ME 343: Mechanical Design-3

Design of Shaft

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Outline

- Practical information
- Shaft design

Instructor

- Lecturer: Dr. Aly Mousaad Aly
- Office: Last floor, Dept. of Mech. Eng.,

  Faculty of Eng., Alexandria University
- Office hours:

  Thursday, 8:30 to 9:35

  Thursday, 11:00 to 12:05

Course Materials

Slides: Available online.

Available at the department copy center.

Course website:

www.engr.uconn.edu/~aly/ME343

References:

- Shigley's Mechanical Engineering Design, Eighth Edition, The McGraw–Hill Companies, Inc., 2006.

Grading

- Class participation
- Assignments
- Reports
- Midterm exam
- Final examination

Policy

- Attendance to lectures and exercises is compulsory.
- We may check the attendance at the beginning of lessons.
- Everybody should attend his scheduled classes according to his name and student number. We will be ***VERY STRICT about this rule.***
- *Come* to lessons about 5 min before the starting time.

Outline

- Practical information
- Shaft design

Definition of shaft?

- It is a rotating member, in general, has a circular cross-section and is used to transmit power.
- The shaft may be solid or hollow. It is supported on bearings and it rotates a set of gears or pulleys for the purpose of power transmission.
- The shaft is generally acted upon by bending moments, torsion and axial forces.

Shaft versus axle and spindle

Axle is a non-rotating member used for supporting rotating wheels, etc., and do not transmit any torque. Spindle is simply defined as a short shaft. However, design method remains the same for axle and spindle as that for a shaft.

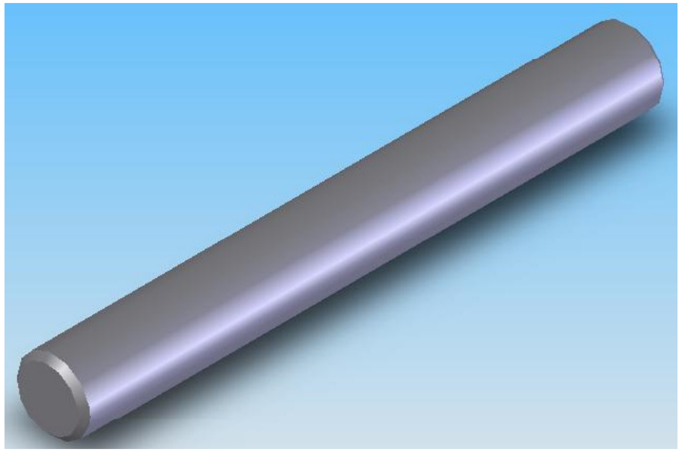
What does it mean “shaft design”?

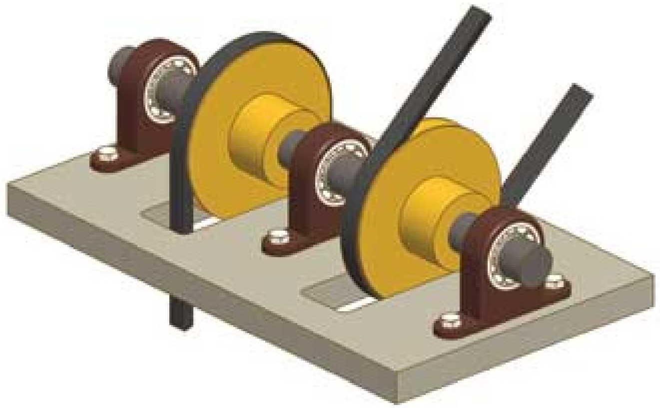
- Material selection
- Geometric layout
- Stress and strength: static and fatigue
- Deflection and rigidity: bending defl., torsional twisting, slope at bearings and shaft-supported elements, and shear deflection due to transverse loading on short shafts.
- Vibration: critical speed

