VEDIC MATHEMATICS TEACHER'S MANUAL

ELEMENTARY LEVEL

Kenneth R. Williams

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PREFACE

This Manual is the first of three self-contained Manuals (Elementary, Intermediate and Advanced) which are designed for adults with a basic understanding of mathematics to learn or teach the Vedic system. So teachers could use it to learn Vedic Mathematics, though it is not suitable as a text for children (for that the Cosmic Calculator Course is recommended). Or it could be used to teach a course on Vedic Mathematics. This Manual is suitable for teachers of children in grades 3 to 7.

The sixteen lessons of this course are based on a series of one week summer courses given at Oxford University by the author to Swedish mathematics teachers between 1990 and 1995. Those courses were quite intensive consisting of eighteen, one and a half hour, lessons.

All techniques are fully explained and proofs are given where appropriate, the relevant Sutras are indicated throughout (these are listed at the end of the Manual) and, for convenience, answers are given after each exercise. Cross-references are given showing what alternative topics may be continued with at certain points.

It should also be noted that in the Vedic system a mental approach is preferred so we always encourage students to work mentally as long as it is comfortable. In the Cosmic Calculator Course pupils are given a short mental test at the start of most or all lessons, which makes a good start to the lesson, revises previous work and introduces some of the ideas needed in the current lesson. In the Cosmic Calculator course there are also many games that help to establish and promote confidence in using the Vedic system.

Some topics will be found to be missing in this text: for example, there is no section on area, only a brief mention. This is because the actual methods are the same as currently taught so that the only difference would be to give the relevant Sutra(s).

CONTENTS

CONTENTS

LESSON 1
COMPLETING THE WHOLE

SUMMARY

1.1 Introduction - background information about Vedic Mathematics.
1.2 The Ten Point Circle – representing numbers on a circle.
1.3 Multiples of Ten
1.4 Deficiency from Ten – relating numbers to multiples of ten.
1.5 Mental Addition
1.6 By Addition and By Subtraction – of numbers near a multiple of ten.

1.1 INTRODUCTION

Vedic Mathematics is the ancient system of mathematics which was rediscovered early last century by **Sri Bharati Krsna Tirthaji** (henceforth referred to as Bharati Krsna).

The Sanskrit word "Veda" means "knowledge". The Vedas are ancient writings whose date is disputed but which date from at least several centuries BC. According to Indian tradition the content of the Vedas was known long before writing was invented and was freely available to everyone. It was passed on by word of mouth. The writings called the Vedas consist of a huge number of documents (there are said to be millions of such documents in India, many of which have not yet been translated) and these have recently been shown to be highly structured, both within themselves and in relation to each other (see Reference 2). Subjects covered in the Vedas include Grammar, Astronomy, Architecture, Psychology, Philosophy, Archery etc., etc.

A hundred years ago Sanskrit scholars were translating the Vedic documents and were surprised at the depth and breadth of knowledge contained in them. But some documents headed "Ganita Sutras", which means mathematics, could not be interpreted by them in terms of mathematics. One verse, for example, said "in the reign of King Kamse famine, pestilence and unsanitary conditions prevailed". This is not mathematics they said, but nonsense.

Bharati Krsna was born in 1884 and died in 1960. He was a brilliant student, obtaining the highest honours in all the subjects he studied, including Sanskrit, Philosophy, English, Mathematics, History and Science. When he heard what the European scholars were saying about the parts of the Vedas which were supposed to contain mathematics he resolved to study the documents and find their meaning. Between 1911 and 1918 he was able to reconstruct the ancient system of mathematics which we now call Vedic Mathematics.

He wrote sixteen books expounding this system, but unfortunately these have been lost and when the loss was confirmed in 1958 Bharati Krsna wrote a single introductory book entitled “Vedic Mathematics”. This is currently available and is a best-seller (see Reference 1).

The present author came across the book “Vedic Mathematics” in 1971 and has been developing the content of that book, and applying the system in other areas not covered by Bharati Krsna, since then. Anything in this book which is not in “Vedic Mathematics” has been developed independently by the author in this way.

There are many special aspects and features of Vedic Mathematics which are better discussed as we go along rather than now because you will need to see the system in action to appreciate it fully. But the main points for now are:

1) The system rediscovered by Bharati Krsna is based on sixteen formulae (or Sutras) and some sub-formulae (sub-Sutras). These Sutras are given in word form: for example *By One More than the One Before* and *Vertically and Crosswise*. In this text they are indicated by italics. The Sutras can be related to natural mental functions such as completing a whole, noticing analogies, generalisation and so on.

2) Not only does the system give many striking general and special methods, previously unknown to modern mathematics, but it is far more coherent and integrated as a system.

3) Vedic Mathematics is a system of mental mathematics (though it can also be written down).

Many of the Vedic methods are new, simple and striking. They are also beautifully interrelated so that division, for example, can be seen as an easy reversal of the simple multiplication method (similarly with squaring and square roots). This is in complete contrast to the modern system. Because the Vedic methods are so different to the conventional methods, and also to gain familiarity with the Vedic system, it is best to practice the techniques as you go along.

“The Sutras (aphorisms) apply to and cover each and every part of each and every chapter of each and every branch of mathematics (including arithmetic, algebra, geometry – plane and solid, trigonometry – plane and spherical, conics- geometrical and analytical, astronomy, calculus – differential and integral etc., etc. In fact, there is no part of mathematics, pure or applied, which is beyond their jurisdiction”
From “Vedic Mathematics”, Page xvi.

1.2 THE TEN POINT CIRCLE

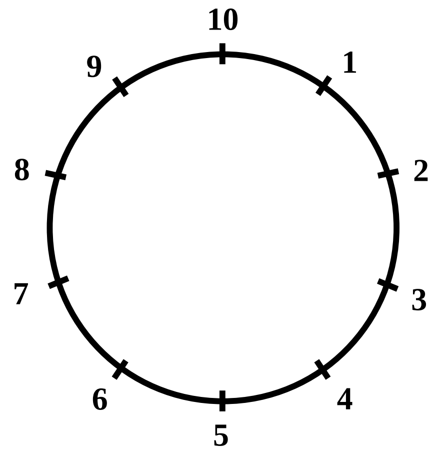
1 2 3 4 5 6 7 8 9 10 . . .

Numbers start with number one.
Then comes number two, then three and so on.
The Sutra *By One More than the One Before* describes the generation of numbers from unity.

Arithmetic is the study of the behaviour of numbers and just as every person is different and special so it is with numbers.
Every number is special and when we get to know numbers they are like friends.

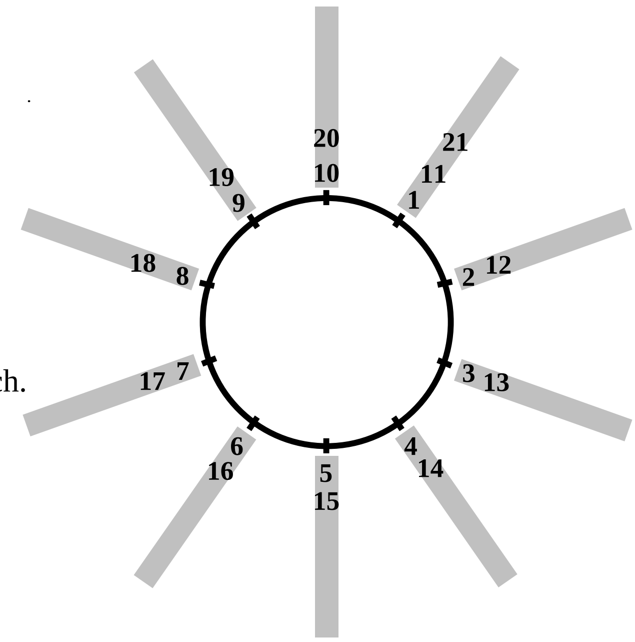
[Some discussion about numbers and where they appear could be introduced here.]

Sometimes it is useful to have the first ten numbers around a circle like this:



We use nine figures, and zero.
For numbers beyond 9 we put two or more of these together to make 10, 11, 12 and so on.

Continuing around the circle we can put 11 where we have 1, but further out on the 1-branch. And number 12 goes next to 2 and so on.

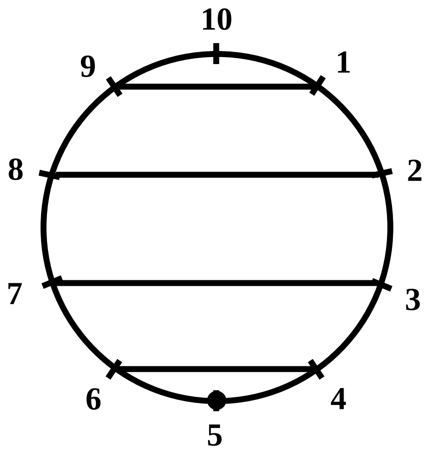


This circle can be used for adding on numbers, and for taking away, just as we use a number line. Notice that the numbers on any branch all end with the same figure and that multiples of ten all appear on the top branch.

1.3 MULTIPLES OF TEN

It is important to know the five pairs of numbers that add up to 10:

1 + 9 = 10, 2 + 8 = 10, 3 + 7 = 10, 4 + 6 = 10, 5 + 5 = 10.



These pairs are shown on the 10-point circle above.
The Sutra *By the Completion or Non-Completion* describes the ability we all have to see and use wholeness.

✐ **Practice A** Complete the following additions:

a 6 + 4	**b** 4 + 16	**c** 5 + 25	**d** 13 + 7	**e** 22 + 8
f 38 + 2	**g** 54 + 6	**h** 47 + 3	**i** 61 + 9	**j** 85 + 5

---

a	**10**	**b**	**20**	**c**	**30**	**d**	**20**	**e**	**30**
f	**40**	**g**	**60**	**h**	**50**	**i**	**70**	**j**	**90**

Completing tens can be done in another way.

1 For example, **24 + 26** is easy because the 4 and 6 make ten.
So **24 + 26 = 50**.

"Little boys come dancing forward with joy and professors ask, 'well, how can the answer be written down without any intermediate steps of working at all?'".
From "Vedic Metaphysics", Page 168.

Practice B Add the following:

a 37 + 23 **b** 42 + 28 **c** 54 + 16 **d** 49 + 21

e 45 + 35 **f** 72 + 18 **g** 38 + 22 **h** 35 + 35

---

a 60 b 70 c 70 d 70
e 80 f 90 g 60 h 70

1.4 DEFICIENCY FROM TEN

The Vedic Sutra *By the Deficiency* relates to the natural ability to see how much something differs from wholeness.

2 You can see that **39** is close to **40** and is **1** short of 40,
and that **58** is close to **60** and is **2** short of 60.

Practice C In the following exercise fill in the missing numbers.

a 37 is close to and is below.

b 49 is close to and is below. **c** 68 is close to and is below.

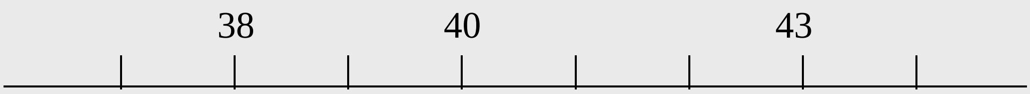
---

a 40, 3 b 50, 1 c 70, 2

<u>DEFICIENCY AND COMPLETION TOGETHER</u>

This makes adding easier because we can complete a whole.

3 **38 + 5 = ?** You know that 38 is close to 40 and is 2 below it.
So take 2 of the 5 to make up to 40 and you have 3 more to add on, which gives **43**.



We can imagine a number line, or draw one out or use the 10-point circle to add numbers like this.

✐ **Practice D**

a 49 + 5	**b** 58 + 3	**c** 37 + 6	**d** 28 + 6
e 79 + 6	**f** 38 + 7	**g** 57 + 7	**h** 69 + 4

---

a 54	**b 61**	**c 43**	**d 34**
e 85	**f 45**	**g 64**	**h 73**

1.5 MENTAL ADDITION

4 When an addition sum has a carry, like **56 + 26** you can add them in your head, like this:
In 56 + 26 you get 7 tens or 70.
Then in the units you have 6 + 6 = 12. And 70 + 12 = 82.
So **56 + 26 = 82**.

$$\begin{array}{r} 5\ 6 \\ +\ \underline{2\ 6} \\ \underline{8\ 2} \\ {\scriptstyle 1\ \ } \end{array}$$

You could also write this as 56 + 26 = 7_12 = 82, writing the 12 as $_12$ to show that the 1 in the 12 has to be carried to the left.

5 Similarly, **48 + 45** = 8_13 = **93**.

You can write the extra step if you like but try to do the whole thing in your head if possible.

✐ **Practice E** Try these:

a 37 + 47	**b** 55 + 28	**c** 47 + 25	**d** 29 + 36
e 56 + 25	**f** 38 + 26	**g** 29 + 44	**h** 35 + 49

---

a 84	**b 83**	**c 72**	**d 65**
e 81	**f 64**	**g 73**	**h 84**

"The Sutras are easy to understand, easy to apply and easy to remember; and the whole work can be truthfully summarised in one word "mental".
From "Vedic Mathematics", Page xvi.

COMPLETING THE WHOLE

In the puzzle below you have to find three numbers that add up to 10.
There are eight answers to this puzzle and one of these is given to you:
1 + 2 + 7 = 10.
But you cannot have 2 + 1 + 7 = 10 as another answer: the numbers must be different.
And you cannot use nought, but you can use a number more than once.

✐ **Practice F** See how many you can find.

$\boxed{1} + \boxed{2} + \boxed{7} = \boxed{10}$	$\boxed{} + \boxed{} + \boxed{} = \boxed{10}$
$\boxed{} + \boxed{} + \boxed{} = \boxed{10}$	$\boxed{} + \boxed{} + \boxed{} = \boxed{10}$
$\boxed{} + \boxed{} + \boxed{} = \boxed{10}$	$\boxed{} + \boxed{} + \boxed{} = \boxed{10}$
$\boxed{} + \boxed{} + \boxed{} = \boxed{10}$	$\boxed{} + \boxed{} + \boxed{} = \boxed{10}$

	2+2+6
1+1+8	**2+3+5**
1+3+6	**2+4+4**
1+4+5	**3+3+4**

Where several numbers are being added it is a good idea to look for whole multiples of 10 (i.e. 10, 20, 30 etc.).

6 For example if you need to find **6 + 7 + 4** you would see that the 6 and 4 make a 10. And you add the 7 on last to get **6 + 7 + 4 = 17**.

7 Also in adding **3 + 6 + 2 + 5** you can see that the 3, 2 and 5 make a 10 so you add these first and add the 6 on last to get **3 + 6 + 2 + 5 = 16**.

✐ **Practice G** Try these:

a 3 + 2 + 8 **b** 9 + 8 + 1 **c** 7 + 2 + 4 + 3

d 4 + 5 + 5 + 7 **e** 8 + 9 + 2 **f** 7 + 6 + 2 + 4

g 8 + 8 + 3 + 2	**h** 7 + 6 + 3 + 4	**i** 4 + 7 + 4 + 2
j 6 + 9 + 2 + 2	**k** 7 + 5 + 1 + 2	**l** 3 + 5 + 4 + 3

---

a	**13**	**b**	**18**	**c**	**16**
d	**21**	**e**	**19**	**f**	**19**
g	**21**	**h**	**20**	**i**	**17**
j	**19**	**k**	**15**	**l**	**15**

You can complete multiples of ten for bigger numbers also.

8 For example given **19 + 8 + 1** you can see that 19 + 1 makes a whole 20 so you add these first and then the 8.
So **19 + 8 + 1 = 28**.

9 Suppose you want **33 + 28 + 4 + 32**.
You notice that the 28 and 32 make a multiple of ten, so you add these first to get 60.
Then adding 33 gives 93, and the 4 makes 97.
So **33 + 28 + 4 + 32 = 97**.

$$\mathbf{33 + 28 + 4 + 32 = 97}$$

✎ **Practice H** Use this method of completing the whole to add the following numbers.

a 29 + 7 +1 + 5	**b** 16 + 3 + 6 + 17	**c** 8 + 51 + 12 + 3
d 37 + 7 + 21 + 13	**e** 13 + 16 + 17 + 24	**f** 12 + 26 + 34 + 8
g 33 + 25 + 22 + 15	**h** 18 + 13 + 14 + 23	**i** 3 + 9 + 5 + 7 + 1
j 27 + 15 + 23	**k** 43 + 8 + 19 + 11	**l** 32 + 15 + 8 + 4
m 24 + 7 + 8 + 6 + 13	**n** 6 + 33 + 24 + 17	**o** 23 + 48 + 27

---

a	**42**	**b**	**42**	**c**	**74**
d	**78**	**e**	**70**	**f**	**80**
g	**95**	**h**	**68**	**i**	**25**
j	**65**	**k**	**81**	**l**	**59**
m	**58**	**n**	**80**	**o**	**98**

COLUMNS OF FIGURES

Another way in which completing tens can be used is in adding columns of figures.

10 For example if you had to find:

$$\begin{array}{rl} 2\ 7 & \\ 3\ 5 & \\ 6\ 3 & \\ 8\ 2 & + \\ \hline \\ \hline \end{array}$$

you look in the units column and see a 7 and 3 there, which makes 10, so that there is a total of 17 altogether in this column.
You put this down, carrying the 1 to the left as shown:

$$\begin{array}{rl} 2\ 7 & \\ 3\ 5 & \\ 6\ 3 & \\ 8\ 2 & + \\ \hline 7 & \\ \hline {\scriptstyle 1}\ \ & \end{array}$$

Then you add the tens column, looking again for tens.
You see 2 + 8 = 10 and so the total is 19.
Adding the carried 1 you get 20 which you put down:

$$\begin{array}{rl} 2\ 7 & \\ 3\ 5 & \\ 6\ 3 & \\ 8\ 2 & + \\ \hline 2\ 0\ 7 & \\ \hline {\scriptstyle 1}\ \ & \end{array}$$

Practice I Try these:

a
$$\begin{array}{rl} 4\ 4 & \\ 2\ 2 & \\ 6\ 5 & \\ 8\ 6 & + \\ \hline \\ \hline \end{array}$$

b
$$\begin{array}{rl} 3\ 5 & \\ 7\ 6 & \\ 4\ 5 & + \\ \hline \end{array}$$

c
$$\begin{array}{rl} 4\ 8 & \\ 3\ 8 & \\ 6\ 2 & \\ 7\ 1 & + \\ \hline \\ \hline \end{array}$$

d
$$\begin{array}{rl} 6\ 3\ 2\ 7 & \\ 5\ 8\ 4 & \\ 7\ 4\ 3 & + \\ \hline \end{array}$$

e
$$\begin{array}{rl} 5\ 4\ 9 & \\ 1\ 8\ 2 & \\ 3\ 1\ 7 & \\ 2\ 4\ 1 & \\ 7\ 2\ 6 & \\ 3\ 2\ 1 & + \\ \hline \\ \hline \end{array}$$

---

a **217** **b** **156** **c** **219** **d** **7654** **e** **2336**

11 Now suppose you have:

$$\begin{array}{rrrrl} & 8 & 2 & 4 & \\ & 6 & 5 & 6 & \\ & & 8 & 5 & \\ & & 3 & 8 & + \\ \hline & & & & \\ \hline \end{array}$$

You immediately see a 10 (4+6) in the first column. And there is also a 13 (5+8). So 13 and 10 give 23 and so you put 3 and carry 2:

$$\begin{array}{rrrrl} & 8 & 2 & 4 & \\ & 6 & 5 & 6 & \\ & & 8 & 5 & \\ & & 3 & 8 & + \\ \hline & & & 3 & \\ \hline & & {\scriptstyle 2} & & \end{array}$$

In the next column you see a 10 (2+8) and also 8 (5+3).
This gives 18 and with the carried 2 we get 20.
So put 0 and carry 2:

$$\begin{array}{rrrrl} & 8 & 2 & 4 & \\ & 6 & 5 & 6 & \\ & & 8 & 5 & \\ & & 3 & 8 & + \\ \hline 1 & 6 & 0 & 3 & \\ \hline & {\scriptstyle 2} & {\scriptstyle 2} & & \end{array}$$

Finally we have 14 in the left column and the carried 2 makes 16, which you put down.

Practice J Try these:

a
$$\begin{array}{rrl} 4 & 7 & \\ 2 & 3 & \\ 3 & 6 & \\ 3 & 6 & + \\ \hline & & \\ \hline \end{array}$$

b
$$\begin{array}{rrl} 3 & 5 & \\ 2 & 8 & \\ 5 & 7 & \\ 3 & 2 & + \\ \hline & & \\ \hline \end{array}$$

c
$$\begin{array}{rrl} 4 & 8 & \\ 3 & 9 & \\ 8 & 8 & \\ 7 & 1 & + \\ \hline & & \\ \hline \end{array}$$

d
$$\begin{array}{rrrrl} 3 & 3 & 2 & 7 & \\ 2 & 5 & 7 & 7 & \\ & 5 & 8 & 5 & \\ & 3 & 8 & 3 & + \\ \hline & & & & \\ \hline \end{array}$$

e
$$\begin{array}{rrrl} 2 & 4 & 2 & \\ 1 & 8 & 8 & \\ 1 & 1 & 5 & \\ 2 & 4 & 3 & \\ 7 & 9 & 6 & \\ 3 & 2 & 1 & + \\ \hline & & & \\ \hline \end{array}$$

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a **142** **b** **152** **c** **246** **d** **6872** **e** **1905**

1.6 BY ADDITION AND BY SUBTRACTION

Numbers like 9, 19, 18, 38, which are just under multiples of ten are particularly easy to add and subtract (take away).

> **12** Suppose you have to find **33 + 9**.
> As 9 is 1 below 10 you can do this by adding 10 and taking 1 away: 33+10–1.
> Adding 10 to 33 gives 43, and taking 1 away leaves 42.
> So **33 + 9 = 42**.

This illustrates the formula *By Addition and By Subtraction*.

Practice K Try some:

a 55 + 9	**b** 64 + 9	**c** 45 + 9	**d** 73 + 9
e 82 + 9	**f** 26 + 9	**g** 67 + 9	**h** 38 + 9

a 64	**b 73**	**c 54**	**d 82**
e 91	**f 35**	**g 76**	**h 47**

> **13** Similarly if you are adding 19, you can add 20 and take 1 away.
> So **66 + 19 = 85**.
> Because you can add 20 to 66 to get 86 and take 1 off to get 85.
>
> **14** And to find **54 + 39** you could add 40 to 54 and take 1 off to get 93.
> So **54 + 39 = 93**.

Practice L

a 44 + 19	**b** 55 + 29	**c** 36 + 49	**d** 73 + 19
e 47 + 39	**f** 26 + 59	**g** 17 + 69	**h** 28 + 29

a 63	**b 84**	**c 85**	**d 92**
e 86	**f 85**	**g 86**	**h 57**

In a similar way you could add 18 to a number by adding 20 and taking 2 away.

Or you could add 38 to a number by adding 40 and taking 2 away.

Or add 37 by adding 40 and taking 3 away.

15 So, for example, **33 + 48 = 81** as you would add 50 to 33 to get 83 and then take 2 away, because 48 is 2 below 50.

✐ **Practice M** Try these:

a	44 + 18	**b**	44 + 27	**c**	55 + 28	**d**	35 + 37
e	62 + 29	**f**	36 + 37	**g**	19 + 19	**h**	28 + 29

a 62 b 71 c 83 d 72
e 91 f 73 g 38 h 57

The sums below are like the ones above except that the number which is just below a multiple of ten is the **first** number in the sum.

16 For example you might have **29 + 55**.
Here you could add 30 to 55 and take 1 off to get **29 + 55 = 84**.

✐ **Practice N** Try a few of these:

a	39 + 44	**b**	33 + 38	**c**	48 + 35	**d**	27 + 34
e	33 + 28	**f**	9 + 73	**g**	18 + 19	**h**	26 + 27

a 83 b 71 c 83 d 61
e 61 f 82 g 37 h 53

SUBTRACTING NUMBERS NEAR A BASE

A similar method can be used for subtracting numbers which are just below a base.

17 For example given **55 – 19** you notice that 19 is 1 below 20.
So take 20 from 55 (to get 35) and add 1 back on.
So **55 – 19 = 36**.

18 And **61 – 38 = 23** because you take 40 from 61 (to get 21) and add 2 back on.

✎ **Practice O** Try these

a 44 – 19	**b** 66 – 29	**c** 88 – 49	**d** 55 – 9
e 52 – 28	**f** 72 – 48	**g** 66 – 38	**h** 81 – 58
i 83 – 36	**j** 90 – 66	**k** 55 – 27	**l** 60 – 57

---

a	**25**	**b**	**37**	**c**	**39**	**d**	**46**
e	**24**	**f**	**24**	**g**	**28**	**h**	**23**
i	**47**	**j**	**24**	**k**	**28**	**l**	**3**

"And we were agreeably astonished and intensely gratified to find that exceedingly tough mathematical problems (which the mathematically most advanced present day Western scientific world had spent huge lots of time, energy and money on and which even now it solves with the utmost difficulty and after vast labour and involving large numbers of difficult, tedious and cumbersome "steps" of working) can be easily and readily solved with the help of these ultra-easy Vedic Sutras (or mathematical aphorisms) contained in the Parishishta (the Appendix-portion) of the ATHARVAVEDA in a few simple steps and by methods which can be conscientiously described as mere "mental arithmetic".

From "Vedic Mathematics", Page xv.

LESSON 2
DOUBLING AND HALVING

SUMMARY

2.1 **Doubling –** multiplying by 2, 4, 8.
2.2 **Halving –** dividing by 2, 4, 8.
2.3 **Extending your Tables –** by using doubling and halving.
2.4 **Multiplying by 5, 50, 25**
2.6 **Dividing by 5, 50, 25**

2.1 DOUBLING

Doubling and halving are very easy to do and can be used to quickly do many simple calculations.

Adding two of the same number is called **doubling**.
It comes under the *Proportionately* formula of Vedic Mathematics.

1 For example to double **34** you can find 34 + 34, which is **68**.
It is the same as multiplying 34 by 2.
34 + 34 = 2 × 34 or 34 × 2.

2 So double **42** is **84**.
Double **35** is **70**.
And double **26** is **52**, because 26 + 26 = 52.

✐ **Practice A** Double the following numbers. Just write down the answer.

a 24	**b** 41	**c** 14	**d** 45	**e** 15	**f** 25
g 36	**h** 27	**i** 18	**j** 29	**k** 34	**l** 48

---

a 48	**b 82**	**c 28**	**d 90**	**e 30**	**f 50**
g 72	**h 54**	**i 36**	**j 58**	**k 68**	**l 96**

3 To double **68** we just think of doubling 60 and 8 and then adding.

Double 60 is 120,
double 8 is 16.
And adding 120 and 16 gives **136**.

4 To double **680** we double 68 and put ‘0’ on the end: **1360**.

In the following exercise just write down the answers to the sums.

Practice B Double the following numbers:

a	58	**b**	61	**c**	73	**d**	65	**e**	66
f	88	**g**	76	**h**	91	**i**	380		

a	**116**	**b**	**122**	**c**	**146**	**d**	**130**	**e**	**132**
f	**176**	**g**	**152**	**h**	**182**	**i**	**760**		

5 To double **273** we double 270 and 3.
So you get 540 + 6 = **546**.

6 To double **636** you can double 600 and 36 to get 1200 and 72.
So the answer is **1272**.

Practice C Double these:

a	362	**b**	453	**c**	612	**d**	319	**e**	707
f	610	**g**	472	**h**	626	**i**	1234	**j**	663

a	**724**	**b**	**906**	**c**	**1224**	**d**	**638**	**e**	**1414**
f	**1220**	**g**	**944**	**h**	**1252**	**i**	**2468**	**j**	**1326**

MULTIPLYING BY 4, 8

You can multiply by 4 by doubling a number twice.
And to multiply by 8, double the number three times.

7 So for **35 × 4** you double 35 to get 70,
and then double again to get 140.
Then **35 × 4 = 140**.

8 For **26 × 8** you double three times.
Doubling 26 gives 52, doubling 52 gives 104, doubling 104 gives 208.
So **26 × 8 = 208**.

✎ **Practice D** Try these:

a 53 × 4	**b** 28 × 4	**c** 33 × 4	**d** 61 × 4
e 18 × 4	**f** 81 × 4	**g** 16 × 4	**h** 16 × 8
i 22 × 8	**j** 45 × 8		

a 212	**b 112**	**c 132**	**d 244**
e 72	**f 324**	**g 64**	**h 128**
i 176	**j 360**		

Doubling halves and quarters is also easy.

9 For **7½ × 8** you double 7½ three times.
You get 15, 30, 60, so **7½ × 8 = 60**.

10 For **2¾ × 8** you double 2¾ three times.
You get 5½, 11, 22, so **2¾ × 8 = 22**.

✎ **Practice E** Multiply the following:

a 8½ × 4	**b** 11½ × 8	**c** 19½ × 4	**d** 2¼ × 4
e 5½ × 8	**f** 9½ × 4	**g** 30½ × 4	**h** 3¼ × 4

a 34	**b 92**	**c 78**	**d 9**
e 44	**f 38**	**g 122**	**h 13**

2.2 HALVING

Halving is the opposite of doubling.

11 So half of **8** is **4**.

Half of **60** is **30**.

Half of **30** is **15**, because two 15's make 30 (or by halving 20 and 10).

✎ **Practice F** Find half of the following numbers:

a 10 **b** 6 **c** 40 **d** 14 **e** 50 **f** 90

a 5 b 3 c 20 d 7 e 25 f 45

12 Also half of **46** is **23** because you can halve the 4 and the 6 to get 2 and 3.

13 Half of **54** is **27** because 54 is 50 and 4.
And halving 50, 4 you get 25, 2,
which make 27.

14 Similarly half of **78** = half of 70 + half of 8 = 35 + 4 = **39**.

✎ **Practice G** Try some, halve these numbers:

a 36 **b** 28 **c** 52 **d** 18 **e** 34

f 86 **g** 56 **h** 32 **i** 62 **j** 98

a 18 b 14 c 26 d 9 e 17
f 43 g 28 h 16 i 31 j 49

SPLITTING NUMBERS

You can halve longer numbers easily by splitting them up.

15 To halve **178** you halve 100, 70 and 8 and add the results.

Half of 100 is 50,
half of 70 is 35
and half of 8 is 4.

So half of 178 is 50 + 35 + 4 = **89**.

Practice H Halve the following numbers. Try to do them in your head.

a 164	**b** 820	**c** 216	**d** 152	**e** 94	**f** 326
g 234	**h** 416	**i** 380	**j** 256	**k** 456	**l** 57

a 82	**b 410**	**c 108**	**d 76**	**e 47**	**f 163**
g 117	**h 208**	**i 190**	**j 128**	**k 228**	**l 28½**

DIVIDING BY 4, 8

Halving numbers is something which can also be repeated.
So if for example you halved a number and then halved again
you would be dividing the number by 4.

16 Divide **72** by **4**.

You halve 72 twice: half of 72 is 36, half of 36 is 18.
So **72 ÷ 4 = 18**.

17 Divide **104** by **8**.

Here you halve three times:
Half of 104 is 52, half of 52 is 26, half of 26 is 13.

So **104 ÷ 8 = 13**.

Practice I Use halving to do the following divisions.

Divide by 4:	**a** 56	**b** 68	**c** 84	**d** 180	**e** 244
Divide by 8:	**f** 120	**g** 440	**h** 248	**i** 216	**j** 44

a	**14**	**b**	**17**	**c**	**21**	**d**	**45**	**e**	**61**
f	**15**	**g**	**55**	**h**	**31**	**i**	**27**	**j**	**5½**

2.3 EXTENDING YOUR TABLES

18 Suppose that you want to find **18 × 3**.
You may think that since you know 9 × 3 = 27,
then 18 × 3 must be double this, which is **54**.

19 Similarly if you don't know **8 × 7**
but you do know that 4 × 7 = 28,
you can just double 28.
So **8 × 7 = 56**.

20 Find **6 × 14**.

Since you know that 6 × 7 = 42, it follows that **6 × 14 = 84**.

The following questions assume you know your tables up to 10 × 10, but if you don't know all these you should still be able to find your way to the answer.

Practice J Find the following:

a 16 × 7	**b** 18 × 6	**c** 14 × 7	**d** 12 × 9
e 4 × 14	**f** 6 × 16	**g** 7 × 18	**h** 9 × 14

a	**112**	**b**	**108**	**c**	**98**	**d**	**108**
e	**56**	**f**	**96**	**g**	**126**	**h**	**126**

21 Find **14 × 18**.

Halving 14 and 18 gives 7 and 9, and since 7 × 9 = 63 you double this twice. That means you double and double again.

You get 126 and 252, so **14 × 18 = 252**.

✎ **Practice K** Find the following:

a 16 × 18 **b** 14 × 16 **c** 12 × 18 **d** 16 × 12

---

a **288** **b** **224** **c** **216** **d** **192**

2.4 MULTIPLYING BY 5, 50, 25

The numbers **2** and **5** are closely related because 2 × 5 = 10 and 10 is a base number.

We can multiply by 5 by multiplying by 10 and halving the result.

22 Find **44 × 5**.

We find half of 440, which is 220. So 44 × 5 = **220**.

23 Find **87 × 5**.

Half of 870 is 435. So **87 × 5 = 435**.

