## Chapter 3

## Digital Logic

Topics in this Chapter:

- Breadboarding
- Electricity
- Gates
- Circuits with Algebra
- Latches and Flip-Flops
- What Is Data?
- A Few More Chips
- Summary


## Introduction

This chapter provides an introduction to digital logic, the basis of microcomputer design. Although it is not critical that you understand everything in this chapter, to gain a full understanding of the microcomputer a background in digital logic is imperative. Each level that we'll cover (digital logic, microcomputer, and software) contains a certain degree of abstraction from the layer following it.

## Breadboarding

We'll perform all the experiments in this chapter using breadboards. To begin, perform the following steps:

1. Find a power supply and snip off one of the hard drive connectors.
2. Strip down the end of the red wire ( 5 volts) and the black wire (ground).
3. Turn the power supply on and check these cables with your multimeter to ensure that they really are 5 volts and ground.
4. On your breadboard, you'll see a series of red and blue lines running the length of the plastic. Connect the power supply cables to the banana plugs, and connect the banana plugs to these lines (see Figure 3.1)

Figure 3.1 Connecting Power and Ground


The holes beside these lines are for power and ground, respectively. A look inside may help us understand how these boards work (see Figure 3.2).

Figure 3.2 Inside the Breadboard


Inside each hole is a metallic socket into which you can plug a wire or component. Every socket in the ground strip is connected; therefore, if you connect one socket to your ground supply, all sockets in the strip are grounded. The same rule applies to the power strip. We have three columns of power and ground strips with nothing connecting them, so we need to use jumper wires across the bottom (see Figure 3.3).

Figure 3.3 Jumpering Power and Ground Lines


To help gain an understanding of the rest of the board, take a look at Figure 3.2 once again. Each horizontal row of five sockets is connected. Therefore, if we wanted to connect three resistors in a series (one after the other), we could configure them as shown in Figure 3.4.

Figure 3.4 Resistors in Series


Next, let's make sure everything's configured correctly by running some tests using the logic probe:

1. Connect your probe's alligator clips to the power and ground lines (see Figure 3.5).

Figure 3.5 Connecting the Logic Probe

2. Touch the point of the probe to any socket on the power line. The "HI" light should come on.
3. Test a socket in each strip by repeating Step 2 .
4. Repeat Step 2 yet another time for the ground sockets; for these, the "LOW" light should come on (see Figure 3.6). The sockets connected to neither power nor ground should produce no light at all.

Figure 3.6 Testing with the Logic Probe


## Electricity

Here you'll find a very brief, hands-on introduction to electronics fundamentals. This chapter provides enough of the basics that you shouldn't have too much difficulty completing the Apple I. However, a more substantial understanding of electronics will serve you well in the future. If you'd like to do some further reading, The Art of Electronics, by Paul Horowitz and Winfield Hill (Cambridge University Press, 1989), is the most widely acclaimed book on the topic but is perhaps too difficult for the beginner. A more appropriate book for the novice is Hands-On Electronics, by Daniel M. Kaplan and Christopher G. White (Cambridge University Press, 2003).

## Voltage and Current

Voltage ( $V$ ) is the potential difference between two points, measured in volts. When you set your multimeter to Volts DC and touch the probes to the breadboard, negative to negative and positive to positive, you'll get a reading of 5 volts. That is related to the amount of energy it takes to move charge from the lower point (ground) to the higher.

Current (I) is used to measure the rate of flow of electrical charge and can be thought of in terms of a water current. Current is measured in amperes $(A)$. Whereas voltage is measured between two points, current measures the rate of flow at a particular point. DC voltage is steady (hook your circuit up to a 5 -volt supply, and 5 volts is what you'll get), but current only "pulls" as much as you need. If
you have a power supply capable of 10 amps and your circuit only needs 5 amps , then 5 amps is all that it will pull.

Power $(P)$ is voltage multiplied by current. Power is measured in watts, which you're probably familiar with seeing on light bulbs. In fact, light bulbs serve as an excellent example. Consider a 20 W and a 50W halogen bulb (see Figure 3.7). Both require 12 volts. The difference is in the current, which we can calculate using $\mathrm{P}=\mathrm{V} \cdot \mathrm{I}$.
$\mathrm{I}=\mathrm{P} / \mathrm{V} \quad 20$ watts $/ 12$ volts $=1.6 \mathrm{amps} \quad 50$ watts $/ 12$ volts $=4.16 \mathrm{amps}$
Appropriately, we see that the brighter light bulb draws significantly more current than the dimmer one. Here's an exercise: Replace all the 40W lights in your house with 200 W bulbs and see what happens to your electricity bill.

Figure 3.7 A Light Bulb Circuit
Light bulb

## Need to Know...

If you have two wires you want to connect in a schematic, place a large dot at the point of connection. If there is no dot, readers (and the schematic software) will assume there is no physical connection between the two lines. See Figure 3.8.

Figure 3.8 Intersections

| No connection |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |

## Resistors and Diodes

Now that we've explained the basics of current and voltage, we can take a more hands-on approach to resistance. Let's wire up a light-emitting diode (LED). A diode is a device that allows current to flow in only one direction. For this experiment, let's use a standard red LED, such as Model \#276-041 from Radio Shack. These LEDs have a recommended voltage of about 2.25 volts, with a maximum voltage of 2.6 volts (you'll find all this information on the packaging). The maximum current is 28 mA . Our power supply provides 5 volts; therefore, if we hook the LED up directly to the power supply, it will probably burn out. (If you have a spare, try this!)

