## **Postgraduate Nuclear Laboratory**

### 2018/2019

# MSc – Nuclear Decommissioning and Waste Management

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# MSc – Physics and Technology of Nuclear Reactors

**Laboratory Manual** 

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### 1 Introduction and Information

### 1.1 General Introduction

Welcome to the Nuclear Laboratory, which takes place in R11 on the first floor, and PB2K in the basement, of the Poynting building. The laboratory module consists of  $\sim$ 6 hours work per week, that you are in the University, and is combined with work outside the laboratory which involves reading about radiation detection, researching the physics of your experiments, analysing your data, and writing up your laboratory reports. The laboratory hours are 10 am to 5 pm on Friday (as per your timetable) with a 1 hour lunch break between 1 pm and 2 pm. PTNR students attend the nuclear laboratory in both the Autumn and Spring Terms, while NDAWM students attend during the Autumn Term only.

The course assessment is via written (word processed) reports (see section 1.4). PTNR students will submit two reports, one at the end of the Autumn Term, and one at the end of the Spring Term. NDAWM students will also submit two reports, one at the beginning of the Spring Term and one during the Easter vacation. The deadlines for these will be announced on the relevant Canvas page and via email. A further contribution to the final mark will be derived from an assessment of the laboratory notebook which should be handed in at the same time as the final laboratory report. Material received after these dates may not be assessed, unless accompanied by an extenuating circumstances note.

Important: The laboratory shares the laboratory facilities with both the MSc courses and the Year 3 laboratories in both Nuclear Physics and Nuclear Engineering in the first semester. Therefore, you should check carefully whether any apparatus has been left on when you begin your session, in case any high voltages are set. When you finish the session, please take care to wind down any high voltages to zero and restore the experiment to how you found it. Be sure to note any settings of the electronics, so that you can reset them on the next laboratory session. Do not move equipment from one experiment to another.

### 1.2 Aims and Objectives

The aims of this laboratory are that:-

- You discover lots of interesting physics which are useful and relevant to your future careers.
- Improve your knowledge and skills in both physics and in written communications.
- Correlate material learnt in lecture courses with the practical applications in the laboratory.
- Develop new skills learnt in other sections of the course, e.g. numerical analysis, statistics, Monte-Carlo etc.

The objectives for this laboratory are that you will acquire the following skills:-

- · A familiarity with basic concepts of nuclear physics.
- An understanding of the physical processes in the interactions of, and detection of, ionizing radiations.
- The use of standard nuclear electronics and simple data acquisition systems, the planning of measurements, and the analysis of data.
- The ability to start from a brief description of the experiment allowing you to plan how to set up the apparatus and to make it work.
- The ability to research properly an unfamiliar topic, by searching books, references, websites etc. when required.
- The important self-discipline to monitor the performance of your apparatus and your command of it in an on-line fashion, ensuring that the results you are obtaining are sensible, and that your analysis of the data is proceeding correctly *during* the experiment.
- The ability to keep a coherent and up-to-date record of your laboratory work and of your data analysis as the experiment is proceeding.
- The ability to extract the maximum physics output from your data, and to compare your results, in a critical fashion, with what is expected from previous work on the topic.
- The ability to write scientific reports.
- The ability to critically examine your own work.

### 1.3 Health and Safety

Your health and safety and the health and safety of those around you are of utmost importance. In addition to the usual safety precautions applicable to all laboratory work, please make a careful note of the following rules that apply in the Nuclear Laboratory. It is your responsibility to adhere to these rules to ensure a safe working environment for everyone in the laboratory.

### 1.3.1 General precautions

- Eating and drinking are not permitted anywhere within the laboratory.
- Wash your hands at the end of the laboratory session and before taking a tea break.

#### 1.3.2 Radioactive sources

- Radioactive sources are issued from a metal cupboard in the source room.
- A radioactive source may only be removed from the cupboard after it has been signed out. You are personally responsible for any source that you have signed out and you must ensure that it is returned at the end of a session.
- A demonstrator must be present when a source is signed out. All sources must also be returned to a demonstrator when you have finished with them.
   Only demonstrators can sign sources in/out of the source cupboard.
- Students may need to remove sources from the main laboratory (e.g. <sup>90</sup>Sr sources for use in PB2K). When checked out, the source location must be recorded.
- Care must be taken not to lose or damage sources. If you damage a source or a source appears to be damaged, inform a member of staff IMMEDIATELY.
- The sources you will use are relatively weak and are quite safe if handled intelligently. A list of available sources is kept in the metal cupboard.
- The majority of our  $\gamma$ -ray sources are sealed in plastic and any  $\alpha$  or  $\beta$  particles that are produced by the source do not get out. Sources of  $\alpha$  or  $\beta$  particles are unsealed, also known as open sources, and these must be handled carefully to avoid transferring radioactive material to hands or clothing. Gloves should be worn and tweezers used when handling open sources.
- Demonstrators will inform you if you need to use an open source. If you are unhappy or unwilling to handle the source yourself, just ask the demonstrator to mount the source in your experiment for you.
- Where stronger sources are in use for a physics experiment, or when apparatus may be counting for a long time, you should avoid sitting near the source for long periods. Adhere to the ALARP principle to ensure that the dose you,

and those around you, receive in the laboratory is As Low As Reasonably Practicable.

 If you leave your experiment running overnight display a notice advising the type and location of source that is present.

#### 1.3.3 Toxic materials

- A small number of materials in use in the laboratory are toxic to the human body. These comprise lead (used as shielding), cadmium (used to absorb thermal neutrons) and bismuth (used as an absorber of γ-rays). Use gloves when handling any of these materials and make a habit of washing your hands after each laboratory session and before taking a tea break.
- If using lead bricks to shield your apparatus, take care not to overload the bench Please adhere to a maximum of 10 bricks per bench.
- Be aware of the potential for crushing injuries when handling lead bricks. Do not carry more than one lead brick at a time. Do not place lead bricks close to the edge of the bench where they may topple onto the floor.

### 1.3.4 High voltage and cables

- Many detectors use High Voltage (HV) power supplies. ALWAYS turn off the
  HV before disconnecting cables or before performing any examination of your
  detector. NEVER turn off the 240 V mains power supply to ANY apparatus
  that is supplying high voltage to a detector without first reducing the HV supply
  setting to zero. Failure to observe this precaution may destroy the delicate
  electronics incorporated in your detector.
- To avoid the risk of electric shock, be careful to use only specially designed HV cables to connect your detector to its power supply. There are four types of connectors used in the laboratory: SHV (Safe High Voltage), BNC (low voltage signal cables used to connect NIM electronics units), LEMO (compact signal connectors used in some electronics units) and MCX (micro coaxial signal connectors used solely on the CAEN digitiser modules). It is easy to confuse SHV and BNC connectors. If in doubt, ask a demonstrator.
- Beware of the potential for trip hazards caused by dangling cables. Choose appropriate cable lengths to connect modules and detectors. Do not allow cables to trail on the floor.

Above all, watch out for the safety of yourself and your fellow students. If you are unsure about any of the safety aspects of your experiment, or you see someone behaving irresponsibly, please consult a demonstrator.

### 1.3.5 Notes on radiation units and dose limits

Units given in italics are now obsolete but may be encountered.

- Activity measure of decay rate. The SI unit of activity is the Becquerel (Bq) = 1 disintegration per second. Activities of the various sources in the laboratory are often quoted in Curies (Ci). 1 Ci = 3.7 × 10<sup>10</sup> Bg
- Absorbed dose energy deposited per unit mass. The SI unit of absorbed dose is the Gray (Gy) = 1 J/kg 1 rad = 0.01 Gy; 1 Gy = 100 rad
- Equivalent dose Absorbed dose × radiation weighting factor. The SI unit of equivalent dose is the Sievert (Sv) The weighting factor of X-rays, gammarays, electrons, positrons and muons of all energies is equal to unity. For neutrons, the weighting factor depends on energy and has a maximum value of 20 in the energy range 100 keV < E < 2 MeV. For alpha particles and heavy nuclei, the weighting factor is also 20. So, 1 Gy of X-rays = 1 Sv, but 1 Gy of alphas = 20 Sv. 1 rem = 0.01 Sv; 1 Sv = 100 rem</li>

The Ionising Radiations Regulations (IRR99) limits the annual whole body effective dose (which is the same as the equivalent dose for whole body exposures) for classified radiation workers to 20 milliSieverts (mSv). For members of the public the recommended annual effective dose limit is 1 mSv. (These limits exclude the dose you receive from natural background sources and medical procedures. For reference, the typical annual dose from background radiation in the UK is around 2 mSv.) Although there are exemptions for students, the University treats them as members of the public. This means that students and demonstrators do not have to wear a personal dosimeter to work in the teaching laboratories.

To ensure that the total dose is below 1 mSv from a full time exposure of 2000 hours per year (40 hour week for 50 weeks per year) requires the average dose rate be less than 1 mSv/2000 =  $0.5~\mu$ Sv/hr. Masters students typically spend 120 hours in the laboratory (6 hours per week for 20 weeks). Undergraduate physics students also spend no more than 120 hours in the laboratory (8 hours per week for 15 weeks). Over this period the average dose rate becomes 1 mSv/120 =  $8.3~\mu$ Sv/hr. No experiment will expose you to an average dose rate larger than this value. In most cases the dose rate will be well below 1  $\mu$ Sv/hr.

