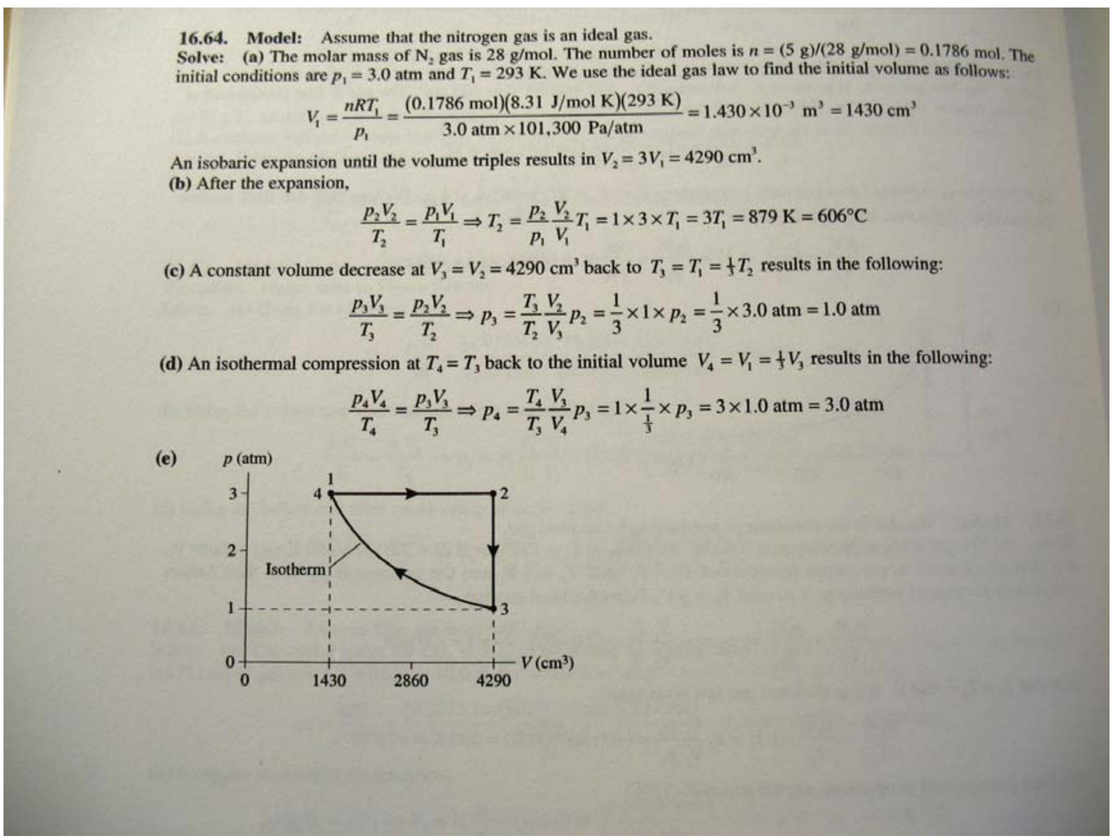
Course PHYSICS260

Assignment 5

Consider ten grams of nitrogen gas at an initial pressure of 6.0 atm and at room temperature.
It undergoes an isobaric expansion resulting in a quadrupling of its volume.
(i) After this expansion, what is the gas volume?
(ii) Determine the gas temperature after this step.
In the next process, the gas pressure is decreased at constant volume until the original temperature is reached.
(iii) After this decrease in gas pressure, what is the value of the pressure?
In the final process, the gas is retured to its initial volume by isothermally compressing it.
(iv) Determine the final gas pressure.
(v) Using appropriate scales on both axes, show the full three-step process on a p − V diagram.



16.64. Model: Assume that the nitrogen gas is an ideal gas.
Solve: **(a)** The molar mass of N_2 gas is 28 g/mol. The number of moles is $n = (5\text{ g})/(28\text{ g/mol}) = 0.1786$ mol. The initial conditions are $p_1 = 3.0$ atm and $T_1 = 293$ K. We use the ideal gas law to find the initial volume as follows:

$$V_1 = \frac{nRT_1}{p_1} = \frac{(0.1786\text{ mol})(8.31\text{ J/mol K})(293\text{ K})}{3.0\text{ atm} \times 101{,}300\text{ Pa/atm}} = 1.430 \times 10^{-3}\text{ m}^3 = 1430\text{ cm}^3$$

An isobaric expansion until the volume triples results in $V_2 = 3V_1 = 4290\text{ cm}^3$.
(b) After the expansion,

$$\frac{p_2V_2}{T_2} = \frac{p_1V_1}{T_1} \Rightarrow T_2 = \frac{p_2}{p_1}\frac{V_2}{V_1}T_1 = 1 \times 3 \times T_1 = 3T_1 = 879\text{ K} = 606°\text{C}$$