Material selection

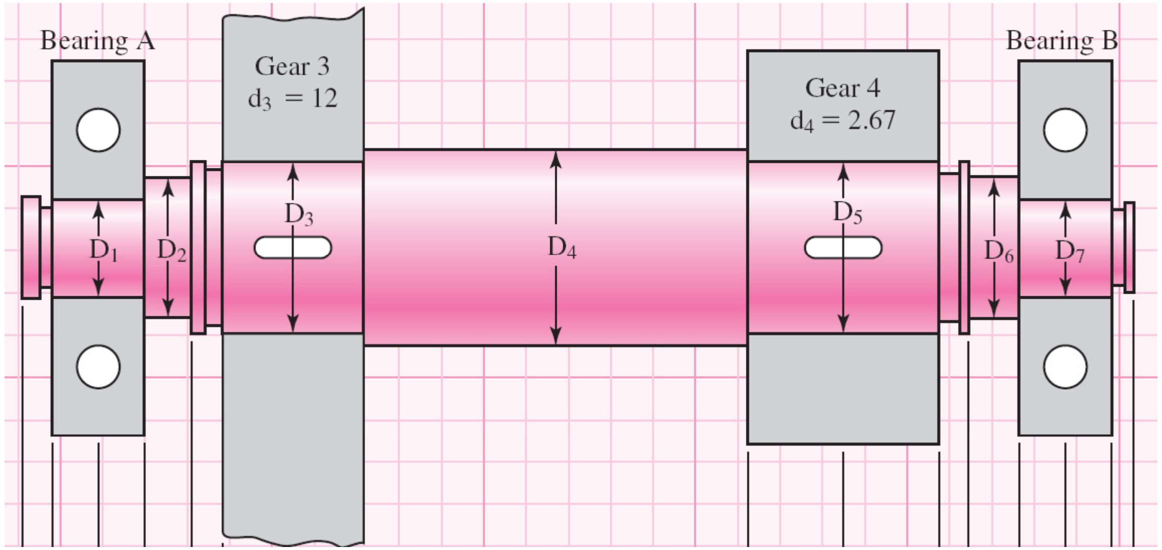
- Many shafts are made from low carbon, cold-drawn or hot-rolled steel.
- Alloy steel: Nickel, chromium and vanadium are some of the common alloying materials. However, alloy steel is expensive.
- Shafts usually don’t need to be surface hardened unless they serve as the actual journal of a bearing surface.
- *Hardening of surface (wear resistant):* case hardening and carburizing ; cyaniding and nitriding.

Geometric layout





Geometric layout



- The geometry of shaft is generally that of stepped cylinder.
- There is no magic formula to give the shaft geometry for any given design situation.

Geometric layout

- The best approach is to learn from similar problems that have been solved and combining the best to solve your own problem.
- A general layout to accommodate shaft elements, e.g. gears, bearings, and pulleys, must be specified early in the design process.
- Shoulders are used for axially locating shaft elements and to carry any thrust loads.
- Common Torque Transfer Elements: keys, set screws, pins, press or shrink fits, tapered fits.

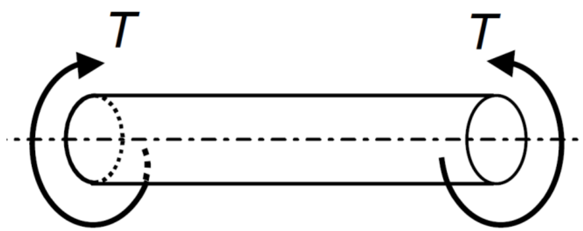
Geometric layout

- Small pinions are often machined onto shafts.
- Sequence of assembly should be thought.
- Use chamfers to ease assembly and avoid interferences.
- Consider stress risers due to grooves and sharp steps in shafts.
- What can fail and how will it happen?

Shaft design based on strength

Design is carried out so that stress at any location of the shaft should not exceed material yielding.

Stress due to torsion:



$$\tau_{xy} = \frac{T \times r}{J} = \frac{16T}{\pi d_o^3 \left(1 - c^4\right)}$$

τ_{xy} *: Shear stress due to torsion*

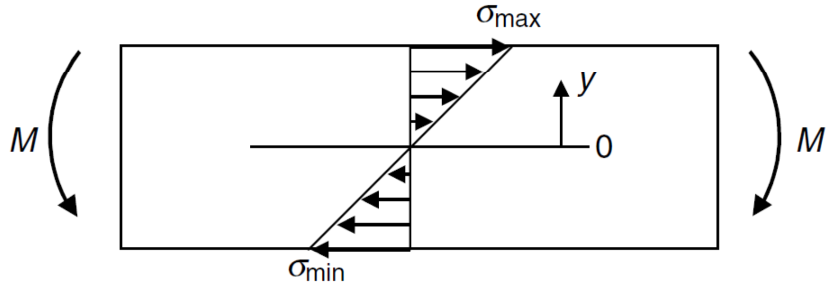
T : Torque on the shaft

Note: $$T \approx \frac{7024 \times hp}{N\,(RPM)} \approx \frac{9549 \times kW}{N\,(RPM)}$$

