mV}}$

(30) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$

$$r_o^{-1} = \frac{dI_C}{dV_{CE}} = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \cdot \frac{1}{V_A} \simeq \frac{I_C}{V_A} \Rightarrow r_o \simeq \frac{V_A}{I_C}$$

$$r_o > 10^{k\Omega} \Rightarrow \frac{V_A}{I_C} > 10^{k\Omega}$$

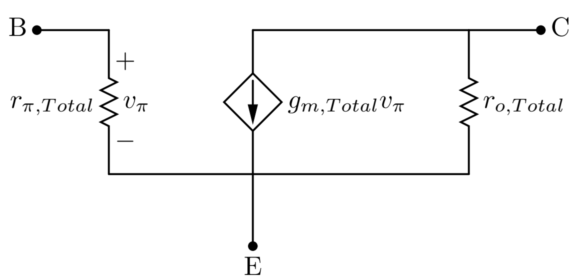
$$\Rightarrow V_A > 10^{k\Omega} \times 2^{mA}$$

$$\Rightarrow \boxed{V_A > 20\,V}$$

4.31

$$
\begin{aligned}
I_C &= I_S e^{V_{BE}/V_T}\left(1+\frac{V_{CE}}{V_A}\right)\\
I_{C,Total} &= nI_C\\
&= nI_S e^{V_{BE}/V_T}\left(1+\frac{V_{CE}}{V_A}\right)\\
g_{m,Total} &= \frac{\partial I_C}{\partial V_{BE}}\\
&= n\frac{I_S}{V_T}e^{V_{BE}/V_T}\\
&\approx n\frac{I_C}{V_T}\\
&= ng_m\\
&= \boxed{n\times 0.4435\text{ S}}\\
I_{B,Total} &= \frac{1}{\beta}I_{C,Total}\\
r_{\pi,Total} &= \left(\frac{\partial I_{B,Total}}{\partial V_{BE}}\right)^{-1}\\
&\approx \left(\frac{I_{C,Total}}{\beta V_T}\right)^{-1}\\
&= \left(\frac{nI_C}{\beta V_T}\right)^{-1}\\
&= \frac{r_\pi}{n}\\
&= \boxed{\frac{225.5\ \Omega}{n}}\text{ (assuming }\beta=100)\\
r_{o,Total} &= \left(\frac{\partial I_{C,Total}}{\partial V_{CE}}\right)^{-1}\\
&\approx \left(\frac{I_{C,Total}}{V_A}\right)^{-1}\\
&= \frac{V_A}{nI_C}\\
&= \frac{r_o}{n}\\
&= \boxed{\frac{693.8\ \Omega}{n}}
\end{aligned}
$$

The small-signal model is shown below.



4.32 (a)

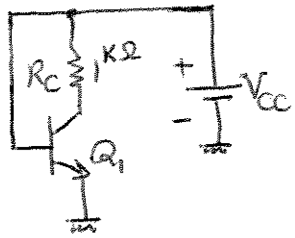
$$\begin{aligned} V_{BE} &= V_{CE} \text{ (for } Q_1 \text{ to operate at the edge of saturation)} \\ V_T \ln(I_C/I_S) &= V_{CC} - I_C R_C \\ I_C &= 885.7\ \mu\text{A} \\ V_B = V_{BE} &= \boxed{728.5\ \text{mV}} \end{aligned}$$

(b) Let I_C', V_B', V_{BE}', and V_{CE}' correspond to the values where the collector-base junction is forward biased by 200 mV.

$$\begin{aligned} V_{BE}' &= V_{CE}' + 200\ \text{mV} \\ V_T \ln(I_C'/I_S) &= V_{CC} - I_C' R_C + 200\ \text{mV} \\ I_C' &= 984.4\ \mu\text{A} \\ V_B' &= 731.3\ \text{mV} \end{aligned}$$

Thus, V_B can increase by $V_B' - V_B = \boxed{2.8\ \text{mV}}$ if we allow soft saturation.

(33)



$I_S = 7\times10^{-16}A$, $V_A = \infty$

$\Downarrow$

$r_o = \infty$

Applying KVL,

$$V_{CC} = R_C I_C + V_{CE} \xrightarrow{V_{CE} = V_{BE} - 0.2^V} R_C I_C + V_{BE} - 0.2^V = V_{CC}$$

$$\Rightarrow R_C I_S e^{\frac{V_{BE}}{V_T}} + V_{BE} - 0.2^V = V_{CC}$$

$$\xrightarrow{V_{BE} = V_{CC}} R_C I_S e^{\frac{V_{CC}}{V_T}} + V_{CC} - 0.2 = V_{CC}$$

$$\Rightarrow R_C I_S e^{V_{CC}/V_T} = 0.2^V$$

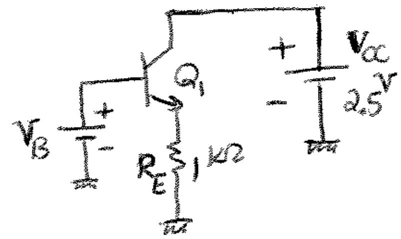
$$\Rightarrow 1^{K\Omega} \times 7\times10^{-16} e^{V_{CC}/26^{mV}} = 0.2^V$$

$$\Rightarrow \boxed{V_{CC} \simeq 686\ mV}$$

4.34

$$\begin{aligned}
V_{BE} &= V_{CC} - I_B R_B \\
V_T \ln(I_C/I_S) &= V_{CC} - I_C R_B/\beta \\
I_C &= 1.67 \text{ mA} \\
V_{BC} &= V_{CC} - I_B R_B - (V_{CC} - I_C R_C) \\
&< 200 \text{ mV} \\
I_C R_C - I_B R_B &< 200 \text{ mV} \\
R_C &< \frac{200 \text{ mV} + I_B R_B}{I_C} \\
&= \frac{200 \text{ mV} + I_C R_B/\beta}{I_C} \\
R_C &< \boxed{1.12 \text{ k}\Omega}
\end{aligned}$$

(35) $I_S = 5\times10^{-16}A$, $V_A = \infty \Rightarrow r_o = \infty$



Soft Saturation $\Rightarrow V_{BC} = 200\,mV$

$$\Rightarrow V_B = V_C + 0.2^V \Rightarrow \boxed{V_B = 2.7\,V}$$

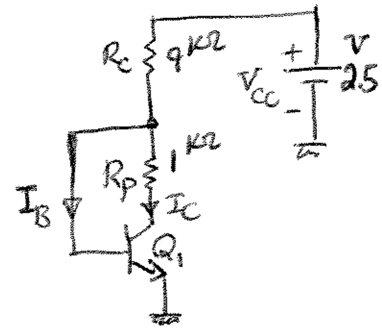
Applying KVL $\Rightarrow V_B = V_{BE} + R_E I_E \xrightarrow{I_E \simeq I_C} V_B = V_{BE} + R_E I_C$

$$\Rightarrow V_{BE} + 1^{k\Omega} \times I_S e^{\frac{V_{BE}}{V_T}} = 2.7^V$$

$$\Rightarrow V_{BE} + 5\times10^{-13} e^{\frac{V_{BE}}{V_T}} = 2.7^V \Rightarrow \boxed{V_{BE} \simeq 754\,mV}$$

$$I_C = I_S e^{V_{BE}/V_T} = 5\times10^{-16} e^{\frac{0.754}{0.026}} \Rightarrow \boxed{I_C \simeq 2\,mA}$$

(36) $\beta = 100, \quad V_A = \infty \Rightarrow r_o = \infty$



$V_{BC} = 0.2\,V \Rightarrow R_P I_C = 0.2\,V$

$$\Rightarrow \boxed{I_C = \frac{0.2\,V}{R_P}}$$

$$V_{BE} = V_{CC} - R_C(I_B + I_C)$$

$$\overset{\beta=100}{\Longrightarrow} V_{BE} = V_{CC} - \frac{\beta+1}{\beta} R_C I_C \Rightarrow \boxed{V_{BE} = V_{CC} - \frac{\beta+1}{\beta}\frac{R_C}{R_P}\times 0.2}$$

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow I_S = I_C \exp\left(-\frac{V_{BE}}{V_T}\right)$$

$$\Rightarrow \boxed{I_S = \frac{0.2}{R_P}\exp\left[\frac{0.2}{V_T}\cdot\frac{\beta+1}{\beta}\cdot\frac{R_C}{R_P} - \frac{V_{CC}}{V_T}\right]}$$

$$\overset{\beta=100}{\Longrightarrow} \boxed{I_S \simeq \frac{0.2}{R_P}\exp\left[\frac{0.2}{V_T}\frac{R_C}{R_P} - \frac{V_{CC}}{V_T}\right]}$$

$$\Rightarrow \boxed{I_S \simeq 4.06 \times 10^{-16}\,A}$$

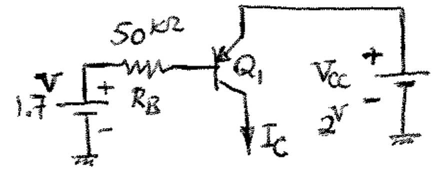
(37) $I_{S_1} = 3I_{S_2} = 6\times10^{-16}\,A$

$$I_1 = I_{S_1}\exp\left(\frac{V_{EB_1}}{V_T}\right) = 6\times10^{-16}\exp\left(\frac{300}{26}\right) \Rightarrow \boxed{I_1 \simeq 6.155\times10^{-11}\,A}$$

$$I_2 = I_{S_2}\exp\left(\frac{V_{EB_2}}{V_T}\right) = 2\times10^{-16}\exp\left(\frac{820}{26}\right) \Rightarrow \boxed{I_2 \simeq 10\,mA}$$

$$I_X = I_1 + I_2 \Rightarrow \boxed{I_X \simeq 10\,mA}$$

(38) $I_S = 2\times10^{-17}\,A \qquad \beta = 100$



Applying KVL,

$$V_{CC} = V_{EB} + R_B I_B + 1.7^V$$

$$\Rightarrow 2^V = V_{EB} + R_B \frac{I_C}{\beta} + 1.7^V$$

$$\Rightarrow 0.3^V = V_{EB} + \frac{50^{k\Omega}}{100} I_C$$

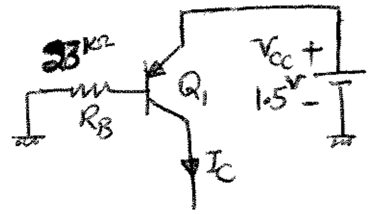
$$\Rightarrow 0.3^V = V_{EB} + 500 \times I_S\, e^{\frac{V_{EB}}{V_T}}$$

$$\Rightarrow 0.3^V = V_{EB} + 10^{-14}\, e^{V_{EB}/26mV} \Rightarrow \boxed{V_{EB} \simeq 0.3\,V}$$

$$I_C = I_S\, e^{\frac{V_{EB}}{V_T}} \Rightarrow I_C = 2\times10^{-17}\, e^{\frac{300}{26}}$$

$$\Rightarrow \boxed{I_C \simeq 2.05\times10^{-12}\,A}$$

(39) $I_C = 3mA$, $\beta = 100$, $R_B = 23\,k\Omega$



Applying KVL,

$$V_{CC} = V_{EB} + R_B I_B \Rightarrow V_{CC} = V_{EB} + R_B \frac{I_C}{\beta}$$

$$\Rightarrow -I_C \frac{R_B}{\beta} + V_{CC} = V_{EB}$$

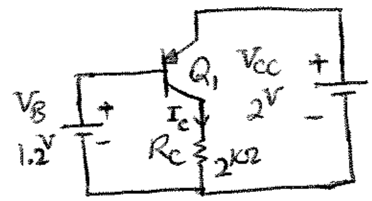
$$I_C = I_S e^{V_{EB}/V_T}$$

$$\Rightarrow I_S = I_C e^{-\frac{V_{EB}}{V_T}}$$

$$\Rightarrow \boxed{I_S = I_C e^{\frac{1}{V_T}\left(\frac{R_B I_C}{\beta} - V_{CC}\right)}}$$

$$\Rightarrow \boxed{I_S \simeq 8.85 \times 10^{-17} A}$$

(40) At the edge of active $\Rightarrow V_{BC} = 0$



$$I_C = \frac{V_B - V_{BC}}{R_C} = \frac{V_B}{R_C}$$

$$\Rightarrow I_C = \frac{1.2^V}{2^{k\Omega}} \Rightarrow \boxed{I_C \simeq 0.6\ mA}$$

$$I_C = I_S \exp\left(\frac{V_{EB}}{V_T}\right) \Rightarrow I_S = I_C \exp\left(-\frac{V_{EB}}{V_T}\right)$$

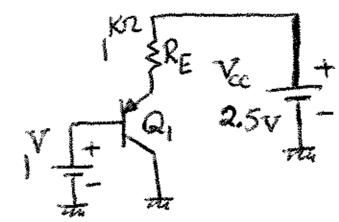
$$\Rightarrow I_S = 0.6 \times 10^{-3} \exp\left(-\frac{800}{26}\right)$$

$$\Rightarrow \boxed{I_S \simeq 2.6 \times 10^{-17}\ A}$$

4.41

$$\begin{aligned}
V_{EB} &= V_{EC} \text{ (for } Q_1 \text{ to operate at the edge of saturation)} \\
V_{CC} - I_B R_B &= V_{CC} - I_C R_C \\
I_C R_B/\beta &= I_C R_C \\
R_B/\beta &= R_C \\
\beta &= R_B/R_C \\
&= \boxed{100}
\end{aligned}$$

(42) $I_S = 3\times10^{-17}$ A



Applying KVL,

$$V_{CC} = R_E I_E + V_{EB} + 1^V \xrightarrow{I_E \simeq I_C} V_{CC} = R_E I_C + V_{EB} + 1^V$$

$$\Rightarrow 2.5 = 1^{k\Omega} \times 3\times10^{-17} e^{V_{EB}/26^{mV}} + V_{EB} + 1$$

$$\Rightarrow V_{EB} + 3\times10^{-14} e^{V_{EB}/26^{mV}} = 1.5^V$$

$$\Rightarrow \boxed{V_{EB} \simeq 800.5 \text{ mV}}$$

$$I_C = I_S e^{\frac{V_{EB}}{V_T}} = 3\times10^{-17} e^{\frac{800.5}{26}} \Rightarrow \boxed{I_C \simeq 0.705 \text{ mA}}$$

(43) $I_S = 3\times10^{-17} A$, $\beta = 100$, $V_A = \infty \Rightarrow \boxed{r_o = \infty}$

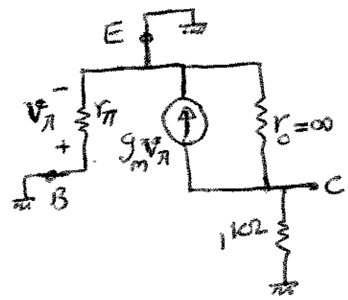
a) $V_{EB} = 2.5 - 1.7 = 0.8\ V$

$I_C = I_S\, exp\left(\frac{V_{EB}}{V_T}\right) = 3\times10^{-17} exp\left(\frac{800}{26}\right) \Rightarrow \boxed{I_C \simeq 0.692^{mA}}$

$V_{EC} = V_{CC} - R_C I_C = 2.5 - 1^{k\Omega} \times 0.692^{mA} \Rightarrow \boxed{V_{EC} \simeq 1.808^{V}}$

$g_m = \frac{I_C}{V_T} = \frac{0.692^{mA}}{26^{mV}} \Rightarrow \boxed{g_m \simeq 26.6 mS}$

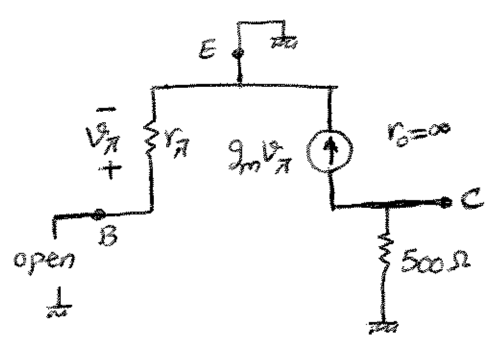
$r_\pi = \frac{\beta}{g_m} = \frac{100}{26.6\times10^{-3}} \Rightarrow \boxed{r_\pi \simeq 3.76\ k\Omega}$



b) $V_{EB} = V_T\, ln\left(\frac{I_C}{I_S}\right) \Rightarrow V_{EB} = V_T\, ln\left(\frac{\beta I_B}{I_S}\right)$

$\Rightarrow V_{EB} = 26^{mV} \times ln\left(\frac{100\times20\times10^{-6}}{3\times10^{-17}}\right)$

$\Rightarrow \boxed{V_{EB} \simeq 827.6\ mV}$



$I_C = \beta I_B \Rightarrow \boxed{I_C = 2mA}$

$V_{EC} = V_{CC} - R_C I_C = 2.5 - 0.5^{k\Omega} \times 2^{mA} \Rightarrow \boxed{V_{EC} = 1.5\ V}$

$g_m = \frac{I_C}{V_T} = \frac{2^{mA}}{26^{mV}} \Rightarrow \boxed{g_m \simeq 77 mS}$ $\quad r_\pi = \frac{\beta}{g_m} \Rightarrow \boxed{r_\pi \simeq 1.3 k\Omega}$

(43) Continued .....

c) Applying KVL,

$$V_{cc} = V_{EB} + (I_C + I_B) \times 2^{k\Omega} \simeq V_{EB} + 2^{k\Omega} \times I_C$$

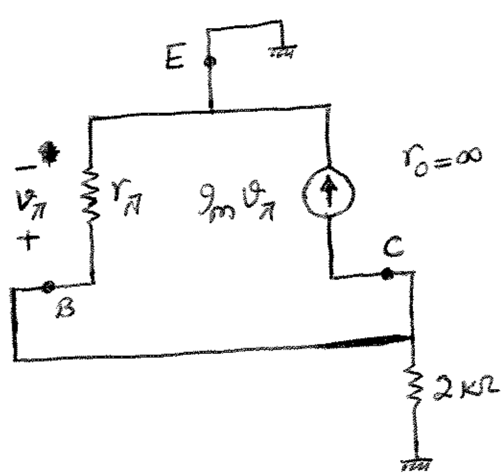
$$\Rightarrow V_{EB} + 2^{k\Omega} \times I_S e^{\frac{V_{EB}}{V_T}} = V_{cc}$$

$$\Rightarrow V_{EB} + 6 \times 10^{-14} e^{V_{EB}/26^{mV}} = 2.5^V \Rightarrow \boxed{V_{EB} \simeq 805 mV}$$

$$I_C \simeq \frac{V_{cc} - V_{EB}}{R} = \frac{2.5 - 0.805}{2^{k\Omega}} \Rightarrow \boxed{I_C \simeq 847.5 \mu A}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.8475 \times 10^{-3}}{0.026} \Rightarrow \boxed{g_m \simeq 32.6 \ mS}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{32.6 \times 10^{-3}} \Rightarrow \boxed{r_\pi \simeq 3068 \ \Omega}$$



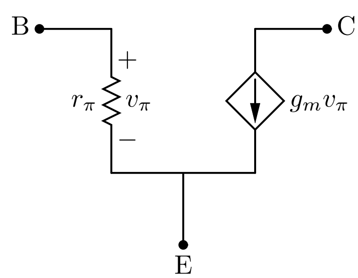
4.44 (a)

$$
\begin{aligned}
I_B &= 2\ \mu\text{A} \\
I_C &= \beta I_B \\
&= \boxed{200\ \mu\text{A}} \\
V_{EB} &= V_T \ln(I_C/I_S) \\
&= \boxed{768\ \text{mV}} \\
V_{EC} &= V_{CC} - I_E(2\ \text{k}\Omega) \\
&= V_{CC} - \frac{1+\beta}{\beta} I_C (2\ \text{k}\Omega) \\
&= \boxed{2.1\ \text{V}}
\end{aligned}
$$

Q_1 is operating in forward active. Its small-signal parameters are

$$
\begin{aligned}
g_m &= I_C/V_T = \boxed{7.69\ \text{mS}} \\
r_\pi &= \beta/g_m = \boxed{13\ \text{k}\Omega} \\
r_o &= \boxed{\infty}
\end{aligned}
$$

The small-signal model is shown below.



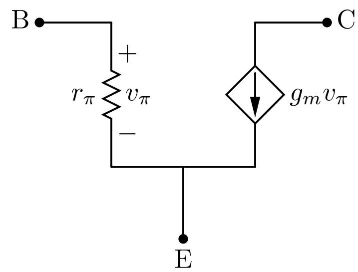
(b)

$$
\begin{aligned}
I_E &= \frac{V_{CC} - V_{EB}}{5\ \text{k}\Omega} \\
\frac{1+\beta}{\beta} I_C &= \frac{V_{CC} - V_T \ln(I_C/I_S)}{5\ \text{k}\Omega} \\
I_C &= \boxed{340\ \mu\text{A}} \\
V_{EB} &= \boxed{782\ \text{mV}} \\
V_{EC} &= V_{EB} = \boxed{782\ \text{mV}}
\end{aligned}
$$

Q_1 is operating in forward active. Its small-signal parameters are

$$
\begin{aligned}
g_m &= I_C/V_T = \boxed{13.1\ \text{mS}} \\
r_\pi &= \beta/g_m = \boxed{7.64\ \text{k}\Omega} \\
r_o &= \boxed{\infty}
\end{aligned}
$$

The small-signal model is shown below.



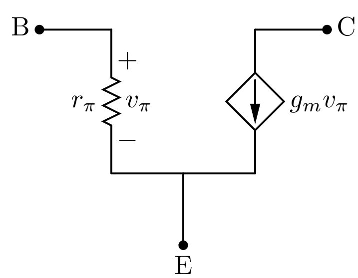
(c)

$$I_E = \frac{1+\beta}{\beta} I_C = 0.5\ \text{mA}$$
$$I_C = \boxed{495\ \mu\text{A}}$$
$$V_{EB} = \boxed{971\ \text{mV}}$$
$$V_{EC} = V_{EB} = \boxed{971\ \text{mV}}$$

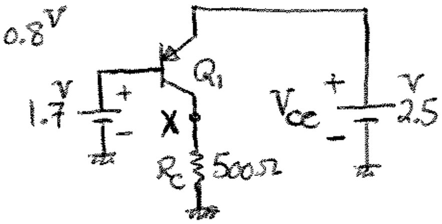
Q_1 is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{19.0\ \text{mS}}$$
$$r_\pi = \beta/g_m = \boxed{5.25\ \text{k}\Omega}$$
$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



(45) $I_S = 5\times10^{-17}A$



a) $V_A = \infty \Rightarrow r_o = \infty$

$$I_c = I_S e^{V_{EB}/V_T} \Rightarrow I_C = 5\times10^{-17} e^{\frac{800}{26}} \Rightarrow \boxed{I_C \simeq 1.15\,mA}$$

$$V_x = R_C I_C = 0.5^{k\Omega} \times 1.15^{mA} \Rightarrow \boxed{V_x \simeq 0.58^V}$$

b) $V_A = 6v$

$$I_C = I_S e^{\frac{V_{EB}}{V_T}}\left(1 + \frac{V_{EC}}{V_A}\right), \quad V_{EC} = V_{CC} - R_C I_C$$

$$\Rightarrow I_C = I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{CC} - R_C I_C}{V_A}\right)$$

$$\Rightarrow I_C = I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{CC}}{V_A}\right) - \frac{I_S R_C}{V_A} e^{\frac{V_{EB}}{V_T}} I_C$$

$$\Rightarrow \boxed{I_C = \frac{I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{CC}}{V_A}\right)}{1 + \frac{I_S R_C}{V_A} e^{V_{EB}/V_T}}} = \frac{5\times10^{-17} e^{\frac{800}{26}}\left(1 + \frac{2.5}{6}\right)}{1 + \frac{5\times10^{-17}\times 0.5^{k\Omega}}{6} e^{800/26}}$$

0.745 V

$$\Rightarrow \boxed{I_C = 1.49\,mA} \qquad V_X = R_C I_C = 500 \times 1.49 \times 10^{-3} \Rightarrow \boxed{V_X = .745^V}$$

(46) $r_o = 60\,k\Omega$, $I_C = 2\,mA$

$$r_o = \frac{V_A}{I_C} \Rightarrow 60\times10^{3}\,\Omega = \frac{V_A}{2\times10^{-3}\,A} \Rightarrow \boxed{V_A = 120\,V}$$

(47) $r_o = 60\ k\Omega$, $I_C = 1\ mA$

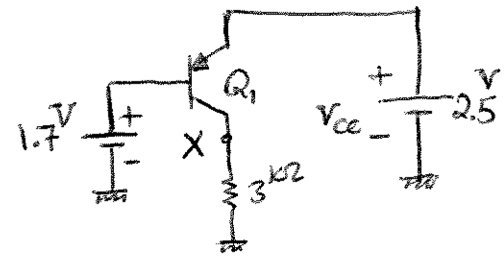
$$r_o = \frac{V_A}{I_C} \Rightarrow \boxed{V_A = r_o \cdot I_C} \Rightarrow V_A \propto I_C$$

$$\Rightarrow V_A = 60^{k\Omega} \times 1^{mA}$$

$$\Rightarrow \boxed{V_A = 60\ V}$$

V_A is half the value in problem 46 as V_A is proportional to I_C.

(48) $V_A = 5\,V$



a) At the edge of active mode

$$\Rightarrow \quad V_X = V_B = 1.7\,V$$

$$I_C = \frac{V_X}{R_C} = \frac{1.7\,V}{3\,k\Omega} \Rightarrow I_C \simeq 0.567\,mA$$

$$I_C = I_S\, e^{\frac{V_{EB}}{V_T}}\left(1 + \frac{V_{EC}}{V_A}\right) \Rightarrow I_S = \frac{I_C\, e^{-\frac{V_{EB}}{V_T}}}{1 + \frac{V_{EC}}{V_A}}$$

$$I_S = \frac{0.567 \times 10^{-3} \times e^{-\frac{800}{26}}}{1 + \frac{2.5 - 1.7}{5}} \Rightarrow I_S \simeq 2.118 \times 10^{-17}\,A$$

b) $V_A = \infty$

$$I_C = I_S\, e^{\frac{V_{EB}}{V_T}} \Rightarrow I_S = I_C\, e^{-\frac{V_{EB}}{V_T}}$$

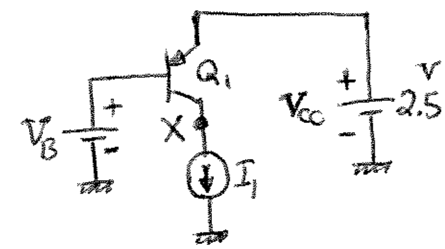
$$I_S = 0.567 \times 10^{-3}\, e^{-\frac{800}{26}} \Rightarrow I_S \simeq 2.457 \times 10^{-17}\,A$$

I_S increases

4.49 The direction of current flow in the large-signal model (Fig. 4.40) indicates the direction of positive current flow when the transistor is properly biased.

The direction of current flow in the small-signal model (Fig. 4.43) indicates the direction of positive change in current flow when the base-emitter voltage v_{be} increases. For example, when v_{be} increases, the current flowing into the collector increases, which is why i_c is shown flowing into the collector in Fig. 4.43. Similar reasoning can be applied to the direction of flow of i_b and i_e in Fig. 4.43.

(50) $I_S = 6\times10^{-16}A$, $V_A = 5V$, $I_1 = 2mA$



a) $I_C = I_S e^{V_{EB}/V_T}\left(1+\frac{V_{EC}}{V_A}\right)$

$$\Rightarrow V_{EB} = V_T \ln\left(\frac{I_C}{I_S\left(1+\frac{V_{EC}}{V_A}\right)}\right)$$

$\xrightarrow[V_{EB}=V_{CC}-V_B]{V_{EC}=V_{CC}-V_X}$ $\boxed{V_B = V_{CC} - V_T \ln\left(\frac{I_C}{I_S\left(1+\frac{V_{CC}-V_X}{V_A}\right)}\right)}$

$$\Rightarrow V_B = 2.5 - 0.026 \ln\left(\frac{2\times10^{-3}}{6\times10^{-16}\left(1+\frac{2.5-1}{5}\right)}\right) \Rightarrow \boxed{V_B \simeq 1.757\,V}$$

b) $I_C = I_S e^{V_{EB}/V_T}\left(1+\frac{V_{EC}}{V_A}\right) \Rightarrow 1+\frac{V_{EC}}{V_A} = \frac{I_C}{I_S} e^{-V_{EB}/V_T}$

$\xrightarrow[V_{EB}=V_{CC}-V_B]{V_{EC}=V_{CC}-V_X}$ $\boxed{V_X = V_{CC} - V_A\left(\frac{I_C}{I_S} e^{-V_{EB}/V_T} - 1\right)}$

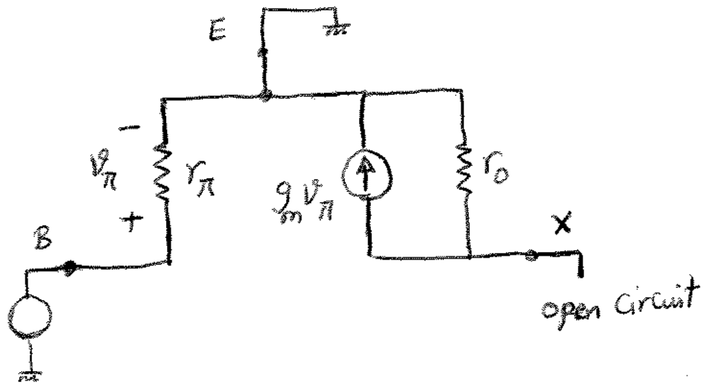
$$\Delta V_X \simeq \frac{dV_X}{dV_{EB}}\Delta V_{EB} \Rightarrow \Delta V_X \simeq \frac{V_A}{V_T}\cdot\frac{I_C}{I_S} e^{-V_{EB}/V_T}\Delta V_{EB}$$

$\xrightarrow{\Delta V_{EB} = -\Delta V_B}$ $\boxed{\Delta V_X \simeq -\frac{V_A}{V_T}\cdot\frac{I_C}{I_S} e^{-V_{EB}/V_T}\Delta V_B}$

$$\Rightarrow \Delta V_X \simeq -\frac{5}{0.026}\times\frac{2\times10^{-3}}{6\times10^{-16}}\exp\left(-\frac{2.5-1.757}{0.026}\right)\times0.1\times10^{-3} \Rightarrow \boxed{\Delta V_X \simeq -24.9\,mV}$$

(50) Continued .....

c)



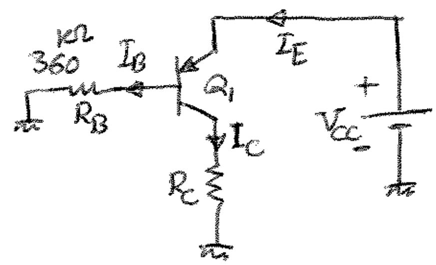
$$r_o = \frac{V_A}{I_C} = \frac{5^V}{2^{mA}} \Rightarrow \boxed{r_o \simeq 2.5\ k\Omega}$$

$$g_m = \frac{I_C}{V_T} = \frac{2^{mA}}{0.026\ V} \Rightarrow \boxed{g_m \simeq 76.9\ mS}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{\frac{2}{26}} \Rightarrow \boxed{r_\pi = 1.3\ k\Omega}$$

(51) $\beta = 100$, $V_A = \infty \Rightarrow r_o = \infty$

$R_B = 360\ k\Omega$



a) given: $V_C = V_B + 0.2^V$

$$\Rightarrow R_C I_C = R_B I_B + 0.2^V$$

$$\Rightarrow R_C I_C = R_B \frac{I_C}{\beta} + 0.2^V \Rightarrow \boxed{I_C = \frac{0.2^V}{R_C - \frac{R_B}{\beta}}} \Rightarrow \boxed{I_C = 0.5^{mA}}$$

$$I_C = I_S e^{+\frac{V_{EB}}{V_T}} \Rightarrow I_S = I_C e^{-\frac{V_{EB}}{V_T}} \Rightarrow I_S = I_C e^{-\left(\frac{V_{CC} - R_B I_B}{V_T}\right)}$$

$$\Rightarrow \boxed{I_S = \left(\frac{0.2}{R_C - \frac{R_B}{\beta}}\right) \exp\left[-\frac{1}{V_T}\left(V_{CC} - R_B \times \frac{0.2^V}{\beta\left(R_C - \frac{R_B}{\beta}\right)}\right)\right]}$$

$$\Rightarrow \boxed{I_S \simeq 10^{-15} A = 1\ fA}$$

b) $g_m = \frac{I_C}{V_T}$

$$\Rightarrow \boxed{g_m = \frac{0.2^V}{V_T\left(R_C - \frac{R_B}{\beta}\right)}} \Rightarrow \boxed{g_m \simeq 19.23\ mS}$$

(52) $I_S = 5\times10^{-16}A$, $\beta = 100$, $V_A = \infty \Rightarrow r_o = \infty$

a) $V_{EB} = 0 \Rightarrow Q_1$ is off $\quad I_C \cong 0$

b) $I_B = 0 \Rightarrow Q_1$ is off

c) Applying KVL : $V_{CC} \cong V_{EB} + 1^{k\Omega} \times I_C$

$$\Rightarrow V_{EB} + 1^{k\Omega} \times I_S e^{\frac{V_{EB}}{V_T}} \simeq V_{CC} \Rightarrow V_{EB} + 5\times10^{-13} e^{\frac{V_{EB}}{26^{mV}}} \simeq 2.5^{V}$$

$$\Rightarrow \boxed{V_{EB} \simeq 751\ mV} \qquad I_C = 5\times10^{-16} e^{\frac{0.751}{0.026}} \Rightarrow \boxed{I_C \simeq 1.8mA}$$

with this current, Transistor is Saturated. Note $V_B < V_C$ Always

(d) $V_{BC} = 0 \Rightarrow$ Transistor is at the edge of Saturation

e) $I_C \cong 0.5mA \Rightarrow V_{EB} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26^{mV} \ln\left(\frac{0.5mA}{5\times10^{-16}}\right)$

$$\Rightarrow \boxed{V_{EB} \simeq 718\ mV}$$

$V_{Collector} = 500^{\Omega} \times I_C \Rightarrow \boxed{V_C = 0.25^V}$

As $V_B = 0$, $V_C = 0.25^V \Rightarrow$ Transistor is soft saturated

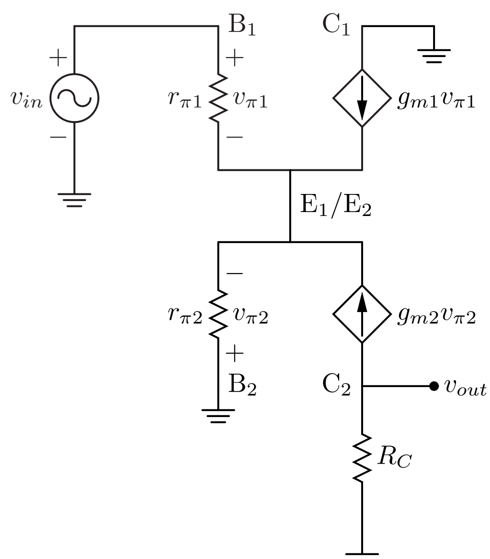
4.53 (a)

$$\begin{aligned}
V_{CB2} &< 200\text{ mV} \\
I_{C2}R_C &< 200\text{ mV} \\
I_{C2} &< 400\ \mu\text{A} \\
V_{EB2} &= V_{E2} \\
&= V_T \ln(I_{C2}/I_{S2}) \\
&< 741\text{ mV} \\
\frac{\beta_2}{1+\beta_2} I_{E2}R_C &< 200\text{ mV} \\
\frac{\beta_2}{1+\beta_2}\frac{1+\beta_1}{\beta_1} I_{C1}R_C &< 200\text{ mV} \\
I_{C1} &< 396\ \mu\text{A} \\
V_{BE1} &= V_T \ln(I_{C1}/I_{S1}) \\
&< 712\text{ mV} \\
V_{in} &= V_{BE1} + V_{EB2} \\
&< \boxed{1.453\text{ V}}
\end{aligned}$$

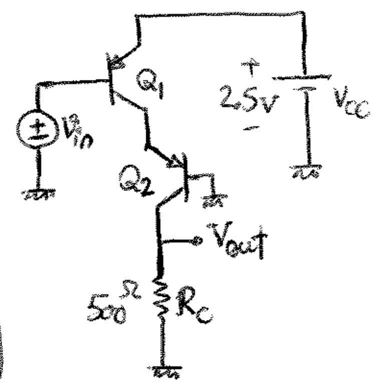
(b)

$$\begin{aligned}
I_{C1} &= 396\ \mu\text{A} \\
I_{C2} &= 400\ \mu\text{A} \\
g_{m1} &= \boxed{15.2\text{ mS}} \\
r_{\pi 1} &= \boxed{6.56\text{ k}\Omega} \\
r_{o1} &= \boxed{\infty} \\
g_{m2} &= \boxed{15.4\text{ mS}} \\
r_{\pi 2} &= \boxed{3.25\text{ k}\Omega} \\
r_{o2} &= \boxed{\infty}
\end{aligned}$$

The small-signal model is shown below.



(54) $I_{S_1} = 3 I_{S_2} = 5\times10^{-16} A$, $\beta_1 = 100$, $\beta_2 = 50$, $V_A = \infty$



a) $V_{B_2} = \phi \xrightarrow[\text{Forward biased by 200mV}]{Q_2 \text{ Base-Collector}} V_{C_2} = 0.2^V$

$\Rightarrow I_{C_{2\,max}} = \frac{V_{C_2 max}}{R_C} = \frac{0.2^V}{500^\Omega} \Rightarrow \boxed{I_{C_2 max} = 0.4 mA}$

As shown: $I_{C_1} \simeq I_{C_2}$ (Note: $I_{C_1} = I_{E_2} = \frac{\beta_2 + 1}{\beta_2} I_{C_2}$ precisely)

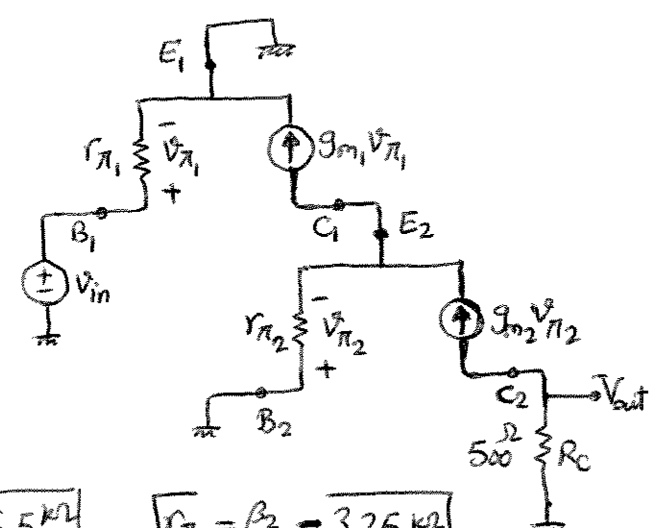
$I_{C_1} \simeq I_{S_1} e^{V_{EB_1}/V_T} \Rightarrow V_{EB_1} = V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right) \Rightarrow V_{CC} - V_{in} = V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right)$

$\Rightarrow \boxed{V_{in} = V_{CC} - V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right)} \Rightarrow V_{in} = 2.5 - 0.026 \ln\left(\frac{4\times10^{-4}}{5\times10^{-16}}\right)$

$\Rightarrow \boxed{V_{in} \simeq 1.787\ V}$ This is minimum acceptable V_{in}

b) $g_{m_1} = \frac{I_{C_1}}{V_T} \simeq \frac{0.4^{mA}}{26^{mV}}$

$g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{0.4^{mA}}{26^{mV}}$



$\Rightarrow \boxed{g_{m_1} = g_{m_2} \simeq 15.4\ mS}$

$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{100}{\frac{0.4}{26}} \Rightarrow \boxed{r_{\pi_1} = 6.5^{k\Omega}}$ $\boxed{r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = 3.25\ k\Omega}$

$V_{EB_2} = V_T \ln\left(\frac{I_{C_2}}{I_{S_2}}\right) = 26^{mV} \ln\left(\frac{0.4\times10^{-3}}{\frac{5}{3}\times10^{-16}}\right) \Rightarrow \boxed{V_{EB_2} \simeq 741\ mV} \Rightarrow \boxed{V_{EC_1} \simeq 1.759\ V}$ (Q_1 in active mode)

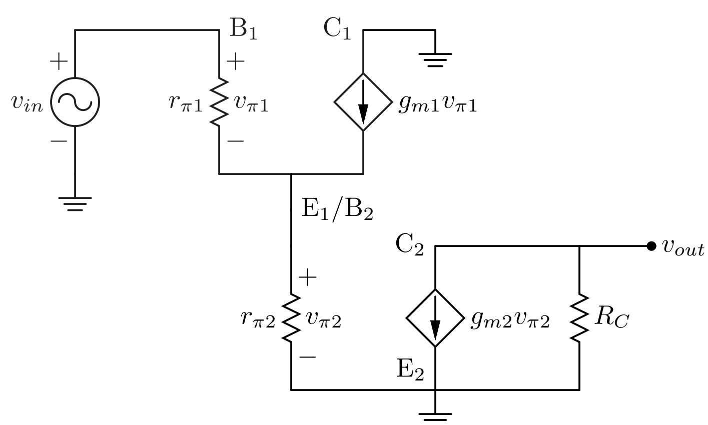
4.55 (a)

$$V_{BC2} < 200\text{ mV}$$
$$V_{BE2} - (V_{CC} - I_{C2}R_C) < 200\text{ mV}$$
$$V_T \ln(I_{C2}/I_{S2}) + I_{C2}R_C - V_{CC} < 200\text{ mV}$$
$$I_{C2} < 3.80\text{ mA}$$
$$V_{BE2} < 799.7\text{ mV}$$
$$I_{E1} = \frac{1+\beta_1}{\beta_1} I_{C1} = I_{B2} = I_{C2}/\beta_2$$
$$I_{C1} < 75.3\ \mu\text{A}$$
$$V_{BE1} < 669.2\text{ mV}$$
$$V_{in} = V_{BE1} + V_{BE2}$$
$$< \boxed{1.469\text{ V}}$$

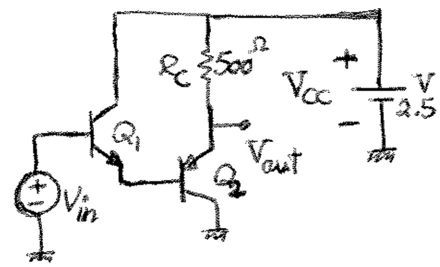
(b)

$$I_{C1} = 75.3\ \mu\text{A}$$
$$I_{C2} = 3.80\text{ mA}$$
$$g_{m1} = \boxed{2.90\text{ mS}}$$
$$r_{\pi 1} = \boxed{34.5\text{ k}\Omega}$$
$$r_{o1} = \boxed{\infty}$$
$$g_{m2} = \boxed{146.2\text{ mS}}$$
$$r_{\pi 2} = \boxed{342\ \Omega}$$
$$r_{o2} = \boxed{\infty}$$

The small-signal model is shown below.



(56) $I_{S_1} = 2I_{S_2} = 6\times10^{-17}\,A, \quad \beta_1 = 80, \quad \beta_2 = 100$



a) $I_{C_2} = 2\,mA$

$$V_{EB_2} = V_T \ln\frac{I_{C_2}}{I_{S_2}} = 26\,mV \ln\left(\frac{2\times10^{-3}}{3\times10^{-17}}\right) \simeq 827.6\,mV$$

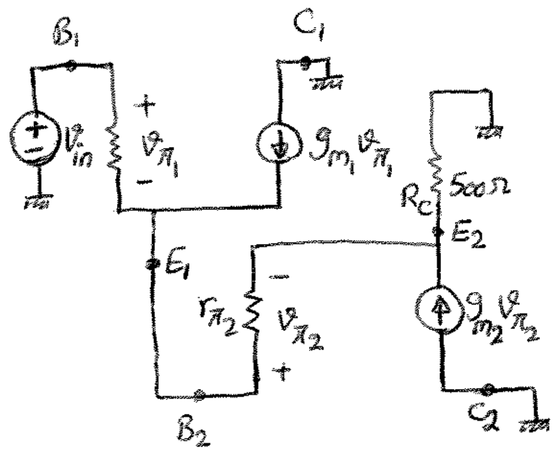
$$V_{BE_1} = V_T \ln\frac{I_{C_1}}{I_{S_1}} = 26\,mV \ln\left(\frac{\frac{2\times10^{-3}}{100}}{6\times10^{-17}}\right) \simeq 689.9\,mV$$

$$\boxed{V_{in} = V_{CC} - R_C I_{C_2} - V_{EB_2} + V_{BE_1}} = 2.5 - 0.5\,k\Omega\times 2\,mA - 0.8276 + 0.6899$$

$$\Rightarrow \boxed{V_{in} \simeq 1.362\,V}$$

b) $g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{2\,mA}{26\,mV} \Rightarrow \boxed{g_{m_2} \simeq 76.9\,mS}$

$g_{m_1} = \frac{I_{C_1}}{V_T} = \frac{2\,mA/100}{26\,mV} \Rightarrow \boxed{g_{m_1} \simeq 769\,\mu S}$



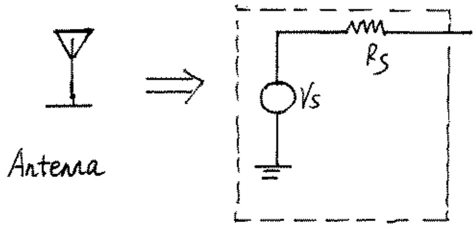
$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{80}{\frac{1}{1300}}$

$\Rightarrow \boxed{r_{\pi_1} = 104\,k\Omega}$

$r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = \frac{100}{2/26} \Rightarrow \boxed{r_{\pi_2} = 1300\,\Omega}$

$V_A = \infty \Rightarrow r_o = \infty$

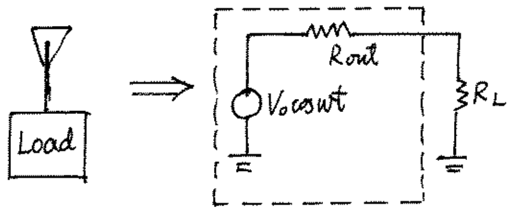
1)



Thevenin Equivalent:

$V_S = V_0 \cos \omega t$

$R_S = R_{out}$



Average power delivered to load $= (I_{RMS})^2 R_L$,

$$I_{RMS} = \frac{V_{RMS}}{R_{out} + R_L}, \quad V_{RMS} = \frac{V_0}{\sqrt{2}} \Rightarrow I_{RMS} = \frac{V_0}{\sqrt{2}(R_{out} + R_L)}$$

$$\text{Average power} = (I_{RMS})^2 R_L = \frac{V_0^2 R_L}{2(R_{out} + R_L)^2} \qquad \text{(Eq. 1)}$$

Plot of Average Power

When R_L is small, Eq. 1 is small.

When R_L is large, Eq. 1 is also small.

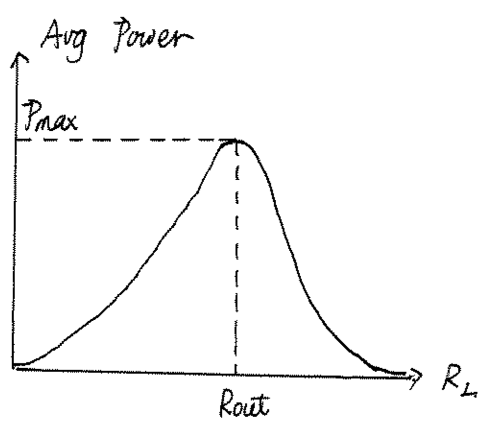
So for some R_L between zero and infinity, the average power will reach its peak. Lets take the derivative of Eq. 1 with respect to R_L to find the optimum R_L.

$$\frac{\partial}{\partial R_L}\left[\frac{V_0^2 R_L}{2(R_{out} + R_L)^2}\right] = \frac{V_0^2}{2(R_{out} + R_L)^2} - \frac{V_0^2 R_L}{(R_{out} + R_L)^3}$$

Setting it to zero and solve for R_L

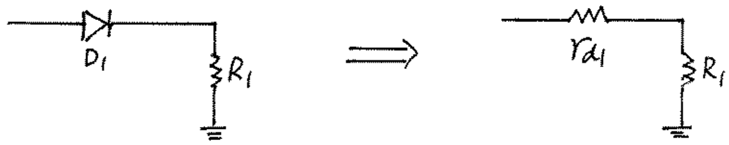
$$\frac{V_0^2}{2(R_{out} + R_L)^2} = \frac{V_0^2 R_L}{(R_{out} + R_L)^3} \Rightarrow \frac{(R_{out} + R_L)}{2} = R_L$$

$$\Rightarrow R_{out} + R_L = 2R_L \Rightarrow R_L = R_{out}$$



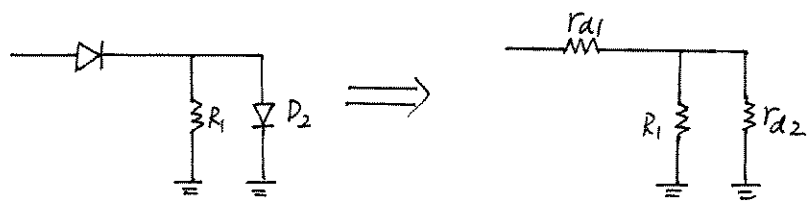
2) In small signal operation, a diode can be replaced by a linear resistor if changes are small.

a).



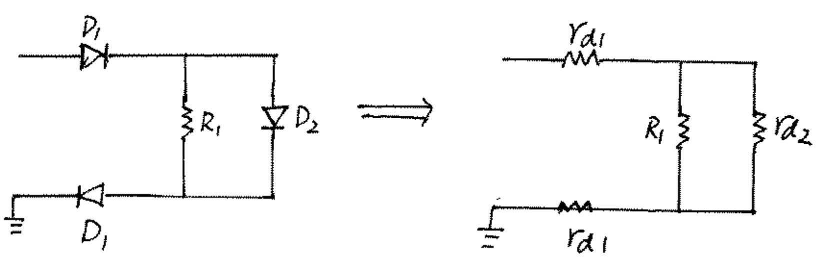
$R_{in} = r_{d1} + R_1$

b).



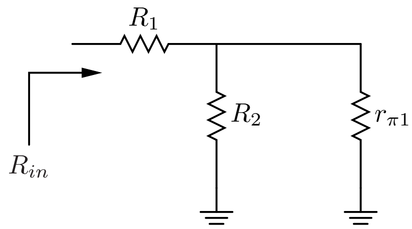
$R_{in} = r_{d1} + R_1 // r_{d2}$ (// means in parallel)

c).



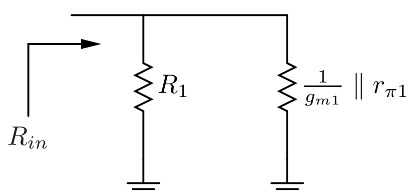
$R_{in} = 2r_{d1} + R_1 // r_{d2}$

5.3 (a) Looking into the base of Q_1 we see an equivalent resistance of $r_{\pi 1}$, so we can draw the following equivalent circuit for finding R_{in}:



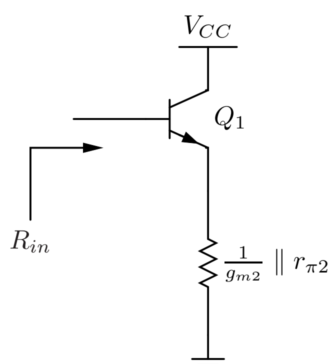
$$R_{in} = \boxed{R_1 + R_2 \parallel r_{\pi 1}}$$

(b) Looking into the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m1}} \parallel r_{\pi 1}$, so we can draw the following equivalent circuit for finding R_{in}:



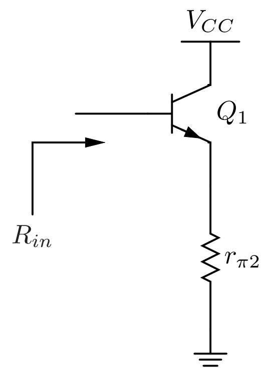
$$R_{in} = \boxed{R_1 \parallel \frac{1}{g_{m1}} \parallel r_{\pi 1}}$$

(c) Looking down from the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:



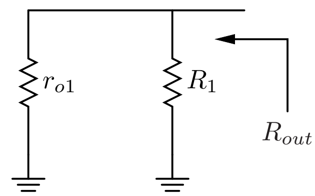
$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

(d) Looking into the base of Q_2 we see an equivalent resistance of $r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:



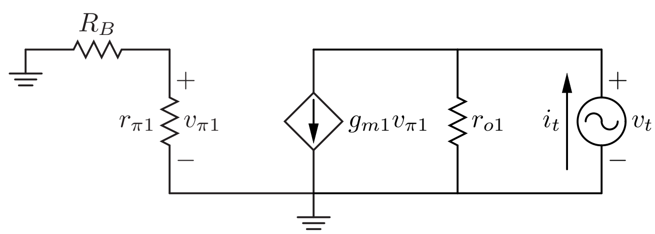
$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1) r_{\pi 2}}$$

5.4 (a) Looking into the collector of Q_1 we see an equivalent resistance of r_{o1}, so we can draw the following equivalent circuit for finding R_{out}:



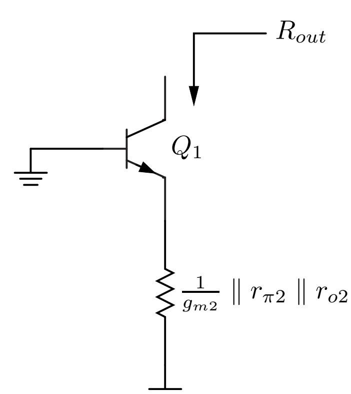
$$R_{out} = \boxed{r_{o1} \parallel R_1}$$

(b) Let's draw the small-signal model and apply a test source at the output.



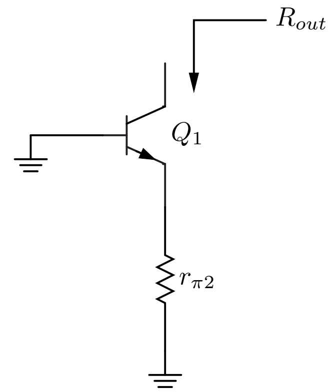
$$\begin{aligned} i_t &= g_{m1}v_{\pi 1} + \frac{v_t}{r_{o1}} \\ v_{\pi 1} &= 0 \\ i_t &= \frac{v_t}{r_{o1}} \\ R_{out} &= \frac{v_t}{i_t} = \boxed{r_{o1}} \end{aligned}$$

(c) Looking down from the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}$, so we can draw the following equivalent circuit for finding R_{out}:



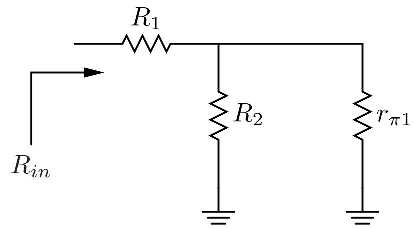
$$R_{out} = \boxed{r_{o1} + (1 + g_{m1}r_{o1})\left(r_{\pi 1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)}$$

(d) Looking into the base of Q_2 we see an equivalent resistance of $r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{out}:



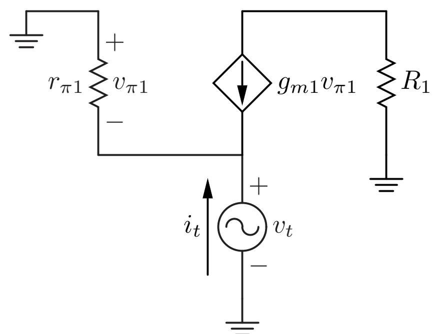
$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) (r_{\pi 1} \parallel r_{\pi 2})}$$

5.5 (a) Looking into the base of Q_1 we see an equivalent resistance of $r_{\pi 1}$, so we can draw the following equivalent circuit for finding R_{in}:



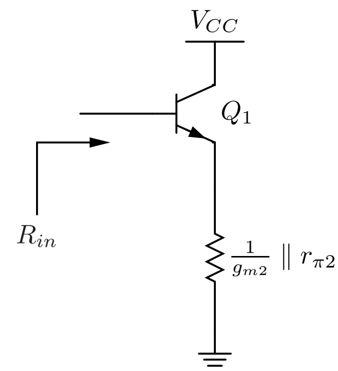
$$R_{in} = \boxed{R_1 + R_2 \parallel r_{\pi 1}}$$

(b) Let's draw the small-signal model and apply a test source at the input.



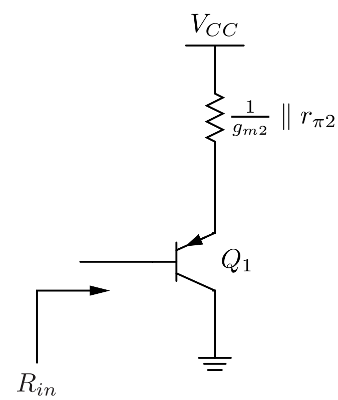
$$\begin{aligned}
i_t &= -\frac{v_{\pi 1}}{r_{\pi 1}} - g_{m1} v_{\pi 1} \\
v_{\pi 1} &= -v_t \\
i_t &= \frac{v_t}{r_{\pi 1}} + g_{m1} v_t \\
i_t &= v_t \left(g_{m1} + \frac{1}{r_{\pi 1}} \right) \\
R_{in} &= \frac{v_t}{i_t} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1}}
\end{aligned}$$

(c) From our analysis in part (b), we know that looking into the emitter we see a resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$. Thus, we can draw the following equivalent circuit for finding R_{in}:



$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

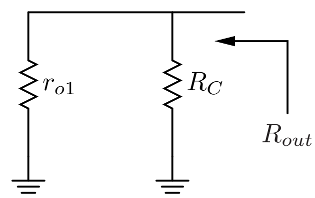
(d) Looking up from the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:



$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

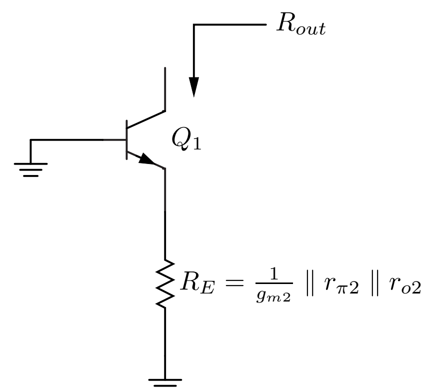
(e) We know that looking into the base of Q_2 we see $R_{in} = \boxed{r_{\pi 2}}$ if the emitter is grounded. Thus, transistor Q_1 does not affect the input impedance of this circuit.

5.6 (a) Looking into the collector of Q_1 we see an equivalent resistance of r_{o1}, so we can draw the following equivalent circuit for finding R_{out}:



$$R_{out} = \boxed{R_C \parallel r_{o1}}$$

(b) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}$, so we can draw the following equivalent circuit for finding R_{out}:



$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) \left(r_{\pi 1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2} \right)}$$

5.7 (a)

$$\begin{aligned}
V_{CC} - I_B(100\ \text{k}\Omega) &= V_{BE} = V_T \ln(I_C/I_S) \\
V_{CC} - \frac{1}{\beta} I_C(100\ \text{k}\Omega) &= V_T \ln(I_C/I_S) \\
I_C &= \boxed{1.754\ \text{mA}} \\
V_{BE} &= V_T \ln(I_C/I_S) = \boxed{746\ \text{mV}} \\
V_{CE} &= V_{CC} - I_C(500\ \Omega) = \boxed{1.62\ \text{V}}
\end{aligned}$$

Q_1 is operating in forward active.

(b)

$$\begin{aligned}
I_{E1} &= I_{E2} \Rightarrow V_{BE1} = V_{BE2} \\
V_{CC} - I_{B1}(100\ \text{k}\Omega) &= 2V_{BE1} \\
V_{CC} - \frac{1}{\beta} I_{C1}(100\ \text{k}\Omega) &= 2V_T \ln(I_{C1}/I_S) \\
I_{C1} &= I_{C2} = \boxed{1.035\ \text{mA}} \\
V_{BE1} &= V_{BE2} = \boxed{733\ \text{mV}} \\
V_{CE2} &= V_{BE2} = \boxed{733\ \text{mV}} \\
V_{CE1} &= V_{CC} - I_C(1\ \text{k}\Omega) - V_{CE2} \\
&= \boxed{733\ \text{mV}}
\end{aligned}$$

Both Q_1 and Q_2 are at the edge of saturation.

(c)

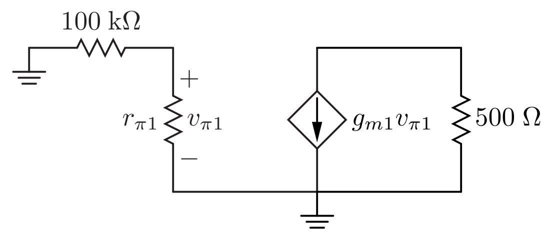
$$\begin{aligned}
V_{CC} - I_B(100\ \text{k}\Omega) &= V_{BE} + 0.5\ \text{V} \\
V_{CC} - \frac{1}{\beta} I_C(100\ \text{k}\Omega) &= V_T \ln(I_C/I_S) + 0.5\ \text{V} \\
I_C &= \boxed{1.262\ \text{mA}} \\
V_{BE} &= \boxed{738\ \text{mV}} \\
V_{CE} &= V_{CC} - I_C(1\ \text{k}\Omega) - 0.5\ \text{V} \\
&= \boxed{738\ \text{mV}}
\end{aligned}$$

Q_1 is operating at the edge of saturation.

5.8 See Problem 7 for the derivation of I_C for each part of this problem.

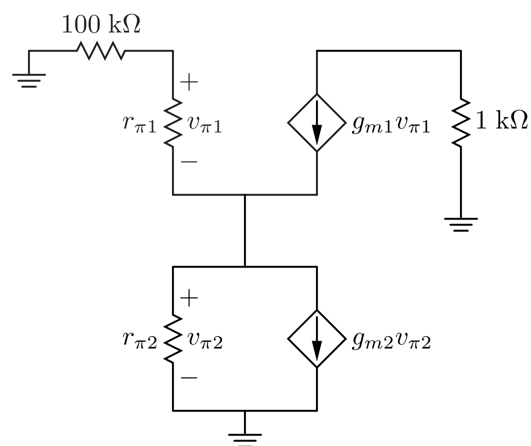
(a)

$$I_{C1} = 1.754 \text{ mA}$$
$$g_{m1} = I_{C1}/V_T = \boxed{67.5 \text{ mS}}$$
$$r_{\pi 1} = \beta/g_{m1} = \boxed{1.482 \text{ k}\Omega}$$



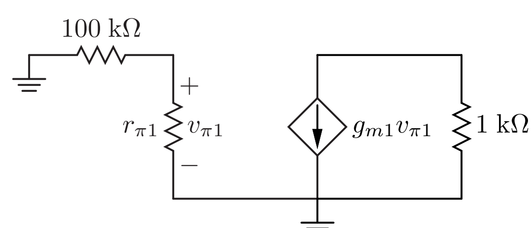
(b)

$$I_{C1} = I_{C2} = 1.034 \text{ mA}$$
$$g_{m1} = g_{m2} = I_{C1}/V_T = \boxed{39.8 \text{ mS}}$$
$$r_{\pi 1} = r_{\pi 2} = \beta/g_{m1} = \boxed{2.515 \text{ k}\Omega}$$



(c)

$$I_{C1} = 1.26 \text{ mA}$$
$$g_{m1} = I_{C1}/V_T = \boxed{48.5 \text{ mS}}$$
$$r_{\pi 1} = \beta/g_{m1} = \boxed{2.063 \text{ k}\Omega}$$



5.9 (a)

$$\frac{V_{CC} - V_{BE}}{34\ \text{k}\Omega} - \frac{V_{BE}}{16\ \text{k}\Omega} = I_B = \frac{I_C}{\beta}$$

$$I_C = \beta\frac{V_{CC} - V_T \ln(I_C/I_S)}{34\ \text{k}\Omega} - \beta\frac{V_T \ln(I_C/I_S)}{16\ \text{k}\Omega}$$

$$I_C = \boxed{677\ \mu\text{A}}$$

$$V_{BE} = \boxed{726\ \text{mV}}$$

$$V_{CE} = V_{CC} - I_C(3\ \text{k}\Omega) = \boxed{468\ \text{mV}}$$

Q_1 is in soft saturation.

(b)

$$I_{E1} = I_{E2}$$

$$\Rightarrow I_{C1} = I_{C2}$$

$$\Rightarrow V_{BE1} = V_{BE2} = V_{BE}$$

$$\frac{V_{CC} - 2V_{BE}}{9\ \text{k}\Omega} - \frac{2V_{BE}}{16\ \text{k}\Omega} = I_{B1} = \frac{I_{C1}}{\beta}$$

$$I_{C1} = \beta\frac{V_{CC} - 2V_T \ln(I_{C1}/I_S)}{9\ \text{k}\Omega} - \beta\frac{2V_T \ln(I_{C1}/I_S)}{16\ \text{k}\Omega}$$

$$I_{C1} = I_{C2} = \boxed{1.72\ \text{mA}}$$

$$V_{BE1} = V_{BE2} = V_{CE2} = \boxed{751\ \text{mV}}$$

$$V_{CE1} = V_{CC} - I_{C1}(500\ \Omega) - V_{CE2} = \boxed{890\ \text{mV}}$$

Q_1 is in forward active and Q_2 is on the edge of saturation.

(c)

$$\frac{V_{CC} - V_{BE} - 0.5\ \text{V}}{12\ \text{k}\Omega} - \frac{V_{BE} + 0.5\ \text{V}}{13\ \text{k}\Omega} = I_B = \frac{I_C}{\beta}$$

$$I_C = \beta\frac{V_{CC} - V_T \ln(I_C/I_S) - 0.5\ \text{V}}{12\ \text{k}\Omega} - \beta\frac{V_T \ln(I_C/I_S) + 0.5\ \text{V}}{13\ \text{k}\Omega}$$

$$I_C = \boxed{1.01\ \text{mA}}$$

$$V_{BE} = \boxed{737\ \text{mV}}$$

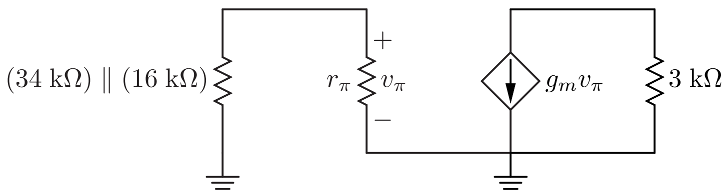
$$V_{CE} = V_{CC} - I_C(1\ \text{k}\Omega) - 0.5\ \text{V} = \boxed{987\ \text{mV}}$$

Q_1 is in forward active.

5.10 See Problem 9 for the derivation of I_C for each part of this problem.

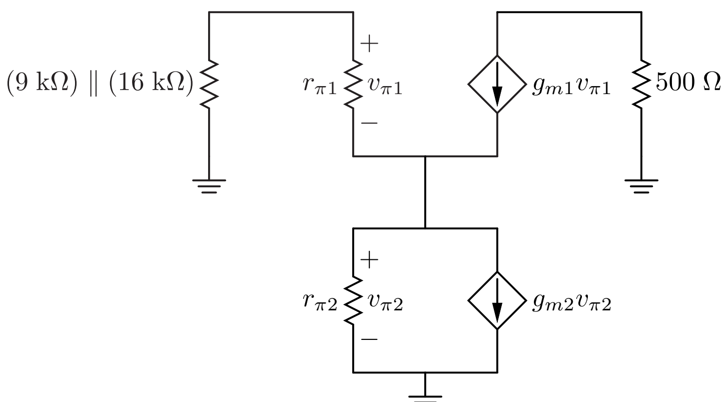
(a)

$$I_C = 677 \ \mu\text{A}$$
$$g_m = I_C/V_T = \boxed{26.0 \text{ mS}}$$
$$r_\pi = \beta/g_m = \boxed{3.84 \text{ k}\Omega}$$



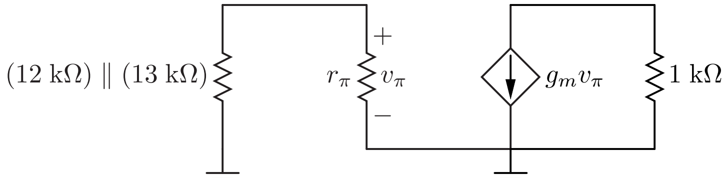
(b)

$$I_{C1} = I_{C2} = 1.72 \text{ mA}$$
$$g_{m1} = g_{m2} = I_{C1}/V_T = \boxed{66.2 \text{ mS}}$$
$$r_{\pi 1} = r_{\pi 2} = \beta/g_{m1} = \boxed{1.51 \text{ k}\Omega}$$



(c)

$$I_C = 1.01 \text{ mA}$$
$$g_m = I_C/V_T = \boxed{38.8 \text{ mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.57 \text{ k}\Omega}$$



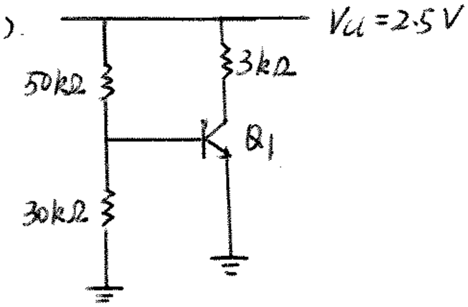
5.11 (a)

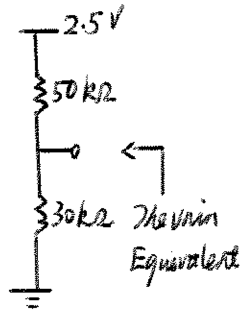
$$V_{CE} \geq V_{BE} \text{ (in order to guarantee operation in the active mode)}$$
$$V_{CC} - I_C(2\text{ k}\Omega) \geq V_{BE}$$
$$V_{CC} - I_C(2\text{ k}\Omega) \geq V_T \ln(I_C/I_S)$$
$$I_C \leq 886\ \mu\text{A}$$
$$\frac{V_{CC} - V_{BE}}{R_B} - \frac{V_{BE}}{3\text{ k}\Omega} = I_B = \frac{I_C}{\beta}$$
$$\frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} - \frac{V_T \ln(I_C/I_S)}{3\text{ k}\Omega} = \frac{I_C}{\beta}$$
$$R_B\left(\frac{I_C}{\beta} + \frac{V_T \ln(I_C/I_S)}{3\text{ k}\Omega}\right) = V_{CC} - V_T \ln(I_C/I_S)$$
$$R_B = \frac{V_{CC} - V_T \ln(I_C/I_S)}{\frac{I_C}{\beta} + \frac{V_T \ln(I_C/I_S)}{3\text{ k}\Omega}}$$
$$R_B \geq \boxed{7.04\text{ k}\Omega}$$

(b)

$$\frac{V_{CC} - V_{BE}}{R_B} - \frac{V_{BE}}{3\text{ k}\Omega} = I_B = \frac{I_C}{\beta}$$
$$I_C = \beta\frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} - \beta\frac{V_T \ln(I_C/I_S)}{3\text{ k}\Omega}$$
$$I_C = 1.14\text{ mA}$$
$$V_{BE} = 735\text{ mV}$$
$$V_{CE} = V_{CC} - I_C(2\text{ k}\Omega) = 215\text{ mV}$$
$$V_{BC} = V_{BE} - V_{CE} = \boxed{520\text{ mV}}$$

12). a).





$V_{th} = 2.5 \times \frac{30}{50+30} = 0.9375$

$R_{th} = \frac{30 \times 50}{30+50} k = 18.75k$

Since $I_C = 0.5mA$, $I_B = \frac{I_C}{\beta} = 0.005mA$.

$I_B = \frac{V_{th} - V_{BE}}{R_{th}} \Rightarrow V_{BE} = V_{th} - I_B \cdot R_{th} = 0.84375$

$I_C = I_S e^{(\frac{V_{BE}}{V_T})} \Rightarrow I_S = \frac{I_C}{e^{(V_{BE}/V_T)}} = 4.03 \times 10^{-15} (mA)$

b). At the edge of saturation means $V_{BE} - V_{CE} = 0$.

(soft saturation not allowed)

$V_{CE} = 2.5 - I_C \cdot (3k)$, in which $I_C = \beta I_B = \beta (\frac{V_{th} - V_{BE}}{R_{th}})$

so $V_{BE} = 2.5 - \beta (\frac{V_{th} - V_{BE}}{R_{th}}) \cdot (3k)$

Solve this equation:

$V_{BE} = 0.83$.

$I_S = \frac{I_C}{e^{(V_{BE}/V_T)}} = \frac{\beta (\frac{V_{th} - V_{BE}}{R_{th}})}{e^{(V_{BE}/V_T)}} = 7.84 \times 10^{-15} (mA)$

5.13 We know the input resistance is $R_{in} = R_1 \parallel R_2 \parallel r_\pi$. Since we want the minimum values of R_1 and R_2 such that $R_{in} > 10\ \text{k}\Omega$, we should pick the maximum value allowable for r_π, which means picking the minimum value allowable for g_m (since $r_\pi \propto 1/g_m$), which is $g_m = 1/260\ \text{S}$.

$$
\begin{aligned}
g_m &= \frac{1}{260}\ \text{S} \\
I_C &= g_m V_T = 100\ \mu\text{A} \\
V_{BE} &= V_T \ln(I_C/I_S) = 760\ \text{mV} \\
I_B &= \frac{I_C}{\beta} = 1\ \mu\text{A} \\
\frac{V_{CC} - V_{BE}}{R_1} - \frac{V_{BE}}{R_2} &= I_B \\
R_1 &= \frac{V_{CC} - V_{BE}}{I_B + \frac{V_{BE}}{R_2}} \\
r_\pi &= \frac{\beta}{g_m} = 26\ \text{k}\Omega \\
R_{in} &= R_1 \parallel R_2 \parallel r_\pi \\
&= \left(\frac{V_{CC} - V_{BE}}{I_B + \frac{V_{BE}}{R_2}} \right) \parallel R_2 \parallel r_\pi \\
&> 10\ \text{k}\Omega \\
R_2 &> \boxed{23.57\ \text{k}\Omega} \\
R_1 &> \boxed{52.32\ \text{k}\Omega}
\end{aligned}
$$

5.14

$$
\begin{aligned}
g_m &= \frac{I_C}{V_T} \geq \frac{1}{26}\ \text{S} \\
r_\pi &= \frac{\beta}{g_m} = 2.6\ \text{k}\Omega \\
R_{in} &= R_1 \parallel R_2 \parallel r_\pi \\
&\leq r_\pi
\end{aligned}
$$

According to the above analysis, R_{in} cannot be greater than 2.6 kΩ. This means that the requirement that $R_{in} \geq 10$ kΩ cannot be met. Qualitatively, the requirement for g_m to be large forces r_π to be small, and since R_{in} is bounded by r_π, it puts an upper bound on R_{in} that, in this case, is below the required 10 kΩ.

5.15

$$
\begin{aligned}
R_{out} &= R_C = R_0 \\
A_v &= -g_m R_C = -g_m R_0 = -\frac{I_C}{V_T} R_0 = -A_0 \\
I_C &= \frac{A_0}{R_0} V_T \\
r_\pi &= \beta \frac{V_T}{I_C} = \beta \frac{R_0}{A_0} \\
V_{BE} &= V_T \ln(I_C/I_S) = V_T \ln\left(\frac{A_0 V_T}{R_0 I_S}\right) \\
\frac{V_{CC} - V_{BE}}{R_1} - \frac{V_{BE}}{R_2} &= I_B = \frac{I_C}{\beta} \\
R_1 &= \frac{V_{CC} - V_{BE}}{\frac{I_C}{\beta} + \frac{V_{BE}}{R_2}} \\
R_{in} &= R_1 \parallel R_2 \parallel r_\pi \\
&= \left(\frac{V_{CC} - V_T \ln\left(\frac{A_0 V_T}{R_0 I_S}\right)}{\frac{I_C}{\beta} + \frac{V_T}{R_2} \ln\left(\frac{A_0 V_T}{R_0 I_S}\right)}\right) \parallel R_2 \parallel \beta \frac{R_0}{A_0}
\end{aligned}
$$

In order to maximize R_{in}, we can let $R_2 \to \infty$. This gives us

$$
R_{in,max} = \boxed{\left(\beta \frac{V_{CC} - V_T \ln\left(\frac{A_0 V_T}{R_0 I_S}\right)}{I_C}\right) \parallel \beta \frac{R_0}{A_0}}
$$

5.16 (a)

$$
\begin{aligned}
I_C &= 0.25\ \text{mA} \\
V_{BE} &= 696\ \text{mV} \\
\frac{V_{CC} - V_{BE} - I_E R_E}{R_1} - \frac{V_{BE} + I_E R_E}{R_2} &= I_B = \frac{I_C}{\beta} \\
R_1 &= \frac{V_{CC} - V_{BE} - \frac{1+\beta}{\beta} I_C R_E}{\frac{I_C}{\beta} + \frac{V_{BE} + \frac{1+\beta}{\beta} I_C R_E}{R_2}} \\
&= \boxed{22.74\ \text{k}\Omega}
\end{aligned}
$$

(b) First, consider a 5 % increase in R_E.

$$
\begin{aligned}
R_E &= 210\ \Omega \\
\frac{V_{CC} - V_{BE} - I_E R_E}{R_1} - \frac{V_{BE} + I_E R_E}{R_2} &= I_B = \frac{I_C}{\beta} \\
\frac{V_{CC} - V_T \ln(I_C/I_S) - \frac{1+\beta}{\beta} I_C R_E}{R_1} - \frac{V_T \ln(I_C/I_S) + \frac{1+\beta}{\beta} I_C R_E}{R_2} &= I_B = \frac{I_C}{\beta} \\
I_C &= 243\ \mu\text{A} \\
\frac{I_C - I_{C,nom}}{I_{C,nom}} \times 100 &= \boxed{-2.6\ \%}
\end{aligned}
$$

Now, consider a 5 % decrease in R_E.

$$
\begin{aligned}
R_E &= 190\ \Omega \\
I_C &= 257\ \mu\text{A} \\
\frac{I_C - I_{C,nom}}{I_{C,nom}} \times 100 &= \boxed{+2.8\ \%}
\end{aligned}
$$

5.17

$$
\begin{aligned}
V_{CE} &\geq V_{BE} \text{ (in order to guarantee operation in the active mode)} \\
V_{CC} - I_C R_C &\geq V_T \ln(I_C/I_S) \\
I_C &\leq 833\ \mu\text{A} \\
\frac{V_{CC} - V_{BE} - I_E R_E}{30\ \text{k}\Omega} - \frac{V_{BE} + I_E R_E}{R_2} &= I_B = \frac{I_C}{\beta} \\
R_2 &= \frac{V_{BE} + I_E R_E}{\frac{V_{CC} - V_{BE} - I_E R_E}{30\ \text{k}\Omega} - \frac{I_C}{\beta}} \\
&= \frac{V_T \ln(I_C/I_S) + \frac{1+\beta}{\beta} I_C R_E}{\frac{V_{CC} - V_T \ln(I_C/I_S) - \frac{1+\beta}{\beta} I_C R_E}{30\ \text{k}\Omega} - \frac{I_C}{\beta}} \\
R_2 &\leq \boxed{20.66\ \text{k}\Omega}
\end{aligned}
$$

5.18 (a) First, note that $V_{BE1} = V_{BE2} = V_{BE}$, but since $I_{S1} = 2I_{S2}$, $I_{C1} = 2I_{C2}$. Also note that $\beta_1 = \beta_2 = \beta = 100$.

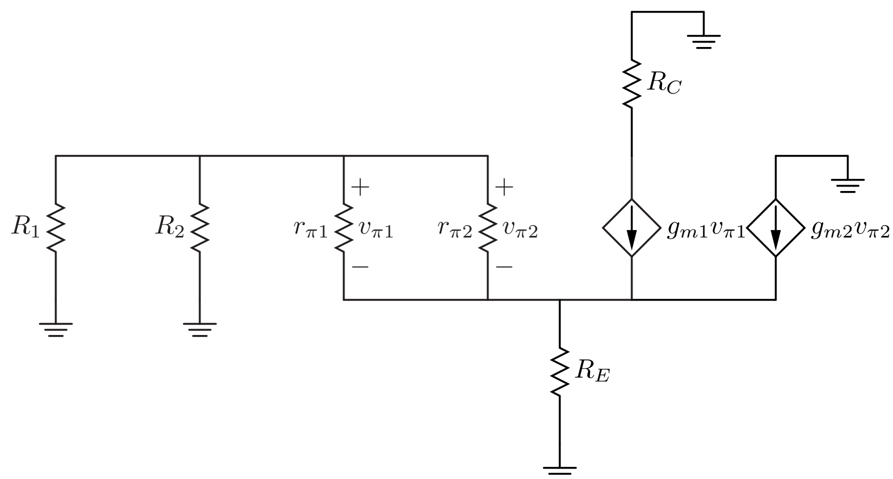
$$I_{B1} = \frac{I_{C1}}{\beta} = \frac{V_{CC} - V_{BE} - (I_{E1} + I_{E2})R_E}{R_1} - \frac{V_{BE} + (I_{E1} + I_{E2})R_E}{R_2}$$

$$I_{C1} = \beta \frac{V_{CC} - V_T \ln(I_{C1}/I_{S1}) - \frac{3}{2}\frac{1+\beta}{\beta} I_{C1} R_E}{R_1} - \frac{V_T \ln(I_{C1}/I_{S1}) + \frac{3}{2}\frac{1+\beta}{\beta} I_{C1} R_E}{R_2}$$

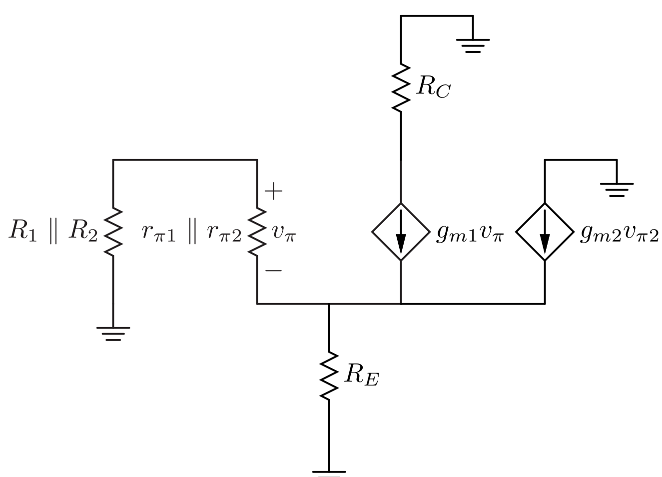
$$I_{C1} = \boxed{707\ \mu\text{A}}$$

$$I_{C2} = \frac{I_{C1}}{2} = \boxed{354\ \mu\text{A}}$$

(b) The small-signal model is shown below.



We can simplify the small-signal model as follows:



$$g_{m1} = I_{C1}/V_T = \boxed{27.2 \text{ mS}}$$
$$r_{\pi 1} = \beta_1/g_{m1} = \boxed{3.677 \text{ k}\Omega}$$
$$g_{m2} = I_{C2}/V_T = \boxed{13.6 \text{ mS}}$$
$$r_{\pi 2} = \beta_2/g_{m2} = \boxed{7.355 \text{ k}\Omega}$$

5.19 (a)

$$I_{E1} = I_{E2} \Rightarrow V_{BE1} = V_{BE2}$$

$$\frac{V_{CC} - 2V_{BE1}}{9\ \text{k}\Omega} - \frac{2V_{BE1}}{16\ \text{k}\Omega} = I_{B1} = \frac{I_{C1}}{\beta_1}$$

$$I_{C1} = \beta_1 \frac{V_{CC} - 2V_T \ln(I_{C1}/I_{S1})}{9\ \text{k}\Omega} - \beta_1 \frac{2V_T \ln(I_{C1}/I_{S1})}{16\ \text{k}\Omega}$$

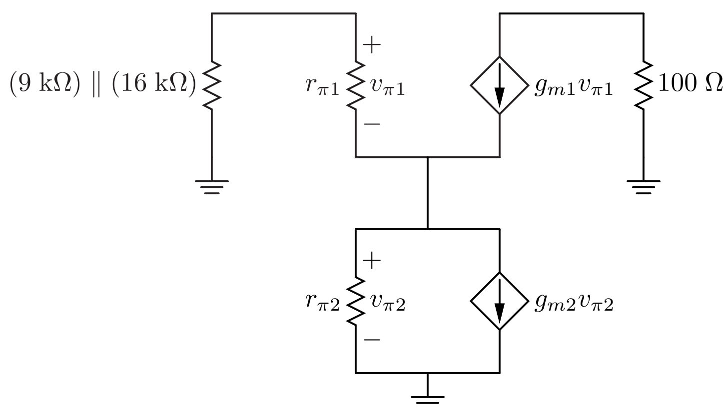
$$I_{C1} = I_{C2} = \boxed{1.588\ \text{mA}}$$

$$V_{BE1} = V_{BE2} = V_T \ln(I_{C1}/I_{S1}) = \boxed{754\ \text{mV}}$$

$$V_{CE2} = V_{BE2} = \boxed{754\ \text{mV}}$$

$$V_{CE1} = V_{CC} - I_{C1}(100\ \Omega) - V_{CE2} = \boxed{1.587\ \text{V}}$$

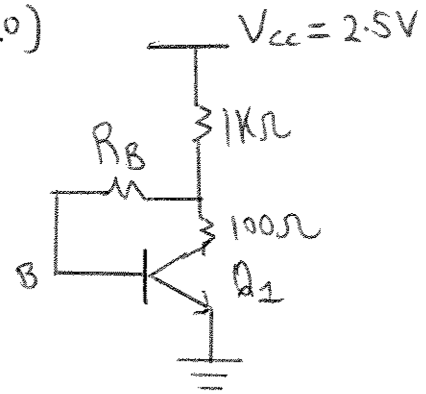
(b) The small-signal model is shown below.



$$g_{m1} = g_{m2} = \frac{I_{C1}}{V_T} = \boxed{61.1\ \text{mS}}$$

$$r_{\pi1} = r_{\pi2} = \frac{\beta_1}{g_{m1}} = \boxed{1.637\ \text{k}\Omega}$$

20)



$I_c = 1mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.750V$

$V_B = 2.5 - (I_E(1K\Omega) + I_B R_B) = 0.750V$

$I_E = 1.01mA$

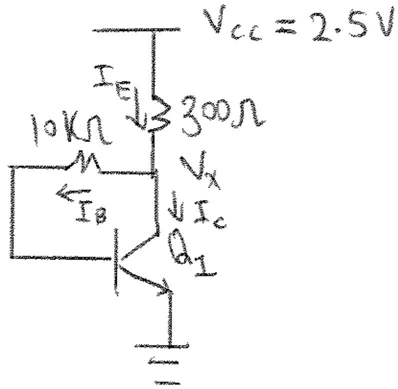
$I_B = 0.01mA$

$V_B = 2.5 - 1.01 - 0.01 R_B = 0.750$

$0.74 = 0.01 R_B$

$R_B = 74K\Omega$

21)



$V_x = 1.1V$

$\beta = 100$

$I_s = ?$

$$I_E = I_B + I_c$$

$$I_E = \frac{2.5 - 1.1}{300\Omega} = 4.67\,mA$$

$$I_B = \frac{I_c}{\beta}$$

$$I_E = \frac{I_c}{\beta} + I_c = 4.67\,mA$$

$$I_c = 4.624\,mA$$

$$I_s = \frac{I_c}{e^{\left(\frac{V_{BE}}{V_T}\right)}} \quad , \quad V_{BE} = 1.1 - \frac{4.624}{100}(10K) = 0.6376V$$

$$I_s = 1.035 \times 10^{-10}\,mA$$

$$I_s = 1.035 \times 10^{-13}\,A$$

5.22

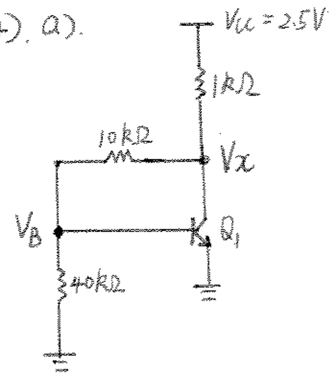
$$
\begin{aligned}
V_{CC} - I_E(500\ \Omega) - I_B(20\ \text{k}\Omega) - I_E(400\ \Omega) &= V_{BE} \\
V_{CC} - \frac{1+\beta}{\beta} I_C(500\ \Omega + 400\ \Omega) - \frac{1}{\beta} I_C(20\ \text{k}\Omega) &= V_T \ln(I_C/I_S) \\
I_C &= \boxed{1.584\ \text{mA}} \\
V_{BE} &= V_T \ln(I_C/I_S) = \boxed{754\ \text{mV}} \\
V_{CE} &= V_{CC} - I_E(500\ \Omega) - I_E(400\ \Omega) \\
&= V_{CC} - \frac{1+\beta}{\beta} I_C(500\ \Omega + 400\ \Omega) = \boxed{1.060\ \text{V}}
\end{aligned}
$$

Q_1 is operating in forward active.

5.23

$$
\begin{aligned}
V_{BC} &\leq 200 \text{ mV} \\
V_{CC} - I_E(1 \text{ k}\Omega) - I_B R_B - (V_{CC} - I_E(1 \text{ k}\Omega) - I_C(500 \ \Omega)) &\leq 200 \text{ mV} \\
I_C(500 \ \Omega) - I_B R_B &\leq 200 \text{ mV} \\
I_B R_B &\geq I_C(500 \ \Omega) - 200 \text{ mV} \\
V_{CC} - I_E(1 \text{ k}\Omega) - I_B R_B &= V_{BE} = V_T \ln(I_C/I_S) \\
V_{CC} - \frac{1+\beta}{\beta} I_C(1 \text{ k}\Omega) - I_C(500 \ \Omega) + 200 \text{ mV} &\leq V_T \ln(I_C/I_S) \\
I_C &\geq 1.29 \text{ mA} \\
R_B &\geq \frac{I_C(500 \ \Omega) - 200 \text{ mV}}{\frac{I_C}{\beta}} \\
&\geq \boxed{34.46 \text{ k}\Omega}
\end{aligned}
$$

24). a).



$I_S = 8 \times 10^{-16} A$

$\beta = 100$

$V_A = \infty$

$V_x = 2.5 - \left(\frac{I_C}{\alpha} + \frac{V_B}{40k}\right) \cdot 1k$

$V_x = \left(\frac{V_B}{40k} + I_B\right) 10k + V_B = \left(\frac{V_B}{40k} + \frac{I_C}{\beta}\right) 10k + V_B$

Equating V_x $\Rightarrow$ $2.5 - \left(V_B + \frac{V_B \cdot 1k}{40k} + \frac{V_B \cdot 10k}{40k}\right) = \frac{I_C}{\alpha} \cdot 1k + \frac{I_C}{\beta} \cdot 10k$

$\Rightarrow I_C = \dfrac{2.5 - 1.275 V_B}{\frac{1k}{\alpha} + \frac{10k}{\beta}}$

Guess $V_B = 0.8$

$I_C = \dfrac{1.48}{\frac{1k}{0.99} + \frac{10k}{100}} = 1.33 m(A)$

Then

$V_B = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.732$, not 0.8.

Reiterate

$I_C = \dfrac{1.5667}{1.11} = 1.4113 (mA)$

$V_B = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.733$

So V_B converges to 0.73V

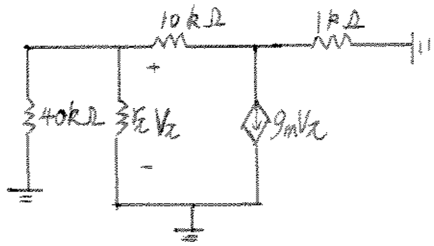
$I_C = 1.41 mA$

$I_B = 14.1 uA$

$V_{CE} = 2.5V - \left(\frac{1.41}{0.99} + \frac{0.73}{40}\right) \times 1V = 1.06V$

$V_{BE} = 0.73V$

24 b) Small Signal



$$g_m = \frac{I_C}{V_T} = 0.054\ S$$

$$r_\pi = \frac{\beta}{g_m} = 1844\Omega$$

5.25 (a)

$$I_{C1} = 1 \text{ mA}$$

$$V_{CC} - (I_{E1} + I_{E2})(500\ \Omega) = V_T \ln(I_{C2}/I_{S2})$$

$$V_{CC} - \left(\frac{1+\beta}{\beta} I_{C1} + \frac{1+\beta}{\beta} I_{C2}\right)(500\ \Omega) = V_T \ln(I_{C2}/I_{S2})$$

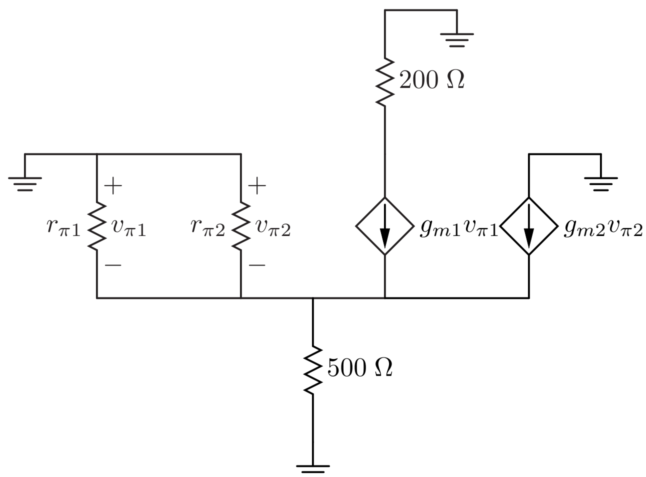
$$I_{C2} = 2.42 \text{ mA}$$

$$V_B - (I_{E1} + I_{E2})(500\ \Omega) = V_T \ln(I_{C1}/I_{S1})$$

$$V_B - \left(\frac{1+\beta}{\beta} I_{C1} + \frac{1+\beta}{\beta} I_{C2}\right)(500\ \Omega) = V_T \ln(I_{C1}/I_{S1})$$

$$V_B = \boxed{2.68 \text{ V}}$$

(b) The small-signal model is shown below.



$$g_{m1} = I_{C1}/V_T = \boxed{38.5 \text{ mS}}$$

$$r_{\pi1} = \beta_1/g_{m1} = \boxed{2.6 \text{ k}\Omega}$$

$$g_{m2} = I_{C2}/V_T = \boxed{93.1 \text{ mS}}$$

$$r_{\pi2} = \beta_2/g_{m2} = \boxed{1.074 \text{ k}\Omega}$$

5.26 (a)

$$
\begin{aligned}
V_{CC} - I_B(60\ \text{k}\Omega) &= V_{EB} \\
V_{CC} - \frac{1}{\beta_{pnp}} I_C(60\ \text{k}\Omega) &= V_T \ln(I_C/I_S) \\
I_C &= \boxed{1.474\ \text{mA}} \\
V_{EB} &= V_T \ln(I_C/I_S) = \boxed{731\ \text{mV}} \\
V_{EC} &= V_{CC} - I_C(200\ \Omega) = \boxed{2.205\ \text{V}}
\end{aligned}
$$

Q_1 is operating in forward active.

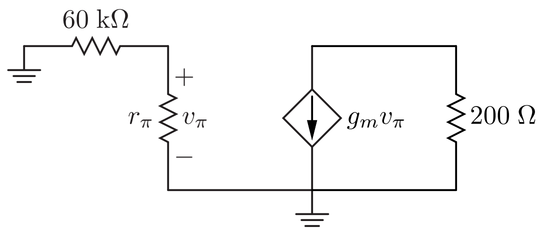
(b)

$$
\begin{aligned}
V_{CC} - V_{BE1} - I_{B2}(80\ \text{k}\Omega) &= V_{EB2} \\
V_{CC} - V_T \ln(I_{C1}/I_S) - I_{B2}(80\ \text{k}\Omega) &= V_T \ln(I_{C2}/I_S) \\
I_{C1} &= \frac{\beta_{npn}}{1+\beta_{npn}} I_{E1} \\
&= \frac{\beta_{npn}}{1+\beta_{npn}} I_{E2} \\
&= \frac{\beta_{npn}}{1+\beta_{npn}} \cdot \frac{1+\beta_{pnp}}{\beta_{pnp}} I_{C2} \\
V_{CC} - V_T \ln\left(\frac{\beta_{npn}}{1+\beta_{npn}} \cdot \frac{1+\beta_{pnp}}{\beta_{pnp}} \cdot \frac{I_{C2}}{I_S}\right) - \frac{1}{\beta_{pnp}} I_{C2}(80\ \text{k}\Omega) &= V_T \ln(I_{C2}/I_S) \\
I_{C2} &= \boxed{674\ \mu\text{A}} \\
V_{BE2} &= V_T \ln(I_{C2}/I_S) = \boxed{711\ \text{mV}} \\
I_{C1} &= \boxed{680\ \mu\text{A}} \\
V_{BE1} &= V_T \ln(I_{C1}/I_S) = \boxed{711\ \text{mV}} \\
V_{CE1} &= V_{BE1} = \boxed{711\ \text{mV}} \\
V_{CE2} &= V_{CC} - V_{CE1} - I_{C2}(300\ \Omega) \\
&= \boxed{1.585\ \text{V}}
\end{aligned}
$$

Q_1 is operating on the edge of saturation. Q_2 is operating in forward active.

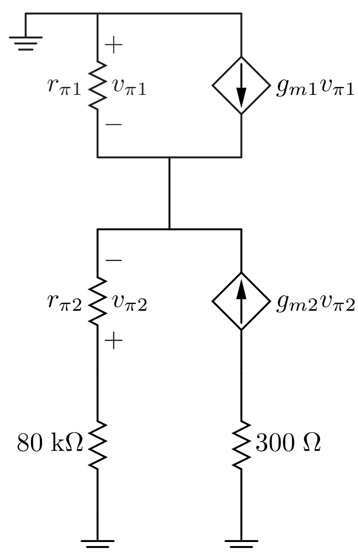
5.27 See Problem 26 for the derivation of I_C for each part of this problem.

(a) The small-signal model is shown below.



$$I_C = 1.474 \text{ mA}$$
$$g_m = \frac{I_C}{V_T} = \boxed{56.7 \text{ mS}}$$
$$r_\pi = \frac{\beta}{g_m} = \boxed{1.764 \text{ k}\Omega}$$

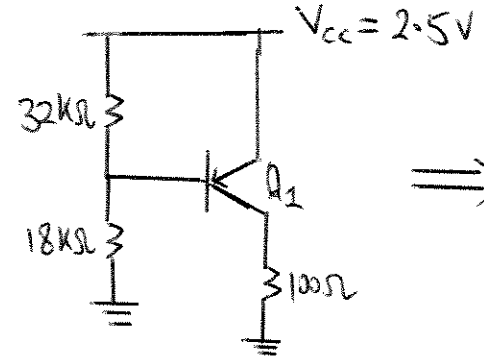
(b) The small-signal model is shown below.



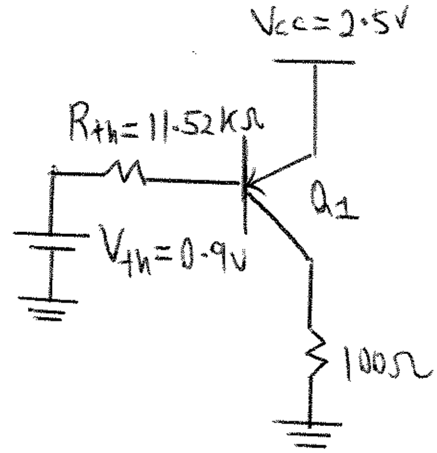
$$I_{C1} = 680 \text{ µA}$$
$$g_{m1} = \frac{I_{C1}}{V_T} = \boxed{26.2 \text{ mS}}$$
$$r_{\pi 1} = \frac{\beta_{npn}}{g_{m1}} = \boxed{3.824 \text{ k}\Omega}$$
$$I_{C2} = 674 \text{ µA}$$
$$g_{m2} = \frac{I_{C2}}{V_T} = \boxed{25.9 \text{ mS}}$$
$$r_{\pi 2} = \frac{\beta_{pnp}}{g_{m2}} = \boxed{1.929 \text{ k}\Omega}$$

28)

a)



⟹



$$I_c = \beta_{pnp}\left(\frac{2.5 - |V_{BE}| - V_{th}}{R_{th}}\right)$$

Guess $|V_{BE}| = 0.7\,V$, $I_c = 3.91\,mA$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.757\,V$

Reiterate, $|V_{BE}| = 0.757V$, $I_c = 3.66\,mA$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.755V$

Reiterate, $|V_{BE}| = 0.755V$, $I_c = 3.67\,mA$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.755V$, Converged!!

$V_c = (3.67mA)(0.1k\Omega) = 0.367V$, $V_B \approx 2.5 - 0.755 = 1.745V$

Q_1 in forward active.

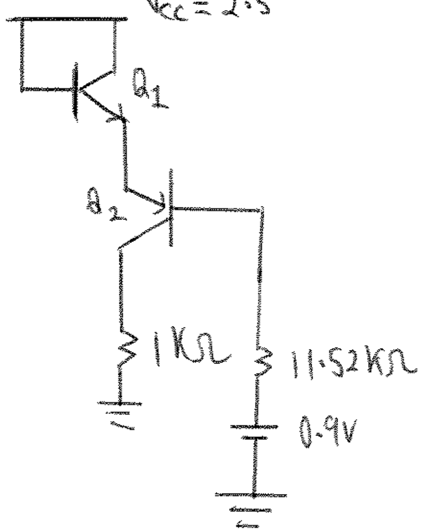
Bias point:

$I_c = 3.67\,mA$ $\quad |V_{BE}| = 0.755$

$I_B = 73.4\,\mu A$ $\quad |V_{CE}| = 2.5 - 0.367 = 2.133V$

28)

b)



$$I_{c2} = \left(\frac{2.5 - (V_{BE1} + V_{BE2}) - 0.9}{11.52K}\right) 50$$

$$I_{c1} = I_{c2}(1.0099)$$

(From β relation)

$$V_{BE1} = V_T \ln\left(\frac{I_{c1}}{I_s}\right)$$

$$|V_{BE2}| = V_T \ln\left(\frac{I_{c2}}{I_s}\right)$$

Guess, $V_{BE1} = V_{BE2} = 0.7V$

$I_{c2} = 0.868\,mA, \quad I_{c1} = 0.877\,mA$

$V_{BE1} = V_T \ln\left(\frac{I_{c1}}{I_s}\right) = 0.718V, \quad |V_{BE2}| = V_T \ln\left(\frac{I_{c2}}{I_s}\right) = 0.717$

Reiterate, $V_{BE1} = 0.718V$, $|V_{BE2}| = 0.717V$

$I_{c2} = 0.716\,mA, \quad I_{c1} = 0.723\,mA$

$V_{BE1} = V_T \ln\left(\frac{I_{c1}}{I_s}\right) = 0.713V, \quad |V_{BE2}| = V_T \ln\left(\frac{I_{c2}}{I_s}\right) = 0.712V$

Reiterate, $V_{BE1} = 0.713V$, $|V_{BE2}| = 0.712V$

$I_{c2} = 0.760\,mA, \quad I_{c1} = 0.767\,mA$

$V_{BE1} = 0.714V, \quad |V_{BE2}| = 0.714V$

28)

b)

Reiterate, $V_{BE_1} = 0.714V$, $|V_{BE_2}| = 0.714V$

$I_{C_2} = 0.747mA$, $I_{C_1} = 0.754mA$

$V_{BE_1} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.714V,$

$|V_{BE_2}| = 0.714\ V$

$$V_{B_2} = \frac{(0.747mA)}{50}(11.52K\Omega) + 0.9 = 1.07\ V$$

$$V_{C_2} = (0.747mA)(1K\Omega) = 0.747\ V$$

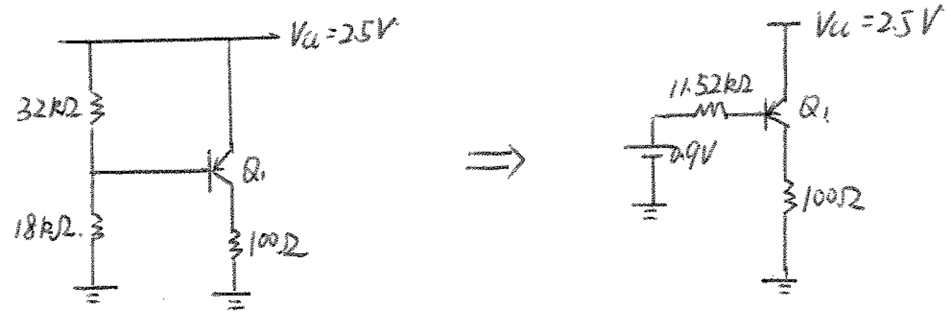
Q_2 is in forward-active region. Q_1 is always in forward-active region.

Bias point:

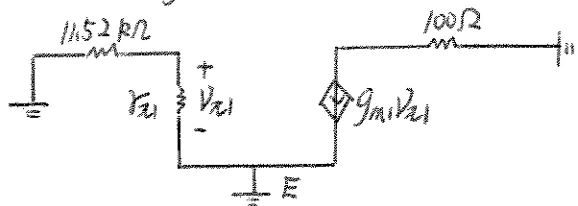
$V_{BE_1} = 0.714V$	$\lvert V_{BE_2}\rvert = 0.714V$
$I_{C_1} = 0.754\ mA$	$I_{C2} = 0.747mA$
$I_{B_1} = 7.54\mu A$	$I_{B2} = 14.94\ \mu A$
$V_{CE_1} = 0.714V$	$\lvert V_{CE_2}\rvert = 2.5 - 0.714 - 0.747 = 1.039V$

29)

a).



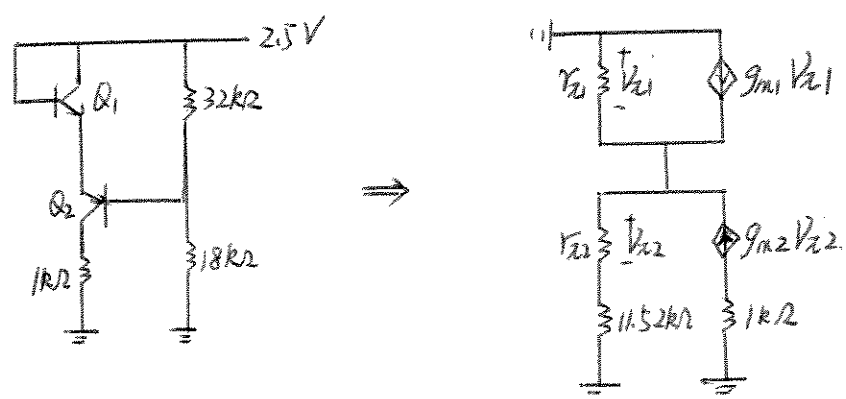
Small Signal:



$$g_{m1} = \frac{3.67mA}{26mV} = 0.141\ S$$

$$r_{\pi} = \frac{50}{0.141}\ \Omega = 354.2\ \Omega$$

b).



$g_{m1} = 0.029\ S$

$r_{\pi 1} = 3448.3\ \Omega$

$g_{m2} = 0.0287\ S$

$r_{\pi 2} = 1740.3\ \Omega$.

5.30

$$
\begin{aligned}
V_{CC} - I_C(1\ \text{k}\Omega) &= V_{EC} = V_{EB} \text{ (in order for } Q_1 \text{ to operate at the edge of saturation)} \\
&= V_T \ln(I_C/I_S) \\
I_C &= 1.761\ \text{mA} \\
V_{EB} &= 739\ \text{mV} \\
\frac{V_{CC} - V_{EB}}{R_B} - \frac{V_{EB}}{5\ \text{k}\Omega} &= I_B = \frac{I_C}{\beta} \\
R_B &= 9.623\ \text{k}\Omega
\end{aligned}
$$

First, let's consider when R_B is 5 % larger than its nominal value.

$$
\begin{aligned}
R_B &= 10.104\ \text{k}\Omega \\
\frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} - \frac{V_T \ln(I_C/I_S)}{5\ \text{k}\Omega} &= \frac{I_C}{\beta} \\
I_C &= 1.411\ \text{mA} \\
V_{EB} &= 733\ \text{mV} \\
V_{EC} &= V_{CC} - I_C(1\ \text{k}\Omega) = 1.089\ \text{V} \\
V_{CB} &= \boxed{-355\ \text{mV}} \text{ (the collector-base junction is reverse biased)}
\end{aligned}
$$

Now, let's consider when R_B is 5 % smaller than its nominal value.

$$
\begin{aligned}
R_B &= 9.142\ \text{k}\Omega \\
\frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} - \frac{V_T \ln(I_C/I_S)}{5\ \text{k}\Omega} &= \frac{I_C}{\beta} \\
I_C &= 2.160\ \text{mA} \\
V_{EB} &= 744\ \text{mV} \\
V_{EC} &= V_{CC} - I_C(1\ \text{k}\Omega) = 340\ \text{mV} \\
V_{CB} &= \boxed{405\ \text{mV}} \text{ (the collector-base junction is forward biased)}
\end{aligned}
$$

5.31

$$\begin{aligned}
\frac{V_{BC} + I_C(5\ \mathrm{k\Omega})}{10\ \mathrm{k\Omega}} - \frac{V_{CC} - V_{BC} - I_C(5\ \mathrm{k\Omega})}{10\ \mathrm{k\Omega}} &= I_B = \frac{I_C}{\beta} \\
V_{BC} &= 300\ \mathrm{mV} \\
I_C &= 194\ \mathrm{\mu A} \\
V_{EB} &= V_T \ln(I_C/I_S) = 682\ \mathrm{mV} \\
V_{CC} - I_E R_E - I_C(5\ \mathrm{k\Omega}) &= V_{EC} = V_{EB} + 300\ \mathrm{mV} \\
V_{CC} - \frac{1+\beta}{\beta} I_C R_E - I_C(5\ \mathrm{k\Omega}) &= V_{EB} + 300\ \mathrm{mV} \\
R_E &= \boxed{2.776\ \mathrm{k\Omega}}
\end{aligned}$$

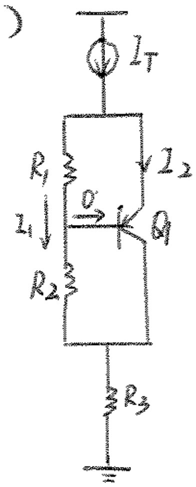
Let's look at what happens when R_E is halved.

$$\begin{aligned}
R_E &= 1.388\ \mathrm{k\Omega} \\
\frac{V_{CC} - I_E R_E - V_{EB}}{10\ \mathrm{k\Omega}} - \frac{V_{CC} - (V_{CC} - I_E R_E - V_{EB})}{10\ \mathrm{k\Omega}} &= I_B = \frac{I_C}{\beta} \\
\beta \frac{V_{CC} - \frac{1+\beta}{\beta} I_C R_E - V_T \ln(I_C/I_S)}{10\ \mathrm{k\Omega}} - \beta \frac{V_{CC} - \left(V_{CC} - \frac{1+\beta}{\beta} I_C R_E - V_T \ln(I_C/I_S)\right)}{10\ \mathrm{k\Omega}} &= I_C \\
I_C &= 364\ \mathrm{\mu A} \\
V_{EB} &= 698\ \mathrm{\mu V} \\
V_{EC} &= 164\ \mathrm{\mu V}
\end{aligned}$$

Thus, when R_E is halved, Q_1 operates in deep saturation.

5.32

$$
\begin{aligned}
V_{CC} - I_B(20\text{ k}\Omega) - I_E(1.6\text{ k}\Omega) &= V_{BE} = V_T \ln(I_C/I_S) \\
V_{CC} - \frac{I_C}{\beta}(20\text{ k}\Omega) - \frac{1+\beta}{\beta} I_C(1.6\text{ k}\Omega) &= V_{BE} = V_T \ln(I_C/I_S) \\
I_S &= \frac{I_C}{e^{\left[V_{CC} - \frac{I_C}{\beta}(20\text{ k}\Omega) - \frac{1+\beta}{\beta} I_C (1.6\text{ k}\Omega)\right]/V_T}} \\
I_C &= 1\text{ mA} \\
I_S &= \boxed{3 \times 10^{-14}\text{ A}}
\end{aligned}
$$

33) 

If Base current is neglected, $I_C = I_E$

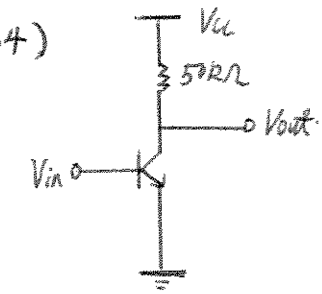
$$I_1 = \frac{V_E - V_C}{R_1 + R_2}$$

$$|V_{BE}| = I_1 R_1 = \frac{V_E - V_C}{R_1 + R_2} R_1 = \frac{|V_{CE}|}{R_1 + R_2} R_1$$

So $$\frac{|V_{CE}|}{|V_{BE}|} = \frac{R_1 + R_2}{R_1}$$

Let $A = \frac{R_1 + R_2}{R_1}$, $|V_{CE}| = A|V_{BE}|$, thus $|V_{BE}|$ is multiplied.

34)



$$A_V = g_m R_C = 20$$

$$\frac{I_C R_C}{V_T} = 20 \Rightarrow I_C = \frac{20 V_T}{R_C}$$

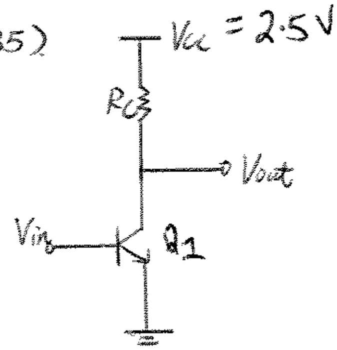
$$I_C = 0.0104 \, mA$$

$$V_{CC} - (50k\Omega)(0.0104 mA) = V_{BE}$$

$$\Rightarrow V_{CC} - 50 \times 0.0104 \, V = 0.8 V$$

$$\Rightarrow V_{CC} = 1.32 V$$

35)



$V_A = 10V$, $r_o = \frac{V_A}{I_c}$, $g_m = \frac{I_c}{V_T}$

$$\left|\frac{V_{out}}{V_{in}}\right| = g_m (R_c // r_o) = g_m \left(\frac{R_c r_o}{R_c + r_o}\right) = \frac{R_c V_A}{V_T (R_c + \frac{V_A}{I_c})}$$

As the equation above shows, a large gain means a large I_c. However, a large I_c will drive Q_1 into saturation. So a tradeoff must be made. The maximum limit for I_c is when it drives Q_1 into the edge of saturation, namely, $V_{BE} = V_{CE}$.

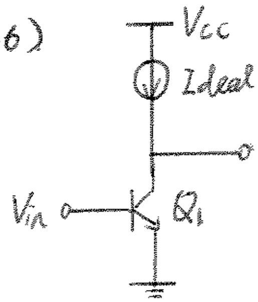
$$V_{CE} = V_{cc} - I_c (1K)$$

$V_{BE} = 0.8V$, $V_{cc} = 2.5V$

$$0.8 = 2.5 - I_c 1K$$

$$I_c = 1.7mA$$

36)



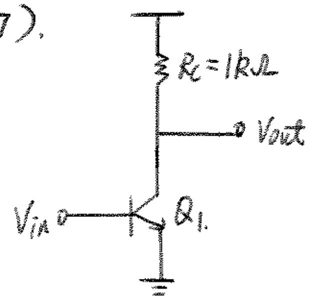
$A_v = 50$

$R_{out} = r_o = 10k\Omega$

$$A_v = g_m R_{out} = \frac{I_c}{V_T} R_{out} = 50$$

$$I_c = 50\left(\frac{V_T}{R_{out}}\right) = 0.13mA$$

37).



$$I_c = I_s \exp\left(\frac{V_{BE}}{2V_T}\right)$$

$$g_m = \frac{\partial I_c}{\partial V_{BE}} = \frac{I_c}{2V_T}$$

$$R_{out} = R_c$$

$$\left|\frac{V_{out}}{V_{in}}\right| = g_m R_{out} = \frac{I_c R_c}{2V_T} = \frac{(1mA)(1K\Omega)}{(2)(0.026V)} = 19.23$$

5.38 (a)

$$A_v = \boxed{-g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{\frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(b)

$$A_v = \boxed{-g_{m1}\left(R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

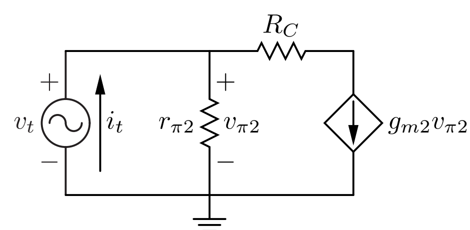
(c)

$$A_v = \boxed{-g_{m1}\left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(d) Let's determine the equivalent resistance seen looking up from the output by drawing a small-signal model and applying a test source.



$$i_t = \frac{v_{\pi 2}}{r_{\pi 2}} + g_{m2}v_{\pi 2}$$

$$v_{\pi 2} = v_t$$

$$i_t = v_t\left(\frac{1}{r_{\pi 2}} + g_{m2}\right)$$

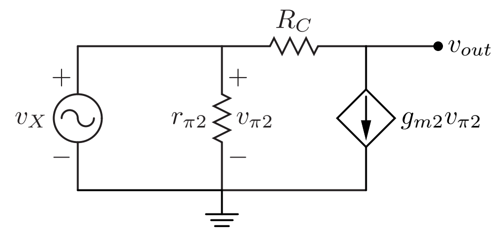
$$\frac{v_t}{i_t} = \frac{1}{g_{m2}} \parallel r_{\pi 2}$$

$$A_v = \boxed{-g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{\frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(e) From (d), we know the gain from the input to the collector of Q_1 is $-g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$. If we find the gain from the collector of Q_1 to v_{out}, we can multiply these expressions to find the overall gain. Let's draw the small-signal model to find the gain from the collector of Q_1 to v_{out}. I'll refer to the collector of Q_1 as node X in the following derivation.

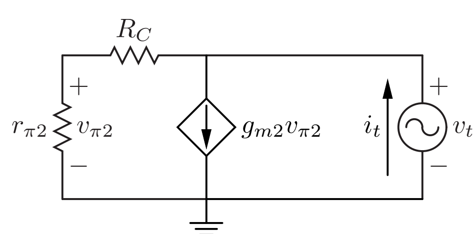


$$\begin{aligned}
\frac{v_X - v_{out}}{R_C} &= g_{m2}v_{\pi 2} \\
v_{\pi 2} &= v_X \\
\frac{v_X - v_{out}}{R_C} &= g_{m2}v_X \\
v_X\left(\frac{1}{R_C} - g_{m2}\right) &= \frac{v_{out}}{R_C} \\
\frac{v_{out}}{v_X} &= 1 - g_{m2}R_C
\end{aligned}$$

Thus, we have

$$\begin{aligned}
A_v &= \boxed{-g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)(1 - g_{m2}R_C)} \\
R_{in} &= \boxed{r_{\pi 1}}
\end{aligned}$$

To find the output resistance, let's draw the small-signal model and apply a test source at the output. Note that looking into the collector of Q_1 we see infinite resistance, so we can exclude it from the small-signal model.



$$
\begin{aligned}
i_t &= g_{m2} v_{\pi 2} + \frac{v_{\pi 2}}{r_{\pi 2}} \\
v_{\pi 2} &= \frac{r_{\pi 2}}{r_{\pi 2} + R_C} v_t \\
i_t &= \left(g_{m2} + \frac{1}{r_{\pi 2}} \right) \frac{r_{\pi 2}}{r_{\pi 2} + R_C} v_t \\
R_{out} &= \frac{v_t}{i_t} \\
&= \boxed{\left(\frac{1}{g_{m2}} \parallel r_{\pi 2} \right) \frac{r_{\pi 2} + R_C}{r_{\pi 2}}}
\end{aligned}
$$

5.39 (a)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)}$$
$$R_{in} = \boxed{r_{\pi 1}}$$
$$R_{out} = \boxed{r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}}$$

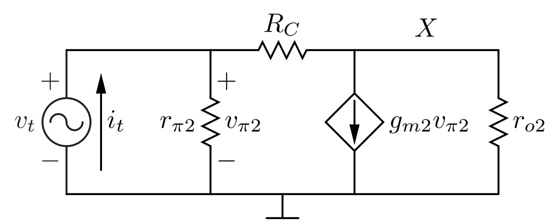
(b)

$$A_v = \boxed{-g_{m1}\left[r_{o1} \parallel \left(R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)\right]}$$
$$R_{in} = \boxed{r_{\pi 1}}$$
$$R_{out} = \boxed{r_{o1} \parallel \left(R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)}$$

(c)

$$A_v = \boxed{-g_{m1}\left[r_{o1} \parallel \left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)\right]}$$
$$R_{in} = \boxed{r_{\pi 1}}$$
$$R_{out} = \boxed{r_{o1} \parallel \left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)}$$

(d) Let's determine the equivalent resistance seen looking up from the output by drawing a small-signal model and applying a test source.



$$i_t = \frac{v_{\pi 2}}{r_{\pi 2}} + \frac{v_t - v_X}{R_C}$$

$$\frac{v_X - v_t}{R_C} + g_{m2} v_{\pi 2} + \frac{v_X}{r_{o2}} = 0$$

$$v_{\pi 2} = v_t$$

$$v_X \left(\frac{1}{R_C} + \frac{1}{r_{o2}} \right) = v_t \left(\frac{1}{R_C} - g_{m2} \right)$$

$$v_X = v_t \left(\frac{1}{R_C} - g_{m2} \right) (r_{o2} \parallel R_C)$$

$$\begin{aligned} i_t &= \frac{v_t}{r_{\pi 2}} + \frac{v_t}{R_C} - \frac{1}{R_C} v_t \left(\frac{1}{R_C} - g_{m2} \right) (r_{o2} \parallel R_C) \\ &= v_t \left[\frac{1}{r_{\pi 2}} + \frac{1}{R_C} - \frac{1}{R_C} \left(\frac{1}{R_C} - g_{m2} \right) (r_{o2} \parallel R_C) \right] \\ &= v_t \left[\frac{1}{r_{\pi 2}} + \frac{1}{R_C} + \left(g_{m2} - \frac{1}{R_C} \right) \frac{r_{o2}}{r_{o2} + R_C} \right] \end{aligned}$$

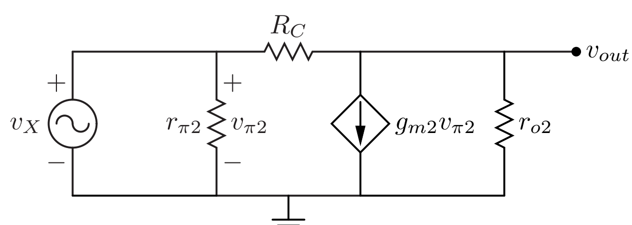
$$\frac{v_t}{i_t} = r_{\pi 2} \parallel R_C \parallel \left[\frac{r_{o2} + R_C}{r_{o2}} \frac{1}{g_{m2} - \frac{1}{R_C}} \right]$$

$$A_v = \boxed{-g_{m1} \left(r_{o1} \parallel r_{\pi 2} \parallel R_C \parallel \left[\frac{r_{o2} + R_C}{r_{o2}} \frac{1}{g_{m2} - \frac{1}{R_C}} \right] \right)}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{r_{o1} \parallel r_{\pi 2} \parallel R_C \parallel \left[\frac{r_{o2} + R_C}{r_{o2}} \frac{1}{g_{m2} - \frac{1}{R_C}} \right]}$$

(e) From (d), we know the gain from the input to the collector of Q_1 is $-g_{m1} \left(r_{o1} \parallel r_{\pi 2} \parallel R_C \parallel \left[\frac{r_{o2}+R_C}{r_{o2}} \frac{1}{g_{m2} - \frac{1}{R_C}} \right] \right)$. If we find the gain from the collector of Q_1 to v_{out}, we can multiply these expressions to find the overall gain. Let's draw the small-signal model to find the gain from the collector of Q_1 to v_{out}. I'll refer to the collector of Q_1 as node X in the following derivation.

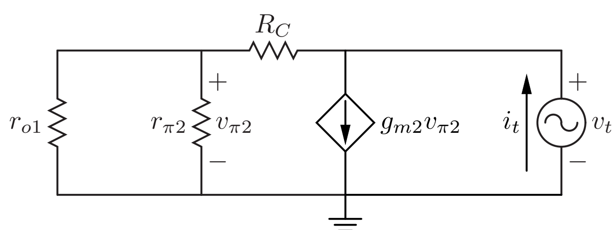


$$\frac{v_{out} - v_X}{R_C} + g_{m2} v_{\pi 2} + \frac{v_{out}}{r_{o2}} = 0$$
$$v_{\pi 2} = v_X$$
$$\frac{v_{out} - v_X}{R_C} + g_{m2} v_X + \frac{v_{out}}{r_{o2}} = 0$$
$$v_{out}\left(\frac{1}{R_C} + \frac{1}{r_{o2}}\right) = v_X\left(\frac{1}{R_C} - g_{m2}\right)$$
$$\frac{v_{out}}{v_X} = \left(\frac{1}{R_C} - g_{m2}\right)(R_C \parallel r_{o2})$$

Thus, we have

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel r_{\pi 2} \parallel R_C \parallel \left[\frac{r_{o2} + R_C}{r_{o2}} \frac{1}{g_{m2} - \frac{1}{R_C}}\right]\right)\left(\frac{1}{R_C} - g_{m2}\right)(R_C \parallel r_{o2})}$$
$$R_{in} = \boxed{r_{\pi 1}}$$

To find the output resistance, let's draw the small-signal model and apply a test source at the output. Note that looking into the collector of Q_1 we see r_{o1}, so we replace Q_1 in the small-signal model with this equivalent resistance. Also note that r_{o2} appears from the output to ground, so we can remove it from this analysis and add it in parallel at the end to find R_{out}.



$$i_t = g_{m2} v_{\pi 2} + \frac{v_{\pi 2}}{r_{\pi 2} \parallel r_{o1}}$$
$$v_{\pi 2} = \frac{r_{\pi 2} \parallel r_{o1}}{r_{\pi 2} \parallel r_{o1} + R_C} v_t$$
$$i_t = \left(g_{m2} + \frac{1}{r_{\pi 2} \parallel r_{o1}}\right)\frac{r_{\pi 2} \parallel r_{o1}}{r_{\pi 2} \parallel r_{o1} + R_C} v_t$$
$$R_{out} = r_{o2} \parallel \frac{v_t}{i_t}$$
$$= \boxed{r_{o2} \parallel \left[\left(\frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o1}\right)\frac{r_{\pi 2} \parallel r_{o1} + R_C}{r_{\pi 2} \parallel r_{o1}}\right]}$$

40)

Gain of a degenerated CE stage ($V_A = \infty$)

$$A_V = \frac{-R_C}{\frac{1}{g_m} + R_E} = \frac{-R_C g_m}{1 + R_E g_m}$$

$$\frac{\partial A_V}{\partial I_C} = R_C \left(\frac{g_m R_E}{(1+R_E g_m)^2} \frac{\partial g_m}{\partial I_C} - \frac{\partial g_m / \partial I_C}{1 + g_m R_E} \right)$$

$$\frac{\partial g_m}{\partial I_C} = \frac{1}{V_T} = \frac{1}{26mV} = 38.46 \left(\frac{1}{V}\right)$$

a) $g_m R_E = 3$

$$\frac{\partial A_V}{\partial I_C} = R_C(-2.404) \quad , \quad \partial I_C = 0.1 I_C$$

$$\partial A_V = -R_C I_C (0.24)$$

$$\text{Relative change in gain} = \frac{\partial A_V}{A_V} = \frac{-0.24(R_C I_C)}{-\frac{R_C I_C}{V_T(1 + R_E g_m)}} = 2.5\%$$

40)

b) $g_m R_E = 7$

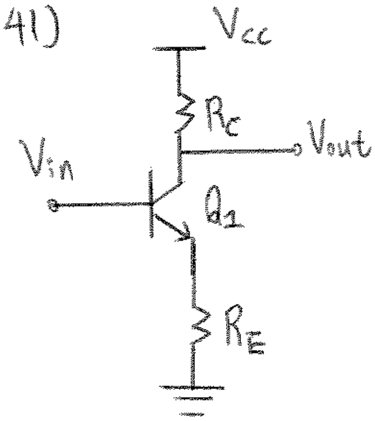
$$\frac{\partial A_v}{\partial I_c} = -R_c\, 0.6$$

$$\partial A_v = -R_c I_c (0.06)$$

Relative change in gain

$$\frac{\partial A_v}{A_v} = \frac{-0.06\,(R_c I_c)}{\frac{-R_c I_c}{V_T(1+R_E g_m)}} = 1.25\%$$

41)



$V_A = \infty$

$R_C I_C = 20 V_T$

$R_E I_C = 5 V_T$

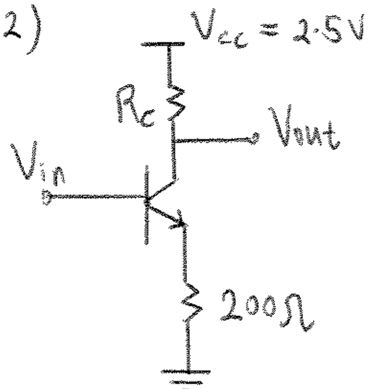
$$|A_v| = \frac{R_C}{R_E + \frac{1}{g_m}} = \frac{R_C}{R_E + \frac{V_T}{I_C}} = \frac{R_C I_C}{R_E I_C + V_T}$$

Assume β is large, so $I_C = I_E$.

$R_C I_C = 20 V_T$, $R_E I_C = 5 V_T$

$$|A_v| = \frac{20 V_T}{5 V_T + V_T} = \frac{20 V_T}{6 V_T} = 3.33$$

42)



$$|A_v| = \frac{R_C I_C}{R_E I_C + V_T} = 10$$

Edge of Saturation

$V_{CE} = V_{BE} = 2.5 - I_C(R_C + R_E)$

$V_{BE} = 0.8V \Rightarrow I_C R_C = 1.7 - I_C 0.2$ (operating point)

$|A_v| = 10 \Rightarrow R_C I_C = 10(R_E I_C + V_T)$ (Gain Equation)

Equating the two equations above $\Rightarrow$

$1.7 - 0.2 I_C = 2 I_C + 0.26 \Rightarrow I_C = 0.655mA$

Check for $V_{BE} \Rightarrow V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.725$, not 0.8, Reiterate

$I_C R_C = 1.775 - I_C 0.2$ (operating point)

$I_C R_C = 2I_C + 0.26$ (Gain equation)

Equating the two equations $\Rightarrow I_C = 0.689mA$

Check for $V_{BE} \Rightarrow V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.727V$, iterate 1 more time

$I_C R_C = 1.773 - I_C 0.2$ (operating point)

$I_C R_C = 2I_C + 0.26$ (Gain equation)

42)

Equating the two equations $\Rightarrow I_c = 0.688\ mA$

Check for $V_{BE} \Rightarrow V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.727V$, converged

$I_c = 0.688mA$

$$R_c = \frac{2I_c + 0.26}{I_c} = \frac{(2)(0.688) + 0.26}{0.688}$$

$R_c = 2.38K\Omega$

$R_{in} = r_\pi + (1+\beta) R_E$

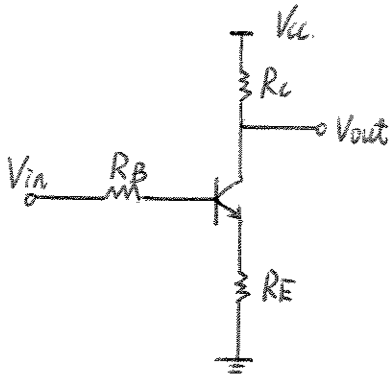
$$R_{in} = \frac{\beta}{g_m} + (101)(0.2) = 24.0K\Omega$$

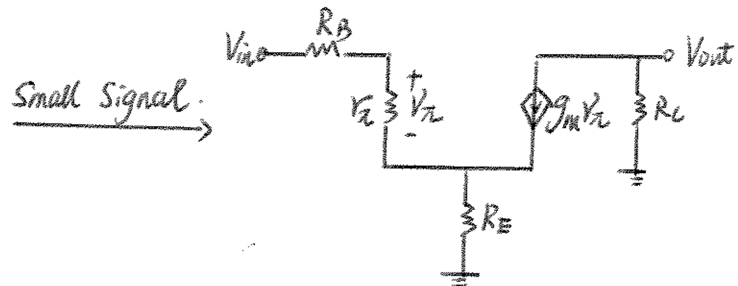
5.43

$$
\begin{aligned}
A_v &= -\frac{R_C}{\frac{1}{g_m} + (200\ \Omega)} \\
&= -\frac{R_C}{\frac{V_T}{I_C} + (200\ \Omega)} \\
&= -100 \\
R_C &= 100\frac{V_T}{I_C} + 100(200\ \Omega) \\
I_C R_C - I_E(200\ \Omega) &= V_{CE} = V_{BE} = V_T \ln(I_C/I_S) \\
I_C\left(100\frac{V_T}{I_C} + 100(200\ \Omega)\right) - \frac{1+\beta}{\beta} I_C(200\ \Omega) &= V_T \ln(I_C/I_S)
\end{aligned}
$$

We can see that this equation has no solution. For example, if we let $I_C = 0$, we see that according to the left side, we should have $V_{BE} = 2.6$ V, which is clearly an infeasible value. Qualitatively, we know that in order to achieve a large gain, we need a large value for R_C. However, increasing R_C will result in a smaller value of V_{CE}, eventually driving the transistor into saturation. When $A_v = -100$, there is no value of R_C that will provide such a large gain without driving the transistor into saturation.

44) $V_A = \infty$





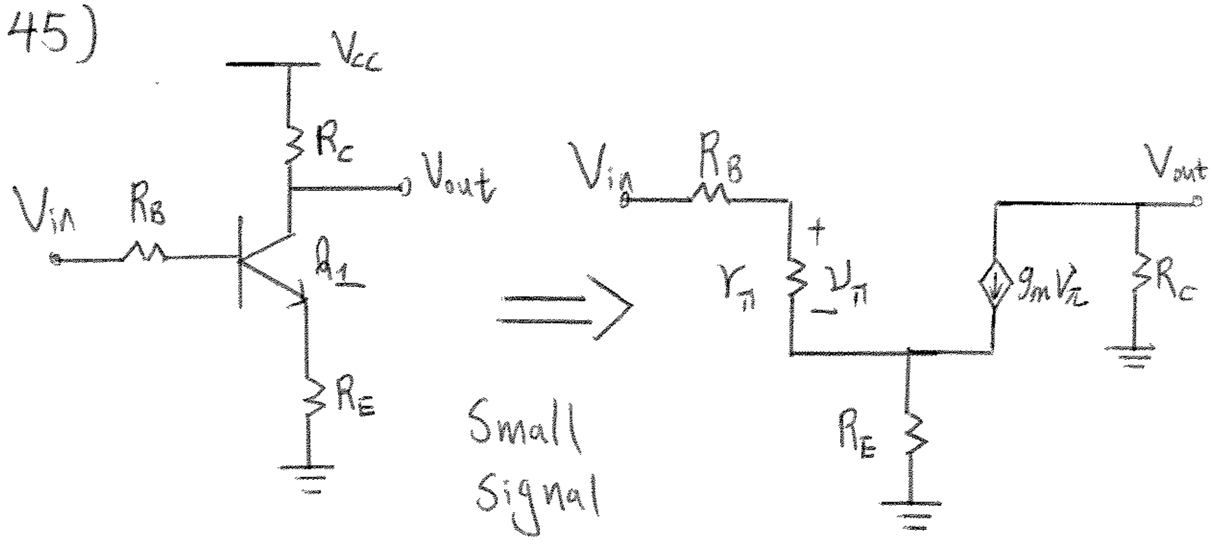
$$V_{out} = -g_m V_\pi R_C$$

$$V_\pi = \frac{V_{in} r_\pi}{R_B + r_\pi + (\beta+1) R_E}$$

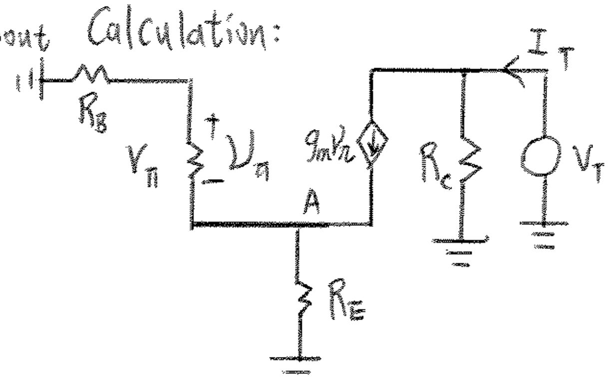
$$V_{out} = \frac{-g_m r_\pi R_C V_{in}}{R_B + r_\pi + (\beta+1) R_E} = \frac{-\beta R_C V_{in}}{R_B + r_\pi + (\beta+1) R_E} = \frac{-R_C V_{in}}{\frac{R_B}{\beta} + \frac{1}{g_m} + \frac{\beta+1}{\beta} R_E}$$

$$\frac{V_{out}}{V_{in}} \approx \frac{-R_C}{\frac{R_B}{\beta+1} + \frac{1}{g_m} + R_E}$$

45)



Rout Calculation:



$V_A = g_m v_\pi (R_E \,//\, R_B + r_\pi)$ (1)

$v_\pi = -\dfrac{V_A r_\pi}{r_\pi + R_B} \Rightarrow V_A = -\dfrac{v_\pi (r_\pi + R_B)}{r_\pi}$ (2)

The only possible solution for 1) and 2) is $v_\pi = V_A = 0$, since 1) is positive and 2) is negative.

$v_\pi = 0 \Rightarrow g_m v_\pi \Rightarrow 0 \Rightarrow \dfrac{V_T}{I_T} = R_C$

Therefore, $R_{out} = R_C$

5.46 (a)

$$A_v = \boxed{-\frac{R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)R_E}$$

$$R_{out} = \boxed{R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(b)

$$A_v = \boxed{-\frac{R_C}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{out} = \boxed{R_C}$$

(c)

$$A_v = \boxed{-\frac{R_C}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{out} = \boxed{R_C}$$

(d)

$$A_v = \boxed{-\frac{R_C}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2} + \frac{R_B}{1+\beta_1}}}$$

$$R_{in} = \boxed{R_B + r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{out} = \boxed{R_C}$$

(e)

$$A_v = \boxed{-\frac{R_C}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2} + \frac{R_B}{1+\beta_1}}}$$

$$R_{in} = \boxed{R_B + r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{out} = \boxed{R_C}$$

5.47 (a)

$$A_v = \boxed{-\frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1) R_E}$$

$$R_{out} = \boxed{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(b)

$$A_v = -\frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E} \cdot \frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

$$= \boxed{-\frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1) R_E}$$

$$R_{out} = \boxed{\frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(c)

$$A_v = \boxed{-\frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3} \| r_{\pi 3}}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1) \left(\frac{1}{g_{m3} \parallel r_{\pi 3}} \right)}$$

$$R_{out} = \boxed{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

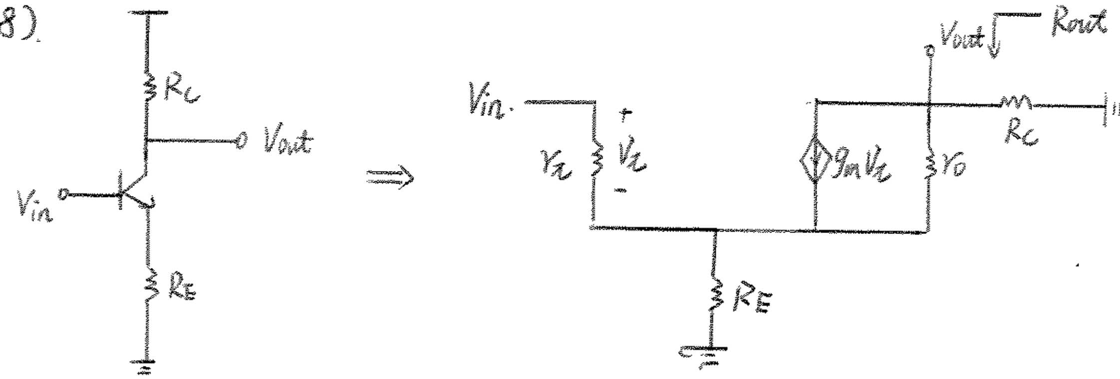
(d)

$$A_v = \boxed{-\frac{R_C \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1) R_E}$$

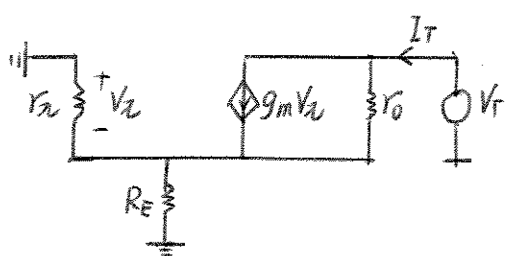
$$R_{out} = \boxed{R_C \parallel r_{\pi 2}}$$

48)



$R_{out} = R_C \parallel R_{eq}$

Solve for R_{eq}.



$$I_T = g_m v_\pi + \frac{(V_T + v_\pi)}{r_o}$$

$$v_\pi = -I_T (r_\pi \parallel R_E)$$

$$I_T = -g_m I_T (r_\pi \parallel R_E) + \frac{(V_T - I_T(r_\pi \parallel R_E))}{r_o}$$

$$\frac{V_T}{I_T} = r_o \left(1 + \frac{(r_\pi \parallel R_E)}{r_o}\right) + g_m (r_\pi \parallel R_E))$$

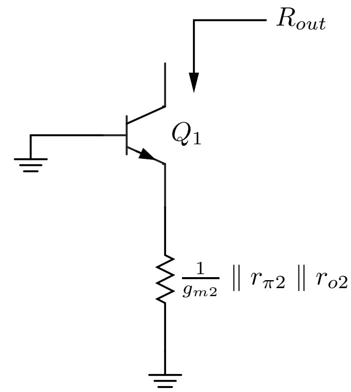
$$\frac{V_T}{I_T} = r_o + (1 + g_m r_o)(r_\pi \parallel R_E)$$

$$R_{eq} = r_o + (1 + g_m r_o)(r_\pi \parallel R_E)$$

$$R_{out} = R_C \parallel r_o + (1 + g_m r_o)(r_\pi \parallel R_E)$$

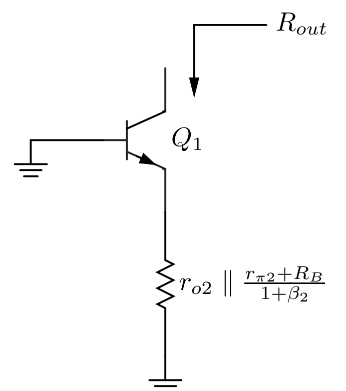
$$R_{out} \approx R_C \parallel r_o (1 + g_m (r_\pi \parallel R_E)) \quad \text{since } g_m r_o \gg 1$$

5.49 (a) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}$, so we can draw the following equivalent circuit for finding R_{out}:



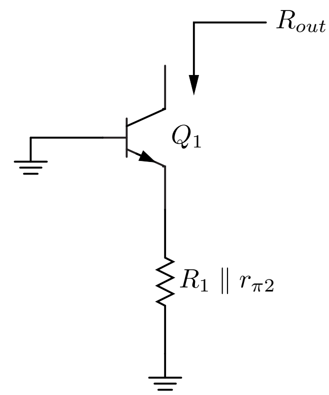
$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) \left(r_{\pi 1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2} \right)}$$

(b) Looking into the emitter of Q_2 we see an equivalent resistance of $r_{o2} \parallel \frac{r_{\pi 2} + R_B}{1+\beta_2}$ (r_{o2} simply appears in parallel with the resistance seen when $V_A = \infty$), so we can draw the following equivalent circuit for finding R_{out}:



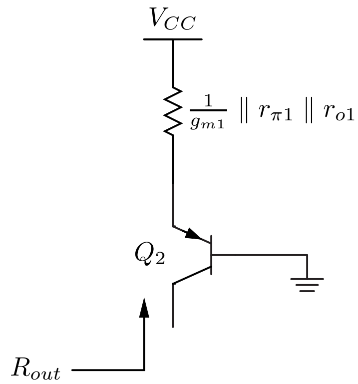
$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) \left(r_{\pi 1} \parallel r_{o2} \parallel \frac{r_{\pi 2} + R_B}{1 + \beta_2} \right)}$$

(c) Looking down from the emitter of Q_1 we see an equivalent resistance of $R_1 \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{out}:



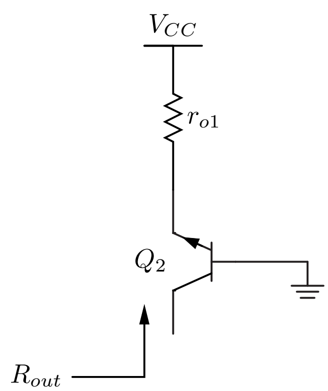
$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1})\,(r_{\pi 1} \parallel R_1 \parallel r_{\pi 2})}$$

5.50 (a) Looking into the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{o1}$, so we can draw the following equivalent circuit for finding R_{out}:



$$R_{out} = \boxed{r_{o2} + (1 + g_{m2} r_{o2}) \left(r_{\pi 2} \parallel \frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{o1} \right)}$$

(b) Looking into the emitter of Q_1 we see an equivalent resistance of r_{o1}, so we can draw the following equivalent circuit for finding R_{out}:



$$R_{out} = \boxed{r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel r_{o1})}$$

Comparing this to the solution to part (a), we can see that the output resistance is larger because instead of a factor of $1/g_{m1}$ dominating the parallel resistors in the expression, $r_{\pi 2}$ dominates (assuming $r_{o1} \gg r_{\pi 2}$).

51). $r_\pi = \beta V_T / I_C$.

$$R_{in} = r_\pi // R_B = \frac{\frac{\beta V_T}{I_C} R_B}{\frac{\beta V_T}{I_C} + R_B} = \frac{V_T R_B}{V_T + \frac{I_C}{\beta} R_B} = \frac{V_T R_B}{V_T + I_B R_B}$$

Since $I_B R_B >> V_T \Rightarrow R_{in} \approx \frac{V_T R_B}{I_B R_B} = \frac{V_T}{I_B} = \frac{V_T}{\frac{I_C}{\beta}} = \frac{\beta V_T}{I_C} \approx r_\pi$

So $R_{in} = r_\pi // R_B \approx r_\pi$.

5.52 (a)

$$V_{CC} - I_B(100\ \text{k}\Omega) - I_E(100\ \Omega) = V_{BE} = V_T \ln(I_C/I_S)$$
$$V_{CC} - \frac{1}{\beta} I_C(100\ \text{k}\Omega) - \frac{1+\beta}{\beta} I_C(100\ \Omega) = V_T \ln(I_C/I_S)$$
$$I_C = 1.6\ \text{mA}$$
$$A_v = -\frac{1\ \text{k}\Omega}{\frac{1}{g_m} + 100\ \Omega}$$
$$g_m = 61.6\ \text{mS}$$
$$A_v = \boxed{-8.60}$$

(b)

$$V_{CC} - I_B(50\ \text{k}\Omega) - I_E(2\ \text{k}\Omega) = V_T \ln(I_C/I_S)$$
$$I_C = 708\ \mu\text{A}$$
$$A_v = -\frac{1\ \text{k}\Omega}{\frac{1}{g_m} + \frac{(1\ \text{k}\Omega)\|(50\ \text{k}\Omega)}{1+\beta}}$$
$$g_m = 27.2\ \text{mS}$$
$$A_v = \boxed{-21.54}$$

(c)

$$I_B = \frac{I_C}{\beta} = \frac{V_{CC} - V_{BE} - I_E(2.5\ \text{k}\Omega)}{14\ \text{k}\Omega} - \frac{V_{BE} + I_E(2.5\ \text{k}\Omega)}{11\ \text{k}\Omega}$$
$$I_C = \beta\frac{V_{CC} - V_T \ln(I_C/I_S) - \frac{1+\beta}{\beta} I_C(2.5\ \text{k}\Omega)}{14\ \text{k}\Omega} - \beta\frac{V_T \ln(I_C/I_S) + \frac{1+\beta}{\beta} I_C(2.5\ \text{k}\Omega)}{11\ \text{k}\Omega}$$
$$I_C = 163\ \mu\text{A}$$
$$A_v = -\frac{10\ \text{k}\Omega}{\frac{1}{g_m} + 500\ \Omega + \frac{(1\ \text{k}\Omega)\|(14\ \text{k}\Omega)\|(11\ \text{k}\Omega)}{1+\beta}}$$
$$g_m = 6.29\ \text{mS}$$
$$A_v = \boxed{-14.98}$$

5.53 (a)

$$
\begin{aligned}
I_C &= \frac{V_{CC} - 1.5\text{ V}}{R_C} \\
&= 4\text{ mA} \\
V_{BE} &= V_T \ln(I_C/I_S) = 832\text{ mV} \\
I_B &= \frac{V_{CC} - V_{BE}}{R_B} = 66.7\ \mu\text{A} \\
\beta &= \frac{I_C}{I_B} = \boxed{60}
\end{aligned}
$$

(b) Assuming the speaker has an impedance of 8 Ω, the gain of the amplifier is

$$
\begin{aligned}
A_v &= -g_m \left(R_C \parallel 8\ \Omega\right) \\
&= -\frac{I_C}{V_T} \left(R_C \parallel 8\ \Omega\right) \\
&= \boxed{-1.19}
\end{aligned}
$$

Thus, the circuit provides greater than unity gain.

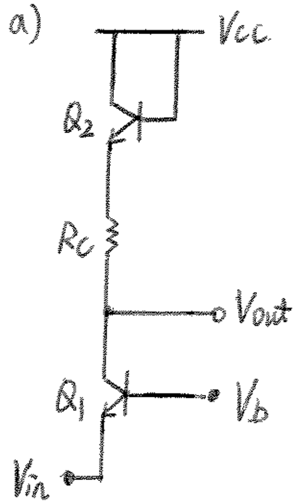
5.54 (a)

$$A_v = g_m R_C$$
$$g_m = \frac{I_C}{V_T} = 76.9 \text{ mS}$$
$$A_v = \boxed{38.46}$$
$$R_{in} = \frac{1}{g_m} \parallel r_\pi$$
$$r_\pi = \frac{\beta}{g_m} = 1.3 \text{ k}\Omega$$
$$R_{in} = \boxed{12.87 \ \Omega}$$
$$R_{out} = R_C = \boxed{500 \ \Omega}$$

(b) Since $A_v = g_m R_C$ and g_m is fixed for a given value of I_C, R_C should be chosen as large as possible to maximize the gain of the amplifier. V_b should be chosen as small as possible to maximize the headroom of the amplifier (since in order for Q_1 to remain in forward active, we require $V_b < V_{CC} - I_C R_C$).

55) $V_A = \infty$

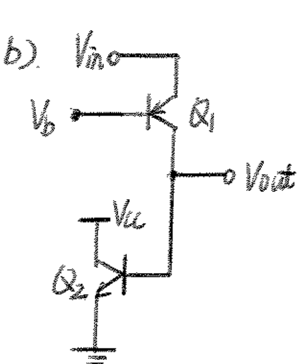
a)



$$|A_v| = \frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}}}$$

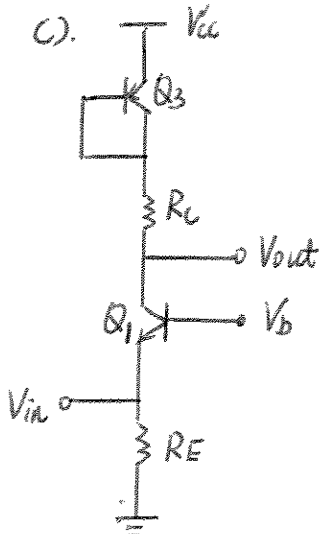
$$= g_{m1}\left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$$

b).



$$|A_v| = \frac{r_{\pi 2}}{\frac{1}{g_{m1}}} = g_{m1} r_{\pi 2}$$

c).

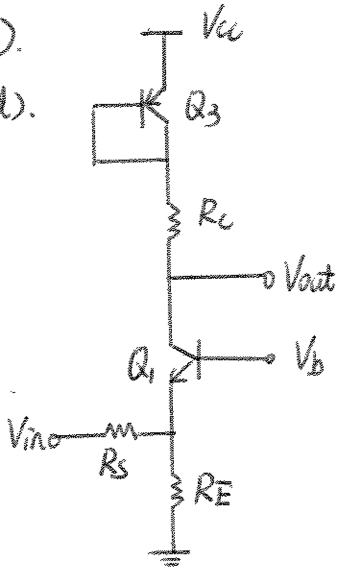


$$|A_v| = \frac{R_C + \frac{1}{g_{m3}} \parallel r_{\pi 3}}{\frac{1}{g_{m1}}}$$

$$= g_{m1}\left(R_C + \frac{1}{g_{m3}} \parallel r_{\pi 3}\right)$$

55).

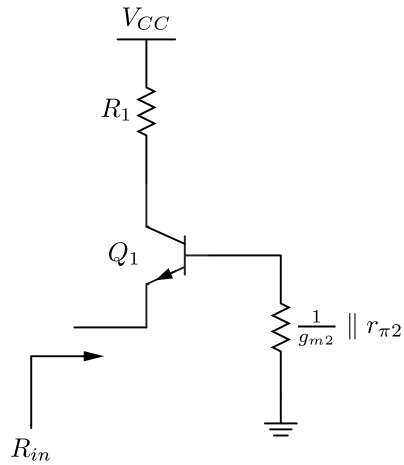
d).



$$|A_v| = \left|\frac{V_{out}}{V_A}\right| \left|\frac{V_A}{V_{in}}\right|$$

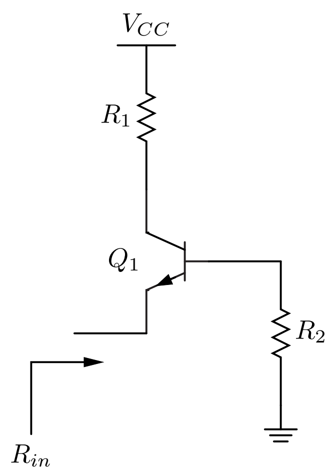
$$= \left[g_{m1} \left(R_C + \frac{1}{g_{m3}} \parallel r_{\pi 3}\right)\right] \left(\frac{R_E \parallel \frac{1}{g_{m1}}}{R_E \parallel \frac{1}{g_{m1}} + R_S} \right)$$

5.56 (a) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:



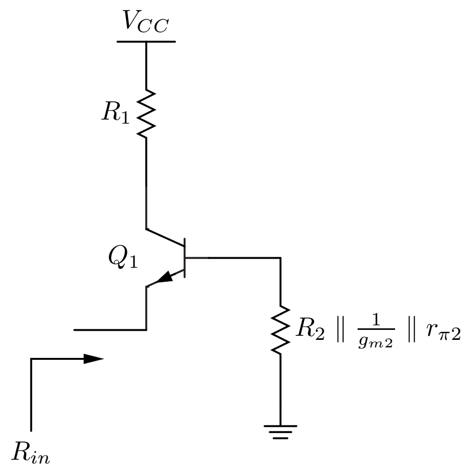
$$R_{in} = \boxed{\frac{r_{\pi 1} + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{1 + \beta_1}}$$

(b) Looking right from the base of Q_1 we see an equivalent resistance of R_2, so we can draw the following equivalent circuit for finding R_{in}:



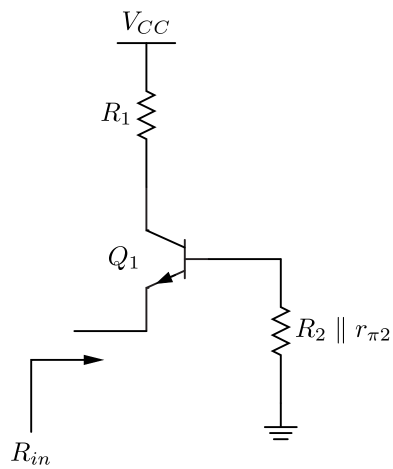
$$R_{in} = \boxed{\frac{r_{\pi 1} + R_2}{1 + \beta_1}}$$

(c) Looking right from the base of Q_1 we see an equivalent resistance of $R_2 \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:



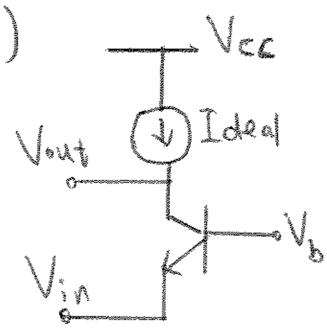
$$R_{in} = \boxed{\frac{r_{\pi 1} + R_2 \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}{1 + \beta_1}}$$

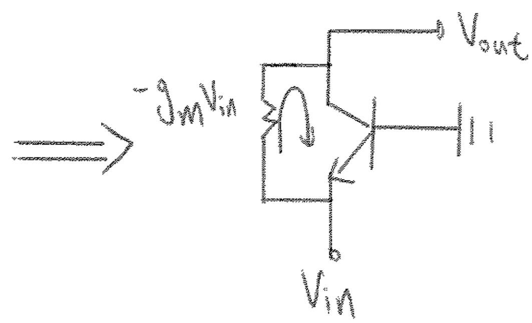
(d) Looking right from the base of Q_1 we see an equivalent resistance of $R_2 \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:



$$R_{in} = \boxed{\frac{r_{\pi 1} + R_2 \parallel r_{\pi 2}}{1 + \beta_1}}$$

57)





Since an ideal current source is an open circuit, the signal current produced by the transistor has no where to go but r_o.

So $V_{out} = -(g_m(0 - V_{in}))r_o + V_{in}$

$V_{out} = g_m r_o V_{in} + V_{in}$

$V_{out} = V_{in}(g_m r_o + 1)$

$$\frac{V_{out}}{V_{in}} = 1 + g_m r_o$$

5.58 (a)

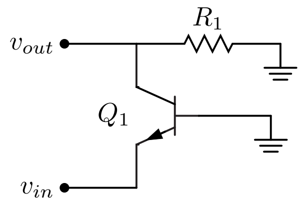
$$
\begin{aligned}
I_B = \frac{I_C}{\beta} &= \frac{V_{CC} - V_{BE} - I_E(400\ \Omega)}{13\ \text{k}\Omega} - \frac{V_{BE} + I_E(400\ \Omega)}{12\ \text{k}\Omega} \\
I_C &= \beta \frac{V_{CC} - V_T \ln(I_C/I_S) - \frac{1+\beta}{\beta} I_C(400\ \Omega)}{13\ \text{k}\Omega} - \beta \frac{V_T \ln(I_C/I_S) + \frac{1+\beta}{\beta} I_C(400\ \Omega)}{12\ \text{k}\Omega} \\
I_C &= \boxed{1.02\ \text{mA}} \\
V_{BE} &= V_T \ln(I_C/I_S) = \boxed{725\ \text{mV}} \\
V_{CE} &= V_{CC} - I_C(1\ \text{k}\Omega) - I_E(400\ \Omega) = \boxed{1.07\ \text{V}}
\end{aligned}
$$

Q_1 is operating in forward active.

(b)

$$
\begin{aligned}
A_v &= g_m(1\ \text{k}\Omega) \\
g_m &= 39.2\ \text{mS} \\
A_v &= \boxed{39.2}
\end{aligned}
$$

5.61 For small-signal analysis, we can draw the following equivalent circuit.



$$A_v = \boxed{g_m R_1}$$

$$R_{in} = \boxed{\frac{1}{g_m} \parallel r_\pi}$$

$$R_{out} = \boxed{R_1}$$

59)

$C_B = 0$

a) Since C_B was not considered during DC analysis, it has no effect on operating point analysis. So it is still the same as 58).

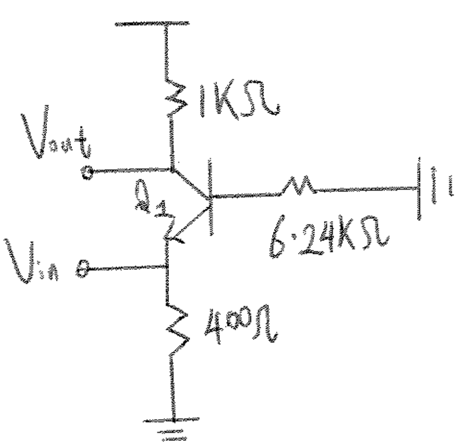
$V_{BE} = 0.725\ V$

$I_C = 1.0163\ mA$

$I_B = 10.163\ \mu A$

$V_{CE} = 1.07\ V$

b) Since capacitor is frequency dependent, the circuit's AC analysis will be different.



$$|A_v| = \frac{1k}{\frac{1}{g_m} + \frac{6.24K\Omega}{\beta+1}} = 11.4$$

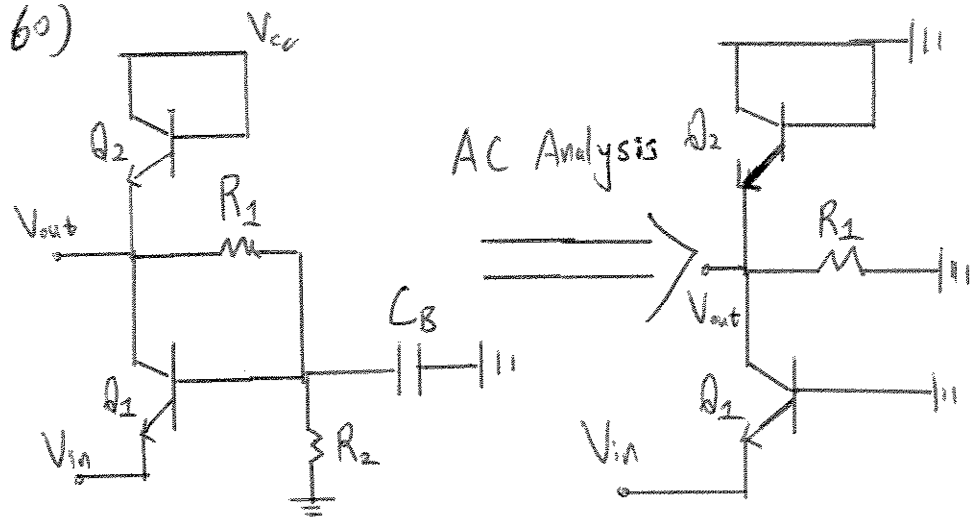
$$R_{in} = 400\Omega \,//\, \left(\frac{1}{g_m} + \frac{6.24K\Omega}{\beta+1}\right)$$

$$R_{in} = 71.7\Omega$$

Note: $6.24K\Omega$ is R_{THEV} of $13K\Omega$ and $12K\Omega$ combination.

$$R_{out} = 1K\Omega$$

60)

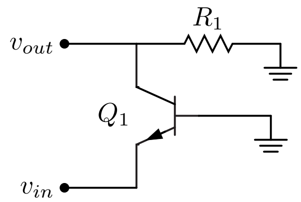


$$R_{out} = \frac{1}{g_{m2}} // r_{\pi 2} // R_1 \approx \frac{1}{g_{m2}} // R_1$$

$$A_v = \left| \frac{V_{out}}{V_{in}} \right| = g_{m1}\left(\frac{1}{g_{m2}} // r_{\pi 2} // R_1 \right) \approx g_{m1}\left(\frac{1}{g_{m2}} // R_1 \right)$$

$$R_{in} = \frac{1}{g_m} // r_{\pi 1} \approx \frac{1}{g_{m1}}$$

5.61 For small-signal analysis, we can draw the following equivalent circuit.

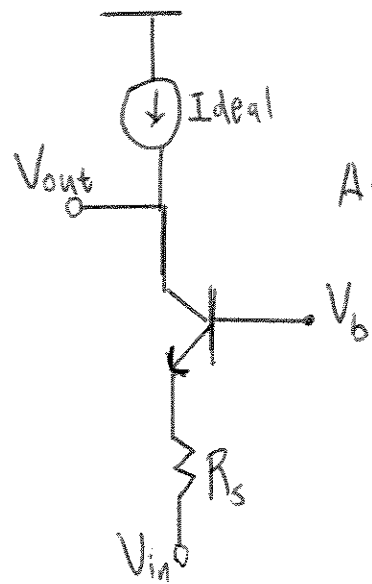


$$A_v = \boxed{g_m R_1}$$

$$R_{in} = \boxed{\frac{1}{g_m} \parallel r_\pi}$$

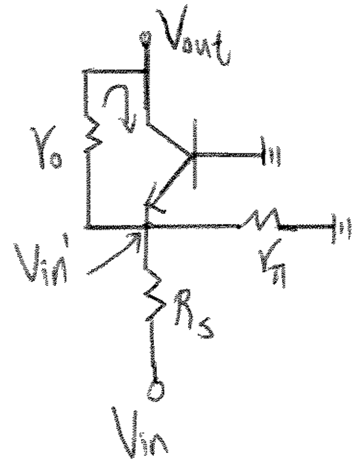
$$R_{out} = \boxed{R_1}$$

62)



$V_A < \infty$

AC Analysis $\Longrightarrow$



$$A_v = \frac{V_{out}}{V_{in}} = \left(\frac{V_{in}'}{V_{in}}\right)\left(\frac{V_{out}}{V_{in}'}\right), \quad \left(\frac{V_{in}'}{V_{in}}\right) = \frac{r_\pi}{r_\pi + R_s}$$

Since V_{out} is float, so looking at emitter and r_o, we will see an infinite impedance.

$$\frac{V_{out}}{V_{in}'} \Rightarrow -g_m(-V_{in}')r_o + V_{in}' = V_{out} \Rightarrow \frac{V_{out}}{V_{in}'} = (g_m r_o + 1)$$

$$A_v = (g_m r_o + 1)\left(\frac{r_\pi}{r_\pi + R_s}\right).$$

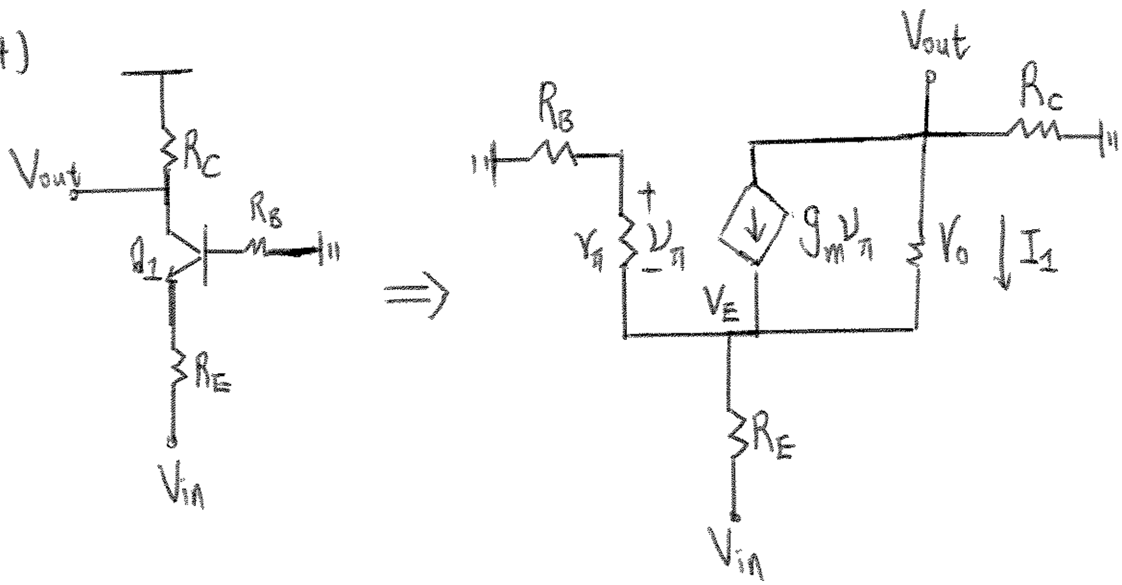
5.63 Since $I_{S1} = 2I_{S2}$ and they're biased identically, we know that $I_{C1} = 2I_{C2}$, which means $g_{m1} = 2g_{m2}$.

$$\frac{v_{out1}}{v_{in}} = g_{m1}R_C = 2g_{m2}R_C$$

$$\frac{v_{out2}}{v_{in}} = g_{m2}R_C$$

$$\Rightarrow \boxed{\frac{v_{out1}}{v_{in}} = 2\frac{v_{out2}}{v_{in}}}$$

64)



$$V_{out} = -(I_1 + g_m v_\pi) R_c \,, \quad I_1 = \frac{V_{out} - V_E}{r_o}$$

$$V_{out} = -\left(\frac{V_{out} - V_E}{r_o} + g_m v_\pi\right) R_c \,, \quad V_E = -\frac{g_m v_\pi}{\beta}(r_\pi + R_B)$$

$$V_{out} = -\left(\frac{V_{out} + \frac{g_m v_\pi (r_\pi + R_B)}{\beta}}{r_o} + g_m v_\pi\right) R_c$$

Rearranging

$$v_\pi = \frac{-\left(1 + \frac{R_c}{r_o}\right)}{\frac{g_m (r_\pi + R_B) R_c}{\beta r_o} + g_m R_c} V_{out} = A V_{out}$$

Summing the voltage at node E.

$$V_E - \left(\left(1 + \frac{1}{\beta}\right) g_m v_\pi + \frac{(V_{out} - V_E)}{r_o}\right) R_E = V_{in} \quad (1)$$

64)

Writing V_E in terms of V_π, and v_π in terms of V_{out}

1) becomes

$$-\frac{g_m A V_{out}}{\beta}(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) - \left(1+\frac{1}{\beta}\right) g_m A V_{out} R_E - \frac{V_{out}}{r_o} R_E = V_{in}$$

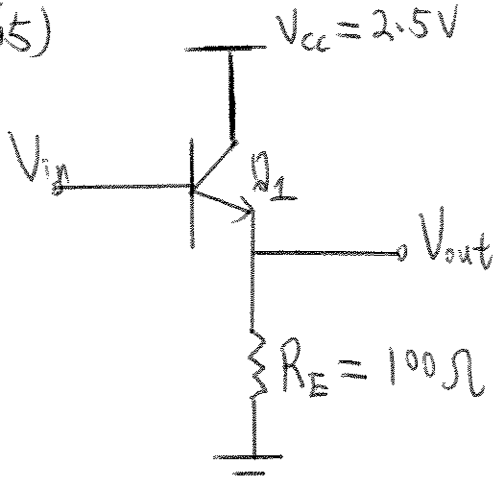
Solving V_{out}/V_{in} =>

$$\frac{V_{out}}{V_{in}} = \frac{1}{-\frac{g_m A}{\beta}(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) - \left(1+\frac{1}{\beta}\right) g_m A R_E - \frac{R_E}{r_o}}$$

subsituting A into equation

$$\frac{V_{out}}{V_{in}} = \frac{\frac{g_m (r_\pi + R_B) R_C}{\beta r_o} + g_m R_C}{g_m\left(1+\frac{R_C}{r_o}\right)(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) + \left(1+\frac{1}{\beta}\right) g_m \left(1+\frac{R_C}{r_o}\right) R_E - \frac{R_E}{r_o}\left(\frac{g_m (r_\pi + R_B) R_C}{\beta r_o} + g_m R_C\right)}$$

65)



$R_E = 100\Omega$

$V_A = \infty$

$|A_v| = 0.8$

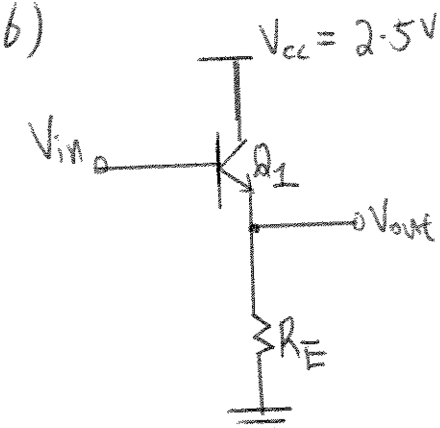
$$|A_v| = \frac{R_E}{R_E + \frac{1}{g_m}} = \frac{R_E I_c}{R_E I_c + V_T} = 0.8$$

$\Rightarrow R_E I_c = 0.8(R_E I_c + V_T)$, $R_E = 100\Omega$

$\Rightarrow 0.1 I_c = 0.08 I_c + 0.0208$ $\Rightarrow$ $0.02 I_c = 0.0208$

$\Rightarrow I_c = 1.04 mA$

6b)



$|A_v| > 0.9$
$R_{in} > 10K\Omega$

$$|A_v| = \frac{R_E I_c}{R_E I_c + V_T} > 0.9 \Rightarrow R_E I_c > 0.9[R_E I_c + V_T]$$

$\Rightarrow R_E I_c > 9V_T = 234\,mV$, Let $R_E I_c = 240\,mV$

$R_{in} = r_\pi + (1+\beta) R_E > 10K \Rightarrow 100 V_T + (101) R_E I_c > 10K\Omega\, I_c$

Subsituting $R_E I_c = 240\,mV \Rightarrow I_c < 2.684\,mA$

Choose I_c to be $2.5\,mA \Rightarrow R_E = 96\Omega$

To Verify:

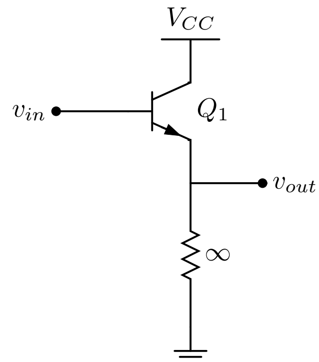
$$R_{in} = 100\frac{(0.026)}{2.5} + (101)0.096 = 10.74K\Omega$$

$$|A_v| = \frac{(0.096)(2.5)}{(0.096)(2.5) + 0.026} = 0.902$$

5.67

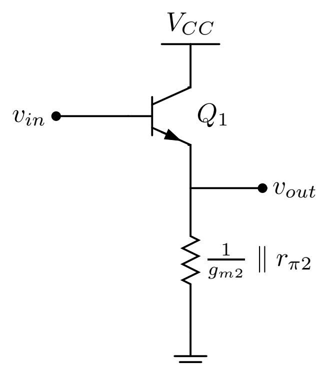
$$
\begin{aligned}
R_{out} &= \frac{r_\pi + R_S}{1+\beta} \\
&= \frac{\beta V_T / I_C + R_S}{1+\beta} \\
&\leq 5\ \Omega \\
I_C &= \frac{\beta}{1+\beta} I_E = \frac{\beta}{1+\beta} I_1 \\
\frac{\frac{\beta(1+\beta)V_T}{\beta I_1} + R_S}{1+\beta} &= \frac{\frac{(1+\beta)V_T}{I_1} + R_S}{1+\beta} \\
&\leq 5\ \Omega \\
I_1 &\geq \boxed{8.61\ \text{mA}}
\end{aligned}
$$

5.68 (a) Looking into the collector of Q_2 we see an equivalent resistance of $r_{o2} = \infty$, so we can draw the following equivalent circuit:



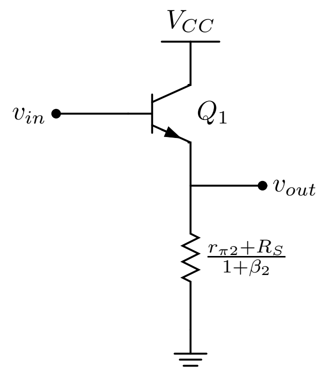
$$A_v = \boxed{1}$$
$$R_{in} = \boxed{\infty}$$
$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1}}$$

(b) Looking down from the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit:



$$A_v = \boxed{\frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$
$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$
$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(c) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{r_{\pi 2} + R_S}{1 + \beta_2}$, so we can draw the following equivalent circuit:

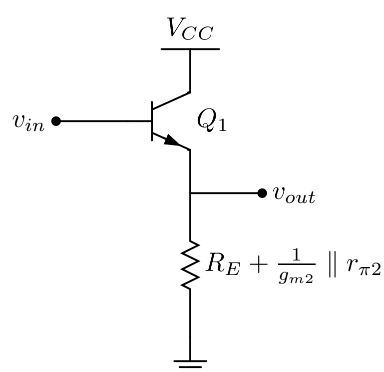


$$A_v = \boxed{\frac{\frac{r_{\pi 2}+R_S}{1+\beta_2}}{\frac{1}{g_{m1}} + \frac{r_{\pi 2}+R_S}{1+\beta_2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(\frac{r_{\pi 2}+R_S}{1+\beta_2}\right)}$$

$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel \left(\frac{r_{\pi 2}+R_S}{1+\beta_2}\right)}$$

(d) Looking down from the emitter of Q_1 we see an equivalent resistance of $R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit:

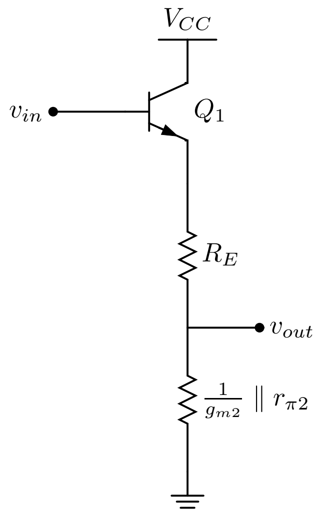


$$A_v = \boxed{\frac{R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(R_E + \frac{1}{g_{m2}}\right)}$$

$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel \left(R_E + \frac{1}{g_{m2}}\right)}$$

(e) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit:



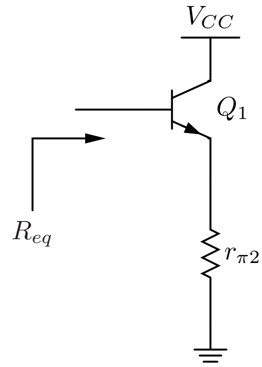
$$A_v = \frac{R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}} \cdot \frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

$$= \boxed{\frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)\left(R_E + \frac{1}{g_{m2} \parallel r_{\pi 2}}\right)}$$

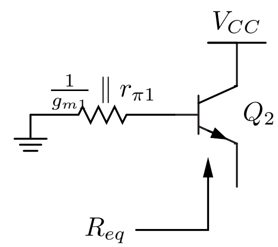
$$R_{out} = \boxed{\left(\frac{1}{g_{m1}} \parallel r_{\pi 1} + R_E\right) \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

5.69 (a) Looking into the base of Q_2 we see an equivalent resistance of $r_{\pi 2}$ (assuming the emitter of Q_2 is grounded), so we can draw the following equivalent circuit for finding the impedance at the base of Q_1:



$$R_{eq} = \boxed{r_{\pi 1} + (1+\beta_1) r_{\pi 2}}$$

(b) Looking into the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m1}} \parallel r_{\pi 1}$ (assuming the base of Q_1 is grounded), so we can draw the following equivalent circuit for finding the impedance at the emitter of Q_2:



$$R_{eq} = \boxed{\frac{r_{\pi 2} + \frac{1}{g_{m1}} \parallel r_{\pi 1}}{1+\beta_2}}$$

(c)

$$\frac{I_{C1} + I_{C2}}{I_{B1}} = \frac{\beta_1 I_{B1} + \beta_2 (1+\beta_1) I_{B1}}{I_{B1}}$$
$$= \boxed{\beta_1 + \beta_2(1+\beta_1)}$$

If we assume that $\beta_1, \beta_2 \gg 1$, then this simplifies to $\beta_1 \beta_2$, meaning a Darlington pair has a current gain approximately equal to the product of the current gains of the individual transistors.

5.70 (a)

$$R_{CS} = \boxed{r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)}$$

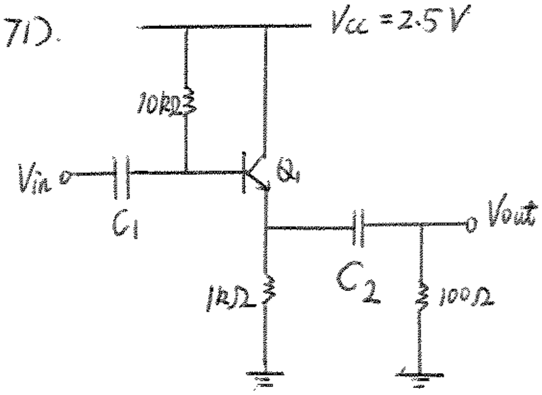
(b)

$$A_v = \boxed{\frac{r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)}{\frac{1}{g_{m1}} + r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1) [r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)]}$$

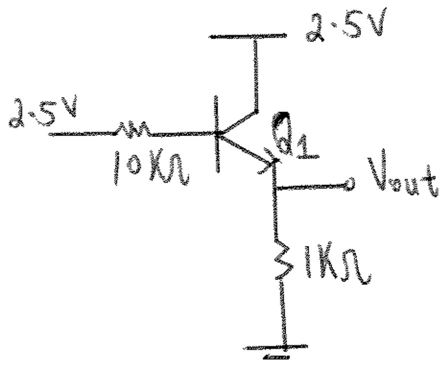
$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel [r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)]}$$

71).



$I_s = 7 \times 10^{-16} A$
$\beta = 100$
$V_A = 5V$

DC Analysis: (Ignore r_o's effect)



$$I_c = \beta \left(\frac{2.5 - (V_{BE} + \frac{I_c}{\alpha} 1K\Omega)}{10K\Omega} \right)$$

rearrange

$$I_c = \frac{2.5 - V_{BE}}{\frac{10K\Omega}{\beta} + \frac{1K\Omega}{\alpha}}$$

Guess: $V_{BE} = 0.7V$, $I_c = 1.621 mA$

check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.740V$, not 0.7, reiterate

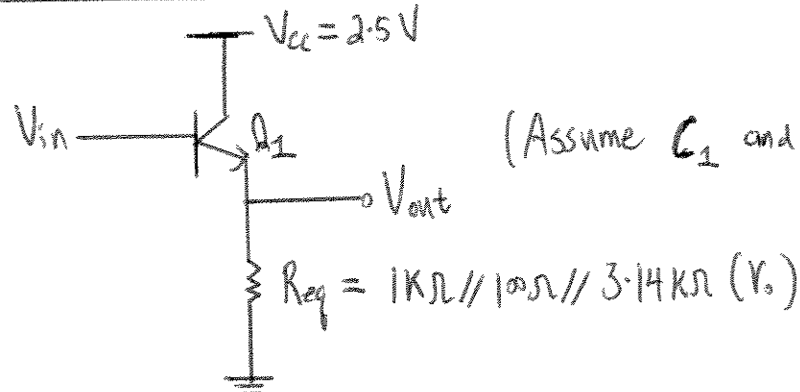
$V_{BE} = 0.740V$, $I_c = 1.59 mA$

Check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.740V$, converged.

So $I_c = 1.59mA$, $g_m = 0.0612 (\frac{1}{\Omega}) S$, $\frac{1}{g_m} = 16.34\Omega$,
$r_o = 3.14K\Omega$

71)

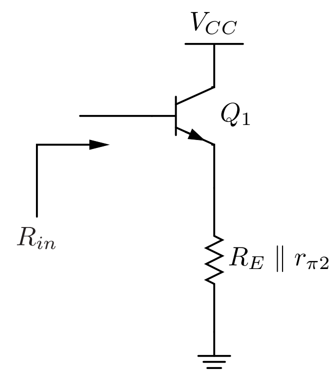
AC Analysis: (Include r_o)



(Assume C_1 and C_2 are large)

$$A_v = \frac{(1K\Omega // 100\Omega // 3.14K\Omega)}{16.34\Omega + (1K\Omega // 100\Omega / 3.14K\Omega)} = 0.84$$

5.72 (a) Looking into the base of Q_2 we see an equivalent resistance of $r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:

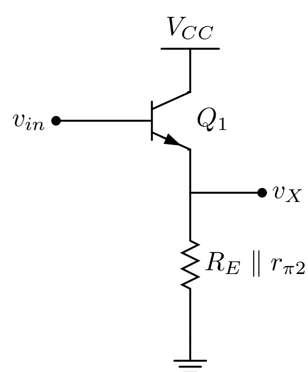


$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)(R_E \parallel r_{o1})}$$

Looking into the collector of Q_2 we see an equivalent resistance of r_{o2}. Thus,

$$R_{out} = \boxed{R_C \parallel r_{o2}}$$

(b) Looking into the base of Q_2 we see an equivalent resistance of $r_{\pi 2}$, so we can draw the following equivalent circuit for finding v_X/v_{in}:

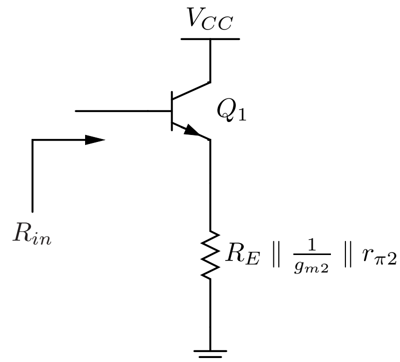


$$\frac{v_X}{v_{in}} = \frac{R_E \parallel r_{\pi 2} \parallel r_{o1}}{\frac{1}{g_{m1}} + R_E \parallel r_{\pi 2} \parallel r_{o1}}$$

We can find v_{out}/v_X by inspection.

$$\begin{aligned}
\frac{v_{out}}{v_X} &= -g_{m2}(R_C \parallel r_{o2}) \\
A_v &= \frac{v_X}{v_{in}} \cdot \frac{v_{out}}{v_X} \\
&= \boxed{-g_{m2}(R_C \parallel r_{o2}) \frac{R_E \parallel r_{\pi 2} \parallel r_{o1}}{\frac{1}{g_{m1}} + R_E \parallel r_{\pi 2} \parallel r_{o1}}}
\end{aligned}$$

5.73 (a) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:

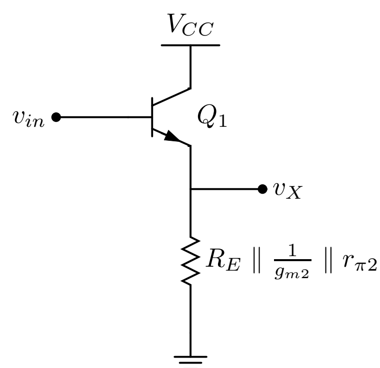


$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)\left(R_E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

Looking into the collector of Q_2, we see an equivalent resistance of ∞ (because $V_A = \infty$), so we have

$$R_{out} = \boxed{R_C}$$

(b) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding v_X/v_{in}:



$$\frac{v_X}{v_{in}} = \frac{R_E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

We can find v_{out}/v_X by inspection.

$$\frac{v_{out}}{v_X} = g_{m2} R_C$$

$$A_v = \frac{v_X}{v_{in}} \cdot \frac{v_{out}}{v_X}$$

$$= \boxed{g_{m2} R_C \frac{R_E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

5.74

$$
\begin{aligned}
R_{out} &= R_C = \boxed{1\ \mathrm{k\Omega}} \\
A_v &= -g_m R_C = -10 \\
g_m &= 10\ \mathrm{mS} \\
I_C &= g_m V_T = 260\ \mu\mathrm{A} \\
\frac{V_{CC} - V_{BE}}{R_B} &= I_B = \frac{I_C}{\beta} \\
R_B &= \beta \frac{V_{CC} - V_T \ln(I_C/I_S)}{I_C} \\
&= \boxed{694\ \mathrm{k\Omega}} \\
R_{in} &= R_B \parallel r_\pi = 9.86\ \mathrm{k\Omega} > 5\ \mathrm{k\Omega}
\end{aligned}
$$

In sizing C_B, we must consider the effect a finite impedance in series with the input will have on the circuit parameters. Any series impedance will cause R_{in} to increase and will not impact R_{out}. However, a series impedance can cause gain degradation. Thus, we must ensure that $|Z_B| = \left|\frac{1}{j\omega C_B}\right|$ does not degrade the gain significantly.