(c) A constant volume decrease at $V_3 = V_2 = 4290\text{ cm}^3$ back to $T_3 = T_1 = \frac{1}{3}T_2$ results in the following:

$$\frac{p_3V_3}{T_3} = \frac{p_2V_2}{T_2} \Rightarrow p_3 = \frac{T_3}{T_2}\frac{V_2}{V_3}p_2 = \frac{1}{3} \times 1 \times p_2 = \frac{1}{3} \times 3.0\text{ atm} = 1.0\text{ atm}$$

(d) An isothermal compression at $T_4 = T_3$ back to the initial volume $V_4 = V_1 = \frac{1}{3}V_3$ results in the following:

$$\frac{p_4V_4}{T_4} = \frac{p_3V_3}{T_3} \Rightarrow p_4 = \frac{T_4}{T_3}\frac{V_3}{V_4}p_3 = 1 \times \frac{1}{\frac{1}{3}} \times p_3 = 3 \times 1.0\text{ atm} = 3.0\text{ atm}$$

(e)

Introduction to the Ideal Gas Law

Description: Practice using the ideal gas law with a series of questions in which all but two gas parameters are held fixed.
Learning Goal: To understand the ideal gas law and be able to apply it to a wide variety of situations.

The absolute temperature T, volume V, and pressure p of a gas sample are related by the *ideal gas law*, which states that

$$pV = nRT.$$

Here n is the number of moles in the gas sample and R is a gas constant that applies to all gases. This empirical law describes gases well only if they are sufficiently dilute and at a sufficiently high temperature that they are not on the verge of condensing.

In applying the ideal gas law, p must be the absolute pressure, measured with respect to vacuum and not with respect to atmospheric pressure, and T must be the absolute temperature, measured in kelvins (that is, with respect to absolute zero). If p is in pascals and V is in cubic meters, use $R = 8.3145\ \mathrm{J/(mol \cdot K)}$. If p is in atmospheres and V is in liters, use $R = 0.08206\ \mathrm{L \cdot atm/(mol \cdot K)}$ instead.

Part A

A gas sample enclosed in a rigid metal container at room temperature (20 $^\circ\mathrm{C}$) has an absolute pressure p_1. The container is immersed in hot water until it warms to 40 $^\circ\mathrm{C}$. What is the new absolute pressure p_2?

Part A.1 How to approach the problem

To find the final pressure, you must first determine which quantities in the ideal gas law remain constant in the given situation. Note that R is always a constant. Determine which of the other four quantities are constant for the process described in this part.
Check all that apply.

ANSWER:
- [ ] p
- [x] V
- [x] n
- [ ] T

Now manipulate the ideal gas law ($pV = nRT$) so that n, R, and V, the constants in this situation, are isolated on the right side of the equation:

$$\frac{P}{T} = \frac{nR}{V}.$$

Since the right side of the equation is a constant in this situation, the quantity P/T, which is always equal to nR/V, must be the same at the beginning and the end of the process. Therefore, set $p_1/T_1 = p_2/T_2$. Plug in the values given in this part and then solve for p_2, the final pressure.

Part A.2 Convert temperatures to kelvins

To apply the ideal gas law, all temperatures must be in absolute units (i.e., in kelvins). What is the initial temperature T_1 in kelvins?

ANSWER:

T_1 = K

- [ ] 0
- [ ] 20
- [ ] 100
- [ ] 273
- [x] 293

The Celsius and Kelvin temperature scales have the same unit size, so to convert from degrees Celsius to kelvins, just add 273.

Express your answer in terms of p_1.

ANSWER: $p_2 = \frac{40+273}{20+273} p_1$

This modest temperature increase (in absolute terms) leads to a pressure increase of just a few percent. Note that it is critical for the temperatures to be converted to absolute units. If you had used Celsius temperatures, you would have predicted that the pressure should double, which is far greater than the actual increase.

Part B

Nitrogen gas is introduced into a large deflated plastic bag. No gas is allowed to escape, but as more and more nitrogen is added, the bag inflates to accommodate it. The pressure of the gas within the bag remains at 1 atm and its temperature remains at room temperature (20 $^\circ\mathrm{C}$). How many moles n have been introduced into the bag by the time its volume reaches 22.4 L?

Hint B.1 How to approach the problem

Rearrange the ideal gas law to isolate n. Be sure to use the value for R in units that are consistent with the rest of the problem and hence will cancel out to leave moles at the end.

Express your answer in moles.

ANSWER: $n = \frac{1 \cdot 22.4}{0.08206(20+273)}$ mol

One mole of gas occupies 22.4 L at STP (standard temperature and pressure: 0 $^\circ\mathrm{C}$ and 1 atm). This fact may be worth memorizing. In this problem, the temperature is slightly higher than STP, so the gas expands and 22.4 L can be filled by slightly less than 1 mol of gas.