### 1.3.6 Radioactive sources requiring special handling

The majority of radioactive sources that you will encounter in the laboratory are gamma-ray sources with individual activities typically less than 500 kBq. No special precautions need to be taken when using these sources, although you should always consider the placement and shielding of sources to ensure that your exposure to radiation is As Low As Reasonably Practicable.

There are a small number of strong sources for which special handling precautions should be taken. These sources are:

- C3N 19/22/1, collimated <sup>137</sup>Cs source used in the Compton Scattering experiment
- C3N 19/65, strong <sup>137</sup>Cs source sometimes used in absorption experiments
- ANS 46, annular <sup>241</sup>Am source used in the X-ray fluorescence experiment

- C3N 19/87, open  $^{241}\mathrm{Am}$  source can be used for X-ray fluorescence or as an  $\alpha$  source.
- AMN 1000, 1 Ci Am-Be neutron source located in the water bath in R11
- ANS 1, 0.3 Ci Am-Be neutron source located in PB2K used in the graphite stack experiment.
- ANS 2, 1 Ci Am-Be neutron source located in PB2K used in the neutron activation experiment.
- ANS 3, 3 Ci Am-Be neutron source located in PB2K used in the graphite stack experiment.

These sources may only be handled by a demonstrator. The demonstrator will instruct you how to use the sources safely and ensure that the source is safely installed in the experiment at the start of the session and removed at the end of the session.

### 1.4 Assessment

Students are expected to work in pairs or threes because the number of separate experiments is limited, but the written reports **must** be written individually.

A good report needs planning; your lab reports should be written in the same structured style as a professional research paper, with a title, abstract, introduction, some sections in the middle, and finally some conclusions. Try to illustrate the text with figures of your results, so that the reader can follow the flow of the project without having to continuously flip to the appendices. Lastly, you must make a critical summary in your conclusions, discussing your results with reference to previous work and to the experimental errors and, perhaps, present some ideas for improvements. Don't forget to include ALL your references (books/journals/www etc.)! Note that references are NOT the same as a bibliography.

The reports should be written so as to be understood by a physicist who does not know in advance anything about the experiment which you have done, so you must explain what the purpose of the experiment was, how you did it, what the results are, and their significance. Reports should be written in clear English and in sentences. Conventionally, descriptions of what you did are generally written in the past tense using a passive style ("The detector was connected to an MCA...")

While you may wish to explain briefly the relevance of your experiment to practical applications, do not spend too much time on background but concentrate on the underlying physics phenomena. You need not give all the details of your measurements or of your calculations, but you should explain what you did in sufficient detail that someone else would be able to reproduce your measurements and get essentially the same results. Some additional details may be given in appendices. When you state results, always give units and always give uncertainties. If you quote a published result (e.g. the accepted value of a particular quantity, or a formula) then give a numbered reference in sufficient detail that the reader can look it up (i.e. if the reference is to a book then give the page number). The report should be word-processed, but neat hand-drawn graphs are also acceptable.

PTNR students are expected to complete approximately seven experiments throughout the year, and present written reports for *two* of them. These reports, along with the laboratory notebook, provide the formal assessment for this laboratory module. The deadlines for submission of the reports will be announced on the relevant Canvas page and by email.

NDAWM students are expected to produce *two* written reports about their investigations. The first will be submitted at the start of semester 2 and will be used to provide formative feedback on what was done well, and what could be improved. The second report will be submitted after the end of semester 2 in the Easter vacation, and will contribute 50% to the module mark.

Material received after these dates may not be assessed, unless accompanied by an extenuating circumstances note approved by Dr Ian Stevens.

### 1.4.1 Laboratory Reports

This should take the form of a formal write-up, normally to be handed in by the deadline given on Canvas. However, you can submit this report at any time during the laboratory. The report should be about 20 pages in length (including figures, tables and appendices). Marks are awarded in three general categories with different weights:

- Theoretical background, research and references (30%): You should make good use of the references cited in this manual and may need to do further reading relevant to your particular experiment. Where possible, you should compare and discuss your experiment and the results with previous measurements or other experiments.
- 2. The account of your experimental work, including marks for presentation and style (40%): You should make it clear how you actually performed the experiment. Marks are allocated to reward conscientious effort and for attention to detail, such as optimisation of shielding in some experiments, or demonstrating your understanding of detectors or electronics. Marks will be awarded for how you have organized your report and presented the information to the reader. You should be careful to label diagrams, and number all figures and tables (and pages); and label axes on histograms.
- 3. Analysis and interpretation of your results (30%): We will be looking for a clear and concise description of the principles of your experiment and the method of your analysis, including some consideration of the experimental uncertainties (e.g. statistical errors). A summary or conclusions section should include your main results with a discussion of the outcome of your experiment in light of your experience. You may wish to suggest ways in which your experiment could be improved.
- 4. The School's policy is that all reports must be prepared electronically, so that they can be submitted to the Turnitin plagiarism checking system. Your report should demonstrate your understanding of the physics involved in the experiment, your competence as an experimental scientist, and should demonstrate your participation in the degree programme as evidenced by background information and methods.

### 1.4.2 Laboratory Notebook

The notebook should be handed in with the final laboratory report. Assessment will be based on the following criteria:

- Completeness of the note taking. The notebook should contain a brief description of the main ideas behind the measurements (e.g. theory and main objectives). The details of the experimental equipment should be complete and should be sufficient for someone else to reconstruct the setup exactly.
- 2. The notebook should contain a clear record of measurements made, the procedure and errors (uncertainties).
- 3. The analysis of the data should be recorded, this would include graphs, calculations, error analysis, fits with models.
- 4. There should be a short discussion on the main conclusions of the measurements, documenting references to any relevant literature used.
- 5. The notebook should have a record of 6–7 experiments for PTNR students, and all experiments for NDAWM students.

### 1.5 Plagiarism

Plagiarism is taken very seriously at the University, and any offenders will face sanctions ranging from loss of marks up to exclusion from the University for more extreme cases. You must reference all material that you have used or adapted from external sources – i.e. anything that you have not produced entirely by yourself – including figures.

You can quote or paraphrase sources, as long as you indicate that you are doing so. However, this does not demonstrate your own understanding of the topic, and is therefore not worth much credit. The lab reports are your opportunity to show your understanding of the phenomena you have investigated, not someone elses!

All reports are submitted electronically to the Turnitin plagiarism checking system to identify plagiarised material. The markers are also very familiar with the relevant sections of the standard textbooks, so will recognize copied or paraphrased text and expect to see appropriate references.

Further information and guidelines can be found in the Plagiarism section in the Practical Skills Module on Canvas.

### 1.6 Notes on Detectors

The general principles of the detectors that you use are described in the comprehensive book by Knoll, *Radiation Detection and Measurement*. The brief information in this manual should be reinforced by further reading from the bibliography in section 1.10.

### 1.6.1 Poisson statistics and energy resolution

It is important that you understand the process by which the signal from the detector is produced, because this affects the properties of the output signals. Detectors use the interactions of ionizing radiation to convert the energy of the radiation into an output signal. You need to distinguish between the particle that you are trying to detect and what creates the signal in the detector.

For example, if you detect a gamma ray with a germanium counter, the *energy*, E, of the gamma ray is converted into fast moving electrons, which ionize atoms in the crystal releasing a *number*,  $N_s$ , of electron-hole (e-h) pairs which are then captured at electrodes. In other types of detection media such as scintillators,  $N_s$  is the number of photons produced by excitation, whilst in gas-filled detectors,  $N_s$  is the number of electron-ion pairs. The number of e-h or ion pairs, or scintillation photons, is determined by the *energy per signal carrier*,  $\omega$ , so that

$$N_s = E/\omega \tag{1}$$

It is  $N_s$ , the number of signal carriers, which is amplified to form the output *analogue* signal, and the amplitude of the output signal is directly proportional to this original number. It is the Poisson statistics of  $N_s$  which determines the ultimate *energy* resolution of the output signal, and hence the width of the peak that you see on the computer display of the energy spectrum.

Following Poisson statistics, the standard deviation of  $N_s$  counts is

$$\sigma(N_s) = N_s^{1/2} \tag{2}$$

so the detector energy resolution must be proportional to  $\sigma(N_s)$  if only statistical factors are involved. For large  $N_s$ , the Poisson distribution tends towards the normal distribution, also called a Gaussian error curve. The resolution of a Gaussian distribution is usually measured as the *Full Width at Half Maximum*, or FWHM, which is given by:

$$FWHM(N_s) = 2\sqrt{2 \ln 2} \ \sigma(N_s) = 2.35 N_s^{1/2}$$
 (3)

We can also see that, for a given normal distribution, the ratio R of the FWHM( $N_s$ ) to its centroid value must be

$$R_{Poisson} = \text{FWHM}(N_s)/N_s = 2.35N_s^{1/2}/N_s = 2.35N_s^{-1/2}$$
 (4)

In the case of semiconductor detectors such as silicon and germanium it is found that the value of R may be 3 or 4 times smaller than expected from Poisson statistics, and a factor F (Fano factor) has been introduced to quantify this effect on the

observed variance. Accordingly, this results in a modified equation

$$R_{Statistical} = \text{FWHM}(N_s)/N_s = 2.35[FN_s]^{1/2}/N_s = 2.35F^{1/2}N_s^{-1/2}$$
 (5)

Typical values of F are about 0.1 or less for Si and Ge.