If we include $|Z_B|$ in the gain expression, we get:

$$
A_v = -\frac{R_C}{\frac{1}{g_m} + \frac{(|Z_B|)\|R_B}{1+\beta}}
$$

Thus, we want $\frac{1}{1+\beta}|Z_B| \ll \frac{1}{g_m}$ to ensure the gain is not significantly degraded.

$$
\begin{aligned}
\frac{1}{1+\beta}\left|\frac{1}{j\omega C_B}\right| &\ll \frac{1}{g_m} \\
\frac{1}{1+\beta}\frac{1}{2\pi f C_B} &= \frac{1}{10}\frac{1}{g_m} \\
C_B &= \boxed{788\ \mathrm{nF}}
\end{aligned}
$$

5.75

$$R_{out} = R_C \leq 500\ \Omega$$

To maximize gain, we should maximize R_C.

$$R_C = \boxed{500\ \Omega}$$
$$V_{CC} - I_C R_C \geq V_{BE} - 400\text{ mV} = V_T \ln(I_C/I_S) - 400\text{ mV}$$
$$I_C \leq 4.261\text{ mA}$$

To maximize gain, we should maximize I_C.

$$I_C = \boxed{4.261\text{ mA}}$$
$$I_B = \frac{I_C}{\beta} = \frac{V_{CC} - V_{BE}}{R_B}$$
$$= \frac{I_C}{\beta} = \frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B}$$
$$R_B = \boxed{40.613\text{ k}\Omega}$$

5.76

$$
\begin{aligned}
R_{out} &= R_C = \boxed{1\ \text{k}\Omega} \\
|A_v| &= g_m R_C \\
&= \frac{I_C R_C}{V_T} \\
&\geq 20 \\
I_C &\geq 520\ \mu\text{A}
\end{aligned}
$$

In order to maximize $R_{in} = R_B \parallel r_\pi$, we need to maximize r_π, meaning we should minimize I_C (since $r_\pi = \frac{\beta V_T}{I_C}$).

$$
\begin{aligned}
I_C &= 520\ \mu\text{A} \\
I_B &= \frac{I_C}{\beta} = \frac{V_{CC} - V_{BE}}{R_B} \\
&= \frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} \\
R_B &= \boxed{343\ \text{k}\Omega}
\end{aligned}
$$

5.77

$$\begin{aligned}
R_{out} &= R_C = \boxed{2\ \text{k}\Omega} \\
A_v &= -g_m R_C \\
&= -\frac{I_C R_C}{V_T} \\
&= -15 \\
I_C &= 195\ \mu\text{A} \\
V_{BE} &= V_T \ln(I_C/I_S) = 689.2\ \text{mV} \\
V_{CE} &\geq V_{BE} - 400\ \text{mV} = 289.2\ \text{mV}
\end{aligned}$$

To minimize the supply voltage, we should minimize V_{CE}.

$$\begin{aligned}
V_{CE} &= 289.2\ \text{mV} \\
\frac{V_{CC} - V_{CE}}{R_C} &= I_C \\
V_{CC} &= 679.2\ \text{mV}
\end{aligned}$$

Note that this value of V_{CC} is less than the required V_{BE}. This means that the value of V_{CC} is constrained by V_{BE}, not V_{CE}. In theory, we could pick $V_{CC} = V_{BE}$, but in this case, we'd have to set $R_B = 0\ \Omega$, which would short the input to V_{CC}. Thus, let's pick a reasonable value for R_B, $R_B = \boxed{100\ \Omega}$.

$$\begin{aligned}
\frac{V_{CC} - V_{BE}}{R_B} &= I_B = \frac{I_C}{\beta} \\
V_{CC} &= \boxed{689.4\ \text{mV}}
\end{aligned}$$

5.78

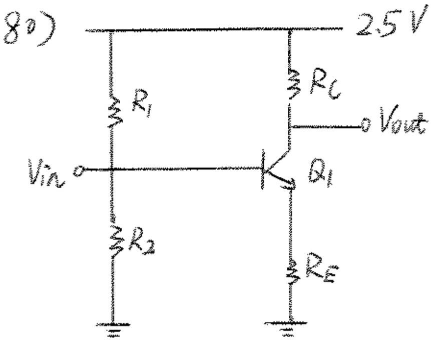
$$
\begin{aligned}
|A_v| &= g_m R_C \\
&= \frac{I_C R_C}{V_T} \\
&= A_0 \\
R_{out} &= R_C \\
A_0 &= \frac{I_C R_{out}}{V_T} \\
I_C &= \frac{A_0 V_T}{R_{out}} \\
P &= I_C V_{CC} \\
&= \boxed{\frac{A_0 V_T}{R_{out}} V_{CC}}
\end{aligned}
$$

Thus, we must trade off a small output resistance with low power consumption (i.e., as we decrease R_{out}, power consumption increases and vice-versa).

5.79

$$
\begin{aligned}
P &= (I_B + I_C)V_{CC} \\
&= \frac{1+\beta}{\beta} I_C V_{CC} \\
&= 1 \text{ mW} \\
I_C &= 396\ \mu\text{A} \\
\frac{V_{CC} - V_{BE}}{R_B} &= I_B = \frac{I_C}{\beta} \\
R_B &= \beta \frac{V_{CC} - V_T \ln(I_C/I_S)}{I_C} \\
&= \boxed{453\ \text{k}\Omega} \\
A_v &= -g_m R_C \\
&= -\frac{I_C R_C}{V_T} \\
&= -20 \\
R_C &= \boxed{1.31\ \text{k}\Omega}
\end{aligned}
$$

80)



$A_V = 5$

$R_{out} = R_c = 500\Omega$

$R_E I_c \approx 300\,mV$

$$A_V = \frac{R_c I_c}{R_E I_c + V_T} = \frac{R_c I_c}{300 + 26} \Rightarrow \begin{array}{l} R_c I_c = 1.63V \Rightarrow I_c = 3.26\,mA \\ R_E I_c \approx 300\,mV \Rightarrow R_E = 92\Omega \end{array}$$

$$R_1 = \frac{2.5 - (V_{BE} + 0.3)}{10\, I_B}, \quad V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.7624$$

$$10 I_B = 0.326\,mA$$

$$R_1 = \frac{2.5 - (0.7624 + 0.3)}{0.326} = 4.41K$$

$$R_2 = \frac{(0.7624 + 0.3)}{(9)(0.0326)} = 3.62K$$

$V_{CE} = 2.5 - 1.63 - 0.3 = 0.57$, $V_{BE} = 0.7624$.

Q_1 is in soft saturation region, so active region characteristics still apply.

$$\begin{array}{l} R_c = 500\Omega \\ R_1 = 4.41K\Omega \\ R_2 = 3.62K\Omega \\ R_E = 92\Omega \end{array} \Rightarrow \begin{array}{l} A_V = 5 \\ R_{out} = 500\Omega \end{array}$$

5.81

$$R_{out} = R_C \geq 1\ \text{k}\Omega$$

To maximize gain, we should maximize R_{out}.

$$R_C = \boxed{1\ \text{k}\Omega}$$
$$V_{CC} - I_C R_C - I_E R_E = V_{CE} \geq V_{BE} - 400\ \text{mV}$$
$$V_{CC} - I_C R_C - 200\ \text{mV} \geq V_T \ln(I_C/I_S) - 400\ \text{mV}$$
$$I_C \leq 1.95\ \text{mA}$$

To maximize gain, we should maximize I_C.

$$I_C = 1.95\ \text{mA}$$
$$I_E R_E = \frac{1+\beta}{\beta} I_C R_E = 200\ \text{mV}$$
$$R_E = \boxed{101.5\ \Omega}$$
$$V_{CC} - 10 I_B R_1 - I_E R_E = V_{BE} = V_T \ln(I_C/I_S)$$
$$R_1 = \boxed{7.950\ \text{k}\Omega}$$
$$9 I_B R_2 - I_E R_E = V_{BE} = V_T \ln(I_C/I_S)$$
$$R_2 = \boxed{5.405\ \text{k}\Omega}$$

5.82

$$
\begin{aligned}
P &= (10I_B + I_C)\,V_{CC} \\
&= \left(10\frac{I_C}{\beta} + I_C\right) V_{CC} \\
&= 5\ \text{mW} \\
I_C &= 1.82\ \text{mA} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 200\ \text{mV} \\
R_E &= \boxed{109\ \Omega} \\
A_v &= -\frac{R_C}{\frac{1}{g_m} + R_E} \\
&= -\frac{R_C}{\frac{V_T}{I_C} + R_E} \\
&= -5 \\
R_C &= \boxed{616\ \Omega} \\
V_{CC} - 10I_B R_1 - 200\ \text{mV} &= V_{BE} = V_T \ln(I_C/I_S) \\
R_1 &= \boxed{8.54\ \text{k}\Omega} \\
9I_B R_2 - 200\ \text{mV} &= V_{BE} = V_T \ln(I_C/I_S) \\
R_2 &= \boxed{5.79\ \text{k}\Omega}
\end{aligned}
$$

5.83

$$
\begin{aligned}
R_{in} &= \frac{1}{g_m} = 50\ \Omega \text{ (since } R_E \text{ doesn't affect } R_{in}) \\
g_m &= 20\ \text{mS} \\
I_C &= g_m V_T = 520\ \mu\text{A} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 260\ \text{mV} \\
R_E &= \boxed{495\ \Omega} \\
A_v &= g_m R_C = 20 \\
R_C &= \boxed{1\ \text{k}\Omega} \\
V_{CC} - 10 I_B R_1 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_1 &= \boxed{29.33\ \text{k}\Omega} \\
9 I_B R_2 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_2 &= \boxed{20.83\ \text{k}\Omega}
\end{aligned}
$$

To pick C_B, we must consider its effect on A_v. If we assume the capacitor has an impedance Z_B and $|Z_B| \ll R_1, R_2$, then we have:

$$
A_v = \frac{R_C}{\frac{1}{g_m} + \frac{|Z_B|}{1+\beta}}
$$

Thus, we should choose $\frac{1}{1+\beta} |Z_B| \ll \frac{1}{g_m}$.

$$
\begin{aligned}
\frac{1}{1+\beta} |Z_B| &= \frac{1}{1+\beta} \frac{1}{2\pi f C_B} = \frac{1}{10} \frac{1}{g_m} \\
C_B &= \boxed{1.58\ \mu\text{F}}
\end{aligned}
$$

5.84

$$R_{out} = R_C = \boxed{500\ \Omega}$$
$$A_v = g_m R_C = 8$$
$$g_m = 16\text{ mS}$$
$$I_C = g_m V_T = 416\ \mu\text{A}$$
$$I_E R_E = \frac{1+\beta}{\beta} I_C R_E = 260\text{ mV}$$
$$R_E = \boxed{619\ \Omega}$$
$$V_{CC} - 10 I_B R_1 - I_E R_E = V_{BE} = V_T \ln(I_C/I_S)$$
$$R_1 = \boxed{36.806\ \text{k}\Omega}$$
$$9 I_B R_2 - I_E R_E = V_{BE} = V_T \ln(I_C/I_S)$$
$$R_2 = \boxed{25.878\ \text{k}\Omega}$$

To pick C_B, we must consider its effect on A_v. If we assume the capacitor has an impedance Z_B and $|Z_B| \ll R_1, R_2$, then we have:

$$A_v = \frac{R_C}{\frac{1}{g_m} + \frac{|Z_B|}{1+\beta}}$$

Thus, we should choose $\frac{1}{1+\beta}|Z_B| \ll \frac{1}{g_m}$.

$$\frac{1}{1+\beta}|Z_B| = \frac{1}{1+\beta}\frac{1}{2\pi f C_B} = \frac{1}{10}\frac{1}{g_m}$$
$$C_B = \boxed{1.26\ \mu\text{F}}$$

5.85

$$\begin{aligned}
R_{out} &= R_C = \boxed{200\ \Omega} \\
A_v &= g_m R_C = \frac{I_C R_C}{V_T} = 20 \\
I_C &= 2.6\ \text{mA} \\
P &= V_{CC}\left(10 I_B + I_C\right) \\
&= V_{CC}\left(10\frac{I_C}{\beta} + I_C\right) \\
&= \boxed{7.15\ \text{mW}}
\end{aligned}$$

5.86

$$
\begin{aligned}
P &= (I_C + 10I_B)\,V_{CC} \\
&= \left(I_C + 10\frac{I_C}{\beta}\right) V_{CC} \\
&= 5\text{ mW} \\
I_C &= 1.82\text{ mA} \\
A_v &= g_m R_C \\
&= \frac{I_C R_C}{V_T} \\
&= 10 \\
R_C &= \boxed{143\ \Omega} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 260\text{ mV} \\
R_E &= \boxed{141.6\ \Omega} \\
V_{CC} - 10I_B R_1 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_1 &= \boxed{8.210\text{ k}\Omega} \\
9I_B R_2 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_2 &= \boxed{6.155\text{ k}\Omega}
\end{aligned}
$$

To pick C_B, we must consider its effect on A_v. If we assume the capacitor has an impedance Z_B and $|Z_B| \ll R_1, R_2$, then we have:

$$
A_v = \frac{R_C}{\frac{1}{g_m} + \frac{|Z_B|}{1+\beta}}
$$

Thus, we should choose $\frac{1}{1+\beta}|Z_B| \ll \frac{1}{g_m}$.

$$
\begin{aligned}
\frac{1}{1+\beta}|Z_B| &= \frac{1}{1+\beta}\frac{1}{2\pi f C_B} = \frac{1}{10}\frac{1}{g_m} \\
C_B &= \boxed{5.52\ \mu\text{F}}
\end{aligned}
$$

5.87

$$
\begin{aligned}
R_{in} &= \frac{1}{g_m} = 50\ \Omega \text{ (since } R_E \text{ doesn't affect } R_{in}) \\
g_m &= 20 \text{ mS} \\
I_C &= g_m V_T = 520\ \mu\text{A} \\
A_v &= g_m R_C = 20 \\
R_C &= \boxed{1\ \text{k}\Omega} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 260 \text{ mV} \\
R_E &= \boxed{495\ \Omega}
\end{aligned}
$$

To minimize the supply voltage, we should allow Q_1 to operate in soft saturation, i.e., $V_{BC} = 400$ mV.

$$
\begin{aligned}
V_{BE} &= V_T \ln(I_C/I_S) = 715 \text{ mV} \\
V_{CE} &= V_{BE} - 400 \text{ mV} = 315 \text{ mV} \\
V_{CC} - I_C R_C - I_E R_E &= V_{CE} \\
V_{CC} &= \boxed{1.095 \text{ V}} \\
V_{CC} - 10 I_B R_1 - I_E R_E &= V_{BE} \\
R_1 &= \boxed{2.308\ \text{k}\Omega} \\
9 I_B R_2 - I_E R_E &= V_{BE} \\
R_2 &= \boxed{20.827\ \text{k}\Omega}
\end{aligned}
$$

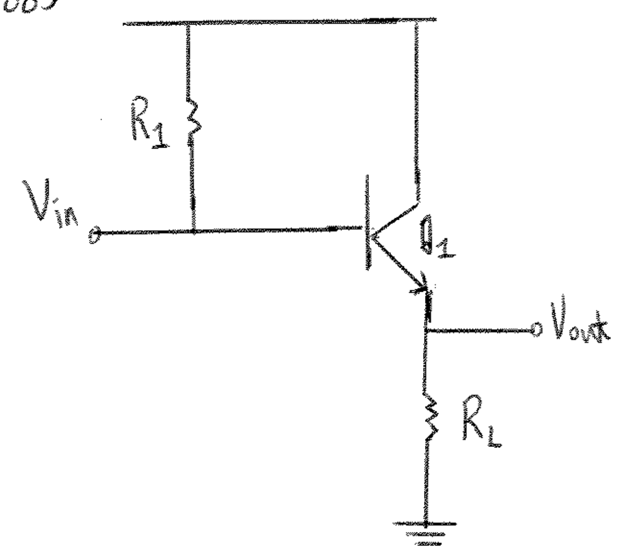
To pick C_B, we must consider its effect on A_v. If we assume the capacitor has an impedance Z_B and $|Z_B| \ll R_1, R_2$, then we have:

$$
A_v = \frac{R_C}{\frac{1}{g_m} + \frac{|Z_B|}{1+\beta}}
$$

Thus, we should choose $\frac{1}{1+\beta} |Z_B| \ll \frac{1}{g_m}$.

$$
\begin{aligned}
\frac{1}{1+\beta} |Z_B| &= \frac{1}{1+\beta} \frac{1}{2\pi f C_B} = \frac{1}{10} \frac{1}{g_m} \\
C_B &= \boxed{1.58\ \mu\text{F}}
\end{aligned}
$$

88)



$A_v = 0.85$
$R_{in} > 10k\Omega$
$R_L = 200\Omega$

$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} = 0.85 \Rightarrow \frac{200}{200 + \frac{1}{g_m}} = 0.85$$

$$\Rightarrow 200 = 0.85\left(200 + \frac{1}{g_m}\right) \Rightarrow \frac{1}{g_m} = 35.294\Omega$$

$$\Rightarrow I_c = \frac{26\,mV}{35.294\Omega} = 0.737\,mA, \quad V_{BE} = V_T \ln\left(\frac{0.737}{6\times10^{-13}}\right) = 0.724V$$

$$R_{in} = R_1 \,//\, (r_\pi + (1+\beta)(200\Omega))$$
$$R_{in} = R_1 \,//\, 23.73K$$
$$R_{in} = \frac{R_1\, 23.73K}{R_1 + 23.73K} > 10K \Rightarrow R_1 > 17.28K \text{ (Input Impedance requirement)}$$

To support an I_c of 0.737, R_1 must be determined.

88)

$$R_1 = \frac{2.5 - (0.724 + (0.737)(0.2)/0.99)}{0.737/100}$$

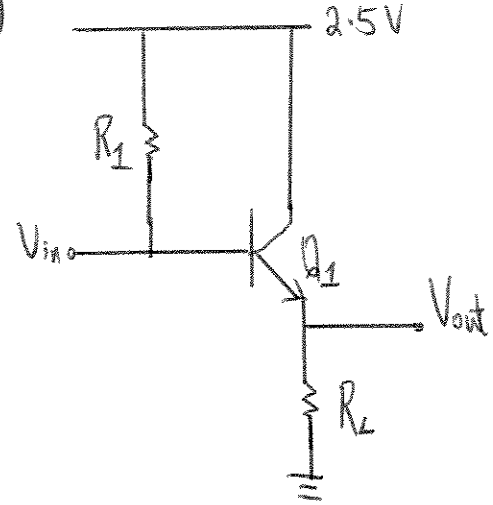
$$R_1 = 220.77K\Omega$$

$R_1 = 220.77K\Omega \Rightarrow R_{in} = 220.77K\Omega // 23.73K\Omega$

$R_{in} = 21.43K\Omega > 10K\Omega$

$R_1 = 220.77K\Omega$, $R_L = 200\Omega$ $\Rightarrow$ $A_v = 0.85$, $R_{in} = 21.43K\Omega$

89)



Power = 5 mW

$A_v = 0.9$

$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} = 0.9 \Rightarrow R_L = 0.9\left(R_L + \frac{1}{g_m}\right)$$

$$R_L = 9\frac{1}{g_m}$$

$$\text{Power} = 2.5\left(I_c + \frac{I_c}{\beta}\right) \Rightarrow I_c = 1.98\,mA$$

$$\frac{1}{g_m} = \frac{V_T}{I_c} = \frac{26mV}{1.98mA} = 13.13\,\Omega$$

$$R_L = (9)(13.13) = 118.17\,\Omega.$$

This is the minimum load resistance, since anything lower will lower the voltage gain.

5.90 As stated in the hint, let's assume that $I_E R_E \gg V_T$. Given this assumption, we can assume that R_E does not affect the gain.

$$I_E R_E = 10V_T = 260 \text{ mV}$$
$$A_v = \frac{R_L}{\frac{1}{g_m} + R_L} = 0.8$$
$$g_m = 80 \text{ mS}$$
$$I_C = g_m V_T = 2.08 \text{ mA}$$
$$\frac{1+\beta}{\beta} I_C R_E = 260 \text{ mV}$$
$$R_E = \boxed{124\ \Omega}$$
$$V_{CC} - I_B R_1 - I_E R_E = V_{BE} = V_T \ln(I_C/I_S)$$
$$R_1 = \boxed{71.6\ \text{k}\Omega}$$

To pick C_1, we must consider its effect on A_v. If we assume the capacitor has an impedance Z_1 and $|Z_1| \ll R_1$, then we have:

$$A_v = \frac{R_E}{\frac{1}{g_m} + R_E + \frac{|Z_1|}{1+\beta}}$$

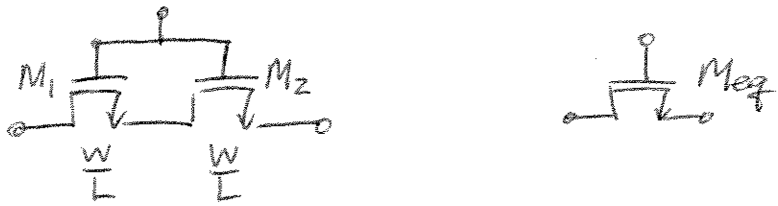
Thus, we should choose $\frac{1}{1+\beta}|Z_1| \ll \frac{1}{g_m}$.

$$\frac{1}{1+\beta}|Z_1| = \frac{1}{1+\beta}\frac{1}{2\pi f C_1} = \frac{1}{10}\frac{1}{g_m}$$
$$C_1 = \boxed{12.6 \text{ pF}}$$

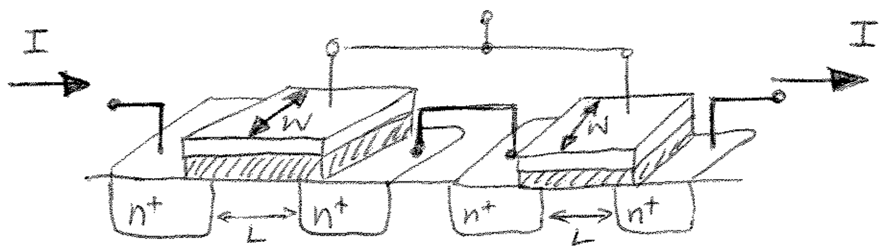
To pick C_2, we must also consider its effect on A_v. Since the capacitor appears in series with R_L, we need to ensure that $|Z_2| \ll R_L$, assuming the capacitor has impedance Z_2.

$$|Z_2| = \frac{1}{2\pi f C_2} = \frac{1}{10} R_L$$
$$C_2 = \boxed{318 \text{ pF}}$$

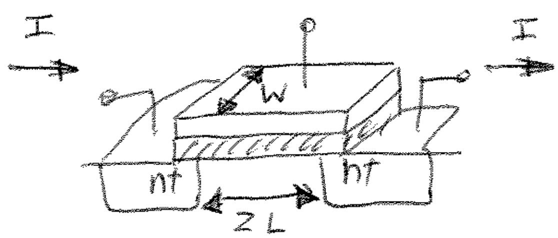
1.



Intuitively, this is similar to having twice of the original channel length:



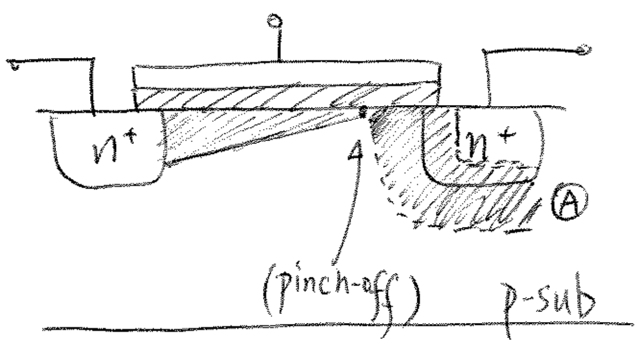
Since current flowing into either non-gate terminals must come out at the other terminal (KCL) and the intermediate node is equipotential, this is as if we have a Meq with width W & length 2L:



This approximation can simplify a lot of calculations.

2. A key point to remember: the charge density APPROACHES zero (not EQUALS) at pinch-off. In other words, Q is never exactly equal to zero (albeit very close.) Another way to view this phenomenon is by observing $I = Q \cdot v$: recognize that v is finite. Since we get some finite value of I at pinch-off, we expect $Q \neq 0$.

Consider the following:



The shaded region, Ⓐ, represents a reverse-biased pn junction. Just as a diode, there exist minority profiles on p & n sides, which $\neq 0$.

Pinch-off implies that the depletion region created no longer has free carriers. The depletion still sweeps all electrons from inversion channel to drain.

3. Given: $C_{ox} = 10\ fF/\mu m^2$ $\quad W = 5\mu m$ $\quad L = 0.1\mu m$

$V_{GS} - V_{TH} = 1V$ $\quad V_{DS} = 0$

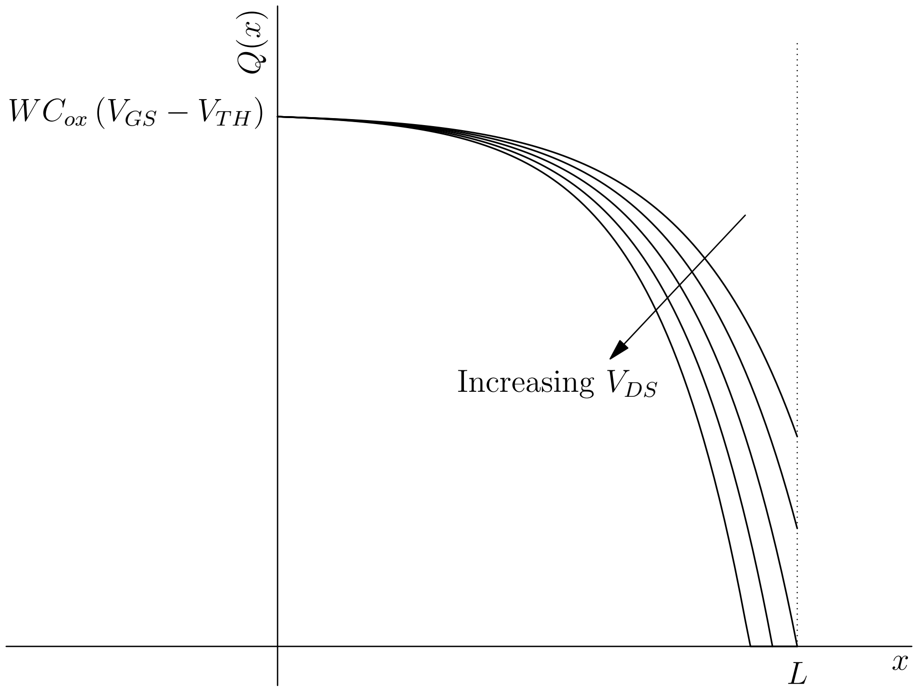
Find: total charge stored in channel, Q_{tot}

$$Q_{tot} = W C_{ox} (V_{GS} - V_{TH}) L$$

$$= (5\mu m)(10\ fF/\mu m^2)(1V)(0.1\mu m) = 5\ fC$$

6.4 (a)

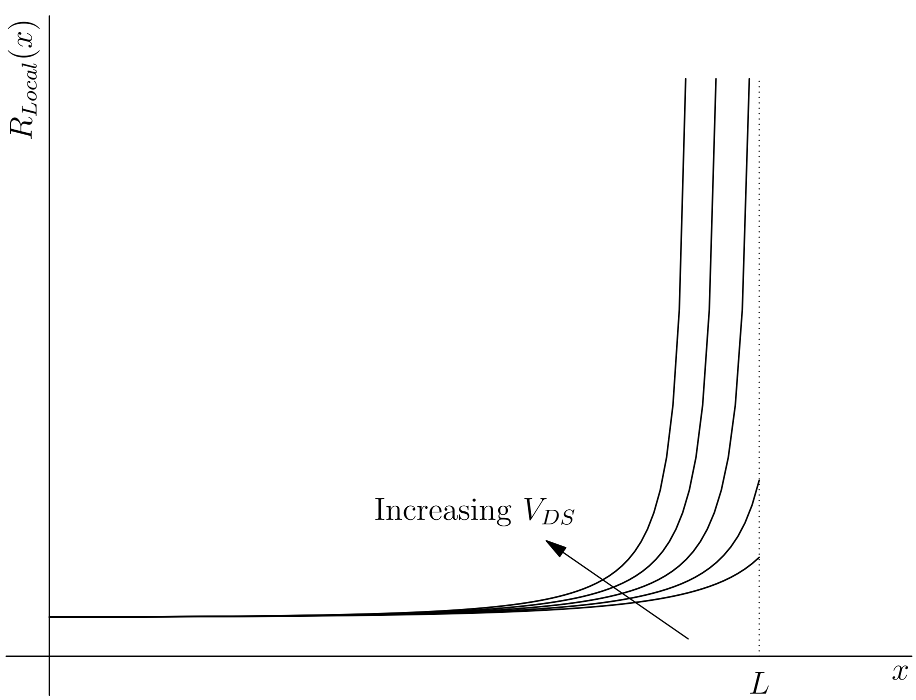
$$Q(x) = WC_{ox}(V_{GS} - V(x) - V_{TH})$$
$$= WC_{ox}(V_{GS} - V_{TH}) - WC_{ox}V(x)$$



The curve that intersects the axis at $x = L$ (i.e., the curve for which the channel begins to pinch off) corresponds to $V_{DS} = V_{GS} - V_{TH}$.

(b)

$$R_{Local}(x) \propto \frac{1}{\mu Q(x)}$$



Note that R_{Local} diverges at $x = L$ when $V_{DS} = V_{GS} - V_{TH}$.

5. $I_D = W C_{ox} [V_{GS} - V(x) - V_{TH}] \mu_n \frac{dV(x)}{dx}$

Define: $A = \frac{I_D}{W C_{ox} \mu_n}$, $B = V_{GS} - V_{TH}$

$$\Rightarrow \quad A = (B - V)\frac{dV}{dx} = \frac{d}{dx}\left(BV - \frac{V^2}{2}\right)$$

Integrating $A = \frac{d}{dx}(BV - V^2/2)$ gives:

$$Ax = BV - V^2/2 \Rightarrow V^2 - 2BV + 2Ax = 0$$

Using quadratic formula:

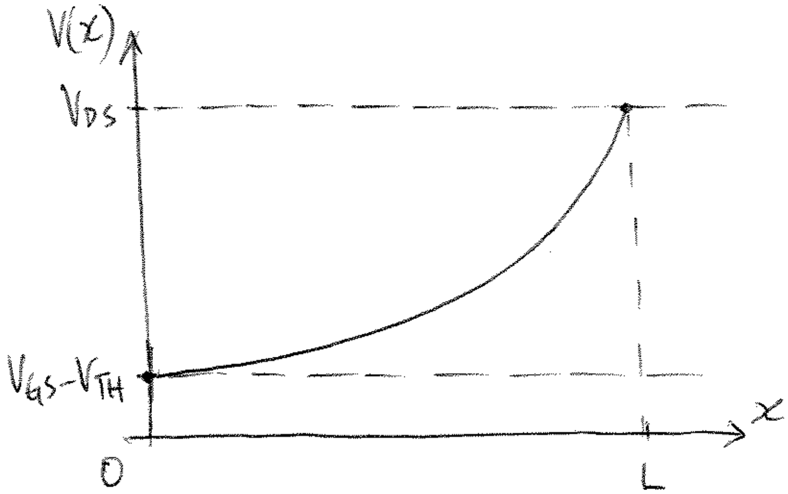
$$V_{+,-} = \frac{2B \pm \sqrt{4B^2 - 4 \cdot 2A}}{2} = B \pm \sqrt{B^2 - 2Ax}$$

$$= B\left(1 \pm \sqrt{1 - 2\left(\frac{A}{B^2}\right)x}\right)$$

$$= (V_{GS} - V_{TH})\left\{1 \pm \sqrt{1 - \left[2 \cdot \frac{I_D}{W C_{ox} \mu_n (V_{GS} - V_{TH})^2}\right] x}\right\}$$

We know that $0 \le V(x) \le V_{GS} - V_{TH}$ (pinch-off), and the term inside the square root is > 0. Therefore, we take V_- as the solution.

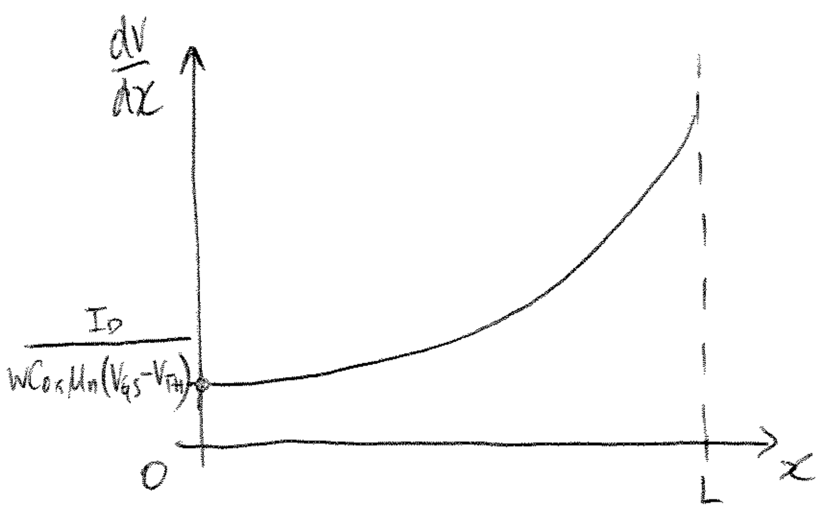
i.e. $V(x) = (V_{GS}-V_{TH})\left\{1-\sqrt{1-\left[\frac{2I_D}{WC_{ox}\mu_n(V_{GS}-V_{TH})^2}\right]x}\right\}$



∵ $I_D \propto W$

⇒ $V(x)$ is independent of W.

$$\frac{dV}{dx} = \frac{I_D}{WC_{ox}\mu_n(V_{GS}-V_{TH})}\cdot\left[1-\frac{2I_D\cdot x}{WC_{ox}\mu_n(V_{GS}-V_{TH})^2}\right]^{-\frac{1}{2}}$$



6. No.

By varying $V_{GS} - V_{TH}$ & V_{DS}, we can only obtain $\mu_n C_{ox} \frac{W}{L}$, but not $\mu_n C_{ox}$ & $\frac{W}{L}$ individually.

7. Given : NMOS $\quad I_D = 1mA \quad V_{GS} - V_{TH} = 0.6V$

$I_D = 1.6mA \quad V_{GS} - V_{TH} = 0.8V$

(triode region) $\quad \mu_n C_{ox} = 200 \frac{\mu A}{V^2}$

Find V_{DS} & W/L.

$$1\,mA = \mu_n C_{ox} \frac{W}{L} \left[(0.6) V_{DS} - V_{DS}^2/2\right] \quad \text{——} \; ①$$

$$1.6\,mA = \mu_n C_{ox} \frac{W}{L} \left[(0.8) V_{DS} - V_{DS}^2/2\right] \quad \text{——} \; ②$$

$$② \div ① : \quad 1.6 = \frac{0.8\,V_{DS} - V_{DS}^2/2}{0.6\,V_{DS} - V_{DS}^2/2} = \frac{1.6 - V_{DS}}{1.2 - V_{DS}}$$

$$\Rightarrow V_{DS} = \frac{1.6(0.2)}{0.6} \approx 0.533 \;V$$

$$\Rightarrow \quad \frac{W}{L} = \frac{I_D}{\mu_n C_{ox} \left[(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2/2\right]}$$

$$= \frac{1\,mA}{200 \frac{\mu A}{V^2} \left[(0.6V)(0.533V) - (0.533V)^2/2\right]}$$

$$\approx 28.$$

8. $I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} [2(V_{GS}-V_{TH}) V_{DS} - V_{DS}^2]$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2} \mu C_{ox} \frac{W}{L} \cdot 2 V_{DS} = \mu C_{ox} \frac{W}{L} V_{DS}$$

$$g_m |_{V_{DS}=0} = 0.$$

Intuitively, when $V_{GS} > V_{TH}$, mobile charges (channel) become available. This determines the on-resistance. But since there is no I_D ($\because V_{DS}=0$), it does not matter if there is an incremental change in V_{GS} (i.e. ∂V_{GS}). Since varying V_{GS} gives no change in I_D, $g_m|_{V_{DS}=0} = 0$.

9. Given: $V_{DD} = 1.8\ V$ $\quad \frac{W}{L} = 20 \quad \mu_n C_{ox} = 200\ \frac{\mu A}{V^2}$

$V_{TH} = 0.4\ V$

Find minimum-on resistance.

$$R_{on} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{DD} - V_{TH})}$$

$$= \frac{1}{\left(200\ \frac{\mu A}{V^2}\right)(20)\ (1.8 - 0.4)V} = 179.\ \Omega$$

10. $500 = \dfrac{1}{\mu_n C_{ox} \frac{W}{L} (1 - V_{TH})}$

$400 = \dfrac{1}{\mu_n C_{ox} \frac{W}{L} (1.5 - V_{TH})}$

For the same NMOS, $\mu_n C_{ox}$ & $\frac{W}{L}$ are fixed

$\Rightarrow 500(1 - V_{TH}) \stackrel{?}{=} 400(1.5 - V_{TH})$

$500(0.6) \neq 400(1.1)$

$\therefore$ This is not possible.

11. $I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2 \right]$

$$r_{DS,tri} \triangleq \left(\frac{\partial I_D}{\partial V_{DS}} \right)^{-1} = \left[\frac{\partial}{\partial V_{DS}} \left\{ \frac{1}{2} \mu C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2 \right] \right\} \right]^{-1}$$

$$= \left[\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) - \mu C_{ox} \frac{W}{L} V_{DS} \right]^{-1}$$

$$= \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH} - V_{DS})}$$

12. When MOS operates as a resistor,

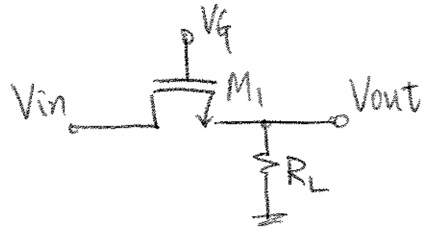
$$R_{on} = \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}$$

$$\Rightarrow \tau = R_{on} C_{GS} = \frac{W L C_{ox}}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})} = \frac{L^2}{\mu (V_{GS} - V_{TH})}$$

To minimize the time constant,

1) use minimum channel length, and
2) maximize overdrive voltage.

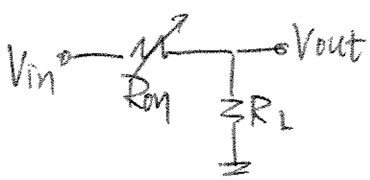
13.



Given $V_{in} \approx 0$
$V_G = 1.8\ V$
$R_L = 100\Omega$

Find $\frac{W}{L}$ such that signal output attenuates by only 5%.

$V_{in} \approx 0$ implies that we can approximate M_1 as a linear resistance controlled by V_G. Therefore, the equivalent circuit becomes a resistive divider:



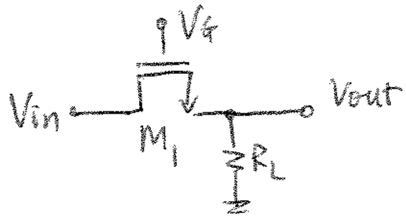
$$V_{out} = 0.95\, V_{in} = \frac{R_L}{R_{on} + R_L} V_{in}$$

$$\Rightarrow R_{on} \cong 5.3\,\Omega$$

$$\therefore \frac{W}{L} = \frac{1}{\mu C_{ox}(V_{GS} - V_{TH}) R_{on}} \approx \frac{1}{200\frac{\mu A}{V^2}(1.8-0.4)(5.3\Omega)}$$

$$= 674.$$

14.



$V_0 \sim$ few mV.

(a) $V_{in} = V_0 \cos\omega t$ $\qquad$ $V_{out} = 0.95 (V_0 \cos\omega t)$

$$V_{out} = \frac{R_L}{R_{ON} + R_L} V_{in} \quad \Rightarrow \frac{R_L}{R_{ON} + R_L} = 0.95 V_0$$

$$R_{ON} = \frac{R_L}{\left(\frac{0.95V_0}{1-0.95V_0}\right)} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_G - V_{TH})}$$

$$\therefore \frac{W}{L} = \frac{0.95V_0/(1-0.95V_0)}{\mu_n C_{ox} R_L (V_G - V_{TH})}$$

(b) $V_{out} = 0.95\ V_{in} = 0.95 (V_0 \cos\omega t + 0.5)$

$\approx 0.95 \times 0.5 = 0.475$

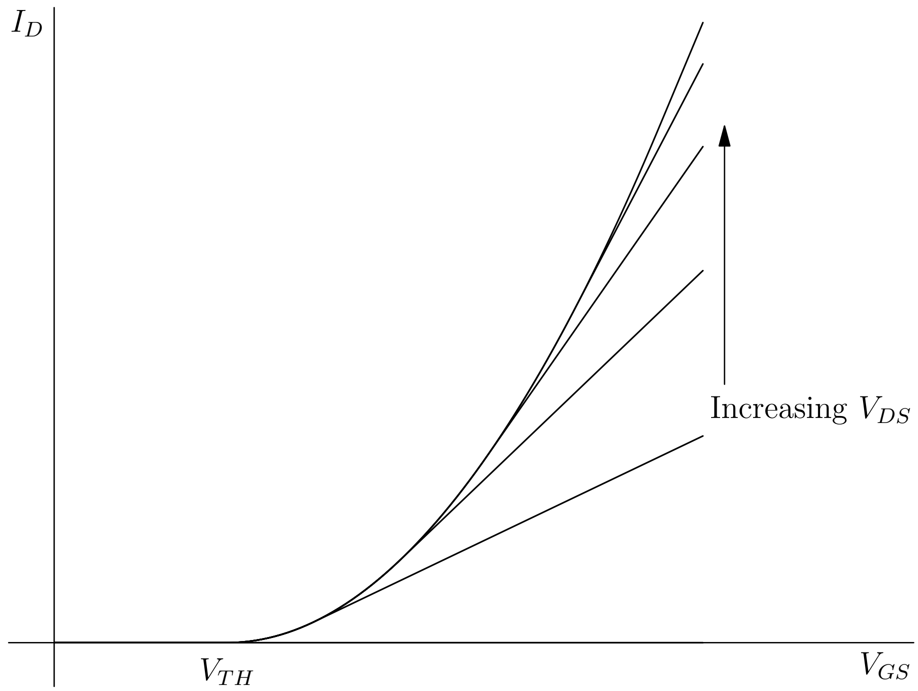
($\because V_0$ is relatively small)

$$\therefore R_{ON} = \frac{R_L}{0.9} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_G - V_{TH})}$$

$$\Rightarrow \frac{W}{L} = \frac{0.9}{\mu_n C_{ox} R_L (V_G - V_{TH})}$$

Results show that if there is no DC voltage as input, the R_{ON} varies with changing sinewave. With a DC bias voltage, R_{ON} becomes more stable (independent of V_o).

6.15



Initially, when V_{GS} is small, the transistor is in cutoff and no current flows. Once V_{GS} increases beyond V_{TH}, the curves start following the square-law characteristic as the transistor enters saturation. However, once V_{GS} increases past $V_{DS} + V_{TH}$ (i.e., when $V_{DS} < V_{GS} - V_{TH}$), the transistor goes into triode and the curves become linear. As we increase V_{DS}, the transistor stays in saturation up to larger values of V_{GS}, as expected.

16. The peak of the parabola signifies pinch-off (i.e. $V_{DS} = V_{GS} - V_{TH}$). This means that (with $\lambda = 0$) I_D cannot be increased further by increasing V_{DS}. Since this curve must be continuous, the peak I_D must originate from the peak of the parabola.

6.17

$$
\begin{aligned}
I_D &= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS}-V_{TH}\right)^{\alpha},\ \alpha < 2 \\
g_m &\triangleq \frac{\partial I_D}{\partial V_{GS}} \\
&= \frac{\alpha}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS}-V_{TH}\right)^{\alpha-1} \\
&= \boxed{\frac{\alpha I_D}{V_{GS}-V_{TH}}}
\end{aligned}
$$

18. $I_D = W C_{ox} (V_{GS} - V_{TH}) v_{SAT}$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = W C_{ox} v_{SAT}$$

19. (a) OFF $\because$ $V_{GS} = 0$

(b) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(c) TRIODE (LINEAR) $\because$ $V_{GS} > V_{TH}$ &
$V_{DS} \ll 2(V_{GS} - V_{TH})$

(d) TRIODE $\because$ $V_{GS} > V_{TH}$ & $V_{DS} < V_{GS} - V_{TH}$
(REMEMBER : MOSFET is symmetric)

(e) TRIODE $\because$ $V_{GS} > V_{TH}$ & $V_{DS} < V_{GS} - V_{TH}$

(f) OFF $\because$ $V_{GS} = 0$

(g) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(h) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(i) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

20. (a) OFF $\because$ $V_{GS} = 0$ ($V_{GS} < V_{TH}$)

(b) OFF $\because$ $V_{GS} = 0$ ($V_{GS} < V_{TH}$)

(c) TRIODE (LINEAR) $\because$ $V_{GS} > V_{TH}$ &

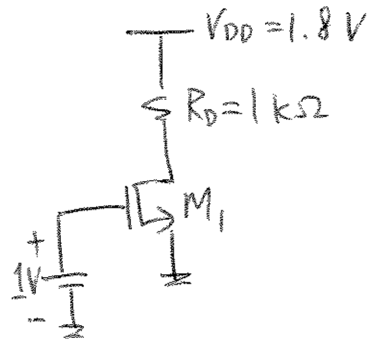
$V_{DS} \ll 2(V_{GS} - V_{TH})$

(d) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

6.21 Since they're being used as current sources, assume M_1 and M_2 are in saturation for this problem. To find the maximum allowable value of λ, we should evaluate λ when $0.99 I_{D2} = I_{D1}$ and $1.01 I_{D2} = I_{D1}$, i.e., at the limits of the allowable values for the currents. However, note that for any valid λ (remember, λ should be non-negative), we know that $I_{D2} > I_{D1}$ (since $V_{DS2} > V_{DS1}$), so the case where $1.01 I_{D2} = I_{D1}$ (which implies $I_{D2} < I_{D1}$) will produce an invalid value for λ (you can check this yourself). Thus, we need only consider the case when $0.99 I_{D2} = I_{D1}$.

$$\begin{aligned}
0.99 I_{D2} &= 0.99 \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_B - V_{TH})^2 (1 + \lambda V_{DS2}) \\
&= I_{D1} \\
&= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_B - V_{TH})^2 (1 + \lambda V_{DS1}) \\
0.99 (1 + \lambda V_{DS2}) &= 1 + \lambda V_{DS1} \\
\lambda &= \boxed{0.02 \text{ V}^{-1}}
\end{aligned}$$

22.



$\lambda = 0$, $V_{TH} = 0.4\,V$

$\mu_n C_{ox} = 200\,\frac{\mu A}{V^2}$

M_1 sits at the edge of saturation when $V_{DS} = V_{GS} - V_{TH}$.

$\Rightarrow V_{DS,edge} = (1 - 0.4)\,V = 0.6\,V$

By KCL, $I_{D_1} = I_{R_D} = \frac{V_{DD} - V_{DS}}{R_D} = \frac{1.2\,V}{1\,k\Omega} = 1.2\,mA$

$\therefore I_{D_1} = 1.2\,mA = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$

$$\Rightarrow \frac{W}{L} = \frac{2 I_{D_1}}{\mu_n C_{ox}(V_{GS} - V_{TH})^2} = \frac{2(1.2\,mA)}{\left(200\,\frac{\mu A}{V^2}\right)\left(1 - 0.4\right)^2 V^2}$$

$\cong 33.$

23. If gate oxide thickness, t_{ox}, doubles, the corresponding capacitance, $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$, is halved.

$\Rightarrow$ $\mu_n C_{ox}$ is also halved

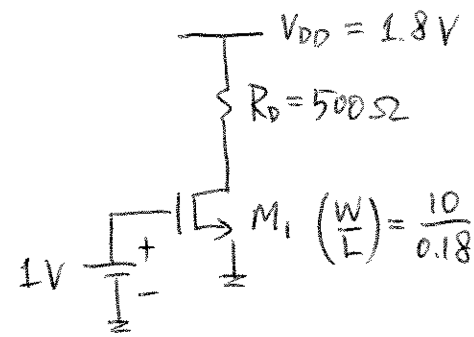
$\Rightarrow$ I_{D_1} is halved $\Rightarrow$ V_{DS} increases

$\Rightarrow$ M_1 stays in saturation ($V_{DS} > V_{GS} - V_{TH}$)

$$I_{D_1} = \frac{1.2\,mA}{2} = 0.6\,mA$$

$$\Rightarrow V_{DS} = (1.8\,V) - (0.6\,mA)(1\,k\Omega) = 1.2\,V$$

24.



$\lambda = 0$

To avoid triode region, $V_{DS} \geq V_{GS} - V_{TH}$.

$\Rightarrow V_{DS} \geq 1 - 0.4 = 0.6\,V$

$$\Rightarrow I_{D_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$$

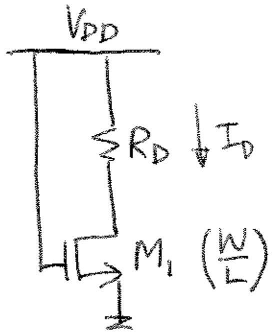
$$= \frac{1}{2}\left(200\,\frac{\mu A}{V^2}\right)\left(\frac{10}{0.18}\right)(0.6)^2 = 2\,mA$$

By KCL, $\frac{V_{DD} - V_{DS}}{R_D} = 2mA$

$\therefore V_{DD} = (2mA)(500\Omega) + 0.6V = 1.6\,V$

Minimum $V_{DD} = 1.6\,V$

25.



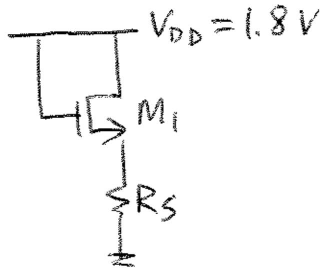
$\lambda = 0$

When M_1 operates at the edge of saturation, $V_{DS} = V_{GS} - V_{TH}$. Also, by KCL:

$$I_{R_D} = I_{D_1} \Rightarrow \frac{V_{DD} - (V_{DD} - V_{TH})}{R_D} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{DD} - V_{TH})^2$$

$$\therefore V_{TH} = R_D \cdot \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{DD} - V_{TH})^2}_{I_D}$$

26.



$\lambda = 0$

Find $\left(\frac{W}{L}\right)$ with bias current $= I_1$.

Since $V_{DS} = V_{GS}$ for M_1, this device always operates in saturation region (given $V_{GS} > V_{TH}$).

By KCL, $I_1 = I_{R_S}$; by Ohm's law, $V_S = I_1 R_S$

$$\Rightarrow \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{DD} - I_1 R_S - V_{TH}\right)^2 = I_1$$

$$\therefore \frac{W}{L} = \frac{2I_1}{\mu_n C_{ox}\left(V_{DD} - I_1 R_S - V_{TH}\right)^2}$$

5.27

$$V_{DD} - I_D R_D = V_{GS} = V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}}$$

$$\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}} = (V_{DD} - V_{TH} - I_D R_D)^2$$

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \left[(V_{DD} - V_{TH})^2 - 2 I_D R_D (V_{DD} - V_{TH}) + I_D^2 R_D^2 \right]$$

We can rearrange this to the standard quadratic form as follows:

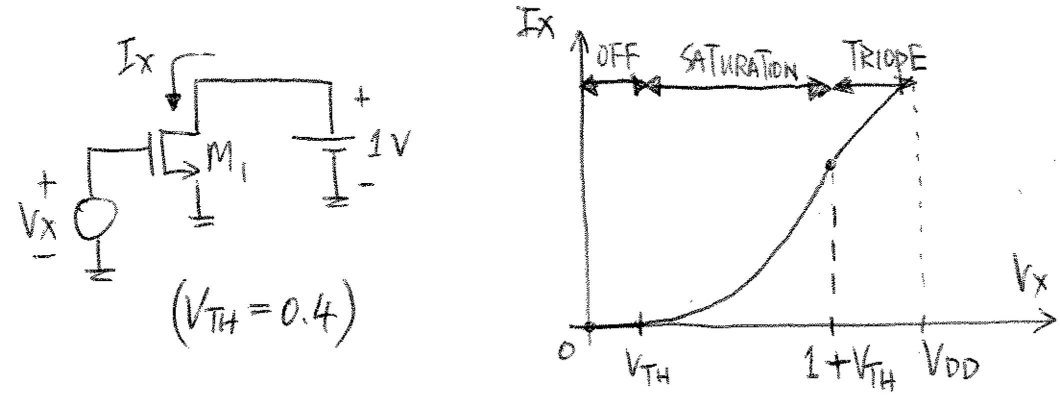
$$\left(\frac{1}{2}\mu_n C_{ox} \frac{W}{L} R_D^2\right) I_D^2 - \left(\mu_n C_{ox} \frac{W}{L} R_D (V_{DD} - V_{TH}) + 1\right) I_D + \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{DD} - V_{TH})^2 = 0$$

Applying the quadratic formula, we have:

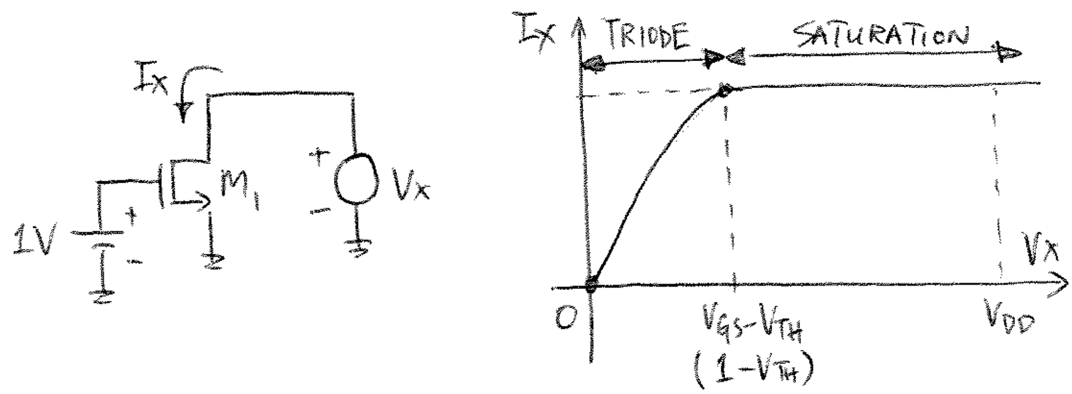
$$\begin{aligned}
I_D &= \frac{\left(\mu_n C_{ox} \frac{W}{L} R_D (V_{DD} - V_{TH}) + 1\right) \pm \sqrt{\left(\mu_n C_{ox} \frac{W}{L} R_D (V_{DD} - V_{TH}) + 1\right)^2 - 4\left(\frac{1}{2}\mu_n C_{ox} \frac{W}{L} R_D (V_{DD} - V_{TH})\right)^2}}{2\left(\frac{1}{2}\mu_n C_{ox} \frac{W}{L} R_D^2\right)} \\
&= \frac{\mu_n C_{ox} \frac{W}{L} R_D (V_{DD} - V_{TH}) + 1 \pm \sqrt{\left(\mu_n C_{ox} \frac{W}{L} R_D (V_{DD} - V_{TH}) + 1\right)^2 - \left(\mu_n C_{ox} \frac{W}{L} R_D (V_{DD} - V_{TH})\right)^2}}{\mu_n C_{ox} \frac{W}{L} R_D^2} \\
&= \boxed{\frac{\mu_n C_{ox} \frac{W}{L} R_D (V_{DD} - V_{TH}) + 1 \pm \sqrt{1 + 2\mu_n C_{ox} \frac{W}{L} R_D (V_{DD} - V_{TH})}}{\mu_n C_{ox} \frac{W}{L} R_D^2}}
\end{aligned}$$

Note that mathematically, there are two possible solutions for I_D. However, since M_1 is diode-connected, we know it will either be in saturation or cutoff. Thus, we must reject the value of I_D that does not match these conditions (for example, a negative value of I_D would not match cutoff or saturation, so it would be rejected in favor of a positive value).

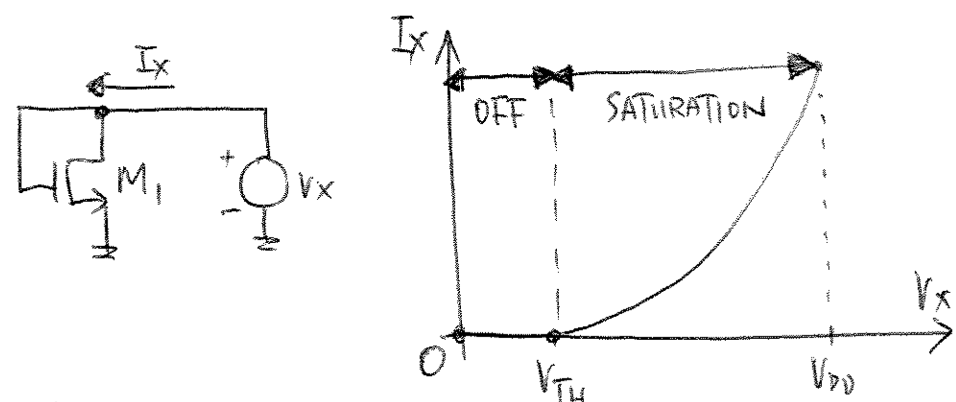
28. (a)


Ix
M1
1V
Vx
(VTH = 0.4)
OFF
SATURATION
TRIODE
0
VTH
1+VTH
VDD

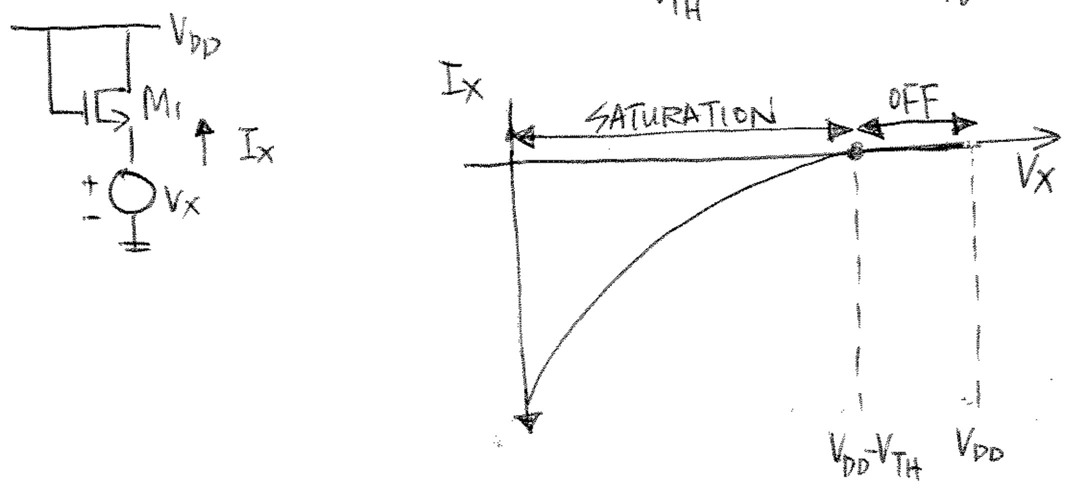
(b)


Ix
M1
1V
Vx
TRIODE
SATURATION
0
VGS-VTH
(1-VTH)
VDD

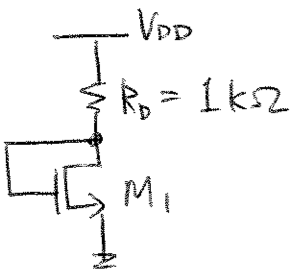
(c)


Ix
M1
Vx
OFF
SATURATION
0
VTH
VDD

(d)


VDD
M1
Ix
Vx
SATURATION
OFF
VDD-VTH
VDD

29.



$\left(\frac{W}{L}\right) = \frac{10}{0.18}$, $\lambda = 0.1\ V^{-1}$

Find I_{D1}

Since M_1 is diode-connected, it operates in saturation.

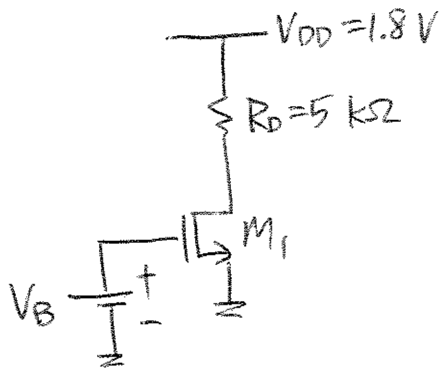
By KCL, $\frac{V_{DD} - V_G}{R_D} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_G - V_{TH})^2(1+\lambda V_G)$

One can solve this by (1) using a graphing calculator, (2) trial-and-error, (3) or iteratively finding V_G.

Using any method gives $V_G \approx 0.807\ V$

$\Rightarrow I_D = \frac{V_{DD} - V_G}{R_D} \approx 1\ mA$

30.



$\frac{W}{L} = \frac{20}{0.18}$, $\lambda = 0.1\ V^{-1}$

At the edge of saturation,

$$I_{D_1} = \frac{V_{DD} - (V_B - V_{TH})}{R_D} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_B - V_{TH})^2(1 + \lambda(V_B - V_{TH}))$$

This equation can be solved by using a graphing calculator, special programs, or iteratively.

Using any method gives $V_B \approx 0.57\ V$
$(I_D \approx 0.33\,mA)$

31. An NMOS device with $\lambda = 0$ must provide a transconductance of $1/50\ \frac{1}{\Omega}$.

(a) Given $I_D = 0.5\ mA$, find W/L.

$$g_m = \frac{1}{50} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\Rightarrow \frac{W}{L} = \frac{g_m^2}{2 \mu_n C_{ox} I_D} = \frac{\left(1/50\ \frac{1}{\Omega}\right)^2}{2\left(200\ \frac{\mu A}{V^2}\right)(0.5 mA)} \approx 2000$$

(b) Given $V_{GS} - V_{TH} = 0.5V$, find W/L.

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$\Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox}(V_{GS} - V_{TH})} = \frac{\left(1/50\ \frac{1}{\Omega}\right)}{\left(200\ \frac{\mu A}{V^2}\right)(0.5\ V)} \approx 200$$

(c) Given $V_{GS} - V_{TH} = 0.5V$, find I_D.

$$\Rightarrow I_D = \frac{g_m (V_{GS} - V_{TH})}{2} = \frac{\left(1/50\ \frac{1}{\Omega}\right)(0.5V)}{2} \approx 5 mA$$

32. (a) $g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$ (I_D constant)

Doubling (W/L) implies a $\sqrt{2}$ times increase in g_m: $g_{m_{NEW}} = \sqrt{2\mu_n C_{ox}\left(2\frac{W}{L}\right) I_D} = \sqrt{2}\, g_m$.

(b) $g_m = \frac{2I_D}{V_{GS}-V_{TH}}$ (I_D constant)

Doubling $(V_{GS}-V_{TH})$ decreases g_m by half:

$$g_{m_{NEW}} = \frac{2I_D}{2(V_{GS}-V_{TH})} = \frac{1}{2} g_m$$

(c) $g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$ (W/L constant)

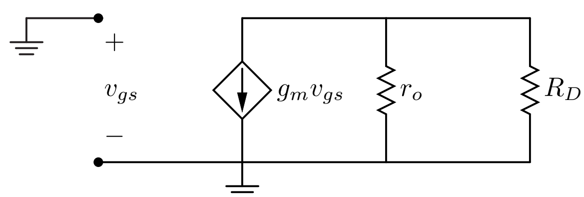
Doubling I_D increases g_m by $\sqrt{2}$ times.

(d) $g_m = \frac{2I_D}{V_{GS}-V_{TH}}$ ($V_{GS}-V_{TH}$ constant)

Doubling I_D increases g_m by 2 times.

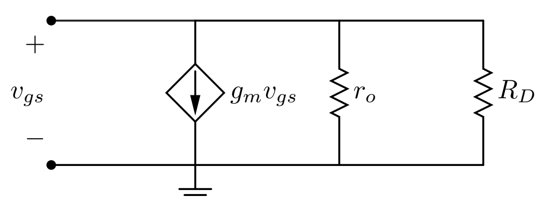
6.33 (a) Assume M_1 is operating in saturation.

$$V_{GS} = 1\text{ V}$$
$$V_{DS} = V_{DD} - I_D R_D = V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2 (1 + \lambda V_{DS}) R_D$$
$$V_{DS} = 1.35\text{ V} > V_{GS} - V_{TH}\text{, which verifies our assumption}$$
$$I_D = 4.54\text{ mA}$$
$$g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH}) = \boxed{13.333\text{ mS}}$$
$$r_o = \frac{1}{\lambda I_D} = \boxed{2.203\text{ k}\Omega}$$



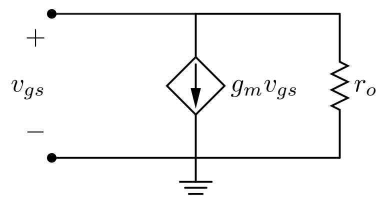
(b) Since M_1 is diode-connected, we know it is operating in saturation.

$$V_{GS} = V_{DS} = V_{DD} - I_D R_D = V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2 (1 + \lambda V_{GS}) R_D$$
$$V_{GS} = V_{DS} = 0.546\text{ V}$$
$$I_D = 251\ \mu\text{A}$$
$$g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH}) = \boxed{3.251\text{ mS}}$$
$$r_o = \frac{1}{\lambda I_D} = \boxed{39.881\text{ k}\Omega}$$



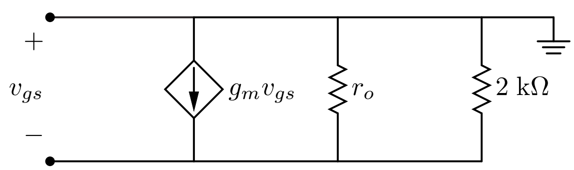
(c) Since M_1 is diode-connected, we know it is operating in saturation.

$$I_D = 1\text{ mA}$$
$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D} = \boxed{6.667\text{ mS}}$$
$$r_o = \frac{1}{\lambda I_D} = \boxed{10\text{ k}\Omega}$$



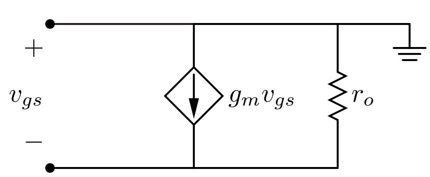
(d) Since M_1 is diode-connected, we know it is operating in saturation.

$$V_{GS} = V_{DS}$$
$$V_{DD} - V_{GS} = I_D(2\text{ k}\Omega) = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2\left(1 + \lambda V_{GS}\right)(2\text{ k}\Omega)$$
$$V_{GS} = V_{DS} = 0.623\text{ V}$$
$$I_D = 588\ \mu\text{A}$$
$$g_m = \mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right) = \boxed{4.961\text{ mS}}$$
$$r_o = \frac{1}{\lambda I_D} = \boxed{16.996\text{ k}\Omega}$$



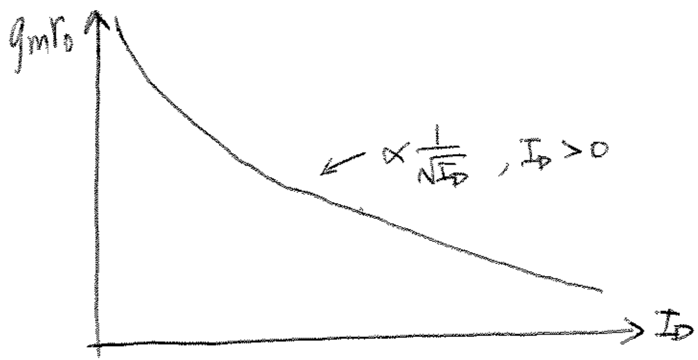
(e) Since M_1 is diode-connected, we know it is operating in saturation.

$$I_D = 0.5\text{ mA}$$
$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \boxed{4.714\text{ mS}}$$
$$r_o = \frac{1}{\lambda I_D} = \boxed{20\text{ k}\Omega}$$



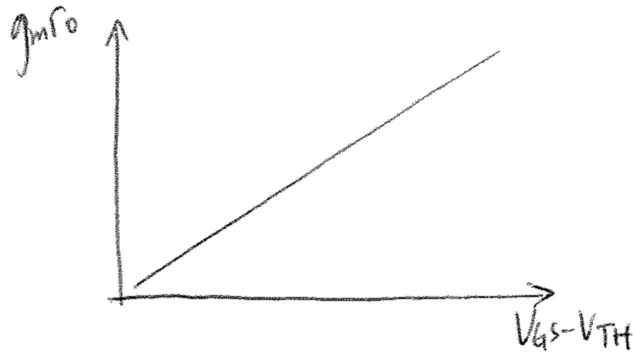
34. $g_m = \sqrt{2\mu C_{ox} \frac{W}{L} I_D}$ $r_o = \left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1} = \frac{1}{\lambda I_D}$

$$g_m r_o = \frac{\sqrt{2\mu C_{ox}(W/L) I_D}}{\lambda I_D} = \frac{1}{\lambda}\sqrt{\frac{2\mu C_{ox}(W/L)}{I_D}}$$

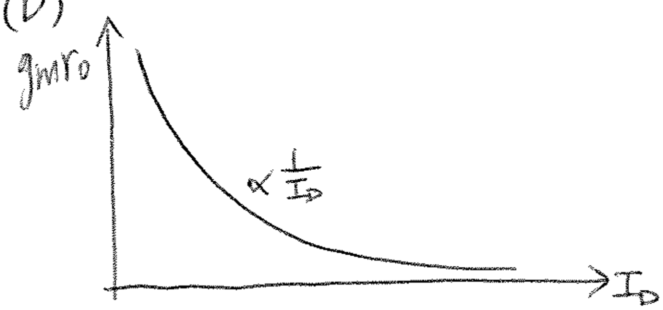


35 (a) $g_m = \mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$ $r_o = \frac{1}{\lambda I_D}$

$$g_m r_o = \frac{\mu C_{ox} (W/L)(V_{GS} - V_{TH})}{\lambda I_D}$$



(b)



36. Given NMOS with $\lambda = 0.1\ V^{-1}$ $g_m r_o = 20$, $V_{DS} = 1.5\ V$

determine W/L if $I_D = 0.5\ mA$.

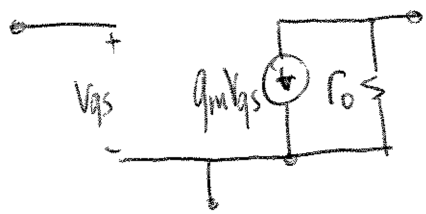
$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\ V^{-1})(0.5\ mA)} = 20\ k\Omega$$

$$\Rightarrow g_m = \frac{20}{20\ k\Omega} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\therefore \frac{W}{L} = \left(\frac{20}{20\ k\Omega}\right)^2 \frac{1}{2 \mu_n C_{ox} I_D}$$

$$= \left(\frac{1}{1\ k\Omega}\right)^2 \frac{1}{2\left(200 \frac{\mu A}{V^2}\right)(0.5\ mA)} \approx 5.$$

37.



Given $\lambda = 0.2\ V^{-1}$

$g_m r_o = 20$

$V_{DS} = 1.5\ V$

$I_D = 0.5\ mA$

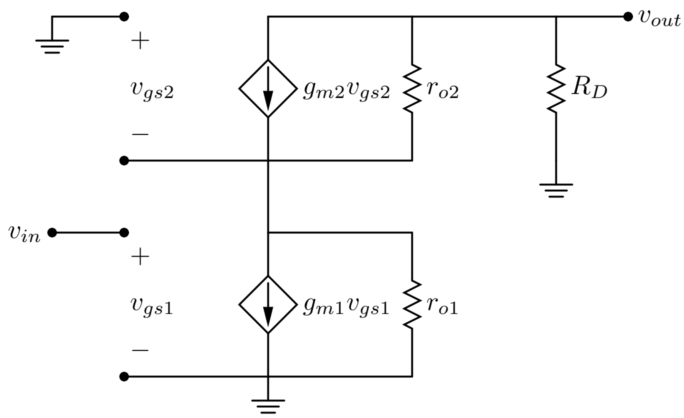
Calculate $\frac{W}{L}$.

$$g_m = \frac{20}{r_o} = 20 \cdot \lambda I_D = 20(0.2V^{-1})(0.5\,mA) = 0.002\ ^1/_\Omega$$

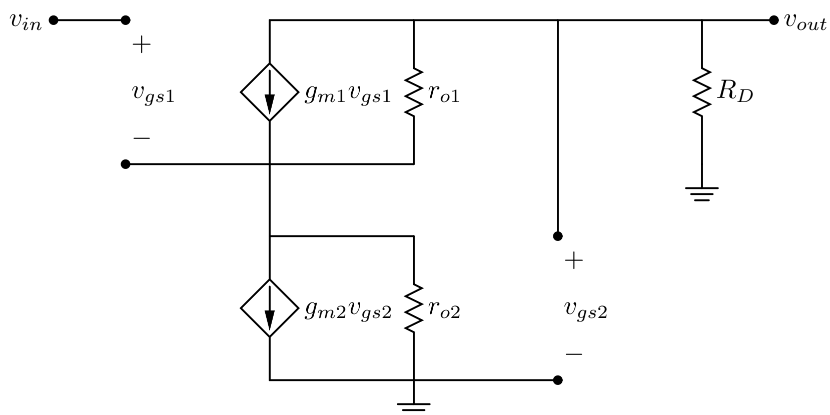
$$\Rightarrow g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\therefore \frac{W}{L} = \frac{g_m^2}{2 \mu_n C_{ox} I_D} = \frac{(0.0002\ ^1/_\Omega)^2}{2\left(200 \frac{\mu A}{V^2}\right)(0.5\,mA)} = 20$$

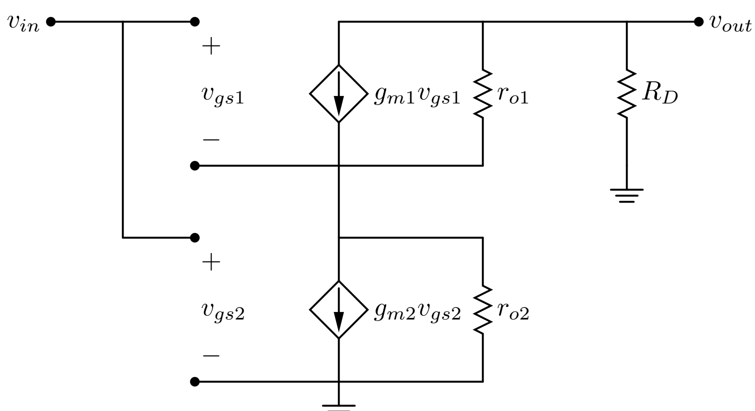
6.38 (a)


v_{out}
$+$
v_{gs2}
$-$
$g_{m2}v_{gs2}$
r_{o2}
R_D
v_{in}
$+$
v_{gs1}
$-$
$g_{m1}v_{gs1}$
r_{o1}

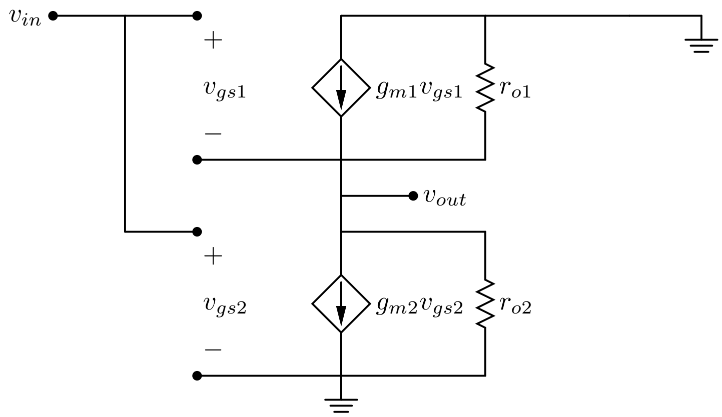
(b)


v_{in}
v_{out}
$+$
v_{gs1}
$-$
$g_{m1}v_{gs1}$
r_{o1}
R_D
$g_{m2}v_{gs2}$
r_{o2}
$+$
v_{gs2}
$-$

(c)

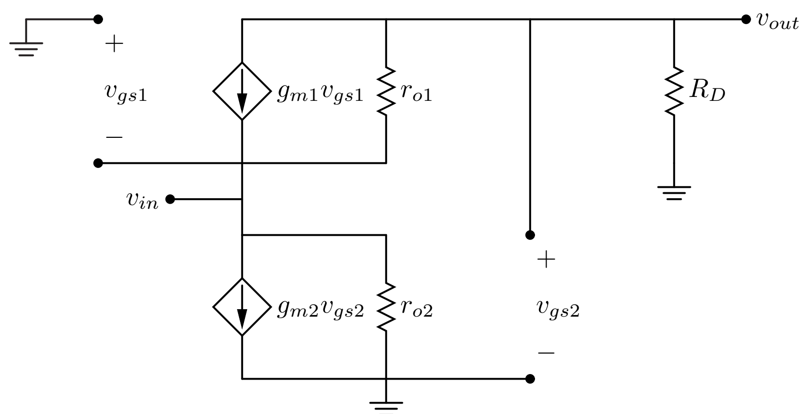

v_{in}
v_{out}
$+$
v_{gs1}
$-$
$g_{m1}v_{gs1}$
r_{o1}
R_D
$+$
v_{gs2}
$-$
$g_{m2}v_{gs2}$
r_{o2}

(d)



v_{in}
+
v_{gs1}
−
$g_{m1}v_{gs1}$
r_{o1}
v_{out}
+
v_{gs2}
−
$g_{m2}v_{gs2}$
r_{o2}

(e)



+
v_{gs1}
−
$g_{m1}v_{gs1}$
r_{o1}
v_{out}
R_D
v_{in}
+
v_{gs2}
−
$g_{m2}v_{gs2}$
r_{o2}

39. (a) OFF $\because |V_{SG}| = 0$

(b) OFF $\because |V_{SG}| < |V_{TH}| = 0.4\,V$

(c) SATURATION $\because |V_{SD}| > |V_{SG}| - |V_{TH}|$

(d) OFF $\because V_{SG} < |V_{TH}|$

40. (a) SATURATION $\because$ $V_{SD} > V_{SG} - |V_{TH}|$

(b) LINEAR (RESISTIVE) $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} \ll 2(V_{SG} - |V_{TH}|)$

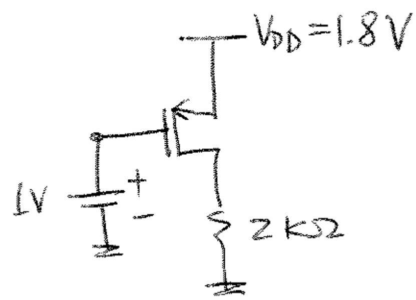
(c) (EDGE OF) SATURATION $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} = V_{SG} - |V_{TH}|$

(d) TRIODE $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} < V_{SG} - |V_{TH}|$

41.



$\lambda = 0$

At the edge of saturation, $V_{SD} = V_{SG} - |V_{TH}|$

$\Rightarrow V_D = 1.4\ V.$

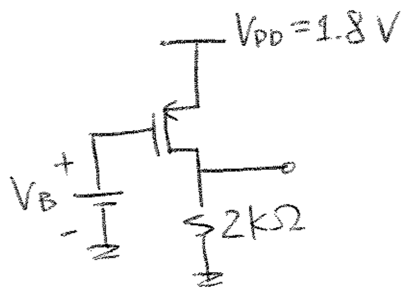
By KCL, $I_{D_1} = I_R$

$$\Rightarrow \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG} - |V_{TH}|)^2 = \frac{V_D}{2k\Omega}$$

$$\therefore \frac{W}{L} = \frac{V_D}{2k\Omega}\cdot\frac{2}{\mu_p C_{ox}(V_{SG} - |V_{TH}|)^2}$$

$$= \frac{1.4V}{2k\Omega}\cdot\frac{2}{100\frac{\mu A}{V^2}(0.8V - 0.4V)^2} \approx 87.5$$

42.



$\lambda = 0$

When $V_B = 1V$, $W/L = 87.5$

When $V_B = 0.8V$,

$$I_D = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG} - |V_{TH}|)^2$$

$$= \frac{1}{2}\left(100\frac{\mu A}{V^2}\right)(87.5)(1-0.4)^2 V^2 \approx 1.6\,mA$$

$\Rightarrow V_D = I_D(2k\Omega) \approx 3.2V$, which exceeds the supply voltage!

$\therefore$ PMOS goes into triode:
($\because I_D$ is too large)

By KCL,

$$\frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[(V_{SG} - |V_{TH}|)\cdot 2V_{SD} - V_{SD}^2\right] = (V_{DD} - V_{SD})/2k\Omega$$

Solving this equation numerically (or trial-and-error) gives $V_{SD} \approx 0.18\ V$

$$\Rightarrow I_D = \frac{V_{DD} - V_{SD}}{2k\Omega} = \frac{(1.8 - 0.18)V}{2k\Omega} \approx 0.81\ mA$$

6.43 (a) Assume M_1 is operating in triode (since $|V_{GS}| = 1.8$ V is large).

$$|V_{GS}| = \boxed{1.8 \text{ V}}$$
$$V_{DD} - |V_{DS}| = |I_D|\,(500\ \Omega) = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[2\left(|V_{GS}| - |V_{TH}|\right)|V_{DS}| - |V_{DS}|^2\right](500\ \Omega)$$
$$|V_{DS}| = \boxed{0.418 \text{ V}} < |V_{GS}| - |V_{TH}|, \text{ which verifies our assumption}$$
$$|I_D| = \boxed{2.764 \text{ mA}}$$

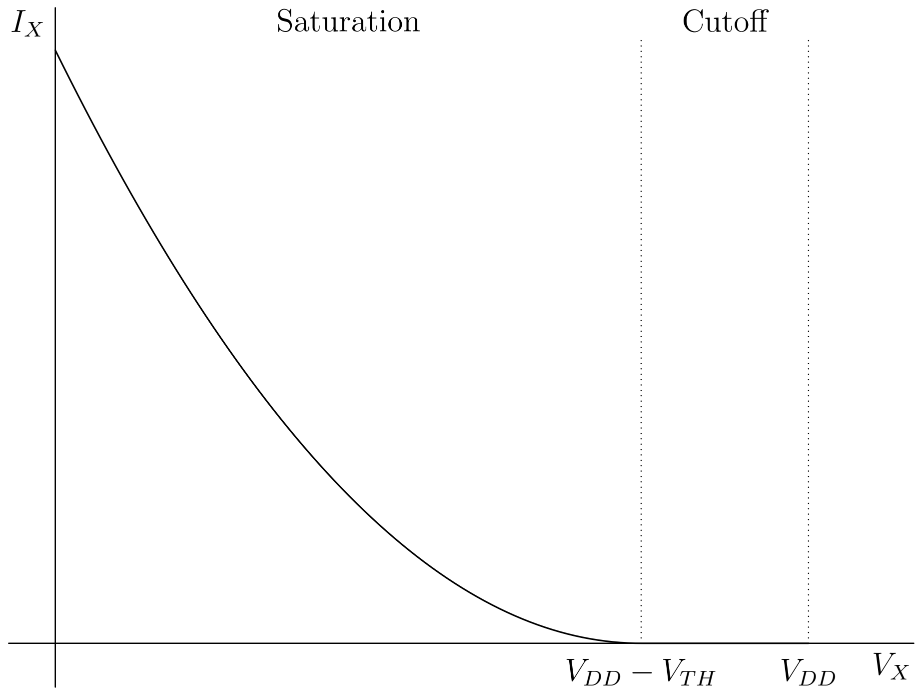
(b) Since M_1 is diode-connected, we know it is operating in saturation.

$$|V_{GS}| = |V_{DS}|$$
$$V_{DD} - |V_{GS}| = |I_D|(1\ \text{k}\Omega) = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(|V_{GS}| - |V_{TH}|\right)^2 (1\ \text{k}\Omega)$$
$$|V_{GS}| = |V_{DS}| = \boxed{0.952 \text{ V}}$$
$$|I_D| = \boxed{848\ \mu\text{A}}$$

(c) Since M_1 is diode-connected, we know it is operating in saturation.

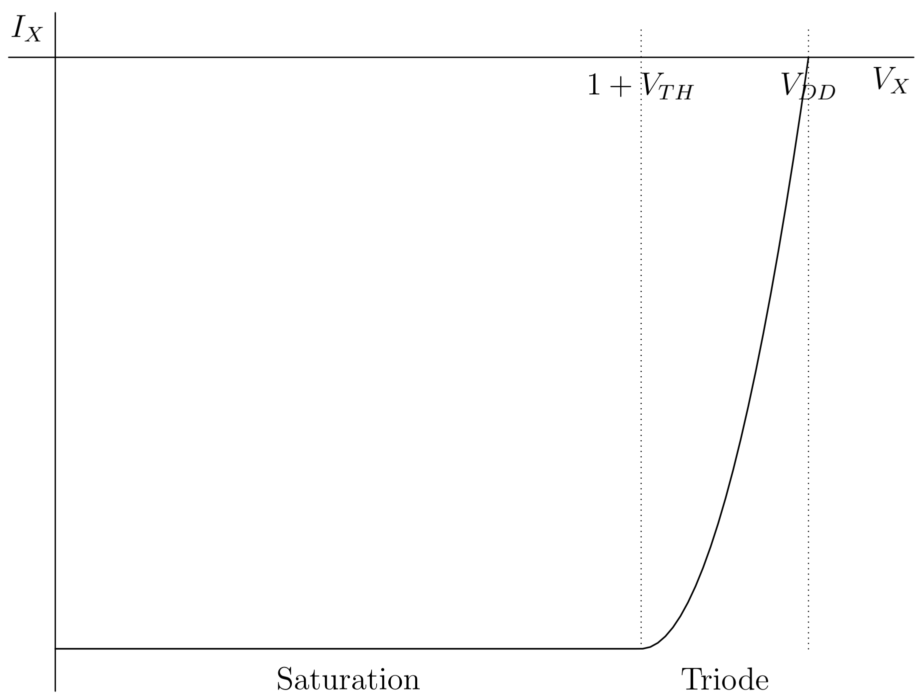
$$|V_{GS}| = |V_{DS}|$$
$$|V_{GS}| = V_{DD} - |I_D|(1\ \text{k}\Omega) = V_{DD} - |I_D|(1\ \text{k}\Omega) = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(|V_{GS}| - |V_{TH}|\right)^2 (1\ \text{k}\Omega)$$
$$|V_{GS}| = |V_{GS}| = \boxed{0.952 \text{ V}}$$
$$|I_D| = \boxed{848\ \mu\text{A}}$$

6.44 (a)



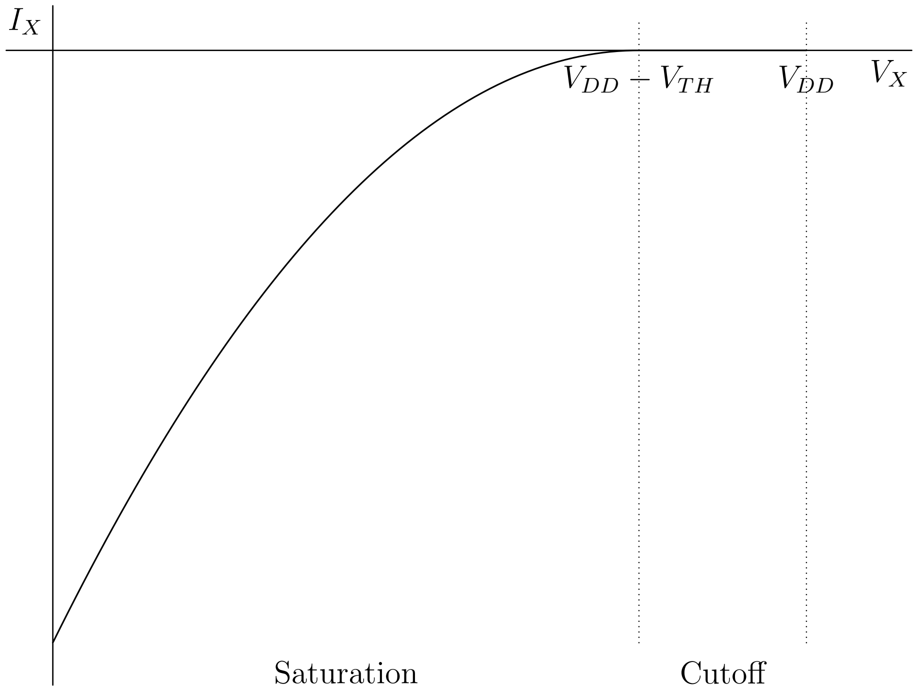
M_1 goes from saturation to cutoff when $V_X = V_{DD} - V_{TH} = 1.4$ V.

(b)



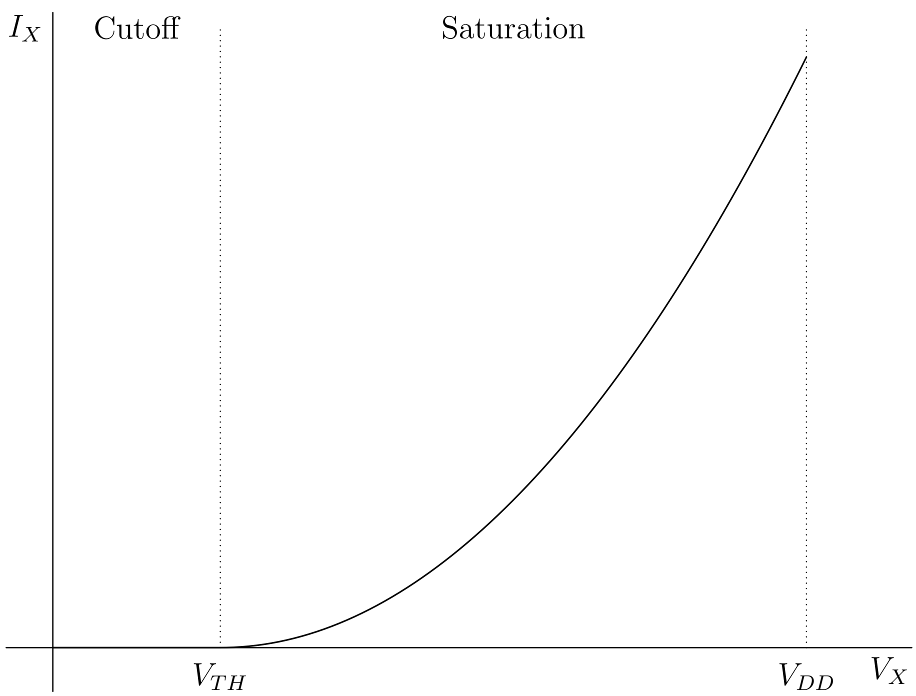
M_1 goes from saturation to triode when $V_X = 1 + V_{TH} = 1.4$ V.

(c)



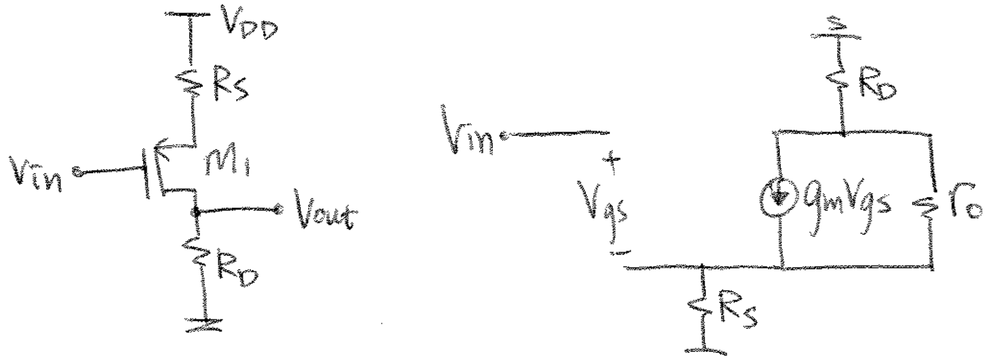
M_1 goes from saturation to cutoff when $V_X = V_{DD} - V_{TH} = 1.4$ V.

(d)



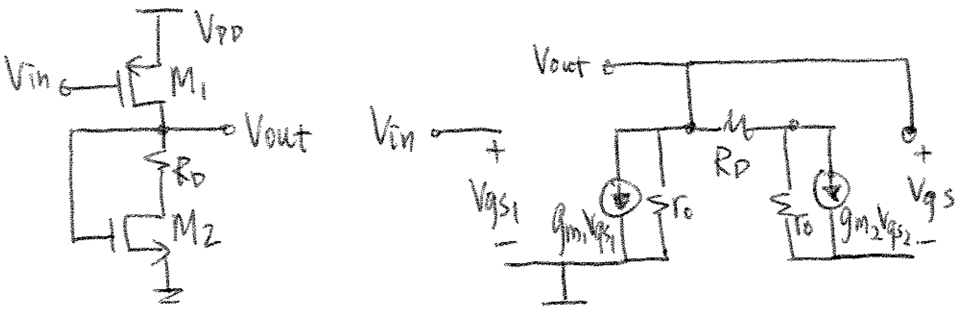
M_1 goes from cutoff to saturation when $V_X = V_{TH} = 0.4$ V.

45. (a)



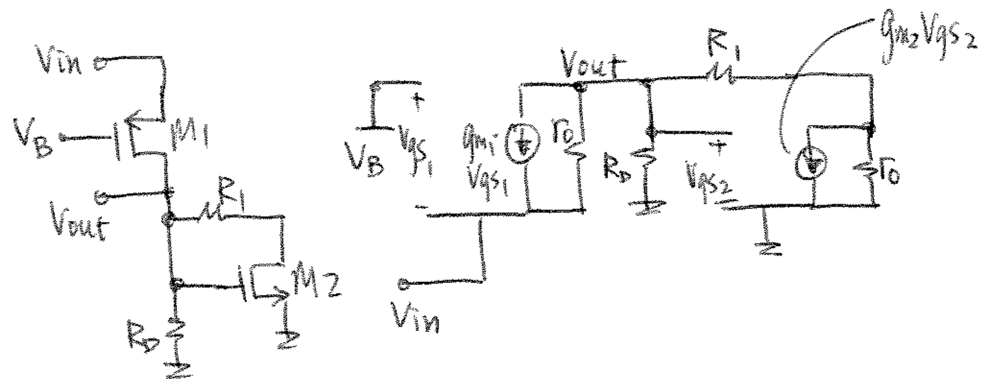
VDD
RS
Vin
M1
Vout
RD
Vin
+
Vgs
−
RS
RD
gmVgs
ro

(b)



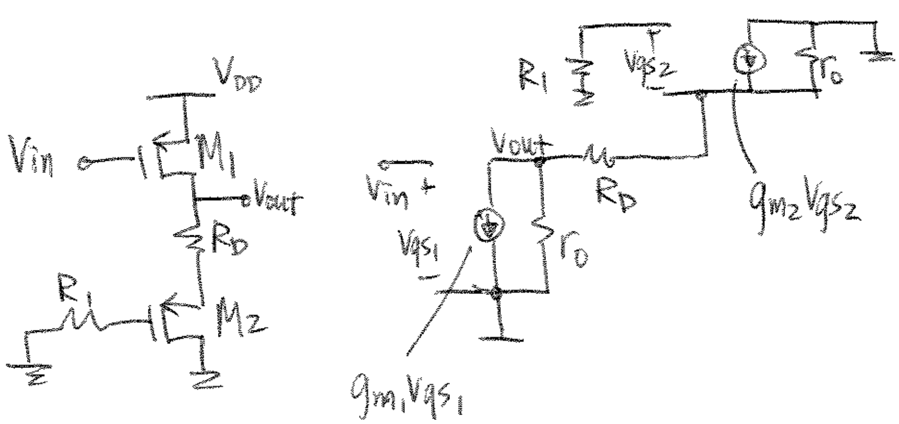
VDD
Vin
M1
Vout
RD
M2
Vout
Vin
+
Vgs1
−
gm1Vgs1
ro
RD
ro
gm2Vgs2
+
Vgs
−

(c)



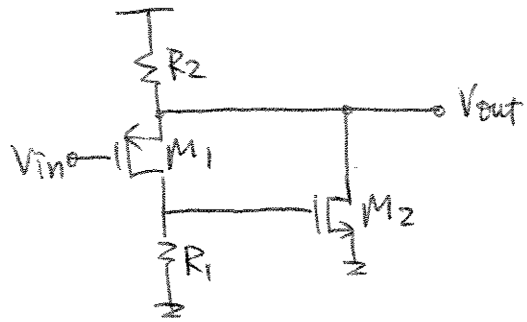
Vin
VB
M1
Vout
R1
M2
RD
Vout
R1
gm2Vgs2
+
VB
Vgs1
−
gm1
Vgs1
ro
RD
+
Vgs2
−
ro
Vin

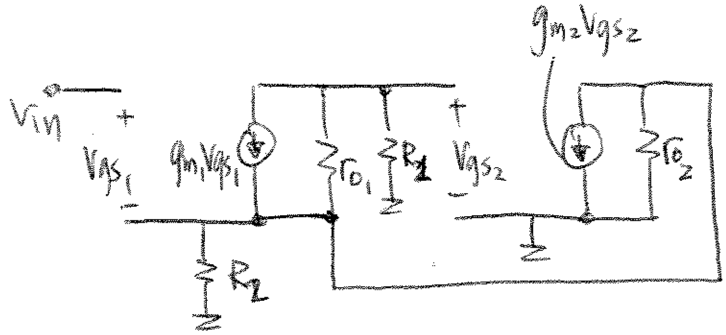
(d)



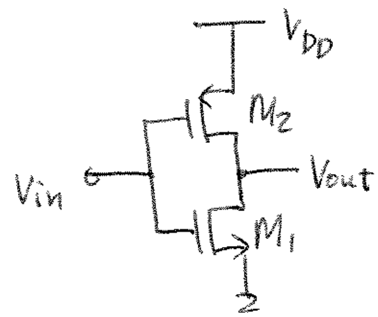
VDD
Vin
M1
Vout
RD
R1
M2
R1
+
Vgs2
−
ro
Vout
RD
gm2Vgs2
Vin +
Vgs1
−
ro
gm1Vgs1

(e)


R2
Vout
Vin
M1
M2
R1

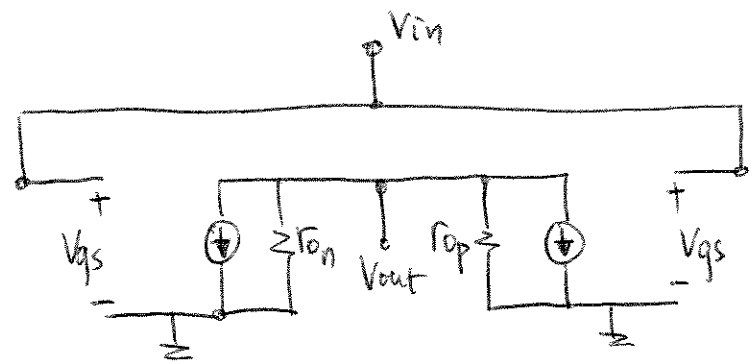

gm2Vgs2
Vin
+
Vgs1
-
gm1Vgs1
ro1
R1
+
Vgs2
-
ro2
R2

46.



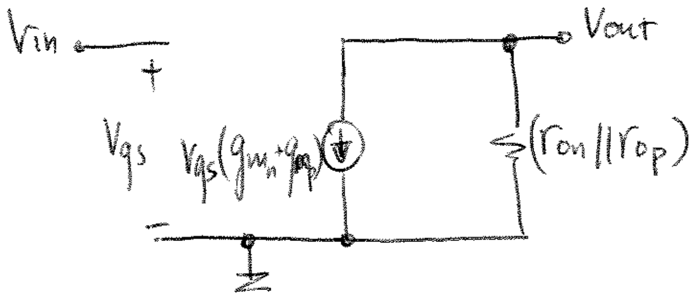
Assume λ_n & λ_p.

(a)



They are in "parallel" because from the small-signal model, both their respective SOURCE and DRAIN nodes are the same.

(b) Assuming both M_1 & M_2 are in saturation, we can combine r_o's & g_m's :



$$\therefore \quad \frac{V_{out}}{V_{in}} = -(g_{mn} + g_{mp})(r_{on} \parallel r_{op})$$

7.1

$$
\begin{aligned}
V_{GS} &= V_{DD} = 1.8\ \text{V} \\
V_{DS} &> V_{GS} - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\
V_{DS} &= V_{DD} - I_D(1\ \text{k}\Omega) \\
&= V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 (1\ \text{k}\Omega) \\
&> V_{GS} - V_{TH} \\
\frac{W}{L} &< \boxed{2.04}
\end{aligned}
$$

② To get $I_{DS} = 1\ mA$,

$$\frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})^2 = 1 \times 10^{-3}\ A.$$

$$\frac{1}{2}(200 \times 10^{-6})\left(\frac{20}{0.18}\right)_1 (V_{GS} - V_{TH})^2 = 10^{-3}$$

$$(V_{GS} - V_{TH})^2 = 0.09$$

$$V_{GS} - V_{TH} = 0.3,$$

$$\text{ie. } V_{GS} = 0.7.,$$

Since

$$V_{GS} = \frac{R_2}{R_1 + R_2} \times 1.8$$

$$0.7 = \frac{R_2}{R_1 + R_2.} \times 1.8$$

$$0.7\ R_1 = \ .\ R_2,$$

$$\therefore \frac{R_1}{R_2} = \frac{11}{7}. \quad \text{———— ①}$$

To get input impedance $\geqslant$ 20k.

$$R_1 \,//\, R_2 \geqslant 20\ k\Omega. \quad \text{———— ②}$$

By inspection, setting $R_1 = 55\ k\Omega$ and $R_2 = 35\ k\Omega$ Will satisfy both ① and ②.

7.3

$$\begin{aligned}
V_{GS} &= V_{DD} - I_D(100\ \Omega) \\
V_{DS} &= V_{DD} - I_D(1\ \text{k}\Omega + 100\ \Omega) \\
&> V_{GS} - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\
V_{DD} - I_D(1\ \text{k}\Omega + 100\ \Omega) &> V_{DD} - I_D(100\ \Omega) - V_{TH} \\
I_D(1\ \text{k}\Omega + 100\ \Omega) &< I_D(100\ \Omega) + V_{TH} \\
I_D(1\ \text{k}\Omega) &< V_{TH} \\
I_D &< 400\ \mu\text{A}
\end{aligned}$$

Since g_m increases with I_D, we should pick the maximum I_D to determine the maximum transconductance that M_1 can provide.

$$\begin{aligned}
I_{D,max} &= 400\ \mu\text{A} \\
g_{m,max} &= \frac{2I_{D,max}}{V_{GS} - V_{TH}} \\
&= \frac{2I_{D,max}}{V_{DD} - I_{D,max}(100\ \Omega) - V_{TH}} \\
&= \boxed{0.588\ \text{mS}}
\end{aligned}$$

④ a). $\because \quad V_{RS} = 200\ mV,$

$\therefore \quad I_{DS}\ R_S = 200\ mV$

$$I_{DS} = \frac{0.2}{100}$$

$$I_{DS} = 2\ mA.$$

For M_1 to stay in saturation,

$$V_{DS} \geqslant V_{GS} - V_{TH}.$$

$\because \quad V_{DS} = V_D - V_s$

$$= [1.8 - (2\times10^{-3})\times500] - 0.2$$

$$= 0.6,$$

$\therefore \quad V_{GS} - V_{TH} \leqslant 0.6,$

Since $\quad I_{DS} = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2,$

$\left(\frac{W}{L}\right)$ is min. when $(V_{GS} - V_{TH})$ is max,

$\therefore$ min $\left(\frac{W}{L}\right)_1$ is when $(V_{GS} - V_{TH}) = 0.6V,$

$$2\times10^{-3} = \frac{1}{2}(200\times10^{-6})\left(\frac{W}{L}\right)_1 (0.6)^2$$

$\therefore$ min. $\left(\frac{W}{L}\right)_1 \approx 56$

b) With $(V_{GS} - V_{TH}) = 0.6$,

$$V_{GS} = 1,$$

$$\therefore V_G = 1 + V_S$$

$$V_G = 1.2V,$$

i.e. $1.8 \times \dfrac{R_2}{R_1 + R_2} = 1.2V,$

$$\frac{R_2}{R_1} = 2 \quad \text{——} ①$$

Input impedance $= R_2 // R_1$,

i.e. $R_2 // R_1 \geqslant 30k\Omega$ —— ②

Set $R_1 = 50k\Omega$ and $R_2 = 100k\Omega$ will satisfy both ① & ②.

7.5

$$I_{D1} = 0.5 \text{ mA}$$

$$V_{GS} = V_{TH} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}}}$$

$$= 0.612 \text{ V}$$

$$V_{GS} = \frac{1}{10} I_{D1} R_2$$

$$R_2 = \boxed{12.243 \text{ k}\Omega}$$

$$V_{GS} = V_{DD} - \frac{1}{10} I_{D1} R_1 - \frac{11}{10} I_{D1} R_S$$

$$R_1 = \boxed{21.557 \text{ k}\Omega}$$

7.6

$$
\begin{aligned}
I_D &= 1 \text{ mA} \\
g_m &= \frac{2I_D}{V_{GS} - V_{TH}} = \frac{1}{100} \\
V_{GS} &= 0.6 \text{ V} \\
V_{GS} &= V_{DD} - I_D R_D \\
R_D &= \boxed{1.2 \text{ k}\Omega}
\end{aligned}
$$

⑦ $I_{DS} = \frac{1}{2}(\mu_n C_{ox})(\frac{W}{L})(V_{GS} - V_{TH})^2$

$0.5 \times 10^{-3} = (100 \times 10^{-6})(\frac{50}{0.18})(V_{GS} - V_{TH})^2$

$\therefore V_{GS} = 0.534V$

$\therefore R_2 = \frac{0.534}{0.05 \times 10^{-3}}$

$R_2 = \underline{\underline{10.68 k\Omega}}$

$\because V_{D1} = 1.8 - (1.1 \times I_{DS} \times 2k\Omega) = 0.1 I_{DS}(R_1 + R_2)$,

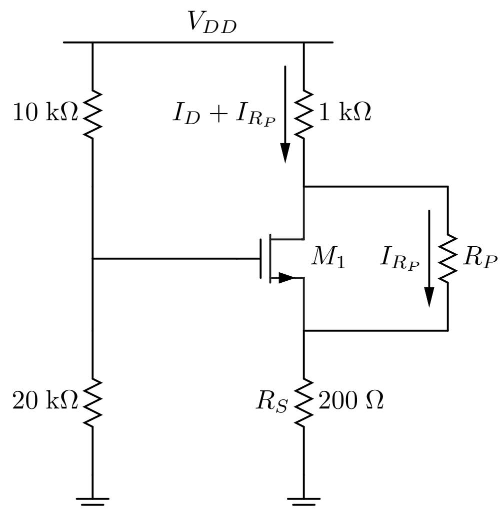
$\therefore 14 k\Omega = R_1 + 10.68 k\Omega$.

$\therefore R_1 = \underline{\underline{3320 \Omega}}$

7.8 First, let's analyze the circuit excluding R_P.

$$\begin{aligned}
V_G &= \frac{20\ \text{k}\Omega}{10\ \text{k}\Omega + 20\ \text{k}\Omega} V_{DD} = 1.2\ \text{V} \\
V_{GS} &= V_G - I_D R_S = V_{DS} = V_{DD} - I_D (1\ \text{k}\Omega + 200\ \Omega) \\
I_D &= 600\ \mu\text{A} \\
V_{GS} &= 1.08\ \text{V} \\
\frac{W}{L} &= \frac{2 I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^2} = 12.9758 \approx \boxed{13}
\end{aligned}$$

Now, let's analyze the circuit with R_P.

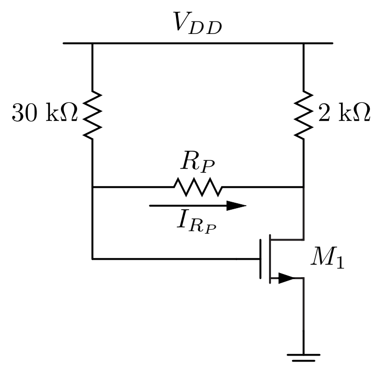


$$\begin{aligned}
V_G &= 1.2\ \text{V} \\
I_D + I_{R_P} &= \frac{V_{DD} - V_{DS}}{1\ \text{k}\Omega + 200\ \Omega} \\
V_{GS} &= V_G - (I_D + I_{R_P}) R_S = V_{DS} + V_{TH} \\
V_G - \frac{V_{DD} - V_{DS}}{1\ \text{k}\Omega + 200\ \Omega} R_S &= V_{DS} + V_{TH} \\
V_{DS} &= 0.6\ \text{V} \\
V_{GS} &= 1\ \text{V} \\
I_D &= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \\
&= 467\ \mu\text{A} \\
I_D + I_{R_P} &= I_D + \frac{V_{DS}}{R_P} = \frac{V_{DD} - V_{DS}}{1\ \text{k}\Omega + 200\ \Omega} \\
R_P &= \boxed{1.126\ \text{k}\Omega}
\end{aligned}$$

7.9 First, let's analyze the circuit excluding R_P.

$$V_{GS} = V_{DD} = 1.8\text{ V}$$
$$V_{DS} = V_{DD} - I_D(2\text{ k}\Omega) = V_{GS} - 100\text{ mV}$$
$$V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2(2\text{ k}\Omega) = V_{GS} - 100\text{ mV}$$
$$\frac{W}{L} = \boxed{0.255}$$

Now, let's analyze the circuit with R_P.



$$V_{GS} = V_{DD} - I_{R_P}(30\text{ k}\Omega)$$
$$I_{R_P} = \frac{V_{GS} - V_{DS}}{R_P} = \frac{50\text{ mV}}{R_P}$$
$$V_{GS} = V_{DD} - (I_D - I_{R_P})(2\text{ k}\Omega) + 50\text{ mV}$$
$$V_{DD} - I_{R_P}(30\text{ k}\Omega) = V_{DD} - \left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 - I_{R_P}\right)(2\text{ k}\Omega) + 50\text{ mV}$$
$$V_{DD} - I_{R_P}(30\text{ k}\Omega) = V_{DD} - \left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{DD} - I_{R_P}(30\text{ k}\Omega) - V_{TH}\right)^2 - I_{R_P}\right)(2\text{ k}\Omega) + 50\text{ mV}$$
$$I_{R_P} = 1.380\ \mu\text{A}$$
$$R_P = \frac{50\text{ mV}}{I_{R_P}} = \boxed{36.222\text{ k}\Omega}$$

(10) For M_1,

$$I_x = \frac{1}{2}(200 \times 100^{-6})\left(\frac{W_1}{0.25}\right)(0.8-0.4)^2 \times (1 + 0.1(0.8))$$

$$10^{-3} = 0.16 \times 10^{-4} \left(\frac{W_1}{0.25}\right)(1.08)$$

$$\therefore W_1 \approx 14.5\mu$$ //

For M_2,

$$0.5 \times 10^{-3} = 0.16 \times 10^{-4}\left(\frac{W_2}{0.25}\right)(1.08)$$

$$\therefore W_2 = 7.25\mu$$ //

Output resistance $= r_o$

$$= \frac{1}{\lambda} \times \frac{1}{I_D}.$$

$$\therefore r_{o1} = \left(\frac{1}{0.1}\right)\left(\frac{1}{10^{-3}}\right)$$

$$= 10 k\Omega$$ //

$$r_{o2} = \left(\frac{1}{0.1}\right)\left(\frac{1}{0.5 \times 10^{-3}}\right)$$

$$= 20 k\Omega$$ //

(11)

$$R_{out} = \frac{1}{\lambda}\left(\frac{1}{I_D}\right)$$

$$= \frac{1}{0.5\times10^{-3}\,\lambda} = 20\,k\Omega$$

$$\therefore \lambda = 0.1\ V^{-1}$$

7.12 Since we're not given V_{DS} for the transistors, let's assume $\lambda = 0$ for large-signal calculations. Let's also assume the transistors operate in saturation, since they're being used as current sources.

$$I_X = \frac{1}{2}\mu_n C_{ox}\frac{W_1}{L_1}\left(V_{B1} - V_{TH}\right)^2 = 0.5 \text{ mA}$$

$$W_1 = \boxed{3.47 \text{ µm}}$$

$$I_Y = \frac{1}{2}\mu_n C_{ox}\frac{W_2}{L_2}\left(V_{B2} - V_{TH}\right)^2 = 0.5 \text{ mA}$$

$$W_2 = \boxed{1.95 \text{ µm}}$$

$$R_{out1} = r_{o1} = \frac{1}{\lambda I_X} = 20 \text{ k}\Omega$$

$$R_{out2} = r_{o2} = \frac{1}{\lambda I_Y} = 20 \text{ k}\Omega$$

Since $I_X = I_Y$ and λ is the same for each current source, the output resistances of the current sources are the same.

7.13 Looking into the source of M_1 we see a resistance of $\frac{1}{g_m}$. Including λ in our analysis, we have

$$\begin{aligned}
\frac{1}{g_m} &= \frac{1}{\mu_p C_{ox} \frac{W}{L} \left(V_X - V_{B1} - |V_{TH}|\right) \left(1 + \lambda V_X\right)} \\
&= \boxed{372\,\Omega}
\end{aligned}$$

(14)

$$I_X = \frac{1}{2}(100\times10^{-6})\left(\frac{20}{0.25}\right)(1-1.8+0.4)^2$$

$$= 0.64\,mA$$

$$I_Y = \frac{1}{2}(100\times10^{-6})\left(2\times\frac{20}{0.25}\right)(1-1.8+0.4)^2$$

$$= 1.28\,mA$$

$$\because r_o \propto \frac{1}{I}$$

and $I_Y = 2I_X$

$$\therefore r_{out,M_1} = 2\,r_{out,M_2}$$

(15) $|I_{DS1}| = |I_{DS2}|$,

$$\frac{1}{2}(200\times 10^{-6})\left(\frac{10}{0.18}\right)(V_B - 0.4)^2(1+0.1\times 0.9)$$

$$= \frac{1}{2}(100\times 10^{-6})(1.8 - V_B - 0.4)^2(1+0.1\times 0.9) \times \left(\frac{30}{0.18}\right)$$

$$2(V_B - 0.4)^2 = 3(1.4 - V_B)^2$$

$$\sqrt{\frac{2}{3}}(V_B - 0.4) = (1.4 - V_B)$$

$$1.816\,V_B = 1.7264$$

$$V_B = 0.95$$

(16) a) For M_1,

$$I_{D1} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{5}{0.18}\right)(V_B - 0.4)^2 (1 + 0.1 \times 0.9)$$

$\therefore V_B \approx 0.806\ V$

b) There are 3 regions of operation:

For $V_X < V_B - V_{TH,1}$, M_1 is in triode and $|I_{DS2}| > |I_{DS1}|$

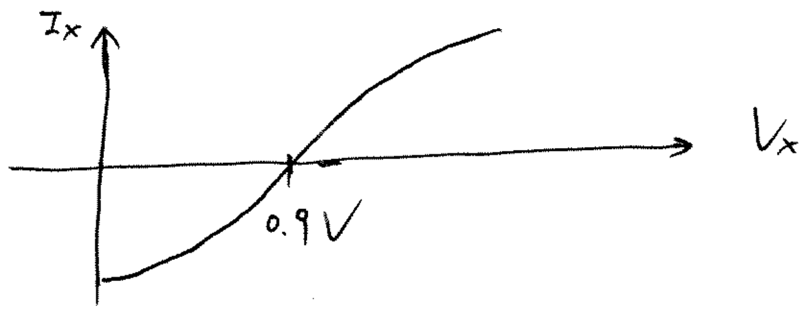
For $|V_X - V_{DD}| > |V_B - V_{DD} - V_{TH2}|$, M_2 is in triode and $I_{DS1} > |I_{DS2}|$

For $V_B - V_{TH} < V_X$ and $|V_X - V_{DD}| < |V_B - V\ \ - V_{TH2}|$

M_1 and M_2 are in saturation.

and $I_{DS1} = |I_{DS2}| = 0.5\,mA$ at $V_X = 0.9\,V$

In all cases, $I_X = I_{DS1} - |I_{DS2}|$



7.17 (a) Assume M_1 is operating in saturation.

$$I_D = 0.5 \text{ mA}$$

$$V_{GS} = V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}}$$

$$= \boxed{0.573 \text{ V}}$$

$$V_{DS} = V_{DD} - I_D R_D = 0.8 \text{ volt} > V_{GS} - V_{TH}, \text{ verifying that } M_1 \text{ is in saturation}$$

(b)

$$A_v = -g_m R_D$$

$$= -\frac{2I_D}{V_{GS} - V_{TH}} R_D$$

$$= \boxed{-11.55}$$

7.18 (a) Assume M_1 is operating in saturation.

$$
\begin{aligned}
I_D &= 0.25 \text{ mA} \\
V_{GS} &= V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} \\
&= \boxed{0.55 \text{ V}} \\
V_{DS} &= V_{DD} - I_D R_D = 1.3 \text{ V} > V_{GS} - V_{TH}, \text{ verifying that } M_1 \text{ is in saturation}
\end{aligned}
$$

(b)

$$
\begin{aligned}
V_{GS} &= 0.55 \text{ V} \\
V_{DS} &> V_{GS} - V_{TH} \text{ (to ensure } M_1 \text{ remains in saturation)} \\
V_{DD} - I_D R_D &> V_{GS} - V_{TH} \\
V_{DD} - \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 R_D &> V_{GS} - V_{TH} \\
\frac{W}{L} &< \frac{2(V_{DD} - V_{GS} + V_{TH})}{\mu_n C_{ox} (V_{GS} - V_{TH})^2 R_D} \\
&= 366.67 \\
&= 3.3 \frac{20}{0.18}
\end{aligned}
$$

Thus, W/L can increase by a factor of $\boxed{3.3}$ while M_1 remains in saturation.

$$
\begin{aligned}
A_v &= -g_m R_D \\
&= -\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) R_D \\
A_{v,max} &= -\mu_n C_{ox} \left(\frac{W}{L}\right)_{max} (V_{GS} - V_{TH}) R_D \\
&= \boxed{-22}
\end{aligned}
$$

7.19

$$\begin{aligned}
P &= V_{DD} I_D < 1 \text{ mW} \\
I_D &< 556\ \mu\text{A} \\
A_v &= -g_m R_D \\
&= -\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\
&= -5 \\
\frac{W}{L} &< \frac{20}{0.18} \\
R_D &> \boxed{1.006\ \text{k}\Omega}
\end{aligned}$$

7.20 (a)

$$\begin{aligned}
I_{D1} &= I_{D2} = 0.5\text{ mA} \\
A_v &= -g_{m1}\left(r_{o1} \parallel r_{o2}\right) \\
&= -\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}\left(\frac{1}{\lambda_1 I_{D1}} \parallel \frac{1}{\lambda_2 I_{D2}}\right) \\
&= -10 \\
\left(\frac{W}{L}\right)_1 &= \boxed{7.8125}
\end{aligned}$$

(b)

$$\begin{aligned}
V_{DD} - V_B &= V_{TH} + \sqrt{\frac{2\left|I_{D2}\right|}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}} \\
V_B &= \boxed{1.1\text{ V}}
\end{aligned}$$

(21) $|A_v| = g_{m1}(r_{o1} \parallel r_{o2})$

$g_{m1} = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{20}{0.18}\right) \times (0.001)}$

(Since V_{ds1} is not given, assume $(1+\lambda_1 V_{DS1})$ has minimal effect on g_{m1})

$= 6.67 \text{ mS.} \quad (S = \Omega^{-1})$

$r_{o1} = \frac{1}{\lambda_1 \times I_{D1}}$

$= \frac{1}{0.1 \times 1mA}$

$= 10 \text{ k}\Omega.$

$r_{o2} \simeq \infty$

$(\because \lambda_2 \ll \lambda_1)$

$\therefore |A_v| = 6.67 \times 10^{-3} \times 10^3 \times 10$

$= 66.7$

7.22 (a) If I_{D1} and I_{D2} remain constant while W and L double, then $g_{m1} \propto \sqrt{(W/L)_1 I_{D1}}$ will not change (since it depends only on the ratio W/L), $r_{o1} \propto \frac{1}{I_{D1}}$ will not change, and $r_{o2} \propto \frac{1}{I_{D2}}$ will not change. Thus, $A_v = -g_{m1}(r_{o1} \| r_{o2})$ $\boxed{\text{will not change}}$.

(b) If I_{D1}, I_{D2}, W, and L double, then $g_{m1} \propto \sqrt{(W/L)_1 I_{D1}}$ will increase by a factor of $\sqrt{2}$, $r_{o1} \propto \frac{1}{I_{D1}}$ will halve, and $r_{o2} \propto \frac{1}{I_{D2}}$ will halve. This means that $r_{o1} \| r_{o2}$ will halve as well, meaning $A_v = -g_{m1}(r_{o1} \| r_{o2})$ will $\boxed{\text{decrease by a factor of } \sqrt{2}}$.

(23). To get higher voltage gain,

(a) is preferred.

For the same dimensions of transistors and same bias current,

(a) has a high "g_m" than (b).

$\because g_{m1} > g_{m2}$.

(since $\mu_n C_{ox} > \mu_p C_{ox}$)

while $(r_{o1} // r_{o2})$ is the same for both cases.

(24)

$$A_v = g_{m2} (r_{o1} // r_{o2})$$

$$r_{o1} = \frac{1}{0.15 \times 0.5mA}$$

$$= 13.3 \ k\Omega.$$

$$r_{o2} = \frac{1}{0.05 \times 0.5mA}$$

$$= 40 \ k\Omega.$$

$$\therefore r_{o1} // r_{o2} = 10 \ k\Omega.$$

$$\therefore 15 = \left[\sqrt{2 \times (100 \times 10^{-6}) \left(\frac{W}{L}\right)_2 0.5mA}\right] \cdot (10 \ k\Omega)$$

$$\left(\frac{W}{L}\right)_2 = 22.5$$

(25) From Eq (7.57)

$$3 = \sqrt{\frac{20/0.18}{(W/L)_2}}$$

$\therefore (W/L)_2 \approx 12.3$ //.

7.26 (a)

$$\begin{aligned}
I_{D1} &= I_{D2} = 0.5\text{ mA} \\
V_{GS1} &= V_{TH} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} \\
&= 0.7\text{ V} \\
V_{DS1} &= V_{GS1} - V_{TH} \text{ (in order of } M_1 \text{ to operate at the edge of saturation)} \\
&= V_{DD} - V_{GS2} \\
V_{GS2} &= V_{DD} - V_{GS1} + V_{TH} = V_{TH} + \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} \\
\left(\frac{W}{L}\right)_2 &= \boxed{4.13}
\end{aligned}$$

(b)

$$\begin{aligned}
A_v &= -\frac{g_{m1}}{g_{m2}} \\
&= -\frac{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}}{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_{D2}}} \\
&= -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}} \\
&= \boxed{-3.667}
\end{aligned}$$

(c) Since $(W/L)_1$ is fixed, we must minimize $(W/L)_2$ in order to maximize the magnitude of the gain (based on the expression derived in part (b)). If we pick the size of M_2 so that M_1 operates at the edge of saturation, then if M_2 were to be any smaller, V_{GS2} would have to be larger (given the same I_{D2}), driving M_1 into triode. Thus, $(W/L)_2$ is its smallest possible value (without driving M_1 into saturation) when M_1 is at the edge of saturation, meaning the gain is largest in magnitude with this choice of $(W/L)_2$.

7.27 (a)

$$\begin{aligned} A_v &= -\frac{g_{m1}}{g_{m2}} \\ &= -\frac{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}}{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_2 I_{D2}}} \\ &= -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}} \\ &= -5 \\ \left(\frac{W}{L}\right)_1 &= \boxed{277.78} \end{aligned}$$

(b)

$$\begin{aligned} V_{DS1} &> V_{GS1} - V_{TH} \text{ (to ensure } M_1 \text{ is in saturation)} \\ V_{DD} - V_{GS2} &> V_{GS1} - V_{TH} \\ V_{DD} - V_{TH} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_2}} &> \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1}} \\ I_{D1} = I_{D2} &< \boxed{1.512 \text{ mA}} \end{aligned}$$

7.28 For this problem, recall that looking into the drain of a transistor with a grounded gate and source we see a resistance of r_o, and looking into either terminal of a diode-connected transistor we see a resistance of $\frac{1}{g_m} \parallel r_o$.

(a)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}\right)}$$

(b)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m3}} \parallel r_{o3}\right)}$$

(c)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m3}} \parallel r_{o3}\right)}$$

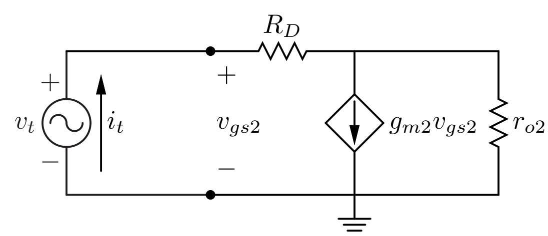
(d)

$$A_v = \boxed{-g_{m2}\left(r_{o2} \parallel r_{o1} \parallel \frac{1}{g_{m3}} \parallel r_{o3}\right)}$$

(e)

$$A_v = \boxed{-g_{m2}\left(r_{o2} \parallel r_{o1} \parallel \frac{1}{g_{m3}} \parallel r_{o3}\right)}$$

(f) Let's draw a small-signal model to find the equivalent resistance seen looking up from the output.



$$i_t = g_{m2}v_{gs2} + \frac{v_t - i_t R_D}{r_{o2}}$$

$$v_{gs2} = v_t$$

$$i_t = g_{m2}v_t + \frac{v_t - i_t R_D}{r_{o2}}$$

$$i_t\left(1 + \frac{R_D}{r_{o2}}\right) = v_t\left(g_{m2} + \frac{1}{r_{o2}}\right)$$

$$\frac{v_t}{i_t} = \frac{1 + \frac{R_D}{r_{o2}}}{g_{m2} + \frac{1}{r_{o2}}} = \frac{r_{o2} + R_D}{1 + g_{m2}r_{o2}}$$

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel \frac{r_{o2} + R_D}{1 + g_{m2}r_{o2}}\right)}$$

7.30 (a) Assume M_1 is operating in saturation.

$$\begin{aligned}
I_D &= 1\ \text{mA} \\
I_D R_S &= 200\ \text{mV} \\
R_S &= 200\ \Omega \\
A_v &= -\frac{R_D}{\frac{1}{g_m} + R_S} \\
&= -\frac{R_D}{\frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} + R_S} \\
&= -4 \\
\frac{W}{L} &= \boxed{1000} \\
V_{GS} &= V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} \\
&= 0.5\ \text{V} \\
V_{DS} &= V_{DD} - I_D R_D - I_D R_S \\
&= 0.6\ \text{V} > V_{GS} - V_{TH}, \text{ verifying that } M_1 \text{ is in saturation}
\end{aligned}$$

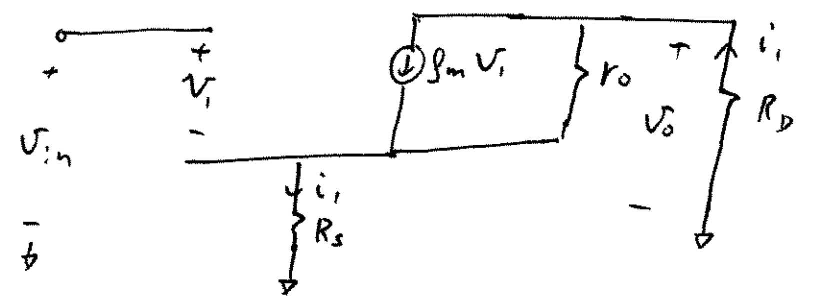
$\boxed{\text{Yes}}$, the transistor operates in saturation.

(b) Assume M_1 is operating in saturation.

$$\begin{aligned}
\frac{W}{L} &= \frac{50}{0.18} \\
R_S &= 200\ \Omega \\
A_v &= -\frac{R_D}{\frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} + R_S} \\
&= -4 \\
R_D &= \boxed{1.179\ \text{k}\Omega} \\
V_{GS} &= V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} \\
&= 0.590\ \text{V} \\
V_{DS} &= V_{DD} - I_D R_D - I_D R_S \\
&= 0.421\ \text{V} > V_{GS} - V_{TH}, \text{ verifying that } M_1 \text{ is in saturation}
\end{aligned}$$

$\boxed{\text{Yes}}$, the transistor operates in saturation.

(31) The small signal model is:



$$v_o = -i_1 R_D \quad \text{——— (1)}$$

$$i_1 = g_m v_1 + \frac{v_o - v_1}{r_o}$$

$$= \frac{(g_m r_o - 1) v_1 + v_o}{r_o}$$

$$i_1 \approx g_m v_1 + \frac{v_o}{r_o}$$

$$\therefore -\frac{v_o}{R_D} = g_m v_1 + \frac{v_o}{r_o} \quad \text{——— (2)}$$

$$v_{in} = v_1 + i_1 R_s$$

$$\therefore v_1 = v_{in} + \frac{v_o}{R_D} R_s \quad \text{——— (3)}$$

(2) combines with (3):

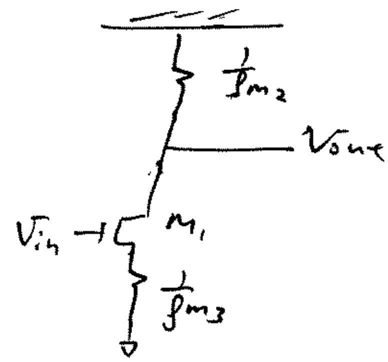
$$-\frac{v_o}{R_D} = g_m v_{in} + g_m v_o \frac{R_s}{R_D} + \frac{v_o}{r_o}$$

$$-v_o \left[\frac{1}{R_D} + g_m \frac{R_s}{R_D} + \frac{1}{r_o} \right] = g_m v_{in}$$

$$\therefore \text{Volt. gain} = \frac{v_o}{v_{in}} = -\left[\frac{g_m}{r_o + g_m R_s r_o + R_D}\right] (r_o R_D)$$

//

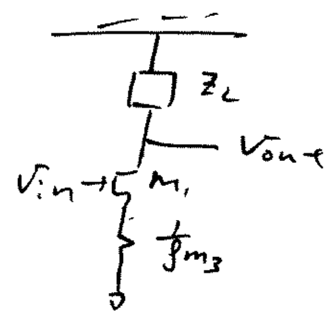
(32) a) Equivalent circuit is:



$$\therefore A_v = -\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}}}$$

b) Similar to Prob. 28 (f),

Equivalent circuit is:

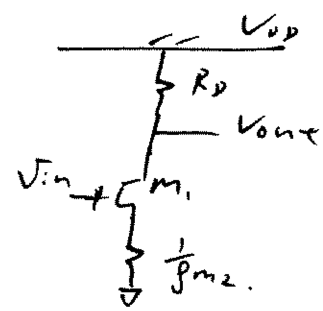


From Prob. 28 (f),

$$Z_L = \frac{1}{g_{m2}} \quad (\text{as } r_{O2} \to \infty)$$

$$\therefore A_v = -\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}}}$$

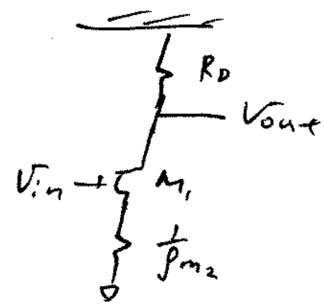
c) Equivalent circuit is:



$$\therefore A_v = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

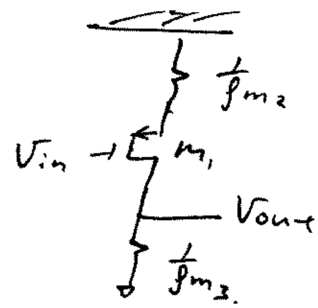
(d). Equivalent circuit is

$$A_v = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$



(e) Equivalent circuit is

$$A_v = \frac{\frac{1}{g_{m3}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$



(33) a) From Eq. (7.71),

$$R_{out} = (1 + g_{m1} r_{o1}) \frac{1}{g_{m2}} + r_{o1}$$

b) From Eq. (7.71),

$$R_{out} = (1 + g_{m1} r_{o1}) \frac{1}{g_{m2}} + r_{o1}$$

c) From Eq. (7.71),

$$R_{out} = (1 + g_{m2} r_{o2}) \left(r_{o1} \parallel \frac{1}{g_{m3}} \right) + r_{o2}$$

d) From Eq. (7.71),

$$R_{out} = (1 + g_{m1} r_{o1}) \left(r_{o2} \parallel \frac{1}{g_{m3}} \right) + r_{o1}$$

(34) To find $\left(\frac{W}{L}\right)$,

$$10^{-3} = \frac{1}{2}(200\times10^{-6})\left(\frac{W}{L}\right)(1-0.4)^2 \times (1+0.1\,V_{DS}),$$

where $V_{DS} = 1.8 - 1k\Omega \times 1mA$

$= 0.8V$,

$\therefore \left(\frac{W}{L}\right) \simeq 25.7$ //

Voltage gain $(A_v) = -g_{m1}(r_{o1} \,//\, R_D)$

$$g_{m1} = \sqrt{2(200\times10^{-6})(25.7)\times10^{-3}\times(1+0.1\times0.8)}$$

$= 3.33$ mS.

$$r_{o1} = \frac{1}{0.1\times10^{-3}}$$

$= 10k\Omega$.

$\therefore A_v = (-3.33\times10^{-3})(10k\Omega \,//\, 1k\Omega)$

$= -3.03$ //

(35) with $\lambda = 0$,

$$10^{-3} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{W}{L}\right)(1 - 0.4)^2$$

$$\therefore \left(\frac{W}{L}\right) \fallingdotseq 27.8$$

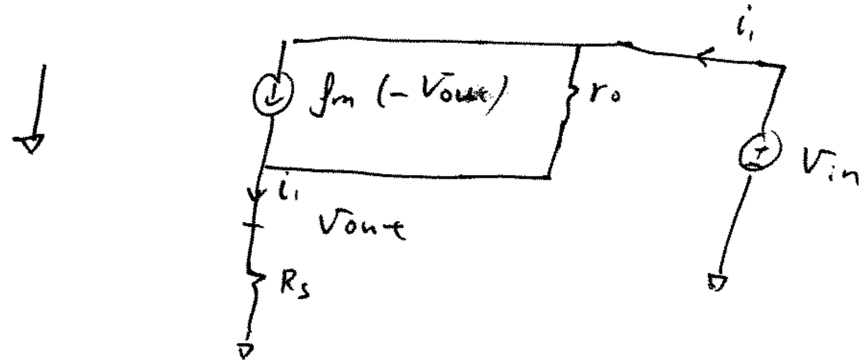
$$A_v = -g_{m1} R_D$$

$$= -\sqrt{2(200 \times 10^{-6})(27.8) \times 10^{-3}} \times 1000$$

$$= -3.33$$

Without r_o, gain increases due mainly to increase in load resistance.

(3 b) The small-signal circuit is:



$$i_1 = \frac{V_{out}}{R_s} \quad \text{—— ①}$$

$$i_1 = g_m(-V_{out}) + \frac{V_{in} - V_{out}}{r_o} \quad \text{—— ②}$$

$$\therefore \frac{V_{out}}{R_s} = -g_m V_{out} + \frac{V_{in}}{r_o} - \frac{V_{out}}{r_o}$$

$$V_{out}\left(\frac{1}{R_s} + g_m + \frac{1}{r_o}\right) = \frac{V_{in}}{r_o}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{r_o}\left(\frac{R_s r_o}{r_o + g_m r_o R_s + R_s}\right)$$

$$= \frac{R_s}{g_m r_o R_s + r_o + R_s}$$

Since $(g_m r_o R_s + r_o) > 0$, the voltage gain < 1.

This is expected: Any variation in V_{in} causes minimal change in the bias current.

$\because V_{out}$ is determined largely by the amount of bias current ($\because V_{out}$ is set by V_{GS1})

$\therefore$ There is almost no variation in V_{out}. (ie. $\frac{V_{out}}{V_{in}} \ll 1$)

(37) a/ $|\text{Voltage gain}| = g_m R_D$

$= 5$

$\therefore g_m = \frac{5}{500}$

$= 10 mS.$

$= \sqrt{2(200 \times 10^{-6})(\frac{W}{L}) \times 10^{-3}}$

$\therefore \frac{W}{L} = 250$

b/ $V_D = 1.8 - 500 \times 10^{-3}$

$= 1.3 V$

To obtain $V_{DS} \geq V_{GS} - V_{TH} + 0.2$,

$V_D \geq V_G - 0.2$

$\therefore V_G \leq 1.5$

Also, $I_{R_1 + R_2} = 0.1 \times 10^{-3} A.$

$\therefore R_1 + R_2 = \frac{1.8}{0.1 \times 10^{-3}}$

$= 18 k\Omega.$

choose $R_2 = 15 k\Omega$ & $R_1 = 3 k\Omega$

c) With twice of (W/L), M_1 will go further away from triode. As (W/L) doubles, & I_{bias} is fixed by the current source, V_{GS} is forced to decrease (so M_1 will have same I_{DS}). Thus, $(V_{GS} - V_{TH})$ decreases, and V_{DS} can be allowed to drop more before M_1 goes into triode.

Gain will be increased by $\sqrt{2}$, because gain $\propto g_m$, and $g_m \propto \sqrt{W/L}$.

(38) a) $V_G = 1.8V.$

$\therefore V_{D,min} = 1.8 - 0.4$ (for M_1 stays in saturation)

$= 1.4V$

$\therefore R_{D,max} = \frac{1.4V}{1mA}$

$= 1.4 k\Omega$

b) $|\text{Voltage gain}| = g_m R_D$

$= 5.$

$\therefore g_m = \frac{5}{R_D}$

$= 3.57 mS.$

$= \sqrt{2 \times (200\times10^{-6}) (\frac{W}{L}) \times 10^{-3}}$

$\therefore (\frac{W}{L}) = 31.9$

(38) To get $R_{in} = 50\Omega$,

$$\frac{1}{g_m} = 50\Omega$$

$$\therefore g_m = 20\,mS.$$

volt gain $(A_v) = g_m R_D$

$$= 4,$$

$$\therefore R_D = \frac{4}{0.02}$$

$$R_D = 200\,\Omega$$

$$\because g_m = \sqrt{2\times(200\times10^{-6})\left(\frac{W}{L}\right)\times0.5\times10^{-3}}$$

$$\therefore \left(\frac{W}{L}\right) = 2000$$

7.42 (a)

$$\begin{aligned}
R_{out} &= R_D = 500\ \Omega \\
V_G &= V_{DD} \\
V_D &> V_G - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\
V_{DD} - I_D R_D &> V_{DD} - V_{TH} \\
I_D &< \boxed{0.8\ \text{mA}}
\end{aligned}$$

(b)

$$\begin{aligned}
I_D &= 0.8\ \text{mA} \\
R_{in} &= \frac{1}{g_m} \\
&= \frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} \\
&= 50\ \Omega \\
\frac{W}{L} &= \boxed{1250}
\end{aligned}$$

(c)

$$\begin{aligned}
A_v &= g_m R_D \\
g_m &= \frac{1}{50}\ \text{S} \\
R_D &= 500\ \Omega \\
A_v &= \boxed{10}
\end{aligned}$$

7.43 (a)

$$
\begin{aligned}
I_D &= I_1 = 1 \text{ mA} \\
V_G &= V_{DD} \\
V_D &= V_G - V_{TH} + 100 \text{ mV} \\
V_{DD} - I_D R_D &= V_G - V_{TH} + 100 \text{ mV} \\
R_D &= \boxed{300\ \Omega}
\end{aligned}
$$

(b)

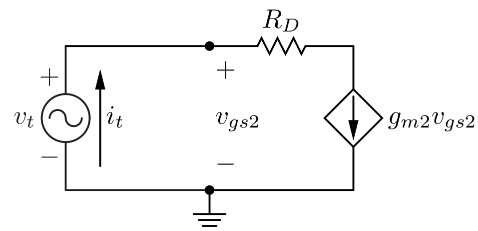
$$
\begin{aligned}
R_D &= 300\ \Omega \\
A_v &= g_m R_D \\
&= \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\
&= 5 \\
\frac{W}{L} &= \boxed{694.4}
\end{aligned}
$$

7.44 For this problem, recall that looking into the drain of a transistor with a grounded gate and source we see a resistance of r_o, and looking into either terminal of a diode-connected transistor we see a resistance of $\frac{1}{g_m} \parallel r_o$.

(a) Referring to Eq. (7.109) with $R_D = \frac{1}{g_{m2}}$ and $g_m = g_{m1}$, we have

$$A_v = \boxed{\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + R_S}}$$

(b) Let's draw a small-signal model to find the equivalent resistance seen looking up from the output.



$$i_t = g_{m2}v_{gs2}$$
$$v_{gs2} = v_t$$
$$i_t = g_{m2}v_t$$
$$\frac{v_t}{i_t} = \frac{1}{g_{m2}}$$
$$A_v = \boxed{\frac{g_{m1}}{g_{m2}}}$$

(c) Referring to Eq. (7.119) with $R_D = \frac{1}{g_{m2}}$, $R_3 = R_1$, and $g_m = g_{m1}$, we have

$$A_v = \boxed{\frac{R_1 \parallel \frac{1}{g_{m1}}}{R_S + R_1 \parallel \frac{1}{g_{m1}}} \frac{g_{m1}}{g_{m2}}}$$

(d)

$$A_v = \boxed{g_{m1}\left(R_D + \frac{1}{g_{m2}} \parallel r_{o3}\right)}$$

(e)

$$A_v = \boxed{g_{m1}\left(R_D + \frac{1}{g_{m2}}\right)}$$

7.45 (a)

$$\begin{aligned}
\frac{v_X}{v_{in}} &= -g_{m1}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right) \\
\frac{v_{out}}{v_X} &= g_{m2}R_{D2} \\
\frac{v_{out}}{v_{in}} &= \frac{v_X}{v_{in}}\frac{v_{out}}{v_X} \\
&= \boxed{-g_{m1}g_{m2}R_{D2}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right)}
\end{aligned}$$

(b)

$$\lim_{R_{D1}\to\infty} -g_{m1}g_{m2}R_{D2}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right) = \boxed{-g_{m1}R_{D2}}$$

This makes sense because the common-source stage acts as a transconductance amplifier with a transconductance of g_{m1}. The common-gate stage acts as a current buffer with a current gain of 1. Thus, the current $g_{m1}v_{in}$ flows through R_{D2}, meaning $v_{out} = -g_{m1}v_{in}R_{D2}$, so that $\frac{v_{out}}{v_{in}} = -g_{m1}R_{D2}$.

This type of amplifier (with $R_{D1} = \infty$) is known as a cascode and will be studied in detail in Chapter 9.

7.40

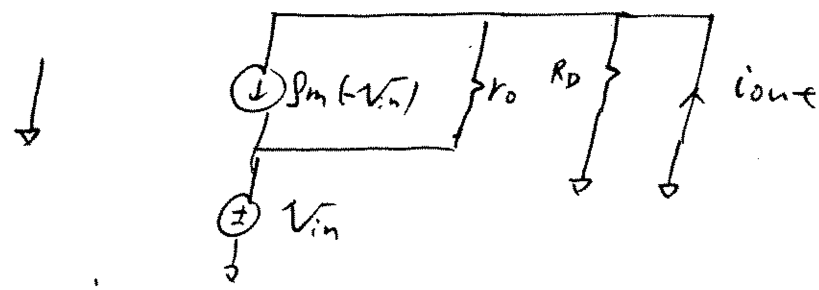
$$
\begin{aligned}
I_D &= 0.5 \text{ mA} \\
R_{in} &= \frac{1}{g_m} \\
&= \frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} \\
&= 50\ \Omega \\
\frac{W}{L} &= \boxed{2000} \\
V_D &> V_G - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\
V_{DD} - I_D R_D &> V_b - V_{TH} \\
R_D &< 2.4\ \text{k}\Omega
\end{aligned}
$$

Since $|A_v| \propto R_D$, we need to maximize R_D in order to maximize the gain. Thus, we should pick $R_D = \boxed{2.4\ \text{k}\Omega}$. This corresponds to a voltage gain of $A_v = -g_m R_D = -48$.

(41) Voltage gain $(A_v) = G_m R_{out}$,

where G_m and R_{out} are the transconductance and output resistance of the circuit respectively.

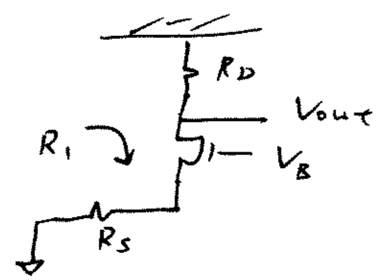
To find G_m:



$$G_m = \frac{i_{out}}{V_{in}} = g_m + \frac{1}{r_o}$$

$$\approx g_m \quad (\because g_m r_o >> 1)$$

To find R_{out}:



$$R_{out} = R_D // R_1$$

$$= R_D // [(1 + g_m r_o) R_S + r_o]$$

(from Eq. (7.110))

$$\approx R_D // (g_m r_o R_S + r_o) \quad (\because g_m r_o >> 1)$$

$$= \frac{g_m r_o R_S R_D + r_o R_D}{R_D + g_m r_o R_S + r_o}$$

$$\therefore \text{Voltage gain} = g_m \left[\frac{g_m r_o R_D R_S + r_o R_D}{R_D + g_m r_o R_S + r_o} \right]$$

7.42 (a)

$$\begin{aligned} R_{out} &= R_D = 500\ \Omega \\ V_G &= V_{DD} \\ V_D &> V_G - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\ V_{DD} - I_D R_D &> V_{DD} - V_{TH} \\ I_D &< \boxed{0.8\ \text{mA}} \end{aligned}$$

(b)

$$\begin{aligned} I_D &= 0.8\ \text{mA} \\ R_{in} &= \frac{1}{g_m} \\ &= \frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} \\ &= 50\ \Omega \\ \frac{W}{L} &= \boxed{1250} \end{aligned}$$

(c)

$$\begin{aligned} A_v &= g_m R_D \\ g_m &= \frac{1}{50}\ \text{S} \\ R_D &= 500\ \Omega \\ A_v &= \boxed{10} \end{aligned}$$

7.43 (a)

$$
\begin{aligned}
I_D &= I_1 = 1 \text{ mA} \\
V_G &= V_{DD} \\
V_D &= V_G - V_{TH} + 100 \text{ mV} \\
V_{DD} - I_D R_D &= V_G - V_{TH} + 100 \text{ mV} \\
R_D &= \boxed{300\ \Omega}
\end{aligned}
$$

(b)

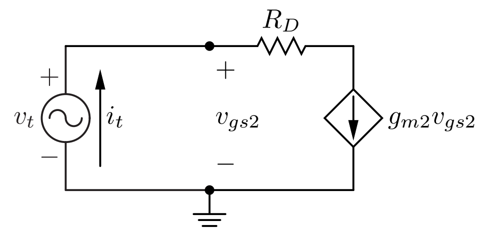
$$
\begin{aligned}
R_D &= 300\ \Omega \\
A_v &= g_m R_D \\
&= \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\
&= 5 \\
\frac{W}{L} &= \boxed{694.4}
\end{aligned}
$$

7.44 For this problem, recall that looking into the drain of a transistor with a grounded gate and source we see a resistance of r_o, and looking into either terminal of a diode-connected transistor we see a resistance of $\frac{1}{g_m} \parallel r_o$.

(a) Referring to Eq. (7.109) with $R_D = \frac{1}{g_{m2}}$ and $g_m = g_{m1}$, we have

$$A_v = \boxed{\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + R_S}}$$

(b) Let's draw a small-signal model to find the equivalent resistance seen looking up from the output.



$$i_t = g_{m2}v_{gs2}$$
$$v_{gs2} = v_t$$
$$i_t = g_{m2}v_t$$
$$\frac{v_t}{i_t} = \frac{1}{g_{m2}}$$
$$A_v = \boxed{\frac{g_{m1}}{g_{m2}}}$$

(c) Referring to Eq. (7.119) with $R_D = \frac{1}{g_{m2}}$, $R_3 = R_1$, and $g_m = g_{m1}$, we have

$$A_v = \boxed{\frac{R_1 \parallel \frac{1}{g_{m1}}}{R_S + R_1 \parallel \frac{1}{g_{m1}}} \frac{g_{m1}}{g_{m2}}}$$

(d)

$$A_v = \boxed{g_{m1}\left(R_D + \frac{1}{g_{m2}} \parallel r_{o3}\right)}$$

(e)

$$A_v = \boxed{g_{m1}\left(R_D + \frac{1}{g_{m2}}\right)}$$

7.45 (a)

$$\frac{v_X}{v_{in}} = -g_{m1}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right)$$

$$\frac{v_{out}}{v_X} = g_{m2}R_{D2}$$

$$\frac{v_{out}}{v_{in}} = \frac{v_X}{v_{in}}\frac{v_{out}}{v_X}$$

$$= \boxed{-g_{m1}g_{m2}R_{D2}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right)}$$

(b)

$$\lim_{R_{D1}\to\infty} -g_{m1}g_{m2}R_{D2}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right) = \boxed{-g_{m1}R_{D2}}$$

This makes sense because the common-source stage acts as a transconductance amplifier with a transconductance of g_{m1}. The common-gate stage acts as a current buffer with a current gain of 1. Thus, the current $g_{m1}v_{in}$ flows through R_{D2}, meaning $v_{out} = -g_{m1}v_{in}R_{D2}$, so that $\frac{v_{out}}{v_{in}} = -g_{m1}R_{D2}$.

This type of amplifier (with $R_{D1} = \infty$) is known as a cascode and will be studied in detail in Chapter 9.

(46) $$\frac{v_x}{v_{in}} = \left(R_{D1} \parallel \frac{1}{g_{m2}}\right) g_{m1}$$

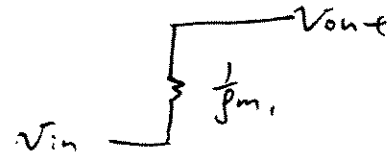
$$\frac{v_{out}}{v_x} = g_{m2} R_{D2}.$$

$$\therefore \frac{v_{out}}{v_{in}} = g_{m1} g_{m2} R_{D2} \left(R_{D1} \parallel \frac{1}{g_{m2}}\right)$$

Similar to prob. (45), voltage gain approaches that of cascode stage as R_{D1} approaches infinity. The gain is $g_{m1} R_{D2}$.

(47) with $\lambda = 0$, M_1 appears as a diode-connected device.

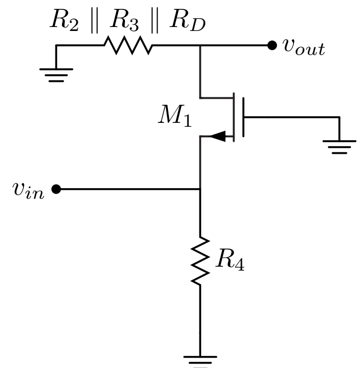
$\therefore$ the circuit becomes:



ie. $\frac{V_{out}}{V_{in}} = 1$

This is not a common-gate (CG) amplifier, because the gate is not fixed. (ie. gate is not at an "a.c. ground")

7.48 For small-signal analysis, we can short the capacitors, producing the following equivalent circuit.



$$A_v = \boxed{g_m (R_2 \parallel R_3 \parallel R_D)}$$

7.49

$$
\begin{aligned}
V_{GS} &= V_{DS} \\
V_{GS} &= V_{DD} - I_D R_S = V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 \left(1 + \lambda V_{GS}\right) R_S \\
V_{GS} &= V_{DS} = 0.7036 \text{ V} \\
I_D &= 1.096 \text{ mA} \\
A_v &= \frac{r_o \parallel R_S}{\frac{1}{g_m} + r_o \parallel R_S} \\
g_m &= \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D} = 6.981 \text{ mS} \\
r_o &= \frac{1}{\lambda I_D} = 9.121 \text{ k}\Omega \\
A_v &= \boxed{0.8628}
\end{aligned}
$$

7.50

$$
\begin{aligned}
A_v &= \frac{R_S}{\frac{1}{g_m} + R_S} \\
&= \frac{R_S}{\frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})} + R_S} \\
&= 0.8 \\
V_{GS} &= 0.64 \text{ V} \\
I_D &= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_{TH}\right)^2 \\
&= 960 \ \mu\text{A} \\
V_G &= V_{GS} + V_S = V_{GS} + I_D R_S \\
&= \boxed{1.12 \text{ V}}
\end{aligned}
$$

(51)

$$A_v = \frac{R_s}{\frac{1}{g_m} + R_s}$$

$$= 0.8$$

$$0.8 = \frac{500}{\frac{1}{g_m} + 500}$$

$$\therefore g_m = 8\,mS.$$

$$\because I_{ds} = \frac{1}{2}\beta\,(V_{GS} - V_t)^2,$$

where $\beta = \left(\frac{W}{L}\right)\mu_n C_{ox}$

and $g_m = \beta(V_{GS} - V_t)$.

$$\therefore I_{ds} = \frac{1}{2} g_m (V_{GS} - V_t)$$

$$= \frac{1}{2} g_m (1.8 - I_{ds}(500) - 0.4)$$

$$I_{ds} = 4\times10^{-3}(1.4 - 500\, I_{ds})$$

$$\therefore I_{ds} = 1.87\,mA.$$

$$\because g_m = \sqrt{2(200\times10^{-6})\frac{W}{L}\times1.87\times10^{-3}}$$

$$\therefore \frac{W}{L} \approx 85.7$$

(52). To get $R_{out} = 100\,\Omega$,

$$\frac{1}{g_m} = 100$$

$$\therefore g_m = 10\,mS.$$

$$\because \quad I_{ds} = \frac{1}{2}\beta(V_{GS} - V_{TH})^2.$$

where $\beta = \mu_n C_{ox} \frac{W}{L}$.

and $g_m = \beta\,(V_{GS} - V_{TH})$

$$\therefore I_{ds} = \frac{1}{2} g_m (V_{GS} - V_{TH})$$

$$= \frac{1}{2}(10\times10^{-3})(0.9-0.4)$$

$$\therefore I_{ds} = 2.5\,mA.$$

$$\because g_m = \sqrt{2\times(200\times10^{-6})(\tfrac{W}{L})(2.5\times10^{-3})}$$

$$\therefore \left(\frac{W}{L}\right) = 100$$

(53). To get $R_{out} = 50\Omega$,

$$\frac{1}{g_m} = 50\Omega$$

$$\therefore g_m = 20\,mS.$$

$$\text{Power}\ (P) = 1.8 \times I_{DS}$$

$$= 2 \times 10^{-3}\,W.$$

$$\therefore I_{DS} = 1.11\,mA.$$

$$\because g_m = \sqrt{2 \times (200 \times 10^{-6})(\frac{W}{L})(1.11mA)}$$

$$\therefore \frac{W}{L} = 900$$

(54)

$$A_v = \frac{R_L}{\frac{1}{g_m} + R_L}$$

$$\therefore 0.8 = \frac{50}{\frac{1}{g_m} + 50}$$

$$g_m = 80 \text{ mS}$$

$$\text{Power } (P) = 1.8 \times I_{DS}$$

$$= 3 \text{ mW}$$

$$\therefore I_{DS} = 1.67 \text{ mA}$$

$$g_m = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) (1.67 \times 10^{-3})}$$

$$\therefore \left(\frac{W}{L}\right) = 9600$$

7.55 For this problem, recall that looking into the drain of a transistor with a grounded gate and source we see a resistance of r_o, and looking into either terminal of a diode-connected transistor we see a resistance of $\frac{1}{g_m} \parallel r_o$.

(a)

$$A_v = \boxed{\frac{r_{o1} \parallel (R_S + r_{o2})}{\frac{1}{g_{m1}} + r_{o1} \parallel (R_S + r_{o2})}}$$

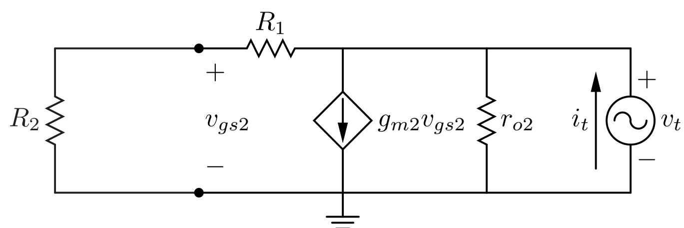
(b) Looking down from the output we see an equivalent resistance of $r_{o2} + (1 + g_{m2} r_{o2}) R_S$ by Eq. (7.110).

$$A_v = \boxed{\frac{r_{o1} \parallel [r_{o2} + (1 + g_{m2} r_{o2}) R_S]}{\frac{1}{g_{m1}} + r_{o1} \parallel [r_{o2} + (1 + g_{m2} r_{o2}) R_S]}}$$

(c)

$$A_v = \boxed{\frac{r_{o1} \parallel \frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + r_{o1} \parallel \frac{1}{g_{m2}}}}$$

(d) Let's draw a small-signal model to find the equivalent resistance seen looking down from the output.



$$i_t = \frac{v_t}{R_1 + R_2} + g_{m2} v_{gs2} + \frac{v_t}{r_{o2}}$$

$$v_{gs2} = \frac{R_2}{R_1 + R_2} v_t$$

$$i_t = \frac{v_t}{R_1 + R_2} + g_{m2} \frac{R_2}{R_1 + R_2} v_t + \frac{v_t}{r_{o2}}$$

$$i_t = v_t \left(\frac{1}{R_1 + R_2} + \frac{g_{m2} R_2}{R_1 + R_2} + \frac{1}{r_{o2}} \right)$$

$$\frac{v_t}{i_t} = (R_1 + R_2) \parallel \left(\frac{R_1 + R_2}{g_{m2} R_2} \right) \parallel r_{o2}$$

$$A_v = \boxed{\frac{r_{o1} \parallel (R_1 + R_2) \parallel \left(\frac{R_1 + R_2}{g_{m2} R_2} \right) \parallel r_{o2}}{\frac{1}{g_{m1}} + r_{o1} \parallel (R_1 + R_2) \parallel \left(\frac{R_1 + R_2}{g_{m2} R_2} \right) \parallel r_{o2}}}$$

(e)

$$A_v = \boxed{\frac{r_{o2} \parallel r_{o3} \parallel \frac{1}{g_{m1}}}{\frac{1}{g_{m2}} + r_{o2} \parallel r_{o3} \parallel \frac{1}{g_{m1}}}}$$

(f) Looking up from the output we see an equivalent resistance of $r_{o2} + (1 + g_{m2}r_{o2})\, r_{o3}$ by Eq. (7.110).

$$A_v = \boxed{\frac{r_{o1} \parallel [r_{o2} + (1 + g_{m2}r_{o2})\, r_{o3}]}{\frac{1}{g_{m1}} + r_{o1} \parallel [r_{o2} + (1 + g_{m2}r_{o2})\, r_{o3}]}}$$

(5 6)

$$\frac{v_x}{v_{in}} = \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}.$$

$$\frac{V_{out}}{v_x} = g_{m2} R_D$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

b) if $g_{m1} = g_{m2}$,

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1} R_D}{2}.$$

(57) $\because R_{out} = 1k\Omega$.

$\therefore R_D = 1k\Omega$.

$\because A_v = 5$

$= g_{m1} R_D$

$\therefore g_{m1} (1000) = 5$

$g_{m1} = 5mS$.

$\because M_1$ is 00 mV away from triode,

$V_D = (V_G - V_{TH}) + 0.1$.

$V_D = (1.8 - 0.4) + 0.1$

$V_D = 1.5V$

$\therefore I_{DS} = \frac{1.8 - 1.5}{R_D} = \frac{0.3}{R_D}$

$= 0.3mA$

$\because g_{m1} = \sqrt{2 \times (200 \times 10^{-6}) (\frac{W}{L}) I_{DS_1}}$

$\therefore (\frac{W}{L}) \approx 208$

$\therefore R_D = 1k\Omega$, $R_G = 10k\Omega$, $(\frac{W}{L}) = 208$

7.58

$$
\begin{aligned}
P &= V_{DD} I_D = 2 \text{ mW} \\
I_D &= 1.11 \text{ mA} \\
R_D I_D &= 1 \text{ V} \\
R_D &= 900\ \Omega \\
A_v &= -g_m R_D \\
&= -\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\
&= -5 \\
\frac{W}{L} &= \boxed{69.44}
\end{aligned}
$$

(58) $|A_v| = g_m R_L$.

$\therefore$ To achieve maximum gain, use maximum R_L

i.e. set $R_D = 500\,\Omega$.

For maximum g_m, use maximum I_{DS}

(while keeping M_1 in saturation),

i.e. $V_D \geq V_G - V_{TH}$

$1.8 - (I_{DS})(500) \geq 1.8 - 0.4$,

$\therefore \quad I_{DS} \leq \frac{0.4}{500}$

$I_{DS,max} = 0.8\,mA$.

<u>Note</u>: Setting a large R_D in this case would force $I_{DS,max}$ to be lower (in order to keep M_1 in saturation).

But since $A_v \propto R_D$, while $A_v \propto \sqrt{I_{DS}}$, sacrificing I_{DS} to get higher R_D would yield a higher gain.

7.60 Let's let R_1 and R_2 consume exactly 5 % of the power budget (which means the branch containing R_D, M_1, and R_S will consume 95 % of the power budget). Let's also assume $V_{ov} = V_{GS} - V_{TH} = 300$ mV exactly.

$$
\begin{aligned}
I_D V_{DD} &= 0.95(2\text{ mW}) \\
I_D &= 1.056\text{ mA} \\
I_D R_S &= 200\text{ mV} \\
R_S &= \boxed{189.5\ \Omega} \\
V_{ov} &= V_{GS} - V_{TH} = 300\text{ mV} \\
I_D &= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{ov}^2 \\
\frac{W}{L} &= \boxed{117.3} \\
A_v &= -\frac{R_D}{\frac{1}{g_m} + R_S} \\
&= -\frac{R_D}{\frac{1}{\sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}} + R_S} \\
&= -4 \\
R_D &= \boxed{1.326\text{ k}\Omega} \\
\frac{V_{DD}^2}{R_1 + R_2} &= 0.05(2\text{ mW}) \\
R_1 + R_2 &= \frac{V_{DD}^2}{0.1\text{ mW}} \\
V_G &= V_{GS} + I_D R_S = V_{ov} + V_{TH} + I_D R_S = 0.9\text{ V} \\
V_G &= \frac{R_2}{R_1 + R_2}V_{DD} \\
&= \frac{R_2}{\frac{V_{DD}^2}{0.1\text{ mW}}} = 0.9\text{ V} \\
R_2 &= \boxed{29.16\text{ k}\Omega} \\
R_1 &= \boxed{3.24\text{ k}\Omega}
\end{aligned}
$$

7.61 Let's let R_1 and R_2 consume exactly 5 % of the power budget (which means the branch containing R_D, M_1, and R_S will consume 95 % of the power budget).

$$
\begin{aligned}
R_D &= 200\ \Omega \\
I_D V_{DD} &= 0.95(6\ \text{mW}) \\
I_D &= 3.167\ \text{mA} \\
I_D R_S &= V_{ov} = V_{GS} - V_{TH} \\
R_S &= \frac{V_{ov}}{I_D} \\
g_m &= \frac{2I_D}{V_{ov}} \\
A_v &= -\frac{R_D}{\frac{1}{g_m} + R_S} \\
&= -\frac{R_D}{\frac{V_{ov}}{2I_D} + \frac{V_{ov}}{I_D}} \\
&= -5 \\
V_{ov} &= 84.44\ \text{mV} \\
R_S &= \boxed{26.67\ \Omega} \\
\frac{W}{L} &= \frac{2I_D}{\mu_n C_{ox} V_{ov}^2} = \boxed{4441} \\
\frac{V_{DD}^2}{R_1 + R_2} &= 0.05(6\ \text{mW}) \\
R_1 + R_2 &= \frac{V_{DD}^2}{0.3\ \text{mW}} \\
V_G &= V_{GS} + I_D R_S = V_{ov} + V_{TH} + I_D R_S = 0.5689\ \text{V} \\
V_G &= \frac{R_2}{R_1 + R_2} V_{DD} \\
&= \frac{R_2}{\frac{V_{DD}^2}{0.3\ \text{mW}}} = 0.5689\ \text{V} \\
R_2 &= \boxed{6.144\ \text{k}\Omega} \\
R_1 &= \boxed{4.656\ \text{k}\Omega}
\end{aligned}
$$

7.62

$$\begin{aligned}
R_{in} &= R_1 = \boxed{20\ \text{k}\Omega} \\
P &= V_{DD} I_D = 2\ \text{mW} \\
I_D &= 1.11\ \text{mA} \\
V_{DS} &= V_{GS} - V_{TH} + 200\ \text{mV} \\
V_{DD} - I_D R_D &= V_{DD} - V_{TH} + 200\ \text{mV} \\
R_D &= 180\ \Omega \\
A_v &= -g_m R_D \\
&= -\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\
&= -6 \\
\frac{W}{L} &= \boxed{2500} \\
V_{GS} &= V_{TH} + \sqrt{\frac{2 I_D}{\mu_n C_{ox} \frac{W}{L}}} \\
&= 0.467\ \text{V} \\
V_{GS} &= V_{DD} - I_D R_S \\
R_S &= \boxed{1.2\ \text{k}\Omega} \\
\frac{1}{2\pi f C_1} &\ll R_1 \\
\frac{1}{2\pi f C_1} &= \frac{1}{10} R_1 \\
f &= 1\ \text{MHz} \\
C_1 &= \boxed{79.6\ \text{pF}} \\
\frac{1}{2\pi f C_S} \parallel R_S &\ll \frac{1}{g_m} \\
\frac{1}{2\pi f C_S} &= \frac{1}{10} \frac{1}{g_m} \\
g_m &= \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = 33.33\ \text{mS} \\
C_S &= \boxed{52.9\ \text{nF}}
\end{aligned}$$

(63). Power $(P) = 2mW$.

$\therefore I_{DS1} = |I_{DS2}| = \frac{2mW}{1.8V} = 1.11mA$.

$r_{o1} = r_{o2} = \frac{1}{\lambda I_{DS}}$

$= \frac{1}{0.1 \times 1.11 \times 10^{-3}}$

$= 9000\,\Omega$.

Gain $(A_v) = g_{m1}(r_{o1} \parallel r_{o2}) = 20$.

$g_{m1}\left(\frac{9000}{2}\right) = 20$.

$\therefore g_{m1} = 4.44\ mS$.

Set V_{DS1} (i.e. V_{out}) $= 1.2V$

(which is $< 1.5V$)

$\therefore V_{IN} = V_{GS1} \leq 1.2 + V_{TH}$

(for M_1 to stay in saturation)

Set $V_{GS1} = 1.2V$

$\therefore g_{m1} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH})$

$\left(\frac{W}{L}\right)_1 = 27.75$

For M_2, $\because M_2$ must be in saturation for $V_{out} \leq 1.5V$,

$\therefore V_{DD} - V_B \leq V_{DD} - 1.5V + V_{TH}$.

$\therefore V_B \geq 1.1V$

Set $V_B = 1.2V$

$$|I_{DS2}| = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (|V_{GS2}| - V_{TH})^2 (1 + \lambda |V_{DS2}|)$$

$$1.11 \times 10^{-3} = \frac{1}{2} \times 100 \times 10^{-6} \left(\frac{W}{L}\right)_2 (0.6 - 0.2)^2 (1 + 0.1 \times (1.8 - 1.5))$$

(assuming $V_{out} = 1.5V$)

$$\therefore \left(\frac{W}{L}\right)_2 \approx 135$$

$\therefore \left(\frac{W}{L}\right)_1 = 27.75 \quad \left(\frac{W}{L}\right)_2 = 135$

$V_{IN} = 1.2 \quad V_b = 1.1$

$I_{DS1} = I_{DS2} = 1.11mA$

7.64 (a)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel R_G \parallel r_{o2}\right)}$$

(b)

$$\begin{aligned}
P &= V_{DD} I_{D1} = 3 \text{ mW} \\
I_{D1} &= |I_{D2}| = 1.67 \text{ mA} \\
|V_{GS2}| &= |V_{DS2}| = V_{DS} = \frac{V_{DD}}{2} \\
|I_{D2}| &= \frac{1}{2}\mu_p C_{ox} \left(\frac{W}{L}\right)_2 \left(|V_{GS2}| - |V_{TH}|\right)^2 \left(1 + \lambda_p |V_{DS2}|\right) \\
\left(\frac{W}{L}\right)_2 &= \boxed{113} \\
A_v &= -g_{m1}\left(r_{o1} \parallel R_G \parallel r_{o2}\right) \\
R_G &= 10\left(r_{o1} \parallel r_{o2}\right) \\
r_{o1} &= \frac{1}{\lambda_n I_{D1}} = 6 \text{ k}\Omega \\
r_{o2} &= \frac{1}{\lambda_p |I_{D2}|} = 3 \text{ k}\Omega \\
R_G &= 10\left(r_{o1} \parallel r_{o2}\right) = \boxed{20 \text{ k}\Omega} \\
A_v &= -\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}\left(r_{o1} \parallel R_G \parallel r_{o2}\right) \\
&= -15 \\
\left(\frac{W}{L}\right)_1 &= \boxed{102.1} \\
V_{IN} &= V_{GS1} = V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(1 + \lambda_n V_{DS1}\right)}} \\
&= \boxed{0.787 \text{ V}}
\end{aligned}$$

(65) a) Impedance looking into drain of M_2

$$= (1 + g_{m2} r_{o2}) R_S + r_{o2}$$

$$= 10\, r_{o1}$$

Assume $g_{m2} r_{o2} \gg 1$,

$\therefore\ g_{m2} r_{o2} R_S + r_{o2} \approx 10 r_{o1}$

$\because\ r_{o1} = r_{o2}$ $(\lambda_1 = \lambda_2$ and $I_{DS1} = |I_{DS2}|)$

$\therefore\ g_{m2} R_S + 1 = 10$

$g_{m2} R_S = 9$ ——— (1)

Given $V_B = 1V$,

set $|V_{GS2}| = 0.6\,V$, (ie. $V_{GS2} - V_{TH} = 0.2V$)

$\therefore\ V_{S2} = 1.6V$.

$\therefore\ V_{R_S} = 1.8V - 1.6V = 0.2V$

$\because$ Power $= 2mW$

$I_{DS1} = |I_{DS2}| = \dfrac{2mW}{1.8V} = 1.11mA$.

$\therefore\ R_S = \dfrac{V_{R_S}}{1.11 \times 10^{-3}} \approx 180\,\Omega$ //

From (1), $g_{m2} = \dfrac{9}{180} = 50\,mS$.

$\because\ g_{m2} = \left(\frac{W}{L}\right)_2 (100 \times 10^{-6}) (V_{GS2} - V_{TH})$

$\therefore\ \left(\frac{W}{L}\right)_2 = 2500$ //

b). $|\text{gain } (A_v)| = g_{m1} (r_{o1} \parallel 10 r_{o1})$

$$30 = g_{m1} (0.909 \, r_{o1})$$

$$r_{o1} = \frac{1}{0.1 \times 1.11 \times 10^{-3}}$$

$$= 9009 \, \Omega$$

$$\therefore g_{m1} = 3.66 \text{ mS.}$$

$$\because g_{m1} = \sqrt{2(\mu_n C_{ox})\left(\frac{W}{L}\right)_1 \times I_{DS1}}$$

$$\therefore \left(\frac{W}{L}\right)_1 \approx 30.2$$

7.66

$$\begin{aligned}
P &= V_{DD} I_{D1} = 1 \text{ mW} \\
I_{D1} &= |I_{D2}| = 556 \ \mu\text{A} \\
V_{ov1} &= V_{GS1} - V_{TH} = \sqrt{2 I_D} \mu_n C_{ox} \left(\frac{W}{L} \right)_1 = 200 \text{ mV} \\
\left(\frac{W}{L} \right)_1 &= \boxed{138.9} \\
A_v &= -\frac{g_{m1}}{g_{m2}} \\
&= -\frac{\sqrt{2 \mu_n C_{ox} \left(\frac{W}{L} \right)_1 I_{D1}}}{\sqrt{2 \mu_n C_{ox} \left(\frac{W}{L} \right)_2 |I_{D2}|}} \\
&= -\sqrt{\frac{\left(\frac{W}{L} \right)_1}{\left(\frac{W}{L} \right)_2}} \\
&= -4 \\
\left(\frac{W}{L} \right)_2 &= \boxed{8.68} \\
V_{IN} &= V_{GS1} = V_{ov1} + V_{TH} = \boxed{0.6 \text{ V}}
\end{aligned}$$

7.67

$$P = V_{DD} I_D = 3 \text{ mW}$$

$$I_D = I_1 = \boxed{1.67 \text{ mA}}$$

$$R_{in} = \frac{1}{g_m} = \frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} = 50\ \Omega$$

$$\frac{W}{L} = \boxed{600}$$

$$A_v = g_m R_D = \frac{1}{50\ \Omega} R_D = 5$$

$$R_D = \boxed{250\ \Omega}$$

7.68

$$
\begin{aligned}
P &= V_{DD} I_D = 2 \text{ mW} \\
I_D &= 1.11 \text{ mA} \\
V_D &= V_G - V_{TH} + 100 \text{ mV} \\
V_{DD} - I_D R_D &= V_G - V_{TH} + 100 \text{ mV} \\
V_G &= V_{DD} \\
A_v &= g_m R_D = \frac{2 I_D}{V_{GS} - V_{TH}} R_D = 4 \\
R_D &= A_v \frac{V_{GS} - V_{TH}}{2 I_D} \\
V_{DD} - I_D A_v \frac{V_{GS} - V_{TH}}{2 I_D} &= V_{DD} - V_{TH} + 100 \text{ mV} \\
V_{GS} &= 0.55 \text{ V} \\
R_D &= \boxed{270\ \Omega} \\
V_S &= V_{DD} - V_{GS} = I_D R_S \\
R_S &= \boxed{1.125 \text{ k}\Omega} \\
\frac{W}{L} &= \frac{2 I_D}{\mu_n C_{ox} \left(V_{GS} - V_{TH}\right)^2} = \boxed{493.8}
\end{aligned}
$$

(69)

$$\text{Power} = 5\,mW$$

$$\therefore I_{DS_1} = \frac{5\times10^{-3}}{1.8} = 2.78\,mA$$

$$\text{Gain}\ (A_v) = g_m R_D = 5$$

$$V_{G_1} = V_{OUT} = 1.8 - IR_D$$

$$V_{S_1} = I R_S$$

$$\text{Let}\quad R_S = \frac{10}{g_{m_1}}$$

$$\therefore V_{S_1} = \frac{10 I}{g_{m_1}}$$

$$\therefore V_{GS_1} = 1.8 - IR_D - \frac{10 I}{g_{m_1}}$$

$$\because I_{DS} = \frac{1}{2} g_m (V_{GS} - V_{TH})$$

$$2.78\times10^{-3} = \frac{g_m}{2}\left(1.8 - 2.78\times10^{-3} R_D - \frac{2.78\times10^{-2}}{g_m}\right)$$

$$= 0.9\,g_m - 1.39\times10^{-3} g_m R_D - 1.39\times10^{-2}$$

$$\because g_m R_D = A_v = 5$$

$$2.78\times10^{-3} = 0.9\,g_m - 6.95\times10^{-3} - 1.39\times10^{-2}$$

$$\therefore g_m \approx 26.3\,mS$$

$$\text{and}\quad R_D = \frac{5}{26.3\times10^{-3}} \approx 190\,\Omega$$

$$R_S = \frac{10}{26.3\times10^{-3}} = 380\,\Omega$$

$$\because g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_{DS_1}} \Rightarrow \left(\frac{W}{L}\right) \approx 622$$

(70) $\because R_s \approx \frac{10}{g_m}$,

$\therefore R_{in} \approx \frac{1}{g_m} = 50\,\Omega$

i.e. $g_m = 20\,mS$ //

$$|\text{gain}(A_v)| = \frac{g_m R_D}{1 + g_m R_s} = 4$$

$$g_m R_D = 4 + 4 g_m R_s$$

$$R_D = \frac{4 + 0.08 R_s}{0.02} = 200 + 4R_s \quad \text{—(1)}$$

$\because R_s I_D + V_{GS} - V_{TH} + 0.25 = 1.8 - I_D R_D$ (given)

and $I_D = \frac{1}{2} g_m (V_{GS} - V_{TH})$

i.e. $V_{GS} - V_{TH} = 100\, I_D$

$\therefore R_s I_D + 100\, I_D + 0.25 = 1.8 - I_D R_D$

From (1):

$$R_s I_D + 100 I_D + 0.25 = 1.8 - 200 I_D - 4 I_D R_s$$

$$5 R_s I_D + 300 I_D = 1.55$$

Set $R_s = \frac{10}{g_m} = 500\,\Omega$

$\therefore 2500 I_D + 300 I_D = 1.55$

$\therefore I_D = 0.554\,mA$ //

$\because I_D = \frac{1}{2} g_m (V_{GS} - V_{TH})$

$$0.554 \times 10^{-3} = \frac{1}{2} \times 20 \times 10^{-3} (V_{GS} - 0.4)$$

$\therefore V_{GS} = 0.455\,V$

To find $(\frac{W}{L})$:

$$g_m = \sqrt{2 (\frac{W}{L}) \mu_n C_{ox} I_{DS}}$$

$$\therefore (\frac{W}{L}) \approx 1805$$

To find R_D:

$$\because R_D = 200 + 4R_S \quad \text{(from (1))}$$

$$R_D = 2200$$

To find R_1 and R_2,

$$\because R_1 + R_2 = 20\,k\Omega$$

and $V_{GS} = V_G - I_D R_S = 0.455V$

i.e. $V_G = 0.732\,V$

$$V_G = \frac{R_1}{R_1 + R_2} \times V_{DD}$$

$$\therefore R_1 = 8133\,\Omega$$

$$R_2 \approx 11.9\,k\Omega.$$

$\therefore R_1 = 8133\,\Omega$, $R_2 = 11.9\,k\Omega$, $R_D = 2200\,\Omega$, $R_S = 500\,\Omega$

$(\frac{W}{L}) = 1805$, $I_{DS} = 0.554\,mA$.