Part C

Some hydrogen gas is enclosed within a chamber being held at 200 $^\circ C$ with a volume of 0.025 m^3. The chamber is fitted with a movable piston. Initially, the pressure in the gas is $1.50 \times 10^5\ Pa$ (about 1.5 atm). The piston is slowly extracted until the pressure in the gas falls to $0.95 \times 10^5\ Pa$. What is the final volume V_2 of the container? Assume that no gas escapes and that the temperature remains at 200 $^\circ C$.

Part C.1 How to approach the problem

To find the final volume, you must first determine which quantities in the ideal gas law remain constant in the given situation. Note that R is always a constant. Determine which of the other four quantities are constant for the process described in this part.

Check all that apply.

ANSWER:

- [ ] p
- [ ] V
- [x] n
- [x] T

Now look at the ideal gas law: $pV = nRT$. Since n, R, and T are all constants in this situation, the quantity pV, which is always equal to nRT, must be the same at the beginning and the end of the process. Therefore, set $p_1V_1 = p_2V_2$. Plug in the values given in this part and then solve for V_2, the final volume.

Enter your answer numerically in cubic meters.

ANSWER: $V_2 = 0.025\frac{1.5}{0.95}$ m^3

Notice how n is not needed to answer this problem and neither is T, although you do make use of the fact that T is a constant.

Part D

Some hydrogen gas is enclosed within a chamber being held at 200 $^\circ C$ whose volume is 0.025 m^3. Initially, the pressure in the gas is $1.50 \times 10^6\ Pa$ (about 15 atm). The chamber is removed from the heat source and allowed to cool until the pressure in the gas falls to $0.95 \times 10^6\ Pa$. At what temperature T_2 does this occur?

Part D.1 How to approach the problem

To find the final temperature, you must first determine which quantities in the ideal gas law remain constant in the given situation. Note that R is always a constant.

Determine which of the other four quantities are constant for the process described in this part.
Check all that apply.

ANSWER:
- [ ] p
- [x] V
- [x] n
- [ ] T

Now manipulate the ideal gas law ($pV = nRT$) so that n, R, and V, the constants in this situation, are isolated on the right side of the equation:

$$\frac{p}{T} = \frac{nR}{V}.$$

Since the right side of the equation is a constant in this part, the quantity p/T, which is always equal to nR/V, must be the same at the beginning and the end of the process. Therefore, set $p_1/T_1 = p_2/T_2$. Plug in the values given in this part and then solve for T_2, the final temperature.
Enter your answer in degrees Celsius.

ANSWER: $T_2 = (200 + 273)\frac{0.95}{1.5} - 273$ °C

This final temperature happens to be close to room temperature. Hydrogen remains a gas to temperatures well below that, but if this question had been about water vapor, for example, the gas would have condensed to liquid water at 100 °C and the ideal gas law would no longer have applied.

Understanding pV Diagrams

Description: Several qualitative and conceptual questions related to pV-diagrams.
Learning Goal: To understand the meaning and the basic applications of *pV* diagrams for an ideal gas.

As you know, the parameters of an ideal gas are described by the equation

$$pV = nRT,$$

where p is the pressure of the gas, V is the volume of the gas, n is the number of moles, R is the universal gas constant, and T is the absolute temperature of the gas. It follows that, for a portion of an ideal gas,

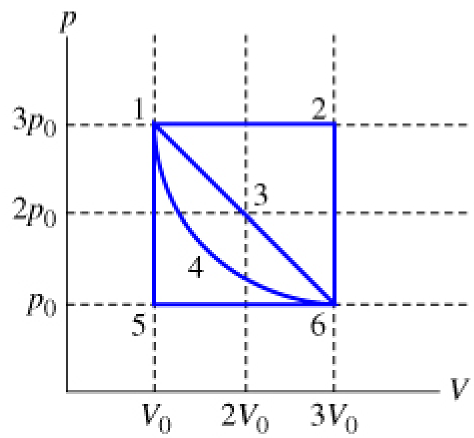
$$\frac{pV}{T} = \text{constant}$$

.

One can see that, if the amount of gas remains constant, it is impossible to change just *one* parameter of the gas: At least one more parameter would also change. For instance, if the pressure of the gas is changed, we can be sure that either the volume or the temperature of the gas (or, maybe, both!) would also change.

To explore these changes, it is often convenient to draw a graph showing one parameter as a function of the other. Although there are many choices of axes, the most common one is a plot of pressure as a function of volume: a *pV* diagram.

In this problem, you will be asked a series of questions related to different processes



shown on a *pV* diagram . They will help you become familiar with such diagrams and to understand what information may be obtained from them.

Part A

Which of the processes are *isobaric*?

Hint A.1 Definition of isobaric

Isobaric comes from the Greek terms *isos* meaning "equal" and *baros* meaning "weight." Isobaric refers to a process in which the pressure does not change.

Check all that apply.

