COMPUTERS & STRUCTURES, INC.

STRUCTURAL AND EARTHQUAKE ENGINEERING SOFTWARE

SAFE® 2016

Design of Slabs, Beams and Foundations
Reinforced and Post-Tensioned Concrete

Verification

COMPUTERS & STRUCTURES, INC. CSi TECHNOLOGY FOR A BETTER WORLD ®

SAFE®

DESIGN OF SLABS, BEAMS AND FOUNDATIONIS
REINFORCED AND POST-TENSIONED CONCRETE

Verification Manual

ISO SAF112816M6 Rev.0
Proudly developed in the United States of America November 2016

.

Copyright

Further information and copies of this documentation may be obtained from:
Computers & Structures, Inc.
www.csiamerica.com

info@csiamerica.com (for general information)
support@csiamerica.com (for technical support)

DISCLAIMER

CONTENTS

Design Examples

SAFE Software Verification Log		
Revision Number	**Date**	**Description**
0	December 02, 2008	Initial release for SAFE v 12.0.0
1	February 19, 2009	Initial release for SAFE v12.1.0. Example 15 and Example 16 were added.
2	December 26, 2009	Revised to reflect results obtained from Version 12.2.0. All examples, including 1 through 16 and all code-specific examples (ACI 318-00, AS 3600-01, BS 8110-97, CSA A23.3-04, Eurocode 2-04, Hong Kong CoP-04, IS 456-00, NZS 3101, and Singapore CP 65-99 – PS-SL, RC-BM, RC-PN, RC-SL)
3	July 12, 2010	Minor changes have been made to the Examples supplied with the software: (1) The documented results for Analysis Examples 1, 4, 5, 7, and 8 have been updated to correct for truncation error in the reported values. The values actually calculated by the software have not changed for these examples. (2) The input data file for Example 16 has been updated to correct the creep and shrinkage parameters used so that they match those of the benchmark example, and the documented results updated accordingly. The behavior of the software has not changed for this example. (3) All Slab Design examples have been updated to report the slab design forces rather than the strip forces. The design forces account for twisting moment in slab, so their values are more meaningful for design. The behavior of the software has not changed for these examples.
4	December 8, 2010	Minor changes have been made to the Examples supplied with the software: (1) The documented results for Analysis Examples 1 to 7 have been updated to include the results from thin-plate and thick-plate formulation. (2) The input data files for Australian AS 3600-2009 have been added. (3) The Eurocode 2-2004 design verification examples now include the verification for all available National Annexes.

SAFE 2014 Software Verification Log		
Revision Number	**Date**	**Description**
0	February 2014	Initial release for SAFE 2014 v14.0.0. New design examples have been added for the following codes: ACI 318-11, Hong Kong CoP-2013, Italian NTC 2008 and

SAFE Software Verification Log		
Revision Number	**Date**	**Description**
		Turkish TS 500-2000 (Incident 63082). Documentation for the punching-shear design examples of the following codes have been corrected for an error in the documented calculation of the punching perimeter: CSA A23.3-04, IS 456-00, and NZS 3101-06. No calculated results have changed. (Incident 46359) Documentation for the beam and slab design examples of the AS 3600-09 code have been updated to account for a change made to the software under Incident 35218 for version 12.3.2 that updated Equation 8.1.3(2). No calculated results have changed. (Incident 46359) Results for the area of reinforcing steel have changed for Eurocode P/T slab example “Eurocode 2-04 PT-SL-001”. (Incident 62486) Documentation for analysis Example 17 has been corrected for an error in the documented cracked width computed by SAFE. No calculated results have changed. (Incident 63153)
1	June 2015	Initial release for SAFE 2014 v14.1.0. New design examples have been added for the following codes: ACI 318-14 (Incident 79838), and CSA A23.3-14 (Incident 71674).

SAFE 2016 Software Verification Log		
Revision Number	**Date**	**Description**
0	November 2016	For Example 5, stiffening elements were updated in the model. Results have changed slightly. For Examples 8-14, stiffening elements were updated in the column areas, and all beam cross-section properties were updated to reflect values calculated from the actual geometry. For Examples 10-14, the models were changed to reflect the fact that the slab should extend to the outside faces of the columns.

SAFE Software Verification Log		
Revision Number	**Date**	**Description**
		The slight increase in slab area has changed reported results. For design examples ACI 318-08 RC-PN-001, ACI 318-11 RC-PN-001 and ACI 318-14 RC-PN-001, incorrectly sized and redundant stiff areas were removed from the models. Reported results have changed slightly. For design examples CSA A23.3-04 RC-SL-001 and CSA A23.3-14 RC-SL-001, mesh size has been changed to 0.25m to be the same as in the rest of the international code examples. Results have changed slightly. For all changed models, reported results that have changed have been updated in the corresponding documentation. Documentation for Example 5 has been updated for incorrect modeling information regarding the column stiff area dimensions. Documentation for Example 14 has been updated for incomplete modeling information in the images. Documentation for Eurocode 2-04 RC-PN-001was updated to reflect changes in how the program is determining K2*Med2 and K3*Med3, which has changed the results. Documentation for all design examples of type RC-SL has had wording updated to reflect the current state of the models.
0	November 2016	Minor typo error for units for modulus of elasticity of concrete (Ec) and modulus of elasticity of steel (Es) and poison ratio value have been updated.

INTRODUCTION

SAFE is a software application, based on the finite element method, for the engineering analysis, design and detailing of reinforced-concrete and post-tensioned slabs, beams and foundations.

This document provides example problems used to test various features and capabilities of the SAFE program. Users should supplement these examples as necessary for verifying their particular application of the software.

METHODOLOGY

A comprehensive series of test problems, or examples, designed to test the various analysis and design features of the program have been created. The results produced by SAFE were compared to independent sources, such as hand calculated results and theoretical or published results. The comparison of the SAFE results with results obtained from independent sources is provided in tabular form as part of each example.

To validate and verify SAFE results, the test problems were run on a PC platform that was an Lenovo ThinkCentre machine with a Core i5, 2.67 GHz processor and 8.0 GB of RAM operating on a Windows 7 operating system.

ACCEPTANCE CRITERIA

The comparison of the SAFE validation and verification example results with independent results is typically characterized in one of the following three ways.

- **Exact:** There is no difference between the SAFE results and the independent results within the larger of the accuracy of the typical SAFE output and the accuracy of the independent result.

- **Acceptable:** For force, moment and displacement values, the difference between the SAFE results and the independent results does not exceed five percent (5%). For internal force and stress values, the difference between the SAFE results and the independent results does not exceed ten percent (10%). For experimental values, the difference between the SAFE results and the independent results does not exceed twenty five percent (25%).

- **Unacceptable:** For force, moment and displacement values, the difference between the SAFE results and the independent results exceeds five percent (5%).

For internal force and stress values, the difference between the SAFE results and the independent results exceeds ten percent (10%). For experimental values, the difference between the SAFE results and the independent results exceeds twenty five percent (25%).

The percentage difference between results is typically calculated using the following formula:

$$\text{Percent Difference} = 100\left(\frac{\text{SAFE Result - Independent Result}}{\text{Maximum of Independent Result}}\right)$$

SUMMARY OF EXAMPLES

Examples 1 through 7 verify the accuracy of the elements and the solution algorithms used in SAFE. These examples compare displacements and member internal forces computed by SAFE with known theoretical solutions for various slab support and load conditions.

Examples 8 through 14 verify the applicability of SAFE in calculating design moments in slabs by comparing results for practical slab geometries with experimental results and/or results using ACI 318-95 recommendations. Examples 15 and 16 verify the applicability of SAFE for temperature loading and cracked deflection analysis for creep and shrinkage by comparing the results from published examples.

Design examples verify the design algorithms used in SAFE for flexural, shear design of beam; flexural and punching shear of reinforced concrete slab; and flexural design and serviceability stress checks of post-tensioned slab, using ACI 318-14, ACI 318-11, ACI 318-08, AS 3600-09, AS 3600-01, BS 8110-97, CSA A23.3-14, CSA A23.3-04, Eurocode 2-02, Hong Kong CP-13, Hong Kong CP-04, IS 456-00, Italian NTC 2008, NZS 31-01-06, Singapore CP 65-99 and Turkish TS 500-2000 codes, by comparing SAFE results with hand calculations.

EXAMPLE 1

Simply Supported Rectangular Plate

PROBLEM DESCRIPTION

A simply supported, rectangular plate is analyzed for three load conditions: uniformly distributed load over the slab (UL), a concentrated point load at the center of the slab (PL), and a line load along a centerline of the slab (LL).

To test convergence, the problem is analyzed employing three mesh sizes, 4 × 4, 8 × 8, and 12 × 12, as shown in Figure 1-2. The slab is modeled using plate elements in SAFE. The simply supported edges are modeled as line supports with a large vertical stiffness. Three load cases are considered. Self weight is not included in these analyses.

To obtain design moments, the plate is divided into three strips — two edge strips and one middle strip — each way, based on the ACI 318-95 definition of design strip widths for a two-way slab system as shown in Figure 1-3.

For comparison with the theoretical results, load factors of unity are used and each load case is processed as a separate load combination.

Closed-form solutions to this problem are given in Timoshenko and Woinowsky (1959) employing a double Fourier Series (Navier's solution) or a single series (Lévy's solution). The numerically computed deflections, local moments, average strip moments, and local shears obtained from SAFE are compared with the corresponding closed form solutions.

SAFE results are shown for both thin plate and thick plate element formulations. The thick plate formulation is recommended for use in SAFE, as it gives more realistic shear forces for design, especially in corners and near supports and other discontinuities. However, thin plate formulation is consistent with the closed-form solutions.

GEOMETRY, PROPERTIES AND LOADING

Plate size,	$a \times b$	=	360 in × 240 in
Plate thickness	T	=	8 inches
Modulus of elasticity	E	=	3000 ksi
Poisson's ratio	v	=	0.3

Load Cases:				
(UL) Uniform load	q	=	100	psf
(PL) Point load	P	=	20	kips
(LL) Line load	q_1	=	1	kip/ft