(7)

$R_{in} = R_G = 10\,k\Omega$

$Power = 2mW$

$\therefore I_{DS} = \frac{2mW}{1.8V} = 1.11mA$

$A_v = \frac{R_S}{\frac{1}{g_m} + R_S} = 0.8$

$\therefore R_S = \frac{4}{g_m}$ —— ①

$\because V_{OUT} = \frac{V_{DD}}{2} = I_{DS} R_S$

$I_{DS} R_S = 0.9$ —— ②

$\because V_G = 1.8V$ and $V_S = 0.9$

$\therefore V_{GS} = 0.9V$

From ②, $\because I_{DS} = 1.11mA$

$R_S = \frac{0.9V}{1.11mA} \approx 810\Omega$

From ①, $g_m = \frac{4}{810\Omega} \approx 4.94\,mS$

$\because g_m = \left(\frac{W}{L}\right)(\mu_n C_{ox})(V_{GS} - V_{TH})$

$\frac{W}{L} \approx 49.4$

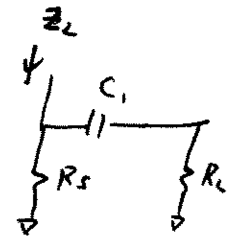
(72) $R_{in} = R_G = 20k\Omega$

$\because$ Power $= 3mW$

$\therefore I_{DS} = \dfrac{3mW}{1.8V} = 1.67mA$

$V_{x,\,at\,DC} = I_{DS} R_S = 0.9V$

$\therefore R_S = 540\Omega$



Load impedance, $Z_L = R_S // \left(\frac{1}{sC_1} + R_L\right)$
(at 100 MHz)

$= 540 // \left(\frac{1}{2\pi \times 10^8 C_1} + 50\right)$

Voltage gain $(A_v) = \dfrac{Z_L}{\frac{1}{g_m} + Z_L}$

$g_m = \dfrac{2 I_{DS}}{V_{GS} - V_{TH}}$

$= \dfrac{2 \times 1.67 \times 10^{-3}}{(1.8 - 0.9) - 0.4}$

$= 6.67 mS.$

$\therefore A_v = \dfrac{Z_L}{+ Z_L} = 0.8$

$Z_L = 120 + Z_L(0.8)$

$\therefore Z_L = 150$

$$\therefore 150 = 540 \,//\, \left(\frac{1}{2\pi\times 10^8 C_1} + 50\right)$$

$$= 540 \,//\, \left[\frac{1 + 50(2\pi\times 10^8 C_1)}{2\pi\times 10^8 C_1}\right]$$

$$= \frac{540 \times \frac{1+50(2\pi\times 10^8 C_1)}{2\pi\times 10^8 C_1}}{540 + \frac{1+50(2\pi\times 10^8 C_1)}{2\pi\times 10^8 C_1}}$$

$$\therefore \frac{1+50(2\pi\times 10^8 C_1)}{2\pi\times 10^8 C_1} \approx 208$$

$$\therefore C_1 \approx 10.1\,pF$$

To find $(\frac{W}{L})$:

$$\because g_m = (\tfrac{W}{L})\mu_n C_{ox}(V_{GS} - V_{TH})$$

$$\frac{W}{L} = 66.7$$

$$\therefore \frac{W}{L} = 66.7, \quad C_1 = 10.1\,pF, \quad R_S = 540\,\Omega.$$

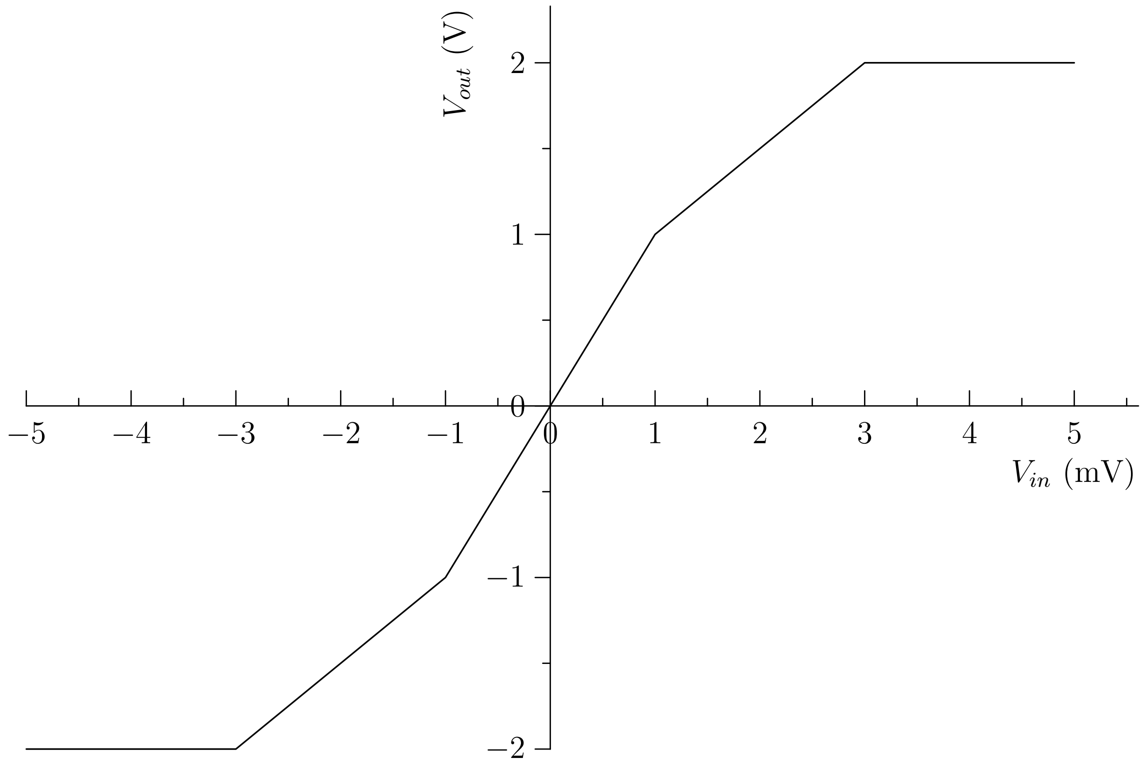
7.73

$$P = V_{DD} I_{D1} = 3\text{ mW}$$
$$I_{D1} = I_{D2} = 1.67\text{ mA}$$
$$A_v = \frac{r_{o1} \parallel r_{o2}}{\frac{1}{g_{m1}} + r_{o1} \parallel r_{o2}}$$
$$= \frac{r_{o1} \parallel r_{o2}}{\frac{1}{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}} + r_{o1} \parallel r_{o2}}$$
$$= 0.9$$
$$r_{o1} = r_{o2} = \frac{1}{\lambda I_{D1}} = 6\text{ k}\Omega$$
$$\left(\frac{W}{L}\right)_1 = \boxed{13.5}$$

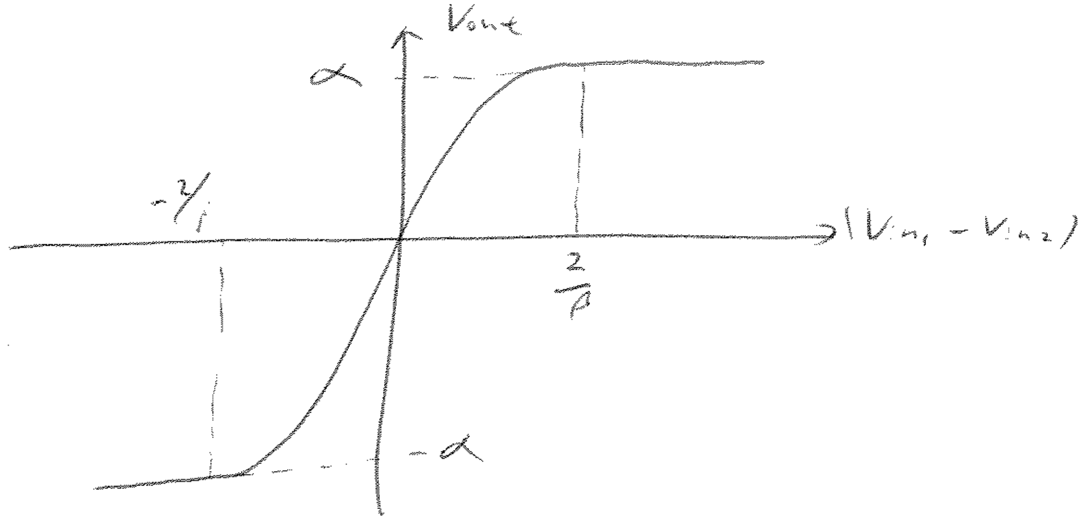
Let $V_{ov2} = V_{GS2} - V_{TH} = 0.3$ V. Let's assume that $V_{OUT} = V_{DS2} = V_{ov2}$.

$$V_{GS2} = V_b = V_{ov2} + V_{TH} = \boxed{0.7\text{ V}}$$
$$\left(\frac{W}{L}\right)_2 = \frac{2 I_{D2}}{\mu_n C_{ox} \left(V_{GS2} - V_{TH}\right)^2 \left(1 + \lambda V_{DS2}\right)}$$
$$= \boxed{161}$$
$$V_{GS1} = V_{TH} + \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(1 + \lambda V_{DS1}\right)}}$$
$$V_{DS1} = V_{DD} - V_{DS2} = 1.5\text{ V}$$
$$V_{GS1} = 1.44\text{ V}$$
$$V_{IN} = V_{GS1} + V_{DS2} = \boxed{1.74\text{ V}}$$

8.1



(2) $V_{out} = \alpha \tanh[\beta(V_{in1} - V_{in2})]$



To find small-signal gain,

$\because \tanh z = z - \frac{1}{3}z^3 + \frac{2}{15}z^5 + \cdots$

$\therefore$ for $\beta(V_{in1} - V_{in2}) \simeq 0$,

$$\frac{d V_{out}}{d(V_{in1}-V_{in2})} \simeq \frac{d}{d(V_{in1}-V_{in2})} \alpha\beta(V_{in1} - V_{in2})$$

$$= \underline{\underline{\alpha\beta}}$$

③ Closed-loop gain $= \left(1 + \frac{R_1}{R_2}\right)$

$$= 8$$

Gain error $= \left(1 + \frac{R_1}{R_2}\right)(A_0)^{-1}$

$$= \frac{8}{2000}$$

$$= \underline{\underline{0.4\%}}$$

④ Closed loop gain $= \left(1 + \frac{R_1}{R_2}\right)$

$$= 4$$

Gain error $= \left(1 + \frac{R_1}{R_2}\right)\left(\frac{1}{A_0}\right)$

$$= 0.1\%$$

$$\therefore \quad 4/A_0 = 0.1\%$$

$$A_0 = \underline{\underline{4000}}$$

⑤ Let $G_o = \left(1 + \frac{R_1}{R_2}\right)$

Desired gain $= \alpha_1$

$$= \frac{A_o}{1 + \frac{R_2}{R_1 + R_2} A_o}$$

$$\therefore \quad \alpha_1 = \frac{A_o}{1 + \frac{A_o}{G_o}}$$

$$1 + \frac{A_o}{G_o} = \frac{A_o}{\alpha_1}$$

$$\frac{1}{G_o} = \frac{1}{\alpha_1} - \frac{1}{A_o}$$

$$G_o = \frac{A_o \alpha_1}{A_o - \alpha_1}$$

$$\therefore \quad \frac{R_2}{R_1 + R_2} = \frac{1}{G_o} = \frac{1}{\alpha_1} - \frac{1}{A_o}$$ //

b) if A_o drops to $0.6 A_o$,

$$\text{Actual gain} = \frac{0.6 A_o}{1 + \left(\frac{1}{\alpha_1} - \frac{1}{A_o}\right) 0.6 A_o}$$

$$= \frac{0.6 A_o}{0.4 + \frac{0.6 A_o}{\alpha_1}}$$

(5) b) (cont'd)

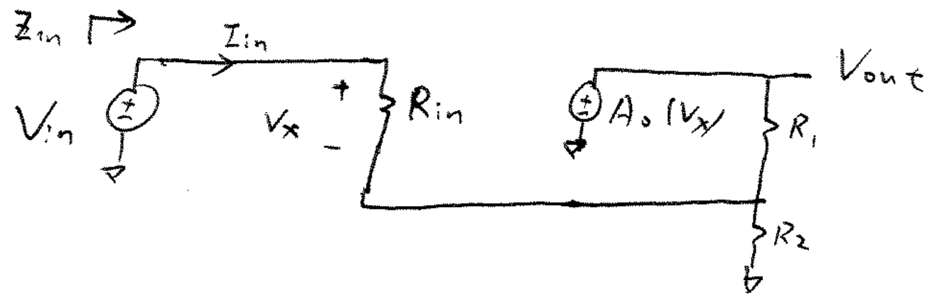
$$\text{Actual gain} = \frac{\alpha_1}{1 + \frac{0.4}{0.6}\frac{\alpha_1}{A_0}}$$

$$\simeq \alpha_1 \left(1 - \frac{0.4}{0.6} \frac{\alpha_1}{A_0}\right)$$

$\therefore$ the gain error $\simeq \frac{0.4}{0.6} \frac{\alpha_1^2}{A_0}$

$$= \frac{2}{3} \frac{\alpha_1^2}{A_0}$$

(6) Using the model in Fig. 8.44,



$$V_x = V_{in} - V_{out}\frac{R_1}{R_1+R_2}$$

$$V_{out} = A_0 V_x$$

$$= A_0\left(V_{in} - V_{out}\frac{R_1}{R_1+R_2}\right)$$

$$A_0 V_{in} = V_{out}\left(1 + A_0\frac{R_1}{R_1+R_2}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + A_0\frac{R_1}{R_1+R_2}} \quad \text{——— (1)}$$

To find input impedance (Z_{in}),

$$I_{in} = \frac{V_x}{R_{in}}$$

$$= \frac{1}{R_{in}}\left(V_{in} - V_{out}\frac{R_1}{R_1+R_2}\right)$$

$$= \frac{V_{in}}{R_{in}}\left(1 - \frac{V_{out}}{V_{in}}\frac{R_1}{R_1+R_2}\right)$$

⑥ (cont'd)

$$I_{in} = \frac{V_{in}}{R_{in}}\left(1 - \frac{A_0}{1 + A_0\frac{R_1}{R_1+R_2}} \frac{R_1}{R_1+R_2}\right)$$

$$= \frac{V_{in}}{R_{in}}\left(1 - \frac{1}{\frac{R_1+R_2}{A_0 R_1} + 1}\right)$$

$$= \frac{V_{in}}{R_{in}}\left(\frac{\frac{R_1+R_2}{A_0 R_1}}{\frac{R_1+R_2}{A_0 R_1} + 1}\right)$$

$$\therefore Z_{in} = \frac{V_{in}}{I_{in}} = R_{in}\left[\frac{1 + \frac{R_1+R_2}{A_0 R_1}}{\frac{R_1+R_2}{A_0 R_1}}\right] \quad \text{— ②}$$

As $A_0 \to \infty$,

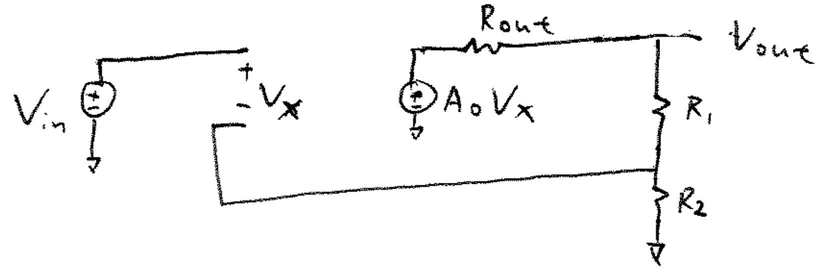
$$\text{gain} = \left.\frac{V_{out}}{V_{in}}\right|_{A_0 \to \infty} \quad [\text{From ①}]$$

$$= 1 + \frac{R_2}{R_1}$$

$$Z_{in} = \left.\frac{V_{in}}{I_{in}}\right|_{A_0 \to \infty} \quad [\text{From ②}]$$

$$= \infty$$

(7)



Similar to Prob. (6),

$$\text{gain} = \frac{V_{out}}{V_{in}}$$

$$V_x = V_{in} - V_{out} \frac{R_2}{R_1 + R_2}$$

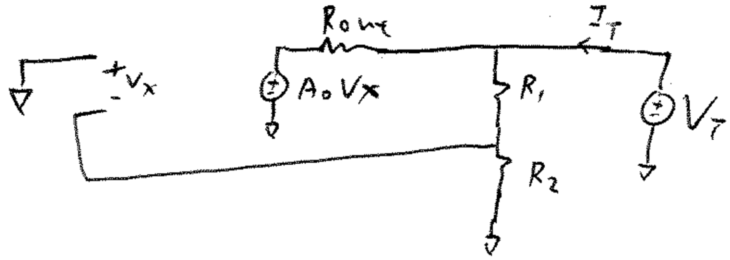
$$V_{out} = A_0 V_x \frac{R_1 + R_2}{R_{out} + R_1 + R_2}$$

$$= A_0 \left(V_{in} - V_{out} \frac{R_2}{R_1 + R_2} \right) \frac{R_1 + R_2}{R_{out} + R_1 + R_2}$$

$$V_{in} A_0 \frac{R_1 + R_2}{R_{out} + R_1 + R_2} = V_{out} \left(1 + \frac{A_0 R_2}{R_{out} + R_1 + R_2} \right)$$

$$\frac{V_{out}}{V_{in}} = \frac{A_0 \frac{R_1 + R_2}{R_{out} + R_1 + R_2}}{1 + \frac{A_0 R_2}{R_{out} + R_1 + R_2}}$$

To find output impedance (Z_{out})



⑦ (cont'd)

$$V_x = \frac{R_2}{R_1 + R_2} V_T$$

$$I_T = \frac{V_T}{R_1 + R_2} + \frac{V_T - A_o V_x}{R_{out}}$$

$$= V_T \left[\frac{R_{out} + R_1 + R_2 - A_o R_2}{(R_{out})(R_1 + R_2)} \right]$$

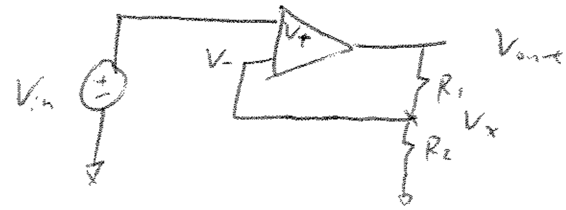
$$Z_{out} = \frac{V_T}{I_T} = \frac{(R_{out})(R_1 + R_2)}{R_{out} + R_1 + R_2 - A_o R_2}$$

As $A_o \to \infty$,

$$\text{gain} = 1 + \frac{R_1}{R_2}$$ //

$$Z_{out} = 0$$ //

(8)



ΔR for now.

$$V_{out} = A_o (V_x)$$

$$V_x = V_{in} - \frac{R_2}{R_1 + R_2} V_{out}$$

$$\therefore \frac{-V_{out}}{A_o} = V_{in} - \frac{R_1}{R_1 + R_2} V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{A_o (R_1 + R_2)}{A_o R_1 - 1} = \text{nominal gain}$$

if $R_2' = \Delta R + R_2$

$$\left(\frac{V_{out}}{V_{in}}\right)' = \frac{A_o (R_1 + \Delta R + R_2)}{A_o R_1 - 1}$$

$$\therefore \text{gain error} = \frac{\left(\frac{V_{out}}{V_{in}}\right)' - \left(\frac{V_{out}}{V_{in}}\right)}{\frac{V_{out}}{V_{in}}}$$

$$= \frac{\Delta R}{A_o R_1 - 1} \times \frac{A_o R_1 - 1}{A_o (R_1 + R_2)}$$

$$= \frac{\Delta R}{A_o (R_1 + R_2)} \quad //$$

(9) Closed-loop gain $\approx \left(1+\frac{R_1}{R_2}\right)\left[1-\left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0}\right]$

$$= 5\left[1-\frac{5}{A_0}\right]$$

$\therefore$ As A_0 decreases to $0.8A_0$, closed-loop gain decreases along (deviating more from the norminal

A_0 drops to $0.8A_0$ when $|V_{in1}-V_{in2}| = 2mV$.

$\because \quad V_{in2} = V_{out}\left(\frac{R_2}{R_1+R_2}\right)$

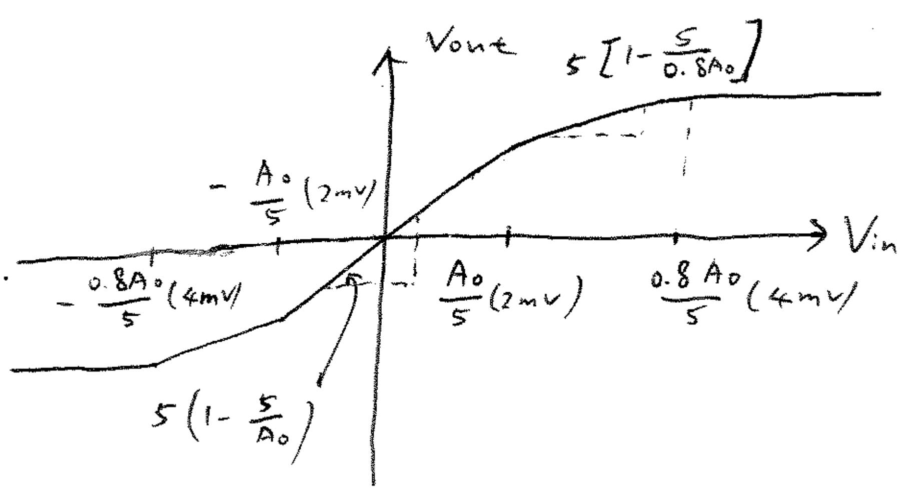
and $\quad V_{out} = 5\left(1-\frac{5}{A_0}\right)V_{in1}$

$\therefore \quad V_{in2} = 5\left(1-\frac{5}{A_0}\right)\left(\frac{1}{5}\right)V_{in1}$

$V_{in1} - V_{in2} = \frac{5}{A_0}V_{in1}$

At $\quad V_{in1} - V_{in2} = 2mV$,

$$V_{in1} = \frac{A_0}{5}(2mV)$$



(10) $$\frac{V_{out}}{V_{in}} = 1 + \frac{R_1}{R_S}$$

$\because V_{in} = 1V,$ $$V_{out} = 1 + \frac{R_1}{R_0 + \alpha W}$$

$$\frac{dV_{out}}{dW} = -R_1 \alpha \left(R_0 + \alpha W \right)^{-2}.$$

$$= \frac{-R_1 \alpha}{(R_0 + \alpha W)^2}$$

8.11

$$\begin{aligned}
V_- &= V_+ = V_{in} \\
V_- &= \frac{R_4 \parallel (R_2 + R_3)}{R_1 + R_4 \parallel (R_2 + R_3)} \frac{R_2}{R_2 + R_3} V_{out} = V_{in} \\
\frac{V_{out}}{V_{in}} &= \left[\frac{R_4 \parallel (R_2 + R_3)}{R_1 + R_4 \parallel (R_2 + R_3)} \frac{R_2}{R_2 + R_3} \right]^{-1} \\
&= \boxed{\frac{(R_2 + R_3) \left[R_1 + R_4 \parallel (R_2 + R_3)\right]}{R_2 \left[R_4 \parallel (R_2 + R_3)\right]}}
\end{aligned}$$

If $R_1 \to 0$, we expect the result to be:

$$\begin{aligned}
V_{in} &= \frac{R_2}{R_2 + R_3} V_{out} \\
\left.\frac{V_{out}}{V_{in}}\right|_{R_1 = 0} &= \frac{R_2 + R_3}{R_2} = 1 + \frac{R_3}{R_2}
\end{aligned}$$

Taking limit of the original expression as $R_1 \to 0$, we have:

$$\begin{aligned}
\lim_{R_1 \to 0} \frac{(R_2 + R_3) \left[R_1 + R_4 \parallel (R_2 + R_3)\right]}{R_2 \left[R_4 \parallel (R_2 + R_3)\right]} &= \frac{(R_2 + R_3) \left[R_4 \parallel (R_2 + R_3)\right]}{R_2 \left[R_4 \parallel (R_2 + R_3)\right]} \\
&= 1 + \frac{R_3}{R_2}
\end{aligned}$$

This agrees with the expected result. Likewise, if $R_3 \to 0$, we expect the result to be:

$$\begin{aligned}
V_{in} &= \frac{R_2 \parallel R_4}{R_1 + R_2 \parallel R_4} V_{out} \\
\left.\frac{V_{out}}{V_{in}}\right|_{R_3 = 0} &= \frac{R_1 + R_2 \parallel R_4}{R_2 \parallel R_4} \\
&= 1 + \frac{R_1}{R_2 \parallel R_4}
\end{aligned}$$

Taking the limit of the original expression as $R_3 \to 0$, we have:

$$\begin{aligned}
\lim_{R_3 \to 0} \frac{(R_2 + R_3) \left[R_1 + R_4 \parallel (R_2 + R_3)\right]}{R_2 \left[R_4 \parallel (R_2 + R_3)\right]} &= \frac{R_2 \left(R_1 + R_2 \parallel R_4\right)}{R_2 \left(R_2 \parallel R_4\right)} \\
&= \frac{R_1 + R_2 \parallel R_4}{R_2 \parallel R_4} \\
&= 1 + \frac{R_1}{R_2 \parallel R_4}
\end{aligned}$$

This agrees with the expected result.

(12) Gain Error $= \frac{1}{A_0}\left(1 + \frac{R_1}{R_2}\right)$

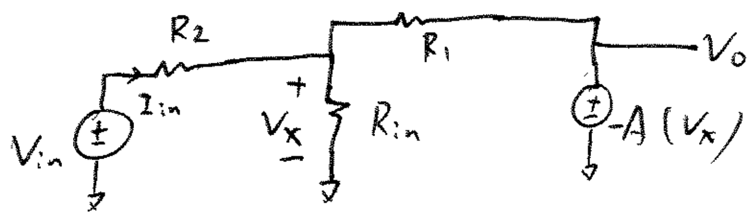
$= \frac{1}{A_0}(1 + 8)$

$= 0.2\%$

$\therefore \frac{1}{A_0}(9) = 0.2\%$

$A_0 = 4500$

(13)



$$V_o = -A V_x \qquad \text{—— (1)}$$

$$\frac{V_{in} - V_x}{R_2} + \frac{V_o - V_x}{R_1} = \frac{V_x}{R_{in}} \qquad \text{—— (2)}$$

Combining (1) and (2),

$$\frac{V_{in}}{R_2} = -\frac{V_o}{R_1} + \frac{V_o}{(-A)}\left(\frac{1}{R_{in}} + \frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{V_{in}}{R_2} = V_o\left[\frac{A R_{in} R_2 + R_1 R_2 + R_{in} R_2 + R_{in} R_1}{(-A) R_{in} R_1 R_2}\right]$$

$$\frac{V_o}{V_{in}} = -\frac{A R_{in} R_1}{R_1 R_2 + R_{in} R_2 + R_{in} R_1 + A R_{in} R_2}$$

Input impedance $(Z_{in}) = \dfrac{V_{in}}{I_{in}}$

$$I_{in} - \frac{V_x}{R_{in}} + \frac{(-A)V_x - V_x}{R_1} = 0$$

$$I_{in} = V_x\left[\frac{1}{R_{in}} + \frac{A+1}{R_1}\right]$$

$$\because \quad V_x = V_{in} - I_{in} R_2$$

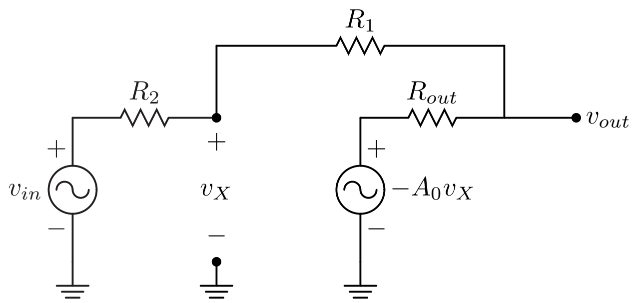
$$I_{in} = \left[V_{in} - I_{in} R_2 \right] \left[\frac{1}{R_{in}} + \frac{A+1}{R_1} \right]$$

$$I_{in} \left[1 + \frac{R_2}{R_{in}} + \frac{R_2}{R_1}(A+1) \right] = V_{in} \left(\frac{1}{R_{in}} + \frac{A+1}{R_1} \right)$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{1 + \frac{R_2}{R_{in}} + \frac{R_2}{R_1}(A+1)}{\frac{1}{R_{in}} + \frac{A+1}{R_1}}$$

//

8.14 We need to derive the closed-loop gain of the following circuit:



$$v_X = (v_{out} - v_{in}) \frac{R_2}{R_1 + R_2} + v_{in}$$

$$v_{out} = (-A_0 v_X - v_{in}) \frac{R_1 + R_2}{R_{out} + R_1 + R_2} + v_{in}$$

$$= \left\{ -A_0 \left[(v_{out} - v_{in}) \frac{R_2}{R_1 + R_2} + v_{in} \right] - v_{in} \right\} \frac{R_1 + R_2}{R_{out} + R_1 + R_2} + v_{in}$$

Grouping terms, we have:

$$v_{out} \left[1 + A_0 \frac{R_2}{R_1 + R_2} \frac{R_1 + R_2}{R_{out} + R_1 + R_2} \right] = v_{in} \left(\frac{R_1 + R_2}{R_{out} + R_1 + R_2} \right) \left[A_0 \frac{R_2}{R_1 + R_2} - A_0 - 1 + \frac{R_{out} + R_1 + R_2}{R_1 + R_2} \right]$$

$$= v_{in} \left(\frac{R_1 + R_2}{R_{out} + R_1 + R_2} \right) \left[\frac{R_{out} + R_1 + R_2}{R_1 + R_2} - A_0 \frac{R_1}{R_1 + R_2} - 1 \right]$$

$$= v_{in} \frac{1}{R_{out} + R_1 + R_2} \left[R_{out} + R_1 + R_2 - A_0 R_1 - R_1 - R_2 \right]$$

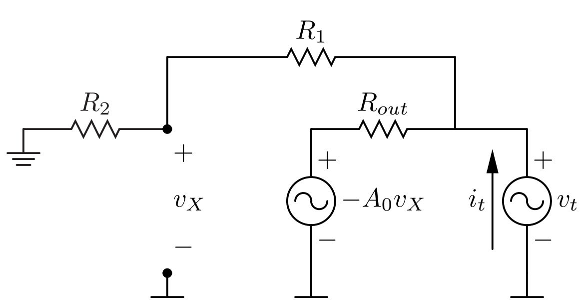
$$= v_{in} \left[1 - \frac{A_0 R_1 + R_1 + R_2}{R_{out} + R_1 + R_2} \right]$$

$$\frac{v_{out}}{v_{in}} = \frac{1 - \frac{A_0 R_1 + R_1 + R_2}{R_{out} + R_1 + R_2}}{1 + \frac{A_0 R_2}{R_{out} + R_1 + R_2}}$$

$$= \frac{R_{out} + R_1 + R_2 - A_0 R_1 - R_1 - R_2}{R_{out} + R_1 + R_2 + A_0 R_2}$$

$$= \boxed{\frac{R_{out} - A_0 R_1}{R_{out} + R_1 + (1 + A_0) R_2}}$$

To find the output impedance, we must find $Z_{out} = \frac{v_t}{i_t}$ for the following circuit:



$$
\begin{aligned}
i_t &= \frac{v_t + A_0 v_X}{R_{out}} + \frac{v_t}{R_1 + R_2} \\
v_X &= \frac{R_2}{R_1 + R_2} v_t \\
i_t &= \frac{v_t + A_0 \frac{R_2}{R_1 + R_2} v_t}{R_{out}} + \frac{v_t}{R_1 + R_2} \\
&= v_t \left(\frac{1}{R_{out}} + \frac{A_0 R_2}{R_{out} (R_1 + R_2)} + \frac{1}{R_1 + R_2} \right) \\
&= v_t \frac{R_1 + (1 + A_0) R_2 + R_{out}}{R_{out} (R_1 + R_2)} \\
Z_{out} &= \frac{v_t}{i_t} = \boxed{\frac{R_{out} (R_1 + R_2)}{R_1 + (1 + A_0) R_2 + R_{out}}}
\end{aligned}
$$

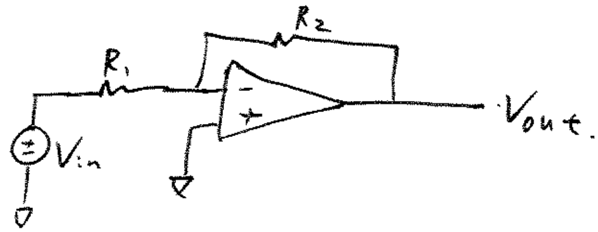
8.15 Refer to the analysis for Fig. 8.42.

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_1}{R_2} = 4$$
$$R_{in} \approx R_2 = 10 \text{ k}\Omega$$
$$R_1 = 4R_2 = 40 \text{ k}\Omega$$

From Eq. (8.99), we have

$$\mathcal{E} = 1 - \frac{A_0 - \frac{R_{out}}{R_1}}{1 + \frac{R_{out}}{R_2} + A_0 + \frac{R_1}{R_2}}$$
$$A_0 = 1000$$
$$R_{out} = 1 \text{ k}\Omega$$
$$\mathcal{E} = \boxed{0.51\ \%}$$

(16)



Nominal gain $= \frac{R_2}{R_1} = 8$ —— ①

$R_2 = 8R_1$

Input impedance $\approx R_1 = 1000\ \Omega$ —— ②

$\therefore R_2 = 8000\ \Omega$.

gain error $= 0.1\%$ —— ③

$\therefore \frac{1}{A_0}\left(1 + \frac{R_2}{R_1}\right) = 0.1\%$

$\frac{1}{A_0}(9) = \frac{0.1}{100}$

$\therefore A_0 = 9000$ //

8.17

$$V_+ = V_- \text{ (since } A_0 = \infty)$$
$$\frac{V_{in}}{R_2} = -\frac{V_{out}}{R_3}\frac{R_3 \parallel R_4}{R_1 + R_3 \parallel R_4}$$
$$\frac{V_{out}}{V_{in}} = \boxed{-\frac{R_3}{R_2}\frac{R_1 + R_3 \parallel R_4}{R_3 \parallel R_4}}$$

If $R_1 \to 0$ or $R_3 \to 0$, we expect the amplifier to reduce to the standard inverting amplifier.

$$\left.\frac{V_{out}}{V_{in}}\right|_{R_1 \to 0} = -\frac{R_3}{R_2}$$
$$\left.\frac{V_{out}}{V_{in}}\right|_{R_3 \to 0} = -\frac{R_1}{R_2}$$

The gain reduces to the expected expressions.

8.18

$$
\begin{aligned}
V_+ &= V_- \text{ (since } A_0 = \infty) \\
V_X &= \frac{R_3}{R_3 + R_4} V_{out} = \frac{R_2}{R_1 + R_2} \left(V_{out} - V_{in} \right) + V_{in} \\
V_{out} \left(\frac{R_3}{R_3 + R_4} - \frac{R_2}{R_1 + R_2} \right) &= V_{in} \left(1 - \frac{R_2}{R_1 + R_2} \right) \\
V_{out} \left[\frac{R_3 \left(R_1 + R_2 \right) - R_2 \left(R_3 + R_4 \right)}{\left(R_1 + R_2 \right) \left(R_3 + R_4 \right)} \right] &= V_{in} \left(\frac{R_1}{R_1 + R_2} \right) \\
\frac{V_{out}}{V_{in}} &= \boxed{\frac{R_1 \left(R_3 + R_4 \right)}{R_3 \left(R_1 + R_2 \right) - R_2 \left(R_3 + R_4 \right)}}
\end{aligned}
$$

(19) From eq. (8.31),

$$V_{out} = -\frac{1}{R_1 C_1} \int V_{in} \, dt$$

$$= -\frac{1}{R_1 C_1} \int V_0 \sin \omega t \, dt$$

$$= \frac{V_0}{R_1 C_1 \omega} \cos \omega t$$

$$\therefore \text{Amplitude of output} = \frac{V_0}{R_1 C_1 \omega}$$

(20) From prob. (19)

Amplification of the integrator $= \frac{1}{R_1 C_1 \omega}$

$$\therefore \quad \frac{1}{R_1 C_1 \omega} = 10$$

$$\frac{1}{\omega} = 10 \times 10\,ns$$

$$\therefore \omega = 10\,MHz.$$

$\therefore$ The frequency of the sinusoid is 10MHz.

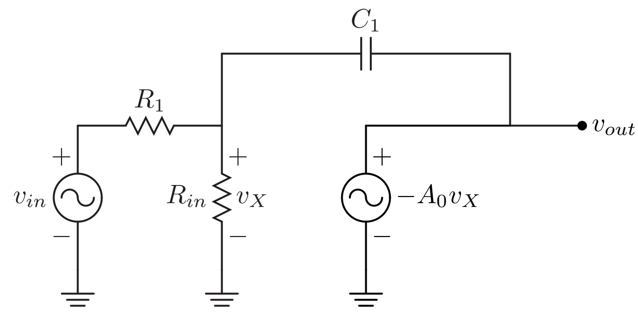
(21) From Eq. (8.37)

$$s_p = \frac{-1}{2\pi (A_0 + 1) R_1 C_1} \leq -1 \text{ Hz}.$$

$$\therefore \quad 2\pi (A_0 + 1)(10\text{ k}\Omega)(1\text{ nF}) \geq 1$$

$$A_0 \geq 15915$$

8.22 We must find the transfer function of the following circuit:



$$
\begin{aligned}
v_{out} &= -A_0 v_X \\
v_X &= v_{out} - \frac{1}{sC_1}\left(\frac{v_X}{R_{in}} + \frac{v_X - v_{in}}{R_1}\right) \\
v_X\left(1 + \frac{1}{sR_{in}C_1} + \frac{1}{sR_1C_1}\right) &= v_{out} + \frac{v_{in}}{sR_1C_1} \\
v_X &= \frac{sR_1R_{in}C_1 v_{out} + R_{in}v_{in}}{sR_1R_{in}C_1 + R_1 + R_{in}} \\
v_{out} &= -A_0\frac{sR_1R_{in}C_1 v_{out} + R_{in}v_{in}}{sR_1R_{in}C_1 + R_1 + R_{in}} \\
v_{out}\left(1 + A_0\frac{sR_1R_{in}C_1}{sR_1R_{in}C_1 + R_1 + R_{in}}\right) &= -A_0 v_{in}\frac{R_{in}}{sR_1R_{in}C_1 + R_1 + R_{in}} \\
\frac{v_{out}}{v_{in}} &= \frac{-A_0R_{in}}{sR_1R_{in}C_1 + R_1 + R_{in}} \cdot \frac{sR_1R_{in}C_1 + R_1 + R_{in}}{sR_1R_{in}C_1 + R_1 + R_{in} + sR_1R_{in}C_1A_0} \\
&= \frac{-A_0R_{in}}{sR_1R_{in}C_1 + R_1 + R_{in} + sR_1R_{in}C_1A_0} \\
&= \frac{-A_0R_{in}}{sR_1R_{in}C_1(1 + A_0) + R_1 + R_{in}} \\
&= \frac{-A_0R_{in}}{1 + s\frac{R_1R_{in}C_1(1+A_0)}{R_1+R_{in}}} \\
&= \boxed{\frac{-A_0R_{in}/(R_1 + R_{in})}{1 + s(R_1 \parallel R_{in})C_1(1 + A_0)}} \\
s_p &= \boxed{-\frac{1}{(R_1 \parallel R_{in})C_1(1 + A_0)}}
\end{aligned}
$$

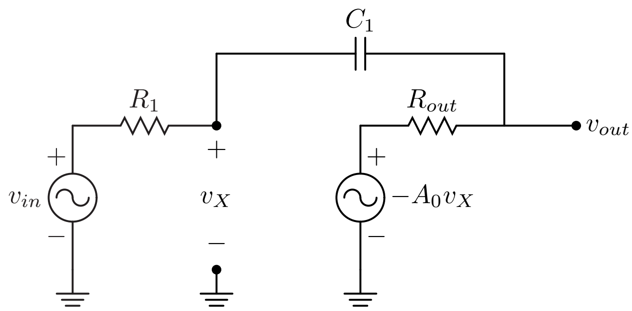
Comparing this to the result in Eq. (8.37), we can see that we can simply replace R_1 with $R_1 \parallel R_{in}$, effectively increasing the pole frequency (since $R_1 \parallel R_{in} < R_1$ for finite R_{in}).

We can also write the result as

$$s_p = -\frac{1}{R_1C_1(1 + A_0)}\left(1 + \frac{R_1}{R_{in}}\right)$$

In this form, it's clear that the pole frequency increases by $1 + R_1/R_{in}$.

8.23 We must find the transfer function of the following circuit:



$$v_{out} = -A_0 v_X + \frac{v_{in} - v_{out}}{R_1 + \frac{1}{sC_1}} R_{out}$$

$$v_X = v_{in} + \frac{R_1}{R_1 + \frac{1}{sC_1}} \left(v_{out} - v_{in}\right)$$

$$v_{out} = -A_0 \left[v_{in} + \frac{R_1}{R_1 + \frac{1}{sC_1}} \left(v_{out} - v_{in}\right)\right] + \frac{v_{in} - v_{out}}{R_1 + \frac{1}{sC_1}} R_{out}$$

$$v_{out} \left[1 + \frac{A_0 R_1 + R_{out}}{R_1 + \frac{1}{sC_1}}\right] = v_{in} \left[-A_0 + \frac{A_0 R_1 + R_{out}}{R_1 + \frac{1}{sC_1}}\right]$$

$$v_{out} \frac{R_1 + \frac{1}{sC_1} + A_0 R_1 + R_{out}}{R_1 + \frac{1}{sC_1}} = v_{in} \frac{-A_0 R_1 - A_0 \frac{1}{sC_1} + A_0 R_1 + R_{out}}{R_1 + \frac{1}{sC_1}}$$

$$v_{out} \left\{1 + sC_1 \left[\left(1 + A_0\right) R_1 + R_{out}\right]\right\} = -v_{in} \left\{A_0 - sC_1 R_{out}\right\}$$

$$\frac{v_{out}}{v_{in}} = \boxed{-\frac{A_0 - sC_1 R_{out}}{1 + sC_1 \left[\left(1 + A_0\right) R_1 + R_{out}\right]}}$$

$$s_p = \boxed{-\frac{1}{C_1 \left[\left(1 + A_0\right) R_1 + R_{out}\right]}}$$

Comparing this to the result in Eq. (8.37), we can see that the pole gets reduced in magnitude due to R_{out}.

(24) $\because A_o = \infty$

$$|A_v| = \frac{R_1}{\frac{1}{\omega C_1}}$$

$$= \omega R_1 C_1$$

$$= 5$$

$$\therefore R_1 C_1 = \frac{5}{\omega}$$

$$= \frac{5}{2\pi \times 10^6}$$

$$= 7.958 \times 10^{-7}$$

(25) From eqn (8.55)

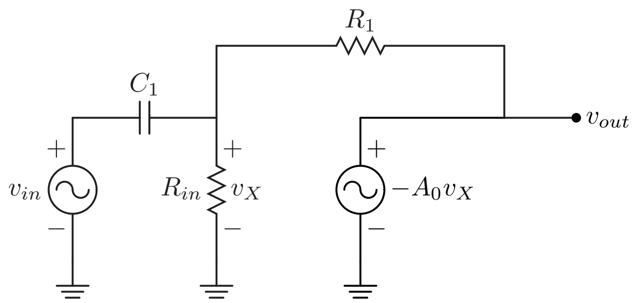
$$S_p = -\frac{A_0+1}{R_1 C_1}$$

$$2\pi \times 100 \times 10^6 = \frac{A_0+1}{1000 \times 10^{-9}}$$

(ie. R_1 and C_1 are chosen at minimum)

$$A_0 \approx 627$$

8.26 We must find the transfer function of the following circuit:



$$v_{out} = -A_0 v_X$$

$$v_X = \left[(v_{in} - v_X) sC_1 - \frac{v_X - v_{out}}{R_1}\right] R_{in}$$

$$v_X \left[1 + sR_{in}C_1 + \frac{R_{in}}{R_1}\right] = v_{in} sR_{in}C_1 + v_{out}\frac{R_{in}}{R_1}$$

$$v_X = \frac{v_{in} sR_{in}C_1 + v_{out}\frac{R_{in}}{R_1}}{1 + sR_{in}C_1 + \frac{R_{in}}{R_1}}$$

$$v_{out} = -A_0 \frac{v_{in} sR_{in}C_1 + v_{out}\frac{R_{in}}{R_1}}{1 + sR_{in}C_1 + \frac{R_{in}}{R_1}}$$

$$v_{out}\left[1 + \frac{A_0\frac{R_{in}}{R_1}}{1 + sR_{in}C_1 + \frac{R_{in}}{R_1}}\right] = -v_{in}\frac{sR_{in}C_1A_0}{1 + sR_{in}C_1 + \frac{R_{in}}{R_1}}$$

$$v_{out}\left[\frac{1 + sR_{in}C_1 + (1 + A_0)\frac{R_{in}}{R_1}}{1 + sR_{in}C_1 + \frac{R_{in}}{R_1}}\right] = -v_{in}\frac{sR_{in}C_1A_0}{1 + sR_{in}C_1 + \frac{R_{in}}{R_1}}$$

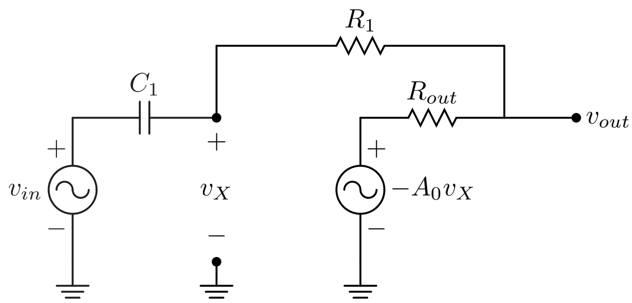
$$v_{out}\left[1 + sR_{in}C_1 + (1 + A_0)\frac{R_{in}}{R_1}\right] = -v_{in}sR_{in}C_1A_0$$

$$\frac{v_{out}}{v_{in}} = \boxed{-\frac{sR_1R_{in}C_1A_0}{R_1 + sR_1R_{in}C_1 + (1 + A_0)R_{in}}}$$

$$\lim_{A_0\to\infty}\frac{v_{out}}{v_{in}} = -sR_1C_1$$

Comparing this to Eq. (8.42), we can see that if we let $A_0 \to \infty$, the result actually reduces to Eq. (8.42).

8.27 We must find the transfer function of the following circuit:



$$v_{out} = -A_0 v_X + \frac{v_{in} - v_{out}}{R_1 + \frac{1}{sC_1}} R_{out}$$

$$v_X = v_{in} + \frac{\frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} (v_{out} - v_{in})$$

$$v_{out} = -A_0 \left[v_{in} + \frac{\frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} (v_{out} - v_{in}) \right] + \frac{v_{in} - v_{out}}{R_1 + \frac{1}{sC_1}} R_{out}$$

$$v_{out} \left[1 + \frac{A_0 \frac{1}{sC_1} + R_{out}}{R_1 + \frac{1}{sC_1}} \right] = v_{in} \left[-A_0 + \frac{A_0 \frac{1}{sC_1} + R_{out}}{R_1 + \frac{1}{sC_1}} \right]$$

$$v_{out} \frac{R_1 + \frac{1}{sC_1} + A_0 \frac{1}{sC_1} + R_{out}}{R_1 + \frac{1}{sC_1}} = v_{in} \frac{-A_0 R_1 - A_0 \frac{1}{sC_1} + A_0 \frac{1}{sC_1} + R_{out}}{R_1 + \frac{1}{sC_1}}$$

$$v_{out} \left\{ 1 + A_0 + sC_1 (R_1 + R_{out}) \right\} = -v_{in} \left\{ sC_1 (A_0 R_1 - R_{out}) \right\}$$

$$\frac{v_{out}}{v_{in}} = \boxed{-\frac{sC_1 (A_0 R_1 - R_{out})}{1 + A_0 + sC_1 (R_1 + R_{out})}}$$

$$\lim_{A_0 \to \infty} \frac{v_{out}}{v_{in}} = -sR_1C_1$$

Comparing this to Eq. (8.42), we can see that if we let $A_0 \to \infty$, the result actually reduces to Eq. (8.42).

8.28

$$\begin{aligned}
v_{out} &= -A_0 v_- \\
v_- &= v_{in} + (v_{out} - v_{in}) \frac{\frac{1}{sC_1} \parallel R_1}{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)} \\
v_{out} &= -A_0 \left[v_{in} + (v_{out} - v_{in}) \frac{\frac{1}{sC_1} \parallel R_1}{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)} \right] \\
v_{out} \left[1 + A_0 \frac{\frac{1}{sC_1} \parallel R_1}{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)} \right] &= -v_{in} A_0 \left[1 - \frac{\frac{1}{sC_1} \parallel R_1}{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)} \right] \\
v_{out} \frac{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right) + A_0 \left(\frac{1}{sC_1} \parallel R_1\right)}{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)} &= -v_{in} A_0 \frac{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right) - \left(\frac{1}{sC_1} \parallel R_1\right)}{\left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)} \\
v_{out} \left\{ (1 + A_0) \left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right) \right\} &= -v_{in} A_0 \left(\frac{1}{sC_2} \parallel R_2\right) \\
\frac{v_{out}}{v_{in}} &= \boxed{-A_0 \frac{\frac{1}{sC_2} \parallel R_2}{(1 + A_0) \left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right)}}
\end{aligned}$$

Unity gain occurs when the numerator and denominator are the same (note that we can drop the negative sign since we only care about the magnitude of the gain):

$$\begin{aligned}
A_0 \left(\frac{1}{sC_2} \parallel R_2\right) &= (1 + A_0) \left(\frac{1}{sC_1} \parallel R_1\right) + \left(\frac{1}{sC_2} \parallel R_2\right) \\
(A_0 - 1) \left(\frac{1}{sC_2} \parallel R_2\right) &= (1 + A_0) \left(\frac{1}{sC_1} \parallel R_1\right) \\
\frac{\left(\frac{1}{sC_2} \parallel R_2\right)}{\left(\frac{1}{sC_1} \parallel R_1\right)} &= \frac{A_0 + 1}{A_0 - 1}
\end{aligned}$$

It is possible to obtain unity gain by choosing the resistors and capacitors according to the above formula.

(29) if $A_0 < \infty$,

Let V_- be the voltage at the negative input terminal of the opamp.

By KCL,

$$\frac{V_{in} - V_-}{R_1 // \frac{1}{sC_1}} = - \frac{V_{out} - V_-}{R_2 // \frac{1}{sC_2}}$$

$$V_{out} = -A_0 V_-,$$

$$\frac{V_{in} + \frac{V_{out}}{A_0}}{R_1 // \frac{1}{sC_1}} = - \frac{V_{out} + \frac{V_{out}}{A_0}}{R_2 // \frac{1}{sC_2}}$$

$$V_{in} = -\left[R_1 // \frac{1}{sC_1}\right] \left[\frac{\left(R_2 // \frac{1}{sC_2}\right) \frac{V_{out}}{A_0} + V_{out} + \frac{V_{out}}{A_0}}{R_2 // \frac{1}{sC_2}}\right.$$

$$\therefore \frac{V_{out}}{V_{in}} = - \frac{R_2 // \frac{1}{sC_2}}{R_1 // \frac{1}{sC_1}} \left[\frac{A_0}{(A_0+1) + \left(R_2 // \frac{1}{sC_2}\right)} \right]$$

To set $\left| \frac{V_{out}}{V_{in}} \right| = 1$,

Let $x = R_1 // \frac{1}{sC_1}$ and $y = R_2 // \frac{1}{sC_2}$.

$$\therefore \text{For } \left| \frac{V_{out}}{V_{in}} \right| = 1, \quad y A_0 = x \left[(A_0+1) + y \right]$$

$$y (A_0 - 1) = x (A_0 + 1)$$

(29) Cont'd

$$\therefore \quad \frac{X}{Y} = \frac{A_0+1}{A_0-1},$$

i.e. we need to set $\frac{R_1 // \frac{1}{sC_1}}{R_2 // \frac{1}{sC_1}} = \frac{A_0+1}{A_0-1}$.

Since A_0 is generally rather large,

$\frac{A_0+1}{A_0-1}$ is a rational fraction,

in which the numerator and the denominator are large, and differ by a small amount.

(e.g. if $A_0 = 1000$, $\frac{A_0+1}{A_0-1} = \frac{1001}{999}$)

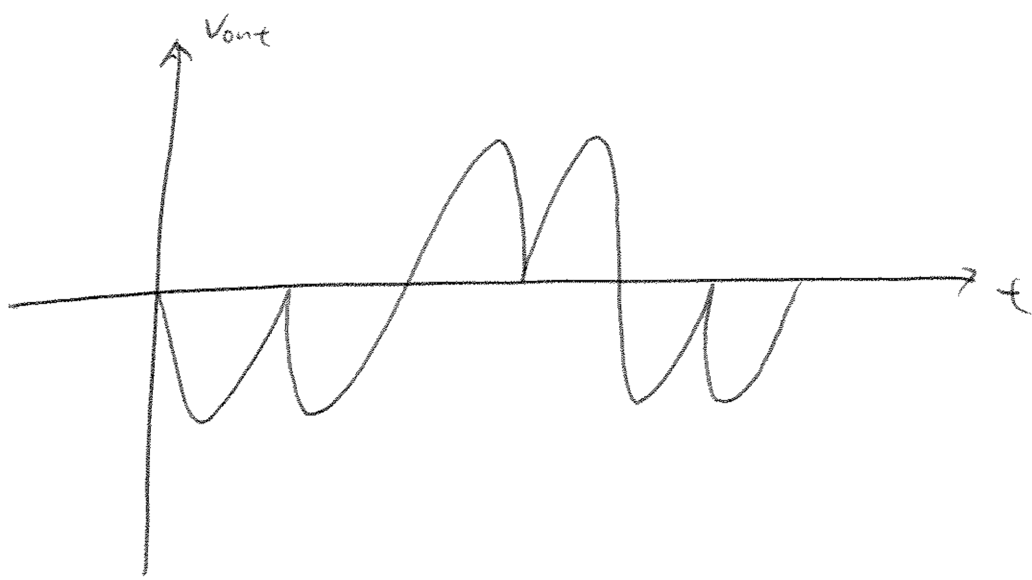
Hence, setting $\left|\frac{V_{out}}{V_{in}}\right|$ to unity is possible in principle, although it would be rather difficult to precisely control A_0.

(30) From eqn (8.63),

$$V_{out} = -R_F \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$$

$\because R_1 = R_2$,

$$V_{out} = -\frac{R_F}{R_1} (V_1 + V_2)$$



8.31

$$\begin{aligned}
v_{out} &= -A_0 v_X \\
\frac{v_1 - v_X}{R_2} + \frac{v_2 - v_X}{R_1} &= \frac{v_X - v_{out}}{R_F} \\
\frac{v_{out}}{R_F} + \frac{v_1}{R_2} + \frac{v_2}{R_1} &= \frac{v_X}{R_1 \parallel R_2 \parallel R_F} \\
v_{out} &= -A_0 \left(R_1 \parallel R_2 \parallel R_F\right) \left(\frac{v_{out}}{R_F} + \frac{v_1}{R_2} + \frac{v_2}{R_1}\right) \\
v_{out} \left[1 + A_0 \frac{\left(R_1 \parallel R_2 \parallel R_F\right)}{R_F}\right] &= -A_0 \left(R_1 \parallel R_2 \parallel R_F\right) \left(\frac{v_1}{R_2} + \frac{v_2}{R_1}\right) \\
v_{out} &= -A_0 \left(R_1 \parallel R_2 \parallel R_F\right) \frac{\frac{v_1}{R_2} + \frac{v_2}{R_1}}{1 + A_0 \frac{\left(R_1 \parallel R_2 \parallel R_F\right)}{R_F}} \\
&= -A_0 R_F \left(R_1 \parallel R_2 \parallel R_F\right) \frac{\frac{v_1}{R_2} + \frac{v_2}{R_1}}{R_F + A_0 \left(R_1 \parallel R_2 \parallel R_F\right)} \\
&= \boxed{-\left(\frac{v_1}{R_2} + \frac{v_2}{R_1}\right) \left[R_F \parallel A_0 \left(R_1 \parallel R_2 \parallel R_F\right)\right]}
\end{aligned}$$

8.32 For $A_0 = \infty$, we know that $v_+ = v_-$, meaning that no current flows through R_P. Thus, R_P will have no effect on v_{out}.

$$v_{out} = \boxed{-R_F\left(\frac{v_1}{R_2} + \frac{v_2}{R_1}\right),\ A_0 = \infty}$$

For $A_0 < \infty$, we have to include the effects of R_P.

$$v_{out} = -A_0 v_X$$

$$v_X = \left(\frac{v_1 - v_X}{R_2} + \frac{v_2 - v_X}{R_1} + \frac{v_{out} - v_X}{R_F}\right) R_P$$

$$v_X\left(\frac{1}{R_P} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_F}\right) = \frac{v_1}{R_2} + \frac{v_2}{R_1} + \frac{v_{out}}{R_F}$$

$$v_X = \left(\frac{v_1}{R_2} + \frac{v_2}{R_1} + \frac{v_{out}}{R_F}\right)(R_1 \parallel R_2 \parallel R_F \parallel R_P)$$

$$v_{out} = -A_0\left(\frac{v_1}{R_2} + \frac{v_2}{R_1} + \frac{v_{out}}{R_F}\right)(R_1 \parallel R_2 \parallel R_F \parallel R_P)$$

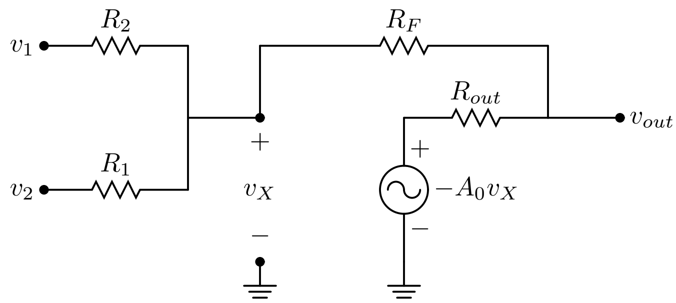
$$v_{out}\left[1 + \frac{A_0}{R_F}(R_1 \parallel R_2 \parallel R_F \parallel R_P)\right] = -A_0\left(\frac{v_1}{R_2} + \frac{v_2}{R_1}\right)(R_1 \parallel R_2 \parallel R_F \parallel R_P)$$

$$v_{out} = -A_0\left(\frac{v_1}{R_2} + \frac{v_2}{R_1}\right)\frac{(R_1 \parallel R_2 \parallel R_F \parallel R_P)}{1 + \frac{A_0}{R_F}(R_1 \parallel R_2 \parallel R_F \parallel R_P)}$$

$$= -\left(\frac{v_1}{R_2} + \frac{v_2}{R_1}\right)\frac{R_F A_0 (R_1 \parallel R_2 \parallel R_F \parallel R_P)}{R_F + A_0 (R_1 \parallel R_2 \parallel R_F \parallel R_P)}$$

$$= \boxed{-\left(\frac{v_1}{R_2} + \frac{v_2}{R_1}\right)\left[R_F \parallel A_0 (R_1 \parallel R_2 \parallel R_F \parallel R_P)\right],\ A_0 < \infty}$$

8.33 We must find v_{out} for the following circuit:



$$v_{out} = -A_0 v_X + \left(\frac{v_1 - v_X}{R_2} + \frac{v_2 - v_X}{R_1}\right) R_{out}$$

$$= -v_X \left(A_0 + \frac{R_{out}}{R_1} + \frac{R_{out}}{R_2}\right) + R_{out}\left(\frac{v_1}{R_2} + \frac{v_2}{R_1}\right)$$

$$v_X = v_{out} + \left(\frac{v_1 - v_X}{R_2} + \frac{v_2 - v_X}{R_1}\right) R_F$$

$$v_X \left(\frac{1}{R_F} + \frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v_{out}}{R_F} + \frac{v_1}{R_2} + \frac{v_2}{R_1}$$

$$v_X = \left(\frac{v_{out}}{R_F} + \frac{v_1}{R_2} + \frac{v_2}{R_1}\right)(R_1 \parallel R_2 \parallel R_F)$$

$$v_{out} = -\left(\frac{v_{out}}{R_F} + \frac{v_1}{R_2} + \frac{v_2}{R_1}\right)(R_1 \parallel R_2 \parallel R_F)\left(A_0 + \frac{R_{out}}{R_1} + \frac{R_{out}}{R_2}\right) + R_{out}\left(\frac{v_1}{R_2} + \frac{v_2}{R_1}\right)$$

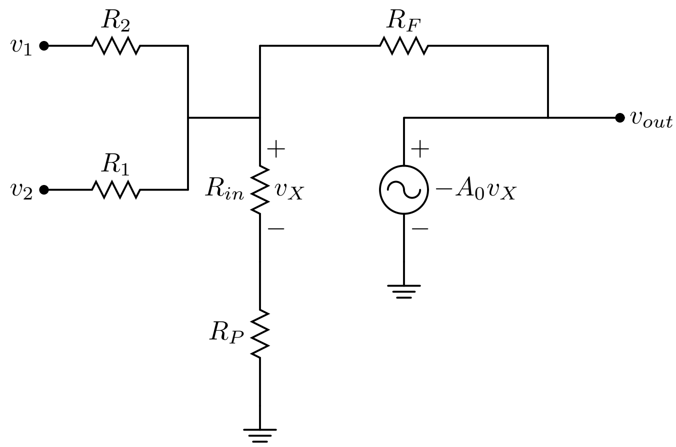
Grouping terms, we have:

$$v_{out}\left[1 + \frac{(R_1 \parallel R_2 \parallel R_F)\left(A_0 + \frac{R_{out}}{R_1 \| R_2}\right)}{R_F}\right] = -\left(\frac{v_1}{R_2} + \frac{v_2}{R_1}\right)(R_1 \parallel R_2 \parallel R_F)\left(A_0 + \frac{R_{out}}{R_1 \parallel R_2}\right) + R_{out}\left(\frac{v_1}{R_2} + \frac{v_2}{R_1}\right)$$

$$= -\left(\frac{v_1}{R_2} + \frac{v_2}{R_1}\right)\left[(R_1 \parallel R_2 \parallel R_F)\left(A_0 + \frac{R_{out}}{R_1 \parallel R_2}\right) + R_{out}\right]$$

$$v_{out} = \boxed{-R_F\left(\frac{v_1}{R_2} + \frac{v_2}{R_1}\right)\frac{R_{out} + (R_1 \parallel R_2 \parallel R_F)\left(A_0 + \frac{R_{out}}{R_1 \| R_2}\right)}{R_F + (R_1 \parallel R_2 \parallel R_F)\left(A_0 + \frac{R_{out}}{R_1 \| R_2}\right)}}$$

8.34 We must find v_{out} for the following circuit:



$$v_{out} = -A_0 v_X$$

$$v_X = \left[\frac{v_1 - v_X\left(1 + \frac{R_P}{R_{in}}\right)}{R_1} + \frac{v_2 - v_X\left(1 + \frac{R_P}{R_{in}}\right)}{R_2} + \frac{v_{out} - v_X\left(1 + \frac{R_P}{R_{in}}\right)}{R_F} \right] R_{in}$$

Grouping terms, we have:

$$v_X \left[\frac{1}{R_{in}} + \left(1 + \frac{R_P}{R_{in}}\right) \frac{1}{R_1 \parallel R_2 \parallel R_F} \right] = \frac{v_1}{R_2} + \frac{v_2}{R_1} + \frac{v_{out}}{R_F}$$

$$v_X \left[\frac{(R_1 \parallel R_2 \parallel R_F) + R_P + R_{in}}{R_{in}(R_1 \parallel R_2 \parallel R_F)} \right] = \frac{v_1}{R_2} + \frac{v_2}{R_1} + \frac{v_{out}}{R_F}$$

$$v_X = \left(\frac{v_1}{R_2} + \frac{v_2}{R_1} + \frac{v_{out}}{R_F} \right) \frac{R_{in}(R_1 \parallel R_2 \parallel R_F)}{(R_1 \parallel R_2 \parallel R_F) + R_P + R_{in}}$$

$$v_{out} = -A_0 \left(\frac{v_1}{R_2} + \frac{v_2}{R_1} + \frac{v_{out}}{R_F} \right) \frac{R_{in}(R_1 \parallel R_2 \parallel R_F)}{(R_1 \parallel R_2 \parallel R_F) + R_P + R_{in}}$$

Grouping terms, we have:

$$v_{out} \left[1 + \frac{A_0}{R_F} \frac{R_{in}(R_1 \parallel R_2 \parallel R_F)}{(R_1 \parallel R_2 \parallel R_F) + R_P + R_{in}} \right] = -\left(\frac{v_1}{R_2} + \frac{v_2}{R_1} \right) \frac{A_0 R_{in}(R_1 \parallel R_2 \parallel R_F)}{(R_1 \parallel R_2 \parallel R_F) + R_P + R_{in}}$$

$$v_{out} \left[\frac{R_F[(R_1 \parallel R_2 \parallel R_F) + R_P + R_{in}] + A_0 R_{in}(R_1 \parallel R_2 \parallel R_F)}{R_F[(R_1 \parallel R_2 \parallel R_F) + R_P + R_{in}]} \right] = -\left(\frac{v_1}{R_2} + \frac{v_2}{R_1} \right) \frac{A_0 R_{in}(R_1 \parallel R_2 \parallel R_F)}{(R_1 \parallel R_2 \parallel R_F) + R_P + R_{in}}$$

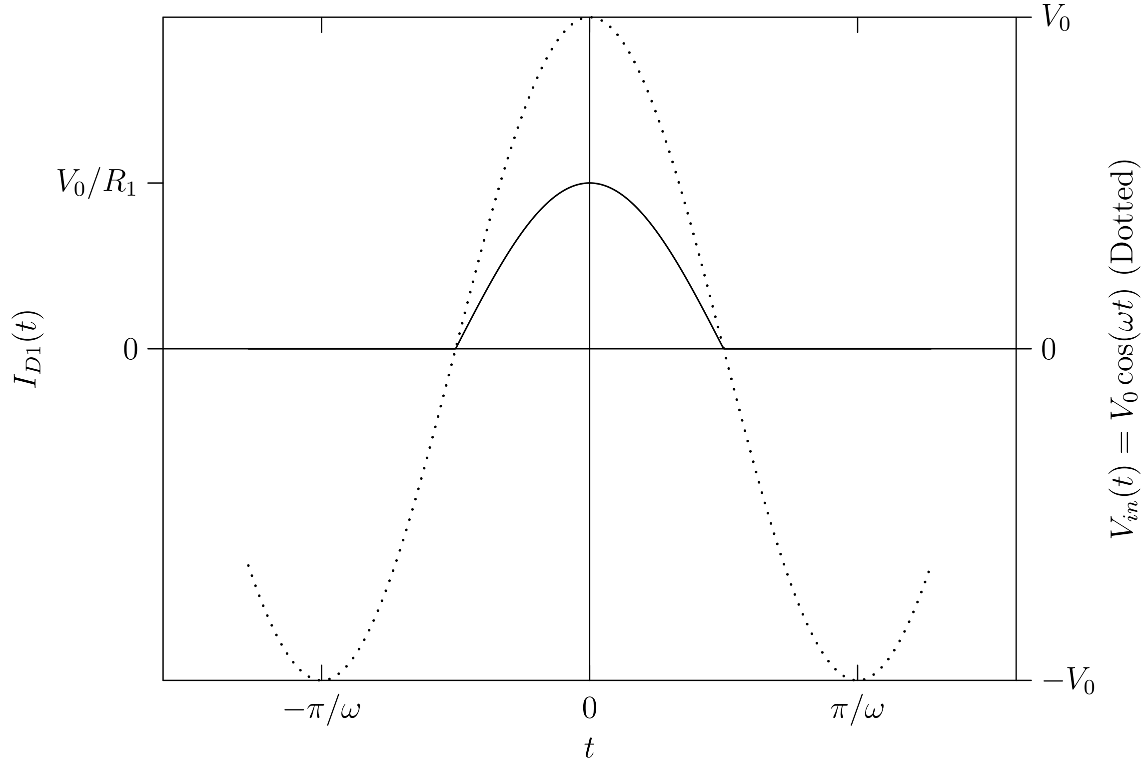
Simplifying, we have:

$$v_{out} = \boxed{-\left(\frac{v_1}{R_2} + \frac{v_2}{R_1} \right) \frac{A_0 R_F R_{in}(R_1 \parallel R_2 \parallel R_F)}{R_F[(R_1 \parallel R_2 \parallel R_F) + R_P + R_{in}] + A_0 R_{in}(R_1 \parallel R_2 \parallel R_F)}}$$

8.35

$$I_{D1} = \begin{cases} \frac{V_{in}}{R_1} & V_{in} > 0 \\ 0 & V_{in} < 0 \end{cases}$$

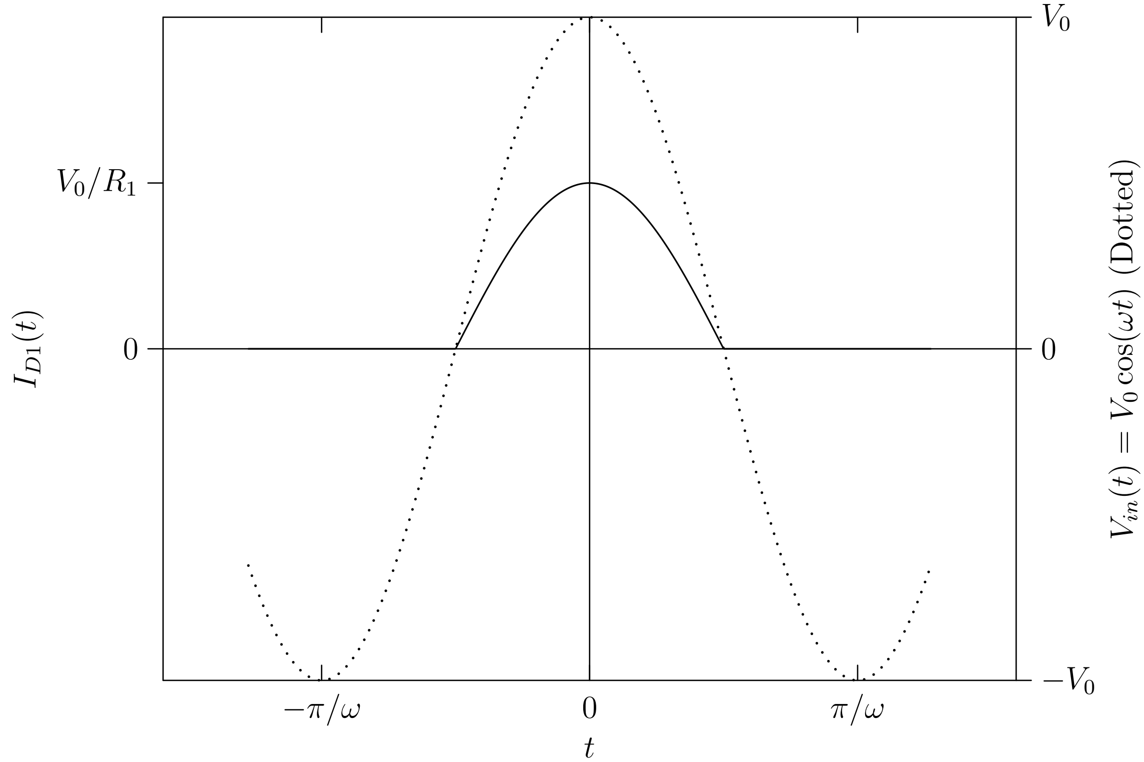
Plotting $I_{D1}(t)$, we have



8.36

$$I_{D1} = \begin{cases} \frac{V_{in}}{R_1} & V_{in} > 0 \\ 0 & V_{in} < 0 \end{cases}$$

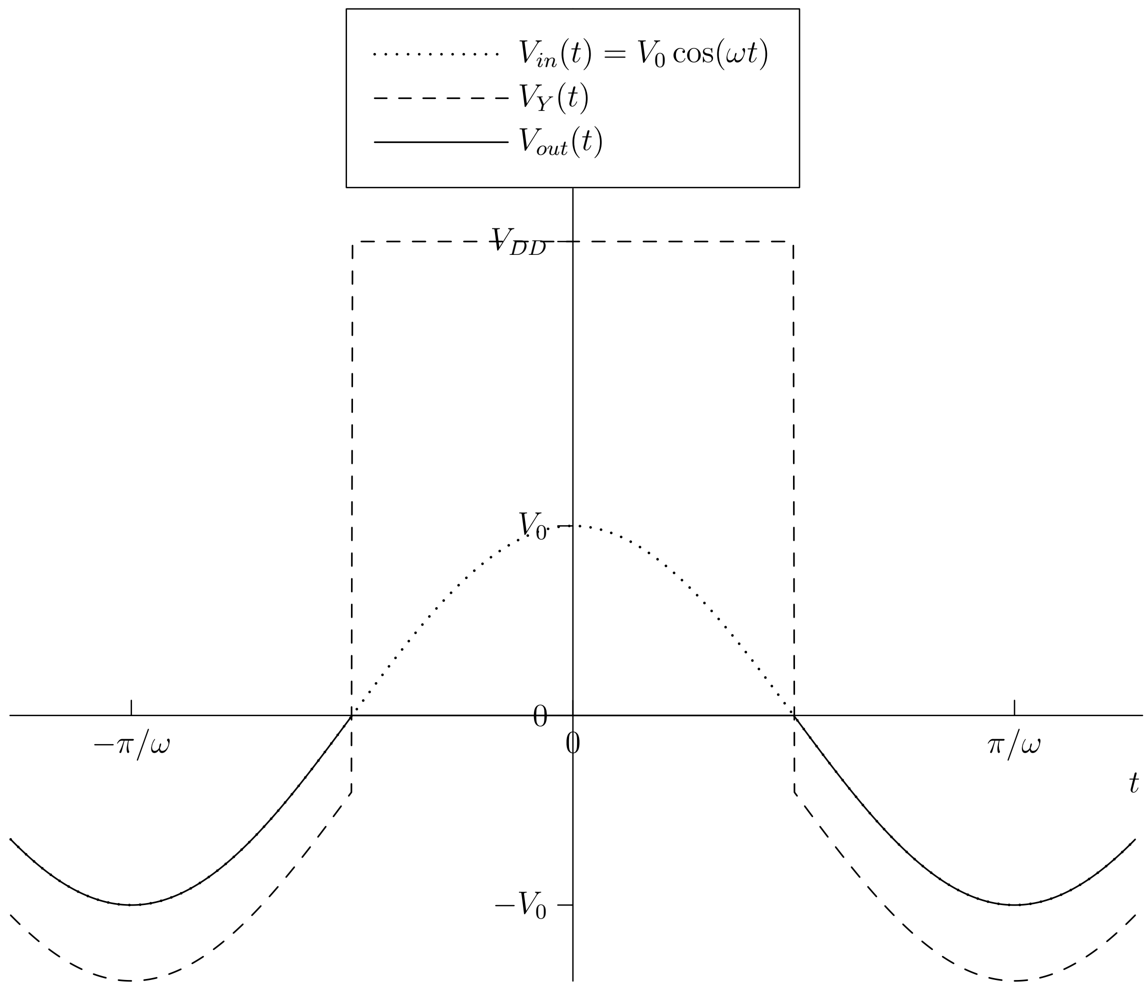
Plotting $I_{D1}(t)$, we have



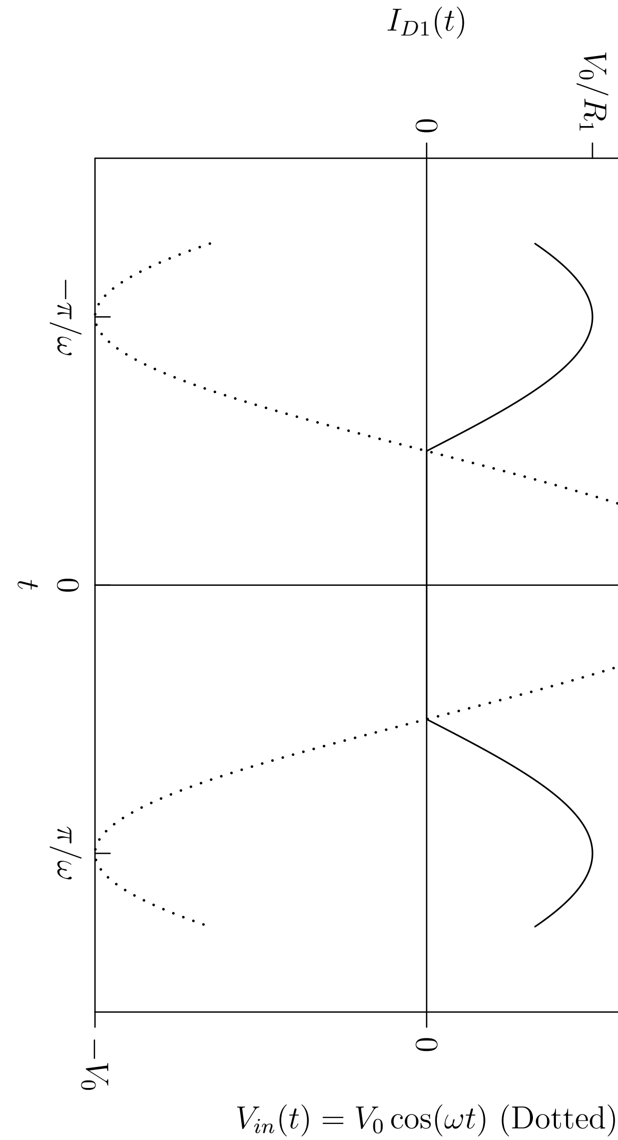
8.37

$$V_Y = \begin{cases} V_{in} - V_{D,on} & V_{in} < 0 \\ V_{DD} & V_{in} > 0 \end{cases} V_{out} = \begin{cases} V_{in} & V_{in} < 0 \\ 0 & V_{in} > 0 \end{cases} I_{D1} = \begin{cases} \frac{V_{in}}{R_1} & V_{in} < 0 \\ 0 & V_{in} > 0 \end{cases}$$

Plotting $V_Y(t)$ and $V_{out}(t)$, we have



Plotting $I_{D1}(t)$, we have:



$I_{D1}(t)$
V_0/R_1
0
t
$-\pi/\omega$
0
π/ω
$-V_0$
0
V_0
$V_{in}(t) = V_0 \cos(\omega t)$ (Dotted)

8.38 Since the negative feedback loop is never broken (even when the diode is off, R_P provides negative feedback), $V_+ = V_-$ will always hold, meaning $V_X = V_{in}$.

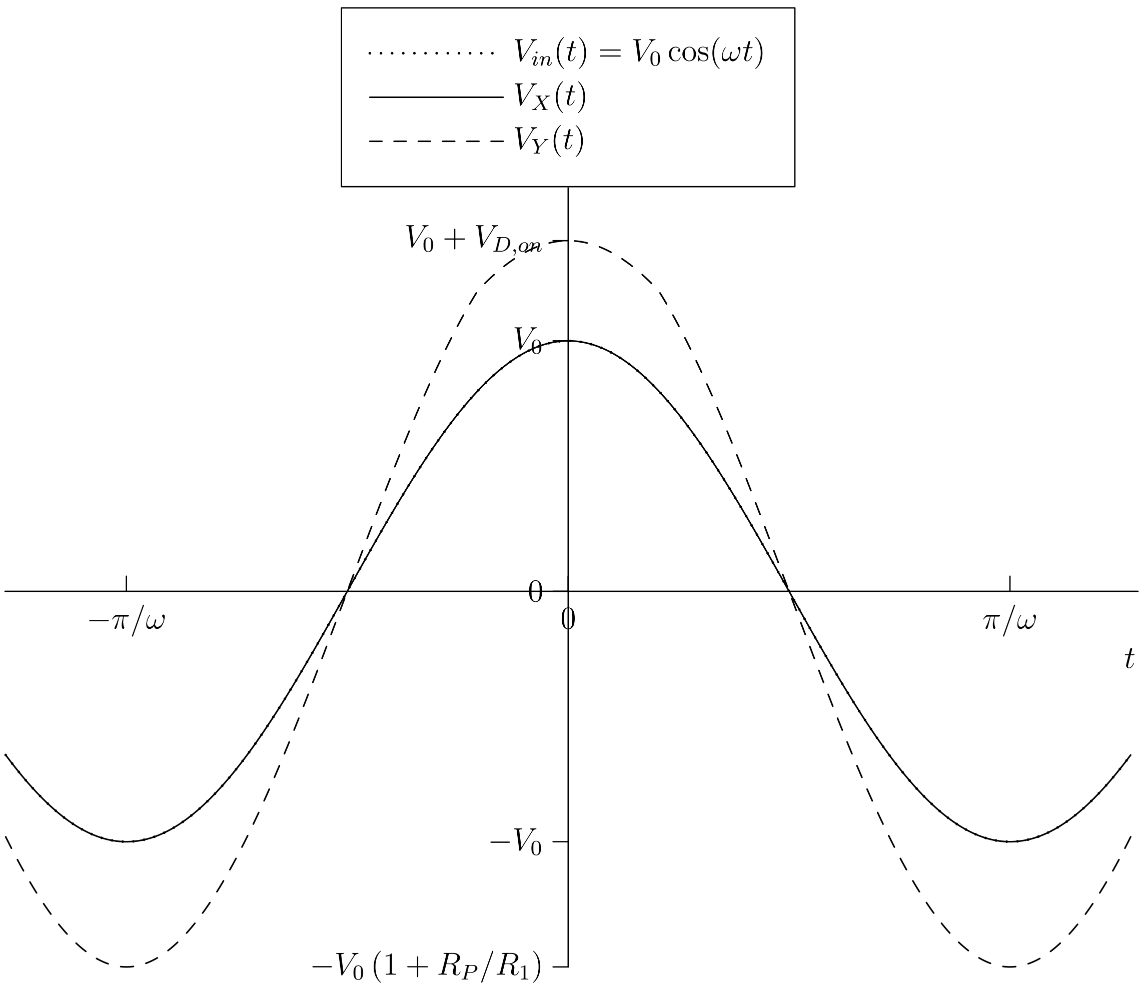
We must determine when D_1 turns on/off to determine V_Y. We know that for $V_{in} < 0$, the diode will be off, and V_X will follow V_{in}. As V_{in} begins to go positive, the diode will remain off until

$$V_{in}\frac{R_P}{R_1} > V_{D,on}$$

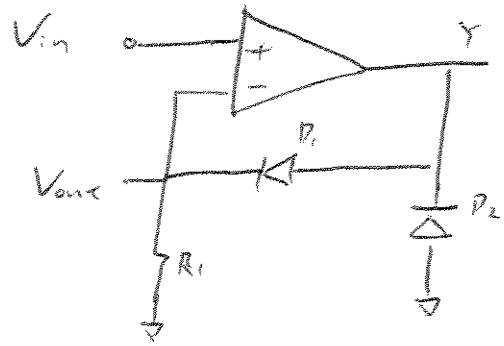
Once the diode turns on, V_Y will be fixed at $V_{in} + V_{D,on}$. Thus, we can write:

$$V_X = V_{in}$$
$$V_Y = \begin{cases} V_{in}\left(1 + \frac{R_P}{R_1}\right) & V_{in} < V_{D,on}\frac{R_1}{R_P} \\ V_{in} + V_{D,on} & V_{in} > V_{D,on}\frac{R_1}{R_P} \end{cases}$$

Plotting $V_Y(t)$ and $V_{out}(t)$, we have

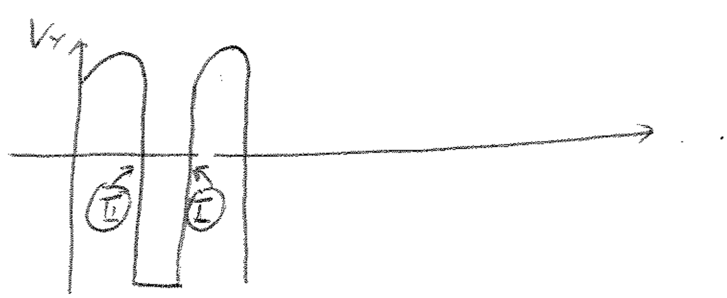


(39) Connecting a diode as below:



D2 allows the parasitic capacitance to charge up faster, right before D1 conducts.

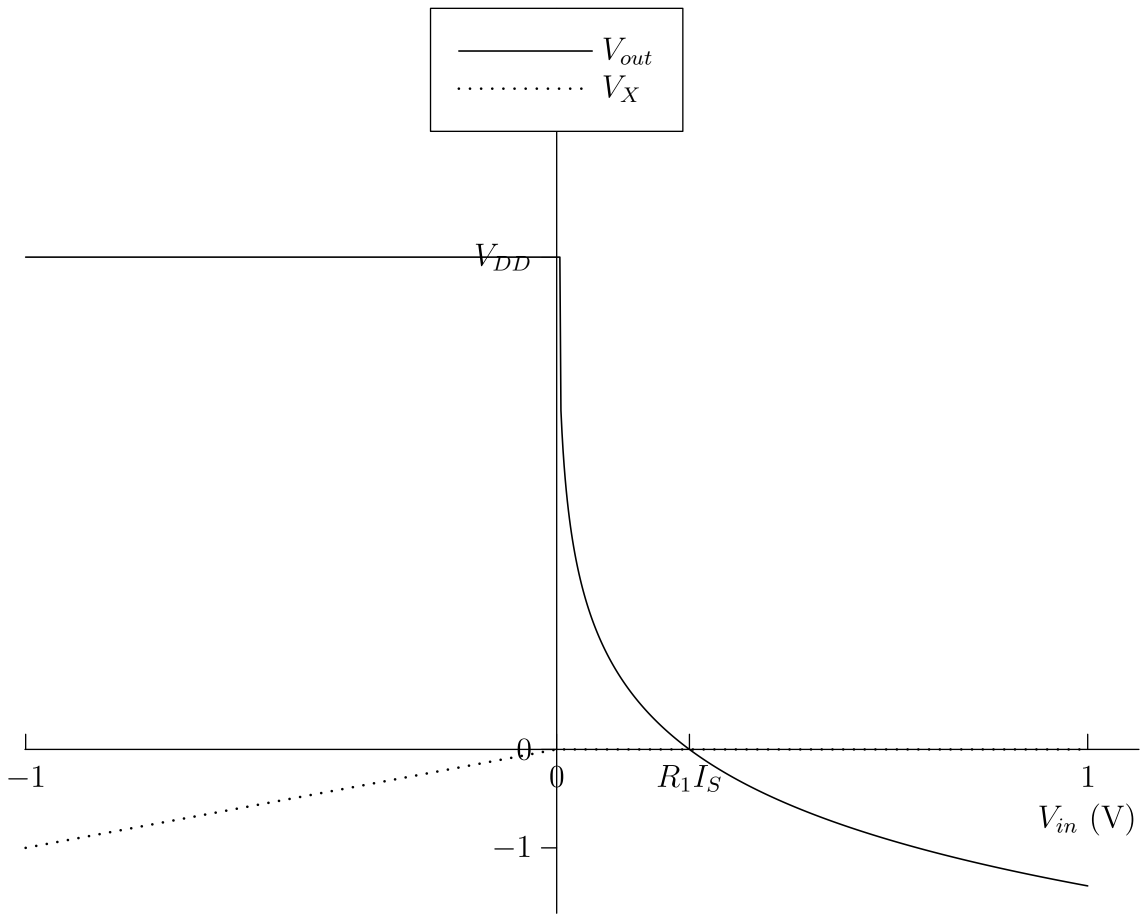
This corresponds to sharpening the transition (I) of V_Y, as shown below



But it will not speed up transition (II).

(which is not critical)

8.40 Note that although in theory the output is unbounded (i.e., by Eq. (8.66), we can take the logarithm of an arbitrarily small positive number), in reality the output will be limited by the positive supply rail, as shown in the following plot.



(41) By KCL,

$$\frac{V_{in} - V_x}{R_1} = I_{R_1}$$

$$\therefore \quad V_{BE} = V_T \ln \frac{\frac{V_{in} - V_x}{R_1}}{I_s}$$

$$= -V_{out}$$

$$\because \ -A_0 V_x = V_{out}$$

$$V_x = -\frac{V_{out}}{A_0}$$

$$\therefore \quad V_{out} = -V_T \ln \frac{V_{in} + \frac{V_{out}}{A_0}}{R_1 I_s}$$

//

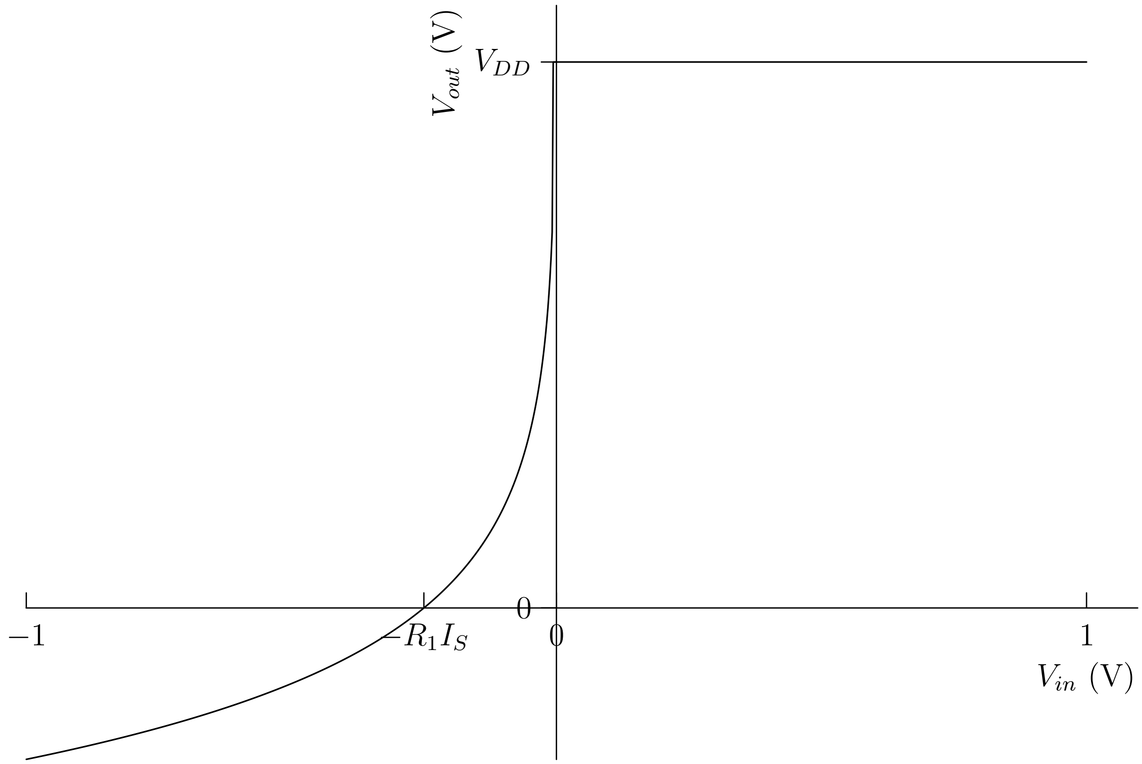
8.42 When $V_{in} > 0$, the feedback loop will be broken, and the output will go to the positive rail.

When $V_{in} < 0$, we have:

$$I_C = -\frac{V_{in}}{R_1} = I_S e^{V_{BE}/V_T} = I_S e^{-V_{out}/V_T}$$

$$V_{out} = \boxed{-V_T \ln\left(-\frac{V_{in}}{R_1 I_S}\right)}$$

This gives us the following plot of V_{out} vs. V_{in}:

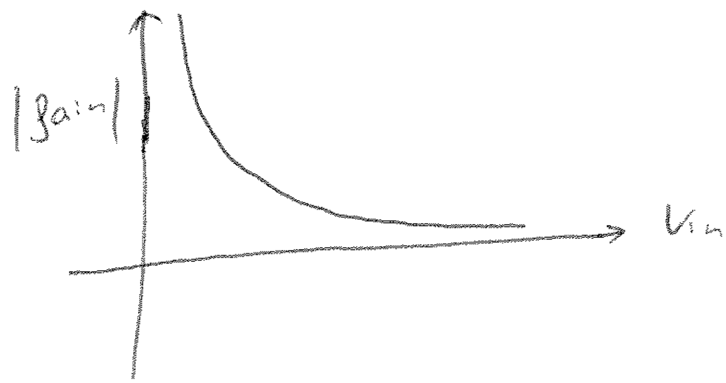


Note that this circuit fails to behave as a non-inverting logarithmic amplifier.

(43)

$$V_{out} = -V_T \ln \frac{V_{in}}{R_1 I_s}$$

$$\frac{dV_{out}}{dV_{in}} = -\frac{V_T}{V_{in}}$$



the gain is compressive, because as Vin increases the magnitude of the gain decreases.

8.44 (a)

$$V_{out} = -V_T \ln\left(\frac{V_{in}}{R_1 I_S}\right)$$

$$-0.2\text{ V} = -V_T \ln\left(\frac{1\text{ V}}{R_1 I_S}\right)$$

$$R_1 I_S = \boxed{456\ \mu\text{V}}$$

(b)

$$A_v = \left.\frac{dV_{out}}{dV_{in}}\right|_{V_{in}=1\text{ V}}$$

$$= -\left.\frac{V_T}{V_{in}}\right|_{V_{in}=1\text{ V}}$$

$$= \boxed{-0.026}$$

8.45 When $V_{in} < V_{TH}$, the output goes to the positive rail. When $V_{in} > V_{TH}$, we have:

$$I_D = \frac{V_{in} - V_{TH}}{R_1}$$

$$V_{GS} = -V_{out} = V_{TH} + \sqrt{\frac{2I_D}{\frac{W}{L}\mu_n C_{ox}}}$$

$$V_{out} = \boxed{-V_{TH} - \sqrt{\frac{2\left(V_{in} - V_{TH}\right)}{R_1 \frac{W}{L}\mu_n C_{ox}}}}$$

$$\frac{dV_{out}}{dV_{in}} = -\frac{1}{2}\sqrt{\frac{R_1 \frac{W}{L}\mu_n C_{ox}}{2\left(V_{in} - V_{TH}\right)}}\frac{2}{R_1 \frac{W}{L}\mu_n C_{ox}}$$

$$= \boxed{-\sqrt{\frac{1}{2R_1 \frac{W}{L}\mu_n C_{ox}\left(V_{in} - V_{TH}\right)}}, V_{in} > V_{TH}}$$

8.46 When $V_{in} > 0$, the output goes to the negative rail. When $V_{in} < 0$, we have:

$$I_D = -\frac{V_{in}}{R_1}$$

$$V_{SG} = V_{out} = |V_{TH}| + \sqrt{\frac{2\,|I_D|}{\frac{W}{L}\mu_p C_{ox}}}$$

$$V_{out} = \boxed{V_{TH} + \sqrt{-\frac{2V_{in}}{R_1 \frac{W}{L}\mu_p C_{ox}}},\ V_{in} < 0}$$

(47) Assume $A_0 = \infty$,

$\therefore V_+ = V_- = V_{in}$

Using voltage divider:

$$V_{in} + V_{os} = V_{out} \frac{R_1}{R_1 + R_2}$$

$$V_{out} = \left(1 + \frac{R_2}{R_1}\right)(V_{in} + V_{os})$$

(48) In Fig. (8.25),

Assuming input is zero,

$$V_x = 10 \times V_{os,A_1}$$

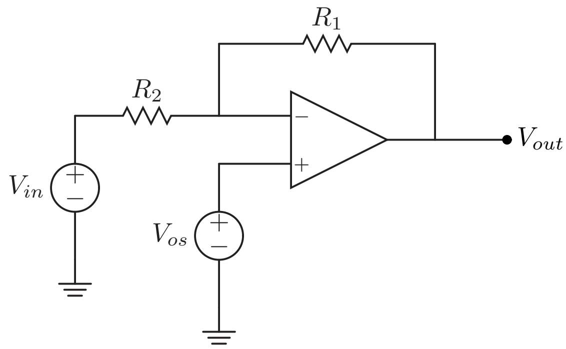
$$= 30mV$$

$$\therefore V_{out} = 10 \times (V_{os,A_2} + V_x)$$

$$= 330mV$$

Thus, the maximum offset error is 330mV.

8.49 We model an input offset with a series voltage source at one of the inputs.



$$
\begin{aligned}
V_{out} &= V_{in} - \frac{V_{in} - V_{os}}{R_2}(R_1 + R_2) \\
&= V_{in}\left(1 - \frac{R_1 + R_2}{R_2}\right) + V_{os}\frac{R_1 + R_2}{R_2} \\
&= \boxed{-\frac{R_1}{R_2}V_{in} + \left(1 + \frac{R_1}{R_2}\right)V_{os}}
\end{aligned}
$$

Note that even when $V_{in} = 0$, $V_{out} = (1 + R_1/R_2)\,V_{os}$.

(50) By eqn (8.72)

$$V_{out} = V_{os}\left(1 + \frac{R_2}{R_1}\right)$$

$$\therefore\ 20\,mV = 3\,mV\left(1 + \frac{R_2}{R_1}\right)$$

$$\frac{17}{3} = \frac{R_2}{R_1} \quad \text{———} (1)$$

$$\because\ \frac{1}{R_2 C_1} \ll 2\pi(1000)$$

and setting $C_1 = 100\,pF$,

$$\frac{1}{R_2 \times 100 \times 10^{-12}} \ll 2\pi(1000)$$

$$\frac{1}{R_2} \ll 6.283 \times 10^{-7}$$

$$\therefore\ R_2 \gg 1.59\,M\Omega$$

choose $R_2 = 17\,M\Omega$.

$R_1 = 3\,M\Omega$. (From (1))

(51) From eqn (8.44)

$$V_{out} \propto \frac{d V_{in}}{dt}$$

(proportional)

Since offset is static (invariant with time)

i.e. $\frac{d V_{os}}{dt} = 0$.

$\therefore$ offset has no effect to V_{out}.

(5-2) From eqn (8.6),

with the presence of offset (V_{os}),

$$V_{out} = -V_T \ln \frac{V_{in} + V_{os}}{R_1 I_s}$$

The effect of offset to V_{out} is very small, because V_{out} is proportional to the log. of ($V_{in} + V_{os}$)

Thus, V_{out} is very insensitive to the magnitude of the offset.

(53). From eqn (8.76),

$V_{out} = R_1 I_{B2}$

∴ V_{out} is independent of I_{B1}

Also I_{B1} will not affect $\frac{V_{out}}{V_{in}}$.

Thus, the small offset (ΔI) in the input bias currents has no effect on V_{out}.

8.54 Let $V_{in} = 0$.

$$\begin{aligned}
V_+ &= -I_{B1}\left(R_1 \parallel R_2\right) = -\left(I_{B2} + \Delta I\right)\left(R_1 \parallel R_2\right) = V_- \\
V_{out} &= V_- + \left(I_{B2} + \frac{V_-}{R_2}\right) R_1 \\
&= -\left(I_{B2} + \Delta I\right)\left(R_1 \parallel R_2\right) + \left(I_{B2} - \frac{\left(I_{B2} + \Delta I\right)\left(R_1 \parallel R_2\right)}{R_2}\right) R_1 \\
&= -\left(I_{B2} + \Delta I\right)\left(R_1 \parallel R_2\right)\left(1 + \frac{R_1}{R_2}\right) + I_{B2} R_1 \\
&= \boxed{-\Delta I R_1}
\end{aligned}$$

If the magnitude of the error must be less than ΔV, we have:

$$\begin{aligned}
\Delta I R_1 &< \Delta V \\
R_1 &< \boxed{\frac{\Delta V}{\Delta I}}
\end{aligned}$$

Note that this does not depend on R_2.

(55) Using eqn. (8.84)

$$\text{Gain} = \frac{A_0}{1 + \frac{s}{\omega_1}}$$

For opamp (a): At 100 MHz:

$$\text{Gain}_{(a)} = \frac{1000}{1 + \frac{2\pi \times 100 \times 10^6}{2\pi \times 50}}$$

$$\approx 5 \times 10^{-4}$$

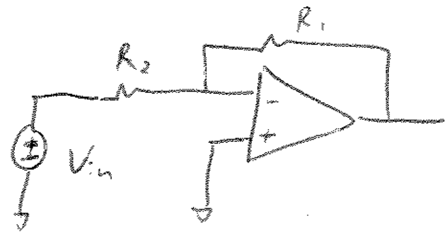
For opamp (b) at 100 MHz,

$$\text{Gain}_{(b)} = \frac{500}{1 + \frac{2\pi \times 100 \times 10^6}{2\pi \times 10}}$$

$$\approx 4.95 \quad > 4$$

∴ opamp (b) is a possible candidate

(5-6)



Using eqn (8.20),

$$\frac{V_{out}}{V_{in}} = -\frac{1}{\frac{R_2}{R_1} + \frac{1}{A_0}\left(1 + \frac{R_2}{R_1}\right)}$$

Here, A_0 becomes $\frac{A_0}{1+\frac{s}{\omega_1}}$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{-1}{\frac{R_2}{R_1} + \frac{A_0}{1+\frac{s}{\omega_1}}\left(1+\frac{R_2}{R_1}\right)}$$

$$= \frac{-\left(1+\frac{s}{\omega_1}\right)}{\left(1+\frac{s}{\omega_1}\right)\frac{R_2}{R_1} + A_0\left(1+\frac{R_2}{R_1}\right)}$$

To find the pole, equate denominator to 0.

i.e. $\left(1+\frac{s}{\omega_1}\right)\frac{R_2}{R_1} + A_0\left(1+\frac{R_2}{R_1}\right) = 0$

$$\left(1+\frac{s}{\omega_1}\right) = -\frac{R_1}{R_2}A_0\left(1+\frac{R_2}{R_1}\right)$$

$$\therefore |\omega_{p,closed}| = \left(1+\frac{R_1}{R_2}A_0\left(1+\frac{R_2}{R_1}\right)\right)\omega_1$$

8.57

$$V_{out} = -\frac{A_0}{1+\frac{s}{\omega_0}} V_-$$

$$V_- = V_{in} + \frac{V_{out} - V_{in}}{R_1 + \frac{1}{sC_1}} R_1$$

$$V_{out} = -\frac{A_0}{1+\frac{s}{\omega_0}} \left(V_{in} + \frac{V_{out} - V_{in}}{R_1 + \frac{1}{sC_1}} R_1 \right)$$

$$V_{out} \left[1 + \frac{A_0}{1+\frac{s}{\omega_0}} \frac{R_1}{R_1 + \frac{1}{sC_1}} \right] = \frac{A_0}{1+\frac{s}{\omega_0}} V_{in} \left[\frac{R_1}{R_1 + \frac{1}{sC_1}} - 1 \right]$$

$$V_{out} \frac{\left(1+\frac{s}{\omega_0}\right)\left(R_1 + \frac{1}{sC_1}\right) + A_0 R_1}{\left(1+\frac{s}{\omega_0}\right)\left(R_1 + \frac{1}{sC_1}\right)} = -V_{in} \frac{A_0 \frac{1}{sC_1}}{\left(1+\frac{s}{\omega_0}\right)\left(R_1 + \frac{1}{sC_1}\right)}$$

$$\frac{V_{out}}{V_{in}} = -\frac{A_0 \frac{1}{sC_1}}{\left(1+\frac{s}{\omega_0}\right)\left(R_1 + \frac{1}{sC_1}\right) + A_0 R_1}$$

$$= -\frac{A_0}{\left(1+\frac{s}{\omega_0}\right)(1 + sR_1C_1) + sA_0R_1C_1}$$

$$= -\frac{A_0}{1 + s\left(R_1C_1 + \frac{1}{\omega_0} + A_0R_1C_1\right) + s^2 \frac{R_1C_1}{\omega_0}}$$

$$= \boxed{-\frac{A_0}{1 + s\left[(1+A_0)R_1C_1 + \frac{1}{\omega_0}\right] + s^2 \frac{R_1C_1}{\omega_0}}}$$

If $\omega_0 \gg \frac{1}{R_1C_1}$, we have:

$$\frac{V_{out}}{V_{in}} = -\frac{1}{\frac{1}{A_0} + s\left[\left(1+\frac{1}{A_0}\right)R_1C_1 + \frac{1}{\omega_0}\right] + s^2 \frac{R_1C_1}{A_0\omega_0}}$$

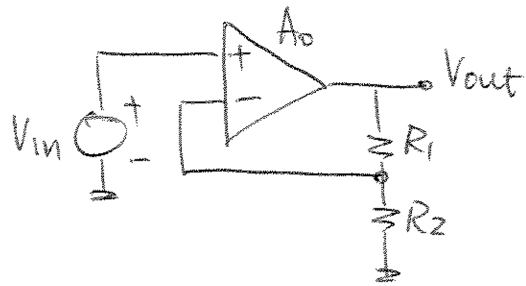
$$= -\frac{1}{\frac{1}{A_0} + s\left(1+\frac{1}{A_0}\right)R_1C_1 + s^2 \frac{R_1C_1}{A_0\omega_0}}$$

$$\approx -\frac{1}{sR_1C_1 + s^2 \frac{R_1C_1}{A_0\omega_0}} \text{ (assuming } A_0 \gg 1)$$

$$= \boxed{-\frac{1}{sR_1C_1\left(1+\frac{s}{A_0\omega_0}\right)}}$$

58.

Nominal Gain = 4
Slew Rate = 1 V/ns
i/p = 0.5 V



$$V_{in}(t) = 0.5 \sin \omega t \Rightarrow V_{out} = 0.5 \times \underbrace{\left(1 + \frac{R_1}{R_2}\right)}_{=4} \sin \omega t.$$

$$\frac{dV_{out}}{dt} = 0.5\left(1 + \frac{R_1}{R_2}\right)\omega \cdot \cos \omega t.$$

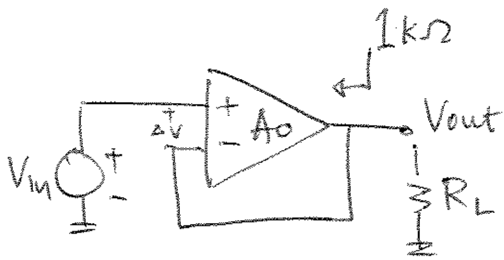
= maximum when $\cos \omega t = 1$

$$\Rightarrow \left.\frac{dV_{out}}{dt}\right|_{max} = 0.5\omega\left(1 + \frac{R_1}{R_2}\right) = 2\omega$$

∴ Highest frequency ⇒ $2\omega = 1\ V/ns$

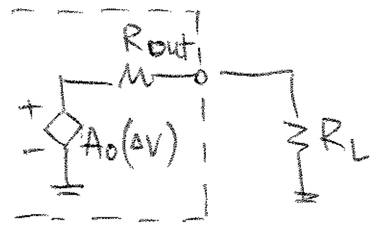
$\Rightarrow \omega = 0.5\ rad/ns \Rightarrow f_{MAX} \simeq 79.6\ MHz$

59.



$R_L = 100\,\Omega$

Gain Error $= 0.5\%$



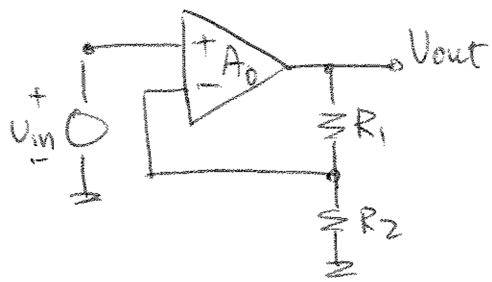
$$(V_{in} - V_{out}) A_0 \times \frac{R_L}{R_{out} + R_L} = V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{R_{out} + R_L}{A_0 R_L}} \simeq 1 - \underbrace{\frac{R_{out} + R_L}{A_0 R_L}}_{= \epsilon}$$

$$\therefore \epsilon = \frac{R_{out} + R_L}{A_0 R_L} \Rightarrow A_0 = \frac{R_{out} + R_L}{\epsilon R_L} = \frac{1000 + 100}{0.5\% \times 100}$$

$$\approx 2200.$$

60.



Nominal Gain = 4

Gain Error = 0.2%

$R_1 + R_2 = 20\,k\Omega$

$$\left[V_{in} - \frac{R_2}{R_1+R_2} \times V_{out}\right] A_0 = V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{A_0}{1 + \frac{R_2}{R_1+R_2} A_0} \simeq \left(1+\frac{R_1}{R_2}\right)\left[1-\left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0}\right]$$

$$(1 + R_1/R_2) = 4 \quad \& \quad (R_1+R_2) = 20\,k\Omega$$

$$\Rightarrow R_1 = 15\,k\Omega, \quad R_2 = 5\,k\Omega.$$

$$0.2\% = \left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0} \Rightarrow A_0 = \left(1+\frac{R_1}{R_2}\right) \times \frac{1}{0.2\%}$$

$$= 2000$$

8.61 Let $\mathcal{E}$ refer to the gain error.

$$\frac{R_1}{R_2} = 8$$

$$R_1 = \boxed{8\text{ k}\Omega}$$

$$R_2 = \boxed{1\text{ k}\Omega}$$

$$\frac{v_{out}}{v_{in}} = -\frac{R_1}{R_2}\frac{A_0 - \frac{R_{out}}{R_1}}{1 + \frac{R_{out}}{R_2} + A_0 + \frac{R_1}{R_2}} \text{ (Eq. 8.99)}$$

$$= -\frac{R_1}{R_2}(1 - \mathcal{E})$$

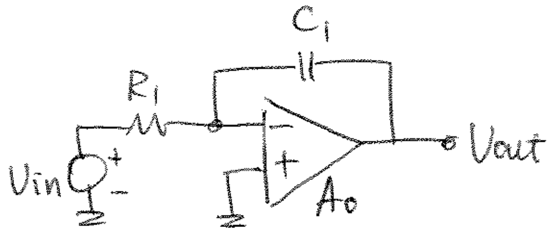
$$\mathcal{E} = 1 - \frac{A_0 - \frac{R_{out}}{R_1}}{1 + \frac{R_{out}}{R_2} + A_0 + \frac{R_1}{R_2}}$$

$$= 0.1\ \%$$

$$A_0 = \boxed{9103}$$

Note that we can pick any R_1, R_2 such that their ratio is 8 (i.e., this solution is not unique). However, A_0 will change depending on the values chosen.

62.



$= 100\ \text{kHz}$

pole $= 100\ \text{Hz}$

$C_{MAX} = 50\ \text{pF}$.

$$\frac{V_{in} - V_{(-)}}{R_1} = (V_{(-)} - V_{out})\, s C_1 \qquad \text{—— ①}$$

$$V_{(-)} \cdot (-A_0) = V_{out} \qquad \text{—— ②}$$

Substitute ② into ①:

$$\frac{V_{out}}{V_{in}} = \frac{-1}{\frac{1}{A_0} + \left(1 + \frac{1}{A_0}\right) R_1 C_1 s}$$

$$\Rightarrow S_p = \frac{-1}{(A_0 + 1) R_1 C_1} = -100\ \text{Hz} \qquad \text{—— ①}$$

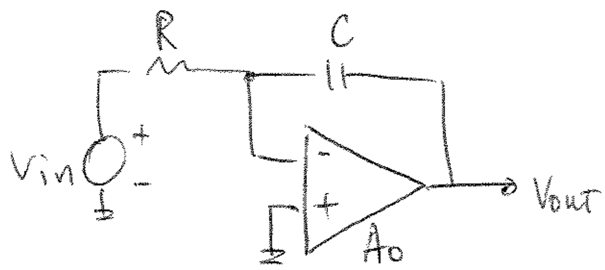
Attenuation above 100 kHz $\Rightarrow \left|\frac{V_{out}}{V_{in}}\right|_{100\text{kHz}} = 1$

$$\Rightarrow \left.\frac{A_0}{\sqrt{1 + \left[(A_0 + 1) R_1 C_1 \omega_z\right]^2}}\right|_{100\text{kHz}} = 1 \qquad \text{—— ②}$$

Substitute ① into ②:

$\Rightarrow A_0 \simeq 1000.$ Choose $C = 50\text{pF} \Rightarrow R \simeq 200\ \text{k}\Omega$.

63.



$V(t) = \alpha t$

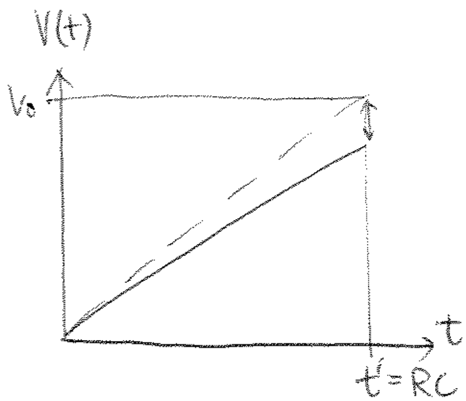
$0 < V(t) < V_o$

where $\alpha = 10\,V/\mu s$

$V_o = 1\,V$

$C_{max} = 20\,pF$

Error $< 0.1\%$



$V_{out}(t) = -\frac{V_o}{RC} t \quad , \quad t \in [0, RC]$

$V(t) = -\alpha t$

At $t = RC$, $\frac{\Delta V}{V_o} = \left.\frac{V_{out}(t) - V(t)}{V_o}\right|_{t=RC} < 0.1\%$

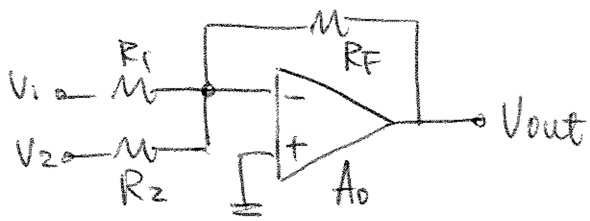
$\Rightarrow \Delta V = V_o \times 0.1\% = 0.001\,V$

$\Rightarrow \left. -\frac{V_o}{RC} \times t + \alpha t \right|_{t=RC} = 0.00\,V \quad (= \Delta V)$

Choose $C = 20\,pF$

$\therefore R = \frac{V_o - \Delta V}{\alpha C} = \frac{1V - 0.001\,V}{10\,V/\mu s \times 20\,pF} = 4995\,\Omega$

64.



$$V_{out} = \alpha_1 V_1 + \alpha_2 V_2$$
($\alpha_1 = 0.5$, $\alpha_2 = 1.5$)

Error of $\alpha \leq 0.5\%$

$r_{in} \geq 10\,k\Omega$.

$$\frac{V_1 - V_{(-)}}{R_1} + \frac{V_2 - V_{(-)}}{R_2} = \frac{V_{(-)} - V_{out}}{R_F} \quad — ①$$

$$V(-A_0) = V_{out} \quad — ②$$

Substitute ② into ① & solve for V_{out}:

$$V_{out} = -\left(\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V_2\right)\cdot\left[\frac{1}{A_0}\left(\frac{R_F}{R_1}+\frac{R_F}{R_2}+1\right)+1\right]^{-1}$$

$$\simeq -\left(\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V_2\right)\cdot\left[1-\frac{1}{A_0}\left(\frac{R_F}{R_1}+\frac{R_F}{R_2}+1\right)\right]$$

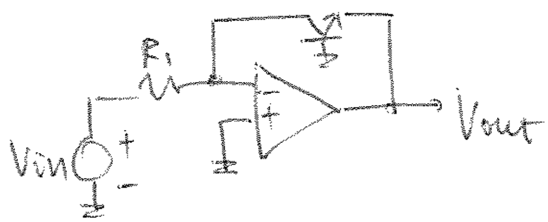
Choose $R_{in,v_2}\ (\simeq R_2) = 10\,k\Omega \Rightarrow R_F = \alpha_2 \times R_2 = 15\,k\Omega$

$\Rightarrow R_1 = R_F/\alpha_1 = 30\,k\Omega \simeq R_{in,v_1}$

$$\Rightarrow \epsilon = 0.5\% = \frac{1}{A_0}\left(\frac{R_F}{R_1}+\frac{R_F}{R_2}+1\right)$$

$$\Rightarrow A_0 = \frac{1}{0.5\%}\left(0.5+1.5+1\right) = 600 \quad \text{(or larger)}$$

65.



$[0.1, 2]V \longmapsto [-0.5, -1]V$

$$V_{out} = -V_T \ln \frac{V_{in}}{I_S R_1}$$

$$-0.5V = -V_T \ln\left[\frac{(0.1)}{I_S R_1}\right] \Rightarrow I_S R_1 = 4.45 \cdot 10^{-10} V \quad —①$$

$$\Rightarrow -V_T \ln\frac{(2)}{I_S R_1} = -0.026V \ln\left(\frac{2}{4.45\cdot 10^{-10}}\right) \approx -0.58V$$

∴ input range of $0.1 \leftrightarrow 2$ V corresponds to output range of $-0.5 \leftrightarrow -0.58$ V

Choose $I_S = 1\times 10^{-16} A \Rightarrow R_1 = 4.45\ M\Omega$.

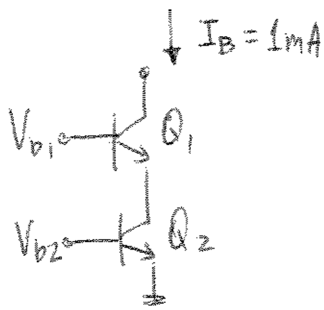
8.66

$$
\begin{aligned}
V_{out} &= -V_T \ln\left(\frac{V_{in}}{R_1 I_S}\right) \\
\frac{dV_{out}}{dV_{in}} &= -V_T \frac{R_1 I_S}{V_{in}} \frac{1}{R_1 I_S} \\
&= -\frac{V_T}{V_{in}}
\end{aligned}
$$

No, it is not possible to satisfy both requirements. As shown above, $\left|\frac{dV_{out}}{dV_{in}}\right| = \frac{V_T}{V_{in}}$, meaning for a specified temperature and input, the gain is fixed. Assuming we could fix the temperature as part of the design, we could still only meet one of the two constraints, since the temperatures at which the constraints are met are not equal.

Ch9

1.



$I_S = 6 \cdot 10^{-17} A$ $\quad \beta = 100$

$V_A \to \infty$ $\quad V_T = \frac{kT}{q}$

$\alpha = \frac{\beta}{\beta + 1}$

(a) $V_{b2} = V_T \ln\left(\frac{I_B/\alpha^2}{I_S}\right) = (0.026V)\ln\left(\frac{1.02mA}{6 \cdot 10^{-17}A}\right)$

$\approx 0.792\ V$

(b) From the configuration,

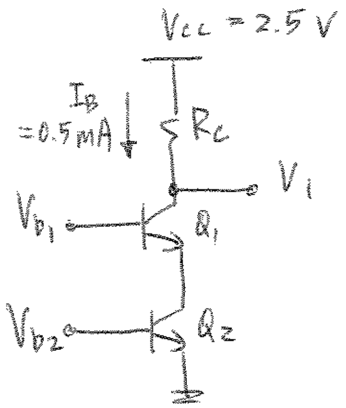
$V_{b1} = V_{CE2} + V_{BE1} = (V_{BE2} - 300mV) + V_{BE1}$

$V_{BE1} = V_T \ln(I_B/I_S) = (0.026V)\ln\left(\frac{1mA}{6 \cdot 10^{-17}A}\right)$

$\approx 0.792\ V$

$\therefore V_{b1} = (0.792 - 0.3) + 0.79 = 1.28\ V$

2.



(a) $V_{b2} = V_{BE2} = V_T \ln\left(\frac{I_B/\alpha^2}{I_s}\right) = (0.026V)\ln\left(\frac{0.51\,mA}{6\cdot 10^{-17}A}\right)$

$\approx 0.774\ V$

$V_{BE1} = V_{b1} - V_{C2} = V_{b1} - (V_{b2} - 300mV)$

$\Rightarrow V_{b1} = V_{BE1} + V_{b2} - 0.3V$

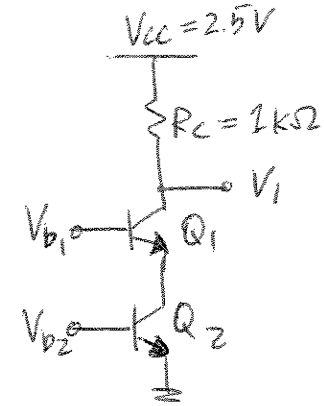
$= (0.026V)\ln\left(\frac{0.5\,mA}{6\cdot 10^{-17}A}\right) + (0.774V) - (0.3V)$

$\approx 1.25\ V$

(b) $V_1 = V_{b1} - 0.3V = 0.95V$

$\therefore\ R_C = \frac{V_{cc} - V_1}{I_B} = \frac{(2.5-0.95)V}{0.5\,mA} \approx 3.1\ k\Omega$

3. From previous experience, assume both V_{BE1} & $V_{BE2} = 0.8\ V$



$$\Rightarrow V_1 = V_{CE_1} + V_{CE_2}$$

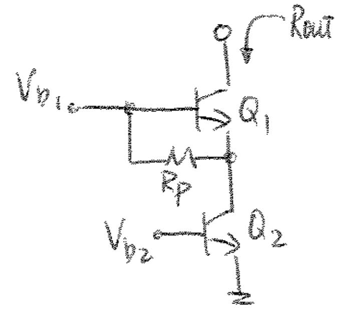
$$= (V_{BE_1} - 200mV) + (V_{b2} - 200mV)$$

$$= 1.2\ V$$

- By KCL, maximum bias current

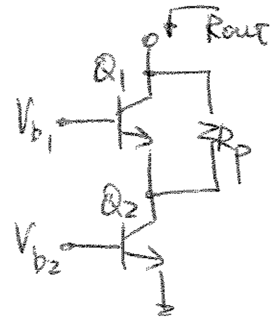
$$\approx \frac{V_{cc} - V_1}{R_c} = \frac{(2.5 - 1.2)V}{1k\Omega} = 1.3\ mA.$$

4. (a) R_p appears in parallel with r_{π_1}



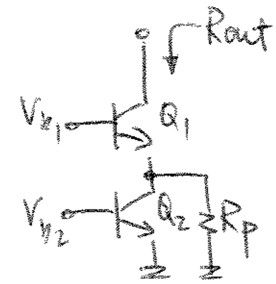
$$\therefore R_{out} = [1 + g_{m_1}(r_{o2} \| r_{\pi_1} \| R_p)] r_{o1} + (r_{o2} \| r_{\pi_1} \| R_p)$$

(b) R_p appears in parallel with r_{o1}



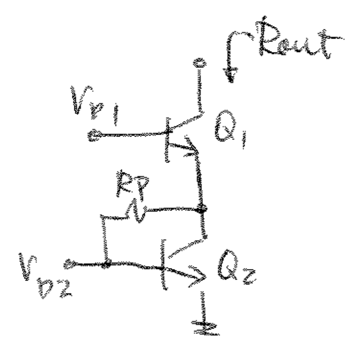
$$\therefore R_{out} = [1 + g_{m_1}(r_{o2} \| r_{\pi_1})](r_{o1} \| R_p) + (r_{o2} \| r_{\pi_1})$$

(c) R_p appears in parallel with r_{o2}



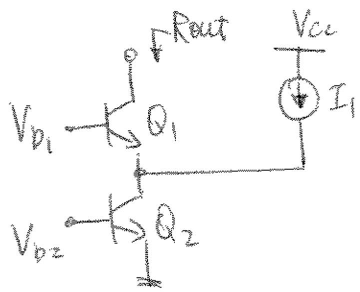
$$\therefore R_{out} = [1 + g_{m_1}(r_{o2} \| r_{\pi_1} \| R_p)] r_{o1} + (r_{o2} \| r_{\pi_1} \| R_p)$$

(d) R_p appears in parallel with r_{o2} (in small-signal) $\because V_{b2}$ is AC GND.



$$\therefore R_{out} = [1 + g_{m_1}(r_{o2} \| r_{\pi_1} \| R_p)] r_{o1} + (r_{o2} \| r_{\pi_1} \| R_p)$$

5.



$I_1 = 0.5\,mA$

$I_{C1} = 0.5\,mA \qquad I_{C2} = 1\,mA.$

$= 2 I_{C1}$

$\beta = 100 \qquad V_A = 5V$

$$R_{out} = g_{m1} r_{o1} (r_{o2} \parallel r_{\pi 1})$$

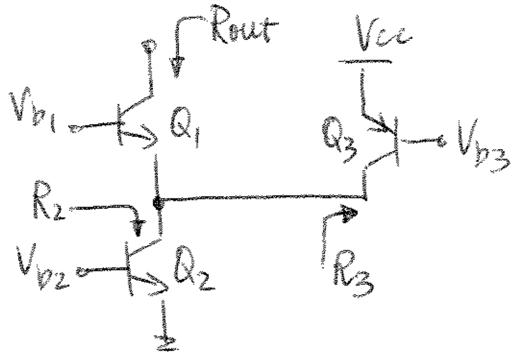
$$= \frac{I_{C1}}{V_T} \cdot \frac{V_A}{I_{C1}} \cdot \frac{V_{A2}/I_{C2} \cdot \beta V_T / I_{C1}}{V_{A2}/I_{C2} + \beta V_T / I_{C1}}$$

$$= \frac{V_A}{V_T} \cdot \frac{\frac{V_{A2}/2}{I_{C1}} \cdot \beta V_T / I_{C1}}{\frac{V_{A2}/2}{I_{C1}} + \beta V_T / I_{C1}} \approx \frac{1}{I_{C1}} \cdot \frac{V_A}{V_T} \cdot \frac{\beta V_A V_T}{V_A + 2\beta V_T}$$

$$= \frac{1}{0.5mA} \cdot \frac{5V}{0.026V} \cdot \frac{100(5V)(0.026V)}{(5V) + 2(100)(0.026V)}$$

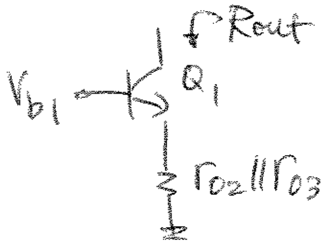
$\therefore$ Rout $\approx$ 490. kΩ.

6.



$R_3 = r_{o3}$ (V_{cc} & V_{b3} are AC GND)

$R_2 = r_{o2}$ (V_{b2} is AC GND)



$$\therefore R_{out} = \left[\left(1+g_{m1}(r_{o2}\|r_{o3}\|r_{\pi1})\right)\right]r_{o1} + (r_{o2}\|r_{o3}\|r_{\pi1})$$

$$\approx g_{m1} r_{o1} (r_{o2}\|r_{o3}\|r_{\pi1})$$

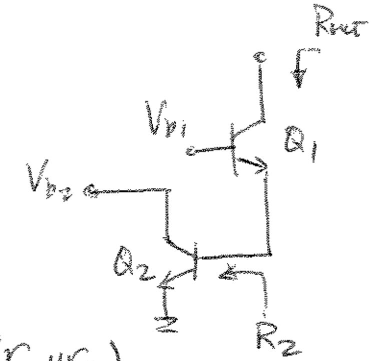
9.7 Let R_2 be the resistance seen looking into the collector of Q_2.

$$R_{out} = r_{o1} + (1 + g_{m1} r_{o1}) (r_{\pi 1} \parallel R_2)$$

Note that this expressoin is maximized as $R_2 \to \infty$. This gives us

$$R_{out,max} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) r_{\pi 1}}$$

8. (a) $R_2 = (r_{\pi_2} \| r_{\pi_1})$



$$\therefore R_{out} = [1 + g_{m_1} R_2] r_{o_1} + R_2$$

$$= [1 + g_{m_1}(r_{\pi_1} \| r_{\pi_2})] r_{o_1} + (r_{\pi_1} \| r_{\pi_2})$$

(b) In part (a), $I_{C2} = \beta I_{C1}$ $(= I_{B_2})$

$$\therefore R_{out(a)} = \left[1 + g_{m_1}\left(\frac{\beta V_T}{I_{C_1}} \| \frac{V_T}{I_{C_1}}\right)\right] r_{o_1} + (r_{\pi_1} \| r_{\pi_2})$$

$$\approx \left(1 + g_{m_1}\frac{V_T}{I_{C_1}}\right) r_{o_1} + \frac{V_T}{I_{C_1}}$$

$$= 2 r_{o_1} + V_T / I_{C_1}$$

$$R_{out,cascode} = [1 + g_{m_1}(r_{o2} \| r_{\pi_1})] r_{o_1} + (r_{o2} \| r_{\pi_1})$$

$$\approx [1 + g_{m_1} r_{\pi_1}] r_{o_1} + r_{\pi_1}$$

$$\approx \beta r_{o_1} + r_{\pi_1} = \beta r_{o_1} + V_A / I_{C_1}$$

Compare term-by-term:

$$\left.\begin{array}{l} 2 r_{o_1} \ll \beta r_{o_1} \\ V_T \ll V_A \end{array}\right\} \Rightarrow R_{out(a)} \ll R_{out,cascode}$$

i.e. using (a) reduces the effect of having a cascode configuration.

9.9

$$
\begin{aligned}
R_{out} &\approx \frac{1}{I_{C1}}\frac{V_A}{V_T}\frac{\beta V_A V_T}{V_A + \beta V_T} \text{ (Eq. 9.9)} \\
&= \frac{1}{I_{C1}}\frac{V_A}{V_T}\beta V_T \\
&= \frac{\beta V_A}{I_{C1}} \\
&= \boxed{\beta r_o}
\end{aligned}
$$

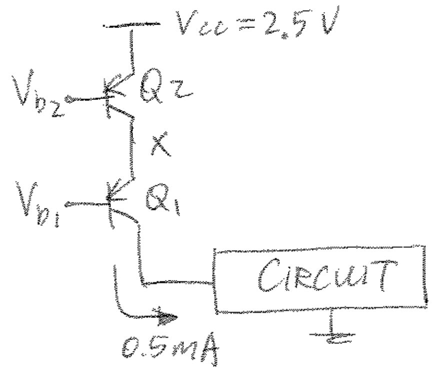
This resembles Eq. (9.12) because the assumption that

$$V_A \gg \beta V_T$$

can be equivalently expressed as

$$
\begin{aligned}
\frac{V_A}{I_C} &\gg \beta\frac{V_T}{I_C} \\
r_o &\gg r_\pi
\end{aligned}
$$

This is the same assumption used in arriving at Eq. (9.12).

10. 

$I_S = 10^{-16}$ A $\quad \beta = 100$

$I_{BIAS} = 0.5$ mA

(a) $I_{BIAS} \approx I_{C2} = 0.5\text{ mA}$

$$\therefore V_{b2} = V_{cc} - |V_{be2}|$$
$$= V_{cc} - V_T \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right)$$
$$= (2.5\text{V}) - (0.026\text{V}) \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right) \approx 1.74\text{ V}$$

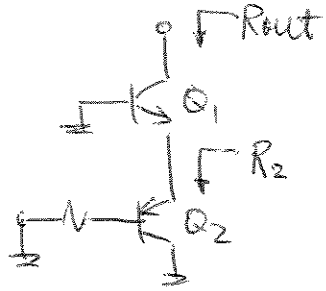
(b) $|V_{CB2}| = V_X - V_{b2} = 200\text{mV}$

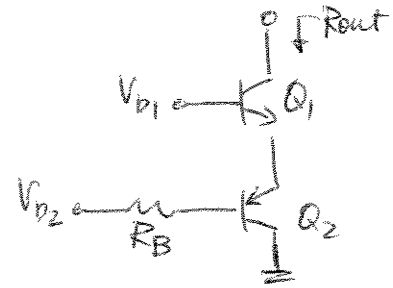
$$\Rightarrow V_{C2} = V_{b2} + |V_{CB2}| = 1.94\text{V}$$

$$\therefore V_{b1} = V_{C2} - |V_{BE1}| = V_{C2} - V_T \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right)$$
$$= (1.94\text{V}) - (0.026\text{V}) \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right) \approx 1.18\text{ V}$$

$\Rightarrow$ Maximum allowable $V_{b1} = 1.18$ V

11. (a)





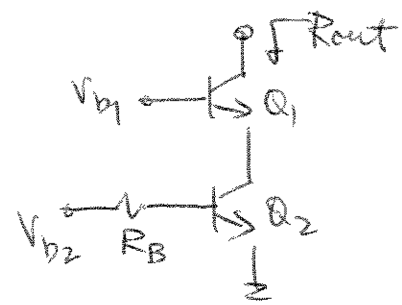
(AC-small signal)

Looking into emitter of Q2,

$$R_2 = \frac{1}{\left(\frac{\beta+1}{R_B + r_{\pi_2}} + \frac{1}{r_{o_2}}\right)}$$

$$\Rightarrow R_{out} = [1 + g_{m_1}(R_2 \parallel r_{\pi_1})] r_{o_1} + (R_2 \parallel r_{\pi_1})$$

(b) R_B does not affect Q_2 in small-signal R_{out}:



$$\therefore R_{out} = [1 + g_{m_1}(r_{o2} \parallel r_{\pi_1})] r_{o_1} + (r_{o2} \parallel r_{\pi_1})$$

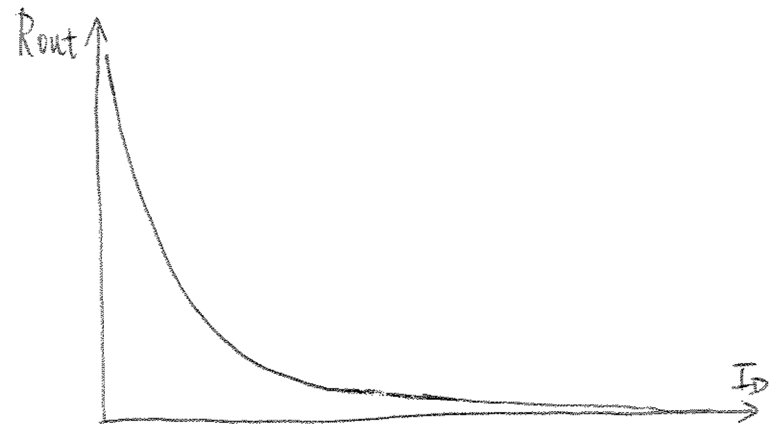
This is a cascode stage.

9.12

$$
\begin{aligned}
I_D &= 0.5 \text{ mA} \\
R_{out} &= r_{o1} + (1 + g_{m1} r_{o1}) \, r_{o2} \\
&= \frac{1}{\lambda I_D} + \left(1 + \sqrt{2 \frac{W}{L} \mu_n C_{ox} I_D} \frac{1}{\lambda I_D}\right) \frac{1}{\lambda I_D} \\
&\geq 50 \text{ k}\Omega \\
\lambda &\leq \boxed{0.558 \text{ V}^{-1}}
\end{aligned}
$$

13. (a) $R_{out} = g_{m2} r_{o1} r_{o2} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$

$= 2\mu_n C_{ox} (W/L) \cdot (I_D)^{-3/2}$

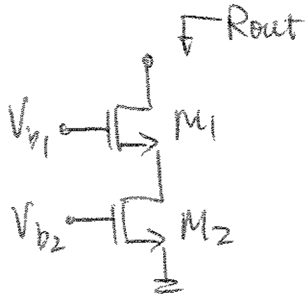


(b) $R_{out}\ (BJT) \propto I_B^{-1}$

$R_{out}\ (MOS) \propto I_B^{-3/2}$

∴ MOS cascode is a stronger function of I in terms of Rout.

14.



$(W/L)_1 = 30/0.18 \qquad (W/L)_2 = 20/0.18$

$I_{BIAS} = 0.5\,mA$

$\mu_n C_{ox} = 100\,\frac{\mu A}{V^2} \qquad V_{TH} = 0.4\,V$

(a) $I_{D2} = I_{BIAS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH})^2$

$$\Rightarrow V_{b2} = \sqrt{\frac{2 I_{BIAS}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} + V_{TH}$$

$$= \sqrt{\frac{2(0.5mA)}{(100\,\frac{\mu A}{V^2})(20/0.18)}} + 0.4V \approx 0.7\,V$$

M_2 operates in saturation as long as

$V_{GS2} - V_{TH} \le V_{DS2} \Rightarrow V_{DS2} \ge 0.3\,V.$

Observe that $V_{GS1} = V_{b1} - V_{DS2}$

$$I_{D1} = I_{BIAS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{DS2} - V_{TH})^2$$

$$\Rightarrow V_{b1} \ge \sqrt{\frac{2 I_{BIAS}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} + 0.4V + 0.3V$$

$$= \sqrt{\frac{2(0.5mA)}{(100\,\frac{\mu A}{V^2})(30/0.18)}} + 0.7V \approx 0.95\,V.$$

$\therefore$ Minimum $V_{b1} = 0.95V.$

(b) $R_{out} = (1 + g_{m1} r_{o2}) r_{o1} + r_{o2}$

$$= \left(1 + \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{BIAS}} \cdot \frac{1}{\lambda I_{BIAS}}\right) \cdot \frac{1}{\lambda I_{BIAS}} + \frac{1}{\lambda I_{BIAS}}$$

$$= \left[1 + \sqrt{2\left(100\frac{\mu A}{V^2}\right)\left(\frac{30}{0.18}\right)(0.5mA)} \cdot \frac{1}{(0.1)(0.5m)}\right] \cdot \frac{1}{(0.1)(0.5m)}$$

$$+ \frac{1}{(0.1)(0.5mA)}$$

$$\approx 1.67\ M\Omega$$

9.15 (a)

$$V_{D1} = V_{DD} - I_D R_D = 1.3\text{ V} > V_{G1} - V_{TH} = V_{b1} - V_{TH}$$
$$V_{b1} < \boxed{1.7\text{ V}}$$

(b)

$$\begin{aligned} V_{b1} &= 1.7\text{ V} \\ V_{GS1} &= V_{b1} - V_X \\ &= V_{TH} + \sqrt{\frac{2I_D}{\left(\frac{W}{L}\right)_1 \mu_n C_{ox}}} \\ &= 0.824\text{ V} \\ V_X &= \boxed{0.876\text{ V}} \end{aligned}$$

9.16 (a) Looking down from the source of M_1, we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{o2}$. Thus, we have

$$R_{out} = \boxed{g_{m1} r_{o1} \left(\frac{1}{g_{m2}} \parallel r_{o2} \right)}$$

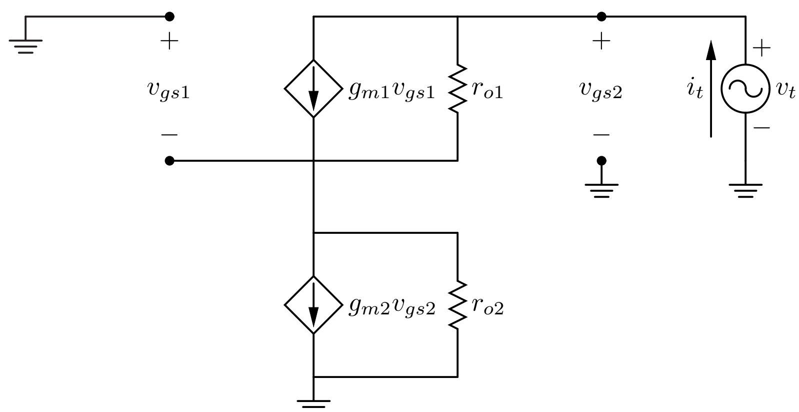
(b)

$$R_{out} = \boxed{g_{m1} r_{o1} r_{o2}}$$

(c) Putting two transistors in parallel, their transconductances will add and their output resistances will be in parallel (i.e., we can treat M_1 and M_3 as a single transistor with $g_m = g_{m1} + g_{m3}$ and $r_o = r_{o1} \parallel r_{o3}$). This can be seen from the small-signal model.

$$R_{out} = \boxed{(g_{m1} + g_{m3})(r_{o1} \parallel r_{o3}) r_{o2}}$$

(d) Let's draw the small-signal model and apply a test source to find R_{out}.

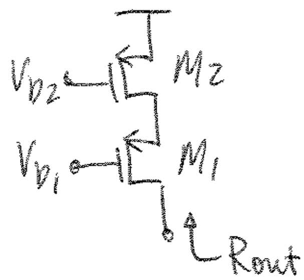


$$\begin{aligned}
i_t &= g_{m2} v_{gs2} - \frac{v_{gs1}}{r_{o2}} = g_{m1} v_{gs1} + \frac{v_{gs2} + v_{gs1}}{r_{o1}} \\
v_{gs1} &= g_{m2} r_{o2} v_t - i_t r_{o2} \\
i_t &= g_{m1} (g_{m2} r_{o2} v_t - i_t r_{o2}) + \frac{v_t + g_{m2} r_{o2} v_t - i_t r_{o2}}{r_{o1}} \\
i_t \left(1 + g_{m1} r_{o2} + \frac{r_{o2}}{r_{o1}}\right) &= v_t \left(g_{m1} g_{m2} r_{o2} + \frac{1 + g_{m2} r_{o2}}{r_{o1}}\right) \\
i_t (g_{m1} r_{o1} r_{o2}) &= v_t (g_{m1} g_{m2} r_{o1} r_{o2}) \\
R_{out} &= \frac{v_t}{i_t} = \boxed{\frac{1}{g_{m2}}}
\end{aligned}$$

9.17

$$\begin{aligned}
I_D &= 0.5 \text{ mA} \\
R_{out} &= r_{o1} + (1 + g_{m1} r_{o1})\, r_{o2} \\
&= \frac{1}{\lambda I_D} + \left(1 + \sqrt{2\left(\frac{W}{L}\right)_1 \mu_p C_{ox} I_D}\,\frac{1}{\lambda I_D}\right)\frac{1}{\lambda I_D} \\
&= 40 \text{ k}\Omega \\
\left(\frac{W}{L}\right)_1 &= \left(\frac{W}{L}\right)_2 = \boxed{8}
\end{aligned}$$

18.



$$R_{out} = g_{m1} r_{o1} r_{o2} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_1 I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$$

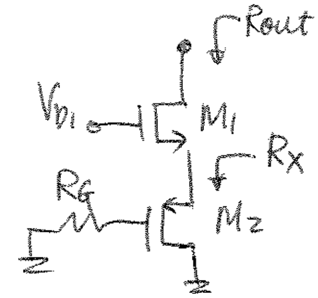
If W_1 & W_2 increase by N times and L_1, L_2, and I_D remain unchanged:

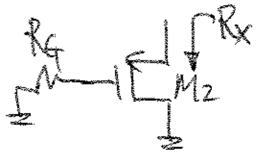
$$R_{out}(New) = \sqrt{2\mu_p C_{ox}\left(\frac{NW}{L}\right) I_D} \cdot \left(\frac{1}{\lambda I_D}\right)^2$$

$$= \sqrt{N}\sqrt{2\mu_p C_{ox}\frac{W}{L} I_D}\left(\frac{1}{\lambda I_D}\right)^2 = \sqrt{N}\, R_{out}$$

$\therefore$ Rout is increased by $\sqrt{N}$ times.

19\. (a) R_X is the input impedence of a common-gate configuration:





"Looking into" the source of M_2,

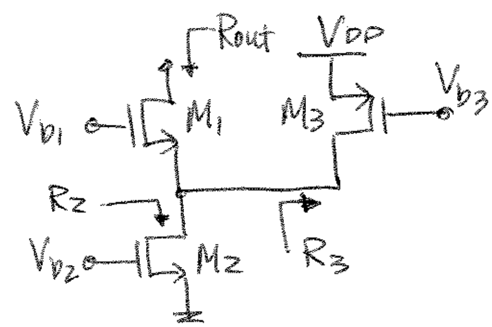
$$R_X = \frac{1}{g_{m2}} \parallel r_{o2}$$

$$\therefore R_{out} = g_{m1} r_{o1} R_X = g_{m1} r_{o1} \left(\frac{1}{g_{m2}} \parallel r_{o2}\right)$$

(b) From observation,

→ $R_3 = r_{o3}$ ($\because V_{SG} = 0$ in AC)

→ $R_2 = r_{o2}$ ($\because V_{SG} = 0$ in AC)

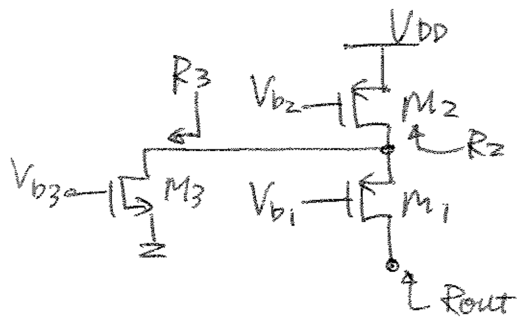


$$\therefore R_{out} = g_{m1} r_{o1} (R_2 \parallel R_3) = g_{m1} r_{o1} (r_{o2} \parallel r_{o3})$$

(c) By observation,

$R_2 = r_{o2}$ ($V_S = V_G = $ AC GND)

$R_3 = r_{o3}$ ($V_S = V_G = $ AC GND)

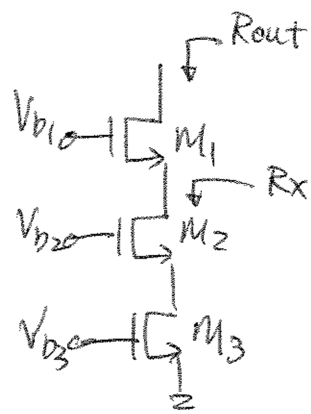


$\therefore R_{out} = g_{m1} r_{o1} (R_2 \| R_3) = g_{m1} r_{o1} (r_{o2} \| r_{o3})$

(d) $R_X = g_{m2} r_{o2} r_{o3}$

$\Rightarrow R_{out} = g_{m1} r_{o1} R_X$

$= g_{m1} g_{m2} r_{o1} r_{o2} r_{o3}$



9.20 (a)

$$G_m = \boxed{g_{m1}}$$

$$R_{out} = \frac{1}{g_{m2}} \parallel r_{o1}$$

$$A_v = \boxed{-g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{o1}\right)}$$

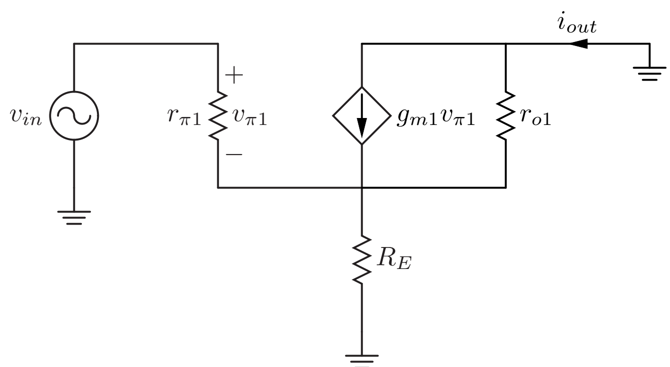
(b)

$$G_m = \boxed{-g_{m2}}$$

$$R_{out} = \frac{1}{g_{m2}} \parallel r_{o2} \parallel r_{o1}$$

$$A_v = \boxed{g_{m2}\left(\frac{1}{g_{m2}} \parallel r_{o2} \parallel r_{o1}\right)}$$

(c) Let's draw the small-signal model to find G_m.



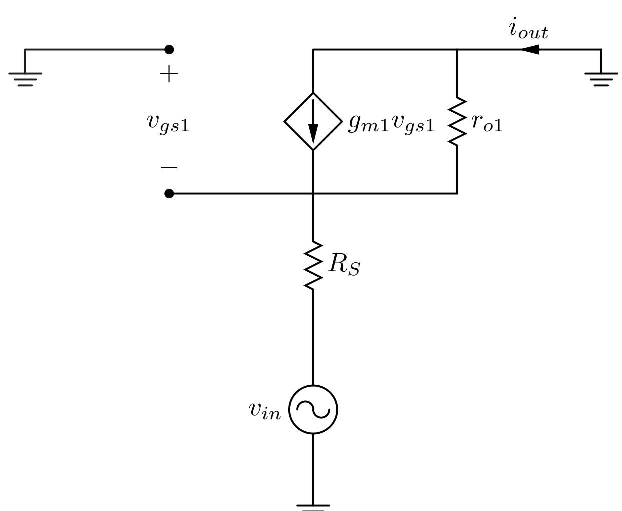
$$\begin{aligned}
i_{out} &= -\frac{v_{\pi 1}}{r_{\pi 1}} + \frac{v_{in} - v_{\pi 1}}{R_E} \\
v_{\pi 1} &= v_{in} + \left(i_{out} - g_{m1} v_{\pi 1}\right) r_{o1} \\
v_{\pi 1}\left(1 + g_{m1} r_{o1}\right) &= v_{in} + i_{out} r_{o1} \\
v_{\pi 1} &= \frac{v_{in} + i_{out} r_{o1}}{1 + g_{m1} r_{o1}} \\
i_{out} &= -\frac{v_{in} + i_{out} r_{o1}}{r_{\pi 1}\left(1 + g_{m1} r_{o1}\right)} + \frac{v_{in}}{R_E} - \frac{v_{in} + i_{out} r_{o1}}{R_E\left(1 + g_{m1} r_{o1}\right)} \\
i_{out}\left[1 + \frac{r_{o1}}{r_{\pi 1}\left(1 + g_{m1} r_{o1}\right)} + \frac{r_{o1}}{R_E\left(1 + g_{m1} r_{o1}\right)}\right] &= v_{in}\left[\frac{1}{R_E} - \frac{1}{r_{\pi 1}\left(1 + g_{m1} r_{o1}\right)} - \frac{1}{R_E\left(1 + g_{m1} r_{o1}\right)}\right] \\
i_{out}\frac{r_{\pi 1} R_E\left(1 + g_{m1} r_{o1}\right) + r_{o1} R_E + r_{o1} r_{\pi 1}}{r_{\pi 1} R_E\left(1 + g_{m1} r_{o1}\right)} &= v_{in}\frac{r_{\pi 1}\left(1 + g_{m1} r_{o1}\right) - R_E - r_{\pi 1}}{r_{\pi 1} R_E\left(1 + g_{m1} r_{o1}\right)} \\
i_{out}\left[r_{\pi 1} R_E\left(1 + g_{m1} r_{o1}\right) + r_{o1} R_E + r_{o1} r_{\pi 1}\right] &= v_{in}\left[r_{\pi 1}\left(1 + g_{m1} r_{o1}\right) - R_E - r_{\pi 1}\right] \\
G_m &= \frac{i_{out}}{v_{in}} \\
&= \boxed{\frac{r_{\pi 1}\left(1 + g_{m1} r_{o1}\right) - R_E - r_{\pi 1}}{r_{\pi 1} R_E\left(1 + g_{m1} r_{o1}\right) + r_{o1} R_E + r_{o1} r_{\pi 1}}} \\
&\approx \frac{g_{m1}}{1 + g_{m1} R_E} \text{ (if } r_{\pi 1}, r_{o1} \text{ are large)} \\
R_{out} &= r_{o2} \parallel \left[r_{o1} + \left(1 + g_{m1} r_{o1}\right)\left(r_{\pi 1} \parallel R_E\right)\right]
\end{aligned}$$

$$A_v = \boxed{-\frac{r_{\pi 1} R_E\left(1 + g_{m1} r_{o1}\right) - R_E - r_{\pi 1}}{r_{\pi 1} R_E\left(1 + g_{m1} r_{o1}\right) + r_{o1} R_E + r_{o1} r_{\pi 1}}\left\{r_{o2} \parallel \left[r_{o1} + \left(1 + g_{m1} r_{o1}\right)\left(r_{\pi 1} \parallel R_E\right)\right]\right\}}$$

(d)

$$\begin{aligned}
G_m &= \boxed{g_{m2}} \\
R_{out} &= r_{o2} \parallel \left[r_{o1} + \left(1 + g_{m1} r_{o1}\right)\left(r_{\pi 1} \parallel R_E\right)\right] \\
A_v &= \boxed{-g_{m2}\left\{r_{o2} \parallel \left[r_{o1} + \left(1 + g_{m1} r_{o1}\right)\left(r_{\pi 1} \parallel R_E\right)\right]\right\}}
\end{aligned}$$

(e) Let's draw the small-signal model to find G_m.



Since the gate and drain are both at AC ground, the dependent current source looks like a resistor with value $1/g_{m1}$. Thus, we have:

$$\begin{aligned}
G_m &= \frac{i_{out}}{v_{in}} = -\frac{1}{R_S + \frac{1}{g_{m1}} \parallel r_{o1}} \\
&= -\frac{1}{R_S + \frac{r_{o1}}{1+g_{m1}r_{o1}}} \\
&= \boxed{-\frac{1 + g_{m1}r_{o1}}{r_{o1} + R_S + g_{m1}r_{o1}R_S}} \\
&\approx -\frac{g_{m1}}{1 + g_{m1}R_S} \text{ (if } r_{o1} \text{ is large)} \\
R_{out} &= \left[r_{o2} + (1 + g_{m2}r_{o2})\,R_E\right] \parallel \left[r_{o1} + (1 + g_{m1}r_{o1})\,R_S\right] \\
A_v &= \boxed{\frac{1 + g_{m1}r_{o1}}{r_{o1} + R_S + g_{m1}r_{o1}R_S} \left\{\left[r_{o2} + (1 + g_{m2}r_{o2})\,R_E\right] \parallel \left[r_{o1} + (1 + g_{m1}r_{o1})\,R_S\right]\right\}}
\end{aligned}$$

(f) We can use the result from part (c) to find G_m here. If we simply let $r_\pi \to \infty$ (and obviously we replace the subscripts as appropriate) in the expression for G_m from part (c), we'll get the result we need here.

$$\begin{aligned}
G_m &= \lim_{r_{\pi 2}\to\infty} \frac{r_{\pi 2}R_E\,(2 + g_{m2}r_{o2}) - R_E - r_{\pi 2}}{r_{\pi 2}R_E\,(2 + g_{m2}r_{o2}) + r_{o2}R_E + r_{o2}r_{\pi 2}} \\
&= \boxed{\frac{g_{m2}r_{o2}}{r_{o2} + R_E + g_{m2}r_{o2}R_E}} \\
&\approx \frac{g_{m2}}{1 + g_{m2}R_E} \text{ (if } r_{o2} \text{ is large)} \\
R_{out} &= \left[r_{o2} + (1 + g_{m2}r_{o2})\,R_E\right] \parallel \left[r_{o1} + (1 + g_{m1}r_{o1})\,R_S\right] \\
A_v &= \boxed{-\frac{g_{m2}r_{o2}}{r_{o2} + R_E + g_{m2}r_{o2}R_E} \left\{\left[r_{o2} + (1 + g_{m2}r_{o2})\,R_E\right] \parallel \left[r_{o1} + (1 + g_{m1}r_{o1})\,R_S\right]\right\}}
\end{aligned}$$

(g) Once again, we can use the result from part (c) to find G_m here (replacing subscripts as appropriate).

$$\begin{aligned}
G_m &= \boxed{\frac{r_{\pi 2}R_E\,(1 + g_{m2}r_{o2}) - R_E - r_{\pi 2}}{r_{\pi 2}R_E\,(1 + g_{m2}r_{o2}) + r_{o2}R_E + r_{o2}r_{\pi 2}}} \\
&\approx \frac{g_{m2}}{1 + g_{m2}R_E} \text{ (if } r_{\pi 2}, r_{o2} \text{ are large)} \\
R_{out} &= R_C \parallel \left[r_{o2} + (1 + g_{m2}r_{o2})\,(r_{\pi 2} \parallel R_E)\right] \\
A_v &= \boxed{-\frac{r_{\pi 2}R_E\,(1 + g_{m2}r_{o2}) - R_E - r_{\pi 2}}{r_{\pi 2}R_E\,(1 + g_{m2}r_{o2}) + r_{o2}R_E + r_{o2}r_{\pi 2}} \left\{R_C \parallel \left[r_{o2} + (1 + g_{m2}r_{o2})\,(r_{\pi 2} \parallel R_E)\right]\right\}}
\end{aligned}$$

21. $A_v = -g_{m1} r_{o1} g_{m1} (r_{o1} \| r_{\pi 2})$

$$= -\frac{I_{C1}}{V_T} \cdot \frac{V_{A1}}{I_{C1}} \cdot \frac{I_{C1}}{V_T} \cdot \frac{1}{\frac{I_{C1}}{V_{A1}} + \frac{I_{C2}}{\beta V_T}}$$

Since $I_{C1} \approx I_{C2}$,

$$A_v \approx -\frac{V_{A1}/V_T^2}{\frac{1}{V_{A1}} + \frac{1}{\beta V_T}} = -\frac{\beta V_A^2}{V_T(V_A + \beta V_T)}$$

9.22

$$
\begin{aligned}
A_v &= -g_{m1}\left[r_{o2} + \left(1 + g_{m2} r_{o2}\right)\left(r_{\pi 2} \parallel r_{o1}\right)\right] \\
I_{C1} &\approx I_{C2} = I_1 \\
V_{A1} &= V_{A2} = V_A \\
A_v &\approx -\frac{I_1}{V_T}\left[\frac{V_A}{I_1} + \left(1 + \frac{V_A}{V_T}\right)\left(\frac{\beta V_T}{I_1} \parallel \frac{V_A}{I_1}\right)\right] \\
&= -500 \\
V_{A1} &= V_{A2} = \boxed{0.618\ \mathrm{V}^{-1}}
\end{aligned}
$$

9.23 (a) Although the output resistance of this stage is the same as that of a cascode, the transconductance of this stage is lower than that of a cascode stage. A cascode has $G_m = g_m$, where as this stage has $G_m = \frac{g_{m2}}{1+g_{m2}r_{o1}}$.

(b)

$$G_m = \boxed{\frac{g_{m2}}{1 + g_{m2}r_{o1}}}$$

$$R_{out} = r_{o2} + (1 + g_{m2}r_{o2})(r_{\pi 2} \parallel r_{o1})$$

$$A_v = -G_m R_{out}$$

$$= \boxed{-\frac{g_{m2}}{1 + g_{m2}r_{o1}}\left[r_{o2} + (1 + g_{m2}r_{o2})(r_{\pi 2} \parallel r_{o1})\right]}$$

9.24

$$G_m = \boxed{-g_{m1}}$$

$$R_{out} = r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel r_{o1})$$

$$A_v = \boxed{g_{m1} [r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel r_{o1})]}$$

9.25 (a)

$$\begin{aligned}
G_m &= g_{m2} \frac{R_P \parallel r_{\pi 1}}{\frac{1}{g_{m1}} + R_P \parallel r_{\pi 1}} \\
R_{out} &= r_{o1} + (1 + g_{m1} r_{o1}) (r_{\pi 1} \parallel r_{o2} \parallel R_P) \\
A_v &= \boxed{-g_{m2} \frac{R_P \parallel r_{\pi 1}}{\frac{1}{g_{m1}} + R_P \parallel r_{\pi 1}} [r_{o1} + (1 + g_{m1} r_{o1}) (r_{\pi 1} \parallel r_{o2} \parallel R_P)]}
\end{aligned}$$

(b)

$$\begin{aligned}
G_m &= g_{m2} \\
R_{out} &= r_{o1} \parallel R_P + [1 + g_{m1} (r_{o1} \parallel R_P)] (r_{\pi 1} \parallel r_{o2}) \\
A_v &= \boxed{-g_{m2} \{r_{o1} \parallel R_P + [1 + g_{m1} (r_{o1} \parallel R_P)] (r_{\pi 1} \parallel r_{o2})\}}
\end{aligned}$$

(c)

$$\begin{aligned}
G_m &= \frac{g_{m2}}{1 + g_{m2} R_E} \\
R_{out} &= r_{o1} + (1 + g_{m1} r_{o1}) [r_{\pi 1} \parallel (r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E))] \\
A_v &= \boxed{-\frac{g_{m2}}{1 + g_{m2} R_E} \{r_{o1} + (1 + g_{m1} r_{o1}) [r_{\pi 1} \parallel (r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E))]\}}
\end{aligned}$$

(d)

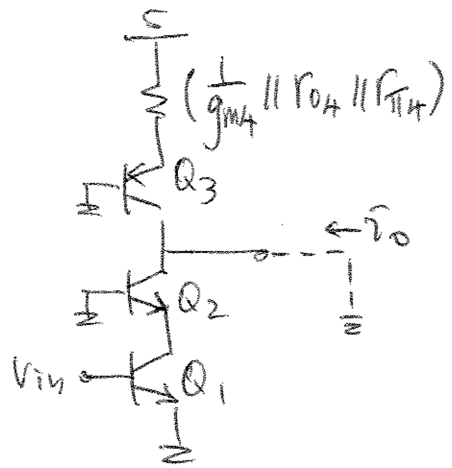
$$\begin{aligned}
G_m &= g_{m2} \\
R_{out} &= r_{o1} + (1 + g_{m1} r_{o1}) (r_{\pi 1} \parallel r_{o2} \parallel r_{o3}) \\
A_v &= \boxed{-g_{m2} [r_{o1} + (1 + g_{m1} r_{o1}) (r_{\pi 1} \parallel r_{o2} \parallel r_{o3})]}
\end{aligned}$$

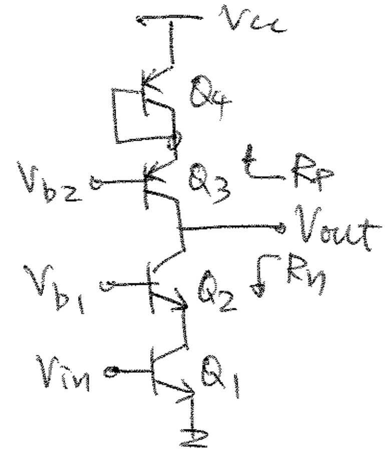
9.26

$$
\begin{aligned}
A_v &= -g_{m1}\left\{\left[r_{o2} + \left(1 + g_{m2} r_{o2}\right)\left(r_{\pi 2} \parallel r_{o1}\right)\right] \parallel \left[r_{o3} + \left(1 + g_{m3} r_{o3}\right)\left(r_{\pi 3} \parallel r_{o4}\right)\right]\right\} \\
&= -\frac{I_C}{V_T}\left\{\left[\frac{V_{A,N}}{I_C} + \left(1 + \frac{V_{A,N}}{V_T}\right)\left(\frac{\beta_N V_T}{I_C} \parallel \frac{V_{A,N}}{I_C}\right)\right] \parallel \left[\frac{V_{A,P}}{I_C} + \left(1 + \frac{V_{A,P}}{V_T}\right)\left(\frac{\beta_P V_T}{I_C} \parallel \frac{V_{A,P}}{I_C}\right)\right]\right\} \\
&= -\frac{I_C}{V_T}\frac{\left[\frac{V_{A,N}}{I_C} + \left(1 + \frac{V_{A,N}}{V_T}\right)\left(\frac{\beta_N V_T}{I_C} \parallel \frac{V_{A,N}}{I_C}\right)\right]\left[\frac{V_{A,P}}{I_C} + \left(1 + \frac{V_{A,P}}{V_T}\right)\left(\frac{\beta_P V_T}{I_C} \parallel \frac{V_{A,P}}{I_C}\right)\right]}{\left[\frac{V_{A,N}}{I_C} + \left(1 + \frac{V_{A,N}}{V_T}\right)\left(\frac{\beta_N V_T}{I_C} \parallel \frac{V_{A,N}}{I_C}\right)\right] + \left[\frac{V_{A,P}}{I_C} + \left(1 + \frac{V_{A,P}}{V_T}\right)\left(\frac{\beta_P V_T}{I_C} \parallel \frac{V_{A,P}}{I_C}\right)\right]} \\
&= -\frac{I_C}{V_T}\frac{\left[\frac{V_{A,N}}{I_C} + \left(1 + \frac{V_{A,N}}{V_T}\right)\frac{\beta_N V_T V_{A,N}}{I_C^2\left(\frac{\beta_N V_T}{I_C} + \frac{V_{A,N}}{I_C}\right)}\right]\left[\frac{V_{A,P}}{I_C} + \left(1 + \frac{V_{A,P}}{V_T}\right)\frac{\beta_P V_T V_{A,P}}{I_C^2\left(\frac{\beta_P V_T}{I_C} + \frac{V_{A,P}}{I_C}\right)}\right]}{\left[\frac{V_{A,N}}{I_C} + \left(1 + \frac{V_{A,N}}{V_T}\right)\frac{\beta_N V_T V_{A,N}}{I_C^2\left(\frac{\beta_N V_T}{I_C} + \frac{V_{A,N}}{I_C}\right)}\right] + \left[\frac{V_{A,P}}{I_C} + \left(1 + \frac{V_{A,P}}{V_T}\right)\frac{\beta_P V_T V_{A,P}}{I_C^2\left(\frac{\beta_P V_T}{I_C} + \frac{V_{A,P}}{I_C}\right)}\right]} \\
&= -\frac{I_C}{V_T}\frac{\frac{1}{I_C^2}\left[V_{A,N} + \left(1 + \frac{V_{A,N}}{V_T}\right)\frac{\beta_N V_T V_{A,N}}{\beta_N V_T + V_{A,N}}\right]\left[V_{A,P} + \left(1 + \frac{V_{A,P}}{V_T}\right)\frac{\beta_P V_T V_{A,P}}{\beta_P V_T + V_{A,P}}\right]}{\frac{1}{I_C}\left[V_{A,N} + \left(1 + \frac{V_{A,N}}{V_T}\right)\frac{\beta_N V_T V_{A,N}}{\beta_N V_T + V_{A,N}}\right] + \frac{1}{I_C}\left[V_{A,P} + \left(1 + \frac{V_{A,P}}{V_T}\right)\frac{\beta_P V_T V_{A,P}}{\beta_P V_T + V_{A,P}}\right]} \\
&= \boxed{-\frac{1}{V_T}\frac{\left[V_{A,N} + \left(1 + \frac{V_{A,N}}{V_T}\right)\frac{\beta_N V_T V_{A,N}}{\beta_N V_T + V_{A,N}}\right]\left[V_{A,P} + \left(1 + \frac{V_{A,P}}{V_T}\right)\frac{\beta_P V_T V_{A,P}}{\beta_P V_T + V_{A,P}}\right]}{\left[V_{A,N} + \left(1 + \frac{V_{A,N}}{V_T}\right)\frac{\beta_N V_T V_{A,N}}{\beta_N V_T + V_{A,N}}\right] + \left[V_{A,P} + \left(1 + \frac{V_{A,P}}{V_T}\right)\frac{\beta_P V_T V_{A,P}}{\beta_P V_T + V_{A,P}}\right]}}
\end{aligned}
$$

The result does not depend on the bias current.

27. Equivalent circuit.





$$G_m = g_{m1} = \frac{i_o}{v_{in}} = \frac{i_{c1}}{v_{in}}$$

$$R_{out} = R_p \| R_n$$

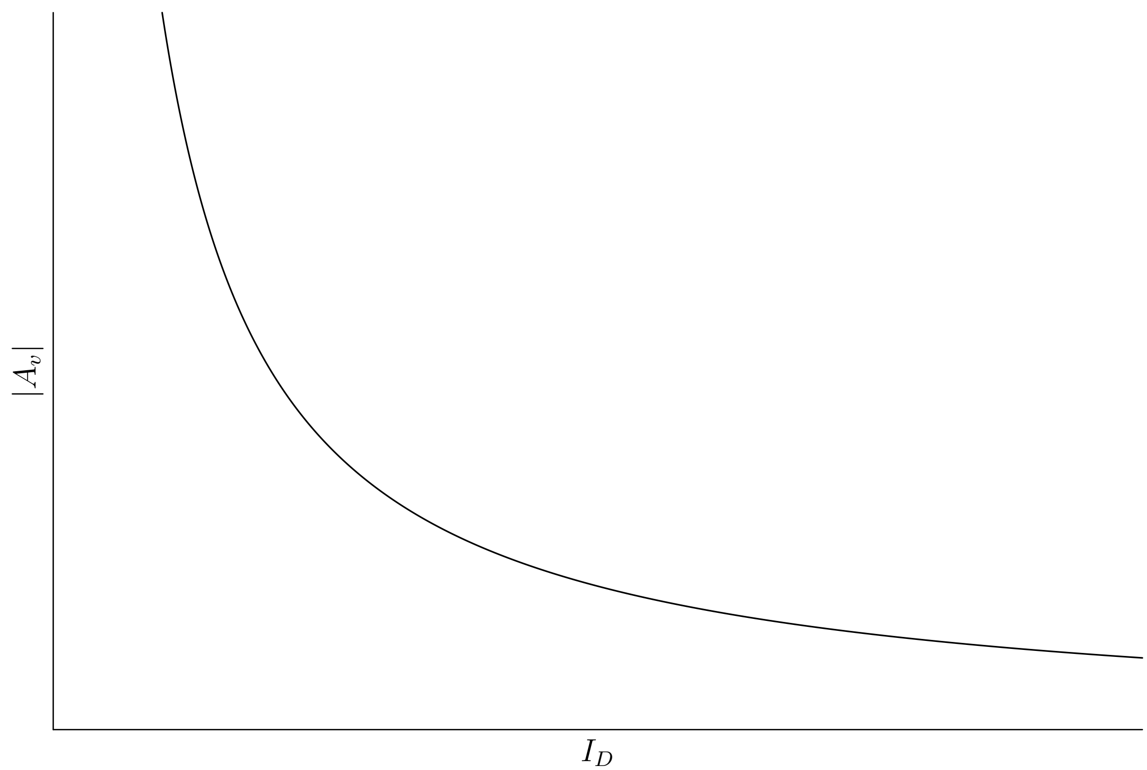
$$R_p = \left[1 + g_{m3}\left(\frac{1}{g_{m4}} \| r_{o4} \| r_{\pi 4} \| r_{\pi 3}\right)\right] r_{o3} + \left[\frac{1}{g_{m4}} \| r_{o4} \| r_{\pi 4} \| r_{\pi 3}\right]$$

$$R_n = \left[1 + g_{m2}(r_{o1} \| r_{\pi 2})\right] r_{o2} + (r_{o1} \| r_{\pi 2})$$

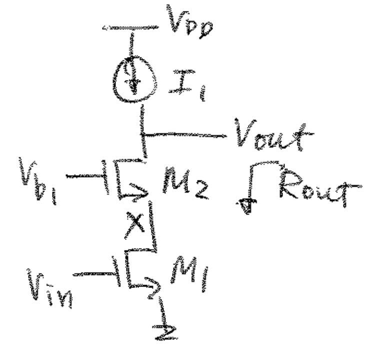
$$\therefore A_v = -G_m R_{out} = -g_{m1}(R_p \| R_n)$$

9.28

$$
\begin{aligned}
A_v &\approx -g_{m1} g_{m2} r_{o1} r_{o2} \text{ (Eq. 9.69)} \\
&= -\sqrt{2\left(\frac{W}{L}\right)_1 \mu_n C_{ox} I_D}\sqrt{2\left(\frac{W}{L}\right)_2 \mu_n C_{ox} I_D}\left(\frac{1}{\lambda I_D}\right)^2 \\
&= -2\mu_n C_{ox} I_D \sqrt{\left(\frac{W}{L}\right)_1 \left(\frac{W}{L}\right)_2}\left(\frac{1}{\lambda I_D}\right)^2 \\
&= \boxed{-2\mu_n C_{ox} \frac{1}{I_D}\frac{1}{\lambda^2}\sqrt{\left(\frac{W}{L}\right)_1 \left(\frac{W}{L}\right)_2}}
\end{aligned}
$$

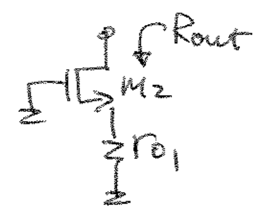


29. $|A_v| = 200$



$\mu_n C_{ox} = 100 \frac{\mu A}{V^2}$ $\lambda = 0.1\, V^{-1}$

Determine $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$



$R_{out} = (1 + g_{m2} r_{o1}) r_{o2} + r_{o1}$

$G_m \cong g_{m1}$ (short-circuit current flows through both M_1 & M_2)

$|A_v| = G_m R_{out} = g_{m1}\left[(1 + g_{m2} r_{o1}) r_{o2} + r_{o1}\right]$

$\approx g_{m1} g_{m2} r_{o1} r_{o2} = (g_m r_o)^2 = 200$

$\left(\because \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 \text{ and } I_{D1} = I_{D2}\right)$

$(g_m r_o)^2 = \left(\frac{2 I_D}{V_{GS} - V_{TH}} \cdot \frac{1}{\lambda I_D}\right)^2 = 200$

$\Rightarrow V_{GS} - V_{TH} = \left(\sqrt{200} \cdot \lambda/2\right)^{-1} = \left[\sqrt{200} \cdot (0.05\, V^{-1})\right]^{-1}$

$\approx 1.41\ V$

$$\Rightarrow \quad I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2$$

$$\therefore \left(\frac{W}{L}\right) = \frac{2 I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^2}$$

$$= \frac{2(1\ mA)}{100 \frac{\mu A}{V^2} (1.41V)^2} \approx 10$$

9.30 From Problem 28, we have

$$A_v = -2\mu_n C_{ox} \frac{1}{I_D} \frac{1}{\lambda^2} \sqrt{\left(\frac{W}{L}\right)_1 \left(\frac{W}{L}\right)_2}$$

If we increase the transistor widths by a factor of N, we will get a new voltage gain A_v':

$$\begin{aligned}
A_v' &= -2\mu_n C_{ox} \frac{1}{I_D} \frac{1}{\lambda^2} \sqrt{N^2 \left(\frac{W}{L}\right)_1 \left(\frac{W}{L}\right)_2} \\
&= -2N\mu_n C_{ox} \frac{1}{I_D} \frac{1}{\lambda^2} \sqrt{\left(\frac{W}{L}\right)_1 \left(\frac{W}{L}\right)_2} \\
&= NA_v
\end{aligned}$$

Thus, the gain increases by a factor of N.

9.31 From Problem 28, we have

$$A_v = -2\mu_n C_{ox} \frac{1}{I_D} \frac{1}{\lambda^2} \sqrt{\left(\frac{W}{L}\right)_1 \left(\frac{W}{L}\right)_2}$$

If we decrease the transistor widths by a factor of N, we will get a new voltage gain A_v':

$$\begin{aligned} A_v' &= -2\mu_n C_{ox} \frac{1}{I_D} \frac{1}{\lambda^2} \sqrt{\frac{1}{N^2} \left(\frac{W}{L}\right)_1 \left(\frac{W}{L}\right)_2} \\ &= -2\frac{1}{N}\mu_n C_{ox} \frac{1}{I_D} \frac{1}{\lambda^2} \sqrt{\left(\frac{W}{L}\right)_1 \left(\frac{W}{L}\right)_2} \\ &= \frac{1}{N} A_v \end{aligned}$$

Thus, the gain decreases by a factor of N.

9.32

$$G_m = -g_{m2}$$
$$R_{out} = r_{o2} \parallel [r_{o3} + (1 + g_{m3} r_{o3}) r_{o4}]$$
$$A_v = \boxed{g_{m2} \{r_{o2} \parallel [r_{o3} + (1 + g_{m3} r_{o3}) r_{o4}]\}}$$

9.33

$$
\begin{aligned}
A_v &= -g_{m1}\left\{\left[r_{o2} + \left(1 + g_{m2} r_{o3}\right) r_{o1}\right] \parallel \left[r_{o3} + \left(1 + g_{m3} r_{o3}\right) r_{o4}\right]\right\} \\
&= -500 \\
g_{m1} = g_{m2} &= \sqrt{2\left(\frac{W}{L}\right)\mu_n C_{ox} I_D} \\
g_{m3} = g_{m4} &= \sqrt{2\left(\frac{W}{L}\right)\mu_p C_{ox} I_D} \\
r_{o1} = r_{o1} &= \frac{1}{\lambda_n I_D} \\
r_{o3} = r_{o4} &= \frac{1}{\lambda_p I_D} \\
I_D &= \boxed{1.15\text{ mA}}
\end{aligned}
$$

9.34 (a)

$$
\begin{aligned}
G_m &= g_{m1} \\
R_{out} &= \left[(r_{o2} \parallel R_P) + (1 + g_{m2}(r_{o2} \parallel R_P))\, r_{o1}\right] \parallel \left[r_{o3} + (1 + g_{m3} r_{o3})\, r_{o4}\right] \\
A_v &= \boxed{-g_{m1}\left\{\left[(r_{o2} \parallel R_P) + (1 + g_{m2}(r_{o2} \parallel R_P))\, r_{o1}\right] \parallel \left[r_{o3} + (1 + g_{m3} r_{o3})\, r_{o4}\right]\right\}}
\end{aligned}
$$

(b)

$$
\begin{aligned}
G_m &= g_{m1} \frac{r_{o1} \parallel R_P}{\frac{1}{g_{m2}} + r_{o1} \parallel R_P} \\
R_{out} &= \left[r_{o2} + (1 + g_{m2} r_{o2})(r_{o1} \parallel R_P)\right] \parallel \left[r_{o3} + (1 + g_{m3} r_{o3})\, r_{o4}\right] \\
A_v &= \boxed{-g_{m1} \frac{r_{o1} \parallel R_P}{\frac{1}{g_{m2}} + r_{o1} \parallel R_P} \left\{\left[r_{o2} + (1 + g_{m2} r_{o2})(r_{o1} \parallel R_P)\right] \parallel \left[r_{o3} + (1 + g_{m3} r_{o3})\, r_{o4}\right]\right\}}
\end{aligned}
$$

(c)

$$
\begin{aligned}
G_m &= g_{m5} \\
R_{out} &= \left[r_{o2} + (1 + g_{m2} r_{o2})(r_{o1} \parallel r_{o5})\right] \parallel \left[r_{o3} + (1 + g_{m3} r_{o3})\, r_{o4}\right] \\
A_v &= \boxed{-g_{m5}\left\{\left[r_{o2} + (1 + g_{m2} r_{o2})(r_{o1} \parallel r_{o5})\right] \parallel \left[r_{o3} + (1 + g_{m3} r_{o3})\, r_{o4}\right]\right\}}
\end{aligned}
$$

(d)

$$
\begin{aligned}
G_m &= g_{m5} \\
R_{out} &= \left[r_{o2} + (1 + g_{m2} r_{o2})\, r_{o1}\right] \parallel \left[r_{o3} + (1 + g_{m3} r_{o3})(r_{o4} \parallel r_{o5})\right] \\
A_v &= \boxed{-g_{m5}\left\{\left[r_{o2} + (1 + g_{m2} r_{o2})\, r_{o1}\right] \parallel \left[r_{o3} + (1 + g_{m3} r_{o3})(r_{o4} \parallel r_{o5})\right]\right\}}
\end{aligned}
$$

35.
$$\frac{R_2}{R_1+R_2} V_{CC} = V_T \ln\left(\frac{I_1}{I_S}\right)$$

$$\Rightarrow I_1 = I_S \cdot \exp\left[\frac{V_{CC}}{V_T} \cdot \frac{R_2}{R_1+R_2}\right]$$

$$\frac{\partial I_1}{\partial V_{CC}} = \frac{I_S}{V_T} \cdot \frac{R_2}{R_1+R_2} \cdot \exp\left[\frac{V_{CC}}{V_T} \cdot \frac{R_2}{R_1+R_2}\right]$$

$$= \frac{I_1}{V_T} \cdot \frac{R_2}{R_1+R_2} = g_m\left(\frac{R_2}{R_1+R_2}\right)$$

Intuitively, we know that an exponential relationship exists between I_C & V_{BE}. Its transconductance is also a function (linear) of I_C. Since V_{BE} comes from a voltage divider (which is also linear), we expect a linear relationship between I_C & V_{CC}.

9.36

$$
\begin{aligned}
I_1 &= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(\frac{R_2}{R_1+R_2}V_{DD} - V_{TH}\right)^2 \quad \text{(Eq. 9.85)} \\
\frac{\partial I_1}{\partial V_{DD}} &= \frac{W}{L}\mu_n C_{ox}\left(\frac{R_2}{R_1+R_2}V_{DD} - V_{TH}\right)\frac{R_2}{R_1+R_2} \\
&= \boxed{\frac{R_2}{R_1+R_2}g_m}
\end{aligned}
$$

Intuitively, we know that g_m is the derivative of I_1 with respect to V_{GS}, or $g_m = \frac{\partial I_1}{\partial V_{GS}}$. Since V_{GS} is linearly dependent on V_{DD} by the relationship established by the voltage divider (meaning $\frac{\partial V_{GS}}{\partial V_{DD}}$ is a constant), we'd expect $\frac{\partial I_1}{\partial V_{DD}}$ to also be proportional to g_m, since $\frac{\partial I_1}{\partial V_{DD}} = \frac{\partial V_{GS}}{\partial V_{DD}} \cdot \frac{\partial I_1}{\partial V_{GS}} = \frac{\partial V_{GS}}{\partial V_{DD}} g_m$.

9.37

$$I_1 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(\frac{R_2}{R_1+R_2}V_{DD} - V_{TH}\right)^2 \quad \text{(Eq. 9.85)}$$

$$\frac{\partial I_1}{\partial V_{TH}} = \boxed{-\mu_n C_{ox}\frac{W}{L}\left(\frac{R_2}{R_1+R_2}V_{DD} - V_{TH}\right)}$$

The sensitivity of I_1 to V_{TH} becomes a more serious issue at low supply voltages because as V_{DD} becomes smaller with respect to V_{TH}, V_{TH} has more control over the sensitivity. When V_{DD} is large enough, it dominates the last term of the expression, reducing the control of V_{TH} over the sensitivity.

9.38 As long as $V_{REF} > 0$, the circuit operates in negative feedback, so that $V_+ = V_- = 0$ V.

$$I_{C1} = I_{S1}e^{-V_1/V_T} = \frac{V_{REF}}{R_1}$$
$$V_1 = -V_T \ln\left(\frac{V_{REF}}{R_1 I_{S1}}\right) = V_{BE2}$$

If $V_{REF} > R_1 I_{S1}$, then we have $V_{BE2} < 0$, and $I_X = 0$. If $V_{REF} < R_1 I_{S1}$, then we have:

$$\begin{aligned} I_X &= I_{S2}e^{-V_T \ln\left(\frac{V_{REF}}{R_1 I_{S1}}\right)/V_T} \\ &= I_{S2}e^{-\ln\left(\frac{V_{REF}}{R_1 I_{S1}}\right)} \\ &= I_{S2}\frac{R_1 I_{S1}}{V_{REF}} \end{aligned}$$

Thus, if $V_{REF} > R_1 I_{S1}$ (which will typically be true, since I_{S1} is typically very small), then we get no output, i.e., $I_X = 0$. When $V_{REF} < R_1 I_{S1}$, we get an inverse relationship between I_X and V_{REF}.

9.39 As long as $V_{REF} > 0$, the circuit operates in negative feedback, so that $V_+ = V_- = 0$ V.

$$I_{C1} = I_{S1} e^{-V_1/V_T} = \frac{V_{REF}}{R_1}$$

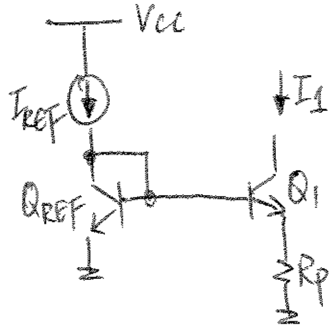
$$V_1 = -V_T \ln\left(\frac{V_{REF}}{R_1 I_{S1}}\right) = -V_{BE2}$$

If $V_{REF} < R_1 I_{S1}$, then we have $V_{BE2} < 0$, and $I_X = 0$. If $V_{REF} > R_1 I_{S1}$, then we have:

$$\begin{aligned} I_X &= I_{S2} e^{V_T \ln\left(\frac{V_{REF}}{R_1 I_{S1}}\right)/V_T} \\ &= I_{S2} \frac{V_{REF}}{R_1 I_{S1}} \\ &= \frac{I_{S2}}{I_{S1}} \frac{V_{REF}}{R_1} \\ &= \frac{I_{S2}}{I_{S1}} I_{C1} \end{aligned}$$

Thus, if $V_{REF} < R_1 I_{S1}$, then we get no output, i.e., $I_X = 0$. When $V_{REF} > R_1 I_{S1}$ (which will typically be true, since I_{S1} is typically very small), we get a current mirror relationship between Q_1 and Q_2 (with I_X copying I_{C1}), where the reference current for Q_1 is $\frac{V_{REF}}{R_1}$ (ensured by the op-amp).

40.



$Q_{REF} = Q_1$
$\beta \to \infty$.

$I_1 = \frac{I_{REF}}{2}$

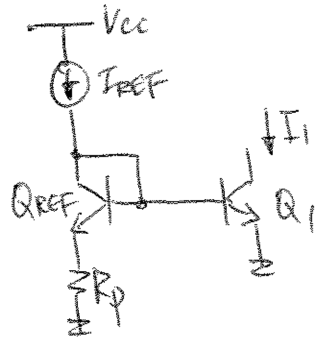
By KVL, $V_{BE,REF} = V_{BE_1} + I_1 R_p$

$$\Rightarrow V_T \ln\left(\frac{I_{REF}}{I_{S,REF}}\right) = V_T \ln\left(\frac{I_{REF}/2}{I_{S,1}}\right) + \frac{I_{REF}}{2} R_p$$

$$V_T \ln(2) = \frac{I_{REF}}{2} R_p$$

$$R_p = 2 \cdot \ln(2) \cdot (V_T / I_{REF})$$

41.



$Q_{REF} = Q_1$
$\beta \to \infty$

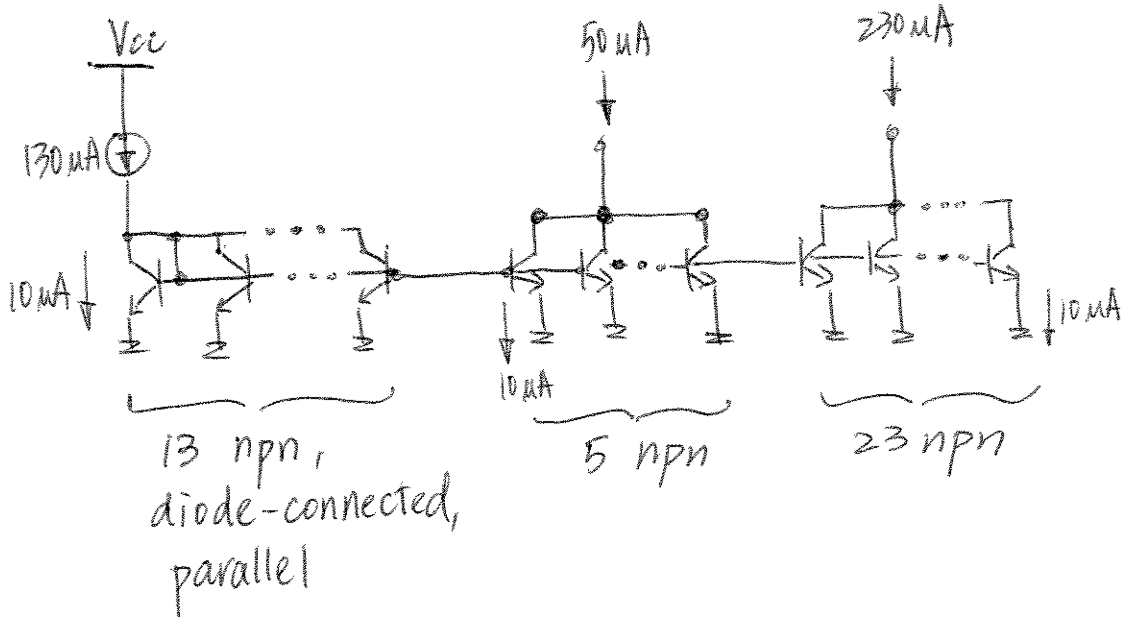
By KVL, $V_{BE,REF} + I_{REF} R_p = V_{BE,1}$

$$\Rightarrow V_T \ln\left(\frac{I_{REF}}{I_{S,REF}}\right) + I_{REF} R_p = V_T \ln\left(\frac{2 I_{REF}}{I_{S,1}}\right)$$

$$I_{REF} R_p = V_T \ln(2)$$

$$R_p = \frac{V_T}{I_{REF}} \ln(2)$$

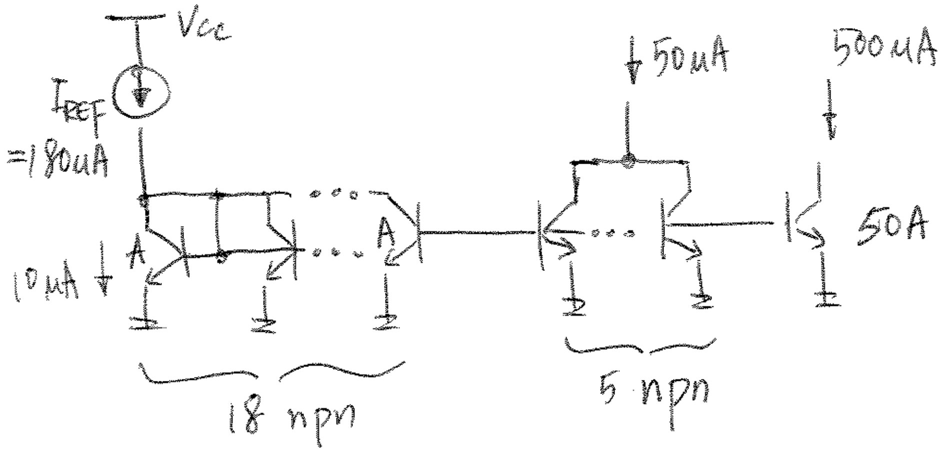
42.

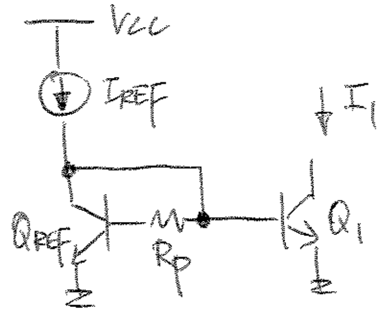


All the bases are the same node.

If the area of BJT is flexible, the 5-npn group can be replaced by one BJT that is 5 times as big in area. Similar concept applies to 23-npn grouping.

43.



44. 

$Q_{REF} = Q_1$

I_1 10% larger. ($I_1 = 1.1\, I_{C,REF}$)
Solve for R_p.

By KVL,

$$V_{BE_{REF}} + \frac{I_{C,REF}}{\beta} \cdot R_p = V_{BE_1}$$

$$\Rightarrow V_T \ln\left(\frac{I_1}{I_S}\right) - V_T \ln\left(\frac{I_{C,REF}}{I_S}\right) = \frac{I_{C,REF}}{\beta} \cdot R_p$$

$$V_T \ln\left(\frac{I_1}{I_{C,REF}}\right) = \frac{I_{C,REF}}{\beta} \cdot R_p$$

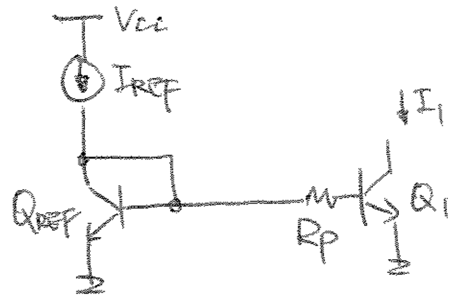
$$\Rightarrow V_T \ln(1.1) = \frac{I_{C,REF}}{\beta} R_p \qquad \Rightarrow I_{C,REF} = \frac{\beta V_T \ln(1.1)}{R_p}$$

By KCL,

$$I_{REF} = I_{C,REF} + I_{C,REF}/\beta + I_1/\beta$$

$$= \frac{\beta}{R_p} V_T \ln(1.1) \cdot \left(1 + 1/\beta\right) + I_1/\beta$$

$$\therefore R_p = \frac{(\beta + 1) V_T \ln(1.1)}{I_{REF} - I_1/\beta}$$

45. 

$I_1 = 0.9 I_{C,REF}$

By KVL, $V_{BE_{REF}} = \frac{I_1}{\beta} R_P + V_{BE_1}$

$$\Rightarrow V_T \ln\left(\frac{I_{C,REF}}{I_1}\right) = \frac{I_1}{\beta} R_P$$

$$V_T \ln\left(\frac{1}{0.9}\right) = 0.9\, I_{C,REF} \frac{R_P}{\beta}$$

$$\Rightarrow I_{C,REF} = \frac{\beta}{0.9 R_P} V_T \ln\left(\frac{1}{0.9}\right)$$

By KCL,

$$I_{REF} = I_{C,REF} + I_{C,REF}/\beta + I_1/\beta$$

$$\therefore I_{REF} - \frac{I_1}{\beta} = \frac{\beta}{0.9 R_P} V_T \ln\left(\frac{1}{0.9}\right)\left(1 + \frac{1}{\beta}\right)$$

$$\Rightarrow R_P = \frac{(\beta+1) V_T \ln(10/9)}{0.9\,(I_{REF} - I_1/\beta)}$$

9.46 (a)

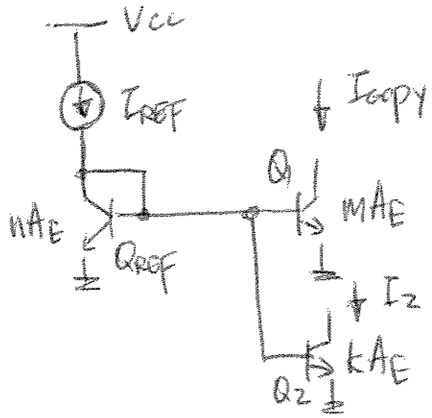
$$\begin{aligned}
I_{copy} &= 5I_{C,REF} \\
I_{REF} &= I_{C,REF} + I_{B,REF} + I_{B1} \\
&= I_{C,REF} + \frac{I_{C,REF}}{\beta} + \frac{I_{copy}}{\beta} \\
&= I_{C,REF} + \frac{I_{C,REF}}{\beta} + \frac{5I_{C,REF}}{\beta} \\
&= I_{C,REF}\left(1 + \frac{1}{\beta} + \frac{5}{\beta}\right) \\
&= \frac{I_{copy}}{5}\left(\frac{6+\beta}{\beta}\right) \\
I_{copy} &= \boxed{\left(\frac{\beta}{6+\beta}\right)5I_{REF}}
\end{aligned}$$

(b)

$$\begin{aligned}
I_{copy} &= \frac{I_{C,REF}}{5} \\
I_{REF} &= I_{C,REF} + I_{B,REF} + I_{B1} \\
&= I_{C,REF} + \frac{I_{C,REF}}{\beta} + \frac{I_{copy}}{\beta} \\
&= I_{C,REF} + \frac{I_{C,REF}}{\beta} + \frac{I_{C,REF}}{5\beta} \\
&= I_{C,REF}\left(1 + \frac{1}{\beta} + \frac{1}{5\beta}\right) \\
&= 5I_{copy}\left(\frac{6+5\beta}{5\beta}\right) \\
I_{copy} &= \boxed{\left(\frac{5\beta}{6+5\beta}\right)\frac{I_{REF}}{5}}
\end{aligned}$$

(c)

$$\begin{aligned}
I_{copy} &= \frac{3}{2} I_{C,REF} \\
I_2 &= \frac{5}{2} I_{C,REF} \\
I_{REF} &= I_{C,REF} + I_{B,REF} + I_{B1} + I_{B2} \\
&= I_{C,REF} + \frac{I_{C,REF}}{\beta} + \frac{I_{copy}}{\beta} + \frac{I_2}{\beta} \\
&= I_{C,REF} + \frac{I_{C,REF}}{\beta} + \frac{3I_{C,REF}}{2\beta} + \frac{5I_{C,REF}}{2\beta} \\
&= I_{C,REF} \left(1 + \frac{1}{\beta} + \frac{3}{2\beta} + \frac{5}{2\beta}\right) \\
&= \frac{2}{3} I_{copy} \left(\frac{10 + 2\beta}{2\beta}\right) \\
I_{copy} &= \boxed{\left(\frac{2\beta}{10 + 2\beta}\right) \frac{3}{2} I_{REF}}
\end{aligned}$$

47. 