24 Similarly **4.6 × 5** = half of 46 = **23**.

✎ **Practice L** Multiply the following:

a 68 × 5 **b** 42 × 5 **c** 36 × 5 **d** 426 × 5

e 8.6 × 5 **f** 5.4 × 5 **g** 4.68 × 5 **h** 0.66 × 5

a	**340**	**b**	**210**	**c**	**180**	**d**	**2130**
e	**43**	**f**	**27**	**g**	**23.4**	**h**	**3.3**

25 Find **27 × 50**.

We multiply 27 by 100, and halve the result. Half of 2700 is 1350.
So **27 × 50 = 1350**.

26 Similarly **5.2 × 50** = half of 520 = **260**.

27 Find **82 × 25**.
25 is half of half of 100, so to multiply a number by 25 we multiply it by 100 and halve twice.
So we find half of half of 8200, which is 2050. **82 × 25 = 2050**.

28 Similarly **6.8 × 25** = half of half of 680 = **170**.

Practice M Multiply the following:

a 46 × 50 **b** 864 × 50 **c** 72 × 25 **d** 85 × 25

e 86.8 × 50 **f** 4.2 × 50 **g** 34.56 × 50 **h** 2.8 × 25

a	**2300**	**b**	**43200**	**c**	**1800**	**d**	**2125**
e	**4340**	**f**	**210**	**g**	**1728**	**h**	**70**

2.5 DIVIDING BY 5, 50, 25

DIVIDING BY 5

29 **85 ÷ 5 = 17**.

For dividing by 5 we can double and then divide by 10.

So 85 is doubled to 170, and dividing by 10 gives **17**.

An alternative method with a different Sutra may be used here (*The Ultimate and Twice the Penultimate*). Since there are two fives in every ten, in the sum 85 ÷ 5 you may decide there are 16 5's in the 80 and therefore 17 5's in 85. In other words you would double the 8 and add 1 on.

30 **665 ÷ 5 = 133** since 665 doubled is 1330.

31 **73 ÷ 5 = 14.6**.
Similarly here double 73 is 146, and dividing by 10 gives **14.6**.

Practice N Divide by 5:

a 65	**b** 135	**c** 375	**d** 470	**e** 505
f 4005	**g** 1235	**h** 7070	**i** 885	**j** 49
k 52	**l** 22.2			

a 13	**b 27**	**c 75**	**d 94**	**e 101**
f 801	**g 247**	**h 1414**	**i 177**	**j 9.8**
k 10.4	**l 4.44**			

DIVIDING BY 50, 25

32 Find **750 ÷ 50**.

Since 50 is half of 100 dividing by 50 involves doubling and dividing by 100.

Doubling 750 gives 1500, and dividing this by 100 gives 15.
So **750 ÷ 50 = 15**.

Again the alternative formula *The Ultimate and Twice the Penultimate* tells us to double the 7 and add on the one extra 50, giving 15 again.

33 **54.32 ÷ 50 = 1.0864**.
Doubling 54.32 gives 108.64, and dividing by 100 gives 1.0864.

34 Find **425 ÷ 25**.

> 25 is a quarter of 100 so to divide by 25 we can double twice and divide by 100.

Doubling 425 gives 850, and doubling this gives 1700.
Dividing by 100 then gives us 17. So **425 ÷ 25 = 17**.

✐ **Practice O** Divide by 50:

a 650	**b** 1250	**c** 3300	**d** 8.8	**e** 44	**f** 77

Divide by 25:

g 225	**h** 550	**i** 44	**j** 137	**k** 6

---

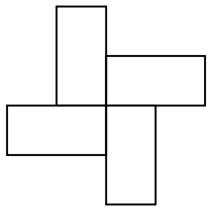
a 13	**b 25**	**c 66**	**d 0.176**	**e 0.88**	**f 1.54**
g 9	**h 22**	**i 1.76**	**j 5.48**	**k 0.24**	

Another application of doubling and halving is shown in Section 4.3

"The Sutras are very short; but, once one understands them and the modus operandi inculcated therein for their practical application, the whole thing becomes a sort of children's play and ceases to be a 'problem'."
From "Vedic Mathematics", Page 13.

LESSON 3
DIGIT SUMS

SUMMARY



3.1 Adding Digits – obtaining digit sums.
3.2 The Nine Point Circle – representing numbers around a circle.
3.3 Casting out Nines – to simplify finding digit sums.
3.4 Digit Sum Puzzles
3.5 The Digit Sum Check – using digit sums to check addition and multiplication sums.
3.6 The Vedic square – characteristics of the nine basic digits.
3.7 Patterns from the Vedic Square – using the Vedic Square to design patterns.
3.8 Number Nine

3.1 ADDING DIGITS

The word **digit** means a single figure number: the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0.
Sum means add.

So 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 are 1-digit numbers.

And numbers 10, 11, 12 . . . . up to 99, are 2-digit numbers.

The **digit sum** of a number is found by adding the digits in the number.

1 To find the digit sum of **17**, for example, you just add the 1 and 7.
1 + 7 = 8, so the digit sum of 17 is **8**.

2 And the digit sum of **123** is 6 because 1+2+3=**6**.

Digit sums can be very useful: for checking calculations (see Sections 3.5, 8.1), in divisibility testing, in finding square roots; and there is an algebraic form too (Section 11.5).

Practice A Find the digit sum of the following numbers:

NUMBER	DIGIT SUM
13	4
241	7
171	9
242	8
303	6
1213	7
900	9

Sometimes two steps are needed to find a digit sum.

The digit sum is found by adding the digits in a number,
and adding again if necessary.

3 So for the digit sum of **19** you add 1 + 9 = 10.
But since 10 is a 2-digit number you add again: 1+0 = **1**.
So for the digit sum of 19 you can write:

19 → 10 → 1

4 Similarly for **39** you get 39 → 12 → 3.
So the digit sum of 39 is **3**.

Practice B Find the digit sum of the following numbers:

NUMBER	DIGIT SUM
83	2
614	2
345	3
5555	2
78	6
2379	3
521832	3
999	9

This means that any number of any size can be reduced to a single digit: just add all the digits, and if you get a 2-figure number, add again.

3.2 THE NINE POINT CIRCLE

The sequence of whole numbers starts at 1 and increases by 1 each time:

1, 2, 3, 4, 5, 6, 7, 8, 9, **10**, 11, 12, 13, 14, 15, 16, 17, 18, 19, **20**, 21 . . . . .

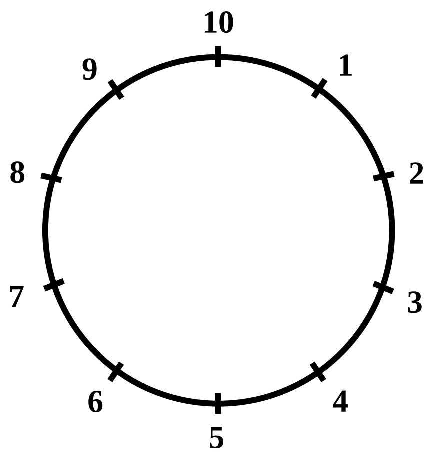
We are very familiar with the cycle of tens in our number system: 10, 20, 30 etc. and we have seen this illustrated neatly in the circle of ten points.

But if we take the digit sums of the counting numbers we get:

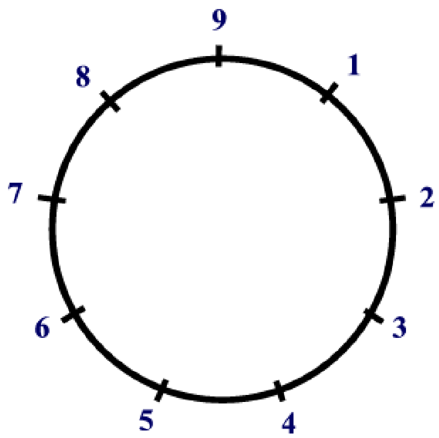
1, 2, 3, 4, 5, 6, 7, 8, 9, **10**, 11, 12, 13, 14, 15, 16, 17, 18, 19, **20**, 21 . . . . .
1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3 . . . . .

and here we see another cycle contained within the cycle of ten: a cycle of nine.

We therefore also need to have a circle of nine points, and this has many uses, as we will see.



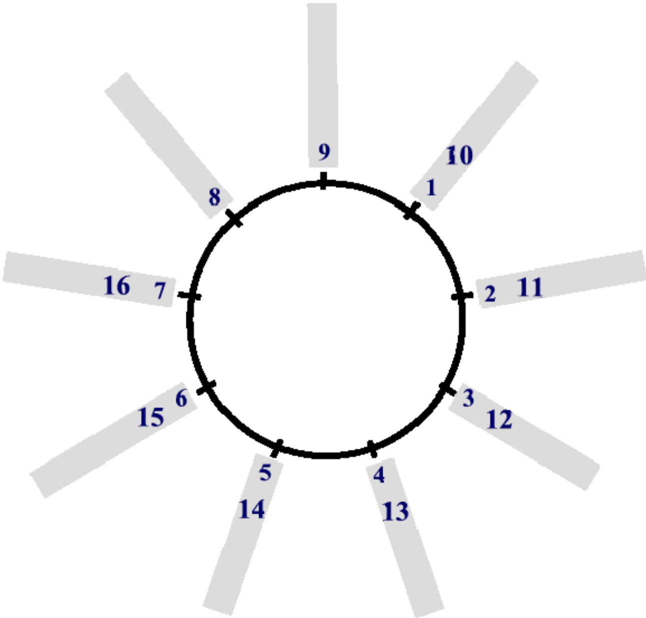
10-point circle



9-point circle

3.3 CASTING OUT NINES

The 9-point circle is a circle whose edge is divided into 9 equal parts and as with the ten-point circle you can continue numbering round the circle as shown below.



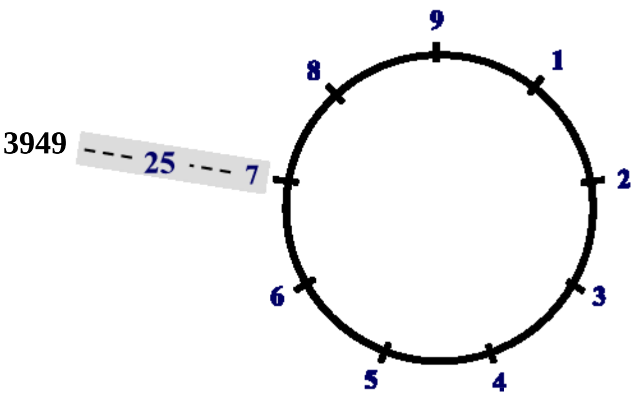
Notice here that on any branch the digit sum of every number is the same. For example on the 1-branch we get 1, 10, 19, 28 etc. all of whose digit sums are 1.

This shows that adding 9 to a number does not affect its digit sum.
And in fact it follows that adding any number of 9's, or subtracting any number of 9's will not affect the digit sum of a number.

> Adding 9 to a number does not affect its digit sum:
>
> so 4, 40, 49, 94, 949 all have a digit sum of 4 for example.

5 To find the digit sum of **3949** you can **cast out the nines** and just add up the 3 and 4.
So the digit sum is **7**.

Or using the longer method you add all the digits: 3+9+4+9 → 25 → 7 again.



Practice C Find the digit sums of the numbers below. Use casting out 9’s.

NUMBER	DIGIT SUM
39	3
93	3
993	3
9993	3
9329	5
941992	7
79896	3

There is another way of **casting out the nines** from a number when you are finding its digit sum:

Any group of figures in a number **that add up to 9** can be "cast out".

6 To find the digit sum of **24701** you see that you have 2 and 7 which add up to 9 and can therefore be cast out.
This leaves only 4 and 1 which add up to 5.
So the digit sum of 24701 is **5**.

7 Similarly with **21035** you see that 1, 3 and 5 add up to 9 and so can be cast out.
This leaves only 2 and so this is the answer.
The digit sum of 21035 is **2**.

Practice D Use casting out 9’s to find the digit sums of the numbers below.

NUMBER	DIGIT SUM
465	6
274	4
3335	5
6193	1
2532	3
819	9 or 0
723	3

NUMBER	DIGIT SUM
2346	6
16271	8
9653	5
36247	4
215841	3
7152	6
9821736	9 or 0

Casting out of 9’s and digits totalling 9 comes under the Sutra *When the Samuccaya is the Same it is Zero*. So in 465, as 4 and 5 total nine, they are cast out and the digit sum is 6: *when the total is the same (as 9) it is zero (can be cast out)*. Cancelling a common factor in a fraction is another example.

3.4 DIGIT SUM PUZZLES

Some simple problems can be given here involving digit sums.

8 The digit sum of a 2-figure number is 8 and the figures are the same, what is the number?
This is clearly **44**.

9 The digit sum of a 2-figure number is 9 and the first figure is twice the second, what is it?
This must be **63**.

10 Give three 2-digit numbers that have a digit sum of 3.
12, 21, 30 . . .

✐ **Practice E** In all of the following puzzles the answer is a 2-figure number.
Some have more than one answer.
You are given the digit sum of the answer and another fact.

DIGIT SUM	OTHER FACT	NUMBER OF ANSWERS	ANSWER(S)
5	difference between the figures is 3	2	14 or 41
6	the figures are the same	1	33
6	first figure is double the second	1	42
7	difference between the figures is 3	2	25, 52
7	one figure is a 4	2	34, 43
6	both figures are odd	3	15, 51, 33
5	the figures are consecutive*	2	23, 32
9	the figures are consecutive*	2	45, 54
3	one figure is double the other	2	12, 21
8	the answer is below 20	1	17
1	number is less than 40	5	10, 19, 28, 37
1	the first figure is a 2	1	28

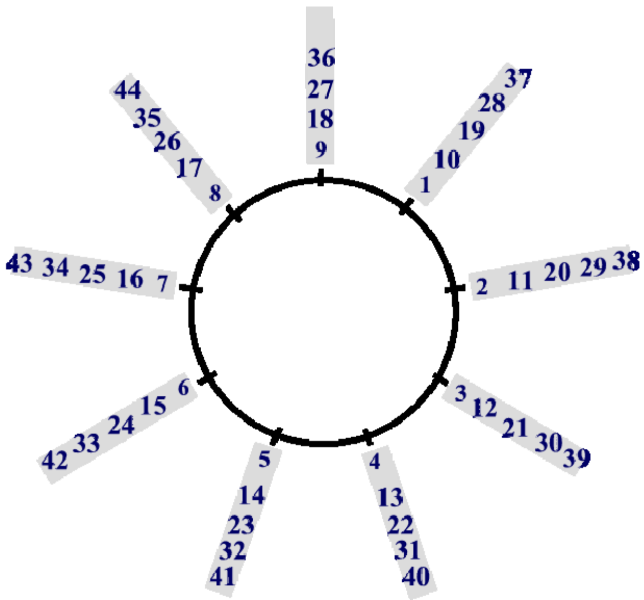
* **Consecutive** means one after the other. E.g. 6 and 7 are consecutive (or 7 and 6).

MORE DIGIT SUM PUZZLES

Harder digit sum problems can be given.

Below is the 9-point circle again but numbered up to 44.
Note that the numbers on each branch have the same digit sum. For example all the numbers on the 3-branch have a digit sum of 3.

> **11** A 2-figure number has a digit sum of 5 and the figures are the same. What is the number?
>
> 5 is an odd number but looking at the 9-point circle we see that 14, which is also on the 5-branch can be split into 7+7. So the number must be **77**.



✐ **Practice F** In the puzzles below you will need to choose the right branch and then select the right answer from the numbers on that branch.
All answers are 2-figure numbers.

DIGIT SUM	OTHER FACT	ANSWER
5	number is between 20 and 30	23
8	answer ends in 5	35
7	first figure is 2	25
2	figures differ by 7	29, 92

1	answer is in the 7 × table	28
3	first figure is 3 times the second	93
4	number is in the 5 × table	40
6	figures are the same	33
8	last figure is 3 times the first	26
5	number is in the 8 × table	32
9	ends in 7	27
3	both figures are odd	57, 75, 39, 93

3.5 THE DIGIT SUM CHECK

You can use digit sums to check that answers are right.

12 Find **32 + 12** and check the answer using digit sums.

$$\begin{array}{rl} 32 & \\ \underline{12} & + \\ \underline{44} & \end{array} \qquad \begin{array}{rl} 5 & \\ \underline{3} & + \\ \underline{8} & \end{array}$$

You get 44 for the answer to the sum.
Then the digit sum of 32 is 5 (3+2=5) and the digit sum of 12 is 3.
The sum (the total) of the digit sums is 5+3=8. If the sum has been done correctly, the digit sum of the answer should also be 8.
44→8; so according to this check the answer is probably correct.

So there are four steps:

1. Do the sum
2. Write down the digit sums of the numbers being added
3. Add the digit sums
4. Check the two answers are the same in digit sums

13 Add **365** and **208** and check the answer.

$$\begin{array}{rl} 365 & \\ \underline{208} & + \\ \underline{573} & \\ {}_{1} & \end{array} \qquad \begin{array}{rl} 5 & \\ \underline{1} & + \\ \underline{6} & \end{array}$$

1. We get 573 for the answer.
2. We find the digit sums of 365, 208 are 5, 1.
3. Adding 5 and 1 gives 6.
4. 573=6 in digit sums, which confirms the answer.

Practice G Add the following and check your answers using the digit sums:

a	66 77 +	**b**	57 29 +	**c**	94 58 +	**d**	304 271 +	**e**	787 176 +
f	389 55 +	**g**	5131 676 +	**h**	456 209 +	**i**	5555 7777 +		

a 143 **3+5=8**	**b 86** **3+2=5**	**c 152** **4+4=8**	**d 575** **7+1=8**	**e 963** **4+5=9**
f 444 **2+1=3**	**g 5807** **1+1=2**	**h 665** **6+2=8**	**i 13332** **2+1=3**	

Here is another example of a digit sum check.

14 Add **77** and **124** and check.

77	5	Here, when we find 5+7 we get 12,
124 +	7 +	but 12 = 3 in digit sums.
201	3	So this confirms the answer.

Practice H Add the following and check your answers using the digit sums:

a	35 47 +	**b**	56 27 +	**c**	35 59 +	**d**	52 24 +	**e**	456 333 +	**f**	188 277 +
g	78 87 +	**h**	66 48 +	**i**	555 77 +	**j**	823 37 +	**k**	3760 481 +		

a 82 **8+2=1**	**b 83** **2+9=2**	**c 94** **8+5=4**	**d 76** **7+6=4**	**e 789** **6+9=6**	**f 465** **8+7=6**
g 165 **6+6=3**	**h 114** **3+3=6**	**i 632** **6+5=2**	**j 860** **4+1=5**	**k 4241** **7+4=2**	

The Vedic formula *The Product of the Sum is the Sum of the Products* applies for all the digit sum checks. For addition it would be *The Total of the Digit Sums is the Digit Sum of the Total*. The formula has many other applications (see Reference 3), for example in finding areas of composite shapes (*The Area of the Whole is the Sum of the Areas*).

CAUTION!

Check the following sum:

$$\begin{array}{r} 279 \\ \underline{121} \; + \\ \underline{490} \end{array}$$

The check is:

$$\begin{array}{r} 9 \\ \underline{4} \; + \\ \underline{4} \end{array}$$

which confirms the answer.

However if you check the addition of the original sum you will find that it is incorrect!
This shows that the digit sum method does not always find an error. It usually works but not always.

We will be meeting other checking devices later on.

MULTIPLICATION CHECK

Multiplying numbers, for example 38 × 3, is a straightforward process. You set the sum out as shown below, and multiply each figure in 38 by 3, starting at the right:

15

Sum:

$$\begin{array}{r} 3\;\;8 \\ \underline{\quad 3} \times \\ \underline{1\;\;1\;\;4} \\ {\scriptstyle 2\;\;} \end{array}$$

Check:

$$\begin{array}{r} 2 \\ \underline{3} \times \\ \underline{6} \end{array}$$

The digit sum check has also been carried out above. The digit sums of the numbers being multiplied are 2 and 3, and when these are **multiplied** you get 6. Since the digit sum of the answer, 114, is also 6 this shows you that the answer is probably correct.

16

$$\begin{array}{r} 6\;\;2 \\ \underline{\quad 4} \times \\ \underline{2\;\;4\;\;8} \end{array}$$

check:

$$\begin{array}{r} 8 \\ \underline{4} \times \\ \underline{5} \end{array}$$

(since 8×4=32 and 3+2=5)

The check here confirms the answer, since the digit sum of 248 is the same as the digit sum of 8×4.

17

$$\begin{array}{r} 3\;\;8\;\;3\;\;9 \\ \underline{\qquad\quad 6} \times \\ \underline{2\;\;3\;\;0\;\;3\;\;4} \\ {\scriptstyle 5\;\;2\;\;5\;\;\;} \end{array}$$

Check:

$$\begin{array}{r} 5 \\ \underline{6} \times \\ \underline{3} \end{array}$$

For the check you get the digit sum of 3839, which is 5 and find that 5 × 6 → 3.
The digit sum of 23034 is 3, so the answer is confirmed.

✐ **Practice I** Multiply the following numbers and check each one using the digit sums:

a 88 × 8 **b** 32 × 3 **c** 73 × 4 **d** 717 × 6

e 234 × 5 **f** 533 × 2 **g** 3115 × 3 **h** 142857×7

---

a 704 (2) **b 96 (6)** **c 292 (4)** **d 4302 (9)**
e 1170 (9) **f 1066 (4)** **g 9345 (3)** **h 999999 (9)**

3.6 THE VEDIC SQUARE

The multiplication table below has many interesting patterns and properties.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

We make the **Vedic Square** by replacing every number in the table above by its digit sum as shown below.

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

Each of the numbers 1 to 9 has its own pattern in the Vedic Square.

> **18** To draw the pattern for the number One, for example, we colour in every square that has a "1" in it.
>
> Alternatively, we can put a dot in the center of each square with a "1" in it and join the dots to make a pleasing pattern.

Practice J Draw the patterns for the nine numbers using the Squares below.

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

3.7 PATTERNS FROM THE VEDIC SQUARE

The Vedic Square is also useful in the design of patterns. Below is the Square again with the nine rows labeled A to I.

A	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
B	**2**	**4**	**6**	**8**	**1**	**3**	**5**	**7**	**9**
C	**3**	**6**	**9**	**3**	**6**	**9**	**3**	**6**	**9**
D	**4**	**8**	**3**	**7**	**2**	**6**	**1**	**5**	**9**
E	**5**	**1**	**6**	**2**	**7**	**3**	**8**	**4**	**9**
F	**6**	**3**	**9**	**6**	**3**	**9**	**6**	**3**	**9**
G	**7**	**5**	**3**	**1**	**8**	**6**	**4**	**2**	**9**
H	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**	**9**
I	**9**	**9**	**9**	**9**	**9**	**9**	**9**	**9**	**9**

To design a pattern we choose a line of the Square, a starting point in that line and an angle of rotation.

19 Suppose we choose line **D (4 8 3 7 2 6 1 5 9**) and start at the beginning. We also choose a rotation of, say, **90°** anticlockwise.

Take a sheet of graph paper and mark a point near the bottom left corner (you will need 2cm to the left of this).

We always start by moving to the right and the numbers in the row we have chosen tell us how many centimetres to move. (It is advisable to use a pencil for this at first)

So now we can draw the design: first we draw a line **4**cm to the right,
then turn **90°** anticlockwise (to the left) and draw a line **8**cm up.
then turn **90°** anticlockwise and draw a line **3**cm long,
then turn **90°** anticlockwise and draw a line **7**cm long,
and so on.

When you come to the end of the row of numbers you start again at the beginning of that row. Eventually you will return to your starting point and the design is complete.

Practice K

a Draw the pattern described above.

b Try another design using row D again (starting at the beginning) but now the rotation angle can be **60°** and so triangular spotty paper can be used instead of graph paper:

(With the long side of your sheet at the bottom mark a dot near the middle of the bottom line.
We start moving to the right again **4**cm.
Then we turn **60°** to the left and draw a line **8**cm long.
Then we turn **60°** to the left and draw a line **3**cm long.
And so on, the same as previously but with a turn of 60° instead of 90°.)

c On another sheet of triangular spotty paper mark a point in the middle, and two rows down from the top of the page. Choose row E this time (starting at the beginning) and a rotation of 120° anticlockwise.
Draw the pattern for this.

(You can also use the columns and diagonals in the Vedic Square as well as the rows, or a combination of them)

The diagram that appears at the beginning of each chapter of this book is formed by using the Vedic Square in this way.

3.8 NUMBER NINE

In our number system the number nine is the largest digit.
The number nine also has many other remarkable properties which make it extremely useful.
You have already seen that it can be used in finding digit sums, and that the digit sum of a number is unchanged if 9 is added to it or subtracted from it.

Now look at the 9-times table:

9 × 1 = 9
9 × 2 = 1 8
9 × 3 = 2 7
9 × 4 = 3 6
9 × 5 = 4 5
9 × 6 = 5 4
9 × 7 = 6 3
9 × 8 = 7 2
9 × 9 = 8 1
9 × 10 = 9 0
9 × 11 = 9 9
9 × 12 =10 8

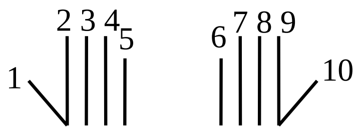
If you look at the answers you will see that in every case the digit sum is 9.

You may also see that if you read the answers as two columns the left column goes up (1, 2, 3, . . .) and the right column goes down (9, 8, 7, . . .).

This makes it easy to get the answers in the 9 times table.

It is also possible to use your fingers to multiply by nine.

Suppose the fingers of your hands are numbered as shown below:



To multiply, say, **4** by **9**, simply fold down the 4th finger.
You will find **3** fingers to the left of the folded finger and **6** fingers to the right.
So **4 × 9 = 36**.
And so on.

See also Russian Peasant Multiplication on Page 69.

LESSON 4
LEFT TO RIGHT

SUMMARY

4.1 Addition: Left to Right
4.2 Multiplication: Left to Right
4.3 Doubling and Halving – converting harder products to easier ones**.**
4.4 Subtraction: Left to Right
4.5 Checking Subtraction Sums – using digit sums.
4.6 More Subtractions – subtracting longer numbers, left to right.

4.1 ADDITION: LEFT TO RIGHT

It is common to do calculations starting at the right and working towards the left.
This is however not always the best way.
Calculating from left to right is often easier, quicker and more useful.

The reason for this is that numbers are written and spoken from left to right.
Also in calculations we often only want the first one, two or three figures of an answer, and starting on the right we would have to do the whole sum and so do a lot of useless work.
This also introduces flexibility into our work, which is a theme of the Vedic system.

In this lesson all the calculations will be done mentally: we write down only the answer.

1 Given the addition sum

$$\begin{array}{cc} 2 & 3 \\ \underline{4} & \underline{5} \; + \\ \hline \end{array}$$

there is no difficulty in finding the answer.
From left to right the columns add up to **6** and **8**.
So the answer is **68**.

2 But in the sum

$$\begin{array}{cc} 4 & 5 \\ \underline{3} & \underline{8} \; + \\ \hline \end{array}$$

the totals we get are **7** and **13**, and 13 is a 2-figure number.
The answer is not 713: the 1 in the 13 must be carried over and added to the 7.
This gives **83** as the answer.

This is easy enough to do mentally, we add the first column and increase this by 1 if there is a carry coming over from the second column. Then we tag the last figure of the second column onto this.

3

6 6	5 5	8 4	5 6
2 8 +	3 5 +	5 8 +	9 6 +
9 4	9 0	1 4 2	1 5 2
8, 14 = 94	8, 10 = 90	13, 12 = 142	14, 12 = 152

We use the curved lines to show which figures are to be combined.

In every case the tens figure in the right-hand column total is carried over to the left-hand column total.

Practice A Add the following mentally from left to right:

a 5 6 6 7 +	**b** 8 8 3 3 +	**c** 4 5 6 7 +	**d** 5 4 6 4 +
e 3 9 4 9 +	**f** 2 7 5 6 +	**g** 7 7 8 8 +	**h** 6 3 7 4 +

a 123 **b** 121 **c** 112 **d** 118
e 88 **f** 83 **g** 165 **h** 137

4 **187 + 446 = 633.**

1 8 7
4 4 6 +

Here the three column totals are **5**, **12** and **13** so two carries are needed.

The 1 in the 12 will be carried over to the 5 making it a 6.
So when the 5 and the 12 are combined we get **62**.
The 1 in the 13 is then carried over and added onto the 2 in 62, making it 63.
So combining 62 and 13 gives the answer, **633**.

It is important to get the idea of doing this mentally from left to right:

First we think of 5, the first total.

Then we have 5, 12 **which we mentally combine into 62.**

Hold this 62 in the mind, and with the third total we have 62, 13

which becomes 633.

5

$$\begin{array}{r} 7\;7\;7 \\ \underline{4\;5\;6} \; + \\ \hline \end{array}$$

The first two columns give 11,12 which becomes 122.

Then with the third column we have 122,13 which is **1233**.

6

$$\begin{array}{r} 5\;5\;5\;5 \\ 3\;1\;3 \\ \underline{6\;2\;4} \; + \\ \hline \end{array}$$

Starting at the left we have 5,14 = 64.

Then 64,8 = 648 (**there is no carry here** as 8 is a single figure).

Finally 648,12 = **6492**.

✐ **Practice B** Add the following sums mentally from left to right:

a $\begin{array}{r} 3\;6\;3 \\ \underline{4\;5\;6} \; + \\ \hline \end{array}$ **b** $\begin{array}{r} 8\;1\;9 \\ \underline{9\;1\;8} \; + \\ \hline \end{array}$ **c** $\begin{array}{r} 7\;7\;7 \\ \underline{4\;4\;4} \; + \\ \hline \end{array}$ **d** $\begin{array}{r} 7\;3\;7 \\ \underline{1\;3\;9} \; + \\ \hline \end{array}$

e $\begin{array}{r} 3\;4\;5 \\ \underline{9\;3\;7} \; + \\ \hline \end{array}$ **f** $\begin{array}{r} 1\;3\;6\;9 \\ \underline{3\;8\;8\;3} \; + \\ \hline \end{array}$ **g** $\begin{array}{r} 9\;6\;3\;1 \\ \underline{8\;7\;0\;9} \; + \\ \hline \end{array}$ **h** $\begin{array}{r} 4\;4\;4\;4 \\ 4\;8\;3\;8 \\ \underline{5\;5\;5} \; + \\ \hline \end{array}$

---

a	**819**	**b**	**1737**	**c**	**1221**	**d**	**876**
e	**1282**	**f**	**5252**	**g**	**18340**	**h**	**9837**

In all these sums the numbers are held in the mind (*On the Flag*) and built up digit by digit until the answer is complete.

Mental mathematics obviously relies more on the memory than conventional methods where every step is written down. Young children have very good memories and mental mathematics helps to strengthen the memory further. (This means that Vedic Mathematics is good for adults too, whose memory may not be so good.) This also gives confidence and teaches self-reliance, showing that we do not need pencil and paper or calculator for every sum but can find an answer without any external help.

4.2 MULTIPLICATION: LEFT TO RIGHT

7 Suppose we have the sum:

$$\begin{array}{r} 2\ 3\ 7 \\ \underline{2} \times \\ \hline \end{array}$$

We multiply each of the figures in 237 by 2 starting at the left.
The answers we get are **4, 6, 14**.

Since the 14 has two figures the 1 must be carried leftwards to the 6.
So 4, 6,14 = **474**.

Again we build up the answer mentally from the left: first 4, then 4,6=46,
then 4, 6,14 = 474.

8 **236 × 7 = 1652.**

$$\begin{array}{r} 2\ 3\ 6 \\ \underline{7} \times \\ \hline \end{array}$$

First we have 14,
then 1 4,2 1 = 161,
then 161,4 2 = **1652**.

9 For **73 × 7** we get 4 9,2 1 = **511**. (because 49+2 = 51)

Practice C Multiply the following from left to right:

a 27 × 3 **b** 76 × 6 **c** 26 × 6 **d** 72 × 7 **e** 78 × 9 **f** 83 × 3

g 642 × 4 **h** 256 × 3 **i** 741 × 3 **j** 223 × 9

k 1059 × 7 **l** 8631 × 4 **m** 5432 × 8 **n** 4097 × 7

---

a	**81**	**b**	**456**	**c**	**156**	**d**	**504**	**e**	**702**	**f**	**249**
g	**2568**	**h**	**768**	**i**	**2223**	**j**	**2007**				
k	**7413**	**l**	**34524**	**m**	**43456**	**n**	**28679**				

Left to right multiplication is continued in Lesson 11.

4.3 DOUBLING AND HALVING

We can use doubling and halving together sometimes.

10 Find **35 × 22**.

We can use doubling and halving in this sum to get a much easier sum.
We double 35 and halve 22 and this gives us 70 × 11 which has the same answer as 35 × 22.

So **35 × 22** = 70 × 11 = **770**.

11 Find **35 × 64**.

Doubling and halving gives us 70 × 32.
So we can use *On the Flag* to find 32 × 7 and put a 0 on the end.

So **35 × 64** = 70 × 32 = **2240**.