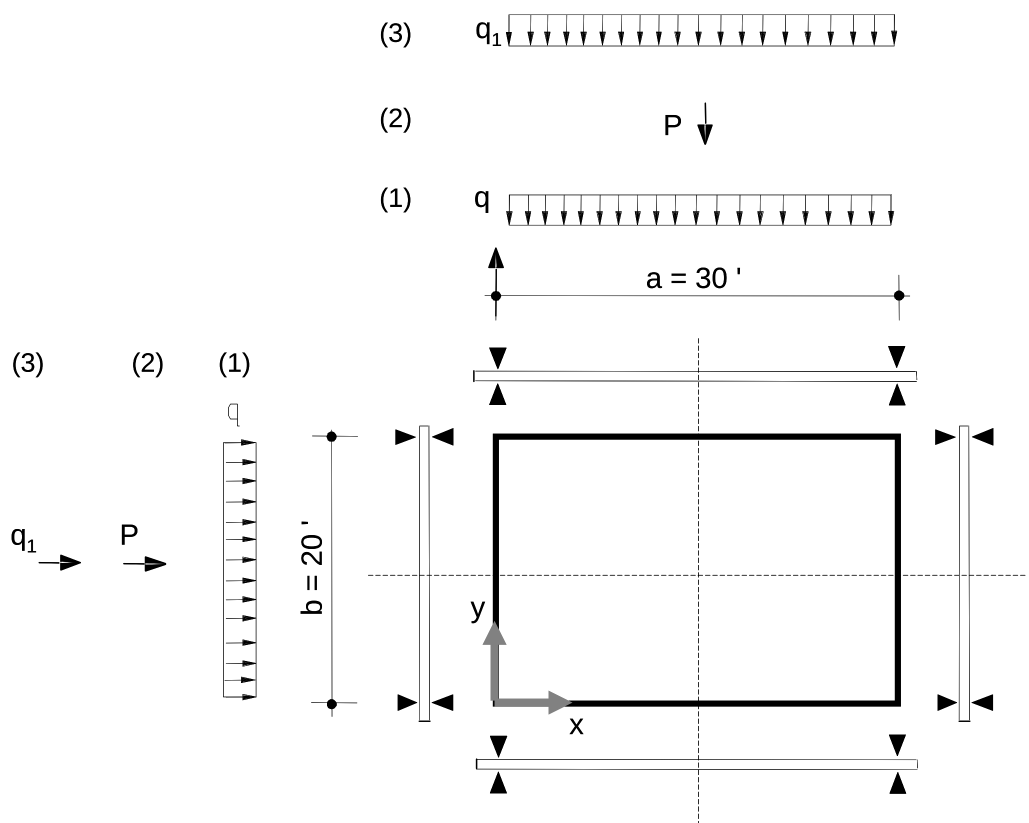


Figure 1-1 Simply Supported Rectangular Plate

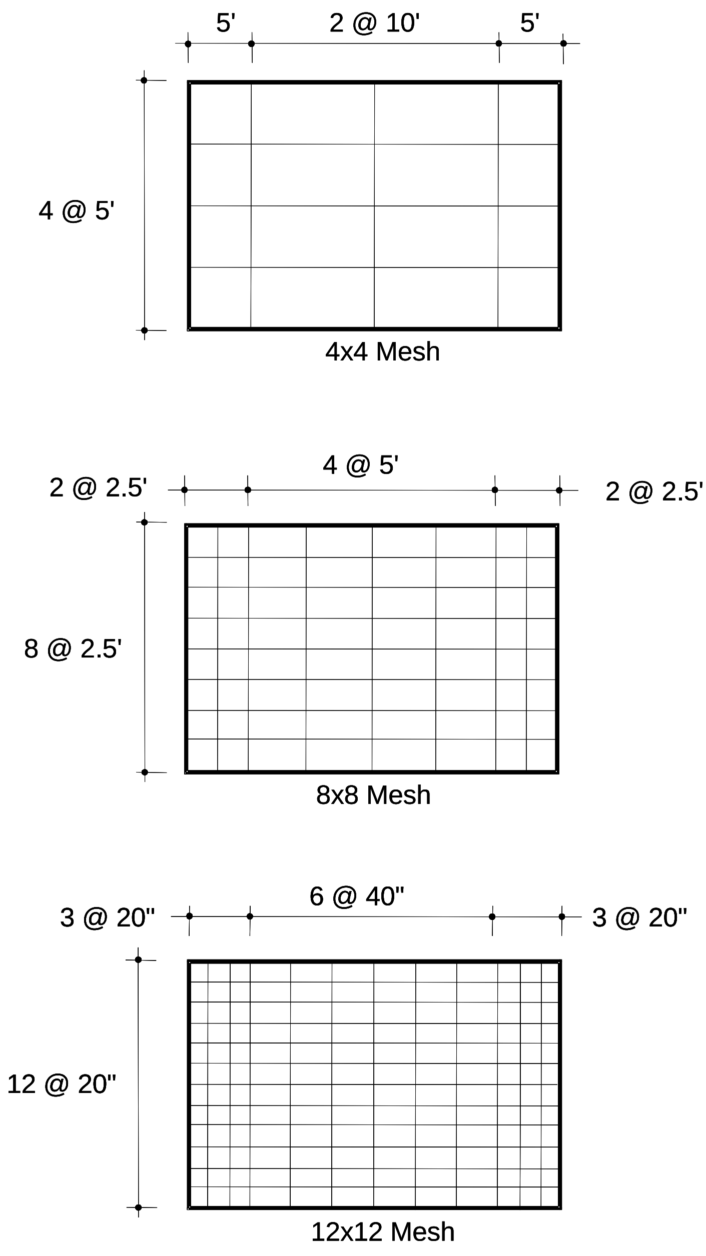


Figure 1-2 SAFE Meshes for Rectangular Plate

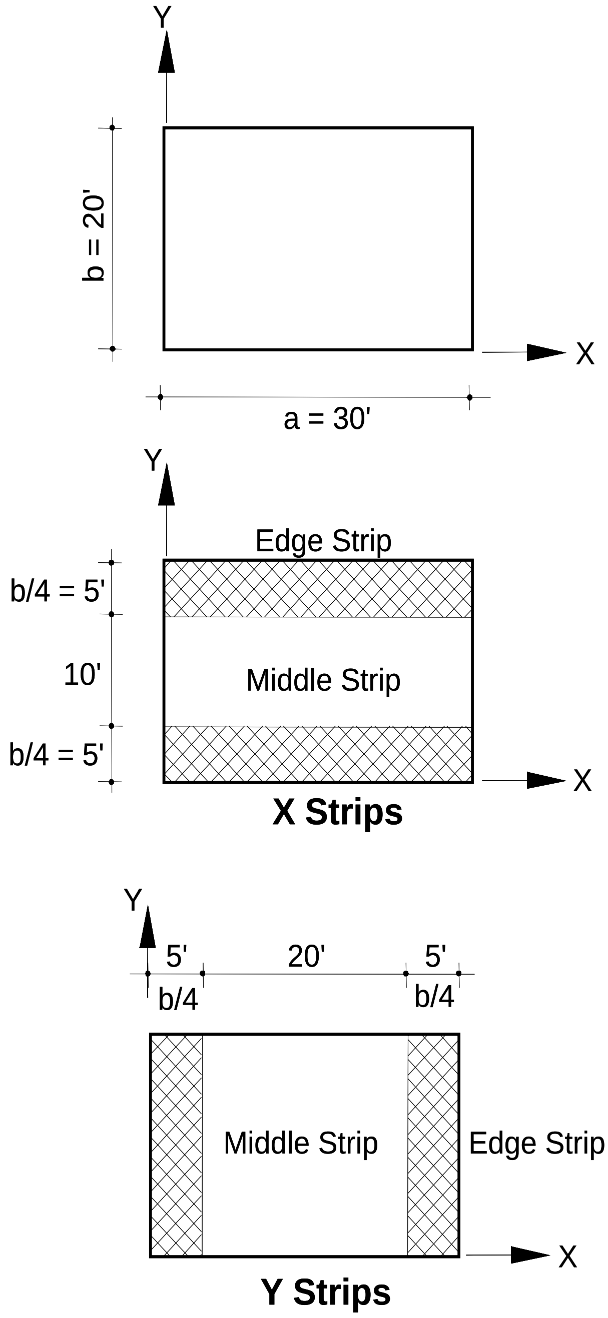


Figure 1-3 SAFE Definition of Design Strips

TECHNICAL FEATURES OF SAFE TESTED

- Deflection of slab at various mesh refinements.
- Local moments, average strip moments, and local shears

RESULTS COMPARISON

Table 1-1 shows the deflections of four different points for three different mesh refinements for the three load cases. The theoretical solutions based on Navier's formulations also are shown for comparison. It can be observed from Table 1-1 that the deflection obtained from SAFE converges monotonically to the theoretical solution with mesh refinement. Moreover, the agreement is acceptable even for the coarse mesh (4×4).

Table 1-2 shows the comparison of the numerically obtained local-moments at critical points with that of the theoretical values. Only results from the 8x8 mesh are reported. The comparison with the theoretical results is acceptable.

Table 1-3 shows the comparison of the numerically obtained local-shears at critical points with that of the theoretical values. The comparison here needs an explanation. The theoretical values were presented for both thin plate and thick plate formulations. The theoretical values are for a thin plate solution where shear strains across the thickness of the plate are ignored. The SAFE results for thick plate are for an element that does not ignore the shear strains. The thin plate theory results in concentrated corner uplift; consideration of the shear strains spreads this uplift over some length of the supports near the corners. The shears reported by SAFE for thick plate are more realistic.

The results of Table 1-3 are plotted in Figures 1-4 to 1-15. In general, it can be seen that the thin plate formulation more closely matches the closed-form solution than does the thick plate solution, as expected. The closed-form solution cannot be used to validate the thick plate shears, since behavior is fundamentally different in the corners. This can be seen clearly in Figures 6, 7, 10, 11, 14 and 15 which show the shear forces trajectories for thin plate and thick plate solutions. The thin plate solution unrealistically carries loads to corners, whereas the thick plate solution carries the load more toward the middle of the sites.

Table 1-4 shows the comparison of the average strip-moments for the load cases with the theoretical average strip-moments. The comparison is excellent. This checks both the accuracy of the finite element analysis and the integration scheme over the elements.

It should be noted that in calculating the theoretical solution, a sufficient number of terms from the series is taken into account to achieve the accuracy of the theoretical solutions.

Table 1-1 Comparison of Displacements

Thin-Plate Formulation

Load Case	Location		SAFE Displacement (in)			Theoretical Displacement (in)
	X (in)	Y (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	
UL	60	60	0.0491	0.0492	0.0493	0.0492961
	60	120	0.0685	0.0684	0.0684	0.0684443
	180	60	0.0912	0.0908	0.0907	0.0906034
	180	120	0.1279	0.1270	0.1267	0.1265195
PL	60	60	0.0371	0.0331	0.0325	0.0320818
	60	120	0.0510	0.0469	0.0463	0.0458716
	180	60	0.0914	0.0829	0.0812	0.0800715
	180	120	0.1412	0.1309	0.1283	0.1255747
LL	60	60	0.0389	0.0375	0.0373	0.0370825
	60	120	0.0593	0.0570	0.0566	0.0562849
	180	60	0.0735	0.0702	0.0696	0.0691282
	180	120	0.1089	0.1041	0.1032	0.1024610

Thick-Plate formulation

Load Case	Location		SAFE Displacement (in)			Theoretical Displacement (in)
	X (in)	Y (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	
UL	60	60	0.0485	0.0501	0.0501	0.0492961
	60	120	0.0679	0.0695	0.0694	0.0684443
	180	60	0.0890	0.0919	0.0917	0.0906034
	180	120	0.1250	0.1284	0.1281	0.1265195
PL	60	60	0.0383	0.0339	0.0330	0.0320818
	60	120	0.0556	0.0474	0.0469	0.0458716
	180	60	0.0864	0.0834	0.0821	0.0800715
	180	120	0.1287	0.1297	0.1293	0.1255747
LL	60	60	0.0387	0.0381	0.0378	0.0370825
	60	120	0.0583	0.0579	0.0574	0.0562849
	180	60	0.0719	0.0710	0.0703	0.0691282
	180	120	0.1060	0.1053	0.1044	0.1024610

Table 1-2 Comparison of Local Moments

Thin-Plate Formulation

Load Case	Location		Moment (kip-in/in)					
			M_{11}		M_{22}		M_{12}	
	X (in)	Y (in)	SAFE 8×8	Analytical (Navier)	SAFE 8×8	Analytical (Navier)	SAFE 8×8	Analytical (Navier)
UL	150	15	0.42	0.45	0.73	0.81	0.31	0.30
	150	45	1.16	1.18	1.95	2.02	0.26	0.26
	150	75	1.66	1.69	2.69	2.77	0.17	0.17
	150	105	1.92	1.95	3.04	3.12	0.06	0.06
PL	150	15	0.37	0.37	0.36	0.36	0.44	0.47
	150	45	1.11	1.13	1.13	1.14	0.48	0.51
	150	75	1.92	1.90	2.16	2.20	0.56	0.59
	150	105	2.81	2.41	3.85	3.75	0.42	0.47
LL	150	15	0.26	0.26	0.34	0.34	0.24	0.24
	150	45	0.77	0.77	1.06	1.08	0.21	0.20
	150	75	1.25	1.25	1.91	1.92	0.14	0.14
	150	105	1.69	1.68	2.94	3.03	0.05	0.05

Thick-Plate Formulation

Load Case	Location		Moment (kip-in/in)					
			M_{11}		M_{22}		M_{12}	
	X (in)	Y (in)	SAFE 8×8	Analytical (Navier)	SAFE 8×8	Analytical (Navier)	SAFE 8×8	Analytical (Navier)
UL	150	15	0.43	0.45	0.74	0.81	0.31	0.30
	150	45	1.16	1.18	1.95	2.02	0.26	0.26
	150	75	1.66	1.69	2.69	2.77	0.17	0.17
	150	105	1.92	1.95	3.04	3.12	0.06	0.06
PL	150	15	0.29	0.37	0.34	0.36	0.43	0.47
	150	45	1.07	1.13	1.14	1.14	0.41	0.51
	150	75	1.91	1.90	2.15	2.20	0.42	0.59
	150	105	2.83	2.41	3.82	3.75	0.22	0.47
LL	150	15	0.27	0.26	0.34	0.34	0.24	0.24
	150	45	0.78	0.77	1.07	1.08	0.21	0.20
	150	75	1.25	1.25	1.91	1.92	0.14	0.14
	150	105	1.68	1.68	2.94	3.03	0.05	0.05

Table 1-3 Comparison of Local Shears

Thin-Plate Formulation

Load Case	Location		Shears (×10^{-3} kip/in)			
			V_{13}		V_{23}	
	X (in)	Y (in)	SAFE (8×8)	Analytical (Navier)	SAFE (8×8)	Analytical (Navier)
UL	15	45	–27.54	–35.2	–5.76	–7.6
	45	45	–16.07	–21.2	–17.19	–21.0
	90	45	–7.31	–10.5	–28.39	–33.4
	150	45	–1.71	–3.0	–36.23	–40.7
PL	15	45	–4.84	–8.7	–2.43	–2.6
	45	45	–6.75	–9.8	–8.57	–8.3
	90	45	–12.45	–13.1	–20.53	–19.2
	150	45	–11.19	–11.2	–34.82	–43.0
LL	15	45	–13.2	–15.7	–4.57	–5.7
	45	45	–10.91	–13.0	–13.47	–16.2
	90	45	–5.76	–7.6	–22.59	–26.5
	150	45	–1.45	–2.2	–29.04	–32.4

Thick-Plate formulation

Load Case	Location		Shears (×10^{-3} kip/in)			
			V_{13}		V_{23}	
	X (in)	Y (in)	SAFE (8×8)	Analytical (Navier)	SAFE (8×8)	Analytical (Navier)
UL	15	45	–21.27	–35.2	24.75	–7.6
	45	45	–7.57	–21.2	–6.35	–21.0
	90	45	–2.30	–10.5	–29.83	–33.4
	150	45	–0.92	–3.0	–43.13	–40.7
PL	15	45	–0.66	–8.7	18.01	–2.6
	45	45	1.83	–9.8	2.33	–8.3
	90	45	–8.01	–13.1	–14.89	–19.2
	150	45	–18.02	–11.2	–48.18	–43.0
LL	15	45	–7.69	–15.7	19.71	–5.7
	45	45	–2.07	–13.0	–4.89	–16.2
	90	45	–1.43	–7.6	–23.51	–26.5
	150	45	–0.63	–2.2	–34.25	–32.4

Table 1-4 Comparison of Average Strip Moments

Thin-Plate Formulation

Load Case	Moment Direction	Strip	SAFE Average Strip Moments (kip-in/in)			Theoretical Average Strip Moments (kip-in/in)
			4×4 Mesh	8×8 Mesh	12×12 Mesh	
UL	$\bar{M}_A$ x = 180"	Column	0.758	0.800	0.805	0.810
		Middle	1.843	1.819	1.819	1.820
	$\bar{M}_B$ y = 120"	Column	0.974	0.989	0.992	0.994
		Middle	2.701	2.769	2.781	2.792
PL	$\bar{M}_A$ x = 180"	Column	0.992	0.958	0.926	0.901
		Middle	3.329	3.847	3.963	3.950
	$\bar{M}_B$ y = 120"	Column	0.440	0.548	0.546	0.548
		Middle	3.514	3.364	3.350	3.307
LL	$\bar{M}_A$ x = 180"	Column	0.547	0.527	0.522	0.519
		Middle	1.560	1.491	1.482	1.475
	$\bar{M}_B$ y = 120"	Column	1.205	1.375	1.418	1.432
		Middle	3.077	3.193	3.213	3.200

Thick-Plate Formulation

Load Case	Moment Direction	Strip	SAFE Average Strip Moments (kip-in/in)			Theoretical Average Strip Moments (kip-in/in)
			4×4 Mesh	8×8 Mesh	12×12 Mesh	
UL	$\bar{M}_A$ *x* = 180"	Column	0.716	0.805	0.799	0.810
		Middle	1.757	1.855	1.832	1.820
	$\bar{M}_B$ *y* = 120"	Column	1.007	0.968	0.984	0.994
		Middle	2.65	2.80	2.805	2.792
PL	$\bar{M}_A$ *x* = 180"	Column	0.969	1.128	1.043	0.901
		Middle	2.481	3.346	3.781	3.950
	$\bar{M}_B$ *y* = 120"	Column	0.763	0.543	0.533	0.548
		Middle	3.149	3.381	3.372	3.307
LL	$\bar{M}_A$ *x* = 180"	Column	0.489	0.526	0.517	0.519
		Middle	1.501	1.520	1.493	1.475
	$\bar{M}_B$ *y* = 120"	Column	1.254	1.338	1.408	1.432
		Middle	2.840	3.205	3.233	3.200