Since E, the energy deposited, is proportional to  $N_s$ , it is obvious that the FWHM resolution in energy units must also be proportional to  $N_s^{1/2}$  and that the ratio  $R_E$  of the energy resolution FWHM(E) to the energy E must be

$$R_E = \text{FWHM}(E)/E = 2.35E^{-1/2}$$
 (6)

assuming that there are no other contributions to the energy resolution from detector or amplifier noise.

Thus a plot of  $\ln(R_E)$  versus  $\ln(E)$  for various energy peaks in your spectra should yield a gradient of -1/2 if the energy resolution is limited by Poisson statistics or a gradient of -1 if the energy resolution is limited by noise.

Note that there are some detectors, such as Geiger Counters, and also Fission Counters for neutrons, whose output signals are almost independent of the energy of the incident radiation, so that usually these detector system emit only a *digital* signal which gives the time of arrival of the radiation, but carries no information about the energy of the particle.

### 1.6.2 Detection efficiency

In order for a quantum of radiation  $(\alpha, \beta, \gamma)$ , neutron etc.) to be counted, it must interact with the detector and produce a sufficient number of signal carriers (ion pairs, electron-hole pairs, scintillation photons etc.) to create an output pulse large enough to be measured. As charged particles, such as  $\alpha$  and  $\beta$  radiation, lose energy rapidly by ionizing or exciting the detector material directly, they can be detected with 100% efficiency. However, neutral radiation such as neutrons or  $\gamma$ -rays must have an interaction within the detector, producing charged secondary radiation that can then be detected. As the probability of such an interaction can be small, there is a chance that some of the incident radiation will not interact at all – producing no output signal – or that only some of the incident energy is deposited in the detector, giving a smaller output signal than expected.

There are two usual measures of how well a detector converts incident radiation into an output signal proportional to the incident energy: peak absolute efficiency,  $\epsilon_{\rm abs}$ , and peak intrinsic efficiency,  $\epsilon_{\rm int}$ . The peak absolute efficiency gives the fraction of all emitted radiation of the energy of interest that deposits its full energy in the detector. So if a source has activity A Bq (i.e. there are A disintegrations per second) and a fraction  $\gamma$  (called the branching ratio or intensity) of these decays results in the radiation of interest, then the rate at which this radiation is produced is  $A\gamma$ . If the measured count rate for the full energy peak is  $R_p$  counts per second, then the peak absolute efficiency is:

$$\epsilon_{\mathsf{abs}} = \frac{R_p}{A\gamma} \tag{7}$$

However, unless the detector completely surrounds the source, not all of the emitted radiation will actually reach the detector, so the peak absolute efficiency depends on the geometry of the setup – basically the distance between the source and detector. The *peak intrinsic efficiency* takes this geometrical factor into account, giving a measure of how effective a particular detector is for capturing the full energy of the incident radiation. If the solid angle covered by the detector is  $\Omega$ , then the fraction of all emitted radiation that can hit the detector is  $\Omega/4\pi$  (assuming an isotropic, point source). Therefore the rate at which the radiation of interest actually hits the detector is  $A\gamma\Omega/4\pi$ . Again, if the measured count rate in the full energy peak is  $R_p$  counts/s, then the peak intrinsic efficiency is:

$$\epsilon_{\rm int} = \frac{R_p}{A\gamma} \frac{4\pi}{\Omega} = \epsilon_{\rm abs} \frac{4\pi}{\Omega} \tag{8}$$

The calculation of  $\Omega$  for most of the circumstances encountered in the laboratory is covered in Knoll [Knoll 118–121].

### 1.6.3 Statistics of counting

If we try to measure the count rate from a detector, we may measure a count value, N, whose error is again governed by Poisson statistics, so that  $\sigma(N)=N^{1/2}$ . If this count is taken over t seconds then the count rate is

$$R = N/t (9)$$

with a standard deviation in R of

$$\sigma(R) = N^{1/2}/t \tag{10}$$

Averaging is good for your experiment. A number of repeated measurements, m, may be taken to obtain an average

$$\overline{N} = \frac{1}{m} \sum_{i=1}^{m} N_i \tag{11}$$

This is valid as long as the statistical error on each of the readings,  $\sigma(N_i)$ , is approximately the same, otherwise a weighted average is more appropriate. An estimate for the error on the mean value,  $\overline{\sigma}$ , may be obtained from the deviation of the individual readings,  $N_i$ , from the average value,  $\overline{N}$ :

$$\overline{\sigma}^2 = \frac{1}{m-1} \sum_{i=1}^{m} (N_i - \overline{N})^2$$
 (12)

Remember that the total area under the standard Gaussian curve is 1.0, whereas the area between the values of  $N+\sigma(N)$  and  $N-\sigma(N)$  is 0.68. This is the probability that the true value of your measurement lies between  $\pm\sigma(N)$  around the value N. It therefore follows that there is a probability of 0.32 (or 1 in 3) that the value will lie outside the  $\pm\sigma$  band, even if there is nothing wrong with the experiment.

#### 1.6.4 Errors on centroids and FWHM

The gamma rays emitted from a source are to a good approximation monoenergetic. The measured peak shape is then due to the statistical and electronic noise added to the signal in the detection process. This measured peak shape can thus be regarded as the probability distribution for measuring that particular gamma ray energy, with each count in the peak being a sample from this distribution.

Using the standard statistical results, the centroid of this distribution gives the best estimate of the average value (in this case the actual energy of the gamma ray), while the standard deviation,  $\sigma_x$ , gives the standard error of a single measurement. The error on the centroid,  $\sigma_C$ , is therefore the same as the error on the mean average of the distribution, which as usual will be given by  $\sigma_C = \sigma_x/\sqrt{A}$ , where A is the net area of the peak, i.e. the total number of measurements in the distribution (the background is not part of the probability distribution, so we must use the net area, not gross area).

Since for a Normal (Gaussian) distribution the FWHM =  $2.35\sigma_x$ , we get that the uncertainty on the centroid is given by:

$$\sigma_C = \frac{\text{FWHM}}{2.35\sqrt{A}} \tag{13}$$

The uncertainty on the FWHM can be estimated from:

$$\frac{\sigma(\text{FWHM})}{\text{FWHM}} \approx \frac{1}{2} \frac{\sigma(A)}{A} \tag{14}$$

i.e. the fractional error in the FWHM is usually about 1/2 the fractional error on the net area.

A much better way to estimate the errors on the centroid and FWHM is to fit a Gaussian lineshape to the data. This can also allow overlapping peaks to be fitted using several Gaussians This can be particularly useful when using detectors with poor resolution, such as gas-filled proportional counters for X-ray detection. Peak fitting software is available in the laboratory, as described below, and you should use it whenever possible to get robust estimates for the uncertainties on the parameters of interest.

### 1.6.5 The ROOT Analyis Framework

A peak fitting macro has been written in C++ to run in a UNIX program called ROOT, which is installed on the Physics UNIX servers. The macros you need (see below) are available from the *LM PH605 Practical Skills for Reactor Physics* section of Canvas and must be downloaded to your ADF user directory or folder where you will run ROOT.

To use the ROOT fitting program, you will need to have saved your spectra in ASCII (human readable) format. To do this, choose file type *ASCII* \*.*Spe* in Maestro, or *Toolkit File* \*.*TKA* in Genie, after selecting **File**/**Save As**. Remember to save your

spectra to the same ADF user directory as the ROOT macro, not to the local disk.

### Accessing the UNIX servers

Instructions on how to access the physics UNIX server from the laboratory, computing cluster or from home, are given in the *Using the Computing Facilities* document in the *LM PH605 Practical Skills for Reactor Physics* section of Canvas.

### Peak Fitting in ROOT

Start the ROOT analysis package by typing **root** at the command prompt in the *SSH Secure Shell* terminal window.

Load the peak fitting macro into ROOT by typing **.L Buffit.C** at the prompt. The macro Buffit.C must be downloaded from the *LM PH605 Practical Skills for Reactor Physics* section of Canvas to the folder where you will run ROOT and where you have saved your MAESTRO or GENIE spectra.

Execute the fitting routine by typing the command **Buffit("MySpectrum")**. Substitute *MySpectrum* with the full filename of your spectrum file: e.g. *Spectrum1.Spe* for Maestro, or *Spectrum1.TKA* for Genie ASCII files. Buffit cannot identify the Genie files just from the name, so you will be prompted to tell Buffit what kind of file it is. Simply enter TKA at the prompt and the spectrum will be loaded.

TIP: If you hit the TAB key while you are writing the filename, ROOT will attempt to autocomplete the filename, which is very handy for long filenames

Zoom in on a peak or group of peaks by typing **ZoomIn()** and follow the on-screen instructions. The range of channels should be wide enough to see the background on either side of the peak(s). You can also provide two arguments to the **ZoomIn()** function specifying the lower and upper bin or channel number, respectively. For example, **ZoomIn(1050,1275)** will display channels in the range 1050 to 1275, inclusive. This feature allows you to zoom consistently to the same region of the histogram if you are doing fits of the same energy peak(s) in several different spectra. You can see the whole spectrum again by typing **ZoomOut()**.

Once you have zoomed in on a peak you can perform a fit by typing the command **Fit()**. You will then be prompted to give a rough value for the FWHM of the peaks and the number of peaks you want to fit (maximum of 4). The underlying background is assumed to be linear. Follow the on-screen instructions and select the peaks by clicking close to the centroid positions of each of the peaks.

TIP: For neighbouring, isolated (not overlapping) peaks you will get a better fit by zooming in on each peak individually. Where neighbouring peaks are overlapping you can fit up to 4 Gaussians simultaneously in an attempt to extract the yield (number of counts) within each peak.