✎ **Practice D** Multiply the following:

a 15 × 18	**b** 15 × 24	**c** 46 × 15
d 82 × 35	**e** 66 × 15	**f** 124 × 45
g 15 × 54	**h** 55 × 16	**i** 75 × 18
j 446 × 15	**k** 132 × 35	**l** 85 × 18
m 16 × 4$\frac{1}{2}$	**n** 24 × 3$\frac{1}{2}$	**o** £4.50 × 32

---

a	**270**	**b**	**360**	**c**	**690**
d	**2870**	**e**	**990**	**f**	**5580**
g	**810**	**h**	**880**	**i**	**1350**
j	**6690**	**k**	**4620**	**l**	**1530**
m	**72**	**n**	**84**	**o**	**£144**

"People who have practical knowledge of the application of the Sutras need not go in or the theory side of it at all. The actual work can be done. Tremendous time is saved. It is a saving not merely of time and energy and money, but more than all, I feel, it is saving the child from tears that very often accompany the study of mathematics.".
From "Vedic Metaphysics", Page 170.

4.4 SUBTRACTION: LEFT TO RIGHT

In this section we show a very easy method of subtracting numbers from left to right that you have probably not seen before.

12 Find **63 – 37**.

You look in the left-hand column and subtract. You get 3. But before writing it down you look in the next column.

$$\begin{array}{r} 6\ \ 3 \\ -\ \underline{3\ \ 7} \\ \underline{} \end{array}$$

Seeing that you cannot take 7 from 3 you therefore put down 2 rather than 3 and put the other one as shown:

$$\begin{array}{r} 6\ {}^{1}3 \\ -\ \underline{3\ \ 7} \\ \underline{2\ \ } \end{array}$$

Then the final step is just 13 – 7 = 6:

$$\begin{array}{r} 6\ {}^{1}3 \\ -\ \underline{3\ \ 7} \\ \underline{2\ \ 6} \end{array}$$

So **63 – 37 = 26**.

So in this method you start at the left, subtract, and write this down if the subtraction in the next column can be done.
If it cannot be done you put down one less and carry 1, and then subtract in the second column.

✎ **Practice E** Try some of these:

a $\begin{array}{r} 6\ 2 \\ -\ \underline{4\ 7} \\ \underline{} \end{array}$ **b** $\begin{array}{r} 7\ 5 \\ -\ \underline{2\ 8} \\ \underline{} \end{array}$ **c** $\begin{array}{r} 5\ 1 \\ -\ \underline{1\ 5} \\ \underline{} \end{array}$ **d** $\begin{array}{r} 6\ 7 \\ -\ \underline{3\ 8} \\ \underline{} \end{array}$

e $\begin{array}{r} 4\ 6 \\ -\ \underline{2\ 5} \\ \underline{} \end{array}$ **f** $\begin{array}{r} 6\ 5 \\ -\ \underline{3\ 7} \\ \underline{} \end{array}$ **g** $\begin{array}{r} 9\ 0 \\ -\ \underline{6\ 2} \\ \underline{} \end{array}$ **h** $\begin{array}{r} 8\ 2 \\ -\ \underline{3\ 8} \\ \underline{} \end{array}$

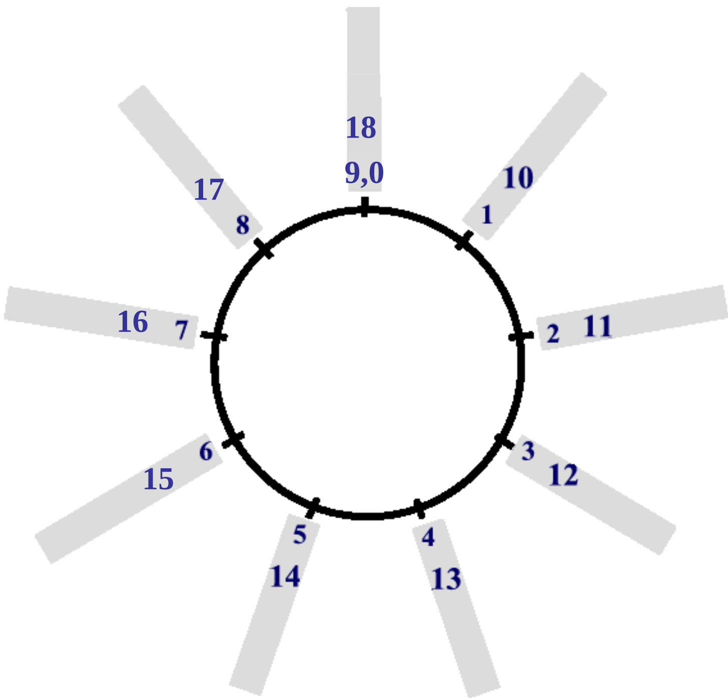
---

a 15 b 47 c 36 d 29
e 21 f 28 g 28 h 44

4.5 CHECKING SUBTRACTION SUMS

Recall the 9-point circle and that 9's in a number can be cast out when finding digit sums.
This means that **in digit sums 9 and 0 are the same**.
You will see them together in the circle below.
You will also remember that it is sometimes useful to use the numbers on the second ring, which are 9 more than those in the inside ring.

Alternatively we can count backwards around the circle: . . 3, 2, 1, 0.



13 Find **69 – 23** and check the answer.

$$\begin{array}{r} 69 \\ -\ \underline{23} \\ \underline{46} \end{array} \qquad \begin{array}{r} 6 \\ -\ \underline{5} \\ \underline{1} \end{array}$$

The answer is 46.
The digit sums of 69 and 23 are 6 and 5.
Then 6 – 5 = 1, which is also the digit sum of 46, so the answer is confirmed.

Note that **you subtract the digit sums**, because this is a subtraction sum.

14

$$\begin{array}{r} 74 \\ -\ \underline{58} \\ \underline{16} \end{array} \qquad \begin{array}{r} 2 \\ -\ \underline{4} \\ \underline{7} \end{array}$$

Here we have 2 – 4 in the digit sum check so we simply add 9 to the upper figure (the 2) and continue: 11 – 4 = 7, which is also the digit sum of 16, so the answer is confirmed. ☞

15

$$\begin{array}{rr} & 5\ 6 \\ - & \underline{2\ 9} \\ & \underline{2\ 7} \end{array} \qquad \begin{array}{rr} & 2 \\ - & \underline{2} \\ & \underline{0} \end{array}$$

In this example, the digit sum of both 56 and 29 is 2 and 2 – 2 = 0.
The digit sum of 27 is 9, but we have already seen that 9 and 0 are the same as digit sums, so the answer is confirmed.

✐ **Practice F** Check your answers to Practice E by using the digit sum check.

---

a	**8-2=6**	**b**	**3-1=2**	**c**	**6-6=9**	**d**	**4-2=2**
e	**1-7=3**	**f**	**2-1=1**	**g**	**9-8=1**	**h**	**1-2=8**

4.6 MORE SUBTRACTIONS

This subtraction method can be extended to the subtraction of numbers of any size.

16 Find **35567 – 11828**.

You set the sum out as normal:
Then starting on the left you subtract in each column.
3 – 1 = 2, but before you put 2 down you check that in the next column the top number is larger.
In this case 5 is larger than 1 so you put 2 down.

$$\begin{array}{rl} & 3\ 5\ 5\ 6\ 7 \\ - & \underline{1\ 1\ 8\ 2\ 8} \\ & \underline{\mathbf{2}\ \ \ \ \ \ \ \ \ } \end{array}$$

In the next column you have 5 – 1 = 4, but looking in the third column you see the top number is not larger than the bottom (5 is less than 8) so instead of putting 4 down you put 3 and the other 1 is placed *On the Flag*, as shown so that the 5 becomes 15.

$$\begin{array}{rl} & 3\ 5\ {}^{1}5\ 6\ 7 \\ - & \underline{1\ 1\ 8\ 2\ 8} \\ & \underline{\mathbf{2\ 3}\ \ \ \ \ \ } \end{array}$$

So now you have 15 – 8 = 7. Checking in the next column you can put this down because 6 is greater than 2.
In the fourth column you have 6 – 2 = 4, but looking at the next column (7 is smaller than 8) you put down only 3 and put the other one *On the Flag* with the 7 as shown.

$$\begin{array}{rl} & 3\ 5\ {}^{1}5\ 6\ {}^{1}7 \\ - & \underline{1\ 1\ 8\ 2\ 8} \\ & \underline{\mathbf{2\ 3\ 7\ 3}\ \_} \end{array}$$

Finally 17 – 8 = **9**:

$$\begin{array}{rl} & 3\ 5\ {}^{1}5\ 6\ {}^{1}7 \\ - & \underline{1\ 1\ 8\ 2\ 8} \\ & \underline{\mathbf{2\ 3\ 7\ 3\ 9}} \end{array}$$

You subtract in each column starting on the left, but before you put an answer down you look in the next column.
If the top is greater than the bottom you put the figure down.
If not, you reduce the figure by 1, put that down and give the other 1 to the smaller number at the top of the next column.
If the figures are the same you look at the next column to decide whether to reduce or not.

✐ **Practice G** Subtract the following from left to right (check your answer):

a 4 4 4 − 1 8 3	**b** 6 3 − 2 8	**c** 8 1 3 − 3 4 5	**d** 6 9 5 − 3 6 8
e 5 1 − 3 8	**f** 3 4 5 6 − 2 8 1	**g** 7 1 1 7 − 1 7 7 1	**h** 8 0 0 8 − 3 8 3 9
i 6 3 6 3 − 3 3 8 8	**j** 5 1 0 1 5 − 2 7 9 8 6	**k** 1 4 2 8 5 − 7 1 4 8	**l** 9 6 3 0 3 6 9 − 3 6 9 0 9 6 3

a 261 **b** 35 **c** 468 **d** 327
e 13 **f** 3175 **g** 5346 **h** 4169
i 2975 **j** 23029 **k** 7137 **l** 5939406

ADVANTAGES OF LEFT TO RIGHT CALCULATIONS

There are many advantages to left to right calculation as we pronounce and write numbers from left to right. Also, sometimes we only need the first two or three significant figures and would waste a lot of time and effort if we found all the figures of a long sum by starting at the right. Division is always done from the left, so all calculations can be done left to right, which means we can combine operations and, for example, find the square root of the sum of two squares in one line (see Manual 2). For finding square roots, trig functions and so on there is no right-hand figure to start from anyway, so there is no option but to start at the left (see Manual 3).

LESSON 5
ALL FROM 9 AND THE LAST FROM 10

SUMMARY

5.1 Applying the Formula
5.2 Subtraction – of numbers from a base.
5.3 Money – an application of subtracting numbers from a base.

5.1 APPLYING THE FORMULA

All From 9 and the Last From 10 is a useful formula, as we will see.

1 If you apply *All From 9 and the Last From 10* to **876**

8	**7**	**6**
↓	↓	↓
1	**2**	**4**

you get **124**,
because you take 8 and 7 from 9 and the 6 from 10.

2

Similarly	3883,	64,	98,	6,	10905,
become	<u>**6117**</u>,	<u>**36**</u>,	<u>**02**</u>,	<u>**4**</u>,	<u>**89095**</u>.

✐ **Practice A** Apply *All from 9 and the Last from 10* to the following:

a 444 **b** 675 **c** 2468 **d** 18276

e 8998 **f** 9888 **g** 1020304 **h** 7

---

a	**556**	**b**	**325**	**c**	**7532**	**d**	**81724**
e	**1002**	**f**	**112**	**g**	**8979696**	**h**	**3**

3 Applying the formula to **470** or any number that ends in 0 we need to be a bit careful.

Ignore the 0 and take 7 as the last figure: apply the formula to 47 and simply put the 0 on afterwards. So you get **530**.

4 Similarly with **28160** you get **71840** (just apply the formula to 2816),
with **4073100** you get **5926900** (just apply the formula to 40731).

✏ **Practice B** Apply the formula to these numbers:

a 3570 **b** 920 **c** 1234560 **d** 3300

---

a 6430 b 80 c 8765440 d 6700

5.2 SUBTRACTION

If you look carefully at the pairs of numbers in Example 2 you may notice that in every case the total of the two numbers is a base number: 10, 100, 1000 etc.

This gives us an easy way to subtract from **base numbers like 10, 100, 1000 . . .**

The formula *All From 9 and the Last From 10*
subtracts numbers from the next highest base number.

5 **1000 – 864 = 136** Just apply *All From 9 and the Last From 10* to 864.
8 from 9 is **1**, 6 from 9 is **3**, 4 from 10 is **6**.

1000 – 307 = 693,

10000 – 6523 = 3477,

100 – 76 = 24,

1000 – 580 = 420. Remember: apply the formula just to 58 here.

In every case here the number is being subtracted from its next highest base number.

Practice C Subtract the following:

a 1000 – 481	**b** 1000 – 309	**c** 1000 – 892	**d** 1000 – 976
e 100 – 78	**f** 100 – 33	**g** 10000 – 8877	**h** 10000 – 9876
i 1000 – 808	**j** 1000 – 710	**k** 10000 – 6300	

---

a 519	**b 691**	**c 108**	**d 24**
e 22	**f 67**	**g 1123**	**h 124**
i 192	**j 290**	**k 3700**	

ADDING ZEROS

In all of the above sums you may have noticed that the number of zeros in the first number is the same as the number of figures in the number being subtracted.
For example 1000–481 has three zeros and 481 has three figures.

6 Suppose you had **1000 – 43**.

This has three zeros, but 43 is only a 2-figure number.

You can solve this by writing **1000 – 043 = 957**.

You put the extra zero in front of 43, and then apply the formula to 043.

7 **10000 – 58**.

Here we need to add two zeros: **10000 – 0058 = 9942**.

In the following exercise you will need to insert zeros, but you can do that mentally.

Practice D Subtract the following:

a 1000 – 86	**b** 1000 – 93	**c** 1000 – 35	**d** 10000 – 678
e 10000 – 353	**f** 10000 – 177	**g** 10000 – 62	**h** 10000 – 85
i 1000 – 8	**j** 10000 – 3		

a	**914**	**b**	**907**	**c**	**965**	**d**	**9322**
e	**9647**	**f**	**9823**	**g**	**9938**	**h**	**9915**
i	**992**	**j**	**9997**				

ONE LESS

8 Now let's look at **600 – 77**.

You have 600 instead of 100.
In fact the 77 will come off one of those six hundreds, so that 500 will be left.

So **600 – 77 = 523**
The 6 is reduced by one to 5, and the *All from 9* . . . formula is applied to 77 to give 23.

9 **5000 – 123 = 4877**. The 5 is reduced by one to 4, and the formula converts 123 to 877.

Practice E Try these:

a 600 – 88 **b** 400 – 83 **c** 900 – 73 **d** 6000 – 762

e 2000 – 979 **f** 50000 – 4334 **g** 70000 – 8012

a	**512**	**b**	**317**	**c**	**827**	**d**	**5238**
e	**1021**	**f**	**45666**	**g**	**61988**		

ONE MORE

Now let's look at another variation.

10 Find **8000 – 4222**.

Considering the thousands, the 8 will be reduced by 5 (one more than 4) because you are taking over 4 thousand away.

All from 9. . . is then applied to the 222 to give 778.

So **8000 – 4222 = 3778**.

When you have a sum like 8000 – 4222 where both numbers have the same number of figures:

> reduce the first figure of the first number by one more than the first figure of the second number to get the first figure of the answer.
> And apply the formula to the remaining figures.

✐ **Practice F** Subtract the following:

a 8000 – 3504	**b** 5000 – 1234	**c** 300 – 132
d 2000 – 1444	**e** 700 – 232	**f** 60,000 – 23,331

---

a	**4496**	**b**	**3766**	**c**	**168**
d	**556**	**e**	**468**	**f**	**36,669**

ONE LESS AGAIN

11 Find **6000 – 32.**

You will see here that you have a 2-figure number to subtract from 6000 which has three zeros.
The sum can be written 6000 – 032.
Then **6000 – 032 = 5968**.
The 6 is reduced to 5, and the formula converts 032 to 968.

12 **30000 – 63** = 30000 – 0063 = **29937**.
The 3 becomes 2, and 0063 becomes 9937.

✐ **Practice G** Subtract the following:

a 5000 – 74	**b** 8000 – 58	**c** 6000 – 94	**d** 4000 – 19
e 80000 – 345	**f** 30000 – 276	**g** 50000 – 44	**h** 700 – 8
i 30000 – 54	**j** 20000 – 222	**k** 30000 – 670	**l** 70000 – 99

---

a	**4926**	**b**	**7942**	**c**	**5906**	**d**	**3981**
e	**79655**	**f**	**29724**	**g**	**49956**	**h**	**692**
i	**29946**	**j**	**19778**	**k**	**29330**	**l**	**69901**

5.3 MONEY

The type of subtraction we have been doing is very useful for checking change.

13 Suppose you buy a computer game for **£7.53** and you pay with a **£10** note.
How much change would you expect to get?

You just apply *All From 9 and the Last From 10* to 753 to get **£2.47**.

14 What change would you expect from a **£20** note when paying **£3.46**?

The change you expect to get is **£16.54** because £3.46 from £10 is £6.54 and there is £10 to add to this.

✏ **Practice H** Do the following money subtractions in a similar way.

a £10 – £2.34	**b** £10 – £6.51	**c** £10 – £5.82	**d** £10 – £9.07
e £20 – £7.44	**f** £20 – £12.78	**g** £20 – £3.18	**h** £20 – £8.40

---

a	**£7.66**	**b**	**£3.49**	**c**	**£4.18**	**d**	**£0.93**
e	**£12.56**	**f**	**£7.22**	**g**	**£16.82**	**h**	**£11.60**

This subtraction method leads to a general subtraction process (see Lesson 9).

The final exercise is a mixture of all the types we have met:

✏ **Practice I** Subtract:

a 100 – 34	**b** 1000 – 474	**c** 5000 – 542	**d** 800 – 72
e 1000 – 33	**f** 5000 – 84	**g** 700 – 58	**h** 9000 – 186
i 10000 – 4321	**j** 200 – 94	**k** 10000 – 358	**l** 400 – 81
m 7000 – 88	**n** 900 – 17	**o** 30000 – 63	**p** 90000 – 899

---

a	**66**	**b**	**526**	**c**	**4458**	**d**	**728**
e	**967**	**f**	**4916**	**g**	**642**	**h**	**8814**
i	**5679**	**j**	**106**	**k**	**9642**	**l**	**319**
m	**6912**	**n**	**883**	**o**	**29937**	**p**	**89101**

LESSON 6
NUMBER SPLITTING

SUMMARY

6.1 Addition
6.2 Subtraction
6.3 Multiplication
6.4 Division

– splitting difficult sums into easy ones, all done from left to right.

6.1 ADDITION

This is a very useful device for splitting a difficult sum into two or more easy ones and comes under the formula *By Alternate Elimination and Retention*.
For quick mental sums number splitting can considerably reduce the work involved in a calculation.

1 Suppose you are given the addition sum:

$$\begin{array}{r} 2\,3\,4\,5 \\ \underline{6\,7\,3\,8} + \\ \underline{\qquad\quad} \end{array}$$

With 4-figure numbers it looks rather hard.
But if you split the sum into two parts, each part can be done easily and mentally (see Sections 1.5, 1.6, 4.1):

$$\begin{array}{cc|cc} 2 & 3 & 4 & 5 \\ 6 & 7 & 3 & 8 \\ \hline 9 & 0 & 8 & 3 \end{array} +$$

On the right we have 45 + 38 which (mentally) is **83**.
So you put this down.
And on the left you have 23 + 67 which is **90**. So **2345 + 6738 = 9083**.

Practice A Add the following (try some of them mentally):

a $\begin{array}{r} 3\,4\,5\,6 \\ \underline{4\,7\,1\,7} \\ \underline{\qquad\quad} \end{array}$ **b** $\begin{array}{r} 1\,8\,1\,9 \\ \underline{1\,7\,1\,6} \\ \underline{\qquad\quad} \end{array}$ **c** $\begin{array}{r} 6\,4\,4\,6 \\ \underline{2\,8\,3\,8} \\ \underline{\qquad\quad} \end{array}$ **d** $\begin{array}{r} 8\,3\,2\,1 \\ \underline{1\,8\,2\,3} \\ \underline{\qquad\quad} \end{array}$

a 81/73 b 35/35 c 92/84 d 101/44

2 Find **481 + 363**.

$$\begin{array}{r|rr} & 4 & 8\ \ 1 \\ + & 3 & 6\ \ 3 \\ \hline & 8 & 4\ \ 4 \\ & 1 & \end{array} \qquad\qquad \begin{array}{rr|r} & 4\ \ 8 & 1 \\ + & 3\ \ 6 & 3 \\ \hline & 8\ \ 4 & 4 \end{array}$$

This example is done in two in ways.
Which is easier?

> You may have to think where to put the line,
> but it is usually best to put it so that there are no carries over the line.

Practice A continued Add the following (try some of them mentally):

e $\begin{array}{r} 7\ 6\ 7 \\ \underline{6\ 1\ 6} \\ \_\_\_ \end{array}$ **f** $\begin{array}{r} 3\ 8\ 3 \\ \underline{3\ 8\ 4} \\ \_\_\_ \end{array}$ **g** $\begin{array}{r} 4\ 4\ 4 \\ \underline{2\ 4\ 6} \\ \_\_\_ \end{array}$ **h** $\begin{array}{r} 8\ 8\ 8 \\ \underline{7\ 0\ 7} \\ \_\_\_ \end{array}$

i $\begin{array}{r} 5\ 5\ 1 \\ \underline{6\ 6\ 2} \\ \_\_\_ \end{array}$ **j** $\begin{array}{r} 4\ 5\ 5\ 4 \\ \underline{3\ 6\ 3\ 6} \\ \_\_\_\_ \end{array}$ **k** $\begin{array}{r} 1\ 2\ 3\ 4 \\ \underline{4\ 9\ 4\ 4} \\ \_\_\_\_ \end{array}$ **l** $\begin{array}{r} 5\ 2\ 3\ 4 \\ \underline{9\ 3\ 9\ 3} \\ \_\_\_\_ \end{array}$

---

e	**13/83**	**f**	**76/7**	**g**	**6/90**	**h**	**15/95**
i	**121/3**	**j**	**81/90**	**k**	**61/78**	**l**	**14/62/7**

6.2 SUBTRACTION

You can also use Number Splitting in subtraction sums.

3 Consider the subtraction sum:

$$\begin{array}{r} 5\ 4\ 5\ 4 \\ -\ \ \underline{1\ 7\ 2\ 6} \\ \_\_\_\_ \end{array}$$

You can split this up into two easy sums:

$$\begin{array}{rr|rrr} & 5 & 4 & 5 & 4 \\ - & 1 & 7 & 2 & 6 \\ \hline & 3 & 7 & 2 & 8 \end{array}$$

First 54 – 26, which is 28,
then 54 – 17, which is 37.

Practice B Subtract the following. Split each sum into two easy ones.

a 3 2 4 3 1 3 1 9	**b** 4 4 4 4 1 8 2 8	**c** 7 0 7 0 1 5 2 6	**d** 3 7 2 1 1 9 0 9
e 6 8 8 9 1 9 3 6	**f** 8 5 2 1 3 9	**g** 7 7 7 5 8 5	**h** 6 6 6 6 2 9 3 8

a 19/24	**b** 26/16	**c** 55/44	**d** 18/12
e 49/53	**f** 7/13	**g** 19/2	**h** 37/28

6.3 MULTIPLICATION

This same splitting technique can be applied in multiplication and division as well.

4 **352 × 2**

You can split this sum like this: 35 / 2 × 2 = **704**. (35 and 2 are easy to double.)

5 Similarly **827 × 2** becomes 8 / 27 × 2 = **1654**,

604 × 7 becomes 6 / 04 × 7 = **4228**,

121745 × 2 becomes 12 / 17 / 45 × 2 = **243490**,

3131 × 5 becomes 3 / 13 / 1 × 5 = **15655**.

You can split the number any way you like, but it is best to:

> split the number so that the parts can be multiplied easily, without a carry.

Practice C Multiply the following:

a 432 × 3	**b** 453 × 2	**c** 626 × 2	**d** 433 × 3	**e** 308 × 6
f 814 × 4	**g** 515 × 5	**h** 919 × 3	**i** 1416 × 4	**j** 2728 × 2
k 3193 × 3	**l** 131415 × 3			

a	**12/96**	**b**	**90/6**	**c**	**12/52**	**d**	**12/99**	**e**	**18/48**
f	**32/56**	**g**	**25/75**	**h**	**27/57**	**i**	**56/64**	**j**	**54/56**
k	**9/57/9**	**l**	**39/42/45**						

6.4 DIVISION

Division sums can also often be simplified by this method.

6 The division sum **2)<u>4 3 2</u>** can be split into: 2)<u>4 / 32</u> = 2/16 = **216**.

because 4 and 32 are both easy to halve.

7 Similarly **2)<u>3 4 5 6</u>** becomes 2)<u>34 / 56</u> = 17/28 = **1728**.

8 And in **3)<u>1266</u>** we notice that 12 and 66 can be divided separately by 3, so:

3)<u>12/66</u> = 4/22 = **422**

✐ **Practice D** Divide the following mentally:

a	2)<u>6 5 6</u>	**b**	2)<u>7 2 6</u>	**c**	3)<u>1 8 9 9</u>	**d**	6)<u>1 2 6 6</u>
e	4)<u>2 0 4 8</u>	**f**	4)<u>2 8 4 4</u>	**g**	3)<u>2 1 3 9</u>	**h**	2)<u>2 6 3 6</u>

a	**3/28**	**b**	**36/3**	**c**	**6/33**	**d**	**2/11**
e	**5/12**	**f**	**7/11**	**g**	**7/13**	**h**	**13/18**

Sometimes we need to be a bit careful and put extra zeros.

9 **6)<u>6 1 2</u>** becomes 6)<u>6 / 12</u> = 1/02 = **102**.
note the 0 here because the 12 takes up two places)

10 **7)<u>2 8 4 9</u>** becomes 7)<u>28 / 49</u> = 4/07 = **407**.

Practice D continued

i 4)2 8 1 6 **j** 4)8 1 2 **k** 6)4 8 1 8 **l** 3)1 2 6 6

m 5)2 0 4 5 **n** 2)3 8 1 4 **o** 7)21014

---

i **704** **j** **203** **k** **803** **l** **422**
m **409** **n** **1907** **o** **3002**

And sometimes we split into three sections.

11 **3) 2 4 4 5 3** becomes 3)24 / 45 / 3 = 8/15/1 = **8151**.

Practice D continued

p 3)9 1 8 2 7 **q** 2)3 8 7 2 5 2 **r** 8)4 0 1 6 8 **s** 5)1 0 3 5 4 5

t 3)1 5 0 1 5 **u** 13)3 9 1 3 5 2

---

p **30609** **q** **193626** **r** **5021** **s** **20709**
t **5005** **u** **30104**

"But, according to the Vedic system, the multiplication tables are not really required above 5×5."
From "Vedic Mathematics", Page 13.

LESSON 7
BASE MULTIPLICATION

SUMMARY

7.1 **Times Tables** – avoiding multiplication tables above 5 × 5.
7.2 **Numbers just Over Ten** – multiplying numbers close to and over ten.
7.3 **Multiplication Table Patterns** – patterns of tables on the 9-point circle.
7.4 **Numbers Close to 100** – multiplying numbers near 100.
7.5 **Larger Numbers** – multiplying larger numbers.
7.6 **Proportionately** – a further extension of the method.
7.7 **Multiplying Numbers near Different Bases**
7.8 **Squaring Numbers near a Base**
7.9 **A summary** – of all multiplication devices so far.

7.1 TIMES TABLES

It is useful to know multiplication tables by heart. If not here is a neat and easy method to use.

1 If you want **7 × 8** you know that 7 is 3 below 10
and 8 is 2 below 10.

So next to 7 put –3 and
next to 8 put –2, like this:

$$\begin{array}{r} 7 - 3 \\ \times\ \underline{8 - 2} \\ \underline{\qquad} \end{array}$$

Then cross-subtract to get the first figure of the answer: 7 – 2 = 5:

$$\begin{array}{r} 7 - 3 \\ \times\ \underline{8 - 2} \\ \underline{5\qquad} \end{array}$$

Or, if you prefer you can subtract the other way:

$$\begin{array}{r} 7 - 3 \\ \times\ \underline{8 - 2} \\ \underline{5\qquad} \end{array}$$

8 – 3 = 5 as well.

Finally, just multiply vertically, 3 × 2, to get 6 for the second part of the answer.

$$\begin{array}{r} 7 - 3 \\ \times\ \underline{8 - 2} \\ \underline{5\quad 6} \end{array}$$

So **7 × 8 = 56**.

So to sum up: 1) put the differences of the numbers from 10: 3 and 2 above,
2) cross-subtract: 7–2 = 5 or 8–3 = 5 and put this down,
3) multiply vertically: 3×2 = 6 and put it down.

This comes under the *Vertically and Crosswise* Sutra.

Sometimes there can be a carry figure, so let's look at this next.

2 To find **6 × 7** we note 6 is 4 below 10 and 7 is 3 below 10.

So we have:

$$\begin{array}{rl} & 6 - 4 \\ \times & \underline{7 - 3} \\ & \underline{} \end{array}$$

Then cross-subtract: 6 **–** 3 = 3 and put this down:

$$\begin{array}{rl} & 6 - 4 \\ \times & \underline{7 - 3} \\ & \underline{3 \quad\;\;} \end{array}$$

Then just multiply 4 × 3 to get 12 for the second part of the answer.
But here, as 12 is a 2-figure number you need to carry the 1 over to the 3:

$$\begin{array}{rl} & 6 - 4 \\ \times & \underline{7 - 3} \\ & \underline{3 \quad 2} = 42 \\ & \quad\;\, {}_{1} \end{array}$$

So **6 × 7 = 42**.

✎ **Practice A** This method is very easy. Try the ones below.

a 7×9 **b** 8×8 **c** 9×6 **d** 7×7 **e** 8×9

f 8×6 **g** 9×9 **h** 6×6 **i** 7×5 **j** 6×5

---

a	**63**	**b**	**64**	**c**	**54**	**d**	**49**	**e**	**72**
f	**48**	**g**	**81**	**h**	**36**	**i**	**35**	**j**	**30**

So in the Vedic system multiplication tables above 9×9 are not essential.
See the note on Russian Peasant Multiplication on Page 69.

7.2 NUMBERS JUST OVER TEN

The method used in the last section can also be used for numbers just over 10 rather than numbers just under 10.

Suppose you want to multiply 12 and 13, which are both close to 10.

3

For **12 × 13** you notice the numbers are close to 10 and that 12 is 2 over ten, and 13 is 3 over ten.

So set the sum out as before except that because the numbers are **over** ten you put a plus instead of a minus:

$$\begin{array}{rl} & 12 + 2 \\ \times & \underline{13 + 3} \\ & \underline{} \end{array}$$

Then you **cross-add**
to get the first part of the answer:
12 + 3 = 15 (or 13 + 2 = 15).

$$\begin{array}{rl} & 12 + 2 \\ \times & \underline{13 + 3} \\ & \underline{15} \end{array}$$

And as before you multiply vertically
to get the last figure: 2 × 3 = 6

$$\begin{array}{rl} & 12 + 2 \\ \times & \underline{13 + 3} \\ & \underline{15\quad 6} \end{array}$$

So **12 × 13 = 156**.

✐ **Practice B** This is the same as before except that we cross-add. Try some.
There is a carry in the sums in the second row.

a	13 × 11	**b**	12 × 12	**c**	11 × 15	**d**	13 × 13	**e**	11 × 11
f	13 × 14	**g**	12 × 16	**h**	14 × 14	**i**	16 × 16	**j**	13 × 18

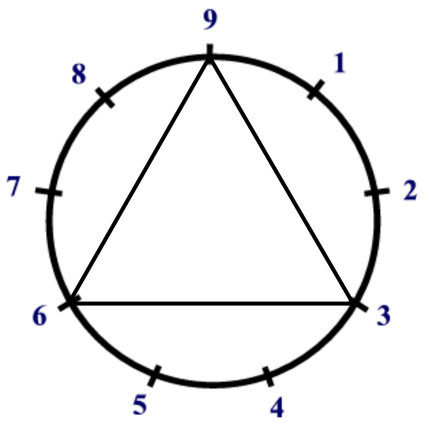
a	**143**	**b**	**144**	**c**	**165**	**d**	**169**	**e**	**121**
f	**182**	**g**	**192**	**h**	**196**	**i**	**256**	**j**	**234**

7.3 MULTIPLICATION TABLE PATTERNS

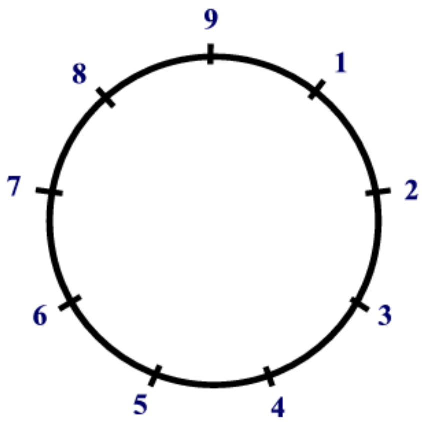
In the 3-times table the answers are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30 . . .
If you find the digit sums of these numbers you get 3, 6, 9, 3, 6, 9, 3, 6, 9 . . .

The same pattern 3, 6, 9 repeats over and over again.
You can show this pattern on the 9-point circle.

3 TIMES TABLE



6 TIMES TABLE



4 Start at 3 and draw a line to the next number, 6
(go over the line in a colour).
Then from 6 draw a line to the next number, 9.
Then from 9 draw a line to the next number 3.

From then on the pattern goes over itself because 3, 6, 9, 3, 6, 9 . . . keeps repeating.