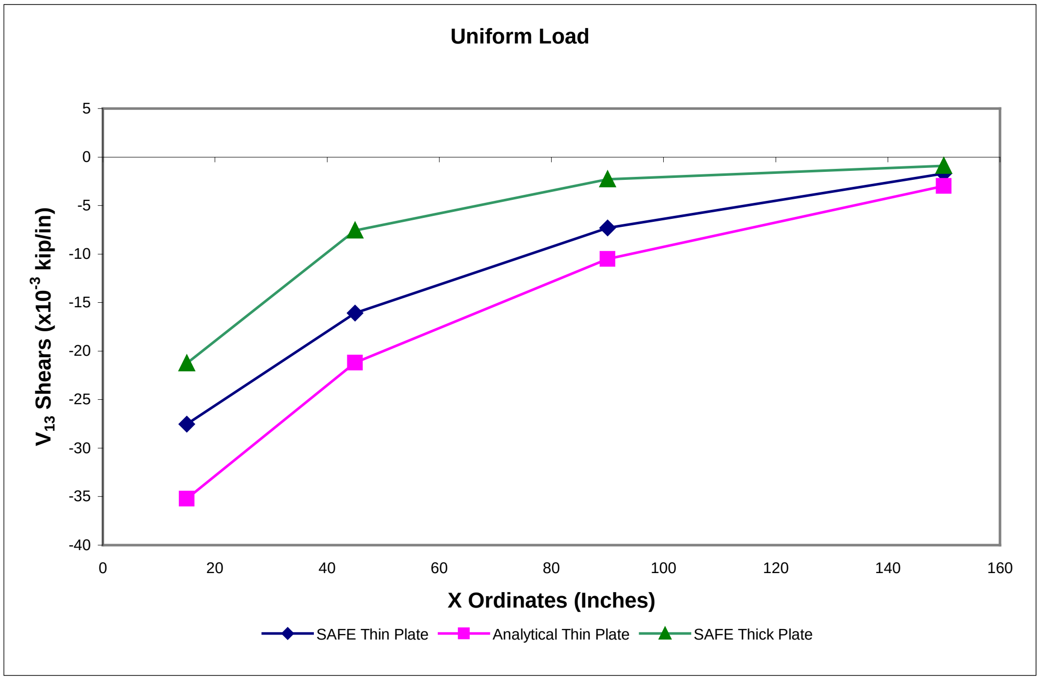


Figure 1-4 V_{12} Shear Force for Uniform Loading

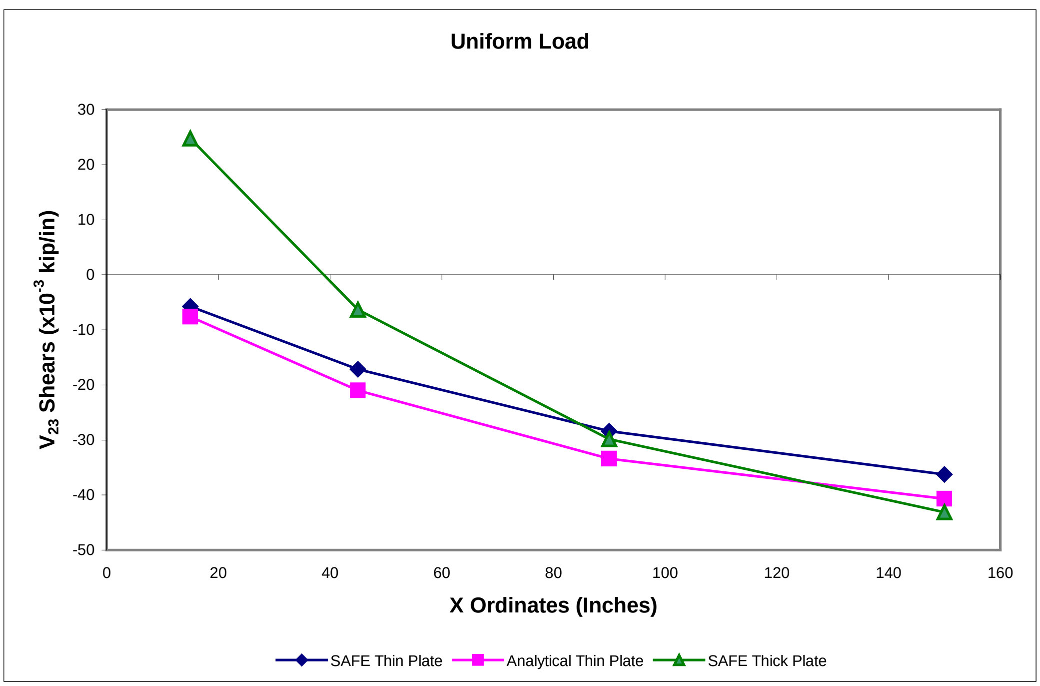


Figure 1-5 V_{13} Shear Force for Uniform Loading

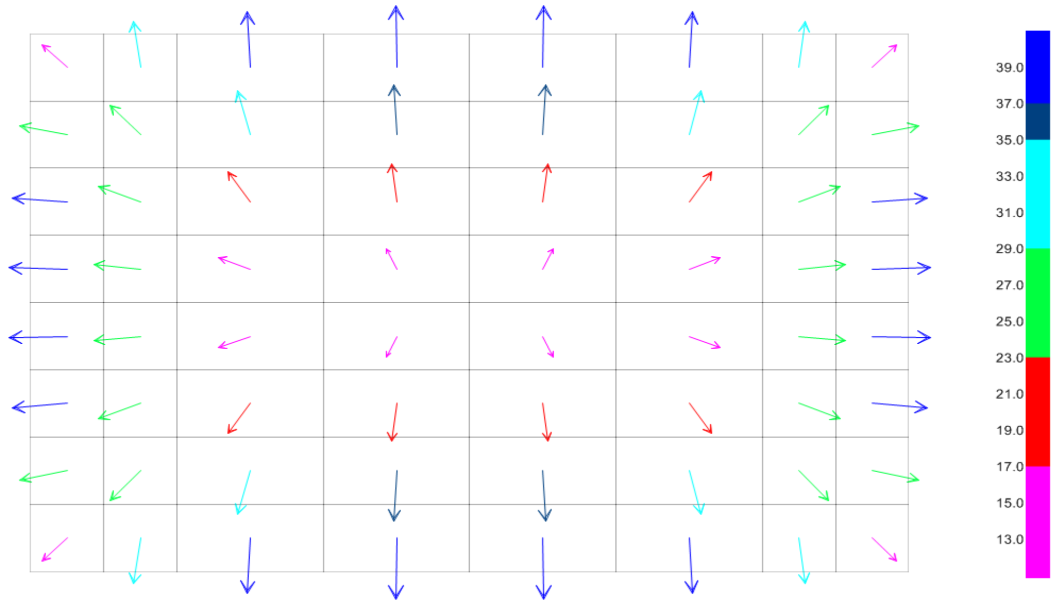


Figure 1-6 **V_{max} for Uniform Load for Thin-Plate Formulation**

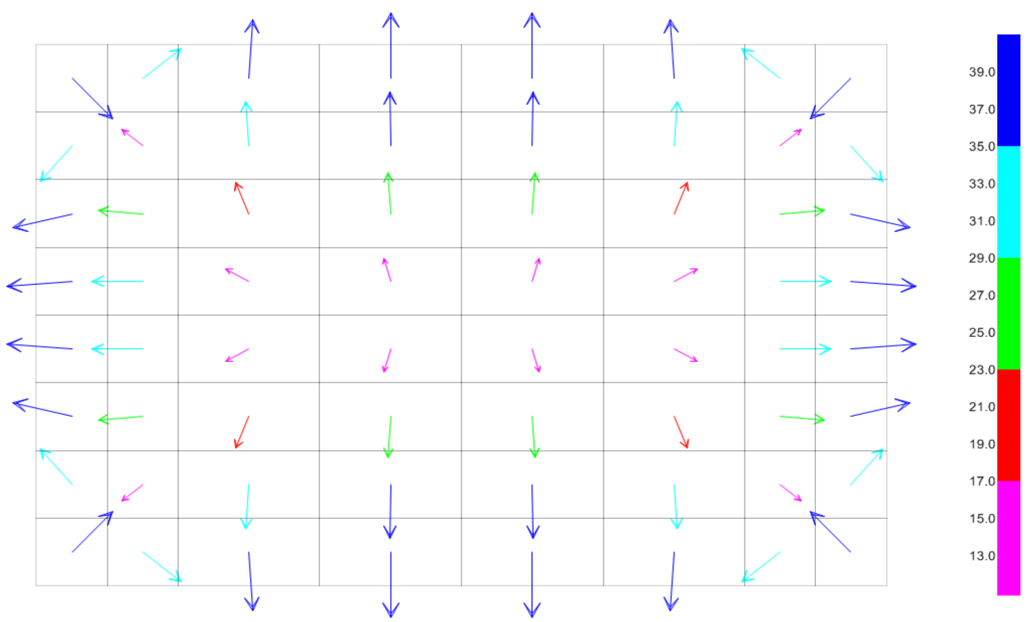


Figure 1-7 **V_{max} for Uniform Load for Thick-Plate Formulation**

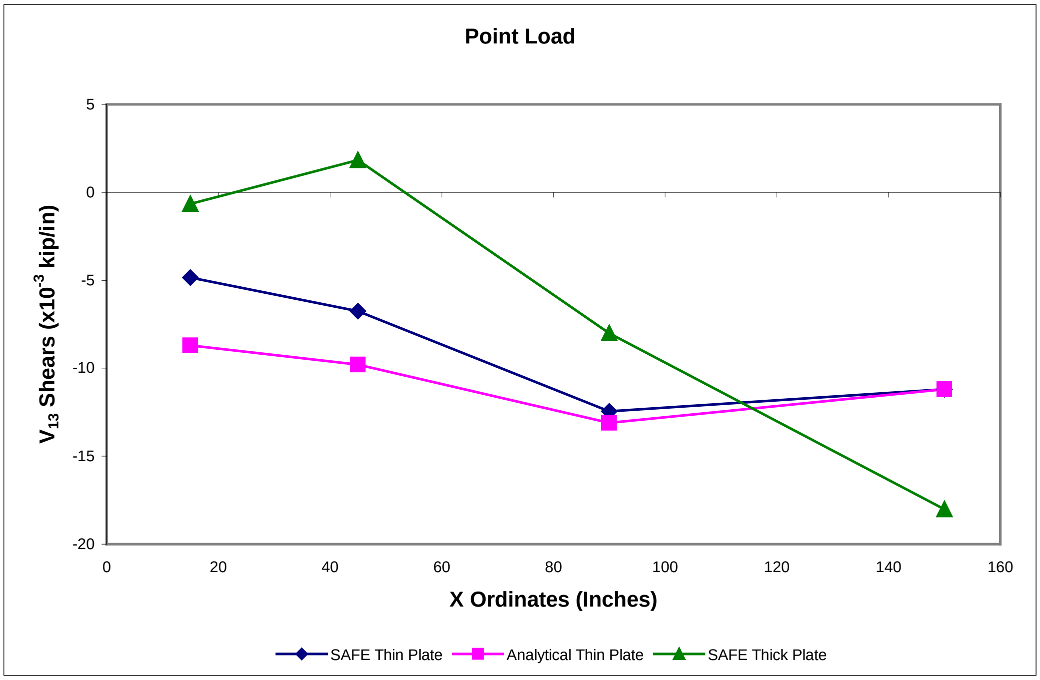


Figure 1-8 V_{12} Shear Force for Point Loading

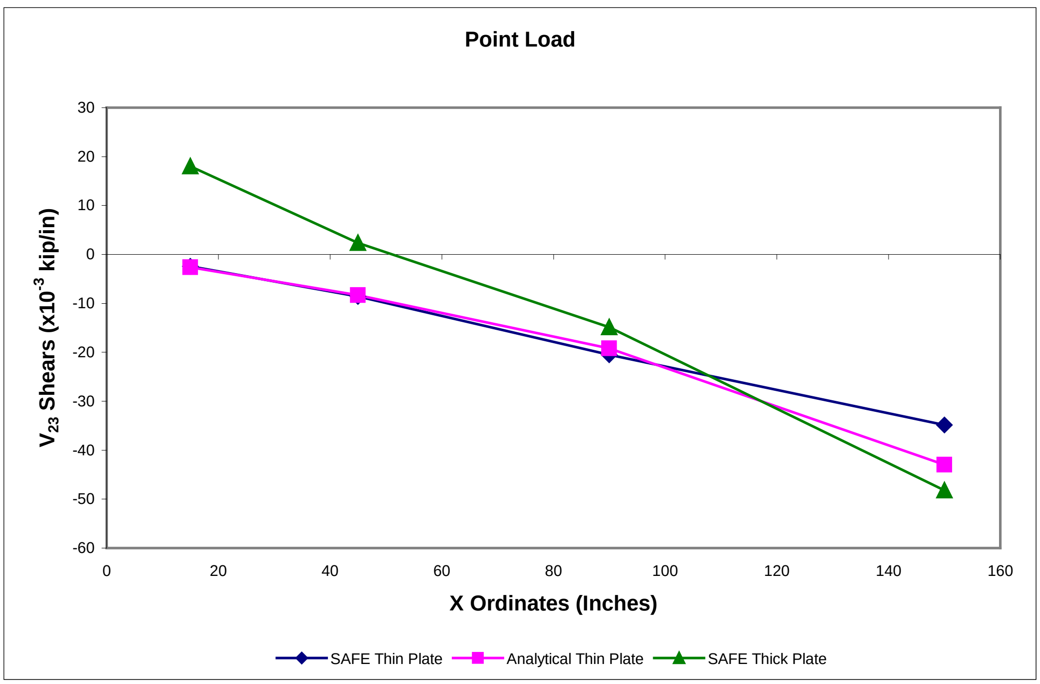


Figure 1-9 V_{13} Shear Force for Point Loading

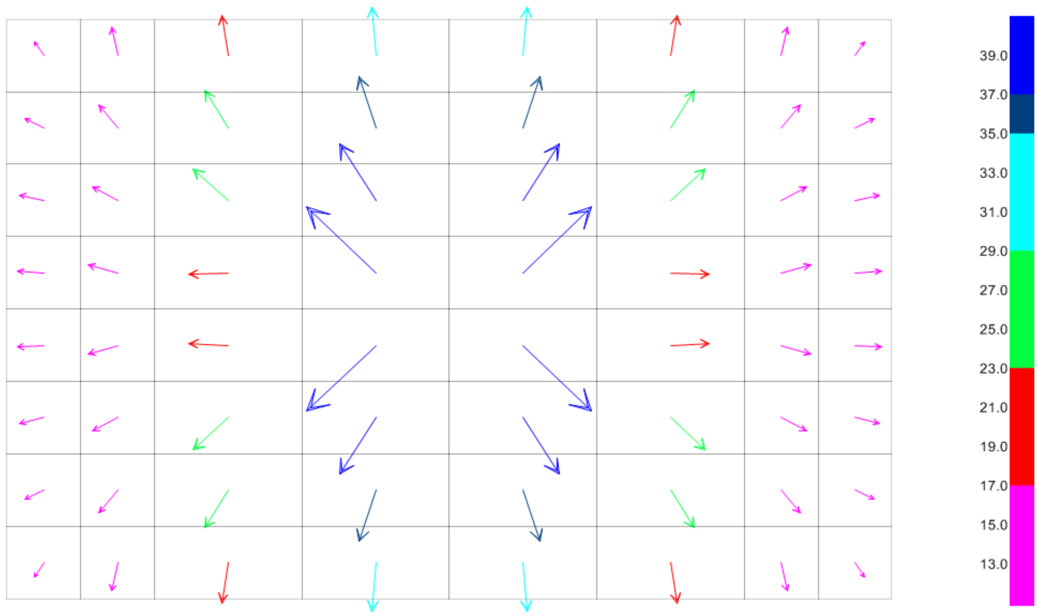


Figure 1-10 **V_{max} for Point Load for Thin-Plate Formulation**

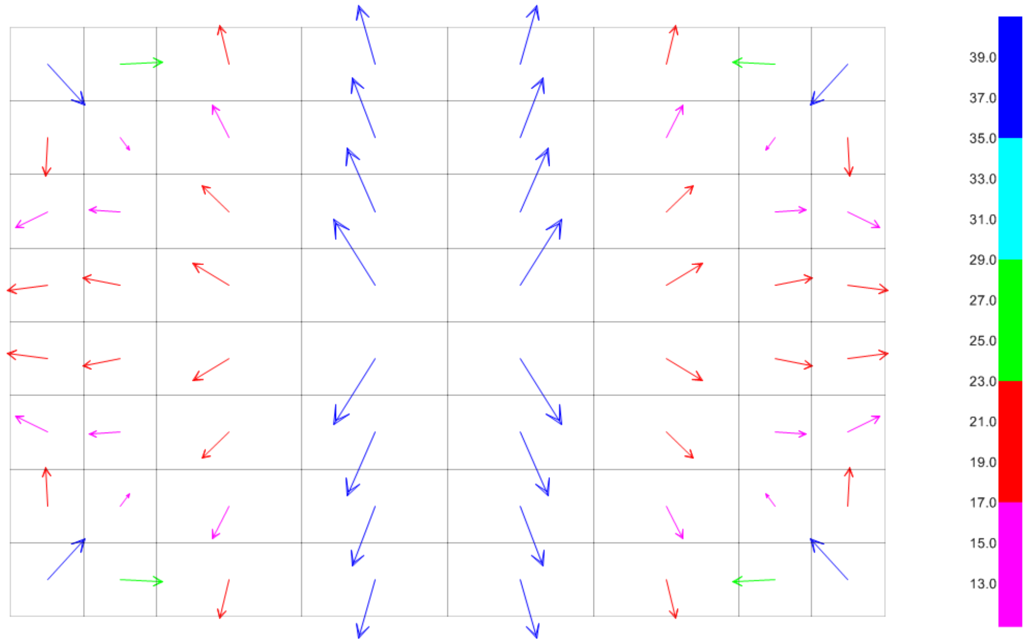


Figure 1-11 **V_{max} for Point Load for Thick-Plate Formulation**

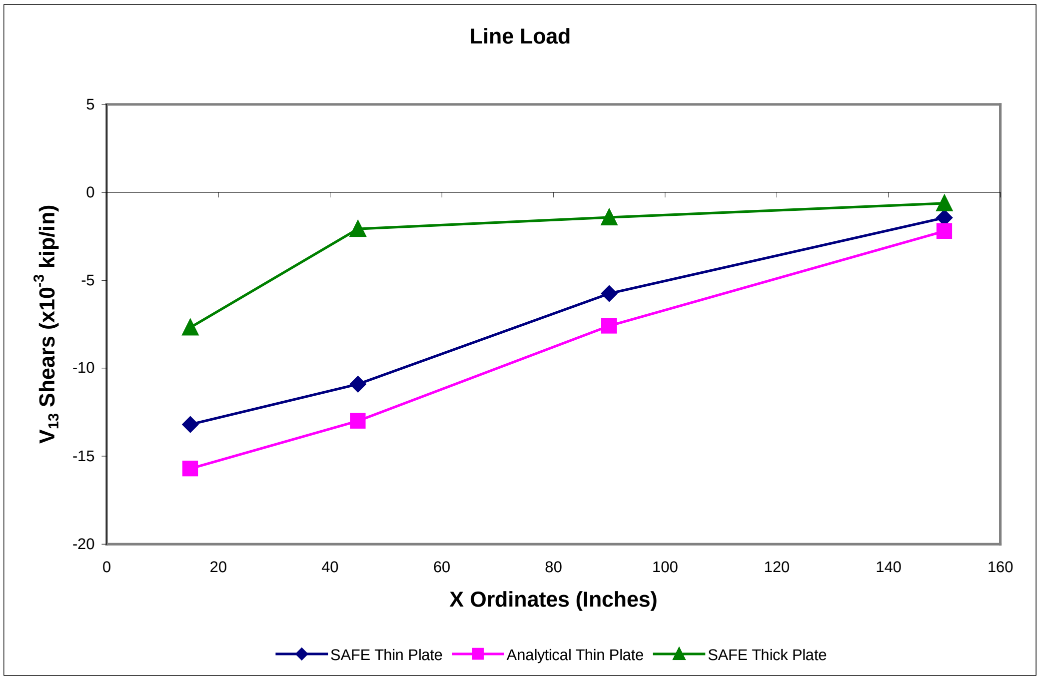


Figure 1-12 V_{12} Shear Force for Line Loading

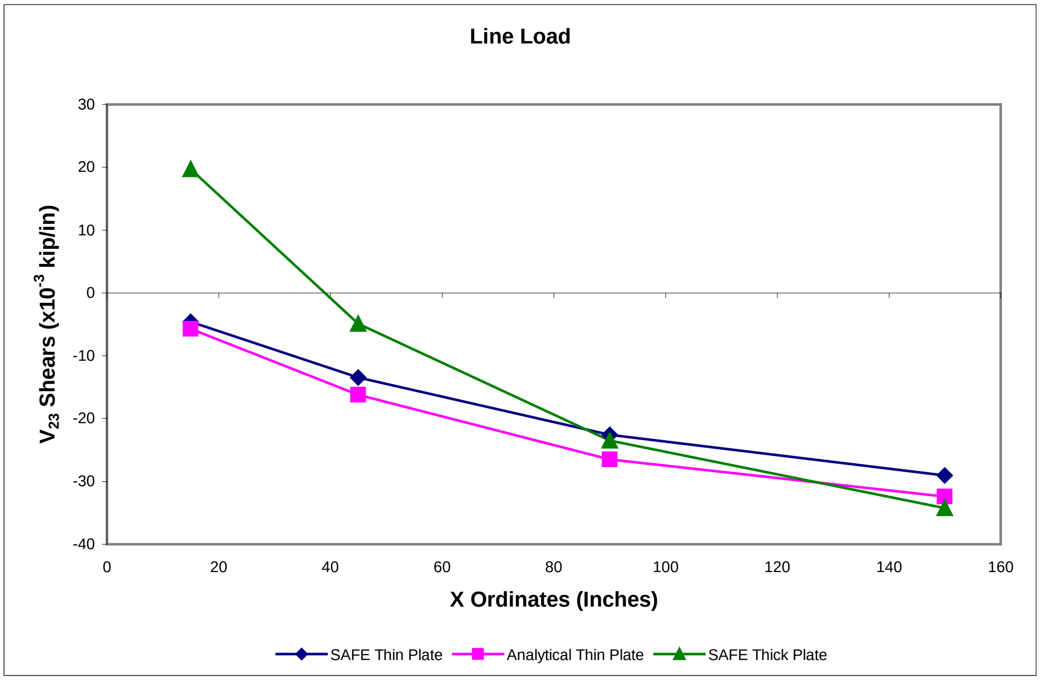


Figure 1-13 V_{13} Shear Force for Point Loading

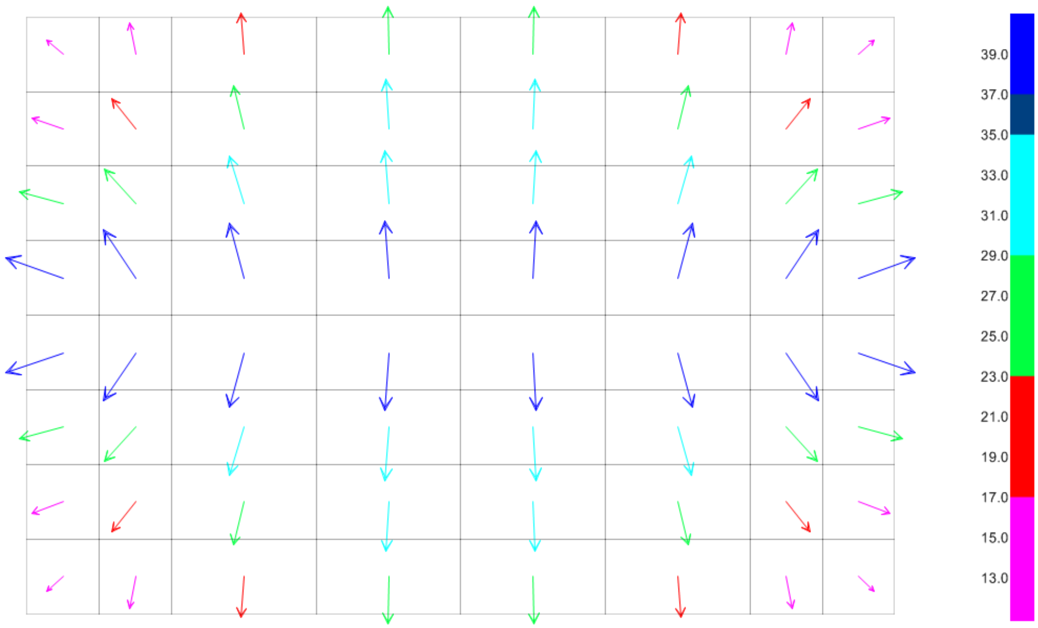


Figure 1-14 **V_{max} for Line Load for Thin-Plate Formulation**

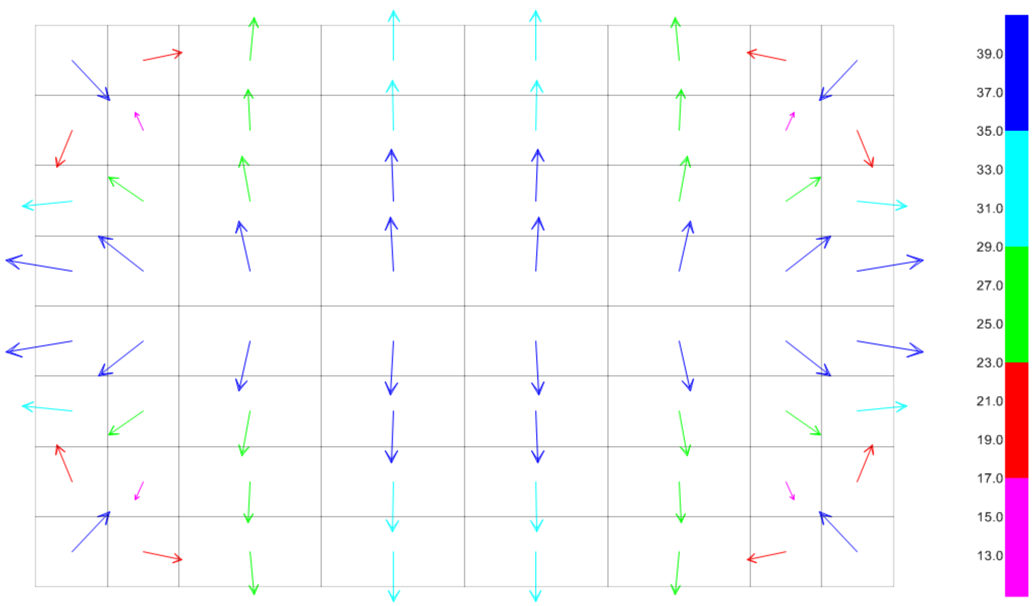


Figure 1-15 **V_{max} for Line Load for Thick-Plate Formulation**

COMPUTER FILE:

S01a-Thin.FDB, S01b-Thin.FDB, S01c-Thin.FDB, S01a-Thick.FDB, S01b-Thick.FDB and S01c-Thick.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

EXAMPLE 2

Rectangular Plate with Fixed Edges

PROBLEM DESCRIPTION

A fully fixed rectangular plate is analyzed for three load conditions. The geometric descriptions and material properties and the load cases are the same as those of Example 1. However, the boundary conditions are different. All edges are fixed, as shown in Figure 2-1. To test convergence, the problem is analyzed using three mesh sizes, as shown in Figure 1-2: 4 × 4, 8 × 8, and 12 × 12. The plate is modeled using plate elements available in SAFE. The fixed edges are modeled as line supports with large vertical and rotational stiffnesses. The self weight of the plate is not included in any of the load cases. The numerical data for this problem are given in the following section.

A theoretical solution to this problem, employing a single series (Lévy's solution), is given in Timoshenko and Woinowsky (1959). The numerically computed deflections obtained from SAFE are compared with the theoretical values.

GEOMETRY, PROPERTIES AND LOADING

Plate size	$a \times b$	=	360" × 240"
Plate thickness	T	=	8 inches
Modulus of Elasticity	E	=	3000 ksi
Poisson's ratio	v	=	0.3

Load Cases:

(UL)	Uniform load	q	=	100	psf
(PL)	Point load	P	=	20	kips
(LL)	Live load	q_1	=	1	kip/ft