By default, Buffit allows the width of each peak to be a free parameter. You can optionally force Buffit to fit each peak using the same FWHM by typing the command **SetFixedFWHM()** before calling Buffit(). This feature may help to deconvolve overlapping peaks. This feature can be turned off by typing **UnSetFixedFWHM()**. Note, although this option uses only one fit parameter for the width, the other peak

widths are deduced by scaling (roughly) by  $\sqrt{E}$ . In this way, all peak widths are constrained in a physical way.

You can save a copy of the fit by selecting **File/Save As...** from the histogram window menu bar. Make sure that the file type is changed to GIF, as this is readily imported into MS Word. The default is PostScript, which is not (but can be imported into LaTeX documents).

### 1.6.6 Deadtime in counting systems

In all counting systems there is a *deadtime*. During this time, following the arrival of a pulse, there is a period of time where the electronics cannot process any pulses. After this period, the electronics reset themselves, and counting can proceed again. For most experiments the main deadtime results from the computer systems, and you will see a figure displayed on the screen which gives you the value of the deadtime as a percentage of the total time. The relationship between the measured count N' and the true count N for a fixed deadtime,  $\tau$ , is:

$$N = N'/(1 - N'\tau) {(15)}$$

with a similar equation relating the true count rate R to the measured rate R'. If you have set the computer to count *livetime*,  $t_{live}$ , rather than *realtime*,  $t_{real}$ , then the computer software subtracts the total deadtime  $N'\tau$  to calculate the livetime

$$t_{live} = t_{real} - N'\tau \tag{16}$$

The deadtime  $N'\tau$  per second is displayed as a percentage. However, this algorithm becomes unreliable at deadtimes above about 20%, so that you should try to keep the deadtime below 20%.

#### 1.6.7 Random rates in coincidence measurements

Some experiments use more than one detector in coincidence to measure, for example, gamma rays which are emitted simultaneously (e.g.  $^{22}$ Na) or successively (e.g.  $^{60}$ Co ) in radioactive decays. In the electronic systems for this type of experiment, you need to use one coincidence unit (see next section) to measure real coincidences and another coincidence unit to measure random coincidences of gamma rays from the source or from background gamma rays or cosmic rays.

If the resolving time of the coincidence unit of the system is  $\tau$ , and the individual count rates in channels 1 and 2 are  $R_1$  and  $R_2$  respectively, then the random coincidence rate is given by

$$R(random) = R_1 \times R_2 \times 2 \times \tau \tag{17}$$

However, if you use a time-to amplitude unit (TAC - see next section) to generate a spectrum of time on the data acquisition system, then the time spectrum will show both the real coincidence events (as a peak) and random events (as a background underneath the peak) in the same spectrum.

### 1.7 Notes on Electronics

The information from the detector (output signal) is carried by short electrical pulses, with widths that vary from a few *nanoseconds* to many *microseconds*. These pulses may be **analogue** pulses, which carry information through both their time (i.e. when they arrive) and their height (i.e. how many Volts they are at their peak), or **digital** pulses, which are always the same height, and carry information only through their time.

The first analogue pulses you encounter are at the output of the detector. In almost all cases, the pulses are fed directly from the detector to a **preamplifier**, which may be used for impedance matching rather than amplification for some detectors. The output of the preamplifier is the most convenient place to look at these pulses. They are usually negative, but this depends on the detector and preamplifier design. The height again depends on the detector, but is usually no more than a few hundred mV and may be much less. The pulses then pass through an **amplifier**, which produces an output pulse of up to 10 V. This output pulse may be a) **unipolar** (positive) or b) **bipolar**:

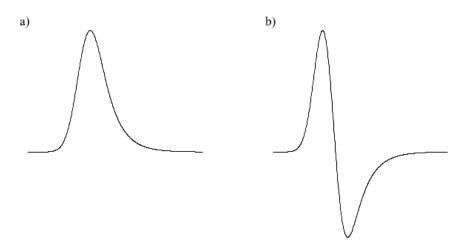


Figure 1: Output pulse shapes. a) Unipolar pulse, b) Bipolar pulse

The unipolar pulse gives the best energy resolution, but the amplifier may need to be adjusted so that the 'pole-zero' compensation is correct [Knoll 633]. The bipolar pulse has some advantages at high count rates; you should use it where possible. Most amplifiers deliver a pulse between 1 and 4  $\mu sec$  wide, which is suitable for most purposes; think whether you need a wider or narrower pulse.

Digital pulses are usually generated by discriminators, or timing single channel analysers. Some units we have in the lab produce negative pulses (usually  $\sim$  0.5 V) and some give positive pulses (usually larger). Make sure that the pulse outputs you have are correct for the device you want to feed them into.

Almost all the electronics you use are packaged in "NIM" modules [Knoll 801], which fit into "NIM bins", which have built-in power supplies. Thus, when a module is plugged into a bin, it is automatically provided with power  $\pm$  12 V,  $\pm$  24 V and sometimes also  $\pm$  6 V. Some units require a  $\pm$  6 V supply, and so are incompatible with those crates having only  $\pm$  12 V and  $\pm$  24 V. The basic operation of any of these devices is obvious enough. But beware – almost all modules have knobs

on the front to change the parameters of the device. The settings of these are important and should be recorded in your notebook; think about the most suitable settings of each module for your experiment. Observe on the oscilloscope how the output signal changes when the settings are varied.

### 1.7.1 Description of the Various Electronics Units

Here is a brief summary of the various units you may encounter in the Laboratory:

- **Preamplifier** Usually a charge-sensitive amplifier which accepts the low-level output signal from a detector and delivers an amplified step-function pulse shape to the output line with a 50  $\Omega$  impedance.
- **Amplifier** Amplifies pulses, usually up to 10 V (unipolar or bipolar), and shapes them (with time constants of between 0.25 to 8  $\mu$ s) so that the duration of the pulses is reasonably short and suitable for input to the Analog to Digital Converter (ADC) which is contained within the PC-based Multi-Channel Analyser (MCA).
- **Discriminator** Gives a digital output pulse when the input signal exceeds a preset threshold.
- **Single-channel analyser (SCA)** Gives a digital output pulse when the input signal lies between two preset levels.
- **Timing SCA** Same as an SCA, but is designed so that the time of the output pulse relative to the input is carefully controlled.
- **Multichannel analyser (MCA)** Rather like a set of many SCAs; the internal ADC sorts the input pulses according to their pulse height and stores a spectrum of these pulse heights usually within a computer based data acquisition system (next section).

If the pulse heights are proportional to the energy of the radiation, (as they are for most detectors), then the spectrum on the MCA is therefore a representation of the energy spectrum of the radiation, and you will need to calibrate the channels in the MCA in terms of the energy of the radiation.

As mentioned in section 1.6.1, Geiger Counters (and Fission Counters), produce a digital output pulse which may be counted by a simple scaler/timer module, making an MCA irrelevant for these types of detector.

**Time-to-Amplitude Converter (TAC)** Also called a Time to Pulse Height Converter. Requires two input logic pulses called "start" and "stop". The unit gives an analogue output pulse whose height is proportional to the time difference between the start and stop pulses. This analogue pulse can then be processed by the computer based MCA, and therefore can display a spectrum which now represents a spectrum of time; you will need to calibrate the channels in the MCA spectrum in terms of time.

### 1.7.2 Trouble-Shooting

Some common problems in operating nuclear physics apparatus are listed below.

1. Detectors always need an applied voltage (HV for a photomultiplier tube, bias voltage for a solid-state detector, etc.).

Be careful to apply the correct polarity for this voltage.

Be careful to raise and lower this voltage **slowly**; some detectors can be damaged by rapid changes in voltage.

Usually, detectors need at least a preamplifier, a main amplifier and either a single-channel analyser (SCA) or a multi-channel analyser (MCA) – see next subsection.

- 2. In a few cases, where timing is more important than energy resolution, the anode signals of a photomultiplier tube are fed directly into a discriminator, without amplifier or preamplifier. In this case, you need to select a discriminator which takes negative input pulses of a few hundred mV, rather than positive pulses up to 10 V.
- 3. Although most electronics are mounted in NIM modules, which plug into bins with built-in power supplies, a few modules (e.g. preamplifiers) need power from a separate source.
- 4. Most signal cables have BNC plugs, but a few units use the smaller LEMO plugs. High-voltage cables usually have different plugs. Some of these are MHV plugs, which look rather like BNC plugs but are not interchangeable (and may be damaged if you try to). Other high-voltage equipment uses SHV plugs. These different plugs are not just to create difficulties, they exist for safety reasons. Remember that Murphy's law applies here; the cable you have to hand never has the right connector on it.
- 5. If you use a vacuum pump, be sure that the valve to allow air into the vacuum equipment is opened immediately after switching off the vacuum pump. Make sure any detector bias is off.
- 6. Observe the safety instructions concerning radioactive sources. Choose your sources carefully to match your experiment; simple calibration procedures rarely need strong sources. You should expect to make extensive use of the reference books in the laboratory to research the decay schemes of sources.

Sources from the cupboard *must* be signed out to you by a demonstrator and a demonstrator must check that the source is returned when you have finished with it.