To reduce the voltage, use a resistor. A resistor is a partial conductor, usually made of carbon. We can calculate resistance using the equation:
$\mathrm{R}=\mathrm{V} / \mathrm{I}$
where $R$ (resistance) is calculated in ohms, $V$ (voltage) in volts, and $I$ (current) in amps.
Our circuit is shown in Figure 3.9. In moving from our 5 -volt supply to ground, we have 5 volts we need to dispose of. The LED uses 2.25 volts, which leaves 2.75 volts for the resistor. This is the value we want to use for $V$ in this equation. Now we can calculate:
$R=(5 v-2.25 v) / .028 A=98$ ohms
If you calculated decimal places, you're taking these numbers too seriously. Next, we need to find a 98 -ohm resistor. A resistor with a smaller value will provide less resistance and potentially damage your LED. One with greater resistance merely means that your LED won't be quite as bright. We're just experimenting, so brightness level isn't a major concern. I used the nearest resistor I had available, which happened to be 300 ohm .

Now it's time to wire it up:

1. Snip off the ends of the resistor, making it fit conveniently into your breadboard.
2. Hook up the resistor and the LED in series. It doesn't matter which one you connect first; however, be aware that the orientation of the LED does matter (see the "LED Polarity" sidebar). The finished circuit is displayed in Figure 3.10.

Try swapping your current resistor with some resistors of varying sizes to see what happens. Also try putting the LED in backward.

Figure 3.9 LED Circuit


Figure 3.10 Wired LED


Need to Know...
An LED typically has two leads projecting from its base (see Figure 3.11). The longer lead is the positive lead (also known as the anode), and the shorter lead is the negative lead (also known as the cathode). The cathode is also denoted by a flat edge on the plastic LED housing. The anode will connect to a positive voltage ( 5 V in this case), and the cathode will connect to ground.

Figure 3.11 LED Diagram


## Need to Know...

Most resistors have four colored bars that will give you the value in ohms, as shown in Table 3.1. A resistor of Yellow, Red, Red, Gold would be 4,200 ohms, with an accuracy of $5 \%$. If we needed a 30 -ohm resistor, we would search for the colors Orange, Black, Black, Gold.

Memorizing Table 3.1 will save you time if you're going to do a lot of work with resistors. At the very least, knowing the values in column three will help prevent you from using an exponentially incorrect resistor. If you don't have the time to memorize the table or have no desire to haul the chart around with you, there are a multitude of Java applets on the Web that will do the conversions for you. There are also many applications for graphing calculators that allow you to complete the conversion without the work, including the RCOL for the HP 48, which is available at hpcalc.org. You can also simply use your multimeter to measure the ohms.

Table 3.1 Resistor Codes

| Color | Bar 1 | Bar 2 | Bar 3 | Bar 4 |
| :--- | :--- | :--- | :--- | :--- |
| Black | 0 | 0 | x1 |  |
| Brown | 1 | 1 | x10 |  |
| Red | 2 | 2 | x100 |  |
| Orange | 3 | 3 | x1,000 |  |
| Yellow | 4 | 4 | x10,000 |  |
| Green | 5 | 5 | x100,000 |  |
| Blue | 6 | 6 | x1,000,000 |  |
| Magenta | 7 | 7 |  |  |
| Gray | 8 | 8 |  |  |
| White | 9 | 9 | $x 0.1$ | $10 \%$ |
| Gold |  |  |  |  |
| Silver |  |  |  |  |

## Capacitors

A capacitor, like those shown in Figure 3.12, stores energy in an electric field. In building our circuits, we'll use these as de-spiking capacitors to filter the power supply. When the output of one of our chips changes, it causes a sudden voltage drop (a negative-going spike). When this occurs, the capacitor will partially discharge its energy to eliminate the severity of this spike.

Figure 3.12 Capacitors


## Gates

Those little black chips that cover circuit boards look extremely complex. Today, most of these integrated circuits, or ICs, are quite intricate, but there are still many chips available of the simpler variety. If you examine the chips in a modern computer, you will notice that many of them have hundreds of pins placed so closely that they appear almost impossible to work with. Open an older computer, and you'll discover chips that look much more accessible, with few pins and great spacing. We'll be working with these older, more "classic" chips throughout this book.

The 7400 series of ICs serves as the basis for digital logic and is very easy to understand. Throughout the rest of this chapter we'll concentrate on building a few very basic digital circuits using 7400 series ICs, resistors, and LEDs.

## AND

IF it is dark AND IF there is a car in the driveway THEN turn on the porch light.
Under what circumstances is the porch light turned on? Only if both statements are true-it must be dark, and there must be a car in the driveway. One of the two is not enough. In a schematic, we'd express this idea as shown in Figure 3.13. The $D$ symbol is a logical $A N D$. We can also use a truth table. Follow along with the truth tables in Table 3.2 and see if they match the aforementioned assertions. You'll note that all three tables contain the same data. True is a logical 1, which is implemented with a 5 -volt signal. False is a logical 0 and is implemented by a 0 -volt signal.

Figure 3.13 An AND Gate
dark out A

Table 3.2 AND Truth Tables

| Input A | Input B | Output C |
| :--- | :--- | :--- |
| False | False | False |
| False | True | False |
| True | False | False |
| True | True | True |

## Continued

Table 3.2 AND Truth Tables

| Input A | Input B | Output C |
| :--- | :--- | :--- |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |


| Input $A$ | Input B | Output C |
| :--- | :--- | :--- |
| 0 v | 0 v | 0 v |
| 0 v | 5 v | 0 v |
| 5 v | 0 v | 0 v |
| 5 v | 5 v | 5 v |

With all this in mind, perform the following steps:

1. Take a 7408 IC and insert it into your breadboard. This chip has four $A N D$ gates, arranged as you see in Figure 3.14. Note the location of the notch in the diagram.
2. Orient all chips so that the notch is at the top, and pin 1 will always be in the upper-left corner.
3. Hook up the supply voltage and ground. Pin 14 is Vcc (Voltage common collector) and should be tied to your 5-volt line. Pin 7, GND, should be connected to ground.

Figure 3.14 A 7408 IC


Need to Know...
Pin numbering always starts in the upper-left corner, goes down the left side, and follows up the right side. Pin 1 is marked either by a circle in the upper-left corner or by a notch in the top center (at which point it's up to you to know left from right). See Figure 3.15.