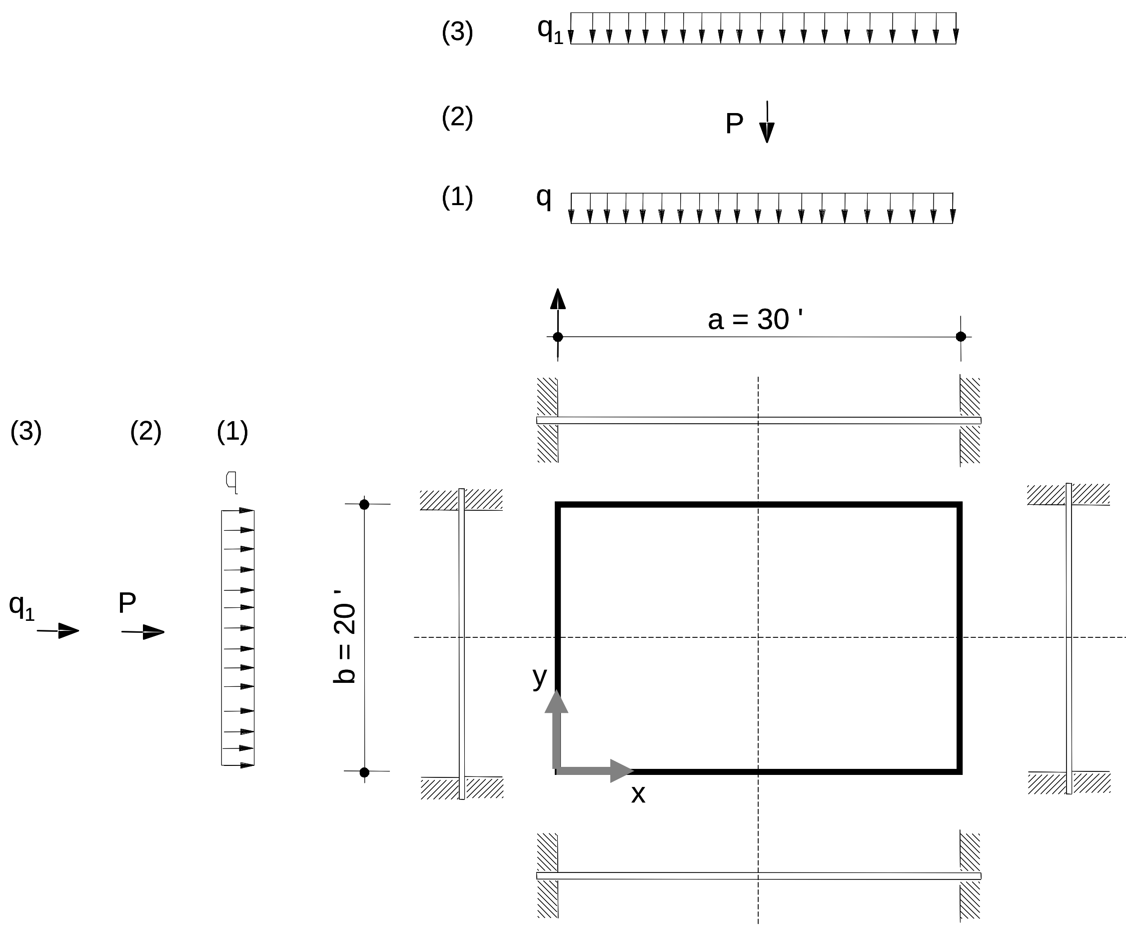


Figure 2-1 Rectangular Plate with All Edges Fixed

TECHNICAL FEATURES OF SAFE TESTED

- Comparison of slab deflection with bench mark solution.

RESULTS COMPARISON

The numerical displacements obtained from SAFE are compared with those obtained from the theoretical solution in Table 2-1. The theoretical results are based on tabular values given in Timoshenko and Woinowsky (1959). A comparison of deflections for the three load cases shows a fast convergence to the theoretical values with successive mesh refinement.

Table 2-1 Comparison of Displacements

Thin Plate Formulation

Load Case	Location		SAFE Displacement (in)			Theoretical Displacement (in)
	X (in)	Y (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	
UL	60	60	0.0098	0.0090	0.0089	
	60	120	0.0168	0.0153	0.0150	
	180	60	0.0237	0.0215	0.0210	
	180	120	0.0413	0.0374	0.0366	0.036036
PL	60	60	0.0065	0.0053	0.0052	
	60	120	0.0111	0.0100	0.0100	
	180	60	0.0315	0.0281	0.0272	
	180	120	0.0659	0.0616	0.0598	0.057453
LL	60	60	0.0079	0.0072	0.0071	
	60	120	0.0177	0.0161	0.0158	
	180	60	0.0209	0.0188	0.0184	
	180	120	0.0413	0.0375	0.0367	

Thick Plate Formulation

Load Case	Location		SAFE Displacement (in)			Theoretical Displacement (in)
	X (in)	Y (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	
UL	60	60	0.0085	0.0093	0.0091	
	60	120	0.0147	0.0156	0.0154	
	180	60	0.0214	0.0219	0.0215	
	180	120	0.0397	0.0381	0.0374	0.036036
PL	60	60	0.0083	0.0056	0.0053	
	60	120	0.0169	0.0101	0.0102	
	180	60	0.0270	0.0283	0.0278	
	180	120	0.0545	0.0600	0.0605	0.057453
LL	60	60	0.0072	0.0073	0.0073	
	60	120	0.0149	0.0165	0.0163	
	180	60	0.0198	0.0191	0.0188	
	180	120	0.0399	0.0382	0.0375	

COMPUTER FILE:

S02a-Thin.FDB. S02b-Thin.FDB, S02c-Thin.FDB, S02a-Thick.FDB. S02b-Thick.FDB, and S02c-Thick.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

EXAMPLE 3

Rectangular Plate with Mixed Boundary

PROBLEM DESCRIPTION

The plate, shown in Figure 3-1, is analyzed for uniform load only. The edges along $x = 0$ and $x = a$ are simply supported, the edge along $y = b$ is free, and the edge along $y = 0$ is fully fixed. The geometrical description and material properties of this problem are the same as those of Example 1. To test convergence, the problem is analyzed employing three mesh sizes, as shown in Figure 1-2: 4×4, 8×8, and 12×12. The plate is modeled using plate elements available in SAFE. The two simply supported edges are modeled as line supports with large vertical stiffnesses. The fixed edge is modeled as a line support with large vertical and rotational stiffnesses. The self weight of the plate is not included in the analysis.