- 7. Geiger counters must first be "plateaued" before use, i.e., find a voltage where the counting rate from a source is independent of bias voltage.
- 8. MOST IMPORTANT always use an oscilloscope to look at the pulses at all points in your electronics. Don't assume that the pulses must be there and have the right size, shape and polarity, just because you connected up the equipment. Make sure that pulses have the shape that you expect at the

outputs of preamplifiers, amplifiers, etc. A 20 MHz oscilloscope is suitable for looking at analogue pulses from amplifiers, but you may need to use a faster oscilloscope (100 MHz) to look at fast NIM logic pulses of a few ns width.

### 1.8 Data acquisition systems

The lab is equipped with PCs, which act as Multi Channel Analysers running the Genie or Maestro software. All the PCs run Windows 10 and have Microsoft Office installed providing word processor (Word) and spreadsheet (Excel) facilities. Data should always be saved onto a flash drive or to your own networked area. Files saved on the local hard disks (C:) will be removed on a regular basis. Flash drives brought into the lab must be checked for viruses.

It is worth spending some time getting familiar with the Maestro or Genie pulse height analysis programs before starting your experiment.

### 1.9 General measurements for gamma spectroscopy

There are several different type of gamma-ray detector around the laboratory, with the two main types being germanium semiconductor (HPGe), or NaI(TI) scintillator detectors. These detectors can be used for a variety of experiments, but here are some general experiments that you should consider for all  $\gamma$ -ray detectors.

- 1. The signal from the preamplifier has a fast leading edge, dictated by the decay time of the signal carriers of the detector. This voltage will reach a maximum and subsequently return to ground with a characteristic exponential decay given by the preamplifier RC network. Use an oscilloscope to observe these pulses and determine the decay time constant of the preamplifier.
- 2. The signals from the preamplifier are fed into a main amplifier, one of whose functions is to replace this long period decay with a much shorter one. As a general rule, the charge integration time should be at least a factor of 4 longer than the preamplifier decay time constant (98% confidence limit in detecting all signal carriers). What is the minimum shaping time you can use note that you may need to adjust the 'pole-zero' compensation [Knoll 633] as the shaping time of the amplifier is reduced and therefore what is the maximum count rate achievable with this system? You may also wish to investigate energy resolution as a function of shaping time.
- 3. Knowing the duration of the pulse from the main amplifier, and assuming this is longer than the dead-time of the MCA (so it is the dominant source of dead-time), estimate the percentage dead-time as a function of count rate. You can investigate this for a range of source activities, or by changing the source to detector distance. What parameters are affected by increased dead-time and why?
- 4. Investigate the energy resolution of the photopeak as a function of bias voltage. Use this information to choose an optimum bias voltage which should remain fixed for the rest of your experiments.
- 5. Measure a statistically significant spectrum from a  $\gamma$ -ray source placed near the detector. Correlate the oscilloscope display with what you observe on the MCA (it may be useful to use a monoenergetic source for clarity). Identify and explain the features in the measured spectrum; as a general rule you should be able to locate around 10 features even for a monoenergetic source (see the detector physics exam questions). Investigate the effect of changing the upper and lower level discriminators in the MCA settings. How significant is electronic noise?
- 6. Usually, you will want to measure the energy of a particular feature in your spectra. Assuming that the detector response is linear with energy measure the positions of the photopeaks for a number of calibrated (known) sources using the peak fitting software of your choice. Perform a (weighted) linear least-squares fit to the data to provide an empirical energy calibration for your detector setup (remember that this is liable to change if you alter any of the detector settings). How linear is the detector response, and how might you test this? Measure a number of other sources and confirm your calibration.

- 7. What is the dynamic range of your detector system, i.e. what are the lowest and highest energies that you can measure? How might you change this range, and why might this be useful experimentally?
- 8. Investigate the energy resolution as a function of  $\gamma$ -ray energy. How does this compare with what you have been taught about Poisson statistics? The energy resolution may have many contributing factors including statistical variation, electronic noise and student error. What is the dominant factor for your detector? Bear in mind that all of the detectors in this laboratory have subtle differences, so a direct comparison to other detectors may be meaningless, even for the same types.
- 9. Investigate the absolute photopeak efficiency as function of source-detector geometry, and of  $\gamma$ -ray energy. How does the absolute efficiency scale with source to detector distance? Use your results to determine the intrinsic efficiency of the detector. How does this scale with source–detector distance, and is this surprising? Can you explain how the efficiency behaves as a function of incident energy, and what does this tell you about gamma interaction mechanisms?

### 1.10 Bibliography

If you haven't yet done a course on Nuclear Physics, you will need to learn the physics background information to the experiments. Most of the experimental details are covered in Knoll's comprehensive book. There is much information about individual NIM modules in the EG&G Ortec manual, whilst the Ortec Application Notes have details of many experiments and how to set them up. Other textbooks that give adequate coverage of detectors are those by Krane and also Burcham and Jobes. Both the texts by Krane and Lilley have excellent chapters on the applications of nuclear science. The classic textbook for nuclear reactors is Glasstone and Sesonske, but the diffusion of neutrons and reactors are also covered in Lilley's book.

#### **General Reference Textbooks**

W E Burcham and M Jobes; Nuclear and Particle Physics, Longman, 1995.

S Glasstone and A Sesonske; *Nuclear Reactor Engineering*, Van Nostrand Rheinhold 1967.

J S Lilley; Nuclear Physics: Principles and Applications, J Wiley, 2001.

G F Knoll; Radiation detection and measurement, 4th edition, J Wiley, 2010.

K S Krane; Introductory nuclear physics, 2nd Edition, J Wiley, 2000.

R D Evans The Atomic Nucleus, McGraw-Hill, 1972.

### **ORTEC** reference material

ORTEC, Experiments in Nuclear Science Laboratory Manuals: http://www.ortec-online.com/Service-Support/Library/Educational-Experiments.aspx

ORTEC Application Notes (AN). For example:

AN58 - How Histogramming and Counting Statistics Affect Peak Position Precision.

AN59 - How Counting Statistics Controls Detection Limits and Peak Precision.

http://www.ortec-online.com/Service-Support/Library/index.aspx

### **Useful online resources**

NIST Physical Reference Data:

http://www.nist.gov/pml/productsservices/physical-reference-data/

This contains information on X-ray and gamma-ray attenuation coefficients.

National Nuclear Data Centre (NNDC): http://www.nndc.bnl.gov

This contains a comprehensive list of nuclear decay schemes for any isotope.

### 1.11 Acknowledgements

This document is the product of the effort of many people over the many years the laboratory has operated. Particular thanks to Peter Jones for the sections on ROOT and the peak fitting routine.

### 2 List of Experiments

The descriptions of the experiments follow in sections 2.1 to 2.7, but for the most part, there is little detail, so that you will need to research these topics in reference works. There are more detailed notes for the neutron diffusion experiment, since the reference material may not be available in some current textbooks.

### The experiments available are:

- 1. Neutron diffusion in graphite (or water tank) using BF<sub>3</sub> or <sup>3</sup>He detectors.
- 2. Coincidence detection of positron annihilation photons.
- 3. X-ray fluorescence analysis using Si(Li) detector.
- 4. Photon spectrometry with proportional counters.
- 5.  $\gamma$ -ray spectrometry of radioactive objects using a Ge detector.
- 6. Neutron activation analysis using Geiger or Ge detectors.
- 7. Attenuation of  $\gamma$ -rays in iron, etc. using a NaI detector.
- 8. Stopping powers, dE/dx, of  $\alpha$  particles in air using a silicon detector.
- 9. Absolute activity measurement using coincidence techniques.
- 10.  $\gamma$ -ray spectroscopy using scintillators.

### 2.1 Neutron Diffusion in Graphite using BF<sub>3</sub> or <sup>3</sup>He detectors

In a nuclear reactor, fast neutrons emitted during fission have to be slowed down by collisions with light nuclei in a moderator, until they attain thermal equilibrium with the atoms of the moderator. They are then described as thermal neutrons. The behaviour of thermal neutrons is analogous to that of gas molecules, and they diffuse from regions of high concentration into regions of lower concentration. There is also a significant probability that a thermal neutron will be captured by a nucleus. The balance of these two processes determines the "diffusion length" of thermal neutrons in a given moderator. Early British reactors used graphite as the moderator.

The aim of this experiment is to determine the diffusion length L of thermal neutrons in graphite, by measuring the relative thermal neutron flux at various points in a large graphite moderator stack using a  $BF_3$  detector.

### There are three stages:-

- 1. Set up and understand the BF<sub>3</sub> detector [Knoll 523–531] using the 0.3 Ci Am/Be neutron source in the stack use the oscilloscope to observe the pulse height variation with HV bias to determine the proportional region of the detector, and then use the SCA to correlate the scaler counts vs SCA setting to determine the energy spectrum of the signals. Use this information to set up the SCA + scaler to count only neutrons.
- 2. Make measurements of the thermal neutron flux at various positions in the stack arising from the 3 Ci Am/Be source at the centre of the base. Because your detector is also sensitive to fast neutrons you will have to make two measurements, with and without the cadmium shield which absorbs thermal neutrons.
- 3. Fit your results in terms of neutron diffusion theory. The relevant equations are given on the next page. Basically, to determine L you must determine the constants  $\alpha$ ,  $\beta$  and  $\gamma$ ;  $\gamma$  can be determined from the vertical variation in flux,  $\alpha$  can be determined from the horizontal variation (this is a non-linear least squares fitting problem). You cannot determine  $\beta$  directly from your measurements but must make an intelligent estimate based on your value of  $\alpha$ .