Figure 3.15 Pin Arrangement


Now let's wire the first $A N D$ gate on the chip:

1. Connect pins 1 and 2 (inputs to the $A N D$ gate) to the supply voltage line.
2. Then, connect pin 3 to the input of the LED that was wired up earlier.
3. Turn on the power, wait a few nanoseconds for the signal to propagate, and the LED should light up (see Figure 3.16). Since both Input A and Input B are high, the output will also be high and the LED will be powered on.

Figure 3.16 The Lit LED

4. Next, take the wire from pin 2 (or pin 1) and move it from supply voltage to ground. To grasp the importance of this step, imagine that instead of moving the cable by hand, it's hooked up to a light sensor that outputs 0 volts when it's light out and 5 volts when it's dark.

## Note

When a chip's inputs are changed, the output does not change instantaneously. The new signals take time to propagate throughout the chip. This propagation varies depending on the speed and complexity of the design, but TTL gates tend to take about 10 nanoseconds (ns) to complete propagation. There are 1 billion nanoseconds in a second.

If you like a challenge, try playing around with the chip a bit more by wiring the inputs of some gates to the outputs of others. Create the equivalent of a four-input $A N D$ gate. You can also examine the logic table in Table 3.3 to find patterns.

Table 3.3 A 4-Input AND Gate

| Input A | Input B | Input C | Input D | Output E |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

## Inverter, NAND

IF it is dark AND IF there is a car in the driveway, THEN turn OFF the porch light.
This statement is almost identical to the one in previous section, even though our goal has changed to produce the opposite effect. The only change that needs to be made is the addition of an inverter (see Table 3.4). The inverter takes whatever signal it is given and outputs the opposite; therefore, if you put 0 volts in, you'll get 5 volts out, and vice versa. In Figure 3.17, you'll see the symbol for an inverter along with an $A N D$ gate. The triangle means "buffer" (replicate the signal), and the small circle means "invert it." Find a 7404 (shown in Figure 3.18) and connect the voltage to supply and ground. Now connect the output of your $A N D$ gate (pin 3) to the input of the 7404 's inverter (pin 1). The output of the inverter (pin 2) goes to the LED to complete our circuit, shown in Figure 3.19.

## Table 3.4 Inverter Truth Table

| Input A | Output B |
| :--- | :--- |
| 0 | 1 |
| 1 | 0 |

Figure 3.17 An Inverter and an AND Gate


Figure 3.18 A 7404 IC


Figure 3.19 AND and NAND Gates, Wired to LED


The inverted $A N D$ is so common that it has its own gate, the NAND (not AND) gate. This gate is functionally equivalent to an $A N D$ followed by an inverter. The $N A N D$ is pictured in Figure 3.20, with its layout in Figure 3.21. Table 3.5 displays its operation, which, hopefully, you were able to surmise.

Figure 3.20 The NAND Gate


Figure 3.21 A 7400 IC


Table 3.5 NAND Truth Table

| Input A | Input B | Output $\mathbf{C}$ |
| :--- | :--- | :--- |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

## OR, NOR

IF it is dark AND IF there is a car in the driveway $O R$ if the light switch is on, THEN turn on the porch light.
Here we take our output from the $A N D$ gate already wired up and $O R$ it with "the light switch is on." The OR gate in TTL logic is the 7432 (see Figure 3.22), which exhibits the characteristics provided in Table 3.6.

Figure 3.22 A 7432 IC


Table 3.6 OR Gate Truth Table

| Input A | Input B | Output C (A OR B) |
| :--- | :--- | :--- |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Perform the following steps:

1. Take your 7432 and connect power and ground.
2. Using the $A N D$ circuit that you already wired, hook the output of the $A N D$ gate to an input of the $O R(\operatorname{pin} 1)$. The other $O R$ input (pin 2) should be connected directly to power or ground to simulate the light switch being on or off.
3. Next, wire the OR's output (pin 3) to the LED (see Figure 3.23). Table 3.7 describes the behavior of this entire circuit. Note that the light comes on for five out of eight possible combinations.

Figure 3.23 AND and $O R$, Wired


Table 3.7 Truth Table for Figure 3.23

| Input A | Input B | A AND B | Input C | Output E ((A AND B) OR C) |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

As with $N A N D$, there is also a $N O R$ gate (see Figure 3.24), which is $O R$ followed by an inverter. You can find NOR gates on the 7402 (see Figure 3.25). Table 3.8 shows a NOR gate truth table.

Figure 3.24 A NOR Gate
\#1 7402


Figure 3.25 A 7402 IC


Table 3.8 NOR Gate Truth Table

| Input A | Input B | Output C (A NOR B) |
| :--- | :--- | :--- |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

## Need to Know...

Let's say you have two AND gates and you want to combine their outputs so that if either AND gate's output is HI , our final output is HI as well. What happens if you connect the two output lines, as in Figure 3.26? When output A goes HI and output B goes LO, the two signals will compete, potentially damaging your chips.

If you're tempted to try this configuration, you're probably looking to use an OR gate, as shown in Figure 3.27.

Figure 3.26 A Bad Circuit


Figure 3.27 A Good Circuit


## XOR

IF it is dark OR IF there is a car in the driveway-but NOT if both-THEN turn on the porch light.
Perhaps this behavior would be desirable if you wanted to discourage visitors after dusk? XOR is short for exclusive OR and means "if one or the other is true, but not if both are true." You can find XOR gates on the 7486 (see Figure 3.28). In Figure 3.29 you see the schematic for this circuit, along with an equivalent circuit that does not use the $X O R$ gate but achieves the same effect. Table 3.9 shows an XOR truth table.