An explicit analytical expression for the deflected surface is given in Timoshenko and Woinowsky (1959). The deflections obtained from SAFE are compared with the theoretical values.

GEOMETRY, PROPERTIES AND LOADING

Plate size	$a \times b$	=	360" × 240"
Plate thickness	T	=	8 inches
Modulus of elasticity	E	=	3000 ksi
Poisson's ratio	v	=	0.3

Load Cases:

Uniform load	q	=	100 psf

TECHNICAL FEATURES OF SAFE TESTED

- Comparison of deflection with bench-mark solution.

RESULTS COMPARISON

The numerical solution obtained from SAFE is compared with the theoretical solution that is given by Lévy (Timoshenko and Woinowsky 1959). Comparison of deflections shows monotonic convergence to the theoretical values with successive mesh refinement as depicted in Table 3-1. It is to be noted that even with a coarse mesh (4×4) the agreement is very good.

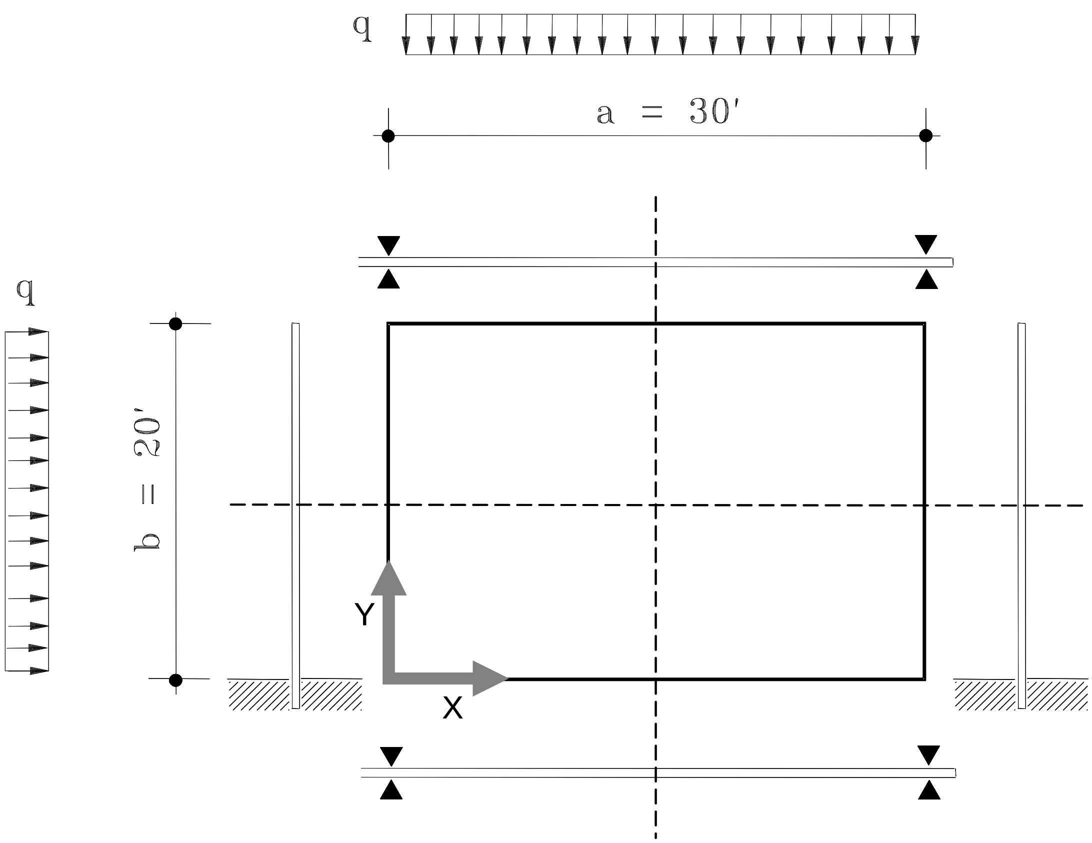


Figure 3-1 Rectangular Plate with Two Edges Simply Supported, One Edge Fixed and One Edge Free

Table 3-1 Comparison of Displacements

Thin Plate Formulation

Location		SAFE Displacement (in)			Theoretical Displacement (in)
X (in)	Y (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	
180	0	0.0000	0.0000	0.0000	0.0000
180	60	0.0849	0.0831	0.0827	0.08237
180	120	0.2410	0.2379	0.2372	0.23641
180	180	0.3971	0.3947	0.3940	0.39309
180	240	0.5537	0.5511	0.5502	0.54908

Thick Plate Formulation

Location		SAFE Displacement (in)			Theoretical Displacement (in)
X (in)	Y (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	
180	0	0.0000	0.0000	0.0000	0.0000
180	60	0.0806	0.0841	0.0839	0.08237
180	120	0.2338	0.2398	0.2392	0.23641
180	180	0.3837	0.3973	0.3970	0.39309
180	240	0.5322	0.5544	0.5542	0.54908

COMPUTER FILE:

S03a-Thin.FDB, S03b-Thin.FDB, S03c-Thin.FDB, S03a-Thick.FDB, S03b-Thick.FDB, and S03c-Thick.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

EXAMPLE 4

Rectangular Plate on Elastic Beams

PROBLEM DESCRIPTION

The plate, shown in Figure 4-1, is analyzed for a uniformly distributed surface load. The edges along $x = 0$ and $x = a$ are simply supported, and the other two edges are supported on elastic beams. It is assumed that the beams resist bending in vertical planes only and do not resist torsion. A theoretical solution to this problem is given in Timoshenko and Woinowsky (1959). The deflections of the plate and the moments and shears of the edge beams are compared with both the theoretical solution and the results obtained using the Direct Design Method as outlined in ACI 318-95 for a relative stiffness factor, λ, equal to 4. The relative stiffness, λ, is the ratio of the bending stiffness of the beam and the bending stiffness of the slab with a width equal to the length of the beam and is given by the following equation.

$$\lambda = \frac{EI_b}{aD}, \text{ where,}$$

$$D = \frac{Eh^3}{12\left(1 - v^2\right)},$$

I_b is the moment of inertia of the beam about the horizontal axis,

a is the length of the beam, which also is equal to the one side of the slab, and

h is the thickness of the slab.

To test convergence of results, the problem is analyzed employing three mesh sizes, as shown in Figure 1-2: 4 × 4, 8 × 8, and 12 × 12. The slab is modeled using plate elements. The simply supported edges are modeled as line supports with a large vertical stiffness and zero rotational stiffness. Beam elements, with no torsional rigidity, are defined on edges $y = 0$ and $y = b$. The flexural stiffness of edge beams is expressed as a λ factor of the plate flexural stiffness.

The subdivision of the plate into column and middle strips and also the definition of tributary loaded areas for shear calculations comply with ACI 318-95 provisions and shown in Figure 4-2. A load factor of unity is used and the self weight of the plate is not included in the analysis.

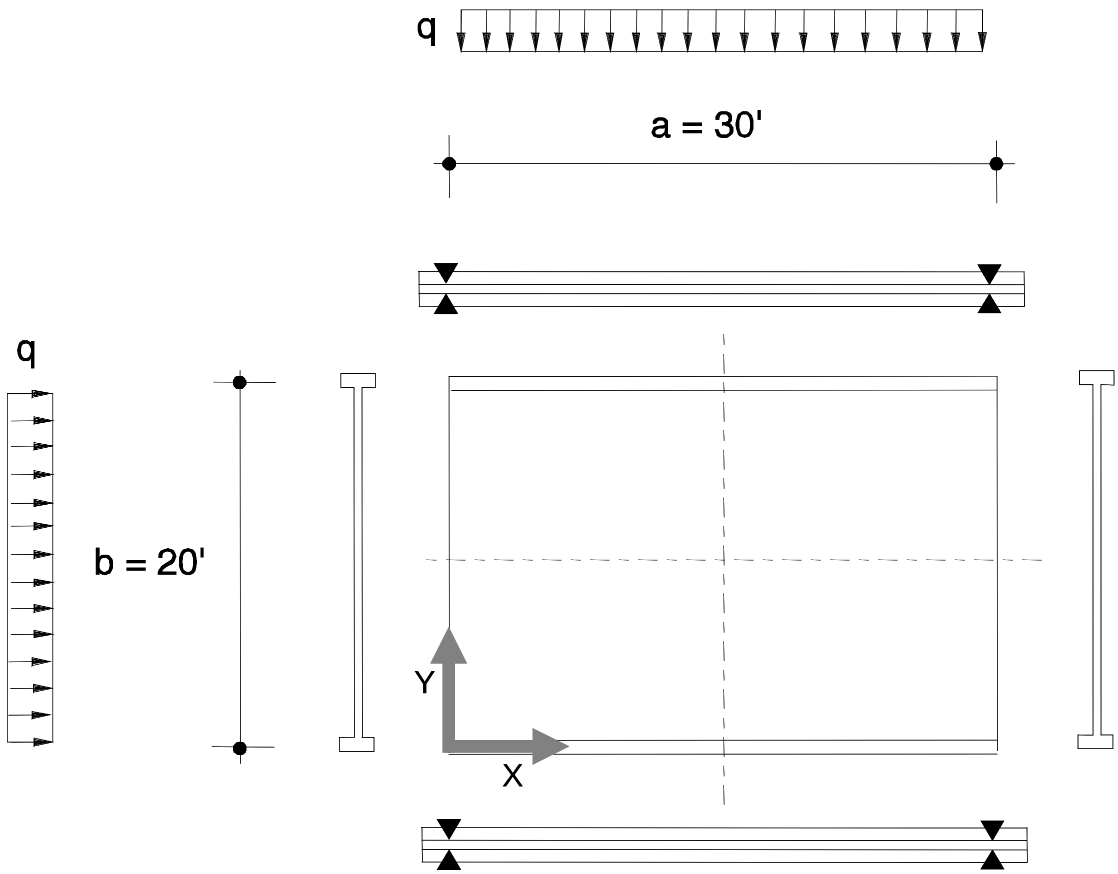
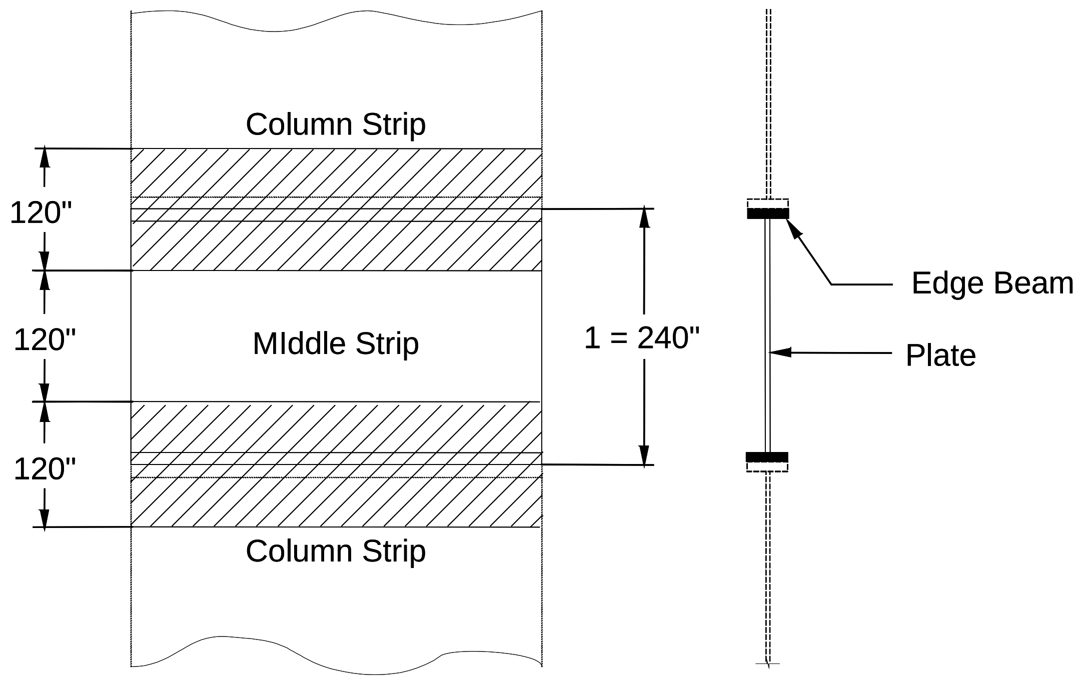
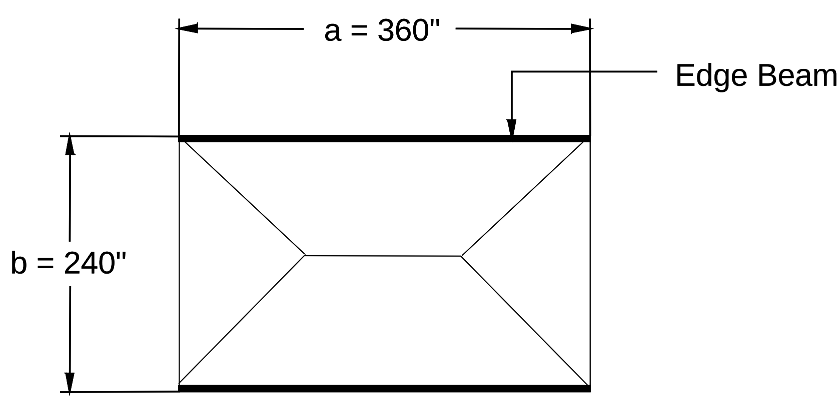


Figure 4-1 Rectangular Plate on Elastic Beams



Definition of Strips



Tributary Loaded Area for Shear on Edge Beams

Figure 4-2 Definition of Slab Strips and Tributary Areas for Shear on Edge Beams

GEOMETRY, PROPERTIES AND LOADING

Plate size	$a \times b$	=	360" × 240"
Plate thickness	T	=	8 inches
Modulus of elasticity	E	=	3000 ksi
Poisson's ration	v	=	0.3
Beam moment of inertia	I_b	=	4
Relative stiffness parameter	λ	=	4
Load Case:	q	=	100 psf (Uniform load)

TECHNICAL FEATURES OF SAFE TESTED

- Comparisons of deflection with benchmark solution.

RESULTS COMPARISON

Table 4-1 shows monotonic convergence of SAFE deflections for $\lambda = 4$ to the theoretical values with successive mesh refinement. Table 4-2 shows the variation of bending moment in the edge beam along its length for $\lambda = 4$. The theoretical solution and the ACI approximation using the Direct Design Method also are shown.

The value of λ is analogous to the ACI ratio $\alpha_1 l_2 / l_1$ (refer to Sections 13.6.4.4 and 13.6.5.1 of ACI 318-95). The correlation between the numerical results from SAFE and the theoretical results is excellent. For design purposes, the ACI approximation (Direct Design Method) compares well with the theory. For the Direct Design Method, the moments are obtained at the grid points. In obtaining SAFE moments, averaging was performed at the grid points.

In obtaining the ACI moments, the following quantities were computed:

$$\alpha_1 = E_{cb} I_b / E_{cs} I_s = 6.59375,$$

$$l_2/l_1 = 240/360 = 0.667,$$

$$\alpha_1 l_2/l_1 = 4.3958,$$

$$\beta_t = 0,$$

$$M_0 = 2700 \text{ k-in.}$$

From ACI section 13.6.4.4 for $l_2/l_1 = 0.667$ and $\alpha_1 l_2/l_1 = 4.3958$, it is determined that the column strip supports 85% of the total positive moment. The beam and slab do not carry any negative moment about the Y-axis because of the simply supported boundary conditions at x = 0 and x = a.

From ACI section 13.6.5.1 for $\alpha_1 l_2 / l_1$ = 4.3958, it is determined that the beam carries 85% of the total column strip moment. Since one beam supports only one-half of the column strip, the maximum beam positive moment is as follows

$$M_{positivebeam} = 0.85 \times 0.85 \times 0.5 \times M_0$$

$$= 0.36125 \times 2700$$

$$= 975.375 \text{ k-in}$$

The beam moments at other locations are obtained assuming a parabolic variation along the beam length.