**Safety notes:** The neutron sources give significant dose rates from both neutrons and gamma rays. The length of the handle on each source gives an indication of the safe working distance and you should remain beyond this distance at all times. Do not remain within twice this distance for prolonged periods. If you are using a source in a bucket it is your responsibility to ensure that others cannot be inadvertently exposed, so place the source as far from everyone else in the lab as possible and mark it with a radiation sign. Do not move the strong (3 Ci) source without permission from a demonstrator. Graphite is very slippery (and messy!) and you are advised to wear gloves when handling it.

### **Neutron diffusion theory**

The thermal neutron flux,  $\phi(\mathbf{r})$ , at point  $\mathbf{r}$  is given by  $n(\mathbf{r})v$ , where  $n(\mathbf{r})$  is the number of thermal neutrons per unit volume at this point, and v is their average speed.

If the flux is non-uniform, diffusion leads to a net flow  ${\bf J}$  of neutrons given by Fick's law

$$\mathbf{J} = -D\nabla\phi\tag{18}$$

where D is the flux diffusion coefficient.

If  ${\bf J}$  is itself non-uniform then the difference between the flow into and out of a given region causes the number of neutrons in that region to vary with time. This can be expressed in differential form as

$$\frac{\partial n}{\partial t} = -\nabla .\mathbf{J} \tag{19}$$

and substitution for J gives the diffusion equation

$$\frac{\partial n}{\partial t} = D\nabla^2 \phi \tag{20}$$

However, the number of neutrons being captured per unit volume per unit time is equal to  $nN\sigma_a v$  where  $\sigma_a$  is the microscopic capture cross section per atom and N is the number of atoms per unit volume in the moderator.

The product  $N\sigma_a$  is referred to as the macroscopic cross section,  $\Sigma_a$ , so that the capture rate per unit volume can be written as  $\Sigma_a \phi$ .

Combining the effect of diffusion with the effect of capture, and neglecting higher order terms, gives the equation:

$$\frac{\partial n}{\partial t} = D\nabla^2 \phi - \Sigma_a \phi \tag{21}$$

which describes the variation in the neutron flux in regions where there are no neutron sources. In the steady state,  $\partial n/\partial t = 0$ , so that the equation reduces to the screened Poisson equation:

$$\nabla^2 \phi = [\Sigma_a/D]\phi \tag{22}$$

The diffusion length, L, is defined as

$$L = [D/\Sigma_a]^{1/2} \tag{23}$$

so that we may finally write:

$$\nabla^2 \phi = \frac{1}{L^2} \phi \tag{24}$$

The boundary conditions are that the flux must be non-negative everywhere and must approach zero at the edges of the graphite stack (the flux is not actually zero at the edge of the stack, as some neutrons leak out – this means that the solutions can be extrapolated to zero a little way outside the stack).

Using Cartesian coordinates (x,y,z), solutions to this equation with appropriate boundary conditions can be found by separation of variables: if  $\phi$  can be written as the product of three functions,  $\phi(\mathbf{r}) = X(x)Y(y)Z(z)$ , then these must satisfy:

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = \frac{1}{L^2}$$
 (25)

and since each term depends only on a single coordinate, each term must be equal to a constant:

$$\frac{1}{X}\frac{d^2X}{dx^2} = A, \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = B, \qquad \frac{1}{Z}\frac{d^2Z}{dz^2} = C$$
 (26)

where

$$A + B + C = \frac{1}{L^2} \tag{27}$$

Thus the solutions are either sinusoidal or exponential, depending on whether the constants are negative or positive respectively. The boundary conditions dictate that in the two horizontal directions (x and y) the flux must approach zero at both sides of the stack, so the solution cannot be exponential in form.

Hence both A and B must be negative, and it is convenient to write  $A=-\alpha^2$  and  $B=-\beta^2$ . Since C must be positive (so that A+B+C is positive), we write  $C=\gamma^2$ .

The general solutions are then:

$$X = P\cos(\alpha x) + Q\sin(\alpha x)$$

$$Y = R\cos(\beta y) + S\sin(\beta y)$$

$$Z = Te^{\gamma z} + Ue^{-\gamma z}$$
(28)

Since we expect the flux to be symmetric about the centre of the stack (x=y=0), we are only interested in the cosine terms for X and Y. Since the flux must decrease with height, we are only interested in the  $e^{-\gamma z}$  term, giving for the neutron flux as a function of position:

$$\phi(x, y, z) = I\cos(\alpha x)\cos(\beta y)e^{-\gamma z}$$
(29)

where I is a combination of P, R and U, and gives an overall scaling factor that depends on the strength of the neutron source.

### 2.2 Positron annihilation detection

The positron emitted in  $\beta^+$  decay slows down by collisions with atomic electrons. Then, when it is almost at rest, it annihilates with an electron, giving two 511 keV photons which are emitted almost exactly 180° apart. Coincident detection of these enables localisation of the source and is the basis of the imaging technique of positron emission tomography.

The aim of this experiment is to use a pair of NaI detectors operating in coincidence to detect the two photons and to demonstrate that they are approximately back-to-back. It is also intended to introduce you to features of coincidence counting, such as the effect of resolving time upon the numbers of real and random coincidences.

You will use a <sup>22</sup>Na source. First use a single NaI(TI) detector [Knoll 239–241], observe the pulse height spectrum with an MCA, investigate the dependence on PM voltage, and set a SCA window on the 511 keV peak. Repeat this for the second detector. Then set up the coincidence circuit and measure coincidence counts versus delay in one arm for detectors at 180°, in order to find the optimum settings, using a strong source as well as the original weak source and observing the contribution of random coincidences [Knoll 688–694]. Then vary the geometry to confirm that the photons are approximately back-to-back.

### 2.3 X-ray fluorescence

Low energy  $\gamma$ -rays interact mainly by photoelectric absorption, knocking out an inner shell electron and creating a vacancy which may subsequently be filled by an outer electron with emission of a characteristic X-ray. This is the phenomenon of X-ray fluorescence (XRF) which can be used to identify the presence of heavy elements within samples. The aim of this experiment is to understand the phenomenon of XRF, its usefulness as an analytical technique and the complications involved in making quantitative measurements. You will use a Si(Li) detector [Knoll 467–485] to identify the emitted X-rays.

You should first investigate the performance of this detector (NB bias must be applied very slowly) using the variable X-ray source (e.g. energy resolution vs bias, and energy resolution vs X-ray energy), and should calibrate the PC MCA system – you should understand the origin of  $K_{\alpha}$  and  $K_{\beta}$  X-rays.

You should then investigate Moseley's Law for K X-ray energies versus atomic number, Z. In 1913, Henry Gwyn Jeffreys Moseley observed that the frequency (energy) of K X-ray transitions increased monotonically with increasing atomic number, Z. In this way, he was able to reorder the periodic table of chemical elements based on increasing atomic number, rather than increasing atomic mass as had been done previously. This laid the foundation for identifying elements using X-ray spectroscopy. You can then use the annular  $^{241}$ Am source to excite fluorescent X-rays from samples of unknown materials (e.g. coinage), and qualitatively analyse their composition via Moseley's Law.

You can then study the attenuation of different X-ray energies in (e.g.) All or Zr foils and should be able to see the difference in absorption either side of the K edge.

**Warning!** Do not switch on any part of this experiment until you have been properly instructed by a demonstrator – the detector can be damaged if voltage is changed too suddenly.

**Safety notes:** The annular sources used for XRF are quite active and care should be taken to avoid unnecessary exposure (but the low energy photons are strongly attenuated by a few mm of lead so there is no need for massive shielding).

### 2.4 Photon spectrometry with proportional counters

Low energy photons interact mainly by photoelectric absorption, where a tightly bound electron is emitted from the absorber atom; this may also be accompanied by characteristic photon emission. The aim of this experiment is to investigate how photoelectric absorption is exploited for radiation detection, and to characterise two similar detectors.

Set up and calibrate both the Kr and Xe filled proportional tubes [Knoll 159–195] using a low energy X-ray source. Investigate peak amplitude and energy resolution as a function of bias voltage. Compare the detection efficiencies which can be obtained for a range of X-ray energies. How does the relative efficiency for each detector compare over the measured energy range? Why do you see a difference? Observe and explain any escape peaks which occur.

### 2.5 Gamma spectrometry of environmental radioactivity

Many everyday materials contain natural radioactivity. The aim of this experiment is to identify and quantify the radioisotopes present by detecting their  $\gamma$ -ray emissions. You will use a germanium detector [Knoll Chapter 12].

You should set up and calibrate the HPGe detector – investigate energy resolution vs bias and  $\gamma$ -ray energy – and should determine the absolute efficiency vs energy both for sources mounted against the detector face and for sources mounted a fixed distance away (using calibrated sources).

You will then be given some samples containing low-level activity to analyse. An interesting exercise is to estimate the natural <sup>40</sup>K activity of the human body, given that a typical adult contains about 140 g of potassium.

**Warning!** The bias voltage **must** be applied and removed **slowly** – the detector can be damaged if voltage is changed too suddenly.

### 2.6 Neutron activation analysis

A thermal neutron may be captured by a stable nucleus, creating a new radioactive isotope, whose subsequent decay can be detected. This is the basis of neutron activation analysis, which provides an accurate way of detecting low levels of certain substances.

The aim of this experiment is to understand neutron activation analysis and to use it to identify a mystery material.