By observing the areas of the BJTs,

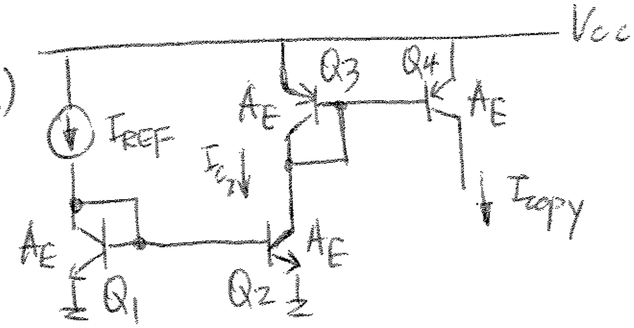
$$I_{C,REF} = \left(\frac{n}{m}\right) I_{copy} = \left(\frac{n}{k}\right) I_2$$

By KCL, $I_{C,REF} = I_{REF} - \frac{I_{C,REF}}{\beta} - \frac{I_{copy}}{\beta} - \frac{I_2}{\beta}$

$$\Rightarrow \quad \frac{n}{m} I_{copy} = I_{REF} - \frac{\left(\frac{n}{m}\right) I_{copy}}{\beta} - \frac{I_{copy}}{\beta} - \frac{\left(\frac{k}{m}\right) I_{copy}}{\beta}$$

$$\therefore \quad I_{copy} = I_{REF}\left[\frac{\beta m}{(\beta+1)n + k + m}\right]$$

48. (a)



$V_{BE_1} = V_{BE_2}$
$\Rightarrow I_{C_1} = I_{C2}$

$V_{BE3} = V_{BE4}$
$\Rightarrow I_{C_3} = I_{C_4}$

First compute $I_{C,2}$:

$$I_{C1} = I_{REF} - \frac{I_{C1}}{\beta} - \frac{I_{C2}}{\beta} \Rightarrow I_{C_2} = \frac{\beta}{\beta+2} \cdot I_{REF}.$$

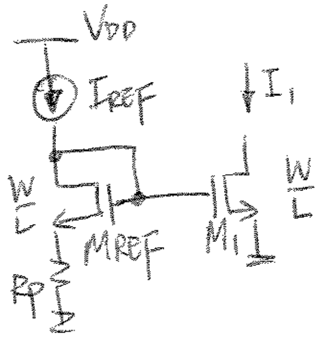
View I_{C2} as the "I_{REF}" for the Q_3–Q_4 current mirror and apply the equation derived.

$$\Rightarrow I_{copy} = \frac{\beta}{\beta+2}\left[\frac{\beta}{\beta+2} \cdot I_{REF}\right] = I_{REF}\left(\frac{\beta}{\beta+2}\right)^2$$

9.49

$$
\begin{aligned}
V_{GS,REF} &= V_{TH} + \sqrt{\frac{2I_{REF}}{\mu_n C_{ox} \frac{W}{L}}} \\
V_{GS1} &= V_{GS,REF} - I_1 R_P \\
&= V_{TH} + \sqrt{\frac{2I_{REF}}{\mu_n C_{ox} \frac{W}{L}}} - I_1 R_P \\
&= V_{TH} + \sqrt{\frac{2I_{REF}}{\mu_n C_{ox} \frac{W}{L}}} - \frac{I_{REF}}{2} R_P \\
I_1 &= \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \left(\sqrt{\frac{2I_{REF}}{\mu_n C_{ox} \frac{W}{L}}} - \frac{I_{REF}}{2} R_P \right)^2 \\
&= \frac{I_{REF}}{2} \\
\sqrt{\frac{2I_{REF}}{\mu_n C_{ox} \frac{W}{L}}} - \frac{I_{REF}}{2} R_P &= \sqrt{\frac{I_{REF}}{\mu_n C_{ox} \frac{W}{L}}} \\
\frac{I_{REF}}{2} R_P &= \sqrt{\frac{2I_{REF}}{\mu_n C_{ox} \frac{W}{L}}} - \sqrt{\frac{I_{REF}}{\mu_n C_{ox} \frac{W}{L}}} \\
&= \left(\sqrt{2} - 1\right) \sqrt{\frac{I_{REF}}{\mu_n C_{ox} \frac{W}{L}}} \\
R_P &= \boxed{\frac{2\left(\sqrt{2} - 1\right)}{\sqrt{I_{REF} \mu_n C_{ox} \frac{W}{L}}}}
\end{aligned}
$$

Given this choice of R_P, I_1 does not change if the threshold voltages of the transistors change by the same amount ΔV. Looking at the expression for I_1 in the derivation above, we can see that it has no dependence on V_{TH} (note that R_P does not depend on V_{TH} either).

50. 

Determine R_p such that $I_1 = 2I_{REF}$.

First calculate V_{GS_1}:

$$V_{GS_1} = \sqrt{\frac{2I_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} + V_{TH} = 2\sqrt{\frac{I_{REF}}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} + V_{TH} \quad -①$$

Assuming I_1 is in saturation:

$$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS,REF} - V_{TH}\right)^2$$

$$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[V_{GS_1} - I_{REF}R_p - V_{TH}\right]^2$$

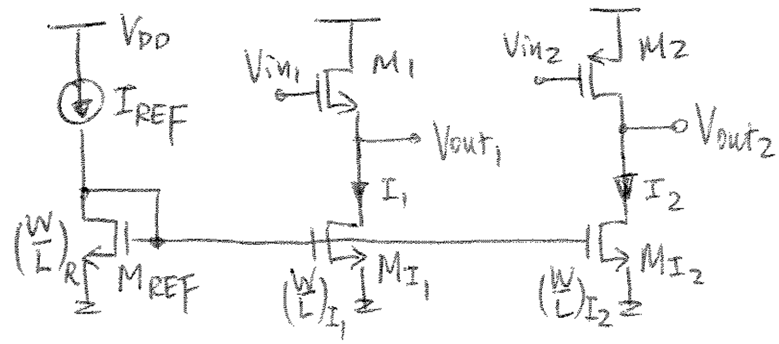
Substitute ① into I_{REF}:

$$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[2\sqrt{\frac{I_{REF}}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} - I_{REF}R_p\right]^2 \quad -②$$

Solve for R_p: $$R_p = \frac{(2-\sqrt{2})}{\sqrt{I_{REF}\cdot\mu_n C_{ox}\left(\frac{W}{L}\right)}}$$

From ②, we find that R_p is <u>independent</u> of any change in V_{TH}, ΔV !!

51.



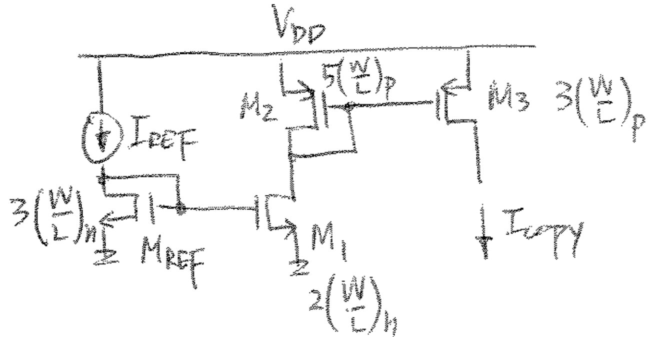
This figure implies that $V_{GS,REF} = V_{GS,I_1} = V_{GS,I_2}$.
Assuming all devices operate in saturation, with $(V_{GS} - V_{TH})$ fixed, $I_D \propto \left(\frac{W}{L}\right)$

$\Rightarrow$ we have $\left(\frac{W}{L}\right)_R = 7\left(\frac{W}{L}\right)$

$$\left(\frac{W}{L}\right)_{I_1} = 4\left(\frac{W}{L}\right)$$

$$\left(\frac{W}{L}\right)_{I_2} = 10\left(\frac{W}{L}\right)$$

52. (a)

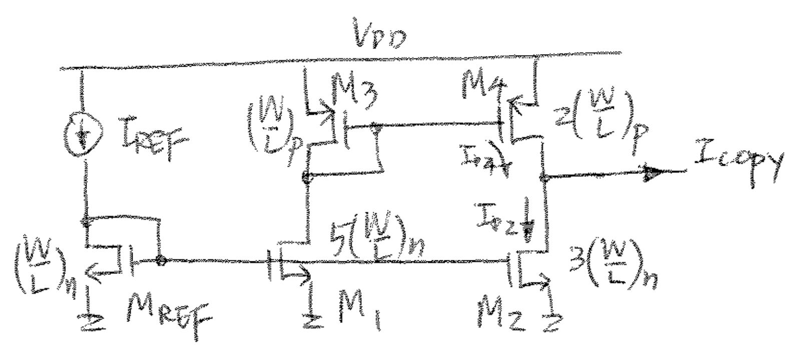


$V_{GS,REF} = V_{GS,1}$: $\Rightarrow I_{D,1} = \frac{2}{3} I_{REF}$

$V_{GS,2} = V_{GS,3}$: $\Rightarrow I_{copy} = \frac{3}{5} I_{D,2} = \frac{3}{5} I_{D,1}$

$= \frac{3}{5} \cdot \left(\frac{2}{3} I_{REF}\right) = \frac{2}{5} I_{REF}$

(b)



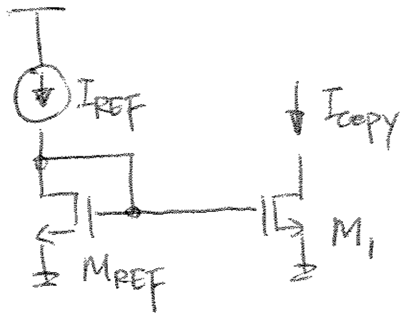
$V_{GS,REF} = V_{GS,1}$: $I_{D,1} = 5 I_{REF}$

$V_{GS,3} = V_{GS,4}$: $I_{D4} = 2 I_{D3} = 2 I_{D,1} = 10 I_{REF}$

$V_{GS,REF} = V_{GS,2}$: $I_{D2} = 3 I_{REF}$

$\therefore I_{copy} = I_{D4} - I_{D2} = 7 I_{REF}$.

53.



$V_{GS,REF} = V_{GS,1} = V_{GS}$

$\lambda \neq 0$

(a) $$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2(1+\lambda V_{GS})$$

$$I_{copy} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2(1+\lambda V_{DS,1})$$

For $I_{REF} = I_{copy} \Rightarrow V_{DS,1} = V_{GS}$

(b) $$\frac{I_{REF}}{I_{copy}} = \frac{1+\lambda V_{GS}}{1+\lambda(V_{GS}-V_{TH})}$$

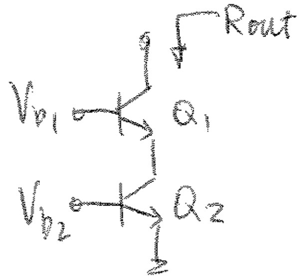
$$\Rightarrow I_{copy} = I_{REF}\left(1 - \frac{\lambda V_{TH}}{1+\lambda V_{GS}}\right)$$

9.54

$$\begin{aligned}
I_{C1} &= 1 \text{ mA} \\
I_{E1}R_E &= \frac{1+\beta_n}{\beta_n} I_{C1} R_E = 0.5 \text{ V} \\
R_E &== 0.5 \text{ V} \\
R_E &= \boxed{495.05\ \Omega} \\
R_{out,a} &= r_{o1} + (1 + g_{m1} r_{o1}) (r_{\pi 1} \parallel R_E) \\
&= 85.49 \text{ k}\Omega \\
R_{out,b} &= r_{o1} + (1 + g_{m1} r_{o1}) (r_{\pi 1} \parallel r_{o2}) \\
&= 334.53 \text{ k}\Omega
\end{aligned}$$

The output impedance of the circuit in Fig. 9.72(b) is significantly larger than the output impedance of the circuit in Fig. 9.72(a) (by a factor of about 4).

55.



$I_{BIAS} = 1mA$
$\beta = 100$

Given $R_{out} = 50\,k\Omega$, $V_{BC_2} = 100\,mV$, determine V_{b1}.

$$R_{out} = [1 + g_{m1}(r_{o2} \| r_{\pi 1})]r_{o1} + (r_{o2} \| r_{\pi 1})$$
$$\approx g_{m1}(r_{o2} \| r_{\pi 1}) r_{o1}$$
$$= \frac{\beta V_A^2}{(V_A + \beta V_T) I_{BIAS}}$$

$$\Rightarrow I_{BIAS} = \left[\frac{R_{out}(V_A + \beta V_T)}{\beta V_A^2}\right]^{-1} = \left[\frac{(50k\Omega)(5V + 100 \cdot 0.026V)}{100(5V)^2}\right]^{-1}$$
$$\approx 6.6\,mA.$$

$$V_{b2} = V_{BE2} = V_T \ln\left(\frac{I_{BIAS}}{I_S}\right) = (0.026V)\ln\left(\frac{6.6mA}{6 \cdot 10^{-16}A}\right)$$
$$\approx 0.78\,V$$

$$\Rightarrow V_{C2} = V_{BE2} - 100mV = 0.68V$$

$$\therefore V_{b1} = V_{C2} + V_{BE1} = V_{C2} + V_T \ln\left(\frac{I_{BIAS}}{I_S}\right)$$
$$= 0.68V + (0.026V)\ln\left(\frac{6.6mA}{6 \cdot 10^{-16}A}\right) \approx 1.46\,V.$$

9.56 (a)

$$R_{out} = r_{o1} + (1 + g_{m1} r_{o1})\, r_{o2} = 200\ \text{k}\Omega$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda I_D}$$

$$g_{m1} = g_{m2} = \sqrt{2 \frac{W}{L} \mu_n C_{ox} I_D}$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \boxed{1.6}$$

(b)

$$V_{b2} = V_{GS2} = V_{TH} + \sqrt{\frac{2 I_D}{\frac{W}{L} \mu_n C_{ox}}}$$

$$= \boxed{2.9\ \text{V}}$$

9.57 (a) Assume $I_{C1} \approx I_{C2}$, since $\beta \gg 1$.

$$\begin{aligned}
A_v &= -g_{m1}\left[r_{o2} + (1 + g_{m2} r_{o2})\,(r_{\pi 2} \parallel r_{o1})\right] \\
g_{m1} &= g_{m2} = \frac{I_1}{V_T} \\
r_{o1} &= r_{o2} = \frac{V_A}{I_1} \\
r_{\pi 1} &= r_{\pi 2} = \beta\frac{V_T}{I_1} \\
A_v &= -\frac{I_1}{V_T}\left[\frac{V_A}{I_1} + \left(1 + \frac{V_A}{V_T}\right)\frac{\beta\frac{V_T}{I_1}\frac{V_A}{I_1}}{\beta\frac{V_T}{I_1} + \frac{V_A}{I_1}}\right] \\
&= -\frac{1}{V_T}\left[V_A + \left(1 + \frac{V_A}{V_T}\right)\frac{\beta V_T V_A}{\beta V_T + V_A}\right] \\
&= -500 \\
V_A &= \boxed{0.618 \text{ V}}
\end{aligned}$$

(b)

$$\begin{aligned}
V_{in} &= V_{BE1} = V_T \ln\left(\frac{I_1}{I_{S1}}\right) \\
&= \boxed{714 \text{ mV}}
\end{aligned}$$

(c)

$$\begin{aligned}
V_{b1} &= V_{BE2} + V_{CE1} \\
&= V_{BE2} + 500 \text{ mV} \\
&= V_T \ln\left(\frac{I_1}{I_{S2}}\right) + 500 \text{ mV} \\
&= \boxed{1.214 \text{ V}}
\end{aligned}$$

9.58 Assume all of the collector currents are the same, since $\beta \gg 1$.

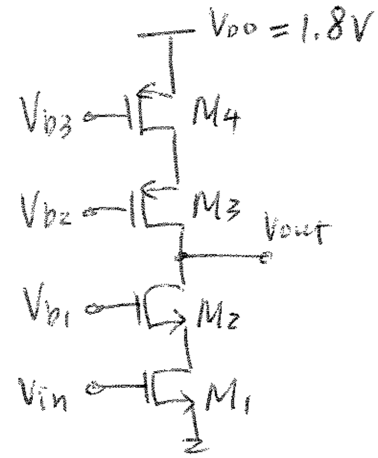
$$
\begin{aligned}
P &= I_C V_{CC} = 2 \text{ mW} \\
I_C &= 0.8 \text{ mA} \\
V_{in} &= V_T \ln\left(\frac{I_C}{I_S}\right) = \boxed{726 \text{ mV}} \\
V_{b1} &= V_{BE2} + V_{CE1} \\
&= V_T \ln\left(\frac{I_C}{I_S}\right) + V_{BE1} - V_{BC1} \\
&= \boxed{1.252 \text{ V}} \\
V_{b3} &= V_{CC} - V_T \ln\left(\frac{I_C}{I_S}\right) = \boxed{1.774 \text{ V}} \\
V_{b2} &= V_{CC} - V_{EC4} - V_{EB3} \\
&= V_{CC} - (V_{EB4} - V_{CB4}) - V_T \ln\left(\frac{I_C}{I_S}\right) \\
&= \boxed{1.248 \text{ V}} \\
A_v &= -g_{m1}\left\{\left[r_{o2} + (1 + g_{m2} r_{o2})\left(r_{\pi 2} \parallel r_{o1}\right)\right] \parallel \left[r_{o3} + (1 + g_{m3} r_{o3})\left(r_{\pi 3} \parallel r_{o4}\right)\right]\right\} \\
&= \boxed{4887}
\end{aligned}
$$

59. Given $A_v = 200$

power budget = 2mW

all $\left(\frac{W}{L}\right) = \frac{20}{0.18}$

$V_{b1} = V_{b2} = 0.9\ V$



$\lambda_n = 0.1\ V^{-1}$

$\lambda_p = 0.2\ V^{-1}$

calculate Vin & V_{b3}

$$A_v \cong -g_{m1}(g_{m2} r_{o1} r_{o2} \parallel g_{m3} r_{o3} r_{o4}) = 200$$

$$power = V_{DD} \times I_{BIAS} \Rightarrow I_{BIAS} = \frac{power}{V_{DD}} = \frac{2mW}{1.8V} \approx 1.11\ mA$$

$$g_{m2} r_{o1} r_{o2} = \sqrt{2 \mu_n C_{ox} \left(\frac{W}{L}\right) I_{BIAS}} \cdot \left(\frac{1}{\lambda_n I_{BIAS}}\right)^2$$

$$= \sqrt{2 \cdot 100\ \mu A \cdot \frac{20}{0.18} \cdot 1.11\ mA} \cdot \left[\frac{1}{(0.1V^{-1})(1.11mA)}\right]^2$$

$$\approx 403\ k\Omega$$

$$g_{m3} r_{o3} r_{o4} \approx 71.\ k\Omega$$

We know that $\frac{|A_v|}{(g_{m2} r_{o1} r_{o2} \parallel g_{m3} r_{o3} r_{o4})} = g_{m1} = \frac{2 I_D}{V_{GS1} - V_{TH}}$

$$\therefore V_{in} = V_{GS1} = V_{TH} + 2 I_D \cdot \frac{g_{m2} r_{o1} r_{o2} \parallel g_{m3} r_{o3} r_{o4}}{A_v}$$

$$= (0.4\,V) + 2(1.11\,mA)\ \frac{(403\,K\Omega \,\|\, 71.\,K\Omega)}{200}$$

$$\approx 1.07\ V$$

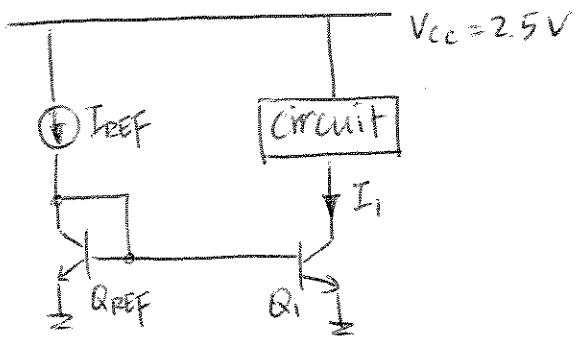
$$g_{m4} = \frac{2I_D}{V_{DD} - V_{D3} - |V_{THp}|} = \sqrt{2\mu_p C_{ox}\frac{W}{L} I_D}$$

$$\therefore V_{D3} = V_{DD} - |V_{THp}| - \frac{2I_D}{\sqrt{2\mu_p C_{ox}\frac{W}{L} I_D}}$$

$$= (1.8\,V) - (0.5\,V) - \frac{2(1.22\,mA)}{\sqrt{2\cdot\left(50\,\frac{\mu A}{V^2}\right)\left(\frac{20}{0.18}\right)(1.22\,mA)}}$$

$$\approx 0.67\ V$$

60.



$I_1 = 0.5mA$

power = 2mW

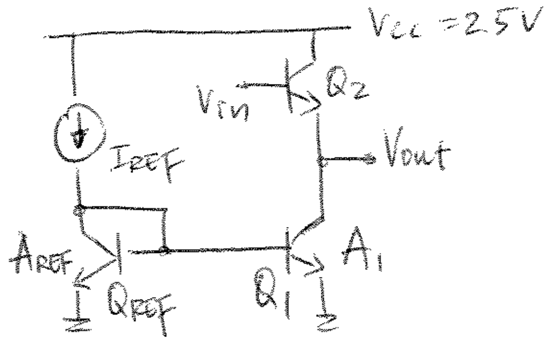
$$\text{Power} = V_{cc}(I_{REF} + I_1)$$

$$\Rightarrow I_{REF} = \frac{\text{Power}}{V_{cc}} - I_1 = \frac{2mW}{2.5V} - 0.5mA = 0.3mA$$

Therefore, if Q_{REF} has area A_E, then Q_1 has area $\frac{5}{3}A_E$ for the currents specified.

i.e. $\frac{A_{REF}}{A_1} = \frac{3}{5}$

61.



power = 3mW
$R_{out} = 50\Omega$

For an emitter follower, $R_{out} = r_{\pi 2} \parallel \frac{1}{g_{m2}}$

$$\Rightarrow R_{out} = 50\Omega = \frac{1}{\frac{I_{C2}}{V_T}\left(1+\frac{1}{\beta}\right)}$$

$$\therefore I_{C2} = \frac{V_T}{R_{out}} \cdot \frac{1}{1+1/\beta} = \frac{0.026}{50} \; \frac{1}{1+0.01} \approx 0.51\ mA$$

Realize that V_{cc} is providing current through I_{REF} & I_{C2}, and we are given

$$\text{power} = V_{cc}\left(I_{REF} + I_{C2}\right) = 3mW$$

$$\Rightarrow I_{REF} = \frac{\text{power}}{V_{cc}} - I_{C2} = \frac{3mW}{2.5V} - 0.51mA \approx 0.69mA$$

$$\Rightarrow \frac{I_{C2}}{I_{REF}} = \frac{A_1}{A_{REF}} = \frac{0.51}{0.69} \approx \frac{5}{7}$$

9.62

$$R_{out} = R_C = \boxed{500\ \Omega}$$

$$A_v = g_{m2} R_C = \frac{I_C R_C}{V_T} = 20$$

$$I_C = 1.04\ \text{mA}$$

$$P = (I_C + I_{REF}) V_{CC} = 3\ \text{mW}$$

$$I_{REF} = \boxed{0.16\ \text{mA}}$$

$$I_C = \frac{A_{E1}}{A_{E,REF}} I_{REF}$$

$$\frac{A_{E1}}{A_{E,REF}} = 6.5$$

$$A_{E,REF} = \boxed{A_E}$$

$$A_{E1} = \boxed{6.5 A_E}$$

9.63

$$\begin{aligned}
I_{copy} &= nI_{C,REF} \\
I_{REF} &= I_{C,REF} + I_{B,REF} + I_{B1} \\
&= I_{C,REF} + \frac{I_{C,REF}}{\beta} + \frac{I_{copy}}{\beta} \\
&= I_{C,REF} + \frac{I_{C,REF}}{\beta} + \frac{nI_{C,REF}}{\beta} \\
&= I_{C,REF}\left(1 + \frac{1}{\beta} + \frac{n}{\beta}\right) \\
&= \frac{I_{copy}}{n}\left(\frac{n+1+\beta}{\beta}\right) \\
I_{copy} &= \left(\frac{\beta}{n+1+\beta}\right) nI_{REF}
\end{aligned}$$

Since nI_{REF} is the nominal value of I_{copy}, the error term, $\frac{\beta}{n+1+\beta}$, must be between 0.99 and 1.01 so that the actual value of I_{copy} is within 1 % of the nominal value. Since the upper constraint (that the error term must be less than 1.01) results in a negative value of n (meaning that we can only get less than the nominal current if we include the error term), we only care about the lower error bound.

$$\begin{aligned}
\frac{\beta}{n+1+\beta} &\geq 0.99 \\
n &\leq 0.0101 \\
I_{REF} &\geq \boxed{50 \text{ mA}}
\end{aligned}$$

We can see that in order to decrease the error term, we must use a smaller value for n (in the ideal case, we have n approaching zero and the error term approaching $\frac{\beta}{1+\beta}$). However, the smaller value of n we use, the larger value we must use for I_{REF}, meaning the more power we must consume. Thus, we have a direct trade-off between accuracy and power consumption.

9.64

$$
\begin{aligned}
I_{C,M} &= \frac{A_{E,M}}{A_{E,REF1}} I_{C,REF1} \\
I_{REF1} &= I_{C,REF1} + I_{B,REF1} + I_{B,M} \\
&= I_{C,REF1} + \frac{I_{C,REF1}}{\beta_n} + \frac{I_{C,M}}{\beta_n} \\
&= I_{C,REF1} + \frac{I_{C,REF1}}{\beta_n} + \frac{A_{E,M} I_{C,REF1}}{A_{E,REF1}\beta_n} \\
&= I_{C,REF1}\left(1 + \frac{1}{\beta_n} + \frac{A_{E,M}}{A_{E,REF1}\beta_n}\right) \\
&= \frac{A_{E,REF1}}{A_{E,M}} I_{C,M}\left(\frac{A_{E,REF1}\beta_n + A_{E,REF1} + A_{E,M}}{A_{E,REF1}\beta_n}\right) \\
I_{C,M} &= \left(\frac{A_{E,REF1}\beta_n}{A_{E,REF1}\beta_n + A_{E,REF1} + A_{E,M}}\right)\frac{A_{E,M}}{A_{E,REF1}} I_{REF}
\end{aligned}
$$

Using a similar derivation to find I_{C2}, we have:

$$
\begin{aligned}
I_{C1} = I_{C2} &= \left(\frac{A_{E,REF2}\beta_p}{A_{E,REF2}\beta_p + A_{E,REF2} + A_{E2}}\right)\frac{A_{E2}}{A_{E,REF2}} I_{C,M} \\
&= \left(\frac{A_{E,REF1}\beta_p}{A_{E,REF1}\beta_p + A_{E,REF1} + A_{E,M}}\right)\left(\frac{A_{E,REF2}\beta_p}{A_{E,REF2}\beta_p + A_{E,REF2} + A_{E2}}\right)\frac{A_{E,M}}{A_{E,REF1}} \cdot \frac{A_{E2}}{A_{E,REF2}} I_{REF}
\end{aligned}
$$

We want the error term to be between 0.90 and 1.10 so that I_{C2} is within 10 % of its nominal value. Since the error term cannot exceed 1 (since we only lose current through the base), we only have to worry about the lower bound.

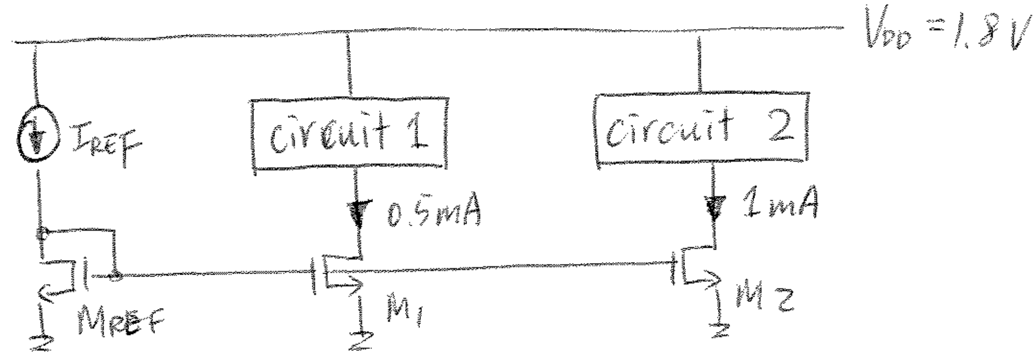
$$
\left(\frac{A_{E,REF1}\beta_n}{A_{E,REF1}\beta_n + A_{E,REF1} + A_{E,M}}\right)\left(\frac{A_{E,REF2}\beta_p}{A_{E,REF2}\beta_p + A_{E,REF2} + A_{E2}}\right) \geq 0.90
$$

Let's let the reference transistors Q_{REF1} and Q_{REF2} have unit size A_E. Then we have:

$$
\left(\frac{\beta_n}{\beta_n + 1 + \frac{A_{E,M}}{A_E}}\right)\left(\frac{\beta_p}{\beta_p + 1 + \frac{A_{E2}}{A_E}}\right) > 0.90
$$

We can pick any $A_{E,M}$ and A_{E2} such that this constraint is satisfied. One valid solution is $A_{E,M} = A_E$, $A_{E2} = 3.466 A_E$, and $I_{REF} = 0.2885$ mA. This gives a nominal value for I_{C2} of 1 mA with an error of 10 %. This solution is not unique (for example, another solution would be $A_{E,M} = A_{E2} = A_E$ and $I_{REF} = 1$ mA, which gives a nominal current of 1 mA and an error of 5.73 %).

65.



power budget = 3 mW.

$$power = V_{DD}(I_{REF} + 0.5mA + 1mA)$$

$$\Rightarrow I_{REF} = \frac{power}{V_{DD}} - 0.5mA - 1mA \approx 0.17mA$$

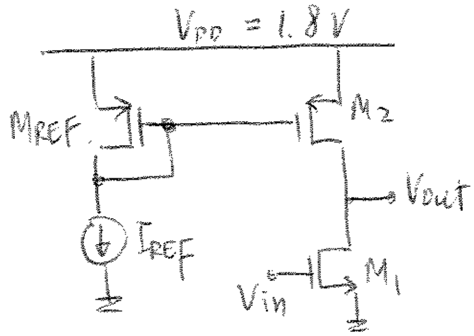
Assuming M_1 & M_2 operate in saturation,

If M_{REF} has $\left(\frac{W}{L}\right)_{REF}$, then

$$\frac{(W/L)_1}{(W/L)_{REF}} = \frac{I_1}{I_{REF}} = \frac{50}{17}$$

$$\frac{(W/L)_2}{(W/L)_{REF}} = \frac{I_2}{I_{REF}} = \frac{100}{17}$$

66.



$A_v = -20$

power = 2mW

$\left(\frac{W}{L}\right)_1 = \frac{20}{0.18}$ $\lambda_n = 0.1V^{-1}$

$\lambda_p = 0.2V^{-1}$

$$R_{out} = r_{o2} \parallel r_{o1} = \frac{1}{\lambda_n I_{D1} + \lambda_p I_{D1}}$$

$$\Rightarrow A_v = -G_m R_{out} = -\frac{g_{m1}}{\lambda_n I_{D1} + \lambda_p I_{D1}} = -\frac{2I_{D1}/(V_{GS1}-V_{TH})}{I_{D1}(\lambda_n+\lambda_p)}$$

$$\Rightarrow -20 = -\frac{2}{(V_{GS1}-V_{TH})(\lambda_n+\lambda_p)}$$

$$\Rightarrow V_{GS1} = \frac{1}{10(\lambda_n+\lambda_p)} + V_{THn}$$

$$= \frac{1}{10(0.1+0.2)V^{-1}} + 0.4V \approx 0.73V$$

$$\Rightarrow I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{THn})^2$$

$$= \frac{1}{2}\left(100\frac{\mu A}{V^2}\right)\left(\frac{20}{0.18}\right)(0.33V)^2 \approx 0.61mA$$

$\because$ power $= V_{DD}(I_{REF} + I_{D1})$

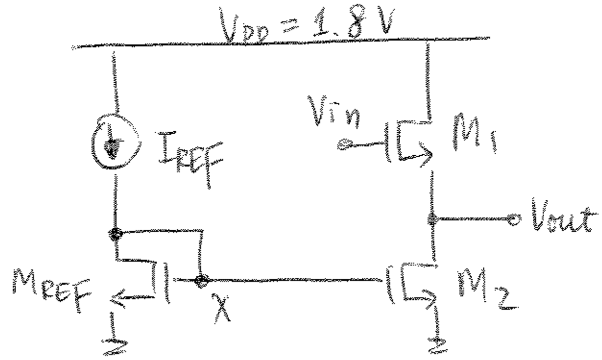
$$\Rightarrow I_{REF} = \frac{power}{V_{DD}} - I_{D1} = \frac{2mW}{1.8V} - 0.61mA$$

$$\approx 0.5mA$$

$\therefore$ if M_{REF} has $\left(\frac{W}{L}\right)_{REF}$, then

$$\frac{(W/L)_2}{(W/L)_{REF}} = \frac{I_{D2}}{I_{REF}} = \frac{61}{50} \approx 1.2$$

67.



Given:

$A_v = 0.85$

$R_{out} = 100\Omega$

$(W/L)_2 = 10/0.18$

$\lambda_n = 0.1V^{-1}$, $\lambda_p = 0.2V^{-1}$

$$R_{out} = r_{o2} \parallel \left(\frac{1}{g_{m1}} \parallel r_{o1}\right) = \frac{1}{g_{m1} + 1/r_{o2} + 1/r_{o1}} = 100$$

For source follower,

$$A_v = \frac{g_{m1}}{g_{m1} + 1/r_{o2} + 1/r_{o1}} = 0.85$$

$$\Rightarrow g_{m1} = \frac{0.85}{100} = 8.5 \cdot 10^{-3} S$$

$$R_{out} = \frac{1}{g_{m1} + 2/r_o} = 100$$

$$\Rightarrow r_o = \frac{200}{1 - 100 g_{m1}} = \frac{200}{1 - 100(8.5 \cdot 10^{-3})}$$

$$\approx 1333\,\Omega$$

$$\Rightarrow I_{D1} = \frac{1}{\lambda_n r_{o1}} = 7.5mA.$$

Assume $V_X \approx 1V$

$$\left(\frac{W}{L}\right)_2 = \frac{2 I_{D1}}{\mu_n C_{ox} (V_X - V_{TH})^2} \approx 416$$

Set $I_{REF} \approx 0.75$ mA.

$$\Rightarrow \left(\frac{W}{L}\right)_{REF} = \left(\frac{W}{L}\right)_{z} \frac{I_{REF}}{I_{D2}} \approx 42.$$

9.68

$$A_v = g_{m1} r_{o3} = g_{m1} \frac{1}{\lambda_p I_{D1}} = 20$$

$$\begin{aligned}
R_{in} &= \frac{1}{g_{m1}} \parallel r_{o2} \\
&= \frac{r_{o2}}{1 + g_{m1} r_{o2}} \\
&= \frac{\frac{1}{\lambda_n I_{D1}}}{1 + g_{m1} \frac{1}{\lambda_n I_{D1}}} \\
&= 50\ \Omega \\
g_{m1} &= 19.5\text{ mS} \\
I_{D1} &= 4.88\text{ mA} \\
g_{m1} &= \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}} \\
\left(\frac{W}{L}\right)_1 &= \boxed{390}
\end{aligned}$$

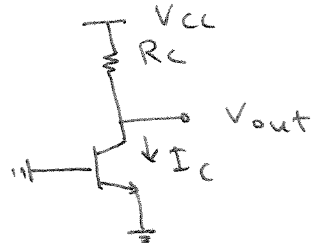
We need to size the rest of the transistors to ensure they provide the correct bias current to the amplifier and to ensure they are all in saturation. V_{G3} will be important in determining how we should bias V_{G5}, since in order for M_5 to be in saturation, we require $V_{G3} > V_{G5} - V_{THn}$, and V_{G3} is fixed by the previously calculated value of I_{D1}.

$$\begin{aligned}
V_{G3} &= V_{DD} - V_{SG3} = V_{DD} - \left(|V_{THp}| + \sqrt{\frac{2I_{D1}}{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}} \right) \\
&= 0.363\text{ V}
\end{aligned}$$

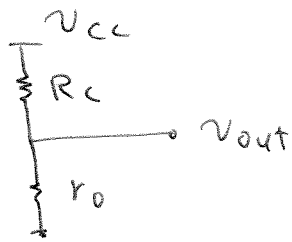
Let's let $I_{REF} = I_{D5} = 1$ mA (which ensures we meet our power constraint, since $P = (I_{REF} + I_{D5} + I_{D1}) V_{DD} = 12.4$ mW) and $V_{GS,REF} = V_{GS5} = 0.5$ V (which ensures M_5 operates in saturation). Then we have

$$\begin{aligned}
I_{REF} &= \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_{REF} (V_{GS,REF} - V_{TH})^2 \\
\left(\frac{W}{L}\right)_{REF} &= \left(\frac{W}{L}\right)_5 = \boxed{\frac{360}{0.18}} \\
\frac{(W/L)_3}{(W/L)_4} &= \frac{I_{D3}}{I_{D4}} \\
\left(\frac{W}{L}\right)_4 &= \boxed{\frac{8.2}{0.18}} \\
\frac{(W/L)_2}{(W/L)_{REF}} &= \frac{I_{D2}}{I_{REF}} \\
\left(\frac{W}{L}\right)_2 &= \boxed{\frac{1756}{0.18}}
\end{aligned}$$

(1)

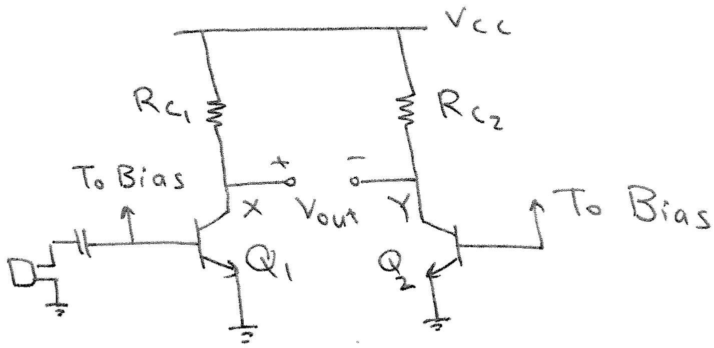


the small signal model is as follow:



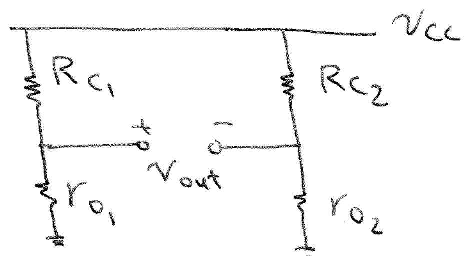
$$\frac{v_{out}}{v_{cc}} = \frac{r_o}{r_o + R_C} = \frac{\frac{V_A}{I_C}}{\frac{V_A}{I_C} + R_C} = \frac{V_A}{V_A + R_C I_C}$$

(2)



$R_{C_1} = R_{C_2}$

The small signal model is:



$r_{o_1} = r_{o_2}$

$R_{C_1} = R_{C_2}$

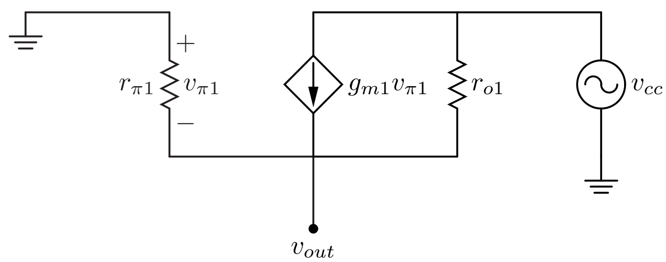
$$\frac{v_{out}}{v_{cc}} = \frac{1}{v_{cc}}\left(\frac{r_{o_1}}{R_{C_1}+r_{o_1}} - \frac{r_{o_2}}{R_{C_2}+r_{o_2}}\right)v_{cc} = 0$$

10.3 (a) Looking into the collector of Q_1, we see an infinite impedance (assuming I_{EE} is an ideal source). Thus, the gain from V_{CC} to V_{out} is $\boxed{1}$.

(b) Looking into the drain of M_1, we see an impedance of $r_{o1} + (1 + g_{m1} r_{o1}) R_S$. Thus, the gain from V_{CC} to V_{out} is

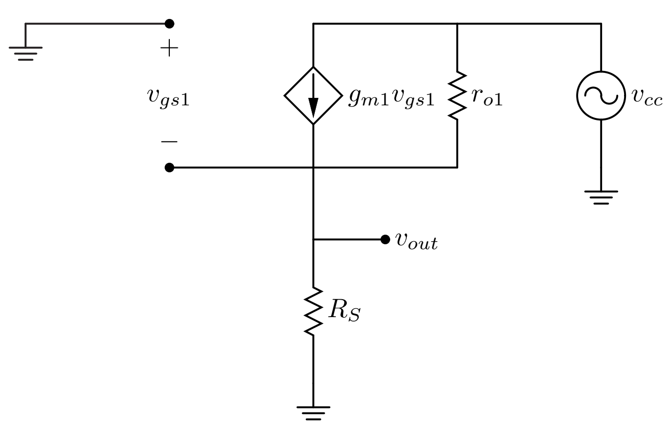
$$\boxed{\frac{r_{o1} + (1 + g_{m1} r_{o1}) R_S}{R_D + r_{o1} + (1 + g_{m1} r_{o1}) R_S}}$$

(c) Let's draw the small-signal model.



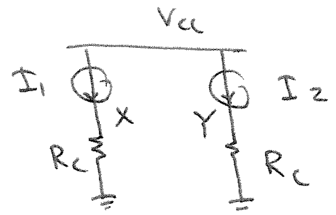
$$\begin{aligned}
v_{out} &= -v_{\pi 1} \\
v_{out} &= \left(g_{m1} v_{\pi 1} + \frac{v_{cc} - v_{out}}{r_{o1}} \right) r_{\pi 1} \\
&= \left(-g_{m1} v_{out} + \frac{v_{cc} - v_{out}}{r_{o1}} \right) r_{\pi 1} \\
v_{out} \left(1 + g_{m1} r_{\pi 1} + \frac{r_{\pi 1}}{r_{o1}} \right) &= v_{cc} \frac{r_{\pi 1}}{r_{o1}} \\
\frac{v_{out}}{v_{cc}} &= \frac{r_{\pi 1}}{r_{o1} \left(1 + \beta + \frac{r_{\pi 1}}{r_{o1}} \right)} \\
&= \boxed{\frac{r_{\pi 1}}{r_{o1} (1 + \beta) + r_{\pi 1}}}
\end{aligned}$$

(d) Let's draw the small-signal model.



$$
\begin{aligned}
v_{out} &= -v_{gs1} \\
v_{out} &= \left(g_{m1} v_{gs1} + \frac{v_{cc} - v_{out}}{r_{o1}} \right) R_S \\
&= \left(-g_{m1} v_{out} + \frac{v_{cc} - v_{out}}{r_{o1}} \right) R_S \\
v_{out} \left(1 + g_{m1} R_S + \frac{R_S}{r_{o1}} \right) &= v_{cc} \frac{R_S}{r_{o1}} \\
\frac{v_{out}}{v_{cc}} &= \frac{R_S}{r_{o1} \left(1 + g_{m1} R_S + \frac{R_S}{r_{o1}} \right)} \\
&= \boxed{\frac{R_S}{r_{o1} \left(1 + g_{m1} R_S \right) + R_S}}
\end{aligned}
$$

④

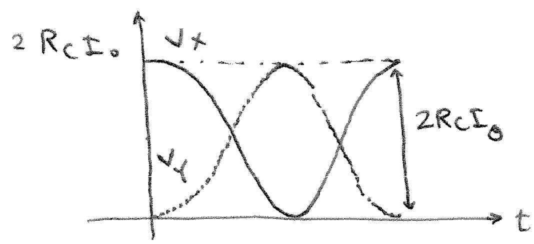


$$I_1 = I_o \cos\omega t + I_o$$

$$I_2 = -I_o \cos\omega t + I_o$$

$$V_X = R_C I_1 = R_C I_o (1 + \cos\omega t)$$

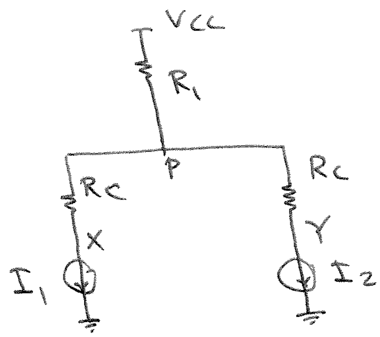
$$V_Y = R_C I_2 = R_C I_o (1 - \cos\omega t)$$



$$V_{X_{p\text{-}p}} = V_{Y_{p\text{-}p}} = 2R_C I_o$$

$$V_{X,CM} = V_{Y,CM} = R_C I_o$$

(5)



$I_1 = I_o \cos\omega t + I_o$

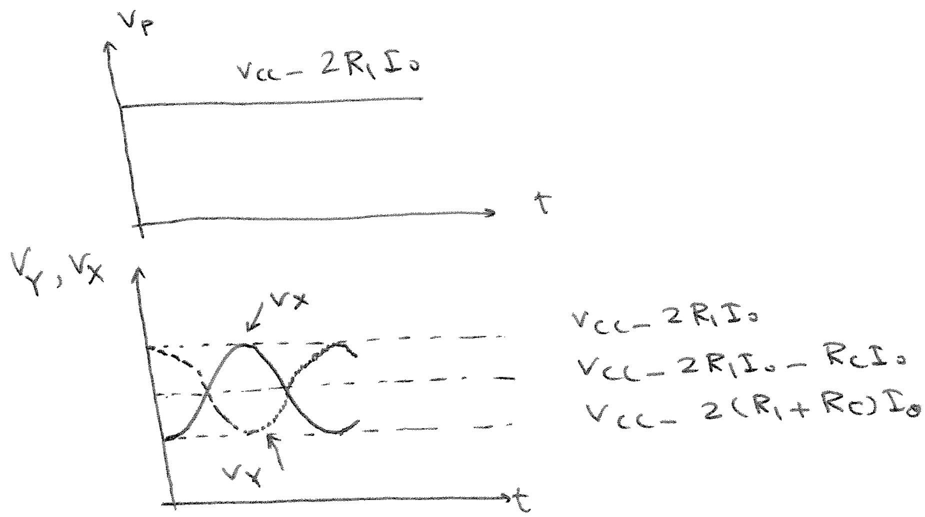
$I_2 = -I_o \cos\omega t + I_o$

$V_p = V_{cc} - R_1 (I_1 + I_2) = V_{cc} - 2 R_1 I_o$

$V_X = V_p - R_C I_1 = V_{cc} - 2R_1 I_o - R_C I_o - R_C I_o \cos\omega t$

$\Rightarrow V_X = V_{cc} - (2R_1 + R_C) I_o - R_C I_o \cos\omega t$

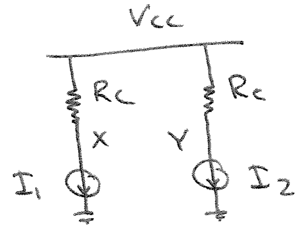
$V_Y = V_p - R_C I_2 = V_{cc} - (2R_1 + R_C) I_o + R_C I_o \cos\omega t$



$V_{X,CM} = V_{Y,CM} = V_{cc} - (2R_1 + R_C) I_o$

$V_{X,P\text{-}P} = V_{Y,P\text{-}P} = 2 R_C I_o$

⑥

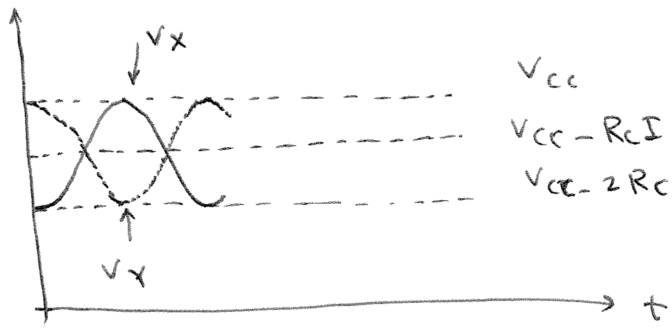


$I_1 = I_0 \cos\omega t + I_0$

$I_2 = -I_0 \cos\omega t + I_0$

$V_X = V_{CC} - R_C I_1 = V_{CC} - R_C I_0 (1 + \cos\omega t)$

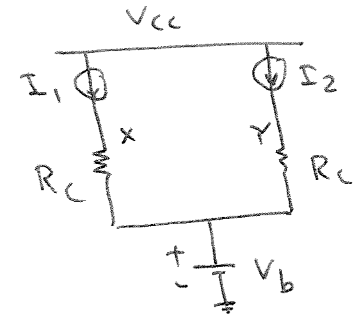
$V_Y = V_{CC} - R_C I_2 = V_{CC} - R_C I_0 (1 - \cos\omega t)$



$V_{X,CM} = V_{Y,CM} = V_{CC} - R_C I_0$

$V_{X,p\text{-}p} = V_{Y,p\text{-}p} = 2 R_C I_0$

⑦

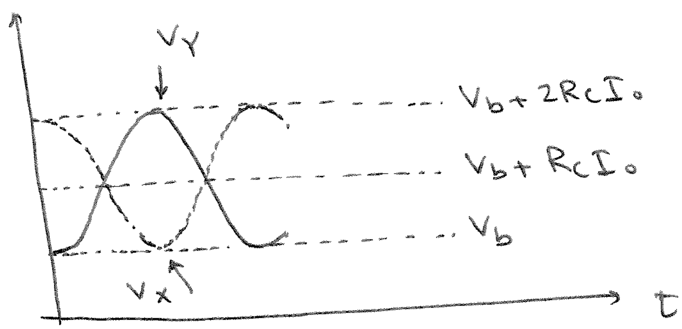


$I_1 = I_0 \cos\omega t + I_0$

$I_2 = -I_0 \cos\omega t + I_0$

$V_X = R_C I_1 + V_b = R_C I_0 (1 + \cos\omega t) + V_b$

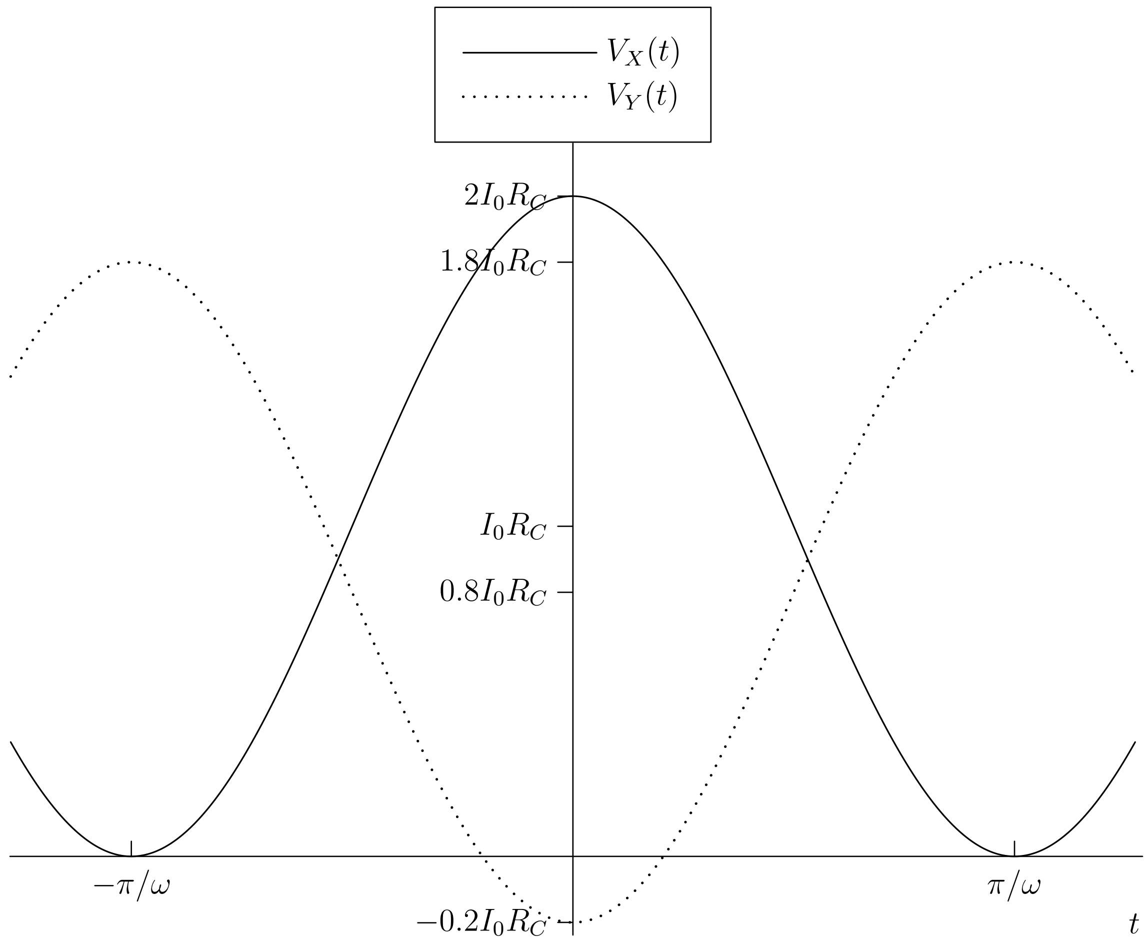
$V_Y = R_C I_2 + V_b = R_C I_0 (1 - \cos\omega t) + V_b$



$V_{X,CM} = V_{Y,CM} = V_b + R_C I_0$

$V_{X,p\text{-}p} = V_{Y,p\text{-}p} = 2 R_C I_0$

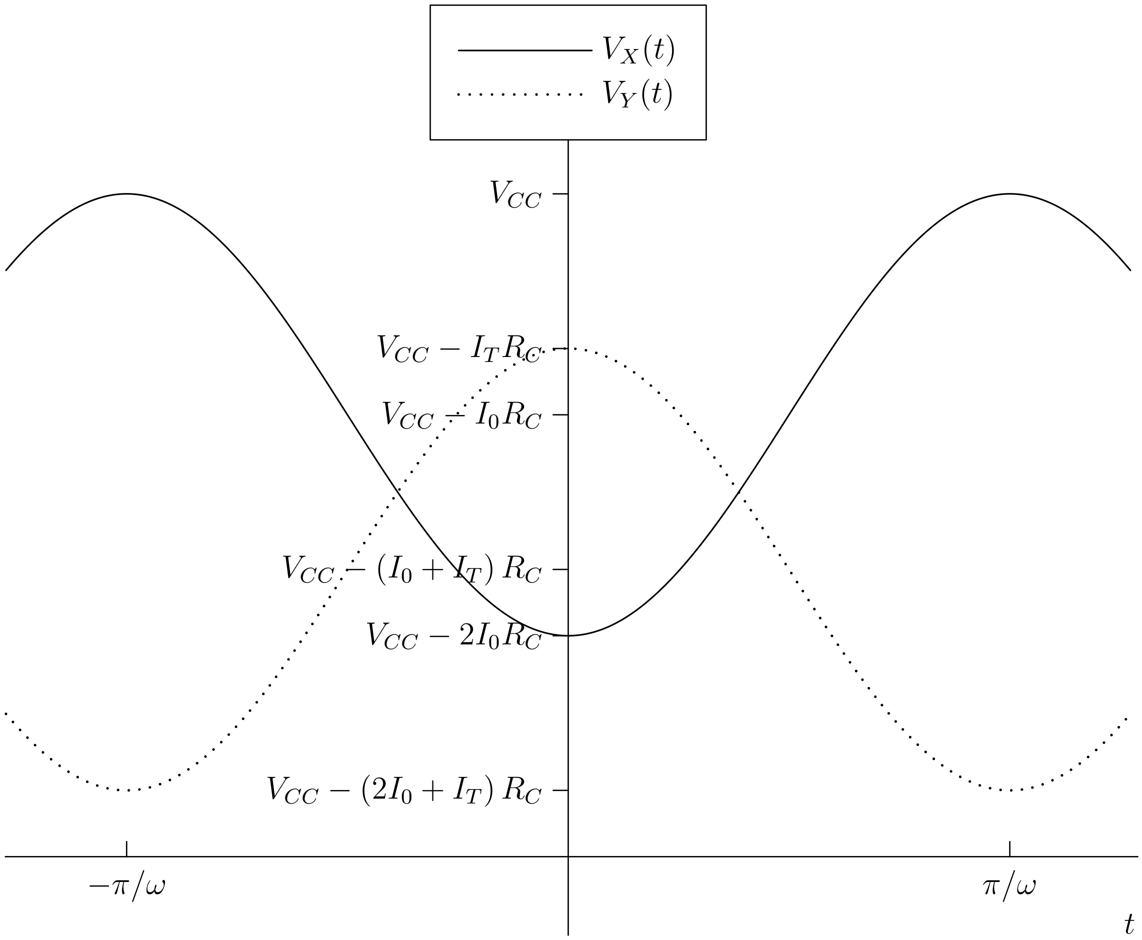
10.8



X and Y are not true differential signals, since their common-mode values differ.

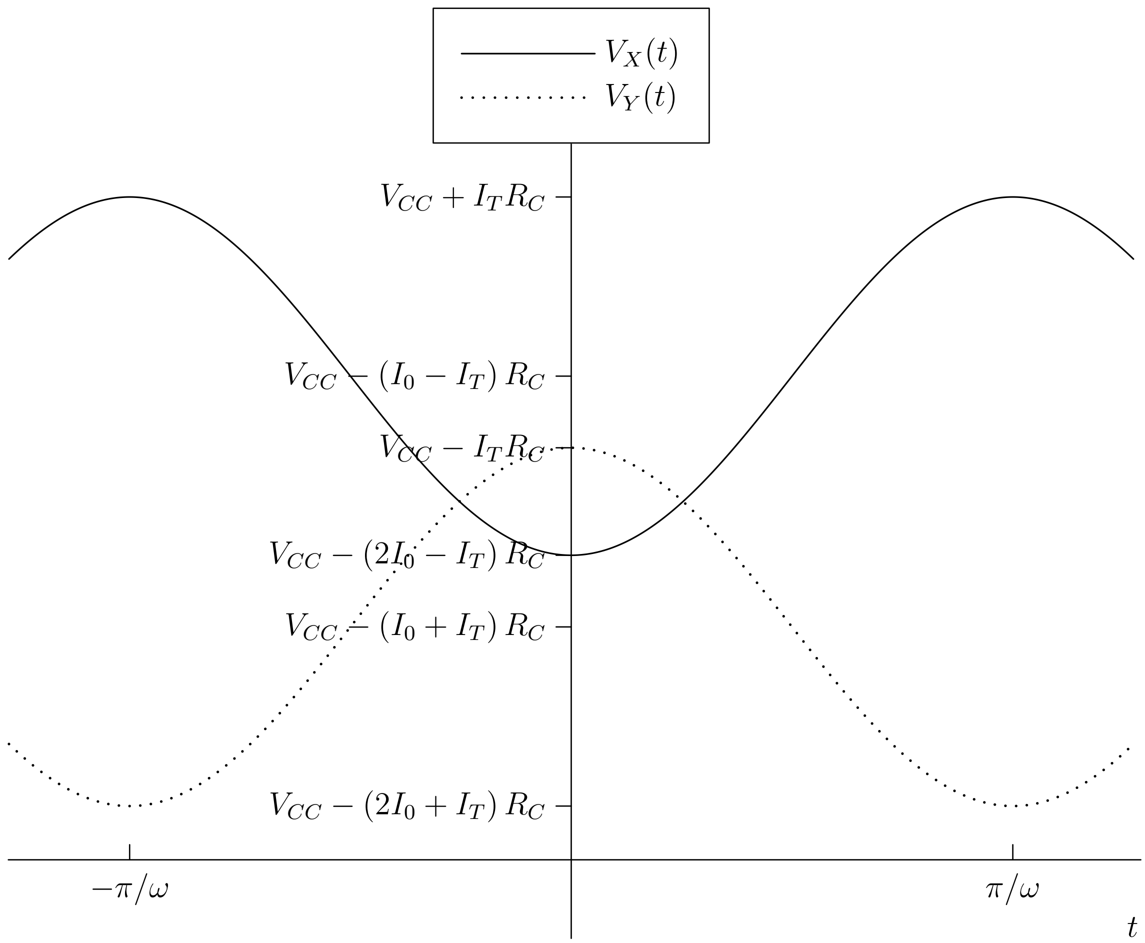
10.9 (a)

$$V_X = V_{CC} - I_1 R_C$$
$$V_Y = V_{CC} - (I_2 + I_T)\, R_C$$



(b)

$$V_X = V_{CC} - (I_1 - I_T)\, R_C$$
$$V_Y = V_{CC} - (I_2 + I_T)\, R_C$$



$V_X(t)$
$V_Y(t)$
$V_{CC} + I_T R_C$
$V_{CC} - (I_0 - I_T)\, R_C$
$V_{CC} - I_T R_C$
$V_{CC} - (2I_0 - I_T)\, R_C$
$V_{CC} - (I_0 + I_T)\, R_C$
$V_{CC} - (2I_0 + I_T)\, R_C$
$-\pi/\omega$
π/ω
t

(c)

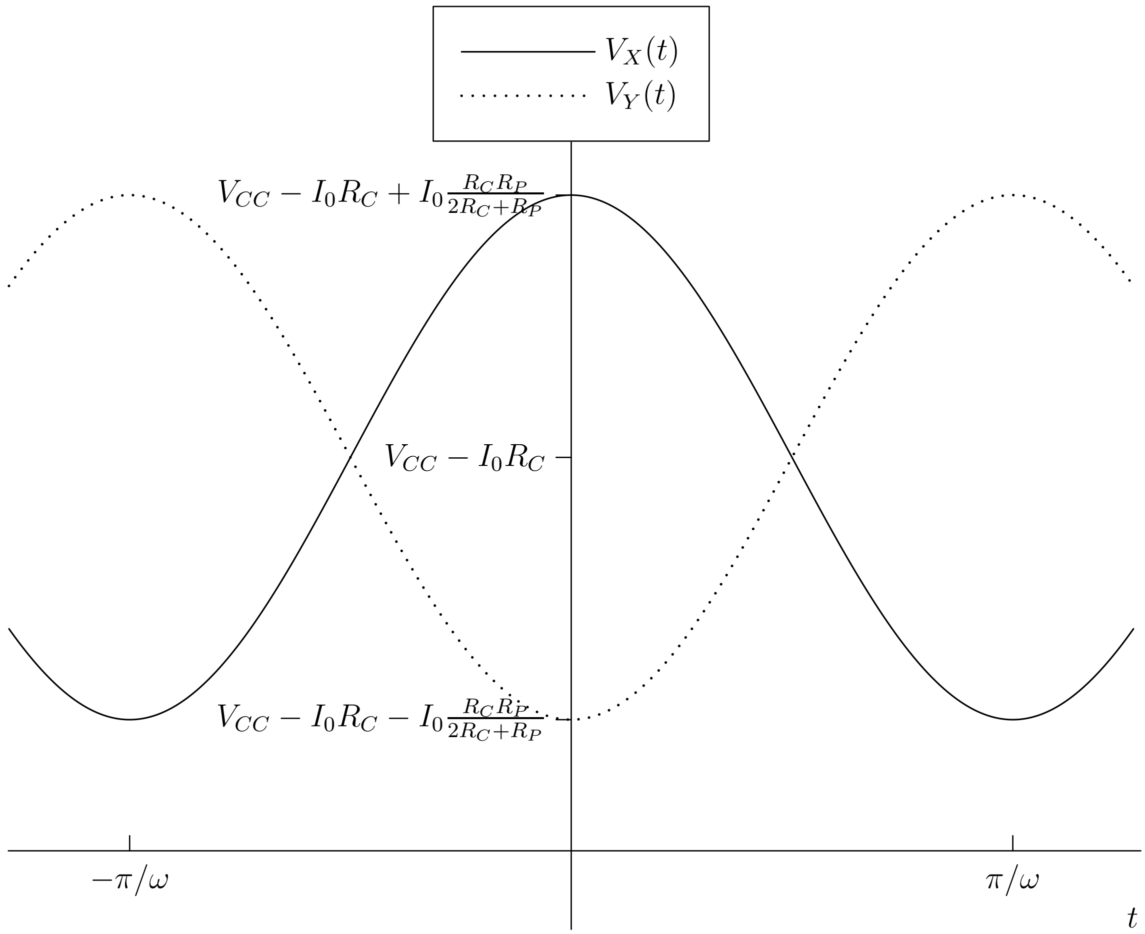
$$
\begin{aligned}
V_X &= V_{CC} - \left(I_1 + \frac{V_X - V_Y}{R_P}\right) R_C \\
V_X\left(1 + \frac{R_C}{R_P}\right) &= V_{CC} - \left(I_1 - \frac{V_Y}{R_P}\right) R_C \\
V_X &= \frac{V_{CC} - \left(I_1 - \frac{V_Y}{R_P}\right) R_C}{1 + \frac{R_C}{R_P}} \\
&= \frac{V_{CC} R_P - \left(I_1 R_P - V_Y\right) R_C}{R_P + R_C} \\
V_Y &= V_{CC} - \left(I_2 + \frac{V_Y - V_X}{R_P}\right) R_C \\
V_Y\left(1 + \frac{R_C}{R_P}\right) &= V_{CC} - \left(I_2 - \frac{V_X}{R_P}\right) R_C \\
V_Y &= \frac{V_{CC} - \left(I_2 - \frac{V_X}{R_P}\right) R_C}{1 + \frac{R_C}{R_P}} \\
&= \frac{V_{CC} R_P - \left(I_2 R_P - V_X\right) R_C}{R_P + R_C} \\
V_X &= \frac{V_{CC} R_P - \left(I_1 R_P - \frac{V_{CC} R_P - (I_2 R_P - V_X) R_C}{R_P + R_C}\right) R_C}{R_P + R_C} \\
&= \frac{V_{CC} R_P - I_1 R_P R_C + \frac{V_{CC} R_P R_C - I_2 R_P R_C^2 + V_X R_C^2}{R_P + R_C}}{R_P + R_C} \\
V_X\left(1 - \frac{R_C^2}{\left(R_P + R_C\right)^2}\right) &= \frac{V_{CC} R_P - I_1 R_P R_C + \frac{V_{CC} R_P R_C - I_2 R_P R_C^2}{R_P + R_C}}{R_P + R_C} \\
V_X\left(\frac{\left(R_P + R_C\right)^2 - R_C^2}{R_P + R_C}\right) &= V_{CC} R_P - I_1 R_P R_C + \frac{V_{CC} R_P R_C - I_2 R_P R_C^2}{R_P + R_C} \\
V_X\left(R_P^2 + 2 R_P R_C\right) &= V_{CC} R_P \left(R_P + R_C\right) - I_1 R_P R_C \left(R_P + R_C\right) + V_{CC} R_P R_C - I_2 R_P R_C^2 \\
V_X &= \frac{V_{CC} R_P \left(R_P + R_C\right) - I_1 R_P R_C \left(R_P + R_C\right) + V_{CC} R_P R_C - I_2 R_P R_C^2}{R_P^2 + 2 R_P R_C} \\
&= \frac{V_{CC} R_P \left(2 R_C + R_P\right) - R_P R_C \left[I_1 \left(R_P + R_C\right) + I_2 R_C\right]}{R_P \left(2 R_C + R_P\right)}
\end{aligned}
$$

Substituting I_1 and I_2, we have:

$$
\begin{aligned}
V_X &= \frac{V_{CC} R_P \left(2 R_C + R_P\right) - R_P R_C \left[\left(I_0 + I_0 \cos(\omega t)\right)\left(R_P + R_C\right) + \left(I_0 - I_0 \cos(\omega t)\right) R_C\right]}{R_P \left(2 R_C + R_P\right)} \\
&= \frac{V_{CC} R_P \left(2 R_C + R_P\right) - R_P R_C \left[I_0 \left(2 R_C + R_P\right) + I_0 \cos(\omega t) R_P\right]}{R_P \left(2 R_C + R_P\right)} \\
&= V_{CC} - I_0 R_C + I_0 \cos(\omega t) \frac{R_C R_P}{2 R_C + R_P}
\end{aligned}
$$

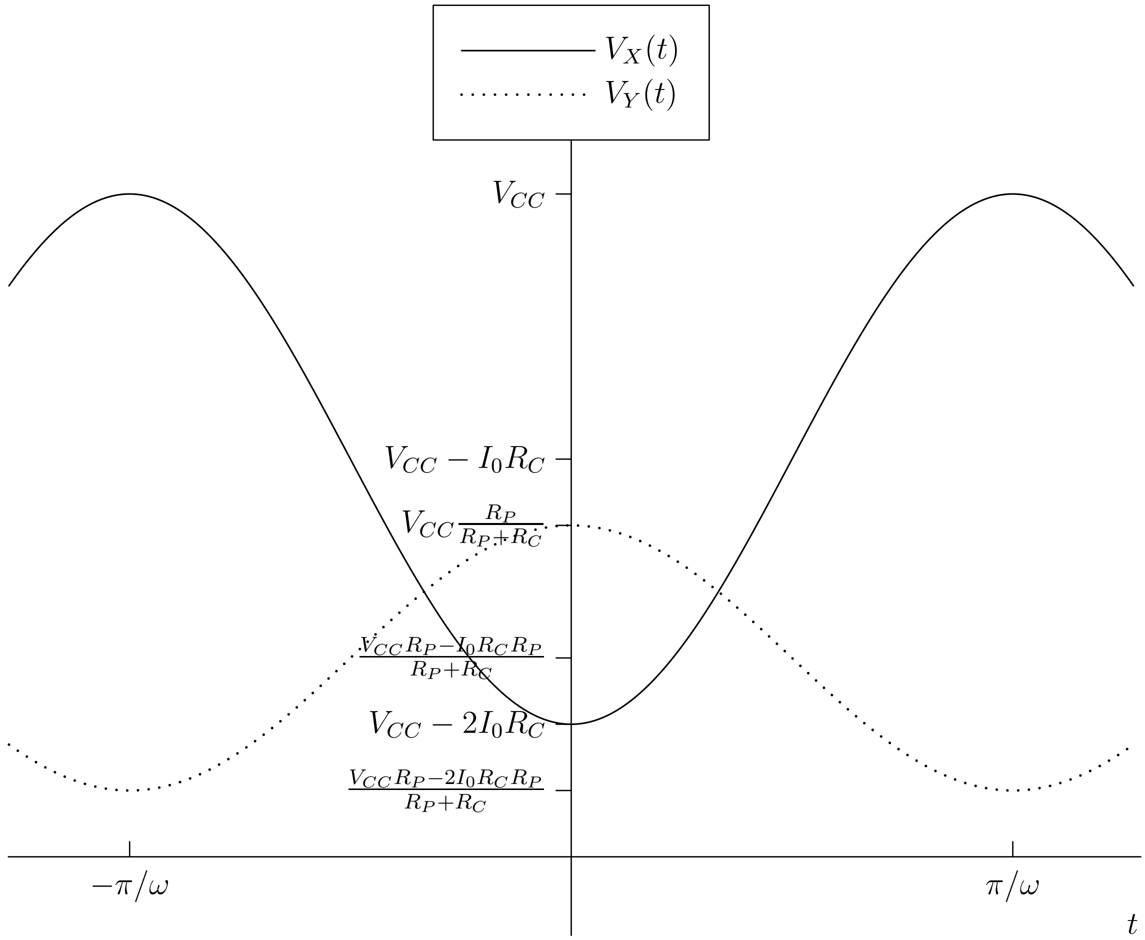
By symmetry, we can write:

$$
V_Y = V_{CC} - I_0 R_C - I_0 \cos(\omega t) \frac{R_C R_P}{2 R_C + R_P}
$$



(d)

$$\begin{aligned}
V_X &= V_{CC} - I_1 R_C \\
V_Y &= V_{CC} - \left(I_2 + \frac{V_Y}{R_P} \right) R_C \\
V_Y \left(1 + \frac{R_C}{R_P} \right) &= V_{CC} - I_2 R_C \\
V_Y &= \frac{V_{CC} - I_2 R_C}{1 + \frac{R_C}{R_P}} \\
&= \frac{V_{CC} R_P - I_2 R_C R_P}{R_P + R_C}
\end{aligned}$$

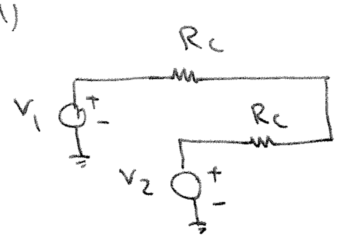


$V_X(t)$
$V_Y(t)$
V_{CC}
$V_{CC} - I_0 R_C$
$V_{CC}\frac{R_P}{R_P+R_C}$
$\frac{V_{CC}R_P - I_0 R_C R_P}{R_P+R_C}$
$V_{CC} - 2I_0 R_C$
$\frac{V_{CC}R_P - 2I_0 R_C R_P}{R_P+R_C}$
$-\pi/\omega$
π/ω
t

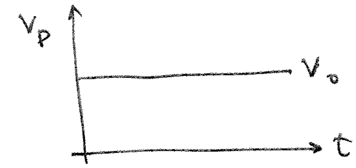
10)

$$V_1 = V_0 \cos\omega t + V_0$$
$$V_2 = -V_0 \cos\omega t + V_0$$

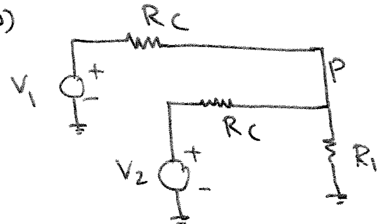
(a)



$$V_P = \frac{V_1 + V_2}{2} = V_0$$



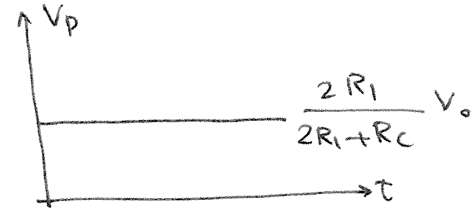
(b)



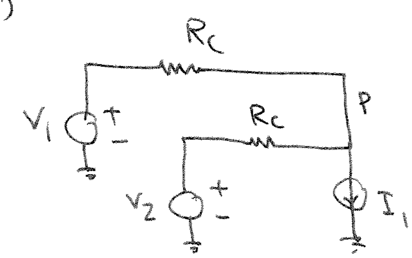
$$\frac{2V_P - V_1 - V_2}{R_C} + \frac{V_P}{R_1} = 0 \Rightarrow$$

$$\frac{2V_P - 2V_0}{R_C} + \frac{V_P}{R_1} = 0 \Rightarrow (2R_1 + R_C)V_P = 2V_0 R_1$$

$$\rightarrow V_P = \frac{2R_1}{2R_1 + R_C} V_0$$



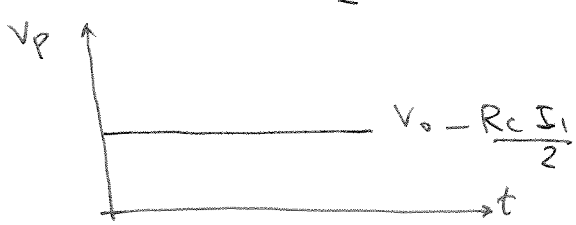
(C)



$$\frac{2V_P - V_1 - V_2}{R_C} + I_1 = 0 \Rightarrow$$

$$\frac{2V_P - 2V_0}{R_C} = -I_1 \Rightarrow$$

$$V_P = V_0 - \frac{R_C I_1}{2}$$

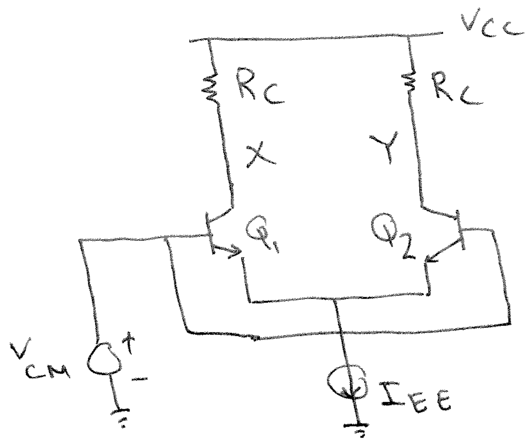


10.11 Note that since the circuit is symmetric and I_{EE} is an ideal source, no matter what value of V_{CC} we have, the current through Q_1 and Q_2 must be $I_{EE}/2$. That means if the supply voltage increases by some amount ΔV, V_X and V_Y must also increase by the same amount to ensure the current remains the same.

$$\Delta V_X = \boxed{\Delta V}$$
$$\Delta V_Y = \boxed{\Delta V}$$
$$\Delta \left(V_X - V_Y\right) = \boxed{0}$$

We can say that this circuit rejects supply noise because changes in the supply voltage (i.e., supply noise) do not show up as changes in the differential output voltage $V_X - V_Y$.

12

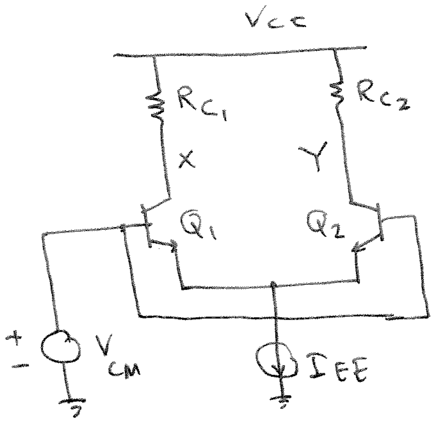


$$\Delta V_X = -\frac{R_C \Delta I_{EE}}{2} = -\frac{R_C \Delta I}{2}$$

$$\Delta V_Y = -\frac{R_C \Delta I_{EE}}{2} = -\frac{R_C \Delta I}{2}$$

$$\Rightarrow \Delta(V_X - V_Y) = 0$$

(13)



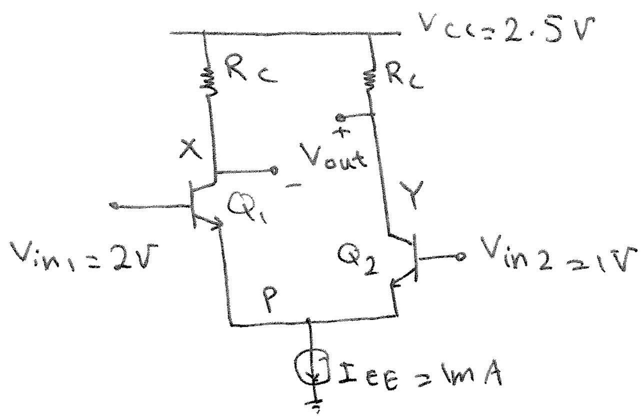
$R_{C1} = R_{C2} + \Delta R$

$$\Delta V_X = -\frac{R_{C1}\,\Delta I}{2} = -\frac{(R_{C2} + \Delta R)\,\Delta I}{2}$$

$$\Delta V_Y = -\frac{R_{C2}\,\Delta I}{2} \Rightarrow$$

$$\Delta(V_X - V_Y) = -\frac{\Delta R\,\Delta I}{2}$$

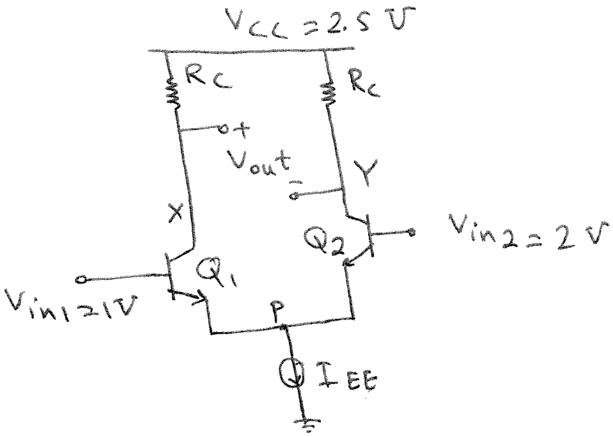
(14)



$$V_X \geq V_{in_1} \Rightarrow V_X \geq 2 \Rightarrow V_{CC} - R_C I_{EE} \geq 2$$

$$\Rightarrow 2.5 - R_C^{(k\Omega)} \geq 2 \Rightarrow R_C \leq 0.5\ k\Omega$$

15

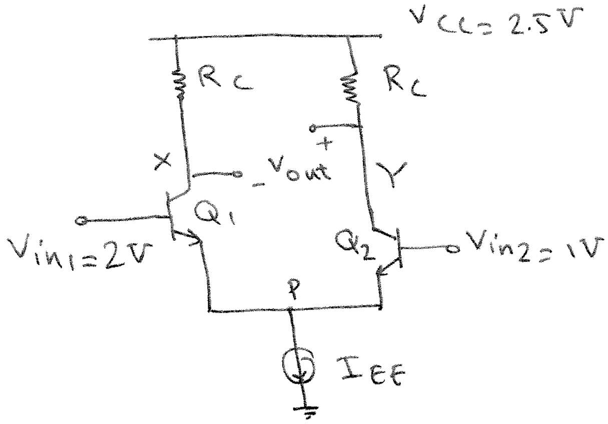


$R_C = 500\Omega$

$$V_Y \geq V_{in2} \Rightarrow V_{CC} - R_C I_{EE} \geq 2 \Rightarrow$$

$$2.5 - 500 I_{EE} \geq 2 \Rightarrow I_{EE} \leq 1mA$$

16



$I_{EE} = 1mA$

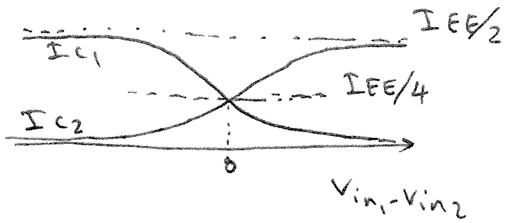
$R_C = 800\Omega$

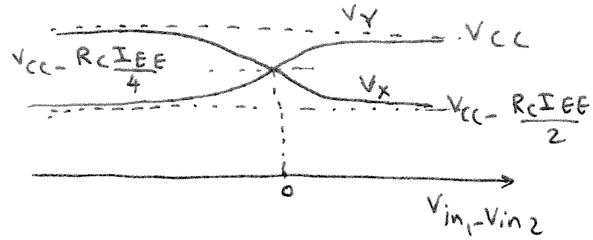
$$V_X = V_{CC} - R_C I_{EE} = 2.5 - 0.8 = 1.7V$$

$\Rightarrow V_X < V_{in1} \Rightarrow Q_1$ is in saturation region.

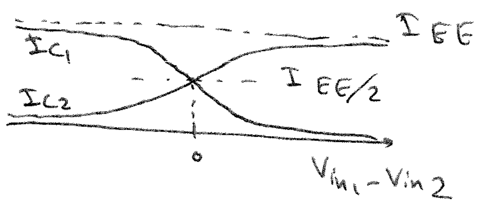
(17)

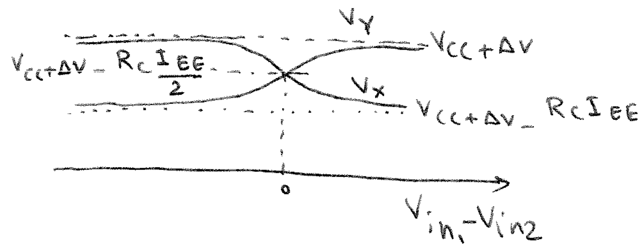
(a)


IEE/2
Ic1
IEE/4
Ic2
0
Vin1-Vin2

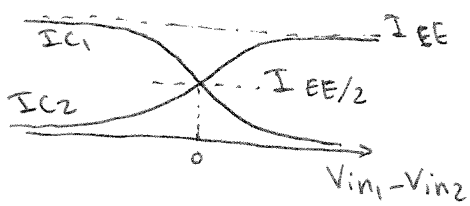

VY
VCC
VCC - RC IEE/4
VX
VCC - RC IEE/2
0
Vin1-Vin2

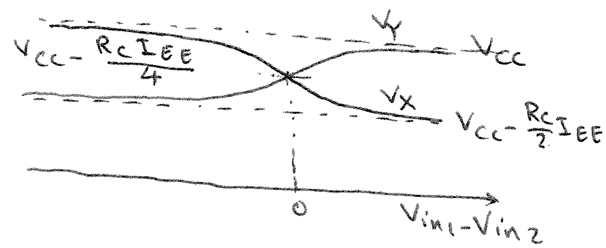
(b)


IEE
Ic1
IEE/2
Ic2
0
Vin1-Vin2

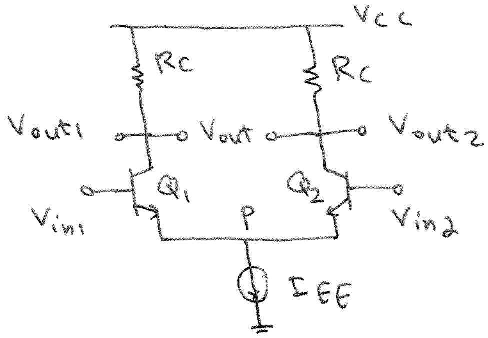

VY
VCC+ΔV
VCC+ΔV - RC IEE/2
VX
VCC+ΔV - RC IEE
0
Vin1-Vin2

(c)


IEE
Ic1
IEE/2
Ic2
0
Vin1-Vin2


VY
VCC
VCC - RC IEE/4
VX
VCC - RC/2 IEE
0
Vin1-Vin2

18



$$\frac{I_{C1}}{I_{C2}} = 5$$

$$V_{in1} - V_{in2} = V_T \ln \frac{I_{C1}}{I_{C2}} = 0.026 \ln 5 = 41.845\,mV$$

at 27°, $V_T = 26mV$ $\Rightarrow$ at 100°C,

$$V_T = \frac{(273+100)}{273+27} 26^{mV} = 32.33\,mV$$

$$\Rightarrow \quad 41.845\,mV = 32.33\,mV \ln \frac{I_{C1}}{I_{C2}} \Rightarrow \frac{I_{C1}}{I_{C2}} = 3.65$$

(19)

$$I_{C2} = I_{C1} = \frac{I_{EE}}{2}$$

if I_{C2} changes by 10% then

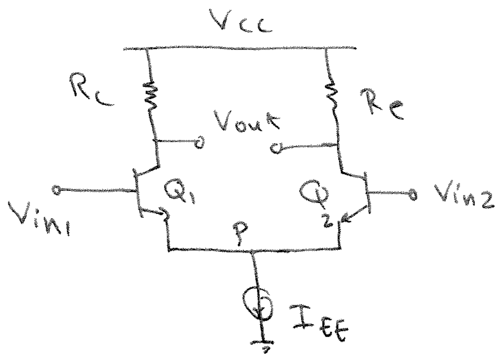
$$1.1 \times I_{C2\,bias} = \frac{I_{EE}}{1 + \exp \frac{V_{in1} - V_{in2}}{V_T}} \Rightarrow$$

$$1.1 \times \frac{I_{EE}}{2} = \frac{I_{EE}}{1 + \exp \frac{V_{in1} - V_{in2}}{V_T}} \Rightarrow$$

$$V_{in1} - V_{in2} = V_T \ln \frac{0.9}{1.1} = -0.2\, V_T = -5.217\, mV$$

So the input differential voltage should change by no more than 5.2 mV.

(20)



$$I_{C2} = \frac{I_{EE}}{1 + \exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)}$$

$$I_{C2,\,bias} = \frac{I_{EE}}{2}$$

if the transconductance of Q_2 drops by a factor of 2, then $I_{C2} = \frac{I_{EE}}{4}$

$$\Rightarrow \quad \frac{I_{EE}}{4} = \frac{I_{EE}}{1 + \exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)} \Rightarrow$$

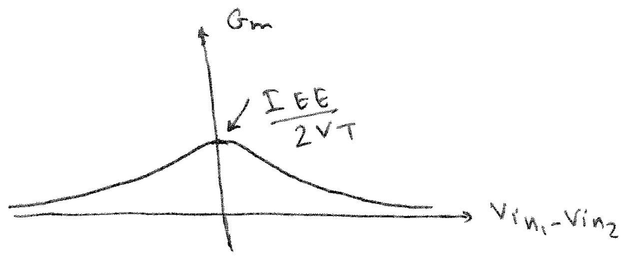
$$V_{in1} - V_{in2} = V_T \ln 3 = 1.0986 V_T = 28.564 \text{ mV}$$

(21) $V_{in1} - V_{in2} = \Delta V_{in}$

$$I_{C1} - I_{C2} = \frac{I_{EE} \exp \frac{\Delta V_{in}}{V_T}}{1 + \exp \frac{\Delta V_{in}}{V_T}} - \frac{I_{EE}}{1 + \exp \frac{\Delta V_{in}}{V_T}}$$

$$\Rightarrow \frac{\partial (I_{C1} - I_{C2})}{\partial (\Delta V_{in})} = I_{EE} \left[\frac{\frac{1}{V_T} \exp(\frac{\Delta V_{in}}{V_T})(1 + \exp \frac{\Delta V_{in}}{V_T}) - \frac{(\exp \frac{\Delta V_{in}}{V_T})^2}{V_T}}{(1 + \exp \frac{\Delta V_{in}}{V_T})^2} + \frac{\frac{1}{V_T} \exp \frac{\Delta V_{in}}{V_T}}{(1 + \exp \frac{\Delta V_{in}}{V_T})^2} \right]$$

$$= \frac{2 I_{EE}}{V_T} \frac{\exp(\frac{V_{in1} - V_{in2}}{V_T})}{(1 + \exp(\frac{V_{in1} - V_{in2}}{V_T}))^2}$$



$\begin{cases} \max G_m = \frac{I_{EE}}{2V_T} \\ \text{At } V_{in1} - V_{in2} = 0 \end{cases}$

if $G_m = \frac{1}{2} G_{m_{max}} = \frac{I_{EE}}{4V_T} \Rightarrow$

$$\frac{\exp(\frac{V_{in1} - V_{in2}}{V_T})}{(1 + \exp(\frac{V_{in1} - V_{in2}}{V_T}))^2} = \frac{1}{8} \Rightarrow V_{in1} - V_{in2} = \pm 1.763 V_T$$

$$= \pm 45.838 \text{ mV}$$

(22)

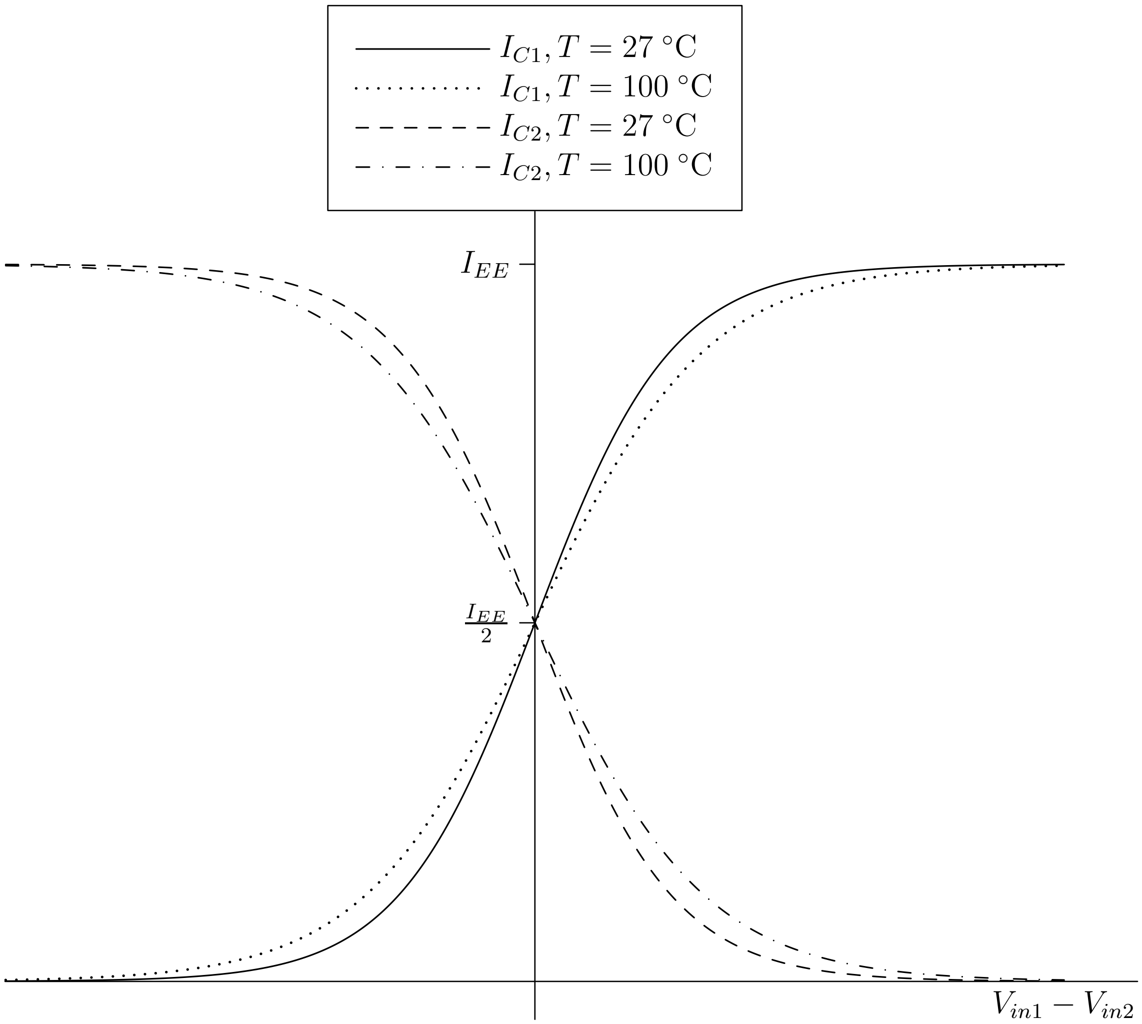
$$V_{out1} - V_{out2} = -R_C I_{EE} \tanh \frac{V_{in1} - V_{in2}}{2V_T}$$

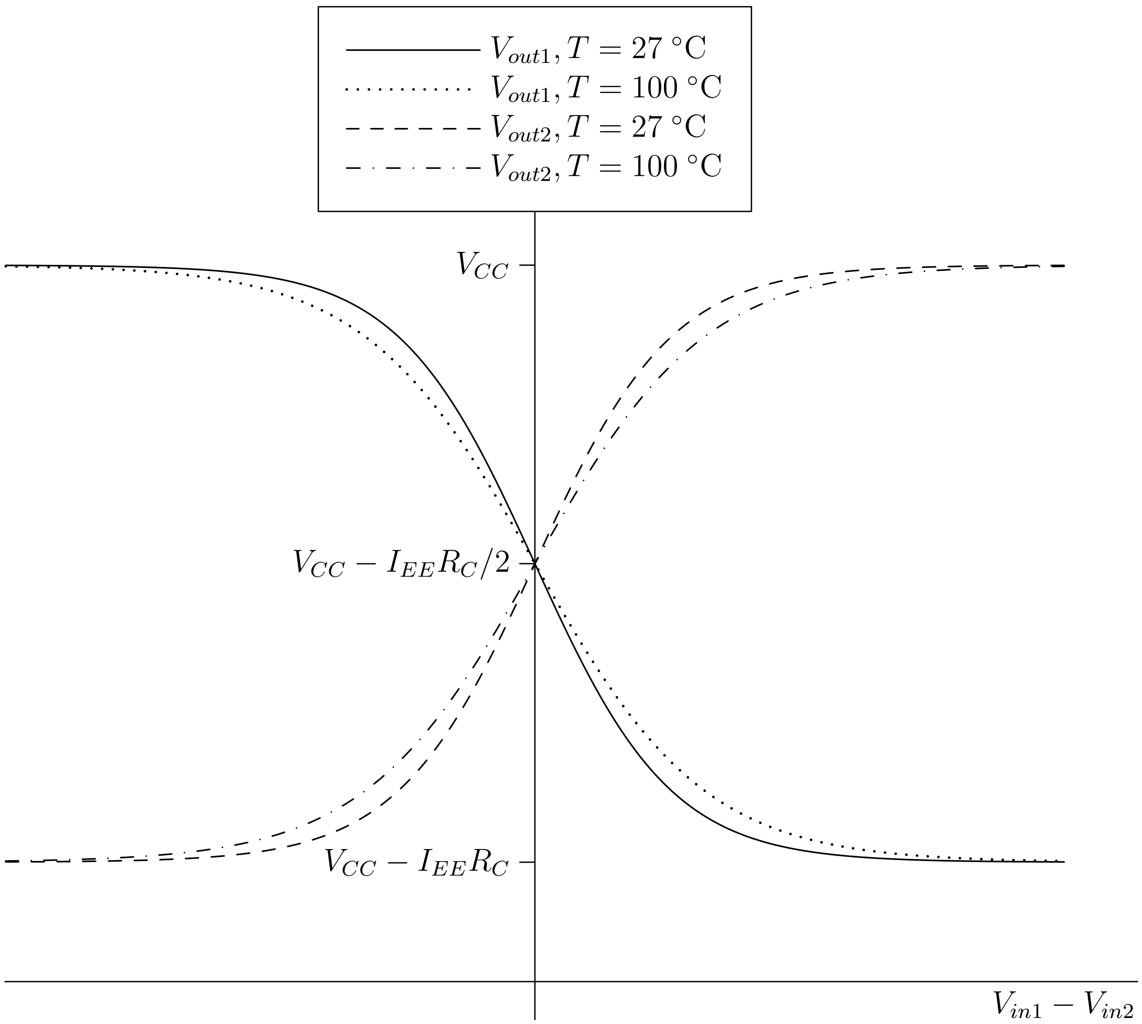
$$A_v = \frac{\partial (V_{out1} - V_{out2})}{\partial (V_{in1} - V_{in2})} = -\frac{2R_C I_{EE}}{V_T} \frac{\exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)}{\left[1 + \exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)\right]^2}$$

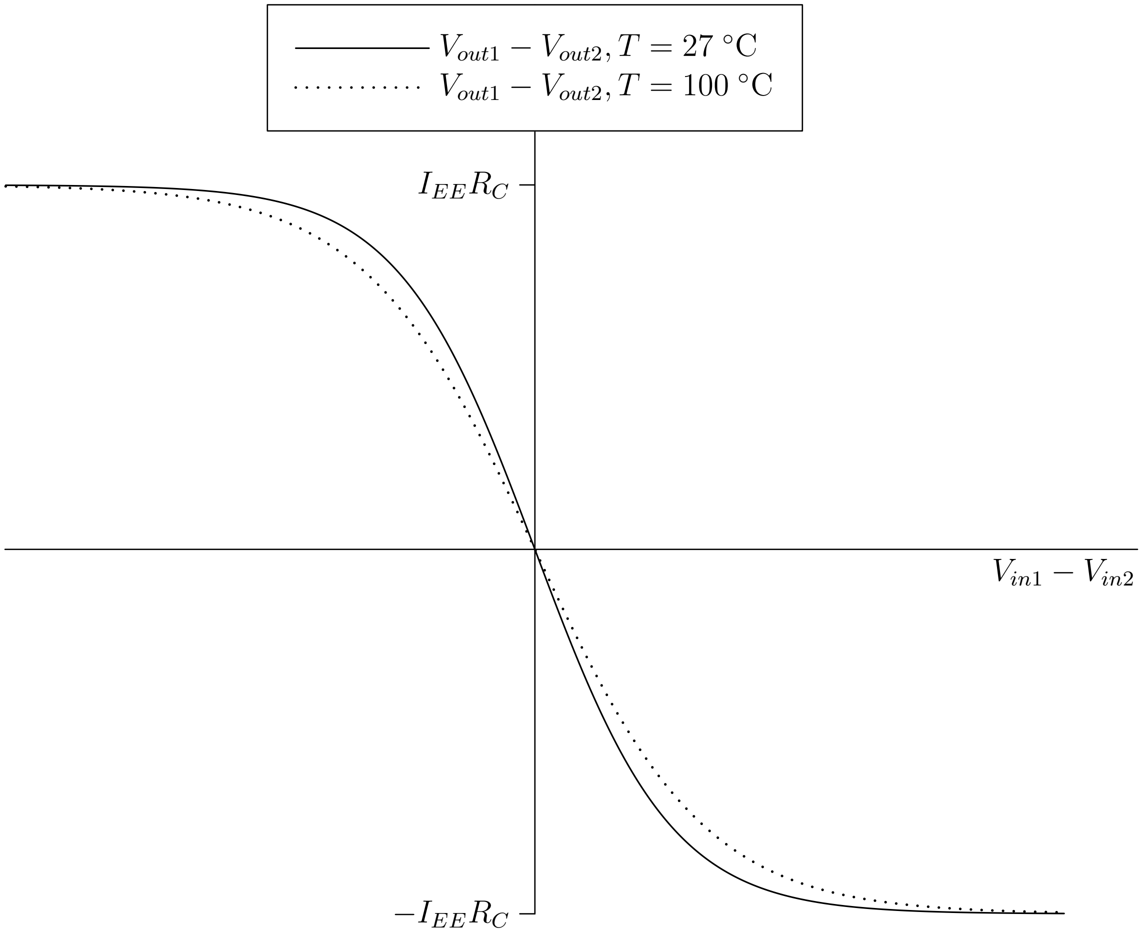
if $V_{in1} - V_{in2} = 30mV \Rightarrow$

$$A_v = -14.02 \; R_C I_{EE}$$

10.23 If the temperature increases from 27 °C to 100 °C, then V_T will increase from 25.87 mV to 32.16 mV. Will will cause the curves to stretch horizontally, since the differential input will have to be larger in magnitude in order to drive the current to one side of the differential pair. This stretching is shown in the following plots.



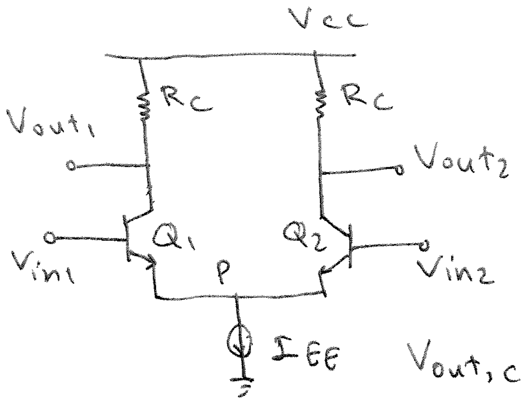

$V_{out1}, T = 27\,°C$
$V_{out1}, T = 100\,°C$
$V_{out2}, T = 27\,°C$
$V_{out2}, T = 100\,°C$
V_{CC}
$V_{CC} - I_{EE}R_C/2$
$V_{CC} - I_{EE}R_C$
$V_{in1} - V_{in2}$



$V_{out1} - V_{out2}, T = 27\ °C$
$V_{out1} - V_{out2}, T = 100\ °C$
$I_{EE}R_C$
$-I_{EE}R_C$
$V_{in1} - V_{in2}$

(24) $R_C = 500\Omega$, $I_{EE} = 1mA$, $V_{CC} = 2.5V$

$V_{in_1} = V_0 \sin\omega t + V_{CM}$ $V_{in_2} = -V_0 \sin\omega t + V_{CM}$, $V_{CM} = 1V$

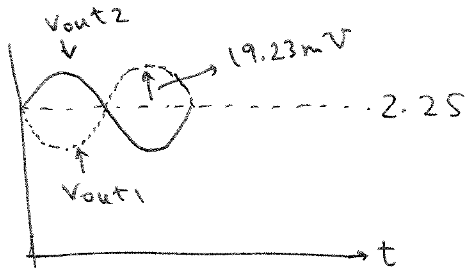


$A_V = -g_m R_C = -\frac{I_{EE}}{2V_T} R_C = -\frac{10^{-3} \times 500}{2 \times 0.026} = -9.615$

$V_{out,CM} = V_{CC} - R_C \frac{I_{EE}}{2} = 2.5 - 0.5 \times 0.5 \Rightarrow$

$V_{out,CM} = 2.25$

(a) $|V_{out}| = |A_V V_{in}| = 9.615 \times 2m = 19.23mV$

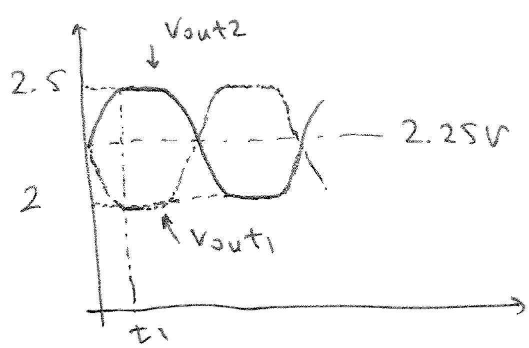


(b) $I_{C_1} = 0.95 I_{EE}$, $I_{C_2} = 0.05 I_{EE}$, $\frac{I_{C_1}}{I_{C_2}} = 19$

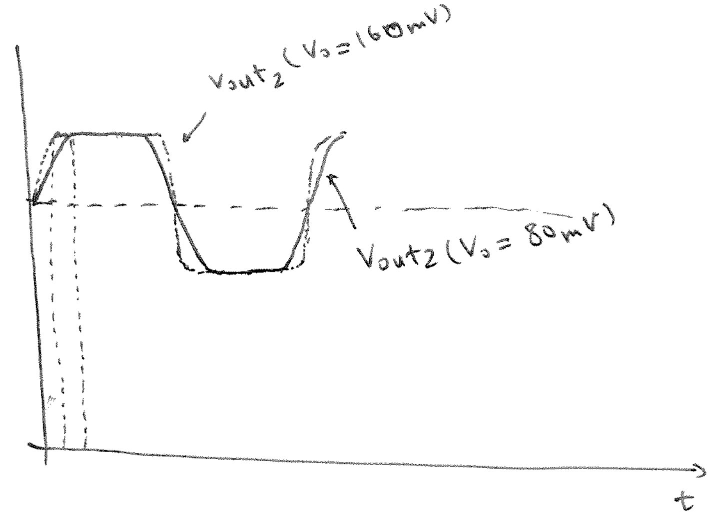
$V_{in_1} - V_{in_2} = V_T \ln \frac{I_{C_1}}{I_{C_2}} = 76.555 mV$

$\frac{V_{in_1} - V_{in_2}}{2} = 50mV \sin\omega t_1 \Rightarrow 38.278mV = 50mV \sin\omega t_1 \Rightarrow$

$t_1 = \frac{0.872}{\omega}$



(25)



The time at which one transistor takes 95% of the tail current source is achievable through:

$$\frac{I_{C_1}}{I_{C_2}} = 19 \Rightarrow V_{in_1} - V_{in_2} = V_T \ln \frac{I_{C_1}}{I_{C_2}} = 76.555 \text{ mV}$$

$$\frac{V_{in_1} - V_{in_2}}{2} = V_o \sin \omega t_1 \Rightarrow t_1 = \frac{\text{Arc Sin } \frac{38.278}{V_o}}{\omega}$$

evidently as V_o increases, t_1 decreases and the output waveform becomes sharper.

$$t_1 (V_o = 80 \text{ mV}) = \frac{0.499}{\omega}$$

$$t_1 (V_o = 160 \text{ mV}) = \frac{0.242}{\omega}$$

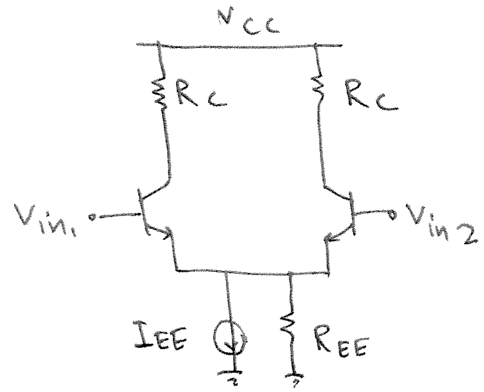
(26) $\omega = 2\pi \times (100\,MHz)$

$$\text{Slope} \approx \frac{V_{cc} - V_{cm}}{t_1} = \frac{0.25\,\omega}{\text{ArcSin}\left(\frac{38.278}{V_o\,(mV)}\right)}$$

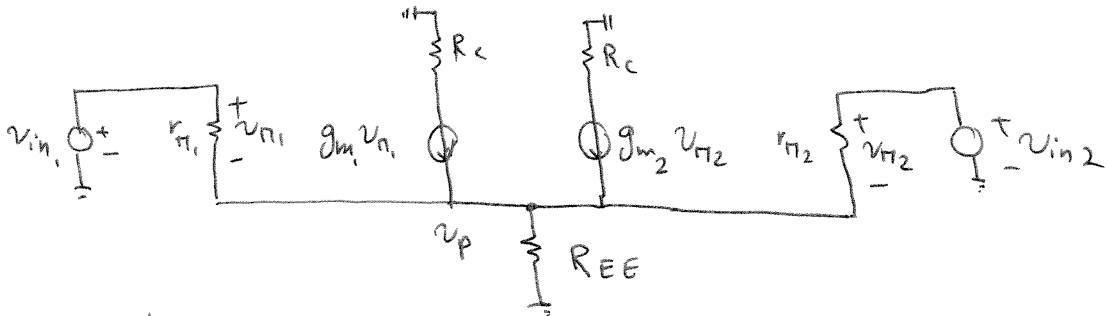
$\Rightarrow$ if $V_o = 80\,mV \Rightarrow \text{Slope} = 3.148 \times 10^8\ V/s$

if $V_o = 160\,mV \Rightarrow \text{Slope} = 6.491 \times 10^8\ V/s$

(27)



The small signal model is:

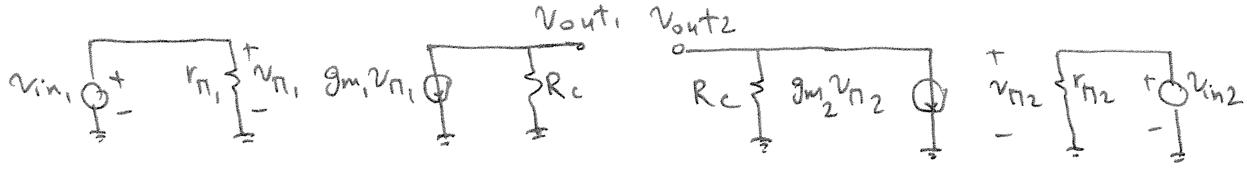


writing the node equation at P we have:

$$\frac{v_p}{R_{EE}} + \frac{v_p - v_{in_1}}{r_{\pi_1}} + g_{m_1}(v_p - v_{in_1}) + \frac{v_p - v_{in2}}{r_{\pi_2}} + g_{m_2}(v_p - v_{in_2}) = 0$$

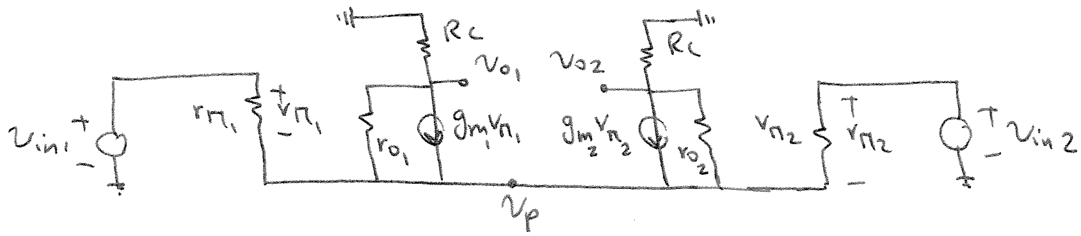
Since $v_{in_1} = -v_{in2}$ and $\begin{cases} r_{\pi_1} = r_{\pi_2} \\ g_{m_1} = g_{m_2} \end{cases}$, the above equation simplifies to:

$$\frac{v_p}{R_{EE}} + \frac{2v_p}{r_{\pi_1}} + 2g_{m_1} v_p = 0 \Rightarrow v_p = 0 \Rightarrow$$ the small signal model is:



$$A_v = \frac{v_{out_1} - v_{out2}}{v_{in_1} - v_{in_2}} = \frac{-g_{m_1} v_{in_1} R_C + g_{m_2} v_{in_2} R_C}{v_{in_1} - v_{in_2}} = -g_{m_1} R_C$$

(28)



$v_{in_1} = -v_{in2} \rightarrow v_{in1} + v_{in2} = 0$

$g_{m_1} = g_{m2}, \; r_{\pi_1} = r_{\pi_2}, \; r_{o_1} = r_{o_2}$

Writing the node equation at v_p:

$$\frac{v_p - v_{in1}}{r_{\pi_1}} + \frac{v_p - v_{o1}}{r_{o_1}} + g_{m_1}(v_p - v_{in_1}) + g_{m_2}(v_p - v_{in2}) +$$

$$\frac{v_p - v_{in2}}{r_{\pi_2}} + \frac{v_p - v_{o2}}{r_{o_2}} = 0 \Rightarrow 2 g_{m_1} v_p + \frac{2v_p - v_{o_1} - v_{o2}}{r_{o_1}} + \frac{v_p \times 2}{r_{\pi_1}} = 0 \quad ①$$

Now the node equations at nodes v_{o_1} and v_{o2} leads to:

$$\begin{cases} \dfrac{v_{o1}}{R_C} + \dfrac{v_{o_1} - v_p}{r_{o_1}} + g_{m_1}(v_{in_1} - v_p) = 0 & ② \\ \dfrac{v_{o2}}{R_C} + \dfrac{v_{o2} - v_p}{r_{o2}} + g_{m_2}(v_{in_2} - v_p) = 0 & ③ \end{cases} \Rightarrow ② + ③ =$$

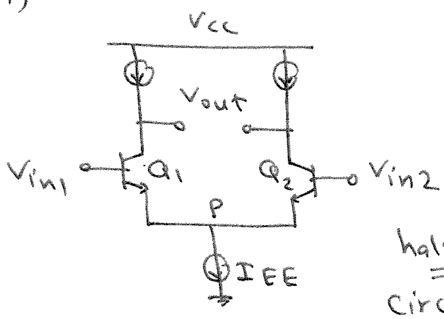
$$(v_{o_1} + v_{o2})\left(\frac{1}{R_C} + \frac{1}{r_{o_1}}\right) = \frac{2v_p}{r_{o_1}} + 2 g_{m_1} v_p \quad ④$$

placing 4 in ① $\Rightarrow$

$$2 g_{m_1} v_p + \frac{1}{r_{o_1}}\left(2v_p - \frac{1}{\frac{1}{R_C} + \frac{1}{r_{o_1}}}\left(\frac{2v_p}{r_{o_1}} + 2 g_{m_1} v_p\right)\right) + \frac{2v_p}{r_\pi} = 0$$

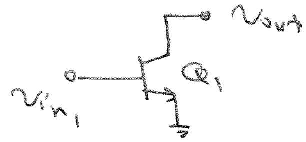
$$\Rightarrow \underline{v_p = 0}$$

(29)



$I_{EE} = 1mA$, $V_A = 5V$

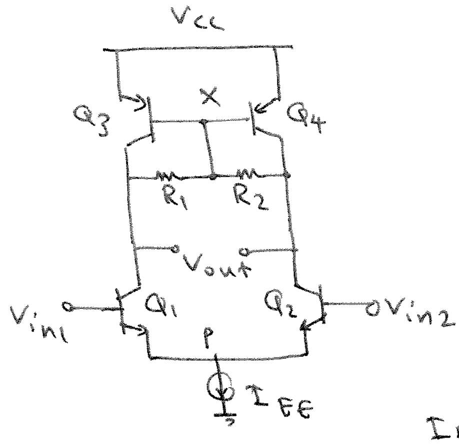
half ⟹ circuit



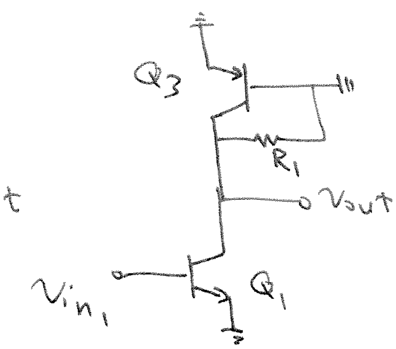
$$A_v = -g_{m_1} \cdot r_{o_1} = -\frac{I_{EE}}{2V_T} \frac{V_A}{\frac{I_{EE}}{2}} = -\frac{V_A}{V_T} = \frac{-5}{0.026}$$

$\rightarrow A_v = -192.31$

(30)



half ⟹ circuit



$I_{EE} = 2mA$, $V_{A,n} = 5V$, $V_{A,p} = 4V$

$$A_v = -g_{m_1}\left(r_{o_1} \,||\, r_{o_3} \,||\, R_1\right)$$

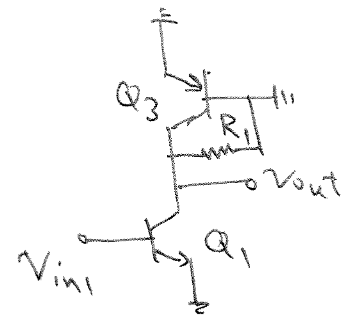
$$\Rightarrow 50 = \frac{I_{EE}}{2V_T}\left(\frac{V_{A,n}}{\frac{I_{EE}}{2}} \,||\, \frac{V_{A,p}}{\frac{I_{EE}}{2}} \,||\, R_1\right) \Rightarrow$$

$$50 = \frac{2}{2 \times 26}\left(\frac{5}{10^{-3}} \,||\, \frac{4}{10^{-3}} \,||\, R_1\right) \rightarrow$$

$$R_1 = 3132.53\,\Omega$$

(31)

The half circuit is:



$$A_v = -g_{m1}(r_{o1} \| r_{o3} \| R_1)$$

$\Rightarrow$

$$50 = \frac{I_{EE}}{2 \times 0.026}\left(\frac{5}{\frac{I_{EE}}{2}} \| \frac{4}{\frac{I_{EE}}{2}} \| 5^K\right) \Rightarrow$$

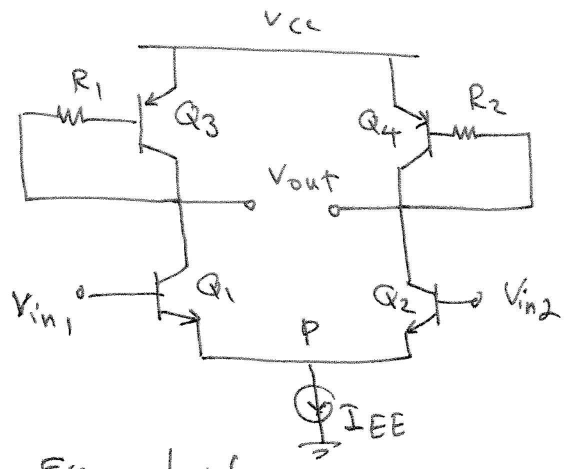
$$50 = \frac{1}{0.052}\left(10 \| 8 \| 5 I_{EE}\right) \Rightarrow$$

(↓ $K\Omega$, ↓ mA)

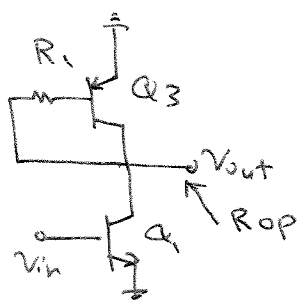
$$I_{EE} = 1.253\ mA$$

(32)

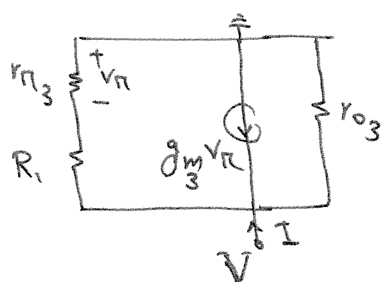
(a)



From half circuit concept we have: $A_v = -g_{m_1}(r_{o_1} \| R_{op})$



To calculate Rop, from small signal model we have:



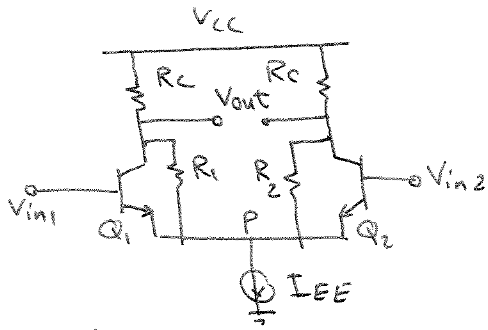
$$I = \frac{V}{r_{o_3}} - g_{m_3} V_\pi + \frac{V}{R_1 + r_{\pi_3}} = V\left[\frac{1}{r_{o_3}} + \frac{1}{R_1 + r_{\pi_3}}\right] + g_{m_3}\frac{r_{\pi_3}}{R_1 + r_{\pi_3}} V$$

$$\rightarrow R_{op} = \frac{V}{I} = r_{o_3} \| (R_1 + r_{\pi_3}) \| \left(\left(1 + \frac{R_1}{r_{\pi_3}}\right)\frac{1}{g_{m_3}}\right)$$

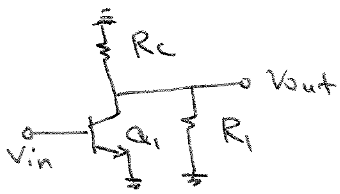
$$\rightarrow A_v = -g_{m_1}\left[r_{o_1} \| r_{o_3} \| (R_1 + r_{\pi_3}) \| \left(\left(1 + \frac{R_1}{r_{\pi_3}}\right)\frac{1}{g_{m_3}}\right)\right]$$

(32)

(b)

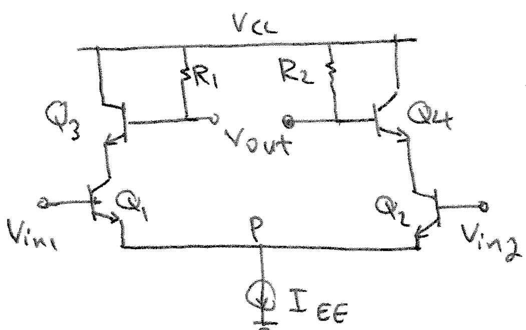


From half circuit concept:

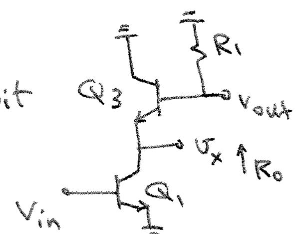


$$\rightarrow A_v = -g_{m_1} (R_1 \| R_C \| r_{o_1})$$

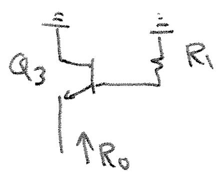
(C)

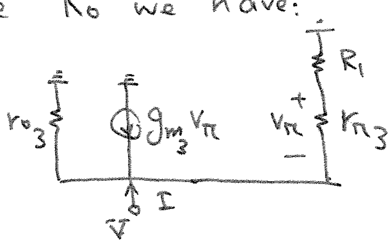


half circuit $\Longrightarrow$



To calculate R_o we have:





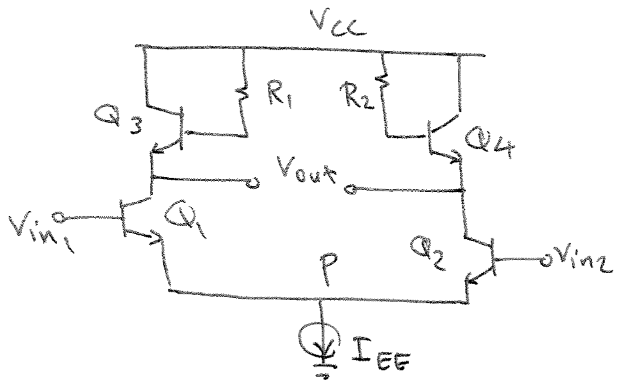
$$R_o = \frac{V}{I}$$

$$I = \frac{V}{r_{o3}} + \frac{V}{R_1 + r_{\pi 3}} + g_{m_3} \frac{r_{\pi 3}}{r_{\pi 3} + R_1} V$$

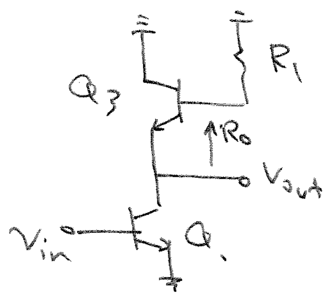
$$\Longrightarrow R_o = r_{o3} \| (R_1 + r_{\pi 3}) \| \left(1 + \frac{R_1}{r_{\pi 3}}\right) \frac{1}{g_{m_3}}$$

$$A_V = \frac{v_{out}}{v_{in}} = \frac{v_x}{v_{in}} \frac{v_{out}}{v_x} = -g_{m_1} (r_{o_1} \| R_o) \frac{R_1}{R_1 + r_{\pi 3}}$$

(32) (d)



From half circuit concept:

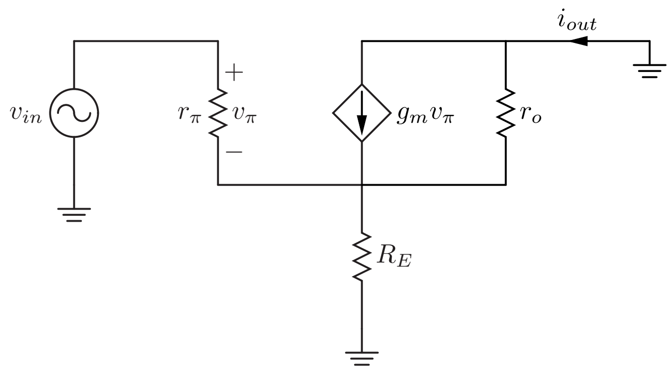


we already proved in part (c) that

$$R_o = r_{o3} \| (R_1 + r_{\pi 3}) \| \left(1 + \frac{R_1}{r_{\pi 3}}\right) \frac{1}{g_{m3}}$$

$$\rightarrow A_v = \frac{v_{out}}{v_{in}} = -g_{m1}(r_{o1} \| R_o)$$

10.33 (a) Treating node P as a virtual ground, we can draw the small-signal model to find G_m.



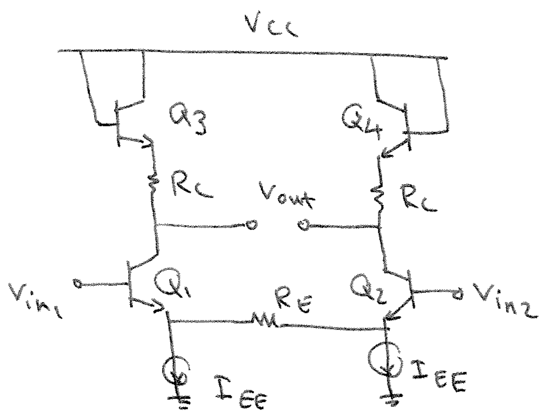
$$
\begin{aligned}
i_{out} &= -\frac{v_\pi}{r_\pi} + \frac{v_{in} - v_\pi}{R_E} \\
v_\pi &= v_{in} - \left(-i_{out} + g_m v_\pi\right) r_o \\
v_\pi \left(1 + g_m r_o\right) &= v_{in} + i_{out} r_o \\
v_\pi &= \frac{v_{in} + i_{out} r_o}{1 + g_m r_o} \\
i_{out} &= -\frac{v_{in} + i_{out} r_o}{r_\pi \left(1 + g_m r_o\right)} + \frac{v_{in}}{R_E} - \frac{v_{in} + i_{out} r_o}{R_E \left(1 + g_m r_o\right)} \\
i_{out}\left(1 + \frac{r_o}{r_\pi \left(1 + g_m r_o\right)} + \frac{r_o}{R_E \left(1 + g_m r_o\right)}\right) &= v_{in}\left(\frac{1}{R_E} - \frac{1}{r_\pi \left(1 + g_m r_o\right)} - \frac{1}{R_E \left(1 + g_m r_o\right)}\right) \\
i_{out}\left(\frac{r_\pi R_E \left(1 + g_m r_o\right) + r_o \left(r_\pi + R_E\right)}{r_\pi R_E \left(1 + g_m r_o\right)}\right) &= v_{in}\left(\frac{r_\pi \left(1 + g_m r_o\right) - R_E - r_\pi}{r_\pi R_E \left(1 + g_m r_o\right)}\right) \\
G_m = \frac{i_{out}}{v_{in}} &= \frac{r_\pi \left(1 + g_m r_o\right) - R_E - r_\pi}{r_\pi R_E \left(1 + g_m r_o\right) + r_o \left(r_\pi + R_E\right)} \\
R_{out} &= R_C \parallel \left[r_o + \left(1 + g_m r_o\right)\left(r_\pi \parallel R_E\right)\right]
\end{aligned}
$$

$$
A_v = \boxed{-\frac{r_\pi \left(1 + g_m r_o\right) - R_E - r_\pi}{r_\pi R_E \left(1 + g_m r_o\right) + r_o \left(r_\pi + R_E\right)} \left\{R_C \parallel \left[r_o + \left(1 + g_m r_o\right)\left(r_\pi \parallel R_E\right)\right]\right\}}
$$

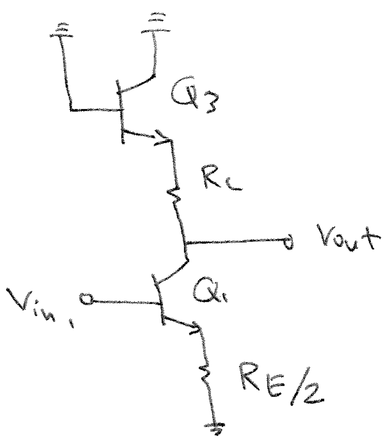
(b) The result is identical to the result from part (a), except R_1 appears in parallel with r_o.

$$
A_v = \boxed{-\frac{r_\pi \left(1 + g_m \left(r_o \parallel R_1\right)\right) - R_E - r_\pi}{r_\pi R_E \left(1 + g_m \left(r_o \parallel R_1\right)\right) + \left(r_o \parallel R_1\right)\left(r_\pi + R_E\right)} \left\{R_C \parallel \left[\left(r_o \parallel R_1\right) + \left(1 + g_m \left(r_o \parallel R_1\right)\right)\left(r_\pi \parallel R_E\right)\right]\right\}}
$$

(34)



The half circuit is shown as:

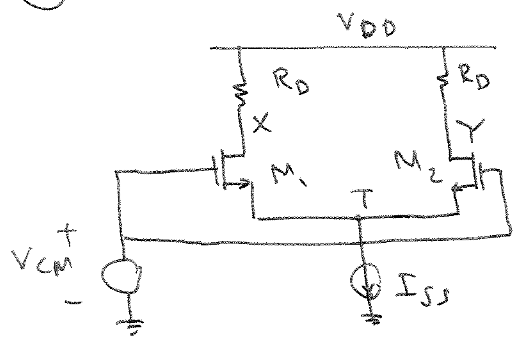


a) $$A_v = \frac{v_{out}}{v_{in_1}} = -\frac{R_C + 1/g_{m3}}{R_E/2 + 1/g_{m1}}$$

b) if $\frac{R_C}{R_E/2} = A$, then if $\frac{1/g_{m3}}{1/g_{m1}} = A$

we conclude $A_v = -A$. So the circuit is very linear.

(35)



$$V_T = V_{CM} - V_{GS1} = V_{CM} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

(a) $$V_T = V_{CM} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{2W}{L}}}$$

The tail voltage increases

(b) $$V_T = V_{CM} - V_{TH} - \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

The tail voltage decreases

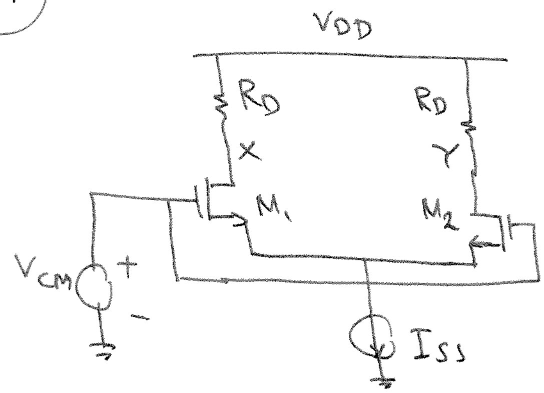
(c) $$V_T = V_{CM} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_n \frac{C_{ox}}{2} \frac{W}{L}}}$$

The tail voltage decreases

10.36

$$
\begin{aligned}
V_{DD} - \frac{I_{SS}R_D}{2} &> V_{CM} - V_{TH,n} \\
V_{DD} &> V_{CM} - V_{TH,n} + \frac{I_{SS}R_D}{2} \\
V_{DD} &> \boxed{1\text{ V}}
\end{aligned}
$$

(37)



$V_{GS} - V_{TH} = 200\,mV$

$\mu_n C_{ox} = 100\ \mu A/V^2$

$\frac{W}{L} = 20/0.18$

$$(V_{GS} - V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \rightarrow$$

$$0.2 = \sqrt{\frac{I_{SS}}{10^{-4} \times \frac{20}{0.18}}} \rightarrow I_{SS} = 0.44\,mA$$

10.38 Let J_D be the current density of a MOSFET, as defined in the problem statement.

$$J_D = \frac{I_D}{W} = \frac{1}{2}\frac{1}{L}\mu_n C_{ox}\left(V_{GS} - V_{TH}\right)^2$$

$$\left(V_{GS} - V_{TH}\right)_{equil} = \sqrt{\frac{2I_D}{\frac{W}{L}\mu_n C_{ox}}}$$

$$= \sqrt{\frac{2J_D}{\frac{1}{L}\mu_n C_{ox}}}$$

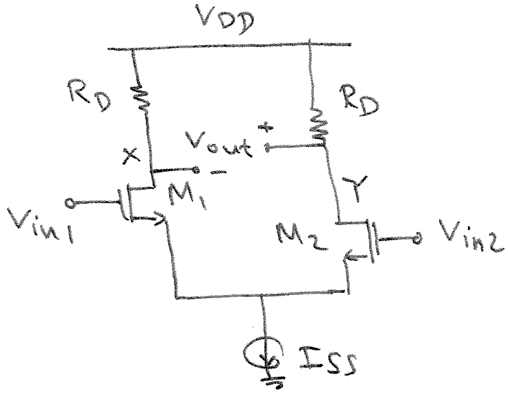
The equilibrium overdrive voltage increases as the square root of the current density.

10.39 Let i_{d1}, i_{d2}, and v_P denote the changes in their respective values given a small differential input of v_{in} ($+v_{in}$ to V_{in1} and $-v_{in}$ to V_{in2}).

$$\begin{aligned} i_{d1} &= g_m\left(v_{in} - v_P\right) \\ i_{d2} &= g_m\left(-v_{in} - v_P\right) \\ v_P &= \left(i_{d1} + i_{d2}\right) R_{SS} \\ &= -2 g_m v_P R_{SS} \\ \Rightarrow v_P &= 0 \end{aligned}$$

Note that we can justify the last step by noting that if $v_P \neq 0$, then we'd have $2g_m R_{SS} = -1$, which makes no sense, since all the values on the left side must be positive. Thus, since the voltage at P does not change with a small differential input, node P acts as a virtual ground.

(40)



$V_{in1} = 1.5$

$V_{in2} = 0.3$

$V_X - V_{TH} > V_{in1} \rightarrow V_{DD} - R_D I_{SS} - V_{TH} > 1.5$

10.41

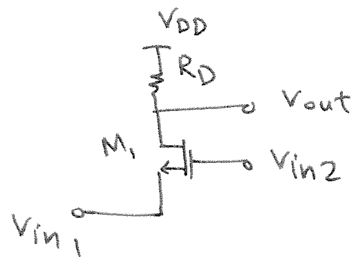
$$
\begin{aligned}
P &= I_{SS} V_{DD} = 2 \text{ mW} \\
I_{SS} &= \boxed{1 \text{ mA}} \\
V_{CM,out} &= V_{DD} - \frac{I_{SS} R_D}{2} = 1.6 \text{ V} \\
R_D &= \boxed{800\ \Omega} \\
|A_v| &= g_m R_D \\
&= \sqrt{2 \left(\frac{W}{L}\right)_1 \mu_n C_{ox} I_D} R_D \\
&= 5 \\
\left(\frac{W}{L}\right)_1 &= \left(\frac{W}{L}\right)_2 = \boxed{390.625}
\end{aligned}
$$

Let's formulate the trade-off between V_{DD} and W/L, let's assume we're trying to meet an output common-mode level of $V_{CM,out}$. Then we have:

$$
\begin{aligned}
I_{SS} &= \frac{P}{V_{DD}} \\
V_{CM,out} &= V_{DD} - \frac{I_{SS} R_D}{2} \\
&= V_{DD} - \frac{P R_D}{2 V_{DD}} \\
R_D &= 2 V_{DD} \left(\frac{V_{DD} - V_{CM,out}}{P}\right) \\
|A_v| &= g_m R_D \\
&= \sqrt{\frac{W}{L} \mu_n C_{ox} I_{SS}} R_D \\
&= \sqrt{\frac{W}{L} \mu_n C_{ox} \frac{P}{V_{DD}}} \left[2 V_{DD} \left(\frac{V_{DD} - V_{CM,out}}{P}\right)\right]
\end{aligned}
$$

To meet a certain gain, W/L and V_{DD} must be adjusted according to the above equation. We can see that if we decrease V_{DD}, we'd have to increase W/L in order to meet the same gain.

(42)



$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{in2} - V_{in1} - V_{TH}\right)^2$$

(1) The current is not an odd function of $(V_{in2} - V_{in1})$. Therefore it is not symmetric around $V_{in1} = V_{in2}$ $\left[(V_{in1} - V_{in2}) = 0\right]$.

(2) The input impedance seen at V_{in1} and V_{in2} are different

(3) The circuit cannot suppress the supply noise. because there is no differential output available.

(43)

$$(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - 2\sqrt{I_{D1} I_{D2}})$$

(a)

$I_{D1} = 0 \Rightarrow$

$$(V_{in1} - V_{in2})^2 = \frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}} \rightarrow V_{in1} - V_{in2} = -\sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

This is the minimum differential input voltage to turn M_1 off.

(b) $I_{D1} = \frac{I_{SS}}{2} \Rightarrow I_{D2} = \frac{I_{SS}}{2}$

$$(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - I_{SS}) = 0 \rightarrow V_{in1} - V_{in2} = 0$$

This is the equilibrium input case.

(c) $I_{D1} = I_{SS} \rightarrow I_{D2} = 0$

$$(V_{in1} - V_{in2})^2 = \frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}} \rightarrow V_{in1} - V_{in2} = +\sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

This is the minimum input differential voltage to turn M_2 off.

(44)

$$I_{D_1} = \frac{I_{SS}}{2} - \frac{1}{4}\sqrt{4I_{SS}^2 - \left[\mu_n C_{ox}\frac{W}{L}(V_{in_1}-V_{in_2})^2 - 2I_{SS}\right]}$$

The analyses which led to the above equation assume that the transistors work in saturation region.

So:

$$-(V_{in_1}-V_{in_2})_{max} \le V_{in_1}-V_{in_2} \le (V_{in_1}-V_{in_2})_{max}$$

$$(V_{in_1}-V_{in_2})_{max} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \Rightarrow$$

$$\mu_n C_{ox}\frac{W}{L}(V_{in_1}-V_{in_2})^2 \le 2I_{SS} \Rightarrow$$

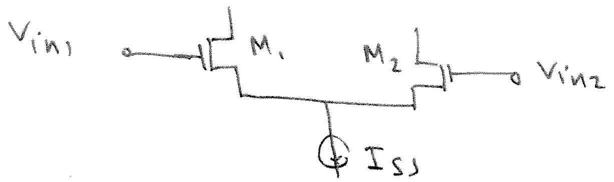
$$-\left[\mu_n C_{ox}\frac{W}{L}(V_{in_1}-V_{in_2})^2 - 2I_{SS}\right] \ge 0 \Rightarrow$$

$$\frac{1}{4}\sqrt{4I_{SS}^2 - \left[\mu_n C_{ox}\frac{W}{L}(V_{in_1}-V_{in_2})^2 - 2I_{SS}\right]} \ge \frac{1}{2}I_{SS}$$

$$\Rightarrow I_{D_1} < 0$$

(45)

$$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4 I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - (V_{in1} - V_{in2})^2}$$



The equilibrium overdrive voltage is:

$$(V_{GS1} - V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} = V_{ov} \Rightarrow$$

$$\mu_n C_{ox} \frac{W}{L} = \frac{I_{SS}}{V_{ov}^2}$$ therefore

$$I_{D1} - I_{D2} = \frac{I_{SS}}{2} \frac{(V_{in1} - V_{in2})}{V_{ov}^2} \sqrt{\frac{4 I_{SS}}{\frac{I_{SS}}{V_{ov}^2}} - (V_{in1} - V_{in2})^2} \Rightarrow$$

$$I_{D1} - I_{D2} = I_{SS} \Rightarrow$$

$$I_{SS} = \frac{I_{SS}}{2} \frac{(V_{in1} - V_{in2})}{V_{ov}^2} \sqrt{4 V_{ov}^2 - (V_{in1} - V_{in2})^2}$$

$$\Rightarrow (V_{in1} - V_{in2})^4 - 4 V_{ov}^2 (V_{in1} - V_{in2})^2 + 4 V_{ov}^4 = 0$$

$$\Rightarrow \left((V_{in1} - V_{in2})^2 - 2 V_{ov}^2 \right)^2 = 0 \Rightarrow$$

$$V_{in1} - V_{in2} = \sqrt{2}\, V_{ov} = \sqrt{2} \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

(46)

$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1} - V_{in2})^2}$$

$$V_{ov} = (V_{GS} - V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \Rightarrow \mu_n C_{ox}\frac{W}{L} = \frac{I_{SS}}{V_{ov}^2}$$

$$\Rightarrow I_{D1} - I_{D2} = \frac{I_{SS}}{2}\frac{(V_{in1} - V_{in2})}{V_{ov}^2}\sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2}$$

$$\Rightarrow G_m = \frac{\partial(I_{D1} - I_{D2})}{\partial(V_{in1} - V_{in2})} =$$

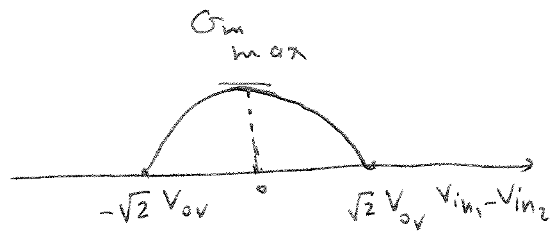
$$\frac{I_{SS}}{2V_{ov}^2}\left[\sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2} - \frac{(V_{in1} - V_{in2})^2}{\sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2}}\right] =$$

$$\frac{I_{SS}}{2V_{ov}^2}\frac{4V_{ov}^2 - 2(V_{in1} - V_{in2})^2}{\sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2}} =$$

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - 2(V_{in1} - V_{in2})^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1} - V_{in2})^2}}$$

$V_{in1} - V_{in2} = 0 \Rightarrow$

$$G_{m\,max} = \sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}$$



(G_m plot)

47

From problem 46:

$$G_{m_{max}} = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} = \sqrt{\frac{I_{SS}}{V_{ov}^2} I_{SS}} = \frac{I_{SS}}{V_{ov}}$$

$\Rightarrow$ if $G_m = \frac{1}{2} \frac{I_{SS}}{V_{ov}}$ we have

$$\frac{1}{2} \frac{I_{SS}}{V_{ov}} = \frac{I_{SS}}{2 V_{ov}^2} \frac{4V_{ov}^2 - 2(V_{in_1} - V_{in_2})^2}{\sqrt{4V_{ov}^2 - (V_{in_1} - V_{in_2})^2}} \Rightarrow$$

$$V_{ov} = \frac{4V_{ov}^2 - 2(V_{in_1} - V_{in_2})^2}{\sqrt{4V_{ov}^2 - (V_{in_1} - V_{in_2})^2}} \Rightarrow$$

$$(4V_{ov}^2 - (V_{in_1} - V_{in_2})^2) V_{ov}^2 = 16 V_{ov}^4 + 4(V_{in_1} - V_{in_2})^4 - 16 V_{ov}^2 (V_{in_1} - V_{in_2})^2$$

$$\Rightarrow 4(V_{in_1} - V_{in_2})^4 - 15 V_{ov}^2 (V_{in_1} - V_{in_2})^2 + 12 V_{ov}^4 = 0$$

$$\Rightarrow (V_{in_1} - V_{in_2})^2 = \frac{15 V_{ov}^2 \pm \sqrt{225 V_{ov}^4 - 192 V_{ov}^4}}{8} =$$

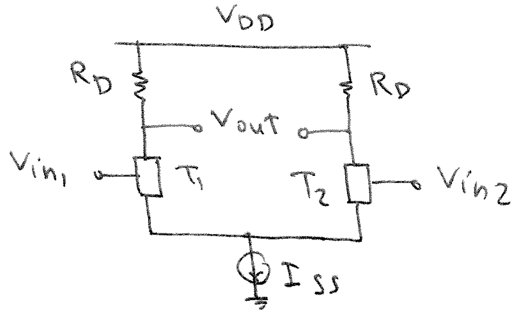
$$\frac{15 V_{ov}^2 \pm \sqrt{33}\, V_{ov}^2}{8}$$ positive sign is not accepted

because $(V_{in_1} - V_{in_2})^2 \leq 2V_{ov}^2 \Rightarrow$

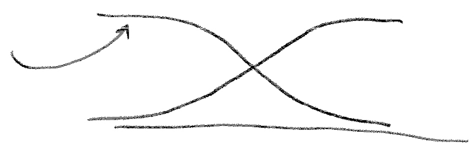
$$V_{in_1} - V_{in_2} = \pm \sqrt{\frac{15 - \sqrt{33}}{8}}\, V_{ov} = \pm 1.0756\, V_{ov}$$

48-

$$I_D = \gamma (V_{GS} - V_{TH})^3$$



(a) The characteristic of $I_{D1} - I_{D2}$ v.s. $V_{in1} - V_{in2}$ is similar to the standard CMOS differential pair. because it has saturation part.



(b) $$I_D = \frac{I_{SS}}{2} = \gamma (V_{GS} - V_{TH})^3 \Rightarrow$$

$$(V_{GS} - V_{TH})_{equil} = \sqrt[3]{\frac{I_{SS}}{2\gamma}}$$

(c) $$I_{D1} = I_{SS} = \gamma (V_{GS1} - V_{TH})^3 \Rightarrow V_{GS1} - V_{TH} = \sqrt[3]{\frac{I_{SS}}{\gamma}}$$

$$I_{D2} = 0 = \gamma (V_{GS2} - V_{TH})^3 \Rightarrow V_{GS2} - V_{TH} = 0$$

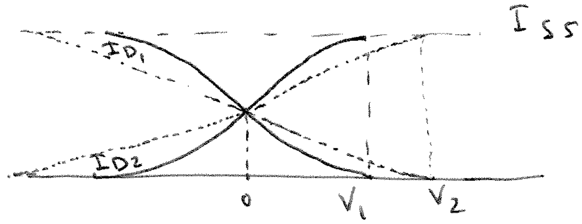
$$\Rightarrow V_{GS1} - V_{GS2} = V_{in1} - V_{in2} = \sqrt[3]{\frac{I_{SS}}{\gamma}} =$$

$$\sqrt[3]{2}\,(V_{GS} - V_{TH})_{equil}$$

(49)

(a)

gate oxide thickness is doubled ⟹ C_{ox} is halved ⟹ $(V_{in_1} - V_{in_2})_{max}$ scales up by $\sqrt{2}$.



$V_2 = \sqrt{2}\, V_1$

so all the curves stretch out to the sides by $\sqrt{2}$ times.

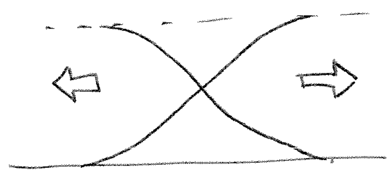
(b) if threshold voltage is halved, nothing will change in the curves. The reason is that the curves depend on $V_{in_1} - V_{in_2}$.

(c) In this case, $(V_{in_1} - V_{in_2})_{max}$ does not change so, all the curves scale half downward because I_{SS} is halved.

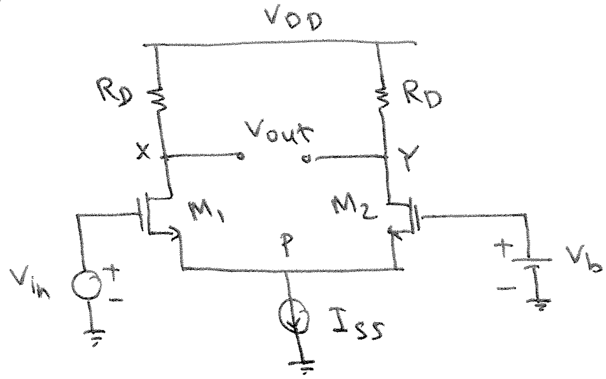
(50)

if mobility falls then $(V_{in1} - V_{in2})_{max}$ will increase because $(V_{in1} - V_{in2})_{max} = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$

So the curves stretch out to the sides.

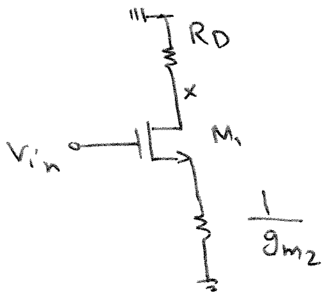


(51)



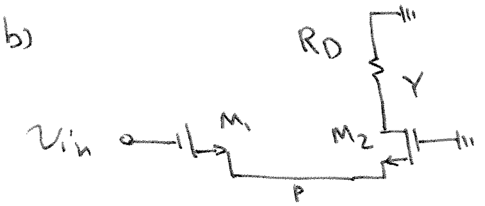
$g_{m1} = g_{m2} = g_m$

(a)



$$v_X = -g_{m1} v_{gs1} R_D = -g_{m1} \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in} R_D = -\frac{g_{m1} g_{m2}}{g_{m1} + g_{m2}} R_D v_{in} = -\frac{g_m}{2} R_D v_{in}$$

(b)



$$v_P = \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in}$$

$$\Rightarrow v_P = \frac{g_{m1}}{g_{m1} + g_{m2}} v_{in} \Rightarrow$$

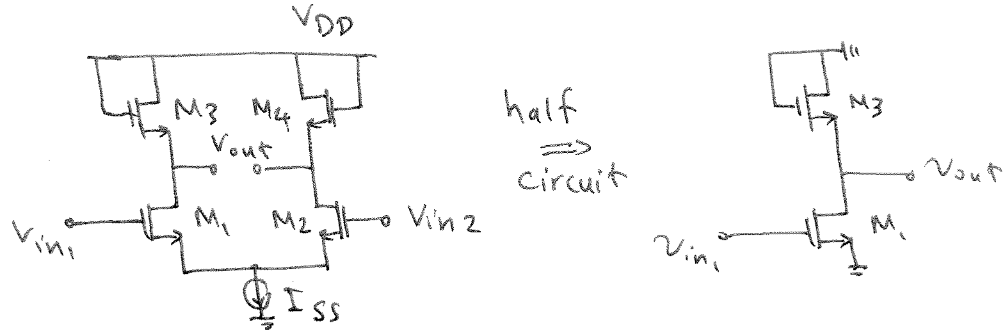
$$v_Y = -g_{m2} v_{gs2} R_D = g_{m2} v_P R_D = \frac{g_{m1} g_{m2}}{g_{m1} + g_{m2}} R_D v_{in}$$

$$\rightarrow v_Y = \frac{g_m}{2} R_D v_{in}$$

(c) $\dfrac{v_X - v_Y}{v_{in}} = -g_m R_D$ This value is equal to the gain of the differential amplifier.

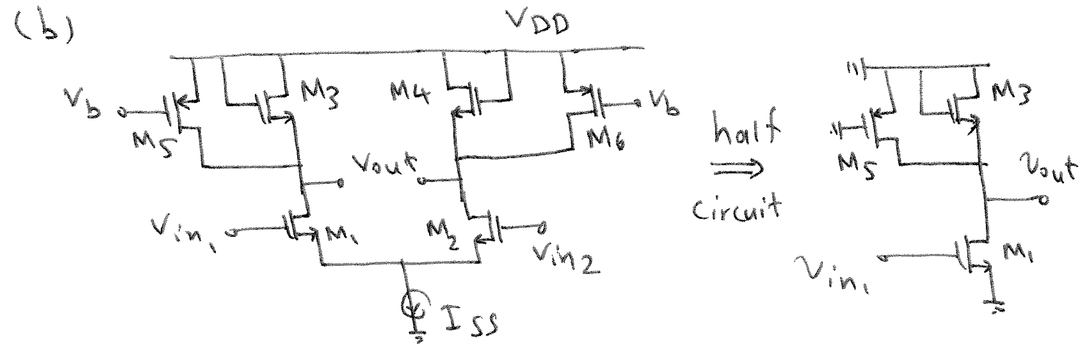
(52)

(a)



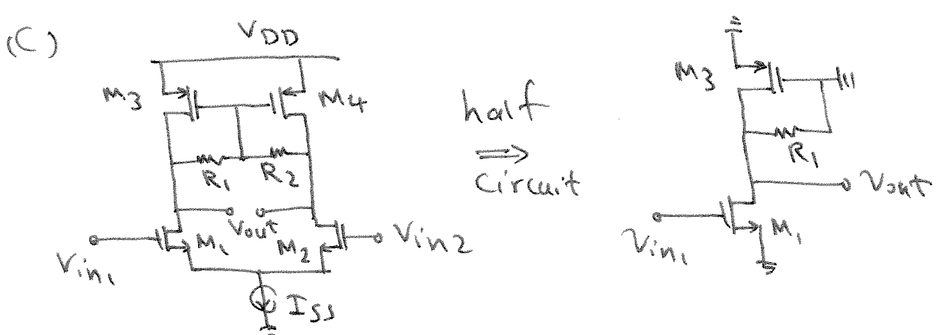
$$A_v = -g_{m_1}\left(r_{o_1} \| r_{o_3} \| \frac{1}{g_{m_3}}\right)$$

(b)



$$A_v = -g_{m_1}\left(r_{o_1} \| r_{o_5} \| \frac{1}{g_{m_3}} \| r_{o_3}\right)$$

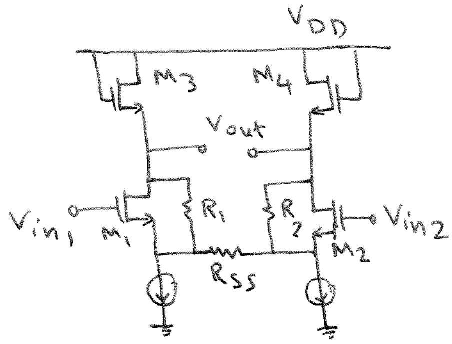
(C)



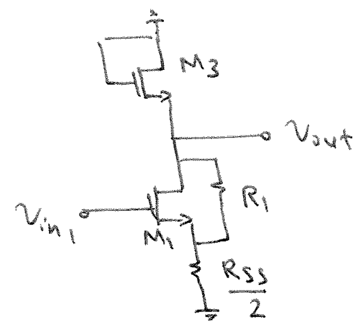
$$A_v = -g_{m_1}\left(r_{o_1} \| r_{o_3} \| R_1\right)$$

(53)

(a)

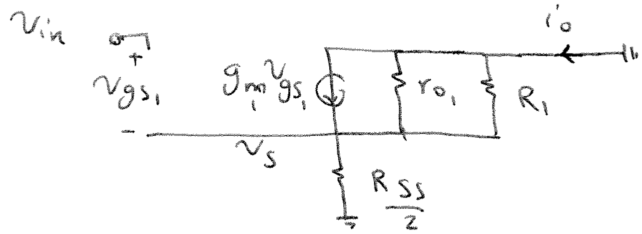


half ⟹ circuit



$$R_{out} = \left(r_{o3} \parallel \frac{1}{g_{m3}}\right) \parallel \left(g_{m1} (R_1 \parallel r_{o1}) \frac{R_{SS}}{2} + \frac{R_{SS}}{2} + R_1 \parallel r_{o1} \right)$$

To calculate G_m:



$$v_{gs1} = v_{in} - v_s$$

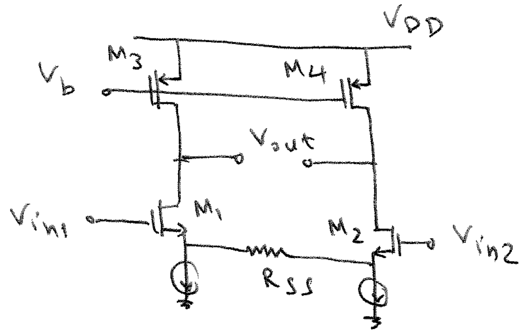
$$\frac{v_s}{\frac{R_{SS}}{2} \parallel R_1 \parallel r_{o1}} + g_{m1} v_s = g_{m1} v_{in} \Rightarrow v_s = \frac{g_{m1} v_{in}}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \parallel R_1 \parallel r_{o1}}}$$

$$i_o = + \frac{v_s}{\frac{R_{SS}}{2}} = + \frac{1}{\frac{R_{SS}}{2}} \frac{g_{m1} v_{in}}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \parallel R_1 \parallel r_{o1}}} \Rightarrow$$

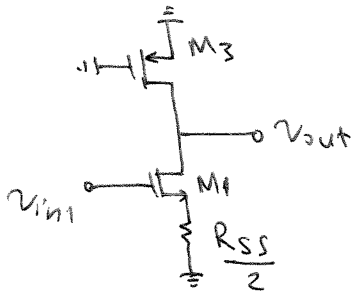
$$G_m = \frac{i_o}{v_{in}} = + \frac{2 g_{m1}}{R_{SS}} \frac{1}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \parallel R_1 \parallel r_{o1}}}$$

$$A_v = -G_m R_{out}$$

53 (b)

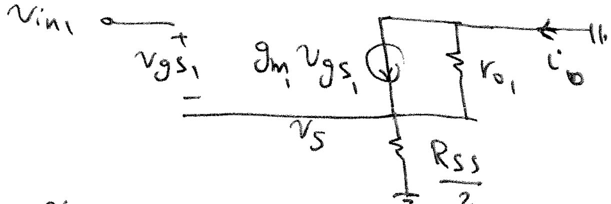


half ⟹ circuit



$$R_{out} = r_{o3} \parallel \left(g_{m1} r_{o1} \frac{R_{SS}}{2} + r_{o1} + \frac{R_{SS}}{2} \right)$$

To calculate G_m:

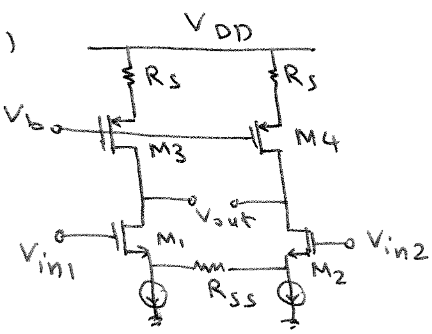


$$\frac{v_s}{r_{o1} \parallel \frac{R_{SS}}{2}} + g_{m1} v_s = g_{m1} v_{in} \Rightarrow v_s = \frac{g_{m1} v_{in}}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \parallel r_{o1}}}$$

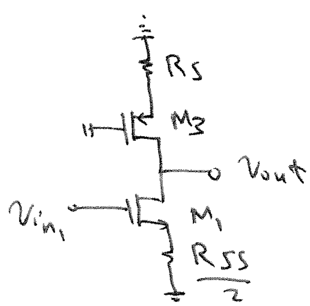
$$G_m = \frac{i_o}{v_{in}} = + \frac{v_s}{\frac{R_{SS}}{2}} \frac{1}{v_{in}} = + \frac{2 g_{m1}}{R_{SS}} \frac{1}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \parallel r_{o1}}}$$

$\rightarrow A_v = -G_m R_{out}$

(c)



half ⟹ circuit

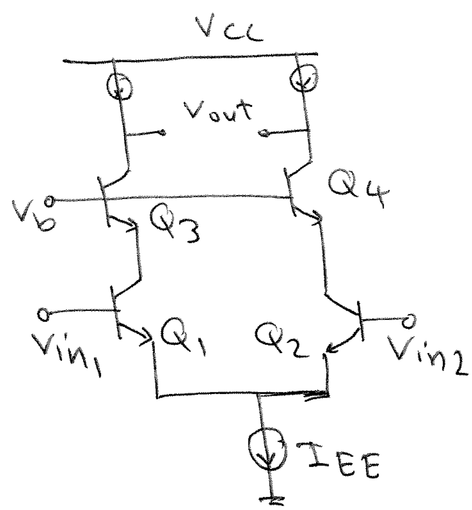


$$R_{out} = (g_{m3} r_{o3} R_s + r_{o3} + R_s) \parallel \left(g_{m1} r_{o1} \frac{R_{SS}}{2} + r_{o1} + \frac{R_{SS}}{2} \right)$$

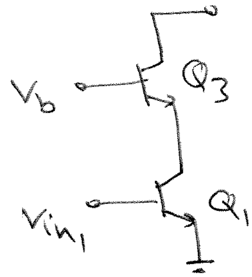
G_m for this circuit is equal to the one for part (b) so:

$$G_m = + \frac{2 g_{m1}}{R_{SS}} \frac{1}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \parallel r_{o1}}} \Rightarrow A_v = -G_m R_{out}$$

(54)



half ⟹ Circuit



$A_v = 4000$
$\beta = 100$

$$A_v = -g_{m_1}\left[g_{m_3}(r_{o_1}\|r_{\pi_3})r_{o_3} + r_{o_3} + r_{o_1}\|r_{\pi_3}\right]$$

$$g_{m_{1-4}} = \frac{I_{EE}}{2V_T} \qquad r_{o_{1-4}} = \frac{2V_A}{I_{EE}} \qquad r_{\pi_3} = \frac{2V_T\beta}{I_{EE}}$$

$$4000 = \frac{I_{EE}}{2V_T}\left[\frac{I_{EE}}{2V_T}\left(\frac{2V_A}{I_{EE}}\Big\|\frac{2V_T\beta}{I_{EE}}\right)\frac{2V_A}{I_{EE}} + \frac{2V_A}{I_{EE}} + \left(\frac{2V_A}{I_{EE}}\Big\|\frac{2V_T\beta}{I_{EE}}\right)\right]$$

$$\Rightarrow 4000 = \frac{1}{2V_T}\left[\frac{V_A}{V_T}(2V_A\|2V_T\beta) + 2V_A + (2V_A\|2V_T\beta)\right] \Rightarrow$$

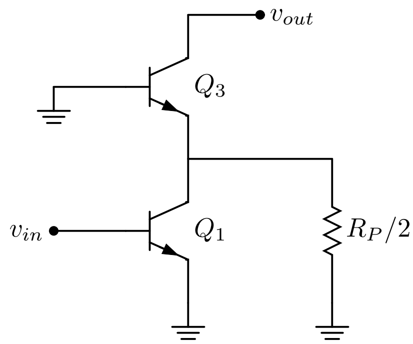
$$4000 = \frac{1}{V_T}\left[\frac{V_A}{V_T}(V_A\|\beta V_T) + V_A + (V_A\|\beta V_T)\right] \Rightarrow$$

$$4000 = \frac{1}{V_T}\left[\frac{\beta V_A^2}{\beta V_T + V_A} + V_A + \frac{\beta V_A V_T}{\beta V_T + V_A}\right] \Rightarrow$$

$$4000 = \frac{1}{0.026}\left[\frac{100 V_A^2}{2.6 + V_A} + V_A + \frac{2.6 V_A}{2.6 + V_A}\right] \Rightarrow$$

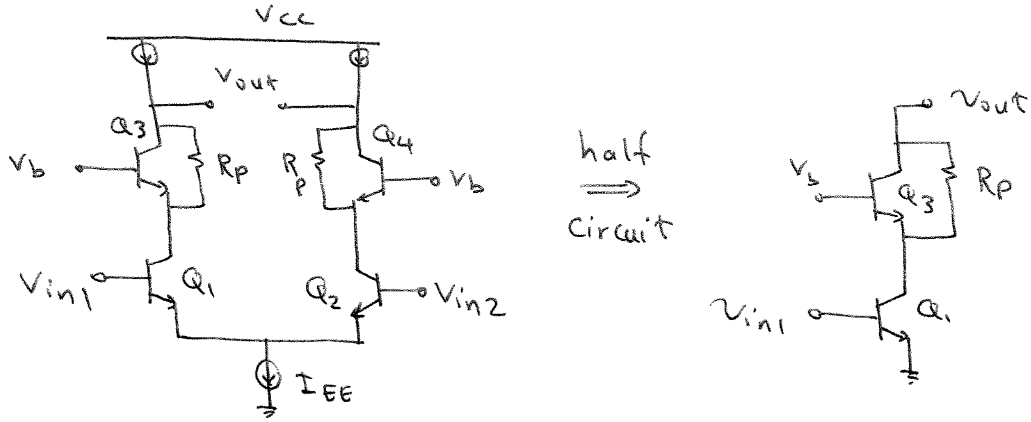
$$V_A = 2.197$$

10.55 Let's draw the half circuit.



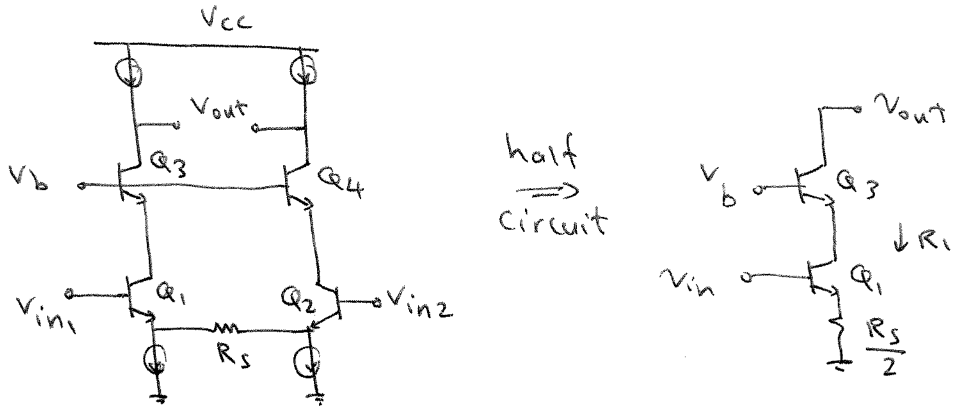
$$
\begin{aligned}
G_m &= g_{m1} \frac{\frac{R_P}{2} \parallel r_{o1} \parallel r_{\pi 3}}{\frac{R_P}{2} \parallel r_{o1} \parallel r_{\pi 3} + \frac{1}{g_{m3}}} \\
&= g_{m1} \frac{g_{m3}\left(\frac{R_P}{2} \parallel r_{o1} \parallel r_{\pi 3}\right)}{1 + g_{m3}\left(\frac{R_P}{2} \parallel r_{o1} \parallel r_{\pi 3}\right)} \\
R_{out} &= r_{o3} + (1 + g_{m3} r_{o3})\left(r_{\pi 3} \parallel \frac{R_P}{2} \parallel r_{o1}\right) \\
A_v &= \boxed{-g_{m1} \frac{g_{m3}\left(\frac{R_P}{2} \parallel r_{o1} \parallel r_{\pi 3}\right)}{1 + g_{m3}\left(\frac{R_P}{2} \parallel r_{o1} \parallel r_{\pi 3}\right)} \left\{ r_{o3} + (1 + g_{m3} r_{o3})\left(r_{\pi 3} \parallel \frac{R_P}{2} \parallel r_{o1}\right)\right\}}
\end{aligned}
$$

(56)



$$A_v = -g_{m_1}\left(g_{m_3}(r_{o3} \| R_P)(r_{o1} \| r_{\pi 3}) + (r_{o3} \| R_P) + (r_{o1} \| r_{\pi 3})\right)$$

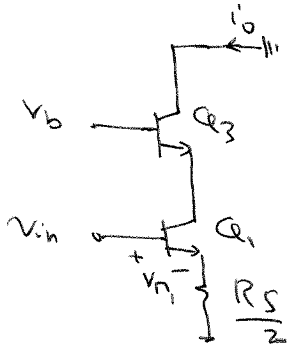
(57)



$$R_1 = g_{m_1} r_{o_1} \left(\frac{R_S}{2} \| r_{\pi_1} \right) + r_{o_1} + \frac{R_S}{2} \| r_{\pi_1}$$

$$R_{out} = g_{m_3} r_{o_3} (R_1 \| r_{\pi_3}) + r_{o_3} + (R_1 \| r_{\pi_3})$$

To calculate G_m:



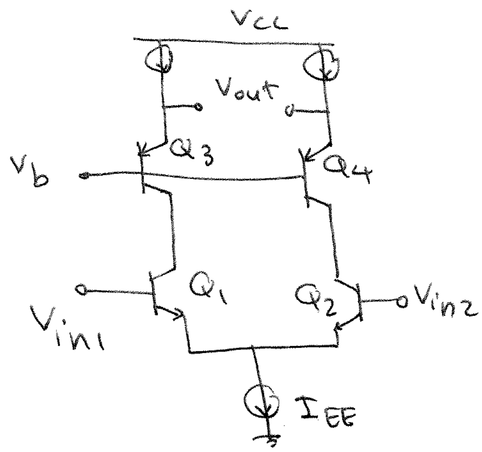
$$v_{\pi_1} \approx \frac{\frac{1}{g_{m_1}}}{\frac{1}{g_{m_1}} + \frac{R_S}{2}} v_{in}$$

$$= \frac{1}{1 + g_{m_1} \frac{R_S}{2}} v_{in}$$

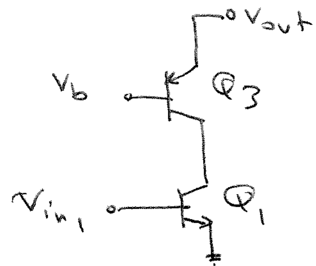
$$G_m = \frac{i_o}{v_{in}} = \frac{+g_{m_1} v_\pi}{v_{in}} = \frac{+g_{m_1}}{1 + g_{m_1} \frac{R_S}{2}}$$

$$A_v = -G_m R_{out}$$

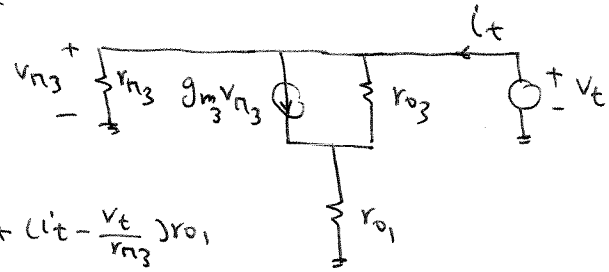
(58)



half $\Longrightarrow$ Circuit



To calculate R_{out}



$$v_{\pi 3} = v_t$$

$$v_t = \left(i_t - g_{m3} v_t - \frac{v_t}{r_{\pi 3}}\right) r_{o3} + \left(i_t - \frac{v_t}{r_{\pi 3}}\right) r_{o1}$$

$$\rightarrow \quad v_t\left(1 + g_{m3} r_{o3} + \frac{r_{o3}}{r_{\pi 3}} + \frac{r_{o1}}{r_{\pi 3}}\right) = i_t (r_{o1} + r_{o3})$$

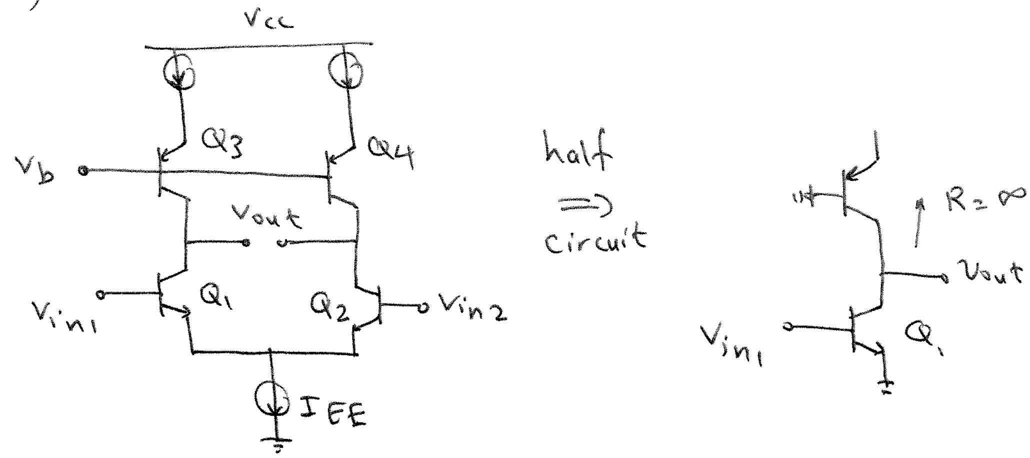
$$\rightarrow \quad \frac{v_t}{i_t} = \frac{r_{o1} + r_{o3}}{1 + g_{m3} r_{o3} + \frac{g_{m3} r_{o3}}{\beta_3} + \frac{g_{m3} r_{o1}}{\beta_3}} = R_{out} \rightarrow$$

$$R_{out} \approx \frac{r_{o1} + r_{o3}}{g_{m3} r_{o3}}$$

$$G_m = +g_{m1} \quad \Longrightarrow$$

$$A_v = -G_m R_{out} = -g_{m1} \frac{r_{o1} + r_{o3}}{g_{m3} r_{o3}}$$

(59)


Vcc
Q3
Q4
Vb
Vout
Q1
Q2
Vin1
Vin2
IEE
half
⟹
circuit
R = ∞
vout
Vin1
Q1

$A_v = -g_{m1} r_{o1}$

10.60 Assume $I_C = \frac{I_{EE}}{2}$ for all of the transistors (since $\beta \gg 1$).

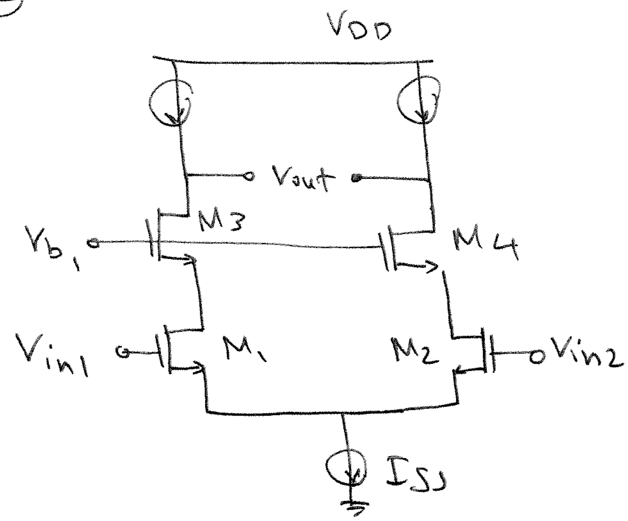
$$\begin{aligned}
A_v &= -g_{m1}\left\{\left[r_{o3} + \left(1 + g_{m3} r_{o3}\right)\left(r_{\pi 3} \parallel r_{o1}\right)\right] \parallel \left[r_{o5} + \left(1 + g_{m5} r_{o5}\right)\left(r_{\pi 5} \parallel r_{o7}\right)\right]\right\} \\
&= -\frac{1}{V_T} \frac{\left[V_{A,n} + \left(1 + \frac{V_{A,n}}{V_T}\right)\frac{\beta_n V_T V_{A,n}}{\beta_n V_T + V_{A,n}}\right]\left[V_{A,p} + \left(1 + \frac{V_{A,p}}{V_T}\right)\frac{\beta_p V_T V_{A,p}}{\beta_p V_T + V_{A,p}}\right]}{\left[V_{A,n} + \left(1 + \frac{V_{A,n}}{V_T}\right)\frac{\beta_n V_T V_{A,n}}{\beta_n V_T + V_{A,n}}\right] + \left[V_{A,p} + \left(1 + \frac{V_{A,p}}{V_T}\right)\frac{\beta_p V_T V_{A,p}}{\beta_p V_T + V_{A,p}}\right]} \\
&= -800 \\
V_{A,n} &= \boxed{2.16 \text{ V}} \\
V_{A,p} &= \boxed{1.08 \text{ V}}
\end{aligned}$$

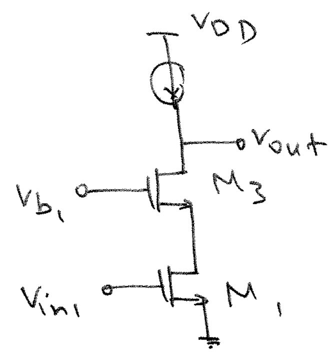
10.61

$$A_v = \boxed{-g_{m1}\left\{\left[r_{o3} + (1 + g_{m3}r_{o3})(r_{\pi3} \parallel r_{o1})\right] \parallel \left[r_{o5} + (1 + g_{m5}r_{o5})\left(r_{\pi5} \parallel \frac{1}{g_{m7}} \parallel r_{\pi7} \parallel r_{o7}\right)\right]\right\}}$$

This topology is not a telescopic cascode. The use of NPN transistors for Q_7 and Q_8 drops the output resistance of the structure from that of the typical telescopic cascode.