ANSWER:

- [x] $1 \rightarrow 2$
- [ ] $1 \rightarrow 3 \rightarrow 6$
- [ ] $1 \rightarrow 5$
- [x] $6 \rightarrow 5$
- [ ] $1 \rightarrow 4 \rightarrow 6$
- [ ] $6 \rightarrow 2$

Isobaric (constant-pressure) processes correspond to the *horizontal* lines on *pV* diagrams.

Part B

Which of the processes are *isochoric*?

Hint B.1 Definition of isochoric

Isochoric comes from the Greek words *isos* meaning "equal" and *chwra* meaning "space." Isochoric refers to a process in which the volume does not change.

Check all that apply.

ANSWER:

- [ ] $1 \rightarrow 2$
- [ ] $1 \rightarrow 3 \rightarrow 6$
- [x] $1 \rightarrow 5$
- [ ] $6 \rightarrow 5$
- [ ] $1 \rightarrow 4 \rightarrow 6$
- [x] $6 \rightarrow 2$

Isochoric (constant-volume) processes correspond to the *vertical* lines on *pV* diagrams.

Part C

Which of the processes may *possibly* be isothermal?

Hint C.1 Definition of isothermal

Isothermal comes from the Greek words *isos* meaning "equal" and *therme* meaning "heat" or *thermos* meaning "hot." Isothermal refers to a process in which the temperature does not change.

Check all that apply.

ANSWER:

- [ ] $2 \rightarrow 1$
- [ ] $6 \rightarrow 3 \rightarrow 1$
- [ ] $1 \rightarrow 5$
- [ ] $5 \rightarrow 6$
- [x] $6 \rightarrow 4 \rightarrow 1$
- [ ] $6 \rightarrow 2$

For isothermal (constant-temperature) processes, $pV = \text{constant}$; that is, pressure is *inversely proportional* to volume, and the graph is a hyperbola. Curve $1 \rightarrow 4 \rightarrow 6$ is the only graph that looks reasonably similar to a hyperbola.

In further questions, assume that process $1 \rightarrow 4 \rightarrow 6$ is, indeed, an isothermal one.

Part D

In which of the processes is the temperature of the gas increasing?

Check all that apply.

ANSWER:

- [ ] $2 \rightarrow 1$
- [ ] $1 \rightarrow 5$
- [x] $5 \rightarrow 6$
- [x] $6 \rightarrow 2$

If the temperature increases, then, to keep the ratio pV/T constant, the product pV must be increasing as well. This should make sense: For instance, if the pressure is constant, the volume is directly proportional to temperature; if the volume is kept constant, the pressure of the heated gas increases directly proportional to the temperature.

Part E

During process $1 \rightarrow 3 \rightarrow 6$, the temperature of the gas __________.

ANSWER:

- ( ) decreases and then increases
- (•) increases and then decreases
- ( ) remains constant

During process $1 \rightarrow 3$, the pressure of the gas decreases *more slowly* than it does in the isothermal process $1 \rightarrow 4$; therefore, its temperature must be *increasing*.

During process $3 \rightarrow 6$, the pressure of the gas decreases *more rapidly* than it does in the isothermal process $4 \rightarrow 6$; therefore, its temperature must be *decreasing*.

A Law for Scuba Divers

Description: Find the increase in the concentration of air in a scuba diver's lungs. Find the number of moles of air exhaled. Also, consider the isothermal expansion of air as a freediver diver surfaces. Identify the proper pV graph. Associated medical conditions discussed.

SCUBA is an acronym for self-contained underwater breathing apparatus. Scuba diving has become an increasingly popular sport, but it requires training and certification owing to its many dangers. In this problem you will explore the biophysics that underlies the two main conditions that may result from diving in an incorrect or unsafe manner.

While underwater, a scuba diver must breathe compressed air to compensate for the increased underwater pressure. There are a couple of reasons for this:

1. If the air were not at the same pressure as the water, the pipe carrying the air might close off or collapse under the external water pressure.

2. Compressed air is also more concentrated than the air we normally breathe, so the diver can afford to breathe more shallowly and less often.

A mechanical device called a regulator dispenses air at the proper (higher than atmospheric) pressure so that the diver can inhale.

Part A

Suppose Gabor, a scuba diver, is at a depth of 15 m. Assume that:

1. The air pressure in his air tract is the same as the net water pressure at this depth. This prevents water from coming in through his nose.
2. The temperature of the air is constant (body temperature).
3. The air acts as an ideal gas.
4. Salt water has an average density of around 1.03 g/cm^3, which translates to an increase in pressure of 1.00 atm for every 10.0 m of depth below the surface. Therefore, for example, at 10.0 m, the net pressure is 2.00 atm.

What is the ratio of the molar concentration of gases in Gabor's lungs at the depth of 15 meters to that at the surface?

The molar concentration refers to $\frac{n}{V}$, i.e., the number of moles per unit volume. So you are asked to calculate $\frac{(n/V)_{15 \text{ m}}}{(n/V)_{\text{surface}}}$.

Part A.1 Find an equation for calculating concentrations

What is the equation for the molar concentration $\frac{n}{V}$ of the air in Gabor's lungs?