The experiment can be performed in two ways. In the first, a Geiger counter is used to detect the induced radioactivity and observe its decay with time and its dependence on activation time. In the second, a germanium detector is used to identify a radioisotope on the basis of the emitted  $\gamma$ -rays. Neutron activation can be carried out using the 1 Ci Am/Be neutron source in the water tank as a source of thermal neutrons.

First plateau the Geiger counter [Knoll 207–215] using a  $^{90}$ Sr beta source. Then activate a sample of indium or silver (approx 5 g pieces are ideal) and detect the induced activity with the Geiger counter – repeated measurements on a single activated sample should enable the half life of the decaying product to be determined (more than one component may be seen). Then study the effect of varying the activation time.

**Safety notes:** The <sup>90</sup>Sr beta sources are potentially unsealed and care should be taken in handling them to avoid any risk of ingesting contamination: do not touch the active area of the source and wash your hands after use.

The dose rate immediately around the water tank (in which samples are activated) is significant so do not linger in this area. When you have finished with an activated sample check whether it is significantly active and if so place it in the bottom of the small source cupboard (by the next day it should no longer be active and may be retrieved).

### 2.7 Gamma-ray attenuation and build up

The aim of this experiment is to observe and understand the processes by which gamma rays are attenuated in passing through matter, and the effects of shielding geometry.

The experiment divides into two parts. In the first, an ionisation chamber is used to measure dose rate as a function of shielding thickness, while in the second a sodium iodide (NaI) detector is used to provide information on the spectrum of transmitted  $\gamma$ -rays.

Start by building a well shielded enclosure and inserting the 7MBq <sup>60</sup>Co source at one end – use the ionisation chamber to measure gamma dose rate at the other end of the enclosure. Place iron plates between, and measure attenuation vs iron thickness, for at least two shielding geometries (well collimated and open), and compare with an exponential falloff to investigate whether build-up is significant [Knoll 53].

Then repeat using a NaI(TI) scintillation detector [KnoIl 239–241] coupled to a photomultiplier tube – first investigate the response of the NaI to weak gamma ray sources (study variations in gain and energy resolution as a function of PM voltage, and variation in energy resolution with  $\gamma$ -ray energy) [KnoIl Chap 10] – then repeat the attenuation measurements in different geometries.

Attenuation of materials other than Fe, can also be studied.

**Safety notes:** The strong <sup>60</sup>Co source used in this experiment should be transported in its carrying brick, always handled remotely and requires appropriate shielding to reduce the dose rate at all exposed areas to less than 2.5 mSv/hr. Do not handle this source until you have been instructed in the use of an appropriate radiation monitor.

### 2.8 Alpha-particle stopping powers in gases

The aim is to determine the stopping power, dE/dx, for alpha particles in various gases as a function of energy, and compare with the predictions of the Bethe-Bloch formula.

You will use an ion-implanted silicon detector [Knoll Chap 11]. Its behaviour should first be investigated in the small vacuum chamber using the Am/Cm/Pu triple alpha source, which can provide a rough energy calibration (do not touch the active surface of the source).

A larger chamber containing a motorised  $^{241}$ Am source should be used to investigate the effect of gas (He or Ar) between source and detector, and determine the energy at the detector as a function of the amount of material traversed. You can then differentiate this function to determine dE/dx as a function of E.

The <sup>241</sup>Am source sealed into the larger chamber only provides monoenergetic alpha particles, so cannot provide an energy calibration by itself. However, the full energy signals from the detector can be simulated by an electronic pulser. As this output pulse amplitude can be precisely attenuated by known amounts, a "match-stick" spectrum can be produced that gives a calibration from just one measured energy.

### The Bethe-Bloch stopping power formula

The Bethe-Bloch formula in SI units

$$\frac{dE}{dx} = (-)\frac{e^4}{(4\pi\epsilon_0)^2} \frac{4\pi z^2 NZ}{m_e c^2 \beta^2} \left\{ \ln\left(\frac{2m_e c^2 \beta^2}{I}\right) - \ln(1 - \beta^2) - \beta^2 \right\}$$
 (30)

relates the stopping power (in  ${\rm Jm}^{-1}$ ) of an ion of charge ze travelling through a medium of atomic number Z to its velocity, v, which appears in the formula as  $\beta=v/c$ . N is the number of atoms per unit volume (in  ${\rm m}^{-3}$ ) in the medium, so that the product NZ simply represents the number of electrons per unit volume;  $m_e$  is the mass of the electron. I is the average ionisation potential for all the electrons in the medium, usually approximated by  $I=11\times Z$  eV (electron volts).

In many books the equation appears with N replaced by  $N_A \rho/A$ , where  $\rho$  is the density of the medium and A its atomic mass. You should note, however, that to give results in SI units the constant  $N_A$  must be  $6\times 10^{26}$ : i.e.  $10^3$  times Avogadro's number.

Safety notes: The  $\alpha$ -particle sources are potentially unsealed and care should be taken in handling them to avoid any risk of ingesting contamination: do not touch the active area of the source and wash your hands after use. The gas cylinders are potentially hazardous and care should be taken to avoid pressurising the chamber – do not operate regulator valves until you have been properly instructed by a demonstrator, and read the document Standard Operating Procedure for Handling Pressurised Gases in the laboratory section of LM PH605 Practical Skills for Reactor Physics on Canvas.

### 2.9 Absolute activity determination

The activity of an unknown source can be estimated easily if a high quality calibration of the detector has been carried out. However, the determinations of the solid angle covered by the detector, and its intrinsic efficiency, introduce the largest sources of error. This experiment introduces the use of coincidence techniques for absolute activity measurements where these are no longer limiting factors.

Set up and calibrate both of the NaI(TI) detectors (study variations in gain and energy resolution as a function of bias voltage) paying particular attention to efficiency measurements. Estimate the activity of unknown sources using the detector efficiency method and estimate the error on these results.

Read about absolute activity measurements using gamma-gamma coincidence in Knoll [Knoll 694–695]. Set up your detectors with suitable single channel scalers as described and connect up the coincidence circuit. Measure single and coincidence (prompt and delayed) count rates for the same sources as above and compare measurements. How do the errors on activity compare using the two methods?

### 2.10 Gamma-ray spectroscopy using scintillators

The teaching laboratory has several different types of scintillator detectors, including sodium iodide (NaI), cerium(III) bromide (CeBr<sub>3</sub>), lanthanum(III) bromide (LaBr<sub>3</sub>) and lanthanum(III) chloride (LaCl<sub>3</sub>) solid state detectors. These are often used for specific experiments, but can be used for other investigations as required. Each type has its own characteristics which can make one preferable over another, and you can investigate the similarities and differences of each:

Investigate PM tube gain as a function of bias voltage. How does the overall multiplication factor scale and what does this tell you about your detector construction?

Find the characteristics of the detectors, as outlined in section 1.9. Measure the energy resolution and efficiency as a function of  $\gamma$ -ray energy, and calibrate your detectors.

Measure the background radiation field in the lab. Ensure that you have a statistically significant sample. What are the main features in this spectrum and where do they originate? You may wish to try this at different times of day or on different days.

Identify the photopeak and Compton continuum for your measured spectra. Does the energy of the Compton edge correlate with what you know about Compton scattering? What about the backscatter peak (if it is easily identifiable). Measure the areas covered by the photopeak and Compton continuum, giving the peak to Compton ratio. How does this behave for different energy sources?

You may wish to compare many of the measurements done using scintillators with similar experiments using other types of detectors in the lab when you change experiments. As always be sure to keep a careful record of what you have measured, and how, so that you can include these results as part of other experiments (and in your lab reports).

### 3 Writing a laboratory report

This section gives some guidelines and suggestions for writing laboratory reports. It is written in the style of a report. The report itself should contain the following sections; abstract, introduction, theory, description of apparatus, method or procedure, results and analysis, discussion and conclusion. As a rule data should be presented in the results section, and appendices may be used if there are very long tables.

### **Abstract**

As a general rule, the abstract should not be longer than about ten sentences, and should contain a concise description of your work, and any important results that you wish to present. This should include numerical results and the associated uncertainties. The abstract should not contain any description of experimental apparatus, unless its success or failure is the actual result, nor should the abstract contain any equations.

### Introduction

The introduction should be just that – a section which introduces the reader to what you have been doing. It may contain some historical information if the experiments have been performed by others in the past, or if there have been other measurements reported in the research literature, or in textbooks, or of course if similar work has been presented on a website. In these cases, report the previous work and refer to the appropriate journal, book or website. The other function of the introduction is to introduce the area of experimentation that you are investigating, and set the framework of the physics involved and the methods you are going to use in order to make your measurements. Remember that you might also have another section called Experimental equipment or Experimental details etc., so do not fill the introduction with great details about the experimental work. If the theory behind the experiment is fairly simple and brief it might be appropriate to put such theory into the Introduction, but if there are lots of equations, it might be better to have a second section which could be called *Theory*, *Theory of the measurements on XYZ*. Similarly, if there is a need to explain carefully about a complicated experimental setup, another section called Background information on measurements XYZ, Detection systems for measurements on XYZ, etc. could be inserted into the text, probably after the *Theory* section.

### Theory

The reports should be written so as to be understood by a physicist who does not know in advance anything about the experiment which you have done, so you must explain what the purpose of the experiment was, how you did it, what the results are, and what their significance is. Reports should be written in clear English, and in sentences, i.e. do not use bullet points. Conventionally, descriptions of what you did are generally written in the past tense using a passive style ("The detector was connected to an MCA...")