Table 4-3 shows the variation of shear in edge beams for $\lambda = 4$. The agreement is good considering that the SAFE element considers shear strains and the theoretical solution does not. The ACI values are calculated based on the definition of loaded tributary areas given in Section 13.6.8.1 of ACI 318-95. The shear forces were obtained at the middle of the grid points. In obtaining SAFE shear, no averaging was required for the shear forces.

Table 4-1 Comparison of Displacements

Thin Plate Formulation

Location		SAFE Displacement (in)			Theoretical Displacement (in)
X (in)	Y (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	
180	120	0.1812	0.1848	0.1854	0.18572
180	60	0.1481	0.1523	0.1530	0.15349
180	0	0.0675	0.0722	0.0730	0.07365

Thick Plate Formulation

Location		SAFE Displacement (in)			Theoretical Displacement (in)
X (in)	Y (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	
180	120	0.1792	0.1856	0.1862	0.18572
180	60	0.1467	0.1529	0.1536	0.15349
180	0	0.0677	0.0721	0.0730	0.07365

Table 4-2 Variation of Average Bending Moment in an Edge Beam (λ = 4)

Thin Plate Formulation

Location		Edge Beam Moment (k-in)				
Y (in)	X (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	ACI	Theoretical
0, 240	0	0.571	0.12	0.05	0	0
	30	—	313.0	—	298.031	313.4984
	60	590.8	591.4	591.5	541.875	591.6774
	120	—	984.9	—	867.000	984.7026
	180	1120.9	1120.8	1120.4	975.375	1120.1518

Thick Plate Formulation

Location		Edge Beam Moment (k-in)				
Y (in)	X (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	ACI	Theoretical
0, 240	0	5.3	31.5	25.2	0	0
	30	—	309.2	—	298.031	313.4984
	60	591.0	586.8	592.1	541.875	591.6774
	120	—	981.3	—	867.000	984.7026
	180	1120.2	1116.4	1118.4	975.375	1120.1518

Table 4-3 Variation of Shear in an Edge Beam (λ = 4)

Thin Plate Formulation

Location		Edge Beam Shear (k)				
Y (in)	X (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	ACI	Theoretical
0, 240	10	—	—	10.58	9.9653	10.6122
	15	—	10.4	—	9.9219	10.4954
	30	9.80	—	9.96	9.6875	9.9837
	45	—	9.26	—	9.2969	9.2937
	50	—	—	9.02	9.1319	9.0336
	80	—	—	7.23	7.7778	7.2458
	90	4.40	6.55	—	7.1875	6.5854
	120	—	—	4.48	5.0000	4.4821
	150	—	2.26	—	2.5000	2.2656
	160	—	—	1.51	1.6667	1.5133

Thick Plate Formulation

Location		Edge Beam Shear (k)				
Y (in)	X (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	ACI	Theoretical
0, 240	10	—	—	8.04	9.9653	10.6122
	15	—	8.31	—	9.9219	10.4954
	30	9.59	—	7.91	9.6875	9.9837
	45	—	7.57	—	9.2969	9.2937
	50	—	—	7.43	9.1319	9.0336
	80	—	—	6.39	7.7778	7.2458
	90	4.32	6.03	—	7.1875	6.5854
	120	—	—	4.06	5.0000	4.4821
	150	—	2.08	—	2.5000	2.2656
	160	—	—	1.38	1.6667	1.5133

COMPUTER FILE:

S04a-Thin.FDB, S04b-Thin.FDB, S04c-Thin.FDB, S04a-Thick.FDB, S04b-Thick.FDB, and S04c-Thick.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

EXAMPLE 5

Infinite Flat Plate on Equidistant Columns

PROBLEM DESCRIPTION

The plate, shown in Figure 5-1, is analyzed for uniform load. The overall dimensions of the plate are significantly larger than the column spacing (*a* and *b* in Figure 5-1). Analysis is limited to a single interior panel because it can be assumed that deformation is identical for all panels in the plate. An analytical solution, based on the foregoing assumption, is given in Timoshenko and Woinowsky (1959).

Three mesh sizes, as shown in Figure 1-2, are used to test the convergence property of the model: 4 × 4, 8 × 8, and 12 × 12. The model consists of a panel of uniform thickness supported at four corners point. The effect of column support within a finite area is not modeled. Due to symmetry, the slope of the deflection surface in the direction normal to the boundaries is zero along the edges and the shearing force is zero at all points along the edges of the panel, except at the corners. To model this boundary condition, line supports with a large rotational stiffness about the support line are defined on all four edges. Additional point supports are provided at the corners. The panel is modeled using plate elements in SAFE. In doing so, the effect of shear distortion is included.

To compare the effects of corner stiffness at the column/slab intersection, a duplicate model of the 12 x 12 mesh was created where this region is approximately modeled. This was done by using a special stiff area section in the region concerned, shown as the 40" × 40" area in Figure 5-2, of which a 20” x 20” portion lies within the modeled region. To obtain design moments, the panel is divided into three strips both ways, two column strips and one middle strip, based on the ACI 318-95 definition of design strip widths, as shown in Figure 5-2 and in Figure 5-3. A load factor of unity is used. The self weight of the panel is not included in the analysis.

Tables 5-1 through 5-3 show the comparison of the numerically computed deflection, local moments, and local shears obtained from SAFE with their theoretical counterparts.

Table 5-4 shows the comparison of the average design strip moments obtained from SAFE with those obtained from the theoretical method and two ACI alternative methods: the Direct Design Method (DDM) and the Equivalent Frame Method (EFM).

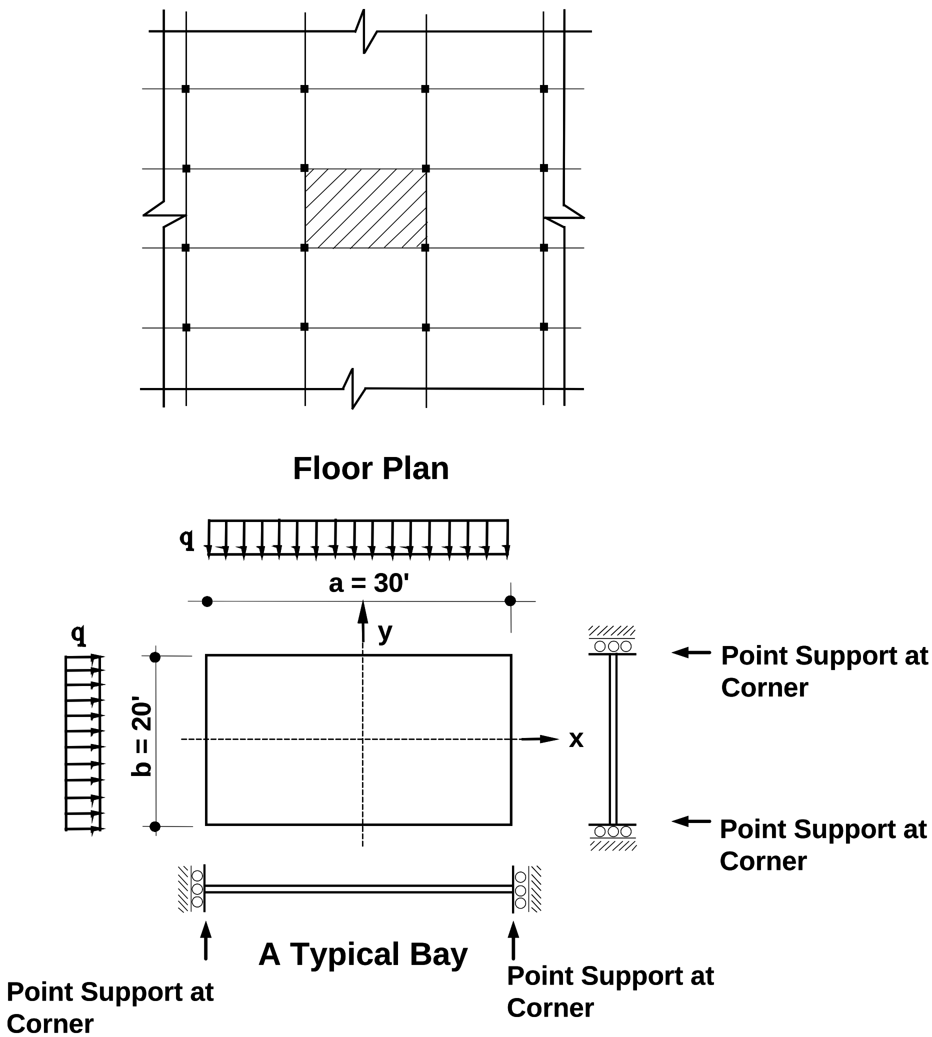
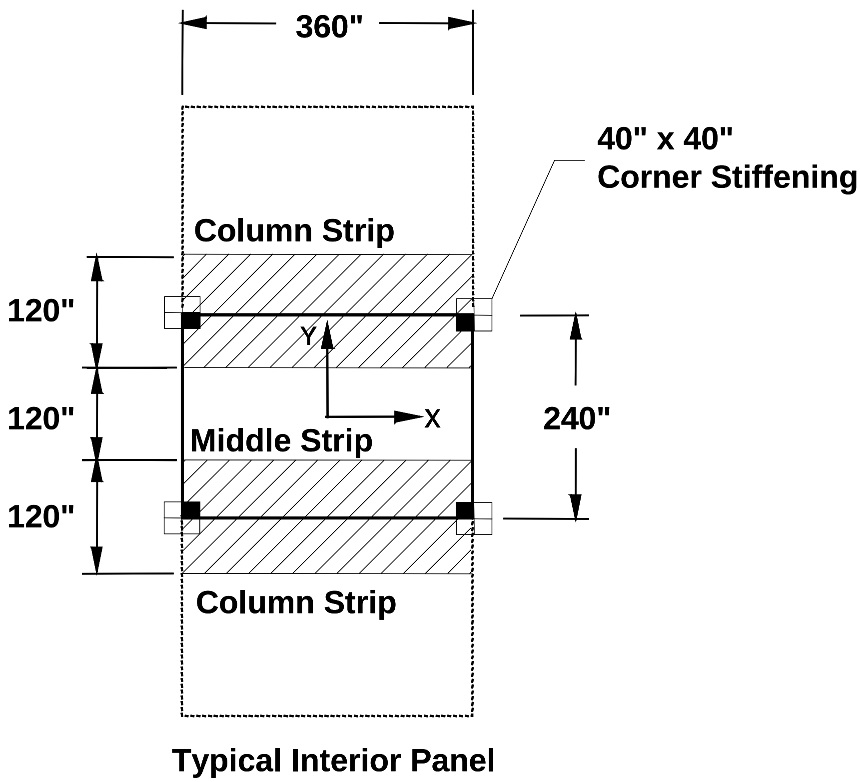


Figure 5-1 Infinite Plate on Equidistant Columns and Detail of Panel used in Analysis

<u>Material Properties and Load</u>

Modulus of Elasticity	= 3000 ksi
Poisson's Ratio	= 0.3
Uniform Load	= 100 psf



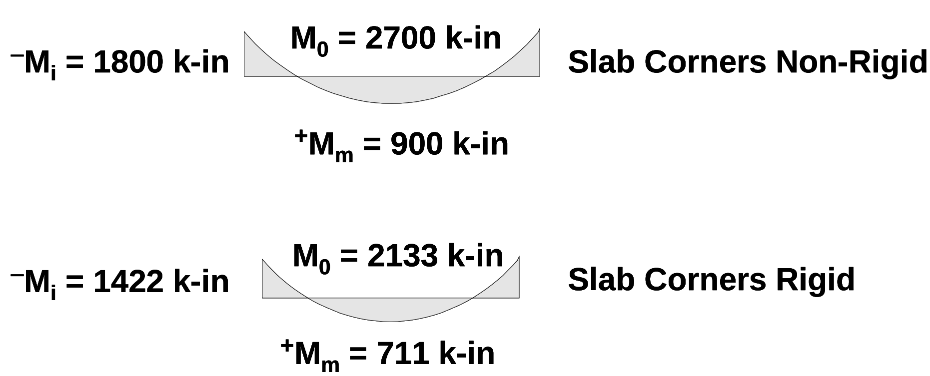


Figure 5-2 Definition of X-Strips
(Moment values obtained by EFM)

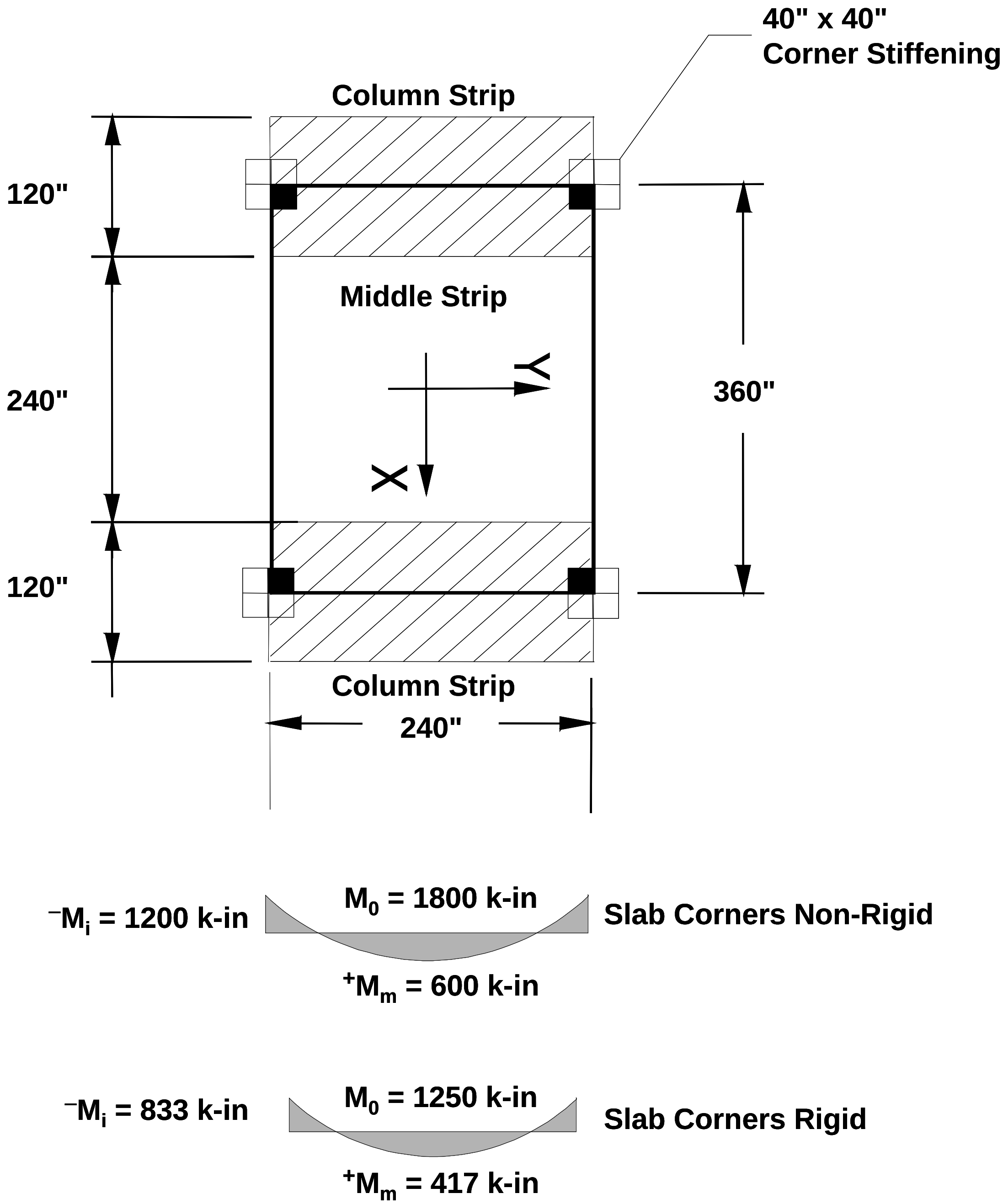


Figure 5-3 Definition of Y-Strips
(Moment values obtained by EFM)

GEOMETRY, PROPERTIES AND LOADING

Plate size	$a \times b$	=	360" × 240"
Plate thickness	T	=	8 inches
Modulus of elasticity	E	=	3000 ksi
Poisson's ration	v	=	0.3
Load Case:	q	=	100 psf (Uniform load)

TECHNICAL FEATURES OF SAFE TESTED

- Comparisons of deflection with benchmark solution.