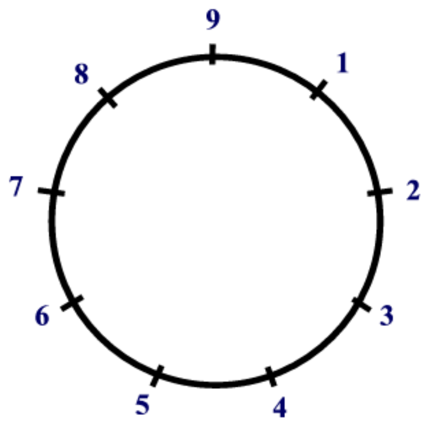
So this is the pattern for the 3-times table and it is shown above.

✎ **Practice C**

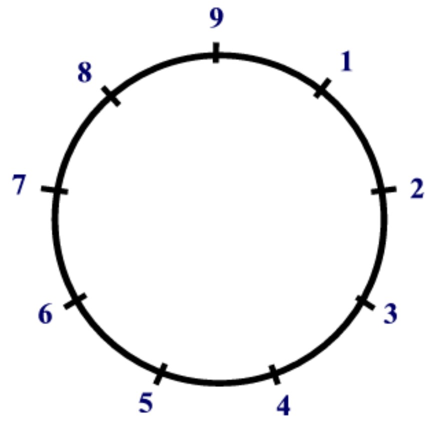
a Draw the pattern for the 6-times table on the right-hand circle above.

b Draw the patterns for the 4 and 5, 1 and 8, 2 and 7 and the 9 times tables on the circles below.

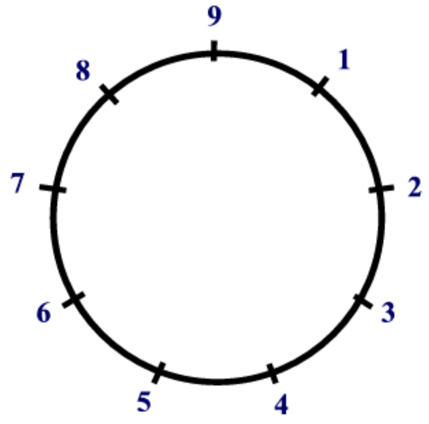
4 TIMES TABLE



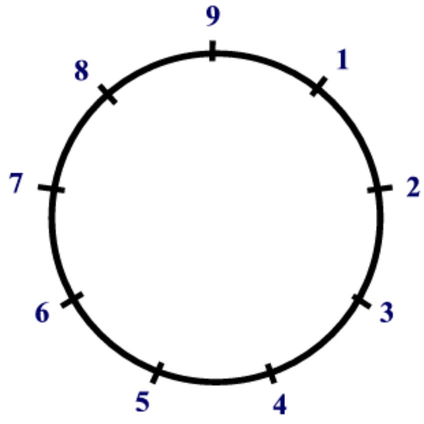
5 TIMES TABLE



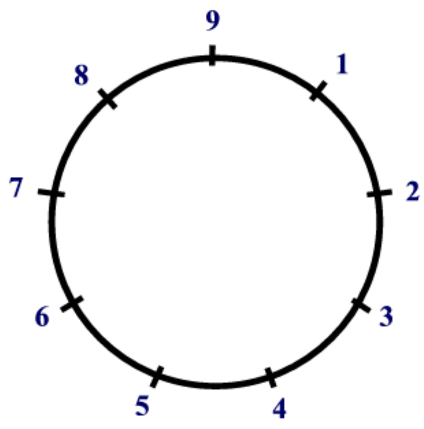
1 TIMES TABLE



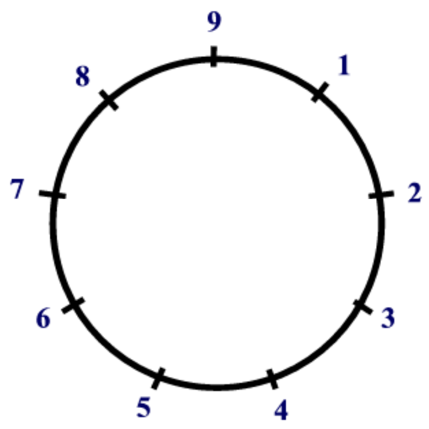
8 TIMES TABLE



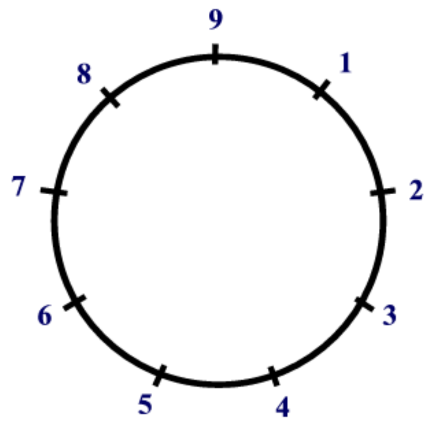
2 TIMES TABLE



7 TIMES TABLE



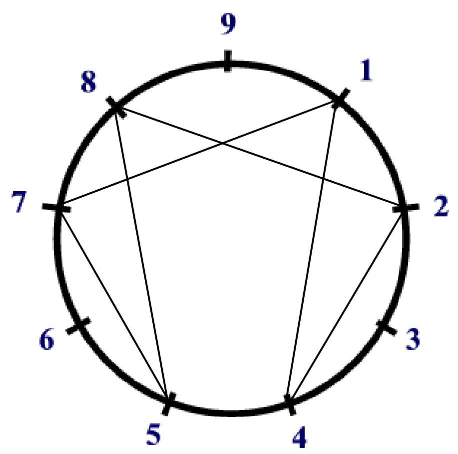
9 TIMES TABLE



RECURRING DECIMALS

This 9-point circle has many uses including representing recurring decimal cycles (see Manual 2 or The Cosmic Calculator, Books 2, 3).

For example: $\frac{1}{7} = 0.\dot{1}4285\dot{7}$
which means the cycle of figures 142857 repeats itself indefinitely.
We draw this pattern by starting at 1 and drawing a line to 4 and so on until we have six lines and the pattern starts to repeat itself. This converts the arithmetic pattern to a geometrical pattern.



In fact any sequence can be represented on the circle: square numbers, triangular numbers, prime numbers, the fibonacci sequence etc.

7.4 NUMBERS CLOSE TO 100

The simple method for multiplying numbers like 7× 8 shown in Section 7.1 can be extended to easily multiply bigger numbers.

Usually a sum like **88 × 98** is considered especially difficult because of the large figures, 8 and 9.

But since the numbers 88 and 98 are close to the base of 100 it is in fact very easy to find the product.

5 **88 × 98 = 8624**.
We set the sum out as shown below:
88 is 12 below 100, so we put –12 next to it,
98 is 2 below 100 so we put –2 next to it.
The answer 8624 is in two parts: 86 and 24.

$$\begin{array}{r} \mathbf{88 - 12} \\ \underline{\mathbf{98 - \ \ 2}} \\ \underline{\mathbf{86 / 24}} \end{array}$$

cross-subtract ← → multiply vertically: 12 × 2 = 24

that is 88 – 2 = 86 or 98 – 12 = 86
(whichever you like),

We call the 12 and 2 **deficiencies** as the numbers 88 and 98 are deficient from the unity of 100 by 12 and 2.

6 For **93 × 96** we get deficiencies of 7 and 4, so

$$\begin{array}{r} \mathbf{93 - 07} \\ \underline{\mathbf{96 - 04}} \\ \underline{\mathbf{89 / 28}} \end{array}$$

93 – 4 = 89 or 96 – 7 = 89,
and 7 × 4 = 28.

7 For **98 × 97**:

$$\begin{array}{r} \mathbf{98 - 02} \\ \underline{\mathbf{97 - \ \ 3}} \\ \underline{\mathbf{95\ / 06}} \end{array}$$

Note the zero inserted here: the numbers being multiplied are near to 100, so two digits are required on the right, as in the other examples.

In fact once we have got the deficiencies we apply the *Vertically and Crosswise* method:
we **cross-subtract** to get the left-hand part of the answer and
we **multiply vertically** in the right-hand column to get the right-hand part of the answer.

✎ **Practice D** Multiply the following:

a 94 × 94 **b** 97 × 89 **c** 87 × 99 **d** 87 × 98 **e** 87 × 95

f 95 × 95 **g** 79 × 96 **h** 98 × 96 **i** 92 × 99 **j** 99 × 99

a	**88/36**	**b**	**86/33**	**c**	**86/13**	**d**	**85/26**	**e**	**82/65**
f	**90/25**	**g**	**75/84**	**h**	**94/08**	**i**	**91/08**	**j**	**9801**

It may happen that there is a carry figure.

8 For 89 × 89:

$$\begin{array}{l} 89 - \ 11 \\ \underline{89 - \ 11} \\ \underline{78 \ /\ _{1}21} = \ \underline{\mathbf{7921}} \end{array}$$

Here the numbers are each 11 below 100, and 11 × 11 = 121, a 3-figure number. The hundreds digit of this is therefore carried over to the left.

✎ **Practice D** continued

k 88 × 88 **l** 97 × 56 **m** 44× 98 **n** 97 × 63

k	**7744**	**l**	**5432**	**m**	**4312**	**n**	**6111**

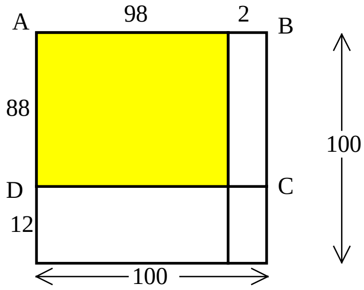
Explanation (based on Example 5 above).

(1)

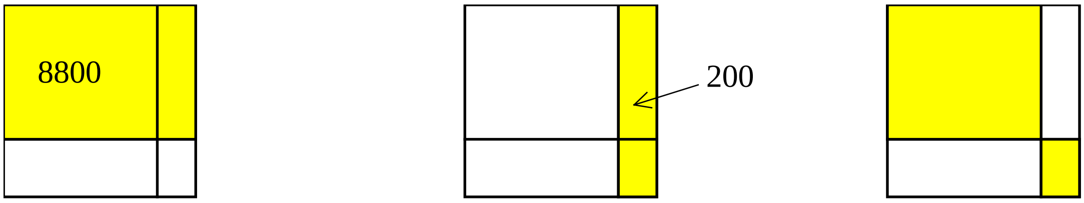
$$\begin{aligned} 88 \times 98 &= 88 \times 100 - 88 \times 2 \\ &= 8800 - (100 \times 2 - 12 \times 2) \\ &= 8800 - 200 + 12 \times 2 \\ &= 8600 + 24 = 8624 \end{aligned}$$

(2) Alternatively consider the following geometrical explanation.

88×98 is the area of a rectangle 88 units by 98 units so we begin with a square of side 100:



You can see the required area shaded in the diagram.
You can also see the deficiencies from 100: 12 and 2.
Now the area ABCD must be 8800 because the base is 100 and the height is 88.



From this we subtract the strip on the right side, the area of which is 200:
so 8800 – 200 = 8600.
This leaves the required area but we have also subtracted the area of the small rectangle shown shaded above on the right. This must therefore be added back on and since its area is 12×2=24 we add 24 to 8600 to get **8624**.

You can probably see that this procedure will work for any product when the numbers are close to 100 and just below it.

(3) An algebraic proof would be: $(x - a)(x - b) = x(x - a - b) + ab$,

where x is the base (in this example 100) and a and b are the deficiencies of the numbers from the base (in this case 12 and 2).
The numbers being multiplied are thus $(x - a)$ and $(x - b)$; $(x - a - b)$ is one number minus the other deficiency; and the x outside the bracket on the RHS has the effect of moving the quantity $(x - a - b)$ to the left as many places as there are zeros in the base.

MENTALLY

Look again at the first example in this section:

$$\begin{array}{r} 88 - 12 \\ \underline{98 - 2} \\ \underline{86 \,/\, 24} \end{array}$$

The most efficient way to do these sums is to take one number and subtract the other number's deficiency from it: 88–2=86, or 98–12=86.
Then multiply the deficiencies together: 12×2=24.
We mentally adjust the first part of the answer if there is a carry figure.

This is so easy it is really just mental arithmetic.

Practice E Multiply these numbers mentally, just write down the answer:

a 87 97	**b** 79 98	**c** 98 93	**d** 94 95
e 96 96	**f** 88 96	**g** 89 98	**h** 93 96
i 93 99	**j** 97 97	**k** 96 67	**l** 95 75

m 8 9
? ?
8 2 7 7 find the missing numbers

a	**84/39**	**b**	**77/42**	**c**	**91/14**	**d**	**89/30**
e	**92/16**	**f**	**84/48**	**g**	**87/22**	**h**	**89/28**
i	**92/07**	**j**	**94/09**	**k**	**64/32**	**l**	**71/25**
m	**93**						

NUMBERS OVER 100

Multiplying numbers that are over 100 is even easier than multiplying numbers just under 100.
Suppose we want 103 × 104.

9 **103 × 104 = 10712**.

103 +03
104 + 4
107 / 12

The method is similar to the previous one.
103 is 3 over 100, so put +3 next to it.
And 104 is 4 over 100 so put +4 next to it.

Then 103 + 4 = **107** or 104 + 3 = **107**,

and 4 × 3 = **12**.

So now we **cross-add,** and multiply vertically.

✐ **Practice F** Multiply mentally:

a 107 × 104	**b** 107 × 108	**c** 133 × 103	**d** 102 × 104
e 123 × 102	**f** 171 × 101	**g** 103 × 111	**h** 125 × 105
i 103 × 103	**j** 111 × 111	**k** 162 × 102	**l** 113 × 105

m 1 0 3
? ? ?
1 0 8 1 5 find the missing numbers

---

a 11128	**b 11556**	**c 13699**	**d 10608**
e 12546	**f 17271**	**g 11433**	**h 13125**
i 10609	**j 12321**	**k 16524**	**l 11865**
m 105			

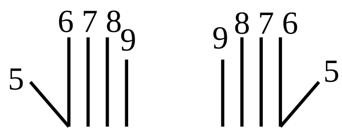
MENTAL MATHS

The Vedic techniques are so easy that the system of Vedic Mathematics is really a system of mental mathematics. This has a number of further advantages as pupils seem to make faster progress and enjoy mathematics more when they are permitted to do the calculation in their head. After all, the objects of mathematics are mental ones, and writing down requires a combination of mental and physical actions, so that the child's attention is alternating between the mental and physical realms. This alternation is an important ability to develop but working only with mental objects also has many advantages.

Mental mathematics leads to greater creativity and the pupils understand the objects of mathematics and their relationships better. They begin to experiment (especially if they are encouraged to do so) and become more flexible. Memory and confidence are also improved through mental mathematics.

RUSSIAN PEASANT MULTIPLICATION

This is using the fingers for multiplication of numbers between 5 and 9 by numbers between 5 and 9, and it is very similar to the Vedic method shown here.



The fingers are numbered as shown with the thumbs counting as 5 and the little fingers as 9. The palms are upward. To multiply, say, 8 by 7, put together the '8 finger' on the left hand and the '7' finger on the right hand. Then count the fingers above the touching fingers: there are **5**, and multiply the number of other fingers on the left hand by the number of other fingers on the right hand: 2 × 3 = **6**.
So **8 × 7 = 56**.

7.5 LARGER NUMBERS

Now, what about numbers close to other bases like 1000 10,000 etc?

10 Find **568 × 998**.

In this sum the numbers are close to 1000, and the deficiencies are 432 and 2.
The deficiency for 568 is found by applying the Sutra: *All from 9 and the Last from 10.*

568 – 432
998 – 2
566 / 864

The method here is just the same, but we allow 3 figures on the right as the base is now 1000.

The differences of the numbers from 1000 are 432 and 2.
Then cross-subtracting:568 – 2 = 566,
And vertically: 432 × 2 = 864.

So **568 × 998 = 566864**.

11 Find **68777 × 99997**.

Even large numbers like this are easily and mentally multiplied by the same method.

68777 – 31223
99997 – 3
68774 / 93669

The number of spaces needed on the right is the number of 0's in the base number.

✐ **Practice G**

Multiply the following mentally:

a 667 × 998	**b** 768 × 997	**c** 989 × 998	**d** 885 × 997
e 883 × 998	**f** 467 × 998	**g** 891 × 989	**h** 8888 × 9996
i 6999 × 9997	**j** 90909 × 99994	**k** 78989 × 99997	**l** 9876 × 9998

a 665/666	**b 765/696**	**c 987/022**	**d 882/345**
e 881/234	**f 466/066**	**g 881/199**	**h 8884/4448**
i 6996/9003	**j 90903/54546**	**k 78986/63033**	**l 9874/0248**

NUMBERS ABOVE THE BASE

Suppose now that the numbers are above the base.

12 **1234 × 1003 = 1237702**. (1234+3=1237, 234×3=702)

13 **10021 × 10002 = 100230042**. (10021+2=10023, 0021×2=0042)

With a base of 10,000 here we need 4 figures on the right.

Practice H

a 1222 × 1003 **b** 1051 × 1007 **c** 1123 × 1002

d 1007 × 1006 **e** 15111 × 10003

---

a 1225/666 **b 1058/357** **c 1125/246**
d 1013/042 **e 15115/5333**

7.6 PROPORTIONATELY

Proportionately just means that you can get an answer by doubling (or trebling etc.) another answer.

We have been doing this quite a lot already.

14 Find **309 × 104**.

You may notice here that **309 is 3 × 103**.
This means we can find 103 × 104 (which have an easy method for) and multiply the answer by 3.

103 × 104 = 10712.

And **10712 × 3 = 32136**.

You can use number splitting to find 10712 × 3: 1/07/12 × 3 = 3/21/36.

15 Find **192 × 92**.

Here we see that if you halve 192 you get 96.
So: find 96 × 92 and double the result.

96 × 92 = 8832, by the easy *Vertical and Crosswise* method,

and so **192 × 92 = 17664**, (by doubling 8832).

Practice I

a 212 × 103 **b** 106 × 208 **c** 182 × 98 **d** 93 × 186

a **21836** **b** **22048** **c** **17836** **d** **17298**

16 Find **47 × 98**.

Here you should double 47 to 94 because both the numbers are then close to 100.
So you find 94 × 98 and halve the answer.

94 × 98 = 9212
And half of 9212 is **4606**.

Again use number splitting: to halve 9212 (think of 92/12).

17 Find **192 × 44**.

Here you can halve 192 and double 44.

This converts the sum to 96 × 88 and there is no doubling or halving to be done to the answer because the halving and doubling cancel each other out.

So 192 × 44 = 96 × 88 = **8448**.

Practice I continued

e 93 × 46 **f** 56 × 104 **g** 306 × 118 **h** 51 × 104

i 206 × 54 **j** 44 × 99 **k** 48 × 184 **l** 228 × 212

e **4278** **f** **5824** **g** **36108** **h** **5304**
i **11124** **j** **4356** **k** **8832** **l** **48336**

ANOTHER APPLICATION OF PROPORTIONATELY

Another way of using the *Proportionately* formula further extends the range of application of this multiplication method.

18 **213 × 203 = 43239**.

$$\begin{array}{rl} & 213 + 13 \\ & \underline{203 + \ \ 3} \\ 2 \times & \underline{216\ /\ 39} = \underline{\mathbf{43239}} \end{array}$$

We see here that the numbers are not near any of the bases used before: 10, 100, 1000 etc.. But they are close to 200, with differences of 13 and 3 as shown above.

The usual procedure gives us 216/39 (213+3=216, 13×3=39).

Now since our base is 200 which is 100×2 we multiply **only the left-hand part** of the answer by 2 to get 43239.

19 **29 × 28 = 812**.

The base is 30 (3×10),
and the deficiencies are –1 and –2.
Cross-subtracting gives 27,
then multiplying vertically on the right we get 2,
and finally 3×27 = 81.

$$\begin{array}{rl} & 29 - 1 \\ & \underline{28 - 2} \\ 3 \times & \underline{27\ /\ 2} = \underline{\mathbf{812}} \end{array}$$

So these are just like the previous sums but with an extra multiplication (of the left-hand side only) at the end.

20 Find **33 × 34**.

In this example there is a carry figure:

$$\begin{array}{rl} & 33 + \ 3 \\ & \underline{34 + \ 4} \\ 3 \times & \underline{37\ /\ _{1}2} = 111\ /_{1}2 = \underline{\mathbf{1122}} \end{array}$$

Note that since the right-hand side does not get multiplied by 3 we multiply the left-hand side by 3 before carrying the 1 over to the left.

✎ **Practice J** Multiply mentally:

a 41 × 42	**b** 204 × 207	**c** 321 × 303	**d** 203 × 208
e 902 × 909	**f** 48 × 47	**g** 188 × 196	**h** 199 × 198
i 189 × 194	**j** 207 × 211	**k** 312 × 307	**l** 5003 × 5108
m 63 × 61	**n** 23 × 24	**o** 79 × 77	

a	**172/2**	**b**	**422/28**	**c**	**972/63**	**d**	**422/24**
e	**8199/18**	**f**	**225/6**	**g**	**368/48**	**h**	**394/02**
i	**366/66**	**j**	**436/77**	**k**	**957/84**	**l**	**25555/324**
m	**3843**	**n**	**552**	**o**	**6083**		

7.7 MULTIPLYING NUMBERS NEAR DIFFERENT BASES

Sometimes we need to multiply numbers that are each near a different base.
In the example below one number is close to 10,000 and the other is close to 100.

21 **9998 × 94 = 9398/12**

Here the numbers are close to different bases: 10,000 and 100,
and the deficiencies are –2 and –6.
We write, or imagine, the sum set out as shown:

$$\begin{array}{l} 9998\ -02 \\ \underline{94\quad -\ 6} \\ \underline{9398\ /\ 12} \end{array}$$

It is important to line the numbers up as shown because the 6 is not subtracted from the 8, as usual, but from the 9 above the 4 in 94. That is, the second column from the left here.

So **9998 becomes 9398**.

Then multiply the deficiencies together: 2×6 = **12**.

Note that the number of figures in the right-hand part of the answer corresponds to the base of the lower number (94 is near 100, therefore there are 2 figures on the right).

You can see why this method works by looking at the sum 9998 × 9400, which is 100 times the sum done above:

$$\begin{array}{l} 9998 - 0002 \\ \underline{9400 - \quad 600} \\ \underline{9398\ \ /\ 1200} \end{array}$$

Now we can see that since 9998 × 9400 = 93981200,
then 9998 × 94 = 939812.

This also shows why the 6 is subtracted in the second column from the left.

✐ **Practice K** Find:

a 97 × 993 **b** 92 × 989 **c** 9988 × 98 **d** 9996 × 988

---

a 963/21 b 909/88 c 9788/24 d 9876/048

In the next example the numbers are close to different bases, but they are over the base rather than under.

22 **10007 × 1003 = 10037021**.

Lining the numbers up:

10007 + 007
1003 + 3
10037 / 021

we see that we need three figures on the right and that the surplus, 3, is added in the 4th column, giving 10037.

✐ **Practice L** Find:

a 103 × 1015 **b** 106 × 1012 **c** 10034 × 102 **d** 1122 × 104

---

a 1045/45 b 1072/72 c 10234/68 d 1166/88

7.8 SQUARING NUMBERS NEAR A BASE

This is especially easy and is for squaring numbers which are near a base.
You will recall that squaring means that a number is multiplied by itself (like 96 × 96).
This method is described by the sub-formula *Reduce (or increase) by the Deficiency and also set up the square.*

23 **96^2 = 92/16**.

96 is 4 below 100, so we reduce 96 by 4, which gives us the first part of the answer, **92**.
The last part is just 4^2 =**16**, as the formula says.

24 **1006^2 = 1012/036**.

Here 1006 is increased by 6 to **1012**, and 6^2 = **36**: but with a base of 1000 we need 3 figures on the right, so we put 036.

Practice M Square the following:

a 94	**b** 103	**c** 108	**d** 1012
e 98	**f** 88	**g** 91	**h** 10006
i 988	**j** 997	**k** 9999	**l** 9989
m 111	**n** 13	**o** 987	

a 8836	**b 10609**	**c 11664**	**d 1024144**
e 9604	**f 7744**	**g 8281**	**h 100120036**
i 976144	**j 994009**	**k 99980001**	**l 99780121**
m 12321	**n 169**	**o 974169**	

25 **304^2 = 3×308/16 = 92416**.

This is similar but because our base is 300 the left-hand part of the answer is multiplied by 3.

Practice N Square the following:

a 206	**b** 212	**c** 302	**d** 601
e 21	**f** 72	**g** 4012	**h** 511

a 424/36	**b 449/44**	**c 912/04**	**d 3612/01**
e 44/1	**f 518/4**	**g 16096/144**	**h 2611/21**

There are many special multiplication methods in the Vedic system: see Lesson 10. And the general method (Lesson 11) is always there if no special method comes to mind.

7.9 A SUMMARY

Here we can summarise the various methods of multiplication and squaring encountered so far.

1. Multiplying by 4, 8 etc. we can just double twice, 3 times etc. E.g. 37×4.
2. We can use doubling to extend the multiplication tables. E.g. 14×8.
3. We can multiply from left to right using *On the Flag*. E.g. 456×3.
4. We can use *All from 9 and the Last from 10* for multiplying numbers near a base.

   E.g. 98×88, 103×104, 203×204.
5. And we can also multiply numbers near different bases. E.g. 998×97.
6. The same Sutra can be used for squaring numbers near a base. E.g. 97^2, 1006^2, 203^2.

✐ **Practice O** The following exercise contains a mixture of all the different types of multiplication we have seen so far:

a 654 × 3	**b** 86 × 98	**c** 97 × 92
d 73 × 4	**e** 7 × 22	**f** 16 × 24
g 798 × 997	**h** 8899 × 9993	**i** 106^2
j 996^2	**k** 103 × 109	**l** 123 × 104
m 203 × 209	**n** 188 × 197	**o** 87 × 97
p 32 × 33	**q** 2004 × 2017	**r** 9997 × 98
s 1023 × 102		

---

a	**1962**	**b**	**8428**	**c**	**8924**
d	**292**	**e**	**154**	**f**	**384**
g	**795606**	**h**	**88927707**	**i**	**11236**
j	**992016**	**k**	**11227**	**l**	**12792**
m	**42427**	**n**	**37036**	**o**	**8439**
p	**1056**	**q**	**4042068**	**r**	**979706**
s	**104346**				

“*all that the student has to do is to look for certain characteristics, spot them out, identify the particular type and apply the formula which is applicable thereto.*”
From “Vedic Mathematics”, Page 106.

LESSON 8
CHECKING AND DIVISIBILITY

SUMMARY

8.1 Digit Sum Check for Division – checking division sums.
8.2 The First by the First and the Last by the Last – more checking devices.
8.3 Divisibility by 4
8.4 Divisibility by 11

8.1 DIGIT SUM CHECK FOR DIVISION

1 Find **3456 ÷ 7**.

$$7\overline{)3\;4^{6}5^{2}6}$$
$$\underline{4\;\;9\;\;3\quad \text{remainder } 5}$$

The division is done in the usual way:
34÷7 = 4 rem 6, placed as shown,
65÷7 = 9 rem 2, placed as shown,
26÷7 = 3 rem 5, as shown.

So **3456 ÷ 7 = 493 rem 5**.

If the above division is correct then **493×7 + 5 = 3456**.
(Just as 7 ÷ 3 = 2 remainder 1 is correct because 2 × 3 + 1 = 7.)

We can check that **493×7 + 5 = 3456** is correct by changing each number to its digit sum:
493 has a digit sum of 7, 3456 has a digit sum of 9.

So **493 × 7 + 5 = 3456**

becomes **7 × 7 + 5 → 9**

and this is true in digit sums because 7×7=49→4, and 4+5 → 9.

(An alternative to the above line would be: 7×7 + 5 = 54, 54→9.)

2 Find **70809 ÷ 6**.

$$6\overline{)7^{1}0^{4}8^{0}0^{0}9}$$

$\underline{1\ \ 1\ \ 8\ \ 0\ \ 1\ \ \text{rem } 3}$ this is the answer and for the

check you show that 11801× 6 + 3 = 70809 is true in digit sums.
This becomes 2×6 + 3 → 6 in digit sums and it is correct
since 2×6=3 in digit sums and 3 + 3 = 6.

✐ **Practice A** Divide the following and check using the digit sums:

a $3\overline{)4\ 6\ 8\ 1}$ **b** $4\overline{)9\ 1\ 3}$ **c** $5\overline{)7\ 0\ 3\ 2}$

d $6\overline{)3\ 2\ 1}$ **e** $7\overline{)2\ 2\ 2}$ **f** $8\overline{)9\ 0\ 8\ 0}$

g $9\overline{)1\ 0\ 0\ 1}$ **h** $2\overline{)3\ 4\ 5\ 6\ 7}$

---

a **1560 r1 (3×3+1→1)** **b** **228 r1 (3×4+1→4)** **c** **1406 r2 (2×5+2→3)**
d **53 r3 (8×6+3→6)** **e** **31 r5 (4×7+5→6)** **f** **1135 r0 (1×8+0→8)**
g **111 r2 (3×9+2→2)** **h** **17283 r1 (3×2+1→7)**

8.2 THE FIRST BY THE FIRST AND THE LAST BY THE LAST

THE FIRST BY THE FIRST

The First by the First and the Last by the Last is useful for giving approximate answers to sums. Sometimes you may only want to find the first figure of an answer and the number of noughts following it, rather than work out the whole sum. Then you can use this method.

3 **32 × 41** is approximately **1000**.

By multiplying the first figure of each number together you find that 32 × 41 is approximately 30 × 40, which is 1200.
So you expect the answer to be about 1000, rounding off to the nearest thousand.

4 Find the approximate value of **641 × 82**.

You want the first figure of the answer and the number of 0's that come after it.
Since 600 × 80 = 48,000 and you know the answer will be more than this you can say the answer is about 50,000 (to the nearest 10,000).

5 Find the approximate value of **39 × 63**.

39 is close to 40 so that *the first by the first* gives 40 × 60 = 2400.
So you can say 2000.

6 Find an approximate value for **383 × 88**.

400 × 90 = 36,000 and the answer must be below this because both 400 and 90 are above the original numbers, so you can say 383 × 88 ≈ 30,000.

Note the symbol ≈ for **approximately equal to**.

So you see that *The First by the First* gives us the first figure of the answer; and the number of figures in the answer is also evident.

You may not always be certain of the first figure (as in the last example) but you will never be more than one out.

✐ **Practice B** Approximate the following:

a 723 × 81 **b** 67 × 82 **c** 4133 × 572

d 38 × 49 **e** 6109 × 377 **f** 3333 × 4444

g 1812 × 1066

---

a 60,000 **b 5000 or 6000** **c 2,000,000**
d 2000 **e 2,000,000** **f 10,000,000**
g 2,000,000

The Sutra (in fact it is a sub-Sutra) *The First by the First and the Last by the Last* is used in many ways. For example in measuring or drawing a line with a ruler (or an angle with a protractor) we line the first point of the line with the first mark on the ruler and note the position of the last point on the ruler.

See also Section 10.4. This Sutra is also useful in recurring decimals, divisibility and factorizing quadratics, cubics etc. (see Reference 3).

THE LAST BY THE LAST

The last figure of a calculation can be seen by looking at the last figures in the sum.

7 **72 × 83** ends in **6**.

by multiplying the last figure of each number together you get the last figure of the answer:
2 × 3 = **6**.

8 **383 × 887** ends in **1**.

since 3 × 7 = 21, which ends with a **1**.

9 **23 × 48 × 63** ends in a **2**.

Because 3×8 ends in a 4 and 4×3=12 ends in a **2**.

✎ **Practice C** What is the last figure in the following sums?

a 456 × 567	**b** 76543 × 97	**c** 67 × 78 × 89
d 789 + 987	**e** 346 × 564	**f** 5328 + 9845

---

a 2 b 7 c 4
d 6 e 4 f 3

8.3 DIVISIBILITY BY 4

The formula *The Ultimate and Twice the Penultimate* can be used to test whether a number can be divided exactly by four.

The **ultimate** means the last figure,

and the **penultimate** is the figure before the last one.

10 So in the number **12376** the formula tells you to add up the 6 and twice the 7.
This gives you 20, and since **4 goes into 20 it will also go exactly into 12376**.

So when using *The Ultimate and Twice the Penultimate*
you add the last figure to twice the one before it,
and if 4 divides into the result then the number is divisible by 4.
Otherwise it is not divisible by 4.

11 In the number **5554** the formula gives us 4 plus twice 5, which is 14.
But 4 will not divide exactly into 14 so **5554 is not divisible by 4**.

✐ **Practice D** For each of the numbers below, write down the totals this formula gives you and then write down whether 4 divides into the number or not.

a 246 **b** 656 **c** 92

d 5573 **e** 7624 **f** 345678

a 14, no **b 16, yes** **c 20, yes**
d 17, no **e 8, yes** **f 22, no**

8.4 DIVISIBILITY BY 11

Testing for divisibility by 11 is particularly easy and comes under the formula *By Addition and by Subtraction.*

12 Is **7282231** divisible by 11?

We add all the digits in the odd positions and all the digits in the even positions and subtract the smaller result from the larger result.
If we end up with 0 or 11 or any multiple of 11 then the number is divisible by 11.

7 2 8 2 2 3 1 in the odd positions: 7 + 8 + 2 + 1 = 18
in the even positions: 2 + 2 + 3 = 7

Since here 18 – 7 = 11 the number **7282231 is divisible by 11**.