Hint A.1.a The ideal gas law

The ideal gas law states that $pV = nRT$. From this, determine $\frac{n}{V}$.

Express your answer in terms of p, R, and T.

ANSWER: $\frac{n}{V} = \frac{p}{RT}$

Now recall that the temperature is assumed to be the same at the surface and at this depth.

Part A.2 Find the pressure underwater

What is the p_{15}, the pressure on the diver at 15 m?

Hint A.2.a How to approach the problem

Using the information given in the problem introduction, find the increase in pressure at an underwater depth of 15 m from that at the surface. Add the pressure increase to the air pressure at the water's surface to find the total pressure.

Part A.2.b Find an expression for the increase in pressure underwater

Given that for every 10 m of depth below the surface, the pressure increases by 1 atm, find the increase in pressure Δp at a given depth d (measured in meters).

Express your answer in terms of the depth d.

ANSWER: $\Delta p = \frac{d}{10}$

Express your answer in atmospheres to three significant digits.

ANSWER: $p_{15} = 2.50 \ \mathrm{atm}$

Express your answer numerically to three significant digits.

ANSWER: $\frac{(n/V)_{15\ \mathrm{m}}}{(n/V)_{\mathrm{surface}}} = 2.5$

The increased concentration of air in the lungs, through diffusion, leads to an increased concentration of air in the bloodstream. While the increase in the oxygen concentration achieved at typical depths is not toxic, the increase in nitrogen concentration can lead to a condition called *nitrogen narcosis.*

"The bends" refer to another condition associated with incorrect scuba diving. If scuba divers rise to the surface from a depth while attempting to hold their breath, or too fast, air bubbles may form in their bloodstream. These bubbles may then get stuck at the joints, causing great pain and possible death. This condition is called *decompression sickness*, often referred to as the bends.

Why it happens: The exact mechanism of formation of these bubbles is not completely understood. However, it is believed that the nature of the mechanism is as follows. When the concentration of air in the lungs decreases very rapidly, a high concentration of air in some parts of the body is produced as the air starts to diffuse back into the lungs. This leads to bubble formation.

Prevention: By surfacing slowly, a diver allows the concentration of gases dissolved in the blood to reduce slowly (through exchange with the lungs), which prevents the formation of bubbles.

Treatment: One of the possible treatments is to repressurize the diver in a pressure chamber, and then slowly decompress him or her over hours, or even days.

Part B

If the temperature of air in Gabor's lungs is 37 $^{\circ}\mathrm{C}$ (98.6 $^{\circ}\mathrm{F}$), and the volume is $6\ \mathrm{L}$, how many moles of air n must be released by the time he reaches the surface? Let the molar gas constant be given by R = 8.31 $\frac{\mathrm{J}}{\mathrm{mol \cdot K}}$.

Hint B.1 How to approach the problem

Compute the number of moles of air in 6 L at the underwater pressure and again at the surface pressure. The difference is the number of moles of air that must be exhaled.

Part B.2 Solve the ideal gas law for moles

If you have a sample of gas occupying volume V at pressure p and temperature T, how many moles of gas do you have?

Express your answer in terms of R, p, T, and V.

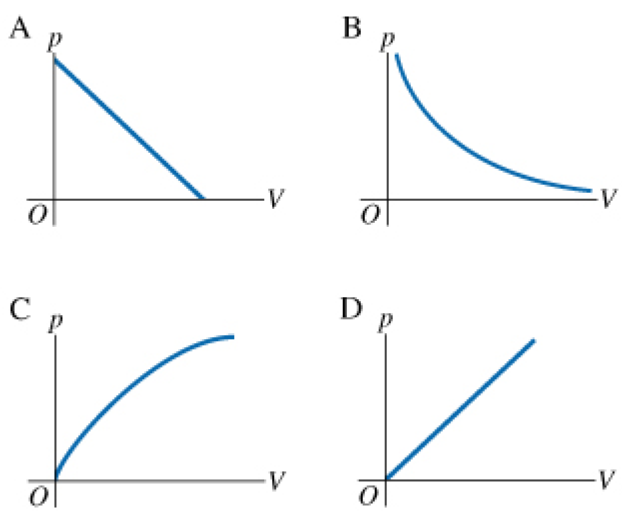
ANSWER: $n = \frac{pV}{RT}$

Express your answer in moles to three significant figures.

ANSWER: n = 0.354 mol

Part C

Now let's briefly discuss diving without gear. If Gabor takes a deep dive without scuba gear, and without inhaling or exhaling, which of the graphs in the figure



best describes the pressure-volume relationship of the air in his lungs?

Again, assume that the temperature of the air is constant (body temperature) and that the air acts as an ideal gas.

Note that the units and scales are not shown on the graphs because they depend on the number of moles of the gas.