(62)





half circuit

$A_v = 300$, $W/L = \frac{20}{0.18}$, $\mu_n C_{ox} = 100\,\mu A/V^2$

$\lambda = 0.1\,V^{-1}$

$$A_v \approx -g_{m3} r_{o3} g_{m1} r_{o1}$$

$$g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \frac{I_{SS}}{2}} \qquad g_{m3} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_3 \frac{I_{SS}}{2}}$$

$$\rightarrow g_{m1} = g_{m3} = \sqrt{10^{-4} \, \frac{20}{0.18} \, I_{SS}}$$

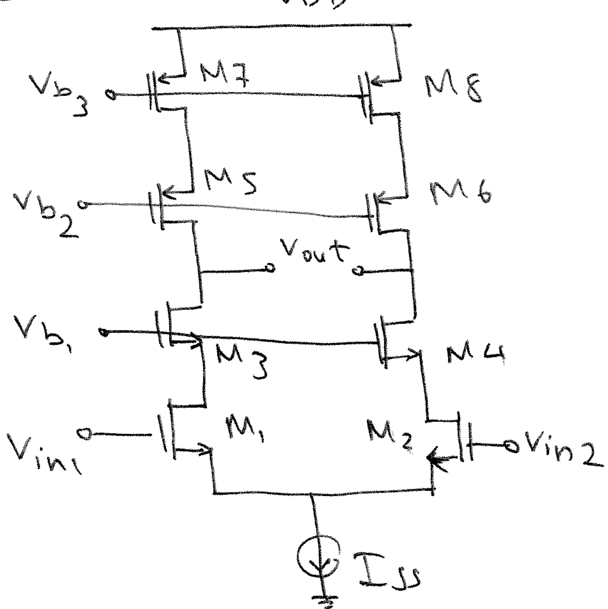
$$r_{o1} = \frac{1}{\lambda \frac{I_{SS}}{2}} , \quad r_{o3} = \frac{1}{\lambda \frac{I_{SS}}{2}} \rightarrow r_{o1} = r_{o3} = \frac{20}{I_{SS}}$$

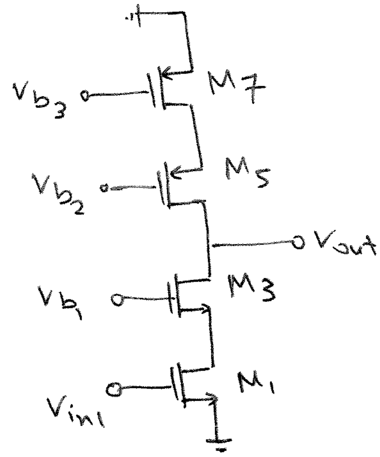
So:

$$300 = \left(10^{-4} \, \frac{20}{0.18} \, I_{SS}\right) \frac{\frac{400}{2}}{I_{SS}} \Rightarrow$$

$I_{SS} = 14.815$ mA

(63)





$A_v = 200$, $I_{SS} = 1mA$, $\mu_n C_{ox} = 100\,\mu A/V^2$

$\mu_p C_{ox} = 50\,\mu A/V^2$, $\lambda_n = 0.1\,V^{-1}$, $\lambda_p = 0.2\,V^{-1}$

$(\frac{W}{L})_1 = \cdots = (W/L)_8 = ?$

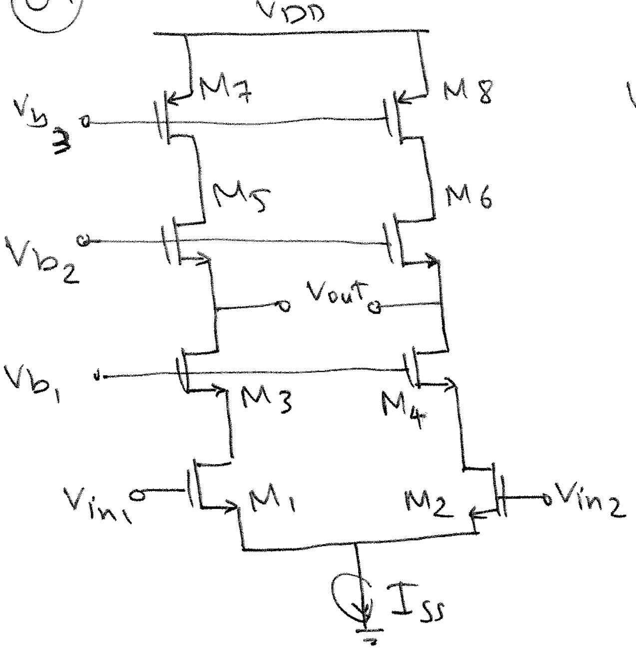
$$A_v \approx -g_{m1}\left[(g_{m3} r_{o3} r_{o1}) \| (g_{m5} r_{o5} r_{o7})\right] \Rightarrow$$

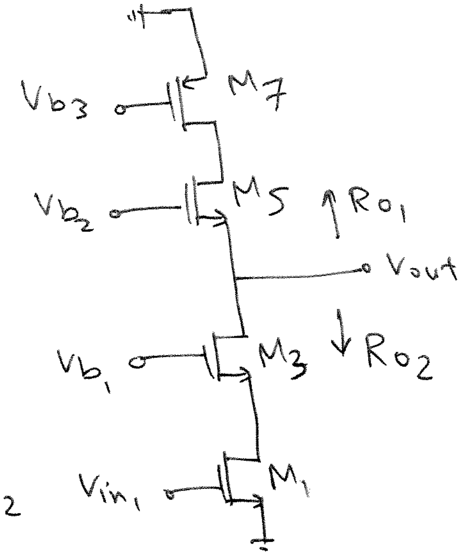
$$200 = \sqrt{\mu_n C_{ox} (\tfrac{W}{L})_1 I_{SS}} \left[\left(\sqrt{\mu_n C_{ox} (\tfrac{W}{L})_3 I_{SS}} \, (\tfrac{2}{\lambda_n I_{SS}})^2\right) \| \left(\sqrt{\mu_p C_{ox} (\tfrac{W}{L})_5 I_{SS}} \, (\tfrac{2}{\lambda_p I_{SS}})^2\right) \right]$$

$$\Rightarrow 200 = \sqrt{10^{-4} (\tfrac{W}{L})_1 10^{-3}} \left[\left(\sqrt{10^{-4} (\tfrac{W}{L}) 10^{-3}} \, (\tfrac{20}{10^{-3}})^2\right) \| \left(\sqrt{0.5\times10^{-4} (\tfrac{W}{L}) 10^{-3}} \, (\tfrac{10}{10^{-3}})^2\right) \right]$$

$$\Rightarrow \frac{W}{L} = 33.28$$

(64)

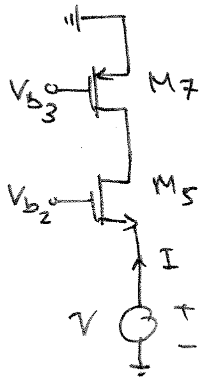




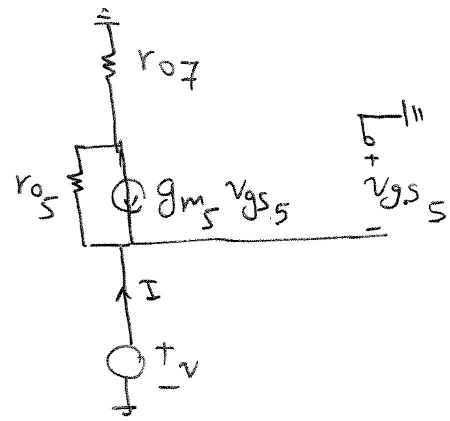
half circuit

$$R_{o2} = g_{m3} r_{o3} r_{o1} + r_{o1} + r_{o3}$$

To calculate R_{o1}, using the small signal model we have:



$\Rightarrow$



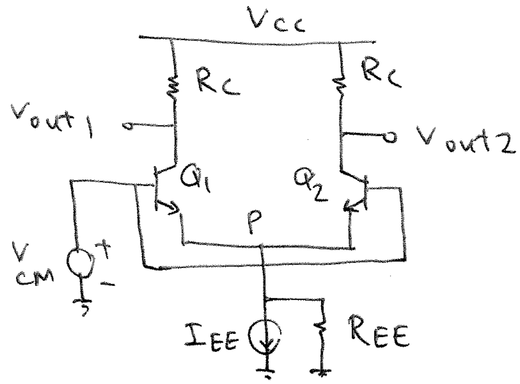
$$v_{gs_5} = -v \rightarrow g_{m_5} v_{gs_5} = -g_{m_5} v$$

From KVL:

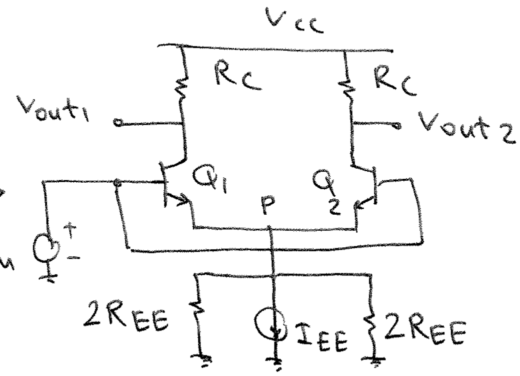
$$v = r_{o5}(I - g_{m_5} v) + r_{o7} I$$

$$\rightarrow \frac{v}{I} = R_{o1} = \frac{r_{o5} + r_{o7}}{1 + g_{m_5} r_{o5}} \Rightarrow A_v = -g_{m_1}(R_{o1} \| R_{o2})$$

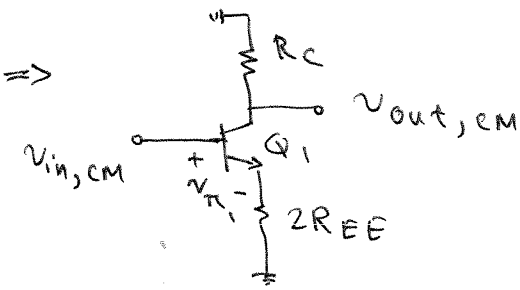
(65)



⟹



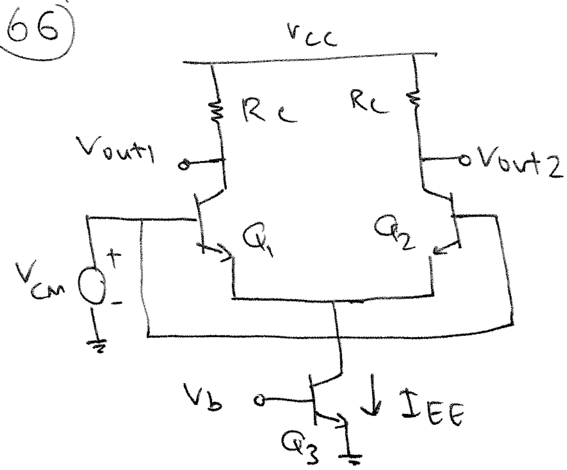
⟹



$$v_{out,cm} = -g_{m1} v_{\pi 1} R_C = -g_{m1} R_C \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + 2R_{EE}} v_{in,cm}$$

$$\Rightarrow \frac{v_{out,cM}}{v_{in,cM}} = -\frac{g_{m1} R_C}{1 + 2R_{EE} g_{m1}} = -\frac{\frac{R_C}{2}}{R_{EE} + \frac{1}{2g_{m1}}}$$

(66)

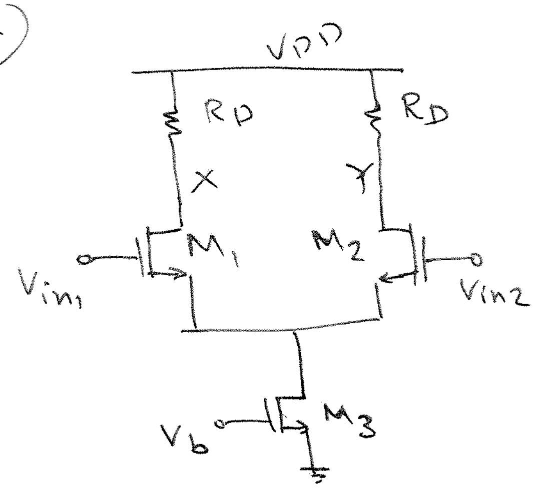


$$A_{CM} = \frac{\Delta V_{out,CM}}{\Delta V_{in,CM}} = -\frac{R_C/2}{\frac{1}{2g_m} + r_{O3}} \Rightarrow$$

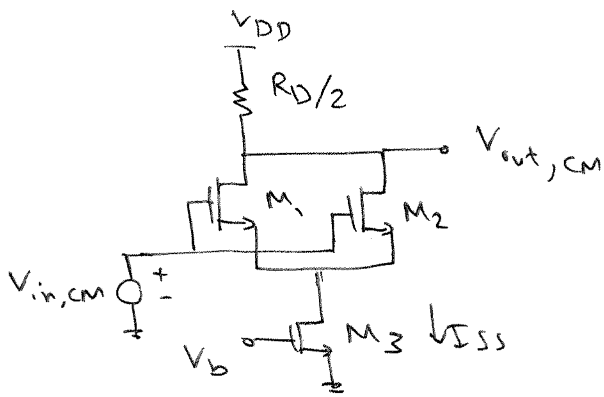
$$A_{CM} \leq 0.01 \Rightarrow \frac{R_C/2}{\frac{1}{2\frac{I_{EE}}{2V_T}} + \frac{V_A}{I_{EE}}} < 0.01 \Rightarrow$$

$$\frac{R_C I_{EE}}{2(V_A + V_T)} < 0.01 \Rightarrow R_C I_{EE} < 0.02(V_A + V_T)$$

(67)



The same value for the inputs common-mode leads to the following circuit:



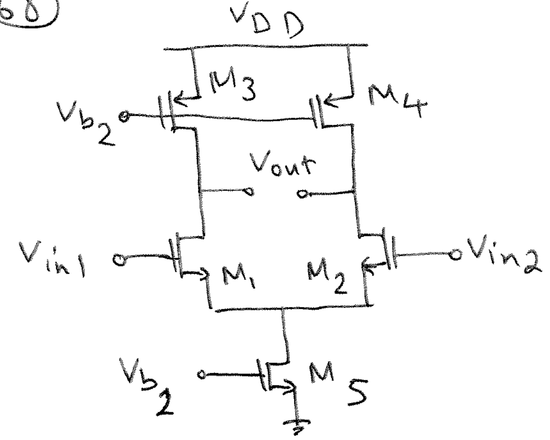
$$g_{m_1} = g_{m_2} = \frac{2\, I_{SS}/2}{(V_{GS}-V_{TH})_{eq}} \Rightarrow = \frac{I_{SS}}{(V_{GS}-V_{TH})_{eq}}$$

$$\frac{\Delta V_{out,CM}}{\Delta V_{in,CM}} = -\frac{R_D/2}{\frac{1}{2g_{m_1}} + r_{o3}}$$

$$= \frac{-R_D}{\frac{1}{g_{m_1}} + 2r_{o3}} = \frac{-R_D}{\frac{(V_{GS}-V_{TH})_{eq}}{I_{SS}} + \frac{2}{\lambda I_{SS}}} \Rightarrow$$

$$A_{CM} = -\frac{R_D\, I_{SS}}{\frac{2}{\lambda} + (V_{GS}-V_{TH})_{eq}}$$

(68)



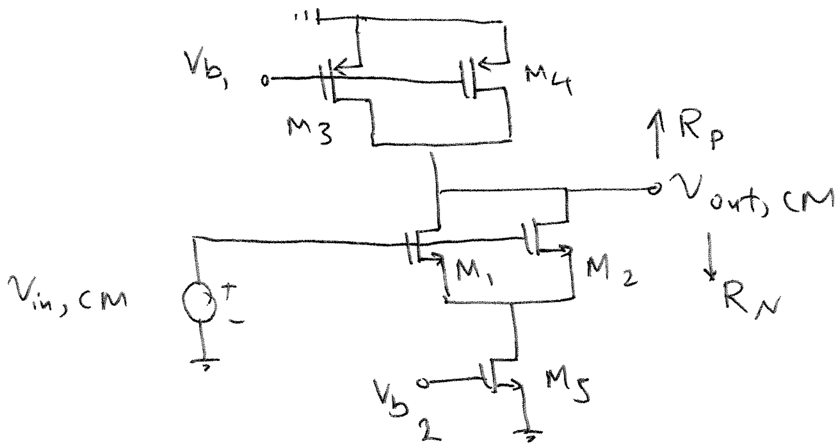
$\lambda > 0$, $g_m r_o \gg 1$

$r_{o3} = r_{o4}$

$r_{o1} = r_{o2}$

$g_{m1} = g_{m2}$

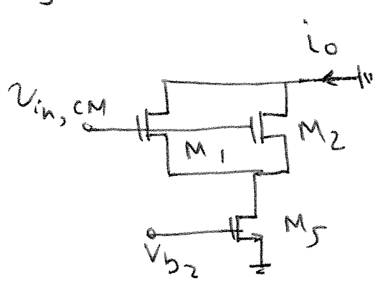
For the common mode input we have:



$$R_P = r_{o3} \parallel r_{o4} = \frac{r_{o3}}{2} = \frac{r_{o4}}{2}$$

$$R_N = r_{o5} + \frac{r_{o1}}{2} + 2 g_{m1} \frac{r_{o1}}{2} r_{o5} =$$

$$g_{m1} r_{o1} r_{o5} + r_{o5} + \frac{r_{o1}}{2}$$



$$\frac{i_o}{v_{in,CM}} = G_m = \frac{2 g_{m1} v_{gs1}}{v_{in,CM}} = 2 g_{m1} \frac{\frac{1}{2 g_{m1}}}{\frac{1}{2 g_{m1}} + r_{o5}}$$

$$\rightarrow G_m = \frac{2 g_{m1}}{1 + 2 g_{m1} r_{o5}} \approx \frac{1}{r_{o5}}$$

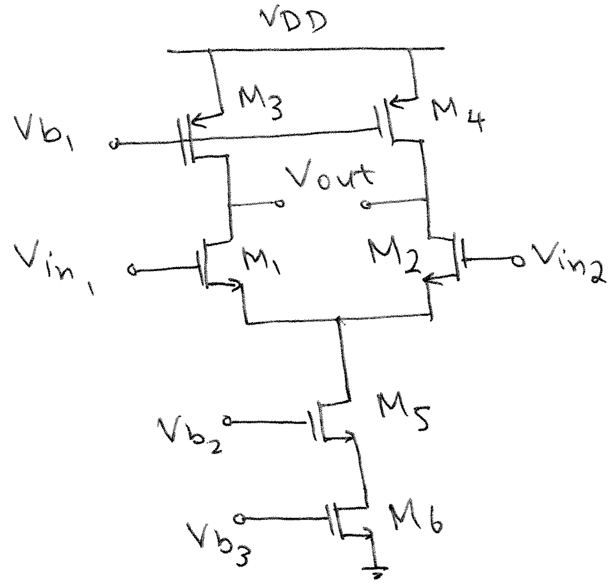
$$\Rightarrow A_{CM} = -G_m R_o$$

$$R_o = R_P \| R_N = \frac{r_{o4}}{2} \| \left(g_{m1} r_{o1} r_{o5} + r_{o5} + \frac{r_{o1}}{2} \right)$$

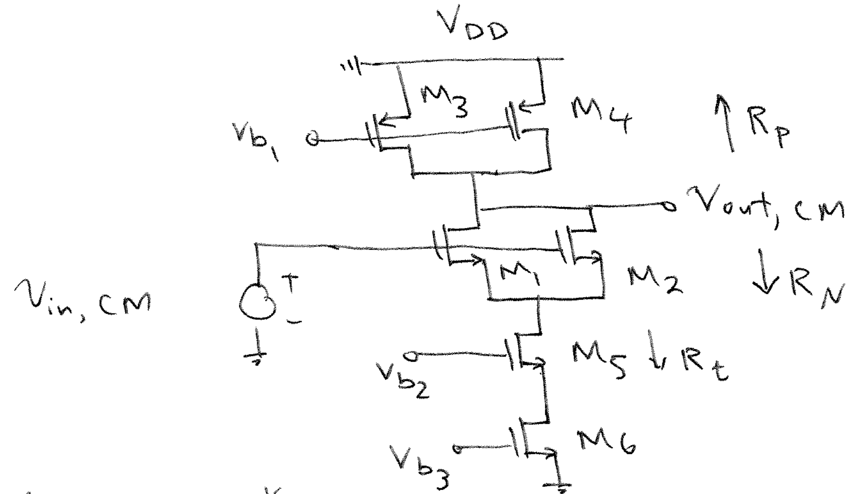
$$\approx \frac{r_{o4}}{2} \| g_{m1} r_{o1} r_{o5} \approx \frac{r_{o4}}{2}$$

$$\rightarrow A_{CM} = -\frac{1}{r_{o5}} \frac{r_{o4}}{2} = -\frac{r_{o4}}{2 r_{o5}}$$

(69) (a)



For the common mode input:



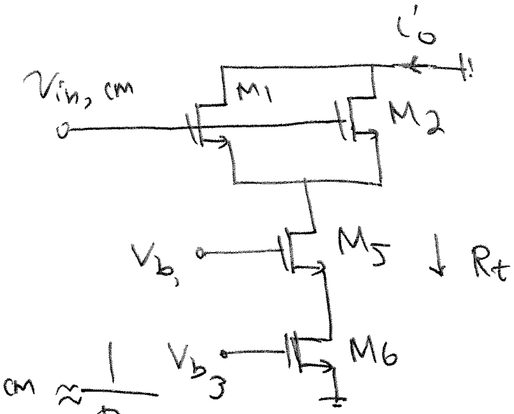
$R_P = r_{o3} \| r_{o4} = \frac{r_{o3}}{2}$

$R_N = \frac{r_{o1}}{2} + R_t + 2g_{m1} \frac{r_{o1}}{2} R_t \simeq g_{m1} r_{o1} R_t \approx$

$g_{m1} r_{o1} g_{m5} r_{o5} r_{o6}$

$R_{out} = R_P \| R_N = \frac{r_{o3}}{2} \| g_{m1} g_{m5} r_{o1} r_{o5} r_{o6} \approx \frac{r_{o3}}{2}$

To calculate G_m:



$$G_m = \frac{i_o}{v_{in,CM}} = \frac{2g_{m_1} v_{gs_1}}{v_{in,CM}}$$

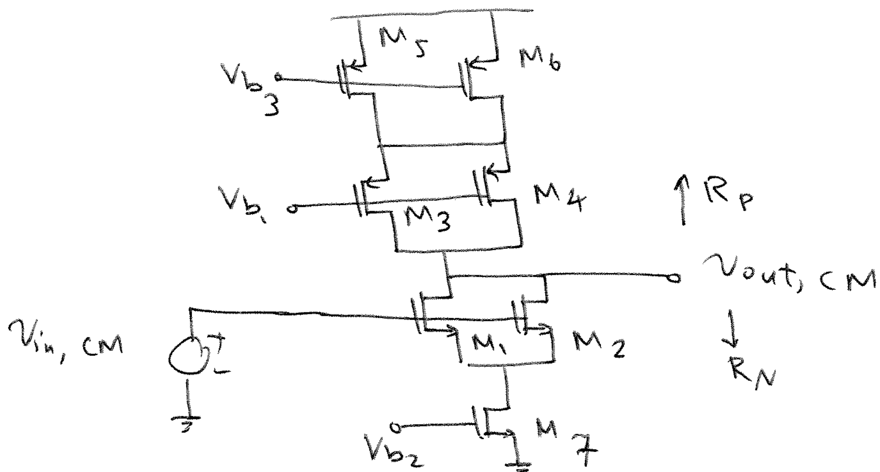
$$= \frac{2g_{m_1}}{v_{in,cm}} \frac{\frac{1}{2g_{m_1}}}{\frac{1}{2g_{m_1}} + R_t} v_{in,cm} \approx \frac{1}{R_t}$$

$$\rightarrow A_{CM} = -G_m R_{out} = -\frac{r_{o_3}}{2R_t} =$$

$$-\frac{r_{o_3}}{2 g_{m_5} r_{o_5} r_{o_6}}$$

(69) (b)

For the common mode input, the circuit is:

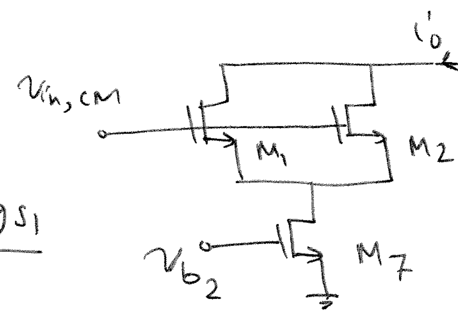


$$R_P = 2g_{m_3}\frac{r_{o3}}{2}\frac{r_{o5}}{2} + \frac{r_{o3}}{2} + \frac{r_{o5}}{2} \approx \frac{g_{m_3} r_{o3} r_{o5}}{2}$$

$$R_N = 2g_{m_1}\frac{r_{o1}}{2} r_{o7} + \frac{r_{o1}}{2} + r_{o7} \approx g_{m_1} r_{o1} r_{o7}$$

$$R_{out} = R_N \| R_P$$

To calculate G_m:



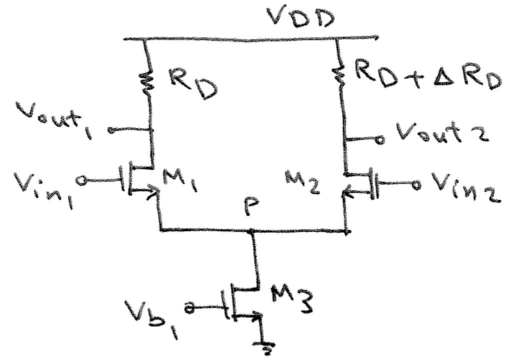
$$G_m = \frac{i_o'}{v_{in,CM}} = \frac{2g_{m_1} v_{gs_1}}{v_{in,CM}}$$

$$= \frac{2g_{m_1}}{v_{in,CM}} \frac{\frac{1}{2g_{m_1}}}{\frac{1}{2g_{m_1}} + r_{o7}} v_{in,CM} \approx \frac{1}{r_{o7}}$$

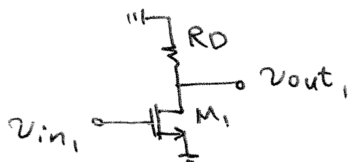
$$\rightarrow A_{CM} = -G_m R_{out}$$

(70)

(a)

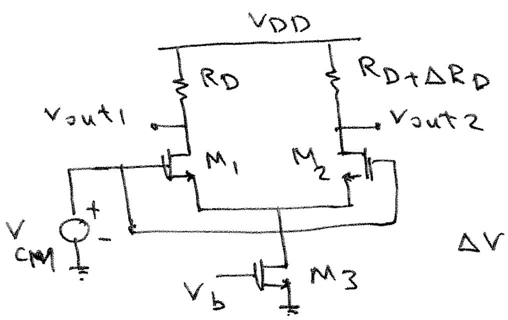


To calculate A_{DM}, using the half circuit:



$$A_{DM} = \frac{v_{out}}{v_{in1}} = -g_{m1} R_D$$

To calculate, A_{CM-DM} we have:



$$A_{CM-DM} = \frac{\Delta V_{out}}{\Delta V_{CM}}$$

$$\Delta V_{CM} = \Delta V_{GS} + 2\Delta I_D r_{o3} \Rightarrow$$

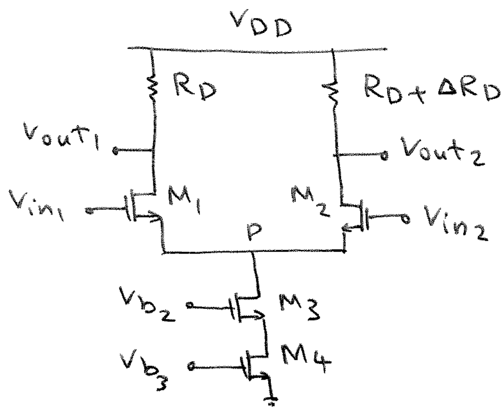
$$\Delta V_{CM} = \Delta I_D \left(\frac{1}{g_{m1}} + 2r_{o3}\right)$$

$$\Delta V_{out} = \Delta V_{out1} - \Delta V_{out2} = -\Delta R_D \, \Delta I_D \Rightarrow$$

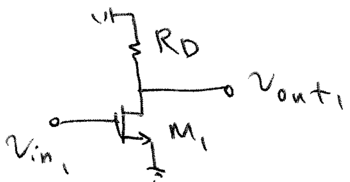
$$A_{CM-DM} = -\frac{\Delta R_D}{\frac{1}{g_{m1}} + 2r_{o3}} \Rightarrow$$

$$CMMR = \frac{A_{DM}}{A_{CM-DM}} = \frac{g_{m1} R_D}{\frac{\Delta R_D}{\frac{1}{g_{m1}} + 2r_{o3}}} = (1 + 2g_{m1} r_{o3}) \frac{R_D}{\Delta R_D}$$

(10) (b)



To calculate A_{DM}, using the half circuit, we have



$$A_{DM} = \frac{v_{out1}}{v_{in1}} = -g_{m1} R_D$$

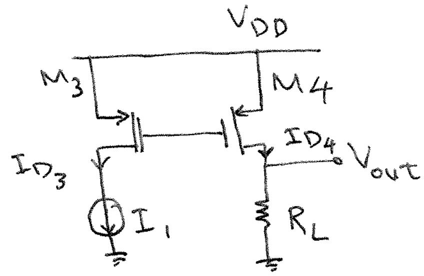
Similar to part (a) we have:

$$A_{CM-DM} = -\frac{\Delta R_D}{\frac{1}{g_{m1}} + 2\left[g_{m3} r_{o3} r_{o4} + r_{o3} + r_{o4}\right]}$$

$$\Rightarrow CMMR = \frac{A_{DM}}{A_{CM-DM}} = \left(1 + 2 g_{m1}\left[g_{m3} r_{o3} r_{o4} + r_{o3} + r_{o4}\right]\right) \frac{R_D}{\Delta R_D}$$

Notice that CMMR of part (b) is much higher than the one for part (a).

(71)



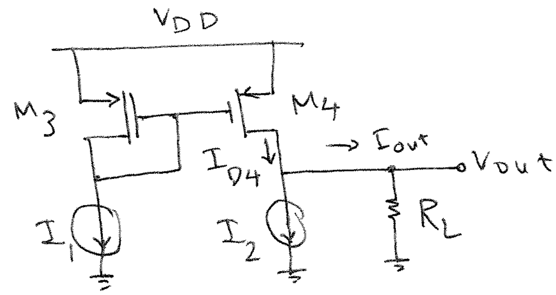
$$\left(\frac{W}{L}\right)_3 = N\left(\frac{W}{L}\right)_4$$

$$\left(\frac{W}{L}\right)_3 = N\left(\frac{W}{L}\right)_4 \Rightarrow I_{D3} = N I_{D4} \Rightarrow \underbrace{i_{d3} = N i_{d4}}_{\text{Small signal}}$$

$$\begin{cases} i_{d3} = i_1 \\ v_{out} = R_L i_{d4} = \dfrac{R_L}{N} i_{d3} = \dfrac{R_L}{N} i_1 \Rightarrow \end{cases}$$

$$\frac{v_{out}}{i_1} = \frac{R_L}{N}$$

(72)



(a) $\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4$

if $I_1 = I_2 = I_0$ $\Rightarrow$ $\begin{cases} V_{out} = I_{out} \times R_L = 0 \quad \text{because} \downarrow \\ I_{out} = I_{D_4} - I_2 = I_1 - I_2 = 0 \end{cases}$

if $I_1 = I_0 + \Delta I \Rightarrow I_{D4} = I_{D3} = I_1 = I_0 + \Delta I$

$I_2 = I_0 - \Delta I \Rightarrow I_{out} = I_{D4} - I_2 = 2\Delta I$

$V_{out} = I_{out} R_L = 2 R_L \Delta I$

(b) $\left(\frac{W}{L}\right)_3 = 2\left(\frac{W}{L}\right)_4$

$\Rightarrow I_{D3} = 2 I_{D4}$

if $I_1 = I_2 = I_0$ then $I_{D3} = I_1 = I_0 \Rightarrow I_{D4} = \frac{I_{D3}}{2}$

$\Rightarrow I_{D4} = \frac{I_0}{2} \Rightarrow I_{out} = I_{D4} - I_2 = -\frac{I_0}{2}$

$\Rightarrow V_{out} = R_L I_{out} = -\frac{R_L I_0}{2}$

if $I_1 = I_0 + \Delta I \Rightarrow I_{D4} = \frac{I_{D3}}{2} = \frac{I_1}{2} = \frac{I_0 + \Delta I}{2}$

$I_2 = I_0 - \Delta I \Rightarrow I_{out} = I_{D4} - I_2 = -\frac{I_0}{2} + \frac{3\Delta I}{2}$

$\Rightarrow V_{out} = R_L I_{out} = + R_L\left(\frac{-I_0}{2} + \frac{3\Delta I}{2}\right)$

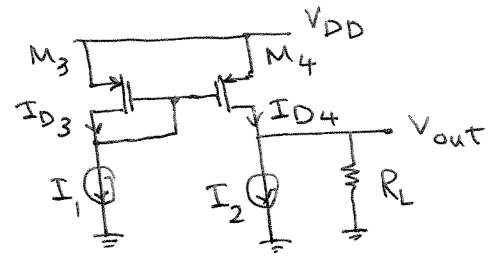
10.73 (a)

$$
\begin{aligned}
V_N &= V_{DD} - V_{SG3} \\
&= V_{DD} - \sqrt{\frac{I_{SS}}{\left(\frac{W}{L}\right)_3 \mu_p C_{ox}}} - |V_{THp}|
\end{aligned}
$$

(b) By symmetry, we know that I_D for M_3 and M_4 is the same, and we also know that their V_{SG} values are the same. Thus, their V_{SD} values must also be equal, meaning $\boxed{V_Y = V_N}$.

(c) If V_{DD} changes by ΔV, then both V_Y and V_N will change by ΔV.

(74)

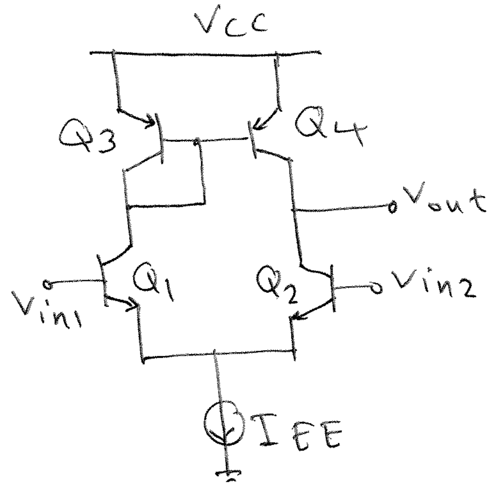


$$I_{D3} = I_{D4} = I_1$$

$$V_{out} = (I_{D4} - I_2) R_L = (I_1 - I_2) R_L$$

$\underset{\text{signal}}{\overset{\text{small}}{\Longrightarrow}}$ $\dfrac{v_{out}}{i_1} = R_L$, $\dfrac{v_{out}}{i_2} = -R_L$

(75)



$V_{A,n} = 5\,V$

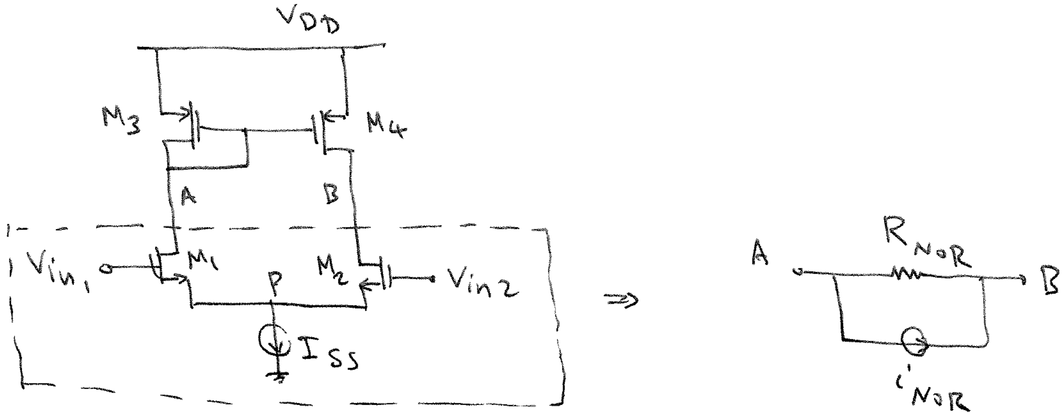
$A_v = 100$

$$\frac{v_{out}}{v_{in1} - v_{in2}} = g_{mN}(r_{oN} \| r_{op}) =$$

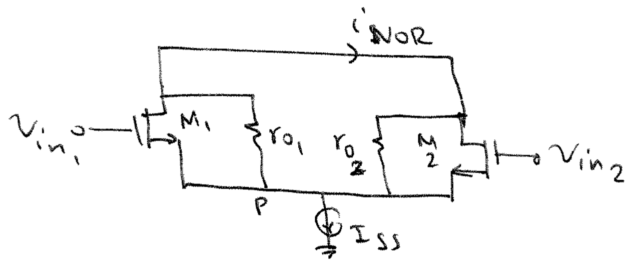
$$\frac{I_{EE}/2}{V_T}\left(\frac{V_{A,n}}{I_{EE}/2} \Big\| \frac{V_{A,p}}{I_{EE}/2}\right) = \frac{V_{A,n}\,V_{A,p}}{(V_{A,n}+V_{A,p})\,V_T}$$

$$\Rightarrow 100 = \frac{5\,V_{A,p}}{(5+V_{A,p})\,0.026} \Rightarrow V_{A,p} = 5.417\,V$$

(76)



To calculate i'_{NOR} we have:

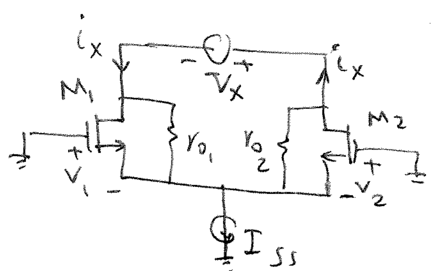


$$r_{o1}(i'_{NOR} + g_{m1} v_{in1}) + r_{o2}(i'_{NOR} - g_{m2} v_{in2}) = 0$$

$$\rightarrow 2 r_{oN} i'_{NOR} = -g_{m1} r_{oN} (v_{in1} - v_{in2}) \Rightarrow$$

$$i'_{NOR} = -\frac{g_{mN}}{2}(v_{in1} - v_{in2})$$

To calculate R_{NOR}:

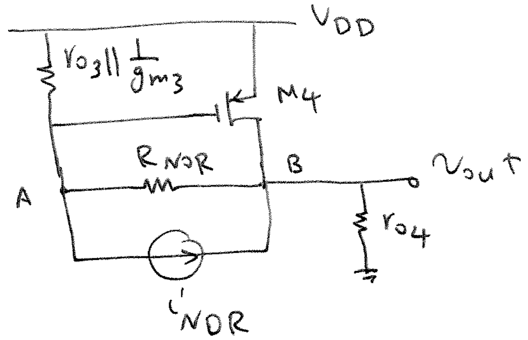


$$v_1 = v_2$$

$$(i_x - g_{m1} v_1) r_{o1} + (i_x + g_{m2} v_2) r_{o2} = v_x$$

$$\Rightarrow R_{NOR} = \frac{v_x}{i_x} = 2 r_{oN}$$

Therefore, utilizing the Norton model we have:



$$\begin{cases} \dfrac{v_A - v_B}{R_{NOR}} + \dfrac{v_A}{r_{o3} \| \frac{1}{g_{m3}}} + i_{NOR} = 0 \Rightarrow v_A = \dfrac{\frac{v_B}{R_{NOR}} - i_{NOR}}{\frac{1}{R_{NOR}} + \frac{1}{r_{o3} \| \frac{1}{g_{m3}}}} \\ \dfrac{v_B - v_A}{R_{NOR}} + \dfrac{v_B}{r_{o4}} - i_{NOR} + g_{m4} v_A = 0 \end{cases}, \quad v_B = v_{out}$$

$$\Rightarrow v_{out}\left(\frac{1}{R_{NOR}} + \frac{1}{r_{o4}}\right) + \left(g_{m4} - \frac{1}{R_{NOR}}\right) \frac{\frac{v_{out}}{R_{NOR}} - i_{NOR}}{\frac{1}{R_{NOR}} + \frac{1}{r_{o3} \| \frac{1}{g_{m3}}}} = i_{NOR}$$

$$\frac{1}{g_{m3}} \ll r_{o3}, \quad \frac{1}{g_{m3}} \ll R_{NOR}, \quad g_{m3} = g_{m4} = g_m, \quad r_{o3} = r_{o4} = r_{op}$$

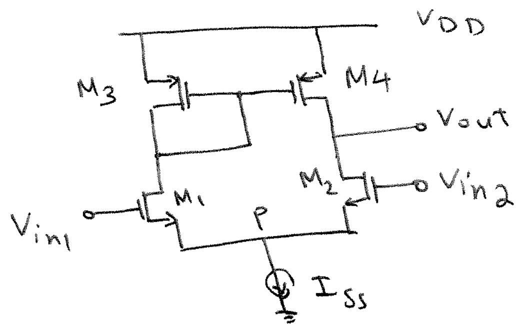
$$\Rightarrow v_{out}\left(\frac{1}{R_{NOR}} + \frac{1}{r_{op}}\right) + g_{m4} \frac{\frac{v_{out}}{R_{NOR}} - i_{NOR}}{g_{m3}} = i_{NOR}$$

$$\Rightarrow v_{out}\left(\frac{1}{R_{NOR}} + \frac{1}{r_{op}}\right) + \frac{v_{out}}{R_{NOR}} = 2 i_{NOR} \Rightarrow$$

$$\frac{2 v_{out}}{R_{NOR}} + \frac{v_{out}}{r_{op}} = 2 i_{NOR} \Rightarrow v_{out}\left(\frac{1}{r_{oN}} + \frac{1}{r_{op}}\right) = -g_{mN}(v_{in1} - v_{in2})$$

$$\Rightarrow \frac{v_{out}}{v_{in1} - v_{in2}} = -g_{mN} \left(r_{oN} \| r_{op}\right)$$

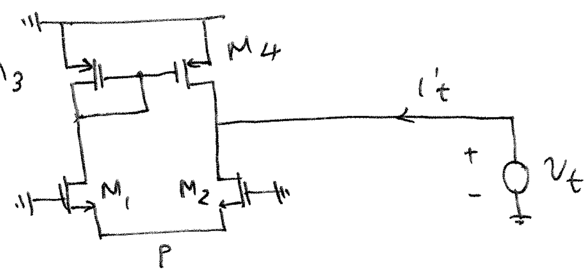
(77)



$g_m r_o >> 1$

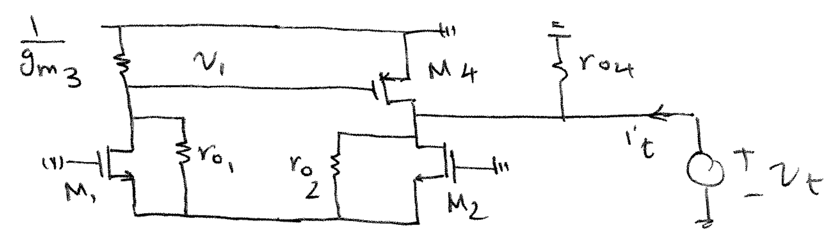
$g_{m1} = g_{m2}$

To calculate the output impedance we have the following circuit:



$R_{out} = \frac{v_t}{i'_t}$

⇓ neglecting r_{o3} ($r_{o3} >> \frac{1}{g_{m3}}$)

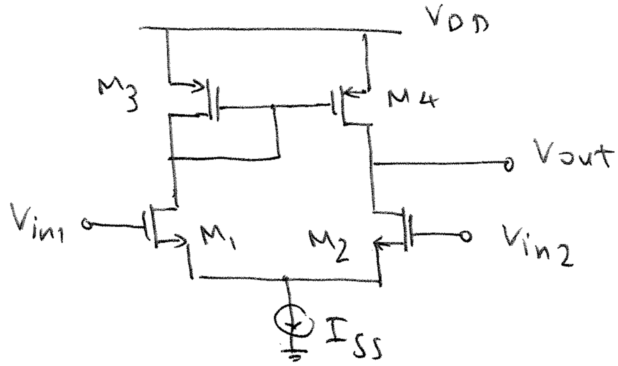


writing node equations of v_1 and v_p:

$$\begin{cases} g_{m3} v_1 + \frac{v_1 - v_p}{r_{o1}} - g_{m1} v_p = 0 \\ 2 g_{m1} v_p + \frac{v_p - v_1}{r_{o1}} + \frac{v_p - v_t}{r_{o2}} = 0 \end{cases} \xrightarrow{g_m r_o >> 1} \begin{cases} g_{m3} v_1 \approx g_{m1} v_p \\ 2 g_{m1} v_p \approx 0 \end{cases}$$

$$\Rightarrow v_p \approx v_1 = 0 \Rightarrow R_{out} = \frac{v_t}{i_t} = r_{o4} \| r_{o2} = r_{oN} \| r_{op}$$

(78)

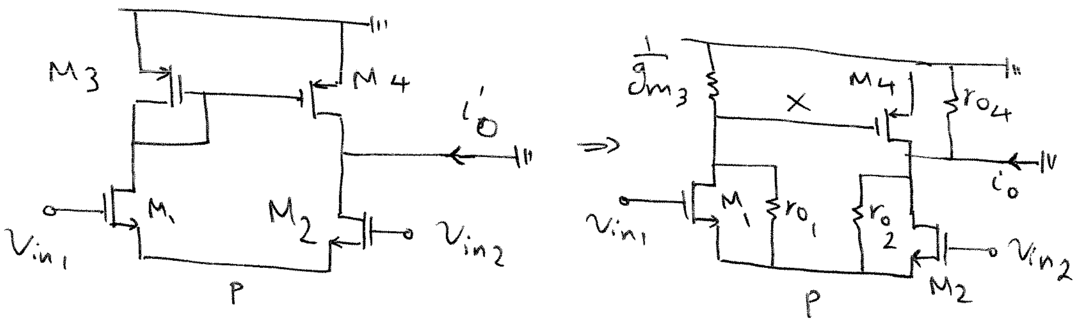


$g_m r_o >> 1$

$g_{m1} = g_{m2}$

$g_{m3} = g_{m4}$

To calculate G_m we have from small signal model:



writing node equations of nodes P and X we have:

$$\begin{cases} g_{m1}(v_P - v_{in1}) + g_{m2}(v_P - v_{in2}) + \dfrac{v_P - v_X}{r_{o1}} + \dfrac{v_P}{r_{o2}} = 0 \\ g_{m3} v_X + g_{m1}(v_{in1} - v_P) + \dfrac{v_X - v_P}{r_{o1}} = 0 \end{cases}$$

Since $g_m r_o >> 1$ we have

$$\begin{cases} g_{m1}(v_P - v_{in1}) + g_{m2}(v_P - v_{in2}) = 0 \Rightarrow v_P = \dfrac{v_{in1} + v_{in2}}{2} \\ v_X = -\dfrac{g_{m1}}{g_{m3}}(v_{in1} - v_P) = -\dfrac{g_{m1}}{g_{m3}}\left(\dfrac{v_{in1} - v_{in2}}{2}\right) \end{cases}$$

$$i'_o = -\frac{v_P}{r_{o2}} + g_{m2}(v_{in2} - v_P) - g_{m4}(-v_X)$$

$$\Rightarrow \quad i_o \approx -g_{m4}(-v_x) + g_{m2}\left(v_{in2} - \frac{v_{in1}+v_{in2}}{2}\right)$$

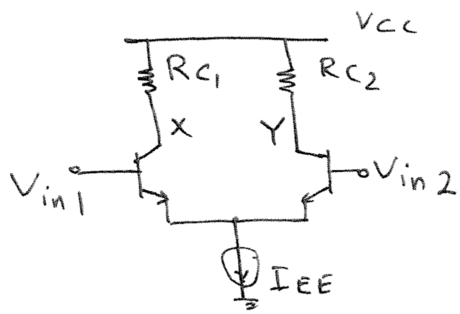
$$= -\left[g_{m4}\ \frac{g_{m1}}{g_{m3}}\left(\frac{v_{in1}-v_{in2}}{2}\right) + g_{m2}\left(\frac{v_{in1}-v_{in2}}{2}\right)\right]$$

$$= -g_{m1}\,(v_{in1} - v_{in2})$$

$$G_m = \frac{i_o}{v_{in1}-v_{in2}} = -g_{m1} = -g_{mN}$$

$$\rightarrow A_v = -G_m R_{out} = g_{mN}\,(r_{oN} \| r_{oP})$$

(79)



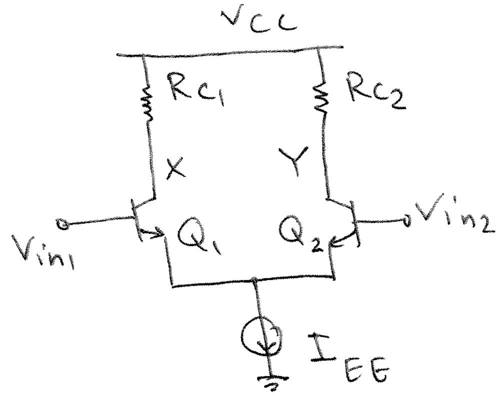
$A_v = 10$,
$P = 2mW$
$V_{cc} = 2.5V$
$V_A = \infty$

$$P = V_{cc} I_{EE} \Rightarrow 2\times10^{-3} = 2.5\, I_{EE} \Rightarrow I_{EE} = 0.8\,mA$$

$$A_v = \frac{v_{XY}}{v_{in_1} - v_{in_2}} = -g_m R_C = -\frac{I_{EE}/2}{V_T} R_C$$

$$\Rightarrow 10 = \frac{0.4\times10^{-3}}{0.026} R_C \Rightarrow R_C = 650\Omega$$

(80)



$V_{in,CM} = 1.2\ V$

$P = 3\ mW$

$V_{CC} = 2.5\ V$

$$P = I_{EE} V_{CC} \Rightarrow 3\times10^{-3} = 2.5\, I_{EE} \Rightarrow I_{EE} = 1.2\,mA$$

$$A_v = -g_{m1} R_C = -\frac{I_{EE}/2}{V_T} R_C = -\frac{R_C I_{EE}}{2V_T}$$

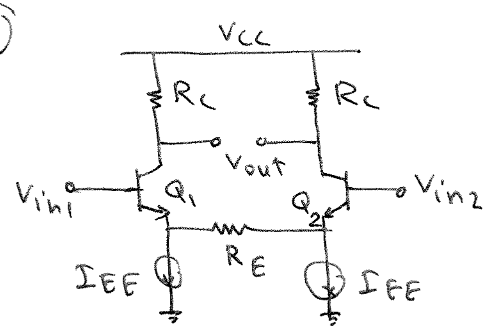
To maximize gain, $R_C I_{EE}$ and therefore R_C should be maximum. However, the upper bound of R_C value is limited by the voltage value of X. because:

$$V_{in,CM} \leq V_X \Rightarrow 1.2 \leq V_{CC} - R_C I_{EE}/2 \Rightarrow$$

$$R_C \leq 2\frac{V_{CC}-1.2}{I_{EE}} \Rightarrow R_C \leq 2\,\frac{2.5-1.2}{1.2\times10^{-3}} \Rightarrow$$

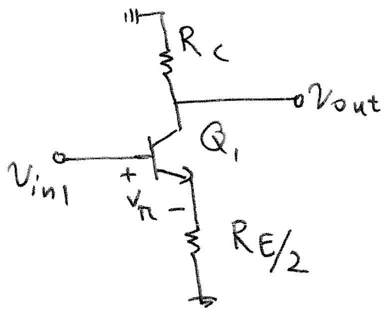
$$R_C \leq 2.167\,k\Omega \Rightarrow \quad R_C = 2.167\,k\Omega$$

(81)



$A_v = 5$
$P = 4mW$
$V_{CC} = 2.5V$
$V_A = \infty$

The half circuit is:



$$A_v = \frac{v_{out}}{v_{in1}} \simeq \frac{-g_{m1} v_\pi R_C}{v_{in1}} = -\frac{g_{m1} R_C}{v_{in1}} \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + \frac{R_E}{2}} v_{in1}$$

$$= -\frac{R_C}{\frac{R_E}{2} + \frac{1}{g_{m1}}}$$

$P = 4mW = 2 I_{EE} V_{CC} = 5 I_{EE} \Rightarrow I_{EE} = 0.8mA$

$g_m = \frac{I_{EE}}{V_T} = 0.03077$

$$A_v = 5 \Rightarrow \frac{R_C}{\frac{R_E}{2} + 32.5} = 5 \quad (1)$$

if I_{EE} increases by 10%, the gain will be:

$$A_v = \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{1.1}} \Rightarrow 5 < \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{1.1}} < 5 \times 1.02 \quad (2)$$

if I_{EE} decreases by 10% then:

$$5 \times 0.98 < \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{0.9}} < 5 \quad (3)$$

The worse case is:

$$\begin{cases} \dfrac{R_C}{\dfrac{R_E}{2} + \dfrac{32.5}{1.1}} = 5 \times 1.02 & (4) \\ \dfrac{R_C}{\dfrac{R_E}{2} + \dfrac{32.5}{0.9}} = 5 \times 0.98 & (5) \end{cases}$$

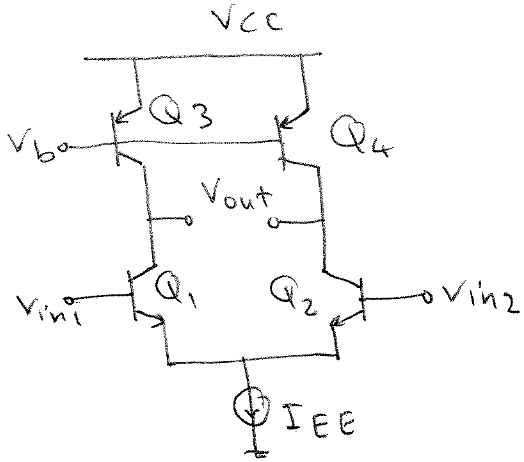
dividing (4) and (5) to (1) leads to:

$$\begin{cases} \dfrac{\dfrac{R_E}{2} + 32.5}{\dfrac{R_E}{2} + \dfrac{32.5}{1.1}} = 1.02 \Rightarrow R_E = 236.36\Omega \\ \dfrac{\dfrac{R_E}{2} + 32.5}{\dfrac{R_E}{2} + \dfrac{32.5}{0.9}} = 0.98 \Rightarrow R_E = 288.89\Omega \end{cases}$$

To ensure less than 2% gain variation for 10% current variation $R_E = 288.89\Omega$

From (1) $R_C = 5\left(\frac{R_E}{2} + 32.5\right) = 884.72\Omega$

(82)



$A_v = 100$

$P = 1mW$

$V_{A,n} = 6$

$V_{CC} = 2.5\ V$

$$P = 1mW = I_{EE} V_{CC} \Rightarrow I_{EE} = \frac{10^{-3}}{2.5} = 0.4mA$$

$$r_{oN} = \frac{V_{An}}{I_{EE}/2} = \frac{6}{0.2 \times 10^{-3}} = 30k\Omega, \quad g_{mN} = \frac{I_{EE}/2}{V_T} = \frac{0.2}{26}\ S$$

$$A_v = -g_{mN}(r_{oN} \| r_{op}) \Rightarrow$$

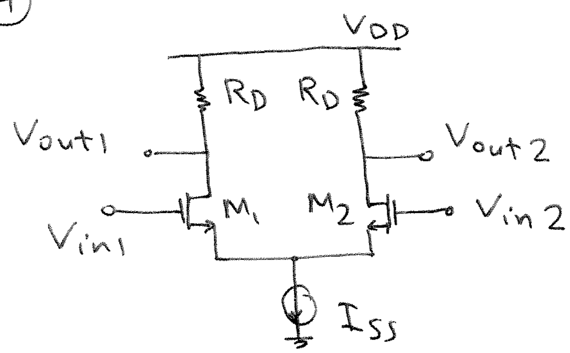
$$100 = \frac{0.2}{26}(30 \times 10^3 \| r_{op}) \Rightarrow r_{op} = 22.94k\Omega$$

$$\Rightarrow V_{A,p} = r_{op} \frac{I_{EE}}{2} = 4.588\ V$$

10.83

$$
\begin{aligned}
P &= V_{CC} I_{EE} = 1\ \text{mW} \\
I_{EE} &= 0.4\ \text{mA} \\
A_v &= -g_{m1} \left(r_{o1} \parallel r_{o3} \parallel R_1 \right) \\
&= -100 \\
R_1 = R_2 &= \boxed{59.1\ \text{k}\Omega}
\end{aligned}
$$

(84)



$\Delta V_{in,max} = 0.3\ V$

$P = 3\ mW$

$R_D = 500\ \Omega$

$\lambda = 0$, $\mu_n C_{ox} = 100\ \mu A/V^2$

$V_{DD} = 1.8\ V$

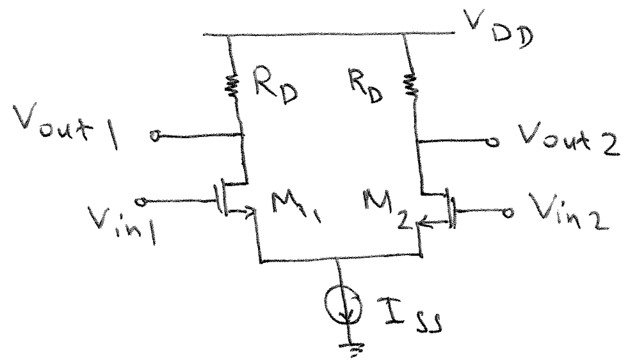
$P = V_{DD} I_{SS} \Rightarrow 3\times10^{-3} = 1.8\, I_{SS} \Rightarrow$

$I_{SS} = 1.67\ mA$

$$\Delta V_{in,max} = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow$$

$$0.3 = \sqrt{\frac{2\times1.67\times10^{-3}}{10^{-4}\times\frac{W}{L}}} \Rightarrow \frac{W}{L} = 370.37$$

(85)



$P = 2mW$

overdrive = 100 mV

$V_{CM} = 1V$

$\lambda = 0$, $\mu_n C_{ox} = 100 \mu A/V^2$

$V_{DD} = 1.8V$

$V_{TH,n} = 0.5$

$$P = I_{SS} V_{DD} \Rightarrow 2\times10^{-3} = 1.8\, I_{SS} \Rightarrow I_{SS} = 1.11\, mA$$

$$V_{GS_1} - V_{TH} = \sqrt{\frac{2I_{D_1}}{\mu_n C_{ox}\frac{W}{L}}} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \Rightarrow$$

$$0.1^2 = \frac{1.11\times10^{-3}}{10^{-4}\times\frac{W}{L}} \Rightarrow \frac{W}{L} = 1111.11$$

$$g_m = \sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS}} = \sqrt{10^{-4}\times1111.11\times1.11\times10^{-3}} = 0.011$$

To place the transistor at the edge of triode region:

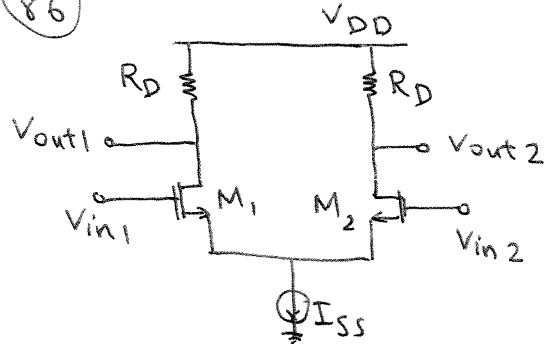
$$V_{in,CM} = V_{out1} + V_{TH,n}$$

$$1 = V_{DD} - R_D \frac{I_{SS}}{2} + 0.5 \Rightarrow$$

$$1 = 1.8 - R_D \frac{1.11\times10^{-3}}{2} + 0.5 \Rightarrow R_D = 2.34\, k\Omega$$

$$A_V = -g_m R_D = -25.74$$

(86)



$A_v = 5$

$P = 1mW$

$(V_{GS} - V_{TH})_{equil} = 150 mV$

$\lambda = 0, \mu_n C_{ox} = 100 \mu A/V^2$

$V_{DD} = 1.8 V$

$P = 1mW = V_{DD} I_{SS} = 1.8 I_{SS} \Rightarrow I_{SS} = 0.556 mA$

$$g_{m_1} = \frac{2 I_{D1}}{(V_{GS} - V_{TH})_{equil}} = \frac{I_{SS}}{(V_{GS} - V_{TH})_{equil}} = \frac{0.556 \times 10^{-3}}{0.15}$$

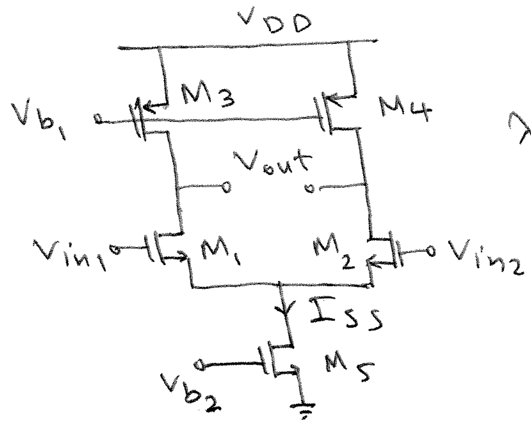
$= 3.704 \ mS$

$A_v = -g_{m_1} R_D \Rightarrow 5 = 3.704 \times 10^{-3} \times R_D \Rightarrow R_D = 1.35 k\Omega$

$$(V_{GS} - V_{TH})_{equil} = \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow$$

$$0.15 = \sqrt{\frac{0.556 \times 10^{-3}}{10^{-4} \times \frac{W}{L}}} \Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 246.91$$

(87)



$A_V = 40$

$(V_{GS} - V_{TH})_{equil} = ?$

$\lambda_n = 0.1\,V^{-1}$ $\lambda_p = 0.2\,V^{-1}$

$\mu_n C_{ox} = 100\,\mu A/V^2$

$\mu_p C_{ox} = 50\,\mu A/V^2$

$V_{DD} = 1.8$

$P = 2\,mW$

$$A_V = -g_{m_N}(r_{o_P} \| r_{o_N}) = -\frac{I_{SS}}{(V_{GS_1} - V_{TH})_{equil}}\left(\frac{1}{\frac{I_{SS}}{2}\lambda_n} \| \frac{1}{\frac{I_{SS}}{2}\lambda_p}\right)$$

$$= -\frac{2}{(V_{GS_1} - V_{TH})_{equil}}\left(\frac{1}{\lambda_n} \| \frac{1}{\lambda_p}\right) \Rightarrow$$

$$\frac{2}{(V_{GS_1} - V_{TH})_{equil}}(10 \| 5) = 40 \Rightarrow (V_{GS_1} - V_{TH})_{equil} = 166.67\,mV$$

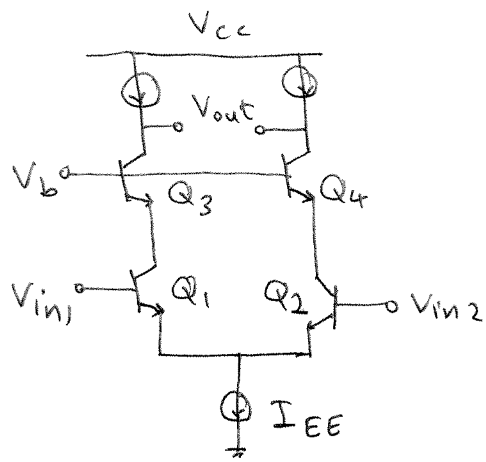
$$P = 2\times 10^{-3} = V_{DD} I_{SS} \Rightarrow I_{SS} = \frac{2\times 10^{-3}}{1.8} = 1.11\,mA$$

$$\left(\frac{W}{L}\right)_{1,2} = \frac{I_{SS}}{\mu_n C_{ox}(V_{GS_1} - V_{TH})^2_{equil}} = \frac{1.11\times 10^{-3}}{10^{-4}\times(0.16667)^2} = 400$$

$$\left(\frac{W}{L}\right)_{3,4} = \frac{I_{SS}}{\mu_p C_{ox}(V_{GS} - V_{TH})_{equil}} = \frac{1.11\times 10^{-3}}{0.5\times 10^{-4}\times(0.16667)^2} = 800$$

$$\left(\frac{W}{L}\right)_5 = \frac{2\,I_{SS}}{\mu_n C_{ox}(V_{GS} - V_{TH})_{equil}} = \frac{2\times 1.11\times 10^{-3}}{10^{-4}\times(0.16667)^2} = 800$$

(88)



$A_v = 4000$

$\beta = 100$

$V_{CC} = 2.5\,V$

$P = 1\,mW$

$$P = I_{EE} V_{CC} = 10^{-3} \Rightarrow I_{EE} = \frac{10^{-3}}{2.5} = 0.4\,mA$$

$$g_{m_{1-4}} = \frac{I_{EE}}{2V_T} = \frac{0.2}{26} = 7.692\,mS$$

$$r_{\pi_{1-4}} = \frac{\beta}{g_{m_1}} = 13\,k\Omega$$

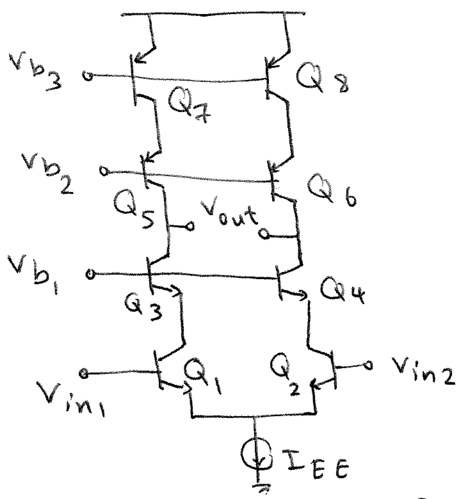
$$r_{o_{1-4}} = \frac{V_A}{\frac{I_{EE}}{2}} = 5\times10^3\,V_A$$

$$A_v = -g_{m_1}\left[g_{m_3}(r_{o_1} \| r_{\pi_3}) r_{o_3} + (r_{o_1} \| r_{\pi_3}) + r_{o_3}\right] \Rightarrow$$

$$4000 = \frac{0.2}{26}\left[\frac{0.2}{26}(5\times10^3 V_A \| 13\times10^3)\, 5\times10^3 V_A + (5\times10^3 V_A) \| 13\times10^3 + 5\times10^3 V_A\right]$$

$$\Rightarrow V_A = 2.197$$

(89)



$A_v = 2000$

$\beta_n = 100$

$\beta_p = 50$

$V_{A,n} = 5V$

$V_{CC} = 2.5V$

$P = 2mW$

$$P = I_{EE} V_{CC} = 2\times10^{-3} \Rightarrow I_{EE} = \frac{2\times10^{-3}}{2.5} = 0.8mA$$

$$g_{m_{1-8}} = \frac{I_{EE}}{2V_T} = \frac{0.4}{26} = 0.0154 \Rightarrow$$

$$r_{\pi_{1-4}} = \frac{\beta_n}{g_{m_1}} = 6.5K\Omega \qquad r_{\pi_{5-8}} = \frac{\beta_p}{g_{m_5}} = 3.25K\Omega$$

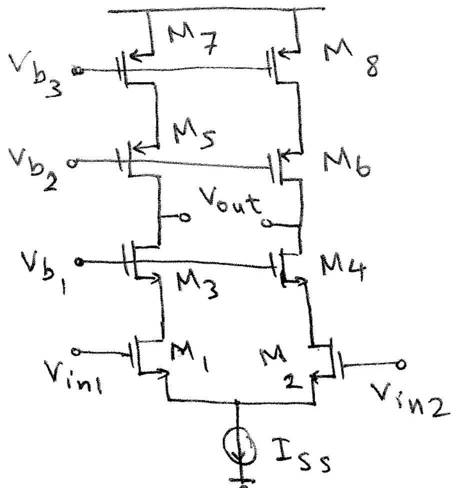
$$r_{o_{1-4}} = \frac{V_{A,n}}{I_{EE}/2} = 12.5K\Omega \qquad r_{o_{5-8}} = \frac{V_{A,p}}{I_{EE}/2}$$

$$A_v \approx -g_{m_1}\left[g_{m_3} r_{o_3}(r_{o_1} \| r_{\pi_3})\right] \| \left[g_{m_5} r_{o_5}(r_{o_7} \| r_{\pi_5})\right]$$

$$\Rightarrow \frac{0.4}{26}\left[\frac{0.4}{26}\times 12.5\times10^{+3}(12.5\times10^{3} \| 6.5\times10^{3})\right] \| \left[\frac{0.4}{26}\,\frac{V_{A,p}}{I_{EE}/2}\left(\frac{V_{A,p}}{\frac{I_{EE}}{2}} \| 3250\right)\right] = 2000$$

$$\Rightarrow V_{A,p} = 2.027 \ V$$

(90)



$A_V = 600$

$P = 4\,mW$

$(V_{GS} - V_{TH})_{NMOS} = 100\,mV$

$(V_{GS} - V_{TH})_{PMOS} = 150\,mV$

$\mu_n C_{ox} = 100\ \mu A/V^2$

$\mu_p C_{ox} = 50\ \mu A/V^2$

$\lambda_n = 0.1\ V^{-1}$

$$A_V \simeq -g_{m1}\left[(g_{m3} r_{o3} r_{o1}) \| (g_{m5} r_{o5} r_{o7})\right] = -600$$

$$P = 4\,mW = I_{SS} V_{DD} \Rightarrow I_{SS} = \frac{4\times10^{-3}}{1.8} = 2.22\,mA$$

$$g_{m1-4} = \frac{2I_{D1}}{(V_{GS}-V_{TH})_{NMOS}} = \frac{I_{SS}}{(V_{GS}-V_{TH})_{NMOS}} = \frac{2.22\times10^{-3}}{0.1} = 22.22\,mS$$

$$g_{m5-8} = \frac{2I_{D5}}{(V_{GS}-V_{TH})_{PMOS}} = \frac{I_{SS}}{(V_{GS}-V_{TH})_{PMOS}} = \frac{2.22\times10^{-3}}{0.15} = 14.815\,mS$$

$$r_{o1-4} = \frac{1}{\lambda_n \frac{I_{SS}}{2}} = \frac{1}{0.1\times\frac{2.22}{2}\times10^{-3}} = 9\,K\Omega$$

$$r_{o5-8} = \frac{1}{\lambda_p \frac{I_{SS}}{2}} = \frac{1}{\lambda_p\times\frac{2.22}{2}\times10^{-3}} = \frac{0.9\times10^{3}}{\lambda_p}$$

$\Rightarrow$ in A_V equation

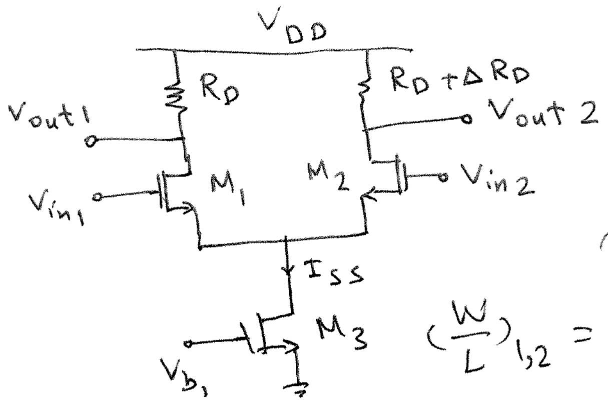
$$22.22\times10^{-3}\left[(22.22\times10^{-3}\times81\times10^{6}) \| (14.815\times10^{-3}\times\frac{0.81\times10^{6}}{\lambda_p^2})\right] = 600 \Rightarrow$$

$\lambda_p = 0.66\ V^{-1}$

$$\left(\frac{W}{L}\right)_{NMOS} = I_{SS}/(\mu_n C_{ox}(V_{GS}-V_{TH})^2_{NMOS}) = 2222.2$$

$$\left(\frac{W}{L}\right)_{PMOS} = I_{SS}/(\mu_p C_{ox}(V_{GS}-V_{TH})^2_{PMOS}) = 1975.31$$

(91)



CMMR = 60dB

$P = 2mW$

$A_{DM} = 5$

$\lambda(M_3) = ?$

$\left(\frac{W}{L}\right)_{1,2} = \frac{10}{0.18}$, $\mu_n C_{ox} = 100\,\mu A/V^2$

$V_{DD} = 1.8$, $\frac{\Delta R_D}{R_D} = 2\%$

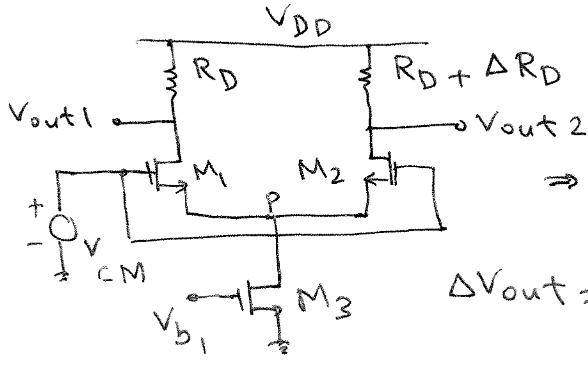
$P = 2mW = I_{SS} V_{DD} \Rightarrow I_{SS} = \frac{2\times10^{-3}}{1.8} = 1.11\,mA$

$A_{DM} = -g_{m1} R_D$

$g_{m1} = \sqrt{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{SS}} = \sqrt{10^{-4} \times \frac{10}{0.18} \times 1.11\times10^{-3}} = 2.4845\,mS$

$\Rightarrow R_D = \frac{|A_{DM}|}{g_{m1}} = \frac{5}{2.4845\times10^{-3}} = 2.012\,k\Omega$

To calculate $A_{CM,DM}$ we have:



$\Delta V_{CM} = \Delta V_{GS} + 2\Delta I_D r_{o3}$

$\Rightarrow \Delta V_{CM} = \Delta I_D \left(\frac{1}{g_{m1}} + r_{o3}\right)$

$\Delta V_{out} = \Delta V_{out1} - \Delta V_{out2} \cong -\Delta R_D I_D$

$$\Rightarrow A_{CM,DM} = \frac{\Delta V_{out}}{\Delta V_{CM}} = -\frac{\Delta R_D / 2}{\frac{1}{2g_{m1}} + r_{o3}}$$

$$\Rightarrow CMMR = \frac{A_{DM}}{A_{CM,DM}} = (1 + 2g_{m1} r_{o3})\frac{R_D}{\Delta R_D}, \quad r_{o3} = \frac{1}{\lambda_3 I_{SS}}$$

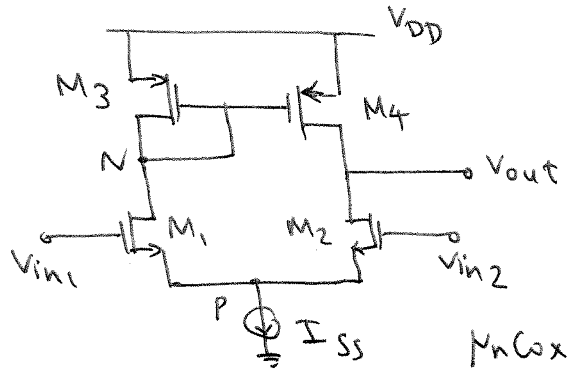
$$\Rightarrow \quad CMMR = 60dB = 10^3 = \left(1 + 2 \times 2.4845 \times 10^{-3} \frac{1}{\lambda_3 \times 1.11 \times 10^{-3}}\right) 50$$

$$\Rightarrow \lambda_3 = 0.2354$$

10.92

$$\begin{aligned} P &= V_{CC} I_{EE} = 3 \text{ mW} \\ I_{EE} &= 1.2 \text{ mA} \\ A_v &= g_{m,n} \left(r_{o,n} \parallel r_{o,p} \right) \\ &= 200 \\ V_{A,n} &= \boxed{15.6 \text{ V}} \\ V_{A,p} &= \boxed{7.8 \text{ V}} \end{aligned}$$

(93)



$A_v = 20$

$P = 1mW$

$V_{DD} = 1.8\ V$

$V_{in,CM} = 1\ V$

$\mu_n C_{ox} = 2\mu_p C_{ox} = 100 \mu A/V^2$

$V_{TH,n} = 0.5V,\ V_{TH,p} = -0.4\ V$

$\lambda_n = \frac{\lambda_p}{2} = 0.1\ V^{-1}$

$$P = V_{DD} I_{SS} \Rightarrow I_{SS} = \frac{10^{-3}}{1.8} = 0.556\ mA$$

$$A_v = + g_{m_N} (r_{o_N} \| r_{o_P}) = 20$$

$$r_{o_N} = \frac{1}{\lambda_n \frac{I_{SS}}{2}} = \frac{1}{0.1\ \frac{0.556 \times 10^{-3}}{2}} = 36\ k\Omega$$

$$r_{o_P} = \frac{1}{\lambda_p \frac{I_{SS}}{2}} = 18\ k\Omega$$

$$g_{m_N} (36^k \| 18^k) = 20 \Rightarrow g_{m_N} = 1.667 m\ S$$

$$\Rightarrow g_{m_N} = \frac{2 I_{D_N}}{(V_{GS} - V_{TH})_{NMOS}} = \frac{I_{SS}}{(V_{GS} - V_{TH})_{NMOS}} \Rightarrow$$

$$(V_{GS} - V_{TH})_{NMOS} = 0.333\ V$$

$$(V_{GS} - V_{TH})_{NMOS} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} (\frac{W}{L})_{NMOS}}} \Rightarrow (\frac{W}{L})_{1,2} = 50$$

$$V_N = V_{in,CM} - V_{TH,n} = 1 - 0.5 = 0.5\ V$$

$$\rightarrow \quad |V_{GS_3}| - |V_{TH,p}| = 1.3 - 0.4 = 0.9\ V$$

$$\rightarrow \quad 0.9 = \sqrt{\frac{I_{SS}}{\mu_p C_{ox} \left(\frac{W}{L}\right)_{PMOS}}} \quad \Rightarrow$$

$$\left(\frac{W}{L}\right)_{3,4} = 13.717$$

11.1

$$\frac{V_{out}}{V_{in}}(j\omega) = -g_m\left(R_D \parallel \frac{1}{j\omega C_L}\right)$$

$$= -\frac{g_m R_D}{1 + j\omega C_L R_D}$$

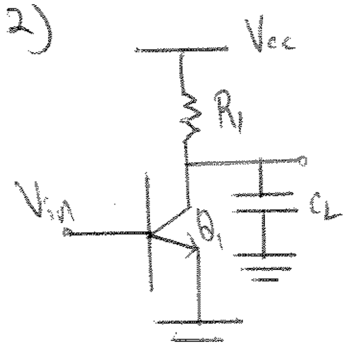
$$\left|\frac{V_{out}}{V_{in}}(j\omega)\right| = \frac{g_m R_D}{\sqrt{1 + (\omega C_L R_D)^2}}$$

$$\frac{g_m R_D}{\sqrt{1 + (\omega_{-1\text{ dB}} C_L R_D)^2}} = 0.9 g_m R_D$$

$$\omega_{-1\text{ dB}} = 4.84 \times 10^8 \text{ rad/s}$$

$$f_{-1\text{ dB}} = \frac{\omega_{-1\text{ dB}}}{2\pi} = \boxed{77.1\text{ MHz}}$$

2)



-3dB bandwidth = 1 GHz

$C_L = 2pF$

Power = 2mW

Low freq gain ?

Power = $2.5V\,I_c$, $I_c = 0.8\,mA$

Dominant Pole at the output = $\frac{1}{R_1 C_L} = 2\pi(1GHz)$

$R_1 = 79.58$ Ohm.

Low freq gain : $-g_m R_1 = \frac{-I_c R_1}{V_T} = \frac{(79.58)(0.8)}{26}$

$A_v\big|_{low\ freq} = -2.45$

11.3 (a)

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right) C_L}}$$

(b)

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(\frac{r_{\pi 2}+R_B}{1+\beta}\right) C_L}} \approx \frac{1}{\left(\frac{1}{g_{m2}} + \frac{R_B}{1+\beta}\right) C_L}$$

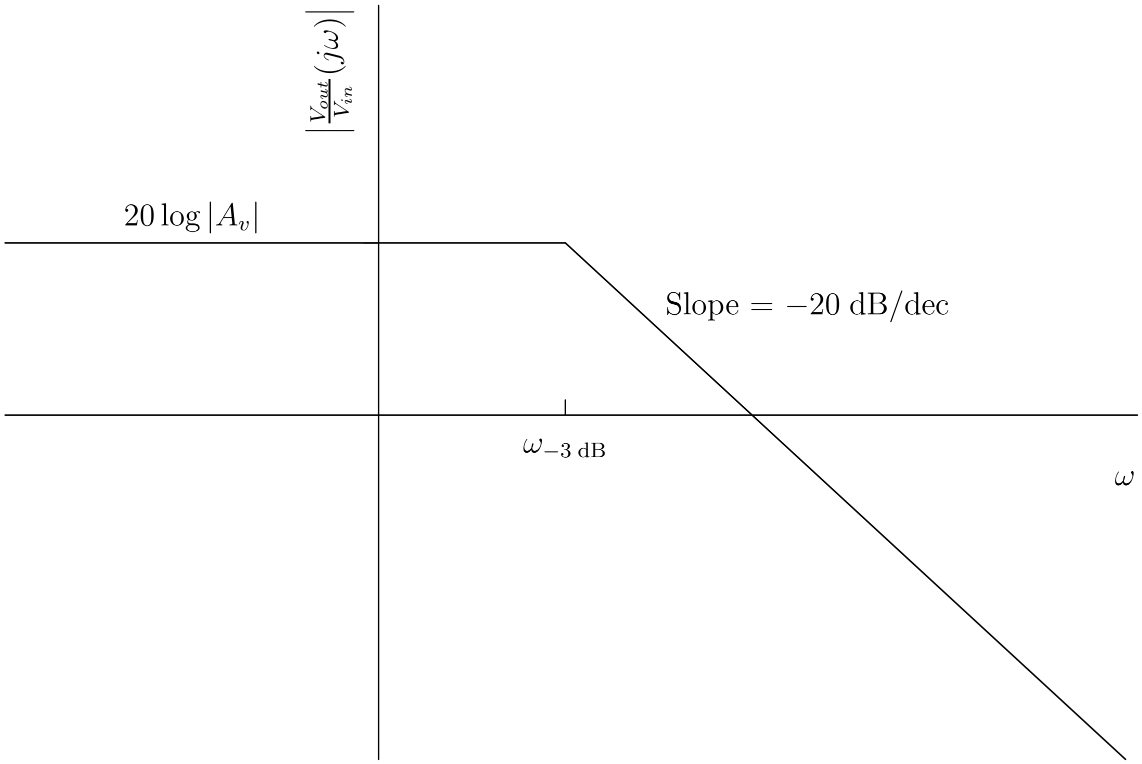
(c)

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(r_{o1} \parallel r_{o2}\right) C_L}}$$

(d)

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}\right) C_L}}$$

11.4 Since all of these circuits are have one pole, all of the Bode plots will look qualitatively identical, with some DC gain at low frequencies that rolls off at 20 dB/dec after hitting the pole at $\omega_{-3\text{ dB}}$. This is shown in the following plot:



For each circuit, we'll derive $|A_v|$ and $\omega_{-3\text{ dB}}$, from which the Bode plot can be constructed as in the figure.

(a)

$$|A_v| = \boxed{g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)C_L}}$$

(b)

$$|A_v| = \boxed{g_{m1}\left(\frac{r_{\pi 2}+R_B}{1+\beta}\right)} \approx g_{m1}\left(\frac{1}{g_{m2}} + \frac{R_B}{1+\beta}\right)$$

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(\frac{r_{\pi 2}+R_B}{1+\beta}\right)C_L}} \approx \frac{1}{\left(\frac{1}{g_{m2}} + \frac{R_B}{1+\beta}\right)C_L}$$

(c)

$$|A_v| = \boxed{g_{m1}\left(r_{o1} \parallel r_{o2}\right)}$$

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(r_{o1} \parallel r_{o2}\right)C_L}}$$

(d)

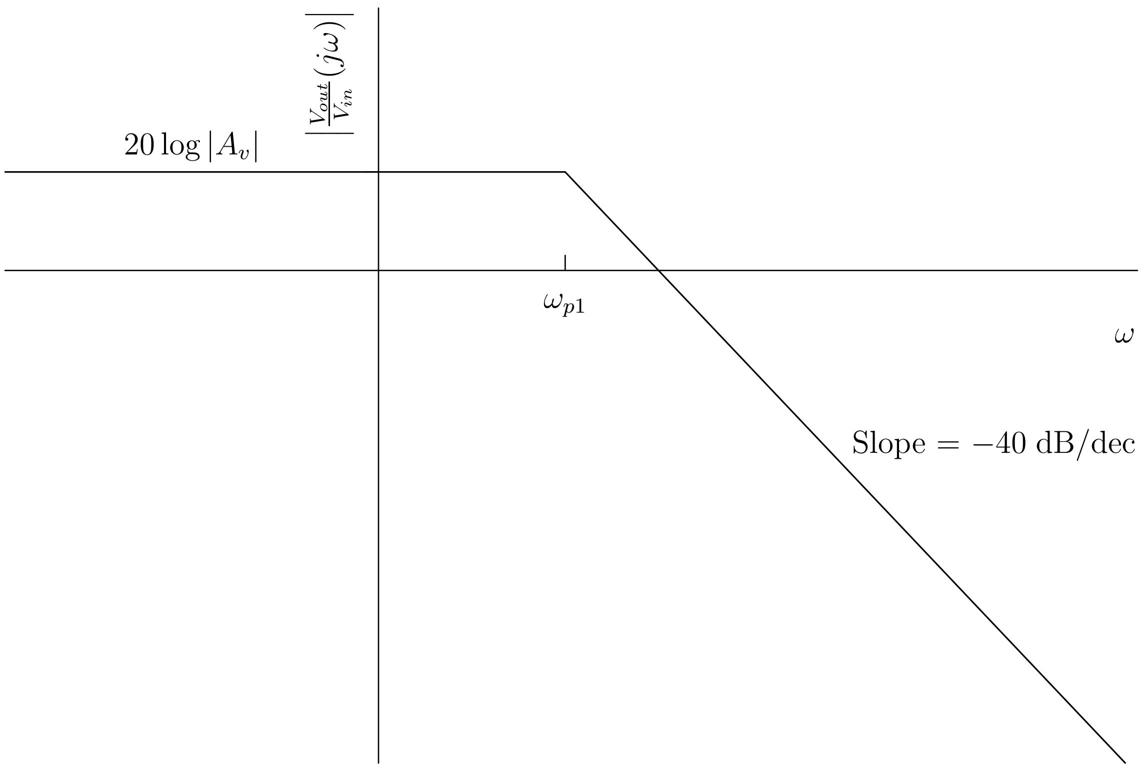
$$|A_v| = \boxed{g_{m1}\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}\right)}$$

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}\right) C_L}}$$

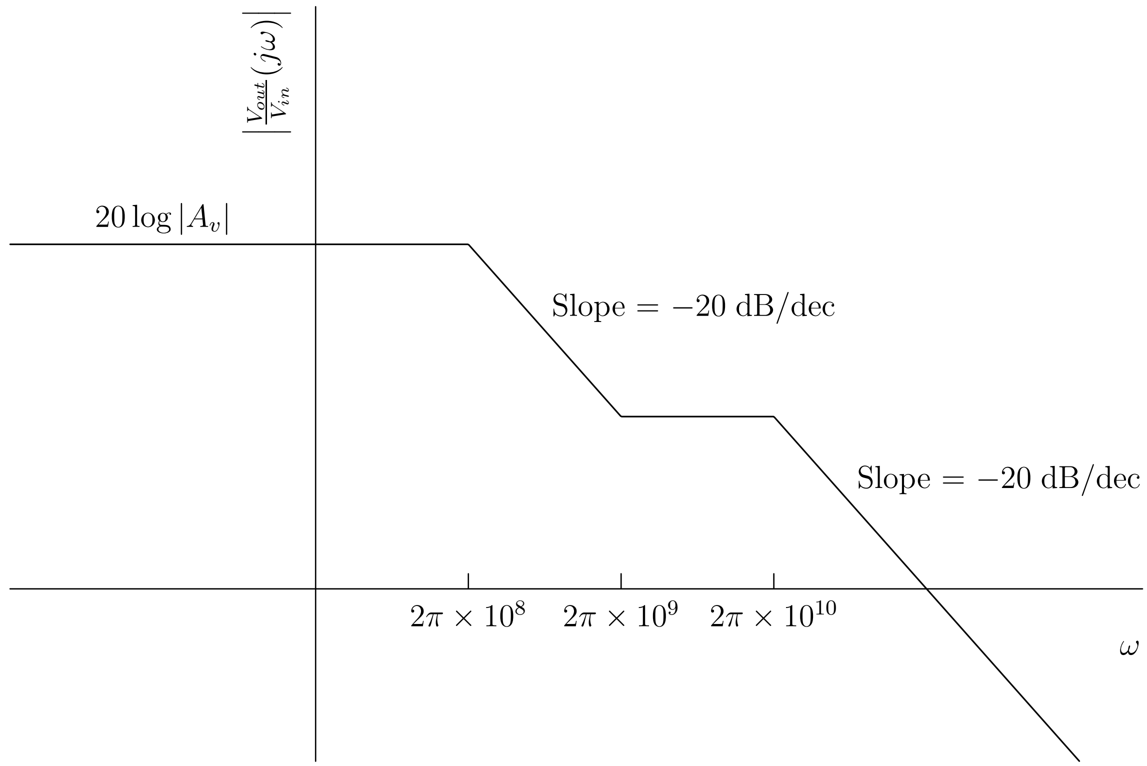
11.5 Assuming the transfer function is of the form

$$\frac{V_{out}}{V_{in}}(j\omega) = \frac{A_v}{\left(1 + j\frac{\omega}{\omega_{p1}}\right)^2}$$

we get the following Bode plot:

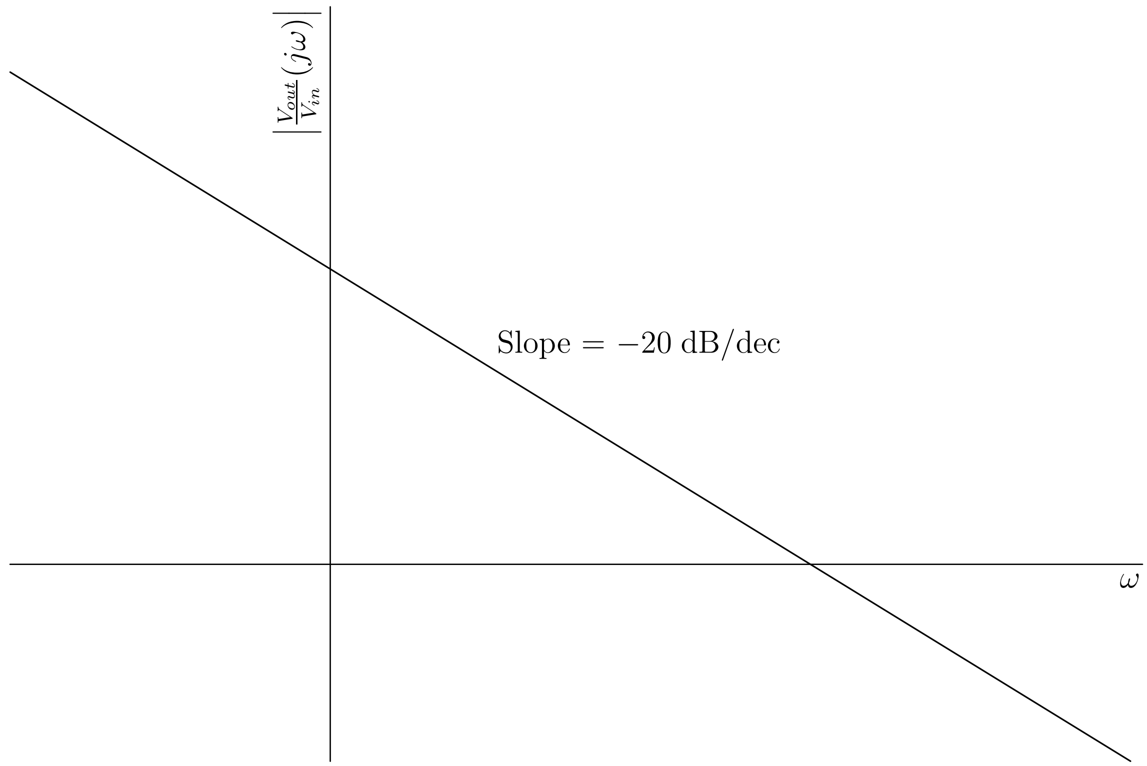


11.6

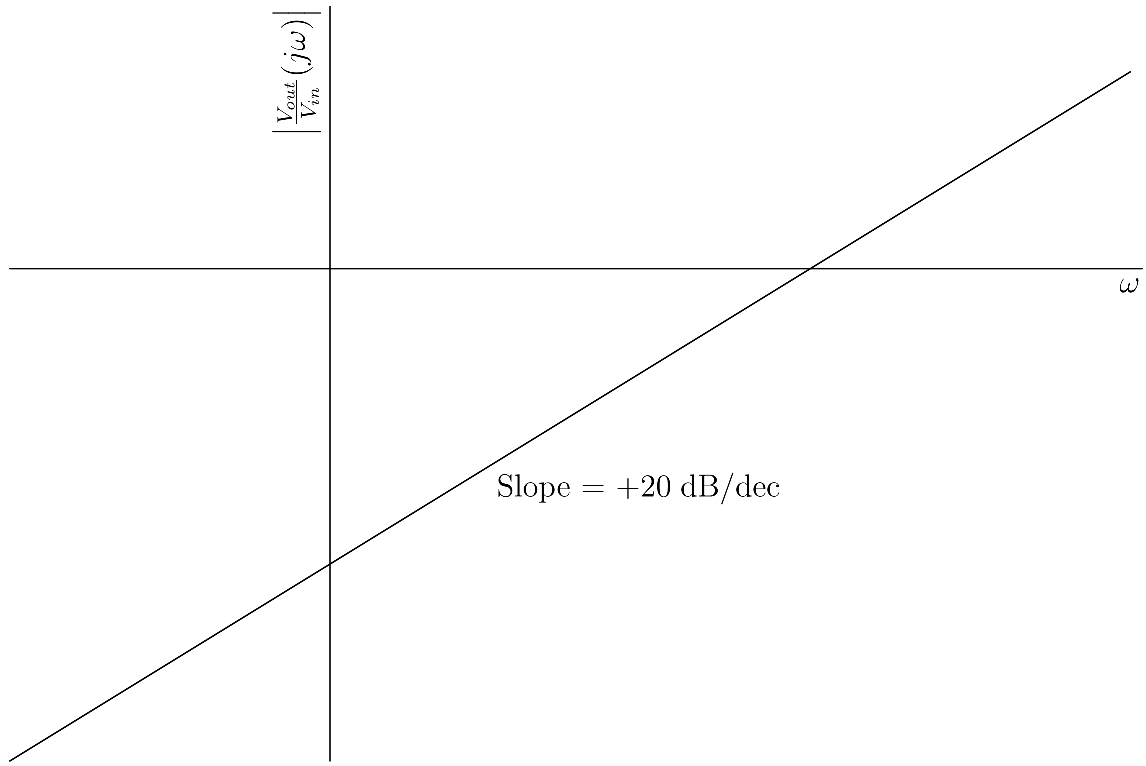


$\left|\frac{V_{out}}{V_{in}}(j\omega)\right|$
$20\log|A_v|$
Slope = −20 dB/dec
Slope = −20 dB/dec
$2\pi \times 10^8$
$2\pi \times 10^9$
$2\pi \times 10^{10}$
ω

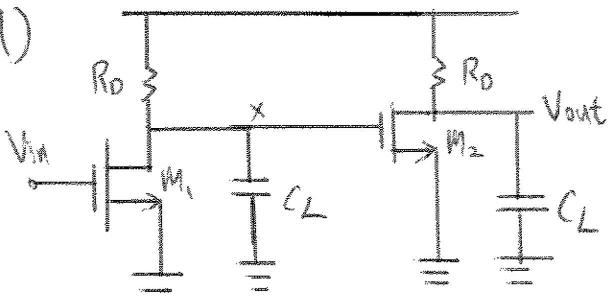
11.7 The gain at arbitrarily low frequencies approaches infinity.



11.8 The gain at arbitrarily high frequencies approaches infinity.



9)



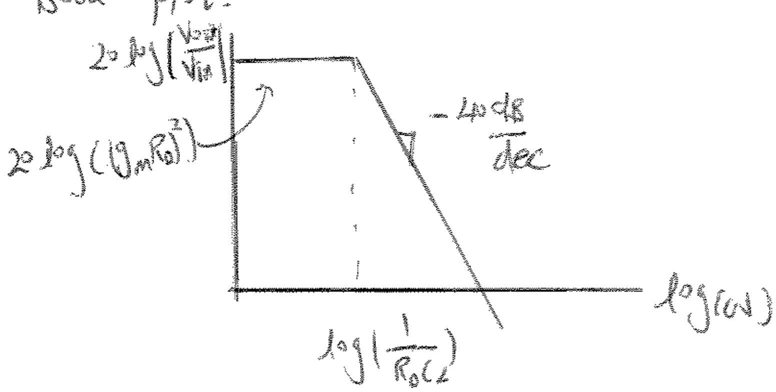
$\lambda = 0$, & neglect other caps.

DC gain: $\frac{V_x}{V_{in}} = -g_m R_D$, $\frac{V_{out}}{V_x} = -g_m R_D$

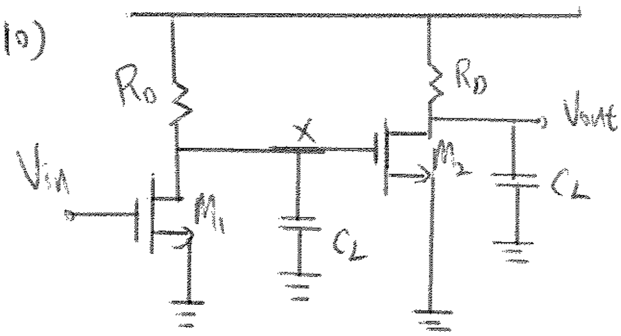
$$\frac{V_{out}}{V_{in}} = (g_m R_D)^2 \quad \text{(At DC)}$$

2 poles at $\frac{1}{R_D C_L}$

Bode plot:



10)



$$\frac{V_x}{V_{in}}(s) = -g_m\left(R_D /\!/ \frac{1}{C_L s}\right), \quad \frac{V_{out}}{V_x}(s) = -g_m\left(\frac{R_D}{R_D C_L s+1}\right)$$

$$= -g_m\left(\frac{R_D}{R_D C_L s+1}\right)$$

$$H(s) = \frac{V_x}{V_{in}}(s)\,\frac{V_{out}}{V_x}(s) = \left(\frac{g_m R_D}{R_D C_L s+1}\right)^2$$

$$s \to \omega j, \quad H(j\omega) = \left(\frac{g_m R_D}{1 + R_D C_L j\omega}\right)^2$$

$$|H(j\omega)| = \frac{(g_m R_D)^2}{1 + (R_D C_L \omega)^2}$$

-3dB Band Width:

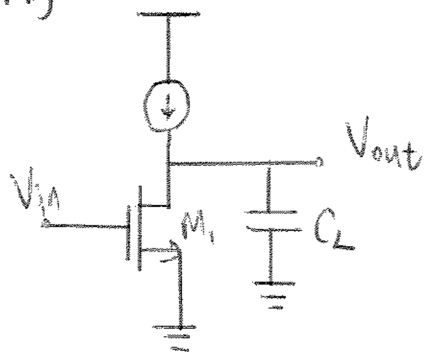
$$\frac{(g_m R_D)^2}{1 + (R_D C_L \omega)^2} = \frac{(g_m R_D)^2}{\sqrt{2}}$$

$$\Rightarrow (R_D C_L \omega)^2 + 1 = \sqrt{2}$$

$$\Rightarrow \omega = \frac{\sqrt{\sqrt{2}-1}}{R_D C_L} = \frac{0.6436}{R_D C_L} \ (\text{rad}/s)$$

$$2\pi f = \frac{0.6436}{R_D C_L} \Rightarrow f = \frac{0.10243}{R_D C_L} \ (Hz)$$

1 a)



$\lambda > 0$

Since $\lambda > 0$, and we have an ideal current source, the impedance looking from out to ground is $r_o // \frac{1}{C_L s}$

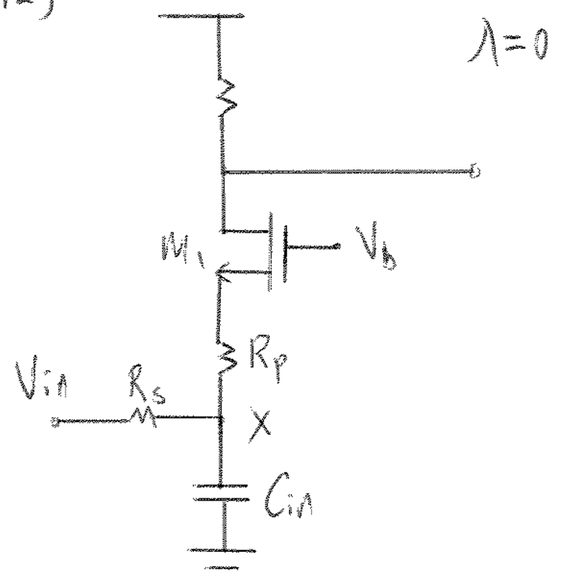
So, $V_{out} = -g_m V_{in}\left(r_o // \frac{1}{C_L s}\right)$

$$H(s) = -g_m\left(\frac{r_o}{r_o C_L s + 1}\right), \quad |H(j\omega)| = \frac{g_m r_o}{\sqrt{(r_o C_L \omega)^2 + 1}}$$

For $\lambda \to 0$, $r_o \to \infty \Rightarrow H(s) \to \frac{-g_m r_o}{r_o C_L s}$

$H(s) = \frac{-g_m}{C_L s}$. A pole at origin, thus operating as an ideal integrator.

12)



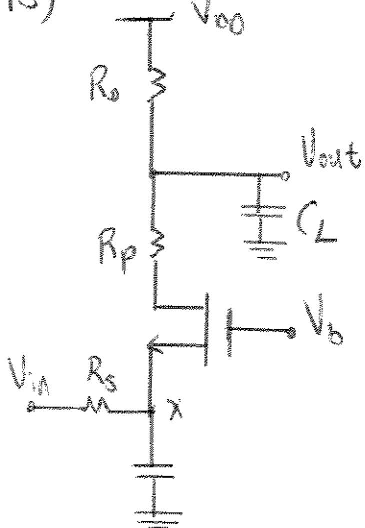
To find input pole, let $V_{in} = 0$ and find the equivalent resistance and capacitance from node X to ground.

$$R_x = R_s // \left(R_p + \frac{1}{g_{m_1}} \right), \quad C_x = C_{in}$$

$$\omega_{p_{in}} = \frac{1}{C_{in}\left[R_s // \left(R_p + \frac{1}{g_m}\right)\right]}$$

$$\omega_{p_{out}} = \frac{1}{R_D C_L}$$

13)



$\lambda = 0$, neglect all other caps.

$R_X = R_S // \frac{1}{g_m}$

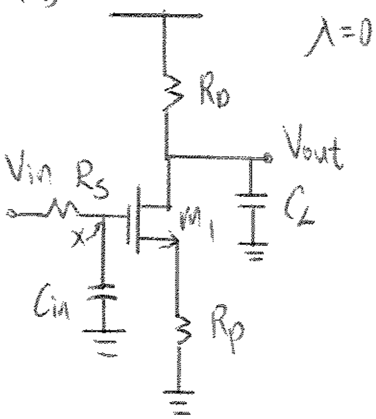
$C_X = C_{in}$

$R_{out} = R_D$ (since $r_o = \infty$)

$C_{out} = C_L$

$$\omega_{pin} = \frac{1}{(R_S // \frac{1}{g_m})\, C_{in}}, \qquad \omega_{pout} = \frac{1}{R_D C_L}$$

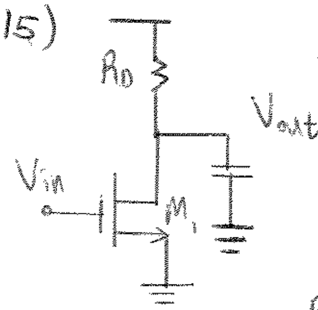
14)



$R_X = R_S$, $R_{out} \approx R_D$

$C_x = C_{in}$, $C_{out} \approx C_L$

$$\omega_{pin} = \frac{1}{R_S C_{in}} , \quad \omega_{pout} = \frac{1}{R_D C_L}$$

15) 

DC Gain: $g_m R_D = \frac{2 I_D R_D}{V_{eff}}$

where $V_{eff} = V_{GS} - V_{th}$

Band Width: $\frac{1}{R_D C_L}$

Power Consumption: $V_{DD} I_D$

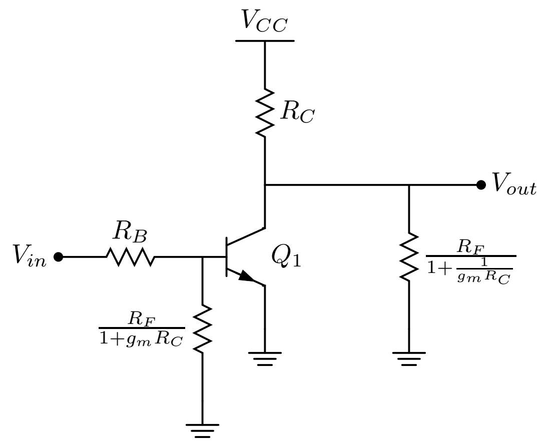
F.O.M. (11.5) $= \frac{\text{Gain} \times \text{Band Width}}{\text{Power Consumption}}$

$$= \frac{\left(\frac{2 I_D R_D}{V_{eff}}\right)\left(\frac{1}{R_D C_L}\right)}{V_{DD} I_D}$$

$$= \frac{2}{V_{eff} V_{DD} C_L}$$

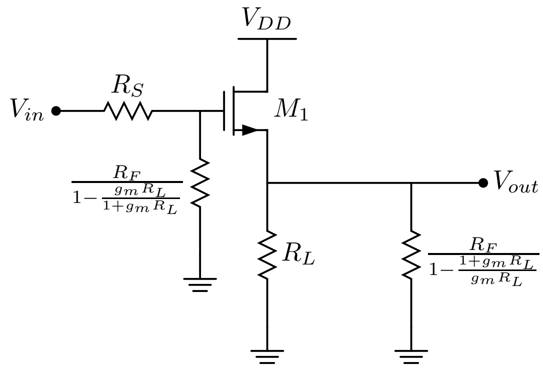
For practical design, $V_{eff} > V_t$, thus bipolar has a larger F.O.M. than MOS.

11.16 Using Miller's theorem, we can split the resistor R_F as follows:



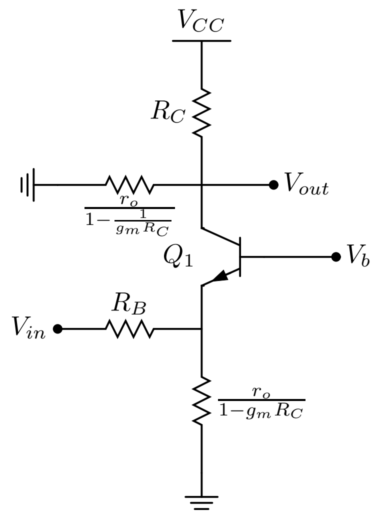
$$A_v = \boxed{-g_m \left(\frac{r_\pi \parallel \frac{R_F}{1+g_m R_C}}{R_B + r_\pi \parallel \frac{R_F}{1+g_m R_C}} \right) \left(R_C \parallel \frac{R_F}{1 + \frac{1}{g_m R_C}} \right)}$$

11.17 Using Miller's theorem, we can split the resistor R_F as follows:



$$A_v = \boxed{\left(\frac{\frac{R_F}{1-\frac{g_m R_L}{1+g_m R_L}}}{R_S + \frac{R_F}{1-\frac{g_m R_L}{1+g_m R_L}}}\right)\left(\frac{g_m\left(R_L \parallel \frac{R_F}{1-\frac{1+g_m R_L}{g_m R_L}}\right)}{1+g_m\left(R_L \parallel \frac{R_F}{1-\frac{1+g_m R_L}{g_m R_L}}\right)}\right)}$$

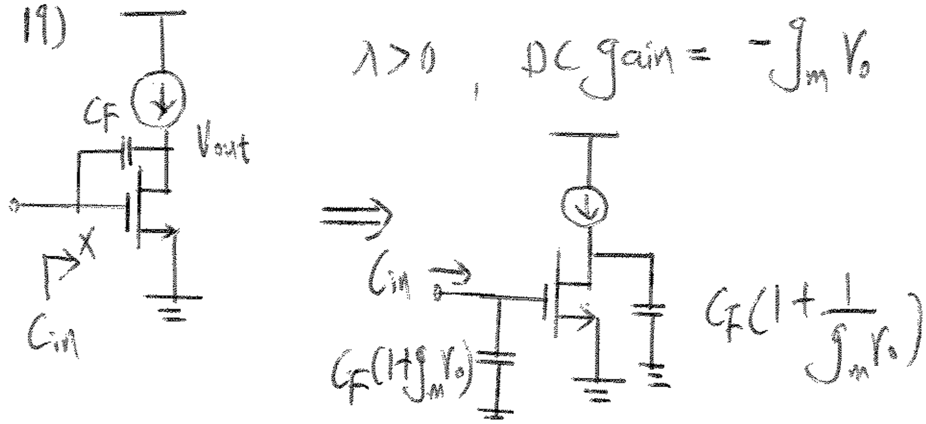
11.18 Using Miller's theorem, we can split the resistor r_o as follows:



$$A_v = \boxed{g_m \left(\frac{\frac{1}{g_m} \parallel r_\pi \parallel \frac{r_o}{1-g_m R_C}}{R_B + \frac{1}{g_m} \parallel r_\pi \parallel \frac{r_o}{1-g_m R_C}} \right) \left(R_C \parallel \frac{r_o}{1 - \frac{1}{g_m R_C}} \right)}$$

19)

$\lambda > 0$, DC gain $= -g_m r_o$

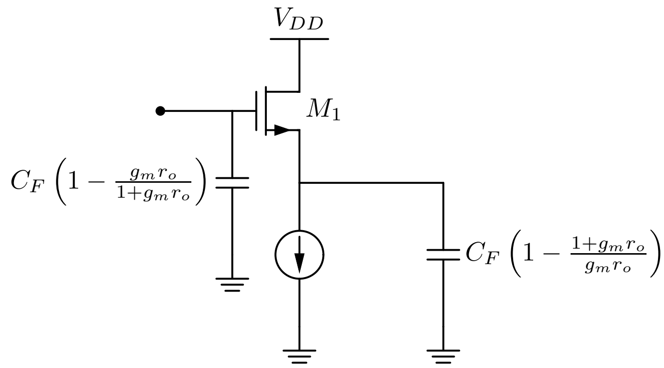


$C_{in} = C_F(1 + g_m r_o)$, neglecting other caps.

As $\lambda \rightarrow 0$, $r_o \rightarrow \infty$, DC gain $\rightarrow \infty$,

$C_{in} \rightarrow \infty$, this bandwidth will $\longrightarrow 0$.

11.20 Using Miller's theorem, we can split the capacitor C_F as follows (note that the DC gain is $A_v = \frac{g_m r_o}{1+g_m r_o}$):

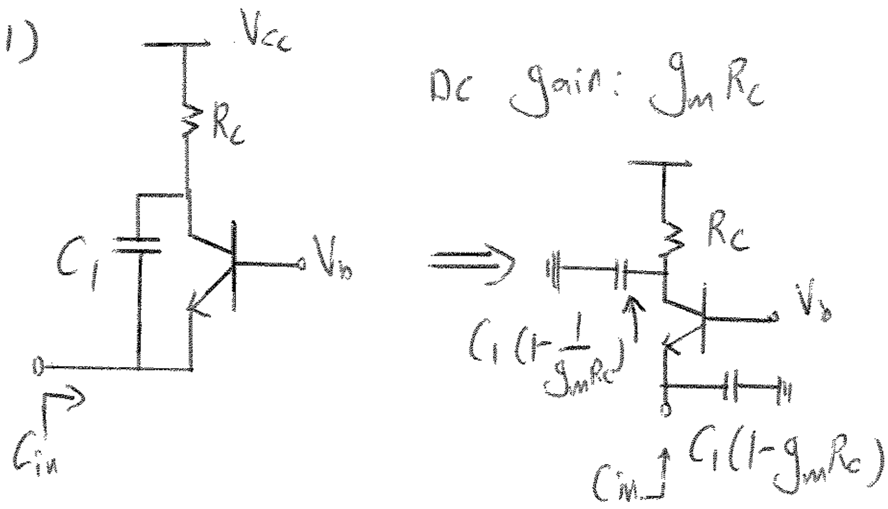


Thus, we have

$$C_{in} = \boxed{C_F\left(1 - \frac{g_m r_o}{1+g_m r_o}\right)}$$

As $\lambda \to 0$, $r_o \to \infty$, meaning the gain approaches 1. When this happens, the input capacitance goes to zero.

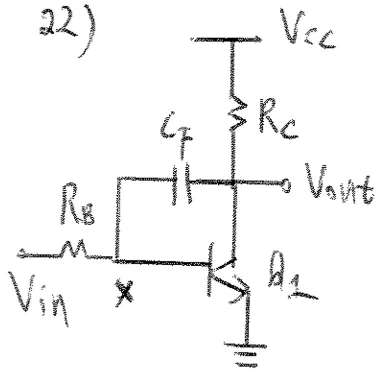
21)



$C_{in} = C_1 (1 - g_m R_c)$.

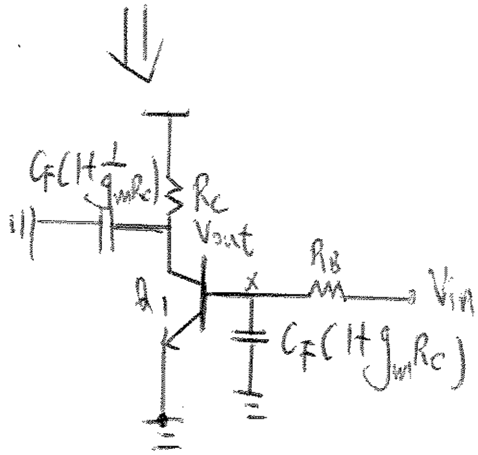
If $g_m R_c$ is designed to be larger than 1, as it normally would, we will have inductive action.

22)



DC gain (from x to out):

$-g_m R_c$



$C_{in} = C_F(1+g_m R_c)$

$R_{in} = R_B \,//\, r_\pi$

$C_{out} = C_F\left(1+\frac{1}{g_m R_c}\right)$

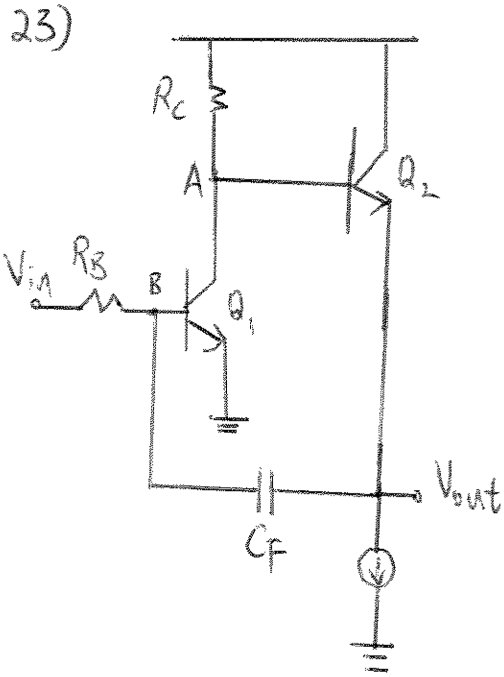
$R_{out} = R_c$

$$\omega_{pin} = \frac{1}{R_B // r_\pi \left[C_F(1+g_m R_c)\right]}$$

$$\omega_{pout} = \frac{1}{R_c C_F\left(1+\frac{1}{g_m R_c}\right)} \approx \frac{1}{R_c C_F}$$

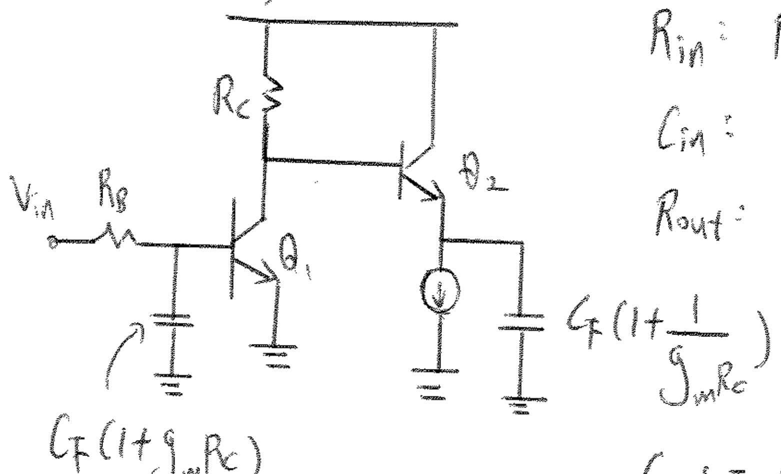
(If $g_m R_c \gg 1$)

23)



The gain from B to A is $-g_m R_C$, from A to out is 1 (since we have an ideal current source). So the gain from B to out is $-g_m R_C$.

⇓



R_{in}: $R_B // r_\pi$

C_{in}: $C_F(1+g_m R_C)$

R_{out}: $\frac{1}{g_m} + \frac{R_C}{\beta+1}$

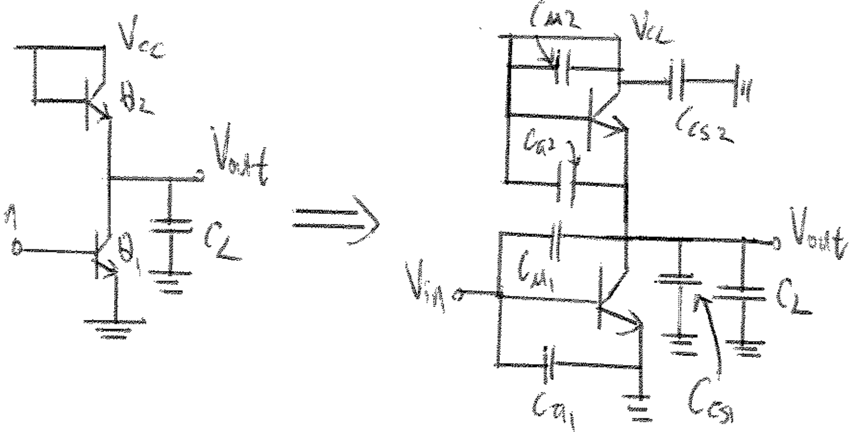
$C_{out} = C_F\left(1+\frac{1}{g_m R_C}\right)$

$$\omega_{pin} = \frac{1}{R_B // r_\pi [C_F(1+g_m R_C)]}$$

$$\omega_{pout} = \frac{1}{\left(\frac{1}{g_m}+\frac{R_C}{\beta+1}\right) C_F\left(1+\frac{1}{g_m R_C}\right)} \approx \frac{1}{\left(\frac{1}{g_m}+\frac{R_C}{\beta+1}\right) C_F}, \quad (g_m R_C \gg 1)$$

24)

a)

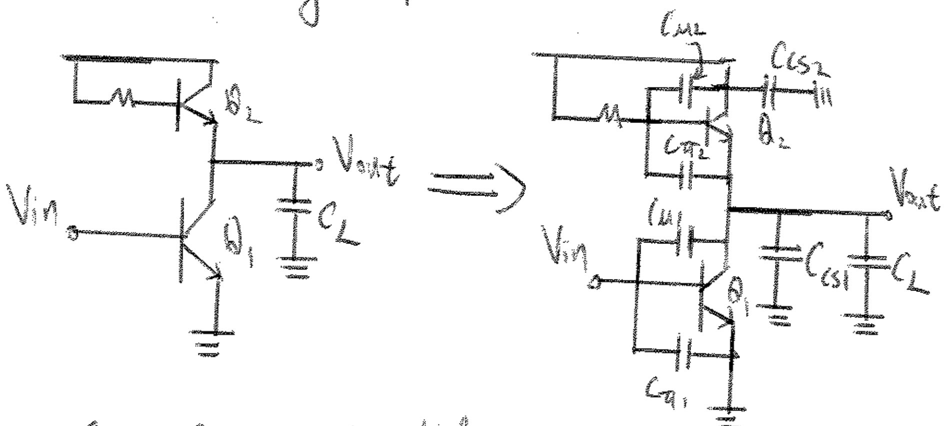


$C_{\pi 2}$, C_{cs1}, C_L are in parallel

$C_{\mu 2}$, C_{cs2} are grounded on both ends.

(and technically in parallel as well)

b)

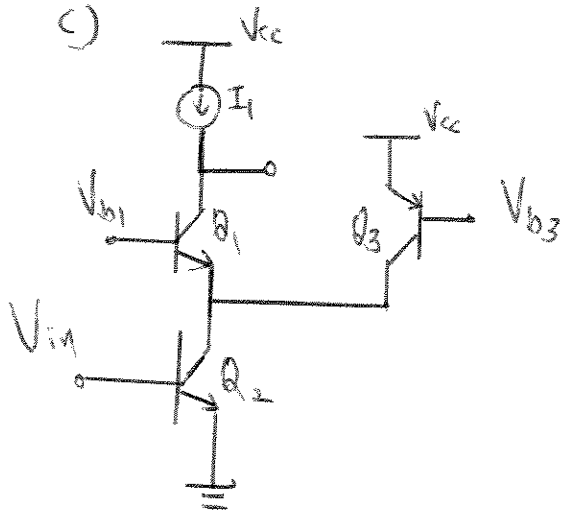


C_{cs1}, C_L are in parallel

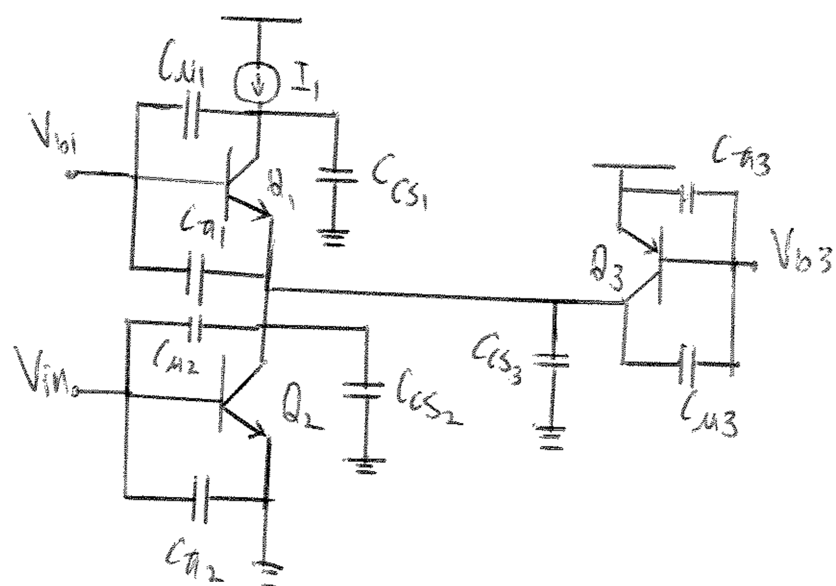
C_{cs2} is grounded on both ends

24)

c)



$\Downarrow$

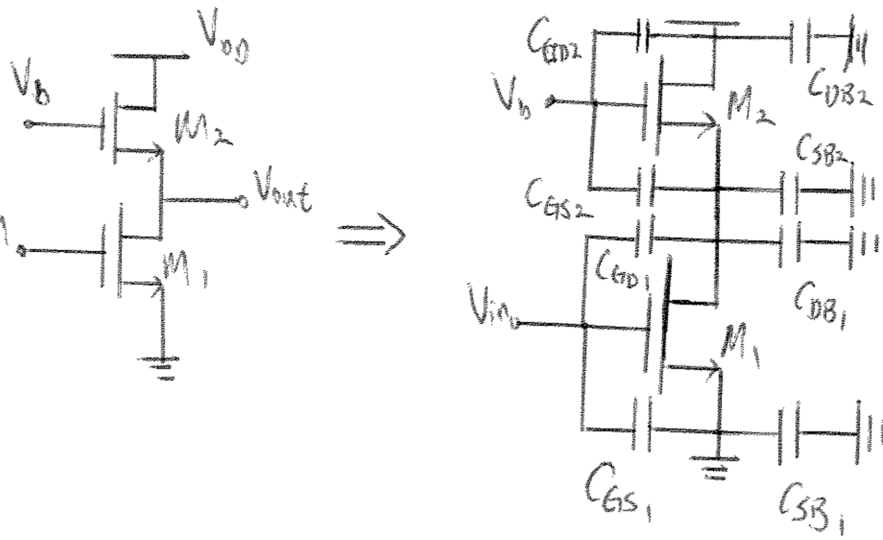


$C_{\pi 1}$, C_{CS_2}, C_{CS_3}, $C_{\mu 3}$ are in parallel
$C_{\mu 1}$, C_{CS_1} are also in parallel

$C_{\pi 3}$ is grounded on both ends

25)

a)

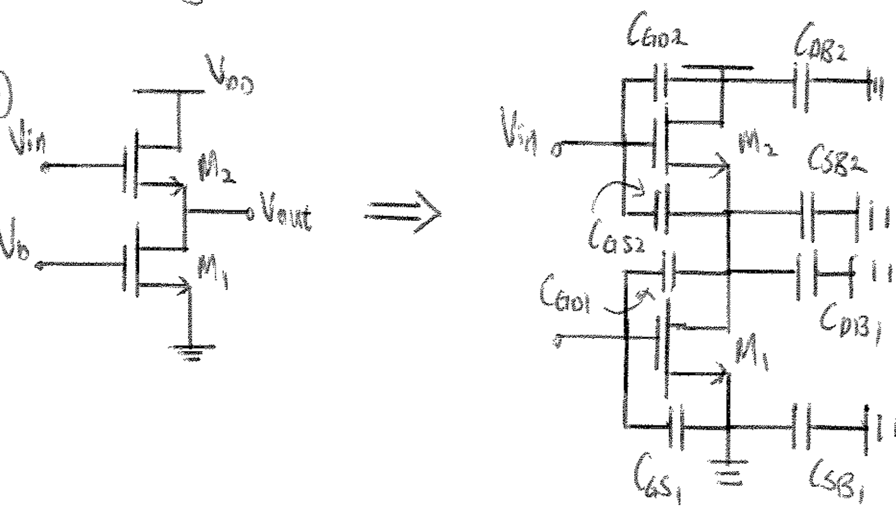


C_{GS2}, C_{SB2}, C_{DB1} are in parallel

C_{GD2}, C_{DB2} are in parallel and grounded on both ends

C_{SB1} is grounded on both ends.

b)



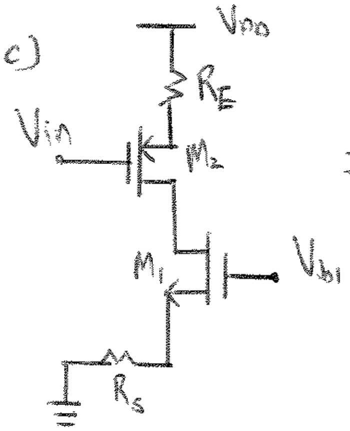
C_{GD1}, C_{DB1}, C_{SB2} are in parallel

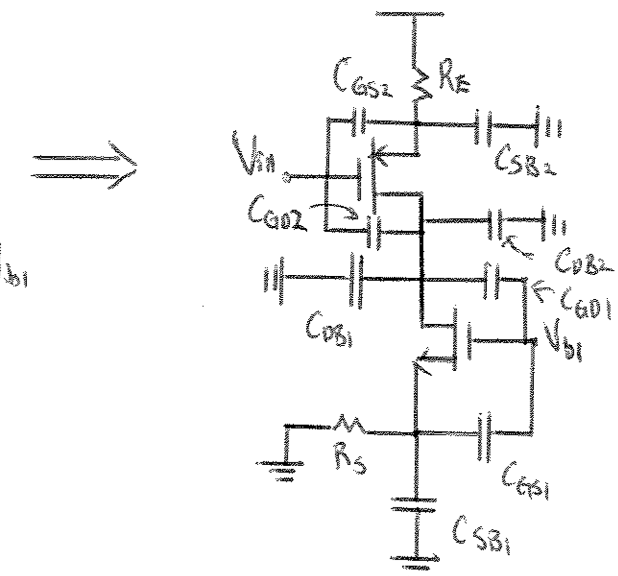
C_{GS1}, C_{SB1} are in parallel and grounded on both ends

C_{DB2} is grounded on both ends.

25)

c)

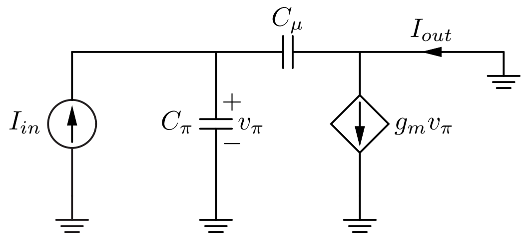




C_{DB2}, C_{GD1}, C_{DB1}, are in parallel

C_{SB1}, C_{GS1} are also in parallel.

11.26 At high frequencies (such as f_T), we can neglect the effects of r_π and r_o, since the low impedances of the capacitors will dominate at high frequencies. Thus, we can draw the following small-signal model to find f_T (for BJTs):



$$
\begin{aligned}
I_{in} &= j\omega v_\pi \left(C_\pi + C_\mu\right) \\
I_\pi &= \frac{I_{in}}{j\omega \left(C_\pi + C_\mu\right)} \\
I_{out} &= g_m v_\pi - j\omega C_\mu v_\pi \\
&= v_\pi \left(g_m - j\omega C_\mu\right) \\
&= \frac{I_{in}}{j\omega \left(C_\pi + C_\mu\right)} \left(g_m - j\omega C_\mu\right) \\
\frac{I_{out}}{I_{in}} &= \frac{g_m - j\omega C_\mu}{j\omega \left(C_\pi + C_\mu\right)} \\
\left|\frac{I_{out}}{I_{in}}\right| &= \frac{\sqrt{g_m^2 + \left(\omega C_\mu\right)^2}}{\omega \left(C_\pi + C_\mu\right)} \\
\frac{\sqrt{g_m^2 + \left(\omega_T C_\mu\right)^2}}{\omega_T \left(C_\pi + C_\mu\right)} &= 1 \\
g_m^2 + \omega_T^2 C_\mu^2 &= \omega_T^2 \left(C_\pi^2 + 2C_\pi C_\mu + C_\mu^2\right) \\
g_m^2 &= \omega_T^2 \left(C_\pi^2 + 2C_\pi C_\mu\right) \\
\omega_T &= \frac{g_m}{\sqrt{C_\pi^2 + 2C_\pi C_\mu}} \\
f_T &= \boxed{\frac{g_m}{2\pi\sqrt{C_\pi^2 + 2C_\pi C_\mu}}}
\end{aligned}
$$

The derivation of f_T for a MOSFET is identical to the derivation of f_T for a BJT, except we have C_{GS} instead of C_π and C_{GD} instead of C_μ. Thus, we have:

$$
f_T = \boxed{\frac{g_m}{2\pi\sqrt{C_{GS}^2 + 2C_{GS}C_{GD}}}}
$$

27)

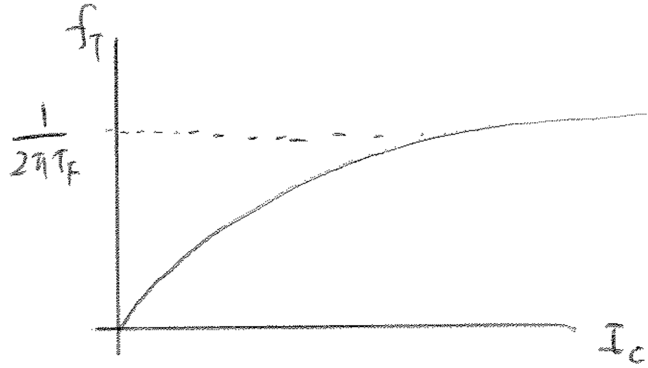
$$C_\pi = g_m \tau_F + C_{je}$$

$$2\pi f_T = \frac{g_m}{C_\pi} = \frac{g_m}{g_m \tau_F + C_{je}}$$

Assume C_{je} to be independent of I_C.

a) $$2\pi f_T = \frac{\frac{I_C}{V_T}}{\frac{I_C}{V_T}\tau_F + C_{je}} \Rightarrow f_T = \frac{I_C}{2\pi(I_C \tau_F + V_T C_{je})}$$

b)



As $I_C \to \infty$, $f_T \to \frac{1}{2\pi\tau_F}$

28)

$$C_{GS} \approx \left(\frac{2}{3}\right) W L C_{ox}$$

$$2\pi f_T = \frac{g_m}{C_{GS}} = \frac{\frac{W}{L}\mu_n C_{ox}(V_{GS}-V_{TH})}{\frac{2}{3}WL\ C_{ox}}$$

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(\ V_{GS} - V_{TH})$$

29)

$$2\pi f_T = \frac{3}{2} \frac{2I_D}{WLC_{ox}} \frac{1}{(V_{GS}-V_{TH})}$$

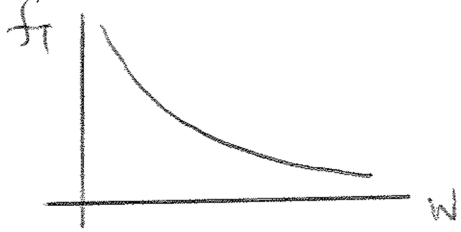
Apparently, f_T decreases with the overdrive. However, when we look closely, I_D is actually proportional to $(V_{GS}-V_{TH})^2$ (In saturation), so f_T is proportional to $(V_{GS}-V_{TH})$.

3c)

a) As $W \uparrow$, $(V_{GS} - V_{TH})$ has to $\downarrow$ by $\frac{1}{\sqrt{W}}$ in order to maintain I_D constant

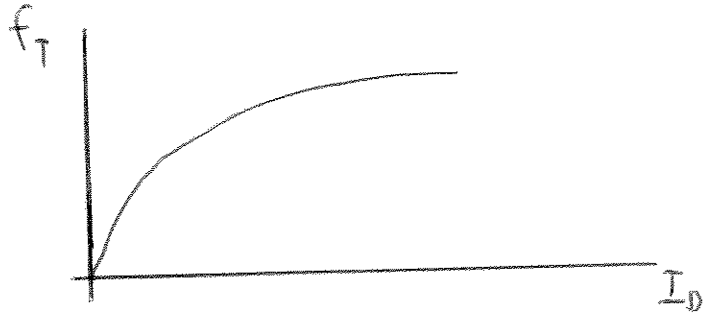
using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$

$2\pi f_T \propto \frac{1}{\sqrt{W}}$



b) $I_D \uparrow$, W constant it means $V_{GS} - V_{TH} \uparrow$ with $\sqrt{I_D}$. Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$

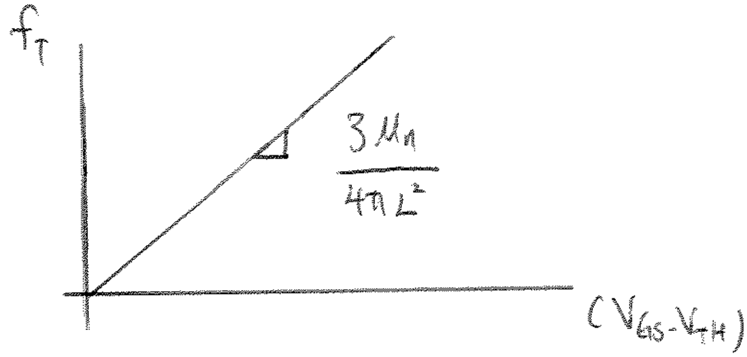
$2\pi f \propto \sqrt{I_D}$



31)

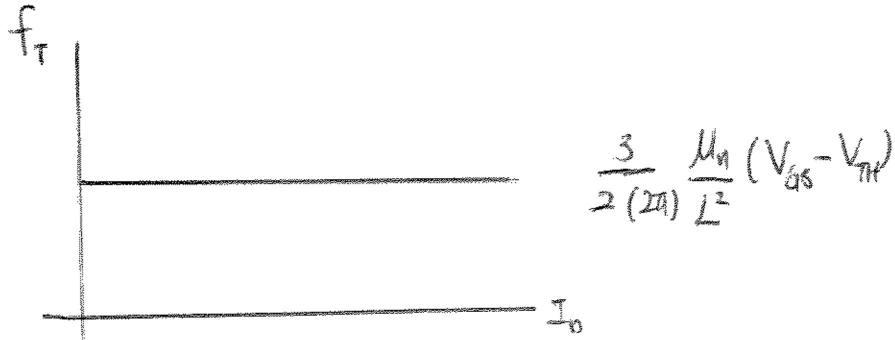
Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$

a) $2\pi f_T \propto (V_{GS}-V_{TH})$



b) Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$

$2\pi f$ = constant for all I_D.

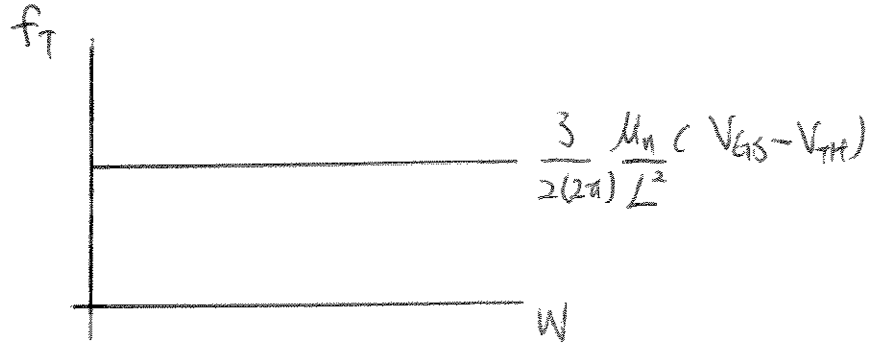


32)

a)

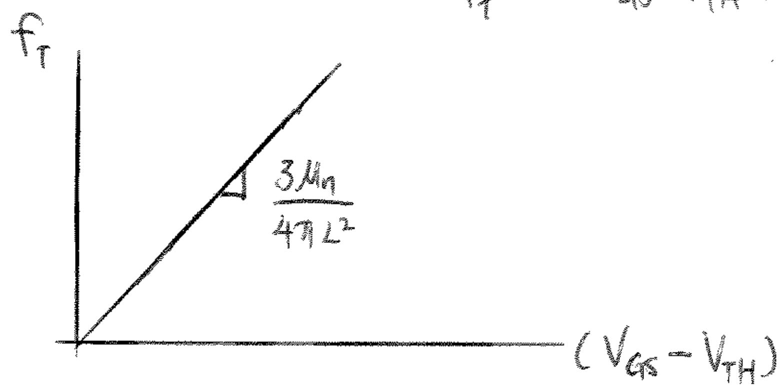
Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$

we know that $2\pi f_T$ is constant for all W.



b) Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$,

we know that $2\pi f_T \propto (V_{GS} - V_{TH})$.



33)

a) $I_D = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{GS}-V_{TH})^2$

As $L\uparrow$, to maintain the same current and overdrive voltage, $W\uparrow$ as well.

So W also 2X.

b) Since $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$, and

L 2X while $(V_{GS}-V_{TH})$ is constant,

$f_T \downarrow$ by $\frac{3}{4}$ or $f_{T_{new}} = \frac{1}{4} f_{T_{old}}$.

34)

a) $V_{GS} - V_{TH} \longrightarrow \frac{1}{2}(V_{GS} - V_{TH})$

Constant I_D and $W\uparrow$ (L constant)

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$$

$$f_{T_{new}} = \frac{f_{T,old}}{2}$$

b) $V_{GS} - V_{TH} \longrightarrow \frac{1}{2}(V_{GS} - V_{TH})$

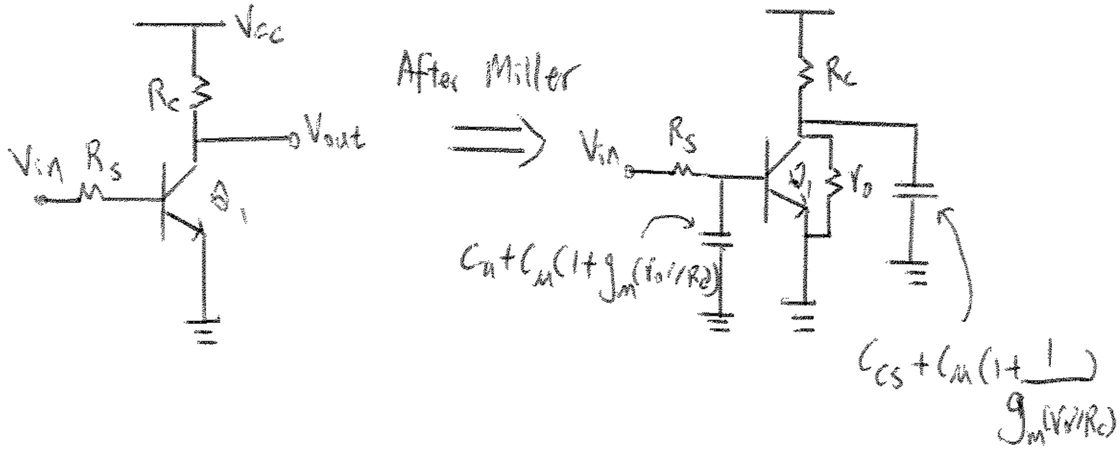
Constant W and $I_D \downarrow$ (L constant)

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$$

$$f_{T_{new}} = \frac{f_{T,old}}{2}$$

35)

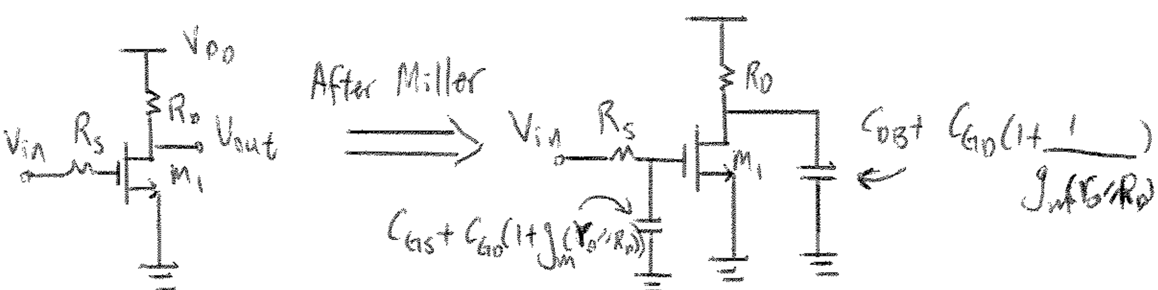
Bipolar CE Stage



$$\omega_{p,in} : \frac{1}{(R_S // r_\pi)[C_\pi + C_\mu(1+g_m(r_o // R_C))]}$$

$$\omega_{p,out} : \frac{1}{(R_C // r_o)[C_{CS} + C_\mu(1+1/g_m(r_o // R_C))]}$$

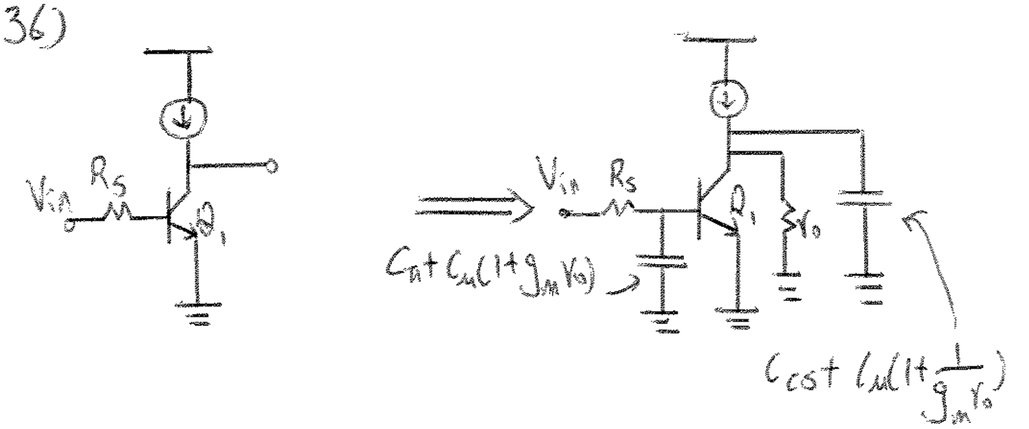
MOS CS Stage



$$\omega_{p,in} : \frac{1}{R_S[C_{GS} + C_{GD}(1+g_m(r_o // R_D))]}$$

$$\omega_{p,out} : \frac{1}{(R_D // r_o)[C_{DB} + C_{GD}(1+1/g_m(r_o // R_D))]}$$

36)



$$\omega_{p_{in}} = \frac{1}{(R_S // r_\pi)[C_\pi + C_\mu(1+g_m r_o)]}$$

$$\omega_{p_{out}} = \frac{1}{r_o[C_{cs} + C_\mu(1+1/g_m r_o)]}$$

$$H(s) = \frac{\text{DC gain}}{\left(1+\frac{s}{\omega_{p_{in}}}\right)\left(1+\frac{s}{\omega_{p_{out}}}\right)}$$

$$H(s) = \frac{g_m r_o (r_\pi/(r_\pi + R_S))}{\left(1+\frac{s}{1/(R_S // r_\pi)[C_\pi + C_\mu(1+g_m r_o)]}\right)\left(1+\frac{s}{1/(r_o[C_{cs}+C_\mu(1+\frac{1}{g_m r_o})])}\right)}$$

11.37 Using Miller's theorem to split $C_{\mu 1}$, we have:

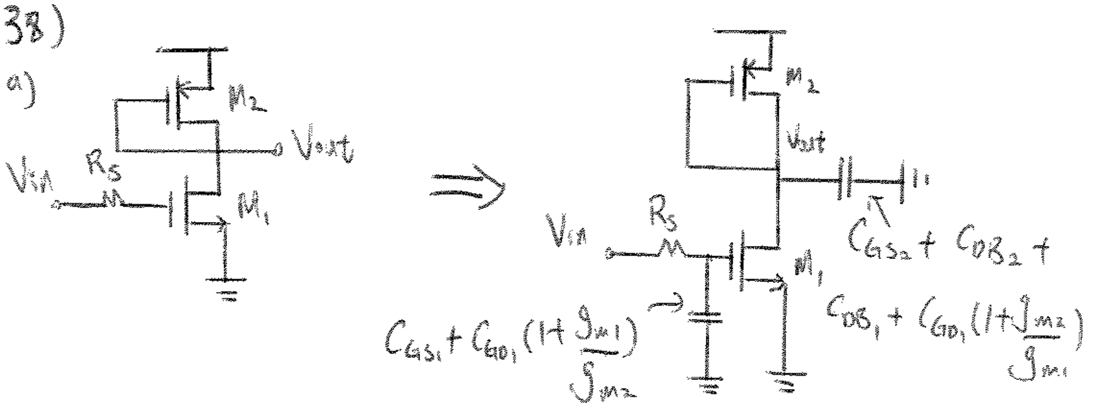
$$\omega_{p,in} = \boxed{\frac{1}{(R_S \parallel r_{\pi 1})\left\{C_{\pi 1} + C_{\mu 1}\left[1 + g_{m1}\left(r_{o1} \parallel r_{o2}\right)\right]\right\}}}$$

$$\omega_{p,out} = \boxed{\frac{1}{(r_{o1} \parallel r_{o2})\left\{C_{\mu 2} + C_{CS1} + C_{CS2} + C_{\mu 1}\left[1 + \frac{1}{g_{m1}(r_{o1} \parallel r_{o2})}\right]\right\}}}$$

$$\frac{V_{out}}{V_{in}}(s) = \boxed{-\frac{g_{m1}\left(\frac{r_{\pi 1}}{r_{\pi 1} + R_S}\right)(r_{o1} \parallel r_{o2})}{\left(1 + \frac{s}{\omega_{p,in}}\right)\left(1 + \frac{s}{\omega_{p,out}}\right)}}$$

38)

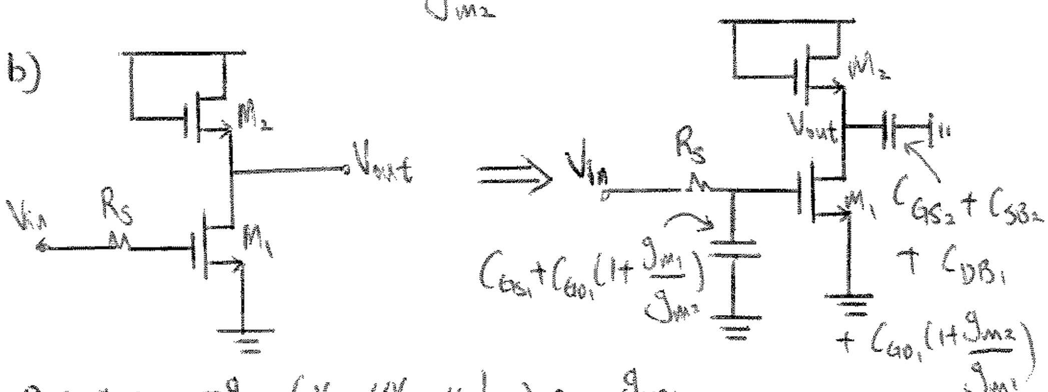
a)



$$\text{DC gain} = -g_{m_1}\left(r_{o_1} \parallel r_{o_2} \parallel \frac{1}{g_{m_2}}\right) \approx -\frac{g_{m_1}}{g_{m_2}}$$

$$\omega_{p_{in}} = \frac{1}{R_s\left(C_{GS_1} + C_{GD_1}\left(1+\frac{g_{m_1}}{g_{m_2}}\right)\right)} \qquad \omega_{p_{out}} = \frac{g_{m_2}}{\left(C_{GS_2} + C_{DB_2} + C_{DB_1} + C_{GD_1}\left(1+\frac{g_{m_2}}{g_{m_1}}\right)\right)}$$

b)



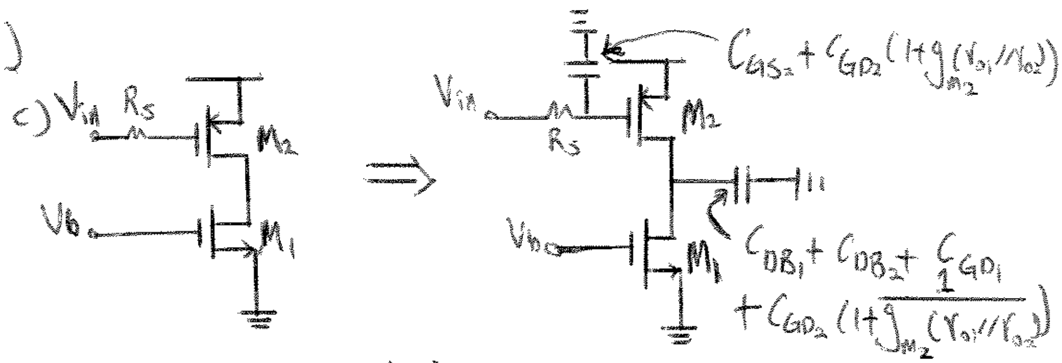
$$\text{DC gain} = -g_{m_1}\left(r_{o_1} \parallel r_{o_2} \parallel \frac{1}{g_{m_2}}\right) \approx -\frac{g_{m_1}}{g_{m_2}}$$

$$\omega_{p_{in}} = \frac{1}{R_s\left(C_{GS_1} + C_{GD_1}\left(1+\frac{g_{m_1}}{g_{m_2}}\right)\right)}$$

$$\omega_{p_{out}} = \frac{g_{m_2}}{C_{SB_2} + C_{GS_2} + C_{DB_1} + C_{GD_1}\left(1+\frac{g_{m_2}}{g_{m_1}}\right)}$$

38)

c)



DC gain: $-g_{m2}(r_{o1}//r_{o2})$

$$\omega_{pin} = \frac{1}{R_S(C_{GS_2} + C_{GD_2}(1 + g_{m_2}(r_{o1}//r_{o2})))}$$

$$\omega_{pout} = \frac{1}{(r_{o1}//r_{o2})\left[C_{DB_1} + C_{DB_2} + C_{GD_1} + C_{GD_2}\left(1 + \frac{1}{g_{m_2}(r_{o1}//r_{o2})}\right)\right]}$$

$$\omega_{pout} \approx \frac{1}{(r_{o1}//r_{o2})[C_{DB_1} + C_{DB_2} + C_{GD_1} + C_{GD_2}]}$$

Since $g_{m2}(r_{o1}//r_{o2}) >> 1$

11.39 (a)

$$\omega_{p,in} = \frac{1}{R_S\left[C_{GS} + C_{GD}\left(1 + g_m R_D\right)\right]} = \boxed{3.125 \times 10^{10} \text{ rad/s}}$$

$$\omega_{p,out} = \frac{1}{R_D\left[C_{DB} + C_{GD}\left(1 + \frac{1}{g_m R_D}\right)\right]} = \boxed{3.846 \times 10^{10} \text{ rad/s}}$$

(b)

$$\frac{V_{out}}{V_{Thev}}(s) = \frac{\left(C_{GD}s - g_m\right)R_D}{as^2 + bs + 1}$$

$$a = R_S R_D \left(C_{GS}C_{GD} + C_{DB}C_{GD} + C_{GS}C_{DB}\right) = 2.8 \times 10^{-22}$$

$$b = \left(1 + g_m R_D\right)C_{GD}R_S + R_S C_{GS} + R_D\left(C_{GD} + C_{DB}\right) = 5.7 \times 10^{-11}$$

Setting the denominator equal to zero and solving for s, we have:

$$s = \frac{-b \pm \sqrt{b^2 - 4a}}{2a}$$

$$|\omega_{p1}| = \boxed{1.939 \times 10^{10} \text{ rad/s}}$$

$$|\omega_{p2}| = \boxed{1.842 \times 10^{11} \text{ rad/s}}$$

We can see substantial differences between the poles calculated with Miller's approximation and the poles calculated from the transfer function directly. We can see that Miller's approximation does a reasonably good job of approximating the input pole (which corresponds to $|\omega_{p1}|$). However, the output pole calculated with Miller's approximation is off by nearly an order of magnitude when compared to ω_{p2}.

11.40 (a) Note that the DC gain is $A_v = -\infty$ if we assume $V_A = \infty$.

$$\omega_{p,in} = \frac{1}{(R_S \parallel r_\pi)\left[C_\pi + C_\mu\left(1 - A_v\right)\right]} = \boxed{0}$$

$$\omega_{p,out} = \boxed{0}$$

(b)

$$\frac{V_{out}}{V_{Thev}}(s) = \lim_{R_L\to\infty} \frac{\left(C_\mu s - g_m\right) R_L}{as^2 + bs + 1}$$

$$a = (R_S \parallel r_\pi) R_L \left(C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS}\right)$$

$$b = (1 + g_m R_L) C_\mu (R_S \parallel r_\pi) + (R_S \parallel r_\pi) C_\pi + R_L \left(C_\mu + C_{CS}\right)$$

$$\lim_{R_L\to\infty} \frac{\left(C_\mu s - g_m\right) R_L}{as^2 + bs + 1} = \frac{C_\mu s - g_m}{\left[(R_S \parallel r_\pi)\left(C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS}\right)\right] s^2 + \left[g_m C_\mu (R_S \parallel r_\pi) + C_\mu + C_{CS}\right] s}$$

$$= \frac{C_\mu s - g_m}{s\left\{(R_S \parallel r_\pi)\left(C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS}\right) s + \left[g_m C_\mu (R_S \parallel r_\pi) + C_\mu + C_{CS}\right]\right\}}$$

$$|\omega_{p1}| = \boxed{0}$$

$$|\omega_{p2}| = \boxed{\frac{g_m C_\mu (R_S \parallel r_\pi) + C_\mu + C_{CS}}{(R_S \parallel r_\pi)\left(C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS}\right)}}$$

We can see that the Miller approximation correctly predicts the input pole to be at DC. However, it incorrectly estimates the output pole to be at DC as well, when in fact it is not, as we can see from the direct analysis.

11.41

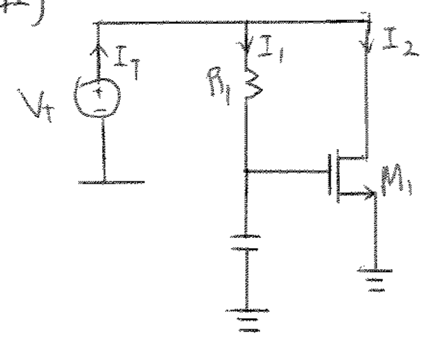
$$|\omega_{p1}| = \lim_{R_L\to\infty} \frac{1}{(1+g_m R_L)\,C_\mu\,(R_S \parallel r_\pi) + (R_S \parallel r_\pi)\,C_\pi + R_L\,(C_\mu + C_{CS})} = \boxed{0}$$

$$|\omega_{p2}| = \lim_{R_L\to\infty} \frac{(R_S \parallel r_\pi)\,R_L\,(C_\pi C_\mu + C_{CS}C_\mu + C_\pi C_{CS})}{(1+g_m R_L)\,C_\mu\,(R_S \parallel r_\pi) + (R_S \parallel r_\pi)\,C_\pi + R_L\,(C_\mu + C_{CS})}$$

$$= \boxed{\frac{(R_S \parallel r_\pi)\,(C_\pi C_\mu + C_{CS}C_\mu + C_\pi C_{CS})}{g_m C_\mu\,(R_S \parallel r_\pi) + C_\mu + C_{CS}}}$$

The dominant-pole approximation gives the same results as analyzing the transfer function directly, as in Problem 40(b).

42)



$\lambda = 0$, and neglect other capacitances.

$$I_T = I_1 + I_2$$

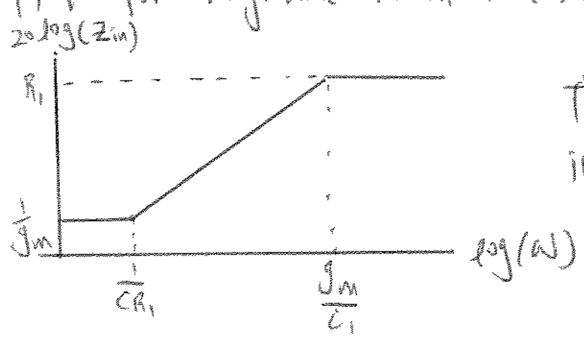
$$I_1 = \frac{V_T}{\left(R_1 + \frac{1}{C_1 s}\right)}, \quad I_2 = \frac{g_{m_1} V_T}{C_1 R_1 s + 1}$$

$$I_T = \frac{C_1 s V_T}{C_1 R_1 s + 1} + \frac{g_{m_1} V_T}{C_1 R_1 s + 1} \Rightarrow \frac{V_T}{I_T} = \frac{C_1 R_1 s + 1}{C_1 s + g_{m_1}}$$

$$s \to j\omega \Rightarrow \frac{C_1 R_1 (j\omega) + 1}{C_1 j\omega + g_{m_1}} = Z_T(j\omega)$$

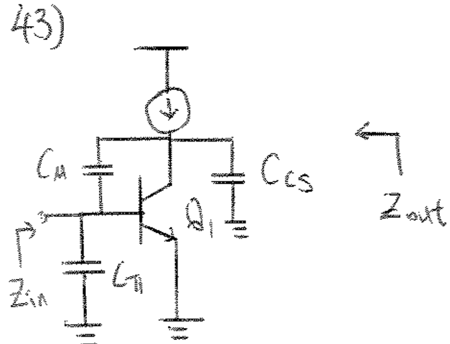
$$|Z_T| = |Z_{in}| = \frac{\sqrt{(C_1 R_1 \omega)^2 + 1}}{\sqrt{(C_1 \omega)^2 + g_{m_1}^2}} = \frac{\sqrt{(C_1 R_1 \omega)^2 + 1}}{g_m \sqrt{\left(\frac{C_1 \omega}{g_{m_1}}\right)^2 + 1}}$$

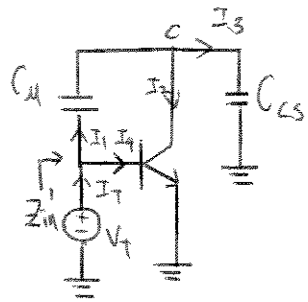
At $\omega = \frac{1}{C_1 R_1}$, we have a zero, at $\omega = \frac{g_{m_1}}{C_1}$, we have a pole. If $R_1 > \frac{1}{g_{m_1}}$, the zero is at a lower frequency than the pole, and the bode-plot for magnitude would look like the following.



The bode-plot shows an impedance that increases with frequency, an inductive behavior.

43)





$$Z_{in} = Z_{in}' \,//\, \frac{1}{C_\pi s}, \quad I_T = I_1 + I_4 = C_\mu s V_{bc} + \frac{g_m V_T}{\beta}$$

$$V_{bc} = V_T - V_c, \quad V_c = (I_1 - g_m V_T)\frac{1}{C_{CS}\, s}$$

$$I_1 = \left[V_T - (I_1 - g_m V_T)\frac{1}{C_{CS}\, s}\right] C_\mu s$$

$$I_1 = V_T\left[C_\mu s + \frac{g_m C_\mu}{C_{CS}}\right] \Big/ \left(1 + \frac{C_\mu}{C_{CS}}\right)$$

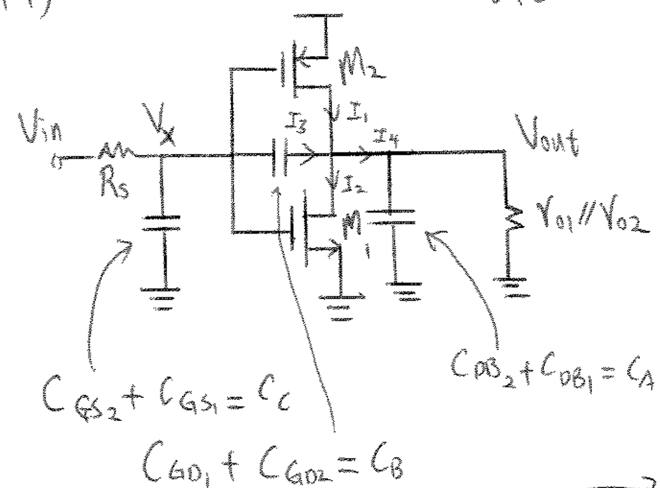
$$I_T = V_T\left[C_\mu s + \frac{g_m C_\mu}{C_{CS}}\right] \Big/ \left(1 + \frac{C_\mu}{C_{CS}}\right) + \frac{g_m V_T}{\beta}$$

$$Z_{in}' = \frac{V_T}{I_T} = \frac{1}{\dfrac{g_m}{\beta} + \dfrac{C_\mu s}{\left(1 + \frac{C_\mu}{C_{CS}}\right)} + \dfrac{g_m \frac{C_\mu}{C_{CS}}}{\left(1 + \frac{C_\mu}{C_{CS}}\right)}}$$

$$Z_{in} = Z_{in}' \,//\, \frac{1}{C_\pi s} = r_\pi \,//\, \frac{1}{\frac{C_{CS} C_\mu}{C_{CS} + C_\mu}\, s} \,//\, \frac{1}{C_\pi s} \,//\, \frac{C_{CS} + C_\mu}{g_m C_\mu}$$

$$Z_{out} = \frac{1}{(C_\mu + C_{CS})\, s}$$

44) $\lambda > 0$



$$V_{out} = I_4 \left(r_{o1} // r_{o2} // \frac{1}{[C_{DB2} + C_{DB1}]s} \right) \rightarrow Z_{out}$$

$$I_4 = I_1 + I_3 - I_2$$

$$I_1 = (0 - V_x) g_{m2}$$

$$I_2 = V_x g_{m1}$$

$$I_3 = (V_x - V_{out})\left((C_{GD1} + C_{GD2}) s \right)$$

$$I_4 = -V_x g_{m2} + (V_x - V_{out}) C_B s - V_x g_{m1}$$

$$V_{out} = Z_{out} \left[-V_x (g_{m2} + g_{m1}) + (V_x - V_{out}) C_B s \right]$$

Writing a node equation at x.

$$\frac{V_x - V_{in}}{R_s} + V_x C_c s + (V_x - V_{out}) C_B s = 0$$

$$V_x = \frac{V_{out} C_B s + V_{in}/R_s}{(1/R_s + C_c s + C_B s)}$$

44)

Subsitute everything and we get

$$V_{out} = Z_{out}\left[-(g_{m1}+g_{m2})\left(\frac{V_{out}C_B s + V_{in}/R_s}{1/R_s + C_c s + C_B s}\right) + \left(\frac{V_{out}C_B s + V_{in}/R_s}{1/R_s + C_c s + C_B s} - V_{out}\right)C_B s\right]$$

collect all the V_{out}'s on one-side and likewise for V_{in}'s, we will get

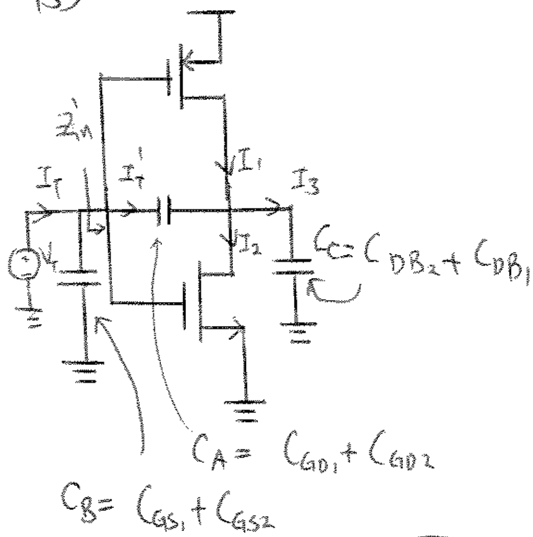
$$\frac{V_{out}}{V_{in}} = \frac{\frac{Z_{out}}{R_s}\left(C_B s - (g_{m1}+g_{m2})\right)}{1/R_s + (C_c+C_B)s + Z_{out}C_B s(g_{m1}+g_{m2}) + Z_{out}C_B s\left(\frac{1}{R_s}+(C_c+C_B)s\right) - Z_{out}C_B^2 s^2}$$

where $Z_{out} = r_{o1} // r_{o2} // \frac{1}{[C_{DB1}+C_{DB2}]s}$

$$C_B = C_{GD1} + C_{GD2}$$

$$C_c = C_{GS1} + C_{GS2}$$

45)



$$Z_{in} = \frac{V_T}{I_T} = \frac{1}{C_B} \,//\, Z_{in}'$$

$$Z_{in}' = \frac{V_T}{I_T'}$$

$$I_T' = \left[V_T - \left(I_3 \frac{1}{C_C s}\right)\right] C_A s$$

$$I_3 = I_T' - V_T g_{m2} - g_{m1} V_T$$

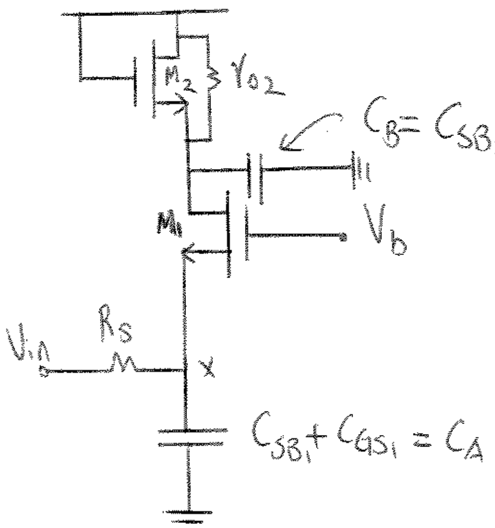
We get $\Rightarrow$ $I_T'\left(1 + \frac{C_A}{C_C}\right) = V_T\left[C_A s + (g_{m1} + g_{m2})\frac{C_A}{C_C}\right]$

$$Z_{in}' = \frac{V_T}{I_T'} = \frac{\left(1 + \frac{C_A}{C_C}\right)}{\left[C_A s + (g_{m1} + g_{m2})\frac{C_A}{C_C}\right]}$$

$$Z_{in} = \frac{1}{[C_{GS1} + C_{GS2}]s} \,//\, \frac{\left(1 + \frac{C_{GD1} + C_{GD2}}{C_{DB1} + C_{DB2}}\right)}{\left[(C_{GD1} + C_{GD2})s + (g_{m1} + g_{m2})\frac{C_{GD1} + C_{GD2}}{C_{DB2} + C_{DB1}}\right]}$$

4b)

a)



$$V_{out} = -(0 - V_x) g_{m_1} \left[\frac{1}{g_{m_2}} \parallel \frac{1}{C_B s} \right] = V_x g_{m_1} \left[\frac{1}{g_{m_2}} \parallel \frac{1}{C_B s} \right]$$

Node equation at X, $\frac{V_x - V_{in}}{R_s} + V_x C_A S - g_m (0 - V_x) = 0$

$$V_x \left(\frac{1}{R_s} + C_A S + g_m \right) = \frac{V_{in}}{R_s} \Rightarrow V_x = \frac{V_{in}}{(1 + R_s C_A S + R_s g_m)}$$

subsitute in V_x and solving for V_{out}/V_{in} $\Rightarrow$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1} \left[1/g_{m_2} \parallel \frac{1}{C_B s} \right]}{(1 + R_s C_A S + R_s g_m)}$$

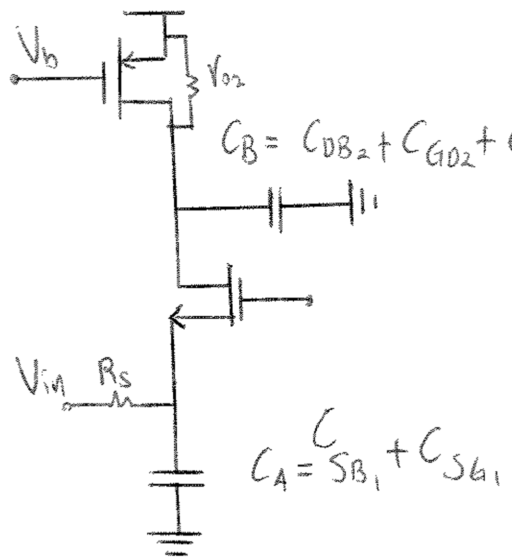
$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1} (1/g_{m_2})}{(C_B (1/g_{m_2}) S + 1)(1 + R_s C_A S + R_s g_{m_1})}$$

Where $C_B = C_{SB_2} + C_{GS_2} + C_{DB_1} + C_{DG_1}$

$C_A = C_{SB_1} + C_{GS_1}$

4b)

b)



Similar to part a), with $\frac{1}{g_{m2}}$ replaced by r_{o2},

and different C_B

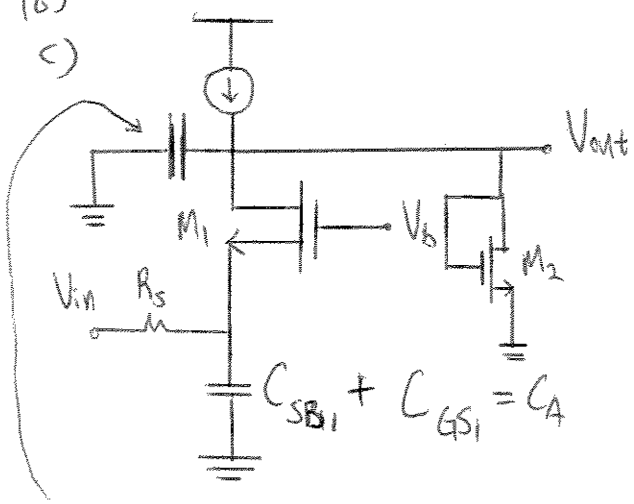
So
$$\frac{V_{out}}{V_{in}} = \frac{g_{m1} r_{o2}}{(C_B r_o S + 1)(1 + R_S C_A S + R_S g_{m1})}$$

Where $C_B = C_{DB2} + C_{GD2} + C_{DB1} + C_{DG1}$

$C_A = C_{SB1} + C_{SG1}$

4b)

c)



$C_B = C_{DB_1} + C_{GD_1} + C_{DB_2} + C_{GS_2}$

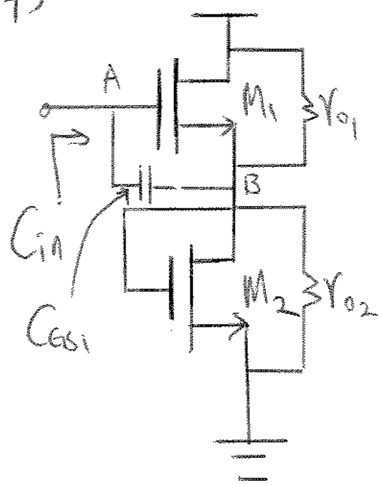
AC-wise, this circuit is very similar to part a). Its transfer function is the same as part a), except for C_B.

$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1}(1/g_{m_2})}{\left(C_B(1/g_{m_2})S + 1\right)\left(1 + R_S C_A S + R_S g_{m_1}\right)}$$

where $C_B = C_{DB_1} + C_{GD_1} + C_{DB_2} + C_{GS_2}$

$C_A = C_{SB_1} + C_{GS_1}$

47)



DC gain from A to B:

$$A_V = \frac{\frac{1}{g_{m2}} // r_{o1} // r_{o2}}{\frac{1}{g_{m2}} // r_{o1} // r_{o2} + \frac{1}{g_{m1}}}$$

$$A_V \approx \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m2}} + \frac{1}{g_{m1}}} = \frac{g_{m1}}{g_{m1} + g_{m2}}$$

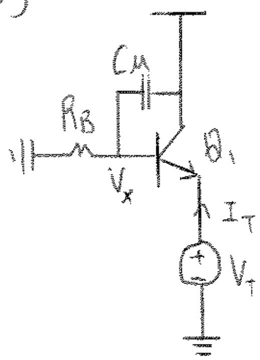
since $g_m r_o >> 1$

Using Miller's Capacitance:

$$C_{in} = C_{GS1}(1 - A_V) = C_{GS1}\left(1 - \frac{g_{m1}}{g_{m1} + g_{m2}}\right)$$

$$C_{in} = C_{GS1}\left(\frac{g_{m2}}{g_{m2} + g_{m1}}\right)$$

48)



$V_A = \infty$,

$\frac{\beta}{\beta+1} \approx 1$, if $\beta >> 1$

$$I_T = -(V_x - V_T) g_m \alpha \approx -(V_x - V_T) g_m$$

$$V_x = \frac{I_T}{\beta}\left(R_B // \frac{1}{C_\mu s}\right)$$

$$I_T = \left(V_T - \frac{I_T}{\beta}\left(R_B // \frac{1}{C_\mu s}\right)\right) g_m$$

$$I_T = g_m V_T - \frac{g_m}{\beta}\left(R_B // \frac{1}{C_\mu s}\right) I_T$$

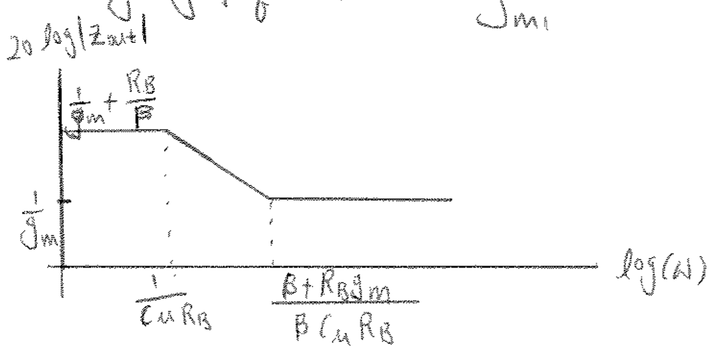
$$I_T\left(1 + \frac{g_m}{\beta}\left(R_B // \frac{1}{C_\mu s}\right)\right) = g_m V_T$$

$$\frac{V_T}{I_T} = \frac{1}{g_m} + \frac{R_B // \frac{1}{C_\mu s}}{\beta} = \frac{\beta C_\mu R_B \left(s + \frac{\beta + R_B g_m}{\beta C_\mu R_B}\right)}{g_m \beta (1 + C_\mu R_B s)}$$

Zero: $\frac{\beta + R_B g_m}{\beta C_\mu R_B}$, Pole: $\frac{1}{C_\mu R_B}$

At DC, $|Z_{out}| = \frac{1}{g_{m1}} + \frac{R_B}{\beta}$

At very high freq: $|Z_{out}| = \frac{1}{g_{m1}}$



11.49

$$\omega_{p1} = \frac{1}{(R_B \parallel r_{\pi 1}) \left\{ C_{\pi 1} + C_{\mu 1} \left[1 + g_{m1} \left(\frac{1}{g_{m2}} \parallel r_{\pi 2} \right) \right] \right\}}$$

$$\approx \frac{1}{(R_B \parallel r_{\pi 1}) \left\{ C_{\pi 1} + C_{\mu 1} \left[1 + \frac{g_{m1}}{g_{m2}} \right] \right\}}$$

$$I_{C1} = 4I_{C2} \Rightarrow g_{m1} = 4g_{m2}$$

$$\omega_{p1} = \boxed{\frac{1}{(R_B \parallel r_{\pi 1})(C_{\pi 1} + 5C_{\mu 1})}}$$

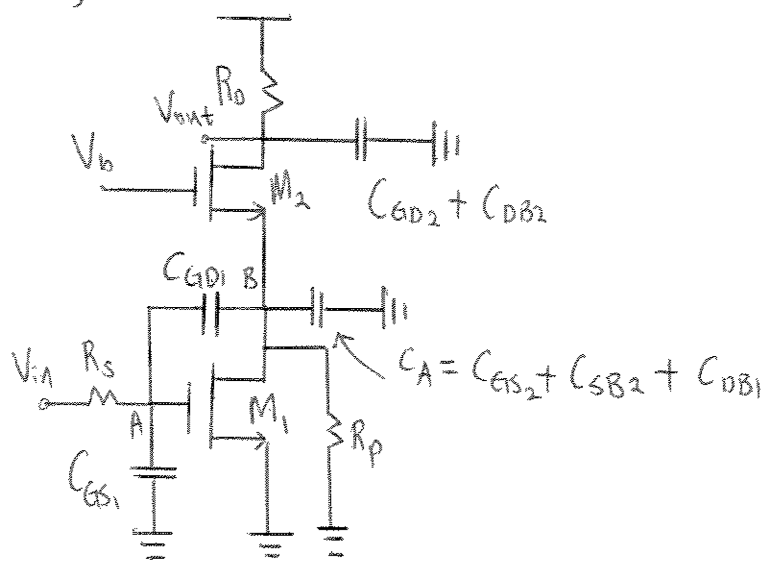
$$\omega_{p2} \approx \frac{1}{\frac{1}{g_{m2}} \left[C_{CS1} + C_{CS3} + C_{\mu 3} + C_{\pi 2} + C_{\mu 1} \left(1 + \frac{g_{m2}}{g_{m1}} \right) \right]}$$

$$= \boxed{\frac{g_{m2}}{C_{CS1} + C_{CS3} + C_{\mu 3} + C_{\pi 2} + \frac{5}{4} C_{\mu 1}}}$$

$$\omega_{p3} = \boxed{\frac{1}{R_C (C_{CS2} + C_{\mu 2})}}$$

Miller's effect is more significant here than in a standard cascode. This is because the gain in the common-emitter stage is increased to four in this topology, where it is about one in a standard cascode. This means that the capacitor $C_{\mu 1}$ will be multiplied by a larger factor when using Miller's theorem.

50)



DC gain from A to B is $-g_{m1}\left(R_P // \frac{1}{g_{m2}}\right)$

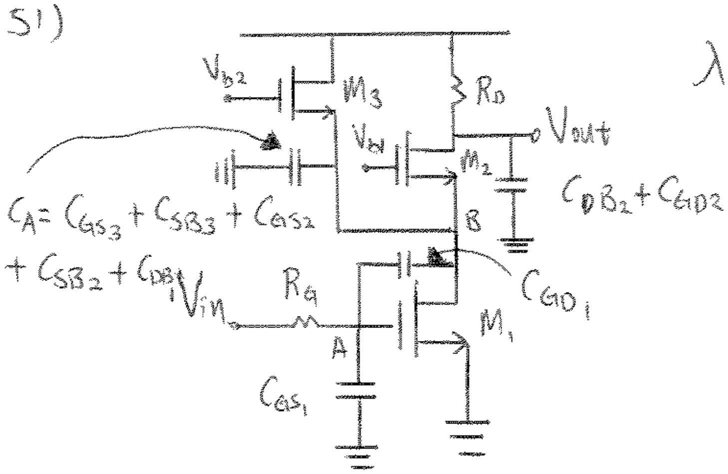
Applying Miller's Theorem:

$$\omega_{pin}\,(\omega_{pA}) = \frac{1}{R_S\left(C_{GS1} + C_{GD1}\left(1 + g_{m1}\left(R_P // \frac{1}{g_{m2}}\right)\right)\right)}$$

$$\omega_{pB} = \frac{1}{R_P // \frac{1}{g_{m2}}\left[C_{GS_2} + C_{SB2} + C_{DB_1} + C_{GD1}\left(1 + 1/g_{m_1}\left(R_P // \frac{1}{g_{m2}}\right)\right)\right]}$$

$$\omega_{pout} = \frac{1}{R_D\left(C_{GD_2} + C_{DB2}\right)}$$

5')



$\lambda = 0$

DC gain from A to B: $-g_{m1}\left(\frac{1}{g_{m3}} \parallel \frac{1}{g_{m2}}\right) = -g_{m1}\left(\frac{1}{g_{m2}+g_{m3}}\right)$

Applying Miller's Theorem:

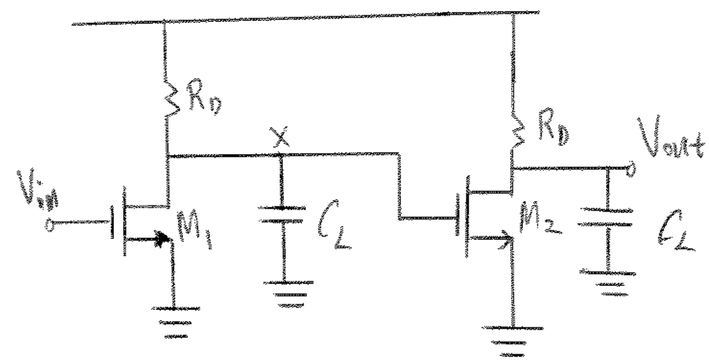
$$\omega_{pin}\ (\omega_{pA}) = \frac{1}{R_G\left(C_{GS1} + C_{GD1}\left(\frac{g_{m1}+g_{m2}+g_{m3}}{g_{m2}+g_{m3}}\right)\right)}$$

$$\omega_{pB} = \frac{g_{m3}+g_{m2}}{\left(C_A + C_{GD1}\left(\frac{g_{m1}+g_{m2}+g_{m3}}{g_{m1}}\right)\right)}$$

$$\omega_{pout} = \frac{1}{R_D\left(C_{DB2} + C_{GD2}\right)}$$

Where $C_A = C_{GS3} + C_{SB3} + C_{GS2} + C_{SB2} + C_{DB1}$

52)



Bias Current = 1mA (each stage)

$C_L = 50fF$

$\mu_n C_{ox} = 100 \mu A/V^2$, $A_V = 20$, $-3dB$: 1GHz

DC gain: $(g_m R_D)^2 = 20$

$-3dB$ bandwidth: $0.10243/(R_D C_L) = 1\,GHz$

Since $C_L = 50fF$, $R_D = 2048.6\,\Omega$

$(g_m R_D)^2 = 20 \Rightarrow g_m = 0.002183 = \frac{2I_D}{V_{eff}} \Rightarrow V_{eff} = 0.916V$

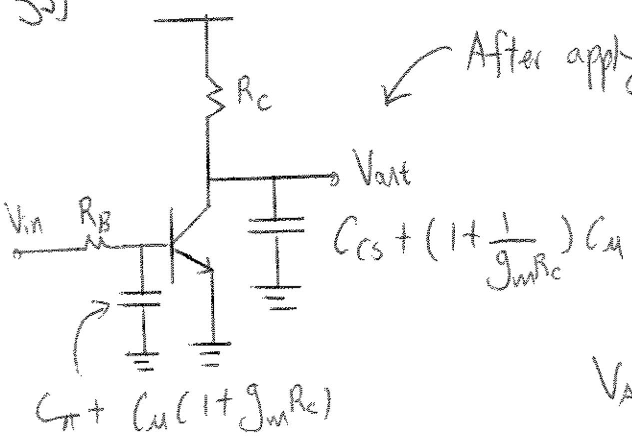
$V_{eff} = V_{GS} - V_{th} = 0.916V$

$g_m = \mu_n C_{ox} \frac{W}{L}(V_{eff}) \Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox}(V_{eff})} = 23.83$

So $R_D = 2.05K$, $C_L = 50fF$

$V_{GS} - V_{th} = 0.916V$, $W/L = 23.83$

53)



After apply Miller's theorem

$\omega_{pin} = (2\pi)(500MHz)$

$\omega_{pout} = (2\pi)(2G)$

$I_c = 1mA$, $C_\pi = 20fF$,

$C_\mu = 5fF$, $C_{CS} = 10fF$

$V_A = \infty$.

Low frequency Voltage gain: $\dfrac{V_{out}}{V_{in}} = \dfrac{-R_c}{1/g_m + \frac{R_B}{\beta+1}}$

$$\omega_{pin} = \frac{1}{(R_B // r_\pi)(C_\pi + C_\mu(1+g_m R_c))} = (2\pi)(500\,MHz)$$

$$\omega_{pout} = \frac{1}{R_c[C_{CS} + (1+1/(g_m R_c))C_\mu]} = (2\pi)(2G)$$

$$\Rightarrow g_m = 2\pi(2G)[g_m R_c C_{CS} + g_m R_c C_\mu + C_\mu]$$

$$\Rightarrow R_c = \left(\frac{g_m}{(2\pi)(2G)} - C_\mu\right) / (g_m(C_{CS} + C_\mu))$$

$$g_m = \frac{I_c}{V_T} = 0.0386\frac{1}{\Omega}, \quad R_c = 5296.53\,\Omega$$

53)

In order to maximize low frequency gain V_{out}/V_{in}, R_B should be as small as possible (restricted by the input pole location). So $R_B \parallel r_\pi \approx R_B$.

$$\omega_{pin} \approx \frac{1}{R_B(C_\pi + C_\mu(1+g_mR_C))} = (2\pi)(500\times10^6)$$

$$g_mR_C = 204.446$$

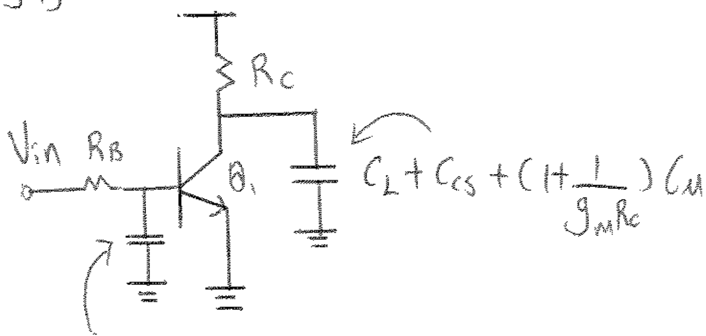
$$R_B = \frac{1}{\omega_{pin}(C_\pi + C_\mu(1+g_mR_C))} \approx 303.95\,\Omega$$

So

$$R_B = 303.95\,\Omega$$

$$R_C = 5296.53\,\Omega$$

54)



Low freq Voltage gain: $\frac{V_{out}}{V_{in}} = \frac{-R_C}{1/g_m + \frac{R_B}{\beta+1}}$

$$\omega_{pout} = \frac{1}{R_C[C_L + C_{CS} + (1+\frac{1}{g_m R_C})C_\mu]} = (2\pi)(2\,GHz)$$

$$g_m = \frac{I_C}{V_T} = 0.0386\ \frac{1}{\Omega}$$

$$g_m = (2\pi)(2\,GHz)[g_m R_C[C_L + C_{CS}] + g_m R_C C_\mu + C_\mu]$$

$$R_C = \left[\frac{g_m}{(2\pi)(2G)} - C_\mu\right] / (g_m[C_L + C_{CS} + C_\mu])$$

$R_C = 2269.94\Omega \approx 2.27\,K\Omega$

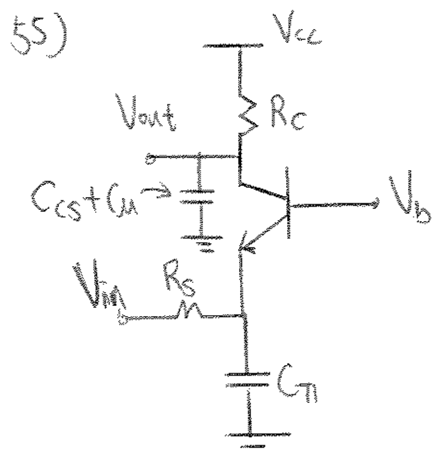
Again, to maximize low freq gain, R_B should be as small as possible, so $R_B // r_\pi \approx R_B$

$$\omega_{pin} \approx \frac{1}{R_B(C_\pi + C_\mu(1+g_m R_C))} = (2\pi)(500\times10^6),\ g_m R_C = 87.62$$

$R_B = 687.35\Omega$

So, $R_C = 2.27K\Omega$, $R_B = 687.35\Omega$

55)



$V_A = \infty$, $I_C = 1\,mA$, $R_S = 50\,\Omega$,
$C_\pi = 20\,fF$, $C_{cs} = 20\,fF$, $C_u = 5\,fF$

-3dB bandwidth: 10 GHz

Since the output node sees a larger capacitance and resistance than the input, (R_c usually large for large gain), dominant pole and thus -3dB bandwidth occurs at the output.

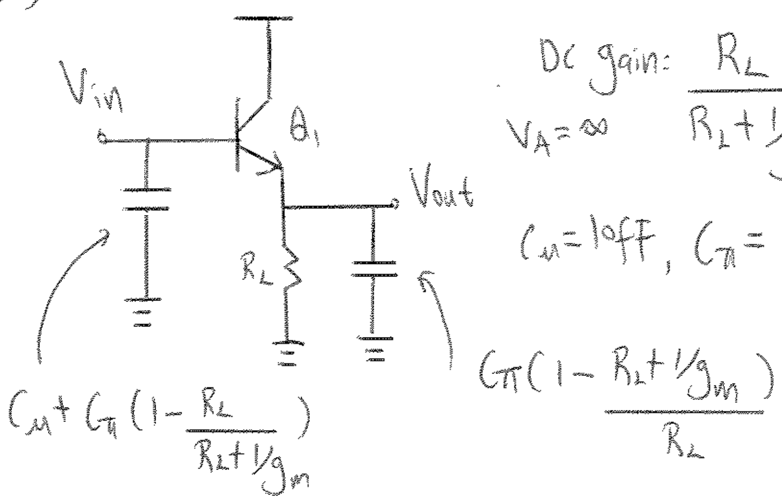
$$\omega_{pout} = \frac{1}{R_c\,[C_u + C_{cs}]} = (2\pi)(10\,GHz)$$

$R_c = 636.62\,\Omega$, $\frac{1}{g_m} = \frac{25.9\,mV}{1\,mA}$

Maximum achievable gain: $\frac{R_c}{R_S + \frac{1}{g_m}} = 8.4$

Here we have a tradeoff between gain and bandwidth.