✎ **Practice E** Test the following numbers for divisibility by 11:

a 5192	**b** 3476	**c** 1358016
d 85547	**e** 570317	**f** 1030607

---

a Yes	**b Yes**	**c Yes**
d Yes	**e Yes**	**f No**

REMAINDER AFTER DIVISION BY 11

You have just seen, in the last exercise, that we find if a number is divisible by 11 by adding alternate figures and subtracting.

E.g. for 727 we get 14–2=**12**.
Since 12 is not a multiple of 11 the number is not divisible by 11.

But this 12 is the remainder after division by 11.
Actually as 12 is 1 more than 11 we can say that the smallest remainder is **1**.
Note that we do the figures in the **odd** positions **minus** the figures in the **even** positions.

> **13** To get the remainder for 38042 we find (3+0+2) – (8+4) = **–7**.
>
> You can add 11 to this –7 to get **4** as the smallest remainder (either –7 or 4 will do here).

✎ **Practice F** Find the remainder from 11 for each of the following numbers:

a 71263	**b** 45678	**c** 203527	**d** 67
e 349	**f** 3817	**g** 1827	**h** 8351
i 481	**j** 34143	**k** 523281	**l** 909192

---

a 5	**b 6**	**c 5**	**d 1**
e 8	**f 0**	**g 1**	**h –9 or 2**
i –3 or 8	**j –1 or 10**	**k 0**	**l –2 or 9**

ANOTHER DIGIT SUM CHECK

You are already familiar with the digit sum check which helps to show if a calculation is correct.

For example, $2434 \times 32 = 77888$ is confirmed by the digit sums because adding the digits gives $4 \times 5 \rightarrow 2$, which is correct in digit sums.

This works because adding the digits in a number gives the remainder of the number after division by 9.

A similar method works by using the remainders of numbers after division by **11** rather than 9.

14 Suppose we want another check for the sum: **2434 × 32 = 77888**.

We find the remainders for each of the 3 numbers as in the exercise above.

Replacing the numbers by their remainders we get: $3 \times 10 \rightarrow 8$ and this is correct in this arithmetic as 30 clearly has a remainder of 8 after division by 11.

✏ **Practice G** Which of the following sums are correct according to the alternative digit sum check?

a 213312 × 45 = 9599040 **b** 234 × 234 = 54756 **c** 3741 × 45 = 186345

d 86 × 68 = 5848 **e** 876 × 333 = 290808 **f** 1011 × 1101 = 1113111

---

a 0×1=0: correct b 3×3=9: correct c 1×1=5: incorrect
d –2×2=7: correct e 7×3=1: incorrect f –1×1=–1: correct

LESSON 9
BAR NUMBERS

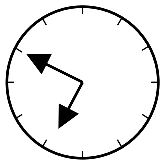
SUMMARY

9.1 **Removing Bar Numbers –** converting numbers containing a negative digit to positive form.
9.2 **Subtraction –** a general subtraction method.
9.3 **Creating Bar Numbers –** removing digits over 5 from a number.
9.4 **Using Bar Numbers –** some applications of bar numbers.

9.1 REMOVING BAR NUMBERS

The number 19 is very close to 20.
And it can therefore be conveniently written in a different way: as $2\bar{1}$

$2\bar{1}$ means 20 – 1, the minus is put on top of the 1.
Similarly $3\bar{1}$ means 30 – 1 or 29.
And $4\bar{2}$ means 38.
This is like telling the time when we say 'ten to seven' or instead of 6:50.



We pronounce $4\bar{2}$ as "four, bar two" because the 2 has a bar on top.

1 $\mathbf{7\bar{2} = 68}$,

$\mathbf{86\bar{1} = 859}$, because $6\bar{1} = 59$ (the 8 is unchanged),

$\mathbf{127\bar{2} = 1268}$, because $7\bar{2} = 68$,

$\mathbf{6\bar{3}0 = 570}$, because we have 600 – 30 (or because $6\bar{3} = 57$).

Practice A Convert the following numbers:

a $6\bar{1}$	**b** $8\bar{2}$	**c** $3\bar{3}$	**d** $5\bar{7}$	**e** $46\bar{2}$
f $999\bar{1}$	**g** $1\bar{2}$	**h** $11\bar{1}$	**i** $12\bar{3}$	**j** $3\bar{4}0$

a **59**	**b** **78**	**c** **27**	**d** **43**	**e** **458**
f **9989**	**g** **8**	**h** **109**	**i** **117**	**j** **260**

Any digit in a number may have a bar on it.

2 How would you remove the bar number in $\mathbf{5\bar{1}3}$?

The best way is to split the number into two parts: $5\bar{1}/3$.
Since $5\bar{1} = 49$, the answer is **493**.

If a number has a bar number in it split the number after the bar.

3 $\mathbf{7\bar{3}1} = 7\bar{3}/1 = \mathbf{671}$,

$\mathbf{52\bar{4}2} = 52\bar{4}/2 = \mathbf{5162}$,

$\mathbf{3\bar{2}15} = 3\bar{2}/15 = \mathbf{2815}$ since $3\bar{2} = 28$,

$\mathbf{5\bar{1}3\bar{2}} = 5\bar{1}/3\bar{2} = \mathbf{4928}$ since $5\bar{1} = 49$ and $3\bar{2} = 28$,

$\mathbf{3\bar{1}3\bar{2}3\bar{3}} = 3\bar{1}/3\bar{2}/3\bar{3} = \mathbf{292827}$.

Practice B Remove the bar numbers:

a $6\bar{1}4$	**b** $4\bar{2}3$	**c** $5\bar{2}5$	**d** $3\bar{1}7$
e $45\bar{2}3$	**f** $333\bar{2}3$	**g** $5\bar{1}32$	**h** $6\bar{2}7\bar{3}$
i $2\bar{1}1$	**j** $41\bar{3}1$	**k** $1\bar{3}15\bar{1}$	**l** $1\bar{3}1$

a	**594**	**b**	**383**	**c**	**485**	**d**	**297**
e	**4483**	**f**	**33283**	**g**	**4932**	**h**	**5867**
i	**191**	**j**	**4071**	**k**	**7149**	**l**	**71**

Next suppose the bar spans more than one digit in a number.

ALL FROM 9 AND THE LAST FROM 10

So far we have only had a bar on a single figure.
But we could have two or more bar numbers together.

4 Remove the bar numbers in $\mathbf{5\overline{33}}$.

The 5 means 500, and $\overline{33}$ means 33 is to be subtracted.
So $5\overline{33}$ means 500 – 33, and we have met sums like this in Lesson 5.

500 – 33 = **467** because the 33 comes off one of the hundreds, so the 5 is reduced to 4.

And applying *All from 9 and the Last from 10* to 33 gives 67.

5 Similarly $\mathbf{7\overline{14}}$ = **686** the 7 reduces to 6 and the Sutra converts 14 to 86,

$\mathbf{26\overline{21}}$ = **2579** 26 reduces to 25,

$\mathbf{7\overline{02}}$ = **698** the Sutra converts 02 to 98,

$\mathbf{50\overline{3}}$ = **497** 50 is reduced to 49 (alternatively, write $50\overline{3}$ as $5\overline{03}$: see previous example),

$\mathbf{4\overline{20}}$ = $4\overline{2}0$ = **380**.

6 $\mathbf{4\overline{23}1}$ = **3771**.

Here we can split the number after the bar: $4\overline{23}/1$.
$4\overline{23}$ changes to 377, and we just put the 1 on the end: $4\overline{23}1$ = **3771**.

7 Similarly $\mathbf{5\overline{12}4}$ = $5\overline{12}/4$ = **4884**,

$\mathbf{3\overline{11}33}$ = $3\overline{11}/33$ = **28933**,

$\mathbf{5\overline{123}}$ = **4877**,

$\mathbf{3\overline{1}4\overline{31}}$ = $3\overline{1}/4\overline{31}$ = **29369**.

Practice C Remove the bar numbers:

a $6\overline{12}$	**b** $7\overline{33}$	**c** $5\overline{11}$	**d** $9\overline{04}$	**e** $72\overline{41}$	**f** $333\overline{22}$
g $6\overline{21}4$	**h** $5\overline{31}22$	**i** $33\overline{22}44$	**j** $7\overline{333}$	**k** $5\overline{104}$	**l** $44\overline{112}$

m $74\overline{031}$ **n** $7\overline{103}1$ **o** $6\overline{3322}$ **p** $3\overline{1}10\overline{2}$ **q** $3\overline{11}4\overline{1}$ **r** $3\overline{2}1\overline{22}$

---

a	**588**	**b**	**667**	**c**	**489**	**d**	**896**	**e**	**7159**	**f**	**33278**
g	**5794**	**h**	**46922**	**i**	**327844**	**j**	**6667**	**k**	**4896**	**l**	**43888**
m	**73969**	**n**	**68971**	**o**	**56678**	**p**	**29098**	**q**	**28939**	**r**	**28078**

ADVANTAGES OF BAR NUMBERS

Bar numbers are an ingenious device which we will be using in later work. Their main advantages are:

1. They give us flexibility: we use the vinculum when it suits us.
2. Large numbers, like 6, 7, 8, 9 can be avoided.
3. Figures tend to cancel each other, or can be made to cancel.
4. 0 and 1 occur twice as frequently as they otherwise would.

9.2 SUBTRACTION

These bar numbers give us an alternative way of subtracting numbers.
Pupils sometimes subtract in each column in a subtraction sum regardless of whether the top is greater than the bottom or not.
This method can however be used to give the correct answer.

8

$$\begin{array}{r} 4\ 4\ 4 \\ \underline{2\ 8\ 6}\ - \\ \hline \end{array}$$

Subtracting in each column we get 4–2 = 2, 4–8 = –4, 4–6 = –2.
Since these negative answers can be written with a bar on top we can write:

$$\begin{array}{r} 4\ 4\ 4 \\ \underline{2\ 8\ 6}\ - \\ \underline{2\ \overline{4}\ \overline{2}} \end{array}$$

and $2\overline{42}$ is easily converted into **158**.

9 Similarly

$$\begin{array}{r} 6\ 7\ 6\ 7 \\ \underline{1\ 9\ 0\ 8}\ - \\ \underline{5\ \overline{2}\ 6\ \overline{1}} \end{array} = \mathbf{4859}$$

Practice D Subtract using bar numbers:

a $\begin{array}{r} 543 \\ \underline{168} \; - \end{array}$	**b** $\begin{array}{r} 567 \\ \underline{279} \; - \end{array}$	**c** $\begin{array}{r} 804 \\ \underline{388} \; - \end{array}$	**d** $\begin{array}{r} 737 \\ \underline{558} \; - \end{array}$	**e** $\begin{array}{r} 6413 \\ \underline{1878} \; - \end{array}$
f $\begin{array}{r} 8024 \\ \underline{5339} \; - \end{array}$	**g** $\begin{array}{r} 6543 \\ \underline{2881} \; - \end{array}$	**h** $\begin{array}{r} 7103 \\ \underline{3991} \; - \end{array}$	**i** $\begin{array}{r} 4545 \\ \underline{1791} \; - \end{array}$	**j** $\begin{array}{r} 3204 \\ \underline{2081} \; - \end{array}$

---

a 375 **b** 288 **c** 416 **d** 179 **e** 4535
f 2685 **g** 3662 **h** 3112 **i** 2754 **j** 1123

9.3 CREATING BAR NUMBERS

We may also need to put numbers **into** bar form.

10

$79 = 8\bar{1}$ because 79 is 1 less than 80,

$239 = 24\bar{1}$ because $39 = 4\bar{1}$,

$7689 = 769\bar{1}$ because $89 = 9\bar{1}$.

$508 = 51\bar{2}$ 08 becomes $1\bar{2}$

Practice E Put the following into bar form:

a 49	**b** 58	**c** 77	**d** 88
e 69	**f** 36	**g** 17	**h** 359
i 848	**j** 7719	**k** 328	**l** 33339
m 609	**n** 708		

---

a $5\bar{1}$ **b** $6\bar{2}$ **c** $8\bar{3}$ **d** $9\bar{2}$
e $7\bar{1}$ **f** $4\bar{4}$ **g** $2\bar{3}$ **h** $36\bar{1}$
i $85\bar{2}$ **j** $772\bar{1}$ **k** $33\bar{2}$ **l** $3334\bar{1}$
m $61\bar{1}$ **n** $71\bar{2}$

One of the main advantages of bar numbers is that we can remove high digits in a number. For example writing **19** as $\mathbf{2\bar{1}}$ means we do not have to deal with the large 9.

11 Remove the large digits from **287**.

Here the 8 and the 7 are large (we say that 6, 7, 8, 9 are large digits).

So we write 287 as $\mathbf{3\overline{13}}$ the 2 at the beginning is increased to 3, and the Sutra *All from 9 and the Last from 10* is applied to 87 to give 13.

You will agree that 287 is 13 below 300, which is what $\mathbf{3\overline{13}}$ says.

12 Similarly **479 =** $\mathbf{5\overline{21}}$,

3888 = $\mathbf{4\overline{112}}$,

292 = $\mathbf{3\bar{1}2}$,

4884 = $\mathbf{5\overline{12}4}$,

77 = $\mathbf{1\overline{23}}$ (you can think of 77 as 077),

and so on.

✎ **Practice F** Remove the large digits from the following:

a 38	**b** 388	**c** 298	**d** 378
e 3991	**f** 3822	**g** 4944	**h** 390
i 299	**j** 98	**k** 87	**l** 888
m 996	**n** 2939	**o** 1849	**p** 7

---

a $4\bar{2}$	**b** $4\overline{12}$	**c** $30\bar{2}$	**d** $4\overline{22}$
e $4\overline{011}$	**f** $4\bar{2}22$	**g** $5\bar{1}44$	**h** $4\bar{1}0$
i $3\overline{01}$ **or** $30\bar{1}$	**j** $1\overline{02}$	**k** $1\overline{13}$	**l** $1\overline{112}$
m $1\overline{004}$	**n** $3\bar{1}4\bar{1}$	**o** $2\bar{2}5\bar{1}$ **or** $2\overline{151}$	**p** $1\bar{3}$

"And, in some very important and striking cases, sums requiring 30, 50, 100 or even more numerous and cumbrous "steps" of working (according to the current Western methods) can be answered in a single and simple step of work by the Vedic method! And little children (of only 10 or 12 years of age) merely look at the sums written on the blackboard (on the platform) and immediately shout out and dictate the answers from the body of the convocation hall (or other venue of demonstration). And this is because, as a matter of fact, each digit automatically yields its predecessor and its successor! and the children have merely to go on tossing off (or reeling off) the digits one after another (forwards or backwards) by mere mental arithmetic (without needing pen or pencil, paper or slate etc)!"

From "Vedic Mathematics", Page xvii.

9.4 USING BAR NUMBERS

Finally here are a few examples showing where bar numbers might be used.

13 **29 + 48 = 77**.

Writing 29 as $3\bar{1}$, or 48 as $5\bar{2}$:

$$\begin{array}{rl} 29 & \\ \underline{5\bar{2}} & + \\ \underline{77} & \end{array} \qquad \begin{array}{rl} 3\bar{1} & \\ \underline{48} & + \\ \underline{77} & \end{array}$$

14 **623 – 188 = 435**.

$$\begin{array}{rl} 623 & \\ \underline{2\bar{1}\bar{2}} & - \\ \underline{435} & \end{array}$$

15 **5032 + 7489 – 2883** = $10\bar{4}38$ = **9638**. We just add up the first digits of the first and second numbers and subtract the first digit of the third number. Similarly with the second, third and fourth digits.

16 **29 × 3 =** $3\bar{1} \times 3 = 9\bar{3}$ = **87**.

17 **87 ÷ 3 =** $9\bar{3} \div 3 = 3\bar{1}$ = **29**.

18 **41 ÷ 7 = 6 remainder** $\bar{\mathbf{1}}$.

These bar numbers can be very useful in more advanced work (see Manuals 2 and 3).

LESSON 10
SPECIAL MULTIPLICATION

SUMMARY

10.1 Multiplication by 11
10.2 By One More than the One Before – a special type of multiplication.
10.3 Multiplication by Nines
10.4 The First by the First and the Last by the Last – a special type of multiplication.
10.5 Using the Average – of numbers to find their product.
10.6 Special Numbers – spotting factors of certain special numbers in a multiplication sum.

If there is an easy way to do a particular sum, rather than using the general method, we call it a special method. For example to multiply a number by 10 we do not use 'long multiplication'. In the Vedic system there are many special methods, which adds to the fun: the general method is always there but there is often a quick way if you can spot it.

The special methods play a large part in encouraging mental mathematics. Everyone likes a short cut, whether it is a quick way to get from one place to another or an easy way of doing a particular calculation. Life is full of special methods: to tackle all similar situations in the same way is not the way most people like to function. Every mathematical calculation invites its own unique method of solution and we should encourage children to look at the special properties of each problem in order to understand it best and decide on the best way forward. This is surely the intelligent way to do mathematics.

10.1 MULTIPLICATION BY 11

The 11 times table is easy to remember, and multiplying longer numbers by 11 is also easy.
If you want, say, 52×11 you want eleven 52's.
This means you want ten 52's and one 52 or 520 + 52:

$$\begin{array}{r} 5\,2\,0 \\ 5\,2\,+ \\ \hline 5\,7\,2 \end{array}$$

note how the 2 and the 5 get added in the middle column.

> **1** Find **52 × 11**.
>
> To multiply a 2-figure number, like 52, by 11 you write down the number being multiplied, and put the total of the figures between the two figures: 572.
>
> So **52 × 11 = 572**, between the 5 and 2 we put 7, which is 5+2.

Practice A Multiply the following by 11:

a 23 × 11 **b** 61 × 11 **c** 44 × 11 **d** 50 × 11

---

a 253 b 671 c 484 d 550

And so we can often quickly tell if a number can be divided exactly by 11.

> **2** Is **473** divisible by 11?
>
> You can see that the middle number is the sum (total) of the outer numbers:
> 4 + 3 = 7.
>
> So the number is divisible by 11.

In the example above you also know how many times 11 divides into 473.
It must be 43 because 43 × 11 = 473.

Just look at the outer numbers 4 and 3.

Practice B Fill in the table below.

Number	Tick if Divisible	No. of Times it Divides
242		
594		
187		
791		
693		

Answers: 22, 54, 17, -, 63

"And as regards the time required by the students for mastering the whole course of Vedic Mathematics as applied to all its branches, we need merely state from our actual experience that 8 months (or 12 months) at an average rate of 2 or 3 hours per day should suffice for completing the whole course of mathematical studies on these Vedic lines instead of 15 or 20 years required according to the existing systems of Indian and also of foreign universities."
From "Vedic Mathematics", Page xvii.

CARRIES

Going back to multiplication by 11, there can sometimes be a carry, as the next example shows.

3 Find **58 × 11**.

The 5 and 8 here add up to 13 so the 1 has to be carried to the left:

58 × 11 = $\mathbf{5_1 38}$ **= 638**.

4 Find **47 × 11**.

The 4 and 7 here add up to 11 so again you carry 1 to the left:

47 × 11 = $\mathbf{4_1 17}$ **= 517**.

Practice C Try these:

a 68 × 11 **b** 79 × 11 **c** 47 × 11

d 86 × 11 **e** 55 × 11 **f** 93 × 11

---

a	**748**	**b**	**869**	**c**	**517**
d	**946**	**e**	**605**	**f**	**1023**

LONGER NUMBERS

This method can be easily extended to longer numbers.

5 Find **234 × 11**.

To multiply this 3-figure number by 11 you put the first and last figures of 234 as the first and last figures of the answer:

$$\begin{array}{cccc} & 2 \quad 3 \quad 4 & \\ 2 & ? \quad ? & 4 \end{array}$$

Then for the second figure you add the first two figures of 234,
And for the third figure you add the last two figures of 234:

$$\begin{array}{ccccccc} & 2 & & 3 & & 4 & \\ 2 & & 5 & & 7 & & 4 \end{array}$$

So **234 × 11 = 2574**.

Practice D Multiply the following by 11:

a 423 × 11 **b** 636 × 11 **c** 534 × 11

d 516 × 11 **e** 706 × 11 **f** 260 × 11

g 444 × 11 **h** 135 × 11 **i** 531 × 11

a	**4653**	**b**	**6996**	**c**	**5874**
d	**5676**	**e**	**7766**	**f**	**2860**
g	**4884**	**h**	**1485**	**i**	**5841**

When you add the first or last two figures you could get a 2-figure number, so that there is a carry figure.

6 Find **777 × 11**.

The method above gives: $7_{1}4_{1}47$ = **8547**. We simply carry the 1's over, as before.

Practice E Multiply by 11:

a 384 × 11 **b** 629 × 11 **c** 888 × 11

d 555 × 11 **e** 393 × 11 **f** 939 × 11

a	**4224**	**b**	**6919**	**c**	**9768**
d	**6105**	**e**	**4323**	**f**	**10329**

This can be extended to numbers of any size and also to multiplying by 111, 1111 etc.
This multiplication is useful in percentages work since if we want to increase a number by 10% we multiply it by 1.1, similarly with other percentage changes (see Manual 2 or The Cosmic Calculator, Book 2).

10.2 BY ONE MORE THAN THE ONE BEFORE

This special type of multiplication is for multiplying numbers whose first figures are the same and whose last figures add up to 10, 100 etc.

For example, 52 × 58, where both numbers start with 5 and 2 + 8 = 10.

7 Suppose we want to find **43 × 47** in which both numbers begin with 4 and the last figures (3 and 7) add up to 10.

Multiply 4 by the number *One More*: 4 × 5 = 20.

Then simply multiply the last figures together: 3 × 7 = 21.

So **43 × 47 = 2021** where 20 = 4×5, 21 = 3×7.

8 Similarly **62 × 68 = 4216** where 42 = 6×7, 16 = 2×8.

9 Find **204 × 206**.

Here both numbers start with 20, and 4 + 6 = 10, so the method applies.

204 × 206 = 42024 (420 = 20×21, 24 = 4×6).

✐ **Practice F** Multiply the following:

a 73 × 77 **b** 58 × 52 **c** 81 × 89 **d** 104 × 106

e 42 × 48 **f** 34 × 36 **g** 93 × 97 **h** 27 × 23

i 297 × 293 **j** 303 × 307

---

a 5621 b 3016 c 7209 d 11024
e 2016 f 1224 g 9021 h 621
i 87021 j 93021

10 **93 × 39** may not look like it comes under this particular type of sum,
but remembering the *Proportionately* formula we notice that 93 = 3×31,
and 31 × 39 does come under this type:

31 × 39 = 1209 (we put 09 as we need double figures here)

so **93 × 39 = 3627** (multiply 1209 by 3)

The thing to notice in the last example is that the 39 needs a 31 for the method to work here: and then we spot that 93 is 3×31.

11 Finally, consider **397 × 303**.

Only the 3 at the beginning of each number is the same, but the rest of the numbers (97 and 03) add up to 100.

So again the method applies, but this time we must expect to have four figures on the right-hand side:

397 × 303 = 120291 where 12 = 3×4, 0291 = 97×3.

✐ **Practice G** Multiply the following:

a 64 × 38	**b** 88 × 46	**c** 33 × 74	**d** 66 × 28
e 36 × 78	**f** 46 × 54	**g** 298 × 202	**h** 391 × 309
i 795 × 705	**j** 401 × 499		

a 2432	**b 4048**	**c 2442**	**d 1848**
e 2808	**f 2484**	**g 60196**	**h 120819**
i 560475	**j 200099**		

10.3 MULTIPLICATION BY NINES

The Vedic formula *By One Less Than the One Before*, which is the converse of the formula *By One More than the One Before* comes in here in combination with *All From 9 and the Last From 10*.

12 **763 × 999 = 762/237**.

The number being multiplied by 9's is first reduced by 1: 763–1 = **762**. This is the first part of the answer.
Then *All From 9 and the Last From 10* is applied to 763 to get **237**, which is the second part of the answer.

13 **1867 × 99999 = 1866/98133**.

Here, as 1867 has 4 figures, and 99999 has 5 figures, we suppose 1867 to be 01867.
This is reduced by 1 to give **1866** for the first part of the answer.
Then applying *All From 9. . .* to 01867 gives **98133** for the last part of the answer.

✐ **Practice H** Find the following:

a 89 × 99	**b** 82 × 99	**c** 19 × 99	**d** 45 × 99
e 778 × 999	**f** 7654 × 9999	**g** 79 × 999	**h** 124 × 9999
i 8989 × 99999	**j** 47 × 99999		

---

a 8811	**b 8118**	**c 1881**	**d 4455**
e 777222	**f 76532346**	**g 78921**	**h 1239876**
i 898891011	**j 4699953**		

10.4 THE FIRST BY THE FIRST AND THE LAST BY THE LAST

Products like 43 × 47 are easy to find because the first figures are the same and the last figures sum to 10.

Similarly products like **27 × 87** are also easy to find because the **last figures are the same** and the **first figures add up to 10**.

This comes under the Vedic formula *The First by the First and the Last by the Last.*

14 **27 × 87 = 23/49**.

The conditions are satisfied here as 2 + 8 = 10
and both numbers end in 7.

So **we multiply the first figure of each number together and add the last figure**: 2 × 8 = 16, 16 + 7 = **23** which is the first part of the answer.

Multiplying the last figures together: 7×7 = **49**: which is the last part of the answer.

15 **69 × 49 = 3381**.

in which **33** = 6×4 + 9, and **81** = 9×9.

✐ **Practice I** Multiply the following by this method:

a 38 × 78	**b** 26 × 86	**c** 91 × 11	**d** 59 × 59

e 63 × 43 **f** 24 × 84 **g** 88 × 28 **h** 29 × 89

i 97 × 17 **j** 64 × 44

The following can also be done like this if you use the *Proportionately* formula as well:

k 31 × 42 **l** 46 × 83 **m** 93 × 71 **n** 88 × 32

a	**2964**	**b**	**2236**	**c**	**1001**	**d**	**3481**
e	**2709**	**f**	**2016**	**g**	**2464**	**h**	**2581**
i	**1649**	**j**	**2816**				
k	**1302**	**l**	**3818**	**m**	**6603**	**n**	**2816**

10.5 USING THE AVERAGE

Here we look at a neat and easy way of multiplying numbers by using their average.
This comes under the formula *Specific General*.

16 Suppose we want to know **29 × 31**.
Since the average of 29 and 31 is 30, we might think that 29 × 31 is 30 × 30, or close to it.
In fact 29 × 31 = **899**
and this is just 1 below 900.

17 Now consider **28 × 32**. Again 30 is their average. 28 × 32 = **896** and this is 4 below 900.

18 For **27 × 33** whose average is also 30: 27 × 33 = **891**, which is 9 below 900.

In fact the rule is:

square the average and
subtract the square of the difference of either number from the average.

19 So **26 × 34** = $30^2 - 4^2$ = 900 – 16 = **884**.

And **58 × 62** = $60^2 - 2^2$ = 3600 – 4 = **3596**.

Similarly **94 × 106** = $100^2 - 6^2$ = 10,000 – 36 = **9964**.

And **37 × 33** = $35^2 - 2^2$ = 1225 – 4 = **1221**. See Section 12.1 for squaring numbers that end in 5.

This method is available for the product of any two numbers. Even if the average is not a very attractive number this method is still often better than multiplying the numbers. For example, for 67 × 69 it is easier to find $68^2 - 1$ than to multiply 67 by 69.

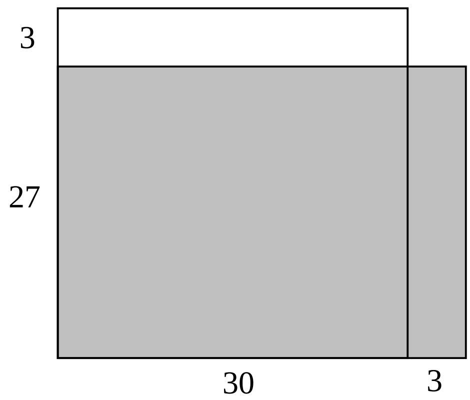
✎ **Practice J** Find:

a 49 × 51	**b** 27 × 33	**c** 57 × 63	**d** 64 × 66
e 85 × 65	**f** 55 × 95	**g** 33 × 47	**h** 91 × 99
i 44 × 48	**j** 43 × 47	**k** 74 × 86	**l** 98 × 102
m 62 × 38	**n** 48 × 72	**o** 73 × 93	**p** 196 × 204

a 2499	**b 891**	**c 3591**	**d 4224**
e 5525	**f 5225**	**g 1551**	**h 9009**
i 2112	**j 2021**	**k 6364**	**l 9996**
m 2356	**n 3456**	**o 6789**	**p 39984**

PROOF

A geometrical explanation for 27 × 33 is shown below.



The shaded rectangle is 27 by 33 and its area is 27 ×33.
The superimposed shape is a 30 by 30 square.
This shows that the square whose area is 30^2 is larger than the required rectangle by 3^2 units, as the top rectangle is 30 × 3 and the right-hand rectangle is 27 × 3, a difference of 3 ×3.

Here is an algebraic proof.

$(a + b)(a - b) = a^2 - b^2$, where a is the average and b the difference of each number from the average. So (a + b) is the higher number and (a – b) is the lower number.

10.6 SPECIAL NUMBERS

REPEATING NUMBERS

Some multiplications are particularly easy.

20 **23 × 101 = 2323**.

To multiply 23 by 101 we need 23 hundreds and 23 ones, which gives 2323.

The effect of multiplying any 2-figure by 101 is simply to make it repeat itself.

21 Similarly **69 × 101 = 6969**.

And **473 × 1001 = 473473**.

Here we have a 3-figure number multiplied by 1001 which makes the 3-figure number repeat itself.

22 **47 × 1001 = 47047**.

Here, because we want to multiply by 1001, we can think of 47 as 047.
So we get 047047, or just 47047.

23 **123 × 101** = 123_123 = **12423**.

Here we have 12300 + 123 so the 1 has to be carried over.

24 **28 × 10101 = 282828**.

✎ **Practice K** Find:

a 46 × 101 **b** 246 × 1001 **c** 321 × 1001

d 439 × 1001 **e** 3456 × 10001 **f** 53 × 10101

g 74 × 1001	**h** 73 × 101	**i** 29 × 1010101
j 277 × 101	**k** 521 × 101	**l** 616 × 101

---

a	**4646**	**b**	**246246**	**c**	**321321**
d	**439439**	**e**	**34563456**	**f**	**535353**
g	**74074**	**h**	**7373**	**i**	**29292929**
j	**27977**	**k**	**52621**	**l**	**62216**

This type of multiplication comes under the Sutra *By Mere Observation*.
Multiplications by 101 etc. are useful in percentages work as we multiply a number by 1.01 to increase it by 1% (see Manual 2 or The Cosmic Calculator, Book 2)

PROPORTIONATELY

25 **43 × 201 = 8643**.

Here we bring in the *Proportionately* formula: because we want to multiply by 201 rather than 101 we must put twice 43 (which is 86) then 43.

26 **31 × 10203 = 316293** we have 31×1, 31×2, 31×3.

Practice L Find:

a 54 × 201	**b** 32 × 102	**c** 333 × 1003	**d** 41 × 10201	**e** 33 × 30201
f 17 × 20102	**g** 13 × 105	**h** 234 × 2001	**i** 234 × 1003	**j** 43 × 203

---

a	**10854**	**b**	**3264**	**c**	**333999**	**d**	**418241**	**e**	**996633**
f	**341734**	**g**	**1365**	**h**	**468234**	**i**	**234702**	**j**	**8729**

DISGUISES

Now it is possible for a sum to be of the above type without it being obvious: it may be disguised.

If we know the factors of some of these special numbers (like 1001, 203 etc.) we can make some sums very easy.

Suppose for example you know that **3 × 67 = 201**.

27 **93 × 67 = 6231**.

$$\begin{aligned} \text{Since} \quad 3 \times 67 &= 201, \\ \text{therefore} \quad 93 \times 67 &= 31 \times (3 \times 67) \\ &= 31 \times 201 \\ &= \mathbf{6231}. \end{aligned}$$

In other words, we recognise that one of the special numbers (201 in this case) is contained in the sum (as 3 × 67).

Now suppose we know that **3 × 37 = 111.**

28 **24 × 37 = 888**.

We know that 3 × 37 = 111, which is a number very easy to multiply.

$$\begin{aligned} \text{So} \quad 24 \times 37 &= 8 \times (3 \times 37) \\ &= 8 \times 111 \\ &= \mathbf{888}. \end{aligned}$$

Also 19×21 = 399 = $40\bar{1}$.

29 **38 × 63 = 2394**.

Since 38 × 63 = 2 × 19 × 3 × 21 = 6 × (19 × 21) = 6 × $40\bar{1}$ = $240\bar{6}$ = **2394**.

If we know the factors of these special numbers we can make good use of them when they come up in a sum, and they arise quite frequently.

Below is a list of a few of these numbers with their factors:

67 × 3 = 201	**17 × 6 = 102**	**11 × 9 = $10\bar{1}$**
43 × 7 = 301	**13 × 8 = 104**	**19 × 21 = $40\bar{1}$**
7 × 11 × 13 = 1001	**29 × 7 = 203**	**23 × 13 = $30\bar{1}$**
3 × 37 = 111	**31 × 13 = 403**	**27 × 37 = $100\bar{1}$**

30 **62 × 39 = 2418**.