Hint C.1 How to approach the problem

The ideal gas law $pV = nRT$ describes the behavior of a gas given the macroscopic parameters of pressure p, volume V, and temperature T, the amount of gas in moles n, and the gas constant R, where $R = 8.314\ \mathrm{J/(K \cdot mol)} = 0.08206\ \mathrm{(L \cdot atm)/(K \cdot mol)}$.
Use the ideal gas law to analyze the behavior of the air in the diver's lungs as he approaches the surface from a given depth.

Part C.2 Express the pressure in terms of the volume and temperature

To determine what the graph should look like, solve the ideal gas law for pressure. Express the answer in terms of n, R, T, and V.

ANSWER: $p = \frac{nRT}{V}$

Hint C.3 Importance of constant temperature

Assuming that the temperature of the gas is constant simplifies the problem because we can then express nRT as a constant c. The ideal gas law becomes $p = c/V$. This inverse relationship determines the basic shape of the *pV* graph.

Choose the letter associated with the graph that best depicts the pressure-volume relationship.

ANSWER:
- ☐ A
- ◉ B
- ☐ C
- ☐ D

If you didn't seal your nose and mouth shut, to keep the water from getting into your nose, you would need the air pressure in your lungs to be the same as the water pressure outside. As the graph shows, this means that your lungs would need to contract. Of course, they cannot do so indefinitely. This is why you may need an air tank or a snorkel even for a short, but deep, dive. For longer dives, you simply need the air for physiological reasons, regardless of depth.

Part D

What type of expansion does the air in the "freediver's" (no gear) lungs undergo as the diver ascends?

Hint D.1 How to approach the problem

Determine which of the thermodynamic properties of the air in the diver's lungs does not change value for this problem. Think about a prefix that means "staying the same."

Part D.2 Determine the proper prefix

Which of the following prefixes means constant or staying the same?

ANSWER:
- (•) iso
- ( ) uni
- ( ) sub
- ( ) trans

Hint D.3 Possible terms for description of processes

The key terms for the macroscopic properties of a gas are -baric for pressure, -choric for volume, and -thermal for temperature.

Give the answer as one word.

ANSWER: isothermal

Rankine Temperature Scale

Description: Calculate the temperature of the triple point of water in the Rankine temperature scale.

Like the Kelvin scale, the Rankine scale is an absolute temperature scale: Absolute zero is zero degrees Rankine (0 $^\circ R$). However, the units of this scale are the same size as those on the Fahrenheit scale ($^\circ F$) rather than the Celsius scale ($^\circ C$).

Part A

Given that water at standard pressure freezes at 0 $^\circ C$, which corresponds to 32 $^\circ F$, and that it boils at 100 $^\circ C$, which corresponds to 212 $^\circ F$, calculate the temperature difference ΔT in degrees Fahrenheit that corresponds to a temperature difference of 1 K on the Kelvin scale.

Hint A.1 Relation of Celsius and Kelvin temperature scales

A temperature increase of one kelvin corresponds to a temperature increase of one degree also on the Celsius scale. The Kelvin temperature scale and the Celsius temperature scale differ only in their zero point.

Give your answer to two significant figures.

ANSWER: $\Delta T = 1\,K$ 1.8 $^\circ F$

Part B

What is the numerical value of the triple-point temperature T_{triple} of water on the Rankine scale?

Hint B.1 Triple-point temperature

On the Kelvin temperature scale, water freezes and coexists in three phases (solid, liquid, and vapor) at 273.16 K at standard pressure. This temperature is known as the triple point.

Give your answer to three significant figures.

ANSWER: T_{triple} = 492 °R

Problem 16.61

Description: m of dry ice (solid CO_2) is placed in a V_1 container, then all the air is quickly pumped out and the container sealed. The container is warmed to 0 degree(s)C, a temperature at which CO_2 is a gas. (a) What is the gas pressure? Give your answer in...

30.0 g of dry ice (solid CO_2) is placed in a 1.00×10^4 cm^3 container, then all the air is quickly pumped out and the container sealed. The container is warmed to 0 °C, a temperature at which CO_2 is a gas.

Part A

What is the gas pressure? Give your answer in atm. The gas then undergoes an isothermal compression until the pressure is 2.20 atm, immediately followed by an isobaric compression until the volume is 1000 cm^3.

ANSWER: $\frac{\frac{m}{0.044}\cdot 8.31\cdot 273}{V_1\cdot 1.013\cdot 10^5}$ atm

Part B

What is the final temperature of the gas?