Figure 3.28 A 7486 IC


Figure 3.29 Schematic of Circuits with and without XOR Gates


Table 3.9 XOR Truth Table

| Input A | Input B | Output C (A XOR B) |
| :--- | :--- | :--- |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

## Circuits with Algebra

Don't worry, this topics is not as bad as it sounds. Using logic expressions, DeMorgan's Laws, and Boolean algebra, you can quickly sketch out basic circuits on paper or even in a simple text editor. Logic expressions will allow you to show gates and lines with symbols and letters. DeMorgan's Laws will allow you to swap gates to get more efficient circuits. Boolean algebra will allow you to simplify your circuits.

## Logic Expressions

Ambiguities in the English language can make it very difficult to precisely express digital logic; therefore, an algebra has been developed for this purpose. Understanding this algebra is merely a matter of getting accustomed to the symbols. Table 3.10 displays these symbols, listed in order of precedence.

Table 3.10 Algebraic Symbols for Expressing Digital Logic

| Symbol | Meaning |
| :--- | :--- |
|  | $N O T$ |
| - | AND |
| + | OR |
| $\oplus$ | XOR |

Figure 3.30 shows a few logic expressions and their equivalent circuits. Take a good look at these and make sure you understand them. Try writing a few of your own.

Figure 3.30 Logic Expressions


## DeMorgan's Laws

The term DeMorgan's Laws sounds dull and tedious, but these laws are a real boon for the improvising hacker. DeMorgan's laws explain how we can substitute different combinations of gates to best use our available resources. They can be expressed as:
$(A+B)^{\prime}=A^{\prime} \cdot B^{\prime}$
$(A \cdot B)^{\prime}=A^{\prime}+B^{\prime}$
Let's take $(A \cdot B)$ ' (a NAND gate), for example. This statement means that $A$ and $B$ are not both true; therefore, at least one of them is false. Consequently, it follows that $A$ is not true or $B$ is not true, which we can write as $A^{\prime}+B^{\prime}$. This looks like a slight change from the first equation, but it can make a huge difference. Consider, for example, that the signals you are receiving may already be inverted or that you may not even have any NAND gates. Also note that each of the chips we've been using contains multiple gates. DeMorgan's Laws allow us to more fully utilize the chips with which we're already working.

There's an easy way to remember DeMorgan's Laws in practice (see Figure 3.31). To find the equivalent for any gate, first swap the symbol (OR to $A N D, A N D$ to $O R$ ). Then look at each input and output. Everywhere there's an inverter, remove it, and everywhere there isn't, add one.

Figure 3.31 DeMorgan's Laws


## Boolean Algebra

Boolean algebra is a method for manipulating logic expressions. It allows us to reduce complex expressions to simpler statements that are logically equivalent. Some useful equivalences are shown in Table 3.11.

Table 3.11 Equivalences

| Equivalence | Equivalence |
| :--- | :--- |
| $A \cdot 1=A$ | $A+0=A$ |
| $A \cdot 0=0$ | $A+1=1$ |
| $A \cdot A=A$ | $A+A=A$ |
| $A \cdot A^{\prime}=0$ | $A+A^{\prime}=1$ |
| $\left(A^{\prime}\right)^{\prime}=A$ | $\left(\left(A^{\prime}\right)^{\prime}\right)^{\prime}=A^{\prime}$ |
| $A \cdot B+A \cdot C=A \cdot(B+C)$ | $(A+B) \cdot(A+C)=A+(B \cdot C)$ |
| $A \cdot(A+B)=A$ | $A+(A \cdot B)=A$ |
| $A \cdot\left(A^{\prime}+B\right)=A \cdot B$ | $A+\left(A^{\prime} \cdot B\right)=A+B$ |
| $(A \cdot B)^{\prime}=A^{\prime}+B^{\prime}$ | $(A+B)^{\prime}=A^{\prime} \cdot B^{\prime}$ |

Make sure you understand why all these equivalences are true. Let's take
$\mathrm{A}+1=1$
as an example. The statement reads "if A is 1 or if 1 is 1 ." Given that 1 is always 1 , this statement is always true and we can simply assert 1.

## All You Need Is NAND

We've already shown that $X O R$ can be expressed as a combination of $A N D$ and $O R$ gates (refer back to Figure 3.29). Likewise, NAND and NOR can be expressed using an inverter along with $A N D$ and $O R$, respectively. Thanks to DeMorgan's Laws, we can use a combination of inverters and $O R$ gates to produce an $A N D$ gate or a combination of inverters and $A N D$ gates to produce an $O R$ gate. At this point, we have a means of re-expressing every gate except the inverter. We can do that with a $N A N D$ gate. Connect both inputs of your NAND gate to the same line (we'll call it $P$ ). A NAND gate is expressed:
$(\mathrm{A} \cdot \mathrm{B})^{\prime}=\mathrm{C}$
Since in our case, $A$ and $B$ are the same line, we can write:
$(P \cdot P)^{\prime}=C$
Using the equivalence
$A \cdot A=A$
given in the previous section, we can reduce this to
$(\mathrm{P})^{\prime}=\mathrm{C}$
which means that our NAND gate now functions as an inverter.
A complete set is a collection of chips that can be used to produce any logical statement. One example of a complete set is the $7408(A N D)$, the $7432(O R)$, and the 7404 (NOT). Another example
is the 7400 (NAND). Any circuit can be built using nothing but NAND. This means that any computer of any complexity could be built using only 7400 ICs. This is not practical when you consider the colossal size such a computer would have to be, but it is interesting to consider the magnitude of the concepts you can express using nothing but NAND.

A second $N A N D$ gate is used to negate the negation, and we're back to an ordinary $A N D$ (Figure 3.32).

Figure 3.32 AND with NANDs


Thanks to DeMorgan's Laws, we know that an $A N D$ gate surrounded by inverters is equivalent to an OR (see Figure 3.33).