We see 31 × 13 contained in this sum:

$$\begin{aligned} 62 \times 39 &= 2\times31 \times 3\times13 \\ &= 2\times3 \times 31\times13 \\ &= 6 \times 403 \\ &= \mathbf{2418}. \end{aligned}$$

Practice M Use the special numbers to find:

a 29 × 28	**b** 35 × 43	**c** 67 × 93	**d** 86 × 63
e 77 × 43	**f** 26 × 77	**g** 34 × 72	**h** 57 × 21
i 58 × 63	**j** 26 × 23	**k** 134 × 36	**l** 56 × 29
m 93 × 65	**n** 54 × 74	**o** 39 × 64	**p** 51 × 42

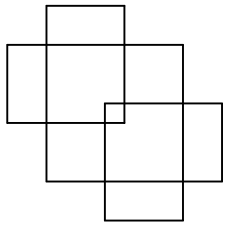
a	**812**	**b**	**1505**	**c**	**6231**	**d**	**5418**
e	**3311**	**f**	**2002**	**g**	**2448**	**h**	**1197**
i	**3654**	**j**	**598**	**k**	**4824**	**l**	**1624**
m	**6045**	**n**	**3996**	**o**	**2496**	**p**	**2142**

"These and many more interesting features there are in the Vedic decimal system, which can turn mathematics for the children from its present excruciatingly painful character to the exhilaratingly pleasant and even funny and delightful character it really bears ."
From "Vedic Mathematics", Page 239.

LESSON 11
GENERAL MULTIPLICATION

SUMMARY

11.1 Revision
11.2 Two-Figure Numbers – multiplying 2-figure numbers in one line, from left to right.
11.3 Moving Multiplier – multiplying long numbers by a 2-figure number.
11.4 Extension – multiplying 3-figure numbers.
11.5 Multiplying Binomials – using the same pattern.
11.6 Multiplying 3-Figure Numbers – extension of previous pattern.
11.7 Written Calculations – from left to right.



11.1 REVISION

We have seen various methods of multiplication but they were all for special cases, where some special condition was satisfied, like both numbers being close to 100 for example.
We come now to the general multiplication technique, by which any two numbers can be multiplied together in one line, by mere mental arithmetic.

First let us briefly revise how we multiply by a single figure number (as in Section 4.2).

You may wish to begin this lesson with written calculations rather than mental: if so go to Section 11.7, but you will need the methods described in Sections 11.2, 11.3, 11.6.

1 Find **74 × 8**.

We multiply each of the figures in 74 by 8 starting at the left:

7 × 8 = **56** and 4 × 8 = **32**.

These are combined by carrying the 3 in 32 over to the 6 in 56: 56,32 = 592.

The inner figures are merged together. So **74 × 8 = 592**.

2 Find **827 × 3**.

The three products are **24**, **6**, **21**.

The first two products are combined: 24,6 = 246 no carry here as 6 is a single figure, then 246 is combined with the 21: 246,21 = 2481. So **827 × 3 = 2481**.

3 Find **77 × 4**.

The products are **28**, **28**.

And 28,28 = 308 (the 28 is increased by 2 to 30). So **77 × 4 = 308**.

Practice A Multiply the following mentally:

a 73 × 3 **b** 63 × 7 **c** 424 × 4 **d** 777 × 3

e 654 × 3 **f** 717 × 8 **g** 876 × 7

---

a 219 **b 441** **c 1 696** **d 2 331**
e 1 962 **f 5 736** **g 6 132**

11.2 TWO-FIGURE NUMBERS

The *Vertically and Crosswise* formula gives us the pattern for multiplying any numbers. For 2-figure numbers it works like this.

4 Find **21 × 23**.

Think of the numbers set out one below the other:

$$\begin{array}{cc} 2 & 1 \\ \underline{2} & \underline{3} \times \\ \hline 4\,8\,3 \end{array}$$

<u>There are 3 steps</u>

A. Multiply vertically in the left-hand column: 2 × 2 = **4**, so 4 is the first figure of the answer.

$$\begin{array}{cc} 2 & 1 \\ 2 & 3 \times \\ \hline 4 & \end{array}$$

B. Multiply crosswise and add: 2 × 3 = 6, 1 × 2 = 2, 6 + 2 = **8**, so 8 is the middle figure of the answer.

$$\begin{array}{cc} 2 & 1 \\ \multicolumn{2}{c}{\times} \\ 2 & 3 \times \\ \hline 4 & 8 \end{array}$$

C. Multiply vertically in the right-hand column: 1 × 3 = **3**, 3 is the last figure of the answer.

$$\begin{array}{ccc} 2 & & 1 \\ 2 & & 3 \times \\ \hline 4 & 8 & 3 \end{array}$$

5 Find **14 × 21**.

$$\begin{array}{l} 1\quad 4 \\ \underline{2\quad 1} \;\times \\ \underline{2\;9\;4} \end{array}$$

A: vertically on the left: 1 × 2 = **2**,
B: crosswise: 1 × 1 = 1, 4 × 2 = 8 and 1 + 8 = **9**,
C: vertically on the right: 4 × 1 = **4**.

This is of course very easy and straightforward and is just mental arithmetic. We should now practice this *vertical and crosswise* pattern to establish the method.

Practice B Multiply mentally:

a 2 2	**b** 2 1	**c** 2 1	**d** 2 2	**e** 6 1	**f** 3 2	**g** 3 1	**h** 1 3
3 1 ×	3 1 ×	2 2 ×	1 3 ×	3 1 ×	2 1 ×	3 1 ×	1 3 ×

a	**682**	**b**	**651**	**c**	**462**	**d**	**286**
e	**1 891**	**f**	**672**	**g**	**961**	**h**	**169**

CARRIES

The previous examples involved no carry figures, so let us consider this next.

6 Find **23 × 41**.

$$\begin{array}{l} 2\quad 3 \\ \underline{4\quad 1} \;\times \\ \underline{9\;4\;3} \end{array}$$

The 3 steps give us: 2 × 4 = **8**,
2 × 1 + 3 × 4 = **14**,
3 × 1 = **3**.

The 14 here involves a carry figure, so in building up the answer mentally from the left we merge these numbers as before.

The mental steps are: 8
8,14 = 94 (the 1 is carried over to the left)
94,3 = 943

So **23 × 41 = 943**.

7 Find **23 × 34**.

$$\begin{array}{r} 2\ \ 3 \\ \underline{3\ \ 4} \times \\ \underline{7\,8\,2} \end{array}$$

The steps are:
6
6,17 = 77
77,12 = **782**

8 Find **33 × 44**.

$$\begin{array}{r} 3\ \ 3 \\ \underline{4\ \ 4} \times \\ \underline{1\,4\,5\,2} \end{array}$$

The steps are:
12
12,24 = 144
144,12 = **1452**

You can now multiply any two 2-figure numbers together in one line.

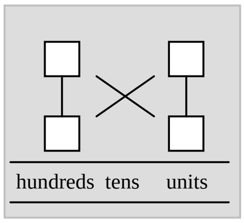
✐ **Practice C** Multiply the following mentally:

a 21 × 47	**b** 23 × 43	**c** 24 × 29	**d** 22 × 28	**e** 22 × 53	**f** 31 × 36
g 22 × 56	**h** 31 × 72	**i** 44 × 53	**j** 33 × 84	**k** 33 × 69	**l** 34 × 42
m 33 × 34	**n** 22 × 52	**o** 34 × 66	**p** 51 × 54	**q** 35 × 67	**r** 55 × 59
s 54 × 64	**t** 55 × 63	**u** 44 × 81	**v** 45 × 81	**w** 48 × 72	**x** 34 × 19

a 987	**b 989**	**c 696**	**d 616**	**e 1 166**	**f 1 116**
g 1 232	**h 2 232**	**i 2 332**	**j 2 772**	**k 2 277**	**l 1 428**
m 1 122	**n 1 144**	**o 2 244**	**p 2 754**	**q 2 345**	**r 3 245**
s 3 456	**t 3 465**	**u 3 564**	**v 3 645**	**w 3 456**	**x 646**

You may have found in this exercise that you prefer to start with the crosswise multiplications, and put the left and right vertical multiplications on afterwards.

EXPLANATION



It is easy to understand how this method works.
The vertical product on the right multiplies units by units and so gives the number of units in the answer. The crosswise operation multiplies tens by units and units by tens and so gives the number of tens in the answer. And the vertical product on the left multiplies tens by tens and gives the number of hundreds in the answer.

So this easy multiplication method, which is quite general, is also easy to understand. It can be done from left to right or right to left (see Section 11.7) it applies to algebraic expressions just as well (see Section 11.5) and it can be reversed to give a simple division method (see Section 16.4).

EXPLANATION OF EARLIER SPECIAL METHOD

We can now explain the special method of multiplication under *By One More than the One Before* from Section 10.2 for multiplying numbers like 72 × 78 in which the first figures are the same and the last figures add up to 10.

Using the present sutra for 72 × 78:

$$\begin{array}{r} 7\ \ 2 \\ \underline{\ \ 7\ \ 8} \\ \underline{5\ \ 6\ {}_{7}1\ {}_{1}6} \end{array}$$

We see that the cross-product is eight 7's and two 7's, that is ten 7's, or 70. The zero here ensures that the 2-digit product 2 × 8 = 16 can go straight into the last two places, and this will always happen when the conditions for this type of product are met. The 7 in 70 means an extra 7 in the left-hand product: so there are eight 7's altogether.

As the method of squaring numbers that end in 5 is a special case of the above (see Section 12.1), this can also be explained this way.

11.3 MOVING MULTIPLIER

In multiplying a long number by a single figure, for example 4321 × 2, we multiply each of the figures in the long number by the single figure. We may think of the 2 moving along the row, multiplying each figure vertically by 2 as it goes.

9 Find **4321 × 32**.

4 3 2 1
3 2

Similarly here we put 32 first of all at the extreme left.
Then vertically on the left, 4 × 3 = **12**.
And crosswise, 4×2 +3×3 = **17**.

4 3 2 1
  3 2

Then move the 32 along and multiply crosswise:
3×2 + 2×3 = **12**.

4 3 2 1
    3 2

Moving the 32 once again:
multiply crosswise, 2×2 + 1×3 = **7**.
Finally the vertical product on the right is 1×2 = **2**.

These 5 results (in bold), 12,17,12,7,2 are combined mentally, as they are obtained, in the usual way:

12,17 = 137
137,12 = 1382
1382,7,2 = **138272**

So we multiply crosswise in every position, but we multiply vertically also at the very beginning and at the very end.

10 Find **31013 × 21**.

Here the 21 takes the positions:

3 1 0 1 3
2 1
	3 1 0 1 3
	  2 1
		3 1 0 1 3
		    2 1
			3 1 0 1 3
			      2 1

The six mental steps give: 6,5,1,2,7,3
so the answer is **651273**.

Practice D Multiply using the moving multiplier method:

a 3 2 1 × 2 1

b 3 2 1 × 2 3

c 4 2 1 × 2 2

d 3 2 1 × 4 1

e 1 2 1 2 × 2 1

f 1 3 3 1 × 2 2

g 1 3 1 3 × 3 1

h 1 1 2 2 1 × 2 2

a 6 741 **b** 7 383 **c** 9 262 **d** 13 161
e 25 452 **f** 29 282 **g** 40 703 **h** 246 862

11.4 EXTENSION

11 Find **123 × 132**.

```
    1 2 3
    1 3 2 ×
1 6 2 3 6
```

The *Vertically and Crosswise* formula can be extended to deal with this, but in fact the previous vertical/crosswise/vertical pattern can be used on this sum also.

We can split the numbers up into 12/3 and 13/2, treating the 12 and 13 as if they were single figures:

```
 12    3
 13    2
162  3 6
```

Vertically 12 × 13 = **156**,
crosswise 12 × 2 + 3 × 13 = **63**,
vertically 3 × 2 = **6**.

Combining these mentally we get: 156
$15\underbrace{6,6}3 = 1623$
1623,6 = **16236**.

Practice E Multiply, treating the numbers as 2-figure numbers:

a 1 1 2
2 0 3

b 1 2 3
1 3 1

c 1 2 3
1 2 2

d 1 1 2
1 2 3

e 4 2 1
2 2

a **22 736** **b** **16 113** **c** **15 006**
d **13 776** **e** **9 262**

12 **304 × 412 = 125248**.

Here we may decide to split the numbers after the first figure: 3/04 × 4/12.

```
 3   04
 4   12
12 52 48
```

When we split the numbers in this way the answer appears **two digits at a time**.

The 3 steps of the pattern are: 3×4 = **12**,
3×12 + 4×4 = **52**,
4×12 = **48**.

These give the 3 pairs of figures in the answer.

Practice F Multiply using pairs of digits:

a $\begin{array}{r} 211 \\ \underline{304} \\ \underline{\quad\quad} \end{array}$	**b** $\begin{array}{r} 307 \\ \underline{407} \\ \underline{\quad\quad} \end{array}$	**c** $\begin{array}{r} 203 \\ \underline{432} \\ \underline{\quad\quad} \end{array}$	**d** $\begin{array}{r} 211 \\ \underline{311} \\ \underline{\quad\quad} \end{array}$
e $\begin{array}{r} 504 \\ \underline{504} \\ \underline{\quad\quad} \end{array}$	**f** $\begin{array}{r} 501 \\ \underline{501} \\ \underline{\quad\quad} \end{array}$	**g** $\begin{array}{r} 712 \\ \underline{112} \\ \underline{\quad\quad} \end{array}$	**h** $\begin{array}{r} 703 \\ \underline{211} \\ \underline{\quad\quad} \end{array}$

---

a **64 144** **b** **124 949** **c** **87 696** **d** **65 621**
e **254 016** **f** **251 001** **g** **79 744** **h** **148 333**

11.5 MULTIPLYING BINOMIALS

In the Vedic system we do not have one method for multiplying numbers and another for multiplying algebraic expressions. The same *Vertically and Crosswise* pattern can be used for both cases.

13 Multiply: **(x + 3)(x + 4)**.

We have to multiply x+3 by x+4.
This means that the x and the 3 in x+3 must both multiply the x and the 4 in x+4.

The best way to do this is to use the *Vertically and Crosswise* method.

Put one binomial under the other:
Multiply vertically on the left: x×x = $\mathbf{x^2}$.
Cross-multiply and add: 4×x + 3×x = **7x**.
Multiply vertically on the right: 3×4 = **12**.

$$\begin{array}{ccc} x & + & 3 \\ \underline{x} & \underline{+} & \underline{4} \\ \multicolumn{3}{c}{\underline{x^2 + 7x + 12}} \end{array}$$

It is just like multiplying two 2-figure numbers together.
Multiply from left to right or right to left: whichever you like.

Practice G Multiply:

a (x + 5)(x + 6) **b** (x + 2)(x + 9) **c** (x + 10)(x + 1) **d** (x + 20)(x + 20)

e (x + 1)(x + 1) **f** (x + 22)(x + 28) **g** (y + 52)(y + 4) **h** $(x + 4)^2$

---

a $\mathbf{x^2+11x+30}$ **b** $\mathbf{x^2+11x+18}$ **c** $\mathbf{x^2+11x+10}$ **d** $\mathbf{x^2+40x+400}$
e $\mathbf{x^2+2x+1}$ **f** $\mathbf{x^2+50x+616}$ **g** $\mathbf{y^2+56y+208}$ **h** $\mathbf{x^2+8x+16}$

14 Multiply **(2x + 5)(3x + 2)**.

$$\begin{array}{l} 2x \quad + 5 \\ \underline{3x \quad + 2} \\ \underline{6x^2 + 19x + 10} \end{array}$$

Vertically on the left: $2x \times 3x = \mathbf{6x^2}$.
Crosswise: $4x + 15x = \mathbf{19x}$.
Vertically on the right: $5 \times 2 = \mathbf{10}$.

15 Multiply **(x + 3y)(5x + 7y)**.

$$\begin{array}{l} x \quad + 3y \\ \underline{5x \quad + 7y} \\ \underline{5x^2 + 22xy + 21y^2} \end{array}$$

On the left: $x \times 5x = \mathbf{5x^2}$.
Crosswise: $7xy + 15xy = \mathbf{22xy}$.
On the right: $3y \times 7y = \mathbf{21y^2}$.

✐ **Practice H** Multiply the following:

a $(2x + 5)(x + 4)$ **b** $(x + 8)(3x + 11)$ **c** $(2x + 1)(2x + 20)$ **d** $(2x + 3)(3x + 7)$

e $(4x + 3)(x + 6)$ **f** $(3x + 17)(3x + 4)$ **g** $(6x + 1)(5x + 1)$ **h** $(2x + 5)(4x + 5)$

i $(3x + 3)(4x + 5)$ **j** $(2x + 3y)(2x + 5y)$ **k** $(5x + 2y)(2x + 5y)$ **l** $(4x + 3y)(7x + y)$

a	$2x^2+13x+20$	**b**	$3x^2+35x+88$	**c**	$4x^2+42x+20$	**d**	$6x^2+23x+21$
e	$4x^2+27x+18$	**f**	$9x^2+63x+68$	**g**	$30x^2+11x+1$	**h**	$8x^2+30x+25$
i	$12x^2+27x+15$	**j**	$4x^2+16xy+15y^2$	**k**	$10x^2+29xy+10y^2$	**l**	$28x^2+25xy+3y^2$

So, unlike the current system, we use the same method for algebraic products as for arithmetic ones.

Next we need to use the methods for combining negative numbers.

16 Multiply **(2x – 3)(3x + 4)**.

This is very similar.
$2x \times 3x = \mathbf{6x^2}$.
Crosswise: $8x - 9x = -1x$ or $\mathbf{-x}$.
And $-3 \times 4 = \mathbf{-12}$.

$$\begin{array}{l} 2x \quad - 3 \\ \underline{3x \quad + 4} \\ \underline{6x^2 - x - 12} \end{array}$$

17 Find **(x – 3)(x – 6)**.

Vertically: $x \times x = \mathbf{x^2}$.
Crosswise: $-6x - 3x = \mathbf{-9x}$.
Vertically: $-3 \times -6 = \mathbf{+18}$.

$$\begin{array}{l} x \quad - 3 \\ \underline{x \quad - 6} \\ \underline{x^2 - 9x + 18} \end{array}$$

✐ **Practice I** Multiply:

a $(x + 3)(x - 5)$ **b** $(x + 7)(x - 2)$ **c** $(x - 4)(x + 5)$ **d** $(x - 5)(x - 4)$

e $(x - 3)(x - 3)$ **f** $(2x - 3)(x + 4)$ **g** $(2x - 3)(3x + 6)$ **h** $(3x - 1)(x + 7)$

---

a $x^2-2x-15$ **b** $x^2+5x-14$ **c** x^2+x-20 **d** $x^2-9x+20$
e x^2-6x+9 **f** $2x^2+5x-12$ **g** $6x^2+3x-18$ **h** $3x^2+20x-7$

THE DIGIT SUM CHECK

The algebraic form of the digit sum check can be used.
If, for example, we wanted to check Example 14 above: $\mathbf{(2x + 5)(3x + 2)} = \mathbf{6x^2 + 19x + 10}$
we check that the product of the sum of the coefficients in the brackets on the left-hand side equals the sum of the coefficients on the right-hand side.

That is $(2 + 5)(3 + 2) = 6 + 19 + 10$.
Since both sides come to 35 this confirms the answer.

11.6 MULTIPLYING 3-FIGURE NUMBERS

18 Find **504 × 321**.

```
  5 0 4
  3 2 1
 161784
```

The extended pattern for multiplying 3-figure numbers is as follows.

A Vertically on the left, 5×3 = **15**.

```
 5 0 4
 |
 3 2 1
15
```

B Then crosswise on the left,
5×2 + 0×3 = 10.
Combining the 15 and 10 as before:
15,10 = **160**.

```
 5 0 4
   ×
 3 2 1
160
```

C Next we take 3 products and add them up,
5×1 + 0×2 + 4×3 = 17. And 160,17 = **1617**.

(actually we are gathering up the hundreds by multiplying hundreds by units, tens by tens and units by hundreds)

```
 5 0 4
 3 2 1
1617
```

D Next we multiply crosswise on the right,
$0 \times 1 + 4 \times 2 = 8$: 1617,8 = **16178**.

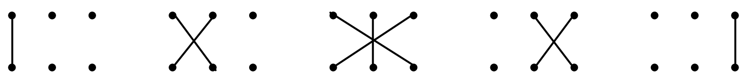
$$\begin{array}{ccc} 5 & 0 & 4 \\ & \times & \\ \underline{3} & \underline{2} & \underline{1} \\ \end{array}$$
$\underline{16178}$

E Finally, vertically on the right,
$4 \times 1 = 4$: 16178,4 = **161784**.

$$\begin{array}{ccc} 5 & 0 & 4 \\ & & | \\ \underline{3} & \underline{2} & \underline{1} \\ \end{array}$$
$\underline{161784}$

Note the symmetry in the 5 steps:
first there is 1 product, then 2, then 3, then 2, then 1.

We may summarise these steps as shown below:



A **B** **C** **D** **E**

19 Find **321 × 321**.

$$\begin{array}{r} 3\ 2\ 1 \\ \underline{3\ 2\ 1} \times \\ \underline{103041} \end{array}$$

The 5 results are 9,12,10,4,1.
The mental steps are 9
$\underset{\smile}{9,1}2$ = 102
$10\underset{\smile}{2,1}0$ = 1030
1030,4,1 = 103041

Sometimes we have a choice about how we multiply.

20 Find **123 × 45**.

This can be done with the moving multiplier method or by the smaller vertical and crosswise pattern, treating 12 in 123 as a single digit.
Alternatively, we can put 045 for 45 and use the extended vertical and crosswise pattern:

$$\begin{array}{r} 1\ 2\ 3 \\ \underline{0\ 4\ 5} \\ \underline{5535} \end{array}$$

For the 5 steps we get 0,4,13,22,15.
Mentally we think 4; 53; 552; 5535.

"We thus follow a process of ascent and descent (going forward with the digits on the upper row and coming rearward with the digits on the lower row)."
From "Vedic Mathematics", Page 42.

Practice J Multiply (there are no carries in the first few sums):

a	1 2 1	**b**	1 3 1	**c**	1 2 1	**d**	3 1 3
	1 3 1		2 1 2		2 2 2		1 2 1
e	2 1 2	**f**	1 2 3	**g**	2 1 2	**h**	2 2 2
	3 1 3		3 2 1		4 1 4		3 3 3
i	2 4 6	**j**	1 0 5	**k**	1 0 6	**l**	5 1 5
	3 3 3		5 0 7		2 2 2		5 5 5
m	4 4 4	**n**	3 2 1	**o**	1 2 3	**p**	1 2 4
	7 7 7		3 2 1		2 7 1		3 5 6

a	**15 851**	**b**	**27 772**	**c**	**26 862**	**d**	**37 873**
e	**66 356**	**f**	**39 483**	**g**	**87 768**	**h**	**73 926**
i	**81 918**	**j**	**53 235**	**k**	**23 532**	**l**	**285 825**
m	**344 988**	**n**	**103 041**	**o**	**33 333**	**p**	**44 144**

11.7 WRITTEN CALCULATIONS

It is also useful to be able to write out our multiplications.
In the Vedic system we can do this from left to right or from right to left.
Here we use the right to left method, but the formula is the same: *Vertically and Crosswise***.**

21 Find **42 × 31**.

The sum is set out as before:

A. We **multiply vertically on the right**: 2×1 = **2**,
and put this down as the right-hand figure of the answer.

$$\begin{array}{cccc} & & 4 & 2 \\ & & 3 & 1 \\ \hline 1 & 3 & 0 & 2 \\ & {}_1 & & \end{array}$$

B. Then we **multiply crosswise** and add to get 4+6 = **10**.
So we put down 0 and carry 1 to the left.

C. Finally we **multiply vertically on the left**: 4×3 = **12**,
12 + the carried 1 makes 13, which we put down.

22 Find **86 × 23**.

A. The method is as above: vertically on the right, 6×3 = **18**, put down 8 carry 1.
B. Crosswise, 24 + 12 = **36**, 36 + carried 1 = 37, put down 7 carry 3.
C. Vertically on the left, 8×2 = **16**, 16 + carried 3 = 19, put down 19.

$$\begin{array}{cccc} & & 8 & 6\\ & & 2 & 3\\ \hline 1 & 9 & 7 & 8\\ & {}_3 & {}_1 & \\ \hline \end{array}$$

23 Find **4321 × 24**.

Here we can use the moving multiplier method.
A. First, vertically on the right, 1×4 = 4, put it down.
B. Crosswise, 8+2 = 10, put down 0, carry 1.
C. Next we cross-multiply the 32 with the 24, this gives 12+4 = 16, 16 + carried 1 gives 17, put down 7 carry 1.
D. Then cross-multiply the 43 with the 24, this gives 16+6 = 22, 22 + carried 1 gives 23, put down 3 carry 2.
E. Vertically on the left, 4×2 = 8, 8 + carried 2 gives 10, put down 10.

$$\begin{array}{cccccc} & & 4 & 3 & 2 & 1\\ & & & & 2 & 4\\ \hline 1 & 0 & 3 & 7 & 0 & 4\\ & {}_2 & {}_1 & {}_1 & & \\ \hline \end{array}$$

24 Find **234 × 234**.

$$\begin{array}{ccccc} & & 2 & 3 & 4\\ & & 2 & 3 & 4\\ \hline 5 & 4 & 7 & 5 & 6\\ {}_1 & {}_2 & {}_2 & {}_1 & \\ \hline \end{array}$$

We simply do the same operations as shown in Section 11.6 but start at the right side:
4×4 = 16, put down 6 and carry 1 to the left.
3×4 + 4×3 = 24, 24 + carried 1 = 25, put down 5 and carry 2.
And so on.

✐ **Practice K** Multiply the following from right to left:

a 31 × 41 **b** 23 × 22 **c** 61 × 42

d 52 × 53 **e** 54 × 45 **f** 78 × 33

g 17 × 71 **h** 88 × 88 **i** 231 × 32

j 416 × 41 **k** 182 × 23 **l** 473 × 37

m 5432 × 32 **n** 6014 × 24 **o** 3333 × 22

p 444 × 333 **q** 543 × 345 **r** 707 × 333

a	**1 271**	**b**	**506**	**c**	**2 562**
d	**2756**	**e**	**2 430**	**f**	**2 574**
g	**1 207**	**h**	**7 744**	**i**	**7 392**
j	**17 056**	**k**	**4 186**	**l**	**17 501**
m	**173 824**	**n**	**144 336**	**o**	**73 326**
p	**147852**	**q**	**187335**	**r**	**235431**

SETTING THE SUMS OUT

In Example 24 each of the five steps had a center of symmetry. The five dots on the right show these five centers and as we move from left to right or right to left through the sum it is as if there is a dot moving through the sum.

$$\begin{array}{rl} \mathbf{2\ \ 3\ \ 4} & \\ \bullet\bullet\bullet\bullet\bullet & \\ \underline{\mathbf{2\ \ 3\ \ 4}} & \times \\ \underline{\mathbf{54756}} & \end{array}$$

In the calculation shown here the units figure of the result of each of the five steps is placed under the dot for that step.

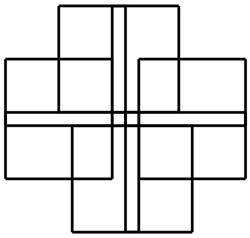
Other ways of setting the sums and answers out are possible and may be preferred.

"On seeing this kind of work actually being performed by the little children, the doctors, professors and other "big-guns" of mathematics are wonder struck and exclaim: "Is this mathematics or magic?" And we invariably answer and say: "It is both. It is magic until you understand it; and it is mathematics thereafter"; and then we proceed to substantiate and prove the correctness of this reply of ours!"

From "Vedic Mathematics", Page xvii.

LESSON 12
SQUARING

SUMMARY



12.1 Squaring Numbers that end in 5
12.2 Squaring Numbers Near 50
12.3 General Squaring – from left to right.
12.4 Number Splitting – to simplify squaring calculations.
12.5 Algebraic Squaring
12.6 Digit Sums of squares – properties of square numbers.
12.7 Square Roots of Perfect Squares – where the answer is a 2-figure number.
12.8 3 and 4-Figure Numbers – squaring bigger numbers.

12.1 SQUARING NUMBERS THAT END IN 5

Squaring is multiplication in which a number is multiplied by itself:
so 75 × 75 is called "75 squared" and is written 75^2.

The formula *By One More Than the One Before* provides a beautifully simple way of squaring numbers that end in 5.

1 In the case of $\mathbf{75^2}$, we simply multiply the 7 (the number before the 5) by the next number up, 8. This gives us 56 as the first part of the answer, and the last part is simply 25 (5^2).

So $\mathbf{75^2 = 56/25}$ where 56=7×8, 25=5^2.

2 Similarly $\mathbf{65^2 = 4225}$ 42=6×7, 25=5^2.

3 And $\mathbf{25^2 = 625}$ where 6=2×3.

4 Also since 4½= 4.5, the same method applies to squaring numbers ending in ½.
So $\mathbf{4\frac{1}{2}^2 = 20\frac{1}{4}}$, where 20 = 4×5 and ¼ = $\frac{1}{2}^2$.

The method can be applied to numbers of any size:

5 $\mathbf{305^2 = 93025}$ where 930 = 30×31.

Even for large numbers like, say, 635, it is still easier to multiply 63 by 64 and put 25 on the end than to multiply 635 by 635.

Algebraic Proof: $(ax + 5)^2 = a(a + 1)x^2 + 25$, where x = 10. See also end of section 11.2.

Practice A Square the following numbers:

a	55	**b**	15	**c**	8½	**d**	95
e	105	**f**	195	**g**	155	**h**	245
i	35	**j**	20½	**k**	8005	**l**	350

What number, when squared, gives:

m 2025 **n** 30¼ **o** 902500

---

a	**3025**	**b**	**225**	**c**	**72¼**	**d**	**9025**
e	**11025**	**f**	**38025**	**g**	**24025**	**h**	**60025**
i	**1225**	**j**	**420¼**	**k**	**64080025**	**l**	**122500**
m	**45**	**n**	**5½**	**o**	**950**		

12.2 SQUARING NUMBERS NEAR 50

Here is another special squaring method.

6 **$53^2 = 2809$**.

The answer is in two parts: 28 and 09.
28 is simply the last figure, 3, increased by 25.
And 09 is just 3^2.

7 Similarly **$52^2 = 2704$** ($2 = 2 + 25$, $04 = 2^2$).

Algebraic Proof: $(50 + a)^2 = 100(25 + a) + a^2$.

✐ **Practice B** Find:

a 54^2	**b** 56^2	**c** 57^2	**d** 58^2	**e** 61^2
f 62^2	**g** 51^2			

a 2916	**b 3136**	**c 3249**	**d 3364**	**e 3721**
f 3844	**g 2601**			

8 **47^2 = 2209**.

Similarly, for numbers below 50 we take the deficiency from 50 (3 here) **from** 50, to get 47 in this case, and put the square of the deficiency, **9**.

In the proof above 'a' would take negative values for numbers below 50.

✐ **Practice C** Square the following numbers by this method:

a 46	**b** 44	**c** 42	**d** 39	**e** 43
f 49	**g** 41	**h** 37		

a 2116	**b 1936**	**c 1764**	**d 1521**	**e 1849**
f 2401	**g 1681**	**h 1369**		

12.3 GENERAL SQUARING

The *Vertically and Crosswise* formula simplifies nicely when the numbers being multiplied are the same, and gives us a very easy method for squaring numbers.

THE DUPLEX

We will use the term **Duplex**, D, as follows:

for 1 figure **D is its square**, e.g. D(4) = 4^2 = **16**;

for 2 figures **D is twice their product**, e.g. D(43) = 2×4×3 = **24**.

Practice D Find the Duplex of the following numbers:

a	5	**b**	23	**c**	55	**d**	2
e	14	**f**	77	**g**	26	**h**	90

---

a	**25**	**b**	**12**	**c**	**50**	**d**	**4**
e	**8**	**f**	**98**	**g**	**24**	**h**	**0**

The square of any number is just the total of its Duplexes, combined in the way we have been using for mental multiplication.

9 **43^2 = 1849**.

Working from left to right there are three duplexes in 43:D(4), D(43) and D(3).

D(4) = **16**, D(43) = **24**, D(3) = **9**,

combining these three results in the usual way we get

16
16,24 = 184
184,9 = **1849**.

10 **64^2 = 4096**.

D(6) = **36**, D(64) = **48**, D(4) = **16**,

So mentally we get

36
36,48 = 408
408,16 = **4096**.

Algebraic proof: $(10a + b)^2 = 100(a^2) + 10(2ab) + b^2$. This method can also be explained by multiplying a number by itself using the general multiplication method.

Practice E Square the following:

a	31	**b**	14	**c**	41	**d**	26
e	23	**f**	32	**g**	21	**h**	66
i	81	**j**	91	**k**	56	**l**	63
m	77	**n**	33				

a	**961**	**b**	**196**	**c**	**1681**	**d**	**676**
e	**529**	**f**	**1024**	**g**	**441**	**h**	**4356**
i	**6561**	**j**	**8281**	**k**	**3136**	**l**	**3969**
m	**5929**	**n**	**1089**				

Duplexes and squares of longer numbers are covered in Section 12.8

12.4 NUMBER SPLITTING

You may recall that we could sometimes group two figures as one when we were multiplying two 2-figure numbers together (see Section 11.4). This also applies to squaring.

11 **123^2 = 15129**.