Shaft design based on strength

Bending stress:

$$\sigma_b = \frac{M \times y}{I} = \frac{32M}{\pi d_o^3 (1 - c)}$$



M : Bending moment at the point of interest

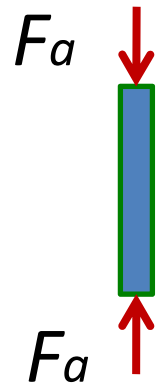
do : Outer diameter of the shaft

c: di/do

Shaft design based on strength

Axial stress:

$$\sigma_a = \frac{F_a}{A} = \frac{4\alpha F_a}{\pi d_o^2 (1 - c^2)}$$



Fa: Axial force (tensile or compressive)

α: Column-action factor(= 1.0 for tensile load)

α arises due to the phenomenon of buckling of long slender members which are acted upon by axial compressive loads.

Shaft design based on strength

Axial stress (continue):

$$\alpha = \frac{1}{1 - 0.0044\lambda}, \qquad (\lambda = L/r) \leq 115$$

$$\alpha = \frac{\lambda^2 s_{yc}}{\pi^2 n E}, \qquad \lambda > 115$$

n = 1.0 for hinged end; n = 2.25 for fixed end

n = 1.6 for ends partly restrained, as in bearing ,
L = shaft length

s_{yc} = *yield stress in compression*

Shaft design based on strength

Maximum shear stress theory (ductile mat.):

Failure occurs when the maximum shear stress at a point exceeds the maximum allowable shear stress for the material. Therefore,

$$\tau_{\max} = \tau_{allowable} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{allowable} = \frac{16}{\pi d_o^3\left(1-c^4\right)}\sqrt{\left(M + \frac{\alpha F_a d_o\left(1+c^2\right)}{8}\right)^2 + T^2}$$

Shaft design based on strength

Maximum normal stress theory (brittle mat.):

$$\sigma_{\max} = \sigma_{allowable} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{allowable} = \frac{16}{\pi d_o^3 \left(1 - c^4\right)} \left[\left(M + \frac{\alpha F_a d_o \left(1 + c^2\right)}{8} \right) + \sqrt{\left(M + \frac{\alpha F_a d_o \left(1 + c^2\right)}{8} \right)^2 + T^2} \right]$$

Shaft design based on strength

Von Mises/ *Distortion-Energy theory:*

$$\sigma_{\max} = \sigma_{allowable} = \sqrt{\sigma_{\mathrm{x}}^2 + 3\tau_{\mathrm{xy}}^2}$$

$$\sigma_{allowable} = \frac{16}{\pi d_o^3 \left(1 - c^4\right)} \sqrt{\left(2M + \frac{\alpha F_a d_o \left(1 + c^2\right)}{4}\right)^2 + 3 \times T^2}$$

Shaft design based on strength

ASME design code (ductile material):

$$\tau_{allowable} = \frac{16}{\pi d_o^3\left(1-c^4\right)}\sqrt{\left(k_m M + \frac{\alpha F_a d_o\left(1+c^2\right)}{8}\right)^2 + \left(k_t T\right)^2}$$

where, k_m and k_t are bending and torsion factors accounts for shock and fatigue. The values of these factors are given in ASME design code for shaft.

Shaft design based on strength

ASME design code (brittle material):

$$\sigma_{allowable} = \frac{16}{\pi d_o^3 \left(1-c^4\right)} \left[\left(k_m M + \frac{\alpha F_a d_o \left(1+c^2\right)}{8} \right) + \sqrt{\left(k_m M + \frac{\alpha F_a d_o \left(1+c^2\right)}{8} \right)^2 + \left(k_t T\right)^2} \right]$$

Shaft design based on strength

ASME design code:

Combined shock and fatigue factors

Type of load	Stationary shaft		Rotating shaft	
	k_m	k_t	k_m	k_t
Gradualy applied load	1	1	1.5	1
Suddenly applied load, minor shock	1.5-2	1.5-2	1.5-2	1-1.5
Suddenly applied load, heavy shock	---	---	2-3	1.5-3

Shaft design based on strength

ASME design code:

Commercial steel shafting

$\tau_{allowable}$ = 55 MPa for shaft without keyway

τallowable = 40 MPa for shaft with keyway

Steel under definite specifications

τallowable = 30% of the yield strength but not over 18% of the ultimate strength in tension for shafts without keyways. These values are to be reduced by 25% for the presence of keyways.