In a theory section, you need to encapsulate all the necessary theory with as few equations as possible; do not include equations that you never use. Be sure to discuss the main physical processes which are involved in your experiment, so that the reader is aware of what is involved, and if necessary can go away and read some background information, hopefully from one of the excellent references that you have listed at the back of your report. Make sure all your equations are numbered and are in a clear font so that they are unambiguous; make sure that numerators and denominators are clearly separated, powers of numbers or variables are in superscript, and indices of variables are in subscript etc. Be very careful to distinguish variables which are vectors from other variables by using a line above the variable or use a bold font.

In between the equations, you need to link, or summarise, the relevance of each equation to the physics and the experiment which you have performed, so that the reader is aware of its importance. Remember that the report is telling a story about your experiment, and you need to keep the reader interested and involved.

### **Background to detection techniques for XYZ**

If your experiment involves a large amount of complicated apparatus, then this section might provide the reader with some valuable background information about the detection system for example. Try to get in some pictures to illustrate what you are planning to use, so that when you come to describe your specific experimental details, the reader will feel familiar with what you are talking about. However, bear in mind that photographs are not necessarily as instructive as schematic diagrams to explain your thinking. Try to link what you are discussing back to the equations in the Theory section if appropriate, but you could also include further equations in this section if they are applicable to the methods or operation of the detection systems. Try and be as specific as possible, e.g. if you used a particular power supply, then state the manufacturer, the model number/name and what the voltage range used was, and why you chose that range.

### **Experimental measurements of XYZ**

Describe the experimental equipment briefly, and then move onto the way in which the experiment was carried out. Try to show that the way you tackled the experiment was actually planned in a reasonable way to produce the correct results. Discuss the results you obtained, illustrating your results with figures within the text, so that the reader does not need to keep flipping over into the appendices. Do not put all your experimental measurements within the text of the report; you may have hundreds of values which can usefully go into an appendix of the report. Remember that one picture is worth a thousand words. So, try and summarise all your results in a few simple figures.

### Results of the detection of XYZ

In this section you should analyse the results, making reference to the equations in the theory section and to the sections on the background and experimental details. One of the best ways to summarise your results is either with a figure, or with a table which contains all the main results. You should also tackle the subject of errors on measurements in general and on your measurements in particular. Try to work out what errors can occur in your measurements and the relevant magnitudes of these errors. Remember that Physicists are the greatest sceptics – there is no truth, just measurement with errors. Try to justify the conclusions that you have drawn from your results, and comment on whether your measured values are consistent with other measurements (within errors).

### **Conclusions**

In this section you need to summarise all your results, and most importantly, discuss your results and compare with other measurements, or previous work. A summary table of your own results compared with other measurements is sometimes useful. Lastly, you could make some critical comments about the experiment and suggest ways in which it might be improved, or ways in which you might have tackled the experiment if you were to do it again. Remember that for some readers, the only pages they may read are the title page with the abstract and the conclusions.

### References

Don't forget to include ALL of your references (books/journals/www etc). There are two schemes of referencing papers, either by numbering[1] or by referring to the authors and the data [Bloggs et al. 1969]

 Bloggs et al., 1969. Referencing techniques for publications, Proceedings of the International Conference on Publications, Lynchburg, Moore County, Tennessee, U.S.A., ed. J. Daniels, Hic! Books Inc, St Louis, Missouri, U.S.A.

### **Bibliography**

Note that references are NOT the same as a bibliography, which should come in a separate section.

### **Appendices**

This section should contain all the tables which are too long to put in the text, or raw data which you might think could be useful for some readers, or bits of theory which are useful but not really important to the flow of the report.

# 4 Gamma-Ray Source Activities

Set	Source	ID	Activity on 21/11/13 (Bq)	Accuracy (%)
	<sup>60</sup> Co	A0732	1640 ± 31	1.9
\	$^{133}$ Ba	A0445	$24800 \pm 1190$	4.8
Year 3 Box 1	$^{137}\mathrm{Cs}$	A0468	$149000 \pm 5520$	3.7
	$^{152}Eu$	1D058	$64700 \pm 3480$	5.4
	<sup>22</sup> Na	1X515	64 ± 2	3.7
Veer O Day O	$^{60}Co$	1U577	$5530\pm105$	1.9
Year 3 Box 2	$^{133}$ Ba	1R479	$52200\pm2500$	4.8
	$^{241}\mathrm{Am}$	1Q532	$374000 \pm 18700$	5.0
	<sup>60</sup> Co	2U270	12100 ± 230	1.9
	$^{133}Ba$	A0877	$23900\pm1150$	4.8
Year 3 Box 3	$^{137}\mathrm{Cs}$	A0697	$142000 \pm 5270$	3.7
	$^{152}Eu$	EW617	$149000 \pm 7460$	5.0
	$^{241}Am$	1Q599	$402000 \pm 20100$	5.0
	<sup>60</sup> Co	1U630	6240 ± 120	1.9
	$^{133}$ Ba	1R545	$45600 \pm 2190$	4.8
MSc Box 1	$^{137}Cs$	1S684	191000 $\pm$ 7080	3.7
	$^{152}Eu$	1D157	$83200 \pm 4160$	5.0
	$^{241}\mathrm{Am}$	1Q598	$415000 \pm 20700$	5.0
	<sup>60</sup> Co	1U629	5850 ± 110	1.9
	$^{133}Ba$	1R548	$45400\pm2180$	4.8
MSc Box 2	$^{137}Cs$	1S681	$184000 \pm 6800$	3.7
	$^{152}Eu$	1D152	$81800 \pm 4090$	5.0
	$^{241}Am$	1Q596	$398000 \pm 19900$	5.0
Individual	<sup>22</sup> Na	C3N.19/91	38100 ± 1960	5.1
Individual	$^{22}$ Na	C3N.19/61	$29000\pm1430$	4.9
Individual	<sup>44</sup> Ti	UOB08121	22800 ± 1190	5.2
Individual	$^{44}Ti$	UOB08122	$22800\pm1200$	5.2
Individual	$^{44}Ti$	UOB08123	$22700 \pm 1190$	5.2
Individual	$^{44}Ti$	UOB08124	$23300\pm1220$	5.2
Individual	<sup>60</sup> Co	C3N.19/18	248000 ± 10500	4.2
Individual	$^{60}{\sf Co}$	C3N.19/62	$35200\pm1500$	4.3
 Individual	<sup>68</sup> Ge	UOB101012	$136000 \pm 7210$	5.3
Individual	<sup>68</sup> Ge	UOB221111	$68900 \pm 3720$	5.4
Individual	<sup>108m</sup> Ag	UOB931201B2	8510 ± 490	5.8
 Individual	<sup>137</sup> Cs	C3N.19/65	5300000 ± 205000	3.9
Individual	$^{137}\mathrm{Cs}$	C3N.19/76	$\textbf{38100} \pm \textbf{2090}$	5.5
 Individual	<sup>207</sup> Bi	C3N.19/89	20300 ± 1030	5.1

#### Summary of Statistical Formulae (+ Knoll ch. 3) 5

### Sample Mean

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

### **Standard Deviation**

$$\sigma(x) \text{ (or } s(x)) = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

(error on a single measurement)

### **Error in Mean**

$$\sigma(\bar{x}) = \frac{\sigma(x)}{\sqrt{n}}$$

### Goodness of Fit

$$\chi^{2} = \sum_{i=1}^{n} \frac{(y_{i}^{obs} - y_{i}^{th})^{2}}{\sigma^{2}(y_{i}^{obs})}$$

 $(0.5 \le \chi^2/(n-p) \le 1.5$  for a good fit to n data points with p free parameters)

### **Propagation of Errors**

1. 
$$F = k.x$$
  $\Rightarrow \sigma(F) = |k|\sigma(x)$ 

2. 
$$F = x^n$$
  $\Rightarrow \frac{\sigma(F)}{F} = |n| \frac{\sigma(x)}{x}$ 

3. 
$$F = x + y + z \Rightarrow \sigma(F) = \sqrt{\sigma^2(x) + \sigma^2(y) + \sigma^2(z)}$$

3. 
$$F = x + y + z \implies \sigma(F) = \sqrt{\sigma^2(x) + \sigma^2(y) + \sigma^2(z)}$$
  
4.  $F = \frac{x \cdot y}{z} \implies \left(\frac{\sigma(F)}{F}\right)^2 = \left(\frac{\sigma(x)}{x}\right)^2 + \left(\frac{\sigma(y)}{y}\right)^2 + \left(\frac{\sigma(z)}{z}\right)^2$ 

5. 
$$F = \frac{x^p \cdot y^q}{z^r}$$
  $\Rightarrow$   $\left(\frac{\sigma(F)}{F}\right)^2 = \left(p\frac{\sigma(x)}{x}\right)^2 + \left(q\frac{\sigma(y)}{y}\right)^2 + \left(r\frac{\sigma(z)}{z}\right)^2$ 

$$\begin{split} \text{In general:} \quad \sigma^2(F) &= \left(\sigma(x)\frac{\partial F}{\partial x}\right)^2 + \left(\sigma(y)\frac{\partial F}{\partial y}\right)^2 + \left(\sigma(z)\frac{\partial F}{\partial z}\right)^2 + \dots \\ &\quad + 2\text{cov}(x,y)\left(\frac{\partial F}{\partial x}\right)\left(\frac{\partial F}{\partial y}\right) + 2\text{cov}(x,z)\left(\frac{\partial F}{\partial x}\right)\left(\frac{\partial F}{\partial z}\right) + \dots \end{split}$$