Here we may think of 123 as 12/3, as if it were a 2-figure number:

D(12) = 12^2 = **144**,
D(12/3) = 2 × 12 × 3 = **72**,
D(3) = 3^2 = **9**.

Combining these: 14 4,7 2 = 1512, and 1512,9 = **15129**.

Practice F Square the following, grouping the first pair of figures together:

a 121 **b** 104 **c** 203 **d** 113

e 116 **f** 108 **g** 111

a **14 641** **b** **10 816** **c** **41 209** **d** **12 769** **e** **13 456** **f** **11 664** **g** **12321**

The other way of splitting the numbers, shown in Section 11.4 can also be used here.

12 **312^2 = 97344**.

Here we can split the number into 3/12 but we must work with **pairs of digits**:

D(3) = **9**, D(3/12) = **72**, D(12) = **144**.

Combining: 9,72 = 972 we can put both figures of 72 after the 9,
97 2,1 44 = **97344**.

Practice G Square the following, grouping the last 2 figures together:

a 211 **b** 412 **c** 304 **d** 902

e 407 **f** 222 **g** 711

a **44 521** **b** **169 744** **c** **92 416** **d** **813 604**
e **165 649** **f** **49 284** **g** **505 521**

12.5 ALGEBRAIC SQUARING

Exactly the same method we have been using for squaring numbers can be used for squaring algebraic expressions.

13 Find $\mathbf{(x + 5)^2}$.

This is just like squaring numbers: we find the duplexes of x, x+5 and 5.

D(x) = $\mathbf{x^2}$, D(x+5) = 2×x×5 = **10x**, D(5) = 5^2 = **25**.

So $\mathbf{(x + 5)^2 = x^2 + 10x + 25}$.

14 Find $\mathbf{(2x + 3)^2}$.

There are three Duplexes: D(2x) = $\mathbf{4x^2}$, D(2x+3) = 2×2x×3 = **12x**, D(3) = **9**.

So $\mathbf{(2x + 3)^2 = 4x^2 + 12x + 9}$.

15 Find $\mathbf{(x - 3y)^2}$.

Similarly: D(x) = $\mathbf{x^2}$, D(x–3y) = 2×x×–3y = **–6xy**, D(–3y) = $\mathbf{9y^2}$.

So $\mathbf{(x - 3y)^2 = x^2 - 6xy + 9y^2}$.

Practice H Square the following:

a (3x + 4) **b** (5y + 2) **c** (2x – 1) **d** (x + 7)

e (x – 5) **f** (x + 2y) **g** (3x + 5y) **h** (2a + b)

i (2x – 3y) **j** (x + y) **k** (x – y) **l** (x – 8y)

a	**$9x^2+24x+16$**	**b**	**$25y^2+20y+4$**	**c**	**$4x^2–4x+1$**	**d**	**$x^2+14x+49$**
e	**$x^2–10x+25$**	**f**	**$x^2+4xy+4y^2$**	**g**	**$9x^2+30xy+25y^2$**	**h**	**$4a^2+4ab+b^2$**
i	**$4x^2–12xy+9y^2$**	**j**	**$x^2+2xy+y^2$**	**k**	**$x^2–2xy+y^2$**	**l**	**$x^2–16xy+64y^2$**

12.6 DIGIT SUMS OF SQUARES

Investigations of square numbers can make interesting and useful lessons, leading for example to the following results.

> Square numbers only have digit sums of 1, 4, 7, 9
> and they only end in 1, 4, 5, 6, 9, 0.

This means that square numbers cannot have certain digit sums and they cannot end with certain figures.

In the exercise below some numbers cannot be square numbers according to the above results.

✐ **Practice I** Which are not square numbers (judging by the above results)?

a 4539 **b** 5776 **c** 6889 **d** 5271

e 104976 **f** 65436 **g** 27478 **h** 75379

a, d, f, g

If a number has a valid digit sum and a valid last figure that does not mean that it is a square number. The last number in the exercise, 75379, is not a square number even though it has an allowed digit sum of 4 and an allowed last figure of 9.

12.7 SQUARE ROOTS OF PERFECT SQUARES

16 Find $\sqrt{\mathbf{6889}}$.

First note that there are two groups of figures, 68’89, so we expect a 2-figure answer.

Next we use *The First by the First and the Last by the Last.* Looking at the 68 at the beginning we can see that since 68 is greater than 64 (8^2) and less than 81 (9^2) the first figure must be 8.

Or looking at it another way 6889 is between 6400 and 8100

$$\begin{aligned} 6400 &= 80^2 \\ \mathbf{6889} &= \mathbf{8?^2} \\ 8100 &= 90^2 \end{aligned}$$

so $\sqrt{6889}$ must be between 80 and 90. I.e. it must be eighty something.

Now we look at the last figure of 6889, which is 9.
Any number ending with 3 will end with 9 when it is squared so the number we are looking for could be 83.

But any number ending in 7 will also end in 9 when it is squared so the number could also be 87.

So is the answer 83 or 87?

There are two easy ways of deciding. One is to use the digit sums.
If $87^2 = 6889$ then converting to digit sums we get $6^2 \rightarrow 4$, which is not correct.
But $83^2 = 6889$ becomes $2^2 \rightarrow 4$, so the answer must be **83**.

The other method is to recall that since $85^2 = 7225$ and 6889 is **below** this
$\sqrt{6889}$ must be **below 85**. So it must be **83**.

To find the square root of a perfect 4-digit square number
we find the first figure by looking at the first figures
and we find two possible last figures by looking at the last figure.
We then decide which is correct either by considering the digit sums
or by considering the square of their mean.

17 Find $\sqrt{\mathbf{5776}}$.

The 57 at the beginning is between 49 and 64, so the first figure must be 7.

The 6 at the end tells us the square root ends in 4 or 6.
So the answer is 74 or 76.

$74^2 = 5776$ becomes $2^2 \rightarrow 7$ which is not true in terms of digit sums, so 74 is not the answer.
$76^2 = 5776$ becomes $4^2 \rightarrow 7$, which is true, so **76** is the answer.

Alternatively to choose between 74 and 76 we note that $75^2 = 5625$ and 5776 is greater than this so the square root must be greater than 75. So it must be **76**.

In the following exercise try to find the answers mentally if you can, writing down only the answers.

✐ **Practice J** Find the square root of:

a 2116	**b** 5329	**c** 1444	**d** 6724
e 3481	**f** 4489	**g** 8836	**h** 361
i 784	**j** 3721	**k** 2209	**l** 4225
m 9604	**n** 5929		

---

a	**46**	**b**	**73**	**c**	**38**	**d**	**82**
e	**59**	**f**	**67**	**g**	**94**	**h**	**19**
i	**28**	**j**	**61**	**k**	**47**	**l**	**65**
m	**98**	**n**	**77**				

As you will have seen, square numbers ending in 5 must have a square root ending in 5, there is only one possibility for the last figure.

12.8 3 AND 4-FIGURE NUMBERS

This follows on from Section 12.3.
As shown before, the duplex of a 1-digit number is its square: e.g. D(4) = 4^2 = 16.
And the duplex of a 2-digit number is twice the product of the digits: e.g. D(35) = 2×3×5 = 30.

We can also find the duplex of 3-digit numbers or bigger.

For 3 digits D is **twice the product of the outer pair + the square of the middle digit**,
e.g. D(137) = 2×1×7 + 3^2 = **23**;

for 4 digits D is **twice the product of the outer pair + twice the product of the inner pair**,
e.g. D(1034) = 2×1×4 + 2×0×3 = **8**;

D(10345) = 2×1×5 + 2×0×4 + 3^2 = **19**;

and so on.

✐ **Practice K** Find the duplex of the following numbers:

a 3	**b** 34	**c** 47	**d** 1	**e** 88
f 234	**g** 282	**h** 111	**i** 304	**j** 270
k 1234	**l** 3032	**m** 7130	**n** 20121	**o** 32104

---

a	**9**	**b**	**24**	**c**	**56**	**d**	**1**	**e**	**128**
f	**25**	**g**	**72**	**h**	**3**	**i**	**24**	**j**	**49**
k	**20**	**l**	**12**	**m**	**6**	**n**	**5**	**o**	**25**

As with 2-figure numbers the square of a number is just the total of its duplexes.

18 **341^2 = 116281**.

Here we have a 3-figure number:

D(3) = 9, D(34) = 24, D(341) = 22, D(41) = 8, D(1) = 1.

Mentally: 9,24 = 114

114,22 = 1162

1162,8,1 = **116281**.

19 **4332^2 = 18766224**.

D(4) = 16, D(43) = 24, D(433) = 33, D(4332) = 34,
D(332) = 21, D(32) = 12, D(2) = 4.

Mentally: 16,24 = 184

184,33 = 1873

1873,34 = 18764

18764,21 = 187661

187661,12 = 1876622

1876622,4 = **18766224**.

Practice L Square the following numbers:

a	212	**b**	131	**c**	204	**d**	513
e	263	**f**	264	**g**	313	**h**	217
i	3103	**j**	2132	**k**	1414	**l**	4144

a	**44 944**	**b**	**17 161**	**c**	**41 616**	**d**	**263 169**
e	**69 169**	**f**	**69 696**	**g**	**97 969**	**h**	**47 089**
i	**9 628 609**	**j**	**4 545 424**	**k**	**1 999 396**	**l**	**17 172 736**

"whatever is consistent with right reasoning should be accepted, even though it comes from a boy or even from a parrot; and whatever is inconsistent therewith ought to be rejected, although emanating from an old man or even from the great sage Shree Shuka himself".
quoted in "Vedic Mathematics", Page 1d.

LESSON 13
EQUATIONS

SUMMARY

13.1 One-step Equations
13.2 Two-Step Equations
13.3 Three-Step Equations

} – mental, one-line solutions.

13.1 ONE-STEP EQUATIONS

Equations like x + 39 = 70, x – 7 = 8, 3x = 15 and $\frac{x}{3} = 7$ are easily solved using the Vedic formula: *Transpose and Apply*.

Transpose means "reverse" and in solving equations *Transpose and Apply* means :

where a number is **added** to the x-term: **subtract**, on the other side
where a number is **subtracted**: **add**,
where the x-term is **multiplied**: **divide**,
where the x-term is **divided**: **multiply**.

✎ **Practice A** Solve the following equations, check each answer to make sure it is right:

a x + 3 = 10 **b** x – 3 = 10 **c** 20 + x = 100 **d** x – 19 = 44

e x + 88 = 100 **f** x – 3½ = 4½ **g** x + 123 = 1000 **h** x + 1.3 = 5

i 5x = 35 **j** 2x = 26 **k** 3x = 960 **l** 2x = 76

m 40x = 120 **n** 2½x = 10 **o** $\frac{x}{7} = 7$ **p** $\frac{x}{4} = 5$

a	**7**	**b**	**13**	**c**	**80**	**d**	**63**
e	**12**	**f**	**8**	**g**	**877**	**h**	**3.7**
i	**7**	**j**	**13**	**k**	**320**	**l**	**38**
m	**3**	**n**	**4**	**o**	**49**	**p**	**20**

This is, of course, just a matter of mental arithmetic, and can be taught as such.

13.2 TWO-STEP EQUATIONS

Sometimes two or more applications of the *Transpose and Apply* formula are needed, as the following examples show.

1 Solve **$2x + 3 = 13$**.

We take 3 from both sides of the equation: this gives $2x = 10$.

Then you can see that **$x = 5$** is the answer.

To check: $2\times\mathbf{5} + 3 = 13$ so it is correct.
There are two applications of *Transpose and Apply* here:
First the +3 indicates that we subtract 3 from 13 (to get 10),
then the 2x indicates that we divide 10 by 2.

2 Solve **$5x - 4 = 36$**.

Using the Sutra we add 4 to 36 to get 40,
then $40\div5 = 8$, so **$x = 8$**.

Check: $5\times\mathbf{8} - 4 = 36$.

Writing the sum out in steps like this is fine,

$$5x - 4 = 36$$
$$5x = 40$$
$$\underline{\mathbf{x = 8}}$$

but students should also be able to put the answer straight down.

3 Solve $\mathbf{\frac{x}{7} + 3 = 5}$.

Here we take 3 from 5 to get 2,
then multiply 2 by 7, so **$x = 14$**.

4 Solve $\mathbf{\frac{2x}{3} = 4}$.

Multiply 3 by 4 to get 12,
then $12\div2=6$, so **$x = 6$**.

5 Solve $\mathbf{\frac{x-3}{4} = 5}$.

Because all the left side is divided by 4 we begin by multiplying 5 by 4, then we add 3 to the result giving **x = 23**.

Practice B Solve the following equations mentally. Check your answers.

a $3x + 7 = 19$ **b** $2x + 11 = 21$ **c** $4x – 5 = 7$ **d** $3x – 8 = 10$

e $\frac{x}{3} + 4 = 6$ **f** $\frac{x}{2} - 8 = 2$ **g** $\frac{2x}{3} = 8$ **h** $\frac{x+4}{7} = 5$

i $\frac{x-21}{10} = 1$ **j** $2x + 1 = 3.8$

---

a 4 b 5 c 3 d 6
e 6 f 20 g 12 h 31
i 31 j 1.4

13.3 THREE-STEP EQUATIONS

Sometimes we need to take three steps to solve an equation. But it still just a matter of mental arithmetic.

6 Solve $\mathbf{\frac{3x}{5} + 4 = 10}$.

First 10 – 4 = 6, then 6×5 = 30, then 30÷3 = 10 so **x = 10**.

7 Solve $\mathbf{\frac{3x+2}{4} = 8}$.

First 8×4 = 32, then 32–2 = 30, then 30÷3 = 10 so **x = 10**.

8 Solve **$2(3x + 4) = 38$**.

The bracket here indicates that $3x + 4$ is being multiplied by the number outside the bracket, which is 2.
So we begin by dividing 38 by 2.

First $38 \div 2 = 19$, then $19 - 4 = 15$, then $15 \div 3 = 5$ so **$x = 5$**.

Alternatively, here, we can multiply the bracket out first:
If $2(3x + 4) = 38$ then $6x + 8 = 38$
and so $38 - 8 = 30$ and $30 \div 6 =$ **5**.

Practice C Solve the following mentally:

a $\frac{2x}{3} + 4 = 8$ **b** $\frac{3x}{5} - 4 = 5$ **c** $\frac{7x}{2} - 10 = 11$ **d** $\frac{3x}{8} + 17 = 20$

e $\frac{2x+1}{3} = 4$ **f** $\frac{2x-3}{5} = 3$ **g** $\frac{5x+2}{3} = 9$ **h** $\frac{6x-1}{7} = 5$

i $3(5x - 2) = 54$ **j** $8(x + 3) = 64$ **k** $3(7x - 3) = 33$ **l** $2(4x + 3) = 102$

a	**6**	**b**	**15**	**c**	**6**	**d**	**8**
e	**5.5**	**f**	**9**	**g**	**5**	**h**	**6**
i	**4**	**j**	**5**	**k**	**2**	**l**	**12**

"The underlying principle behind all of them is Paravartya Yojayet which means: 'Transpose and adjust'. The applications, however, are numerous and splendidly useful."
From "Vedic Mathematics", Page 103.

LESSON 14
FRACTIONS

SUMMARY

14.1 Vertically and Crosswise – addition and subtraction of fractions.
14.2 A Simplification
14.3 Comparing Fractions
14.4 Unification of Operations: +, –, ×, ÷ of fractions are all simply related.

14.1 VERTICALLY AND CROSSWISE

Addition and subtraction of fractions are usually found to be very difficult as the method is complicated and hard to remember. But the *Vertically and Crosswise formula* gives the answer immediately.

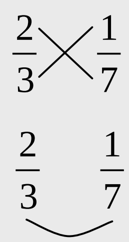
1 Find $\mathbf{\frac{2}{3}+\frac{1}{7}}$

We multiply crosswise and add the get the numerator:

$2\times7 + 1\times3 = \mathbf{17}$,

then multiply the denominators to get the denominator:

$3\times7 = \mathbf{21}$.



So $\mathbf{\frac{2}{3}+\frac{1}{7}=\frac{17}{21}}$

The reason why this works is that in order to add the fractions we must get the denominators to be equal, and we do this by multiplying top and bottom of $\frac{2}{3}$ by 7 (to get a denominator of 21) and the top and bottom of $\frac{1}{7}$ by 3 (to get the same denominator of 21). So each numerator gets multiplied by the other denominator, and this is exactly what we did.

2 Find $\mathbf{7\frac{4}{5}+2\frac{1}{3}}$.

$\mathbf{7\frac{4}{5}+2\frac{1}{3}} = 9\frac{17}{15} = \mathbf{10\frac{2}{15}}$. Here we can add the whole parts and the fractions separately: for the whole ones 7+2 = 9 and for the fractions: $4\times3 + 1\times5 = 17$, the numerator, and $5\times3 = 15$, the denominator.

For subtractions we use the same pattern.

3 Find **a** $\frac{6}{7}-\frac{1}{4}$ **b** $5\frac{4}{5}-1\frac{3}{4}$ **c** $4\frac{1}{3}-1\frac{2}{5}$.

a Subtraction is the same except we cross-multiply and **subtract** rather than add. Be sure to start at the top left.

$$\frac{6}{7}-\frac{1}{4}=\frac{6\times 4 - 1\times 7}{7\times 4}=\frac{17}{28}$$

b $5\frac{4}{5}-1\frac{3}{4}=4\frac{4\times 4 - 3\times 5}{5\times 4}=4\frac{1}{20}$ Similarly here but deal with the whole parts first.

c $4\frac{1}{3}-1\frac{2}{5}=3\frac{1\times 5 - 2\times 3}{3\times 5}=3\frac{\bar{1}}{15}=2\frac{14}{15}$ Here we get a negative numerator, but it is easily dealt with by taking $\frac{1}{15}$ from one of the whole ones.

Alternatively, to avoid the minus number here, put both fractions into top-heavy form and subtract. This will mean dealing with larger numbers however.

Practice A Combine the following , cancelling down your answer or leaving as mixed numbers where necessary:

a $\frac{2}{5}+\frac{1}{4}$ **b** $\frac{3}{8}+\frac{2}{5}$ **c** $\frac{1}{2}+\frac{2}{5}$ **d** $1\frac{1}{3}+2\frac{1}{4}$

e $3\frac{3}{4}+2\frac{1}{3}$ **f** $\frac{3}{5}-\frac{2}{7}$ **g** $\frac{8}{9}-\frac{1}{2}$ **h** $\frac{3}{4}-\frac{1}{20}$

i $5\frac{3}{5}-2\frac{1}{2}$ **j** $10\frac{2}{3}-1\frac{4}{5}$ **k** $\frac{5}{12}+\frac{7}{18}$

---

a $\frac{13}{20}$ **b** $\frac{31}{40}$ **c** $\frac{9}{10}$ **d** $3\frac{7}{12}$
e $6\frac{1}{12}$ **f** $\frac{11}{35}$ **g** $\frac{7}{18}$ **h** $\frac{7}{10}$
i $3\frac{1}{10}$ **j** $8\frac{13}{15}$ **k** $\frac{29}{36}$

Algebraic proof: $\frac{a}{b}+\frac{c}{d}=\frac{ad+cb}{bd}$.

Exactly the same pattern can be used for algebraic fractions as is used for numerical fractions.

\# We may note here that fractions are often written horizontally, for example $\frac{2}{3}$ is written 2/3. This is more consistent with the ratio notation (2:3) and place value. If fractions are written in this way then crosswise and horizontally (see Example 1) becomes crosswise and vertically.
So for $\frac{2}{3}+\frac{1}{7}$:

$$\begin{array}{r} 2/3 \\ \underline{1/7} + \\ \underline{17/21} \end{array}$$

14.2 A SIMPLIFICATION

In the last question of the last exercise you did (and in question **h**) the numbers were rather large and some cancelling had to be done at the end. Where the denominators of two fractions are not relatively prime the working can be simplified as shown in the next example.

4 The denominators in $\frac{5}{12}+\frac{7}{18}$ are not relatively prime: there is a common factor of 6.
We divide both denominators by this common factor and put these numbers below the denominators:

$$\frac{5}{\underset{(2)}{12}} + \frac{7}{\underset{(3)}{18}} = \frac{5\times3+\ 7\times2}{12\times3} = \mathbf{\frac{29}{36}}$$

So we put 2 and 3 below 12 and 18.
Then when cross-multiplying we use the 2 and 3 rather than the 12 and 18.
For the denominator of the answer we cross-multiply in the denominators:
either 12×3 or 18×2, both give 36.

Subtraction of fractions with denominators which are not relatively prime is done in just the same way, except we subtract in the numerator as before.

✎ **Practice B** Use this simplification to add or subtract the following:

a $\frac{1}{3}+\frac{4}{9}$ **b** $\frac{3}{8}+\frac{1}{6}$ **c** $\frac{3}{5}+\frac{3}{10}$ **d** $\frac{5}{6}-\frac{3}{4}$

e $\frac{5}{6}+\frac{3}{4}$ **f** $\frac{5}{18}-\frac{1}{27}$ **g** $3\frac{3}{4}-1\frac{1}{8}$ **h** $\frac{7}{36}-\frac{11}{60}$

a $\frac{7}{9}$ **b** $\frac{13}{24}$ **c** $\frac{9}{10}$ **d** $\frac{1}{12}$
e $1\frac{7}{12}$ **f** $\frac{13}{54}$ **g** $2\frac{5}{8}$ **h** $\frac{1}{90}$

14.3 COMPARING FRACTIONS

Sometimes we need to know whether one fraction is greater or smaller than another, or we may have to put fractions in order of size.

5 Put the fractions $\mathbf{\frac{4}{5}}$, $\mathbf{\frac{2}{3}}$, $\mathbf{\frac{5}{6}}$ in ascending order.

Looking at the first two fractions we cross-multiply and subtract as if we wanted to subtract the fractions.
If we find the subtraction is possible without going into negative numbers then the first fraction must be greater: since 4×3 is greater than 2×5, $\frac{4}{5}$ must be greater than $\frac{2}{3}$.

Doing this with $\frac{2}{3}$ and $\frac{5}{6}$ we find that 2×6 is less than 5×3, so $\frac{5}{6}$ is greater than $\frac{2}{3}$.

If we now cross-multiply $\frac{4}{5}$ with $\frac{5}{6}$ we find that $\frac{5}{6}$ is greater.

So in ascending order the fractions are: $\mathbf{\frac{2}{3}}$, $\mathbf{\frac{4}{5}}$, $\mathbf{\frac{5}{6}}$.

✐ **Practice C** Put the following fractions in ascending order:

a $\frac{1}{3}$, $\frac{2}{5}$ **b** $\frac{3}{4}$, $\frac{8}{11}$ **c** $\frac{2}{3}$, $\frac{7}{12}$, $\frac{3}{4}$ **d** $\frac{5}{6}$, $\frac{5}{8}$, $\frac{6}{7}$

a $\frac{1}{3}, \frac{2}{5}$ **b** $\frac{8}{11}, \frac{3}{4}$ **c** $\frac{7}{12}, \frac{2}{3}, \frac{3}{4}$ **d** $\frac{5}{8}, \frac{5}{6}, \frac{6}{7}$

14.4 UNIFICATION OF OPERATIONS

Multiplying and dividing fractions is also very easy.

6 Find **a** $\frac{1}{2} \times \frac{3}{4}$ **b** $\frac{3}{4} \div \frac{2}{5}$

a $\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$ We simply multiply the numerators to get the numerator of the answer, and multiply the denominators to get the denominator of the answer.

b $\frac{3}{4} \div \frac{2}{5} = \frac{3 \times 5}{2 \times 4} = \frac{15}{8} = 1\frac{7}{8}$ We simply cross-multiply and put the first product over the second product.

The four operations, addition, subtraction, multiplication and division are now seen to have a much more unified relation.

We can summarise these as follows:

Addition	Subtraction	Multiplication	Division
$\frac{4}{5} \quad \frac{1}{3}$	$\frac{4}{5} \quad \frac{1}{3}$	$\frac{4}{5} \quad \frac{1}{3}$	$\frac{4}{5} \quad \frac{1}{3}$

LESSON 15
SPECIAL DIVISION

SUMMARY

15.1 Division by 9
15.2 Division by 8 etc.
15.3 Division by 99, 98 etc.
15.4 Divisor Below a Base Number
15.5 Divisor Above a Base Number

15.1 DIVISION BY 9

As you have seen before, the number 9 is special and there is a very easy way to divide by 9.

1 Find **23 ÷ 9**.

$$9\overline{)\,2\ \ 3}$$
$$\mathbf{2\ \ r\,5}$$

The first figure of 23 is the answer: 2.
And adding the figures of 23 gives the remainder: 2 + 3 = 5.

So **23 ÷ 9 = 2 remainder 5**.

It is easy to see why this works because every 10 contains a 9 with 1 left over.
So 2 tens contains **2** nines with **2** left over.
The answer is the same as the remainder, 2.
And that is why we add 2 to 3 to get the remainder.

✐ **Practice A** Divide by 9:

a $9\overline{)51}$ **b** $9\overline{)34}$ **c** $9\overline{)17}$ **d** $9\overline{)44}$

e $9\overline{)60}$ **f** $9\overline{)71}$ **g** $9\overline{)26}$ **h** $9\overline{)46}$

---

a 5 r6 b 3 r7 c 1 r8 d 4 r8
e 6 r6 f 7 r8 g 2 r8 h 4 r10 = 5 r1

It can happen that there is another nine in the remainder, as occurred in the last question of the last exercise and as the next example shows.

2 Find **66 ÷ 9**.

$$\begin{array}{r|l} 9) & \underline{6 \quad 6} \\ & \underline{6 \quad r\,12 = \mathbf{7\ r\ 3}} \end{array}$$

We get 6 remainder 12 and there is another nine in the remainder of 12.

So we add the one extra nine to the 6, which becomes 7.
And the remainder is reduced to 3 (take 9 from 12).

3 Find **58 ÷ 9.**

$$\begin{array}{r|l} 9) & \underline{5 \quad 8} \\ & \underline{5 \quad r\,13 = \mathbf{6\ r\ 4}} \end{array}$$

Remember you are trying to find the number of nines in 66 and the first answer you get is 6 remainder 12. So there are 6 nines with 12 remaining. Since there is another nine in the 12 you therefore have 7 nines altogether and 3 remaining.

You can also get the final remainder, 3, by adding the digits in 12.

✎ **Practice B** Divide the following by 9:

a 9)57 **b** 9)77 **c** 9)58 **d** 9)49

e 9)64 **f** 9)88 **g** 9)96

---

a 6 r3 b 8 r5 c 6 r4 d 5 r4
e 7 r1 f 9 r7 g 10 r6

The unique property of number nine, that it is one unit below ten leads to many of the very easy Vedic methods, as in Sections 15.2, 15.3, 15.4 following. See also the methods of converting fractions to their recurring decimal form in Manual 2 (or References 1 and 3), as well as corresponding algebraic applications.

LONGER NUMBERS

This can be easily extended to longer numbers.

4 **2301 ÷ 9 = 255 remainder 6**.

The sum can be set out like this: 9)2 3 0 1

The **2** at the beginning of 2301 is brought straight down into the answer:

9) 2 3 0 1
2

This 2 is then added to the 3 in 2301, and **5** is put down:

9) 2 3 0 1
2 5

This 5 is then added to the 0 in 2301, and **5** is put down:

9) 2 3 0 1
2 5 5

This 5 is then added to 1 to give the remainder, **6**:

9) 2 3 0 1
2 5 5 r6

The first figure of the number being divided is the first figure of the answer,
and each figure in the answer is added to the next figure in the dividend
to give the next figure of the answer.
The last number we write down is the remainder.

Practice C Divide the following:

a 9)2 1 2 3 **b** 9)3 1 2 **c** 9)1 1 2 0 2

d 9)4 3 1 **e** 9)5 0 3 0 **f** 9)7 0 7

g 9)2 0 3 0 1 0 **h** 9)1 6 4 **i** 9)3 1 0 3 2

a	**235 r8**	**b**	**34 r6**	**c**	**1244 r6**
d	**47 r8**	**e**	**558 r8**	**f**	**78 r5**
g	**22556 r6**	**h**	**18 r2**	**i**	**3448 r0**

CARRIES

In the method of division by 9 which you have used it can happen that a 2-figure number appears in the answer.

5 Find **3172 ÷ 9**.

$$\begin{array}{r} 9)\underline{3\quad 1\quad 7\quad\ 2} \\ \underline{3\quad 4\quad {}_{1}1\ \ r\,13} \end{array} = \mathbf{352\ remainder\ 4}$$

Here you find you get an 11 and a 13:
the first 1 in the 11 must be carried over to the 4, giving 351,
and there is also another 1 in the remainder so we get **352** remainder **4**.

Practice D Divide the following:

a 9)6 1 5 3	**b** 9)3 2 8 2	**c** 9)5 5 5
d 9)8 2 5 2	**e** 9)6 6 1	**f** 9)4 7 4 1
g 9)1 2 3 4 5	**h** 9)4 7 4 7	**i** 9)2 0 0 8 2

a	**683 rem 6**	**b**	**364 rem 6**	**c**	**61 rem 6**
d	**916 rem 8**	**e**	**73 rem 4**	**f**	**526 rem 7**
g	**1371 r6**	**h**	**527 r4**	**i**	**2231 r3**

A SHORT CUT

We can avoid the double figures that crop up in some of these sums.
Let us do Example 5 above again.

6 Find **3172 ÷ 9**.

We can avoid the build-up of large numbers like 11 and 13.
In the last example we may notice, before we put the 4 down, that the next step will give a 2-figure number and so we put 5 down instead:

$$\begin{array}{r} 9)\underline{3\quad 1\quad 7\quad 2} \\ \underline{\mathbf{3\quad 5\quad 2\ \ r\,4}} \end{array}$$

Then add 5 to 7 to get 12, but as the 1 has already been carried over we only put the 2 down. Finally, 2+2 = 4.

7 Find **777 ÷ 9**.

$$9)\underline{7\quad 7\quad 7}$$
$$\underline{\mathbf{8\quad 6\ r\,3}}$$

If we put 7 for the first figure we get 14 at the next step, so we put 8.
8+7 = 15 and the 1 has already been carried over.

Now, if we put the 5 down we see a 2-figure number coming in the next step, so we put 6 down.

6+7 = 13 and the 1 has been carried over, so just put down the 3.

✐ **Practice E** Divide the following by 9:

a 6153 **b** 3272 **c** 555 **d** 8252

e 661 **f** 4741 **g** 5747 **h** 2938

i 12345 **j** 75057 **k** 443322 **l** 1918161

a	**683 rem 6**	**b**	**363 rem 5**	**c**	**61 rem 6**	**d**	**916 rem 8**
e	**73 rem 4**	**f**	**526 rem 7**	**g**	**638 rem 5**	**h**	**326 rem 4**
i	**1371 rem 6**	**j**	**8339 rem 6**	**k**	**49258**	**l**	**213129**

15.2 DIVISION BY 8 ETC.

This easy way to divide by 9 can be extended for 8, 7 etc.

8 Suppose we want to divide **31** by **8**.

$$\mathbf{8)\underline{3\quad 1}}$$
$$\underline{\mathbf{3\ r\ 7}}$$

We bring the first **3** down into the answer.
Then instead of adding this to the 1 as we do when dividing by 9,
we add double 3 to the 1 to get **7** for the remainder.

We **double** the 3 because 8 is **2** below 10.

✎ **Practice F** Try some of these:

a 8) 2 2 **b** 8) 1 5 **c** 8) 2 5 **d** 8) 5 1

a 2 r6 **b** 1 r7 **c** 3 r1 **d** 6 r3

9 Similarly for **211** divided by **8**:

8) 2 1 1
2 5 r 11 = 26 r3

We bring down the first 2,
add double this to the 1 in the next column and put down 5,
then add double the 5 to the 1 in the last column and put down 11 as the remainder.
Since this remainder contains another 8 we convert our answer to **26 rem 3**.

✎ **Practice G** Try the following.

a 8) 1 1 1 **b** 8) 1 5 1 **c** 8) 1 0 0 **d** 8) 2 1 4 **e** 8) 1 1 2 1

a 13 r7 **b** 18 r7 **c** 12 r4 **d** 26 r6 **e** 140 r1

10 Now, in dividing by 7 which is 3 below 10 we must **treble** the last answer figure at each step.

7) 1 1
1 r 4

and

7) 1 2 3
1 5 r18 = 17 r4

✎ **Practice H** Try these:

a 7) 1 3 **b** 7) 3 1 **c** 7) 2 3 **d** 7) 4 0

e 7) 1 0 3 **f** 7) 1 1 1 **g** 7) 1 0 0

a 1 r6 **b** 4 r3 **c** 3 r2 **d** 5 r5
e 14 r5 **f** 15 r6 **g** 14 r2

15.3 DIVISION BY 99, 98 ETC.

11 Suppose we want to divide the number **121314 by 99**.

This is very similar to division by 9, but because 99 has two 9's we can get the answer **two digits at a time**.

Think of the number split into pairs: 12/13/14 where the last pair is part of the remainder.
Then put down the 12 as the first part of the answer:

99) 12 / 13 / 14
12

Then add the 12 to the 13 and put down 25 as the next part:

99) 12 / 13 / 14
12 / 25

Finally add the 25 to the last pair and put down 39 as the remainder:

99) 12 / 13 / 14
12 / 25 / 39

So the answer is **1225 remainder 39**.