56)



DC gain: $\frac{R_L}{R_L + 1/g_m}$

$V_A = \infty$

$C_\mu = 10fF$, $C_\pi = 100fF$

$$C_{in} < 50fF \Rightarrow C_\mu + C_\pi\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 50fF$$

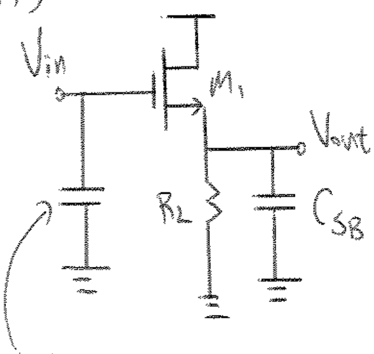
$$10fF + 100fF\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 50fF$$

$$100fF\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 40fF$$

$$\left(\frac{\frac{1}{g_m}}{R_L + 1/g_m}\right) < 0.4$$

$$R_L > \frac{3}{2g_m} = 38.85\Omega$$

57)



$C_{GD} + C_{GS}(1-0.8)$

$R_L = 100\Omega$, $I_D = 1mA$

$A_V = \frac{V_{out}}{V_{in}} = 0.8$ $\mu_n C_{ox} = 100 \mu A/V^2$

$L = 0.18\mu m$, $\lambda = 0$, $C_{GD} \approx 0$,

$C_{SB} \approx 0$, $C_{GS} = (2/3) WL C_{ox}$

$C_{ox} = 12 fF/(\mu m)^2$

$C_{in} = C_{GD} + C_{GS}(0.2)$, $C_{in} = C_{GS}(0.2) = C_{in(min)}$

$A_V = \frac{R_L}{R_L + 1/g_m} = 0.8$, $\frac{1}{g_m} = 25 = \frac{V_{eff}}{2I_D}$

$V_{eff} = 50mV$, $I_D = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{eff})^2 \Rightarrow W = 1440$

$C_{in,min} = 0.2\,C_{GS} = 0.2\left(\frac{2}{3}\right) WL\,C_{ox} = 414.72fF$

or $C_{in,min} = 0.415pF$

11.58

$$I_D = \frac{1}{2}\left(\frac{W}{L}\right)_1 \mu_n C_{ox} V_{ov}^2 = 0.5\text{ mA}$$

$$(W/L)_1 = (W/L)_2 = \boxed{250}$$

$$W_1 = W_2 = 45\ \mu\text{m}$$

$$g_{m1} = g_{m2} = \frac{W}{L}\mu_n C_{ox} V_{ov} = 5\text{ mS}$$

$$C_{GD1} = C_{GD2} = C_0 W = 9\text{ fF}$$

$$C_{GS1} = C_{GS2} = \frac{2}{3} W L C_{ox} = 64.8\text{ fF}$$

$$\omega_{p,in} = \frac{1}{R_G\left\{C_{GS1} + C_{GD1}\left(1 + \frac{g_{m1}}{g_{m2}}\right)\right\}} = 2\pi \times 5\text{ GHz}$$

$$R_G = \boxed{384\ \Omega}$$

$$\omega_{p,out} = \frac{1}{R_D C_{GD2}} = 2\pi \times 10\text{ GHz}$$

$$R_D = \boxed{1.768\text{ k}\Omega}$$

$$A_v = -g_{m1} R_D = \boxed{-8.84}$$

59)

$W_2 = 4W_1$, $V_{eff_2} = \frac{V_{eff_1}}{2}$ (To maintain the current constant)

$V_{eff_1} = 200mV$, $V_{eff_2} = 100mV$ (Assume V_{eff_1} is not changed)

DC gain: $-\frac{g_{m_1}}{g_{m_2}} = \frac{-g_{m_1}}{2g_{m_1}} = -\frac{1}{2}$

$$\omega_{pin} = \frac{1}{R_G[\frac{2}{3}WLC_{ox} + (0.2)W(\frac{1}{2})]} = (5\times10^9)(2\pi)$$

$W = 45\mu$

$\Rightarrow R_G = 459.32\Omega$

$$R_D = \frac{1}{(10\times10^9)(2\pi)(0.2)(4)(45)} = 442.097\Omega$$

DC gain: $|g_{m_1}R_D| = \frac{2I_D}{V_{eff_1}}R_D = 2.2105$

12.1 (a)

$$\begin{aligned} Y &= A_1\left(X - KA_2Y\right) \\ Y\left(1 + KA_1A_2\right) &= A_1X \\ \frac{Y}{X} &= \boxed{\frac{A_1}{1 + KA_1A_2}} \end{aligned}$$

(b)

$$\begin{aligned} Y &= X - KY - A_1\left(X - KY\right) \\ Y\left(1 + K - A_1K\right) &= X\left(1 - A_1\right) \\ \frac{Y}{X} &= \boxed{\frac{1 - A_1}{1 + K\left(1 - A_1\right)}} \end{aligned}$$

(c)

$$\begin{aligned} Y &= A_2X - A_1\left(X - KY\right) \\ Y\left(1 - A_1K\right) &= X\left(A_2 - A_1\right) \\ \frac{Y}{X} &= \boxed{\frac{A_2 - A_1}{1 - A_1K}} \end{aligned}$$

(d)

$$\begin{aligned} Y &= X - \left(KY - Y\right) - A_1\left[X - \left(KY - Y\right)\right] \\ Y &= X - KY + Y - A_1X + KA_1Y - A_1Y \\ Y\left[A_1\left(1 - K\right) + K\right] &= X\left(1 - A_1\right) \\ \frac{Y}{X} &= \boxed{\frac{1 - A_1}{A_1\left(1 - K\right) + K}} \end{aligned}$$

2. (a) $W = A_2 Y = A_2\left[(X - KW)A_1\right]$

$$\Rightarrow \frac{W}{X} = \frac{A_1 A_2}{1 + A_1 A_2 K}$$

(b) $W = A_1 \times E = A_1\left[X - K\left(\frac{W}{A_1} - W\right)\right]$

$$\Rightarrow \frac{W}{X} = \frac{A_1}{1 + K(1 - A_1)}$$

(c) $W = A_1 E = A_1\left[X - (A_2 X - W)K\right]$

$$\Rightarrow \frac{W}{X} = \frac{A_1(1 - A_2 K)}{(1 - A_1 K)}$$

(d) $W = A_1 E = A_1\left[X - \left\{\left(\frac{W}{A_1} - W\right)K - \left(\frac{W}{A_1} - W\right)\right\}\right]$

$$\Rightarrow \frac{W}{X} = \frac{A_1}{1 + (K-1)(1 - A_1)}$$

3. (a) $E = X - kA_2A_1E$

$$\Rightarrow \frac{E}{X} = \frac{1}{1 + kA_2A_1}$$

(b) $E = X - k[E - A_1E]$

$$\Rightarrow \frac{E}{X} = \frac{1}{1 + k(1 - A_1)}$$

(c) $E = X - k[A_2X - A_1E]$

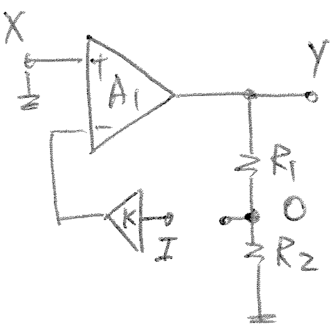
$$\Rightarrow \frac{E}{X} = \frac{1 - A_2k}{1 - A_1k}$$

(d) $E = X - \{k[E - A_1E] - [E - A_1E]\}$

$$\Rightarrow \frac{E}{X} = \frac{1}{1 + (k - 1)(1 - A_1)}$$

4.

(a)

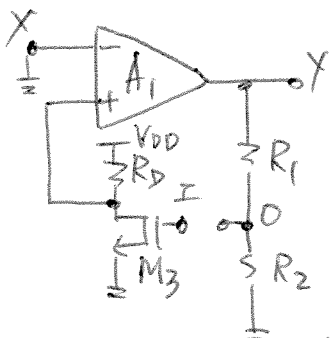


(X is grounded in loop-gain calculation)

$$O = Y\frac{R_2}{R_1+R_2} = (-IK)A_1\left(\frac{R_2}{R_1+R_2}\right)$$

$$\Rightarrow -\frac{O}{I} = \text{Loop Gain} = +KA_1\left(\frac{R_2}{R_1+R_2}\right)$$

(b)

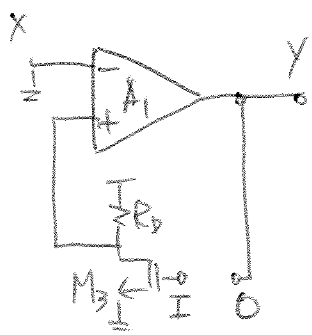


(X is grounded)

$$O = Y\left(\frac{R_2}{R_1+R_2}\right) = -Ig_{m3}R_D A_1\left(\frac{R_2}{R_1+R_2}\right)$$

$$\Rightarrow -\frac{O}{I} = \text{Loop Gain} = +g_{m3}R_D A_1\left(\frac{R_2}{R_1+R_2}\right)$$

(c)

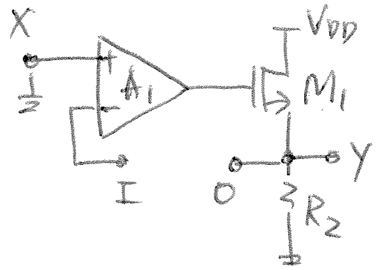


(X is grounded)

$$O = Y = -Ig_{m3}R_D A_1$$

$$\Rightarrow -\frac{O}{I} = \text{Loop Gain} = +g_{m3}R_D A_1$$

(d)



(X is grounded)

$$O = Y = -I \times \frac{g_{m1} R_2}{1 + g_{m1} R_2} \times A_1$$

$$\Rightarrow -\frac{O}{I} = \text{Loop Gain}$$

$$= + A_1 \frac{g_{m1} R_2}{1 + g_{m1} R_2}$$

12.5 The loop gains calculated in Problem 4 are used.

(a)

$$A_{OL} = A_1$$
$$A_{loop} = KA_1\left(\frac{R_2}{R_1+R_2}\right)$$
$$\frac{Y}{X} = \boxed{\frac{A_1}{1+KA_1\left(\frac{R_2}{R_1+R_2}\right)}}$$

(b)

$$A_{OL} = -A_1$$
$$A_{loop} = g_{m3}R_DA_1\left(\frac{R_2}{R_1+R_2}\right)$$
$$\frac{Y}{X} = \boxed{-\frac{A_1}{1+g_{m3}R_DA_1\left(\frac{R_2}{R_1+R_2}\right)}}$$

(c)

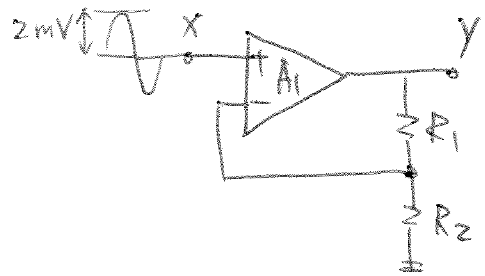
$$A_{OL} = -A_1$$
$$A_{loop} = g_{m3}R_DA_1$$
$$\frac{Y}{X} = \boxed{-\frac{A_1}{1+g_{m3}R_DA_1}}$$

(d)

$$A_{OL} = A_1\left(\frac{g_{m1}R_2}{1+g_{m1}R_2}\right)$$
$$A_{loop} = A_1\left(\frac{g_{m1}R_2}{1+g_{m1}R_2}\right)$$
$$\frac{Y}{X} = \boxed{\frac{A_1\left(\frac{g_{m1}R_2}{1+g_{m1}R_2}\right)}{1+A_1\left(\frac{g_{m1}R_2}{1+g_{m1}R_2}\right)}}$$

6. $A_1 = 500$

$R_1/R_2 = 7$



$$\frac{y}{x} \approx 1 + \frac{R_1}{R_2} = 8$$

$$\Rightarrow \frac{R_2}{R_1+R_2} = \frac{1}{8} = K$$

$$E = \frac{x}{1+KA_1} = \frac{2\,mV}{1+500/8} \approx 0.031\ mV$$

$\therefore$ Amplitude of feedback waveform

$= X - E \approx 1.969\ mV$

Amplitude of output waveform

$$= X\frac{A_1}{1+KA_1} \simeq 15.75\ mV$$

7. $$A_{CL} = \frac{A_1}{1+A_1K}$$

$$\frac{dA_{CL}}{dA_1} = \frac{1}{(1+A_1K)^2} \Rightarrow dA_{CL} = \frac{dA_1}{(1+A_1K)^2}$$

$$\Rightarrow \frac{dA_{CL}}{A_{CL}} = \frac{dA_{CL}}{\left(\frac{A_1}{1+A_1K}\right)} = dA_1\left(\frac{1+A_1K}{A_1}\right)\left(\frac{1}{(1+A_1K)^2}\right)$$

$$= \frac{(dA_1/A_1)}{(1+A_1K)}$$

This equation implies that for a fractional change in A_{CL}, it is reduced by $(1+A_1K)$ compared to a fractional change in A_1.

$$\Rightarrow 0.01 > \frac{0.2}{1+A_1K} \Rightarrow A_1K > 19$$

12.8

$$
\begin{aligned}
A_{OL} &= -g_m r_o \\
&= -\sqrt{2\frac{W}{L}\mu_n C_{ox} I_D}\frac{1}{\lambda I_D} \\
&= -\frac{1}{\lambda\sqrt{I_D}}\sqrt{2\frac{W}{L}\mu_n C_{ox}} \\
\frac{V_{out}}{V_{in}} &= \frac{A_{OL}}{1+KA_{OL}}
\end{aligned}
$$

We want to look at the maximum and minimum deviations that $\frac{V_{out}}{V_{in}}$ will have from the base value given the variations in λ and $\mu_n C_{ox}$. First, let's consider what happens when λ decreases by 20 % and $\mu_n C_{ox}$ increases by 10 %. This causes A_{OL} to increase in magnitude by a factor of $\frac{\sqrt{1.1}}{0.8} = 1.311$. We want $\frac{V_{out}}{V_{in}}$ to change by less than 5 % given this deviation in A_{OL}.

$$
\begin{gathered}
\frac{1.311A_{OL}}{1+1.311KA_{OL}} < 1.05\frac{A_{OL}}{1+KA_{OL}} \\
KA_{OL} > 3.982
\end{gathered}
$$

Next, let's consider what happens when λ increases by 20 % and $\mu_n C_{ox}$ decreases by 10 %. This causes A_{OL} to decrease in magnitude by a factor of $\frac{\sqrt{0.9}}{1.2} = 0.7906$. We want $\frac{V_{out}}{V_{in}}$ to change by less than 5 % given this deviation in A_{OL}.

$$
\begin{gathered}
\frac{0.7906A_{OL}}{1+0.7906KA_{OL}} < 0.95\frac{A_{OL}}{1+KA_{OL}} \\
KA_{OL} > 4.033
\end{gathered}
$$

Thus, to satisfy the constraints on both the maximum and minimum deviations, we require $KA_{OL} > \boxed{4.033}$.

9. From the question,

$$(1-10\%)A_o = |A(j\omega')| \quad \text{where } \omega' = -1\text{-dB bandwidth frequency}$$

$$0.9A_o = \frac{A_o}{\left|1+j\frac{\omega'}{\omega_o}\right|} = \frac{A_o}{\sqrt{1+\left(\frac{\omega'}{\omega_o}\right)^2}}$$

$\Rightarrow \omega' \cong 0.48\,\omega_o$

$\Rightarrow$ This is the open-loop -1dB bandwidth.

Similarly,

$$0.9\,\frac{A_o}{1+LG} = \left|\frac{Y}{X}(j\omega'')\right| \quad \text{where } \omega'' = -1\text{dB bandwidth frequency}$$

LG = Loop Gain

$$0.9\,\frac{A_o}{1+LG} = \frac{\frac{A_o}{1+LG}}{\left|1+j\frac{\omega''}{\omega_o(1+LG)}\right|} = \frac{\frac{A_o}{1+LG}}{\sqrt{1+\left[\frac{\omega''}{\omega_o(1+LG)}\right]^2}}$$

$\Rightarrow \omega'' \cong 0.48\,\omega_o\,(1+LG)$

$\therefore$ -1-dB bandwidth is boosted (expected) by $(1+LG)$ in closed-loop measurement.

12.10

$$
\begin{aligned}
A_{OL} &= -g_m \left(r_o \parallel \frac{1}{sC_L} \right) \\
&= -\frac{g_m r_o}{1 + s r_o C_L} \\
\frac{V_{out}}{V_{in}} &= \frac{A_{OL}}{1 + K A_{OL}} \\
&= \frac{-\frac{g_m r_o}{1 + s r_o C_L}}{1 - K \frac{g_m r_o}{1 + s r_o C_L}} \\
&= -\frac{g_m r_o}{1 + s r_o C_L - K g_m r_o}
\end{aligned}
$$

Setting the denominator equal to zero and solving for s gives us the bandwidth B.

$$
\begin{aligned}
B &= \frac{K g_m r_o - 1}{r_o C_L} \\
K &= \boxed{\frac{1 + B r_o C_L}{g_m r_o}}
\end{aligned}
$$

12.11 (a)

- Feedforward system: M_1 and R_D (which act as a common-gate amplifier)
- Sense mechanism: C_1 and C_2 (which act as a capacitive divider)
- Feedback network: C_1 and C_2
- Comparison mechanism: M_1 (which amplifies the difference between the fed back signal and the input)

(b)

$$A_{OL} = \boxed{g_m R_D}$$

$$A_{loop} = g_m R_D \left(\frac{C_1}{C_1 + C_2} \right)$$

$$\frac{v_{out}}{v_{in}} = \boxed{\frac{g_m R_D}{1 + g_m R_D \left(\frac{C_1}{C_1 + C_2} \right)}}$$

(c)

$$R_{in,open} = \boxed{\frac{1}{g_m}}$$

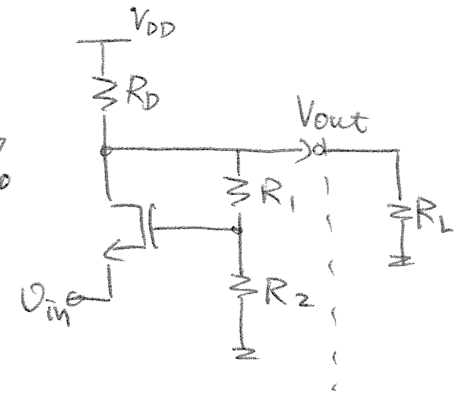
$$R_{in,closed} = \boxed{\frac{1 + g_m R_D \left(\frac{C_1}{C_1 + C_2} \right)}{g_m}}$$

$$R_{out,open} = \boxed{R_D}$$

$$R_{out,closed} = \boxed{\frac{R_D}{1 + g_m R_D \left(\frac{C_1}{C_1 + C_2} \right)}}$$

12.

Given: $$\frac{\text{Gain}_{\text{loaded}} - \text{Gain}_{\text{loaded}}}{\text{Gain}_{\text{unloaded}}} = 10\% = 0.1$$



$$\text{Gain}_{\text{unloaded}} \cong \frac{g_m R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D} \quad (\text{assume } R_1 + R_2 \gg R_D)$$

$$\text{Gain}_{\text{loaded}} = \frac{g_m (R_D \| R_L)}{1 + \frac{R_2}{R_1+R_2} g_m (R_D \| R_L)}$$

$$\therefore 0.1 = \frac{\dfrac{g_m R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D} - \dfrac{g_m (R_D \| R_L)}{1 + \frac{R_2}{R_1+R_2} g_m (R_D \| R_L)}}{\dfrac{g_m R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D}}$$

After solving for R_L:

$$R_L = \frac{9 R_D}{1 + \left(\frac{R_2}{R_1+R_2}\right) g_m R_D}$$

13. Gain at $x_1 = \dfrac{500}{1+500K}$

Gain at $x_2 = \dfrac{420}{1+420K}$

$$\Rightarrow \frac{\dfrac{500}{1+500K} - \dfrac{420}{1+420K}}{\dfrac{500}{1+500K}} < 0.05$$

$$\Rightarrow K > \frac{11}{2100}$$

$$A_{x_1} = \frac{500}{1+500(K)} = \frac{2625}{19} \approx 138.16$$

$$A_{x_2} = \frac{420}{1+420(K)} = \frac{525}{4} \cong 131.25$$

14. $y = \alpha_1 x - \alpha_3 x^3$

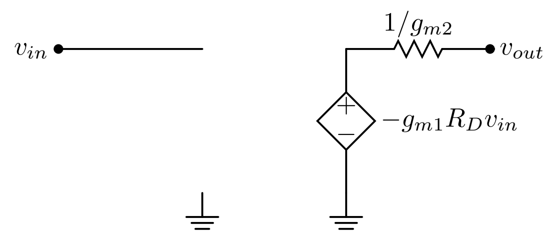
(a) $\dfrac{\partial y}{\partial x} = \alpha_1 - 3\alpha_3 x^2$

$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = \alpha_1 \qquad \left.\frac{\partial y}{\partial x}\right|_{x=\Delta x} = \alpha_1 \quad (\text{around } x=0)$$

(b) $\text{Closed-loop}\big|_{x=0} = \dfrac{\alpha_1}{1+\alpha_1 K}$

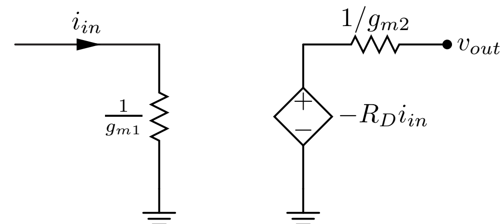
$\text{Closed-loop}\big|_{x=\Delta x} = \dfrac{\alpha_1}{1+\alpha_1 K} \quad (\text{around } x=0)$

12.15 (a)



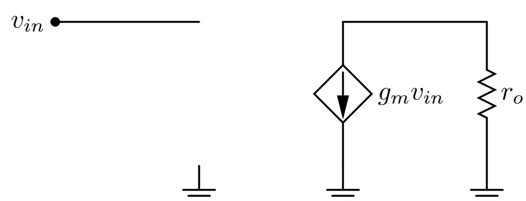
v_{in}
$1/g_{m2}$
v_{out}
$-g_{m1}R_Dv_{in}$

(b)



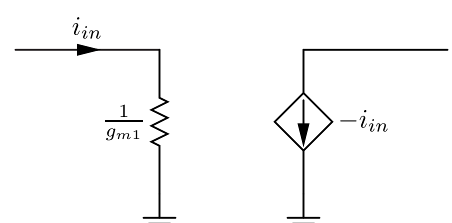
i_{in}
$\frac{1}{g_{m1}}$
$1/g_{m2}$
v_{out}
$-R_Di_{in}$

(c)



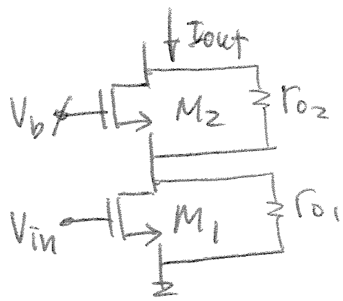
v_{in}
g_mv_{in}
r_o

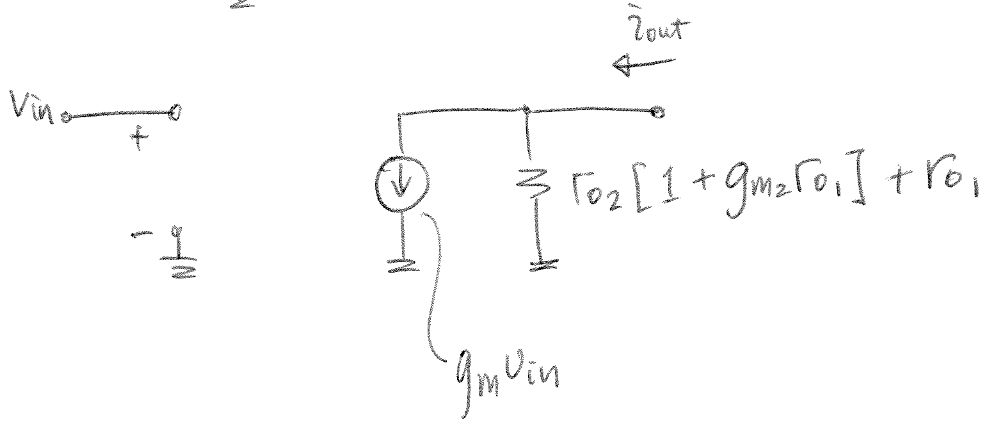
(d)



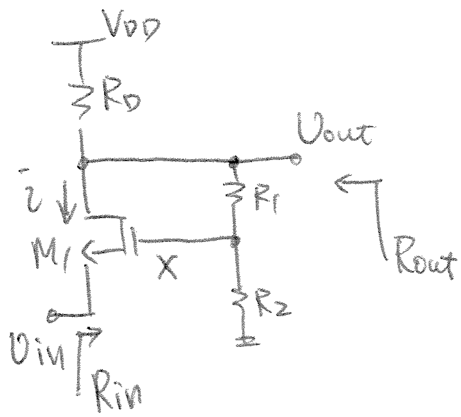
i_{in}
$\frac{1}{g_{m1}}$
$-i_{in}$

16. $\lambda > 0$





17.

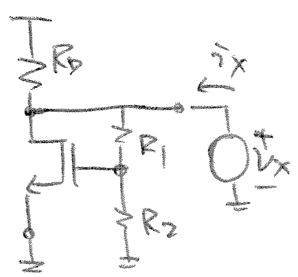


$$-v_{out} = i\,[R_D \,\|\, (R_1+R_2)]$$

$$i = g_{m1}(v_X - v_{in}) = g_{m1}\left(v_{out} \times \frac{R_2}{R_1+R_2} - v_{in}\right)$$

Combining the equations above yields:

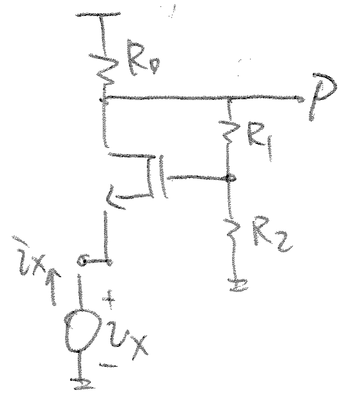
$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}[R_D \| (R_1+R_2)]}{1 + \frac{R_2}{R_1+R_2} g_{m1}[R_D \| (R_1+R_2)]} \triangleq A_v$$



By KCL,

$$i_X = \frac{v_X}{R_1+R_2} + \frac{v_X}{R_D} + g_{m1}\left(v_X \frac{R_2}{R_1+R_2}\right)$$

$$\Rightarrow \frac{v_X}{i_X} = R_{out} = [(R_1+R_2) \| R_D]\left[1 + g_{m1}\frac{R_2}{R_1+R_2}(R_D \| (R_1+R_2))\right]$$



By KCL,

$$i_x = g_{m1}\left(v_x - v_P \frac{R_2}{R_1+R_2}\right)$$

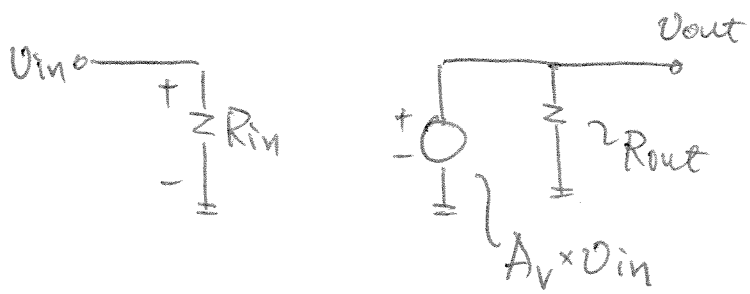
$$\Rightarrow v_P = \left(v_x - \frac{i_x}{g_{m1}}\right)\left(\frac{R_1+R_2}{R_2}\right) \quad \text{—(1)}$$

$$i_x = \frac{v_P}{R_D \parallel (R_1+R_2)} \quad \text{—(2)}$$

Substitute (1) into (2) & solve for $\frac{v_x}{i_x}$:

$$\frac{v_x}{i_x} = R_{in} = \frac{1}{g_{m1}}\left[1 + g_{m1}\{R_D \parallel (R_1+R_2)\}\frac{R_2}{R_1+R_2}\right]$$

Model:



12.18 (a)
- Sense mechanism: Voltage at the source of M_3
- Return mechanism: Voltage at the gate of M_2

(b)
- Sense mechanism: Voltage at the source of M_3
- Return mechanism: Voltage at the gate of M_2

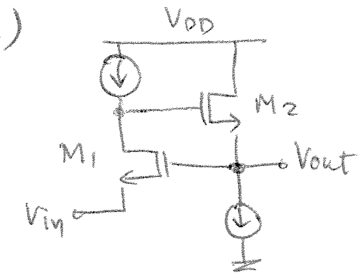
(c)
- Sense mechanism: Current Ωowing through R_1
- Return mechanism: Voltage at the gate of M_2

(d)
- Sense mechanism: Current Ωowing through R_1
- Return mechanism: Voltage at the gate of M_2

(e)
- Sense mechanism: Voltage divider formed by R_1 and R_2
- Return mechanism: Voltage at the gate of M_2

(f)
- Sense mechanism: Voltage at the source of M_3
- Return mechanism: Voltage at the gate of M_2

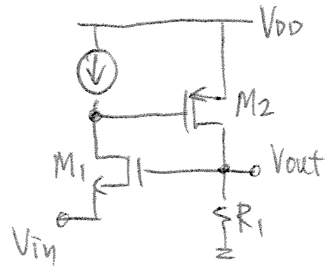
19. (a)



Sense Mechanism:
Voltage sensing at Vout.

Return Mechanism:
Voltage to Gate of M_1.

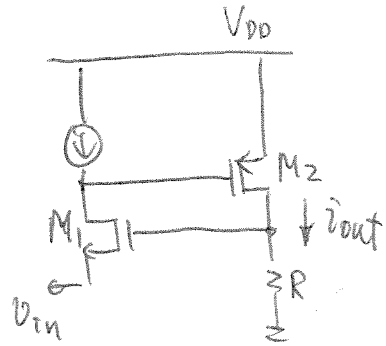
(b)



Sense Mechanism:
Voltage output from M_2.

Return Mechanism:
Voltage to Gate of M_1.

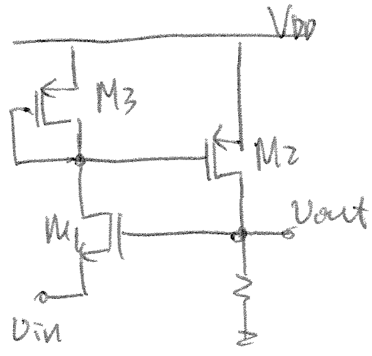
(c)



Sense Mechanism:
R_1

Return Mechanism:
Voltage to Gate of M_1.

(d)



Sense Mechanism:
Voltage output of M_2

Return Mechanism:
Voltage to Gate of M_1

12.20 (a)
- Sense mechanism: Voltage at the gate of M_2
- Return mechanism: Current through M_2

(b)
- Sense mechanism: Voltage at the gate of M_2
- Return mechanism: Current through M_2

(c)
- Sense mechanism: Voltage at the source of M_2
- Return mechanism: Current through M_2

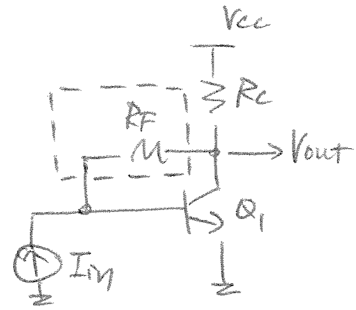
(d)
- Sense mechanism: Voltage at the gate of M_2
- Return mechanism: Current through M_2

21. (a) Sense Mechanism:

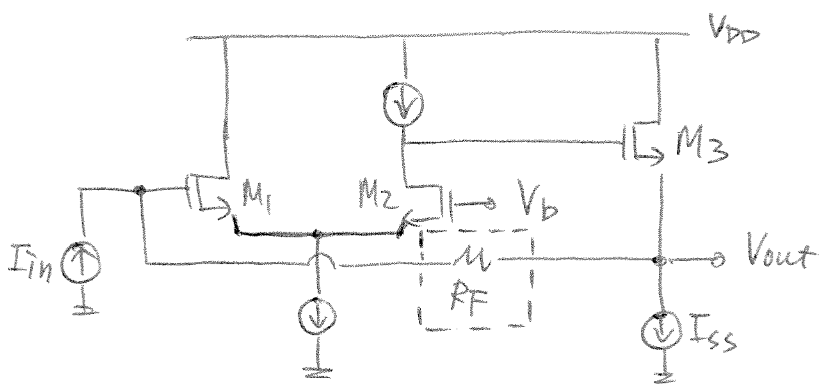
Resistor (R_F) — Voltage

Return Mechanism:

Current through R_F.



(b)



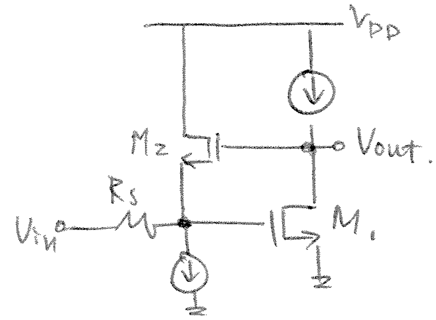
Sense Mechanism:

Resistor (R_F) — Voltage

Return Mechanism:

Current through R_F

22. First, recognize that
(a) both input & output are voltages.



* V_{in} primarily drives the Gate of M_1.

Sequence: Suppose V_{in} increases by ΔV_{in}

$\Rightarrow$ V_{out} drops by $+g_{m1} \Delta V_{in} \times r_{o1}$ (Common-Source)

$\Rightarrow$ Source of M_2 decreases by same amount (Source follower)

$\therefore$ $V_{in} \uparrow \Rightarrow V_{M_1,D} \downarrow \Rightarrow V_{M_1,G} \downarrow$

$\Rightarrow$ effective V_{in} driving $M_{1,G} \downarrow$

$\Rightarrow$ negative feedback

(b) $v_{in} \uparrow \Rightarrow v_{out} \downarrow \Rightarrow v_{M_2,G} \uparrow$

$\Rightarrow$ effective v_{in} driving $M_{1,G} \uparrow$

$\Rightarrow$ positive feedback.

(c) $v_{in} \uparrow \Rightarrow v_{out} \downarrow \Rightarrow v_{M1,G} \downarrow$

$\Rightarrow$ effective v_{in} driving $M_{1,G} \downarrow$

$\Rightarrow$ negative feedback.

(d) $v_{in} \uparrow \Rightarrow v_{out} \uparrow$ (common-base, M_1)

$\Rightarrow v_{M1,S} \downarrow$

$\Rightarrow$ effective v_{in} driving $M_{1,S} \downarrow$

$\Rightarrow$ negative feedback.

12.23 If I_{in} increases, then the voltage at the gate of M_1 will increase, meaning I_{D1} will increase. This will cause the drain voltage of M_1 to decrease, meaning I_{D2} will decrease and V_{out} will increase. This will cause the voltage at the gate of M_1 to decrease, which counters the original increase, meaning there is $\boxed{\text{negative feedback}}$.

Fig. 12.83 (a) $V_{in} \uparrow, V_{S1} \uparrow, V_{G3} \uparrow, V_{out} \downarrow, V_{G3} \downarrow \Rightarrow$ $\boxed{\text{negative feedback}}$.

(b) $V_{in} \uparrow, V_{S1} \uparrow, V_{G3} \uparrow, V_{out} \downarrow, V_{G3} \uparrow \Rightarrow$ $\boxed{\text{positive feedback}}$.

(c) Same as (b), $\boxed{\text{positive feedback}}$.

(d) Same as (a), $\boxed{\text{negative feedback}}$.

(e) $V_{in} \uparrow, V_{S1} \uparrow, V_{out} \uparrow, V_{G2} \uparrow, V_{out} \downarrow \Rightarrow$ $\boxed{\text{negative feedback}}$.

(f) $V_{in} \uparrow, V_{S1} \uparrow, V_{G3} \uparrow, V_{out} \downarrow, V_{G3} \uparrow \Rightarrow$ $\boxed{\text{positive feedback}}$.

Fig. 12.84 (a) $V_{in} \uparrow, V_{G2} \uparrow, V_{out} \uparrow, V_{G2} \downarrow \Rightarrow$ $\boxed{\text{negative feedback}}$.

(b) $V_{in} \uparrow, V_{G2} \uparrow, V_{out} \downarrow, V_{G2} \uparrow \Rightarrow$ $\boxed{\text{positive feedback}}$.

(c) Same as (b), $\boxed{\text{positive feedback}}$.

(d) $V_{in} \uparrow, V_{G2} \uparrow, V_{out} \downarrow, V_{G2} \uparrow \Rightarrow$ $\boxed{\text{positive feedback}}$.

Fig. 12.85 (a) $I_{in} \uparrow, V_{G1} \uparrow$ (consider I_{in} Ωows through an equivalent small-signal resistance of $1/g_{m2}$ at the gate of M_1), $V_{out} \downarrow, V_{G1} \downarrow \Rightarrow$ $\boxed{\text{negative feedback}}$.

(b) $I_{in} \uparrow, V_{G1} \uparrow, V_{out} \downarrow, V_{G1} \uparrow \Rightarrow$ $\boxed{\text{positive feedback}}$.

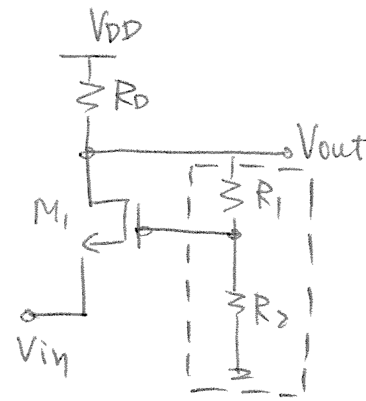
(c) $I_{in} \uparrow, V_{G1} \uparrow, V_{out} \downarrow, V_{G1} \downarrow \Rightarrow$ $\boxed{\text{negative feedback}}$.

(d) $I_{in} \uparrow, V_{S1} \uparrow, V_{out} \uparrow, V_{S1} \downarrow \Rightarrow$ $\boxed{\text{negative feedback}}$.

Fig. 12.86 (a) $I_{in} \uparrow, V_{BE1} \uparrow, V_{out} \downarrow, V_{BE1} \downarrow \Rightarrow$ $\boxed{\text{negative feedback}}$.

(b) $I_{in} \uparrow, V_{G3} \uparrow, V_{out} \uparrow, V_{G3} \uparrow \Rightarrow$ $\boxed{\text{positive feedback}}$.

25.



$R_1 + R_2 \gg R_D$

$\lambda = 0$

(Without feedback)

$$\frac{v_{out}}{v_{in}} = A_{O.L.} = g_m R_D$$

Feedback factor, k:

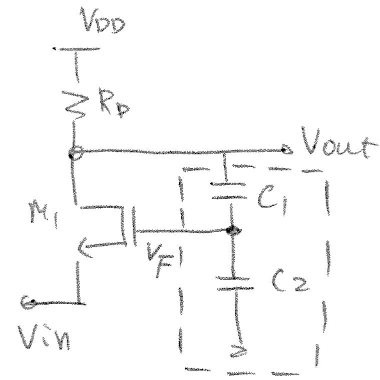
$$k = \frac{R_2}{R_1 + R_2}$$

$$\Rightarrow A_{C.L.} = \left.\frac{v_{out}}{v_{in}}\right|_{C.L.} = \frac{A_{O.L}}{1 + A_{O.L} \cdot K} = \frac{g_m R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D}$$

$$R_{in,CLOSED} = \frac{1}{g_{m1}}\left(1 + \frac{R_2}{R_1+R_2} g_m R_D\right)$$

$$R_{out,CLOSED} = \frac{R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D}$$

26.



λ = 0

C_1, C_2 small.

(Without feedback)

$$A_{O.L.} = g_m R_D$$

Feedback factor, k :

$$k = \frac{C_1}{C_1 + C_2}$$

$$\Rightarrow A_{C.L.} = \left.\frac{V_{out}}{V_{in}}\right|_{C.L.} = \frac{A_{O.L}}{1 + A_{O.L.} k} = \frac{g_m R_D}{1 + \frac{C_1}{C_1+C_2} g_m R_D}$$

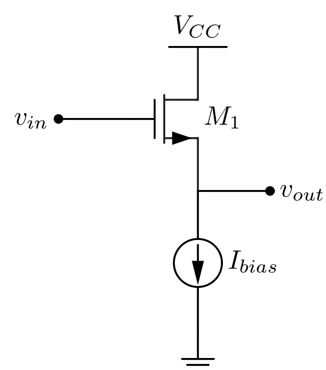
$$R_{in,CLOSED} = \frac{1}{g_m}\left[1 + \frac{C_1}{C_1+C_2} g_m R_D\right]$$

$$R_{out,CLOSED} = \frac{R_D}{1 + \frac{C_1}{C_1+C_2} g_m R_D}$$

12.27

$$
\begin{aligned}
A_{OL} &= g_{m1}\left(r_{o2} \parallel r_{o4}\right)\left(\frac{g_{m5}r_{o5}}{1+g_{m5}r_{o5}}\right) \\
K &= 1 \text{ (since the output is fed back directly to the inverting input)} \\
\frac{v_{out}}{v_{in}} &= \boxed{\frac{g_{m1}\left(r_{o2} \parallel r_{o4}\right)\left(\frac{g_{m5}r_{o5}}{1+g_{m5}r_{o5}}\right)}{1+g_{m1}\left(r_{o2} \parallel r_{o4}\right)\left(\frac{g_{m5}r_{o5}}{1+g_{m5}r_{o5}}\right)}} \\
R_{out,open} &= \frac{1}{g_{m5}} \parallel r_{o5} \\
R_{out,closed} &= \boxed{\frac{\frac{1}{g_{m5}} \parallel r_{o5}}{1+g_{m1}\left(r_{o2} \parallel r_{o4}\right)\left(\frac{g_{m5}r_{o5}}{1+g_{m5}r_{o5}}\right)}}
\end{aligned}
$$

Let's recall the gain and output impedance of a simple source follower, as shown in the following diagram.



$$
\begin{aligned}
A_v &= \frac{g_{m1}r_{o1}}{1+g_{m1}r_{o1}} \\
R_{out} &= \frac{1}{g_{m1}} \parallel r_{o1}
\end{aligned}
$$

We can see that the gain of the circuit in Fig. 12.90 is the gain of a simple source follower multiplied by a factor of

$$
\frac{g_{m1}\left(r_{o2} \parallel r_{o4}\right)}{1+g_{m1}\left(r_{o2} \parallel r_{o4}\right)\left(\frac{g_{m5}r_{o5}}{1+g_{m5}r_{o5}}\right)}
$$

This factor is less than 1, which means that the gain is reduced. However, we do get an improvement in output resistance, which is reduced by a factor of

$$
1+g_{m1}\left(r_{o2} \parallel r_{o4}\right)\left(\frac{g_{m5}r_{o5}}{1+g_{m5}r_{o5}}\right)
$$

12.28 (a) $V_{in} \uparrow, V_{G5} \uparrow, V_{out} \downarrow, V_{G5} \uparrow \Rightarrow$ positive feedback.

(b)

$$A_{loop} = \boxed{-g_{m1}g_{m5}\left(r_{o2} \parallel r_{o4}\right)r_{o5}}$$

Since the loop gain is negative, the feedback is positive.

12.29

$$A_{OL} = g_{m1} g_{m5} (r_{o1} \parallel r_{o3}) r_{o5}$$
$$K = 1$$
$$\frac{v_{out}}{v_{in}} = \boxed{\frac{g_{m1} g_{m5} (r_{o1} \parallel r_{o3}) r_{o5}}{1 + g_{m1} g_{m5} (r_{o1} \parallel r_{o3}) r_{o5}}}$$
$$R_{in,open} = R_{in,closed} = \boxed{\infty}$$
$$R_{out,open} = r_{o5}$$
$$R_{out,closed} = \boxed{\frac{r_{o5}}{1 + g_{m1} g_{m5} (r_{o1} \parallel r_{o3}) r_{o5}}}$$

Like the circuit in Problem 12.27, the closed loop gain is approximately (but slightly less than) 1. Looking at the equations, the closed loop gain of this circuit will typically be larger than the closed loop gain of the circuit in Problem 12.27.

The output impedance of this circuit is not quite as small as the output impedance of the circuit in Problem 12.27. Despite the loop gain being larger, the open loop output impedance is significantly higher than that of Problem 12.27, so that overall, the output impedance is slightly higher in this circuit.

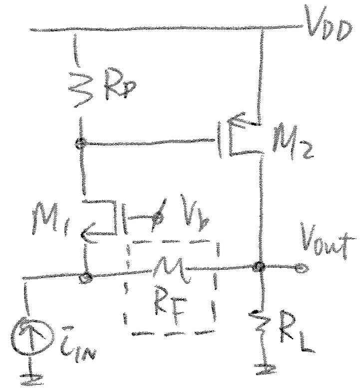
12.30 (a) $I_{in} \uparrow, V_{G2} \uparrow, V_{out} \uparrow, V_{S1} \uparrow, V_{G2} \uparrow \Rightarrow$ positive feedback.

(b)

$$A_{loop} = \boxed{-\frac{g_{m1} g_{m2} R_D \left(R_F + \frac{1}{g_{m1}}\right)}{\left[1 + g_{m2}\left(R_F + \frac{1}{g_{m1}}\right)\right](1 + g_{m1} R_F)}}$$

Since the loop gain is negative, the feedback is positive.

31.



$\lambda = 0$

$R_F \gg 1.$

(a) $i_{IN} \uparrow \Delta \Rightarrow \Delta i_{IN}$ mostly flows in $\frac{1}{g_{m1}} \Rightarrow V_{G,M2} \uparrow$
(Common Gate)
$\Rightarrow V_{out} \downarrow$ (Common Source)
$\Rightarrow R_F$ momentarily demands more current from i_{IN}
$\Rightarrow$ Negative feedback.

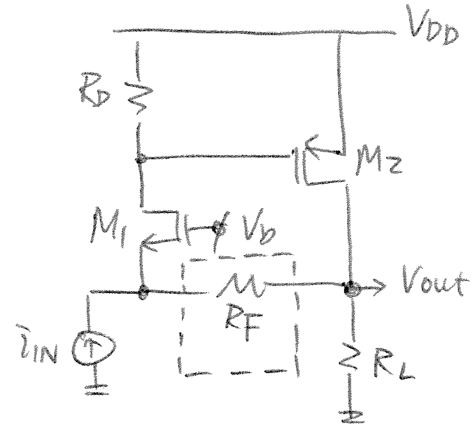
(b) $R_{O.L.} = \left.\frac{V_{out}}{i_{IN}}\right|_{O.L.} = -R_D \times g_{m2} R_L$

(c) K (feedback factor) $= \frac{-1}{R_F}$

$$\Rightarrow R_{C.L.} = \frac{R_{O.L}}{1 + R_{O.L.} \times K} = \frac{-R_D \times g_{m2} R_L}{1 + \frac{R_D}{R_F} g_{m2} R_L}$$

32.

$$R_{C.L.} = \frac{-g_{m2} R_D R_L}{1 + \frac{g_{m2} R_D R_L}{R_F}}$$



$$\text{loop gain} = \frac{g_{m2} R_D R_L}{R_F}$$

$$r_{in} \approx \frac{1}{g_{m1}}$$

$$r_{out} \approx R_L \qquad (R_F \text{ large})$$

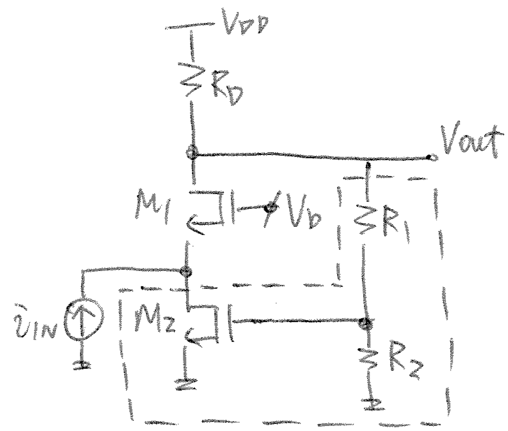
$$\Rightarrow r_{in}\Big|_{C.L.} = \frac{1/g_{m1}}{1 + \frac{g_{m2} R_D R_L}{R_F}}$$

$$r_{out}\Big|_{C.L.} = \frac{R_L}{1 + \frac{g_{m2} R_D R_L}{R_F}}$$

33.

$$R_{O.L.} = \frac{V_{out}}{i_{IN}} \text{ (no feedback)}$$

$$= R_D$$



$\lambda = 0$

$R_1 + R_2 \gg 1.$

K (feedback factor)

$$= g_{m2} \times \frac{R_2}{R_1 + R_2}$$

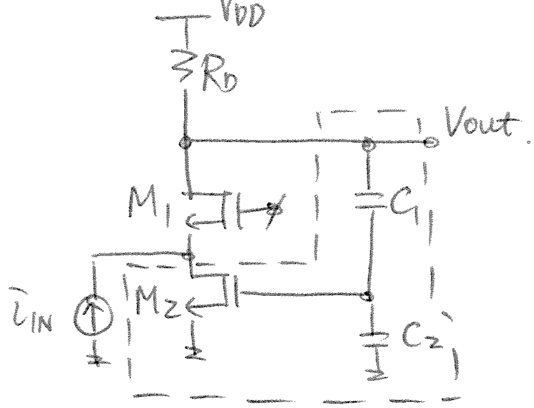
$$\Rightarrow R_{C.L.} = \frac{V_{out}}{i_{IN}} = \frac{R_D}{1 + R_D \times g_{m2}\frac{R_2}{R_1+R_2}}$$

$$r_{in}|_{C.L.} = \frac{1/g_{m1}}{1 + R_D \times g_{m2}\frac{R_2}{R_1+R_2}}$$

$$r_{out}|_{C.L.} = \frac{R_D}{1 + R_D \times g_{m2}\frac{R_2}{R_1+R_2}}$$

34.

$$R_{O.L.} = \frac{v_{out}}{i_{IN}} \text{ (no feedback)}$$
$$= R_D$$



K (feedback factor)

$$= g_{m2} \times \frac{C_1}{C_1 + C_2}$$

$$\Rightarrow R_{C.L.} = \frac{v_{out}}{i_{IN}} = \frac{R_D}{1 + R_D \times g_{m2} \frac{C_1}{C_1 + C_2}}$$

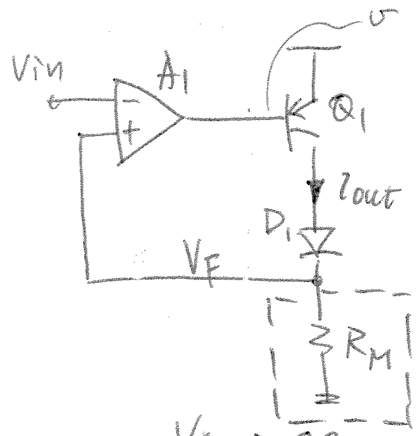
$$r_{in}|_{C.L.} = \frac{1/g_{m1}}{1 + R_D \times g_{m2} \frac{C_1}{C_1 + C_2}}$$

$$r_{out}|_{C.L.} = \frac{R_D}{1 + R_D \times g_{m2} \frac{C_1}{C_1 + C_2}}$$

35.

(a) $G_{O.L.} = \frac{i_{out}}{v_{in}} = g_{m1} A_1$

(common emitter)



(b) K (feedback factor)

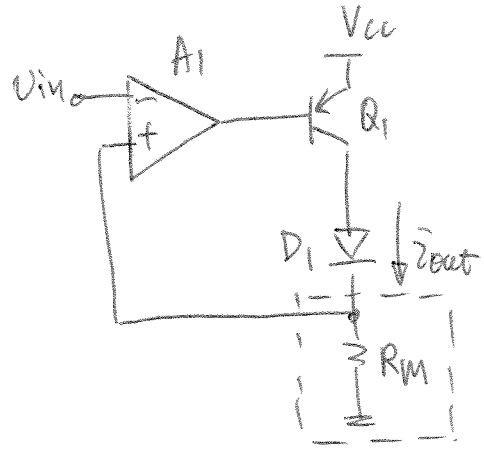
$\Rightarrow V_F = i_{out} \times R_M$

$\Rightarrow K = \frac{V_F}{i_{out}} = R_M$

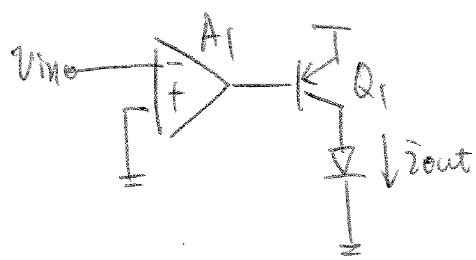
$\therefore$ Loop Gain $= G_{OL} K = g_{m1} A_1 R_M$

$$G_{C.L.} = \frac{G_{OL}}{1 + G_{OL} K} = \frac{g_{m1} A_1}{1 + g_{m1} A_1 R_M}$$

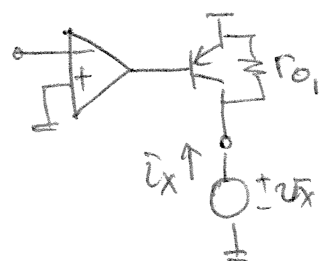
36.



Since R_M is small, the following circuit results:



$\Rightarrow$



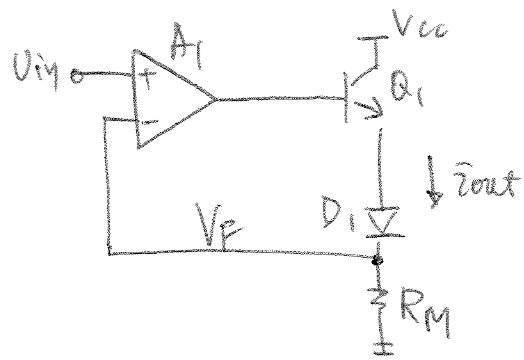
$$\therefore \; R_{out,OPEN} = \frac{v_x}{i_x} = r_{o1}$$

$$G_{OL} = \frac{i_{out}}{v_{in}} = A_1 g_{m1} \qquad K = R_M$$

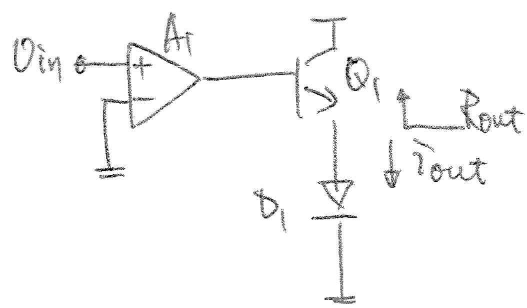
$$\therefore \; R_{out,CLOSED} = R_{out,OPEN}(1 + G_{OL}K)$$

$$= r_{o1}(1 + A_1 g_{m1} R_M)$$

37.



Since R_M is small, the open-loop equivalent becomes the following:



$$G_{OL} = \frac{i_{out}}{v_{in}} \approx A_1 g_{m1}$$

$$R_{out} = \frac{r_\pi}{\beta+1} \approx \frac{1}{g_{m1}}$$

$$K = \frac{V_F}{i_{out}} = R_M$$

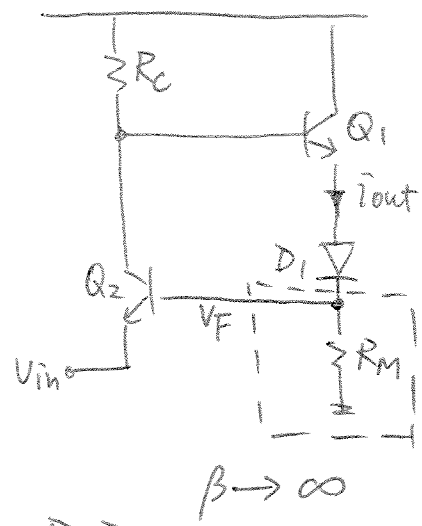
$$\Rightarrow G_{C.L.} = \frac{G_{OL}}{1+G_{OL}K} = \frac{A_1 g_{m1}}{1+g_{m1}A_1R_M}$$

$$\text{Loop Gain} = G_{OL}K = g_{m1}A_1R_M$$

$$R_{out,\,CLOSED} = \frac{1}{g_{m1}}(1+g_{m1}A_1R_M)$$

This circuit provides a much lower output resistance which in general is non-desirable (ideally any current source should have high impedance.)

38.



Using procedure in Ex 12.21

$G_{O.L.} = \frac{i_{out}}{V_{in}} = g_{m2} R_C \times g_{m1}$

K (feedback factor)

$= \frac{V_F}{i_{out}} = R_M.$

$\Rightarrow$ loop gain $= G_{O.L.} \times K = g_{m1} g_{m2} R_C R_M$

$\Rightarrow$ closed-loop gain $G_{CL} = \frac{g_{m1} g_{m2} R_C}{1 + g_{m1} g_{m2} R_C R_M}$

Using procedure in Ex. 12.22

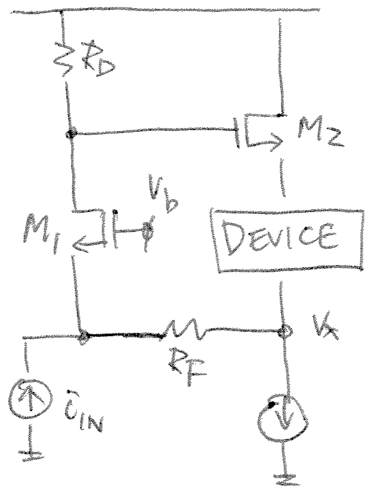
$G_{OL} = g_{m1} g_{m2} R_C$ $\qquad K = R_M$

$r_{in}|_{O.L.} = \frac{1}{g_{m1}}$ $\qquad r_{out}|_{O.L.} \cong \frac{1}{g_{m2}}$

$r_{in}|_{C.L.} = \frac{1}{g_{m1}}(1 + g_{m1} g_{m2} R_C R_M)$

$r_{out}|_{C.L.} = \frac{1}{g_{m2}}(1 + g_{m1} g_{m2} R_C R_M)$

39.



(a) i_{IN} ↑ Δ ⇒ Most of i_{IN} flows into $1/g_{m1}$

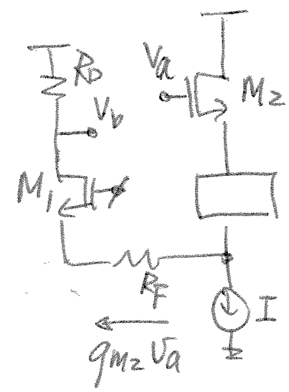
⇒ $V_{G,M2}$ ↑ (Common Gate)

⇒ V_x ↑ (Source Follower)

⇒ R_F momentarily provides more current to Source of M_1

⇒ $V_{G,M2}$ ↑ ⇒ Positive feedback.

(b)



$$v_a \times g_{m2} \times R_D = v_b$$

$$\Rightarrow \text{loop gain} = -\frac{v_b}{v_a} = -g_{m2}R_D.$$

Since loop gain is negative, feedback is positive.

12.40 (a)

$$A_{OL} = g_{m2}\left(R_C \parallel r_{\pi 2}\right)$$

$$A_{loop} = \frac{g_{m1} g_{m2}\left(R_F \parallel R_M\right)\left(R_C \parallel r_{\pi 2}\right)}{1 + g_{m1} R_F}$$

$$= \frac{g_{m2}\left(R_F \parallel R_M\right)\left(R_C \parallel r_{\pi 2}\right)}{\frac{1}{g_{m1}} + R_F}$$

$$\approx g_{m2}\left(R_C \parallel r_{\pi 2}\right)\frac{R_F \parallel R_M}{R_F} \text{ (since } R_F \text{ is very large)}$$

$$\frac{i_{out}}{i_{in}} = \boxed{\frac{g_{m2}\left(R_C \parallel r_{\pi 2}\right)}{1 + g_{m2}\left(R_C \parallel r_{\pi 2}\right)\frac{R_F \parallel R_M}{R_F}}}$$

$$R_{in,open} = \frac{1}{g_{m1}} \parallel r_{\pi 1}$$

$$R_{in,closed} = \boxed{\frac{\frac{1}{g_{m2}} \parallel r_{\pi 1}}{1 + g_{m2}\left(R_C \parallel r_{\pi 2}\right)\frac{R_F \parallel R_M}{R_F}}}$$

$$R_{out,open} = R_{out,closed} = \boxed{\infty} \text{ (since } V_A = \infty\text{)}$$

(b)

$$A_{OL} = -g_{m2} R_M\left(R_C \parallel r_{\pi 2}\right)$$

$$A_{loop} \approx g_{m2}\left(R_C \parallel r_{\pi 2}\right)\frac{R_F \parallel R_M}{R_F} \text{ (same as (a))}$$

$$\frac{v_{out}}{i_{in}} = \boxed{-\frac{-g_{m2} R_M\left(R_C \parallel r_{\pi 2}\right)}{1 + g_{m2}\left(R_C \parallel r_{\pi 2}\right)\frac{R_F \parallel R_M}{R_F}}}$$

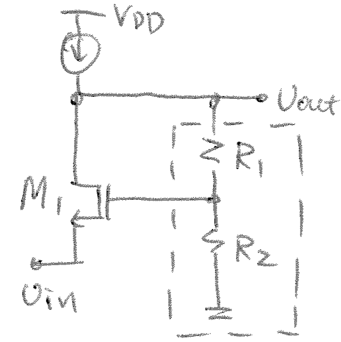
$$R_{in,open} = \frac{1}{g_{m1}} \parallel r_{\pi 1}$$

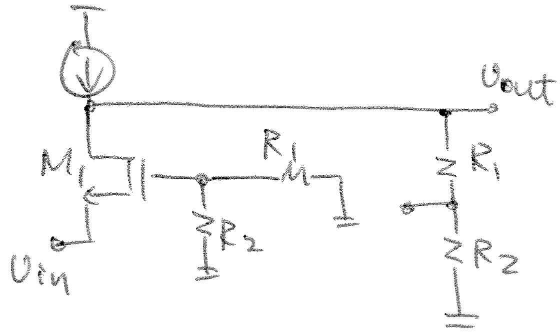
$$R_{in,closed} = \boxed{\frac{\frac{1}{g_{m1}} \parallel r_{\pi 1}}{1 + g_{m2}\left(R_C \parallel r_{\pi 2}\right)\frac{R_F \parallel R_M}{R_F}}}$$

$$R_{out,open} = R_M \parallel R_F$$

$$R_{out,closed} = \boxed{\frac{R_M \parallel R_F}{1 + g_{m2}\left(R_C \parallel r_{\pi 2}\right)\frac{R_F \parallel R_M}{R_F}}}$$

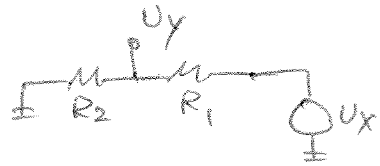
41. Breaking the feedback network results in the following circuit:





Feedback factor

$$= K = \frac{V_Y}{V_X} = \frac{R_2}{R_1+R_2}$$



$$A_{O.L.} = +g_{m1}(R_1+R_2)$$

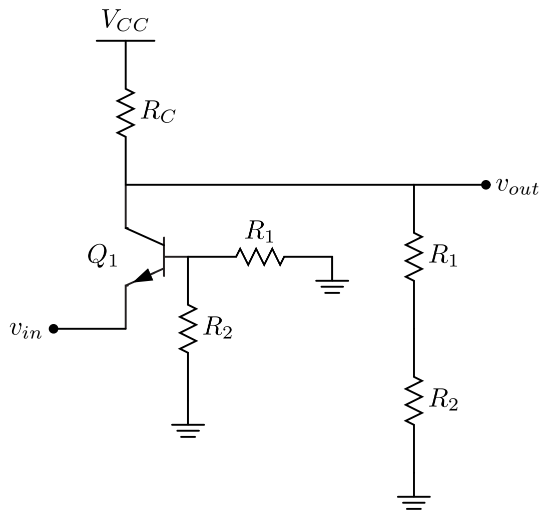
$$\text{Loop Gain} = A_{O.L.}K = g_{m1}R_2$$

$$\therefore A_{C.L.} = \frac{A_{OL}}{1+A_{OL}K} = \frac{g_{m1}(R_1+R_2)}{1+g_{m1}R_2}$$

$$R_{in,CLOSED} = \frac{1}{g_{m1}}(1+g_{m1}R_2)$$

$$R_{out,CLOSED} = \frac{R_1+R_2}{1+g_{m1}R_2}$$

12.42 We can break the feedback network as shown here:



$$A_{OL} = \frac{R_C \parallel (R_1 + R_2)}{\frac{1}{g_{m1}} + \frac{R_1 \| R_2}{1+\beta}}$$

$$K = \frac{R_2}{R_1 + R_2}$$

$$\frac{v_{out}}{v_{in}} = \boxed{\frac{\frac{R_C \| (R_1+R_2)}{\frac{1}{g_{m1}} + \frac{R_1 \| R_2}{1+\beta}}}{1 + \frac{R_2}{R_1+R_2} \frac{R_C \| (R_1+R_2)}{\frac{1}{g_{m1}} + \frac{R_1 \| R_2}{1+\beta}}}}$$

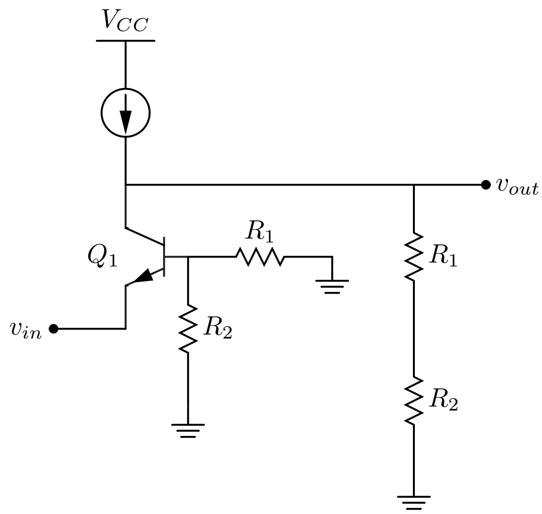
$$R_{in,open} = \frac{r_{\pi 1} + R_1 \parallel R_2}{1+\beta}$$

$$R_{in,closed} = \boxed{\left(\frac{r_{\pi 1} + R_1 \parallel R_2}{1+\beta}\right)\left(1 + \frac{R_2}{R_1 + R_2} \frac{R_C \parallel (R_1 + R_2)}{\frac{1}{g_{m1}} + \frac{R_1 \| R_2}{1+\beta}}\right)}$$

$$R_{out,open} = R_C \parallel (R_1 + R_2)$$

$$R_{out,closed} = \boxed{\frac{R_C \parallel (R_1 + R_2)}{1 + \frac{R_2}{R_1+R_2} \frac{R_C \| (R_1+R_2)}{\frac{1}{g_{m1}} + \frac{R_1 \| R_2}{1+\beta}}}}$$

12.43 We can break the feedback network as shown here:



$$A_{OL} = \frac{R_1 + R_2}{\frac{1}{g_{m1}} + \frac{R_1 \| R_2}{1+\beta}}$$

$$K = \frac{R_2}{R_1 + R_2}$$

$$\frac{v_{out}}{v_{in}} = \boxed{\frac{\frac{R_1+R_2}{\frac{1}{g_{m1}} + \frac{R_1\|R_2}{1+\beta}}}{1 + \frac{R_2}{\frac{1}{g_{m1}} + \frac{R_1\|R_2}{1+\beta}}}}$$

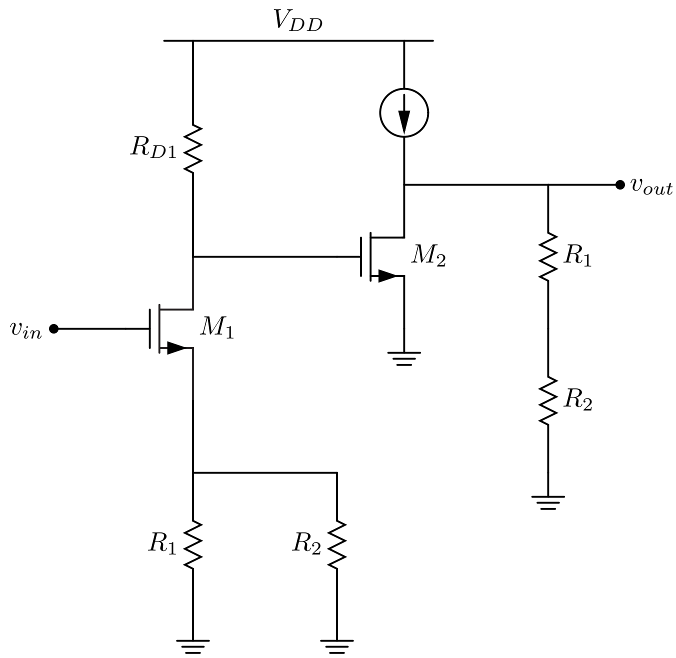
$$R_{in,open} = \frac{r_{\pi 1} + R_1 \parallel R_2}{1+\beta}$$

$$R_{in,closed} = \boxed{\left(\frac{r_{\pi 1} + R_1 \parallel R_2}{1+\beta}\right)\left(1 + \frac{R_2}{\frac{1}{g_{m1}} + \frac{R_1\|R_2}{1+\beta}}\right)}$$

$$R_{out,open} = R_1 + R_2$$

$$R_{out,closed} = \boxed{\frac{R_1 + R_2}{1 + \frac{R_2}{\frac{1}{g_{m1}} + \frac{R_1\|R_2}{1+\beta}}}}$$

12.44 We can break the feedback network as shown here:



$$A_{OL} = \frac{g_{m1} g_{m2} R_{D1} (R_1 + R_2)}{1 + g_{m1} (R_1 \parallel R_2)}$$

$$K = \frac{R_2}{R_1 + R_2}$$

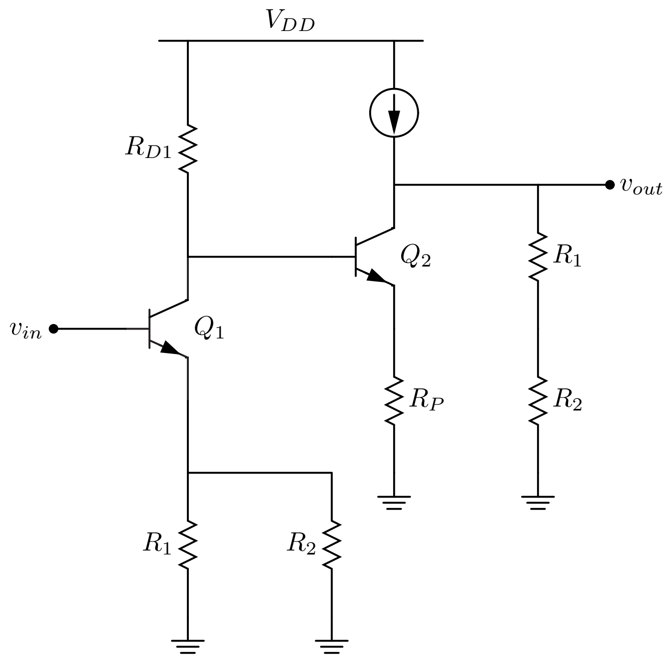
$$\frac{v_{out}}{v_{in}} = \boxed{\frac{\frac{g_{m1} g_{m2} R_{D1} (R_1 + R_2)}{1 + g_{m1} (R_1 \parallel R_2)}}{1 + \frac{g_{m1} g_{m2} R_{D1} R_2}{1 + g_{m1} (R_1 \parallel R_2)}}}$$

$$R_{in,open} = R_{in,closed} = \boxed{\infty}$$

$$R_{out,open} = R_1 + R_2$$

$$R_{out,closed} = \boxed{\frac{R_1 + R_2}{1 + \frac{g_{m1} g_{m2} R_{D1} R_2}{1 + g_{m1} (R_1 \parallel R_2)}}}$$

12.45 We can break the feedback network as shown here:



$$A_{OL} = \frac{g_{m1} g_{m2} \left[R_{D1} \parallel \left(r_{\pi 2} + (1+\beta) R_P \right) \right] (R_1 + R_2)}{\left[1 + g_{m1} (R_1 \parallel R_2) \right] (1 + g_{m2} R_P)}$$

$$K = \frac{R_2}{R_1 + R_2}$$

$$\frac{v_{out}}{v_{in}} = \boxed{\frac{\frac{g_{m1} g_{m2} [R_{D1} \| (r_{\pi 2} + (1+\beta) R_P)] (R_1 + R_2)}{[1 + g_{m1} (R_1 \| R_2)] (1 + g_{m2} R_P)}}{1 + \frac{g_{m1} g_{m2} [R_{D1} \| (r_{\pi 2} + (1+\beta) R_P)] R_2}{[1 + g_{m1} (R_1 \| R_2)] (1 + g_{m2} R_P)}}}$$

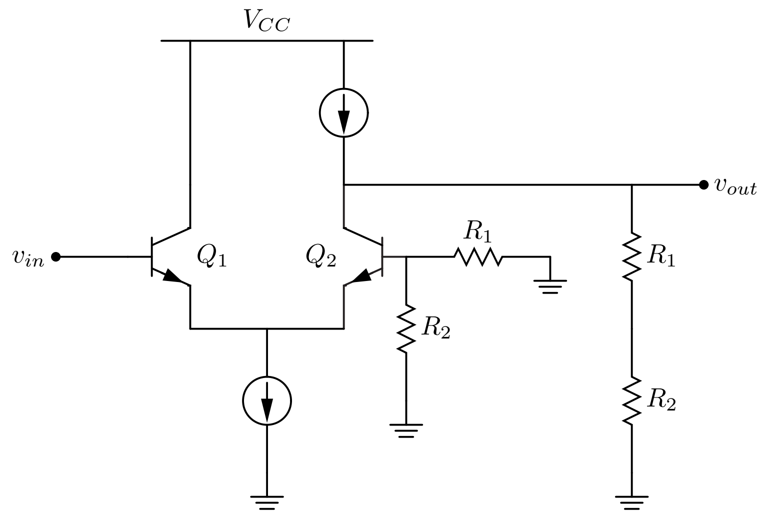
$$R_{in,open} = r_{\pi 1} + (1+\beta)(R_1 \parallel R_2)$$

$$R_{in,closed} = \boxed{\left\{ r_{\pi 1} + (1+\beta)(R_1 \parallel R_2) \right\} \left\{ 1 + \frac{g_{m1} g_{m2} \left[R_{D1} \parallel \left(r_{\pi 2} + (1+\beta) R_P \right) \right] R_2}{\left[1 + g_{m1} (R_1 \parallel R_2) \right] (1 + g_{m2} R_P)} \right\}}$$

$$R_{out,open} = R_1 + R_2$$

$$R_{out,closed} = \boxed{\frac{R_1 + R_2}{1 + \frac{g_{m1} g_{m2} [R_{D1} \| (r_{\pi 2} + (1+\beta) R_P)] R_2}{[1 + g_{m1} (R_1 \| R_2)] (1 + g_{m2} R_P)}}}$$

12.46 We can break the feedback network as shown here:



$$A_{OL} = \left(\frac{\frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}}{\frac{1}{g_{m1}} + \frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}} \right) \left(\frac{R_1 + R_2}{\frac{1}{g_{m2}} + \frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}} \right)$$

$$K = \frac{R_2}{R_1 + R_2}$$

$$\frac{v_{out}}{v_{in}} = \boxed{\frac{\left(\frac{\frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}}{\frac{1}{g_{m1}} + \frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}} \right) \left(\frac{R_1 + R_2}{\frac{1}{g_{m2}} + \frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}} \right)}{1 + \left(\frac{\frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}}{\frac{1}{g_{m1}} + \frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}} \right) \left(\frac{R_2}{\frac{1}{g_{m2}} + \frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}} \right)}}$$

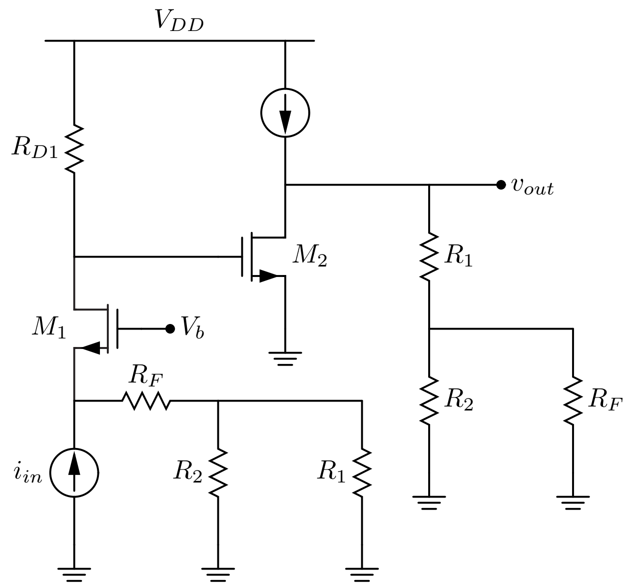
$$R_{in,open} = r_{\pi 1} + (1+\beta_1) \left(\frac{r_{\pi 2} + R_1 \parallel R_2}{1+\beta_2} \right)$$

$$R_{in,closed} = \boxed{\left[r_{\pi 1} + (1+\beta_1) \left(\frac{r_{\pi 2} + R_1 \parallel R_2}{1+\beta_2} \right) \right] \left[1 + \left(\frac{\frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}}{\frac{1}{g_{m1}} + \frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}} \right) \left(\frac{R_2}{\frac{1}{g_{m2}} + \frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}} \right) \right]}$$

$$R_{out,open} = R_1 + R_2$$

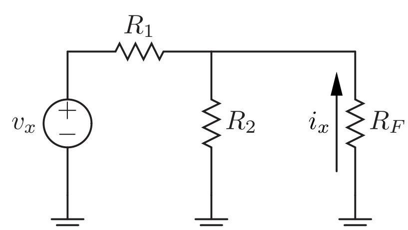
$$R_{out,closed} = \boxed{\frac{R_1 + R_2}{1 + \left(\frac{\frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}}{\frac{1}{g_{m1}} + \frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}} \right) \left(\frac{R_2}{\frac{1}{g_{m2}} + \frac{r_{\pi 2}+R_1 \| R_2}{1+\beta_2}} \right)}}$$

12.47 We can break the feedback network as shown here:



$$A_{OL} = -g_{m2}\left[r_{o2} \parallel (R_1 + R_2 \parallel R_F)\right] R_{D1} \frac{R_F + R_1 \parallel R_2}{\frac{1}{g_{m1}} + R_F + R_1 \parallel R_2}$$

To find the feedback factor K, we can use the following diagram:



$$K = \frac{i_x}{v_x} = -\frac{R_2}{(R_1 + R_2 \parallel R_F)(R_2 + R_F)} = -\frac{R_2 \parallel R_F}{R_F (R_1 + R_2 \parallel R_F)}$$

$$\frac{v_{out}}{i_{in}} = \boxed{-\frac{g_{m2}\left[r_{o2} \parallel (R_1 + R_2 \parallel R_F)\right] R_{D1} \frac{R_F + R_1 \parallel R_2}{\frac{1}{g_{m1}} + R_F + R_1 \parallel R_2}}{1 + \left\{g_{m2}\left[r_{o2} \parallel (R_1 + R_2 \parallel R_F)\right] R_{D1} \frac{R_F + R_1 \parallel R_2}{\frac{1}{g_{m1}} + R_F + R_1 \parallel R_2}\right\}\left\{\frac{R_2 \parallel R_F}{R_F (R_1 + R_2 \parallel R_F)}\right\}}}$$

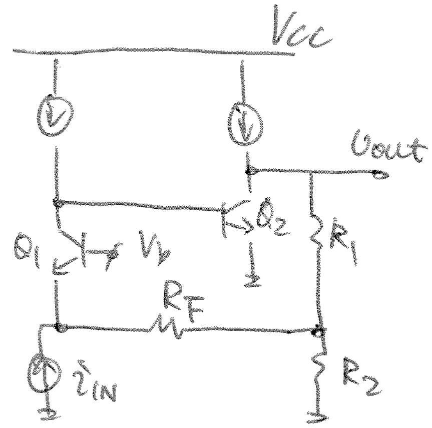
$$R_{in,open} = \frac{1}{g_{m1}} \parallel (R_F + R_1 \parallel R_2)$$

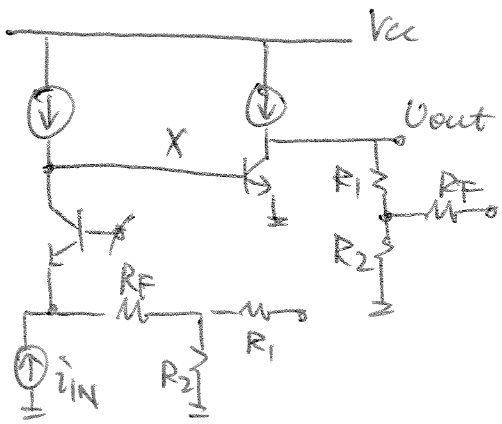
$$R_{in,closed} = \boxed{\frac{\frac{1}{g_{m1}} \parallel (R_F + R_1 \parallel R_2)}{1 + \left\{g_{m2}\left[r_{o2} \parallel (R_1 + R_2 \parallel R_F)\right] R_{D1} \frac{R_F + R_1 \parallel R_2}{\frac{1}{g_{m1}} + R_F + R_1 \parallel R_2}\right\}\left\{\frac{R_2 \parallel R_F}{R_F (R_1 + R_2 \parallel R_F)}\right\}}}$$

$$R_{out,open} = r_{o2} \parallel (R_1 + R_2 \parallel R_F)$$

$$R_{out,closed} = \boxed{\frac{r_{o2} \parallel (R_1 + R_2 \parallel R_F)}{1 + \left\{g_{m2}\left[r_{o2} \parallel (R_1 + R_2 \parallel R_F)\right] R_{D1} \frac{R_F + R_1 \parallel R_2}{\frac{1}{g_{m1}} + R_F + R_1 \parallel R_2}\right\}\left\{\frac{R_2 \parallel R_F}{R_F (R_1 + R_2 \parallel R_F)}\right\}}}$$

48. Breaking the feedback network results in the following circuit:

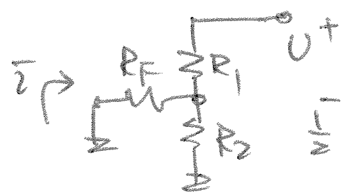




$$R_{OL} = \frac{v_{out}}{i_{IN}} = \frac{v_{out}}{v_X} \times \frac{v_X}{i_{IN}} = \left[-g_{m2}(R_1+R_2)\right] \times \left[g_{m1} r_{\pi 2} \left\{\frac{1}{g_{m1}} \,\|\, (R_F+R_2)\right\}\right]$$

$$R_{in,OPEN} = \frac{1}{g_{m1}} \,\|\, (R_F+R_2) \qquad R_{out,OPEN} = R_1+R_2$$

$$K = \frac{\bar{i}}{v} = -\frac{(R_2 \| R_F)/R_F}{R_1+(R_2 \| R_F)}$$

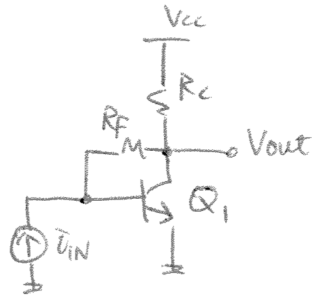


$$\therefore\ R_{CL} = \frac{R_{OL}}{1+R_{OL}K}$$

$$R_{in,CLOSED} = \frac{\frac{1}{g_{m1}} \,\|\, (R_F+R_2)}{1+R_{OL}K}$$

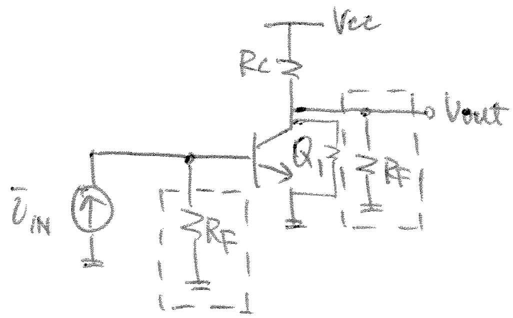
$$R_{out,CLOSED} = \frac{R_1+R_2}{1+R_{OL}K}$$

49. The feedback network consists of R_F.



$(V_A < \infty)$

Using the method discussed in lecture, break the circuit as follows:



This is the open-loop circuit with consideration of I/O loading.

- By inspection,

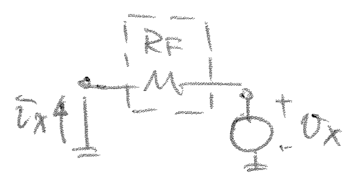
$$v_{out} = i_c \times (R_C \parallel R_F \parallel r_o)$$
$$= -g_m (i_{IN} \times (R_F \parallel r_\pi)) \times (R_C \parallel R_F \parallel r_o)$$

$$\Rightarrow R_{O.L.} = \frac{v_{out}}{i_{IN}} = -g_m (R_F \parallel r_\pi)(R_C \parallel R_F \parallel r_o) \quad — (1)$$

$R_{in,open} = (R_F \parallel r_\pi)$ $\qquad R_{out,open} = (R_C \parallel R_F \parallel r_o)$

- Feedback factor K:

$$K = \frac{i_x}{v_x} = -\frac{1}{R_F}$$

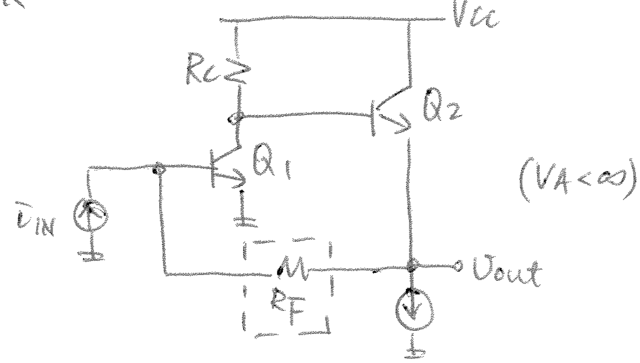


$$\therefore \quad R_{C.L.} = \frac{R_{O.L.}}{1 + R_{O.L.} \times K} = \frac{-g_m (R_F \| r_\pi)(R_C \| R_F \| r_o)}{1 + \frac{g_m (R_F \| r_\pi)(R_C \| R_F \| r_o)}{R_F}}$$

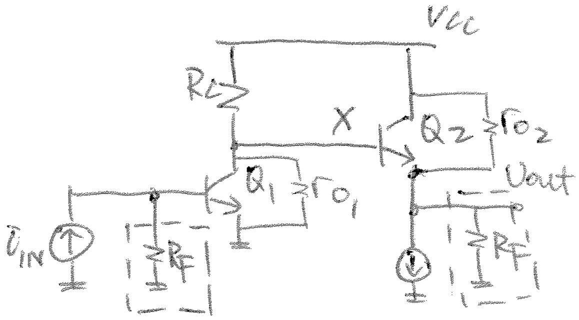
$$R_{in,CLOSED} = \frac{(R_F \| r_\pi)}{1 - \frac{R_{O.L.}}{R_F}} \qquad R_{out,CLOSED} = \frac{(R_C \| R_F \| r_o)}{1 - \frac{R_{O.L.}}{R_F}}$$

where $R_{O.L.}$ is given by (1).

50. The feedback network consists of R_F.



Using the method discussed in lecture, break the circuit as follows:



This is the open-loop circuit with consideration of I/O loading.

- Gain of common-emitter stage:

$$\frac{v_X}{i_{IN}} = -g_{m1}(R_F \| r_{\pi 1}) \times \left\{ R_C \| r_{o1} \| \left[r_{\pi 2} + (\beta_2+1)(R_F \| r_{o2}) \right] \right\}$$

- Gain of emitter-follower stage:

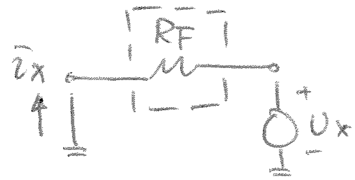
$$\frac{v_{out}}{v_X} = \frac{g_{m2}(R_F \| r_{o2})}{1 + g_{m2}(R_F \| r_{o2})}$$

$$\Rightarrow R_{O.L.} = \frac{v_X}{i_{IN}} \cdot \frac{v_{out}}{v_X} \qquad \text{—— (1)}$$

$R_{in,OPEN} = R_F \| r_{\pi 1}$ $\qquad$ $R_{out,OPEN} \cong R_F \| r_{o2} \| \frac{1}{g_{m2}}$

- Feedback factor k :

$$k = \frac{v_x}{i_x} = -\frac{1}{R_F}$$



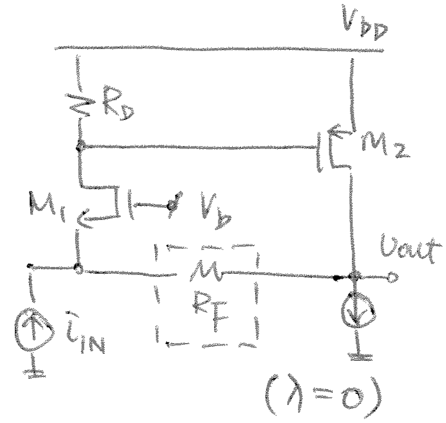
$$\therefore \; R_{C.L.} = \frac{R_{O.L.}}{1 + R_{O.L.} \times K} = \frac{R_{O.L.}}{1 - R_{O.L.}/R_F}$$

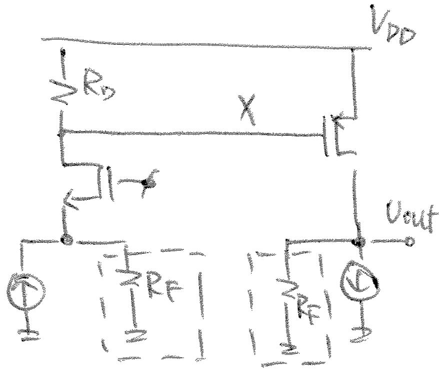
$$R_{in,CLOSED} = \frac{(R_F \parallel r_{\pi 1})}{1 - \frac{R_{O.L.}}{R_F}} \qquad R_{out,CLOSED} = \frac{R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}}{1 - \frac{R_{O.L.}}{R_F}}$$

where $R_{O.L.}$ is given by (1).

51.

(a) Breaking the feedback loop results in the following circuit:



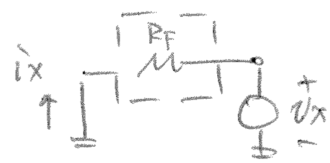


$$R_{O.L.} = \frac{v_X}{i_{IN}} \cdot \frac{v_{out}}{v_X}$$

$$= g_{m_1} R_D \left(\frac{1}{g_{m_1}} \| R_F\right) \times (-g_{m_2} R_F)$$

$$R_{in,OPEN} = \frac{1}{g_{m_1}} \| R_F \qquad R_{out,OPEN} = R_F$$

- Feedback factor K:



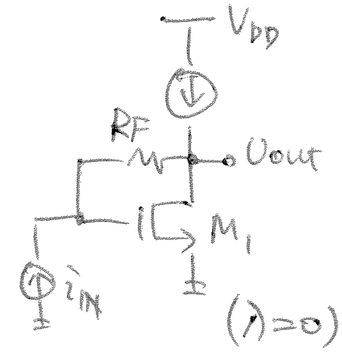
$$K = \frac{v_X}{i_X} = -\frac{1}{R_F}$$

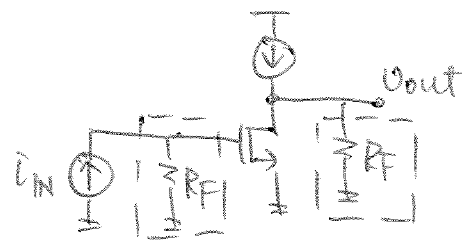
$$\Rightarrow R_{C.L.} = \frac{R_{O.L.}}{1 + R_{O.L.} \times K} = \frac{-g_{m_1} g_{m_2} R_D R_F \left(\frac{1}{g_{m_1}} \| R_F\right)}{1 + g_{m_1} g_{m_2} R_D \left(\frac{1}{g_{m_1}} \| R_F\right)}$$

$$R_{in,CLOSED} = \frac{\left(\frac{1}{g_{m_1}} \| R_F\right)}{1 + g_{m_1} g_{m_2} R_D \left(\frac{1}{g_{m_1}} \| R_F\right)}$$

$$R_{out,CLOSED} = \frac{R_F}{1 + g_{m_1} g_{m_2} R_D \left(\frac{1}{g_{m_1}} \| R_F\right)}$$

(b) Breaking the feedback loop results in the following circuit:

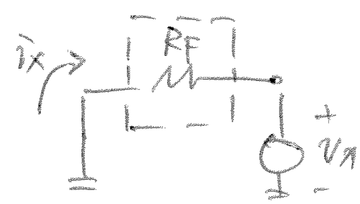




$$R_{O.L.} = \frac{v_{out}}{i_{IN}} = -g_m R_F R_F = -g_m R_F^2$$

$R_{in,OPEN} = R_F$ $\quad$ $R_{out,OPEN} = R_F$

- Feedback factor K:



$$K = \frac{v_x}{i_x} = -\frac{1}{R_F}$$

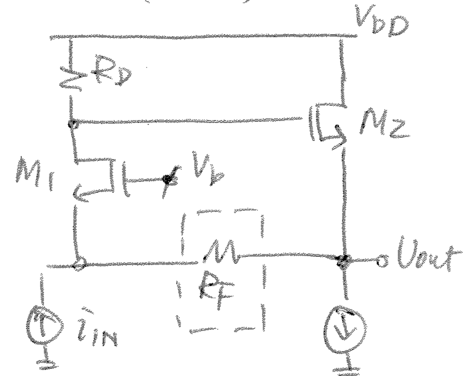
$$\Rightarrow R_{C.L.} = \frac{-g_m R_F^2}{1 + g_m R_F}$$

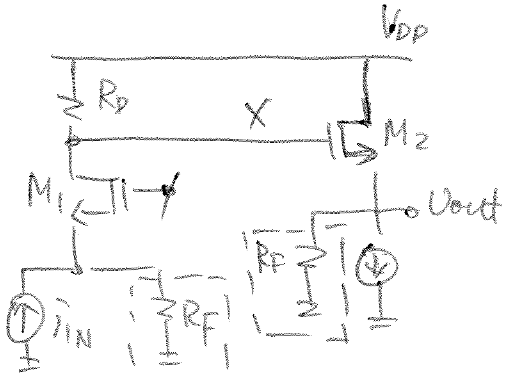
$$R_{in,CLOSED} = \frac{R_F}{1 + g_m R_F}$$

$$R_{out,CLOSED} = \frac{R_F}{1 + g_m R_F}$$

(c) Breaking the feedback loop results in the following circuit:

$(\lambda = 0)$





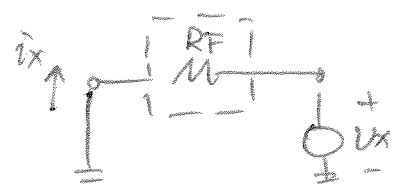
$$R_{O.L.} = \frac{v_{out}}{i_{IN}} = \frac{v_x}{i_{IN}} \cdot \frac{v_{out}}{v_x}$$

$$= g_{m1} R_D \left(\frac{1}{g_{m1}} \| R_F\right) \times g_{m2}\left(R_F \| \frac{1}{g_{m2}}\right)$$

$$R_{in,OPEN} = \left(\frac{1}{g_{m1}} \| R_F\right) \qquad R_{out,OPEN} = \left(R_F \| \frac{1}{g_{m2}}\right)$$

- Feedback factor K:

$$K = \frac{v_x}{i_x} = -\frac{1}{R_F}$$



(Note: Feedback is positive.)

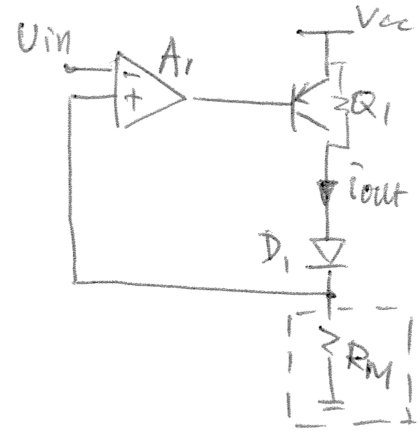
$$\Rightarrow R_{C.L.} = \frac{R_{O.L.}}{1 + R_{O.L.} \times K}$$

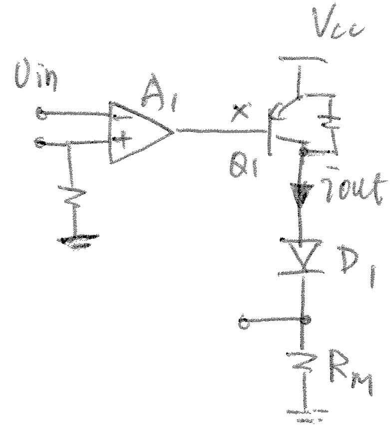
$$= \frac{g_{m1} g_{m2} R_D \left(1/g_{m1} \| R_F\right)\left(1/g_{m2} \| R_F\right)}{1 - g_{m1} g_{m2} \left(\frac{R_D}{R_F}\right)\left(\frac{1}{g_{m1}} \| R_F\right)\left(1/g_{m2} \| R_F\right)}$$

$$R_{IN,CLOSED} = \frac{\left(1/g_{m1} \| R_F\right)}{1 - \frac{R_{O.L.}}{R_F}} \qquad R_{out,CLOSED} = \frac{\left(1/g_{m2} \| R_F\right)}{1 - \frac{R_{O.L.}}{R_F}}$$

52. Breaking the feedback network (i.e. R_M) results in the following circuit:

($V_A < \infty$)

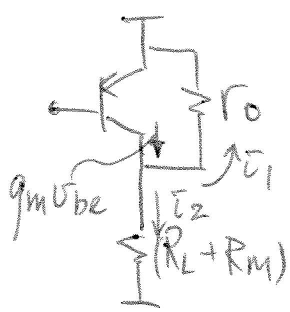




$$G_{O.L.} = \frac{i_{out}}{v_{in}} = \frac{i_{out}}{v_x} \times \frac{v_x}{v_{in}} \quad \text{—(1)}$$

$$= \underbrace{g_{m1} \times \frac{[(R_L + R_M) \| r_{o1}]}{(R_L + R_M)}}_{\text{(current division)}} \times (-A_1)$$

Note: current ($g_m v_{be}$) splits between r_o & [R_L (impedance of D_1) + R_M]



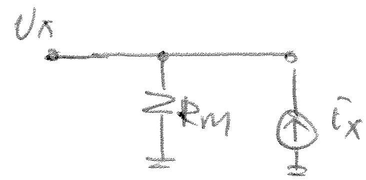
$g_m v_{be} = i_1 + i_2$

$R_{in,\,open} \to \infty$

$R_{out,\,open} = r_{o1} + R_M$

- Feedback factor K:

$K = \frac{v_x}{i_x} = R_M$



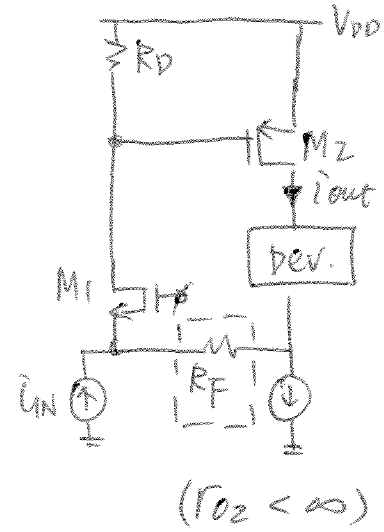
$$\therefore\ G_{C.L.} = \frac{G_{O.L.}}{1 + G_{O.L.} \times K} = \frac{G_{O.L.}}{1 + G_{O.L.} \times R_M}$$

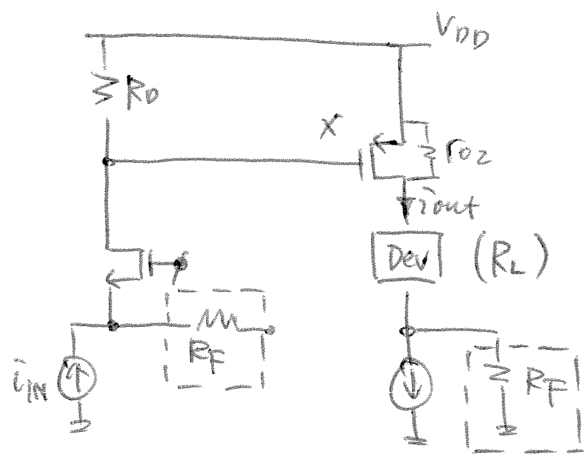
$$R_{in,\,CLOSED} \rightarrow \infty$$

$$R_{out,\,CLOSED} = (r_{O1} + R_M)(1 + G_{O.L.} \times R_M)$$

where $G_{O.L.}$ is given by (1)

53. Breaking the feedback loop results in the following circuit:





$$A_{I,O.L.} = \frac{i_{out}}{i_{IN}} = \frac{i_{out}}{v_X} \times \frac{v_X}{i_{IN}}$$

$$= -g_{m2} \times \frac{(R_L + R_F) \| r_{O2}}{(R_L + R_F)} \times R_D$$

$$R_{in,OPEN} = \frac{1}{g_{m1}} \qquad R_{out,OPEN} = r_{O2} + R_F$$

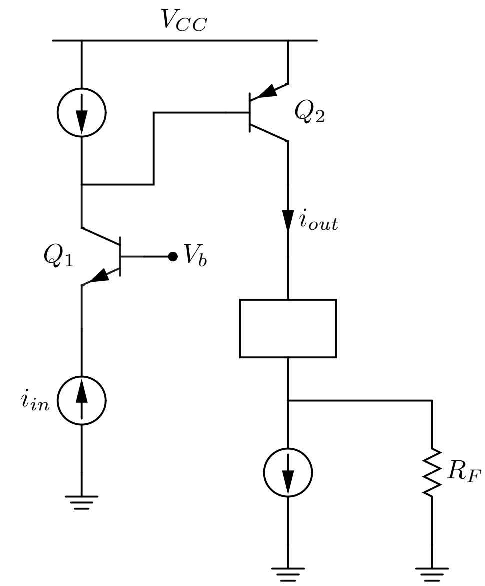
- Feedback factor K:

$$K = \frac{i_Y}{i_X} = -1$$

$$\Rightarrow A_{I,C.L.} = \frac{A_{I,O.L.}}{1 + A_{I,OL} \times K} = \frac{A_{I,OL}}{1 - A_{I,OL}}$$

$$R_{in,CLOSED} = \frac{1/g_{m1}}{1 - A_{I,OL}} \qquad R_{out,CLOSED} = (r_{O2} + R_F)(1 - A_{I,OL})$$

12.54 We can break the feedback network as shown here:



$$A_{OL} = -\beta_2$$

$$K = -1 \text{ (by inspection)}$$

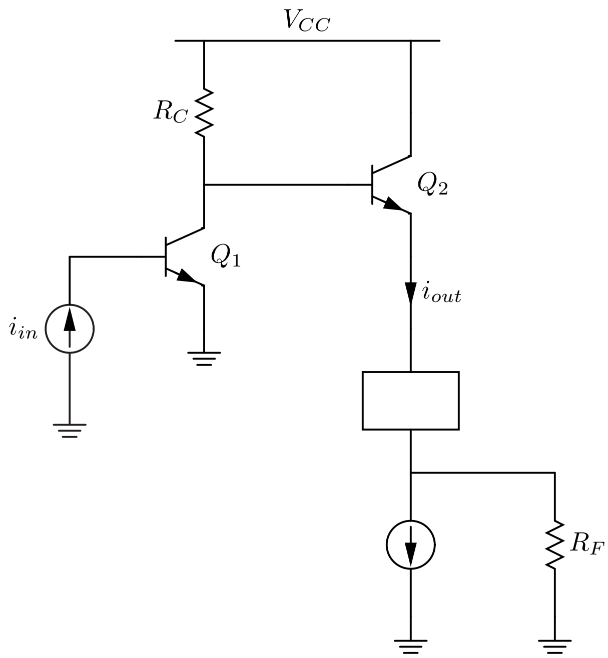
$$\frac{i_{out}}{i_{in}} = \boxed{-\frac{\beta_2}{1+\beta_2}}$$

$$R_{in,open} = \frac{1}{g_{m1}} \parallel r_{\pi 1}$$

$$R_{in,closed} = \boxed{\frac{\frac{1}{g_{m1}} \parallel r_{\pi 1}}{1+\beta_2}}$$

$$R_{out,open} = R_{out,closed} = \boxed{\infty} \text{ (since } V_A = \infty\text{)}$$

12.55 We can break the feedback network as shown here:



We can find $A_{OL} = \frac{i_{out}}{i_{in}}$ by using current dividers to determine how much of i_{in} goes to i_{out}. Let's assume the device has some small-signal resistance R_L.

$$A_{OL} = -\beta_1\beta_2 \frac{R_C}{R_C + r_{\pi 2} + (1+\beta_2)(R_L + R_F)}$$

$$K = -1 \text{ (by inspection)}$$

$$\frac{i_{out}}{i_{in}} = \boxed{-\frac{\beta_1\beta_2 \frac{R_C}{R_C + r_{\pi 2} + (1+\beta_2)(R_L+R_F)}}{1 + \beta_1\beta_2 \frac{R_C}{R_C + r_{\pi 2} + (1+\beta_2)(R_L+R_F)}}}$$

$$R_{in,open} = r_{\pi 1}$$

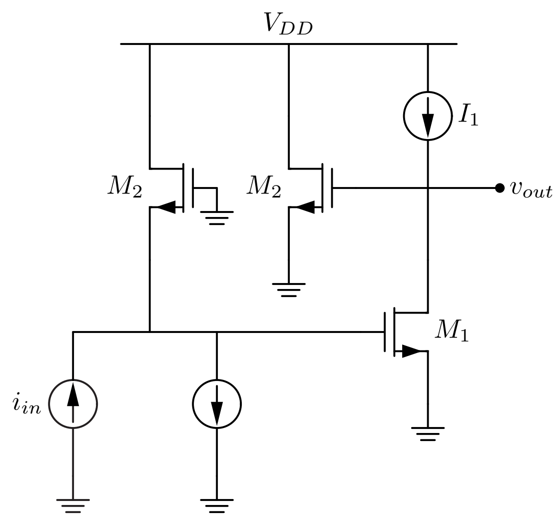
$$R_{in,closed} = \boxed{\frac{r_{\pi 1}}{1 + \beta_1\beta_2 \frac{R_C}{R_C + r_{\pi 2} + (1+\beta_2)(R_L+R_F)}}}$$

$$R_{out,open} = \frac{r_{\pi 2} + R_C}{1+\beta_2} + R_F$$

$$\approx \frac{1}{g_{m2}} + \frac{R_C}{1+\beta_2} + R_F$$

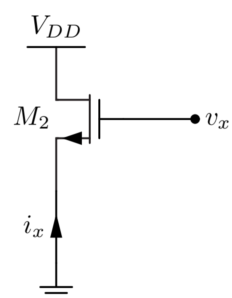
$$R_{out,closed} = \boxed{\left(\frac{r_{\pi 2} + R_C}{1+\beta_2} + R_F\right)\left\{1 + \beta_1\beta_2 \frac{R_C}{R_C + r_{\pi 2} + (1+\beta_2)(R_L+R_F)}\right\}}$$

12.56 (a) We can break the feedback network as shown here:



$$A_{OL} = -g_{m1}r_{o1}\left(\frac{1}{g_{m2}} \parallel r_{o2}\right)$$

To find the feedback factor K, we can use the following diagram:



$$K = \frac{v_x}{i_x} = -g_{m2}$$

$$\frac{v_{out}}{i_{in}} = \boxed{-\frac{g_{m1}r_{o1}\left(\frac{1}{g_{m2}} \parallel r_{o2}\right)}{1 + g_{m1}g_{m2}r_{o1}\left(\frac{1}{g_{m2}} \parallel r_{o2}\right)}}$$

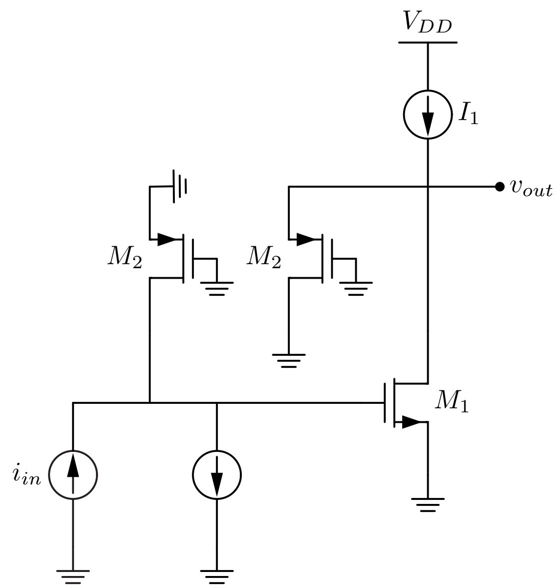
$$R_{in,open} = \frac{1}{g_{m2}} \parallel r_{o2}$$

$$R_{in,closed} = \boxed{\frac{\frac{1}{g_{m2}} \parallel r_{o2}}{1 + g_{m1}g_{m2}r_{o1}\left(\frac{1}{g_{m2}} \parallel r_{o2}\right)}}$$

$$R_{out,open} = r_{o1}$$

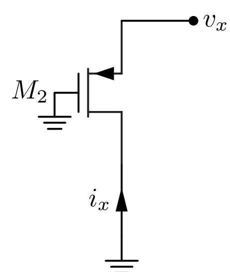
$$R_{out,closed} = \boxed{\frac{r_{o1}}{1 + g_{m1}g_{m2}r_{o1}\left(\frac{1}{g_{m2}} \parallel r_{o2}\right)}}$$

(b) We can break the feedback network as shown here:



$$A_{OL} = -g_{m1} r_{o2} \left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2} \right)$$

To find the feedback factor K, we can use the following diagram:



$$K = \frac{v_x}{i_x} = -g_{m2}$$

$$\frac{v_{out}}{i_{in}} = \boxed{-\frac{g_{m1} r_{o2}\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}\right)}{1 + g_{m1} g_{m2} r_{o2}\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}\right)}}$$

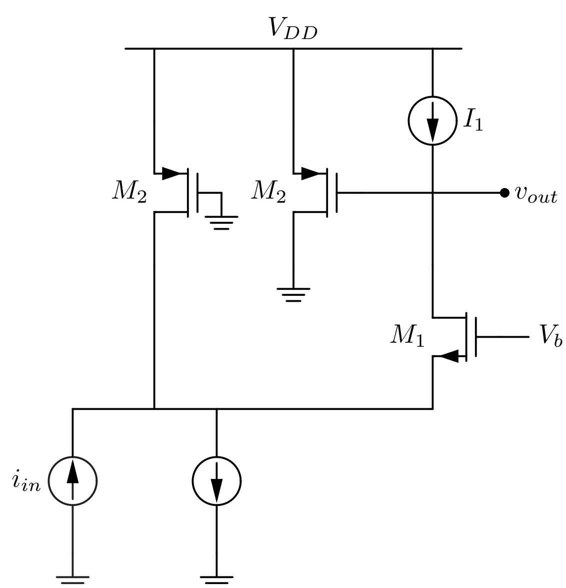
$$R_{in,open} = r_{o2}$$

$$R_{in,closed} = \boxed{\frac{r_{o2}}{1 + g_{m1} g_{m2} r_{o2}\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}\right)}}$$

$$R_{out,open} = r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}$$

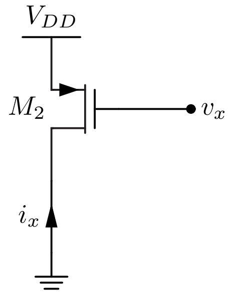
$$R_{out,closed} = \boxed{\frac{r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}}{1 + g_{m1} g_{m2} r_{o2}\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}\right)}}$$

(c) We can break the feedback network as shown here:



$$A_{OL} = g_{m1} r_{o1}\left(\frac{1}{g_{m1}} \parallel r_{o2}\right)$$

To find the feedback factor K, we can use the following diagram:



$$K = \frac{i_x}{v_x} = g_{m2}$$

$$\frac{v_{out}}{i_{in}} = \boxed{\frac{g_{m1} r_{o1} \left(\frac{1}{g_{m1}} \parallel r_{o2}\right)}{1 + g_{m1} g_{m2} r_{o1} \left(\frac{1}{g_{m1}} \parallel r_{o2}\right)}}$$

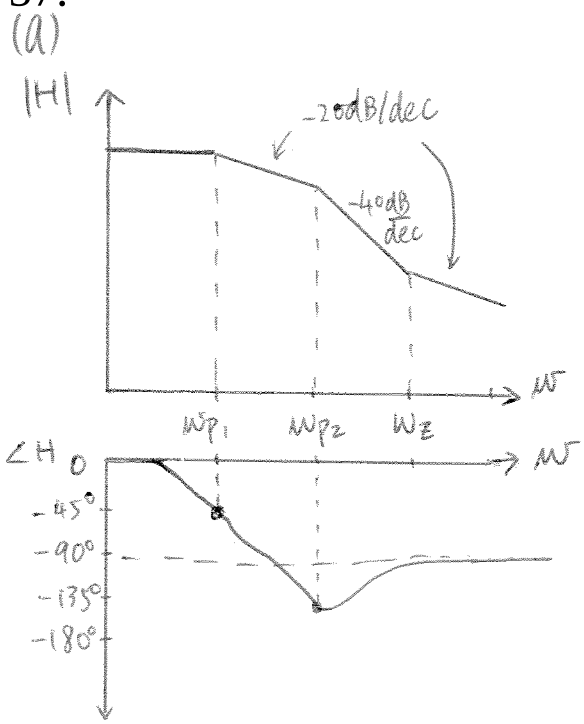
$$R_{in,open} = \frac{1}{g_{m1}} \parallel r_{o2}$$

$$R_{in,closed} = \boxed{\frac{\frac{1}{g_{m1}} \parallel r_{o2}}{1 + g_{m1} g_{m2} r_{o1} \left(\frac{1}{g_{m1}} \parallel r_{o2}\right)}}$$

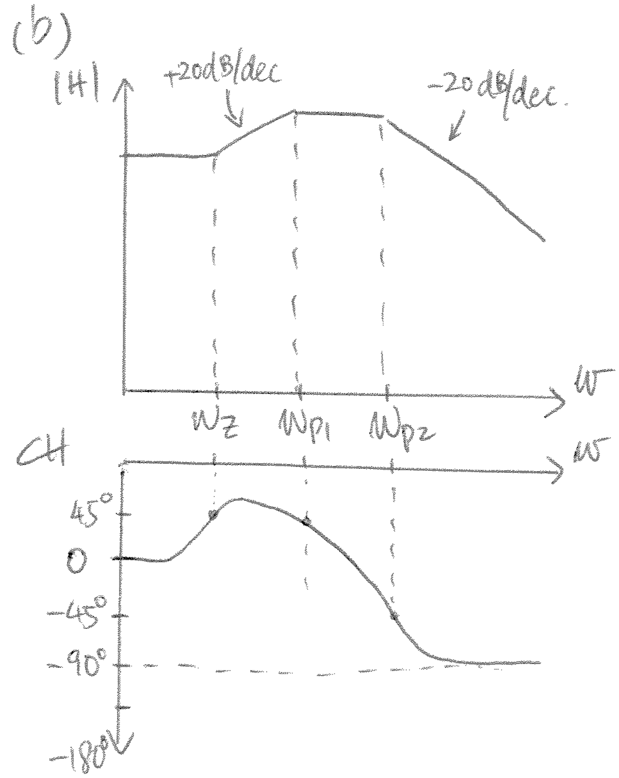
$$R_{out,open} = r_{o1} + (1 + g_{m1} r_{o1})\, r_{o2}$$

$$R_{out,closed} = \boxed{\frac{r_{o1} + (1 + g_{m1} r_{o1})\, r_{o2}}{1 + g_{m1} g_{m2} r_{o1} \left(\frac{1}{g_{m1}} \parallel r_{o2}\right)}}$$

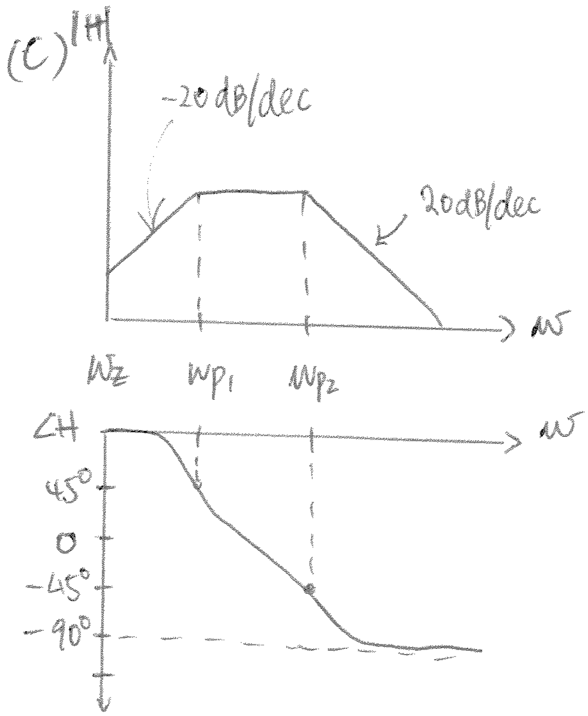
57.



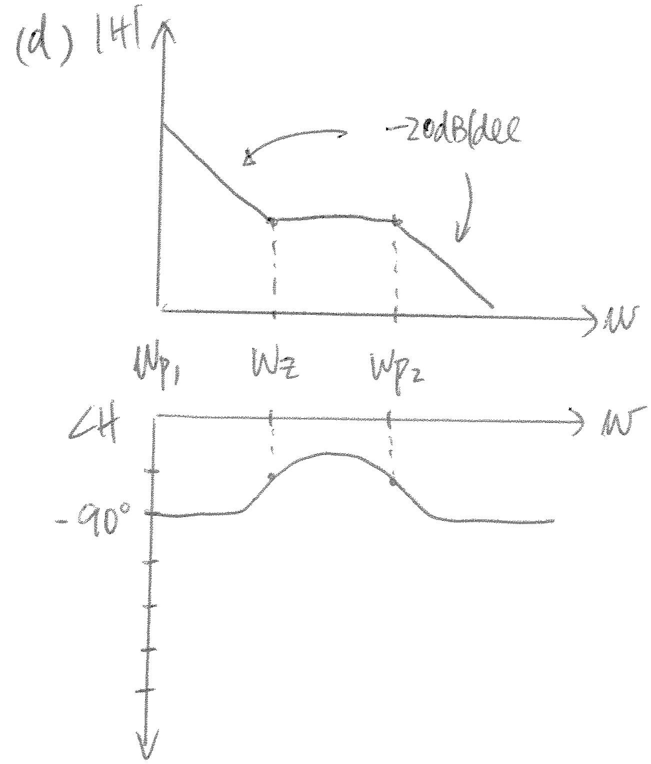
(a)
|H|
-20dB/dec
-40dB/dec
ω
ωp1
ωp2
ωz
∠H
0
-45°
-90°
-135°
-180°
ω



(b)
|H|
+20dB/dec
-20dB/dec
ω
ωz
ωp1
ωp2
∠H
ω
45°
0
-45°
-90°
-180°



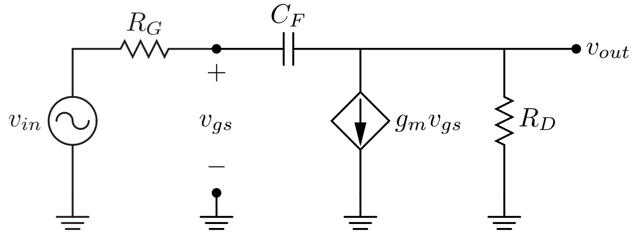
(c)
|H|
-20dB/dec
20dB/dec
ω
ωz
ωp1
ωp2
∠H
ω
45°
0
-45°
-90°



(d)
|H|
-20dB/dec
ω
ωp1
ωz
ωp2
∠H
ω
-90°

58. As w_z comes closer to w_{p1} or w_{p2}, it cancels out the effect (i.e. −20dB/dec decrease) — pole-zero cancellation. It would appear as if nothing occured at that overlapping frequency.

12.59 Let's draw the small-signal model and find $\frac{v_{out}}{v_{in}}(s)$.



$$\frac{v_{in} - v_{gs}}{R_G} = (v_{gs} - v_{out})\, sC_F$$

$$(v_{gs} - v_{out})\, sC_F = g_m v_{gs} + \frac{v_{out}}{r_{o1}}$$

$$v_{gs}\,(sC_F - g_m) = v_{out}\left(\frac{1}{r_{o1}} + sC_F\right)$$

$$v_{gs} = \frac{1 + sC_F r_{o1}}{r_{o1}\,(sC_F - g_m)}$$

$$\frac{v_{in}}{R_G} = v_{gs}\left(\frac{1}{R_G} + sC_F\right) - v_{out} sC_F$$

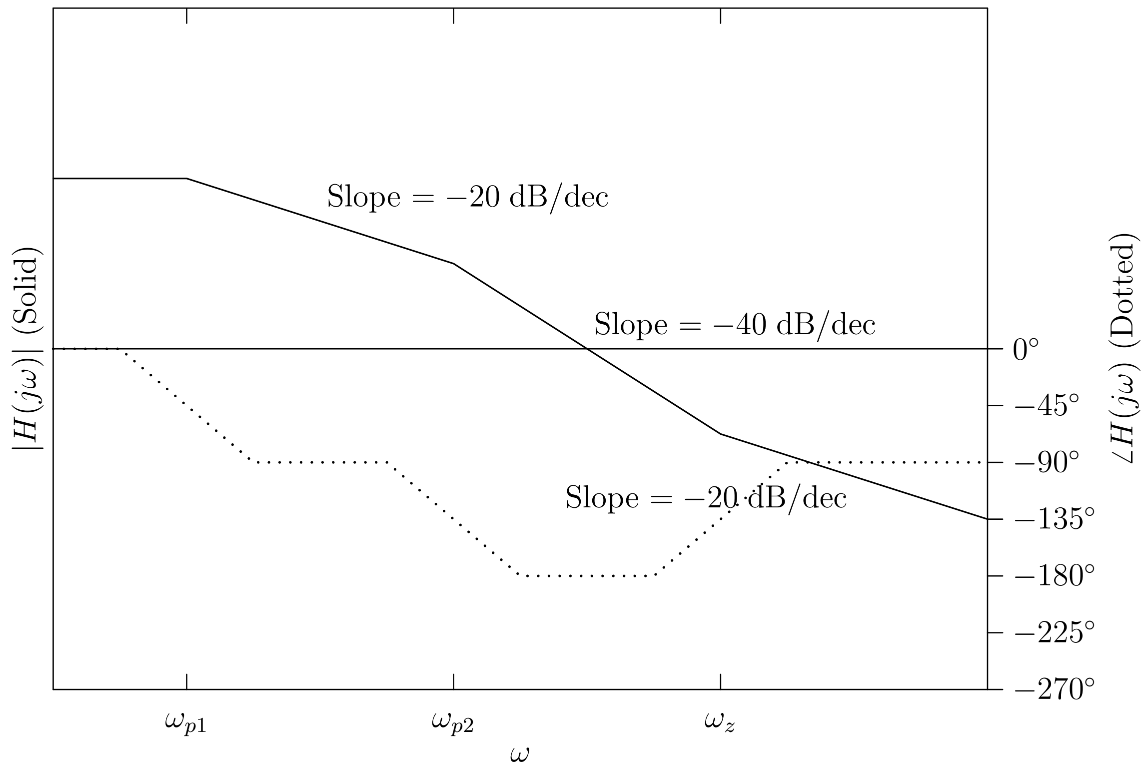
$$\frac{v_{in}}{R_G} = v_{out}\left[\left(\frac{1 + sC_F r_{o1}}{r_{o1}\,(sC_F - g_m)}\right)\left(\frac{1}{R_G} + sC_F\right) - sC_F\right]$$

$$v_{in} = v_{out}\left[\left(\frac{1 + sC_F r_{o1}}{r_{o1}\,(sC_F - g_m)}\right)(1 + sC_F R_G) - sC_F R_G\right]$$

$$v_{in} = v_{out}\left[\frac{(1 + sC_F r_{o1})\,(1 + sC_F R_G) - sC_F R_G r_{o1}\,(sC_F - g_m)}{r_{o1}\,(sC_F - g_m)}\right]$$

$$\frac{v_{out}}{v_{in}}(s) = \boxed{\frac{r_{o1}\,(sC_F - g_m)}{(1 + sC_F r_{o1})\,(1 + sC_F R_G) - sC_F R_G r_{o1}\,(sC_F - g_m)}}$$

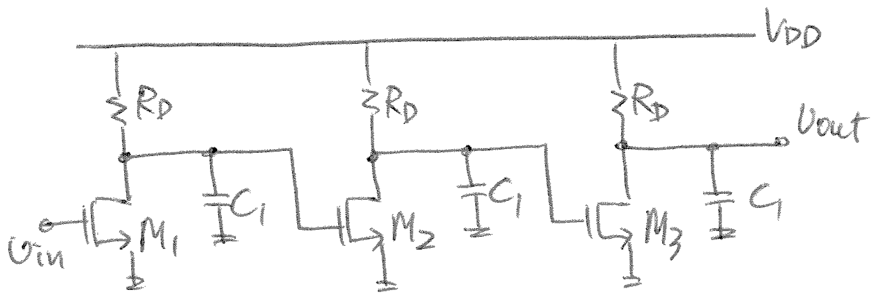
From the transfer function, we can see that we'll have one zero and two poles (since the numerator is of degree 1 and the denominator is of degree 2).



Slope = −20 dB/dec
Slope = −40 dB/dec
Slope = −20 dB/dec
$|H(j\omega)|$ (Solid)
$\angle H(j\omega)$ (Dotted)
0°
−45°
−90°
−135°
−180°
−225°
−270°
ω_{p1}
ω_{p2}
ω_z
ω

60. By Nyquist Criterion, decreasing k ($k \to 0$) eventually leads to $|KH| < 1$ at $\angle H = -180°$, which implies stability.

61.



$$H(s) = \frac{V_{out}}{V_{in}}(s) = \frac{(-g_m R_D)^3}{\left(1 + \frac{s}{\omega_p}\right)^3} \quad \text{where } \omega_p = \frac{1}{R_D C_1}$$

$$\Rightarrow \angle H(j\omega) = \angle(-g_m R_D)^3 - \angle\left(1 + j\frac{\omega}{\omega_p}\right)^3$$

$$= 0 - 3\,\mathrm{TAN}^{-1}\left(\frac{\omega}{\omega_p}\right)$$

$$\therefore \angle H\Big|_{\omega = 0.1\omega_p} = -3\,\mathrm{TAN}^{-1}\left(\frac{0.1\,\omega_p}{\omega_p}\right) \cong -17.1^\circ$$

62. $H(s) = \dfrac{(-g_m R_D)^3}{\left(1 + \frac{s}{\omega_p}\right)^3}$ $\quad (M_1 = M_2 = M_3)$

$$\Rightarrow |H|\Big|_{\omega=\omega_p} = \frac{|g_m R_D{}^3|}{\left|\left(1 + j\frac{\omega_p}{\omega_p}\right)^3\right|} = \frac{(g_m R_D)^3}{\left(\sqrt{1+1}\right)^3} = \frac{(g_m R_D)^3}{\sqrt{8}}$$

$$\Rightarrow 20\log|H|\Big|_{\omega=\omega_p} = 20\log(g_m R_D)^3 - 20\log\sqrt{8}$$
$$\cong 20\log(g_m R_D)^3 - (9\text{ dB})$$

$\therefore$ $|H|$ falls by 9 dB due to the three coincident poles.

12.63 We'll drop the negative sign in $H(s)$ as done in Example 12.38.

$$H(s) = \frac{(g_m R_D)^3}{\left(1 + \frac{s}{\omega_p}\right)^3}$$

$$\angle H(j\omega) = -3\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$

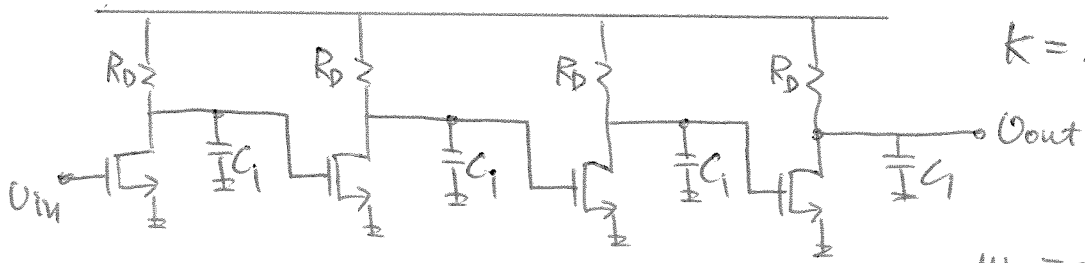
$$-3\tan^{-1}\left(\frac{\omega_{PX}}{\omega_p}\right) = -180$$

$$\omega_{PX} = \sqrt{3}\omega_p$$

$$|KH(j\omega_{PX})| = 0.1\frac{(g_m R_D)^3}{\left[\sqrt{1 + \left(\frac{\omega_{PX}}{\omega_p}\right)}\right]^3} < 1$$

$$g_m R_D < \sqrt[3]{80} = \boxed{4.31}$$

64.

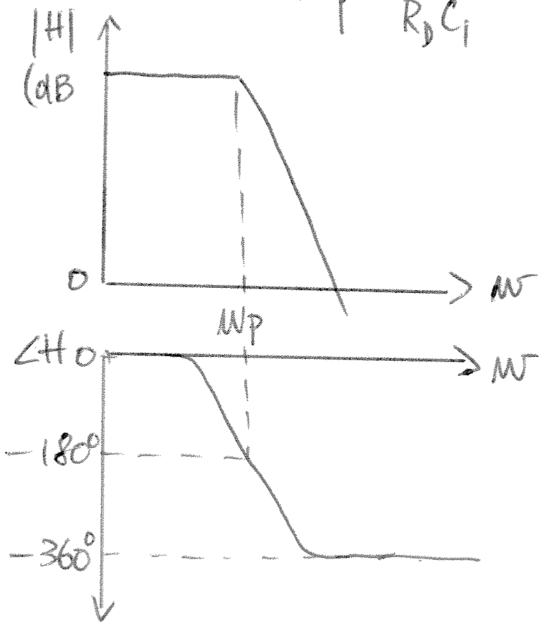


$K = 1$

$$\omega_p = \frac{1}{R_D C_1}$$

$$H(j\omega) = \frac{(g_m R_D)^4}{\left(1 + j\frac{\omega}{\omega_p}\right)^4}$$

$$\angle H = -4\,\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$



- To guarantee stability, $|KH| < 1$ when $\angle H = -180°$.

$$\angle H = -180° = -4\,\tan^{-1}\left(\frac{\omega}{\omega_p}\right) \Rightarrow \omega = \omega_p$$

$$|KH|\Big|_{\omega=\omega_p} = \frac{(g_m R_D)^4}{(\sqrt{1+1})^4} < 1$$

$$\Rightarrow g_m R_D < \sqrt{2}$$

This four-pole system implies a lower upper-limit ($=\sqrt{2}$) on $g_m R_D$, which makes sense since $|H|$ drops faster here.

12.65

$$
\begin{aligned}
H(s) &= \frac{A_0}{1+\frac{s}{\omega_0}} \\
|KH(\omega_{GX})| &= \frac{A_0}{\sqrt{1+\left(\frac{\omega_{GX}}{\omega_0}\right)^2}} = 1 \\
\omega_{GX} &= \omega_0\sqrt{A_0^2-1} \\
\angle H(j\omega_{GX}) &= -\tan^{-1}\left(\frac{\omega_{GX}}{\omega_0}\right) \\
&= -\tan^{-1}\left(\frac{\omega_0\sqrt{A_0^2-1}}{\omega_0}\right) \\
&= -\tan^{-1}\left(\sqrt{A_0^2-1}\right) \\
\text{Phase Margin} &= \angle H(j\omega_{GX}) + 180^\circ \\
&= \boxed{180^\circ - \tan^{-1}\left(\sqrt{A_0^2-1}\right)}
\end{aligned}
$$

The phase margin can be anything from 90° to 180°, depending on the value of A_0 (smaller A_0 means larger phase margin).

12.66

$$
\begin{aligned}
H(s) &= \frac{A_0}{1+\frac{s}{\omega_0}} \\
|KH(\omega_{GX})| &= 0.5\frac{A_0}{\sqrt{1+\left(\frac{\omega_{GX}}{\omega_0}\right)^2}} = 1 \\
\omega_{GX} &= \omega_0\sqrt{\left(\frac{A_0}{2}\right)^2 - 1} \\
\angle H(j\omega_{GX}) &= -\tan^{-1}\left(\frac{\omega_{GX}}{\omega_0}\right) \\
&= -\tan^{-1}\left(\frac{\omega_0\sqrt{\left(\frac{A_0}{2}\right)^2 - 1}}{\omega_0}\right) \\
&= -\tan^{-1}\left(\sqrt{\left(\frac{A_0}{2}\right)^2 - 1}\right) \\
\text{Phase Margin} &= \angle H(j\omega_{GX}) + 180^\circ \\
&= \boxed{180^\circ - \tan^{-1}\left(\sqrt{\left(\frac{A_0}{2}\right)^2 - 1}\right)}
\end{aligned}
$$

The phase margin can be anything from 90° to 180°, depending on the value of A_0 (smaller A_0 means larger phase margin).

67. All three scenarios will become stable eventually (depending on how far w_{Gx} is from w_{Px}, & $w_{Gx} < w_{Px}$.)

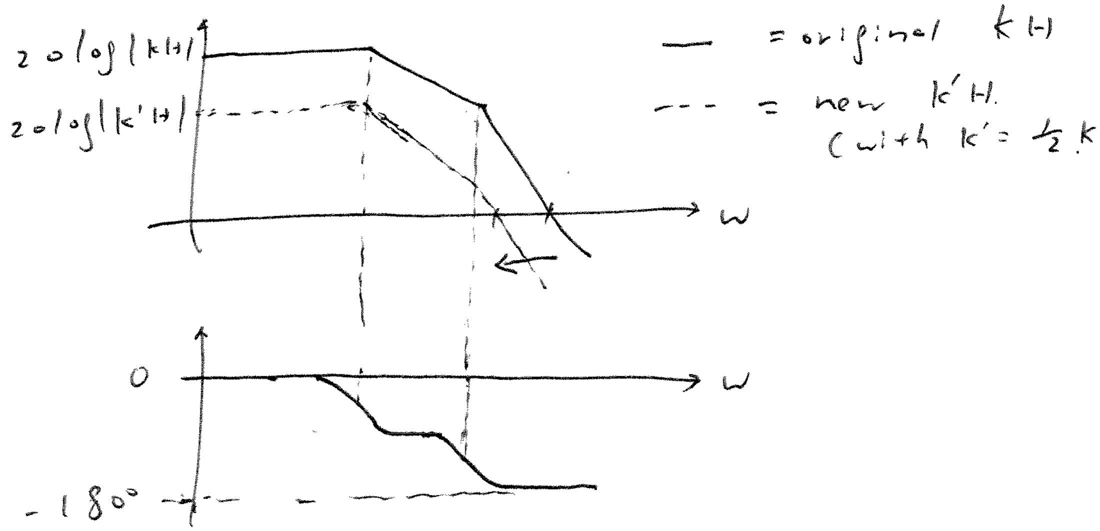
12.68 With a factor of $K = 0.5$, the magnitude Bode plot of KH will simply be the magnitude plot of H shifted down by 6 dB (since $20 \log 0.5 = -6$ dB). Since the slope of the magnitude plot between ω_{p1} and ω_{p2} is -20 dB/dec, this means that ω_{GX} will be shifted left by $\frac{6}{20} = 0.3$ decades, or a factor of $10^{0.3} = 2$.

Thus, the new value of ω_{GX}, which we'll call ω'_{GX}, is $\omega'_{GX} = \frac{\omega_{GX}}{2} = \frac{\omega_{p2}}{2}$.

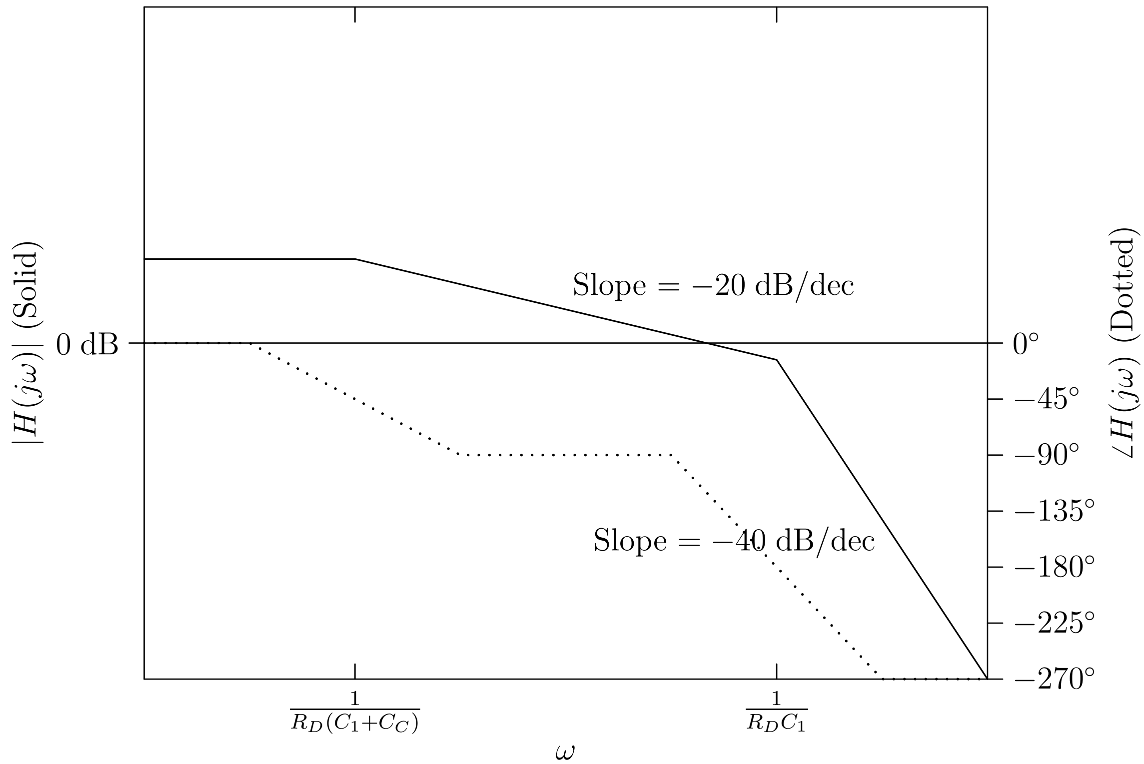
Now, we need to find $\angle H(j\omega_{GX})$.

$$\begin{aligned}
\angle H(j\omega) &= -\tan^{-1}\left(\frac{\omega}{\omega_{p1}}\right) - \tan^{-1}\left(\frac{\omega}{\omega_{p2}}\right) \\
\angle H(j\omega_{GX}) &= \angle H\left(j\frac{\omega_{p2}}{2}\right) \\
&= -\tan^{-1}\left(\frac{\omega_{p2}}{2\omega_{p1}}\right) - \tan^{-1}\left(\frac{\omega_{GX}}{2\omega_{p2}}\right) \\
&= -90^\circ - \tan^{-1}(0.5) \\
&= -116^\circ \\
\text{Phase Margin} &= 180^\circ + \angle H(j\omega_{GX}) \\
&= 180^\circ - 116^\circ \\
&= \boxed{63^\circ}
\end{aligned}$$

69. When K drops by a factor of 2, the phase margin improves. This is because a lower K corresponds to shifting the amplitude part of KH down by 6 dB. (The phase ∠KH remains unchanged, since phase is only depends on pole location and is independent of amplitude of KH.) Thus, the gain |KH| drops to 0 dB at a lower frequency. This results in a larger phase margin.



12.70 The compensation capacitor allowsus to push the pole associated with that node to a lower frequency (while the other poles do not change). This will cause the gain to start dropping sooner, so that ω_{GX} decreases. By adjusting C_C properly, we can reduce ω_{GX} enough so that the phase is at $-135°$ at ω_{GX}. This results in the following Bode plots:



12.71

$$
\begin{aligned}
A_{OL} &\approx g_{m1}\left(r_{o2} \parallel r_{o4}\right) \\
&= g_{m1}\left(\frac{2}{\lambda_n I_{SS}} \parallel \frac{2}{\lambda_p I_{SS}}\right) \\
&= 50 \\
g_{m1} &= 3.75 \text{ mS} \\
K &= \frac{R_2}{R_1+R_2} = \frac{R_2}{10\left(r_{o2} \parallel r_{o4}\right)} \\
\frac{v_{out}}{v_{in}} &= \frac{g_{m1}\left(r_{o2} \parallel r_{o4}\right)}{1+g_{m1}\frac{R_2}{10(r_{o2}\parallel r_{o4})}\left(r_{o2} \parallel r_{o4}\right)} \\
&= 4 \\
R_2 &= \boxed{30.667 \text{ k}\Omega} \\
R_1 &= \boxed{102.667 \text{ k}\Omega}
\end{aligned}
$$

72. Open loop gain, $A_o = g_m R_D$

(assuming $R_1 + R_2$ is very large.)

i.e. $g_m R_D = 10$

$$\text{Closed-loop gain} = \frac{g_m R_D}{1 + \left(\frac{R_2}{R_1 + R_2}\right) g_m R_D}$$

$$= 2$$

$$\therefore \quad \frac{10}{1 + \left(\frac{R_2}{R_1 + R_2}\right) \times 10} = 2$$

$$\frac{R_2}{R_1 + R_2} = 0.4$$

$$\text{Closed-loop input impedance} = \frac{1}{g_m}\left[1 + \frac{R_2}{R_1 + R_2} \times 10\right]$$

$$= 50\,\Omega.$$

$$\therefore \quad \frac{1}{g_m} \times 5 = 50$$

$$g_m = 0.1\,S$$

$$\therefore \quad R_D = 100\,\Omega.$$

$$\therefore \quad R_1 + R_2 = 10 \times 100\,\Omega$$

$$= 1\,k\Omega.$$

$$\therefore \quad R_2 = 400\,\Omega$$

$$R_1 = 600\,\Omega$$

12.73

$$
\begin{aligned}
A_{OL} &= -g_{m2} R_{D1} R_{D2} = -10 \text{ k}\Omega \\
K &= -\frac{1}{R_F} \\
\frac{v_{out}}{i_{in}} &= -\frac{g_{m2} R_{D1} R_{D2}}{1 + \frac{g_{m2} R_{D1} R_{D2}}{R_F}} \\
&= -\frac{10 \text{ k}\Omega}{1 + \frac{10 \text{ k}\Omega}{R_F}} \\
&= -1 \text{ k}\Omega \\
R_F &= \boxed{1.111 \text{ k}\Omega} \\
R_{in,open} &= \frac{1}{g_{m1}} \\
R_{in,closed} &= \frac{1}{g_{m1}} \left(1 + \frac{g_{m2} R_{D1} R_{D2}}{R_F}\right)^{-1} \\
&= \frac{1}{g_{m1}} \left(1 + \frac{10 \text{ k}\Omega}{1.111 \text{ k}\Omega}\right)^{-1} \\
&= 50 \ \Omega \\
g_{m1} &= \boxed{2 \text{ mS}} \\
R_{out,open} &= R_{D2} \\
R_{out,closed} &= \frac{R_{D2}}{1 + \frac{g_{m2} R_{D1} R_{D2}}{R_F}} \\
&= \frac{R_{D2}}{1 + \frac{10 \text{ k}\Omega}{1.111 \text{ k}\Omega}} \\
&= 200 \ \Omega \\
R_{D2} &= \boxed{2 \text{ k}\Omega} \\
g_{m2} &= \frac{A_{OL}}{R_{D1} R_{D2}} \\
&= \boxed{5 \text{ mS}}
\end{aligned}
$$

74. Assuming R_F is very large,

open-loop gain $= R_D\,(g_{m2} R_L)$

$= 10\,k\Omega$

closed-loop gain $= \dfrac{10\,k\Omega}{1 + \dfrac{10\,k\Omega}{R_F}}$

$= 1\,k\Omega.$

$\therefore R_F = 1.11\,k\Omega$ //

closed-loop input impedance $= \frac{1}{g_{m1}} (1+\beta)^{-1} = 50\,\Omega.$

$g_{m1} = 2\,mS.$ //

closed-loop output impedance $= \dfrac{R_L}{10}$

$\therefore R_L = 2000\,\Omega.$ //

$R_D = 1\,k\Omega.$

$\therefore g_{m2} = 5\,mS$ //

75. a) open-loop gain $= R_c\,(g_{m2} R_m)$

$= 20\ k\Omega.$

$g_{m2} = \frac{I}{V_T}$

$\therefore g_{m2} = \frac{1mA}{26mV} = 38.5\ mS$

$\therefore R_c R_m = \frac{20\ k\Omega}{38.5\ mS}$

open-loop output impandance $= R_m \quad (\because r_o = \infty)$

$\therefore R_m = 500\Omega$ //

$R_c = 1040\Omega$ //

---

b) closed-loop gain $= \frac{20\ k\Omega}{1 + \frac{20\ k\Omega}{R_F}}$

$= 1\ k\Omega$

$\therefore R_F = 1053\ \Omega$ //

c) Closed-loop input impedance $= \frac{\frac{1}{38.5\ mS}}{1 + \frac{20\ k}{1053}}$

$= 1.30\Omega$ //

closed-loop output impedance $\approx (500)\left(\frac{1}{20}\right)$

$= 25\Omega$ //

12.76 See Problem 44 for derivations of the following expressions.

$$
\begin{aligned}
A_{OL} &= \frac{g_{m1} g_{m2} R_{D1} (R_1 + R_2)}{1 + g_{m1} (R_1 \parallel R_2)} = 20 \\
\frac{v_{out}}{v_{in}} &= \frac{\frac{g_{m1} g_{m2} R_{D1} (R_1 + R_2)}{1 + g_{m1} (R_1 \| R_2)}}{1 + \frac{g_{m1} g_{m2} R_{D1} R_2}{1 + g_{m1} (R_1 \| R_2)}} \\
&= \frac{20}{1 + 20 \left(\frac{R_2}{R_1 + R_2} \right)} \\
&= 4 \\
\frac{R_2}{R_1 + R_2} &= 0.2 \\
R_{out,open} &= R_1 + R_2 = 2 \text{ k}\Omega \\
R_2 &= \boxed{400\ \Omega} \\
R_1 &= \boxed{1.6\text{ k}\Omega}
\end{aligned}
$$

Lacking any additional constraints, we can pick any g_{m1}, g_{m2}, and R_{D1} so that $A_{OL} = 20$. Let's pick $g_{m1} = g_{m2} = \boxed{2\text{ mS}}$. This gives us $R_{D1} = \boxed{4.1\text{ k}\Omega}$.

If we are also required to minimize the power consumption of the amplifier, we need to minimize the current consumption of each stage. This requires minimizing g_{m1} and g_{m2} and maximizing R_{D1} while keeping all transistors in saturation.

12.77 See Problem 46 for derivations of the following expressions.

$$A_{OL} = \frac{g_{m1} g_{m2} \left(\frac{1}{g_{m2}} + \frac{R_1 \| R_2}{\beta_2 + 1} \right)}{1 + g_{m1} \left(\frac{1}{g_{m2}} + \frac{R_1 \| R_2}{\beta_2 + 1} \right)} (R_1 + R_2) = 2$$

$$g_{m1} = g_{m2} = \frac{I_{SS}}{2V_T} = \frac{1}{52} \text{ S}$$

$$K = \frac{R_2}{R_1 + R_2}$$

$$R_{out,closed} = \frac{R_1 + R_2}{1 + K A_{OL}}$$

$$= \frac{R_1 + R_2}{1 + \frac{2R_2}{R_1 + R_2}}$$

$$= \frac{(R_1 + R_2)^2}{1 + 3R_2}$$

Looking at this expression for $R_{out,closed}$, we can see that it will be minimized for very small values of R_1. This will force R_2 to be larger in order to meet the required A_{OL}, but since R_{out} depends more strongly on R_1 than R_2, we should focus on minimizing R_1.

In fact, we can actually set $R_1 = \boxed{0}$. We can then solve the A_{OL} equation to find $R_2 = \boxed{208\ \Omega}$, which means $R_{out} = 69.33\ \Omega$.

12.78 See Problem 50 for derivations of the following expressions. Assume $\beta = 100$.

$$A_{OL} = -\frac{g_{m1} g_{m2} \left(R_F \parallel r_{\pi 1}\right) R_F \left\{R_C \parallel \left[r_{\pi 2} + \left(1+\beta\right) R_F\right]\right\}}{1 + g_{m2} R_F}$$

$$\frac{v_{out}}{i_{in}} = \frac{A_{OL}}{1 - \frac{A_{OL}}{R_F}} = -1\ \text{k}\Omega$$

$$R_{in} = \frac{R_F \parallel r_{\pi 1}}{1 - \frac{A_{OL}}{R_F}} = 50\ \Omega$$

$$g_{m1} = g_{m2} = \frac{1}{26}\ \Omega$$

$$r_{\pi 1} = r_{\pi 2} = \frac{\beta}{g_m} = 2.6\ \text{k}\Omega$$

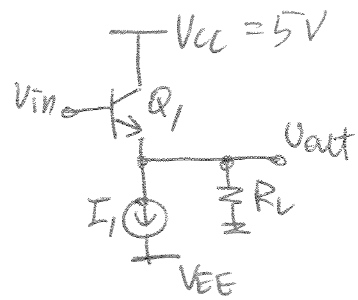
We have two equations ($\frac{v_{out}}{i_{in}} = -1\ \text{k}\Omega$ and $R_{in} = 50\ \Omega$) and two unknowns (R_F and A_{OL}). Solving, we get:

$$R_F = \boxed{1.071\ \text{k}\Omega}$$

$$A_{OL} = 15167$$

$$R_C = \boxed{535.2\ \Omega}$$

1. $A_v = \dfrac{g_{m1} R_L}{1 + g_{m1} R_L}$



$P_{LOAD} = 0.5\,W$

$R_L = 8\Omega$

(Assume Vout biased at $V_{BE(ON),1} \approx 800\,mV$

(a) $0.8 = \dfrac{g_{m1}(8\Omega)}{1 + g_{m1}(8\Omega)}$

$\Rightarrow g_{m1} = 0.5 = \dfrac{I_c}{V_T} = \dfrac{I_1}{V_T}$

$\therefore I_1 = 13\,mA$

(b) When $V_{in} = V_p = V_{cc}$, $V_{out} \approx V_{cc} - V_{BE(ON),1}$

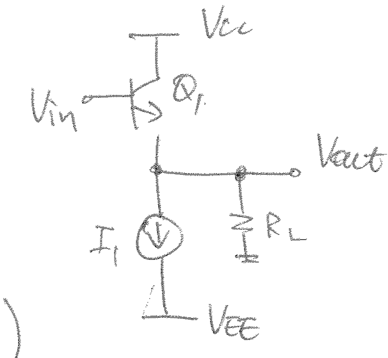
$I_{C1} = I_1 + \dfrac{V_{out}}{R_L} \Rightarrow I_{C1} = I_1 + \dfrac{5 - 0.8}{8} \approx 0.54\,A$

$\Rightarrow g_{m1} = \dfrac{I_{C1}}{V_T} = \dfrac{0.54A}{0.026V} = 20.8\,S$

$\Rightarrow A_v\Big|_{V_{in}=V_p} = \dfrac{g_{m1} R_L}{1 + g_{m1} R_L} = \dfrac{(20.8\,S)(8\Omega)}{1 + (20.8\,S)(8\Omega)} \approx 0.99$

2.

(a) $I_1 = V_p / R_L \qquad V_p \gg V_T$



$$A_V = \frac{I_C R_L}{I_C R_L + V_T}$$

$$= \frac{\frac{I_C}{I_1} V_p}{\frac{I_C}{I_1} V_p + V_T} = \frac{V_p}{V_p + V_T} \quad (\approx 1)$$

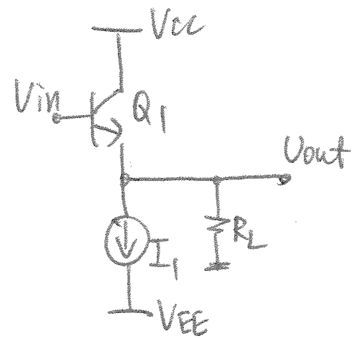
(b) When $V_{out} = V_p$, $I_{C_1} = I_1 + \frac{V_{out}}{R_L} = \frac{V_p}{R_L} + \frac{V_p}{R_L}$

$$= \frac{2V_p}{R_L}$$

$$\therefore A_V = \frac{\left(2\frac{V_p}{R_L}\right) R_L}{\left(\frac{2V_p}{R_L}\right) R_L + V_T} = \frac{2V_p}{2V_p + V_T} \left(\approx \frac{2V_p}{2V_p} = 1.\right)$$

$$\Delta A_V = \frac{\frac{2V_p}{2V_p + V_T} - \frac{V_p}{V_p + V_T}}{\frac{V_p}{V_p + V_T}} = \frac{V_T}{2V_p + V_T} \left(\approx \frac{V_T}{V_p}\right)$$

3. $A_v = 0.7$ $R_L = 4\Omega$



Q_1 shuts off when:

$$I_1 = \frac{V_P}{R_L}$$

- Suppose $V_{out} = V_p \sin\omega t$. $\left(\omega = \frac{2\pi}{T}\right)$

$$P_{R_L,AVG} = \frac{1}{T}\int_0^T \frac{(V_{out})^2}{R_L}\,dt = \frac{1}{T}\int_0^T \frac{V_p^2 \sin^2\omega t}{R_L}\,dt = \frac{1}{2} V_P^2/R_L$$

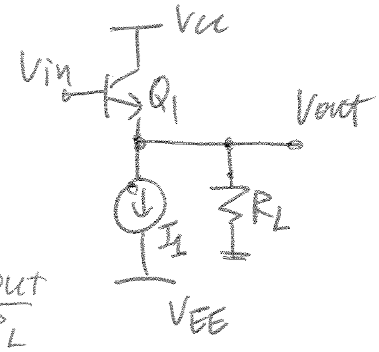
$\therefore$ Largest power (average) $= \frac{1}{2}\frac{(I_1 R_L)^2}{R_L}$

$$A_v = 0.7 = \frac{g_{m_1} R_L}{1 + g_{m_1} R_L} \Rightarrow g_{m_1} = \frac{A_v}{(1-A_v)R_L} = \frac{0.7}{(1-0.7)(4)} = 0.58\,S$$

$$\Rightarrow I_{C_1}\ (= I_1) = g_{m_1} V_T = 0.015\ A$$

$$\therefore P_{AV,MAX} = \frac{1}{2} I_1^2 R_L = \frac{1}{2}(0.015\,A)^2(4\Omega) = 0.45\,W$$

4. $A_v = \dfrac{g_{m1} R_L}{1 + g_{m1} R_L}$ $\left(g_m = \dfrac{I_C}{V_T}\right)$



- Q_1 shuts off when $I_1 = -\dfrac{V_{out}}{R_L}$

$\Rightarrow V_p = I_1 \times R_L$

$$g_{m1} = \frac{A_v}{(1-A_v)R_L} = \frac{I_{C1}}{V_T} \Rightarrow I_{C1} = \frac{V_T A_v}{R_L(1-A_v)} \; (= I_1)$$

- Power delivered to R_L:

$$P_{R_L} = \frac{1}{T}\int_0^T \frac{V_{out}^2}{R_L}\,dt = \frac{1}{T}\int_0^T \frac{V_p^2 \sin^2\omega t}{R_L}\,dt$$

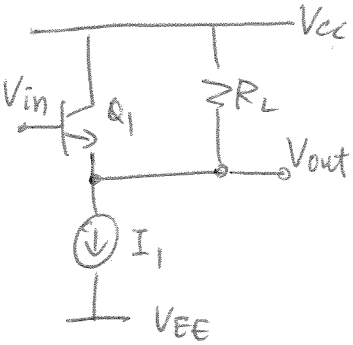
$$= \frac{1}{2}\frac{V_p^2}{R_L}$$

$\therefore$ Maximum power $= \dfrac{1}{2}\dfrac{(I_1 R_L)^2}{R_L}$

$$= \frac{1}{2}\left[\frac{V_T A_v}{(1-A_v)}\right]^2 \cdot \frac{1}{R_L}$$

5.

Assume $V_{cc} = 5V$



$I_{S_1} = 5 \cdot 10^{-17} A$

$R_L = 8 \Omega$

$I_1 = 20 mA.$

(a) By KCL,

$$I_1 = I_{S_1} \cdot \exp\left(\frac{V_{in} - V_{out}}{V_T}\right) + \frac{V_{cc} - V_{out}}{R_L}$$

$$\Rightarrow V_{in} = V_{out} + V_T \ln\left(\frac{I_1}{I_{S_1}} - \frac{V_{cc} - V_{out}}{I_{S_1} R_L}\right)$$

$= 0$ (X) — no solution

$\therefore V_{out} \approx 5 - I_1 R_L = 4.84 V$

(i.e. Q_1 is off.)

(b) $(0.01) I_1 = I_1 - \frac{V_{cc} - V_{out}}{R_L}$

$\Rightarrow V_{out} = 4.84 V$

$$I_{C_1} = (0.01) I_1 = I_{S_1} \exp\left(\frac{V_{in} - V_{out}}{V_T}\right)$$

$$\Rightarrow V_{in} = V_{out} + V_T \ln\left(0.01 \frac{I_1}{I_S}\right)$$

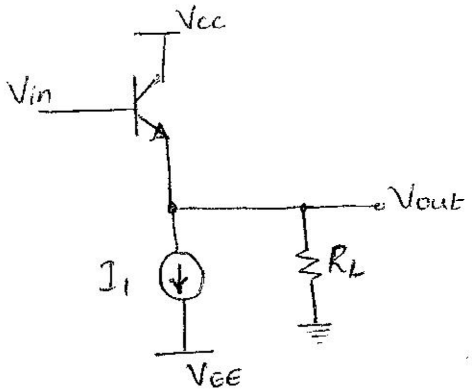
$$= 4.84 + (0.026) \ln\left(0.01 \frac{20mA}{5 \cdot 10^{-17} A}\right)$$

$$\approx 5.59 V$$

(exceeds V_{cc})

6.

(a) Calculate V_{BE} for



$I_{S1} = 6 \cdot 10^{-17} A$

$R_L = 8 \Omega$

$I_1 = 25 mA$

$V_p = 1V$

<u>$V_{in} = 1V$:</u>

$$I_{C1} = I_1 + \frac{V_{out}}{R_L}$$

$$\Rightarrow I_{S1} \exp\left(\frac{V_{in} - V_{out}}{V_T}\right) = I_1 + \frac{V_{out}}{R_L}$$

Solving by iteration for V_{out} gives:

$$V_{out} \approx 0.113V$$

$$\therefore V_{BE}\Big|_{V_{in}=1V} = V_{in} - V_{out} = 1 - 0.113$$

$$= 0.887V$$

<u>$V_{in} = -1V$:</u>

$$I_{C1} = I_1 + -\frac{V_{out}}{R_L} \Rightarrow I_{S1} \exp\left(\frac{V_{in} - V_{out}}{V_T}\right) = I_1 - \frac{V_{out}}{R_L}$$

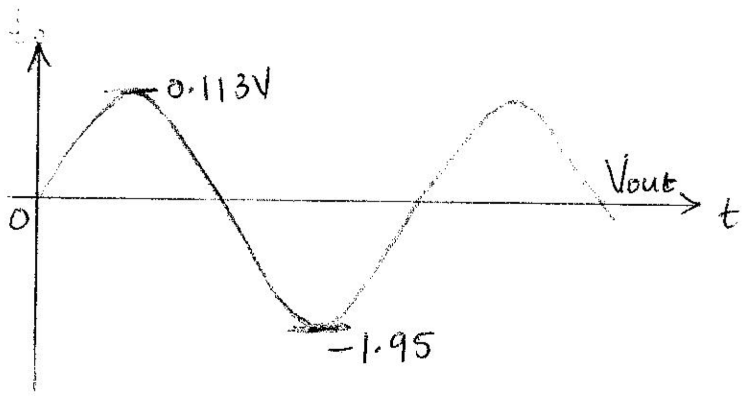
Solving by iteration for V_{out} gives:

$$V_{out} \approx -1.95V$$

$$\therefore V_{BE}\Big|_{V_{in}=-1V} = V_{in} - V_{out} = -1 - (-1.95)$$

$$= 0.95V$$

(b)



7. Determine V_p such that

$$V_{BE}\Big|_{V_{in}=+V_p} - V_{BE}\Big|_{V_{in}=-V_p} = 10\,mV$$

$$\Rightarrow \left(V_p^+ - V_{out,+}\right) - \left(V_p^- - V_{out,-}\right) = 10\,mV$$

$$I_s \exp\left(\frac{V_p^+ - V_{out,+}}{V_T}\right) = I_1 + \frac{V_{out,+}}{R_L} \quad \text{——} \ (1)$$

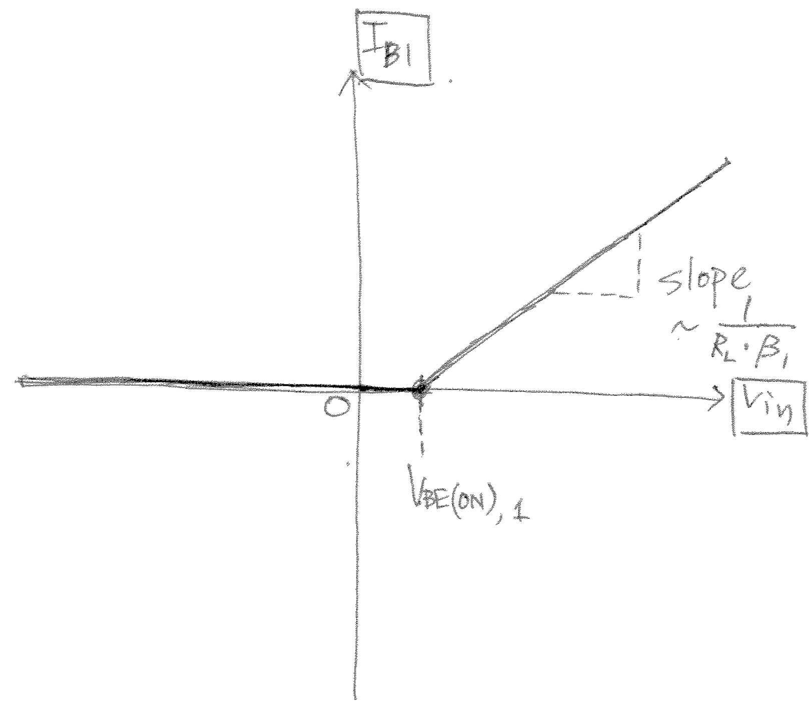
$$I_s \exp\left(\frac{V_p^- - V_{out,-}}{V_T}\right) = I_1 - \frac{V_{out,-}}{R_L} \quad \text{——} \ (2)$$

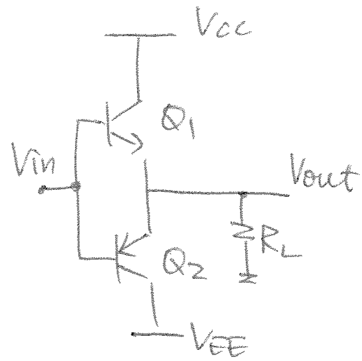
Iterate (1) & (2). This gives:

$V_p \approx 0.7\ V$

$$\Rightarrow \text{Nonlinearity} = \frac{10\,mV}{0.7 \times 2} = 0.007.$$

8.





β_1 : current gain of Q_1

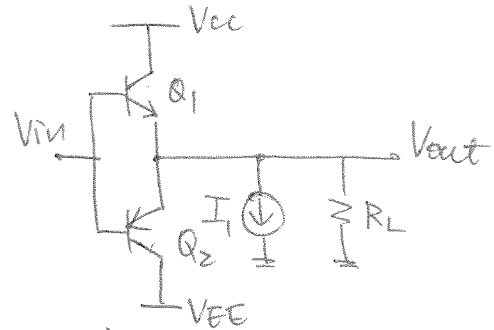
- Q_1 is on whenever $V_{in} \geq V_{BE(ON),1}$. In this region,

$$V_{out} = V_{in} - V_{BE(ON)1} \qquad I_{C_1} = \frac{V_{out}}{R_L}$$

$$\therefore I_{B_1} = \frac{I_{C1}}{\beta} = \frac{V_{out}}{\beta R_L} = \frac{V_{in} - V_{BE(ON)1}}{\beta R_L}.$$

9.

(a) To guarantee Q_1 on,



$I_{S2} = 6\cdot 10^{-17}A$

$R_L = 8\Omega$

- $V_{out} \approx V_{in} - V_{BE(ON)1}$
$= -800\,mV$

$\Rightarrow I_{C1} = I_1 + \frac{V_{out}}{R_L}$ (Q_2 is off)

- $I_{C1} \geq 0 \Rightarrow I_1 + \frac{V_{out}}{R_L} \geq 0$

$\Rightarrow I_1 + \frac{-800mV}{R_L} \geq 0$

$\therefore I_1 R_L \geq 800\,mV$ ——— ①

(b) When Q_2 turns on,

$-\frac{V_{out}}{R_L} - I_1 = I_{C2}$

$\Rightarrow -\frac{V_{out}}{R_L} - \left(\frac{800\,mV}{R_L}\right) = I_{S2}\exp\left(\frac{V_{BE2}}{V_T}\right)$

$\Rightarrow V_{out} = -R_L I_{S2}\cdot\exp\left(\frac{V_{BE2}}{V_T}\right) - 0.8$

$= -(8\Omega)(6\times10^{-17}A)\exp\left(\frac{0.8}{0.026}\right) - 0.8$

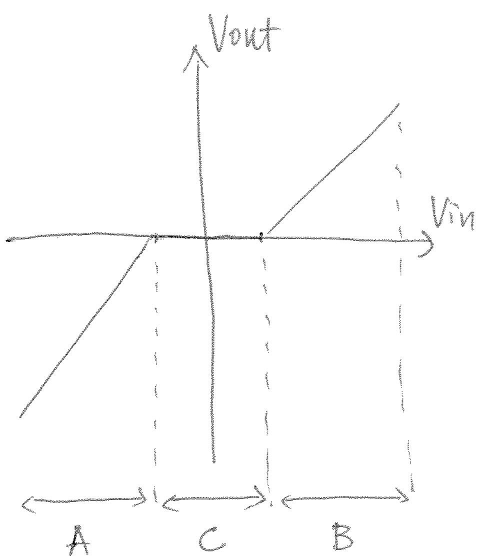
$\approx -0.81\ V$

$\therefore V_{in} = V_{out} - |V_{BE(ON)2}| = -0.81 - 0.8 = -1.61\ V$

10. Consider two scenarios:

- In gain regions ($|V_{in}| \geq |V_{BE(ON)}|$), V_{out} tracks V_{in}.
- In dead zone, both transistors shut off.

In both cases, V_{out} has an important role. Current source I_1 affects the input/output characteristic by modulating V_{out}:



I/O characteristic of Push-pull stage.

Consider region A:

$$I_{C2} + I_1 = \frac{-V_{out}}{R_L}$$

$\therefore I_1 \uparrow\downarrow \Rightarrow |V_{BE2}| = |V_{out} - V_{in}|$ stays relatively constant.

(Q_2 absorbs/sinks all the currents from I_1 in order to have the same $|V_{BE2}|$)

Consider region B:

$$I_{C1} = I_1 + \frac{V_{out}}{R_L}$$

$\therefore I_1 \uparrow\downarrow \Rightarrow |V_{BE1}| = |V_{in} - V_{out}|$ stays relatively constant.

(Q_1 provides/sources current to I_1 in order to have $|V_{BE1}|$ constant.)

Consider region C: (Dead zone).

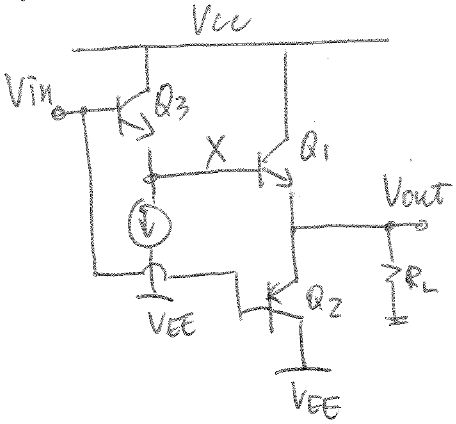
$I_1 = -\frac{V_{out}}{R_L}$ (Both transistors off)

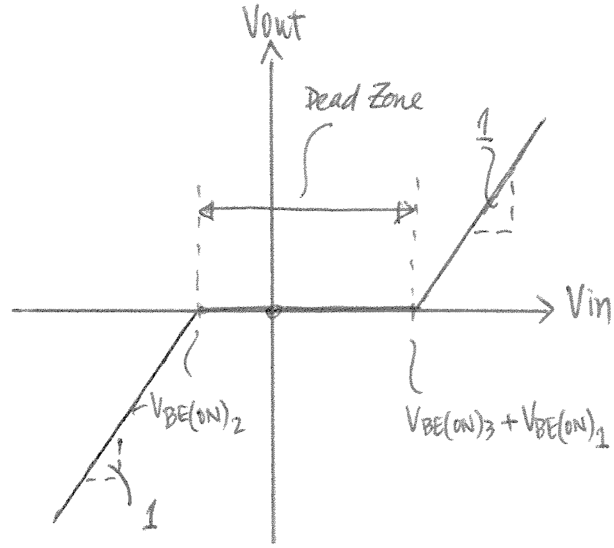
$\therefore I_1 \uparrow \Rightarrow V_{out} \downarrow$

$I_1 \downarrow \Rightarrow V_{out} \uparrow$

i.e. In the dead zone, Vout is predominantly controlled by I_1. One can use this to control Vout and effectively shift the region of dead zone.
($\because V_{out}|_{V_{in}=0} \neq 0$ anymore)

11.





Dead Zone

$= |V_{BE(ON)2}| + V_{BE(ON)3} + V_{BE(ON)1}$

Analysis

- $(0 < V_{in} < V_{BE(ON)3} + V_{BE(ON)1})$:

  Q_1 is OFF ($V_{in} < V_{BE(ON)1}$)

  Q_2 is OFF (V_{BE2} reverse-biased)

  $\Rightarrow V_{out} = 0$

- $(-|V_{BE(ON)2}| < V_{in} < 0)$:

  Q_1, Q_2 OFF. } $V_{out} = 0$

- $(V_{BE(ON)3} + V_{BE(ON)1} < V_{in} < V_{cc})$

  Q_1 ON

  Q_2 OFF

  } $V_{out} = V_{in} - V_{BE(ON)3} - V_{BE(ON)1}$

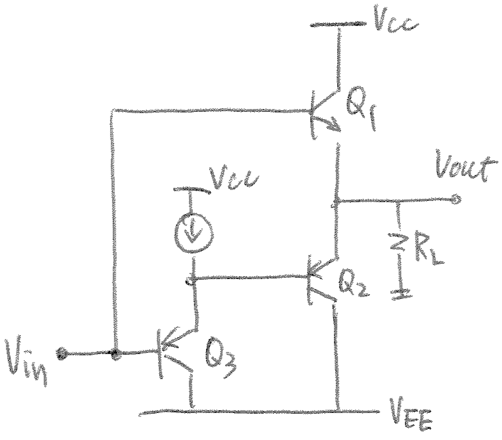
- $(-|V_{EE}| < V_{in} < -|V_{BE(ON)2}|)$

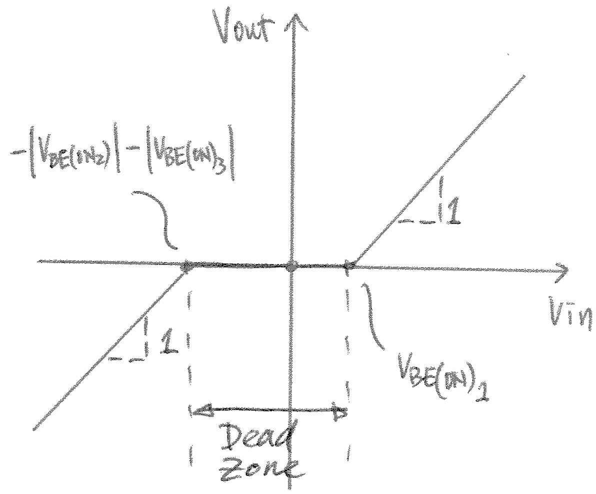
  Q_2 ON

  Q_1 OFF

  } $V_{out} = V_{in} + |V_{BE(ON)2}|$

12.





$-V_{EE} < V_{in} < -\left(|V_{BE(ON)_2}| + |V_{BE(ON)_3}|\right)$:

$\Rightarrow$ Q_2, Q_3 ON, Q_1 OFF $\Big\}$ $V_{out} = V_{in} + |V_{BE(ON)_3}| + |V_{BE(ON)_2}|$

$-\left(|V_{BE(ON)_2}| + |V_{BE(ON)_3}|\right) < V_{in} < V_{BE(ON)_1}$:

$\Rightarrow$ Q_1, Q_2 OFF $\Rightarrow$ $V_{out} \cong 0$

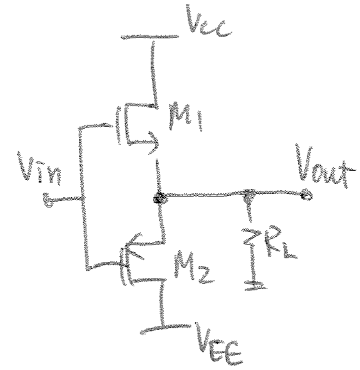
$V_{BE(ON)_1} < V_{in} < V_{cc}$:

$\Rightarrow$ Q_1 ON, Q_2, Q_3 OFF $\Big\}$ $V_{out} = V_{in} - V_{BE(ON)_1}$

$$\text{Dead Zone} = V_{BE(ON)_1} + |V_{BE(ON)_2}| + |V_{BE(ON)_3}|$$

13.

(a)



Ignore body effect.

$-|V_{EE}| < V_{in} < -|V_{t,p}|$:

$\Rightarrow$ M_1 OFF, M_2 ON (saturation) $\}$ $V_{out} = V_{in} + V_{SG,2}$

$V_{CC} > V_{in} > V_{t,n}$:

$\Rightarrow$ M_1 ON, M_2 OFF $\}$ $V_{out} = V_{in} - V_{GS,1}$

$\Rightarrow$ M_1 & M_2 can never on at the same time.

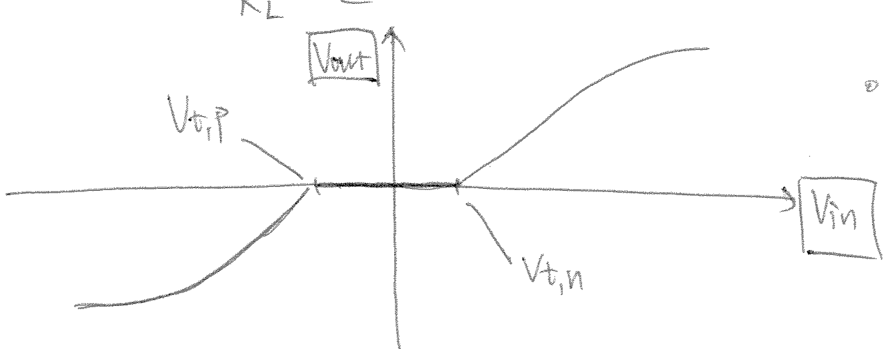
$-|V_{t,p}| < V_{in} < V_{t,n}$:

M_1, M_2 OFF $\Rightarrow V_{out} = 0$

For MOS, $I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}(|V_{GS}| - |V_t|)^2$ — saturation region.

$\Rightarrow$ M_1 ON: $\frac{V_{out}}{R_L} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{t,n})^2$, $V_{out} > 0$

M_2 ON: $-\frac{V_{out}}{R_L} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{out} - V_{in} - V_{t,p})^2$, $V_{out} < 0$

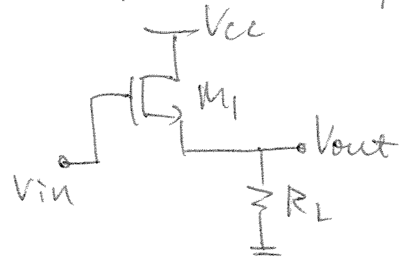


- Solve for Vout in both cases.

(b) Outside dead zone

⇒ either M_1 or M_2 is on.

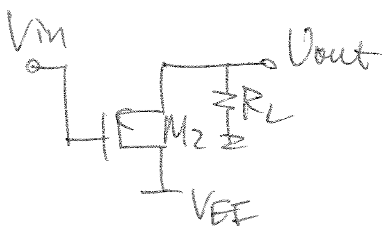
• For positive inputs:



Source follower:

$$\therefore \frac{v_{out}}{v_{in}} = \frac{g_{m1}}{1+g_{m1}R_L}$$

• For negative inputs:

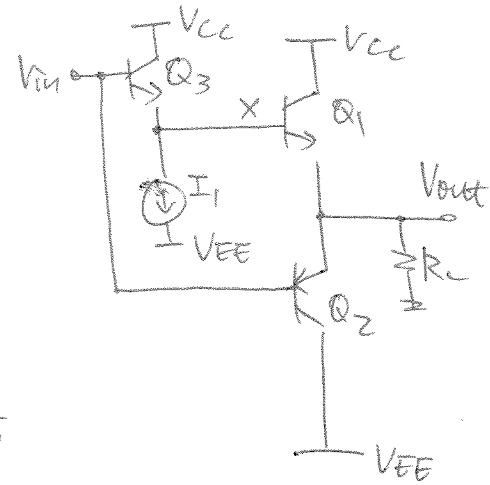


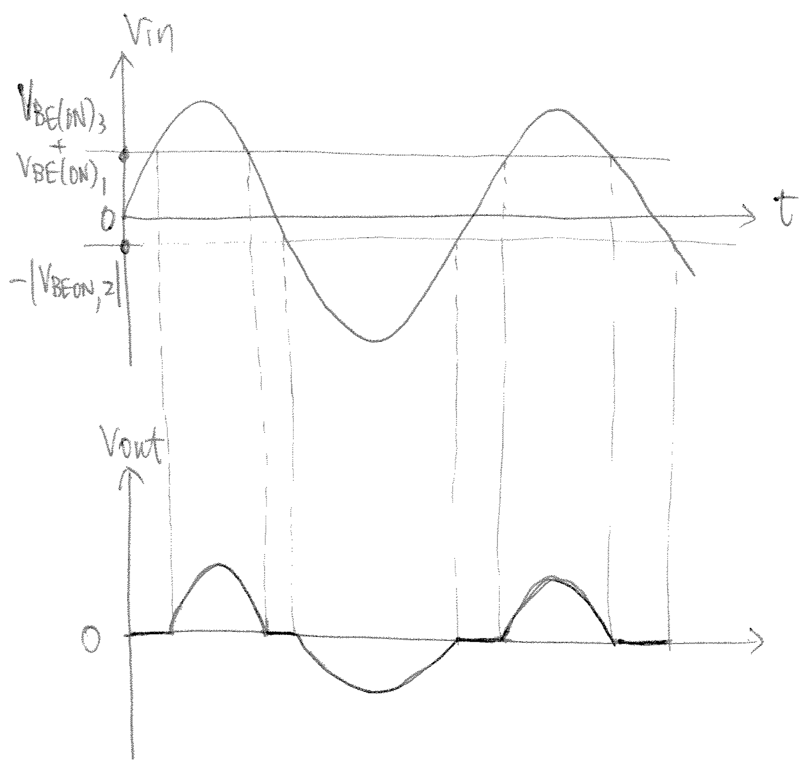
Source follower:

$$\therefore \frac{v_{out}}{v_{in}} = \frac{g_{m2}}{1+g_{m2}R_L}$$

14. Dead zone :

$$V_{out} \in \left[-|V_{BE(ON),2}| \, , \; V_{BE(ON)3} + V_{BE(ON)1} \right]$$

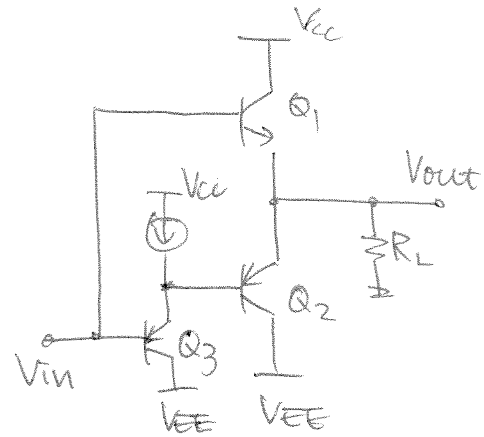


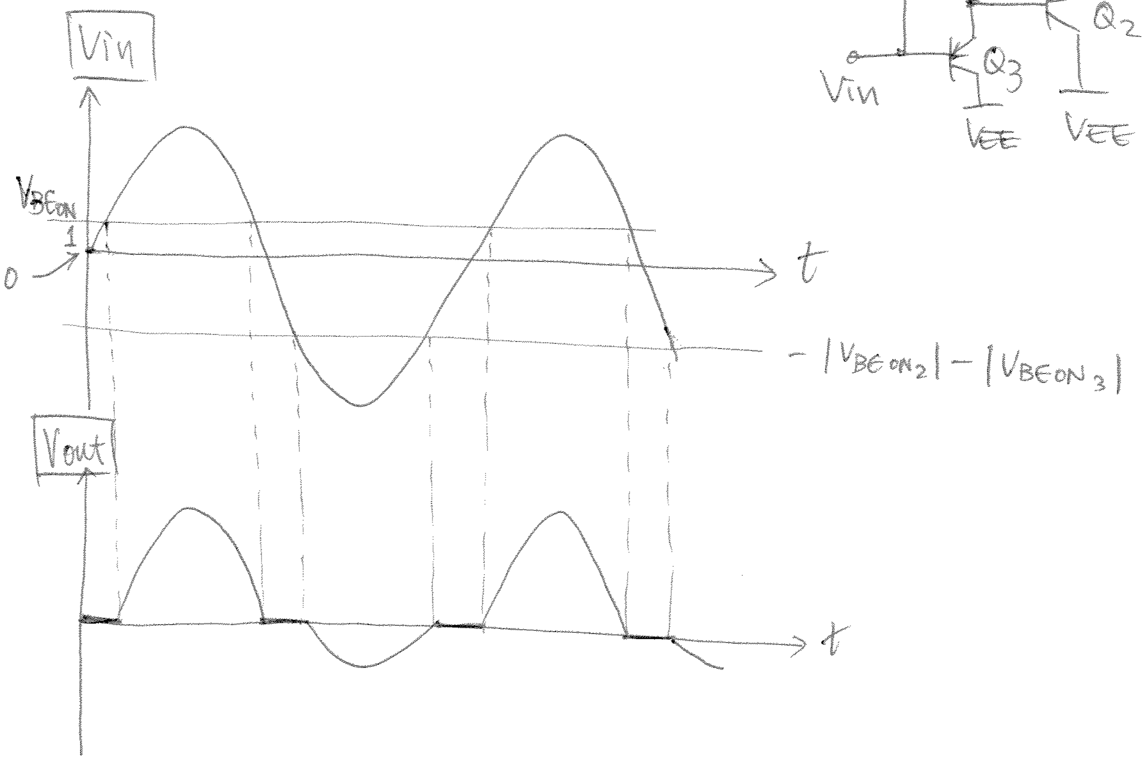


15.

Dead zone:

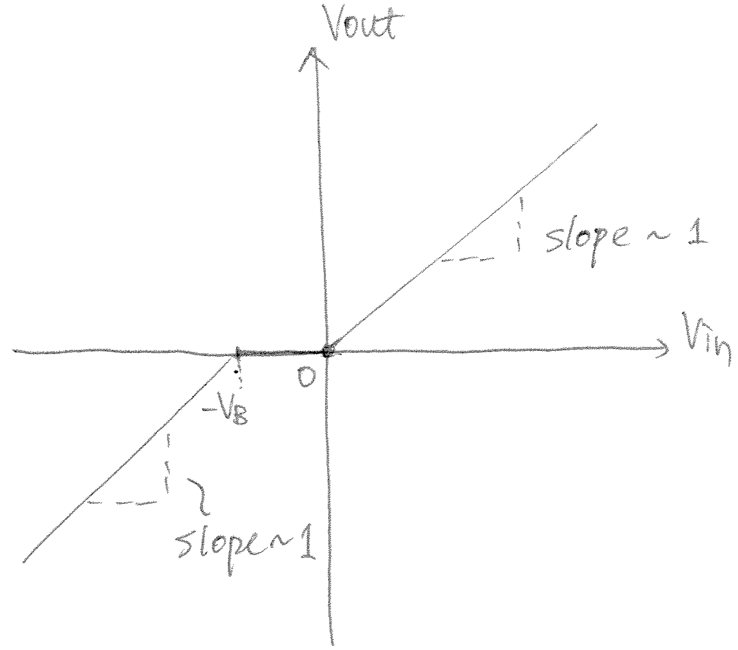
$V_{out} \in \left[-\left(|V_{BE,ON,2}| + |V_{BE,ON,3}|\right), V_{BE,ON,1}\right]$



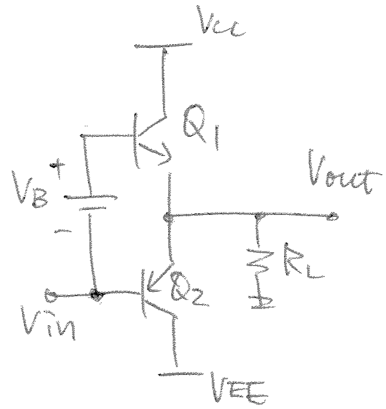


16.

(a)



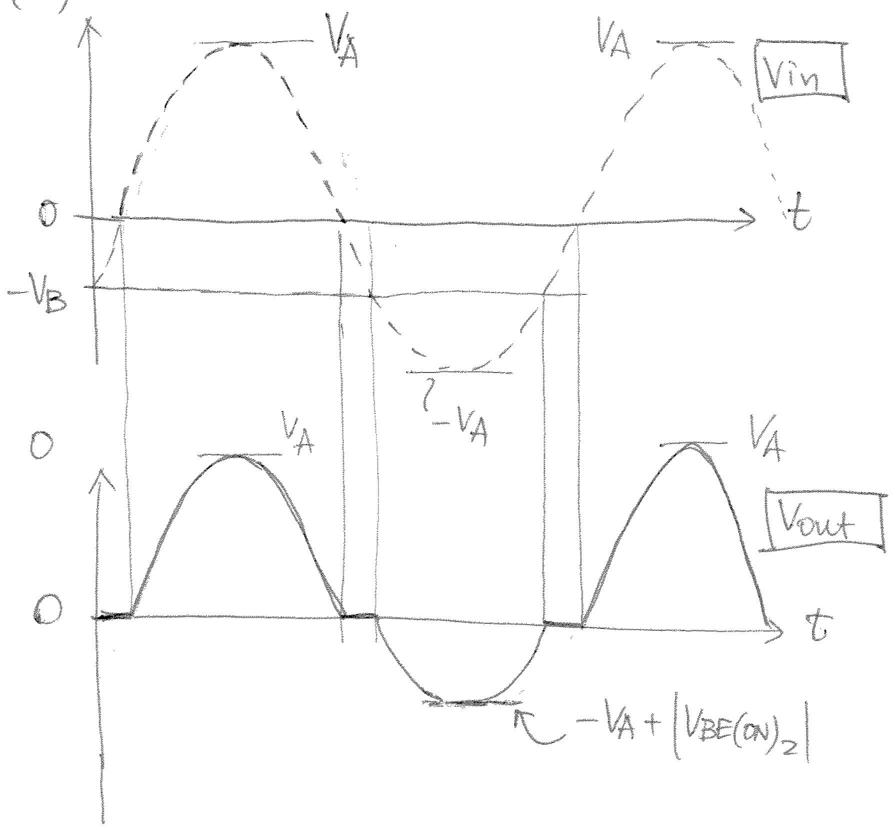
Vout
Vin
0
$-V_B$
slope ~ 1
slope ~ 1



Vcc
Q_1
V_B
+
−
Vout
R_L
Q_2
Vin
VEE

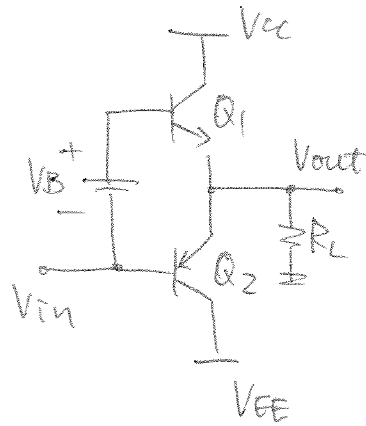
$(V_B = V_{BE})$

(b)



V_A
V_A
Vin
0
t
$-V_B$
$-V_A$
0
V_A
V_A
Vout
0
t
$-V_A + |V_{BE(ON)_2}|$

17.