Figure 3.33 OR with NANDs


An XOR gate can be expressed as $\left(A \cdot B^{\prime}\right)+\left(A^{\prime} \cdot B\right)$. Using that equivalency and the previous examples as parts, we can express an XOR using nine NAND gates (see Figure 3.34). This circuit, by the way, can be reduced to five NAND gates pretty easily, and there's also a way to do it with just four.

Figure 3.34 XOR with NANDs


## Latches and Flip-Flops

You might have noticed a shortcoming in the circuits we've described thus far. What if we want the porch light to stay on after it detects a car in the driveway? The circuits we've covered have no memory. In this section we discuss ways to add memory to our circuits, using only the gates we've covered in previous sections.

## SR Latch

A set-reset (SR) latch allows us to set a bit to 1 or reset it to 0 (see Figure 3.35). We can do this using ordinary logic gates. The trick is to loop the output back into the input:

Figure 3.35 An SR Latch

$$
\text { \#1 } 7402
$$



Grab a 7402 and wire up this circuit. Connect an LED to the output Q (see Figure 3.36).
Figure 3.36 Connecting an LED to the Output Q


This circuit is confusing because our outputs are determining our inputs. Consequently, the first thing we need to do when we power this circuit on is to reset it so that we have a circuit with known outputs. Looking at the \#2 NOR gate, we can see that the input $R$ alone being high is enough to make the output Q low. In your circuit, wire $R$ to high and $S$ to low. The output Q will be low. Now check the inputs to the \#1 NOR gate. Both of these are low, making Q' high. Q' is an input to the lower NOR gate. This means that even when Reset goes low, the circuit will stay in the reset state.

Now, let's make $R$ low and make $S$ high. The output of the \#1 NOR becomes 0 , which means both the inputs to the $\# 2$ NOR are now 0 , making the $Q$ output 1. Take $S$ low and $Q$ stays high, because $Q$ is being looped back into the input of the $\# 1$ NOR gate. $Q$ will remain high until the circuit is reset.

Table 3.12 is the complete table of operation. What's most important to realize here is that all it takes is a momentary high on the set or reset lines to permanently set or reset the output Q .

Table 3.12 Latch Operation

| $\mathbf{S}$ | $\mathbf{R}$ | $\mathbf{Q}$ |
| :--- | :--- | :--- |
| 0 | 0 | Previous value of $\mathbf{Q}$ |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | Not used |

## Flip-Flop

Imagine we have a situation where we want to grab a value at a certain instant and then hold onto it. For example, every hour, we want check to see if the water level in a container is above a certain indicator. If it is, sound the alarm for a full hour, then check again. We can accomplish this using a $D$ flip-flop.

Figure 3.37 shows the circuit to help you build it on your breadboard. We're not going to examine its operation, but if you want to figure it out, follow the levels in your mind or with a logic probe. Normally, when you see a flip-flop in a schematic, it'll look like Figure 3.38.

If you don't want to wire up all those NAND gates, use a 7474 TTL flip-flop, which is shown in Figure 3.39. Notice the inverters at the inputs to the Preset and Clear lines. Due to their presence, we use a high signal instead of a low to deactivate these lines.

Figure 3.37 A Flip-Flop in Logic Gates


Figure 3.38 A D Flip-Flop


Figure 3.39 A 74LS74 IC


To use the circuit, perform the following steps:

1. Connect the clock input to low and the D input to whatever value you want (let's say 1 , in this case).
2. Now we want to pulse the clock. Disconnect the clock from low, touch it to high, then move it back to low. That's a pulse. In a D flip-flop, a pulse is detected at the moment of the change from low to high.
3. You can leave the clock connected to high for as long as you like and it is still only seen as a single pulse.

## What Is Data?

Before we look at more complex chips that are used in the Apple I, we need to understand what all this data is that we're passing around.

## Counting in Binary and Hexadecimal

Digital logic has only two states, high or low; so our numbers need to be expressed in a binary (base
2) format. A binary digit, like a digital signal, is always either 1 or 0 . Converting from decimal to binary is tedious, but you can calculate the values quickly by adding up each place value.

Consider that in decimal we have place values $1,10,100$, and so on, such that we could calculate the number 203 by multiplying the digit by the place value:
$2 \times 100+0 \times 10+3 \times 1=203$
In binary, we can do the same thing, except that our place values are now $1,2,4,8$, and so on (see Table 3.13). If we take 1101, we can calculate:
$1 \times 8+1 \times 4+0 \times 2+1 \times 1=13$

Table 3.13 Values Expressed as Decimals, Binary Numbers, and Hexadecimals

| Decimal | Binary | Hexadecimal |
| :--- | :--- | :--- |
| $1,000,000$ | 64 | $16,777,216$ |
| 100,000 | 32 | $1,048,576$ |
| 10,000 | 16 | 65,536 |
| 1,000 | 8 | 4,096 |
| 100 | 4 | 256 |
| 10 | 2 | 16 |
| 1 | 1 | 1 |

## Note

If we write 10 , is it a decimal 10 or a binary 2 ? To distinguish binary numbers from decimal numbers, append $a b$ to the end so that it reads 10b. Hexadecimal numbers are prefixed with a dollar sign (\$) or sometimes with 0x; hence, a hexadecimal 10 would look like this: $\$ 10$ or $0 \times 10$.

Each digit in binary is called a bit. With 10 bits, we can express 0 through 1023 in binary. Note that you also have 10 fingers. Counting on your fingers, with each finger corresponding to a bit, is a good way to get used to the binary system (see Figure 3.40). As you count, you'll also notice some patterns, which are also apparent in Table 3.14. The ones column alternates 0 and 1 with each row; the twos column alternates every two rows, and so on.