✐ **Practice I** Divide by 99:

a 121416 **b** 213141 **c** 332211 **d** 282828 **e** 363432

f 11221122 (this has 4 pairs, but the method is the same) **g** 3456 (this has 2 pairs)

a **1226 r42** **b** **2152 r93** **c** **3355 r66** **d** **2856 r84** **e** **(3670 r102) 3671 r3**
f **113344 r66** **g** **34 r90**

Dividing by 98 is similar.

12 **121314 ÷ 98 = 1237 remainder 88**.
.
This is the same as before, but because 98 is **2** below 100 we **double** the last part of the answer before adding it to the next part of the sum.

So we begin as before by bringing 12 down into the answer:

98) 12 / 13 / 14
12

Then we double 12 and add this to 13 to get 37:

98) 12 / 13 / 14
12 / 37

Finally double 37 and add it to 14:

98) 12 / 13 / 14
12 / 37 / 88 = 1237 remainder 88

Practice J Divide by 98:

a 112203 **b** 102010 **c** 131313 **d** 200202 **e** 2131

---

a 1144 r91 **b 1040 r90** **c 1339 r91** **d 2042 r86** **e 21 r 73**

In a similar way we can divide by numbers like 97 and 999.

15.4 DIVISOR BELOW A BASE NUMBER

Dividing by 9 is easy, as you have seen.
It is similarly easy to divide by numbers near other base numbers: 100, 1000 etc.

Suppose we want to divide **235 by 88** (which is close to 100).

We need to know how many times 88 can be taken from 235 and what the remainder is.
Since every 100 must contain an 88 there are clearly two 88's in 235.
And the remainder will be two 12's (because 88 is 12 short of 100) plus the 35 in 235.
So the answer is **2 remainder 59** (24+35=59).

A neat way of doing the division is as follows.

8 8)	**2**	**3**	**5**

We separate the two figures on the right because 88 is close to 100 (which has 2 zeros).

Then since 88 is 12 below 100 we put 12 below 88, as shown below.

8 8)	**2**	**3**	**5**
1 2		2	4
	2	**5**	**9**

We bring down the initial 2 into the answer.

This 2 then multiplies the flagged 12 and the 24 is placed under the 35 as shown.
We then simply add up the last 2 columns.

Note that the deficiency of 88 from 100 is given by the formula *All from 9 and the Last from 10*.

Note also that **the position of the vertical line is always determined by the number of noughts in the base number**: if the base number has 4 noughts then the vertical line goes 4 digits from the right, and so on.

This is easily understood since when we bring the initial 2 down into the answer we are expecting to find two 88's in 235. And as there is one 88 in every hundred and 12 left over, in two hundreds there will be two 88's and two 12's remainder, which must be added to the 35 to give 59 as the full remainder.

14 Divide **31313 by 7887**.

We set the sum out as before: **7 8 8 7) 3 | 1 3 1 3**

7 8 8 7)	**3**	**1**	**3**	**1**	**3**
2 1 1 3		6	3	3	9
	3	**7**	**6**	**5**	**2**

Applying *All From 9 and the Last From 10* to 7887 gives 2113.
Bring the 3, (the first figure of 31313) down into the answer.

We now multiply this by the flagged 2113 and put 6339 in the middle row.
Then adding up the last four columns gives the remainder of 7652.

Practice K Divide the following (do as many mentally as you can):

a 88)1 2 1 **b** 76)2 1 1 **c** 83)1 3 2

d 98)3 3 3 **e** 887)1 2 2 3 **f** 867)1 5 1 3

g 779)2 2 2 2 **h** 765)3 0 0 1 **i** 8907)1 3 1 0 3

j 7999)1 2 3 2 1 **k** 7789)2 1 0 1 2 **l** 8888)4 4 3 4 4

a	**1/33**	**b**	**2/59**	**c**	**1/49**
d	**3/39**	**e**	**1/336**	**f**	**1/646**
g	**2/664**	**h**	**3/706**	**i**	**1/4196**
j	**1/4322**	**k**	**2/5434**	**l**	**4/8792**

TWO-FIGURE ANSWERS

Here we consider the case where the answer consists of more than one digit.

15 **1108 ÷ 79** = 13 remainder 81 = **14 remainder 2**.

We set the sum out marking off two figures on the right and leave two rows as there are to be two answer figures:

7 9)	**1**	**1**	**0**	**8**

7 9)	**1**	**1**	**0**	**8**
2 1		2	1	
			6	3
	1	**3**	**8**	**1**

Bring the first 1 down into the answer.
Multiply the flagged 21 by this 1 and put the answer (2 1) as shown in the second row.

Adding in the second column we get 3 which we put down and then multiply the 21 by this 3 to get 63, which we place as shown in the third row.

Add up the last two columns, but since the remainder, 81, is greater than the divisor, 79, there is another 79 contained in 81 so there are 14 79's in 1108 with 2 remaining.

16 Find **1121123 ÷ 8989**.

8 9 8 9)	**1**	**1**	**2**	**1**	**1**	**2**	**3**
1 0 1 1		1	0	1	1		
			2	0	2	2	
				4	0	4	4
	1	**2**	**4**	**6**	**4**	**8**	**7**

The initial 1 comes down into the answer and multiplies the flagged 1011.
This is placed as shown in the second row.

Adding in the second column we put 2 down in the answer and then multiply the 1011 by it. Put 2022 in the third row.

Adding in the third column we get 4 which we put down and also multiply by 1011. So we put 4044 in the fourth row and then add up the last four columns to get the remainder.

> Once the vertical line has been drawn in you can see the number of lines of working needed: this is the number of figures to the left of this line (3 figures and therefore 3 lines of working in Example 16 above).

✐ **Practice L** Divide the following:

a 8 9)1 0 2 1 **b** 8 8) 1 1 2 2 **c** 7 9)1 0 0 1

d 8 8)2 1 1 1 **e** 9 7) 1 1 1 1 **f** 8 8 8) 1 0 0 1 1

g 8 8 7)1 1 2 4 3 **h** 8 9 9)2 1 2 1 2 **i** 9 8 8)3 0 1 2 5

j 8 8 9 9)2 0 1 0 2 0

---

a	**11/42**	**b**	**12/66**	**c**	**12/53**
d	**23/87**	**e**	**11/44**	**f**	**11/243**
g	**12/599**	**h**	**23/535**	**i**	**30/485**
j	**22/5242**				

A SIMPLIFICATION

In these examples (and in the ones in the next section) the lines of working can be dispensed with by using the *Vertically and Crosswise* formula. We use the vertical and crosswise products in the flag and answer digits.

In Example 15 we have 21 flagged and the first answer figure is 1:

2 1
1 -

The first vertical product here gives 2×1=2 which is to be added in the second column of 1108 to give 3 as the second answer figure:

2 1
1 3

So now we take the cross-product 2×3 + 1×1 = 7 and add this to the 0 in 1108 to give 7 as the first remainder figure.
Finally the vertical product on the right in

2 1
1 3

gives 1×3=3 to be added to the last figure of 1108 which makes 11 and gives the full remainder of 7_11 = 81.

Similarly longer sums like Example 16 can also be dealt with in this way.

15.5 DIVISOR ABOVE A BASE NUMBER

A very similar method, but under the formula *Transpose and Apply* allows us to divide numbers which are close to but above a base number.

17 **1489 ÷ 123 = 12 remainder 13**.

Here we see that 123 is close to the base of 100 so we mark 2 figures off on the right.
In fact the method is just as before except that we write the flagged numbers as bar numbers:

$$\begin{array}{l|cc|cc}
1\,2\,3\,) & 1 & 4 & 8 & 9 \\
\quad\bar{2}\,\bar{3} & & \bar{2} & \bar{3} & \\
 & & & \bar{4} & \bar{6} \\ \hline
 & 1 & 2 & 1 & 3
\end{array}$$

Bring the initial 1 down into the answer.
Multiply this 1 by the flagged $\bar{2}\,\bar{3}$ and write down $\bar{2}$, $\bar{3}$.
Add in the second column and put down 2.
Multiply this 2 by the $\bar{2}\,\bar{3}$ and put $\bar{4}$, $\bar{6}$.
Then add up the last two columns.

The Sutra in use is *Transpose and* Apply, as stated above, because we are actually subtracting from the digits 4, 8 and 9.

✎ **Practice M** Divide the following:

a 1 2 3)1 3 7 7 **b** 1 3 1)1 4 8 1 **c** 1 2 1)2 5 6

d 1 3 2)1 3 6 6 **e** 1 2 1 2)1 3 5 4 5 **f** 1 6 1)1 7 8 1

g 1 0 0 3)3 2 1 9 8 7 **h** 1 1 1)7 9 9 9 9

---

a **11/24** **b** **11/40** **c** **2/14**
d **10/46** **e** **11/213** **f** **11/10**
g **321/24** **h** **720/79**

Two other variations, where negative numbers come into the answer or remainder are worth noting next.

18 **10121 ÷ 113 = 89 remainder 64**.

$$
\begin{array}{r|rrr|rr}
113\,) & 1 & 0 & 1 & 2 & 1 \\
\bar{1}\,\bar{3} & & \bar{1} & \bar{3} & & \\
 & & & {\scriptstyle 1} & {\scriptstyle 3} & \\
 & & & & {\scriptstyle 1} & {\scriptstyle 3} \\
\hline
 & 1 & \bar{1} & \bar{1} & 6 & 4
\end{array}
$$

When we come to the second column we find we have to bring $\bar{1}$ down into the answer, multiplying this by the flagged $\bar{1}\,\bar{3}$ means we add 13 in the third row (two minuses make a plus).

The answer $1\bar{1}\bar{1}$ we finally arrive at is the same as 100 – 11 which is 89.

19 Find **2211 ÷ 112**.

$$
\begin{array}{r|rr|rr}
112\,) & 2 & 2 & 1 & 1 \\
\bar{1}\,\bar{2} & & \bar{2} & \bar{4} & \\
 & & & {\scriptstyle 0} & {\scriptstyle 0} \\
\hline
 & 2 & 0 & \bar{3} & 1
\end{array}
= 20 \text{ rem } \overline{29} \text{ or } \mathbf{19 \text{ rem } 83}
$$

20 remainder –29 means that 2211 is 29 short of 20 112's.
This means there are only 19 112's in 2211, so **we add 112 to –29** to get 19 remainder 83.

✐ **Practice N** Divide the following:

a 1 1 2)1 2 3 4 **b** 1 2 1)3 9 9 3 **c** 1 0 3)4 3 2

d 1 0 1 2)2 1 3 1 2 **e** 1 2 2)3 3 3 3 **f** 1 2 3)2 5 8 4

g 1 1 3)1 3 6 9 6 **h** 1 2 1 2)1 3 7 9 8 7 **i** 1 1 1)7 9 9 9 9

j 1 2 1)2 6 5 2 **k** 1 2 3 1)3 3 0 3 3

---

a	**11/02**	**b**	**33/00**	**c**	**4/20**
d	**21/060**	**e**	**27/39**	**f**	**21/01**
g	**121/23**	**h**	**113/1031**	**i**	**720/79**
j	**21/111**	**k**	**26/1027**		

"We go on, at last, to the long-promised Vedic process of STRAIGHT (AT SIGHT) DIVISION which is a simple and easy application of the URDHVA-TIRYAK Sutra which is capable of immediate application to all cases and which we have repeatedly been describing as the 'CROWNING GEM of all' for the very simple reason that over and above the universality of its application, it is the most supreme and superlative manifestation of the Vedic ideal of the at-sight mental-one-line method of mathematical computation."
From "Vedic Mathematics", Page 240.

LESSON 16
THE CROWNING GEM

SUMMARY

16.1 Single Figure on the Flag – one-line division by 2-figure numbers.
16.2 Short Division Digression – choosing the remainder you want.
16.3 Longer Numbers – dividing numbers of any size.
16.4 Negative Flag Digits – using bar numbers to simplify the work.
16.5 Decimalising the Remainder

16.1 SINGLE FIGURE ON THE FLAG

The general division method, also called straight division, allows us to divide numbers of any size by numbers of any size, in one line. Sri Bharati Krsna Tirthaji, the man who rediscovered the Vedic system, called this ”the crowning gem of Vedic Mathematics”.
It comes under the *Vertically and Crosswise* Sutra.

1 Suppose we want to divide **209 by 52**.

We need to know how many 52's there are in 209.
Looking at the first figures we see that since 5 goes into 20 four times we can expect four 52's in 209.

We now take four 52's from 209 to see what is left.
Taking four 50's from 209 leaves 9 and we need to take four 2's away as well.
This leaves a remainder of 1.

We set the sum out like this:

$$\begin{array}{r|cc|c} 2 & 2 & 0 & {}_{0}9 \\ 5 & & & \\ \hline & & 4 & 1 \end{array}$$

The **divisor**, **52**, is written with the 2 raised up, *On the Flag*, and a vertical line is drawn one figure from the right-hand end to separate the answer, 4, from the remainder, 1.

The steps are:
A. 5 into 20 goes **4** remainder 0, as shown.

B. Answer digit 4 multiplied by the flagged 2 gives 8, and this 8 taken from 09 leaves the remainder of **1**, as shown.

2 Divide **321 by 63**.

We set the sum out as before:

$$\begin{array}{c|cc|c} 3 & 3 & 2 & {}_{2}1 \\ 6 & & & \\ \hline & & 5 & 6 \end{array} \quad \mathbf{= 5 \text{ remainder } 6}$$

Then 6 into 32 goes **5** remainder 2, as shown, and answer, 5, times the flagged 3 gives 15 which we take from the 21 to leave the remainder of **6**.

What we are doing here is subtracting five 60's from 321, which leaves 21 and then subtracting five 3's from the 21. That means we have subtracted five 63's and 6 is left.

In the following exercise set the sums out as shown above.

✎ **Practice A** Divide the following:

a 103 ÷ 43 **b** 234 ÷ 54 **c** 74 ÷ 23 **d** 504 ÷ 72

e 444 ÷ 63 **f** 543 ÷ 82 **g** 567 ÷ 93

---

a 2r17 b 4r18 c 3r5 d 7r0
e 7r3 f 6r51 g 6r9

16.2 SHORT DIVISION DIGRESSION

Suppose we want to divide 3 into 10.
The answer is clearly 3 remainder 1: 3) 1 0
3 rem 1

But other answers are possible: 3) 1 0 or 3) 1 0 or even 3) 1 0
2 rem 4 1 rem 7 4 rem $\bar{2}$

Since all of these are correct we can select the one which is best for a particular sum.

Practice B Copy each of the following sums and replace the question mark with the correct number:

a 5) 2 1
3 rem ?

b 7) 5 1
6 rem ?

c 4) 3 0
6 rem ?

d 3) 2 2
? rem 4

e 5) 4 2
6 rem ?

f 6) 3 9
4 rem ?

g 5) 2 4
5 rem ?

h 7) 2 6
4 rem ?

---

a **6** **b** **9** **c** **6** **d** **6**
e **12** **f** **15** **g** $\bar{1}$ **h** $\bar{2}$

Find **503 ÷ 72**.

If we proceed as before:

$$\begin{array}{r|rr|l} \mathbf{2} & \mathbf{5} & \mathbf{0} & {}_{\mathbf{1}}\mathbf{3} \\ \mathbf{7} & & & \\ \hline & & \mathbf{7} & \end{array}$$

We find we have to take 14 from 13, which means the answer is 7 rem $\bar{1}$.

If a negative number is not acceptable however we can say that dividing 7 into 50 in the sum above is not 7 rem 1, but 6 rem 8:

$$\begin{array}{r|rr|l} \mathbf{2} & \mathbf{5} & \mathbf{0} & {}_{\mathbf{8}}\mathbf{3} \\ \mathbf{7} & & & \\ \hline & & \mathbf{6} & \mathbf{71} \end{array}$$

Then we find we can take 12 from 83 to get the positive remainder 71.

This reducing of the answer figure by 1 or 2 is sometimes necessary if negative numbers are to be avoided. But it worth noting that when the answer figure is reduced by 1 the remainder is increased by the first figure of the divisor. So in the answer above the 7 rem 1 is replaced by 6 rem 8: the remainder is increased by 7, the first figure of 72.

Continuing the above example with the first method we would get:

$$\begin{array}{r|rr|l} 2 & 5 & 0 & {}_{1}3 \\ 7 & & & \\ \hline & & 7 & \bar{1} \end{array} = \text{ 6 rem 71.}$$

The 7 we get in the answer represents seven 72's, so we take one of these (leaving 6 of them) and add it to the negative remainder to get 72 + $\bar{1}$ = 71 for the remainder.

✎ **Practice C** Divide the following:

a 97 ÷ 28	**b** 184 ÷ 47	**c** 210 ÷ 53	**d** 373 ÷ 63	**e** 353 ÷ 52
f 333 ÷ 44	**g** 267 ÷ 37	**h** 357 ÷ 59	**i** 353 ÷ 59	

a **3r13**	**b** **3r43**	**c** **3r51**	**d** **5r58**	**e** **6r41**
f **7r25**	**g** **7r8**	**h** **6r3**	**i** **5r58**	

16.3 LONGER NUMBERS

17496 ÷ 72 = 243 remainder 0.

The procedure is just the same as before and goes in cycles.

We set the sum out in the usual way:

$$\begin{array}{c|cccc|c} 2 & 1 & 7 & 4 & 9 & 6 \\ 7 & & & & & \\ \hline & & & & & \end{array}$$

Then we divide 7 into 17 and put down 2 remainder 3, as shown:

$$\begin{array}{c|cccc|c} 2 & 1 & 7 & {}_{3}4 & 9 & 6 \\ 7 & & & & & \\ \hline & & 2 & & & \end{array}$$

Note the diagonal of numbers: 2, 3, 4.
Next we multiply the answer figure by the flag figure: 2×2=4, take this from the 34 to get 30, and then divide by 7 again, to get 4 remainder 2, as shown.

$$\begin{array}{c|cccc|c} 2 & 1 & 7 & {}_{3}4 & {}_{2}9 & 6 \\ 7 & & & & & \\ \hline & & 2 & 4 & & \end{array}$$

Then we repeat: multiply the last answer figure by the flag to get 8, take this from 29 to get 21, then 7 into 21 goes 3 remainder 0, as shown.

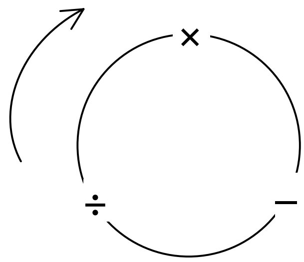
$$\begin{array}{c|cccc|c} 2 & 1 & 7 & {}_{3}4 & {}_{2}9 & {}_{0}6 \\ 7 & & & & & \\ \hline & & 2 & 4 & 3 & 0 \end{array}$$

Finally we again multiply the last answer figure by the flag to get 6 and take this from the 6 to get a remainder of 0.

It is important to note that we proceed in cycles as shown in the diagrams above.
Each cycle is completed as each diagonal goes down.

> Each cycle consists of :
> **A.** multiplying the last answer figure by the flag,
> **B.** taking this from the number indicated by the top two figures of the diagonal,
> **C.** dividing the result by the first figure of the divisor and putting down the answer and remainder.

That is: (divide), multiply, subtract, divide;
multiply, subtract, divide; . . .



5 **50607 ÷ 123 = 411 rem 54**.

Although the divisor has three digits here dividing by 12 is not a problem and so we can use the same procedure:

3	5	0	$_{2}6$	$_{2}0$	$_{5}7$
12					
		4	1	1	54

✐ **Practice D** Divide the following (the remainder is zero for the first four sums, so you will know if it is correct):

a 19902 ÷ 62	**b** 44749 ÷ 73	**c** 1936 ÷ 88	**d** 4032 ÷ 72
e 4154 ÷ 92	**f** 23824 ÷ 51	**g** 92054 ÷ 63	**h** 142857 ÷ 61
i 12233 ÷ 53	**j** 9018 ÷ 71	**k** 8910 ÷ 72	**l** 23658 ÷ 112
m 40000000 ÷ 61	**n** 14018 ÷ 64	**o** 4712 ÷ 45	**p** 22222 ÷ 76
q 651258 ÷ 82	**r** 301291 ÷ 56	**s** 511717 ÷ 73	**t** 360293 ÷ 46

a	**321**	**b**	**613**	**c**	**22**	**d**	**56**
e	**45r14**	**f**	**467r7**	**g**	**1461r11**	**h**	**2341r56**
i	**230r43**	**j**	**127r1**	**k**	**123r54**	**l**	**211r26**
m	**655737r43**	**n**	**219r2**	**o**	**104r32**	**p**	**292r30**
q	**7942r14**	**r**	**5380r11**	**s**	**7009r60**	**t**	**7832r21**

16.4 NEGATIVE FLAG DIGITS

When the flag number is large we often need to reduce more frequently. It is possible to avoid these reductions however by using negative flag digits.

6 **97 ÷ 28 = 3 remainder 13**.

If we proceed as usual we get:

$$\begin{array}{cc|c|c} & \mathbf{8} & \mathbf{9} & {}_{3}\mathbf{7} \\ \mathbf{2} & & & \\ \hline & & \mathbf{3} & \mathbf{13} \end{array}$$

We have to reduce the answer digit from 4 to 3 so that the remainder is big enough.

These reductions occur more frequently when the flag number is large (8 here).
This can be avoided however by rewriting 28 as $3\bar{2}$:

$$\begin{array}{cc|c|c} & \bar{\mathbf{2}} & \mathbf{9} & {}_{0}\mathbf{7} \\ \mathbf{3} & & & \\ \hline & & \mathbf{3} & \mathbf{13} \end{array}$$

3 into 9 goes 3 remainder 0.
We then multiply the $\bar{2}$ by 3 to get $\bar{6}$ and this is to be subtracted from 7.
But subtracting a negative number means adding it, so we get $7-\bar{6}=13$ for the remainder.

This is much easier and it means that:

> whenever we use a bar number on the flag we add the product at each step instead of subtracting it.

✐ **Practice E** Divide the following, giving answer and remainder:

a 373 ÷ 58 **b** 357 ÷ 48 **c** 300 ÷ 59 **d** 321 ÷ 47

e 505 ÷ 78 **f** 543 ÷ 68

---

a 6r25 b 7r21 c 5r5 d 6r39
e 6r37 f 7r67

MULTIPLICATION REVERSED

Straight division can also be demonstrated by reversing the vertically and crosswise multiplication method.

Given 4032÷72 for example:

$$\begin{array}{cccc} & & p & q \\ & & 7 & 2 \\ \hline 4 & 0 & 3 & 2 \\ \hline \end{array}$$

We need the values of p and q so that the number pq multiplied by 72 gives 4032.
We see p must be 5 because p multiplied by 7 must account for the 40 in 4032 (or most of it).
And since 5×7=35 there is a remainder of 5.

So now we have:

$$\begin{array}{cccc} & 5 & & q \\ & 7 & & 2 \\ \hline 4 & 0 & {}_{5}3 & 2 \\ \hline \end{array}$$

We are left with 532 to be accounted for by the crosswise multiplication and the vertical product on the right. Considering the crosswise part we see we have 5×2=10 and we can take this off the 53 in 532 to leave 43: to be produced by the other part of the crosswise product, 7×q. This tells us that q must be 6 and there is a remainder of 1 from the 53:

$$\begin{array}{cccc} & 5 & & 6 \\ & 7 & & 2 \\ \hline 4 & 0 & {}_{5}3 & {}_{1}2 \\ \hline \end{array}$$

The 12 now in the right-hand place is then fully accounted for by the vertical product on the right, so there is no remainder.
All divisions can be done in this way, as a reversal of the multiplication process, and the *on the flag* method in this chapter can be derived from it.

16.5 DECIMALISING THE REMAINDER

We can continue the division when the remainder is reached and give the answer to as many decimal places as required.

7 Find **40342 ÷ 73** to 5 decimal places.

$$\begin{array}{r|l} 3 & 4\ 0\ 3\ 4\ 2\ .0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0 \\ 7 & \quad\ \ {\scriptstyle 5\ \ 3\ \ 5\ \ 4\ \ 1\ \ \ 1\ \ 3\ \ 6\ \ 2} \\ \hline & \ \ 5\ \ 5\ \ 2.\ 6\ \ 3\ \ 0\ \ 1\ \ 3\ \ 7 \end{array}$$

To give an answer correct to 5 decimal places we should find 6 figures after the point in case we need to round up. So we put a decimal point and six zeros after 40342.

The decimal point in the answer goes where the vertical line went before, one figure to the left of the last figure of the dividend.
We proceed as usual: multiply by the flag, subtract, divide by 7 for each cycle.

So the answer is **552.63014** to 5 decimal places.

8 Find **23.1 ÷ 83** to 3 decimal places.

The answer is clearly less than 1 because 23 is less than 83.

$$\begin{array}{r|l} 3 & 2\ \ 3.\ 1\ \ 0\ \ 0\ \ 0 \\ 8 & \quad\ \ {\scriptstyle 7\ \ 9\ \ 5\ \ 2} \\ \hline & 0\ .2\ \ 7\ \ 8\ \ 3 \end{array}$$

As before the decimal point goes one figure to the left in the answer, which is **0.278**.

✐ **Practice F** Find to 2 decimal places:

a 108 ÷ 31 **b** 4050 ÷ 73 **c** 9876 ÷ 94 **d** 25.52 ÷ 38

e 78 ÷ 49 **f** 6.7 ÷ 88 **g** 19 ÷ 62 **h** 62 ÷ 19

---

a	**3.48**	**b**	**55.48**	**c**	**105.06**	**d**	**0.67**
e	**1.59**	**f**	**0.08**	**g**	**0.31**	**h**	**3.26**

This straight division method is developed further in Manual 2 (or see References 1, 3, 5).

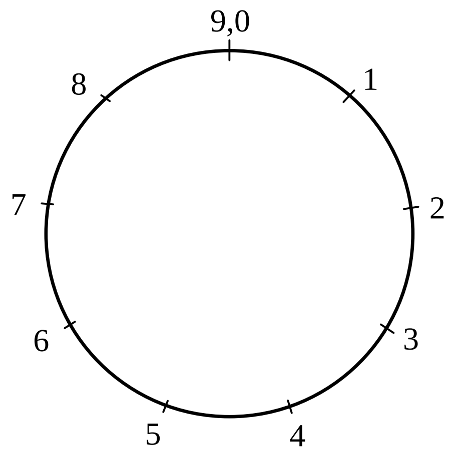
VEDIC MATHEMATICS SUTRAS

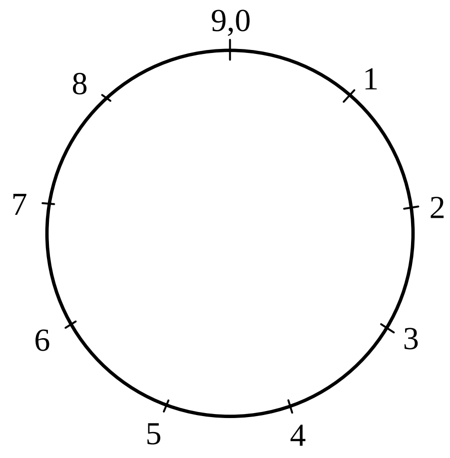
1	एकाधिकेन पूर्वेन Ekādhikena Pūrveṇa	*By One More than the One Before*
2	निखिलं नवतश्चरमं दशतः Nikhilaṃ Navatascaramaṃ Dasataḥ	*All from 9 and the Last from 10*
3	ऊर्ध्वतिर्यग्भ्यामं Ūrdhva Tiryagbhyāṃ	*Vertically and Crosswise*
4	परावर्त्य योजयेत् Parāvartya Yojayet	*Transpose and Apply*
5	शून्यं साम्यसमुच्चये Sūnyaṃ Sāmyasamuccaya	*If the Samuccaya is the Same it is Zero*
6	आनुरूप्ये शून्यं अन्यत् (Ānurūpya) Sūnyamanyat	*If One is in Ratio the Other is Zero*
7	संकलन व्यवकलनाभ्यां Saṅkalana Vyavakalanābhyām	*By Addition and by Subtraction*
8	पूरणापूरणाभ्यां Pūraṇāpūraṇābhyām	*By the Completion or Non-Completion*
9	चलनकलनाभ्याम् Calana Kalanābhyām	*Differential Calculus*
10	यावदूनं Yāvadūnam	*By the Deficiency*
11	व्यष्टिसमष्टिः Vyaṣṭisamaṣṭiḥ	*Specific and General*
12	शेषाण्यङ्केन चरमेण Seṣaṇyaṅkena Carameṇa	*The Remainders by the Last Digit*
13	सोपान्त्यद्वयमन्त्यं Sopāntyadvayamantyam	*The Ultimate and Twice the Penultimate*
14	एकन्यूनेन पूर्वेन Ekanūnena Pūrveṇa	*By One Less than the One Before*
15	गुणितसमुच्चयः Guṇitsamuccayaḥ	*The Product of the Sum*
16	गुणकसमुच्चयः Guṇakasamuccayaḥ	*All the Multipliers*

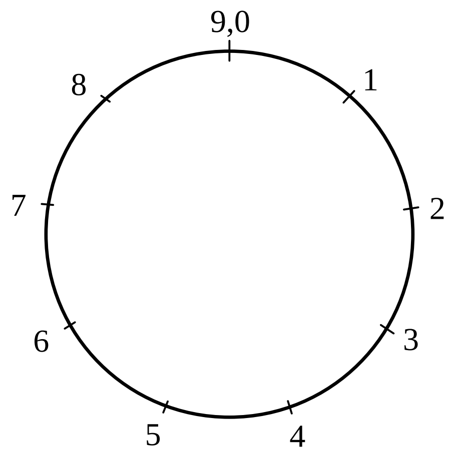
SUB-SUTRAS

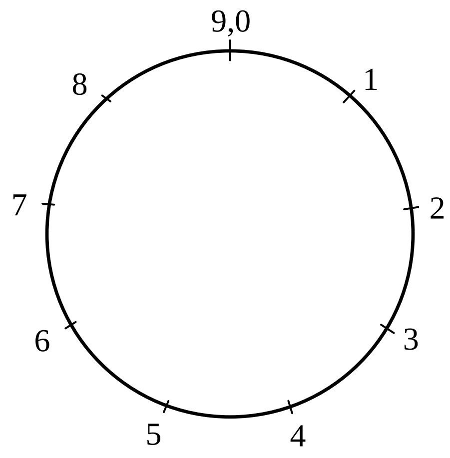
1	आनुरूप्येण Ānurūpyeṇa	*Proportionately*
2	शिष्यते शेषसंज्ञः Siṣyate Seṣaṃjñaḥ	*The Remainder Remains Constant*
3	आद्यमाद्येनान्त्यमन्त्येन Ādyamādyenāntyamantyena	*The First by the First and the Last by the Last*
4	केवलैः सप्तकं गुण्यात् Kevalaiḥ Saptakaṃ Guṇyāt	*For 7 the Multiplicand is 143*
5	वेष्टनम् Veṣṭanaṃ	*By Osculation*
6	यावदूनं तावदूनं Yāvadūnaṃ Tāvadūnaṃ	*Lessen by the Deficiency*
7	यावदूनं तावदूनीकृत्य वर्गं च योजयेत् Yāvadūnaṃ Tāvadūnīkṛtya Vargañca Yojayet	*Whatever the Deficiency lessen by that amount and set up the Square of the Deficiency*
8	अन्त्ययोर्दशकेऽपि Antyayordasake'pi	*Last Totalling 10*
9	अन्त्ययोरेव Antyayoreva	*Only the Last Terms*
10	समुच्चयगुणितः Samuccayagunitaḥ	*The Sum of the Products*
11	लोपनस्थापनाभ्यां Lopanasthāpanābhyām	*By Alternate Elimination and Retention*
12	विलोकनं Vilokanam	*By Mere Observation*
13	गुणितसमुच्चयः समुच्चयगुणितः Guṇitsamuccayaḥ Samuccayaguṇitaḥ	*The Product of the Sum is the Sum of the Products*
14	ध्वजाङ्क Dhvajāṅka	*On the Flag*

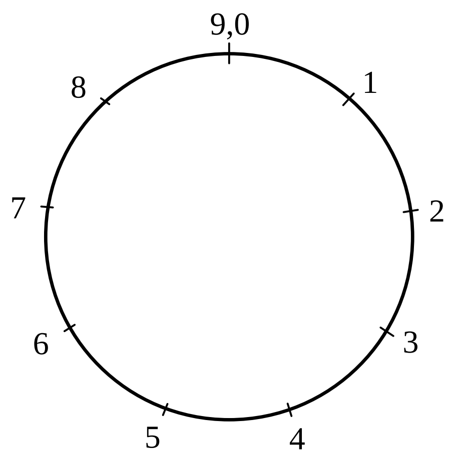
9-POINT CIRCLES

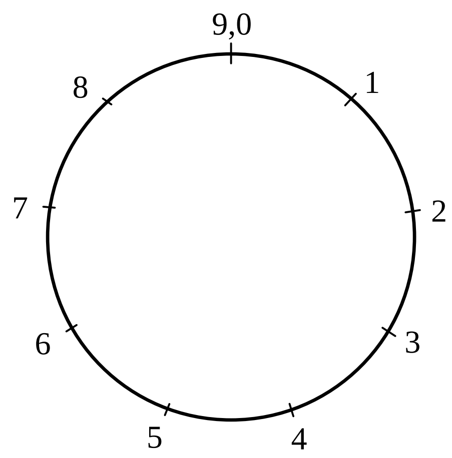












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INDEX OF THE VEDIC FORMULAE

SUTRAS

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SUB-SUTRAS

INDEX

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