RESULTS COMPARISON

Table 5-1 shows the comparison of the numerical and the theoretical deflections. The data indicates monotonic convergence of the numerical solution to the theoretical values with successive mesh refinement.

The SAFE results for local moment and shear also compare closely with the theoretical values, as shown in Table 5-2 and Table 5-3, respectively.

In Table 5-4 average strip moments obtained from SAFE are compared with both the ACI and the theoretical values. EFM is used to calculate the interior span moments as depicted in Figure 5-2 and Figure 5-3. The agreement between the SAFE and the theoretical solution is excellent. ACI approximations, employing either DDM or EFM, however, deviate from the theory. It should be noted that, regardless of the method used, the absolute sum of positive and negative moments in each direction equals the total static moment in that direction.

Table 5-5 shows the effect of corner rigidity. Comparisons with the EFM method are shown.

Table 5-1 Comparison of Displacements

Thin Plate Formulation

Location		SAFE Displacement (in)			Theoretical Displacement (in)
X (in)	Y (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	
0	0	0.263	0.278	0.280	0.280
0	60	0.264	0.274	0.275	0.275
0	120	0.266	0.271	0.271	0.270
120	0	0.150	0.153	0.153	0.152
120	120	0.101	0.101	0.100	0.098
180	0	0.114	0.108	0.106	0.104
180	60	0.072	0.069	0.067	0.065
180	120	0.000	0.000	0.000	0.000

Thick Plate Formulation

Location		SAFE Displacement (in)			Theoretical Displacement (in)
X (in)	Y (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	
0	0	0.249	0.279	0.284	0.280
0	60	0.252	0.276	0.280	0.275
0	120	0.252	0.273	0.275	0.270
120	0	0.139	0.155	0.157	0.152
120	120	0.082	0.101	0.103	0.098
180	0	0.094	0.109	0.110	0.104
180	60	0.052	0.069	0.070	0.065
180	120	0.000	0.000	0.000	0.000

Table 5-2 Comparison of Local Moments

Thin Plate Formulation

Location		Moments (k-in/in)			
		M_{11}		M_{22}	
X (in)	Y (in)	SAFE (8×8)	Theoretical	SAFE (8×8)	Theoretical
30	15	3.093	3.266	1.398	1.470
30	105	3.473	3.610	0.582	0.580
165	15	–2.948	–3.142	1.887	1.904
165	105	–9.758	–9.804	–7.961	–7.638

Thick Plate Formulation

Location		Moments (k-in/in)			
		M_{11}		M_{22}	
X (in)	Y (in)	SAFE (8×8)	Theoretical	SAFE (8×8)	Theoretical
30	15	3.115	3.266	1.394	1.470
30	105	3.446	3.610	0.583	0.580
165	15	–2.977	–3.142	1.846	1.904
165	105	–9.686	–9.804	–7.894	–7.638

Table 5-3 Comparison of Local Shears

Thin Plate Formulation

Location		Shears ($\times 10^{-3}$ k)			
		V_{13}		V_{23}	
X (in)	Y (in)	SAFE (8×8)	Theoretical	SAFE (8×8)	Theoretical
30	45	20.9	17.3	8.2	2.2
30	105	21.2	23.5	3.1	5.4
165	15	17.3	14.7	19.1	23.8
165	105	357.1	329.0	350.4	320.0

Thick Plate Formulation

Location		Shears ($\times 10^{-3}$ k)			
		V_{13}		V_{23}	
X (in)	Y (in)	SAFE (8×8)	Theoretical	SAFE (8×8)	Theoretical
30	45	20.2	17.3	8.7	2.2
30	105	24.3	23.5	8.1	5.4
165	15	26.7	14.7	24.7	23.8
165	105	287.5	329.0	277.6	320.0

Table 5-4 Comparison of Average Strip Moments

Thin Plate Formulation

Average Moment	Location	Strip	SAFE Moments (k-in/in) 4 × 4 Mesh	8 × 8 Mesh	12 × 12 Mesh	Theoretical (k-in/in)	ACI 318-95 (k-in/in) DDM	EFM
$\bar{M}_A$	x = 180"	Column	4.431	3.999	3.922	3.859	4.725	4.500
		Middle	4.302	3.805	3.711	3.641	3.150	3.000
$\bar{M}_A$	x = 360"	Column	–10.184	–10.865	–10.971	–11.091	–10.968	–11.250
		Middle	–3.524	–3.777	–3.843	–3.891	–3.656	–3.750
$\bar{M}_B$	y= 120"	Column	2.265	2.028	1.971	1.925	3.150	3.000
		Middle	1.674	1.561	1.547	1.538	1.050	1.000
$\bar{M}_B$	y = 240"	Column	–8.236	–8.902	–9.000	–9.139	–7.313	–7.500
		Middle	–0.551	–0.449	–0.442	–0.430	–1.219	–1.250

Thick Plate Formulation

Average Moment	Location	Strip	SAFE Moments (k-in/in) 4 × 4 Mesh	8 × 8 Mesh	12 × 12 Mesh	Theoretical (k-in/in)	ACI 318-95 (k-in/in) DDM	EFM
$\bar{M}_A$	x = 180"	Column	4.802	4.079	3.952	3.859	4.725	4.500
		Middle	3.932	3.726	3.682	3.641	3.150	3.000
$\bar{M}_A$	x = 360"	Column	–8.748	–10.691	–10.993	–11.091	–10.968	–11.250
		Middle	–4.965	–3.954	–3.823	–3.891	–3.656	–3.750

Thick Plate Formulation

Average Moment	Location	Strip	SAFE Moments (k-in/in) 4 × 4 Mesh	SAFE Moments 8 × 8 Mesh	SAFE Moments 12 × 12 Mesh	Theoretical (k-in/in)	ACI 318-95 (k-in/in) DDM	ACI 318-95 EFM
$\bar{M}_B$	y= 120"	Column	2.361	2.078	2.000	1.925	3.150	3.000
		Middle	1.628	1.537	1.533	1.538	1.050	1.000
$\bar{M}_B$	y = 240"	Column	–6.321	–8.670	–9.025	–9.139	–7.313	–7.500
		Middle	–1.514	–0.567	–0.431	–0.430	–1.219	–1.250

Table 5-5 Comparison of Average Strip Moments : Effect of Corner Rigidity

Thin Plate Formulation

Average Moment	Location	Strip	SAFE Moments (12×12 Mesh) (k-in/in) Slab Corner Non-Rigid	SAFE Moments Slab Corner Rigid	ACI 318-95 (EFM Method) (k-in/in) Slab Corner Non-Rigid	ACI 318-95 Slab Corner Rigid
$\bar{M}_A$	x = 180"	Column	3.922	3.472	4.500	3.555
		Middle	3.711	3.285	3.000	2.370
$\bar{M}_A$	x = 360"	Column	–10.971	–8.110	—	–8.887
		Middle	–3.843	–2.863	—	–2.962
$\bar{M}_B$	y= 120"	Column	1.971	1.470	3.000	2.085
		Middle	1.547	1.361	1.000	0.695
$\bar{M}_B$	y = 240"	Column	–4.807	–5.489	—	–5.206
		Middle	–0.272	–0.347	—	–0.867

Thick Plate Formulation

Average Moment	Location	Strip	SAFE Moments (12×12 Mesh) (k-in/in) Slab Corner Non-Rigid	SAFE Moments (12×12 Mesh) (k-in/in) Slab Corner Rigid	ACI 318-95 (EFM Method) (k-in/in) Slab Corner Non-Rigid	ACI 318-95 (EFM Method) (k-in/in) Slab Corner Rigid
$\bar{M}_A$	x = 180"	Column	3.952	3.459	4.500	3.555
		Middle	3.682	3.219	3.000	2.370
$\bar{M}_A$	x = 360"	Column	–10.993	–8.249	—	–8.887
		Middle	–3.823	–2.806	—	–2.962
$\bar{M}_B$	y= 120"	Column	2.000	1.456	3.000	2.085
		Middle	1.533	1.327	1.000	0.695
$\bar{M}_B$	y = 240"	Column	–9.025	–5.742	—	–5.206
		Middle	–0.431	–0.263	—	–0.867

COMPUTER FILE:

S05a-Thin.FDB, S05b-Thin.FDB, S05c-Thin.FDB, S05d.FDB, S05a-Thick.FDB, S05b-Thick.FDB, S05c-Thick.FDB, and S05d-Thick.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

EXAMPLE 6

Infinite Flat Plate on Elastic Subgrade

PROBLEM DESCRIPTION

An infinite plate resting on elastic subgrade and carrying equidistant and equal loads, *P*, is shown in Figure 6-1. Each load is assumed to be distributed uniformly over the area $u \times v$ of a rectangle. A theoretical double series solution to this example is given in Timoshenko and Woinowsky (1959).

The numerically computed deflections and local moments obtained from SAFE are compared to the theoretical values, as shown in Table 6-1 and Table 6-2.

Analysis is confined to a single interior panel. To model the panel, three mesh sizes, as shown in Figure 1-2, are used: 4 × 4, 8 × 8, and 12 × 12. The slab is modeled using plate elements and the elastic support is modeled as a surface support with a spring constant of *k*, the modulus of subgrade reaction. The edges are modeled as line supports with a large rotational stiffness about the support line. Point loads $P/4$ are defined at the panel corners. In the theoretical formulation (Timoshenko and Woinowsky 1959), each column load *P* is assumed to be distributed over an area $u \times v$ of a rectangle, as shown in Figure 6-1. To apply the theoretical formulation to this problem, concentrated corner loads are modeled as a uniformly distributed load acting over a very small rectangular area where *u* and *v* are very small.

GEOMETRY, PROPERTIES AND LOADING

Plate size	$a \times b$	=	360" × 240"
Plate thickness	T	=	15 inches
Modulus of elasticity	E	=	3000 ksi
Poisson's ratio	v	=	0.2
Modulus of subgrade reaction	k	=	1 ksi/in

Loading: Point Load	P	=	400 kips

(assumed to be uniformly distributed over an area $u \times v$)

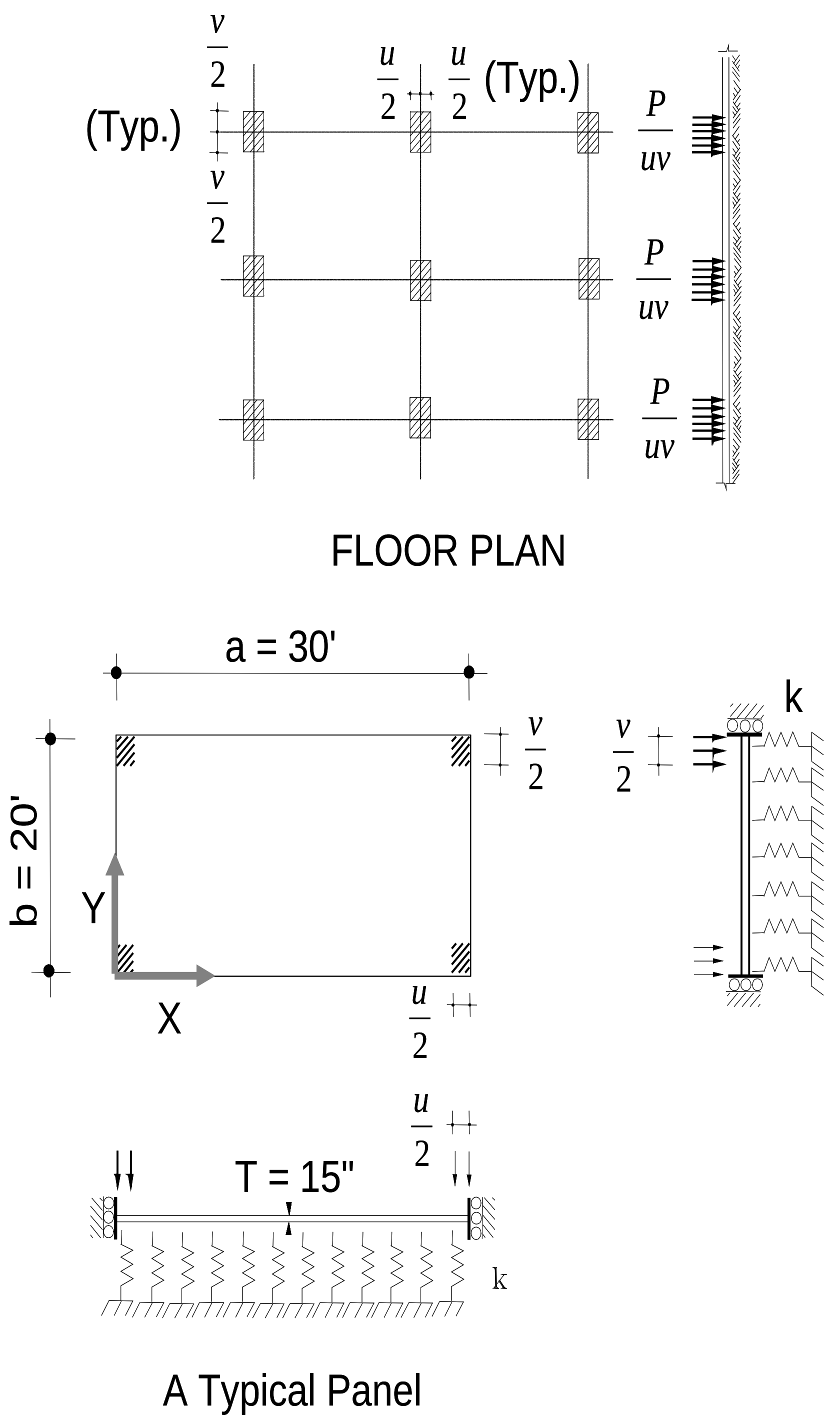


Figure 6-1 Rectangular Plate on Elastic Subgrade

TECHNICAL FEATURES OF SAFE TESTED

- Comparison of deflection on elastic foundation.

RESULTS COMPARISON

Good agreement has been found between the numerical and theoretical deflection for k = 1 ksi/in, as shown in Table 6-1, except near the concentrated load. The consideration of shear strains in the SAFE element makes it deflect more near the concentrated load. As the modulus k is changed, the distribution of pressure between the plate and the subgrade changes accordingly. The particular case, as k approaches 0, corresponds to a uniformly distributed subgrade reaction, i.e., to the case of a "reversed flat slab" uniformly loaded with $q = P/ab$. In fact the problem changes to that of Example 5, with the direction of vertical axis reversed. In Example 5, for a uniform load of 100 psf (P = 60 kips), the maximum relative deflection is calculated as 0.280. Applying the formulation used here with $k = 1 \times 10^{-6}$ yields a deflection value of 0.279". Table 6-2 shows the comparison of the SAFE local moments using the 12 × 12 mesh with the theoretical results. The results agree well.