- $V_{out} = 0$:

$\Rightarrow I_{C_1} = I_{C_2} = I_{BIAS}$

$$\Rightarrow I_{S_1} \exp\left(\frac{V_{in}+V_B-V_{out}}{V_T}\right) = I_{S_2} \exp\left(\frac{|V_{out}-V_{in}|}{V_T}\right)$$

$$\ln\left(\frac{I_{S1}}{I_{S2}}\right) + \frac{V_{in}+V_B-V_{out}}{V_T} = \frac{V_{out}-V_{in}}{V_T}$$

$I_{S_1} = 5 \cdot 10^{-17} A$

$I_{S2} = 8 \cdot 10^{-17} A$

$I_{BIAS} = 5 mA$
$(V_{out} = 0)$

- For $V_{out} = 0$, $V_T = 0.026\ V$:

$$\Rightarrow \ln\left(\frac{5}{8}\right) + \frac{V_{in}+V_B}{0.026} = +\frac{V_{in}}{0.026}$$

- Given $I_{C2} = 5mA$

$$\Rightarrow I_{S2} \exp\left(\frac{-V_{in}}{0.026}\right) = 5mA \Rightarrow V_{in} = -0.83\ V$$

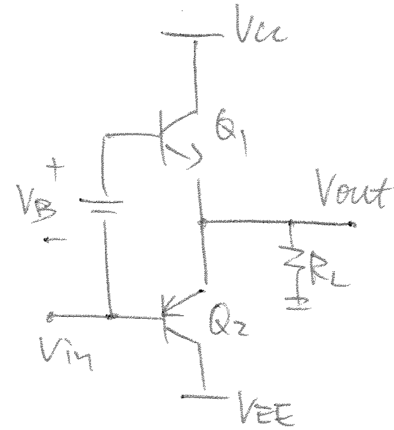
$$I_{C_1} = I_{S_1} \exp\left(\frac{V_{in}+V_B-V_{out}}{V_T}\right) = (5 \cdot 10^{-17} A) \exp \frac{-0.83+V_B}{V_T}$$

$$\Rightarrow V_B = 0.83 + 0.026 \ln\left(\frac{5mA}{5 \cdot 10^{-17} A}\right)$$

$$\approx 1.67\ V.$$

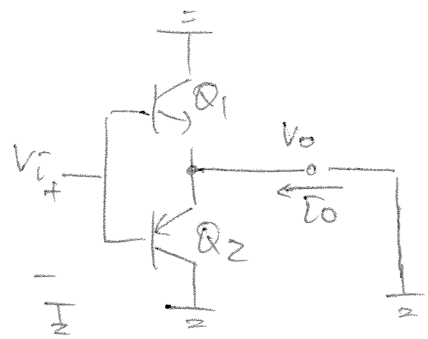
18.

(a) Equivalent circuit (small-signal) around $V_{out}=0$:



$V_{P_{in}} = 2\,V$

$R_L = 8\,\Omega$



$$i_o = -g_{m1} v_i + (-v_i) g_{m2}$$
$$= -(g_{m1} + g_{m2}) v_i$$

$$\therefore G_m = \frac{i_o}{v_i} = -(g_{m1} + g_{m2})$$

$$\therefore A_V = \frac{v_o}{v_i} = \frac{i_o \times R_L}{v_i} = -(g_{m1} + g_{m2}) R_L$$

(b) $A_V = -(g_{m1} + g_{m2}) R_L = -\left(\frac{I_{C1}}{V_T} + \frac{I_{C2}}{V_T}\right) R_L$

$$= -\left(\frac{5mA}{0.026V} + \frac{5mA}{0.026V}\right)(8\,\Omega) = -3.08$$

$$\Rightarrow |V_o|_p = |v_i A_V|_p = |(2V)(-3.08)| = 6.16\,V$$

(Assume Vcc is large enough)

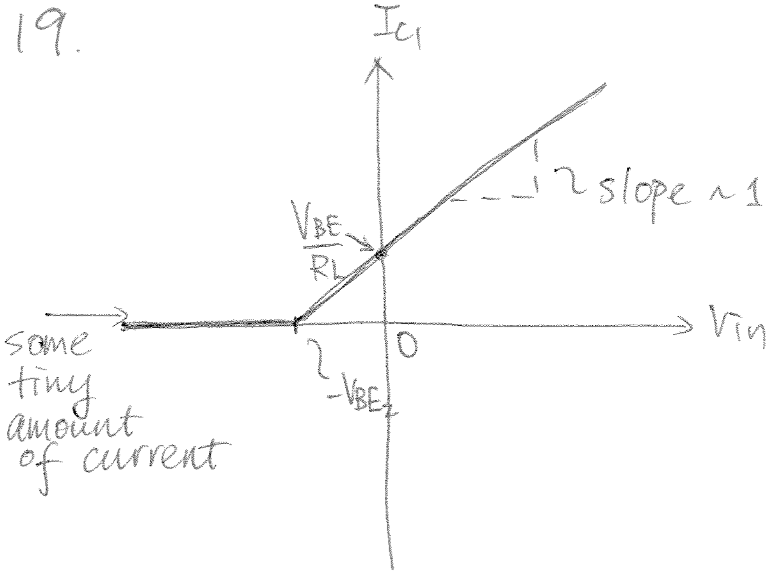
(c) $I_{C1} = I_{C2} + \frac{V_{out}}{R_L}$

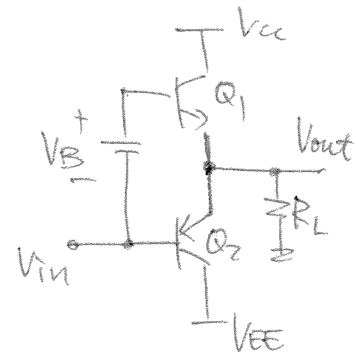
$$I_{C1,peak} = I_{C2} + \frac{V_P}{R_L}$$

$$= 5mA + \frac{6.16V}{8\Omega}$$

$$= 775 mA$$

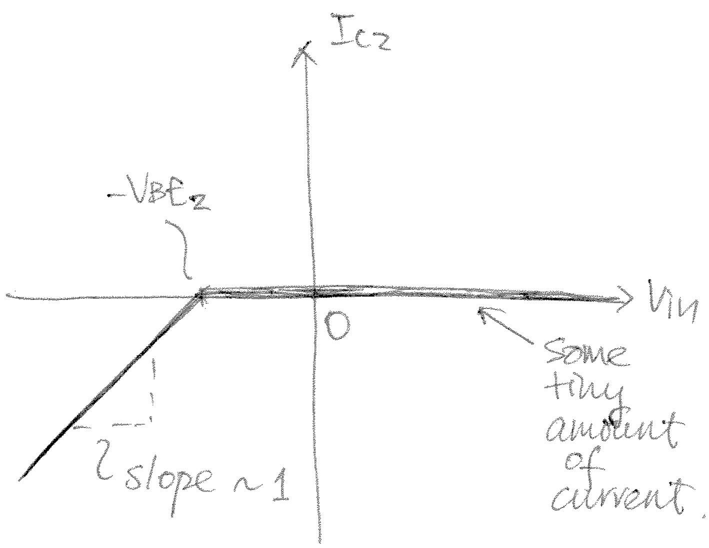
19.



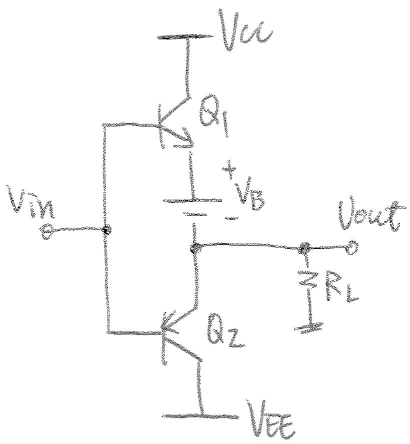


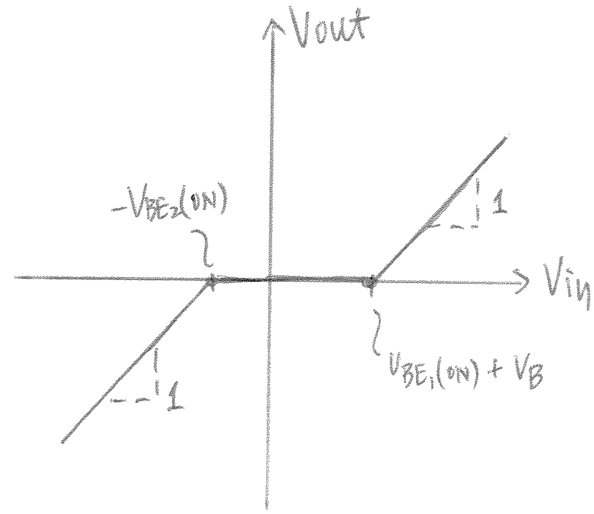
$$V_{out} = V_{in} + |V_{BE2}|$$

$$\Rightarrow I_{C1} = I_{C2} + \frac{V_{out}}{R_L}$$



20.





- To analyze such circuit, assume $V_{out} = 0$:

$\Rightarrow \quad -V_{BE2(ON)} < V_{in} < V_{BE1(ON)} + V_B$.

$(V_{BE1(ON)} + V_B) < V_{in}$: $\quad V_{out} = V_{in} - V_{BE1(ON)} - V_B$

$V_{in} < -V_{BE2(ON)}$: $\quad V_{out} = V_{in} + |V_{BE2(ON)}|$

21. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow V_T \left[\ln \frac{I_{C_1}}{I_{S_{Q1}}} + \ln \frac{I_{C_2}}{I_{S_{Q2}}} \right] = V_T \left[\ln \frac{I_{D_1}}{I_{S_{D_1}}} + \ln \frac{I_{D_2}}{I_{S_{D_2}}} \right]$$

$$\Rightarrow \frac{I_{C_1} I_{C_2}}{I_{S_{Q_1}} I_{S_{Q_2}}} = \frac{I_{D_1} I_{D_2}}{I_{S_{D_1}} I_{S_{D_2}}}$$

$\therefore$ If $I_{S_{Q_1}} I_{S_{Q_2}} = I_{S_{D_1}} I_{S_{D_2}}$,

then $I_{C_1} I_{C_2} = I_{D_1} I_{D_2}$

22. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow V_T \ln\left(\frac{I_{C_1} I_{C_2}}{I_{S,Q_1} I_{S,Q_2}}\right) = V_T \ln\left(\frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}}\right)$$

$$\Rightarrow \frac{I_{C_1} I_{C_2}}{I_{S,Q_1} I_{S,Q_2}} = \frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}} \quad \text{———— (1)}$$

$I_1 = I_{D_1} = I_{D_2} = 1\text{mA}$; $I_{S,Q} = 16 I_{S,D}$

$V_{out} = 0 \Rightarrow I_{C_1} = I_{C_2}$ ———— (2)

Substitute all into (1):

$$\frac{I_{C_1} I_{C_1}}{(16 I_{S,D})^2} = \frac{(1\text{mA})^2}{(I_{S,D})^2} \Rightarrow I_{C_1} = I_{C_2} = 16\text{ mA}$$

23. $V_{BE_1} + |V_{BE2}| = V_{D_1} + V_{D2}$

$$\Rightarrow \frac{I_{C_1} I_{C2}}{I_{S,Q_1} I_{S,Q_2}} = \frac{I_{D_1} I_{D2}}{I_{S,D_1} I_{S,D_2}} \quad \text{———} \quad (1)$$

$$I_{C_1} = I_{C_2} = 5mA \quad \text{———} \quad (2)$$

$$I_{S,Q} = 8 I_{S,D} \quad \text{———} \quad (3)$$

Substitute all into (1):

$$\frac{(5mA)^2}{(8 I_{S,D})^2} = \frac{I_{D_1} I_{D_2}}{(I_{S,D})^2} \Rightarrow I_1 = I_D = 0.625 \, mA$$

24. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow \frac{I_{C1} I_{C2}}{I_{S,Q_1} I_{S,Q_2}} = \frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}} \quad \text{———} \quad (1)$$

$I_1 = I_D = 2mA$

$I_{S,Q_1} = 8 I_{S,D_1}$; $I_{S,Q_2} = 16 I_{S,D_2}$

Substitute all into (1):

$$\frac{I_{C1} I_{C2}}{(8 I_{S,D_1})(16 I_{S,D_2})} = \frac{(2mA)^2}{I_{S,D_1} I_{S,D_2}}$$

$\Rightarrow I_{C_1} = I_{C_2} \cong 22.6\ mA$

25. $V_{BE_1} + |V_{BE_2}| = V_{D_1} + V_{D_2}$

$$\Rightarrow \frac{kT_Q}{q}\left[\ln\left(\frac{I_{C_1} I_{C_2}}{I_{S,Q_1} I_{S,Q_2}}\right)\right] = \frac{kT_D}{q}\left[\ln\left(\frac{I_{D_1} I_{D_2}}{I_{S,D_1} I_{S,D_2}}\right)\right]$$

Suppose $T_D = (T_Q + \Delta T)$:

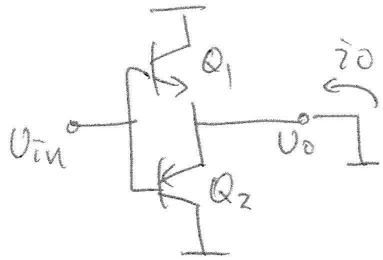
$$\Rightarrow T_Q\left[\ln \frac{I_{C_1} I_{C_2}}{I_{SQ_1} I_{SQ_2}} - \ln \frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}}\right] = \Delta T \cdot \ln \frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}}$$

$$\Rightarrow I_{C_1} \cdot I_{C_2} = I_{SQ_1} \cdot I_{SQ_2} \cdot \left(\frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}}\right)^{1 + \frac{\Delta T}{T_Q}}$$

Typically, $\frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}} > 1$

$\Rightarrow$ A ΔT introduces a factor $\left(\frac{I_{D_1} I_{D_2}}{I_{SD_1} I_{SD_2}}\right)^{\frac{\Delta T}{T_Q}} < 0$, implying that the $I_{C_1} I_{C_2}$ product drops corresponding to a change (positive) in temperature.

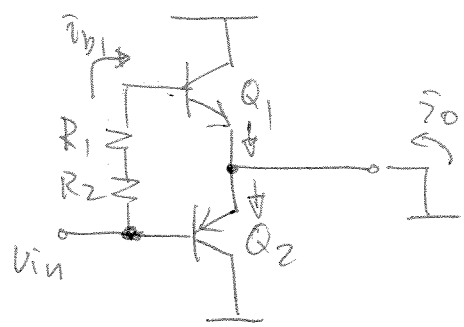
26. Small Signal:



$$G_m = \frac{i_o}{v_{in}} = -(g_{m1} + g_{m2})$$

$$\Rightarrow \frac{v_o}{v_{in}} = \frac{i_o R_L}{v_{in}} = +(g_{m1} + g_{m2}) R_L$$

27. Small-Signal:



$$i_o = -g_{m1} v_{be1} + g_{m2}|v_{be2}| \qquad (i_o = i_{c2} - i_{c1})$$

$$|v_{be2}| = v_{in}$$

$$v_{be1} = v_{in} - i_{b1}(R_1+R_2) = v_{in} - \frac{i_{c1}}{\beta_1}(R_1+R_2)$$

$$= v_{in} - \frac{i_{c2} - i_o}{\beta_1}(R_1+R_2)$$

$$= v_{in} + \frac{g_{m2} v_{in} + i_o}{\beta_1}(R_1+R_2)$$

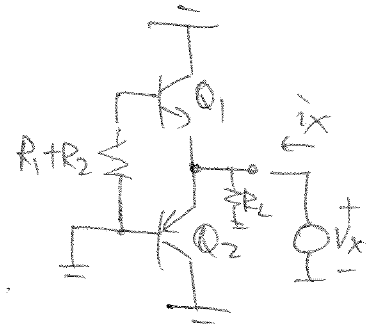
$$\therefore \; g_{m1}\left[v_{in} + \frac{g_{m2} v_{in} + i_o}{\beta_1}(R_1+R_2)\right] + i_o = -g_{m2} v_{in}$$

Solving for $\frac{i_o}{v_{in}}$ gives:

$$G_m = \frac{i_o}{v_{in}} = \frac{-\left[g_{m1} + \frac{g_{m1} g_{m2}}{\beta_1}(R_1+R_2) + g_{m2}\right]}{1 + \frac{g_{m1}(R_1+R_2)}{\beta_1}}$$

Rout:

$$\frac{v_X}{i_X} = R_{out} = \left(r_{\pi 2} \parallel \frac{1}{g_{m2}}\right) \parallel \left[(r_{\pi 1} + R_1 + R_2) \parallel \frac{1}{g_{m1}}\right] \parallel R_L$$



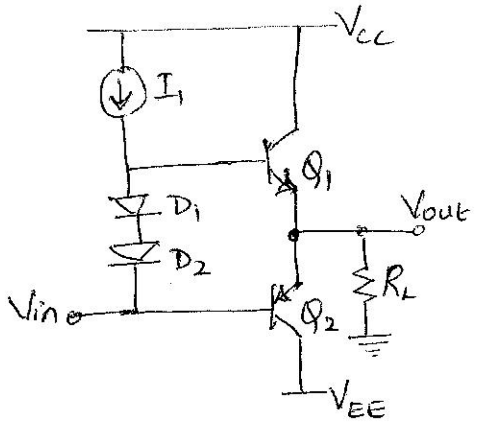
$$\therefore A_v = G_m R_{out}$$

$$= -\left[\frac{g_{m1} + \dfrac{g_{m1} g_{m2} (R_1 + R_2)}{\beta_1} + g_{m2}}{1 + \dfrac{g_{m1}(R_1 + R_2)}{\beta_1}}\right] \cdot \left\{\left[r_{\pi 2} \parallel \frac{1}{g_{m2}}\right] \parallel \left[(r_{\pi 1} + R_1 + R_2) \parallel \frac{1}{g_{m1}}\right] \parallel R_L\right\}$$

(28) Small Signal gain around $V_{out} = 0$:

$$A_V = +(g_{m1} + g_{m2}) R_L$$

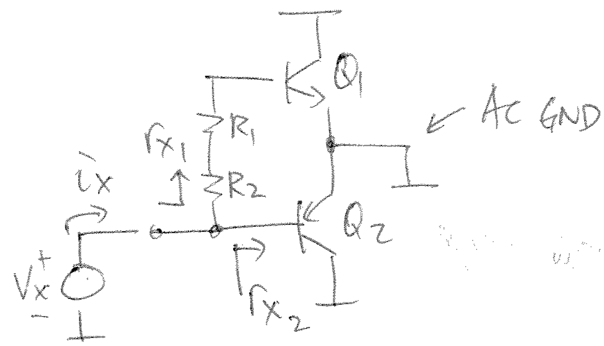
$$0.8 = (I_{C1} + I_{C2}) \frac{R_L}{V_T}$$

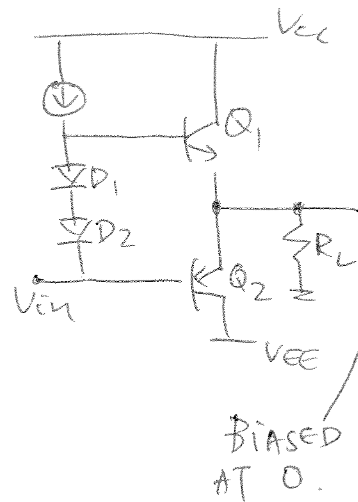


If $I_{C1} = I_{C2} = I_{BIAS}$, then

$$I_C = \frac{0.8}{2} \times \frac{V_T}{R_L} = 0.4 \frac{V_T}{R_L} = \frac{0.0104}{R_L} = 1.3\,mA$$

29. Small-signal equivalent:





$$R_{in} = \frac{v_x}{i_x} = r_{x_1} \parallel r_{x_2}$$

$$= (R_1 + R_2 + r_{\pi_1}) \parallel r_{\pi_2}$$

- R_1 & R_2 can be neglected when $r_{\pi_1} \gg (R_1 + R_2)$

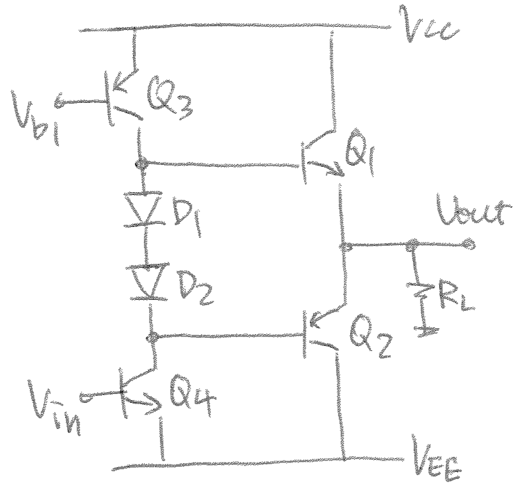
30. $I_{C_1} = I_{C_2} = 10\ mA$

$I_{C_3} = I_{C_4} = 1\ mA$

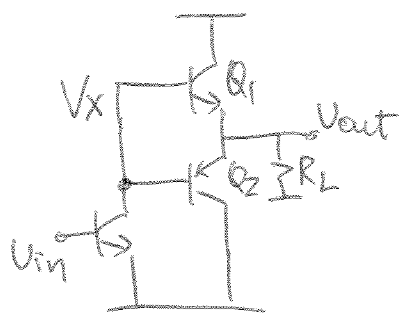
$\beta_1 = 40 \quad \beta_2 = 20$

$R_L = 8\Omega$

$R_{D_1} = R_{D_2} = 0$



Small-signal



$$A_V = \frac{v_{out}}{v_x} \cdot \frac{v_x}{v_{in}}$$

$$= -g_{m4}\left[(g_{m1}+g_{m2})(r_{\pi_1} \| r_{\pi_2})R_L + (r_{\pi_1} \| r_{\pi_2})\right] \times \frac{R_L}{R_L + \frac{1}{g_{m1}+g_{m2}}}$$

$$= -g_{m4}(r_{\pi_1} \| r_{\pi_2})(g_{m1}+g_{m2})R_L$$

$$\therefore A_V = -\frac{I_{C4}}{V_T}\left(\frac{\beta_1 V_T}{I_{C1}} \| \frac{\beta_2 V_T}{I_{C2}}\right)\left(\frac{I_{C1}}{V_T} + \frac{I_{C2}}{V_T}\right)R_L$$

$$= -\frac{1mA}{0.026}[35]\cdot\left(2\times\frac{10mA}{V_T}\right)(8)$$

$$\approx -8.3$$

31.

$$\frac{v_{out}}{v_{in}} = -g_{m4}(r_{\pi 1} \| r_{\pi 2})(g_{m1}+g_{m2})R_L \qquad \left(r_\pi = \frac{\beta}{g_m}\right)$$

When $g_{m1} \approx g_{m2}$: $(\Rightarrow r_\pi$

$$\frac{v_{out}}{v_{in}} \approx -g_{m4}R_L(2g_{m1})\left(\frac{\beta_1}{g_{m1}} \| \frac{\beta_2}{g_{m1}}\right)$$

$$= -g_{m4}R_L(2g_{m1})\left[\frac{1}{g_{m1}} \cdot \frac{\beta_1\beta_2}{\beta_1+\beta_2}\right]$$

$$= -\frac{2\beta_1\beta_2}{\beta_1+\beta_2} g_{m4}R_L$$

(32) From eqn. (13.23), Small-Signal gain of the output stage is:

$$\left|\frac{V_{out}}{V_{in}}\right| = +g_{m4}\left(r_{\pi 1} \,||\, r_{\pi 2}\right)\left(g_{m1}+g_{m2}\right)R_L$$

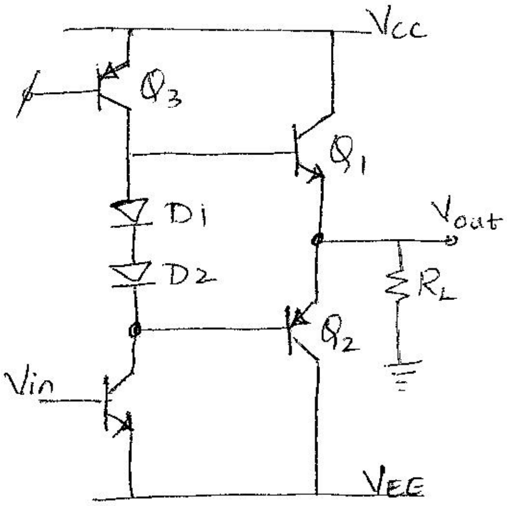
$$\approx +g_{m4} R_L \times \frac{2\beta_1 \beta_2}{\beta_1+\beta_2}$$

$$\Rightarrow 4 = +\frac{I_{C4}}{V_T}(8\Omega) * \frac{2(40)(20)}{40+20}$$

$$\Rightarrow I_{C4} \approx I_{C3}$$

$$= \frac{4V_T}{(8\Omega)} \cdot \frac{40+20}{2(40)(20)}$$

$$= \underline{0.49\,mA}$$



$A_V = \frac{V_{out}}{V_{in}} = 4$

$\beta_1 = 40$

$\beta_2 = 20$

$R_L = 8\Omega$

33) From equation 13.27,

$$\frac{V_x}{I_x} = \frac{1}{g_{m1} + g_{m2}} + \frac{r_{O3} \parallel r_{O4}}{(g_{m1} + g_{m2})(r_{\pi 1} \parallel r_{\pi 2})}$$

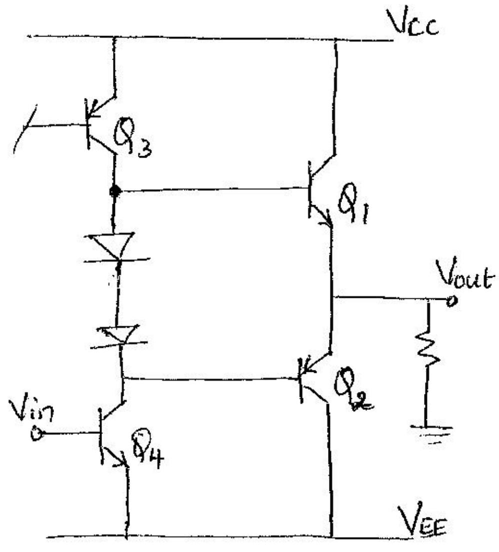
If $g_{m1} \approx g_{m2} = g_m$:

$$\frac{V_x}{I_x} \approx \frac{1}{2g_m} + \frac{r_{O3} \parallel r_{O4}}{2g_m \left(\frac{\beta_1}{g_m} \parallel \frac{\beta_2}{g_m}\right)}$$

$$= \frac{1}{2g_m} + \frac{r_{O3} \parallel r_{O4}}{2g_m \left(\frac{1}{g_m} \frac{\beta_1 \beta_2}{\beta_1 + \beta_2}\right)}$$

$$= \frac{1}{2g_m} + \frac{r_{O3} \parallel r_{O4}}{2\beta_1 \beta_2} (\beta_1 + \beta_2)$$

34)



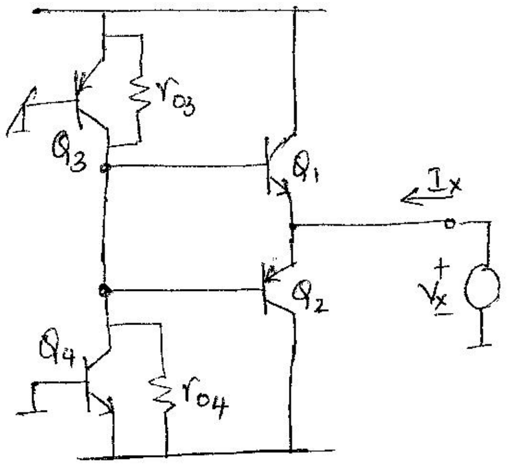
$I_3 = I_4 = 1mA$

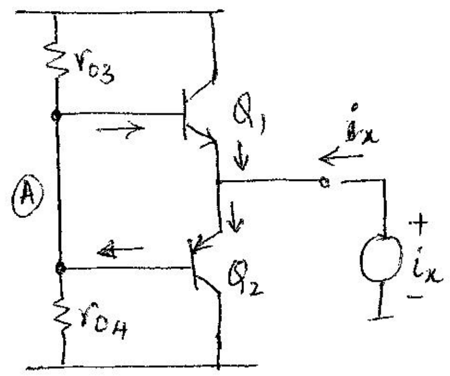
$I_1 = I_2 = 8mA$

$V_{A3} = 10V$

$V_{A4} = 15V$

(a) Small-Signal Equivalent





$$V_{eb} = V_x \frac{(r_{\pi 1} \| r_{\pi 2})}{(r_{\pi 1} \| r_{\pi 2}) + (r_{o3} \| r_{o4})} \qquad V_{be} = V_A - V_x$$

$$i_x + i_{c1} = i_{e2} \Rightarrow i_x = i_{c2} - i_{c1} = g_{m2} V_{eb} - g_{m1} V_{be}$$

$$\therefore i_x = [g_{m2} + g_{m1}] V_x \frac{(r_{\pi 1} \| r_{\pi 2})}{(r_{\pi 1} \| r_{\pi 2}) + (r_{o3} \| r_{o4})}$$

$$\Rightarrow \frac{V_x}{i_x} = R_{out} = \frac{(r_{\pi 1} \| r_{\pi 2}) + (r_{o3} \| r_{o4})}{[g_{m1} + g_{m2}) \ (r_{\pi 1} \| r_{\pi 2})}$$

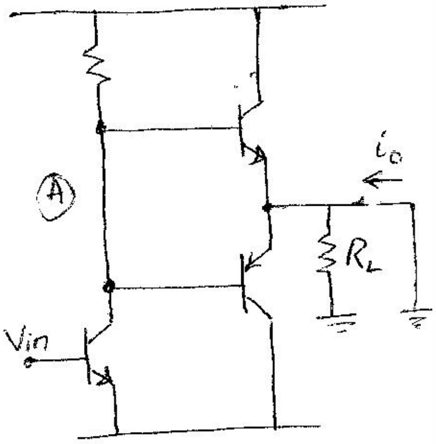
$$r_{\pi 1} = \frac{\beta_1 V_T}{I_{C_1}} = 130\Omega \qquad r_{\pi 2} = \frac{\beta_2 V_T}{I_{C2}} = 65\Omega$$

$$r_{o3} = \frac{V_{A3}}{I_{C3}} = 10k\Omega \qquad r_{o4} = \frac{V_{A4}}{I_{C4}} = 15k\Omega$$

$$g_{m_1} = 0.31\,S \qquad g_{m_2} = 0.31\,S$$

$$\Rightarrow R_{out} = \frac{43.3 + 6000}{(0.62)(43.3)} \approx 225.1\Omega$$

(b) Effective $R_{out} = R_{out,a} \| 8\Omega \approx 8\Omega$



$$G_m = \frac{i_o}{V_A} \cdot \frac{V_A}{V_{in}}$$

$$= -g_{m4}\left(r_{\pi 1} \| r_{\pi 2} \| r_{o3}\right) \cdot \left(g_{m_1} + g_{m_2}\right)$$

$$\therefore A_V = G_m R_{out}$$

$$= -g_{m4}\left(r_{\pi 1} \| r_{\pi 2} \| r_{o3}\right)\left(g_{m_1} + g_{m_2}\right) R_{out}$$

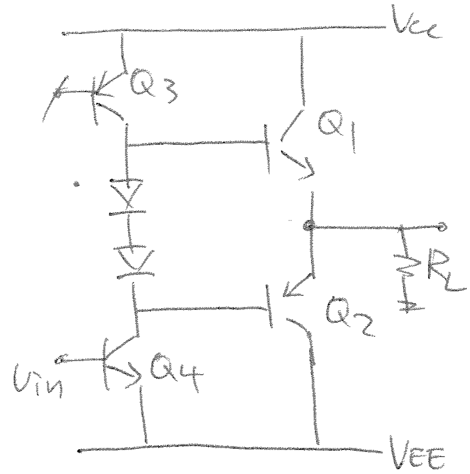
$$= -0.038\left[130 \| 65 \| 10k\right]\left[0.62\right](8)$$

$$\approx -8.1$$

$$g_{m4} = \frac{I_{C4}}{V_T} = 0.038\,S$$

35. Max current delivered by $Q_1 = I_{C3} \cdot \beta_1 = 1mA \cdot 40$

$= 40\ mA$. (Q_4 off)



$I_{C3} = I_{C4} = 1mA$

$\beta_1 = 40 \quad \beta_2 = 20$

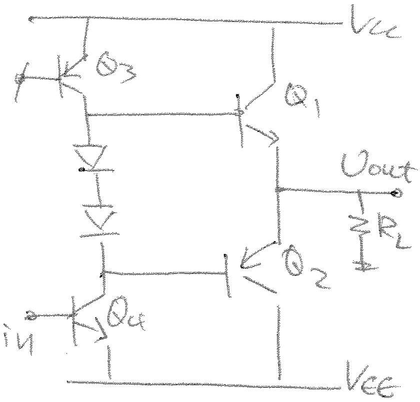
Max current delivered by $Q_2 = I_{C4} \cdot \beta_2$

$= 1mA \cdot 20$

$= 20\ mA$. (Q_3 off)

36. $P = 0.5\,W$ $R_L = 8\,\Omega$

$\beta_1 = 40$ $\beta_2 = 20.$



$$P_{AVG} = \frac{1}{2}\frac{V_P^2}{R_L} = 0.5$$

$$\Rightarrow V_P^2 = 2(0.5)R_L$$

$$\Rightarrow V_P = \sqrt{R_L} = 2\sqrt{2}$$

At positive V_P, $I_{C_1} = \frac{V_P}{R_L} = \frac{2\sqrt{2}}{8} = 0.35\,A.$

At negative V_P, $I_{C_2} = \frac{V_P}{R_L} \Rightarrow I_{C_2} = 0.35\,A.$

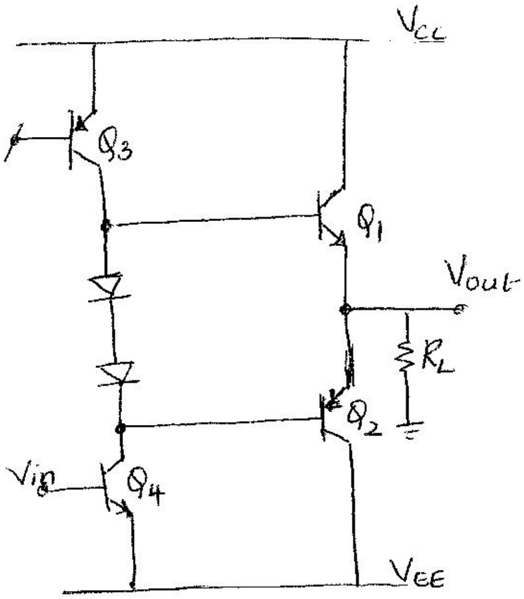
- At $+V_P$, all of I_{C_3} supports the base current of Q_1

$$\Rightarrow I_{C_3} = I_{B_1} = \frac{I_{C_1}}{\beta_1} = \frac{0.35A}{40} = 8.75\,mA$$

- At $-V_P$, all of I_{C_4} supports the base current of Q_2

$$\Rightarrow I_{C_4} = I_{B_2} = \frac{I_{C_2}}{\beta_2} = \frac{0.35A}{20} = 17.5\,mA$$

37) $P_{AVG} = 0.5W \quad R_L = 8\Omega \quad V_{CC} = 5V$



$$\Rightarrow 0.5W = \frac{1}{2}\frac{V_P^2}{R_L}$$

$$\Rightarrow V_P = 2\sqrt{2}\ V$$

$$P_{Q1} = \frac{1}{T}\int_0^{T/2} I_{C1} V_{CE1}\, dt$$

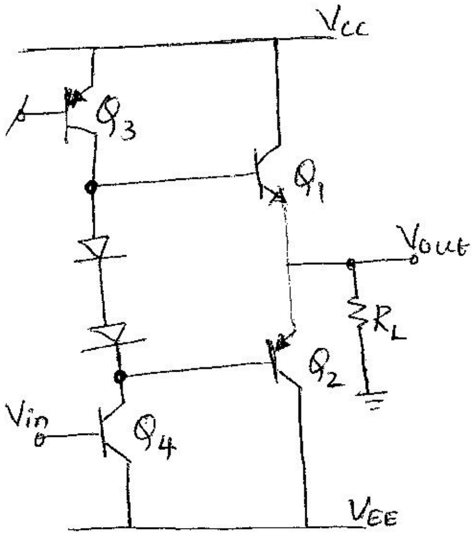
$$= \frac{1}{T}\int_0^{T/2}\left(\frac{V_P \sin\omega t}{R_L}\right)\left(V_{CC} - V_P \sin\omega t\right) dt$$

$$= \frac{1}{T}\int_0^{T/2}\left[\frac{V_{CC} V_P}{R_L}\sin\omega t\right] dt - \frac{V_P^2}{2R_L}$$

$$= \frac{V_P}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_P}{4}\right) = \frac{2\sqrt{2}}{8}\left(\frac{5}{\pi} - \frac{2\sqrt{2}}{4}\right)$$

$$\approx 0.31\ W$$

38) $P_{Q,MAX} = 0.75W$, $R_L = 8\Omega$, $V_{CC} = 5V$



- Out of all 4 transistors, Q_1 & Q_2 must sustain the most currents

$$P_{Q,MAX\,(INST)} = V_{CE} \times I_{C1,MAX} = (V_{CC} - V_{OUT}) I_{C1,MAX}$$

$$\Rightarrow P_{Q,MAX\,(AVG)} = \frac{1}{T}\int_0^{T/2} \frac{V_P \sin\omega t}{R_L} \cdot (V_{CC} - V_P \sin\omega t)\, dt$$

$$= \frac{1}{T}\int_0^{T/2} \left(\frac{V_{CC} V_P}{R_L} \sin\omega t\right) dt - \frac{V_P^2}{2R_L}$$

$$= \frac{V_P}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_P}{4}\right)$$

$$\Rightarrow \frac{dP_Q}{dV_P} = \frac{V_{CC}}{\pi R_L} - \frac{V_P}{2R_L} = 0 \quad \text{when } V_P = \frac{2V_{CC}}{\pi} = 3.18\,V$$

$$P_Q /_{V_P} = \frac{2V_{CC}}{\pi} = 0.32W$$

$$\therefore P_{R_L,MAX} = \frac{1}{2}\frac{V_P^2}{R_L} = 0.63W$$

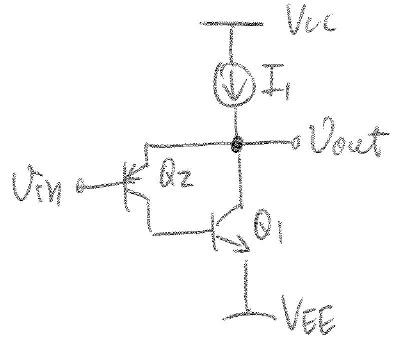
39. $P_{Q_1,MAX} = \left(\frac{V_{CC}}{\pi} - \frac{2V_{CC}}{4\pi}\right) \cdot \frac{2V_{CC}}{\pi R_L} \leq 0.75\ W$

$\Rightarrow V_{CC}|_{MAX} = 7.7\ V$

$\Rightarrow V_{P,MAX} = \frac{2V_{CC\,MAX}}{\pi} = 4.9\ V$

$\Rightarrow P_{R_L,MAX} = \frac{1}{2}\frac{V_{P\,MAX}^2}{R_L} = 1.5\ W$

40.



$I_1 = 5\,mA$

$\beta_1 = 40$

$\beta_2 = 50.$

$$
\begin{aligned}
I_1 &= I_{C1} + I_{E2} \\
&= I_{C1} + \frac{\beta_1 + 1}{\beta_1} I_{C2} \\
&= I_{C1} + \frac{\beta_1 + 1}{\beta_1} I_{B1} \\
&= \beta_1 I_{B1} + \frac{\beta_1 + 1}{\beta_1} I_{B1}
\end{aligned}
$$

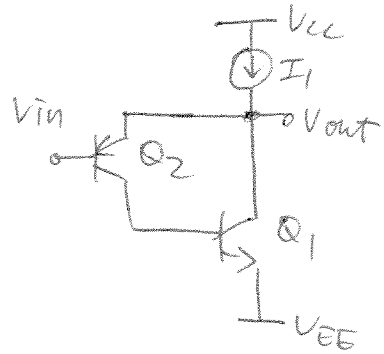
$$\Rightarrow I_{B1} = \frac{I_1}{\beta_1 + \frac{\beta_1 + 1}{\beta_1}} = \frac{0.005}{40 + \frac{41}{40}}$$

$$\approx 0.12\,mA$$

$$\Rightarrow I_{B2} = \frac{I_{C2}}{\beta_2} = \frac{I_{B1}}{\beta_2} = 0.0024\,mA$$

41. $V_{in} = 0.5\,V$

$I_{S2} = 6 \cdot 10^{-17} A$.



$I_{B_1} = I_{C2} = 0.12\,mA$

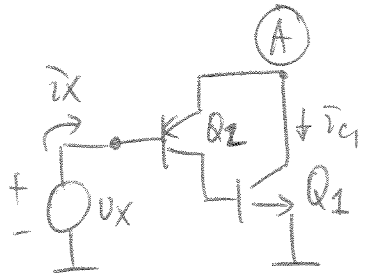
$\Rightarrow I_{C2} = I_{S2} \cdot \exp\left(\frac{V_{out} - V_{in}}{V_T}\right)$

$\therefore V_{out} = V_T \ln\left(\frac{I_{C2}}{I_{S2}}\right) + V_{in}$

$= 0.026 \ln\left(\frac{0.12\,mA}{6 \cdot 10^{-17} A}\right) + 0.5$

$\approx 1.24\,V$

42.



$i_{c2} = i_x \beta_2$

$i_{c1} = -g_{m2} v_{eb2} = i_{e2} = i_{c2} + i_{b2} = i_x(\beta_2 + 1)$

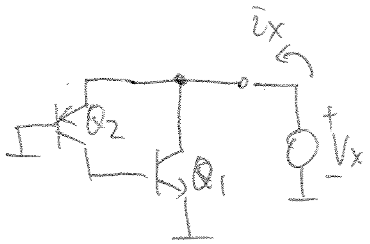
$v_{eb2} = v_A - v_x$

where $v_A = v_x - i_x r_{\pi 2}$

$\therefore\ i_{b1} = -g_{m2}(i_x \cdot r_{\pi 2})$

$i_{c1} = i_{b1} + i_{b1} \cdot \beta_1 = -g_m i_x \cdot r_{\pi 2}(1+\beta_1)$

$\Rightarrow \frac{v_x}{i_x} \to \infty$ (Rin)



$i_x = i_{e2} + i_{c1}$

$= i_{e2} + i_{b1}\beta_1$

$= i_{e2} + i_{c2}\beta_1$

$= i_{c2} + i_{b2} + i_{c2}\beta_1$

$= i_{c2}\left(1+\beta_1+\frac{1}{\beta_1}\right)$

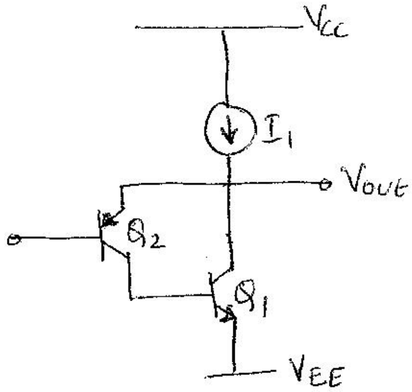
$= v_x g_{m2}\left(1+\beta_1+\frac{1}{\beta_1}\right)$

$g_{m2} = \frac{I_{B2}\beta_2}{V_T} = 4.6\ S$

$\Rightarrow R_{out} = \frac{v_x}{i_x} = \frac{1}{g_{m2}\left(1+\beta_1+\frac{1}{\beta_1}\right)}$

$= 0.005\ \Omega.$

43) $R_{out} = 1\,\Omega$ $\beta_1 = 40$ $\beta_2 = 50$



$$R_{out} = 1 = \frac{1}{g_{m2}\left(1+\beta_1+\frac{1}{\beta_1}\right)}$$

$$\Rightarrow g_{m2} = 0.024\,S = \frac{I_{B2}\beta_2}{V_T}$$

$$\Rightarrow I_{B2} = 0.012\,mA$$

$$I_1 = I_{C1} + I_{E2} = I_{B1}\beta_1 + (I_{C2} + I_{B2})$$

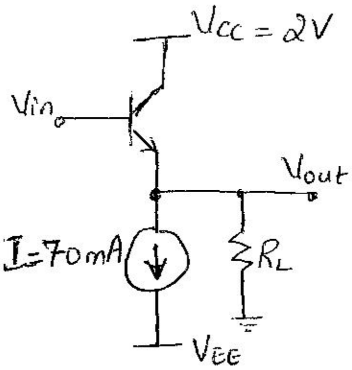
$$= I_{C2}\beta_1 + I_{B2}(\beta_2+1)$$

$$= I_{B2}\beta_2\beta_1 + I_{B2}(\beta_2+1)$$

$$= 0.012\,[50 \times 40 + 50 + 1]$$

$$= 24.6\,mA$$

44)



$V_p = 0.5\,V, \quad R_L = 8\,\Omega$

$$P_{R_L} = \frac{V_p^2}{2R_L} = \frac{0.25}{16} = 0.0156\,W$$

$$P_I = -I \times V_{EE} = 0.14\,W$$

$$P_{Q_1} = I_1\left(V_{CC} - \frac{V_p}{2}\right) = 0.1225\,W$$

$$\therefore \eta = \frac{P_{R_L}}{P_{R_L} + P_I + P_{Q_1}} = \frac{0.0156}{0.2781} = 5.6\%$$

45. $P_{R_L} = \dfrac{V_P^2}{2R_L} = \dfrac{(V_{CC}-V_{BE})^2}{2R_L}$

$P_{Q_1} = I_1\left(V_{CC} - \dfrac{V_{CC}-V_{BE}}{2}\right)$

$P_I = +I_1|V_{EE}|$

- Assume $|V_{CC}| = |V_{EE}|$,

$I_1 = V_P/R_L = \dfrac{V_{CC}-V_{BE}}{R_L}$

$$\therefore \eta = \frac{P_{R_L}}{P_{R_L}+P_{Q_1}+P_I} = \frac{\dfrac{(V_{CC}-V_{BE})^2}{2R_L}}{\dfrac{(V_{CC}-V_{BE})^2}{2R_L} + I_1\left[V_{CC} - \dfrac{V_{CC}-V_{BE}}{2} + |V_{EE}|\right]}$$

$$= \frac{\dfrac{1}{2R_L}}{\dfrac{1}{2R_L} + \dfrac{3V_{CC}+V_{BE}}{2R_L(V_{CC}-V_{BE})}}$$

$$= \frac{1}{1 + \dfrac{3V_{CC}-V_{BE}}{V_{CC}-V_{BE}}} \approx \frac{V_{CC}-V_{BE}}{3V_{CC}-V_{BE}}$$

46. $$\eta = \frac{\frac{V_p^2}{2R_L}}{\frac{V_p^2}{2R_L} + \frac{2V_p}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_p}{4}\right)}$$

$$= \frac{\pi}{4}\frac{V_p}{V_{CC}}$$

$$\Rightarrow \eta\Big|_{V_p = V_{CC} - V_{BE}} = \frac{\pi}{4} - \frac{\pi}{4}\cdot\frac{V_{BE}}{V_{CC}}$$

47. $$\eta = \frac{\frac{(V_p/2)^2}{2R_L}}{\frac{(V_p/2)^2}{2R_L} + \frac{2(V_p/2)}{R_L}\left(\frac{V_{cc}}{\pi} - \frac{V_p/2}{4}\right)}$$

$$= \frac{V_p^2/8R_L}{\frac{V_p^2}{8R_L} + \frac{V_p}{R_L}\left(\frac{V_{cc}}{\pi} - \frac{V_p}{8}\right)} = \frac{1/8R_L}{1/8R_L + \frac{1}{R_L}\left(\frac{V_{cc}}{V_p\pi} - \frac{1}{8}\right)}$$

$$= \frac{1}{1 + \left(\frac{8V_{cc}}{V_p\pi} - 1\right)} = \frac{\pi}{8}\frac{V_p}{V_{cc}} \approx 39\%.$$

48.

$V_{cc} = 3\,V \qquad P_{R_L} = 0.2\,W \qquad R_L = 8\,\Omega$

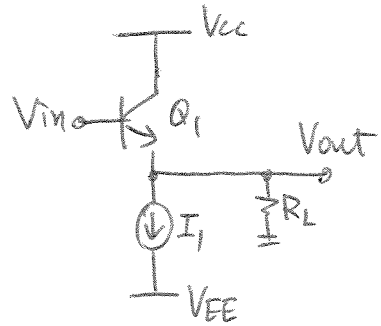
$$P_{R_L} = \frac{1}{2}\frac{V_p^2}{R_L} \Rightarrow V_p = \sqrt{2P_{R_L} \times R_L} = 1.8\,V$$

$$\therefore \eta = \frac{P_{R_L}}{P_{R_L} + \frac{2V_p}{R_L}\left(\frac{V_{cc}}{\pi} - \frac{V_p}{4}\right)} = \frac{0.2}{0.2 + \frac{3.6}{8}\left(\frac{3}{\pi} - \frac{1.8}{4}\right)}$$

$$\approx 46.8\,\%$$

49. Power = 1 W

$R_L = 8\Omega$



$$P_{LOAD} = \frac{1}{2}\frac{V_p^2}{R_L} = 1W$$

$$\Rightarrow V_p = 4\,V \Rightarrow I_1 = \frac{V_p}{R_L} = 0.5\text{ mA}$$

(Note: the problem does not specify small-signal voltage gain, so choose $V_p = I_1 R_L$)

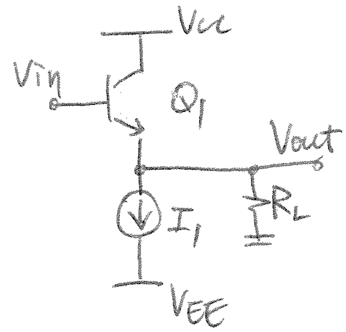
$$P_{Q_1}\text{(power rating)} = I_1(V_{CC})$$

$$= (0.5\text{mA})(5V)$$

$$= 2.5\text{ mW}$$

50. $A_V = 0.8$

$R_L = 4\Omega$



$$A_V = \frac{R_L}{R_L + \frac{1}{g_{m1}}} = \frac{4}{4 + \frac{0.026}{I_{C1}}} = 0.8$$

$\Rightarrow I_{C1} = 26 \text{ mA}$

$\therefore I_1 = I_{C1} = 26\text{mA}$ (Vout biased at 0 V.)

Max Output Swing $= I_1 R_L$

$\approx (26\text{mA})(8\Omega)$

$= 0.208\text{V}$

P_{Q_1} (power rating) $= I_1 V_{CC}$ ($V_p = 0$)

$= (26 \text{ mA})(5\text{V}) = 130 \text{ mW}$

51. $A_v = 0.6$

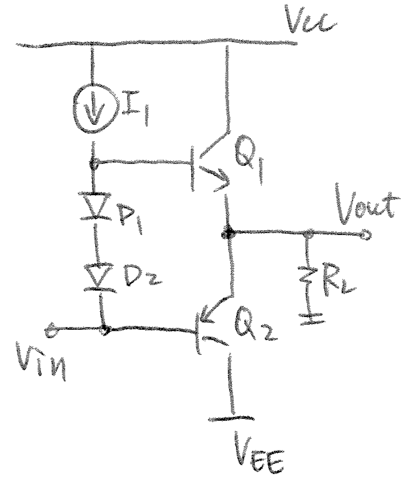
$R_L = 8\ \Omega$

$r_{D_1} = r_{D_2} = 0$

$$A_v = \frac{R_L}{R_L + \frac{1}{g_{m_1}}} = \frac{(8\Omega)}{(8\Omega) + \frac{0.026\,V}{I_{Q_1}}}$$

$= 0.6$

$\Rightarrow I_{Q_1} = I_{Q_2} = 4.8\ mA$



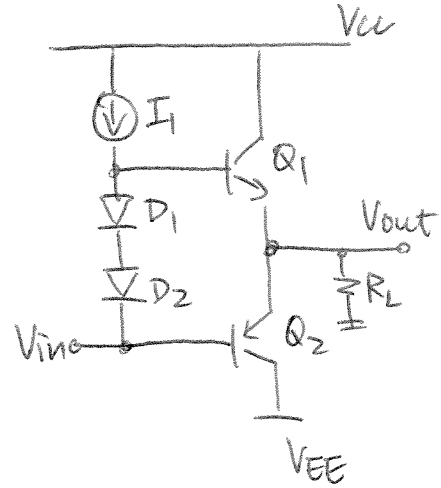
(Vout biased at 0V.)

52. Power = 1 W (to load)

$R_L = 8\,\Omega$

$|V_{BE}| \approx 0.8$ V

$\beta_1 = 40$



$$P_L = \frac{1}{2}\frac{V_p^2}{R_L} = 1\text{ W}$$

$\Rightarrow V_p = 4$ V

$\therefore$ Min allowable supply voltage $= V_p + |V_{BE}| = 4.8$ V

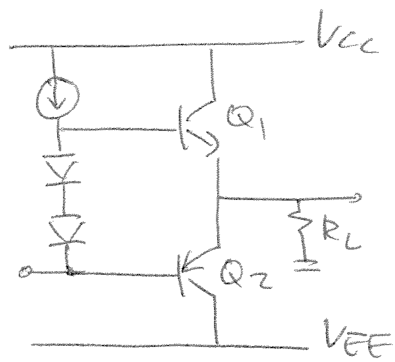
- At $+V_p$, all of I_1 goes to base of Q_1

$$\Rightarrow I_1 = I_{B_1} = \frac{I_{C_1}}{\beta_1} = \frac{V_p}{R_L}\cdot\frac{1}{\beta_1} \quad (Q_2 \text{ off})$$

$$= \frac{4}{8}\cdot\frac{1}{40} = \frac{1}{80} = 12.5\text{ mA}.$$

53. $P_{Q,MAX} = 2\,W$

$R_L = 8\,\Omega$.



For this circuit,

$$P_{AVG,MAX} = \frac{V_{CC}^2}{\pi^2 R_L} \quad \left(V_p = \frac{2V_{CC}}{\pi}\right)$$

$$= 2\,W$$

$$\Rightarrow V_{CC}\Big|_{MAX} = 12.6\ V \quad \Rightarrow V_p\Big|_{max} = \frac{25.2}{\pi} = 8.02\ V$$

$$\therefore P_{R_L\,MAX} = \frac{V_{P\,max}^2}{2R_L} = \frac{(8.02)^2}{2 \cdot 8} = 4.02\ W$$

54. For this circuit,

$P_{Q,MAX} = 2W$

$R_L = 4\Omega$

$$P_{AVG,MAX} = \frac{V_{CC}^2}{\pi^2 R_L} \qquad \left(V_P = \frac{2V_{CC}}{\pi}\right)$$

$$\Rightarrow V_{CC\,max} = \sqrt{\frac{\pi^2 R_L P_{Q\,MAX}}{1}} = 8.9\ V$$

$$\Rightarrow V_{P\,max} = \frac{2V_{CC\,max}}{\pi} = 5.6\ V$$

$$\therefore P_{R_L,MAX} = \frac{V_{P,MAX}^2}{2R_L} = \frac{32}{2(4)} = 4W$$

55) $A_V = 4$ $\quad R_L = 8\Omega$ $\quad I_{C1} \approx I_{C2}$ $\quad \beta_1 = 40$ $\quad \beta_2 = 20$

Suppose we want 1st-Stage (CE amplifier) to have gain = 5 $\Rightarrow$ 2nd Stage gain = 0.8

$$\Rightarrow 0.8 = \frac{R_L}{R_L + \frac{1}{g_{m1}+g_{m2}}}$$

$$0.8 = \frac{8}{8 + \frac{1}{2g_m}} \Rightarrow g_{m1} = \tfrac{1}{4}\,S \Rightarrow I_{C1} = I_{C2} = 6.5\,mA$$

$$r_{\pi 1} \parallel r_{\pi 2} = \frac{\beta_1 V_T}{I_{C1}} \parallel \frac{\beta_2 V_T}{I_{C2}} = \frac{40(0.026)}{6.5\,mA} \parallel \frac{20(0.026)}{6.5\,mA} \approx 133\,\Omega$$

- $A_V = 4 = g_{m4}\,(r_{\pi 1} \parallel r_{\pi 2})\,(g_{m1} + g_{m2})\,R_L$

$$= \frac{I_{C4}}{V_T}(133)(0.5)8$$

$$\Rightarrow I_{C4} = I_{C3} = \frac{4\,V_T}{8(133)(0.5)} = 0.488\,mA$$

Max I_{Q1} when all of I_{C3}/I_{C4} Supports base current of Q_1

$$\Rightarrow I_{Q1\,MAX} = I_{C4} = 0.488\,mA$$

56) $A_V = 4 \qquad R_L = 4\,\Omega \qquad I_{C1} \approx I_{C2}$

$\beta_1 = 40 \qquad \beta_2 = 20$

1st Stage gain = 5 (CE amplifier).

2nd Stage gain = 0.8

$$* \quad 0.8 = \frac{R_L}{R_L + \frac{1}{g_{m1} + g_{m2}}} = \frac{4}{4 + \frac{1}{2 g_{m1}}}$$

$$\Rightarrow g_{m1} = 0.5\,S \implies I_{C1} = I_{C2} = 13\,mA$$

$$r_{\pi 1} \parallel r_{\pi 2} = \frac{\beta_1 V_T}{I_{C1}} \parallel \frac{\beta_2 V_T}{I_{C2}} = 80 \parallel 40 = 26.7\,\Omega$$

$$* \quad A_V = 4 = g_{m4} (r_{\pi 1} \parallel r_{\pi 2}) (g_{m1} + g_{m2}) R_L$$

$$= \frac{I_{C4}}{V_T} (26.7)(1)(4)$$

$$\Rightarrow I_{C4} = I_{C3} = \frac{4 V_T}{(26.7)(1)(4)} = 0.974\,mA$$

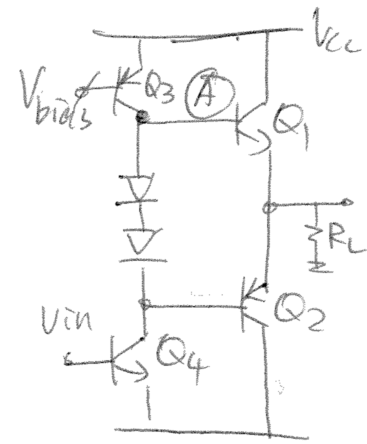
* Max I_{Q1} ($I_{Q1\,MAX}$) when $I_{C4} = I_{Q1MAX} = 0.974\,mA$

* For a reduction of 2x the R_L, we have to provide ~2x current to base of Q_1 $\left(\frac{0.974}{0.488} \approx 2\right)$

57. $P_{R_L} = 2W$ $\quad \beta_1 = 40$

$R_L = 8\,\Omega$ $\quad \beta_2 = 20$

$|V_{BE}| = 0.8V$



(a) $P_{R_L} = \frac{1}{2}\frac{V_p^2}{R_L} \Rightarrow V_p \approx 5.6\ V$

- At $+V_p$, $V_A = V_p + |V_{BE}|$.
- For Q_3 in active region, $V_A \le V_{bias}$

$\Rightarrow V_{CC} \ge V_{bias} + |V_{BE}| = V_p + 2|V_{BE}|$

$\ge 5.6 + 1.6 = 7.2\ V.$

(b) $I_p = \frac{V_p}{R_L} = 0.7\ A.\ (= I_{E_1}),\ (= I_{E_2})$

$\Rightarrow I_{B_1} = \frac{I_{E_1}}{1+\beta_1} = 17\ mA.$

$\therefore$ We bias Q_3 & Q_4 with $I_C = 17\ mA$.

(c) $P_{AV} = \frac{V_P}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_P}{4}\right)$

$= \frac{5.6}{8}\left(\frac{5}{\pi} - \frac{5.6}{4}\right) = 3.66\ W$

(d) $P_{IQ3} = 2V_{CC} \times I_{Q3} = 10 \times 17mA = 170\,mW$

$P_{AV,Q_1} = \frac{V_P}{R_L}\left(\frac{V_{CC}}{\pi} - \frac{V_P}{4}\right) = 3.66\ W$

$P_{R_L} = 2W$

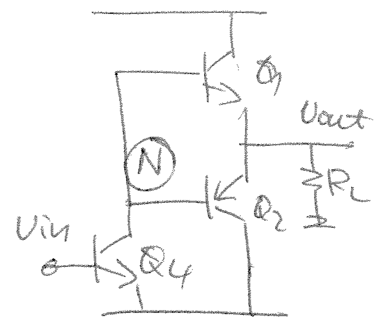
$\Rightarrow \eta = \frac{P_{R_L}}{P_{IQ3} + 2 \cdot P_{AV,Q_1} + P_{R_L}}$

$= \frac{2}{170m + 3.66 \times 2 + 2} = 0.21 = 21\%$

58.

(a) $A_v = 5 \quad R_L = 4\Omega \quad \beta_1 = 40 \quad \beta_2 = 20.$

Assume $I_{C1} \approx I_{C2}$.



$$\frac{v_{out}}{v_N} = \frac{R_L}{(g_{m1}+g_{m2})^{-1} + R_L} = 0.8$$

$$\Rightarrow 2g_{m1}^{-1} = 1 \Rightarrow I_{C1} = 2V_T$$

$$= 0.052\ A.$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = +g_{m4}(r_{\pi1} \| r_{\pi2})(g_{m1}+g_{m2})R_L = 5$$

Assume $g_{m1} \approx g_{m2}$:

$$\Rightarrow I_{C4} = V_T \frac{5}{(r_{\pi1} \| r_{\pi2})(g_{m1}+g_{m2})R_L}$$

$$= V_T \times \frac{5}{(r_{\pi1} \| r_{\pi2})(g_{m1} \times 2)R_L}$$

$$= 0.026 \frac{5}{(6.7\Omega)(2\times 2)(4)}$$

$$\approx 1.2\ mA.$$

$\Rightarrow$ Max I by $Q_1 = \beta_1 \times I_{C4} = 48\ mA$

$\Rightarrow P_{R_L} = \frac{1}{2} I^2 R_L = 24 \times 4\ mW = 96\ mW$, BELOW requirement!

(b) $P = 5\,W = \frac{1}{2}\frac{V_P^2}{R_L} \Rightarrow V_P = 6.3\,V$

$\Rightarrow I_P = \frac{V_P}{R_L} = 1.6\,A$

$\Rightarrow I_{B2,MAX} = \frac{I_P}{\beta_2} = \frac{1.6}{20} = 79\,mA$

$\Rightarrow I_{C4}$ must equal 79 mA to allow max output swing V_P

$\Rightarrow g_{m4} = \frac{I_{C4}}{V_T} = 3.04\,S$

Suppose 2nd stage gain $= 0.8$ $(I_{C1} \simeq I_{C2})$

$\Rightarrow \frac{v_{out}}{v_N} = \frac{R_L}{R_L + \frac{1}{g_{m1}+g_{m2}}} = 0.8 \Rightarrow g_{m1} = 0.5\,S$

$\Rightarrow I_{C1} = I_{C2} = 13\,mA.$

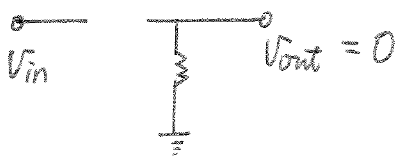
$r_{\pi 1} \parallel r_{\pi 2} = \frac{\beta_1 V_T}{I_{C1}} \parallel \frac{\beta_2 V_T}{I_{C2}} = 26.7\,\Omega.$

$\therefore \frac{v_{out}}{v_{in}} = -(3.04)(26.7\,\Omega)(0.5+0.5)4$

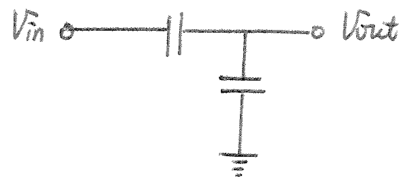
$= -324$!! (Huge! Impractical)

- Even when the 2nd stage gets close to 1, we still need huge gain from first stage.

1. (a)



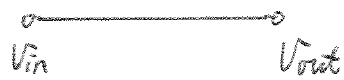
(Low freq.)



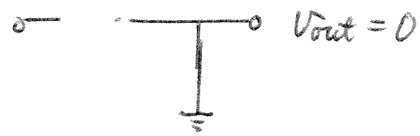
(High freq.)

This is a high pass filter.

(b).



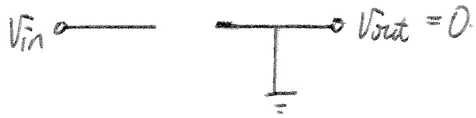
(Low freq.)



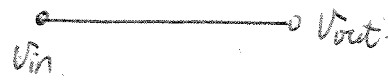
(High freq.)

This is a low pass filter.

(c).



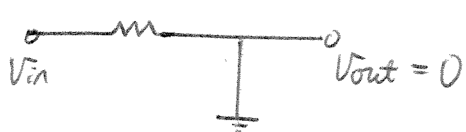
(Low freq)



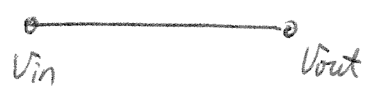
(High freq.).

This is a high pass filter.

(d)



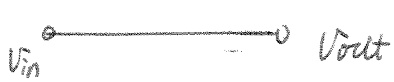
(Low freq.)



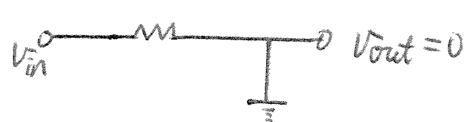
(High freq.)

This is a high pass filter.

(e)



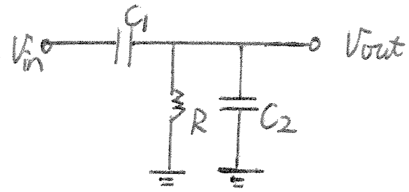
(Low freq.).



(High freq.)

This is a low pass filter.

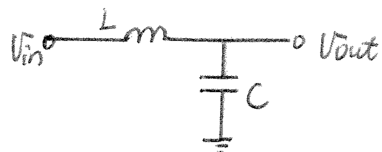
2. (a).



$$V_{out} = \frac{R // \frac{1}{sC_2}}{\frac{1}{sC_1} + R // \frac{1}{sC_2}} V_{in}.$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{C_1}{C_1+C_2} s}{s + \frac{1}{R(C_1+C_2)}}, \quad zero = 0; \quad pole = -\frac{1}{R(C_1+C_2)}$$

(b)

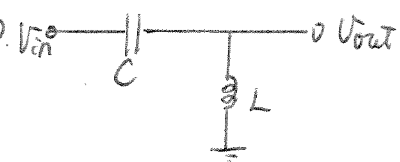


$$V_{out} = \frac{\frac{1}{sC}}{sL + \frac{1}{sC}} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{1/(LC)}{s^2 + \frac{1}{LC}}$$

zero: No finite zero; poles $= \pm i \frac{1}{\sqrt{LC}}$

(c)

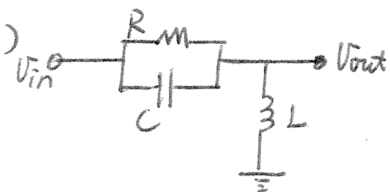


$$V_{out} = \frac{sL}{sL + \frac{1}{sC}} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{s^2}{s^2 + \frac{1}{LC}}$$

zeros: Two zeros at 0; poles $= \pm i \frac{1}{\sqrt{LC}}$

(d)



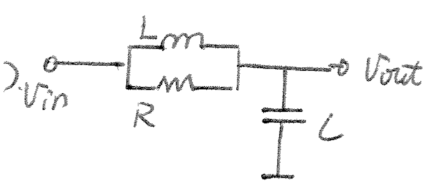
$$V_{out} = \frac{sL}{sL + R // \frac{1}{sC}} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{s(s + \frac{1}{RC})(RC)^2}{s^2 + \frac{s}{RC} + \frac{1}{LC}}.$$

zeros: $0, -\frac{1}{RC}$

poles $= \frac{-\frac{1}{RC} \pm \sqrt{(\frac{1}{RC})^2 - \frac{4}{LC}}}{2}$

(e)



$$V_{out} = \frac{\frac{1}{sC}}{\frac{1}{sC} + sL // R} V_{in}$$

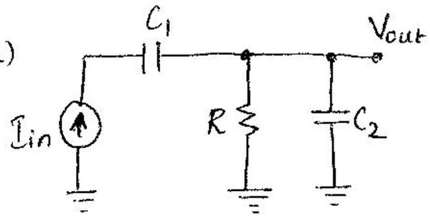
$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{RC}(s + \frac{R}{L})}{s^2 + \frac{s}{RC} + \frac{1}{LC}}, \quad zero = -\frac{R}{L}; \quad poles = \frac{-\frac{1}{RC} \pm \sqrt{(\frac{1}{RC})^2 - \frac{4}{LC}}}{2}.$$

3.

Since $\frac{V_{out}}{V_{in}} = \frac{1}{(s+a)(s+b)}$, where a and b are real and positive, the transfer function contains no finite zero and two real poles on the left hand plane. But, after reviewing Problem #2 we discover that NONE of the networks yield this case.

(4)

(a)

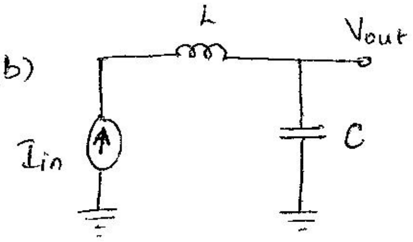


$$\frac{V_{out}}{I_{in}} = \frac{1/C_2}{s + 1/(RC_2)}$$

Zero: No finite zero

Pole: $-1/(RC_2)$

(b)

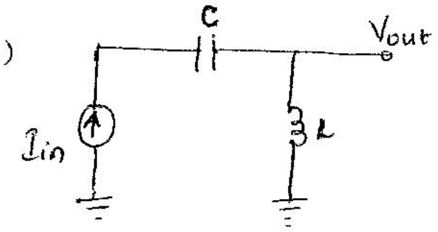


$$\frac{V_{out}}{I_{in}} = \frac{1}{sC}$$

Zero: No finite zero

Pole: 0

(c)

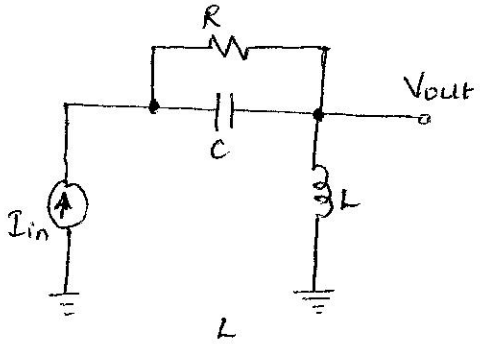


$$\frac{V_{out}}{I_{in}} = sL$$

Zero: 0

Pole: No finite Pole

(d)

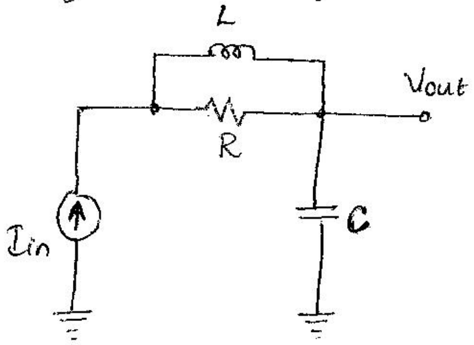


$$\frac{V_{out}}{I_{in}} = sL$$

Zero: 0

Pole: No finite Pole

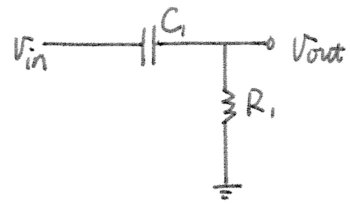
(e)



$$\frac{V_{out}}{I_{in}} = \frac{1}{sC}$$

Zero: No finite zero

Pole: 0

5). 

$$\frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 + \frac{1}{C_1 S}} = \frac{S}{S + \frac{1}{R_1 C_1}}$$

zero $= 0$; pole $= -\frac{1}{R_1 C_1}$.

$$\frac{dP}{dC_1} = \frac{1}{(R_1 C_1)^2} \cdot R_1 = \frac{1}{R_1 C_1^2}.$$

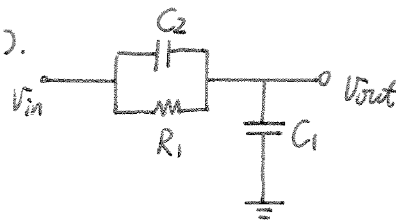
$$S_{C_1}^{P} = \frac{\frac{dP}{P}}{\frac{dC_1}{C_1}} = \frac{dP}{dC_1} \cdot \frac{C_1}{P} = -\frac{1}{R_1 C_1^2} \cdot C_1 \cdot (R_1 C_1) = -1.$$

Similarly

$$S_{R_1}^{P} = -1.$$

As for the sensitivity of zero, since the zero is at 0, which is indepedant of R_1 and C_1, $S_{R_1}^{z} = S_{C_1}^{z} = 0$.

6).



$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{C_1 S}}{\frac{1}{C_1 S} + R_1 // \frac{1}{SC_2}}$$

$$= \frac{1}{R_1 C_2} \cdot \frac{S + \frac{1}{R_1 C_2}}{R_1(C_1+C_2)\left[S + \frac{1}{R_1(C_1+C_2)}\right]}$$

$$\text{zero} = -\frac{1}{R_1 C_2}, \quad \text{pole} = -\frac{1}{R_1(C_1+C_2)}$$

$$\frac{dP}{dR_1} = [R_1(C_1+C_2)]^{-2} \cdot (C_1+C_2) = -\frac{C_1+C_2}{R_1(C_1+C_2)} \cdot P. = -\frac{P}{R_1}$$

$$S_{R_1}^{P} = \frac{\frac{dP}{P}}{\frac{dR_1}{R_1}} = \frac{dP}{dR_1} \cdot \frac{R_1}{P} = -1.$$

$$\frac{dP}{dC_1} = [R_1(C_1+C_2)]^{-2} \cdot R_1 = -\frac{P}{C_1+C_2}$$

$$S_{C_1}^{P} = \frac{\frac{dP}{P}}{\frac{dC_1}{C_1}} = \frac{dP}{dC_1} \cdot \frac{C_1}{P} = -\frac{C_1}{C_1+C_2}$$

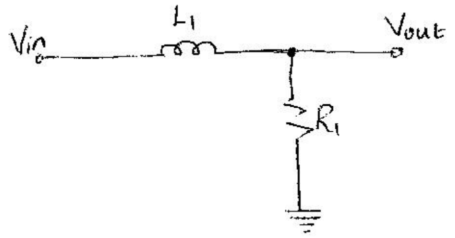
Conversely,

$$S_{C_2}^{P} = -\frac{C_2}{C_1+C_2}.$$

From Problem 5).

$$S_{R_1}^{z} = S_{C_2}^{z} = -1.$$

7)



$$\frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 + L_1 s} = \frac{R_1/L_1}{s + R_1/L_1}$$

$$\text{Pole} = -\frac{R_1}{L_1}$$

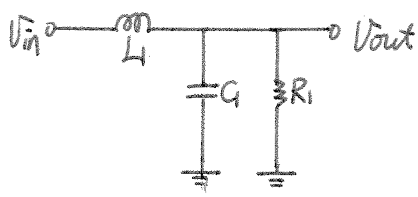
$$dP = \frac{\partial P}{\partial R_1} \cdot dR_1 + \frac{\partial P}{\partial L_1} dL_1 = -\frac{1}{L_1} dR_1 + \frac{R_1}{L_1^2} dL_1$$

$$\Rightarrow \frac{dP}{P} = \frac{dR_1}{R_1} - \frac{dL_1}{L_1}$$

$$\left|\frac{dP}{P}\right| \leq 5\% \quad , \quad \text{and} \quad \left|\frac{dR_1}{R_1}\right| \leq 3\%$$

$$\Rightarrow \left|\frac{dL_1}{L_1}\right| \leq 2\%$$

8).



a). $V_{out} = V_{in} \cdot \dfrac{R_1 // \frac{1}{C_1 S}}{R_1 // \frac{1}{C_1 S} + SL_1}$

$= V_{in} \cdot \dfrac{\frac{R_1}{R_1 C_1 S + 1}}{\frac{R_1}{R_1 C_1 S + 1} + SL_1}$

$$\frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 C_1 L_1 S^2 + L_1 S + R_1} = \frac{1}{L_1 C_1} \cdot \frac{1}{S^2 + \frac{1}{R_1 C_1} S + \frac{1}{L_1 C_1}}$$

b). poles $= \dfrac{-\frac{1}{R_1 C_1} \pm \sqrt{\left(\frac{1}{R_1 C_1}\right)^2 - \frac{4}{L_1 C_1}}}{2}$.

For them to be real $\Rightarrow \left(\frac{1}{R_1 C_1}\right)^2 - \frac{4}{L_1 C_1} \geq 0$

$\Rightarrow \dfrac{1}{R_1 C_1} \geq \dfrac{2}{\sqrt{L_1 C_1}}$

14.8

(c)

$$\omega_{p1,2} = \frac{1}{2}\left[\frac{-1}{R_1C_1} \pm \sqrt{\left(\frac{1}{R_1C_1}\right)^2 - \frac{4}{L_1C_1}}\right]$$

$$\frac{\partial\omega_{p1,2}}{\partial R_1} = -\frac{1}{2}\left[\frac{1}{R_1^2C_1} \pm \frac{-2}{R_1^3C_1^2}\frac{1/2}{\sqrt{\left(\frac{1}{R_1C_1}\right)^2 - \frac{4}{L_1C_1}}}\right]$$

$$\frac{\partial\omega_{p1,2}}{\omega_{p1,2}} = \frac{-\frac{1}{2}\frac{\partial R_1}{R_1}\left[\frac{1}{R_1C_1} \pm \frac{-1}{R_1^2C_1^2\sqrt{\left(\frac{1}{R_1C_1}\right)^2 - \frac{4}{L_1C_1}}}\right]}{\frac{1}{2}\left[-\frac{1}{R_1C_1} \pm \sqrt{\left(\frac{1}{R_1C_1}\right)^2 - \frac{4}{L_1C_1}}\right]}$$

$$\Rightarrow S_{R_1}^{\omega_{p1,2}} = -\frac{\frac{1}{R_1C_1} \pm \frac{-1}{R_1^2C_1^2\sqrt{(1/R_1C_1)^2 - 4/(L_1C_1)}}}{-\frac{1}{R_1C_1} \pm \sqrt{\left(\frac{1}{R_1C_1}\right)^2 - \frac{4}{L_1C_1}}}.$$

$$\frac{\partial\omega_{p1,2}}{\partial C_1} = -\frac{1}{2}\left[\frac{1}{R_1C_1^2} \pm \left(\frac{-2}{R_1C_1^3} + \frac{4}{L_1C_1^2}\right)\frac{1/2}{\sqrt{\left(\frac{1}{R_1C_1}\right)^2 - \frac{4}{L_1C_1}}}\right]$$

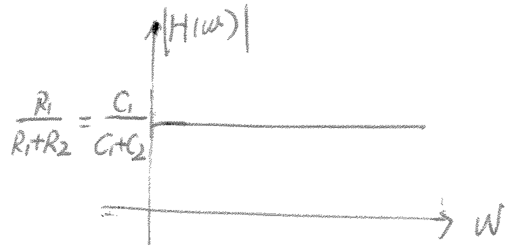
$$\frac{\partial\omega_{p1,2}}{\omega_{p1,2}} = \frac{-\frac{1}{2}\frac{\partial C_1}{C_1}\left[\frac{1}{R_1C_1} \pm \left(\frac{-2}{R_1C_1^2} + \frac{4}{L_1C_1}\right)\frac{1/2}{\sqrt{(1/R_1C_1)^2 - 4/(L_1C_1)}}\right]}{\frac{1}{2}\left[-\frac{1}{R_1C_1} \pm \sqrt{\left(\frac{1}{R_1C_1}\right)^2 - \frac{4}{L_1C_1}}\right]}$$

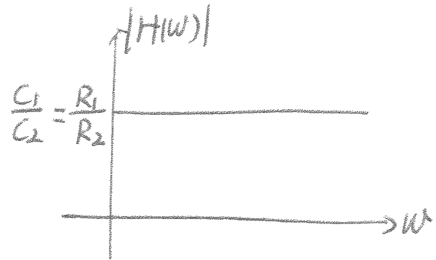
$$\Rightarrow S_{C_1}^{\omega_{p1,2}} = -\frac{\frac{1}{R_1C_1} \pm \left(\frac{-1}{R_1C_1^2} + \frac{2}{L_1C_1}\right)\frac{1}{\sqrt{(1/R_1C_1)^2 - 4/(L_1C_1)}}}{-\frac{1}{R_1C_1} \pm \sqrt{\left(\frac{1}{R_1C_1}\right)^2 - \frac{4}{L_1C_1}}}$$

$$\frac{\partial\omega_{p1,2}}{\partial L_1} = \pm\frac{1}{4}\left(\frac{4}{L_1^2C_1}\right)\frac{1}{\sqrt{\left(\frac{1}{R_1C_1}\right)^2 - \frac{4}{L_1C_1}}} \Rightarrow \frac{\partial\omega_{p1,2}}{\omega_{p1,2}} = \pm\frac{\partial L_1}{L_1}\frac{1}{L_1C_1}\frac{1}{\sqrt{\left(\frac{1}{R_1C_1}\right)^2 - \frac{4}{L_1C_1}}}$$

$$\Rightarrow S_{L_1}^{\omega_{p1,2}} = \pm\frac{\frac{1}{L_1C_1}\frac{1}{\sqrt{(1/R_1C_1)^2 - 4/(L_1C_1)}}}{-\frac{1}{R_1C_1} \pm \sqrt{\left(\frac{1}{R_1C_1}\right)^2 - \frac{4}{L_1C_1}}}$$

9). If the zero and pole coincide, they will neutralize each other, and also render the transfer function flat.





10).

$$H(s) = \frac{\alpha s^2 + \beta s + \gamma}{s^2 + \frac{w_n}{Q} s + w_n^2}$$

$$P_{1,2} = -\frac{w_n}{2Q} \pm j w_n \sqrt{1 - \frac{1}{4Q^2}}$$

If $Q = \frac{1}{2}$, then $P_{1,2} = -\frac{w_n}{2Q}$.

11.

$$|H(j\omega)|^2 = \frac{r^2}{(\omega_n^2-\omega^2)^2+\left(\frac{\omega_n}{Q}\omega\right)^2}$$

No peaking means no local minimum for $(\omega_n^2-\omega^2)^2+\left(\frac{\omega_n}{Q}\omega\right)^2$, which is also known as $D(\omega)$.

A local min exists if $\frac{\partial D(\omega)}{\partial \omega} = 0$.

$$\frac{\partial D(\omega)}{\partial \omega} = \left(\frac{\partial D(\omega)}{\partial \omega^2}\right)\left(\frac{\partial \omega^2}{\partial \omega}\right), \quad \frac{\partial D(\omega)}{\partial \omega^2} = -2(\omega_n^2-\omega^2) + \left(\frac{\omega_n}{Q}\right)^2$$

$$\frac{\partial \omega^2}{\partial \omega} = 2\omega, \text{ so } \frac{\partial D(\omega)}{\partial \omega} = 2\omega\left[2(\omega^2-\omega_n^2)+\left(\frac{\omega_n}{Q}\right)^2\right]=0$$

Solving for ω, we have $\omega = 0, \pm\sqrt{\omega_n^2-\frac{1}{2}\left(\frac{\omega_n}{Q}\right)^2}$

Will bring $D(\omega)$ to its min value.

At $\omega=0$, we have the DC value of the transfer function. However if $Q^2<\frac{1}{2}$ or $Q<\frac{1}{\sqrt{2}}$, $\omega_n^2-\frac{1}{2}\left(\frac{\omega_n}{Q}\right)^2$ becomes negative, which is not physical. Therefore, there is no peaking for $Q<\frac{1}{\sqrt{2}}$. And at $Q=\frac{1}{\sqrt{2}}$, we have $\omega=\pm 0$, which corresponding to the DC value of the transfer function, not peaking. Therefore, the only option left is for $Q>\frac{1}{\sqrt{2}}$, and that is the condition for peaking.

12). $|H(j\omega)|^2 = \dfrac{\gamma^2}{(\omega_n^2-\omega^2)^2 + (\frac{\omega_n}{Q}\omega)^2}$

If $Q > \sqrt{2}/2$, it will peak at $\omega_0 = \omega_n\sqrt{1-1/(2Q)^2}$

$$H(j\omega) = \frac{\gamma}{\sqrt{(\omega_n^2-\omega^2)^2 + (\frac{\omega_n}{Q}\omega)^2}}$$

$$\Rightarrow H(j\omega_0) = \frac{\gamma}{\sqrt{[\omega_n^2-(\omega_n\sqrt{1-\frac{1}{2Q^2}})^2]^2 + (\frac{\omega_n}{Q}\omega_n\sqrt{1-\frac{1}{2Q^2}})^2}}$$

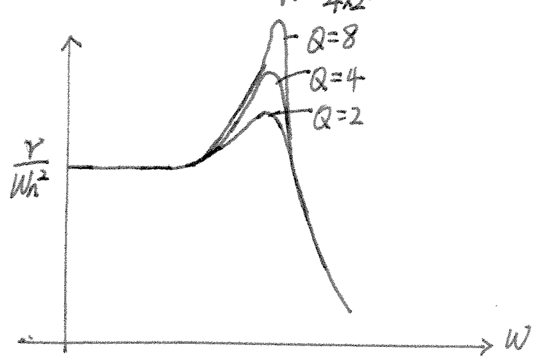
$$= \frac{\gamma}{\sqrt{(\omega_n^2\cdot\frac{1}{2Q^2})^2 + \frac{\omega_n^4}{Q^2}(1-\frac{1}{2Q^2})}}$$

$$= \frac{\gamma}{\sqrt{\frac{\omega_n^4}{4Q^4} + \frac{\omega_n^4}{Q^2} - \frac{\omega_n^4}{2Q^4}}}$$

$$= \frac{Q\gamma}{\omega^2\sqrt{1-\frac{1}{4Q^2}}}$$

Normalize to passband $\Rightarrow \dfrac{Q}{\sqrt{1-\frac{1}{4Q^2}}}$

$Q=2$, peak $= \dfrac{2}{\sqrt{1-\frac{1}{4\times 2^2}}} = 2.07$; $Q=4$, peak $= 4.03$; $Q=8$, peak $= 8.02$



13)

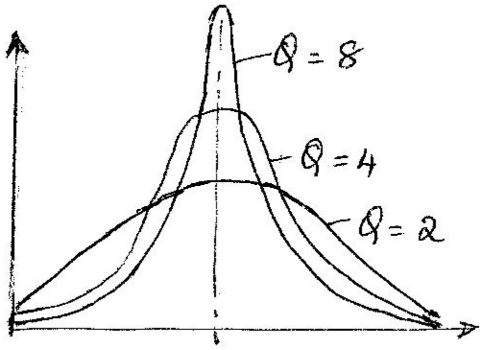
$$H(s) = \frac{\beta s}{s^2 + \frac{\omega_n}{Q} s + \omega_n^2} \quad , \quad H(j\omega) = \frac{j\beta\omega}{\omega_n^2 + j\frac{\omega_n}{Q}\omega - \omega^2}$$

$$|H(j\omega)| = \frac{\beta\omega}{\sqrt{(\omega_n^2 - \omega^2) + (\omega_n/Q\,\omega)^2}}$$

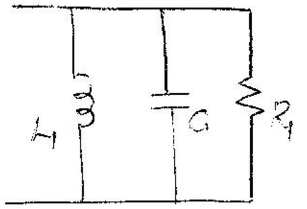
At $\omega = \omega_n$,

$$|H(j\omega_n)| = \frac{\beta\omega_n}{\frac{\omega_n}{Q}\omega_n} = \frac{Q}{\omega_n}\beta .$$

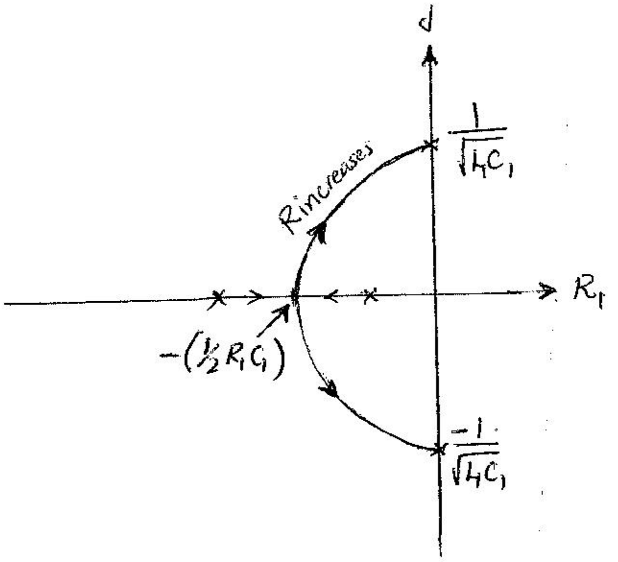
So if we normalize to β, we get $\frac{Q}{\omega_n}$



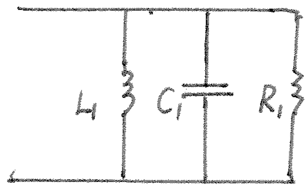
14)

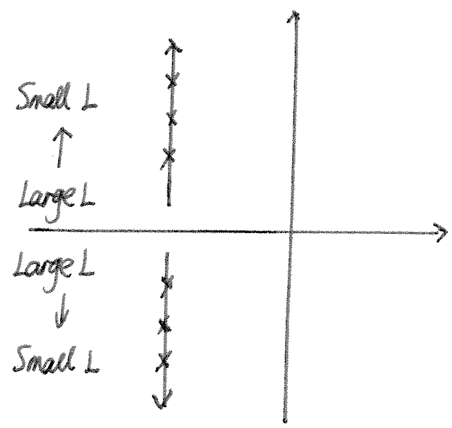


Assume R is never negative



15).





16.

1 dB peaking $\Rightarrow \dfrac{Q^2}{\left(1-\frac{1}{4Q^2}\right)} = (1.1)^2 = 1.21$

$$Q^2 = 1.21\left(1-\frac{1}{4Q^2}\right) \Rightarrow 4Q^4 - 4(1.1)^2 Q^2 + (1.1)^2 = 0$$

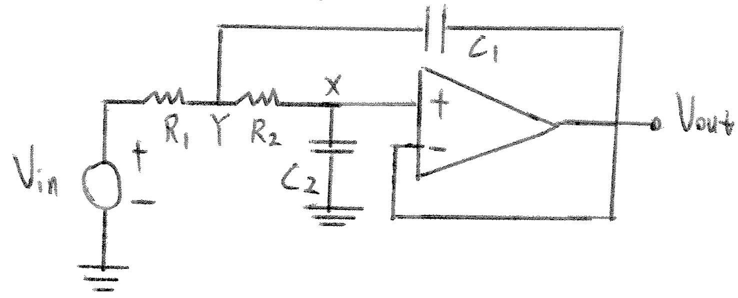
$$Q^2 = 0.85704,\ 0.35296,\ Q = 0.925765,\ 0.59410$$

$Q = 0.925765$, since $Q > \frac{1}{\sqrt{2}}$ for peaking

$$Q = \frac{\omega_n}{\beta} = \frac{RC}{\sqrt{LC}} = R\sqrt{\frac{C}{L}} = 0.925765$$

17.

Sallen and Key filter



$$H(s) = \frac{1}{R_1R_2C_1C_2s^2 + (R_1+R_2)C_2s + 1}, \quad Q = \frac{1}{R_1+R_2}\sqrt{\frac{R_1R_2C_1}{C_2}},$$

$$\omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}},$$

$$H(s) = \left(s^2 + \frac{(R_1+R_2)C_2}{R_1R_2C_1C_2}s + \frac{1}{R_1R_2C_1C_2}\right)^{-1}$$

$$P_{1,2} = -\frac{(R_1+R_2)}{R_1R_2C_1} \pm \sqrt{\left(\frac{R_1+R_2}{R_1R_2C_1}\right)^2 - \frac{4}{R_1R_2C_1C_2}}$$

$$P_{1,2} = -\frac{1}{2(R_1//R_2)C_1} \pm \sqrt{\left(\frac{1}{(R_1//R_2)C_1}\right)^2 - \frac{4}{R_1C_1R_2C_2}}$$

Assuming $\frac{4}{R_1C_1R_2C_2} > \frac{1}{[(R_1//R_2)C_1]^2}$

$$P_{1,2} = -\frac{1}{2(R_1//R_2)C_1} \pm j2\sqrt{\frac{1}{R_1C_1R_2C_2} - \frac{1}{4[(R_1//R_2)C_1]^2}}$$

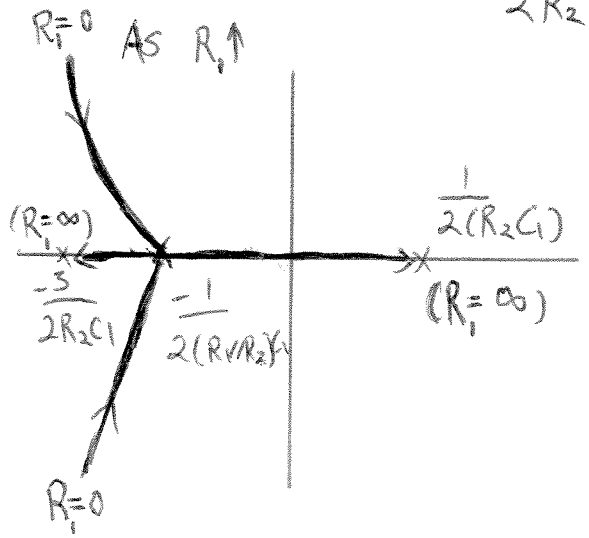
17.

a) R_1: $0 \to \infty$

When $R_1 = 0$, Poles are at $\pm\infty$, so no finite poles. As $R_1 \uparrow$, $\frac{1}{R_1C_1R_2C_2}$ approaches 0, and $\frac{1}{4[(R_1//R_2)C_1]^2}$ approaches $\frac{1}{4[R_2C_1]^2}$. There exists a R_1 such that $\frac{1}{R_1C_1R_2C_2} = \frac{1}{4[(R_1//R_2)C_1]^2}$ $\Rightarrow$

$$P_{1,2} = -\frac{1}{2(R_1//R_2)C_1}$$

As $R_1 \to \infty$, $P_{1,2} = -\frac{1}{2R_2C_1} \pm \frac{1}{R_2C_1} = -\frac{3}{2R_2C_1}, \frac{1}{2R_2C_1}$



17. b)

R_2 from $0 \to \infty$

when $R_2 = 0$, $P_{1,2}$ are at $\pm\infty$

As $R_2 \uparrow$, $\frac{-1}{2(R_1 // R_2)C_1} \longrightarrow -\frac{1}{2R_1C_1}$

$\frac{1}{R_1C_1R_2C_2} \to 0$, and $\frac{1}{4[(R_1//R_2)C_1]^2} \to \frac{1}{4[R_1C_1]^2}$

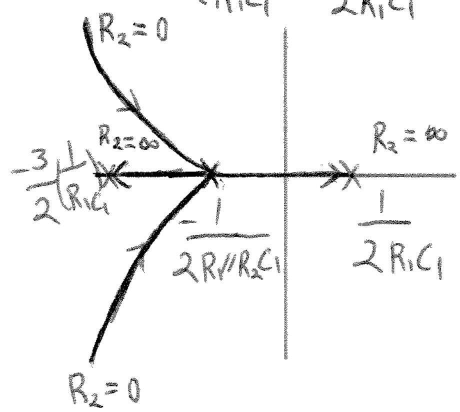
For a certain R_2, $\frac{1}{R_1C_1R_2C_2} = \frac{1}{4[(R_1//R_2)C_1]^2}$

and $P_{1,2} = \frac{-1}{2(R_1//R_2)C_1}$

Finally, when $R_2 = \infty$,

$$P_{1,2} = -\frac{1}{2R_1C_1} \pm 2\sqrt{\frac{1}{4[R_1C_1]^2}} = -\frac{1}{2R_1C_1} \pm \frac{1}{R_1C_1}$$

$$P_{1,2} = -\frac{3}{2}\frac{1}{R_1C_1}, \quad \frac{1}{2R_1C_1}$$

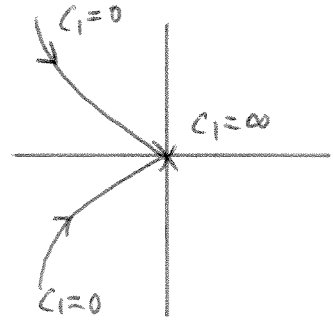


17. c)

As $C_1 : 0 \to \infty$

When $C_1 = 0$, Poles are at $\pm\infty$

As $C_1 \uparrow$, Poles approach 0.

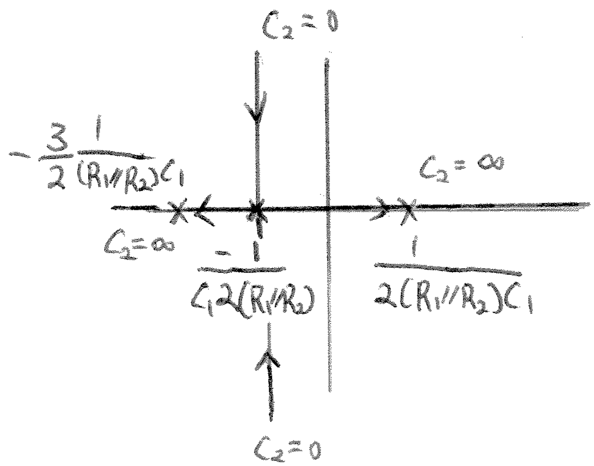


d) As $C_2 : 0 \to \infty$

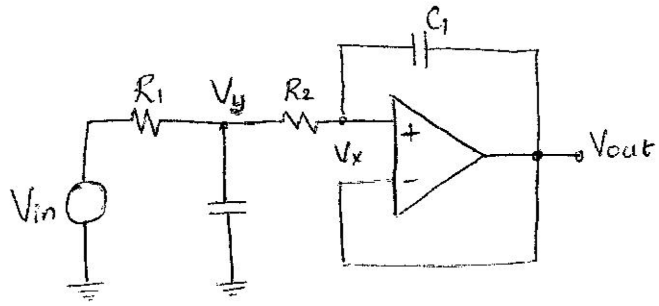
When $C_2 = 0$, Poles are at $\pm\infty$

When $C_2 = \infty$, poles: $-\frac{1}{2(R_1 // R_2)C_1} \pm \frac{1}{R_1 // R_2 C_1} = -\frac{3}{2}\left(\frac{1}{R_1 // R_2 C_1}\right)$

$+ \frac{1}{2 R_1 // R_2 C_1}$ (note, real part doesn't depend on C_2).



18)

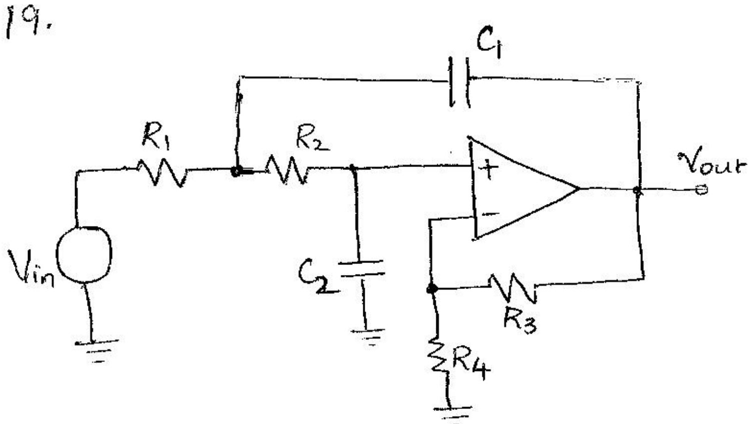


Assuming an ideal op-amp, $V_x = V_{out}$. Therefore, no current will flow through C_1. Moreover, since the input impedance of an op-amp (ideal) is infinite, no current will flow through R_2 as well, which means $V_y = V_x = V_{out}$

$$\Rightarrow \quad V_y = V_{out} = \frac{1/C_2 s}{R_1 + 1/C_2 s} * V_{in}$$

Not very useful since it's only a simple single pole low-Pass filter. We can implement it with passive components, instead of op-amp.

19.



$K = 4, \quad C_1 = C_2$

$Q = 4$

$K = 1 + \frac{R_3}{R_4} = 4 \Rightarrow \frac{R_3}{R_4} = 3, \quad \frac{C_1}{C_2} = 1$

$$\frac{1}{Q} = \sqrt{\frac{R_1 C_1}{R_2 C_1}} + \sqrt{\frac{R_2}{R_1}\frac{C_2}{C_1}} - \sqrt{\frac{R_1 C_1}{R_2 C_2}}\frac{R_3}{R_4}$$

$$\frac{1}{Q} = \sqrt{\frac{R_1}{R_2}} + \sqrt{\frac{R_2}{R_1}} - 3\sqrt{\frac{R_1}{R_2}} \Rightarrow \sqrt{\frac{R_2}{R_1}} - 2\sqrt{\frac{R_1}{R_2}} = \frac{1}{Q}$$

$$\frac{1}{Q} = \left(\frac{R_1}{R_2}\right)^{-1/2} - 2\left(\frac{R_1}{R_2}\right)^{1/2}$$ ⇒ Squaring both sides ⇒

$$\frac{1}{Q^2} = 4\left(\frac{R_1}{R_2}\right) - 4 + \left(\frac{R_1}{R_2}\right)^{-1} \Rightarrow \frac{1}{16} = 4\left(\frac{R_1}{R_2}\right) - 4 + \left(\frac{R_2}{R_1}\right)$$

$$\left(\frac{1}{16} + 4\right)\frac{R_1}{R_2} = \frac{R_1}{R_2}\left(4\frac{R_1}{R_2} + \frac{R_2}{R_1}\right) \Rightarrow 4.0625\frac{R_1}{R_2} = 4\left(\frac{R_1}{R_2}\right)^2 + 1$$

$$4\left(\frac{R_1}{R_2}\right)^2 - 4.0625\left(\frac{R_1}{R_2}\right) + 1 = 0, \quad \frac{R_1}{R_2} = 0.41908,$$ 0.5~~9~~655

This leads to a negative Q.

$$S_{R_1}^{Q} = -\frac{1}{2}\left[\sqrt{\frac{R_1 C_2}{R_2 C_1}} - \sqrt{\frac{R_2 C_2}{R_1 C_1}} - (K-1)\sqrt{\frac{R_1 C_1}{R_2 C_2}}\right] Q$$

$$S_{R_1}^{Q} = -\frac{1}{2}\left[\sqrt{0.41908} - \sqrt{1/0.41908} - 3\sqrt{0.41908}\right] 4$$

$$S_{R_1}^{Q} = 5.68$$

20)

$$H(s) = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1+R_2) C_2 s + 1}$$

$$Q = \frac{1}{R_1 + R_2}\sqrt{R_1 R_2 \frac{C_1}{C_2}} \quad , \quad \omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$\frac{Q}{\sqrt{1-\frac{1}{4Q^2}}} = 1.122 \Rightarrow Q^2 = (1.122)^2 \left(1 - \frac{1}{4Q^2}\right)$$

$$\Rightarrow 3.3058 Q^4 - 4Q^2 + 1 = 0$$

$$Q^2 = 0.85704 \, , \, 0.35296$$

$$Q = \pm 0.925765, \pm 0.5941$$

In order to peak, $Q > \frac{1}{\sqrt{2}} \Rightarrow Q = 0.925765$

$$\Rightarrow \frac{1}{R_1+R_2}\sqrt{R_1 R_2 \frac{C_1}{C_2}} = 0.925765$$

let $\frac{C_1}{C_2} = 1 \Rightarrow \frac{1}{R_1+R_2}\sqrt{R_1 R_2} = 0.925765$

$$\Rightarrow R_1 R_2 = (0.925765)^2 (R_1 + R_2)^2$$

$$\frac{R_1 R_2}{R_1 + R_2} = R_1 \| R_2 = 0.85704 \, (R_1 + R_2)$$

$$R_1 \| R_2 = 0.85704 \, (R_1 + R_2)$$

Only if $C_1/C_2 = 1$

21)

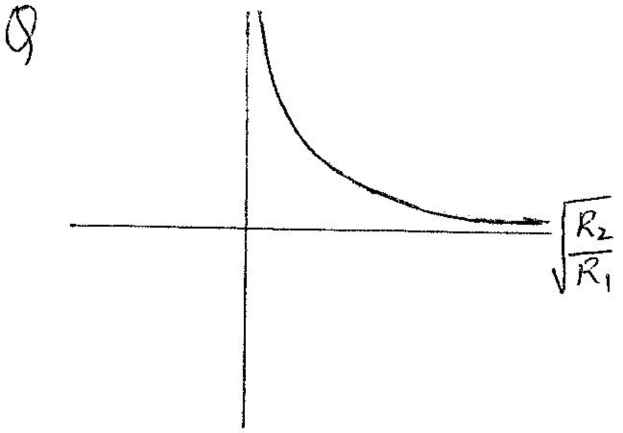
$$S_{R_1}^{Q} = 2\ , \quad C_2 = C_1\ , \quad Q = f\left(\sqrt{\frac{R_2}{R_1}}\right)$$

Range of Q and $\sqrt{R_2/R_1}$

$$S_{R_1}^{Q} = -\frac{1}{2}\left[\sqrt{\frac{R_1 C_2}{R_2 C_1}} - \sqrt{\frac{R_2 C_2}{R_1 C_1}} - (K-1)\sqrt{\frac{R_1 C_1}{R_2 C_2}}\right] Q$$

$$S_{R_1}^{Q} = -\frac{1}{2} + Q\sqrt{\frac{R_2 C_2}{R_1 C_1}} \Rightarrow 2 = -\frac{1}{2} + Q\sqrt{\frac{R_2}{R_1}}$$

$$\Rightarrow 2.5 = Q\sqrt{\frac{R_2}{R_1}}\ , \quad Q = \frac{2.5}{\sqrt{R_2/R_1}}$$



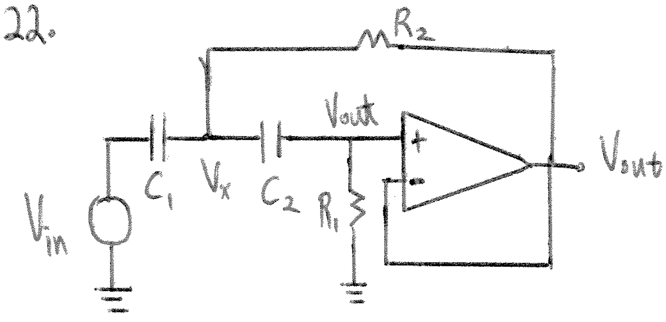
$\left(\frac{R_2}{R_1}\text{ can't be negative}\right)$

Range: $0 < Q < \infty$

$0 < \sqrt{\frac{R_2}{R_1}} < \infty$

If the transfer function does not want to experience peaking, then $0 < Q \le \frac{1}{\sqrt{2}} \Rightarrow 3.5356 \le \sqrt{\frac{R_2}{R_1}} < \infty$

22.



Assuming an ideal Op amp.

1) $(V_x - V_{in})C_1 s + (V_x - V_{out})\left(C_2 s + \frac{1}{R_2}\right) = 0$, nodal equation at V_x.

2) $(V_x - V_{out})C_2 s - \frac{V_{out}}{R_1} = 0$, nodal equation at V_{out}.

2) $\Rightarrow V_x = V_{out}\left[\frac{C_2 s + 1/R_1}{C_2 s}\right]$ (A) The stuff in the bracket becomes "A"

1) $\Rightarrow (AV_{out} - V_{in})C_1 s + (AV_{out} - V_{out})\left[C_2 s + \frac{1}{R_2}\right] = 0$

$\Rightarrow AV_{out}C_1 s + V_{out}(A-1)(C_2 s + 1/R_2) = V_{in}C_1 s$

$A - 1 = \frac{1}{R_1 C_2 s}$, $A = \frac{C_2 s + 1/R_1}{C_2 s}$

Subsitute (A-1) and A into 1) $\Rightarrow$

$$\left(\frac{C_2 s + 1/R_1}{C_2 s}\right)C_1 s V_{out} + \left(C_2 s + \frac{1}{R_2}\right)\frac{1}{R_1 C_2 s}V_{out} = V_{in}C_1 s$$

22.

$$\frac{V_{out}}{V_{in}} = \frac{C_1 S}{\frac{C_1}{C_2}\left(C_2 S + \frac{1}{R_1}\right) + \frac{1}{R_1 C_2 S}\left(C_2 S + \frac{1}{R_2}\right)}$$

Rearranging

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{S^2}{S^2 + \left(\frac{C_1 + C_2}{C_2 R_1 C_1}\right) S + \frac{1}{R_2 C_2 R_1 C_1}}$$

$$\omega_n^2 = \frac{1}{R_2 C_2 R_1 C_1}, \quad \frac{\omega_n}{Q} = \frac{C_1 + C_2}{C_2 R_1 C_1}$$

$$\omega_n = \frac{1}{\sqrt{R_2 C_2 R_1 C_1}}, \quad Q = \sqrt{\frac{C_2 C_1 R_1}{R_2}} \left(\frac{1}{C_1 + C_2}\right)$$

23.

$$Q = \frac{1}{C_1 + C_2}\sqrt{\frac{C_2 C_1 R_1}{R_2}} \Rightarrow \frac{1}{Q} = (C_1 + C_2)\sqrt{\frac{R_2}{C_2 C_1 R_1}}$$

1) $$\frac{d}{dQ}\left[\frac{1}{Q}\right] = -\frac{1}{Q^2} \Rightarrow d\left[\frac{1}{Q}\right] = -\frac{1}{Q^2}\, dQ$$

2) $$\frac{d\left[\frac{1}{Q}\right]}{dR_2} = \frac{1}{2}\frac{C_1 + C_2}{\sqrt{C_2 C_1 R_1 R_2}} \Rightarrow d\left[\frac{1}{Q}\right] = \frac{1}{2}\frac{C_1 + C_2}{\sqrt{C_2 C_1 R_1 R_2}}\, dR_2$$

Equating 1) and 2) and multiple 2) by $\frac{R_2}{R_2}$

$$-\frac{dQ}{Q^2} = \frac{1}{2}\frac{(C_1 + C_2) R_2}{\sqrt{C_2 C_1 R_1 R_2}}\,\frac{dR_2}{R_2}$$

$$\frac{dQ}{Q} \Big/ \frac{dR_2}{R_2} = -\frac{Q}{2}(C_1 + C_2)\sqrt{\frac{R_2}{C_1 C_2 R_1}}$$

$$S_{R_2}^{Q} = -\frac{Q}{2}(C_1 + C_2)\sqrt{\frac{R_2}{C_1 C_2 R_1}} = -\frac{1}{2}$$

$$\frac{1}{Q} = (C_1 + C_2)\sqrt{\frac{R_2}{C_2 C_1 R_1}} = C_1\sqrt{\frac{R_2}{C_2 C_1 R_1}} + C_2\sqrt{\frac{R_2}{C_2 C_1 R_1}}$$

$$\frac{\partial\left(\frac{1}{Q}\right)}{\partial C_1} = \frac{1}{2}\sqrt{\frac{R_2}{C_2 R_1 C_1}} - \frac{C_2}{2C_1}\sqrt{\frac{R_2}{C_2 R_1 C_1}}, \quad \frac{d\left(\frac{1}{Q}\right)}{dQ} = -\frac{1}{Q^2}$$

Rearranging $\Rightarrow$ $$-\frac{\partial Q}{Q^2} = \frac{\partial C_1}{C_1}\left(\frac{C_1 - C_2}{2}\sqrt{\frac{R_2}{C_2 R_1 C_1}}\right)$$

$$\frac{\partial Q}{Q} \Big/ \frac{\partial C_1}{C_1} = S_{C_1}^{Q} = -Q\left(\frac{C_1 - C_2}{2}\sqrt{\frac{R_2}{C_2 R_1 C_1}}\right)$$

23. Similarly:

$$S_{C_2}^{Q} = -Q\left(\frac{C_2 - C_1}{2}\sqrt{\frac{R_2}{C_2 R_1 C_1}}\right)$$

$$S_{R_1}^{Q} = Q\left(\frac{C_1 + C_2}{2}\sqrt{\frac{R_2}{C_2 C_1 R_1}}\right) = \frac{1}{2}$$

24.

$$\frac{V_{out}}{V_{in}}(s) = \frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

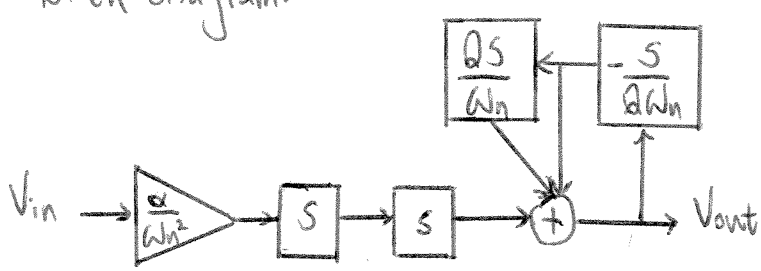
Cross-multiply.

$$V_{out}s^2 + V_{out}\frac{\omega_n}{Q}s + V_{out}\omega_n^2 = V_{in}\alpha s^2$$

Rearranging

$$V_{out} = V_{in}\frac{\alpha}{\omega_n^2}s^2 - V_{out}\frac{s^2}{\omega_n^2} - V_{out}\frac{s}{Q\omega_n}$$

Block diagram:



25.

$Q = 2, \quad \omega_n = (2\pi)(2\times10^6)$

$R_6 = R_3, \quad R_1 = R_2, \quad C_1 = C_2$

$10\,pF < \text{Total } C < 1\,nF, \quad 1K\Omega < \text{Total } R < 50K\Omega$

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4+R_5}\left(\frac{1}{R_1C_1}\right), \quad \omega_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)$$

Since $R_6 = R_3 \Rightarrow \omega_n^2 = \left(\frac{1}{R_1C_1}\right)^2 = (2\pi\times2\times10^6)^2$

$$\frac{1}{R_1C_1} = 2\pi\times2\times10^6 = \omega_n$$

$$Q = \frac{R_4+R_5}{R_4} = 2 \Rightarrow R_5 = R_4$$

Let $C_1 = C_2 = 100\,pf$, $R_1 = \frac{1}{(2\pi)(2\times10^6)(100\,pf)} = 795.77\Omega$

So $R_1 = R_2 = 795.77\Omega$, $C_1 = C_2 = 100\,pf$.

Since R_3, R_4, R_5, R_6 don't affect Q and ω_n, let them be 500Ω each.

Total R: $(4)(500) + (2)(795.77) = 3.6K\Omega$

Total C: $100\,pf + 100\,pf = 200\,pf$.

26.

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4+R_5}\,\frac{1}{R_1C_1}, \quad \omega_n^2 = \frac{R_6}{R_3}\cdot\frac{1}{R_1R_2C_1C_2}$$

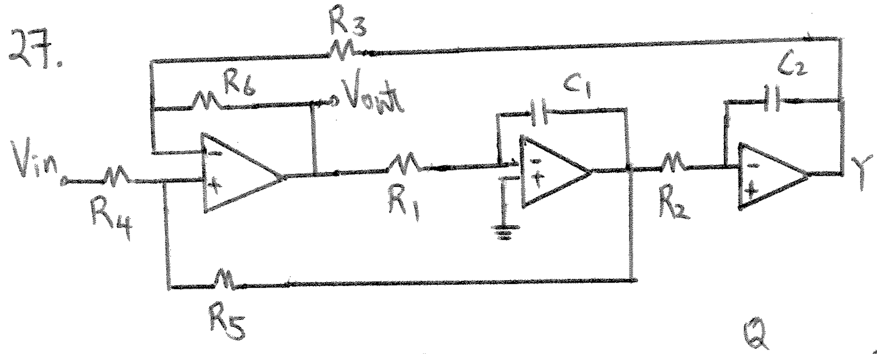
$$Q = \omega_n\left(\frac{R_4+R_5}{R_4}\right)R_1C_1, \quad \omega_n = \sqrt{\frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)}$$

$$Q = \sqrt{\frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)}\left(\frac{R_4+R_5}{R_4}\right)R_1C_1$$

$$Q = \sqrt{\frac{R_6}{R_3}}\sqrt{\frac{R_1C_1}{R_2C_2}}\left(\frac{R_4+R_5}{R_4}\right)$$

If $R_6 = R_3$, Q doesn't depend on R_6 and R_3, hence zero sensitivity.

27.



Low pass, low freq gain of 2. $S^{Q}_{R_3,R_6} = 0$

$$\frac{V_Y}{V_{in}} = \left(\frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}\right)\left(\frac{1}{R_1R_2C_1C_2 s^2}\right)$$ Low pass transfer function

Low freq gain: $\dfrac{\alpha}{\omega_n^2 R_1R_2C_1C_2}$, where $\alpha = \dfrac{R_5}{R_4+R_5}\left(1+\dfrac{R_6}{R_3}\right)$

and $\omega_n^2 = \dfrac{R_6}{R_3}\left(\dfrac{1}{R_1R_2C_1C_2}\right)$

Therefore, Low freq gain: $\dfrac{\alpha}{\frac{R_6}{R_3}} = \dfrac{R_5}{R_4+R_5}\left(1+\dfrac{R_6}{R_3}\right)\left(\dfrac{R_3}{R_6}\right)$

$$S^{Q}_{R_3,R_6} = \frac{Q}{2}\,\frac{|R_3-R_6|}{1+R_5/R_4}\sqrt{\frac{R_2C_2}{R_3R_6R_1C_1}}$$

To obtain $S^{Q}_{R_3,R_6} = 0$, $R_3 = R_6$, however this makes the low freq gain: $2\left(\dfrac{R_5}{R_4+R_5}\right) \neq 2$.

Therefore, it's impossible a low freq gain of 2 if $S^{Q}_{R_3,R_6} = 0$.

28.

Peaking : 1 dB, $R_3 = R_6$

Normalized Peak Value: $\dfrac{Q}{\sqrt{1-(4Q^2)^{-1}}} = 1.1$

solving for Q^2: 0.8570, ~~0.3380~~ (not possible for peaking)

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4+R_5}\left(\frac{1}{R_1C_1}\right), \quad \left(\frac{\omega_n}{Q}\right)^2 = \left(\frac{R_4}{R_4+R_5}\right)^2\left(\frac{1}{R_1C_1}\right)^2$$

$$\omega_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right) = \frac{1}{R_1R_2C_1C_2} \quad (\text{since } R_6 = R_3)$$

Therefore, $Q^2 = \left(\dfrac{R_1C_1}{R_2C_2}\right)\left(\dfrac{R_4+R_5}{R_4}\right)^2 = 0.8570$

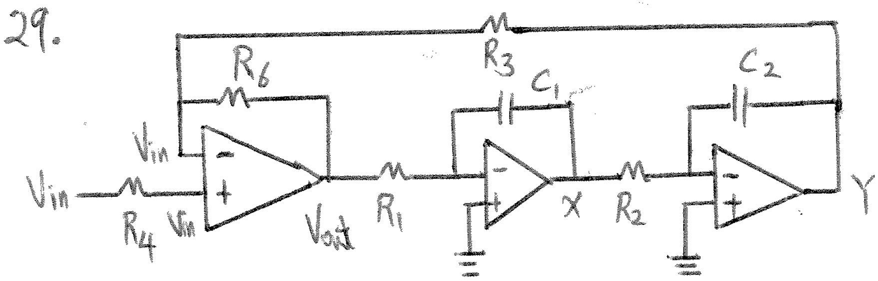
Low pass gain: $2\dfrac{R_5}{R_4+R_5} = \alpha$ (Since $R_6 = R_3$)

So $\dfrac{\alpha}{2} = \dfrac{R_5}{R_4+R_5}$, $\dfrac{R_4}{R_4+R_5} = 1-\dfrac{\alpha}{2} \Rightarrow \dfrac{R_4+R_5}{R_4} = \left(1-\dfrac{\alpha}{2}\right)^{-1}$

So $\left(\dfrac{R_1C_1}{R_2C_2}\right)\left(1-\dfrac{\alpha}{2}\right)^{-2} = 0.8570$, if $\alpha = 1 \Rightarrow \dfrac{R_1C_1}{R_2C_2} = 0.214$.

However, can't go down any further without knowing more information

29.



$$V_X = -\frac{V_{out}}{R_1}\left(\frac{1}{C_1 S}\right), \quad V_Y = -\frac{V_X}{R_2}\left(\frac{1}{C_2 S}\right), \quad V_{out} = V_{in} - \frac{(V_Y - V_{in})R_6}{R_3}$$

Subsituting V_x into V_y $\Rightarrow$ $V_y = \frac{V_{out}}{R_1}\left(\frac{1}{C_1 S}\right)\left(\frac{1}{R_2 C_2 S}\right)$

Subsituting V_y into V_{out} and rearranging:

$$\frac{V_{out}}{V_{in}} = \frac{(R_1C_1)(R_2C_2)S^2\left(1+\frac{R_6}{R_3}\right)}{(R_1C_1)(R_2C_2)S^2 + \frac{R_6}{R_3}}$$

simplifying

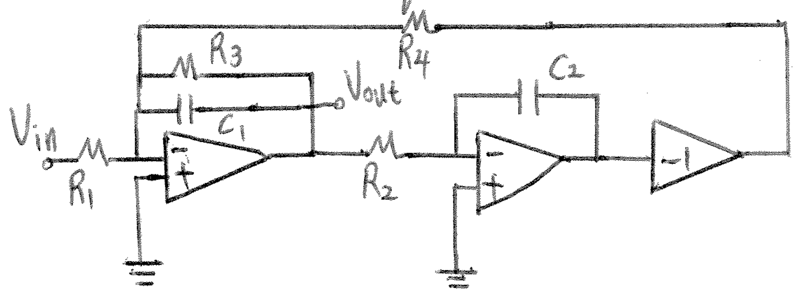
$$\frac{V_{out}}{V_{in}} = \frac{S^2\left(1+\frac{R_6}{R_3}\right)}{S^2 + \frac{R_6}{R_3}\left(\frac{1}{R_1C_1R_2C_2}\right)}$$

$$\omega_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1C_1R_2C_2}\right), \quad Q = \infty$$

$$\alpha = \left(1 + \frac{R_6}{R_3}\right)$$

30.

Tow-Thomas Biquad:



$$\omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}, \quad Q^{-1} = \frac{1}{R_3}\sqrt{\frac{R_2 R_4 C_2}{C_1}}$$

$$\frac{\partial \omega_n}{\partial R_2} = -\frac{1}{2}\frac{1}{R_2\sqrt{R_2 R_4 C_1 C_2}} = -\frac{1}{2}\frac{\omega_n}{R_2}$$

$$\frac{\partial \omega_n}{\omega_n} \Big/ \frac{\partial R_2}{R_2} = S_{R_2}^{\omega_n} = -\frac{1}{2}$$

Since R_2, R_4, C_1, C_2 are equivalent in ω_n's definition, all of their sensitivities $= -\frac{1}{2}$

Sensitivities of Q:

$$\frac{\partial Q}{\partial R_3} = \sqrt{\frac{C_1}{R_2 R_4 C_2}}\left(\frac{R_3}{R_3}\right) \Rightarrow \frac{\partial Q}{Q} = \frac{\partial R_3}{R_3} \Rightarrow S_{R_3}^{Q} = 1$$

$$\frac{\partial Q}{\partial C_1} = \frac{1}{2} R_3 \left(\frac{C_1}{R_2 R_4 C_2}\right)^{-\frac{1}{2}}\left(\frac{1}{R_2 R_4 C_2}\right)\frac{C_1}{C_1} \Rightarrow \frac{\partial Q}{Q} = \frac{1}{2}\frac{\partial C_1}{C_1} \Rightarrow S_{C_1}^{Q} = \frac{1}{2}$$

$$\frac{\partial Q}{\partial R_2} = -\frac{1}{2} R_3 \left(\frac{C_1}{R_2 R_4 C_2}\right)^{-\frac{1}{2}}\frac{C_1}{R_4 C_2}\left(\frac{1}{R_2^2}\right) \Rightarrow \frac{\partial Q}{Q} = -\frac{1}{2}\frac{\partial R_2}{R_2} \Rightarrow S_{R_2}^{Q} = -\frac{1}{2}$$

30.

Since R_2, R_4 and C_2 are equivalent in the expression

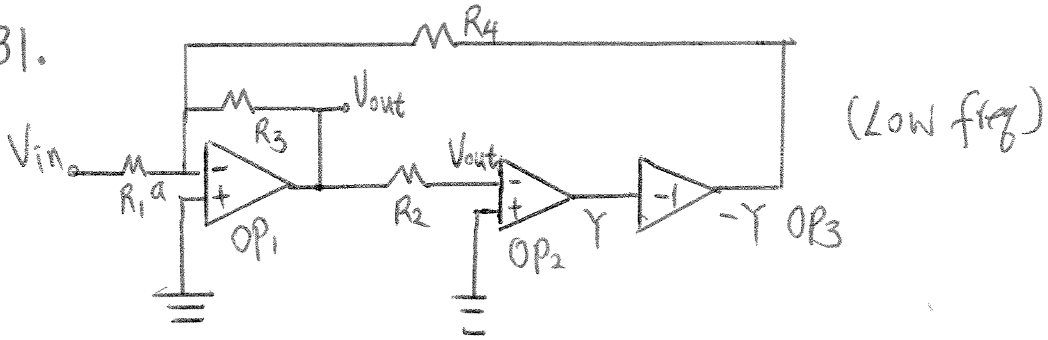
$$S^{Q}_{R_2,R_4,C_2} = -\frac{1}{2}$$

So, $S^{\omega_n}_{R_2,R_4,C_1,C_2} = -\frac{1}{2}$, $S^{\omega_n}_{R_1,R_3} = 0$

$$S^{Q}_{R_2,R_4,C_2} = -\frac{1}{2}, \quad S^{Q}_{C_1} = \frac{1}{2}, \quad S^{Q}_{R_3} = 1$$

$$S^{Q}_{R_1} = 0$$

31.



Vout equals zero because of OP2's negative feedback.
Likewise, Va equals to zero as well.
So, summing all the currents thru R3, we have

$$-\left(\frac{0-V_Y}{R_4}+\frac{V_{in}}{R_1}\right)R_3 = V_{out} = 0$$

$$\Rightarrow \frac{V_{in}}{R_1} = \frac{V_Y}{R_4} \Rightarrow \frac{V_Y}{V_{in}} = \frac{R_4}{R_1}$$

32.

$$\frac{V_Y}{V_{in}} = \frac{R_3 R_4}{R_1}\left(\frac{1}{R_2 R_3 R_4 C_1 C_2 S^2 + R_2 R_4 C_2 S + R_3}\right)$$

$\omega_n = (2\pi)(10\text{ MHz})$, $R_3 = 1K$, $R_2 = R_4$, $C_1 = C_2$

$$\omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}, \quad Q^{-1} = \frac{1}{R_3}\sqrt{\frac{R_2 R_4 C_2}{C_1}}$$

Peaking: 1dB

$$\frac{Q}{\sqrt{1-(4Q^2)^{-1}}} = 1.1, \quad Q^2 = 0.8570$$

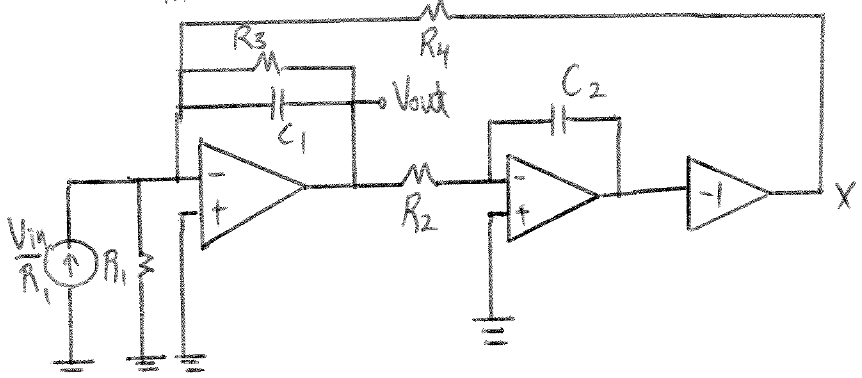
$$\omega_n = \frac{1}{\sqrt{(R_2 C_1)^2}} = (2\pi)(10\times10^6) \Rightarrow \frac{1}{R_2 C_1} = (2\pi)(10\times10^6)$$

$$\frac{1}{Q} = \frac{1}{1000}\sqrt{R_2^2} = \frac{1}{Q} = \frac{R_2}{1000} \Rightarrow \begin{array}{l} R_2 = 1166.86\,\text{Ohm} \\ R_2 = 1.2\,K\Omega \end{array}$$

Solving for C_1 we have: $C_1 = 13.64\,pf$.

33.

$$\frac{V_Y}{V_{in}} = \frac{R_3 R_4}{R_1}\left(\frac{1}{R_2R_3R_4C_1C_2s^2 + R_2R_4C_2s + R_3}\right)$$



When R_1 and V_{in} are replaced with its Norton equivalent, we see that the "upper" terminal of R_1 is at virtual ground. Since R_1's two terminals are at the same potential, no current will flow throw it, therefore it can be seen as an open. So R_1 is not in the signal path, and therefore will not affect the frequency response. However, since its magnitude is embedded in the Norton current source, it will affect the DC gain.

34.

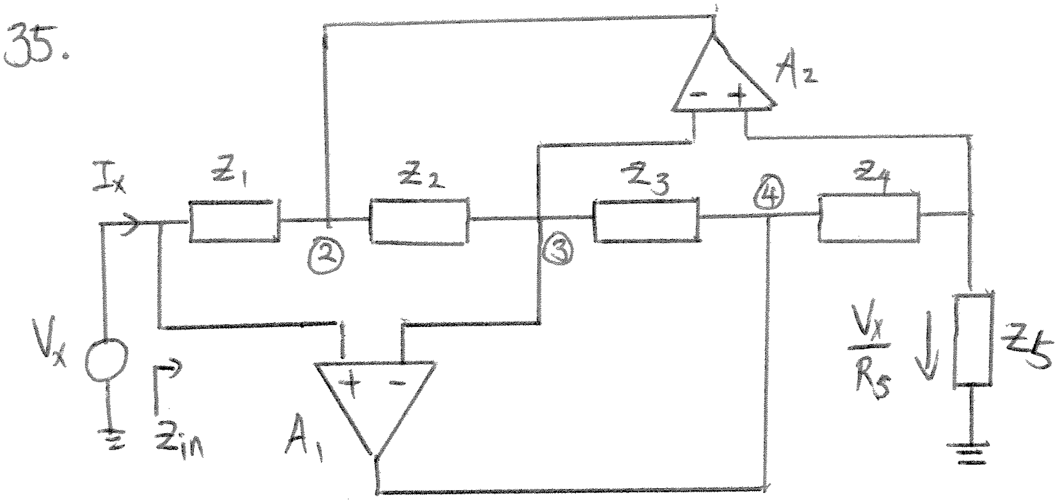
$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$ (for circuit diagram, please refer to Problem # 35)

For Z_{in} to be inductive, the following combinations will work.

1	2	3
$Z_5 = R$	$Z_5 = R$	$Z_5 = R$
$Z_4 = R$	$Z_4 = C$	$Z_4 = C$
$Z_3 = R$	$Z_3 = R$	$Z_3 = R$
$Z_2 = C$	$Z_2 = C$	$Z_2 = R$
$Z_1 = R$	$Z_1 = C$	$Z_1 = R$

Any other combination will result in DC path blockage at a node. Moreover, in #2 it's assumed that the input can provide a DC bias.

35.



$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

For Z_{in} to be Capacitive the following combinations can be used.

1	2	3	4	5
$Z_5 = C$	$Z_5 = C$	$Z_5 = R$	$Z_5 = R$	$Z_5 = R$
$Z_4 = R$	$Z_4 = R$	$Z_4 = R$	$Z_4 = R$	$Z_4 = C$
$Z_3 = R$	$Z_3 = R$	$Z_3 = R$	$Z_3 = C$	$Z_3 = C$
$Z_2 = R$	$Z_2 = C$	$Z_2 = R$	$Z_2 = R$	$Z_2 = R$
$Z_1 = R$	$Z_1 = C$	$Z_1 = C$	$Z_1 = R$	$Z_1 = C$

Any other combination results in a DC path blockage at a node. Moreover, in # 2, 3, 5, it is assumed that the input node will produce a DC bias.

36.

$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

$$Z_5 = R_x + \frac{1}{CS}, \quad Z_4 = R_x, \quad Z_3 = R_x, \quad Z_2 = R_x,$$

$$Z_1 = \frac{1}{CS}$$

$$Z_{in} = \frac{\frac{R_x}{CS}\left(R_x + \frac{1}{CS}\right)}{R_x^2} = \frac{1}{CS R_x}\left(R_x + \frac{1}{CS}\right)$$

$$Z_{in} = \frac{1}{CS} + \frac{1}{C^2 S^2 R_x}$$

$$V_{out} = \frac{V_{in}\left[S^2[C^2 R_x] + CS\right]}{\left[S^2[C^2 R_x] + SC + R_1[S^3 C^3 R_x]\right]}$$

$$\frac{V_{out}}{V_{in}} = \frac{SCR_x + 1}{S^2 R_1 R_x C^2 + SCR_x + 1}$$

37.

$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$ (for circuit diagram, please refer to problem # 35)

Let Z_5 be a capacitor, Z_2 and Z_4 be large resistors and Z_1 and Z_3 be small resistors compared to Z_2 and Z_4.

For example, let Z_1 and Z_3 equal 50Ω and Z_2 and Z_4 equal $5K\Omega$. Then there's a $(100)^2 = 10000$ multiplication factor onto C_5.

38.

Butterworth filter: roll-off of 1dB @ $\omega = 0.9\,\omega_0$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(0.9)^{2n}}} = 0.9 \Rightarrow 2n = \frac{\log(0.2345679)}{\log(0.9)}$$

$n = 6.88$

So we need a 7th order.

39.

$$|H(j\omega)| = \frac{1}{\sqrt{1+(0.9)^{2n}}}$$

$n=0 \Rightarrow -3\,dB$

$n=1 \Rightarrow -2.577\,dB$

$n=3 \Rightarrow -1.851\,dB$

$n=5 \Rightarrow -1.299\,dB$

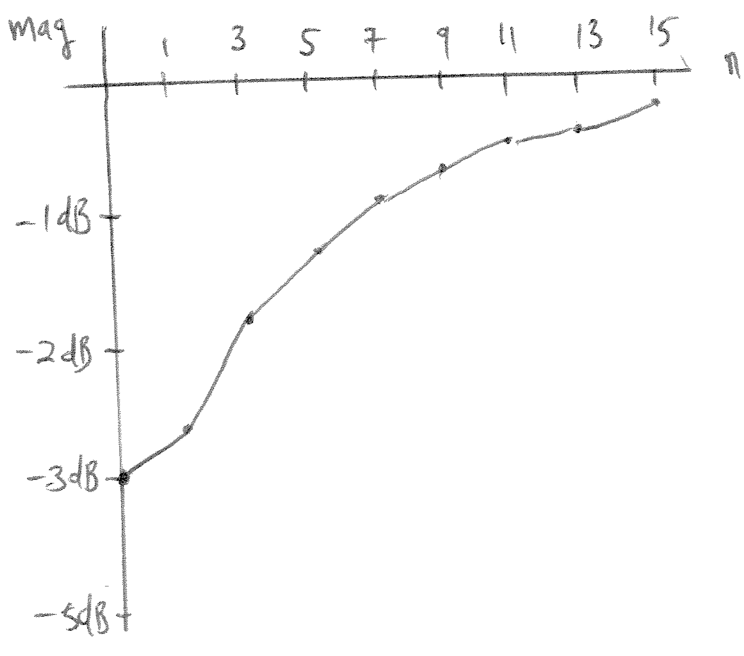
$n=7 \Rightarrow -0.895\,dB$

$n=9 \Rightarrow -0.607\,dB$

$n=11 \Rightarrow -0.408\,dB$

$n=13 \Rightarrow -0.272\,dB$

$n=15 \Rightarrow -0.180\,dB$



40.

$1.1\omega_0$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(1.1)^{2n}}} = 0.1 \Rightarrow 2n = \frac{\log(99)}{\log(1.1)}$$

$n = 24.106$ so needs $n = 25$.

$n=0 \Rightarrow -3$ dB

$n=1 \Rightarrow -3.4439$ dB

$n=3 \Rightarrow -4.427$ dB

$n=5 \Rightarrow -5.555$ dB

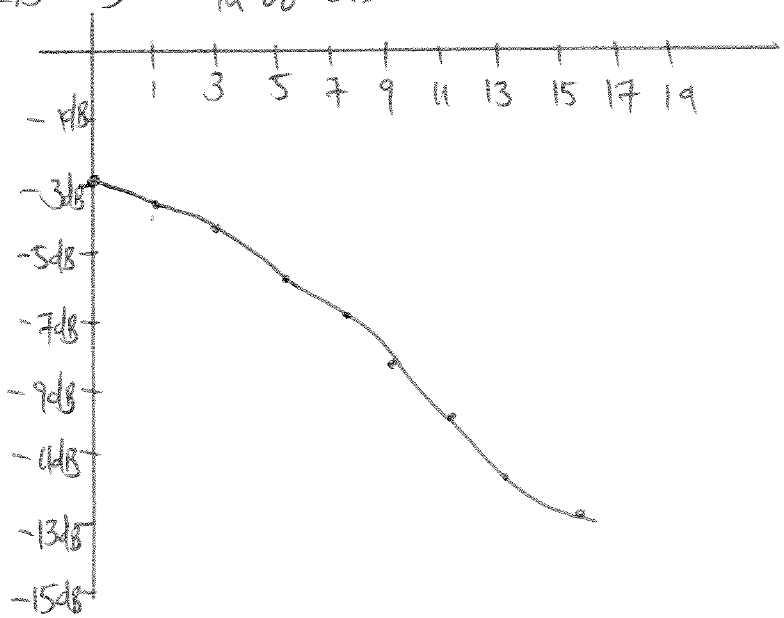
$n=7 \Rightarrow -6.810$ dB

$n=9 \Rightarrow -8.169$ dB

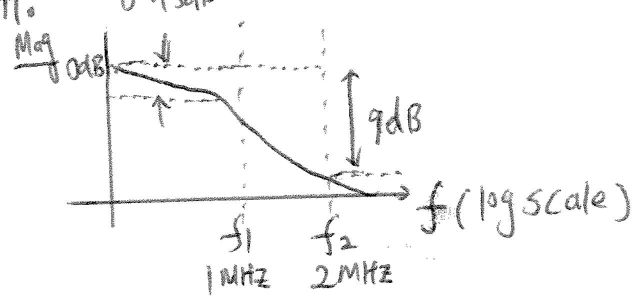
$n=11 \Rightarrow -9.61$ dB

$n=13 \Rightarrow -11.112$ dB

$n=15 \Rightarrow -12.66$ dB



41.



$$|H(f)| = \frac{1}{\left(1 + \left(\frac{2\pi f}{\omega_0}\right)^6\right)^{\frac{1}{2}}}$$

$|H(5\text{MHz})| = 0.02438$

Suppression: $20 \log (0.02438) = -32.26$ dB

42.

Low-pass Butterworth: Passband flatness of 0.5 dB

$f_1 = 1\,\text{MHz}$, $f_2 = 2\,\text{MHz}$, order < 5

$$-0.5\,\text{dB} = 20\log(x) \Rightarrow x = 0.944$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}} = 0.944 \Rightarrow \frac{1}{\left(1+\left(\frac{1}{f_0}\right)^{2n}\right)} = (0.944)^2$$

@ $f = 1\,\text{MHz}$

$$\Rightarrow 1 + \frac{1}{(f_0)^{2n}} = \frac{1}{(0.944)^2} \Rightarrow f_0 = 10^{\frac{0.91306}{2n}}$$

for $n = 1$, $f_0 \approx 2.86\,\text{MHz}$

for $n = 5$, $f_0 \approx 1.234\,\text{MHz}$

Therefore, for greatest attenuation $n = 5$

So
$$H(2\,\text{MHz}) = \frac{1}{\sqrt{1+\left(\frac{2}{1.234}\right)^{10}}} = 0.089$$

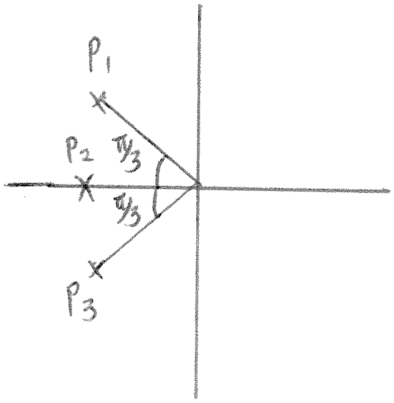
$$20\log(0.089) = -21.0\,\text{dB} \text{ at } n = 5$$

43.

$$P_K = \omega_0 \exp\left(\frac{j\pi}{2}\right) \exp\left(j\,\frac{2K-1}{2n}\pi\right), \quad K = 1, 2, \cdots n$$

The poles lie on a circle because all of their magnitude, which is the distance from the origin to the poles, are the same (ω_0) with each K; only the phase, which is the angle the poles make with the positive real axis, differ. Therefore, a circle is formed.

44.



$$P_1 = 2\pi(1.45\,MHz)\left[\cos\left(\frac{2\pi}{3}\right) + j\sin\left(\frac{2\pi}{3}\right)\right]$$

$$P_2 = (2\pi)(1.45\,MHz)$$

$$P_3 = (2\pi)(1.45\,MHz)\left[\cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right)\right]$$

$$H(s) = \frac{(-P_1)(-P_3)}{(s-P_1)(s-P_3)} = \frac{[2\pi(1.45\,MHz)]^2}{s^2 - [4\pi(1.45\,MHz)\cos(2\pi/3)]s + [2\pi(1.45\,MHz)]^2}$$

KHN Low pass Transfer function:

$$\frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2} \cdot \frac{1}{R_1R_2C_1C_2s^2} = \frac{\alpha/(R_1R_2C_1C_2)}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

$$\frac{\alpha}{R_1R_2C_1C_2} = (2\pi \times 1.45 \times 10^6)^2, \quad \omega_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right) = [2\pi \times 1.45 \times 10^6]^2$$

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4+R_5}\left(\frac{1}{R_1C_1}\right) = -(4\pi \times 1.45 \times 10^6 \times \cos(2\pi/3))$$

$$\frac{\alpha}{R_1R_2C_1C_2} = \frac{R_5}{R_4+R_5}\left(1+\frac{R_6}{R_3}\right)\left(\frac{1}{R_1R_2C_1C_2}\right)$$

Let $R_6 = R_3$, $R_2 = 4R_1$, $C_1 = C_2$

$$\omega_n^2 = \left(\frac{1}{4R_1C_1}\right)^2 = (2\pi \times 1.45 \times 10^6)^2 \Rightarrow \frac{1}{2R_1C_1} = 2\pi \times 1.45 \times 10^6$$

44.

Let $R_1 = 5K \Rightarrow C_1 = 10.98pf$, $R_2 = 20K$, $C_2 = 10.98pf$

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4+R_5}\left(\frac{1}{R_1C_1}\right) = 9110618.7$$

$$\Rightarrow \frac{R_4}{R_4+R_5} = \frac{1}{2}$$

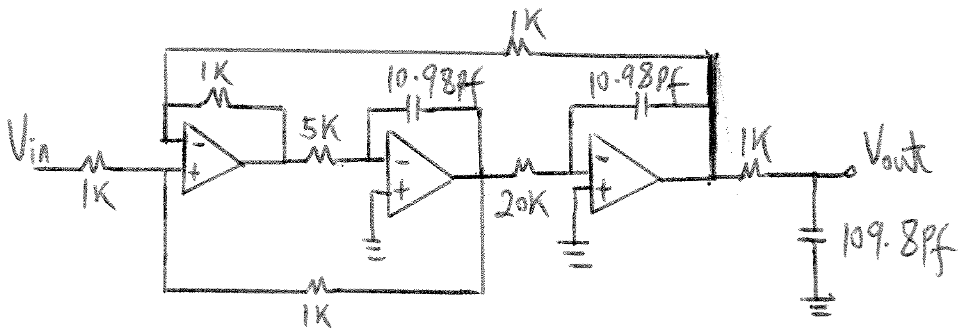
$$\frac{R_5}{R_4+R_5}\left(1+\frac{R_6}{R_3}\right)\left(\frac{1}{R_1R_2C_1C_2}\right) = \left(\frac{1}{2}\right)(2)\left(2\pi \times 1.45\times10^6\right)^2 = \left(2\pi\times1.45\times10^6\right)^2$$

let R_5 and R_4 be 1K apiece.

So $R_5 = R_4 = R_6 = R_3 = 1K$

$R_1 = 5K$, $R_2 = 20K$

$C_1 = C_2 = 10.98pf$



45.

Tow-Thomas Biquad

$$\frac{V_Y}{V_{in}} = \frac{R_3R_4}{R_1}\left(\frac{1}{R_2R_3R_4C_1C_2s^2 + R_2R_4C_2s + R_3}\right)$$

$$\frac{V_Y}{V_{in}} = \frac{1/(R_1R_2C_1C_2)}{s^2 + 1/(R_3C_1)s + 1/(R_2R_4C_1C_2)}$$

$$\frac{V_Y}{V_{in}} = \frac{(2\pi \times 1.45\times 10^6)^2}{s^2 - (4\pi \times 1.45\times10^6 \times \cos(2\pi/3))s + (2\pi\times1.45\times10^6)^2}$$

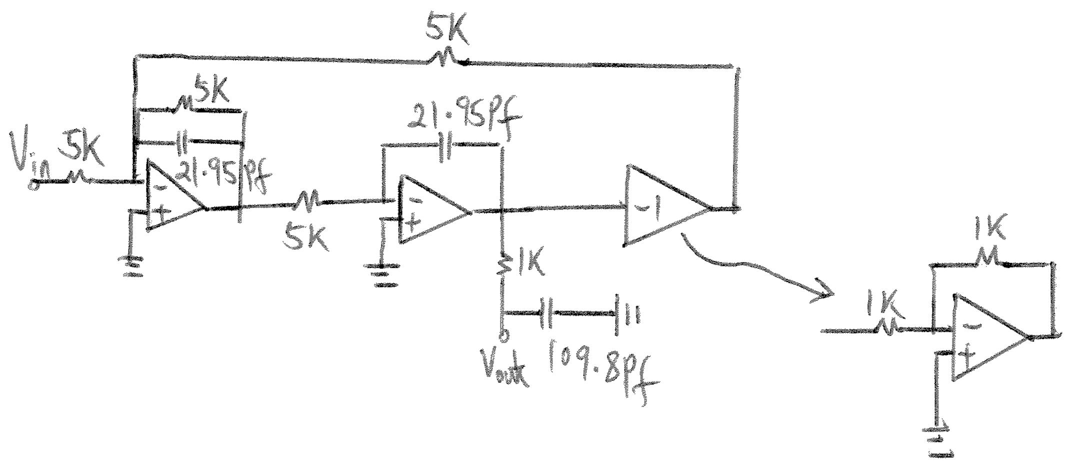
$$\frac{1}{R_1R_2C_1C_2} = (2\pi\times1.45\times10^6)^2, \quad \frac{1}{R_2R_4C_1C_2} = (2\pi\times1.45\times10^6)^2$$

$$\frac{1}{R_3C_1} = 2\pi\times1.45\times10^6$$

Let $R_1 = R_2 = R_3 = R_4$, $C_1 = C_2$

let $R_3 = 5K \Rightarrow C_1 = 21.95pf$

So $R_1 = R_2 = R_3 = R_4 = 5K$, and $C_1 = C_2 = 21.95pf$



46.

$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2 C_n^2\left(\frac{\omega}{\omega_0}\right)}} \qquad n=4, \quad \epsilon = 0.2$$

$$C_n\left(\frac{\omega}{\omega_0}\right) = \cos\left(n\cos^{-1}\frac{\omega}{\omega_0}\right) = \cos\left(4\cos^{-1}\frac{\omega}{\omega_0}\right)$$

$$C_n^2\left(\frac{\omega}{\omega_0}\right) = \cos^2\left(n\cos^{-1}\frac{\omega}{\omega_0}\right) = \frac{1}{2}\left(1+\cos\left(2n\cos^{-1}\frac{\omega}{\omega_0}\right)\right)$$

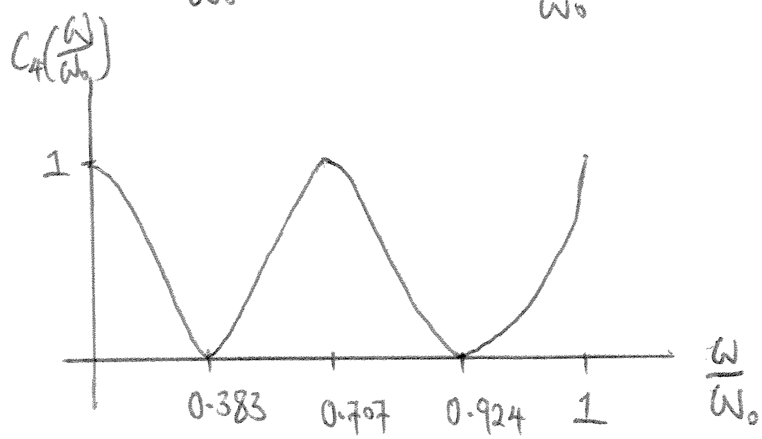
$$2n\cos^{-1}\frac{\omega}{\omega_0} = \pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0.924$$

$$2n\cos^{-1}\frac{\omega}{\omega_0} = 3\pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0.383$$

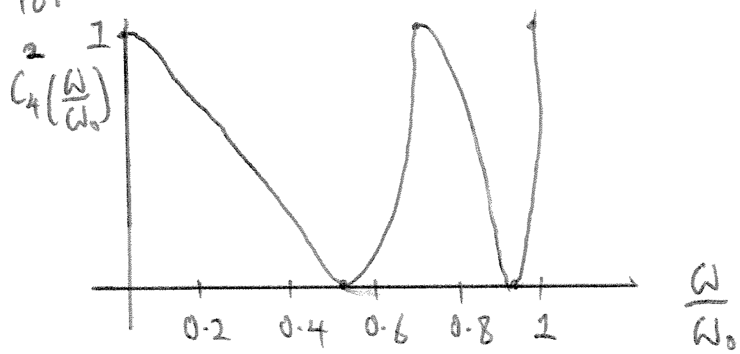
$$2n\cos^{-1}\frac{\omega}{\omega_0} = 0, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 1$$

$$2n\cos^{-1}\frac{\omega}{\omega_0} = 2\pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0.707$$

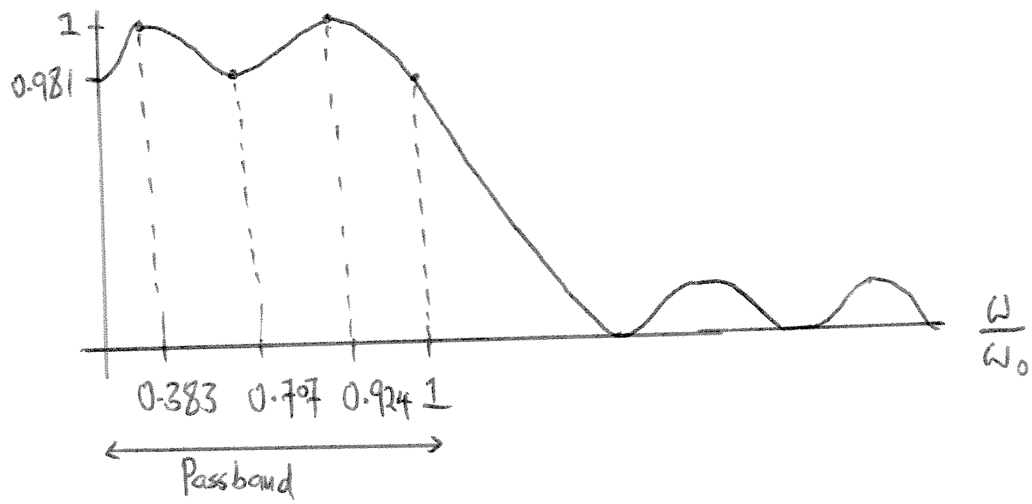
$$2n\cos^{-1}\frac{\omega}{\omega_0} = 4\pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0$$



46.



$$H(j\omega) = \frac{1}{\sqrt{1 + (0.2)^2\left[C_4^2\left(\frac{\omega}{\omega_0}\right)\right]}}$$



47. Chebyshev: 25 dB at 5 MHz.

$n = 5, \quad W_0 = 2\text{MHz}, \quad \frac{W}{W_0} = \frac{5}{2}.$

$$\frac{1}{\sqrt{1+\epsilon^2 \cosh^2(n\cosh^{-1}\frac{5}{2})}} = -25\,dB = 0.056234$$

$$\Rightarrow \frac{1}{1+\epsilon^2 (1.5939\times10^6)} = 0.0031622771$$

$$\Rightarrow \epsilon^2 = 1.9777\times10^{-4}$$

$\Rightarrow$ Minimum Ripple

$$\frac{1}{\sqrt{1+(1.9777\times10^{-4})}} = 0.99990 = -8.6\times10^{-4}\,dB.$$

48. $n = 6$

$$\cosh^2\left(6\cos^{-1}\left(\frac{5}{2}\right)\right) = 36590401$$

$$\frac{1}{\sqrt{1+\epsilon^2(36590401)}} = 0.056234$$

$$\epsilon^2 = 8.615 \times 10^{-6}$$

$$\text{Minimum Ripple} = \frac{1}{\sqrt{1+8.615\times10^{-6}}} = -3.74\times10^{-5}\ \text{dB}$$

Smaller than when $n=5$.

49. $\epsilon = 0.509$, $n = 4$

$$P_{1,4} = -0.140w_0 \pm 0.983jw_0$$

$$P_{2,3} = -0.337w_0 \pm 0.407jw_0$$

$$H_{1,4}(w) = \frac{0.986 w_0^2}{s^2 + 0.28w_0 s + 0.986 w_0^2} = \frac{\alpha/(R_1R_2C_1C_2)}{s^2 + \frac{w_n}{Q}s + w_n^2}$$

$$Q = 3.55$$

$$w_n = (2\pi)(4.965\text{ MHz})$$

$$w_n^2 = [(2\pi)(4.965\times10^6)]^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)$$

$$\frac{w_n}{Q} = (0.28)(5\text{MHz})(2\pi) = (1.4\text{MHz})(2\pi) = \frac{R_4}{R_4+R_5}\cdot\left(\frac{1}{R_1C_1}\right)$$

$$\frac{R_5}{R_4+R_5}\left(1+\frac{R_6}{R_3}\right)\left(\frac{1}{R_1R_2C_1C_2}\right) = \frac{R_6}{R_3}\left(\frac{1}{R_1R_2C_1C_2}\right)$$

$$\Rightarrow \frac{R_5}{R_4+R_5}\left(1+\frac{R_6}{R_3}\right) = \frac{R_6}{R_3}$$

$$\Rightarrow \frac{R_5}{R_4+R_5} = \frac{\frac{R_6}{R_3}}{1+\frac{R_6}{R_3}}$$

$$\Rightarrow 1-\alpha = \frac{\frac{R_6}{R_3}}{1+\frac{R_6}{R_3}}, \quad \alpha = \frac{R_4}{R_4+R_5}$$

$$\Rightarrow \frac{R_6}{R_3} = \frac{1-\alpha}{\alpha}$$

Let $\alpha = 0.5 \Rightarrow \frac{R_6}{R_3} = 1$

49.

$$\Rightarrow W_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1 C_1 R_2 C_2}\right) = \frac{1}{R_1 C_1 R_2 C_2} \quad (*)$$

Since $\frac{W_n}{Q} = \alpha\left(\frac{1}{R_1 C_1}\right) = 1.4 \times 10^6 \times 2\pi$, $\alpha = 0.5$

$$\Rightarrow \frac{1}{R_1 C_1} = 1.76 \times 10^7 \quad (1)$$

Consider (*)

$$\Rightarrow \frac{1}{R_2 C_2} = W_n^2 \cdot R_1 C_2 = 5.53 \times 10^7 \quad (2)$$

$R_6 = R_3 = R_5 = R_4 = 1K$. According to (1).(2), choose

$R_1 = 5K$, $C_1 = 11.368p$

$R_2 = 5K$, $C_2 = 3.62p$

For $H_{2,3}(S) = \dfrac{0.279 W_0^2}{S^2 + 0.674 W_0 S + 0.279 W_0^2}$

$W_n = (2\pi)(2.64 \times 10^6)$

$\frac{W_n}{Q} = (2\pi)(0.674 \times 5 \times 10^6) = (2\pi)(3.37 \times 10^6)$

Let $\alpha = 0.5$

49. $\frac{W_n}{Q} = (\alpha)\left(\frac{1}{R_1C_1}\right)$

$$\Rightarrow \frac{1}{R_1C_1} = \frac{W_n}{Q} \cdot \frac{1}{\alpha} = 4.23 \times 10^7 \quad ③$$

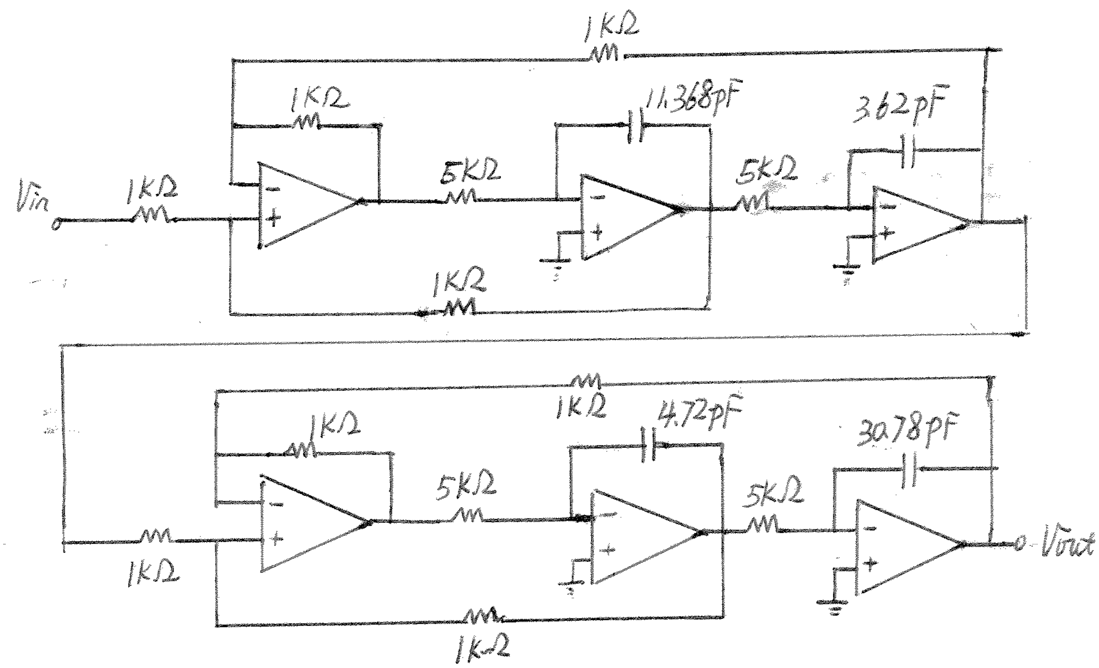
$$\frac{R_6}{R_3} = 1, \quad W_n^2 = \frac{R_6}{R_3}\left(\frac{1}{R_1C_1R_2C_2}\right) = \frac{1}{R_1C_1R_2C_2}$$

$$\Rightarrow \frac{1}{R_2C_2} = W_n^2 \cdot R_1C_1 = 6.50 \times 10^6 \quad ④$$

Consider ③ ④, choose

$R_1 = 5K$, $C_1 = 4.72p$; $R_2 = 5k$, $C_2 = 30.78p$

$R_6 = R_3 = R_5 = R_4 = 1K$.



50. Tow Thomas

Low Pass Transfer Function

$$P_{1,4} = \frac{V_4}{V_{in}} = \frac{\frac{1}{R_1R_2C_1C_2}}{S^2 + \frac{1}{R_3C_1}S + \frac{1}{R_2R_4C_1C_2}} = \frac{0.986\,W_o^2}{S^2 + 0.28\,W_oS + 0.986\,W_o^2}$$

$$\frac{1}{R_1R_2C_1C_2} = \frac{1}{R_2R_4C_1C_2} = [(2\pi)(4.965\times10^6)]^2 \quad (*)$$

$$\frac{1}{R_3C_1} = 2\pi \times 1.4\times10^6$$

Let $R_3 = 5K$, $C_1 = 22.736P$

Let $C_1 = C_2$, $C_2 = 22.736P$

$R_1 = R_2 \overset{(*)}{\Rightarrow} R_1 = R_2 = R_4 = 1.4K$

$R_3 = 5K$, $C_1 = C_2 = 22.736P$

} for $P_{1,4}$

$$P_{2,3} = \frac{0.279\,W_o^2}{S^2 + 0.674W_oS + 0.279W_o^2} = \frac{\frac{1}{R_1R_2C_1C_2}}{S^2 + \frac{1}{R_3C_1}S + \frac{1}{R_2R_4C_1C_2}}$$

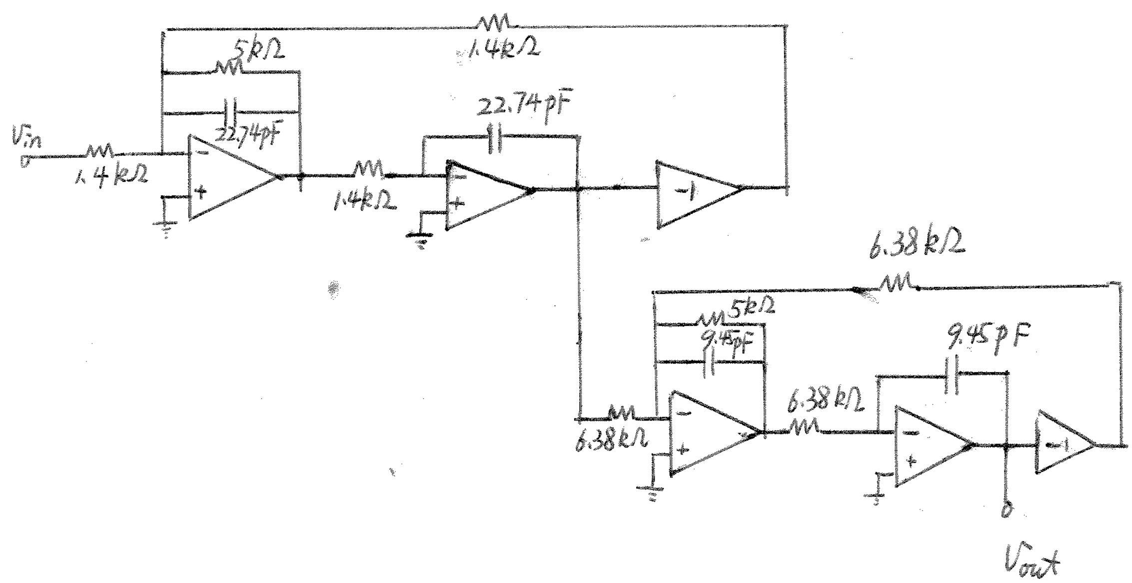
$$\frac{1}{R_1R_2C_1C_2} = \frac{1}{R_2R_4C_1C_2} = [(2\pi)(2.64\times10^6)]^2 \quad (*)$$

$$\frac{1}{R_3C_1} = (2\pi)(3.37\times10^6)$$

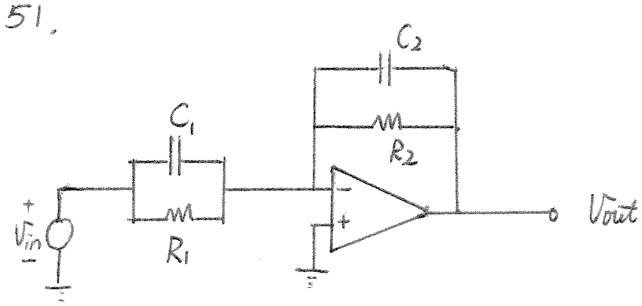
50. Let $R_3 = 5K$, $C_1 = 9.45p$

Let $C_1 = C_2 = 9.45p$, $R_1 = R_2 \overset{(*)'}{\Longrightarrow} R_1 = R_2 = R_4 = 6.38K$.

For $P_{1,4}$. $\begin{cases} R_1 = R_2 = R_4 = 6.38K, \quad R_3 = 5K \\ C_1 = C_2 = 9.45p. \end{cases}$

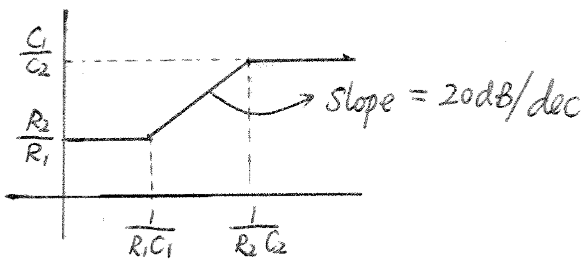


51.



High pass, 1MHz ⇒ 10dB atten

$f > 5\text{MHz}$, gain = 1



$$\frac{1}{R_2C_2} = (5\text{MHz})(2\pi)$$

Let $\frac{C_1}{C_2} = 1$, $\frac{R_2}{R_1} = -10\,dB = 0.316$

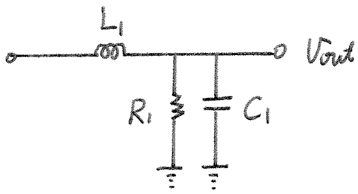
So $\frac{1}{0.316} = 3.1623$ $\quad \frac{5\text{MHz}}{3.1623} = 1.58\text{ MHz}$

$$\Rightarrow \frac{1}{R_1C_1} = (1.58\text{ MHz})(2\pi)$$

Choose $C_2 = 31.83\text{pF} \Rightarrow R_2 = 1\text{K}\Omega$

$C_1 = 31.83\text{pF} \Rightarrow R_1 = 3.16\text{k}\Omega$

52.



Peaking : 1dB

bandwidth : 100MHz

$L_1 < 100nH$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{L_1C_1}}{S^2+\frac{1}{R_1C_1}S+\frac{1}{L_1C_1}} = \frac{\gamma}{S^2+\frac{W_n}{Q}S+W_n^2}\bigg|_{S=jw} = \frac{\gamma}{(jw)^2+\frac{W_n}{Q}(jw)+W_n^2}$$

$$H(jw) = \frac{\gamma}{(W_n^2-W^2)+\frac{W_n}{Q}Wj}$$

$$|H(jw)| = \frac{\gamma}{\sqrt{(W_n^2-W^2)^2+(\frac{W_n}{Q}W)^2}}$$

At W_1, $$|H(jW_1)| = \frac{\gamma}{\sqrt{(W_n^2-W_1^2)^2+(\frac{W_n}{Q}W_1)^2}} = \frac{\gamma}{W_n^2\sqrt{2}}$$

$$\Rightarrow \sqrt{\frac{(W_n^2-W_1)^2+(\frac{W_n}{Q}W_1)^2}{W_n^4}} = \sqrt{2}.$$

$$\Rightarrow (W_n^2-W_1)^2+(\frac{W_n}{Q}W_1)^2 = 2W_n^4 \qquad (*).$$

$$\frac{Q}{\sqrt{1-(4Q^2)^{-1}}} = 1.1 \Rightarrow Q = 0.9258,\ 0.5941 (< \tfrac{1}{\sqrt{2}}, \text{ can't produce peaking})$$

So $Q = 0.9258$.

Solve (*) gives $W = \sqrt{1.5}\,W_n$.

$$W = \sqrt{1.5}\frac{1}{\sqrt{L_1C_1}} = (2\pi)(100\times10^6) \qquad (1)$$

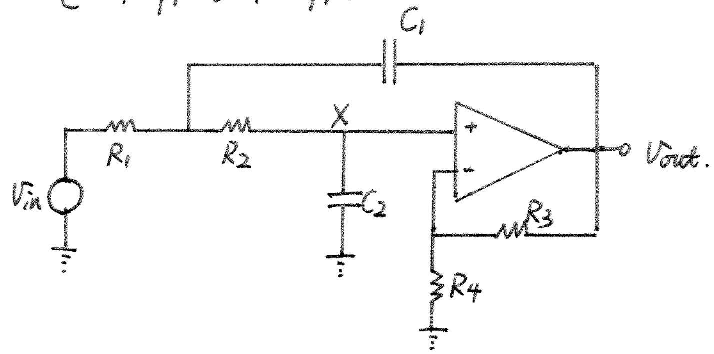
52.

$$\frac{\omega_n}{Q} = \frac{1}{R_1 C_1} \Rightarrow Q = R_1 C_1 \frac{1}{\sqrt{L_1 C_1}} = R_1 \sqrt{\frac{C_1}{L_1}} = 0.9258 \quad ②$$

$$\text{Let } L_1 = 90\,nH \overset{①}{\Longrightarrow} C_1 = 42.22\,pF \overset{②}{\Longrightarrow} R_1 = 42.74\,\Omega.$$

53. $W_n = (2\pi)(50MHz)$, $Q = 1.5$, Low frequency gain $= 2$.

$C = 10pF$ to $100pF$.



$$\frac{V_{out}}{V_{in}} = \frac{1+\frac{R_3}{R_4}}{R_1R_2C_1C_2S^2 + (R_1C_2 + R_2C_2 - R_1\frac{R_3}{R_4}C_1)S + 1}$$

$$= \frac{(1+\frac{R_3}{R_4})/(R_1R_2C_1C_2)}{S^2 + \frac{(R_1C_2 + R_2C_2 - \frac{R_1R_3}{R_4}C_1)S}{R_1R_2C_1C_2} + \frac{1}{R_1R_2C_1C_2}}$$

$$W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad \frac{W_n}{Q} = \frac{R_1C_2 + R_2C_2 - R_1\frac{R_3}{R_4}C_1}{R_1R_2C_1C_2.}$$

Low frequency gain $(1+\frac{R_3}{R_4}) = 2$, Let $\frac{R_3}{R_4} = 1$.

$$\frac{1}{\sqrt{R_1R_2C_1C_2}} = W_n = (2\pi)(50\times10^6) \quad ①$$

$$R_1C_2 + R_2C_2 - R_1C_1 = \frac{W_n}{Q}(R_1R_2C_1C_2) = \frac{1}{(1.5)(2\pi)(50\times10^6)} \quad ②$$

Let $C_1 = C_2 = 10pF$

② $\Rightarrow R_2 = \frac{1}{(1.5)(2\pi)(50\times10^6)} \cdot \frac{1}{C_2} = 212.2\ (\Omega)$

① $\Rightarrow R_1 = 1/[(2\pi)^2(50\times10^6)^2 \cdot R_2C_1C_2] = 477.5\ (\Omega)$

Let $R_3 = R_4 = 1k\Omega$.

54. $\omega_{3dB} = (30\times10^6)(2\pi)$, gain = 2, sensitivities no greater than 1.

$$H(s) = \frac{K\omega_n^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}, \quad s = j\omega \Rightarrow$$

$$H(j\omega) = \frac{K\omega_n^2}{\omega_n^2 - \omega^2 + \frac{\omega_n}{Q}\omega j}$$

$$|H(j\omega)| = \frac{K\omega_n^2}{\sqrt{(\omega_n^2-\omega^2)^2 + (\frac{\omega_n}{Q}\omega)^2}}$$

$$|H(j\omega)| = \frac{K}{\sqrt{2}} \Rightarrow \frac{\omega_n^2}{\sqrt{(\omega_n^2-\omega^2)^2 + (\frac{\omega_n}{Q}\omega)^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow (\omega_n^2 - \omega^2)^2 + (\frac{\omega_n}{Q}\omega)^2 = 2\omega_n^4$$

$$\Rightarrow \omega_n^4(1 - \frac{\omega^2}{\omega_n^2})^2 + \omega_n^4\left[(\frac{1}{Q})^2\cdot(\frac{\omega}{\omega_n})^2\right] = 2\omega_n^4$$

$$\Rightarrow \left[1 - (\frac{\omega}{\omega_n})^2\right]^2 + (\frac{1}{Q})^2(\frac{\omega}{\omega_n})^2 = 2$$

$$\Rightarrow (\frac{\omega}{\omega_n})^4 + \left[(\frac{1}{Q})^2 - 2\right](\frac{\omega}{\omega_n})^2 - 1 = 0$$

$S^{\omega_n}_{R_2, C_1, C_2, R_1} = -\frac{1}{2}$ (sensitivities of ω_n all < 1)

$$S^Q_{R1} = -S^Q_{R2} = -\frac{1}{2} + Q\sqrt{\frac{R_2C_2}{R_1C_1}}$$

$$S^Q_{C1} = -S^Q_{C2} = -\frac{1}{2} + Q\left(\sqrt{\frac{R_1C_2}{R_2C_1}} + \sqrt{\frac{R_2C_2}{R_1C_1}}\right) = \frac{1}{2} + Q\sqrt{\frac{R_1C_1}{R_2C_2}}$$

$$S^Q_K = QK\sqrt{\frac{R_1C_1}{R_2C_2}} = 2Q\sqrt{\frac{R_1C_1}{R_2C_2}}$$

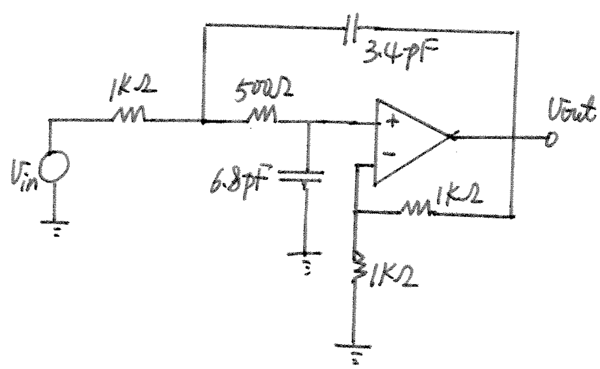
Let $\sqrt{\frac{R_1C_2}{R_2C_2}} = 1$, and $Q = \frac{1}{2}$,

$S_K^Q = 2\cdot(\frac{1}{2}) = 1$, $\quad S_{C_1}^Q = \frac{1}{2} + \frac{1}{2} = 1$

$S_{C_2}^Q = -1$, $\quad S_{R_1}^Q = -\frac{1}{2} + \frac{1}{2} = 0$, $\quad S_{R_2}^Q = 0$

Since $Q = \frac{1}{2}$.,

$$(\frac{w}{w_n})^4 + 2(\frac{w}{w_n})^2 - 1 = 0$$



$$\Rightarrow (\frac{w}{w_n})^2 = 0.4142$$

$$\Rightarrow w = \sqrt{0.4142}\, w_n.$$

Since $R_1C_1 = R_2C_2$,

$$w_n = \frac{1}{\sqrt{(R_1C_1)^2}} = \frac{1}{R_1C_1}$$

$$\Rightarrow \sqrt{0.4142}\, w_n = \frac{\sqrt{0.4142}}{R_1C_1} = (2\pi)(30\times10^6) \quad ①$$

Also $\frac{1}{Qw_n} = R_1C_2 + R_2C_2 - R_1C_1 = R_1C_2 \quad ②$

$$\Rightarrow \frac{R_1C_1}{R_1C_2} = \frac{\frac{1}{w_n}}{\frac{1}{Qw_n}} = Q = \frac{1}{2}.$$

$$\Rightarrow \frac{C_1}{C_2} = \frac{1}{2}.$$

Let $R_1 = 1K\Omega \overset{①}{\Rightarrow} C_1 = \frac{\sqrt{0.4142}}{(2\pi)(30\times10^6)\cdot R_1} = 3.4pF.$

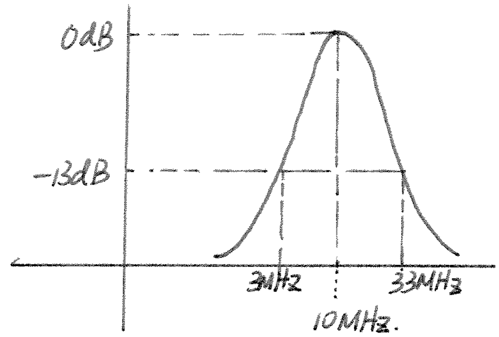
$$\Rightarrow C_2 = 2C_1 = 6.8pF.$$

$$\Rightarrow R_2 = R_1\frac{C_1}{C_2} = 500\Omega.$$

And as before, $R_3 = R_4 = 1K\Omega$.

55. 10MHz, Gain = 1 (peak), $R_6 = R_3$, −13 dB @ 3MHz, 33MHz.

$$\frac{V_x}{V_{in}} = \frac{\alpha S^2}{S^2 + \frac{W_n}{Q}S + W_n^2} \cdot \frac{-1}{R_1C_1S}$$



$$1 = \left(\frac{\alpha}{R_1C_1}\right)\cdot\frac{Q}{W_n}, \quad \alpha = \frac{R_5}{R_4+R_5}\left(1+\frac{R_6}{R_3}\right), \quad \frac{W_n}{Q} = \frac{R_4}{R_4+R_5}\cdot\frac{1}{R_1C_1}.$$

since $R_6 = R_3$, $\alpha = 2\frac{R_5}{R_4+R_5} \Rightarrow \frac{\alpha}{2} = \frac{R_5}{R_4+R_5}$

$$\Rightarrow \frac{R_4}{R_4+R_5} = 1 - \frac{\alpha}{2} \Rightarrow \frac{Q}{W_n} = \frac{R_1C_1}{1-\frac{\alpha}{2}}.$$

$$\Rightarrow \left(\frac{\alpha}{R_1C_1}\right)\cdot\left(\frac{R_1C_1}{1-\frac{\alpha}{2}}\right) = 1 \Rightarrow \alpha = 1 - \frac{\alpha}{2}.$$

$$\Rightarrow \alpha = \frac{2}{3}, \quad \frac{R_5}{R_4+R_5} = \frac{1}{3}$$

$$\Rightarrow R_5 = \frac{1}{2}R_4.$$

$$\left.\begin{array}{l} \frac{W_n}{Q} = \left(\frac{2}{3}\right)\left(\frac{1}{R_1C_1}\right) \\ W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}} \end{array}\right\} \Rightarrow \frac{3}{2}\frac{R_1C_1}{\sqrt{R_1R_2C_1C_2}} = Q$$

$$\Rightarrow \frac{3}{2}\sqrt{\frac{R_1C_1}{R_2C_2}} = Q$$

Let $R_1C_1 = R_2C_2 \Rightarrow Q = \frac{3}{2}, \quad W_n = \frac{1}{R_1C_1}.$

$$H(jw) = \frac{\frac{2}{3}w^2}{\frac{w}{w_n}\sqrt{(w_n^2-w^2)^2+(\frac{2}{3}w_n w)^2}} = \frac{\frac{2}{3}w^2}{\frac{w}{w_n}\sqrt{w_n^4-\frac{14}{9}(w_n w)^2+w^4}}$$

$$H(jw) = 1$$

$$\Rightarrow \frac{4}{9}w^2 = w_n^2 - \frac{14}{9}w^2 + \frac{w^4}{w_n^2}$$

$$\Rightarrow w_n^4 - 2w^2 w_n^2 + w^4 = 0$$

$$\Rightarrow w_n^2 = w^2 = [(2\pi)(10MHz)]^2$$

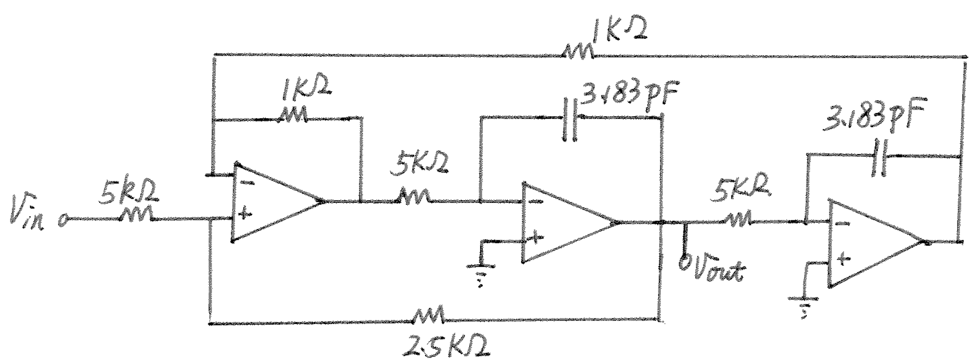
As derived, $w_n = \frac{1}{R_1 C_1}$

Let $R_1 = 5K\Omega \Rightarrow C_1 = 3.183pF$.

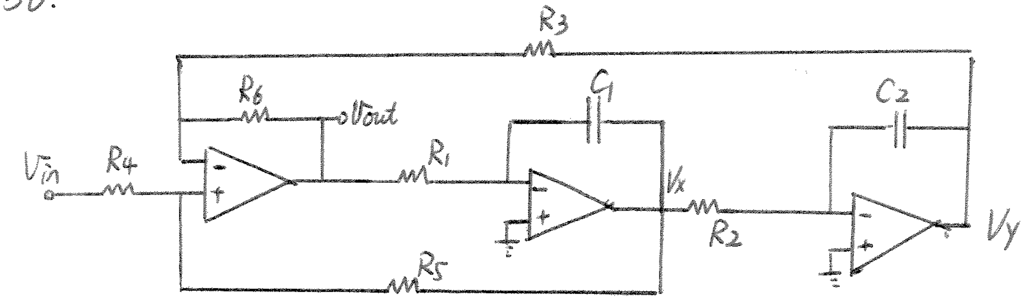
Let $R_2 = R_1 = 5k\Omega \Rightarrow C_2 = C_1 = 3.183pF$

Let $R_4 = 5K\Omega \Rightarrow R_5 = \frac{1}{2}R_4 = 2.5k\Omega$.

Let $R_3 = R_6 = 1K\Omega$.



56.



Low pass, $\frac{V_y}{V_{in}} = \frac{\alpha S^2}{S^2 + \frac{W_n}{Q}S + W_n^2} \cdot \frac{1}{R_1R_2C_1C_2S^2}$

$$= \frac{\alpha}{(S^2 + \frac{W_n}{Q}S + W_n^2)(R_1R_2C_1C_2)}$$

$$H(S) = \frac{\alpha/(R_1R_2C_1C_2)}{S^2 + \frac{W_n}{Q}S + W_n^2}$$

$$H(jW) = \frac{\alpha/(R_1R_2C_1C_2)}{(W_n^2 - W^2) + j\frac{W_n}{Q}W}$$

$$|H(jW)| = \frac{\alpha/(R_1R_2C_1C_2)}{\sqrt{(W_n^2 - W^2)^2 + (\frac{W_nW}{Q})^2}}$$

$$|H(jW_{3dB})| = \frac{\alpha/(R_1R_2C_1C_2)}{\sqrt{(W_n^2 - W_{3dB}^2)^2 + (\frac{W_nW_{3dB}}{Q})^2}} = \frac{\alpha}{W_n^2(R_1R_2C_1C_2)\sqrt{2}}$$

$$\Rightarrow \frac{\sqrt{(W_n^2 - W_{3dB}^2)^2 + (\frac{W_nW_{3dB}}{Q})^2}}{W_n^2} = \sqrt{2}$$

$$\Rightarrow 1 - 2\left(\frac{W_{3dB}}{W_n}\right)^2 + \left(\frac{W_{3dB}}{W_n}\right)^4 + \frac{1}{Q^2}\left(\frac{W_{3dB}}{W_n}\right)^2 = 2$$

$$\Rightarrow \left(\frac{W_{3dB}}{W_n}\right)^4 + \left(\frac{1}{Q^2} - 2\right)\left(\frac{W_{3dB}}{W_n}\right)^2 - 1 = 0$$

$$Q = 1.5 \Rightarrow \left(\frac{W_{3dB}}{W_n}\right)^4 - 1.556\left(\frac{W_{3dB}}{W_n}\right)^2 - 1 = 0$$

$$\Rightarrow \left(\frac{W_{3dB}}{W_n}\right)^2 = 2.0446, \quad -0.4891 \text{ (impossible)}$$

$$\Rightarrow W_{3dB} = 1.43\, W_n$$

$\Rightarrow$ Low pass corner = 14.3 MHz.