ANSWER: $\frac{V_3\cdot 273}{\frac{\frac{m}{0.044}\cdot 8.31\cdot 273}{p_2}} - 273$ °C

16.61. Model: Assume CO_2 gas is an ideal gas.

Solve: **(a)** The molar mass for CO_2 is M_{mol} = 44 g/mol, so a 10 g piece of dry ice is 0.2273 mol. This becomes 0.227 mol of gas at 0°C. With $V_1 = 10{,}000\ cm^3 = 0.010\ m^3$ and $T_1 = 0°C = 273\ K$, the pressure is

$$p_1 = \frac{nRT_1}{V_1} = \frac{(0.2273\ mol)(8.31\ J/mol\ K)(273\ K)}{0.010\ m^3} = 5.156\times10^4\ Pa = 0.509\ atm$$

(b) From the isothermal compression,

$$p_2V_2 = p_1V_1 \Rightarrow V_2 = V_1\frac{p_1}{p_2} = (0.010\ m^3)\left(\frac{0.509\ atm}{3.0\ atm}\right) = 1.70\times10^{-3}\ m^3 = 1700\ cm^3$$

From the isobaric compression,

$$T_3 = T_2\frac{V_3}{V_2} = (273\ K)\left(\frac{1000\ cm^3}{1700\ cm^3}\right) = 161\ K = -112°C$$

Problem 16.46

Description: An electric generating plant boils water to produce high- pressure steam. The steam spins a turbine that is connected to the generator. (a) How many liters of water must be boiled to fill a 5.0 m^3 boiler with 50 atm of steam at 400 degree(s)C? (b)...

An electric generating plant boils water to produce high- pressure steam. The steam spins a turbine that is connected to the generator.

Part A

How many liters of water must be boiled to fill a $5.0\ \mathrm{m}^3$ boiler with 50 atm of steam at 400 $^\circ$C?

ANSWER: 81.5 L

Part B

The steam has dropped to 2.0 atm pressure at 150 $^\circ$C as it exits the turbine. How much volume does it now occupy?

ANSWER: 78.6 m^3

16.46. Model: Assume that the steam is an ideal gas.

Solve: (a) The volume of water is

$$V = \frac{M}{\rho} = \frac{nM_{mol}}{\rho} = \frac{pV}{RT}\frac{M_{mol}}{\rho} = \frac{50(1.013\times10^5\ \mathrm{Pa})(5.0\ \mathrm{m}^3)(0.018\ \mathrm{kg/mol})}{(8.31\ \mathrm{J/mol\ K})(673\ \mathrm{K})(1000\ \mathrm{kg/m}^3)} = 0.0815\ \mathrm{m}^3 = 81.5\ \mathrm{L}$$

(b) Using the before-and-after relationship of an ideal gas,

$$\frac{p_2V_2}{T_2} = \frac{p_1V_1}{T_1} \Rightarrow V_2 = \frac{T_2}{T_1}\frac{p_1}{p_2}V_1 = \left(\frac{(273+150)\ \mathrm{K}}{673\ \mathrm{K}}\right)\left(\frac{50\ \mathrm{atm}}{2.0\ \mathrm{atm}}\right)(5.0\ \mathrm{m}^3) = 78.6\ \mathrm{m}^3$$

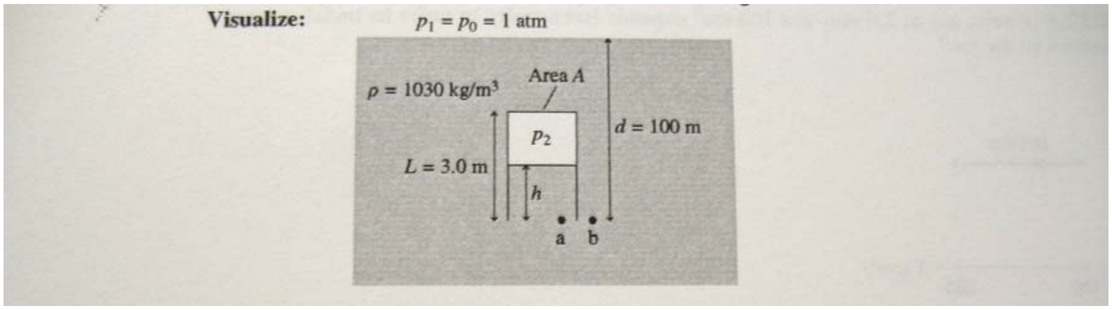
Problem 16.71

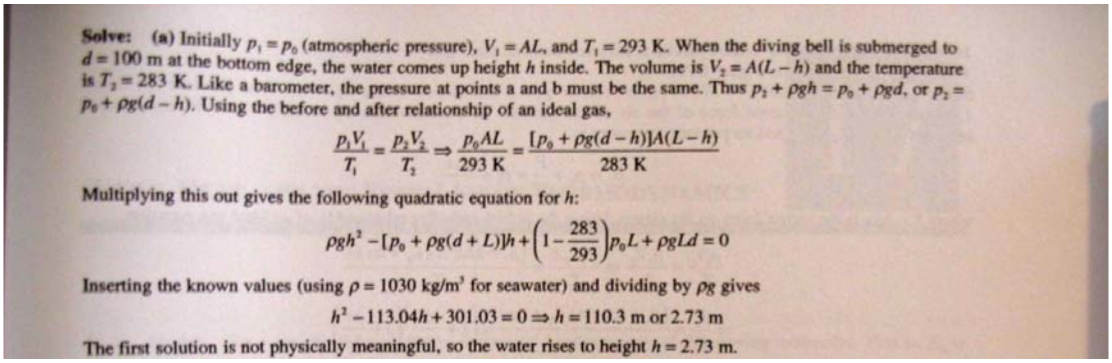
A diving bell is a 4.60 m-tall cylinder closed at the upper end but open at the lower end. The temperature of the air in the bell is 30.0 $^\circ$C. The bell is lowered into the ocean until its lower end is 100 m deep. The temperature at that depth is 10 $^\circ$C.