Figure 3.40 Counting Binary Numbers with Your Fingers


Table 3.14 Patterns in Values Expressed as Decimals, Binary Numbers, Exponents, and Hexadecimals

| Decimal | Binary | Exponential | Hexadecimal |
| :---: | :---: | :---: | :---: |
| 0 | 0000 0000b | 0 | \$00 |
| 1 | 0000 0001b | 20 | \$01 |
| 2 | 0000 0010b | 21 | \$02 |
| 3 | 0000 0011b | $2^{1}+2^{0}$ | \$03 |
| 4 | 0000 0100b | $2^{2}$ | \$04 |
| 5 | 0000 0101b | $2^{2}+2^{0}$ | \$05 |
| 6 | 0000 0110b | $2^{2}+2^{1}$ | \$06 |
| 7 | 0000 0111b | $2^{2}+2^{1}+2^{0}$ | \$07 |
| 8 | 0000 1000b | $2^{3}$ | \$08 |
| 9 | 0000 1001b | $2^{3}+2^{0}$ | \$09 |
| 10 | 0000 1010b | $2^{3}+2^{1}$ | \$0A |
| 11 | 0000 1011b | $2^{3}+2^{1}+2^{0}$ | \$0B |
| 12 | 0000 1100b | $2^{3}+2^{2}$ | \$0C |
| 13 | 0000 1101b | $2^{3}+2^{2}+2^{0}$ | \$0D |
| 14 | 0000 1110b | $2^{3}+2^{2}+2^{1}$ | \$0E |
| 15 | 0000 1111b | $2^{3}+2^{2}+2^{1}+2^{0}$ | \$0F |
| 16 | 0001 0000b | $2^{4}$ | \$10 |
| 17 | 0001 0001b | $2^{4}+2^{0}$ | \$11 |
| 18 | 0001 0010b | $2^{4}+2^{1}$ | \$12 |
| 19 | 0001 0011b | $2^{4}+2^{1}+2^{0}$ | \$13 |
| 20 | 0001 0100b | $2^{4}+2^{2}$ | \$14 |
| 32 | 0010 0000b |  | \$20 |
| 64 | 0100 0000b |  | \$40 |
| 128 | 1000 0000b |  | \$80 |
| 255 | 1111 1111b |  | \$FF |
| 256 | 000000010000 0000b |  | \$0100 |
| 1000 | 000000111110 1000b |  | \$03 E8 |
| 1023 | 000000111111 1111b |  | \$03 FF |
| 1024 | 000001000000 0000b |  | \$0400 |
| 14287 | 001101111100 1111b |  | \$37 CF |
| 65535 | 111111111111 1111b |  | \$FF FF |

Binary numbers are difficult to read and write, not to mention pronounce. Fortunately, they are very easy to translate into the more convenient hexadecimal (base 16). Note that each binary number in the table is divided into groups of four bits. With 4 bits, we can count up to 15 , which correlates perfectly to hexadecimal. Since we only have 10 symbols ( $0-9$ ) in decimal, we need to create six more for hexadecimal. For these last six symbols, the letters $A$ through $F$ are used. For example, $B$ is 11, $C$ is 12 , and $F$ is 15 . Let's take a look at a couple of hexadecimal examples using place-value calculations:
$\$ C F=12 \times 16+15 \times 1=207$
$\$ 10=1 \times 16+0 \times 1=16$
$\$$ FFFF $=15 \times 4,096+15 \times 256+15 \times 16+15 \times 1=65,535$

## Bytes

There's not much that can be expressed with a single bit; so, computers examine a collection of bits at one time. This collection of bits is called a word. Modern computers have 32- or 64-bit words.
Traditionally, personal computers such as the Apple I have used an 8-bit word, which is why you'll often hear computers from the 1970 s and early ' 80 s referred to as " 8 -bit microcomputers." This 8 -bit word is known as a byte.

Eight bits can be used to represent any letter in the alphabet, a computer instruction, a number, the color of a pixel, or data in countless other formats. A byte is not significantly harder to work with than a single bit. An 8-bit flip-flop (called a register), for example, is just eight flip-flops connected to the same clock (see Figure 3.41).

Figure 3.41 An 8-Bit Flip-Flop (Register)


## Chapter 3-Digital Logic

When we work with eight bits, it can become very repetitive drawing identical objects one after the other, eight times. A shortcut is therefore adopted whereby we use a single (usually thicker) line to represent multiple lines. The circuit in Figure 3.42 is equivalent to that in Figure 3.41, but it takes less space and is quicker to read. If you're using schematic software such as McCAD, it may forgo the slash and merely use a thicker line.

Figure 3.42 A Compact 8-Bit Flip-Flop


A selection of lines (such as D0 through D7) treated as a group is called a bus. Since our Apple I circuit is based on the byte, we will be using the 8 -bit bus quite extensively. A bus can be either parallel or serial. If you're familiar with the parallel and serial ports used on the PC and the Apple II, you probably already have an idea of the difference between the two. In a parallel bus, which you've see in Figures 3.41 and 3.42 , every bit is on its own line and arrives at the same time. At each clock pulse, a new byte of data arrives. This bus is the simplest to use and to understand.

The other option, the serial bus, uses a single data line. All data arrives over this single line, one bit after the next. Each time the clock pulses, the next bit arrives and must be saved by the receiving device. Once the eighth bit arrives, the recipient can examine the entire byte. Upon the next clock pulse, the first bit of the next byte arrives.

## Note

A nibble is 4 bits—half a byte. The term is not often used, but I mention it here for completeness.