High pass:

$$\frac{V_{out}}{V_{in}}(S) = \frac{\alpha S^2}{S^2 + \frac{W_n}{Q} S + W_n^2}$$

$$|H(jw)| = \frac{\alpha w^2}{\sqrt{(W_n^2 - W^2)^2 + \left(\frac{W W_n}{Q}\right)^2}}$$

$$|H(jW_{3dB})| = \frac{\alpha W_{3dB}^2}{\sqrt{(W_n^2 - W_{3dB}^2)^2 + \left(\frac{W_{3dB} W_n}{Q}\right)^2}} = \frac{\alpha}{\sqrt{2}}$$

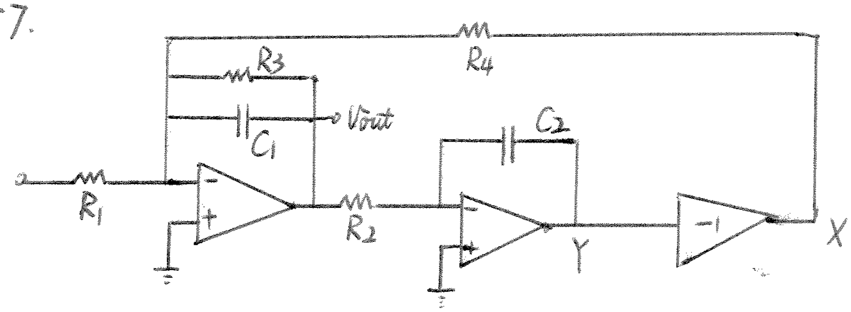
$$\Rightarrow \left(\frac{W_n}{W_{3dB}}\right)^4 + \left(\frac{1}{Q^2} - 2\right)\left(\frac{W_n}{W_{3dB}}\right)^2 - 1 = 0$$

Since $Q = 1.5$

$$\Rightarrow \left(\frac{W_n}{W_{3dB}}\right)^2 = 2.0446 \Rightarrow W_{3dB} = \frac{W_n}{1.43}$$

$\Rightarrow W_{3dB} = 7\text{MHz}$. (high pass corner)

57.



$W_n = 10\,MHz$,

$-13dB = 3MHz$, $33MHz$.

$$\frac{V_{out}}{V_{in}} = -\frac{R_2 R_3 R_4}{R_1}\left(\frac{C_2 S}{R_2 R_3 R_4 C_1 C_2 S^2 + R_2 R_4 C_2 S + R_3}\right)$$

Same as in #55, $Q = \frac{10}{6.684} = 1.5$.

$$\frac{V_{out}}{V_{in}} = -\frac{\frac{1}{R_1 C_1} S}{S^2 + \frac{1}{R_3 C_1} S + \frac{1}{R_2 R_4 C_1 C_2}}$$

$$\frac{W_n}{Q} = \frac{1}{R_3 C_1}, \quad W_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}.$$

$$Q = \frac{R_3 C_1}{\sqrt{R_2 R_4 C_1 C_2}} = R_3 \sqrt{\frac{C_1}{R_2 R_4 C_2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{-\beta S}{S^2 + \frac{W_n}{Q} S + W_n^2};$$

At $W = W_n \Rightarrow |H(jW_n)| = 1 = \frac{\beta Q}{W_n}$.

$$\frac{\beta Q}{W_n} = \left(\frac{1}{R_1 C_1}\right)(R_3 C_1) = \frac{R_3}{R_1} = 1$$

$$Q = R_3\sqrt{\frac{C_1}{R_2 R_4 C_2}} = 1.5, \qquad W_n = \frac{1}{\sqrt{R_2 R_4 C_2 C_1}} = (10\times 10^6)(2\pi)$$

Let $R_2 = R_4 = 1K\Omega$.

$$\frac{1}{\sqrt{10^6 \times C_1 C_2}} = (10\times 10^6)(2\pi). \Rightarrow C_1 C_2 = 2.533\times 10^{-22}$$

Let $C_1 = C_2 = 15.9pF$

$$R_3\sqrt{\frac{1}{1000\times 1000}} = 1.5$$

$\Rightarrow R_3 = 1.5\,k\Omega = R_1$

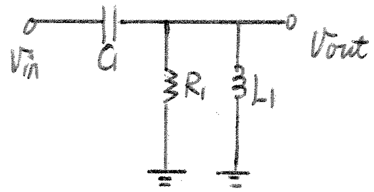
So : $R_1 = 1.5\,K\Omega$, $R_2 = 1K\Omega$, $R_3 = 1.5K\Omega$, $R_4 = 1K\Omega$.

$C_1 = C_2 = 15.9pF$.

58. Peaking : 1dB @ 7MHz.

Corner : 3.69 MHz

−13.6dB @ 2MHz.



$$\frac{V_{out}}{V_{in}} = \frac{s^2}{s^2 + \frac{1}{R_1C_1}s + \frac{1}{L_1C_1}}$$

Peaking 1dB ⇒ $Q = 0.926$.

$$\frac{\omega_n}{\sqrt{1-1/(2Q^2)}} = (2\pi)(7MHz) \Rightarrow \omega_n = (2\pi)(4.52MHz)$$

$$\frac{\omega_n}{Q} = \frac{(2\pi)(4.52MHz)}{0.926} = (2\pi)(4.88MHz) = \frac{1}{R_1C_1}$$

$$\omega_n^2 = [(2\pi)(4.52MHz)]^2 = \frac{1}{L_1C_1}$$

Let $C_1 = 100pF$, $L_1 = 12.4\mu H$, $R_1 = 326.1\Omega$.

With simulated inductor

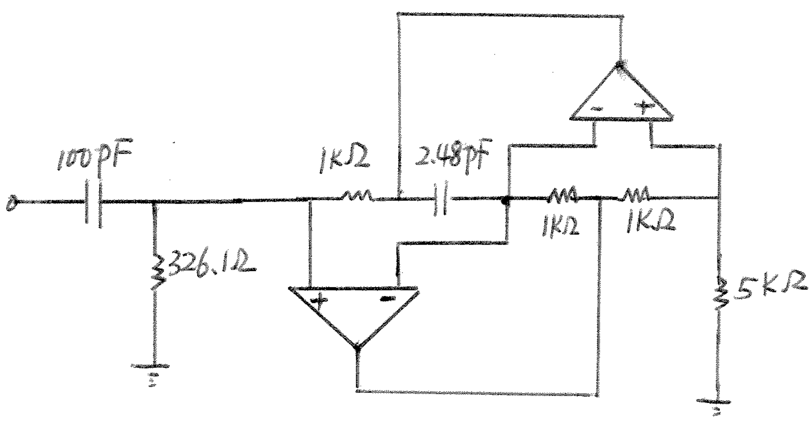
$$Z_{in} = \left(\frac{Z_1Z_3}{Z_2Z_4}\right)Z_5 = R_Y R_X C_s$$

Let $Z_1 = Z_3 = Z_4 = R_Y$, $Z_5 = R_X$, $Z_2 = C_s^{-1}$

Let $R_Y = 1K\Omega$, $R_X = 5K\Omega$.

$12.4 \times 10^{-6} = (1000)(5000)\,C$.

⇒ $C = 2.48pF$ to simulate an L of $12.4\mu H$.


100pF
326.1Ω
1KΩ
2.48pF
1KΩ
1KΩ
5KΩ

59. Corner @ 16.38 MHz., Peaking 0.5 dB @ 8 MHz.

5.9 dB ≈ 6 dB attenuation @ 20 MHz.

$$\frac{V_{out}}{V_{in}} = \frac{1}{R_1 R_x C^2 S^2 + R_1 C S + 1} = \frac{1/(R_1 R_x C^2)}{S^2 + \frac{S}{R_x C} + \frac{1}{R_1 R_x C^2}}$$

0.5 dB $\Leftrightarrow$ 1.05292.

$$\frac{Q}{\sqrt{1-\frac{1}{4Q^2}}} = 1.05292 \Rightarrow Q = 0.8636$$

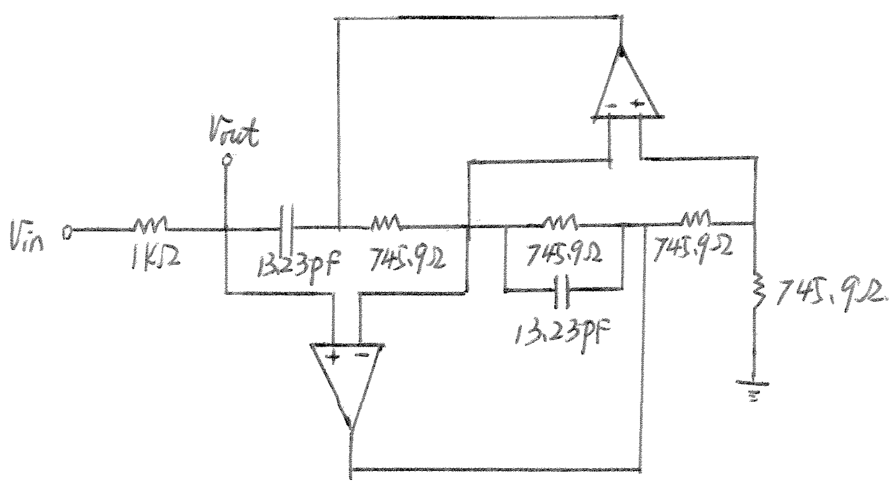
$$W_n \sqrt{1-\frac{1}{2Q^2}} = (2\pi)(8\times10^6)$$

$$\Rightarrow W_n = (2\pi)(13.934\times10^6)$$

$$\frac{1}{R_1 R_x C^2} = W_n^2, \quad \frac{1}{R_x C} = \frac{W_n}{Q} = (2\pi)(16.134\times10^6)$$

$$\Rightarrow \frac{1}{R_1 C} = 7.56\times10^7$$

Let $R_1 = 1k\Omega$, $C = 13.23pF$, $R_x = 745.9\Omega$.



60. Butterworth

a) Passband 0.5dB @ 1MHz , $-0.5dB \Leftrightarrow 0.944$

Attenuation 12dB @ 2.5MHz. , $-12dB \Leftrightarrow 0.2512$.

$$|H(j\omega)|^2_{1MHz} = \frac{1}{1+\left[\frac{(2\pi)(10^6)}{W_0}\right]^{2n}} = 0.944^2 \quad ①$$

$$|H(j\omega)|^2_{2.5MHz} = \frac{1}{1+\left(\frac{2\pi\times 2.5\times 10^6}{W_0}\right)^{2n}} = 0.2512^2 \quad ②$$

$$① \Rightarrow 1 = (0.944)^2\left[\left(\frac{2\pi\times 10^6}{W_0}\right)^{2n} + 1\right]$$

$$\Rightarrow W_0^{2n} = 8.186 \times (2\pi\times 10^6)^{2n} \quad ③$$

$$② \Rightarrow 1 = (0.2512)^2\left[\frac{(2\pi\times 2.5\times 10^6)^{2n}}{8.186\times(2\pi\times 10^6)^{2n}} + 1\right]$$

$$\Rightarrow n = 2.62$$

So choose $n = 3$. $\overset{③}{\Rightarrow}$ $W_0 = 2\pi \times 1.42$MHz.

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{W}{2\pi\times 1.42\times 10^6}\right)^6}}$$

b). Passband: 0.1 dB @ 1 MHz

$$\frac{1}{1+\left(\frac{2\pi\times10^6}{W_0}\right)^{2n}} = (0.98855)^2 \qquad ①$$

Stopband attenuation: 12 dB @ 2.5 MHz

$$\frac{1}{1+\left(\frac{2\pi\times2.5\times10^6}{W_0}\right)^{2n}} = (0.2512)^2 \qquad ②$$

$$① \Rightarrow W_0^{2n} = 42.931 \times (2\pi\times10^6)^{2n}$$

$$② \Rightarrow n = 3.52$$

$$\text{Choose } n=4 \overset{③}{\Longrightarrow} W_0 = 2\pi \times 1.6\,\text{MHz}.$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{2\pi\times1.6\times10^6}\right)^8}}$$

c). Passband 1 dB @ 1 MHz $\Rightarrow \frac{1}{1+\left(\frac{2\pi\times10^6}{W_0}\right)^{2n}} = (0.90)^2 \qquad ④$

Attenuation 18 dB @ 2.5 MHz $\Rightarrow \frac{1}{1+\left(\frac{2\pi\times2.5\times10^6}{W_0}\right)^{2n}} = (0.1259)^2 \qquad ⑤$

$$④ \Rightarrow W_0^{2n} = 4.263 \times (2\pi\times10^6)^{2n} \qquad ⑥$$

$$⑤ \Rightarrow n = 3.0$$

$$\text{Choose } n=3 \overset{⑥}{\Longrightarrow} W_0 = 2\pi \times 1.27\,\text{MHz}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{2\pi\times1.27\times10^6}\right)^6}}$$

d) Passband: 0.5dB @ 1MHz $\Rightarrow \frac{1}{1+\left(\frac{2\pi \times 10^6}{W_0}\right)^{2n}} = 0.944^2$ ①

Attenuation: 18dB @ 2.5MHz $\Rightarrow \frac{1}{1+\left(\frac{2\pi \times 2.5 \times 10^6}{W_0}\right)^{2n}} = 0.1259^2$ ②

① $\Rightarrow W_0^{2n} = 8.186 \times (2\pi \times 10^6)^{2n}$ ③

② $\Rightarrow n = 3.4$

Choose $n = 4 \overset{③}{\Longrightarrow} W_0 = 2\pi \times 1.3\,MHz$

$$|H(jw)| = \frac{1}{\sqrt{1+\left(\frac{w}{2\pi \times 1.3 \times 10^6}\right)^8}}$$

Chebyshev

a) Passband 0.5dB @ 1MHz $\Rightarrow 0.5 = 20 \log(\sqrt{1+\epsilon^2})$

$\Rightarrow \epsilon = 0.3493$, $W_0 = 1MHz$.

Attenuation 12dB @ 2.5MHz

$\Rightarrow \frac{1}{\sqrt{1+\epsilon^2 \cosh^2[n \cosh^{-1}(\frac{w}{W_0})]}} = 0.2512$, when $w = 2.5 \times 10^6 \times 2\pi$.

Since w, w_0, ϵ, known

$\Rightarrow n = 1.9733$

Choose $n=2$, $|H(jw)| = \frac{1}{\sqrt{1+0.3493^2 C_2^2(\frac{w}{W_0})}}$, $W_0 = 2\pi \times 1MHz$

b). Passband 0.1 dB @ 1MHz , W_0 = 1MHz

$\Rightarrow 0.1 = 20\log(\sqrt{1+\epsilon^2}) \Rightarrow \epsilon = 0.1526.$

Attenuation 12dB @ 2.5MHz

$$\Rightarrow \frac{1}{1+0.1526^2\cosh^2[n\cosh^{-1}(2.5)]} = 0.2512^2$$

$\Rightarrow n = 2.5$

Choose n=3, $|H(jw)| = \dfrac{1}{\sqrt{1+0.1526^2 C_3^2(\frac{W}{W_0})}}$, $W_0=(2\pi)(1MHz)$

c). Passband 1dB@1MHz, W_0 = 1MHz

$\Rightarrow 1 = 20\log\sqrt{1+\epsilon^2} \Rightarrow \epsilon = 0.5089.$

Attenuation 18dB @ 2.5MHz

$$\Rightarrow \frac{1}{1+0.5089^2\cosh^2[n\cosh^{-1}(2.5)]} = 0.1259^2 \Rightarrow n = 2.19$$

Choose n = 3, $|H(jw)| = \dfrac{1}{\sqrt{1+0.5089^2 C_3^2(\frac{W}{W_0})}}$, $W_0=(2\pi)(1MHz)$

d). Passband 0.5dB@1MHz $\Rightarrow \epsilon = 0.3493$

Attenuation 18dB @ 2.5MHz

$$\Rightarrow \frac{1}{1+0.3493^2\cosh^2[n\cosh^{-1}(2.5)]} = 0.1259^2 \Rightarrow n = 2.43$$

Choose n=3, $|H(jw)| = \dfrac{1}{\sqrt{1+0.3493^2 C_3^2(\frac{W}{W_0})}}$, $W_0=(2\pi)(1MHz)$

61. a) Butterworth in Sallen and Key

$n=3,\ W_0 = (2\pi)(1.42MHz)$

$P_K = W_0 \cdot \exp(\frac{j\pi}{2})\ \exp(j\frac{2K-1}{2n}\pi),\ K=1,2,3.$

$P_1 = W_0 \exp(j\frac{2\pi}{3}) = (2\pi)(1.42MHz)\times(\cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3})$

$P_2 = W_0 \exp(j\pi) = -(2\pi)(1.42MHz)$

$P_3 = W_0 \exp(j\frac{4\pi}{3}) = (2\pi)(1.42MHz)\times(\cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3})$

$$H_{P_{1,3}}(S) = \frac{(-P_1)(-P_3)}{(S-P_1)(S-P_3)}$$

$$= \frac{[2\pi\times(1.42MHz)]^2}{S^2-[4\pi\times(1.42MHz)\cos\frac{2\pi}{3}]S + [2\pi\times(1.42MHz)]^2}$$

$W_n = 2\pi\times 1.42MHz\ (= \frac{1}{\sqrt{R_1R_2C_1C_2}})$

$\frac{W_n}{Q} = 2\pi\times 1.42MHz\cdot\cos(\frac{2\pi}{3}) \Rightarrow Q = \frac{-1}{2\cos(\frac{2\pi}{3})}\left(= \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}\right)$

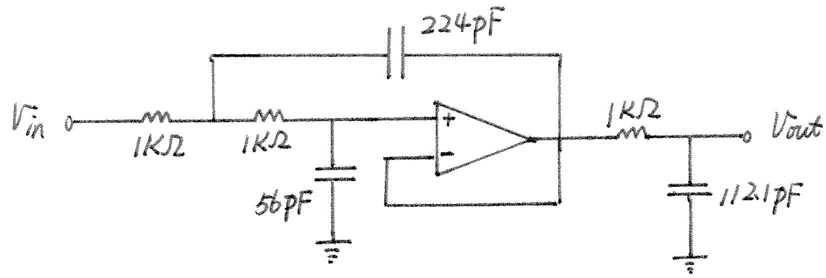
Let $C_1 = 4C_2$, $R_1 = R_2$, so that it satisfies $Q = \frac{-1}{2\cos(\frac{2\pi}{3})} = 1$

Also $\frac{1}{\sqrt{R_1R_2C_1C_2}} = 2\pi\times 1.42MHz$

Let $R_1 = R_2 = 1K\Omega \Rightarrow C_1 = 224pF,\ C_2 = 56pF$

$P_2 = -W_0, \quad \frac{1}{R_3C_3} = (2\pi)(1.42\,MHz)$

Let $R_3 = 1K\Omega \Rightarrow C_3 = 112.1\,pF$.



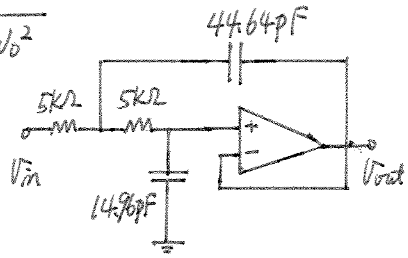
Chebyshev in Sallen and Key

$P_k = -W_0 \sin\frac{(2K-1)\pi}{2n}\sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right) + jW_0\cos\frac{(2K-1)\pi}{2n}\cosh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right)$

$n=2,\ W_0 = (2\pi)(1\,MHz),\ \epsilon = 0.3493,\ K=1,2.$

$P_{1,2} = -0.7128W_0 \pm j1.0041W_0$

$H_{SK}(s) = \frac{(-P_1)(-P_2)}{(S-P_1)(S-P_2)} = \frac{(1.2314)^2W_0^2}{S^2+1.4256W_0S+(1.2314)^2W_0^2}$



$W_n = 1.2314W_0 = (2\pi)(1.2314\,MHz)$

$\frac{W_n}{Q} = 1.4256W_0 \Rightarrow Q = \frac{1.2314}{1.4256} = 0.8638.$

$W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}$

Let $R_1 = R_2 \quad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 2.9844$

$\frac{1}{\sqrt{R_1R_2C_1C_2}} = W_n = 2\pi(1.2314\,MHz), \Rightarrow \frac{1}{R_1C_2\cdot\sqrt{2.9844}} = 2\pi(1.2314\,MHz)$

Let $R_1 = R_2 = 5K\Omega \Rightarrow C_2 = 14.96\,pF,\ C_1 = 44.64\,pF$

b). Butterworth with SK

$n = 4, \quad W_0 = (2\pi)(1.6\,MHz)$

$P_K = W_0 \exp(j\frac{\pi}{2}) \exp(j\frac{2K-1}{2n}\pi), \quad K = 1, 2, 3, 4.$

$P_1 = W_0 \exp(j\frac{5\pi}{8}), \; P_2 = W_0 \exp(j\frac{7\pi}{8}), \; P_3 = W_0 \exp(-j\frac{5\pi}{8}), \; P_4 = W_0 \exp(-j\frac{7\pi}{8})$

$$H_{SK1,4}(S) = \frac{(-P_1)(-P_4)}{(S-P_1)(S-P_4)} = \frac{[(2\pi)(1.6\times10^6)]^2}{S^2 - [4\pi\times(1.6\times10^6)\cos(\frac{5\pi}{8})]S + [2\pi\times(1.6\times10^6)]^2}$$

$W_n = 2\pi \times 1.6\times10^6$

$\frac{W_n}{Q} = (4\pi)(1.6\times10^6)\cos(\frac{5\pi}{8}) \Rightarrow Q = 1.31.$

$W_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}.$

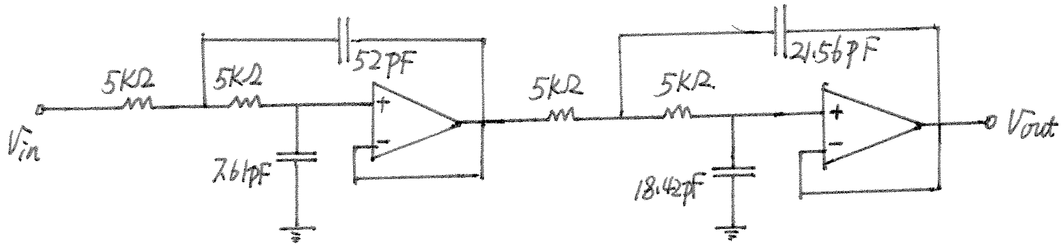
Let $R_1 = R_2$, $Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 6.83.$

$W_n = \frac{1}{\sqrt{6.83}\,R_1C_2} = 2\pi\times1.6\times10^6$

Let $R_1 = R_2 = 5K\Omega \Rightarrow C_2 = 7.61\,pF, \; C_1 = 52\,pF.$

Similarly, $H_{SK2,3}(S) = \frac{(-P_2)(-P_3)}{(S-P_2)(S-P_3)}$, it can be derived for $H_{SK2,3}$.

$R_1 = R_2 = 5K\Omega, \; C_2 = 18.42\,pF, \; C_1 = 21.55\,pF.$



(b) Chebyshev in SK.

$n = 3, \quad \omega_0 = (2\pi)(1\times10^6), \quad \epsilon = 0.1526$

$P_1 = -\omega_0(0.9694)\sin\left(\frac{1}{6}\pi\right) + j\omega_0(1.3927)\cos\left(\frac{\pi}{6}\right) = -0.4847\omega_0 + j1.2061\omega_0$

$P_2 = -\omega_0(0.9694)\sin\left(\frac{3}{6}\pi\right) + j\omega_0(1.3927)\cos\left(\frac{3\pi}{6}\right) = -0.9496\omega_0$

$P_3 = -\omega_0(0.9694)\sin\left(\frac{5}{6}\pi\right) + j\omega_0(1.3927)\sin\left(\frac{5\pi}{6}\right) = -0.4847\omega_0 - j1.2061\omega_0$

$$H_{SK}(s) = \frac{(-P_1)(-P_3)}{(s-P_1)(s-P_3)} = \frac{1.3^2\omega_0^2}{s^2 + 0.9694\omega_0 s + (1.3)^2\omega_0^2}$$

$\omega_n = 1.3\omega_0$

$\frac{\omega_n}{Q} = 0.9694\omega_0 \Rightarrow Q = 1.3410$

$\omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\,C_1/C_2}$

Let $R_1 = R_2 \Rightarrow Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = (2Q)^2 = 7.1931$

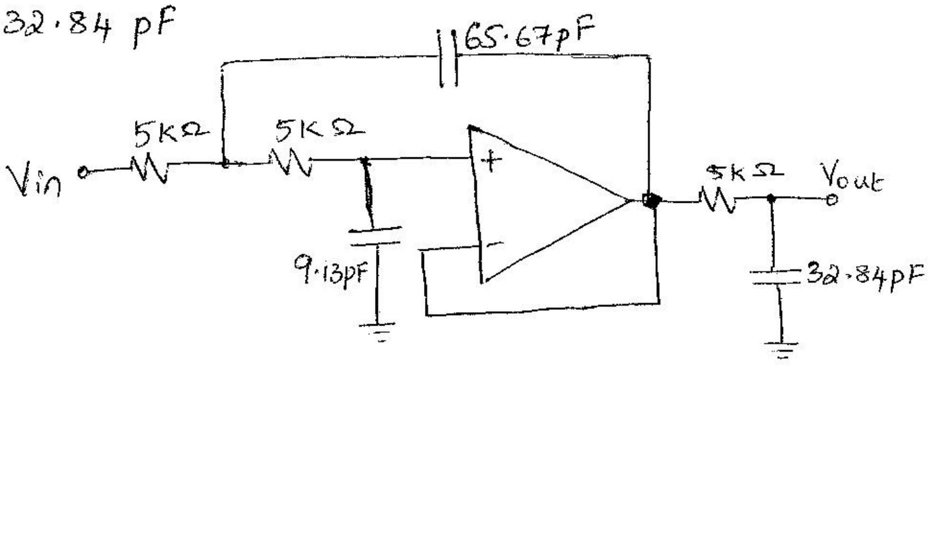
$\omega_n = \frac{1}{\sqrt{7.1931}\,R_1C_2} = (1.3)(2\pi)(1\times10^6)$

Let $R_1 = R_2 = 5\,k\Omega \Rightarrow C_1 = 9.13\,pF \Rightarrow C_1 = 65.67\,pF$

$P_2 = (2\pi)(0.9694\times10^6)$, and

$\frac{1}{R_3C_3} = P_2$

Let $R_3 = 5\,k\Omega \Rightarrow C_3 = 32.84\,pF$



c). Butterworth in SK.

$n=3$, $w_0=(2\pi)(1.27\times10^6)$

$P_K = w_0 \exp(j\frac{\pi}{2}) \exp(j\frac{2K-1}{2n}\pi)$, $K=1,2,3.$

$$H_{P_{1,3}}(s) = \frac{(-P_1)(-P_3)}{(s-P_1)(s-P_3)} = \frac{[(2\pi)(1.27\times10^6)]^2}{s^2-[4\pi\times(1.27\times10^6)\cos(\frac{2\pi}{3})]s+[2\pi(1.27\times10^6)]^2}$$

$w_n=(2\pi)(1.27\times10^6)$

$\frac{w_n}{Q}=(4\pi)(1.27\times10^6)\cos(\frac{2\pi}{3}) \Rightarrow Q=1.$

$w_n=\frac{1}{\sqrt{R_1R_2C_1C_2}}$, $Q=\frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}$.

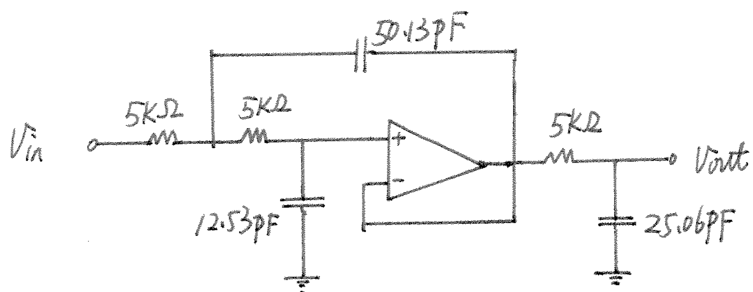
Let $C_1=4C_2$, $R_1=R_2$

$w_n=\frac{1}{2R_1C_2} = (2\pi)(1.27\times10^6)$

Let $R_1=R_2=5K\Omega \Rightarrow C_2=12.53pF$, $C_1=50.13pF$.

$P_2=-w_0=(2\pi)(1.27\times10^6)=\frac{1}{R_3C_3}$

Let $R_3=5K\Omega \Rightarrow C_3=25.06pF$



C). Chebyshev in SK.

$n=3, \ \epsilon \doteq 0.5089, \ \omega_0=(2\pi)(10^6).$

$P_1 = -0.2470\omega_0 + j0.9660\omega_0$

$P_2 = -0.4941\omega_0$

$P_3 = -0.2470\omega_0 - j0.9660\omega_0$

$$H_{P_{1,3}}(S) = \frac{(-P_1)(-P_3)}{(S-P_1)(S-P_3)} \doteq \frac{[(2\pi)(0.9971\times10^6)]^2}{S^2+(0.4940)(2\pi)(10^6)S+(2\pi\times0.9971\times10^6)^2}$$

$\omega_n = (2\pi)(0.9971\times10^6)$

$\frac{\omega_n}{Q} = (0.4940)(2\pi\times10^6) \Rightarrow Q = 2.02.$

$\omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}.$

Let $R_1=R_2 \Rightarrow Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 16.296.$

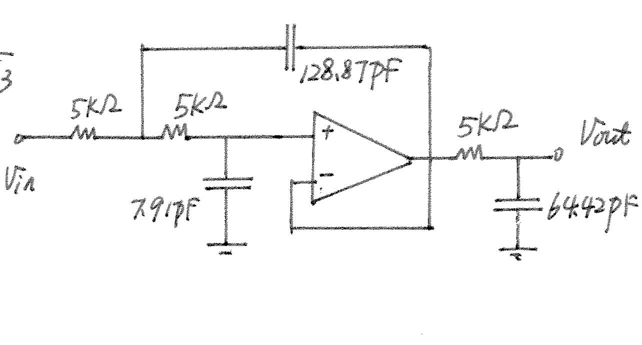
$\omega_n = \frac{1}{\sqrt{16.296}\,R_1C_2} = (2\pi)(0.9971\times10^6)$

Let $R_1=R_2=5K\Omega \Rightarrow C_2 = 7.91pF \Rightarrow C_1 = 128.87pF$

$P_2 = 2\pi\times0.4941\times10^6 = \frac{1}{R_3C_3}$

Let $R_3 = 5K\Omega$

$\Rightarrow C_3 = 64.42pF$



d). Butterworth in SK.

$n=4, \quad w_0=(2\pi)(1.3\times10^6)$

$$P_k = w_0 \exp(j\tfrac{\pi}{2})\exp(j\tfrac{2k-1}{2n}\pi), \quad k=1,2,3,4.$$

$$H_{SK_{1,4}}(s) = \frac{(-P_1)(-P_4)}{(s-P_1)(s-P_4)} = \frac{w_0^2}{s^2-[2w_0\cos(\frac{5\pi}{8})]s+w_0^2}$$

$$w_n = w_0 = (2\pi)(1.3\times10^6)$$

$$\frac{w_n}{Q} = 4\pi(1.3\times10^6)\cos(\tfrac{5\pi}{8}) \Rightarrow Q = 1.31$$

$$w_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \qquad Q = \frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}.$$

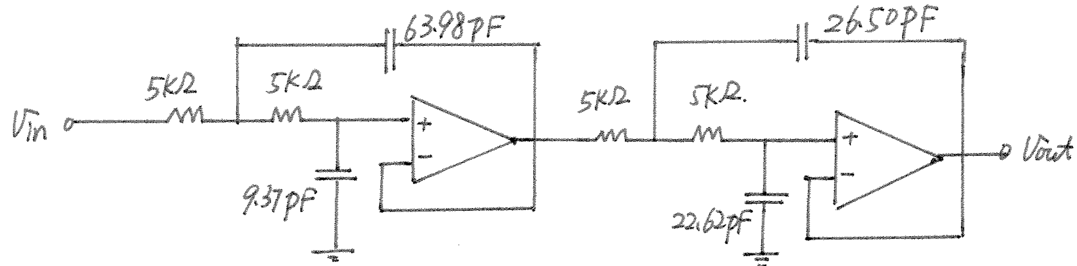
Let $R_1=R_2 \Rightarrow Q=\frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 6.828$

$$w_n = \frac{1}{\sqrt{6.828}\,R_1C_2} = (2\pi)(1.3\times10^6)$$

Let $R_1=R_2=5K\Omega \Rightarrow C_2 = 9.37pF \Rightarrow C_1 = 63.98\,pF$

Similarly, $H_{SK_{2,3}}(s) = \frac{(-P_2)(-P_3)}{(s-P_2)(s-P_3)}$. It can be derived that

$R_1=R_2=5K\Omega$, $C_2=22.62pF$, $C_1=26.50pF$.



d). Chebyshev in SK.

$n=3, \quad \epsilon=0.3493, \quad \omega_0=(2\pi)(1\times10^6)$

$p_1 = -\omega_0 0.6265 \sin(\frac{1}{6}\pi) + j\omega_0(1.11800)\cos(\frac{1}{6}\pi)$

$p_2 = -\omega_0 0.6265$

$p_3 = -\omega_0 0.6265 \sin(\frac{5}{6}\pi) + j\omega_0(1.1800)\cos(\frac{5}{6}\pi)$

$$H_{p_{1,3}} = \frac{(-p_1)(-p_3)}{(s-p_1)(s-p_3)} = \frac{[2\pi\times1.069\times10^6]^2}{s^2+(0.6265)(2\pi\times10^6)s+(2\pi\times1.069\times10^6)^2}$$

$\omega_n=(2\pi)(1.069\times10^6)$

$\frac{\omega_n}{Q}=(0.6265)(2\pi\times10^6) \Rightarrow Q=1.7063$

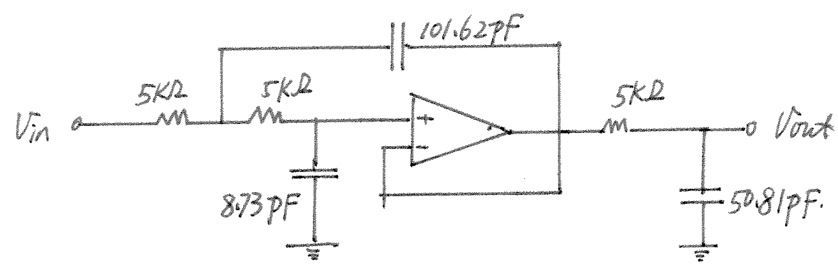
Let $R_1=R_2 \Rightarrow Q=\frac{1}{R_1+R_2}\sqrt{R_1R_2\frac{C_1}{C_2}}=\frac{1}{2}\sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2}=4Q^2=11.6459.$

$\omega_n=\frac{1}{\sqrt{R_1R_2C_1C_2}}=\frac{1}{\sqrt{11.6459}\,R_1C_2}=(2\pi)(1.069\times10^6)$

Let $R_1=R_2=5k\Omega \Rightarrow C_2=8.73pF \Rightarrow C_1=101.62pF.$

$-p_2=(0.6265)(2\pi\times10^6)=\frac{1}{R_3C_3}$

Let $R_3=5k\Omega \Rightarrow C_3=50.81pF.$



62) a). Butterworth TT

$n=3$, $W_0 = (2\pi)(1.42\times10^6)$

$P_1 = W_0 \exp(j\frac{2\pi}{3})$, $P_2 = -W_0$, $P_3 = W_0 \exp(-j\frac{2\pi}{3})$.

$$H_{P_{1,3}} = \frac{[2\pi\times(1.42\times10^6)]^2}{s^2 - [4\pi\times(1.42\times10^6)\cos(\frac{2\pi}{3})]s + [2\pi\times1.42\times10^6)^2}$$

$W_n = (2\pi)(1.42\times10^6)$.

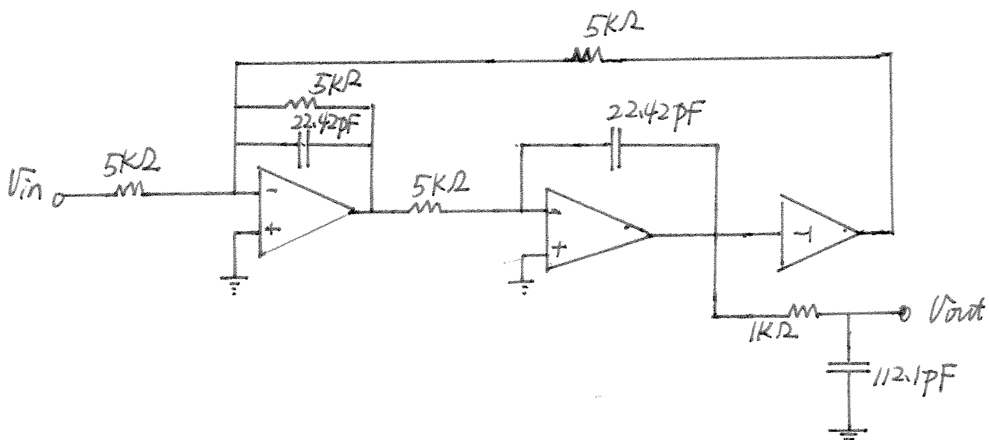
$$Q = \frac{-1}{2\cos(\frac{2\pi}{3})} = 1$$

$W_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}$, $Q = R_3\sqrt{\frac{C_1}{C_2R_2R_4}}$.

Let $R_2 = R_4 = R_3 = 5K\Omega$, $C_1 = C_2 = 22.42pF$

$-P_2 = +(2\pi)(1.42\times10^6) = \frac{1}{R_3C_3}$.

Let $R_3 = 1K\Omega \Rightarrow C_3 = 112.1\,pF$



$R_1 = R_2 = R_4$, to match low frequency gain requirement.

a) Chebyshev II

$n=2,\ w_0=(2\pi)(1MHz),\ \epsilon=0.3493$

$$Hp_{1,2} = \frac{(1.2314)^2 w_0^2}{s^2+1.4256 w_0 s+(1.2314)^2 w_0^2}$$

$$w_n = (2\pi)(1.2314\times10^6)$$

$$Q = \frac{1.2314}{1.4256} = 0.8638$$

$$Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}}, \qquad w_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}$$

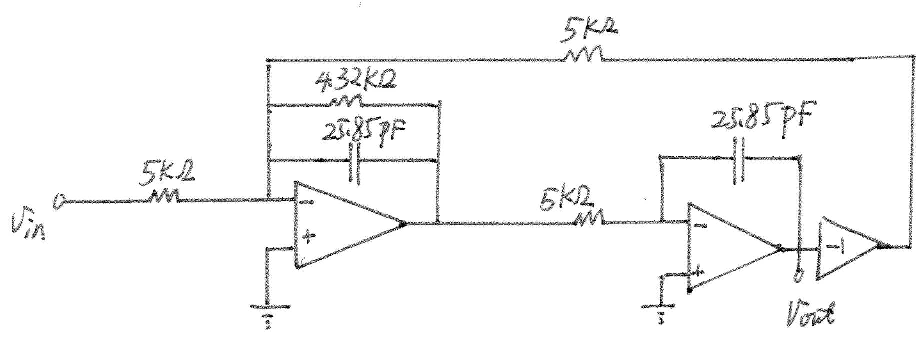
Let $C_1=C_2,\ R_2=R_4$

$$w_n = \frac{1}{R_2C_1} = (2\pi)(1.2314\times10^6)$$

Let $R_2=R_4=5K\Omega,\ C_1=C_2=25.85pF$

$R_1=R_2=R_4=5K\Omega$, to match low frequency gain of unity.

$$Q=\frac{R_3}{R_2} \Rightarrow R_3 = 4.32K\Omega.$$



b). Butterworth with TT

$n=4,\ w_0=(2\pi)(1.6\times10^6)$

$P_1 = w_0 \exp(j\frac{5\pi}{8})$, $P_4 = w_0 \exp(-j\frac{5\pi}{8})$

$P_2 = w_0 \exp(j\frac{7\pi}{8})$, $P_3 = w_0 \exp(-j\frac{7\pi}{8})$

$$H_{P_{1,4}} = \frac{w_0^2}{s^2-[2w_0\cos(\frac{5\pi}{8})]s+w_0^2}$$

$W_n = W_0 = (2\pi)(1.6\times10^6)$

$\frac{W_n}{Q} = (4\pi)(1.6\times10^6)\cos(\frac{5\pi}{8}) \Rightarrow Q = 1.31$

$W_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}$, $Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}}$

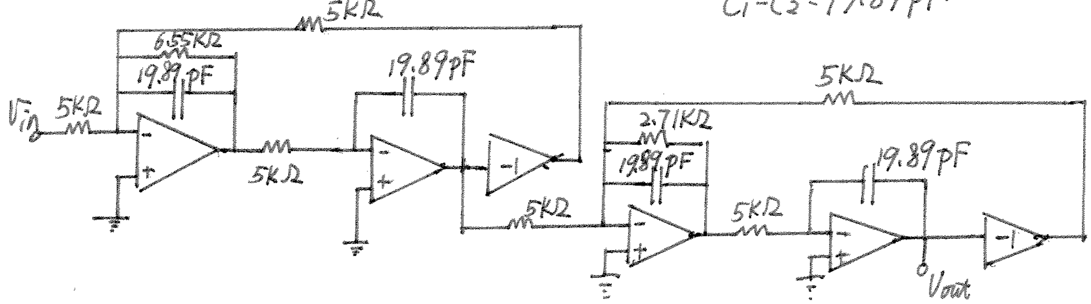
Let $R_2=R_4,\ C_1=C_2 \Rightarrow W_n=\frac{1}{R_2C_1}=(2\pi)(1.6\times10^6)$

Let $R_2=R_4=5K\Omega \Rightarrow C_1=C_2=19.89pF$.

$R_1=R_2=R_4=5K\Omega$, to obtain a low-frequency gain of unity.

$Q=\frac{R_3}{R_2}=1.31 \Rightarrow R_3=1.31R_2=6.55K\Omega$.

Similarly for $H_{P_{2,3}}$, it can be derived that $R_1=R_2=R_4=5K\Omega, R_3=2.7K\Omega$
$C_1=C_2=19.89pF$.



b). Chebyshev with TT

$n=3, \quad \epsilon=0.1526, \quad W_0=(2\pi)(1\times10^6)$

$P_{1,3}=0.4847W_0 \pm j1.2061W_0$

$P_2=-0.9694W_0$

$$H_{P_{1,3}}(S)=\frac{(1.3)^2W_0^2}{S^2+0.9694W_0S+(1.3)^2W_0^2}$$

$W_n=1.3W_0, \quad \frac{W_n}{Q}=0.9694W_0$

$$Q=\frac{1.3}{0.9694}=1.3410$$

$$Q=R_3\sqrt{\frac{C_1}{R_2R_4C_2}}, \quad W_n=\frac{1}{\sqrt{R_2R_4C_1C_2}}$$

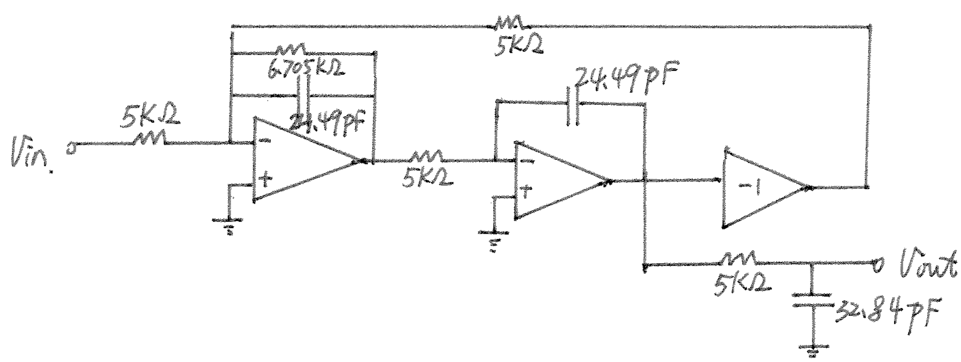
Let $R_2=R_4$, $C_1=C_2$, $\frac{1}{R_2C_1}=(1.3)(2\pi)(10^6)$

Let $R_2=R_4=5K\Omega$, $\Rightarrow C_1=C_2=24.49pF$.

$R_1=R_2=R_4=5K\Omega$, to obtain Low-frequency gain of unity.

$R_3=QR_2=6.705K\Omega$.

$-P_2=(2\pi)(0.9694\times10^6)=\frac{1}{R_5C_5}$, Let $R_5=5K\Omega \Rightarrow C_5=32.84pF$.



c) Butterworth with TT

$n=3,\ w_0 = (2\pi)(1.27\times10^6)$

$P_1 = w_0 \exp(j\frac{2\pi}{3}),\quad P_3 = w_0 \exp(-j\frac{2\pi}{3}),\quad P_2 = -w_0.$

$$H_{P_{1,3}}(s) = \frac{[(2\pi)(1.27\times10^6)]^2}{s^2 - [(4\pi)(1.27\times10^6)\cos(\frac{2\pi}{3})]s + [(2\pi)(1.27\times10^6)]^2}$$

$w_n = (2\pi)(1.27\times10^6),\quad \frac{w_n}{Q} = (4\pi)(1.27\times10^6)\cos(\frac{2\pi}{3})$

$Q = -\frac{1}{2\cos(\frac{2\pi}{3})} = 1.$

$Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}},\quad w_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}$

Let $R_2 = R_4,\ C_1 = C_2,\ \frac{1}{R_2C_1} = (2\pi)(1.27\times10^6)$

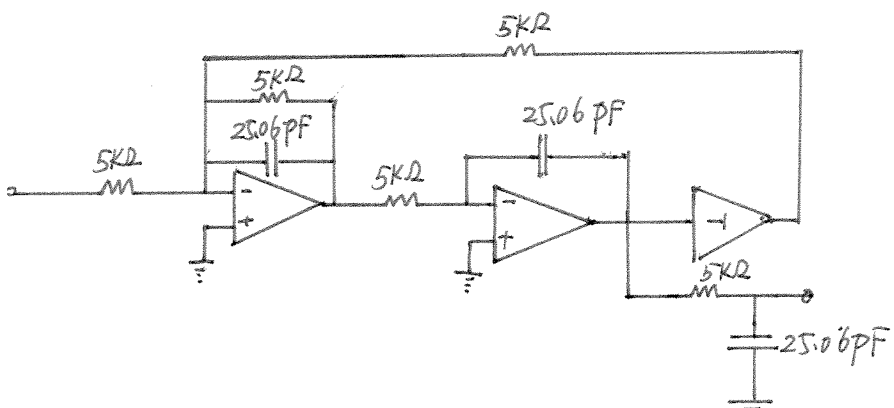
Let $R_2 = R_4 = 5K\Omega \Rightarrow C_1 = C_2 = 25.06\ pF.$

$R_1 = R_2 = R_4 = 5K\Omega$, to obtain Low-fequency gain of unity.

$R_3 = Q R_2 = 5K\Omega.$

$-P_2 = w_0 \Rightarrow \frac{1}{R_5C_5} = (2\pi)(1.27\times10^6)$

Let $R_5 = 5K\Omega \Rightarrow C_5 = 25.06pF$



c) Chebyshev TT

$n=3, \ \epsilon=0.5089, \ w_0=(2\pi)(1\times10^6)$

$P_{1,3}=-0.2470\,w_0 \pm j\,0.9660\,w_0 \ , \ P_2=-0.4941\,w_0$

$$H_{P_{1,3}}(s)=\frac{[(2\pi)(0.9971\times10^6)]^2}{s^2+(0.4940)(2\pi\times10^6)s+[2\pi\times0.9971\times10^6]^2}$$

$W_n=(2\pi)(0.9971\times10^6)$

$\frac{W_n}{Q}=(0.4940)(2\pi\times10^6) \Rightarrow Q=2.02$

$Q=R_3\sqrt{\frac{C_1}{R_2R_4C_2}} \ , \quad W_n=\frac{1}{\sqrt{R_2R_4C_1C_2}}$

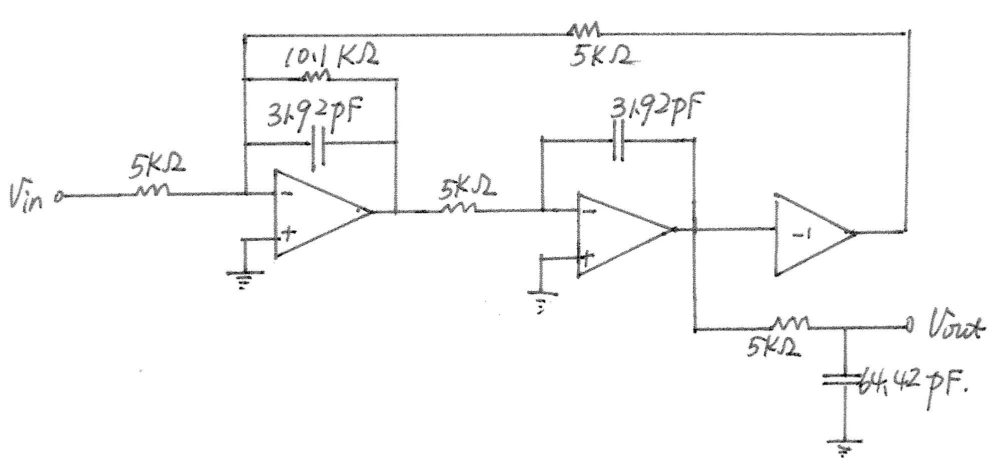
Let $R_2=R_4,\ C_1=C_2 \Rightarrow \frac{1}{R_2C_1}=(2\pi)(0.9971\times10^6)$

Let $R_2=R_4=5K\Omega \Rightarrow C_1=C_2=31.92pF$

$R_1=R_2=R_4=5K\Omega$, to obtain Low-frequency gain of unity.

$R_3=QR_2=10.1\,K\Omega$.

$-P_2=(2\pi)(0.4941\times10^6)=\frac{1}{R_5C_5}$. Let $R_5=5K\Omega \Rightarrow C_5=64.42pF$.



d). Butterworth in TT

$n=4, \quad W_0 = (2\pi)(1.3\times10^6)$

$P_{1,4} = W_0 \exp(\pm j\frac{5\pi}{8}), \quad P_{2,3} = W_0 \exp(\pm j\frac{7\pi}{8})$

$$H_{P_{1,4}}(S) = \frac{W_0^2}{S^2 - [2W_0\cos(\frac{5\pi}{8})]S + W_0^2}$$

$W_n = (2\pi)(1.3\times10^6)$

$\frac{W_n}{Q} = (4\pi)(1.3\times10^6)\cos(\frac{5\pi}{8}) \Rightarrow Q = -\frac{1}{2\cos(\frac{5\pi}{8})} = 1.31.$

$W_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}, \quad Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}}$

Let $R_2 = R_4$, $C_1 = C_2$, $W_n = \frac{1}{\sqrt{R_2^2C_1^2}} = \frac{1}{R_2C_1} = (2\pi)(1.3\times10^6)$

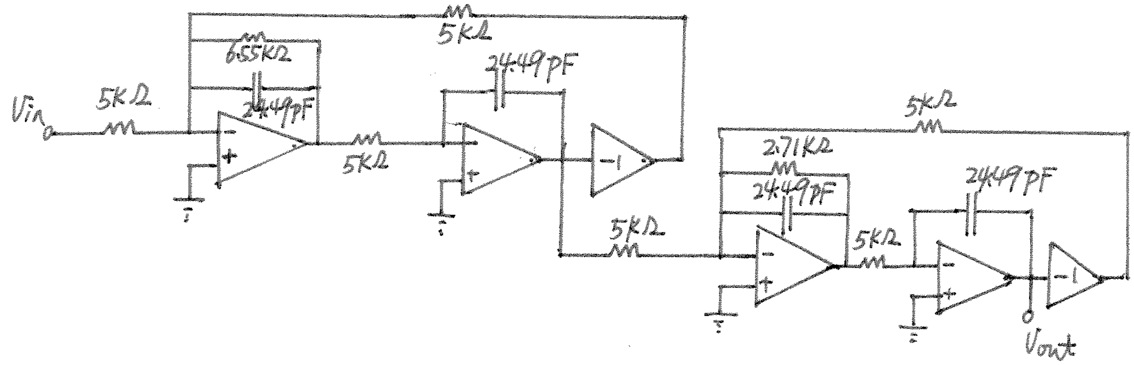
Let $R_2 = R_4 = 5K\Omega \Rightarrow C_1 = C_2 = 24.49\,pF.$

$R_1 = R_2 = R_4 = 5K\Omega$, to obtain a low-frequency gain of unity.

$R_3 = QR_2 = 6.55K\Omega.$

Similarly, $H_{P_{2,3}}(S) = \frac{W_0^2}{S^2 - [2W_0\cos(\frac{7\pi}{8})]S + W_0^2}$

It can be obtained that $R_1=R_2=R_4=5K\Omega$, $R_3 = 2.71K\Omega$, $C_1=C_2=24.49\,pF$.



d). Chebyshev TT

$n=3, \quad \epsilon = 0.3493, \quad \omega_0 = (2\pi)(1\times10^6)$

$P_{1,3} = -0.3133\,\omega_0 \pm j1.022\,\omega_0, \quad P_2 = -0.6265\,\omega_0.$

$$H_{P_{1,3}} = \frac{[2\pi\times1.069\times10^6]^2}{s^2 + (0.6265)(2\pi\times10^6)s + (2\pi\times1.069\times10^6)^2}$$

$\omega_n = (2\pi)(1.069\times10^6)$

$\frac{\omega_n}{Q} = (0.6265)(2\pi\times10^6) \Rightarrow Q = \frac{1.069}{0.6265} = 1.7063$

$\omega_n = \frac{1}{\sqrt{R_2R_4C_1C_2}}, \quad Q = R_3\sqrt{\frac{C_1}{R_2R_4C_2}}$

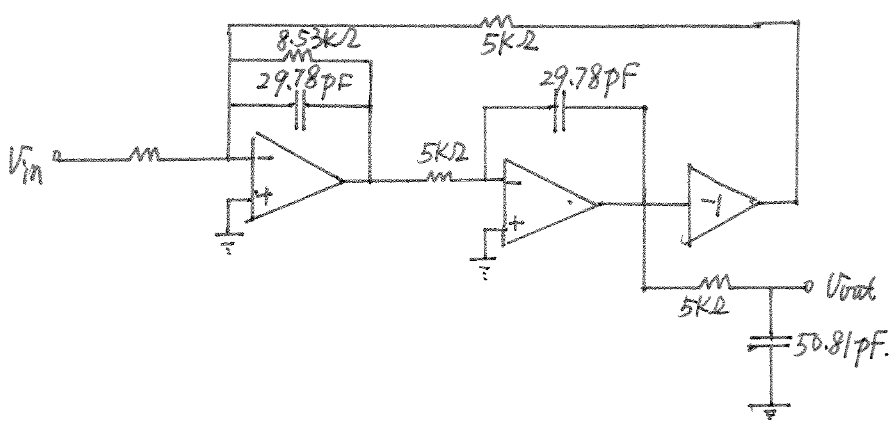
Let $R_2 = R_4$, $C_1 = C_2$, $\frac{1}{R_2C_1} = (2\pi)(1.069\times10^6)$

Let $R_2 = R_4 = 5K\Omega \Rightarrow C_1 = C_2 = 29.78pF$.

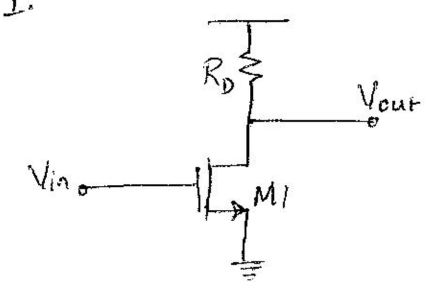
$R_1 = R_2 = R_4 = 5K\Omega$, to obtain a low-frequency gain of unity.

$R_3 = QR_2 = 8.53K\Omega$.

$-P_2 = 0.6265\times(2\pi\times10^6) = \frac{1}{R_5C_5}$, Let $R_5 = 5K\Omega \Rightarrow C_5 = 50.81pF$



1.



M_1 operates in the triode region

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right]$$

$$V_{out} = V_{DD} - R_D I_D$$

$R_D = 10K$

$$\left(W/L\right)_1 = 3/0.18$$

$V_{out,min} = ?$ when $V_{in} = V_{DD}$

$$V_{out,min} = V_{DD} - R_D I_{D,max}$$

$$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right] \times R_D$$

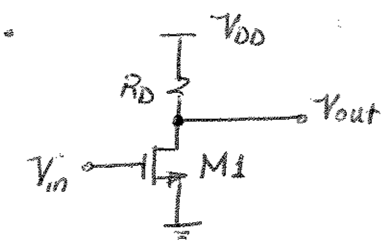
If the second term in the square brackets is neglected. Then

$$V_{out,min} \approx \frac{V_{DD}}{1+\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH})R_D}$$

$$= \frac{1.8}{1+100\times10^{-6} * 3/0.18 * (1.8-0.4)\times10^5}$$

$$\boxed{V_{out,min} \approx 7.7\,mV}$$

2.



$V_{out,min} \leq 100\,mV$

$R_D = 5K\Omega$

$(W/L)_{1,min} = ?$

Output low level establishes for $V_{in} = V_{DD}$, driving M_1 into the triode region.

$$I_{D max} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right]$$

$$V_{out,min} = V_{DD} - R_D \times I_{D,max}$$

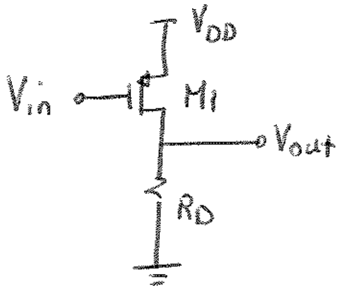
$$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right] \times R_D$$

$$\left(\frac{W}{L}\right)_1 = \frac{V_{DD} - V_{out,min}}{\frac{1}{2}\mu_n C_{ox}\left[2(V_{DD}-V_{TH})V_{out,min} - V_{out,min}^2\right] \times R_D}$$

$$\left(\frac{W}{L}\right)_{1,min} = \frac{1.8 - 100\times10^{-3}}{\frac{1}{2}\times 100\times10^{-6}\left[2(1.8-0.4)100\times10^{-3} - (100\times10^{-3})^2\right]\times 5\times10^3}$$

$$\boxed{\left(\frac{W}{L}\right)_{1,min} = 25}$$

3.



$\left(\frac{W}{L}\right)_1 = 20/0.18$, $R_D = 5K$

$V_{OL}, V_{OH} = ?$

(1) $V_{in} = V_{DD} \longrightarrow M_1$ off $\longrightarrow I_D = 0 \longrightarrow \boxed{V_{out} = V_{OL} = 0}$

(2) $V_{in} = 0 \longrightarrow M_1$ operates in the triode region

$$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{SG} - |V_{THP}|)V_{SD} - V_{SD}^2\right]$$

$$I_{D,max} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD} - |V_{THP}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right] \quad (1)$$

$$I_{D,max} = \frac{V_{out}}{R_D} \quad (2)$$

Equating (1) and (2) and neglecting the second order term in the brackets

$$\frac{V_{out}}{R_D} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 \times 2(V_{DD} - |V_{THP}|)(-V_{out} + V_{DD})$$

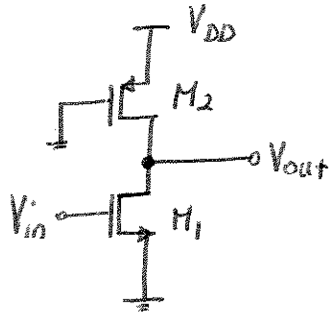
$$V_{out}\left[\frac{1}{R_D} + \mu_p C_{ox}\left(\frac{W}{L}\right)_1(V_{DD} - |V_{THP}|)\right] = \mu_p C_{ox}\left(\frac{W}{L}\right)_1(V_{DD} - |V_{THP}|) \times V_{DD}$$

$$V_{out} = \frac{R_D}{R_D + \frac{1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_1(V_{DD} - |V_{THP}|)}} V_{DD}$$

$$V_{out} = \frac{5000}{5000 + \frac{1}{50\times10^{-6} \times \left(\frac{20}{0.18}\right) \times (1.8 - 0.5)}} \times 1.8$$

$$\boxed{V_{out} = V_{OH} = 1.75\ V}$$

4.



$\left(\frac{W}{L}\right)_1 = 3/0.18 \quad \left(W/L\right)_2 = 2/0.18$

(a) if $V_{in} = V_{DD}$, M_2 Saturated $\rightarrow V_{OL} = ?$

(b) if $V_{in} = V_{out} \longrightarrow V_{in} = ?$

(a) $I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{SG} - |V_{THP}|\right)^2$

$I_{D2} = \frac{1}{2} \times 50 \times 10^{-6} \times \left(\frac{2}{0.18}\right)\left(1.8 - 0.5\right)^2$, Note that $V_{SG} = V_{DD}$

$I_{D2} = 4.7 \times 10^{-4}\ A$

$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2\left(V_{GS} - V_{THN}\right) V_{DS} - V_{DS}^2\right]$

$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2\left(V_{DD} - V_{THN}\right) V_{OL} - V_{OL}^2\right]$

However $I_{D1} = I_{D2}$

$4.7 \times 10^{-4} = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{3}{0.18}\right)\left[2(1.8 - 0.4) V_{OL} - V_{OL}^2\right]$

Neglecting the second-order term yields:

$\boxed{V_{OL} = 0.2\ V}$

As $(V_{in} - V_{THN}) = (V_{DD} - V_{THN}) = (1.8 - 0.4) = 1.4 > V_{DS_1} = V_{OL} = 0.2\ V$

the assumption of M_1 being in Triode region is correct

We define, $V_X = V_{in} - V_{TH,N} \rightarrow V_{in} = V_X + V_{TH,N}$

$$\frac{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2} V_X^2 = 2(V_{DD} - |V_{TH,P}|)(V_{DD} - V_{TH,N} - V_X) - (V_{DD} - V_{TH,N} - V_X)^2$$

$$\frac{100}{50} \times \frac{3/0.18}{2/0.18} V_X^2 = 2(1.8-0.5)(1.8-0.4-V_X) - (1.8-0.4-V_X)^2$$

$$3V_X^2 = 2.6(1.4 - V_X) - (1.4 - V_X)^2$$

$$3V_X^2 = 3.64 - 2.6V_X - 1.96 + 2.8V_X - V_X^2$$

$$4V_X^2 - 0.2V_X - 1.68 = 0$$

$$V_X = \frac{0.2 \pm \sqrt{0.2^2 + 4 \times 4 \times 1.68}}{8} \longrightarrow \boxed{V_X = 0.67\ V}$$

$$V_{in} = V_X + V_{TH,N} = 0.67 + 0.4 \rightarrow \boxed{V_{in} = V_{out} = 1\ V}$$

This value of V_{out} guarantees that M_2 operates in the triode region.

Now, let's investigate the region of operation of M_2

$V_{SD2} = V_{DD} - V_{out}$

$= 1.8 - 0.2$

$V_{SD2} = 1.6\ V$

$V_{SG2} - |V_{THP}| = V_{DD} - |V_{THP}|$

$= 1.8 - 0.5$

$V_{SG2} - |V_{THP}| = 1.3$

As $V_{SD2} > V_{SG2} - |V_{THP}|$, M_2 operates in the saturation region and the initial assumption is valid.

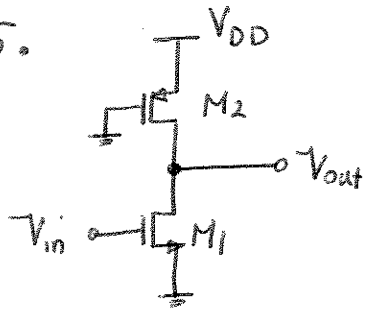
(b) As $V_{in} = V_{out} \rightarrow M_1$ is saturated.

We assume that M_2 is in the triode region and check the validity of this assumption

$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{THN})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{THP}|) \times (V_{DD} - V_{in}) - (V_{DD} - V_{in})^2\right]$$

5.



$V_{OL} \leq 100\ mV$

$(W/L)_2 = 3/0.18$

$\left(\frac{W}{L}\right)_{1,min} = ?$

$V_{in} = V_{DD} \rightarrow$ M_1 operates in the triode region and M_2 in the saturation.

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{SG} - |V_{TH,p}|\right)^2$$

$$= \frac{1}{2} \times 50\times10^{-6} \times \left(\frac{3}{0.18}\right) \times (1.8 - 0.5)^2$$

$$I_{D2} = 7.041\times10^{-4} A$$

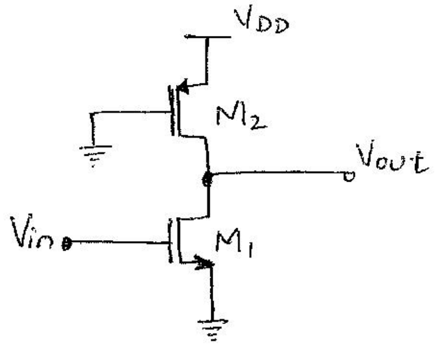
$$I_{D1} = I_{D2} = 7.041\times10^{-4} A$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{GS} - V_{TH,N})V_{DS} - V_{DS}^2\right]$$

$$7.041\times10^{-4} = \frac{1}{2}\times 100\times10^{-6}\left(\frac{W}{L}\right)_1 \left[2(1.8-0.4)0.1 - (0.1)^2\right]$$

$$\boxed{\left(\frac{W}{L}\right)_{1,min} = 52.16}$$

6)



$V_{OL} \leqslant 80\ mV$

$\left(\frac{W}{L}\right)_1 = 2/0.18$

$\left(\frac{W}{L}\right)_{2,max}$

$V_{in} = V_{DD} \longrightarrow M_1$ operates in the triode region and M_2 in the Saturation

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2\left(V_{GS} - V_{TH,N}\right)V_{DS} - V_{DS}^2\right]$$

$$I_{D1} = \frac{1}{2} \times \left(100 \times 10^{-6}\right) \times \left(\frac{2}{0.18}\right) \times \left[2\left(1.8 - 0.4\right)0.08 - 0.08^2\right]$$

$$I_{D1} = 1.2 \times 10^{-4}\ A$$

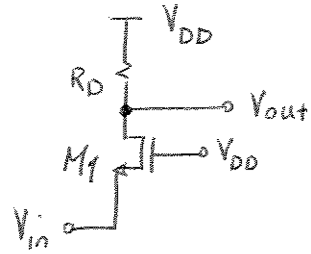
$$I_{D2} = I_{D1} = 1.2 \times 10^{-4}\ A$$

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[V_{GS} - |V_{TH2,P}|\right]^2$$

$$1.2 \times 10^{-4} = \frac{1}{2} \times 50 \times 10^{-6}\left(\frac{W}{L}\right)_2\left(1.8 - 0.5\right)^2$$

$$\boxed{\left(\frac{W}{L}\right)_{2,max} = 2.84}$$

7.



(a) If $V_{in} = 0, V_{DD}, V_{out} = ?$

If $V_{in} = 0 \longrightarrow M_1$ operates in the triode region.

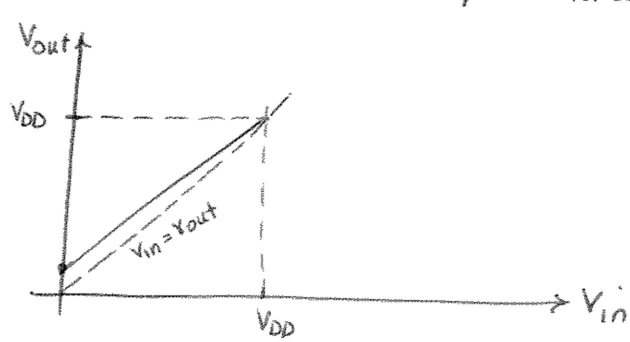
$$R_{on1} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH,N})}$$

$$V_{out} \cong \frac{R_{on1}}{R_{on1} + R_D} \times V_{DD} \longrightarrow V_{out} \cong \frac{1}{1 + \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH,N}) R_D} \times V_{DD}$$

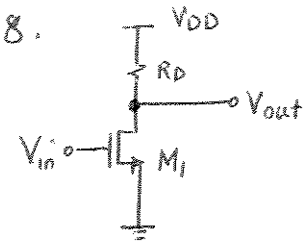
If $V_{in} = V_{DD} \longrightarrow V_{out} = V_{DD}$

No, this circuit does not invert.

(b) A trip point cannot be found for this circuit because $V_{out} = V_{in}$ line does not intersect the transfer characteristic of this buffer.



8.



$$\left(\frac{W}{L}\right)_1 = 5/0.18$$

$$R_D = 2K\Omega$$

$$NM_L , NM_H = ?$$

Small signal gain of the circuit is equal to $-g_m R_D$

and $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N})$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N}) R_D = 1 \quad , \quad V_{GS} = V_{IL}$$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH,N}) R_D = 1$$

$$V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{TH} = \frac{1}{100 \times 10^{-6} \times \frac{5}{0.18} \times 2000} + 0.4$$

$$\boxed{V_{IL} = 0.58\ V}$$

To determine NM_H, we note that V_{in} drives M_1 into the triode region

$$V_{out} = V_{DD} - R_D I_D$$

$$= V_{DD} - \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2 (V_{in} - V_{TH,N}) V_{out} - V_{out}^2 \right] R_D \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2 (V_{in} - V_{TH,N}) \frac{\partial V_{out}}{\partial V_{in}} - 2 V_{out} \frac{\partial V_{out}}{\partial V_{in}} + 2 V_{out} \right] R_D$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1 \ @ \ V_{IH}$$

$$-1 = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[-2 (V_{in} - V_{TH,N}) + 4 V_{out} \right] R_D$$

$$1 = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[-V_{in} + V_{TH,N} + 2V_{out} \right] R_D$$

$$\frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} = -(V_{in} - V_{TH,N}) + 2V_{out}$$

$$V_{out} = \frac{1}{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + \frac{V_{in} - V_{TH,N}}{2} \longrightarrow V_{out} = 0.5V_{in} - 0.11$$

Substituting this in (1) yields:

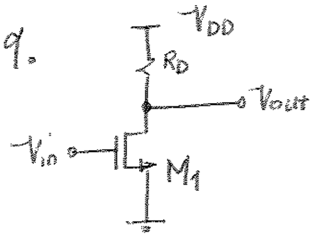
$$0.5V_{in} - 0.11 = 1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \left[2(V_{in} - 0.4)(0.5V_{in} - 0.11) - (0.5V_{in} - 0.11)^2 \right]$$

$$0.75V_{in}^2 - 0.33V_{in} - 0.6117 = \emptyset$$

$$V_{in} = V_{IH} = 1.15$$

$$NM_H = V_{DD} - V_{IH} = 1.8 - 1.15 \longrightarrow \boxed{NM_H = 0.65\,V}$$

9.



Small signal gain of the inverter is equal to $-g_m R_D$

and $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N})$

$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N}) R_D = 1$, $V_{GS} = V_{IL}$

$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH,N}) R_D = 1 \longrightarrow V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{TH,N}$

If we double the value of $\left(\frac{W}{L}\right)_1$ or R_D

$$V_{IL} = \frac{1}{100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \times 2} + 0.4 \longrightarrow \boxed{V_{IL} = 0.49}$$

To determin NM_H, we note that V_{in} drives M_1 into the triode region

$V_{out} = V_{DD} - R_D I_D$

$$= V_{DD} - \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH}) V_{out} - V_{out}^2\right] R_D. \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2 V_{out} + 2(V_{in} - V_{TH}) \frac{\partial V_{out}}{\partial V_{in}} - 2 V_{out} \frac{\partial V_{out}}{\partial V_{in}}\right]$$

$\frac{\partial V_{out}}{\partial V_{in}} = -1$ @ V_{IH}

$$V_{out} = \frac{1}{2 \mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + \frac{V_{in} - V_{TH,N}}{2}$$

Doubling $\left(\frac{W}{L}\right)_1$ or R_D leads to $\boxed{V_{out} = 0.5V_{in} - 0.155}$

Substituting in (1) yields:

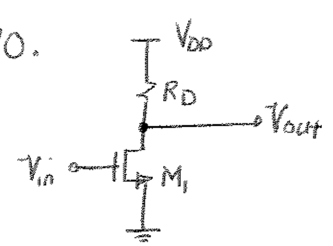
$$0.5V_{in} - 0.155 = 1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000 \times 2 \left[2(V_{in} - 0.4)(0.5V_{in} - 0.155) - (0.5V_{in} - 0.155)^2\right]$$

$$0.75V_{in}^2 - 0.465V_{in} - 0.251925 = 0$$

$V_{in} = 0.967\,V \longrightarrow NM_H = 1.8 - 0.967$

$$\boxed{NM_H = 0.833\,V}$$

10.



$\left(\frac{W}{L}\right)_1 = \frac{5}{0.18}$

$R_D = 2K$

NM_L and $NM_H = ?$ if $\frac{\partial V_{out}}{\partial V_{in}} = -0.5$ instead of -1

Small signal gain of the inverter is equal to "$-g_m R_D$"

and $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH,N})$

$$\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{IL} - V_{TH,N}) R_D = 0.5$$

$$V_{IL} = \frac{1}{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + V_{TH,N} = \frac{1}{2 \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000} + 0.4$$

$\boxed{V_{IL} = 0.49}$ which is less than 0.58 obtained in problem 8.

To determine NM_H, note that M_1 operates in the triode region

$$V_{out} = V_{DD} - R_D I_D$$

$$= V_{DD} - \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH}) V_{out} - V_{out}^2\right] R_D \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2V_{out} + 2(V_{in} - V_{TH}) \frac{\partial V_{out}}{\partial V_{in}} - 2V_{out} \frac{\partial V_{out}}{\partial V_{in}}\right] R_D$$

$\frac{\partial V_{out}}{\partial V_{in}} = -0.5$ @ V_{IH}

$$-0.5 = -\frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[-(V_{in} - V_{TH,N}) + 3V_{out}\right] R_D$$

$$V_{out} = \frac{1}{3\mu_n C_{ox} \left(\frac{W}{L}\right)_1 R_D} + \frac{V_{in} - V_{THN}}{3} \longrightarrow V_{out} = -73.33 \times 10^{-3} + 0.33 V_{in}^{o}$$

or $V_{out} = -\frac{0.22}{3} + \frac{V_{in}}{3}$

Substituting in (1) yields:

$$-\frac{0.22}{3} + \frac{V_{in}}{3} = 1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times \frac{5}{0.18} \times 2000\left[2\left(V_{in} - 0.4\right)\left(-\frac{0.22}{3} + \frac{V_{in}}{3}\right) - \left(-\frac{0.22}{3} + \frac{V_{in}}{3}\right)^2\right]$$

$$5V_{in}^2 - 2.2V_{in} - 5.59 = 0$$

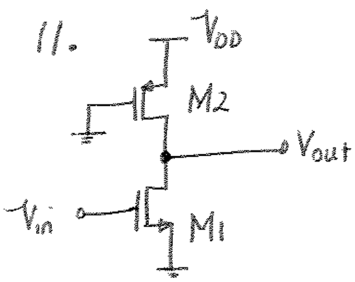
$$V_{in} = V_{IH} = 1.3$$

$$NM_H = 1.8 - 1.3$$

$$\boxed{NM_H = 0.5\,V}$$

less than 0.65 V obtained in problem 8 because V_{IH} is now further pushed up toward V_{DD}.

11.



$$\left(\frac{W}{L}\right)_1 = \frac{4}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{9}{0.18}$$

To calculate V_{IL}, we assume that M_1 and M_2 operate in saturation and triode region respectively.

$$I_{D1} = I_{D2} \qquad (1)$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH,N})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH,P}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{THN}) = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH,P}|)\left(-\frac{\partial V_{out}}{\partial V_{in}}\right) - 2(V_{DD} - V_{out})\left(-\frac{\partial V_{out}}{\partial V_{in}}\right)\right]$$

By substituting $\frac{\partial V_{out}}{\partial V_{in}}$ with "-1" in the above relationship:

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH,N}) = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH,P}|) - 2(V_{DD} - V_{out})\right]$$

$$V_{out} = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}(V_{in} - V_{THN}) + |V_{THP}| = \frac{100\times10^{-6}\times 4/0.18}{50\times10^{-6}\times 9/0.18}(V_{in} - 0.4) + 0.5$$

$V_{out} = 0.144 + 0.88\,V_{in}^{o}$ or $\boxed{V_{out} = \frac{8}{9}V_{in}^{o} + \frac{1.3}{9}}$

Substituting V_{out} in (1) by the derivation versus V_{in} gives:

$$136\,V_{in}^2 - 108.8\,V_{in}^{o} - 115.13 = 0$$

$$\boxed{V_{in}^{o} = V_{IL} = NM_L = 1.4\,V}$$

To calculate V_{IH}, we assume that M_1 and M_2 operate in the triode and saturation region respectively.

$$I_{D1} = I_{D2}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{THN})V_{out}-V_{out}^2\right] = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-|V_{THP}|\right)^2 \quad (2)$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{THN})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right] = \emptyset$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}-2(V_{in}-V_{THN})+2V_{out}\right] = \emptyset$$

$V_{out} = \frac{V_{in}-V_{THN}}{2}$ supstituted in (2) yields:

$$V_{in} = \sqrt{\frac{3}{2}}\left(V_{DD}-|V_{THP}|\right)+V_{THN}$$

$V_{in} = 2 \longrightarrow V_{out} = 0.8$ This value of V_{out} puts M_2 into the triode region so our initial assumption is not correct

Now we assume that both M_1 and M_2 operate in the triode region.

$$I_{D1} = I_{D2}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{THN})V_{out}-V_{out}^2\right] = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD}-|V_{THP}|)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right] \quad (3)$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2V_{out} + 2(V_{in} - V_{THN})\frac{\partial V_{out}}{\partial V_{in}} - 2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right] = \mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times$$

$$\left[2(V_{DD} - |V_{THP}|)\left(-\frac{\partial V_{out}}{\partial V_{in}}\right) - 2(V_{DD} - V_{out})\left(-\frac{\partial V_{out}}{\partial V_{in}}\right)\right]$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2V_{out} - 2(V_{in} - V_{THN}) + 2V_{out}\right] = \mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times$$

$$\left[2(V_{DD} - |V_{THP}|) - 2(V_{DD} - V_{out})\right]$$

$$V_{out} = \frac{\dfrac{\mu_n C_{ox}(\frac{W}{L})_1}{\mu_p C_{ox}(\frac{W}{L})_2}(V_{in} - V_{THN}) - |V_{THP}|}{2\dfrac{\mu_n C_{ox}(\frac{W}{L})_1}{\mu_p C_{ox}(\frac{W}{L})_2} - 1}$$

$$V_{out} = \frac{8}{7}V_{in} - 1.1$$

After substituting in (3) it leads to:

$$2.1769\, V_{in}^2 - 4.19\, V_{in} + 0.576 = 0$$

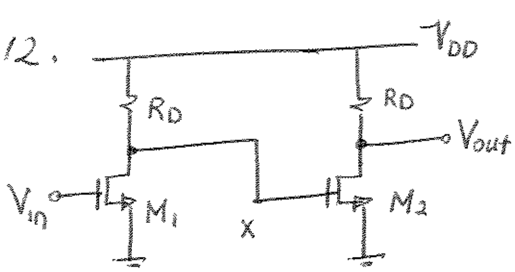
$$\boxed{V_{in} = 1.77\,V}$$

$V_{out} = 0.93\,V \longrightarrow$ The assumption is correct

$V_{IH} = 1.77\,V \longrightarrow NM_H = 1.8 - 1.77$

$$\boxed{NM_H = 0.03\,V}$$

12.



The small signal gain of the circuit is equal to $-g_m R_D$ and since

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{THN})$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{IL} - V_{THN}) R_D = 1$$

$$V_{IL} = \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D} + V_{THN} = \frac{2}{S} + 0.4 \quad ; \quad \left(\frac{W}{L}\right)_{1,2} = S$$

Now we calculate the output of M_1 for $V_{in} = V_{DD}$:

$$V_{DD} - R_D I_D = V_{out}$$

$$V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{THN})V_{out} - V_{out}^2\right] R_D = V_{out} \quad ; \quad \left(\frac{W}{L}\right)_{1,2} = S$$

$$1.8 - \frac{1}{2} \times 100 \times 10^{-6} \times S \left[2(1.8 - 0.4)\left(\frac{2}{S} + 0.4\right) - \left(\frac{2}{S} + 0.4\right)^2\right] \times 5000 = \left(\frac{2}{S} + 0.4\right)$$

$$1.8 - 0.25 \times \left[2.8(2 + 0.4S) - S\left(\frac{2}{S} + 0.4\right)^2\right] = \frac{2}{S} + 0.4$$

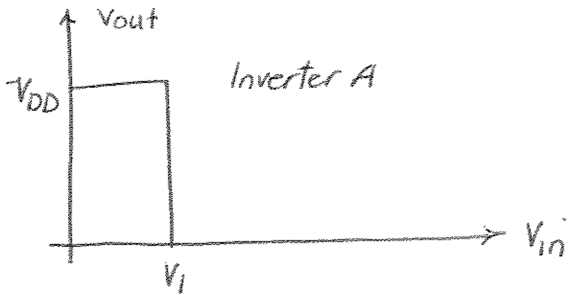
$$1.8S - 0.25 \times \left[2.8(2S + 0.4S^2) - S^2\left(\frac{2}{S} + 0.4\right)^2\right] = 2 + 0.4S$$

$$1.8S - 0.25 \times \left[5.6S + 1.12S^2 - 4 - 1.6S - 0.16S^2\right] = 2 + 0.4S$$

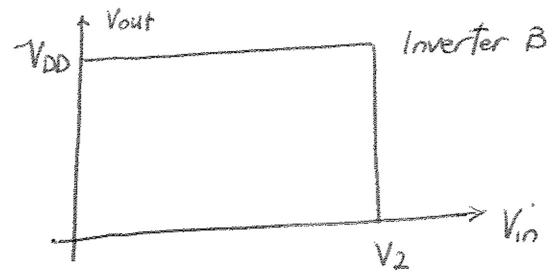
$$0.24S^2 - 0.4S + 1 = \emptyset$$

$$\Delta < \emptyset !$$

13.

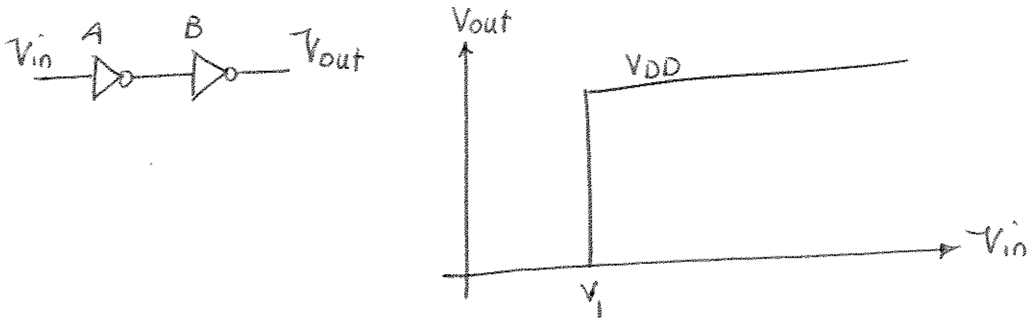


Vout
VDD
Inverter A
Vin
V1



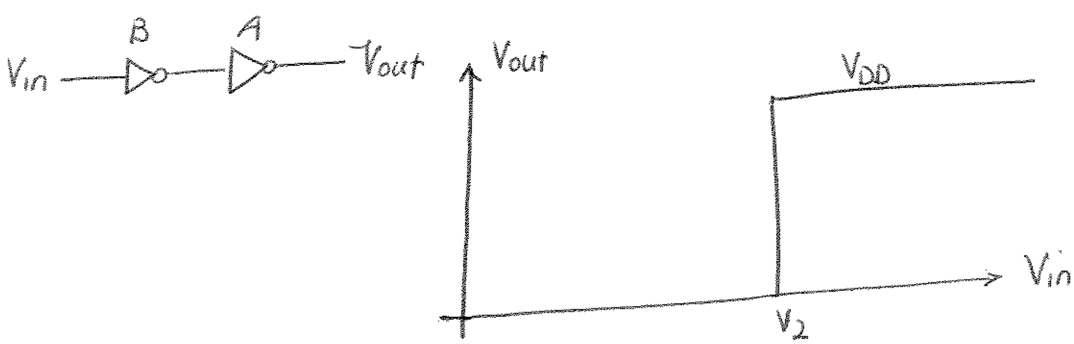
Vout
VDD
Inverter B
Vin
V2

(a)



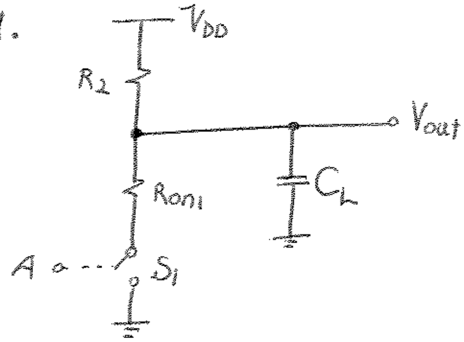
Vin
A
B
Vout
Vout
VDD
Vin
V1

(b)



Vin
B
A
Vout
Vout
VDD
Vin
V2

14.



$R_{on1} << R_2 \longrightarrow V_{out,min} \approx 0$

(a) $V_{out}(t) = V_{out}(0^-) + \left[V_{DD} - V_{out}(0^-)\right]\left(1 - \exp\frac{-t}{R_2 C_L}\right) \quad t > 0$

Note that $V_{out}(0^-) = 0$, $V_{out}(\infty) = V_{DD}$

$$V_{out}(t) = V_{DD} \times \left(1 - \exp\frac{-t}{R_2 C_L}\right) \quad t > 0$$

$$0.95\, V_{DD} = V_{DD} \times \left(1 - \exp\frac{-T_{95\%}}{R_2 C_L}\right)$$

$$\boxed{T_{95\%} = 3 R_2 C_L}$$

(b) $V_{out}(t) = V_{out}(0^-) + \left[V_{out}(\infty) - V_{out}(0^-)\right] \times \left(1 - \exp\frac{-t}{R_2 C_L}\right)$

$$V_{out}(t) = V_{DD} + \left[0 - V_{DD}\right] \times \left(1 - \exp\frac{-t}{R_2 C_L}\right)$$

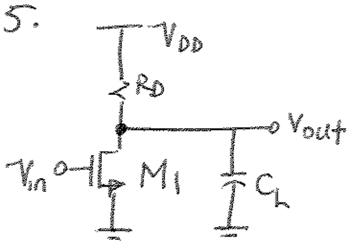
$$V_{out}(t) = V_{DD} \exp\frac{-t}{R_2 C_L}$$

$$0.05\, V_{DD} = V_{DD} \exp\frac{-T_{0.05}}{R_2 C_L}$$

$$\boxed{T_{5\%} = 3 R_2 C_L}$$

If $R_{on1} << R_2$, inverter exhibits equal rise and fall time (or low-to-high and high-to-low delay) at the output.

15.



$C_L = 50\,fF$

$T_R = 100\,pS$

$T_R = 3\,\tau_{out}$

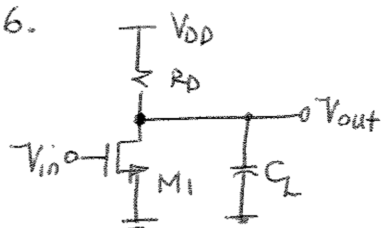
$R_{D,max} = ?$

$$T_R = 3 R_D C_L = 100\,pS$$

$$R_D \leqslant \frac{100\,pS}{3 \times 50\,fF}$$

$$\boxed{R_D \leqslant 666.67\ \Omega}$$

16.



$C_L = 100fF$

$V_{out,min} = 50mV$

$T_R = 200pS$

$R_D,\ \left(\frac{W}{L}\right)_1 = ?$

$T_R = 3\tau_{out}$

$T_R = 3R_DC_L$

$200\times10^{-12} = 3\times R_D\times100\times10^{-15}$

$$\boxed{R_D = 666.667\ \Omega}$$

$V_{in} = V_{DD}$ places M_1 in the triode region

$$V_{out,min} = V_{DD} - R_D I_{D,max}$$

$$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D\left[2(V_{DD}-V_{THN})V_{out,min} - V_{out,min}^2\right]$$

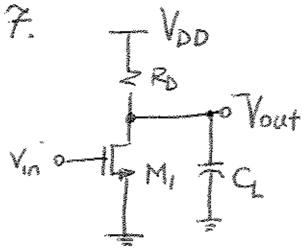
Neglecting the 2^{nd}-order term in the square brackets yields:

$$V_{out,min} = \frac{V_{DD}}{1+\mu_n C_{ox}\left(\frac{W}{L}\right)_1 R_D(V_{DD}-V_{TH})}$$

$$50\times10^{-3} = \frac{1.8}{1+100\times10^{-6}\times\left(\frac{W}{L}\right)_1\times666.7\times(1.8-0.4)}$$

$$\boxed{\left(\frac{W}{L}\right)_1 = 375}$$

17.



$C_L = 100\,fF$

$I_{D,max} \leq 1mA$

$V_{out,min} \approx 0$

$T_{R,min} = ?$

$$I_{D,max} = \frac{V_{DD} - V_{out,min}}{R_D}$$

$$10^{-3} = \frac{1.8 - 0}{R_D}$$

$$\boxed{R_D = 1.8\,K\Omega}$$

$$V_{out}(t) = V_{out}(0^-) + \left[V_{out}(\infty) - V_{out}(0^-)\right]\left(1 - \exp\frac{-t}{R_D C_L}\right) \quad t > 0$$

$$V_{out}(t) = V_{out,min} + \left[V_{DD} - V_{out,min}\right] \times \left(1 - \exp\frac{-t}{R_D C_L}\right) \quad t > 0$$

$$V_{out}(t) = V_{DD}\left(1 - \exp\frac{-t}{R_D C_L}\right) \quad t > 0$$

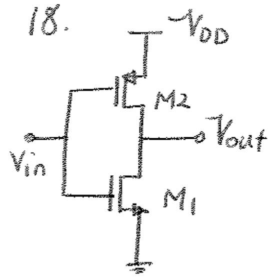
$$0.1 V_{DD} = V_{DD}\left(1 - \exp\frac{-T_{10\%}}{R_D C_L}\right) \longrightarrow T_{10\%} = 0.105\, R_D C_L$$

$$0.9 V_{DD} = V_{DD}\left(1 - \exp\frac{-T_{90\%}}{R_D C_L}\right) \longrightarrow T_{90\%} = 2.3\, R_D C_L$$

$$T_R = T_{90\%} - T_{10\%} = 2.197\, R_D C_L = 2.197 \times 1.8 \times 10^{3} \times 100 \times 10^{-15}$$

$$\boxed{T_R = 395.5\,pS}$$

18.



$$\left(\frac{W}{L}\right)_1 = \frac{2}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{3}{0.18}$$

$I_{D1} = I_{D2}$

At the trip point $V_{in} = V_{out}$; therefore, both M_1 and M_2 operate in the Saturation region.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{in}^{o} - V_{THN}\right)^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in}^{o} - |V_{THP}|\right)^2$$

$$V_{in}^{o} = \frac{V_{DD} - |V_{THP}| + \sqrt{\frac{\mu_n C_{ox} (W/L)_1}{\mu_p C_{ox} (W/L)_2}} \times V_{THN}}{1 + \sqrt{\frac{\mu_n C_{ox} (W/L)_1}{\mu_p C_{ox} (W/L)_2}}}$$

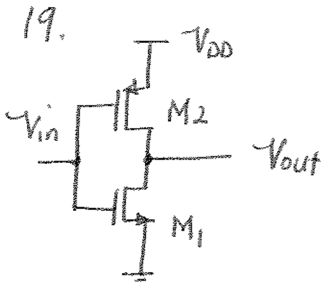
$$V_{in}^{o} = \frac{1.8 - 0.5 + \left(\frac{100 \times 2}{50 \times 3}\right)^{1/2} \times 0.4}{1 + \left(\frac{100 \times 2}{50 \times 3}\right)^{1/2}}$$

$$\boxed{V_{in} = V_{out} = 0.82\,V}$$

$$I_{D1} = I_{D2} = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right)\left(0.82 - 0.4\right)^2$$

$$\boxed{I_{D1} = I_{D2} = 97\,\mu A}$$

19.



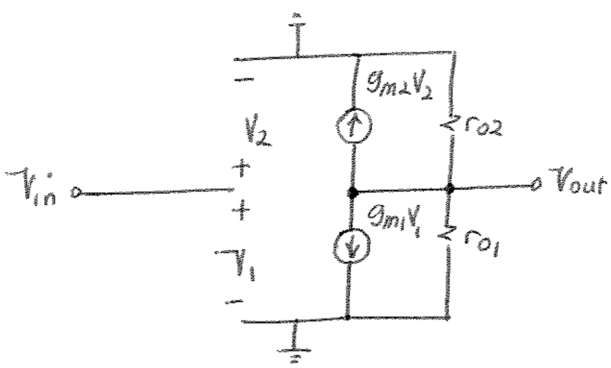
$$\left(\frac{W}{L}\right)_1 = \frac{2}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{3}{0.18}$$

$$\lambda_N = 0.1\,V^{-1}$$

$$\lambda_P = 0.2\,V^{-1}$$

Replacing M_1 and M_2 with their small-signal model in the saturation region yields:



$$V_{out} = (-g_{m1}V_1 - g_{m2}V_2)(r_{o1} \| r_{o2})$$

$$V_1 = V_2 = V_{in}$$

$$V_{out} = -(g_{m1} + g_{m2})(r_{o1} \| r_{o2})V_{in}$$

$$\frac{V_{out}}{V_{in}} = -(g_{m1} + g_{m2})(r_{o1} \| r_{o2})$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{THN})^2$$

$$g_{m1} = \frac{\partial I_{D1}}{\partial V_{in}} = \mu_n C_{ox}\left(\frac{W}{L_1}\right)_1 (V_{in} - V_{THN}) = \frac{2I_{D1}}{V_{in} - V_{THN}}$$

$$g_{m1} = \frac{2 \times 9.7 \times 10^{-5}}{(0.817 - 0.4)} \rightarrow \boxed{g_{m1} = 4.641 \times 10^{-4}\ \mho}$$

$$g_{m2} = \frac{2 I_{D2}}{(V_{SG} - |V_{THP}|)} = \frac{2 \times 9.7 \times 10^{-5}}{(1.8 - 0.817 - 0.5)} \rightarrow \boxed{g_{m2} = 4.02 \times 10^{-4}\ \mho}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$$

$$g_o = \frac{\partial I_D}{\partial V_{DS}} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2 \lambda \simeq \lambda I_D$$

$$r_o \cong \frac{1}{\lambda I_D}$$

$$r_{oN} \simeq \frac{1}{0.1 \times 9.7 \times 10^{-5}} = 103.17\,K\Omega$$

$$r_{op} \simeq \frac{1}{0.2 \times 9.7 \times 10^{-5}} = 51.58\,K\Omega$$

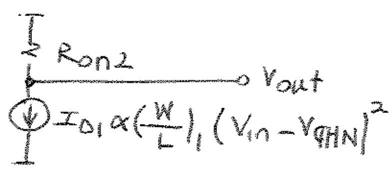
$$Gain = \frac{V_{out}}{V_{in}} = -(4.641 \times 10^{-4} + 4.02 \times 10^{-4})(51.58K \| 103.17K)$$

$$\boxed{Gain = -29.8}$$

20.

(a) Length of M_1 is increased

Let's assume that $V_{in} < V_{TH1}$, as a result M_1 is off and M_2 is on operating in the triode region. As V_{in} increases beyond V_{TH1}, M_1 starts pulling current (Conducting) in the saturation region while M_2 is still in the triode region. operating as a resistor; therefore, CMOS inverter Can be modelled as follows:

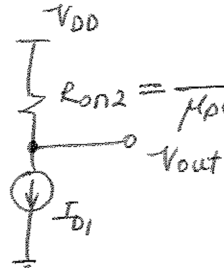


By increasing L_1, I_{D1} is weakened due to the inverse proportionality; as a result, an excess V_{in} is required to drop V_{out} to the point where $V_{out} = V_{in} + |V_{TH2}|$ and M_2 is placed at the edge of Saturation. Therefore Characteristic is shifted to the right and it will be steeper at the gain region where both M_1 and M_2 are in Saturation region.

(b) Length of M_2 is increased

Again if we assume that $V_{in} < V_{TH1}$, M_1 is off and M_2 is operating in the triode region with no current. By increasing

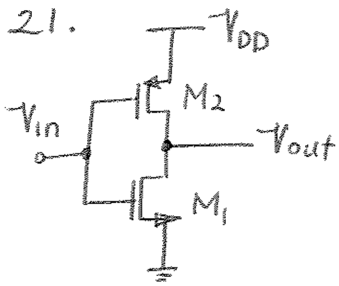
V_{in} above V_{TH1}, M_1 conducts in the saturation region while M_2 is operating in the triode region. Using the same models as used in part (a) yields:



$$R_{on2} = \frac{1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{THP}|\right)}$$

By increasing L_2, R_{on2} becomes larger; as a result, lower value of I_{D1} causes comparable voltage drop at the output. This will drive M_2 into the saturation with lower current (I_{D1}) and, hence, lower value of V_{in}. Therefore, characteristic is shifted to the left and small signal gain will be higher.

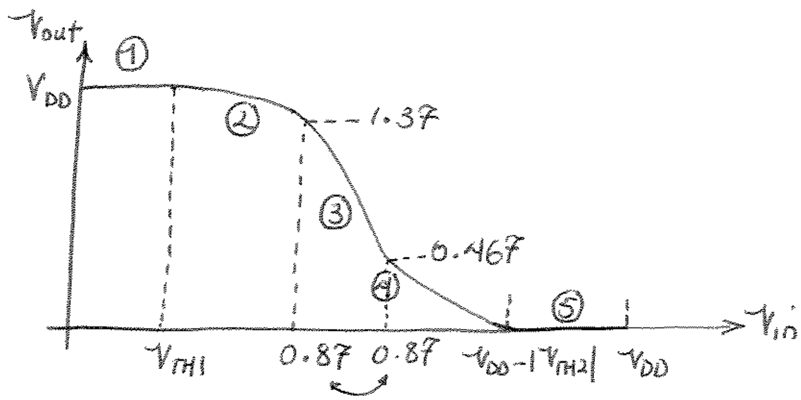
21.



$$\left(\frac{W}{L}\right)_1 = \frac{3}{0.18}$$

$$\left(\frac{W}{L}\right)_2 = \frac{7}{0.18}$$

VTC looks like the following figure



① M_1 off, M_2 in triode region

$$I_{D1} = \emptyset$$

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2\left(V_{DD}-V_{in}-|V_{TH2}|\right)V_{SD} - V_{SD}^2\right] = \emptyset$$

$$V_{SD} = \emptyset \rightarrow V_{out} = V_{DD} \quad (1)$$

② M_1 in saturation, M_2 in triode region

$$I_{D1} = I_{D2}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2\left(V_{DD}-V_{in}-|V_{THP}|\right)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right]$$

$$\frac{1}{2}\times 100\times 10^{-6}\times \frac{3}{0.18}\times (V_{in}-0.4)^2 = \frac{1}{2}\times 50\times 10^{-6}\times \frac{7}{0.18}\times \Big[2(1.8-V_{in}-0.5)\times(1.8-V_{out}) - (1.8-V_{out})^2\Big]$$

$$6(V_{in}-0.4)^2 = 7\Big[2(1.3-V_{in})(1.8-V_{out}) - (1.8-V_{out})^2\Big] \quad (2)$$

If V_{out} falls significantly, M_2 enters saturation. That is $V_{out} = V_{in} + |V_{TH2}|$.

then M_2 is about to exit the triode region.

Replacing V_{out} by $V_{in} + |V_{TH2}|$ in (2) leads to:

$$6(V_{in}-0.4)^2 = 7\Big[2(1.3-V_{in})(1.8-V_{in}-0.5) - (1.8-V_{in}-0.5)^2\Big]$$

$$6(V_{in}-0.4)^2 = 7(1.3-V_{in})^2 \rightarrow \sqrt{\frac{6}{7}}\,(V_{in}-0.4) = (1.3-V_{in})$$

$V_{in} = 0.867\ V$, $V_{out} = 1.37$

③ $$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2(1+\lambda_1 V_{out}) = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD}-V_{in}-|V_{TH2}|)^2 \times \big[1+\lambda_2(V_{DD}-V_{out})\big]$$

$$V_{out} = \frac{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD}-V_{in}-|V_{TH2}|)^2 - \mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2}{\lambda_2\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD}-V_{in}-|V_{TH2}|)^2 + \lambda_1\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2}$$

$$V_{out} = \frac{7(1.3-V_{in})^2 - 6(V_{in}-0.4)^2}{7\lambda_2(1.3-V_{in})^2 + 6\lambda_1(V_{in}-0.4)^2} \quad (3)$$

in region ③ M_1 and M_2 are both in saturation.

④ M_1 in triode region, M_2 in saturation

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \times \left(V_{DD}-V_{in}-|V_{TH2}|\right)^2 \quad (A)$$

$$6\left[2(V_{in}-0.4)V_{out}-V_{out}^2\right]=7(1.3-V_{in})^2$$

If V_{out} falls sufficiently, M_1 enters the triode region. That is, if $V_{in}=V_{out}+V_{TH1}$. Then M_1 is about to enter the triode region.

By substituting V_{in} with $V_{out}+0.4$ in (A), we have:

$$6\left[2V_{out}^2-V_{out}^2\right]=7(0.9-V_{out})^2$$

$V_{out}=0.467$, $V_{in}=0.867$

As channel length modulation has been neglected in this calculation tha value of input voltage that makes CMOS inverter transition from region ② to ③ is the same as that which makes inverter transition from region ③ to ④.

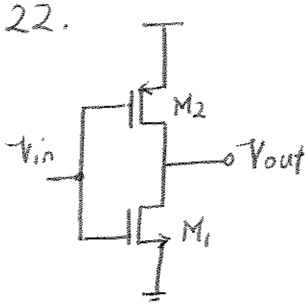
The slope in region ③ is infinit; however, we assume a finite slope in that region to emphasize the behavior of inverter as to producing a high gain.

⑤ M_1 in triode region, M_2 off

$I_{D2}=0$; $I_{D1}=0$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=0 \longrightarrow V_{out}=0$$

22.



$V_{in} = V_{out} = 0.5\,V$

M_1 and M_2 are both in saturation region

$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 (0.5 - 0.4)^2 = \frac{1}{2} \times 50 \times 10^{-6} \times \left(\frac{W}{L}\right)_2 (1.8 - 0.5 - 0.5)^2$$

$$\left(\frac{W}{L}\right)_1 \Big/ \left(\frac{W}{L}\right)_2 = 32$$

23. The value of the trip point has to be larger than the threshold voltage of NMOS transistor, 0.4 V. Therefore, 0.3 V Cannot be the trip point of such an inverter.

24.

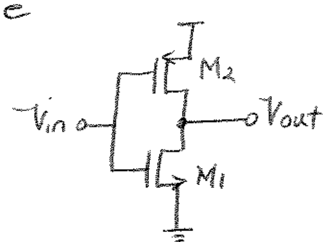
(a) If the inverter exhibits a very high voltage gain around the trip point, the range of input voltage values which guarantees that M_1 and M_2 are in saturation region is very narrow. Therefore this range can be fairly approximated with only one value of input voltage.

(b) $(W/L)_1 = 3/0.18$ and $(W/L)_2 = 7/0.18$

To calculate the minimum input voltage at which both transistors operate in saturation we assume

M_1 saturation

M_2 triode



$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - V_{in} - |V_{TH2}|)(V_{out} - V_{DD}) - (V_{out} - V_{DD})^2\right]$$

$V_{out} = V_{in} + |V_{TH2}|$ places M_2 at the edge of saturation

$$2 \times 3 \times (V_{in} - 0.4)^2 = 7 \times \left[2(1.8 - V_{in} - 0.5)(1.8 - V_{in} - 0.5) - (1.8 - V_{in} - 0.5)^2\right]$$

$$\boxed{V_{in,min} = 0.867}$$

To calculate $V_{in,max}$, we assume that M_1 and M_2 are in triode and saturation region respectively

$I_{D1} = I_{D2}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{in} - V_{TH1}) V_{out} - V_{out}^2\right] = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2$$

When M_1 is just going to leave the saturation and enters the triode region

$$V_{in} = V_{out} + 0.4^{(V_{TH1})}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \frac{3}{0.18} \times \left[2(V_{out} + V_{TH1} - V_{TH1}) V_{out} - V_{out}^2\right] = \frac{1}{2} \times 50 \times 10^{-6} \times \frac{7}{0.18} \times (V_{DD} - V_{in} - |V_{TH2}|)^2$$

$$\frac{6}{7} V_{out}^2 = (V_{DD} - V_{out} - V_{TH1} - |V_{TH2}|)^2$$

$$\frac{6}{7} V_{out}^2 = (0.9 - V_{out})^2$$

$V_{out} = 0.467\,V$, $\boxed{V_{in,max} = 0.867}$

To find the trip point, M_1 and M_2 are assumed to be in saturation.

$$I_{D1} = I_{D2}$$

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2$$

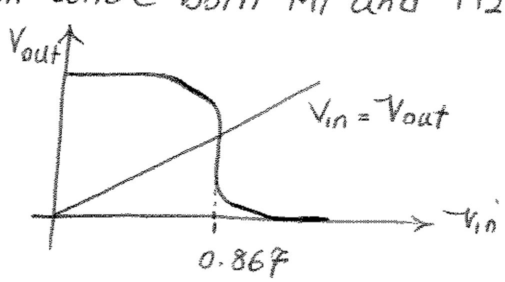
$$2 \times 3 \times (V_{in} - 0.4)^2 = 7 \times (1.8 - V_{in} - 0.5)^2$$

$\boxed{V_{in}^{0} = 0.867 \text{ @ trip point}}$

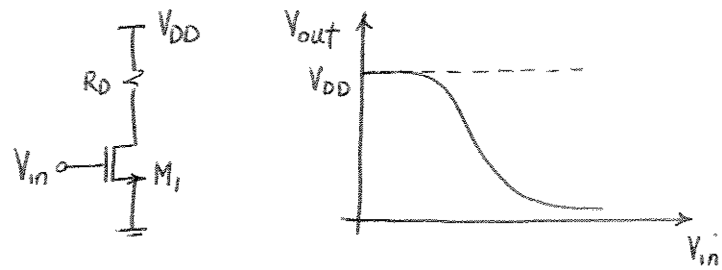
$$V_{in,trip} - V_{in,min} = \emptyset$$

$$V_{in,max} - V_{in,trip} = \emptyset$$

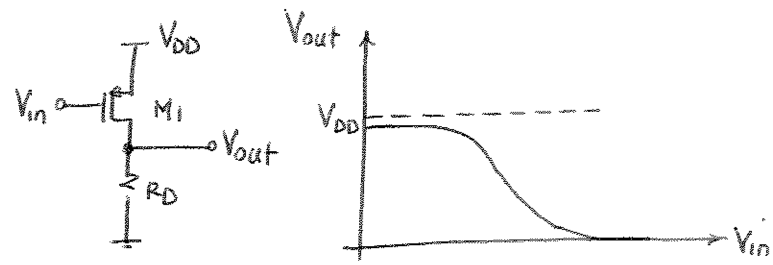
This result is not surprising because VTC of inverter has infinite slope at the region where both M_1 and M_2 are in saturation region



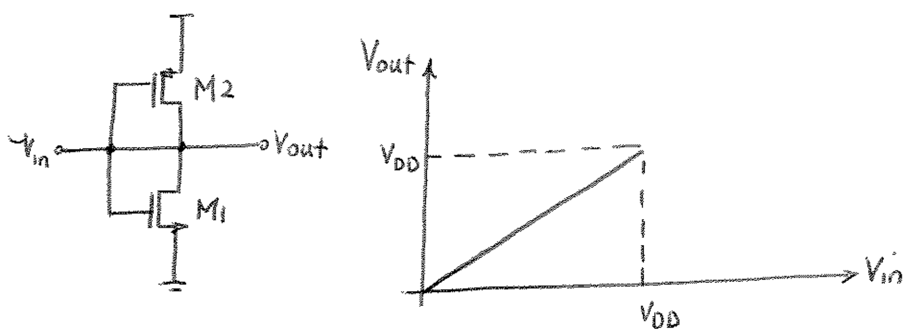
25.



VDD
RD
Vin
M1
Vout
VDD
Vin

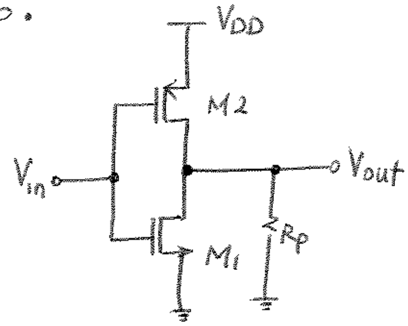


VDD
Vin
M1
Vout
RD
Vout
VDD
Vin



M2
Vin
Vout
M1
Vout
VDD
Vin
VDD

26.



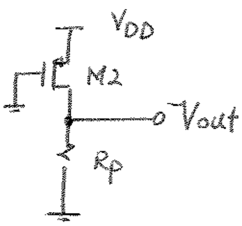
$R_p = 2K$

$\left(\frac{W}{L}\right)_1 = 3/0.18$

$\left(\frac{W}{L}\right)_2 = 5/0.18$

$V_{OL}, V_{OH}, V_{in,trip} = ?$

To calculate V_{OH}, V_{in} is assumed to be 0V



$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$I_{D2} = \frac{V_{out}}{R_p}$$

$$\frac{V_{out}}{R_p} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$V_{out}^2 - 0.28V_{out} - 1.44 = 0$$

$$\boxed{V_{out} = V_{OH} = 1.348\ V}$$

$\boxed{V_{OL} = 0}$ because M_2 is off for $V_{in} = V_{DD}$

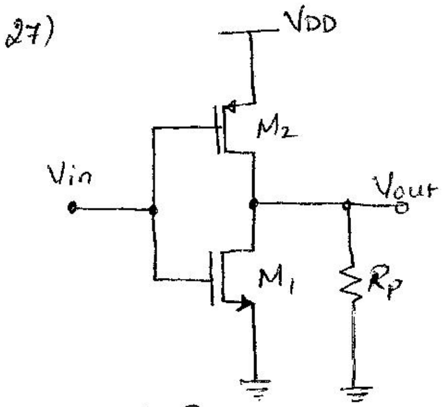
(a) trip point $V_{in} = V_{out}$

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 + \frac{V_{out}}{2000}$$

$$0.05V_{out}^2 + 0.59V_{out} - 0.3745 = 0 \longrightarrow \boxed{V_{in} = V_{out} = 0.6\ V}$$

27)



$R_p = 2k$

$\left(W/L\right)_1 = 3/0.18$

$\left(W/L\right)_2 = 5/0.18$

$V_{in} = V_{out} = 0.6V$ @ trip point

With R_p

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{TH2}|\right)^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{in} - V_{TH1}\right)^2 + \frac{V_{out}}{2000}$$

$$0.05\, V_{out}^2 + 0.59\, V_{out} - 0.3745 = 0$$

$$V_{in} = V_{out} = 0.6\,V$$

$$I_{D1} = \frac{1}{2} \times 100 \times 10^{-6} \times \frac{3}{0.18} \times (0.6 - 0.4)^2$$

$$\boxed{I_{D1} = 3.33 \times 10^{-5}\,A}$$

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p} = 3.33 \times 10^{-5} + \frac{0.6}{2000}$$

$$\boxed{I_{D2} = 3.35 \times 10^{-4}\,A}$$

$$g_{m1} = \frac{2 I_{D1}}{V_{eff1}} = \frac{2 \times 3.33 \times 10^{-5}}{(0.6 - 0.4)} \rightarrow \boxed{g_{m1} = 333\,\mu℧}$$

$$g_{m2} = \frac{2 I_{D2}}{V_{eff2}} = \frac{2 \times 3.35 \times 10^{-4}}{(1.8 - 0.6 - 0.5)} \rightarrow \boxed{g_{m2} = 957\,\mu℧}$$

$$A_v = -(g_{m1} + g_{m2}) * R_p$$

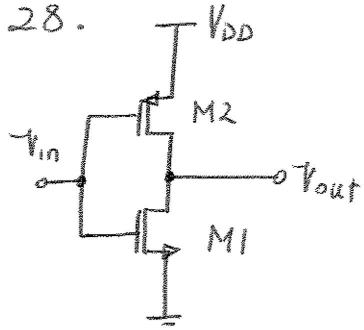
$$A_v = -(333 \times 10^{-6} + 957 \times 10^{-6}) \times 2000$$

$$\boxed{A_v = -2.58}$$

Without R_p

$$A_v \longrightarrow -\infty$$

28.



$(W/L)_1 = 5/0.18$

$(W/L)_2 = 11/0.18$

NM_L and $NM_H = ?$

To calculate NM_L, M_1 and M_2 are assumed to operate in the saturation and triode region respectively.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD}-V_{in}-|V_{TH2}|)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right] \quad (1)$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1}) = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[-2(V_{DD}-V_{out}) - 2(V_{DD}-V_{in}-|V_{TH2}|)\times\frac{\partial V_{out}}{\partial V_{in}} + 2(V_{DD}-V_{out})\frac{\partial V_{out}}{\partial V_{in}}\right]$$

$\frac{\partial V_{out}}{\partial V_{in}} = -1$, $V_{in} = V_{IL}$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{IL}-V_{TH1}) = \mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2V_{OH} - V_{IL} - |V_{TH2}| - V_{DD}\right] \quad (2)$$

Obtaining V_{OH} from (2) and substituting in (1) yields:

$$V_{IL} = \frac{2\sqrt{a}\,(V_{DD}-V_{TH1}-|V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD}-aV_{TH1}-|V_{TH2}|}{a-1}$$

$$a = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2} = \frac{100}{50}\times\frac{5}{11} = \frac{10}{11}$$

$$V_{IL} = \frac{2\sqrt{10/11}\,(1.8-0.4-0.5)}{(10/11-1)\sqrt{10/11+3}} - \frac{1.8-(10/11)\times 0.4-0.5}{10/11-1}$$

$$\boxed{V_{IL} = 0.7516\ V}$$

To determine NM_H, M_1 and M_2 are assumed to operate in the triode and saturation region respectively.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-|V_{TH2}|-V_{in}\right)^2 \quad (3)$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{TH1})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right]=-\mu_p C_{ox}\left(\frac{W}{L}\right)_2\times\left(V_{DD}-V_{in}-|V_{TH2}|\right)$$

$\frac{\partial V_{out}}{\partial V_{in}}=-1$ yields

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[V_{out}-V_{in}+V_{TH1}+V_{out}\right]=-\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left(V_{DD}-V_{in}-|V_{TH2}|\right)$$

$$100\times 5\times\left[2V_{out}-(V_{in}-0.4)\right]=-50\times 11\times(1.8-0.5-V_{in})$$

$$V_{out}=1.05V_{in}-0.915 \quad (4)$$

Substituting (4) in (3) yields an equation versus V_{in} as follows:

$$10\times\left[2(V_{in}-0.4)(1.05V_{in}-0.915)-(1.05V_{in}-0.915)^2\right]=11(1.3-V_{in})^2$$

$$1.025V_{in}^2-21.115V_{in}+19.64225=0$$

$$V_{in}=V_{IH}=0.9765\ V$$

$$NM_H=V_{DD}-V_{IH}$$

$$\boxed{NM_H=0.823\ V}$$

29. $NM_L = 0.6\,V$

$$(W/L)_1 / (W/L)_2 = ?$$

$$V_{IL} = \frac{2\sqrt{a}\,(V_{DD} - V_{TH1} - |V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD} - aV_{TH1} - |V_{TH2}|}{a-1}$$

$$a = \frac{\mu_n C_{ox} \left(\frac{W}{L}\right)_1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2}$$

$$0.6 = \frac{2\sqrt{a}\,(1.8 - 0.4 - 0.5)}{(a-1)\sqrt{a+3}} - \frac{1.8 - a\,0.4 - 0.5}{a-1}$$

$$a = 3\sqrt{\frac{a}{a+3}} - \frac{1.3 - 0.4a}{0.6} + 1$$

$$a = \frac{a+3}{9} \times \left[a - 1 + \frac{1.3 - 0.4a}{0.6}\right]^2$$

$$\boxed{a = 1}$$

$$(W/L)_1 / (W/L)_2 = \frac{\mu_p C_{ox}}{\mu_n C_{ox}} = \frac{1}{2}$$

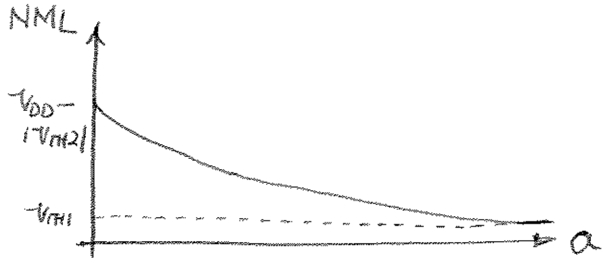
$$\boxed{(W/L)_1 / (W/L)_2 = \frac{1}{2}}$$

30. $$V_{IL} = \frac{2\sqrt{a}\,(V_{DD} - V_{TH1} - |V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD} - aV_{TH1} - |V_{TH2}|}{a-1}$$

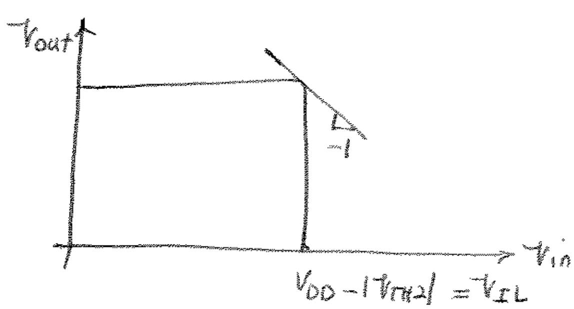
$$a = \frac{\mu_n \left(\frac{W}{L}\right)_1}{\mu_p \left(\frac{W}{L}\right)_2}$$

$a \to 0 \quad V_{IL} = V_{DD} - |V_{TH2}|$

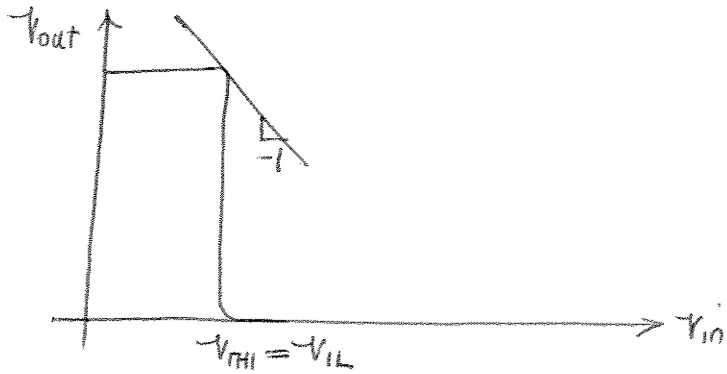
$a \to \infty \quad V_{IL} = V_{TH1}$



If $a = \frac{\mu_n}{\mu_p} \times \frac{(W/L)_1}{(W/L)_2} \to 0$, it implies that PMOS transistor is extremely stronger than NMOS. Therefor, as V_{in} increases from 0V, the output of inverter stays at V_{DD} until input reaches $V_{DD} - |V_{TH2}|$. At that point, PMOS is cut off and V_{out} sharply drops to 0V.



When $a \to \infty$, NMOS is prevailing and once input voltage hits the threshold voltage of NMOS, output voltage falls sharply to 0V.



Vout
-1
Vin
VTH = VIL

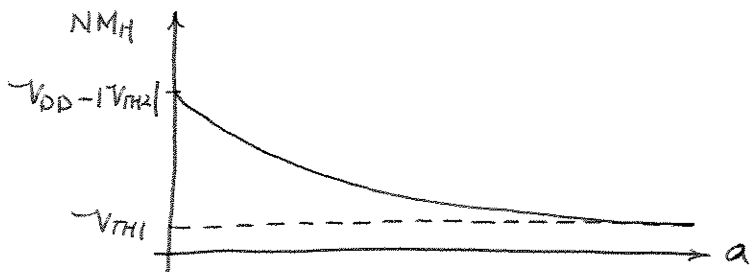
31.

$$NM_H = V_{DD} - \frac{2a\,(V_{DD} - V_{TH1} - |V_{TH2}|)}{(a-1)\sqrt{1+3a}} + \frac{V_{DD} - aV_{TH1} - |V_{TH2}|}{a-1}$$

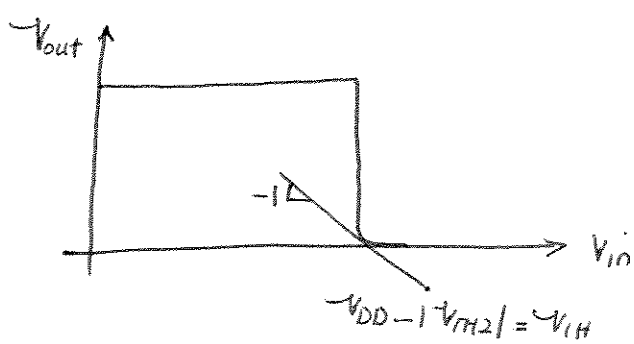
$$a = \frac{\mu_n \left(\frac{W}{L}\right)_1}{\mu_p \left(\frac{W}{L}\right)_2}$$

$a \to 0 \quad NM_H = |V_{TH2}|\ , \quad V_{IH} = V_{DD} - |V_{TH2}|$

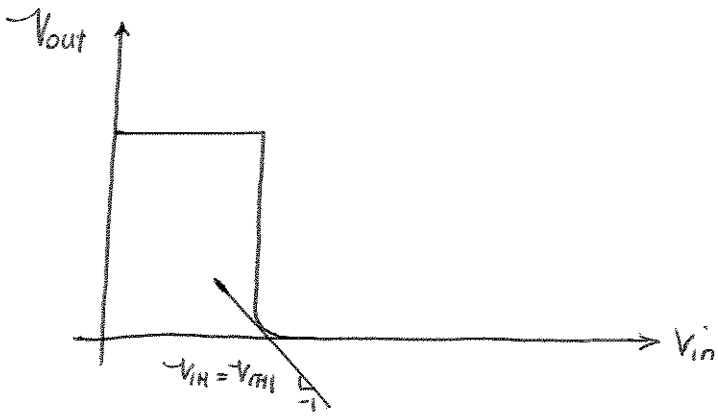
$a \to \infty \quad NM_H = V_{DD} - V_{TH1}\ , \quad V_{IH} = V_{TH1}$



If $a = \frac{\mu_n}{\mu_p} \times \frac{(W/L)_1}{(W/L)_2} \to 0$, it implies that PMOS transistor is much stronger than NMOS. Therefor, as V_{in} increases from 0 V, the output of inverter remains at V_{DD} until input reaches $V_{DD} - |V_{TH2}|$. At that point, PMOS is cutoff and V_{out} sharply drops to 0 V.

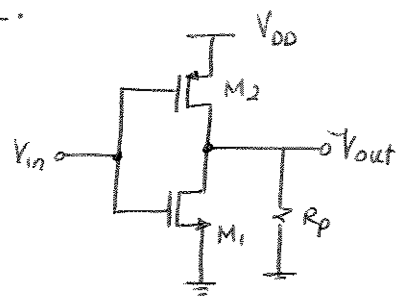


When "a" approaches infinity, NMOS is prevailing and once input Voltage hits the threshold voltage of NMOS, output voltage falls sharply to 0V.



Note that the separation between V_{IH} and V_{IL} depends on the slope of VTC in the transition region. If "a" approaches either "0" or infinity, VTC exhibits infinite gain in its transition region. Therefore V_{IL} and V_{IH} Coincide.

32.



$R_p = 2K$

$(W/L)_1 = 3/0.18$

$(W/L)_2 = 5/0.18$

$NM_L,\ NM_H = ?$

To calculate NM_L, M_1 and M_2 are assumed to be in the saturation and triode region respectively.

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p} \quad (V_{in} = V_{IL})$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD}-V_{in}-|V_{TH2}|)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right] =$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2 + \frac{V_{out}}{R_p} \quad (1)$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -1, \quad V_{in} = V_{IL}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[-2(V_{DD}-V_{out}) - 2(V_{DD}-V_{in}-|V_{TH2}|)\frac{\partial V_{out}}{\partial V_{in}} + 2(V_{DD}-V_{out})\frac{\partial V_{out}}{\partial V_{in}}\right] =$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1}) + \frac{1}{R_p}\frac{\partial V_{out}}{\partial V_{in}}$$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{IL}-V_{TH1}) - \frac{1}{R_p} = \mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2V_{OH} - V_{IL} - |V_{TH2}| - V_{DD}\right] \quad (2)$$

$$\boxed{V_{OH} = 101\, V_{IL} + 0.73}$$

Replacing V_{out} in (1) with its equivalent versus V_{IL} obtained from (2) yields:

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD}-V_{IL}-|V_{TH2}|)(V_{DD}-1.1V_{IL}-0.73)-(V_{DD}-1.1V_{IL}-0.73)^2\right]=$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{IL}-V_{TH1})^2+\frac{1.1V_{IL}+0.73}{R_p}$$

$$\frac{1}{2}\times 50\times 10^{-6}\times\frac{5}{0.18}\left[2(1.8-V_{IL}-0.5)(1.8-1.1V_{IL}-0.73)-(1.8-1.1V_{IL}-0.73)^2\right]=$$

$$\frac{1}{2}\times 100\times 10^{-6}\times\frac{3}{0.18}(V_{IL}-0.4)^2+\frac{1.1V_{IL}+0.73}{2000}$$

$$-52.5\times 10^{-3}V_{IL}-0.6195V_{IL}+0.229875=0$$

$$\boxed{V_{IL}=NM_L=0.36\,V}\quad < V_{TH1}\qquad \text{Not Acceptable !}$$

This is less than threshold voltage of M_1; therefore, this answer is not acceptable. It means that M_1 is off and should be left out in this calculation.

$$I_{D1}=0,\quad I_{D2}=\frac{V_{out}}{R_p}$$

$$\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2V_{OH}-V_{IL}-|V_{TH2}|-V_{DD}\right]=-\frac{1}{R_p}$$

$$50\times 10^{-6}\times\frac{5}{0.18}\times\left[2V_{OH}-V_{IL}-0.5-1.8\right]=-\frac{1}{2000}$$

$$\boxed{V_{OH}=V_{out}=0.5V_{IL}+0.97}\quad (3)$$

$$\frac{1}{2}\times 50\times 10^{-6}\times\frac{5}{0.18}\times\left[2(1.8-V_{IL}-0.5)(1.8-0.5V_{IL}-0.97)-(1.8-0.5V_{IL}-0.97)^2\right]=$$

$$\frac{0.5V_{IL}+0.97}{2000}$$

$$0.1875V_{IL}^2-0.6225V_{IL}+0.192675=0$$

$$\boxed{V_{IL}=NM_L=0.345\,V}$$

To determine NM_H, M_1 and M_2 are assumed to operate in the triode and saturation region respectively.

$$I_{D2} = I_{D1} + \frac{V_{out}}{R_p} \quad (V_{in} = V_{IH})$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{TH2}|\right)^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2\left(V_{in} - V_{TH1}\right)V_{out} - V_{out}^2\right] + \frac{V_{out}}{R_p} \quad (4)$$

$$-\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{TH2}|\right) = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2V_{out} + 2\left(V_{in} - V_{TH1}\right)\frac{\partial V_{out}}{\partial V_{in}} - 2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right] + \frac{\partial V_{out}}{\partial V_{in}}\frac{1}{R_p}$$

$$-\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} - |V_{TH2}|\right) = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[V_{out} - V_{in} + V_{TH1} + V_{out}\right] - \frac{1}{R_p}$$

$$-50\times10^{-6}\times\frac{5}{0.18}\times\left(1.8 - V_{in} - 0.5\right) = 100\times10^{-6}\times\frac{3}{0.18}\times\left(2V_{out} - V_{in} + 0.4\right) - \frac{1}{2000}$$

$$\boxed{V_{out} = \frac{0.1V_{in} + 0.59}{1.2}} \quad (5)$$

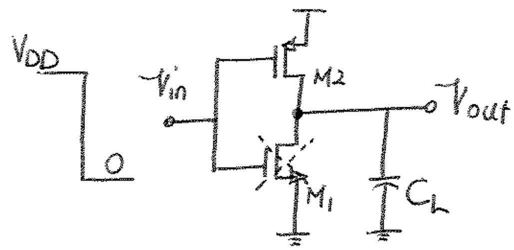
Combining equs (4) and (5) yields:

$$\frac{1}{2}\times50\times10^{-6}\times\frac{5}{0.18}\left(1.8 - V_{in} - 0.5\right)^2 = \frac{1}{2}\times100\times10^{-6}\times\frac{3}{0.18}\left[2\left(V_{in} - 0.4\right)\frac{0.1V_{in}+0.59}{1.2} - \frac{\left(0.1V_{in}+0.59\right)^2}{1.2^2}\right] + \frac{0.1V_{in}+0.59}{1.2\times2000}$$

$$-0.291V_{in}^2 + 1.3182V_{in} - 0.75531 = 0$$

$$V_{in} = V_{IH} = 0.673\,V \longrightarrow \boxed{NM_H = V_{DD} - V_{IH} = 1.127\,V}$$

33.



$V_{out}(t=0)=0$

$(W/L)_2 = 6/0.18$

$C_L = 50fF$

$0 < V_{out} < |V_{TH2}|$; M_2 in the saturation

$$C_L \frac{dV_{out}}{dt} = I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD}-V_{in}-|V_{TH2}|)^2 = \frac{1}{2}\times 50\times10^{-6}\times\frac{6}{0.18}(1.8-0.5)^2$$

$$= 1.4\times10^{-3}A$$

$$V_{out}(t) = \frac{I_{D2}}{C_L}\times t$$

$$|V_{TH2}| = \frac{I_{D2}}{C_L}\cdot T_1 \longrightarrow T_1 = \frac{C_L\times|V_{TH2}|}{I_{D2}} = 50\times10^{-15}\times(1.4\times10^{-3})^{-1}\times0.5$$

$T_1 = 17.75\ pS$

$|V_{TH2}| < V_{out} < V_{DD}/2$, M_2 in triode

$$C_L\frac{dV_{out}}{dt} = I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD}-|V_{TH2}|)(-V_{out}+V_{DD})-(V_{DD}-V_{out})^2\right]$$

$$\frac{dV_{out}}{2(V_{DD}-|V_{TH2}|)(V_{DD}-V_{out})-(V_{DD}-V_{out})^2} = \frac{1}{2}\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt$$

$$\frac{1}{(V_{DD}-V_{out})\left[2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})\right]} = \frac{1}{2(V_{DD}-|V_{TH2}|)}\left[\frac{1}{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}+\frac{1}{V_{DD}-V_{out}}\right]$$

$$\frac{1}{\cancel{2}(V_{DD}-|V_{TH2}|)}\left[\frac{dV_{out}}{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}+\frac{dV_{out}}{V_{DD}-V_{out}}\right]=\frac{1}{\cancel{2}}\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt$$

$$\ln\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}}=\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)t+C$$

$$\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}}=K\cdot\exp\left[\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)t\right]$$

Time origin is assumed to be at $t=T_1=17.75\,pS$

$V_{out}(t=0)=|V_{TH2}| \longrightarrow K=1$

$$\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}}=e^{\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)t}$$

ⓐ $V_{out}=V_{DD}/2$ $\quad T_2=\dfrac{\ln(3-4|V_{TH2}|/V_{DD})}{\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)}$

$$=\frac{\ln(3-4\times0.5/1.8)}{50\times10^{-6}\times\frac{1}{50\times10^{-15}}\times\frac{6}{0.18}\times(1.8-0.5)}$$

$$\boxed{T_2=1.467\times10^{-11}}$$

$T_{0\to V_{DD}/2}=T_1+T_2=17.75+14.67$

$$\boxed{T_{0\to V_{DD}/2}=32.43\,pS}$$

34. $|V_{TH2}| < V_{out} < 0.95 V_{DD}$ M_2 in Triode

$$\frac{2(V_{DD}-|V_{TH2}|)-(V_{DD}-V_{out})}{V_{DD}-V_{out}} = e^{\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD}-|V_{TH2}|)t}$$

ⓐ $V_{out} = 0.95 V_{DD}$, $T_2 = \dfrac{\ln(39-40|V_{TH2}|/V_{DD})}{\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD}-|V_{TH2}|)}$

$$= \frac{\ln(39-40\times 0.5/(1.8))}{50\times10^{-6} \times \frac{1}{50\times10^{-15}} \times \frac{6}{0.18} \times (1.8-0.5)}$$

$$\boxed{T_2 = 7.68\times10^{-11}}$$

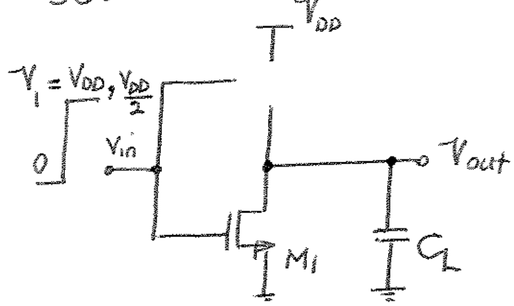
$T_1 = 17.75$ ps from previous problem

$T_{0\to 0.95 V_{DD}} = T_1 + T_2 = 17.75 + 76.8$

$$\boxed{T_{0\to 0.95 V_{DD}} = 94.55 \text{ ps}}$$

$$\boxed{(T_{0\to 0.95 V_{DD}})/(T_{0\to V_{DD}/2}) \approx 3}$$

35.



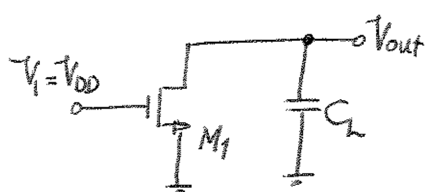
$V_{out}(t=0) = V_{DD}$ $\qquad C_L = 30fF$

$\left(\frac{W}{L}\right)_1 = 1/0.18$ $\qquad T_{V_{DD} \to V_{DD}/2} = ?$

$\left(\frac{W}{L}\right)_2 =$ Not necessary $\qquad V_1 = V_{DD}$ & $V_{DD}/2$

(a) $V_1 = V_{DD}$

$V_{DD} - V_{TH1} \leqslant V_{out} \leqslant V_{DD}$ $\quad M_1$ saturation



$\frac{V_{DD}}{2} \leqslant V_{out} \leqslant V_{DD} - V_{TH1}$ $\quad M_1$ Triode

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18}(1.8 - 0.4)^2 = 5.44 \times 10^{-4} A$$

$$dV_{out} = -\frac{I_{D1}}{C_L} \cdot dt$$

$$V_{out}(t) - V_{DD} = -\frac{I_{D1}}{C_L} t \longrightarrow V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L} t$$

$$T_{V_{DD} \to V_{DD} - V_{TH1}} = \frac{V_{TH1} \times C_L}{I_{D1}} = \frac{0.4 \times 30 \times 10^{-15}}{5.44 \times 10^{-4}} = 2.2 \times 10^{-11} S$$

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1})V_{out} - V_{out}^2\right]$$

$$\frac{dV_{out}}{2(V_{DD} - V_{TH1})V_{out} - V_{out}^2} = -\frac{1}{2}\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 dt$$

$$\frac{1}{\left[2(V_{DD} - V_{TH1}) - V_{out}\right]V_{out}} = \frac{1}{2(V_{DD} - V_{TH1})}\left[\frac{1}{2(V_{DD} - V_{TH1}) - V_{out}} + \frac{1}{V_{out}}\right]$$

$$\frac{1}{\cancel{2}(V_{DD}-V_{TH1})}\left[\frac{dV_{out}}{2(V_{DD}-V_{TH1})-V_{out}}+\frac{dV_{out}}{V_{out}}\right]=-\frac{1}{\cancel{2}}\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 dt$$

$$-Ln\left[2(V_{DD}-V_{TH1})-V_{out}\right]+Ln\,V_{out}=-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})t+C$$

$$\frac{V_{out}}{2(V_{DD}-V_{TH1})-V_{out}}=K\cdot\exp\left[-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})t\right]$$

$V_{out}(t=0)=V_{DD}-V_{TH1}$ Note that time origin is assumed to be 2.2×10^{-11}

$K=1 \longrightarrow$

$$\frac{V_{out}}{2(V_{DD}-V_{TH1})-V_{out}}=e^{-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})t}$$

$$V_{out}=V_{DD}/2\longrightarrow\frac{V_{DD}/2}{2(V_{DD}-V_{TH1})-V_{DD}/2}=e^{-\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})T_{(V_{DD}-V_{TH1})\rightarrow V_{DD}/2}}$$

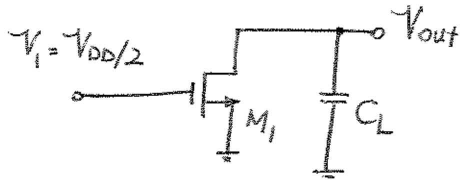
$$T_{(V_{DD}-V_{TH1})\rightarrow V_{DD}/2}=\frac{Ln\left(3-\frac{4V_{TH1}}{V_{DD}}\right)}{\mu_n\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1(V_{DD}-V_{TH1})}$$

$$=\frac{Ln(3-4\times0.4/1.8)}{100\times10^{-6}\times\frac{1}{30\times10^{-15}}\times\frac{1}{0.18}\times(1.8-0.4)}=2.88\times10^{-11}S$$

$$T_{V_{DD}\rightarrow V_{DD}/2}=T_{V_{DD}\rightarrow V_{DD}-V_{TH1}}+T_{(V_{DD}-V_{TH1})\rightarrow V_{DD}/2}$$

$$\boxed{T_{V_{DD}\rightarrow V_{DD}/2}=5\times10^{-11}=50.86\,pS}$$

(b) $V_1 = V_{DD}/2$



$V_{DD}/2 - V_{TH1} < V_{out} < V_{DD}$ M_1 in Saturation

$V_{DD}/2 < V_{out} < V_{DD}$ M_1 in Saturation

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{DD}/2 - V_{TH1}\right)^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18}(0.9-0.4)^2$$

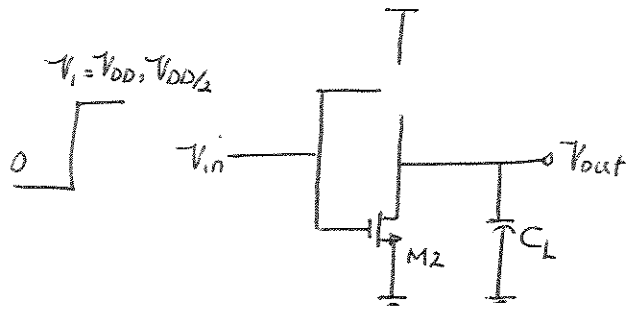
$$= 6.944 \times 10^{-5}$$

$$V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L} \times t$$

$$V_{DD}/2 = V_{DD} - \frac{I_{D1}}{C_L} \times T_{(V_{DD} \to V_{DD}/2)} \rightarrow T_{(V_{DD} \to V_{DD}/2)} = \frac{(V_{DD}/2) \times C_L}{I_{D1}}$$

$$\boxed{T_{(V_{DD} \to V_{DD}/2)} = 3.888 \times 10^{-10}}$$

36.



$V_{out}(0) = V_{DD}$

$(W/L)_1 = 1/0.18$

$C_L = 30fF$

$T_{V_{DD} \to 0.05 V_{DD}} = ?$

(a) $V_1 = V_{DD}$ $\quad V_{DD} - V_{TH1} < V_{out} < V_{DD}$ M_1 in saturation

$0.05 V_{DD} < V_{out} < V_{DD} - V_{TH1}$ M_1 in Triode

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2 = -\frac{1}{2} \times 100 \times 10^{-6} \times \frac{1}{0.18}(1.8 - 0.4)^2$$
$$= 5.44 \times 10^{-4} A$$

$$V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L} \times t$$

$$T_{V_{DD} \to V_{DD} - V_{TH1}} = \frac{V_{TH1} \times C_L}{I_{D1}} = \frac{0.4 \times 30 \times 10^{-15}}{5.44 \times 10^{-4}} = 2.2 \times 10^{-11} S$$

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1})V_{out} - V_{out}^2\right]$$

$$\frac{1}{2(V_{DD} - V_{TH1})}\left[\frac{dV_{out}}{2(V_{DD} - V_{TH1}) - V_{out}} + \frac{dV_{out}}{V_{out}}\right] = -\frac{1}{2}\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 dt$$

$V_{out}(t = 0) = V_{DD} - V_{TH1}$ Note that time origin is assumed to be $2.2 \times 10^{-11} S$

$$\frac{V_{out}}{2(V_{DD} - V_{TH1}) - V_{out}} = e^{-\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1}) t}$$

$V_{out} = 0.05 V_{DD}$

$$\frac{0.05 V_{DD}}{2(V_{DD}-V_{TH1}) - 0.05 V_{DD}} = e^{-\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1}) T_{(V_{DD}-V_{TH1}) \to 0.05 V_{DD}}}$$

$$T_{(V_{DD}-V_{TH1}) \to 0.05V_{DD}} = \frac{Ln(39 - 40 V_{TH1}/V_{DD})}{\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1})}$$

$$= \frac{Ln(39 - 40 \times 0.4/1.8)}{100\times10^{-6} \times \frac{1}{30\times10^{-15}} \times \frac{1}{0.18}(1.8-0.4)}$$

$$T(V_{DD}-V_{TH1}) \to 0.05V_{DD} = 131.33\ pS$$

$$T_{(V_{DD}\to 0.05V_{DD})} = T_{(V_{DD}\to V_{DD}-V_{TH1})} - T_{(V_{DD}-V_{TH1}\to 0.05V_{DD})}$$

$$= 2.2\times10^{-11} + 1.3133\times10^{-10}$$

$$\boxed{T_{(V_{DD}\to 0.05V_{DD})} = 153.33\ pS}$$

(b) $V_1 = V_{DD}/2$ $\quad V_{DD}/2 - V_{TH1} < V_{out} < V_{DD}$ M_1 in Saturation

$0.05 V_{DD} < V_{out} < V_{DD}/2 - V_{TH1}$ M_1 in Triode

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{DD}/2 - V_{TH1}\right)^2 = -\frac{1}{2}\times100\times10^{-6}\times\frac{1}{0.18}(0.9-0.4)^2$$

$$= 6.944\times10^{-5} A$$

$$V_{out}(t) = V_{DD} - \frac{I_{D1}}{C_L}\times t$$

$$V_{DD}/2 - V_{TH1} = V_{DD} - \frac{I_{D1}}{C_L} T_{(V_{DD} \to V_{DD}/2 - V_{TH1})}$$

$$T_{(V_{DD} \to V_{DD}/2 - V_{TH1})} = \frac{(V_{DD}/2 + V_{TH1}) \times C_L}{I_{D1}}$$

$$T_{(V_{DD} \to V_{DD}/2 - V_{TH1})} = 5.616 \times 10^{-10}$$

<u>for $0.05V_{DD} < V_{out} < V_{DD}/2 - V_{TH1}$</u>

$$C_L \frac{dV_{out}}{dt} = -I_{D1} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{DD}/2 - V_{TH1})V_{out} - V_{out}^2\right]$$

$$\frac{V_{out}}{2(V_{DD}/2 - V_{TH1}) - V_{out}} = e^{-\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD}/2 - V_{TH1}) t}$$

$$V_{out} = 0.05V_{DD} \to \frac{0.05V_{DD}}{2(V_{DD}/2 - V_{TH1}) - 0.05V_{DD}} = e^{-\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD}/2 - V_{TH1}) \times T}$$

$$T_{(V_{DD}/2 - V_{TH1} \to 0.05V_{DD})} = \frac{\ln(19 - 40V_{TH1}/V_{DD})}{\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 (V_{DD}/2 - V_{TH1})}$$

$$= \frac{\ln(19 - 40 \times 0.4/1.8)}{100 \times 10^{-6} \times \frac{1}{30 \times 10^{-15}} \times \frac{1}{0.18}(0.9 - 0.4)}$$

$$= 2.5 \times 10^{-10}$$

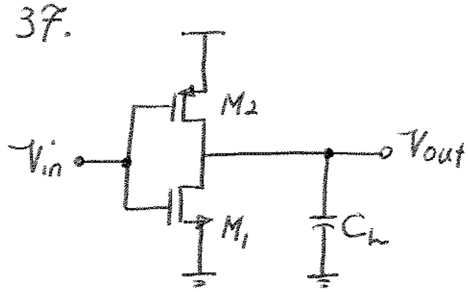
$$T_{(V_{DD} \to 0.05V_{DD})} = T_{(V_{DD} \to V_{DD}/2 - V_{TH1})} + T_{(V_{DD}/2 - V_{TH1} \to 0.05V_{DD})}$$

$$\boxed{T_{(V_{DD} \to 0.05V_{DD})} = 5.616 \times 10^{-10} + 2.5 \times 10^{-10} = 811.5\,ps}$$

By decreasing V_{in} from V_{DD} to $V_{DD}/2$, the time it takes the output to reach $0.05 V_{DD}$ will be 5.3 time larger!

$$\frac{T(V_{DD} \rightarrow 0.05V_{DD})(V_{in} = V_{DD})}{T(V_{DD} \longrightarrow 0.05V_{DD})(V_{in} = V_{DD}/2)} = \frac{811.5p}{153.33p} \approx 5.3$$

37.



$\left(\frac{W}{L}\right)_1 = 1/0.18$

$\left(\frac{W}{L}\right)_2 = 3/0.18$

$C_L = 80fF$

$T_{PHL}, T_{PLH} = ?$

To calculate T_{PLH}

$0 < V_{out} < |V_{TH2}|$ M_2 in Saturation

$|V_{TH2}| < V_{out} < V_{DD}/2$ M_2 in Triode

$$|I_{D2}| = \frac{1}{2}\mu_P C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2$$

$$V_{out}(t) = \frac{|I_{D2}|}{C_L} t$$

$$= \frac{1}{2}\mu_P \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2 t.$$

$$V_{out}(T_{PLH1}) = |V_{TH2}|$$

$$T_{PLH1} = \frac{|V_{TH2}| \times C_L}{\frac{1}{2}\mu_P C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2}.$$

for M_2 operating in Triode region

$$C_L \frac{dV_{out}}{dt} = \frac{1}{2}\mu_P C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right]$$

$$\frac{dV_{out}}{2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2} = \frac{1}{2}\mu_P \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt.$$

Defining $V_{DD} - V_{out} = u$ and noting that $\int \frac{du}{au - u^2} = \frac{1}{a} \ln \frac{u}{a-u}$,

$$\frac{-1}{2(V_{DD} - |V_{TH2}|)} \ln \frac{V_{DD} - V_{out}}{V_{DD} - 2|V_{TH2}| + V_{out}} \Bigg|_{V_{out} = |V_{TH2}|}^{V_{out} = V_{DD}/2} = \frac{1}{2} \mu_p \frac{C_{ox}}{C_L} \left(\frac{W}{L}\right)_2 T_{PLH2}$$

$$T_{PLH2} = \frac{C_L}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \ln \left(3 - 4 \frac{|V_{TH2}|}{V_{DD}}\right)$$

$$T_{PLH} = T_{PLH1} + T_{PLH2} = \frac{C_L}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + \ln \left(3 - 4 \frac{|V_{TH2}|}{V_{DD}}\right)\right]$$

$$T_{PLH} = \frac{80 \times 10^{-15}}{50 \times 10^{-6} \times \frac{3}{0.18} (1.8 - 0.5)} \left[\frac{2 \times 0.5}{1.8 - 0.5} + \ln \left(3 - 4 \frac{0.5}{1.8}\right)\right]$$

$$\boxed{T_{PLH} = 1.0377 \times 10^{-10}}$$

To calculate T_{PHL} $\quad V_{DD} - V_{TH1} < V_{out} < V_{DD}$ $\quad M_1$ in Saturation

$V_{DD}/2 < V_{out} < V_{DD} - V_{TH1}$ $\quad M_1$ in Triode

$$T_{PHL1} = \frac{-\Delta V_{out} \times C_L}{I_{D1}} = \frac{V_{TH1} \times C_L}{\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2}$$

after this point in time.

$$C_L \frac{dV_{out}}{dt} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2(V_{DD} - V_{TH1}) V_{out} - V_{out}^2\right]$$

$$V_{out}(t=0) = V_{DD} - V_{TH1}$$

$$\frac{1}{2(V_{DD}-V_{TH1})} Ln \frac{V_{out}}{2(V_{DD}-V_{TH1})-V_{out}} \Bigg|_{V_{out}=V_{DD}-V_{TH1}}^{V_{out}=V_{DD}/2} = -\frac{1}{2}\mu_n \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_1 T_{HL2}$$

$$T_{PHL2} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1})} \times Ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)$$

$$T_{PHL} = T_{PHL1} + T_{PHL2} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1})} \times \left[\frac{2V_{TH1}}{V_{DD}-V_{TH1}} + Ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$T_{PHL} = \frac{80\times10^{-15}}{100\times10^{-6}\times\frac{1}{0.18}(1.8-0.4)} \times \left[\frac{2\times0.4}{1.8-0.4} + Ln\left(3-4\frac{0.4}{1.8}\right)\right]$$

$$\boxed{T_{PHL} = 1.3563\times10^{-10}}$$

38. $V_{DD} = 1.8 + 1.8 \times 0.1 = 1.98$

$$T_{PLH} = \frac{C_L}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right) \right]$$

$$= \frac{80 \times 10^{-15}}{50 \times 10^{-6} \times \frac{3}{0.18} \times (1.98 - 0.5)} \times \left[\frac{2 \times 0.5}{1.98 - 0.5} + \ln\left(3 - 4 \times \frac{0.5}{1.98}\right) \right]$$

$$\boxed{T_{PLH} = 8.846 \times 10^{-11}}$$

$$\text{Decrease in } T_{PLH} = \left| \frac{8.846 \times 10^{-11} - 1.0377 \times 10^{-10}}{1.0377 \times 10^{-10}} \right| \times 100$$

$$= 14.75\%$$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \times \left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right) \right]$$

$$= \frac{80 \times 10^{-15}}{100 \times 10^{-6} \times \frac{1}{0.18} \times (1.98 - 0.4)} \left[\frac{2 \times 0.4}{1.98 - 0.4} + \ln\left(3 - 4\frac{0.4}{1.98}\right) \right]$$

$$\boxed{T_{PHL} = 1.1767 \times 10^{-10}}$$

$$\text{Decrease in } T_{PHL} = \left| \frac{1.1767 \times 10^{-10} - 1.3563 \times 10^{-10}}{1.3563 \times 10^{-10}} \right| \times 100$$

$$= 13.24\%$$

39. $V_{DD} = 0.9\,V$

$$C_L \frac{dV_{out}}{dt} = I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)^2$$

$$T_{PLH} = \frac{\Delta V_{out} \times C_L}{I_{D2}} = \frac{\left(V_{DD}/2\right) \times C_L}{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)^2}$$

$$= \frac{0.45 \times 80 \times 10^{-15}}{\frac{1}{2} \times 50 \times 10^{-6} \times \frac{3}{0.18} \times (0.9 - 0.5)^2}$$

$$\boxed{T_{PLH} = 5.4 \times 10^{-10} = 540\,pS}$$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{DD} - V_{TH1}\right)} \left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$= \frac{80 \times 10^{-15}}{100 \times 10^{-6} \times \frac{1}{0.18} \times (0.9 - 0.4)} \times \left[\frac{2 \times 0.4}{0.9 - 0.4} + \ln\left(3 - 4\frac{0.4}{0.9}\right)\right]$$

$$\boxed{T_{PHL} = 5.186 \times 10^{-10} = 518.6\,pS}$$

$$\text{Increase in } T_{PLH} = \left|\frac{5.4 \times 10^{-10} - 1.0377 \times 10^{-10}}{1.0377 \times 10^{-10}}\right| \times 100$$

$$= 420.38\,\%$$

$$\text{Increase in } T_{PHL} = \left|\frac{5.186 \times 10^{-10} - 1.3563 \times 10^{-10}}{1.3563 \times 10^{-10}}\right| \times 100$$

$$= 282.36\,\%$$

40. $T_{PLH} = T_{PHL} = 80\,pS$

$C_L = 50\,fF$

$(W/L)_1 \;,\; (W/L)_2 = ?$

$$T_{PLH} = \frac{C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - |V_{TH2}|\right)}\left[\frac{2|V_{TH2}|}{V_{DD}-|V_{TH2}|} + \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right)\right]$$

$$80\times10^{-12} = \frac{50\times10^{-15}}{50\times10^{-6}\times(1.8-0.5)\times\left(\frac{W}{L}\right)_2}\times\left[\frac{2\times0.5}{1.8-0.5} + \ln\left(3-4\times\frac{0.5}{1.8}\right)\right]$$

$$\boxed{\left(\frac{W}{L}\right)_2 = \frac{2.4}{0.18}}$$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(V_{DD} - V_{TH1}\right)}\times\left[\frac{2V_{TH1}}{V_{DD}-V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$$80\times10^{-12} = \frac{50\times10^{-15}}{100\times10^{-6}\times(1.8-0.4)\times\left(\frac{W}{L}\right)_1}\times\left[\frac{2\times0.4}{1.8-0.4} + \ln\left(3-4\times\frac{0.4}{1.8}\right)\right]$$

$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{1}{0.18}}$$

41.

$$\tau_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD}-V_{TH1})}\left[\frac{2V_{TH1}}{V_{DD}-V_{TH1}} + \ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$V_{TH1} = 0.4$

$$\frac{2V_{TH1}}{V_{DD}-V_{TH1}} = \ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right) \longrightarrow V_{DD} = V_{TH1}\left[1+\frac{2}{\ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)}\right]$$

$V_{TH1} = 0.4 \longrightarrow \boxed{V_{DD} = 1.57}$

$$\frac{2V_{TH1}}{V_{DD}-V_{TH1}} = 0.1 \times \ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right) \longrightarrow V_{DD} = V_{TH1}\left[1+\frac{20}{\ln\left(3-4\frac{V_{TH1}}{V_{DD}}\right)}\right]$$

$V_{TH1} = 0.4 \longrightarrow \boxed{V_{DD} = 8.16}$

42. $\left(\frac{W}{L}\right)_1 = 1/0.18$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})}\left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$T_{PHL} = 100\,pS$

$C_L = 80\,fF$

$V_{DD} = ?$

$$100\times10^{-12} = \frac{80\times10^{-15}}{100\times10^{-6}\times\frac{1}{0.18}\times(V_{DD}-0.4)}\times\left[\frac{2\times0.4}{V_{DD}-0.4} + \ln\left(3-4\frac{0.4}{V_{DD}}\right)\right]$$

$$V_{DD} = 0.4 + 1.44\left[\frac{0.8}{V_{DD}-0.4} + \ln\left(3 - \frac{1.6}{V_{DD}}\right)\right]$$

$$\boxed{V_{DD} = 2.22}$$

43. $T_{PHL} = 120\ ps$ $\quad (W/L)_1 = ?$

$C_L = 90\ fF$ $\quad V_{TH1} = ?$

$V_{DD} = 1.8$

$T_{PHL} = 160\ ps$

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})}\left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$V_{DD} = 1.5\ V$

$C_L = 90\ fF$

$$120 \times 10^{-12} = \frac{90 \times 10^{-15}}{100 \times 10^{-6}\left(\frac{W}{L}\right)_1 (1.8 - V_{TH1})} \times \left[\frac{2V_{TH1}}{1.8 - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{1.8}\right)\right] \quad (1)$$

$$160 \times 10^{-12} = \frac{90 \times 10^{-15}}{100 \times 10^{-6}\left(\frac{W}{L}\right)_1 (1.5 - V_{TH1})} \times \left[\frac{2V_{TH1}}{1.5 - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)\right] \quad (2)$$

Dividing Equations (1) and (2) yields:

$$0.75 = \frac{1.5 - V_{TH1}}{1.8 - V_{TH1}} \times \frac{\frac{2V_{TH1}}{1.8 - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{1.8}\right)}{\frac{2V_{TH1}}{1.5 - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)}$$

$$V_{TH1} = 1.8 - \left(\frac{1.5 - V_{TH1}}{0.75}\right) \times \frac{\frac{2V_{TH1}}{1.8 - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{1.8}\right)}{\frac{2V_{TH1}}{1.5 - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)}$$

This equation does not lead to a real Value for V_{TH1} so we use another derivation

$$V_{TH1} = 0.45 \times \left\{3 - e^{\left[0.75\frac{1.8 - V_{TH1}}{1.5 - V_{TH1}} \times \left[\frac{2V_{TH1}}{1.5 - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{1.5}\right)\right] - \frac{2V_{TH1}}{1.8 - V_{TH1}}\right]}\right\}$$

$$V_{TH1} = 0.39$$

$$\left(\frac{W}{L}\right)_1 = \frac{1.26}{0.18}$$

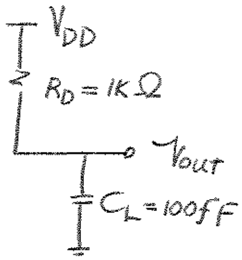
44.

$$T_{PHL} = \frac{C_L}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})}\left[\frac{2V_{TH1}}{V_{DD} - V_{TH1}} + \ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)\right]$$

$\ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)$ is meaningless if $V_{DD} < 4V_{TH1}/3$.

Let's consider the case where $V_{DD} = \frac{4}{3}V_{TH1}$; then, T_{PHL} is the time it takes for the output to drop from $V_{DD} = \frac{4}{3}V_{TH1}$ to $\frac{V_{DD}}{2} = \frac{2}{3}V_{TH1}$. However, $\left(V_{in}^{o} = V_{DD} = \frac{4}{3}V_{TH1}\right) - \left(V_{out} = \frac{2}{3}V_{TH1}\right) = \frac{2}{3}V_{TH1} < V_{TH1}$. In other words, M_1 never enters the triode region in the region where T_{PHL} is calculated. The logrithmic term is derived from equation in which M_1 was assumed to be in Triode region. Therefore the logarithmic term is meaningless for $V_{DD} < \frac{4}{3}V_{TH1}$.

45.



$$V_{R_D} = (V_{DD} - V_{out})$$

$$I_{R_D} = C_L \frac{dV_{out}}{dt}$$

$$P_{R_D}(t) = V_{R_D} \cdot I_{R_D} = C_L (V_{DD} - V_{out}) \frac{dV_{out}}{dt}$$

$$E_{R_D} = \int_{t=0}^{\infty} P_{R_D}(t)\, dt = \int_{V_{out}=0}^{V_{DD}} (V_{DD} - V_{out})\, dV_{out} = \frac{1}{2} C_L V_{DD}^2$$

$$= \frac{1}{2} \times 100 \times 10^{-15} \times (1.8)^2$$

$$\boxed{E_{R_D} = 0.162\ pJ}$$

46. 10^6 Gates

$f = 2\,GHz$

20% of gates switch in every clock cycle

$C_L = 20fF$ for each gate

$P_{av} = ?$

$$P_{av,\,gate} = f_{in} C_L V_{DD}^2$$

$$P_{av,\,total} = 0.2 \times 10^6 \times f_{in} C_L V_{DD}^2$$

$$= 0.2 \times 10^6 \times 2 \times 10^9 \times 20 \times 10^{-15} \times (1.8)^2$$

$$\boxed{P_{av,\,total} = 25.92\ W}$$

47. $f = 2\ GHz$

5×10^6 Transistors with $W = 1\ \mu m$, $L = 0.18\ \mu m$, $C_{ox} = 10\ fF/\mu m^2$

$$C_{gate} = WLC_{ox}$$

$$C_{Load} = 5\times10^6\ C_{gate}$$

$$= 5\times10^6 \times WLC_{ox}$$

$$= 5\times10^6 \times 1\ \mu m \times 0.18\ \mu m \times 10\ fF/\mu m^2$$

$$\boxed{C_{Load} = 9\ pF}$$

$$P_{av} = f_{in} C_L V_{DD}^2$$

$$= 2\times10^9 \times 9\times10^{-9} \times (1.8)^2$$

$$\boxed{P_{av} = 58.32\ W}$$

48.

$$V_{DD} = V_{DD} + 0.1\,V_{DD} = 1.98$$

$$\left(\frac{W}{L}\right)_1 = 2/0.18$$

$$\left(\frac{W}{L}\right)_2 = 4/0.18$$

$$I_{Peak}\Big|_{V_{DD}=1.8} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left(\frac{V_{DD}}{2} - V_{TH1}\right)^2\left(1 + \lambda_1\frac{V_{DD}}{2}\right)$$

$$= \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right)\left(0.9 - 0.4\right)^2$$

$$I_{Peak}\Big|_{V_{DD}=1.8} = 1.388 \times 10^{-4}$$

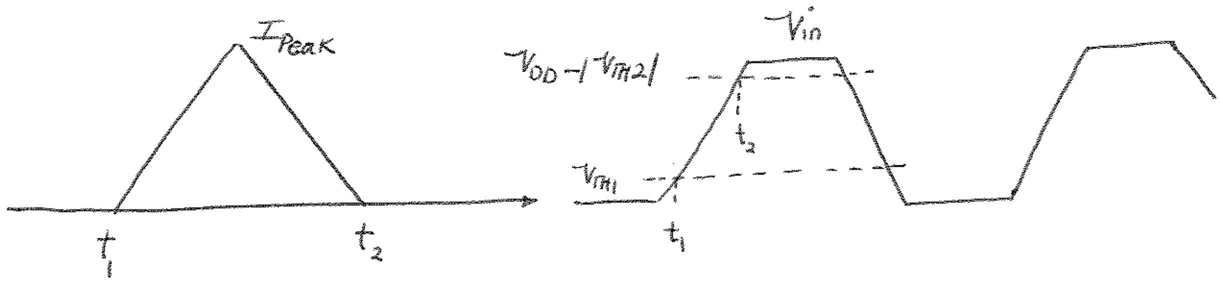
$$I_{Peak}\Big|_{V_{DD}=1.98} = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{2}{0.18}\right)\left(0.99 - 0.4\right)^2$$

$$I_{Peak}\Big|_{V_{DD}=1.98} = 1.9338 \times 10^{-4}$$

$$\text{Change in Crowbar Current} = \frac{1.9338 \times 10^{-4} - 1.388 \times 10^{-4}}{1.388 \times 10^{-4}}$$

Change in Crowbar Current = 39.24%

49.



Total Energy drawn from V_{DD} during the interval $[t_1, t_2]$ is:

$$E = V_{DD} \times I_{Peak} \times \frac{t_2 - t_1}{2}$$

In a periode the total energy is:

$$E_{tot} = 2 \times V_{DD} \times I_{Peak} \times \frac{t_2 - t_1}{2}$$

$$P_{av} = V_{DD} I_{Peak} (t_2 - t_1) f_1$$

$$\text{Slope of input voltage} = \frac{0.9V_{DD} - 0.1V_{DD}}{t_r} = \frac{0.8V_{DD}}{t_r}$$

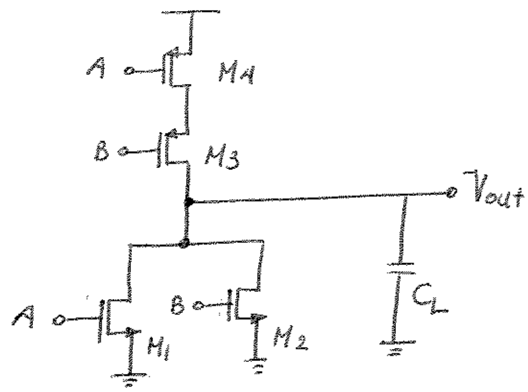
$$(t_2 - t_1) = \frac{(V_{DD} - V_{TH1} - |V_{TH2}|)}{0.8V_{DD}} \times t_r$$

$$P_{av} = V_{DD} \times \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(\frac{V_{DD}}{2} - V_{TH1}\right)^2 \times \frac{(V_{DD} - V_{TH1} - |V_{TH2}|)}{0.8V_{DD}} t_r \times f_1$$

$$P_{av} = \frac{1}{1.6} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(\frac{V_{DD}}{2} - V_{TH1}\right)^2 (V_{DD} - V_{TH1} - |V_{TH2}|) f_1 \cdot t_r$$

$$\boxed{P_{av} = 1.4 \times 10^{-5} \left(\frac{W}{L}\right)_1 \times t_r \times f_1}$$

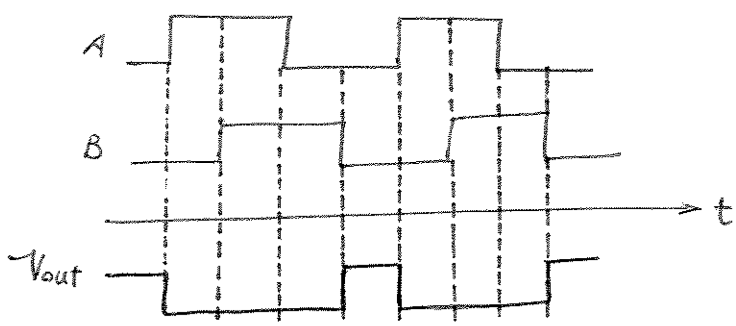
50.



$C_L = 20\,fF$

$f_1 = 500\,MHz$

$P_{av} = ?$

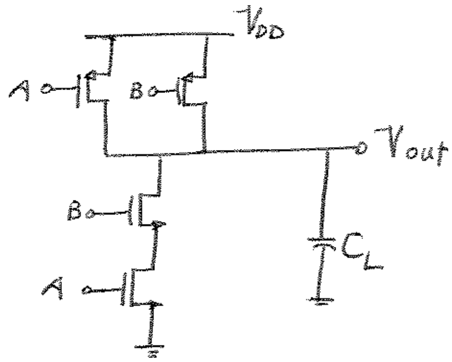


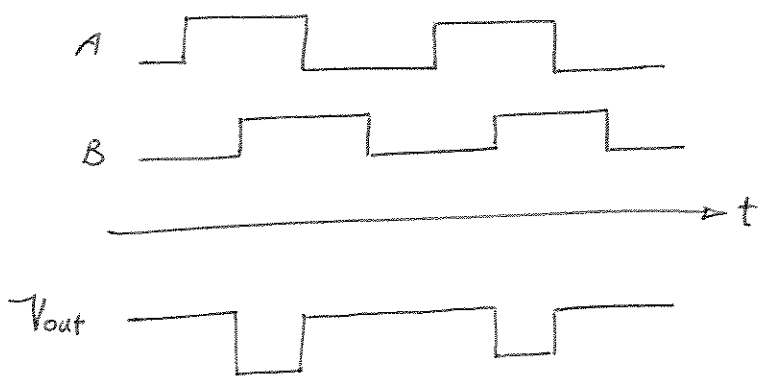
$$P_{av} = f_{in}\, C_L\, V_{DD}^2$$

$$= 500\times10^{6} \times 20\times10^{-15} \times (1.8)^2$$

$$\boxed{P_{av} = 3.24\times10^{-5}\,W}$$

51.



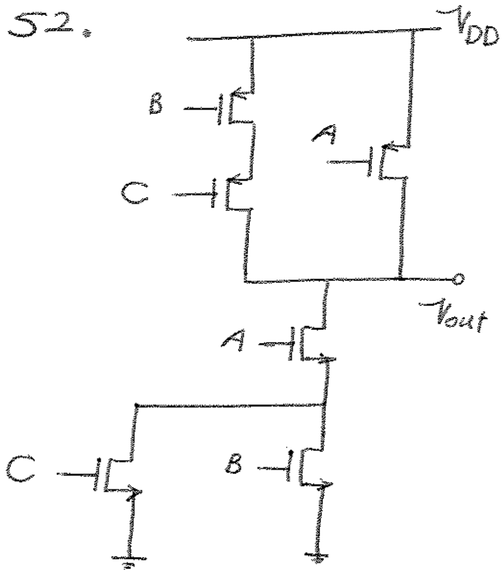


$$P_{av} = f_{in} C_L V_{DD}^2$$

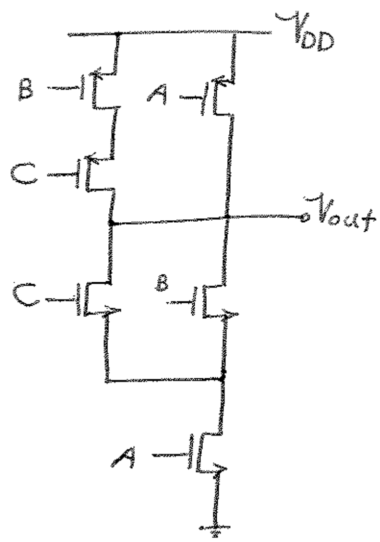
$$= 500 \times 10^6 \times 20 \times 10^{-15} \times (1.8)^2$$

$$\boxed{P_{av} = 3.24 \times 10^{-5}\,W}$$

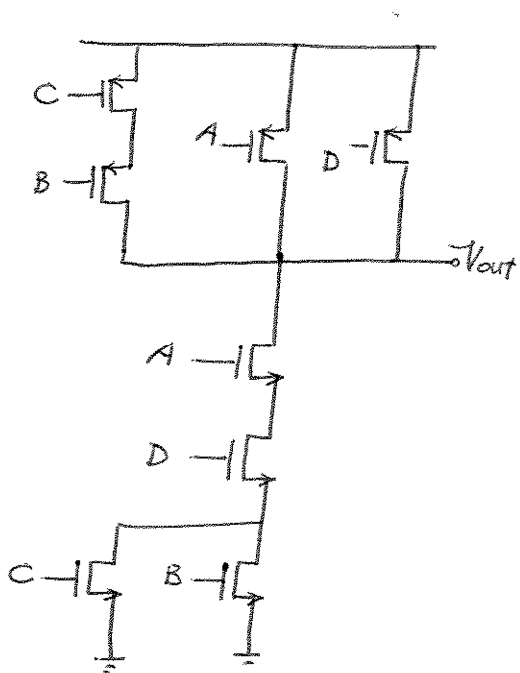
52.



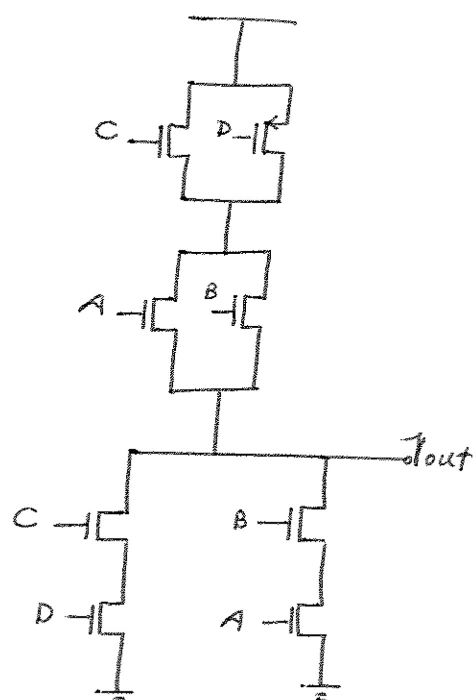
$V_{out} = \overline{(B+C)A}$



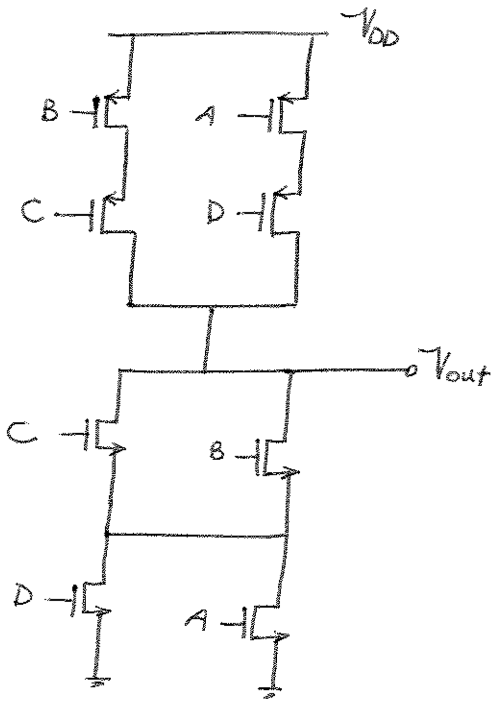
$V_{out} = \overline{(B+C)\cdot A}$



$V_{out} = \overline{(B+C)\,D\cdot A}$

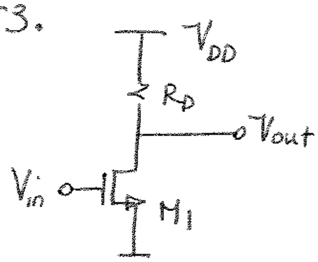


$V_{out} = \overline{A\cdot B + C\cdot D}$



$$V_{out} = \overline{(A+D)\cdot(B+C)}$$

53.



$P_{static} = 0.5\,mW$

$V_{OL} = 100\,mV$

$$\frac{(V_{DD}-V_{OL})^2}{R_D} + V_{OL} \times \frac{V_{DD}-V_{OL}}{R_D} = 0.5 \times 10^{-3}$$

$$\frac{(1.8-0.1)^2}{R_D} + 0.1 \times \frac{1.8-0.1}{R_D} = 0.5 \times 10^{-3}$$

$$\frac{1}{R_D} \times 3.06 = 0.5 \times 10^{-3}$$

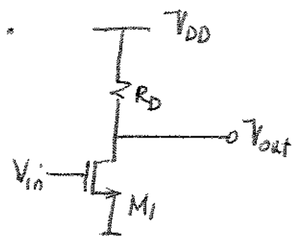
$$\boxed{R_D = 6120\,\Omega}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{DD}-V_{TH1})V_{OL} - V_{OL}^2\right] = \frac{V_{DD}-V_{OL}}{R_D}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 \times \left[2(1.8-0.4)0.1 - 0.1^2\right] = \frac{1.8-0.1}{6120}$$

$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{3.7}{0.18}}$$

54.



$P_{Static} = 0.25\ mW$

$NM_L = 600\ mV$

Small signal gain $= -g_m R_D$

$g_m = \mu_n C_{ox} (W/L)(V_{GS} - V_{TH})$

$\mu_n C_{ox} \frac{W}{L} (V_{IL} - V_{TH1}) R_D = 1$

$$V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) R_D} + V_{TH1}$$

$$NM_L = V_{IL} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) R_D} + V_{TH1}$$

$$\frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) R_D} = (NM_L - V_{TH1}) \rightarrow \left(\frac{W}{L}\right) R_D = \frac{1}{\mu_n C_{ox} (NM_L - V_{TH1})}$$

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) \left[2(V_{DD} - V_{TH1}) V_{OL} - V_{OL}^2 \right] = \frac{V_{DD} - V_{OL}}{R_D}$$

$$\frac{1}{2} \mu_n C_{ox} \frac{1}{\mu_n C_{ox} (NM_L - V_{TH1})} \times \left[2(V_{DD} - V_{TH1}) V_{OL} - V_{OL}^2 \right] = (V_{DD} - V_{OL})$$

$$2(V_{DD} - V_{TH1}) V_{OL} - V_{OL}^2 = 2(NM_L - V_{TH1})(V_{DD} - V_{OL})$$

$$V_{OL}^2 - 2(V_{DD} - V_{TH1}) V_{OL} - 2(NM_L - V_{TH1}) V_{OL} + 2(NM_L - V_{TH1}) V_{DD} = 0$$

$$V_{OL}^2 - 2(V_{DD} + NM_L - 2V_{TH1}) V_{OL} + 2(NM_L - V_{TH1}) V_{DD} = 0$$

$$V_{OL}^2 - 3.2 V_{OL} + 0.72 = 0$$

$$\boxed{V_{OL} = 0.2435}$$

$$\frac{(V_{DD} - V_{OL})^2}{R_D} + V_{OL} \times \frac{V_{DD} - V_{OL}}{R_D} = 0.25 \times 10^{-3}$$

$$\frac{(1.8 - 0.24)^2 + 0.24 \times (1.8 - 0.24)}{R_D} = 0.25 \times 10^{-3}$$

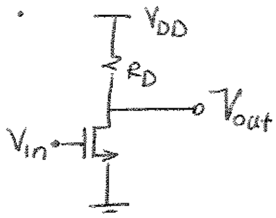
$$\boxed{R_D = 11206.55\ \Omega}$$

$$\left(\frac{W}{L}\right) = \frac{1}{\mu_n C_{ox} (NM_L - V_{TH1}) R_D}$$

$$\left(\frac{W}{L}\right) = \frac{1}{100 \times 10^{-6} (0.6 - 0.4)\, 11206.55}$$

$$\boxed{\left(\frac{W}{L}\right) = \frac{0.8}{0.18}}$$

55.



$V_{OL} = 100mV$ $\quad P_{av} = 0.25mW$

$$\frac{(V_{DD}-V_{OL})^2}{R_D} + \frac{V_{OL}(V_{DD}-V_{OL})}{R_D} = P_{av}$$

$$\frac{(1.8-0.1)^2 + 0.1 \times (1.8-0.1)}{0.25 \times 10^{-3}} = R_D$$

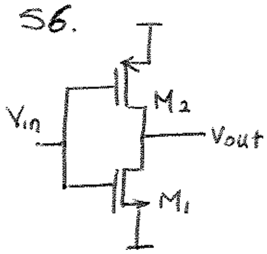
$$\boxed{R_D = 12240}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[2(V_{DD}-V_{TH})V_{OL} - V_{OL}^2\right] = \frac{V_{DD}-V_{OL}}{R_D}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right) \times \left[2(1.8-0.4) \times 0.1 - 0.1^2\right] = \frac{1.8-0.1}{12240}$$

$$\boxed{\left(\frac{W}{L}\right) = \frac{1.85}{0.18}}$$

56.



$V_{in} = V_{out} = 0.8\,V$, $I_{D1} = I_{D2} = 0.5\,mA$

$\lambda_n = 0.1\,V^{-1}$

$\lambda_p = 0.2\,V^{-1}$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 (1 + \lambda_N V_{out}) = I_{D1}$$

$$\frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 (0.8 - 0.4)^2 (1 + 0.1 \times 0.8) = 0.5 \times 10^{-3}$$

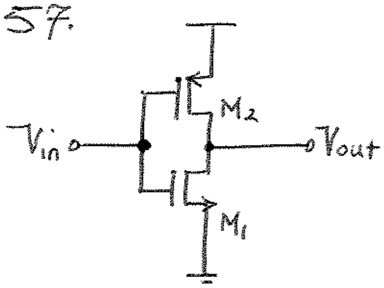
$$\boxed{\left(\frac{W}{L}\right)_1 = \frac{10.4}{0.18}}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} - |V_{TH2}|)^2 \left[1 + \lambda_p (V_{DD} - V_{out})\right] = I_{D2}$$

$$\frac{1}{2} \times 50 \times 10^{-6} \times \left(\frac{W}{L}\right)_2 (1.8 - 0.8 - 0.5)^2 \left[1 + 0.2 \times (1.8 - 0.8)\right] = 0.5 \times 10^{-3}$$

$$\boxed{\left(\frac{W}{L}\right)_2 = \frac{12}{0.18}}$$

57.



$NM_L = NM_H = 0.7\,V$

NM_L : M_1 in Saturation and M_2 in triode

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left[2(V_{DD}-V_{in}-|V_{TH2}|)(V_{DD}-V_{out}) - (V_{DD}-V_{out})^2\right] \quad (1)$$

Differentiating both sides with respect to V_{in}

$$2\mu_n\left(\frac{W}{L}\right)_1 (V_{in}-V_{TH1}) = \mu_p\left(\frac{W}{L}\right)_2 \left[-2(V_{DD}-V_{out}) - 2(V_{DD}-V_{in}-|V_{TH2}|)\frac{\partial V_{out}}{\partial V_{in}} + 2(V_{DD}-V_{out})\frac{\partial V_{out}}{\partial V_{in}}\right]$$

(a) $V_{in} = V_{IL}$, $\frac{\partial V_{out}}{\partial V_{in}} = -1$

$$\mu_n\left(\frac{W}{L}\right)_1 (V_{IL}-V_{TH1}) = \mu_p\left(\frac{W}{L}\right)_2 \left(2V_{OH} - V_{IL} - |V_{TH2}| - V_{DD}\right) \quad (2)$$

obtaining V_{OH} from (2), substituting in (1), we arrive at

$$V_{IL} = \frac{2\sqrt{a}\,(V_{DD}-V_{TH1}-|V_{TH2}|)}{(a-1)\sqrt{a+3}} - \frac{V_{DD}-aV_{TH1}-|V_{TH2}|}{a-1}$$

$$a = \frac{\mu_n\left(\frac{W}{L}\right)_1}{\mu_p\left(\frac{W}{L}\right)_2}$$

NM_H, M_1 in triode and M_2 in Saturation

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[2(V_{in}-V_{TH1})V_{out}-V_{out}^2\right]=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2(V_{DD}-V_{in}-|V_{TH2}|)^2$$

Differentiating both sides with respect to V_{in}:

$$\mu_n\left(\frac{W}{L}\right)_1\left[2V_{out}+2(V_{in}-V_{TH1})\frac{\partial V_{out}}{\partial V_{in}}-2V_{out}\frac{\partial V_{out}}{\partial V_{in}}\right]=2\mu_p\left(\frac{W}{L}\right)_2\times(V_{in}-V_{DD}-|V_{TH2}|)$$

Assuming $\frac{\partial V_{out}}{\partial V_{in}}=-1$, $V_{in}=V_{IH}$, and $V_{out}=V_{OL}$ obtaining

$$V_{IH}=\frac{2a(V_{DD}-V_{TH1}-|V_{TH2}|)}{(a-1)\sqrt{1+3a}}-\frac{V_{DD}-aV_{TH1}-|V_{TH2}|}{a-1}$$

$$V_{IL}=NM_L=0.7$$

$$V_{IH}=V_{DD}-NM_H=1.8-0.7=1.1$$

$$0.7=\frac{2\sqrt{a}(1.8-0.4-0.5)}{(a-1)\sqrt{a+3}}-\frac{1.8-0.4a-0.5}{a-1}$$

$$0.7(a-1)=\frac{1.8\sqrt{a}}{\sqrt{a+3}}-\frac{1.3-0.4a}{1}$$

$$0.7a-0.7+1.3-0.4a=\sqrt{\frac{a}{a+3}}\times 1.8$$

$$\frac{0.6+0.3a}{1.8}=\sqrt{\frac{a}{a+3}}\longrightarrow a^3+7a^2-20a+12=0$$

$$a=\begin{cases}-9.3\\1.3\\1\end{cases}\longrightarrow \boxed{a=1.3}$$

$$1.1 = \frac{2a(1.8-0.4-0.5)}{(a-1)\sqrt{1+3a}} - \frac{1.8-0.4a-0.5}{a-1}$$

$$1.1(a-1) = \frac{1.8a}{\sqrt{1+3a}} - 1.3 + 0.4a$$

$$1.1a - 1.1 + 1.3 - 0.4a = \frac{1.8a}{\sqrt{1+3a}}$$

$$0.2 + 0.7a = \frac{1.8a}{\sqrt{1+3a}}$$

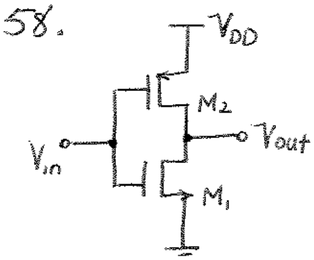
$$147a^3 - 191a^2 + 40a + 4 = 0 \longrightarrow \begin{cases} a_1 = 1 \\ a_2 = 0.37 \\ a_3 = -0.073 \end{cases} \longrightarrow \boxed{a = 0.37}$$

No it is not possible to design a CMOS inverter with $NM_L = NM_H = 0.7$.

The reason is that each value of $a = \frac{\mu_n C_{ox} (W/L)_1}{\mu_p C_{ox} (W/L)_2}$ specifies a unique set of noise margins (NM_L, NM_H).

Remember, the relative strength of NMOS and PMOS determines the noise margins interdependently.

58.



$T_{PLH} = T_{PHL} = 100\,ps$

$C_L = 50\,fF$

T_{PLH}

$$|I_{D2}| = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2$$

$$V_{out}(t) = \frac{|I_{D2}|}{C_L}t$$

$$= \frac{1}{2}\mu_p \frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2 t.$$

$$T_{PLH1} = \frac{2|V_{TH2}|C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2}.$$

$$|I_{D2}| = C_L \frac{dV_{out}}{dt}$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2\right] = C_L\frac{dV_{out}}{dt}$$

$$\frac{dV_{out}}{2(V_{DD} - |V_{TH2}|)(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2} = \frac{1}{2}\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 dt$$

$$\frac{-1}{2(V_{DD} - |V_{TH2}|)} \ln \frac{V_{DD} - V_{out}}{V_{DD} - 2|V_{TH2}| + V_{out}}\Bigg|_{V_{out} = |V_{TH2}|}^{V_{out} = V_{DD}/2} = \frac{1}{2}\mu_p\frac{C_{ox}}{C_L}\left(\frac{W}{L}\right)_2 T_{PLH2}$$

$$T_{PLH2} = \frac{C_L}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right)$$

$$T_{PLH} = T_{PLH1} + T_{PLH2}$$

$$= \frac{C_L}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)} \left[\frac{2|V_{TH2}|}{V_{DD} - |V_{TH2}|} + Ln\left(3 - 4\frac{|V_{TH2}|}{V_{DD}}\right) \right]$$

$$100 \times 10^{-12} = \frac{50 \times 10^{-15}}{50 \times 10^{-6} \left(\frac{W}{L}\right)_2 (1.8 - 0.5)} \left[\frac{2 \times 0.5}{1.8 - 0.5} + Ln\left(3 - 4\frac{0.5}{1.8}\right) \right]$$

$$\boxed{\left(\frac{W}{L}\right)_2 = \frac{1.9}{0.18}}$$

T_{PHL}

$$T_{PHL1} = \frac{2 V_{TH1} C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})^2}$$

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[2 (V_{DD} - V_{TH1}) V_{out} - V_{out}^2 \right] = -C_L \frac{dV_{out}}{dt}$$

$$\frac{-1}{2(V_{DD} - V_{TH1})} Ln \frac{V_{out}}{2(V_{DD} - V_{TH1}) - V_{out}} \Bigg|_{V_{out} = V_{DD} - V_{TH1}}^{V_{out} = V_{DD}/2} = \frac{1}{2} \mu_n C_{ox}/C_L \left(\frac{W}{L}\right)_1 T_{PHL2}.$$

$$T_{PHL2} = \frac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right)$$

$$T_{PHL} = T_{PHL1} + T_{PHL2} = \frac{C_L}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{DD} - V_{TH1})} \left[\frac{2 V_{TH1}}{V_{DD} - V_{TH1}} + Ln\left(3 - 4\frac{V_{TH1}}{V_{DD}}\right) \right]$$

$$100 \times 10^{-12} = \frac{50 \times 10^{-15}}{100 \times 10^{-6} \times \left(\frac{W}{L}\right)_1 \times (1.8 - 0.4)} \times \left[\frac{2 \times 0.4}{1.8 - 0.4} + Ln\left(3 - 4 \times \frac{0.4}{1.8}\right) \right], \quad \boxed{\left(\frac{W}{L}\right)_1 = \frac{0.85}{0.18}}$$

Razavi 1e – Fundamentals of Microelectronics

CHAPTER 16 SOLUTIONS MANUAL

***For **Chapter 16** solutions, please refer to **Chapter 7** as the questions are identical in each chapter.