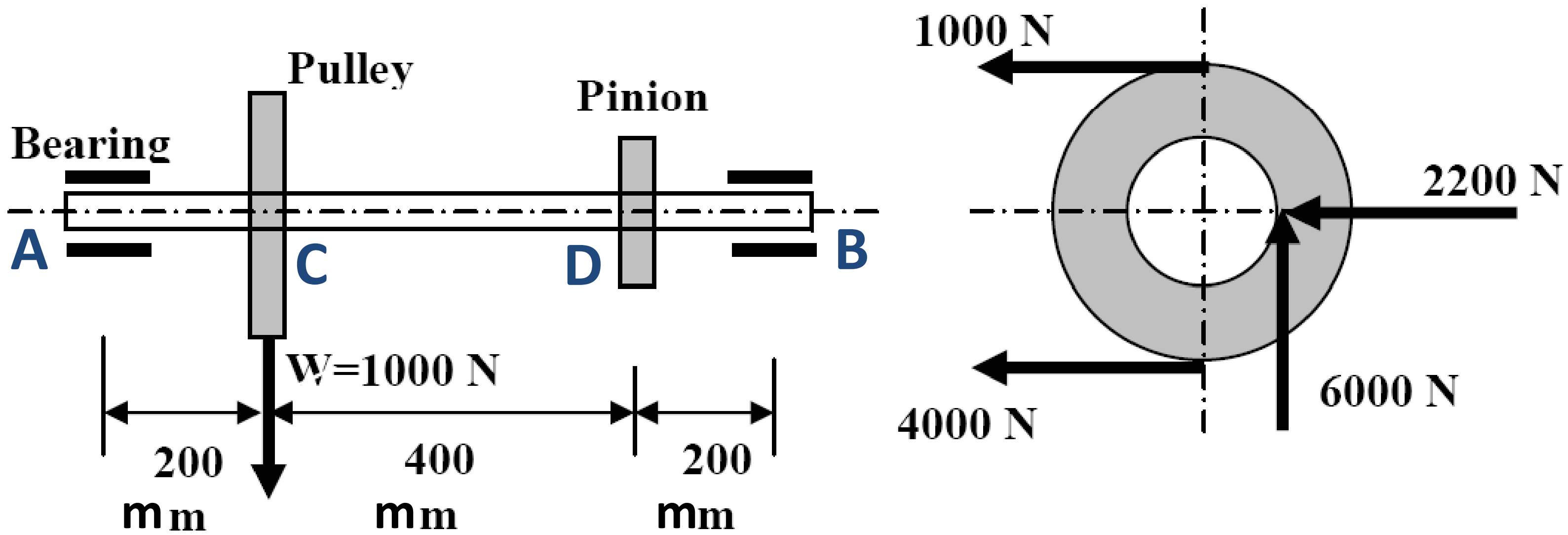
Standard sizes of shafts

Typical sizes of solid shaft that are available in the market are:

diameter	increments
up to 25 mm	0.5 mm
25 to 50 mm	1.0 mm
50 to 100 mm	2.0 mm
100 to 200 mm	5.0 mm

Example: problem

A pulley drive is transmitting power to a pinion, which in turn is transmitting power to some other machine element. Pulley and pinion diameters are 400 mm and 200 mm respectively. Shaft has to be designed for minor to heavy shock.



Example: solution

Torsion:

$T_D = 6000 \times (D_{pinion}/2)$

$= 6000 \times (200/2)$

$= 6 \times 10e5$ N.mm

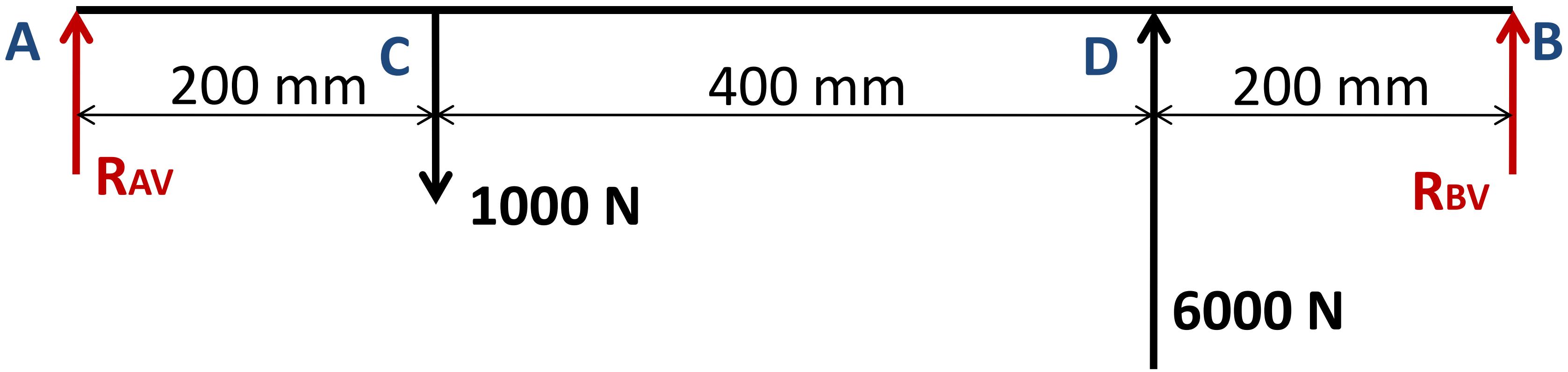
OR

$T_C = (4000 - 1000) \times (D_{pulley}/2)$

$= 3000 \times (400/2) = 6 \times 10e5$ N.mm

Example: solution

Bending (vertical plane):



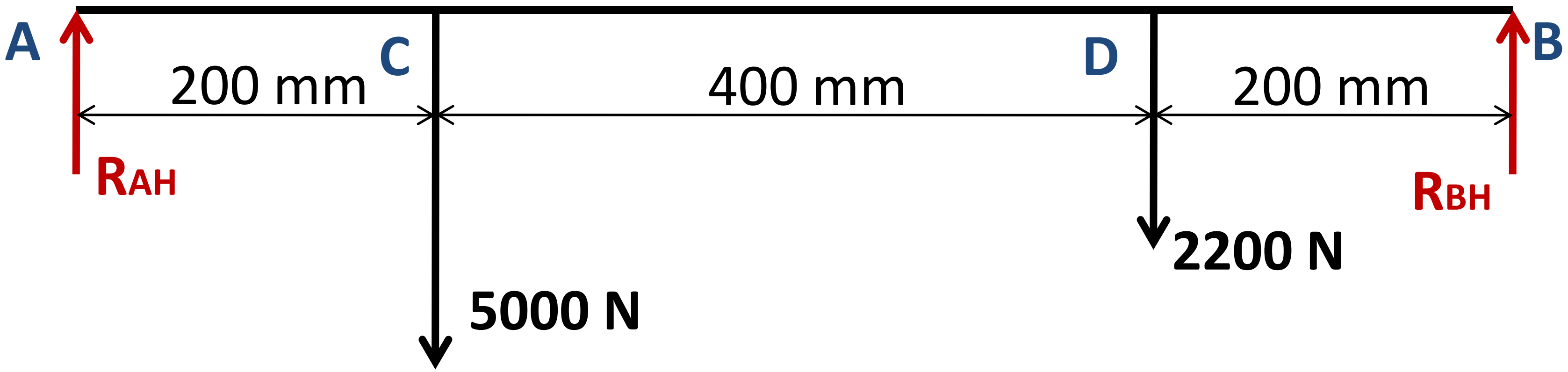
R_{BV}= (1000x200- 6000x(400+200))/(200+400+200) = -4250 N

M_{DV}= -4250x200 = -8.5e5 N.mm

M_{CV}= 6000x400 - 4250x600 = -1.5e5 N.mm

Example: solution

Bending (horizontal plane):



R_{BH} = (5000x200 + 2200x(400+200))/(200+400+200)

= 2900 N

M_{DH} = 2900x200 = 5.8e5 N.mm

M_{CH} = 2900x600 - 2200x400 = 8.6e5 N.mm

Example: solution

Bending (resultant):

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2}$$

$$= 10.29 \times 10^5 \, N.mm$$

Similarly,

$$M_C = \sqrt{\left(1.5 \times 10^5\right)^2 + \left(8.6 \times 10^5\right)^2}$$

$$= 8.73 \times 10^5 \, N.mm$$

Since $T_C = T_D$ and $M_D > M_C$, section-D is critical.

Example: solution

ASME code:

Under minor to heavy shock, let us consider $k_m = 2$ and $k_t = 1.5$. Also let us assume the shaft will be fabricated from commercial steel, i.e. $\tau_{allowable}$ = 40 Mpa.

$$d_o^3 = \frac{16}{40 \times \pi} \sqrt{\left(2 \times 10.29 \times 10^5\right)^2 + \left(1.5 \times 6 \times 10^5\right)^2}$$

$$d_o = 65.88 \ mm$$

The value of standard shaft diameter is 66 mm.