## ASCII and the Alphabet

A byte has 256 possible combinations, more than enough to represent the entire alphabet. The American National Standards Institute (ANSI) developed a standardized code in the 1960s to facilitate the exchange of information between different computers. Called the American Standard Code for Information Interchange (ASCII), it remains in common use today. ASCII uses 7 bits to produce the 128 characters enumerated in Table 3.15.
Table 3.15 ASCII Chart

| Dec | Hex | Char | Dec | Hex | Char | Dec | Hex | Char | Dec | Hex | Char |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | NUL (null) | 32 | 20 | Space | 64 | 40 | @ | 96 | 60 |  |
| 1 | 1 | SOH (start of heading) | 33 | 21 | ! | 65 | 41 | A | 97 | 61 | a |
| 2 | 2 | STX (start of text) | 34 | 22 | " | 66 | 42 | B | 98 | 62 | b |
| 3 | 3 | ETX (end of text) | 35 | 23 | \# | 67 | 43 | C | 99 | 63 | c |
| 4 | 4 | EOT (end of trans.) | 36 | 24 | \$ | 68 | 44 | D | 100 | 64 | d |
| 5 | 5 | ENQ (enquiry) | 37 | 25 | \% | 69 | 45 | E | 101 | 65 | e |
| 6 | 6 | ACK (acknowledge) | 38 | 26 | \& | 70 | 46 | D | 102 | 66 | f |
| 7 | 7 | BEL (bell) | 39 | 27 |  | 71 | 47 | G | 103 | 67 | g |
| 8 | 8 | BS (backspace) | 40 | 28 | ( | 72 | 48 | H | 104 | 68 | h |
| 9 | 9 | TAB (horizontal tab) | 41 | 29 | ) | 73 | 49 | 1 | 105 | 69 | i |
| 10 | A | LF (line feed) | 42 | 2A | * | 74 | 4A | J | 106 | 6A | j |
| 11 | B | VT (vertical tab) | 43 | 2B | + | 75 | 4B | K | 107 | 6B | k |
| 12 | C | FF (form feed) | 44 | 2 C | , | 76 | 4C | L | 108 | 6C | । |
| 13 | D | CR (carriage return) | 45 | 2D | - | 77 | 4D | M | 109 | 6D | m |
| 14 | E | SO (shift out) | 46 | 2 E | . | 78 | 4 E | N | 110 | 6E | n |
| 15 | F | SI (shift in) | 47 | 2 F | 1 | 79 | 4 F | O | 111 | 6F | - |
| 16 | 10 | DLE (data link escape) | 48 | 30 | 0 | 80 | 50 | P | 112 | 70 | p |
| 17 | 11 | DC1 (device control 1) | 49 | 31 | 1 | 81 | 51 | Q | 113 | 71 | q |
| 18 | 12 | DC2 (device control 2 ) | 50 | 32 | 2 | 82 | 52 | R | 114 | 72 | r |
| 19 | 13 | DC3 (device control 3) | 51 | 33 | 3 | 83 | 53 | S | 115 | 73 | s |
| 20 | 14 | DC4 (device control 4) | 52 | 34 | 4 | 84 | 54 | T | 116 | 74 | t |
| 21 | 15 | NAK (neg. acknowledge) | 53 | 35 | 5 | 85 | 55 | U | 117 | 75 | $u$ |
| 22 | 16 | SYN (synchronous idle) | 54 | 36 | 6 | 86 | 56 | v | 118 | 76 | $v$ |
| 23 | 17 | ETB (end of trans.) | 55 | 37 | 7 | 87 | 57 | w | 119 | 77 | w |
| 24 | 18 | CAN (cancel) | 56 | 38 | 8 | 88 | 58 | x | 120 | 78 | $x$ |
| 25 | 19 | EM (end of medium) | 57 | 39 | 9 | 89 | 59 | Y | 121 | 79 | y |
| 26 | 1A | SUB (substitute) | 58 | 3A | : | 90 | 5A | Z | 122 | 7A | z |

Table 3.15 ASCII Chart

| Dec | Hex | Char | Dec | Hex | Char | Dec | Hex | Char | Dec | Hex | Char |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 027 | 1B | ESC (escape) | 59 | $3 B$ | $;$ | 91 | $5 B$ | $[$ | 123 | $7 B$ | $\{$ |
|  |  |  |  |  |  |  |  |  |  |  |  |
| 28 | 1C | FS (file separator) | 60 | $3 C$ | $<$ | 92 | $5 C$ | $\backslash$ | 124 | $7 C$ | $\mid$ |
| 29 | 1D | GS (group separator) | 61 | $3 D$ | $=$ | 92 | $5 D$ | $]$ | 125 | $7 D$ | $\}$ |
| 30 | 1E | RS (record separator) | 62 | 3 E | $>$ | 94 | 5 E | $\wedge$ | 126 | 7 E | $\sim$ |
| 31 | 1F | US (unit separator) | 63 | 3 F | $?$ | 95 | 5 F | - | 127 | 7 F | Del |

## A Few More Chips

We will now discuss shift registers, buffers, tri-state buffers, encoders, and decoders.

## Shift Register

A shift register is a collection of flip-flops hooked up in a row such that the output of one is the input to the next (see Figure 3.43). This allows us to store a series of data, as it comes in-1 bit for every clock pulse. Let's assume that at every pulse of the clock, a new bit of data arrives on the input line, which is, in fact, usually the case. This data will be:

## 101100000...

Though most shift registers have a "clear" input, to reset all the flip-flops to 0 , ours does not; therefore, we're going to assume that the content of the flip-flops before we enter our data is unknown (either 0 or 1). This will be represented in our table by an $x$. We also use $t$ to represent time. For example, $t 0$ is when we start, $t 1$ is immediately after the first clock pulse, $t 2$ after the second pulse, and so on.

Figure 3.43 Shift Register with D Flip-Flops


At $t 0$ we don't know the contents of any of the flip-flops, but some time before the first clock pulse a 1 arrives on the D input line. At $t 1$ the clock pulses, and this 1 is loaded into the first flopflop. As soon as it is loaded into that flip-flop, it becomes present on the $Q$ output. At $t 2$, the clock pulses again. At this instant, 1 is loaded into flip-flop \#2 and the next 0 is loaded into flip-flop \#1. You can see the full continuation of this process in Table 3.16.