Part A

How high does the water rise in the bell after enough time has passed for the air to reach thermal equilibrium?

ANSWER: $h - h\frac{1.013\cdot 283}{11.107 T_1}$ m




Solve: (a) Initially $p_1 = p_0$ (atmospheric pressure), $V_1 = AL$, and $T_1 = 293$ K. When the diving bell is submerged to $d = 100$ m at the bottom edge, the water comes up height h inside. The volume is $V_2 = A(L-h)$ and the temperature is $T_2 = 283$ K. Like a barometer, the pressure at points a and b must be the same. Thus $p_2 + \rho gh = p_0 + \rho gd$, or $p_2 = p_0 + \rho g(d-h)$. Using the before and after relationship of an ideal gas,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \Rightarrow \frac{p_0AL}{293\text{ K}} = \frac{[p_0 + \rho g(d-h)]A(L-h)}{283\text{ K}}$$

Multiplying this out gives the following quadratic equation for h:

$$\rho gh^2 - [p_0 + \rho g(d+L)]h + \left(1 - \frac{283}{293}\right)p_0L + \rho gLd = 0$$

Inserting the known values (using $\rho = 1030$ kg/m^3 for seawater) and dividing by ρg gives

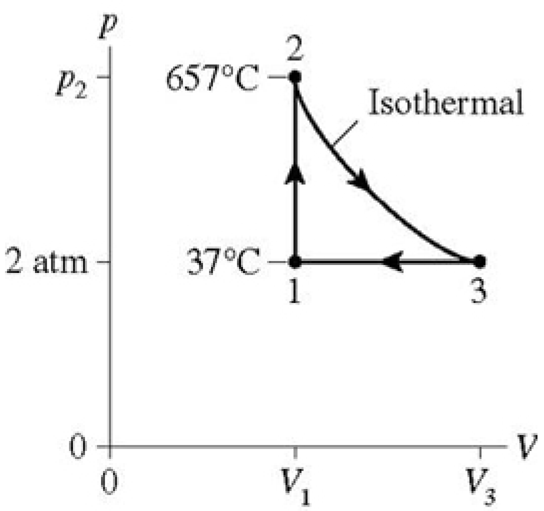
$$h^2 - 113.04h + 301.03 = 0 \Rightarrow h = 110.3\text{ m or } 2.73\text{ m}$$

The first solution is not physically meaningful, so the water rises to height $h = 2.73$ m.

Problem 16.56

Description: 8.0 g of helium gas follows the process 1 rightarrow 2 rightarrow 3 shown in the figure . (a) Find the value of V_1. (b) Find the value of V_3. (c) Find the value of p_2. (d) Find the value of T_3.

8.0 g of helium gas follows the process $1 \rightarrow 2 \rightarrow 3$ shown in the figure



.

Part A

Find the value of V_1.

ANSWER: 25.4 L

Part B

Find the value of V_3.

ANSWER: 76.3 L

Part C

Find the value of p_2.

ANSWER: 6.00 atm

Part D

Find the value of T_3.

ANSWER: 657 °C

16.56. Model: Assume that the helium gas is an ideal gas.

Visualize: Please refer to Figure P16.56. Process 1 → 2 is isochoric, process 2 → 3 is isothermal, and process 3 → 1 is isobaric.

Solve: The number of moles of helium is

$$n = \frac{M}{M_{mol}} = \frac{8.0\text{ g}}{4\text{ g/mol}} = 2.0\text{ mol}$$

Using the ideal-gas equation,

$$V_1 = \frac{nRT_1}{p_1} = \frac{(2.0\text{ mol})(8.31\text{ J/mol K})[(273+37)\text{ K}]}{2(1.013\times10^5\text{ Pa})} = 0.0254\text{ m}^3$$

For the isochoric process $V_2 = V_1$, and

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow p_2 = p_1\frac{T_2}{T_1} = (2\text{ atm})\left(\frac{657+273}{37+273}\right) = 6\text{ atm}$$

For the isothermal process, the equation $p_3V_3 = p_2V_2$ is

$$V_3 = V_2\frac{p_2}{p_3} = (0.0254\text{ m}^3)\left(\frac{6\text{ atm}}{2\text{ atm}}\right) = 0.0762\text{ m}^3$$

For the isothermal process, $T_3 = T_2 = 657°\text{C}$.