Table 3.16 Shift Register Output

|  | t0 | t1 | t2 | t3 | t4 | t5 | t6 | t7 | t8 | t9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| FF \#1 | x | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| FF \#2 | x | x | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| FF \#3 | x | x | x | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| FF \#4 | x | x | x | x | 1 | 0 | 1 | 1 | 0 | 0 |

## Buffer and Tri-State Buffer

On occasion, we want to use the output of one gate as the input to quite a few other gates, but we are limited by the fact that a TTL gate can drive only 10 other gates. To alleviate this problem, we have the buffer. As you can see from Table 3.17, a buffer is like an inverter that doesn't invert-it just replicates the signal. Each buffer we have on an output can drive 10 more gates.

Table 3.17 Buffer Truth Table

| Input A | Output Y |
| :--- | :--- |
| 0 | 0 |
| 1 | 1 |

The tri-state buffer is even more interesting. When we discussed OR gates, we looked at how a gate's output is either high or low, 0 or 1.The tri-state buffer adds a third state-off. In this third state, "floating," no output at all comes from the chip and it will not interfere with other signals on the line.

Examine Figure 3.44 and Table 3.18. You can think of line $G$ as a valve that turns on the flow of electricity. So long as $G$ is low (in other words, the flow is turned off), you can put any signal you want on the output lines without harming the chip. This allows us to create circuits such as that in Figure 3.45. Beware, though, if both gate's outputs are turned on at the same time, you'll damage your chips. If you'd like to try wiring some of these yourself, you can use the 74LS367 (see Figure 3.46).

Figure 3.44 A Tri-State Buffer


Table 3.18 Tri-State Buffer Truth Table

| Input A | Input G | Output Y |
| :--- | :--- | :--- |
| 0 | 0 | X |
| 0 | 1 | 0 |
| 1 | 0 | X |
| 1 | 1 | 1 |

Figure 3.45 A Circuit with Tri-State Buffers


Figure 3.46 A 74LC367 IC


## Encoders and Decoders

We have eight inputs. Line six is high. Let's say we want to send the number 6 to our computer or to some output device such as a numeric display. The receiving device is going to expect this data in binary format. If you've paid attention to the patterns in binary digits, a binary encoder is not very difficult to make. To express numbers between 0 and 7 in binary, we need 3 bits. Examine the patterns in Table 3.14 earlier in this chapter and you'll see that the lowest bit (bit 0 ) alternates between 0 and

1 such that bit 0 is high for $1,3,5$, and 7 . The second lowest bit (bit 1 ) alternates every two lines between 0 and 1 , so it's high for $2,3,6$, and 7 . Finally, bit 3 alternates every four lines, so it's high for $4,5,6$, and 7.This circuit is expressed in Figure 3.47. The same circuit is shown in Figure 3.48 as an ordinary encoder.

Figure 3.47 Encoder with Gates


Figure 3.48 A 1-of-8 Encoder


Is it possible to go in the reverse from binary to single-line output? Let's design a 1 -of- 8 decoder (see Figure 3.49). First step, fill out a truth table (see Table 3.19). Consider writing this out by hand before looking ahead. Remember that your inputs ( $x 0, x 1, x 2$ ) represent the binary digits and $z 0$ through $z 7$ are your outputs, so when you're filling out the table, first fill in all the inputs before solving the outputs, line by line.

Figure 3.49 A 1-of-8 Decoder


Table 3.19 Decoder Truth Table

| $\mathbf{x 2}$ | $\mathbf{x 1}$ | $\mathbf{x 0}$ | $\mathbf{z 0}$ | $\mathbf{z 1}$ | $\mathbf{z 2}$ | z3 | $\mathbf{z 4}$ | $\mathbf{z 5}$ | $\mathbf{z 6}$ | $\mathbf{z 7}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

Next, write logic statements based on this truth table. For example, $z 0$ is high only when $x 2$ is not high, $x 1$ is not high, and $x 0$ is not high; hence, we write $z 0=x 0^{\prime} \bullet x 1^{\prime} \cdot x 2^{\prime}$. The complete set of logic statements is:
$\mathrm{z0}=\mathrm{x} 0^{\prime} \cdot \mathrm{x} 1^{\prime} \cdot \mathrm{x} 2^{\prime}$
$z 1=x 0 \cdot x 1^{\prime} \cdot x 2^{\prime}$
$z 2=x 0^{\prime} \cdot x 1 \cdot x 2^{\prime}$
$\mathrm{z} 3=x 0 \cdot x 1 \cdot x 2^{\prime}$
$z 4=x 0^{\prime} \cdot x 1^{\prime} \cdot x 2$
$\mathrm{z} 5=\mathrm{x} 0 \cdot \mathrm{x} 1^{\prime} \cdot \mathrm{x} 2$
$\mathrm{z} 6=x 0^{\prime} \cdot x 1 \cdot x 2$
$\mathrm{z7}=\mathrm{x} 0 \cdot \mathrm{x} 1$ • x 2
Finally, we draw a circuit based on these logic gates (see Figure 3.50). (You might notice that this circuit uses three more inverters than it needs to. There are two ways to cut the number of inverters in half, by using DeMorgan's law or by simply rearranging some wires.)

Figure 3.50 A Decoder with Gates


## Summary

We've covered a lot of ground in this chapter. Each layer-digital logic, microcomputer, and soft-ware-can stand on its own, so there's no need to be too concerned if you didn't grasp everything; that doesn't mean you won't be able to understand the upcoming chapters. If you'd like to learn even more about digital logic, I would encourage you to look at the following list of resources:

Books:
The Art of Electronics, by Horowitz and Hill (Cambridge University Press, 1989)
Hands-On Electronics, by Kaplan and White (Cambridge University Press, 2003)
Web sites:
CAL for Digital Logic: www.ee.surrey.ac.uk/Projects/Labview/
Digital Logic: www.play-hookey.com/digital/
Discover Circuits: www.discovercircuits.com/D/digital.htm
How Electronic Gates Work: http://electronics.howstuffworks.com/digital-electronics.htm
TTL Data Book: http://upgrade.cntc.ac.kr/data/ttl/