Table 6-1 Comparison of Displacements

Thin Plate Formulation

Location		SAFE Displacement (in)			Theoretical Displacement (in)
X (in)	Y (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	
0	0	–0.0493	–0.05410	–0.05405	–0.05308
180	60	0.00091	0.00076	0.00080	0.00096
180	120	0.00040	0.00060	0.00064	0.00067

Thick Plate Formulation

Location		SAFE Displacement (in)			Theoretical Displacement (in)
X (in)	Y (in)	4×4 Mesh	8×8 Mesh	12×12 Mesh	
0	0	–0.0436	–0.06011	–0.06328	–0.05308
180	60	0.00130	0.00074	0.00076	0.00096
180	120	–0.0019	0.00050	0.00059	0.00067

Table 6-2 Comparison of Local Moments

Thin Plate Formulation

Location		Moments (kip-in/in)			
		M_{11}		M_{22}	
X (in)	Y (in)	SAFE (12×12)	Theoretical	SAFE (12×12)	Theoretical
10	10	37.99	35.97	37.97	35.56
10	50	7.38	7.70	–6.74	–6.87
10	110	–0.30	–0.27	–5.48	–5.69
80	10	–6.52	–6.89	1.98	1.72
80	50	–3.58	–3.78	–0.93	–1.02
80	110	–0.88	–0.98	–1.86	–1.69

Thick Plate Formulation

Location		Moments (kip-in/in)			
		M_{11}		M_{22}	
X (in)	Y (in)	SAFE (12×12)	Theoretical	SAFE (12×12)	Theoretical
10	10	36.77	35.97	36.73	35.56
10	50	7.13	7.70	–6.37	–6.87
10	110	–0.21	–0.27	–5.17	–5.69
80	10	–6.11	–6.89	2.05	1.72
80	50	–3.56	–3.78	–0.82	–1.02
80	110	–0.87	–0.98	–1.86	–1.69

COMPUTER FILE:

S06a-Thin.FDB, S06b-Thin.FDB, S06c-Thin.FDB, S06a-Thick.FDB, S06b-Thick.FDB and S06c-Thick.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

EXAMPLE 7

Skewed Plate with Mixed Boundary

PROBLEM DESCRIPTION

A skewed plate under uniform load, as shown in Figure 7-1, is analyzed for two different support configurations. In the first case, all the edges are assumed to be simply supported. In the second case, the edges $y = 0$ and $y = b$ are released, i.e., the plate is assumed to be supported on its oblique edges only. A theoretical solution to this problem is given in Timoshenko and Woinowsky (1959). In both cases, the maximum deflection and the maximum moment are compared with the corresponding theoretical values.

An 8 × 24 base mesh is used to model the plate, as shown in Figure 7-1. A large vertical stiffness is defined for supports, and support lines are added on all four edges for the first case and along the skewed edges only for the second case. A load factor of unity is used. The self weight of the plate is not included in the analysis.

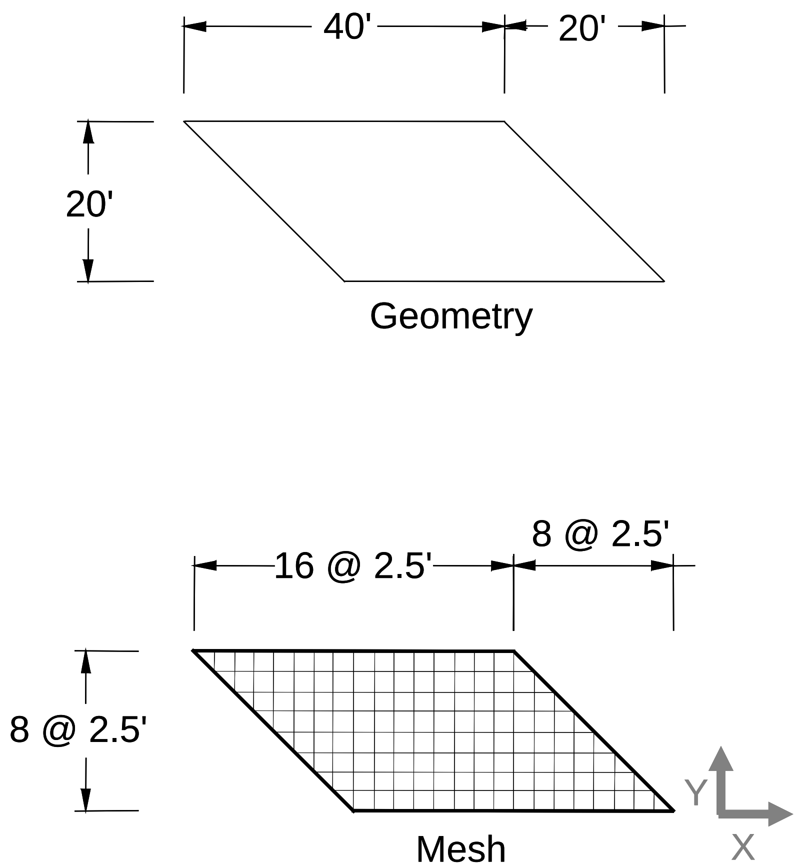
GEOMETRY, PROPERTIES, AND LOADING

Plate size	$a \times b$	=	480" × 240"
Plate thickness	T	=	8 inches
Modulus of elasticity	E	=	3,000 ksi
Poisson's ratio	v	=	0.2

Load Cases:	Uniform load, q	=	100 psf

TECHNICAL FEATURES OF SAFE TESTED

- Comparison of deflection and moments on skewed plate.



Support Conditions:

(1) Simply supported on all edges

(2) Simply supported on oblique edges

Figure 7-1 Skew Plate

RESULTS COMPARISON

Under the simply supported boundary condition, maximum deflection occurs at the plate center and the maximum principal moment acts nearly in the direction of the short span. Under the simply supported condition on the oblique edges and free boundary conditions on the other two edges, maximum deflection occurs at the free edge as expected.

Table 7-1 Comparison of Deflections and Bending Moments

Boundary Condition	Responses	SAFE		Theoretical
		Thin Plate	Thick Plate	
Simply supported on all edges	Maximum displacement (inches)	0.156	0.160	0.162
	Maximum bending moment (k-in)	3.66	3.75	3.59
Simply supported on oblique edges	Maximum displacement at the free edges (in)	1.51	1.52	1.50
	Maximum bending moment of the free edges (k-in)	12.03	12.28	11.84
Simply supported on oblique edges	Displacement at the center (in)	1.21	1.23	1.22
	Maximum bending moment at the center (k-in)	11.78	11.81	11.64

COMPUTER FILES

S07a-Thin.FDB, S07b-Thin.FDB, S07a-Thick.FDB and S07b-Thick.FDB

CONCLUSION

The comparison of SAFE and the theoretical results is acceptable, as shown in Table 7-1.

EXAMPLE 8

ACI Handbook Flat Slab Example 1

PROBLEM DESCRIPTION

The flat slab system, arranged three-by-four, is shown in Figure 8-1. The slab consists of twelve 7.5-inch-thick 18' × 22' panels. Edge beams on two sides extend 16 inches below the slab soffit. Details are shown in Figure 8-2. There are three sizes of columns and in some locations, column capitals. Floor to floor heights below and above the slab are 16 feet and 14 feet respectively. A full description of this problem is given in Example 1 of ACI 340.R-97 (ACI Committee 340, 1997). The total factored moments in an interior E-W design frame obtained from SAFE are compared with the corresponding results obtained by the Direct Design Method, the Modified Stiffness Method, and the Equivalent Frame Method.

The computational model uses a 10 × 10 mesh of elements per panel, as shown in Figure 8-3. The mesh contains gridlines at column centerlines, column faces, and the edges of column capitals. The grid lines extend to the slab edges. The regular slab thickness is 7.5". A slab thickness of 21.5" is used to approximately model a typical capital. The slab is modeled using plate elements. The columns are modeled as point supports with vertical and rotational stiffnesses. Stiffness coefficients used in the calculation of support flexural stiffness are all reproduced from ACI Committee 340 (1997). Beams are defined on two slab edges, as shown in Figure 8-1.

The model is analyzed for a uniform factored load of 0.365 ksf ($w_u = 1.4w_d + 1.7\ w_t$) in total, including self weight. To obtain factored moments in an E-W interior design frame, the slab is divided into strips in the X-direction (E-W direction), as shown in Figure 8-4. An interior design frame consists of one column strip and two halves of adjacent middle strips.

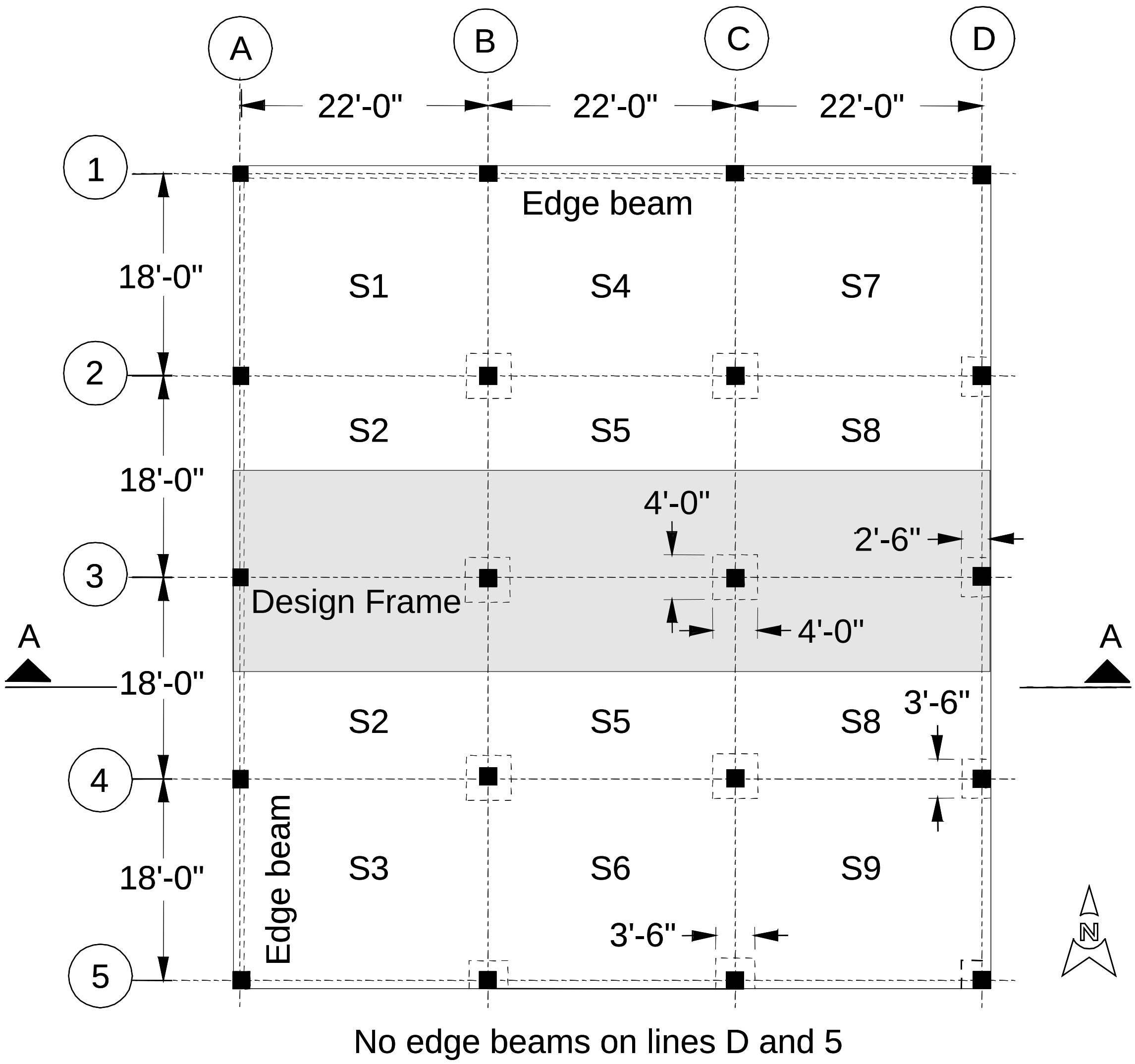
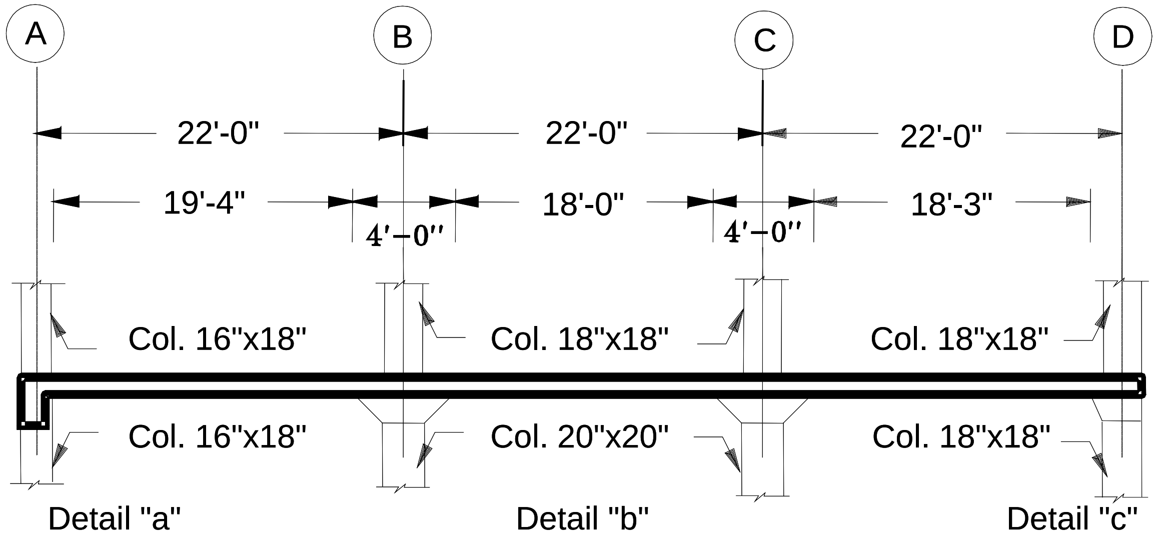


Figure 8-1 Flat Slab from ACI Handbook



SECTION A—A

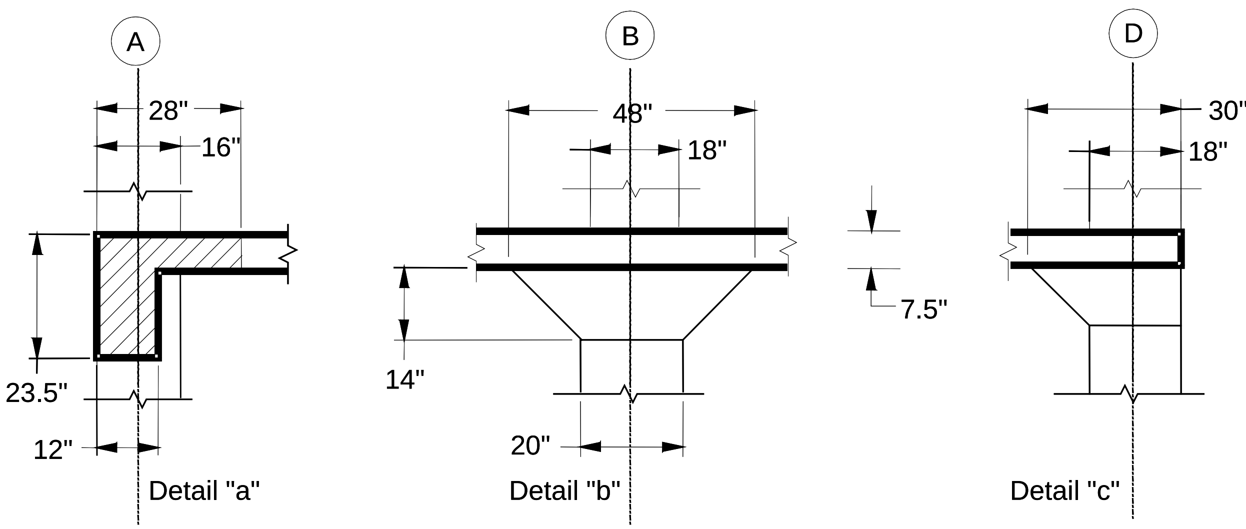


Figure 8-2 Sections and Details of ACI Handbook Flat Slab Example

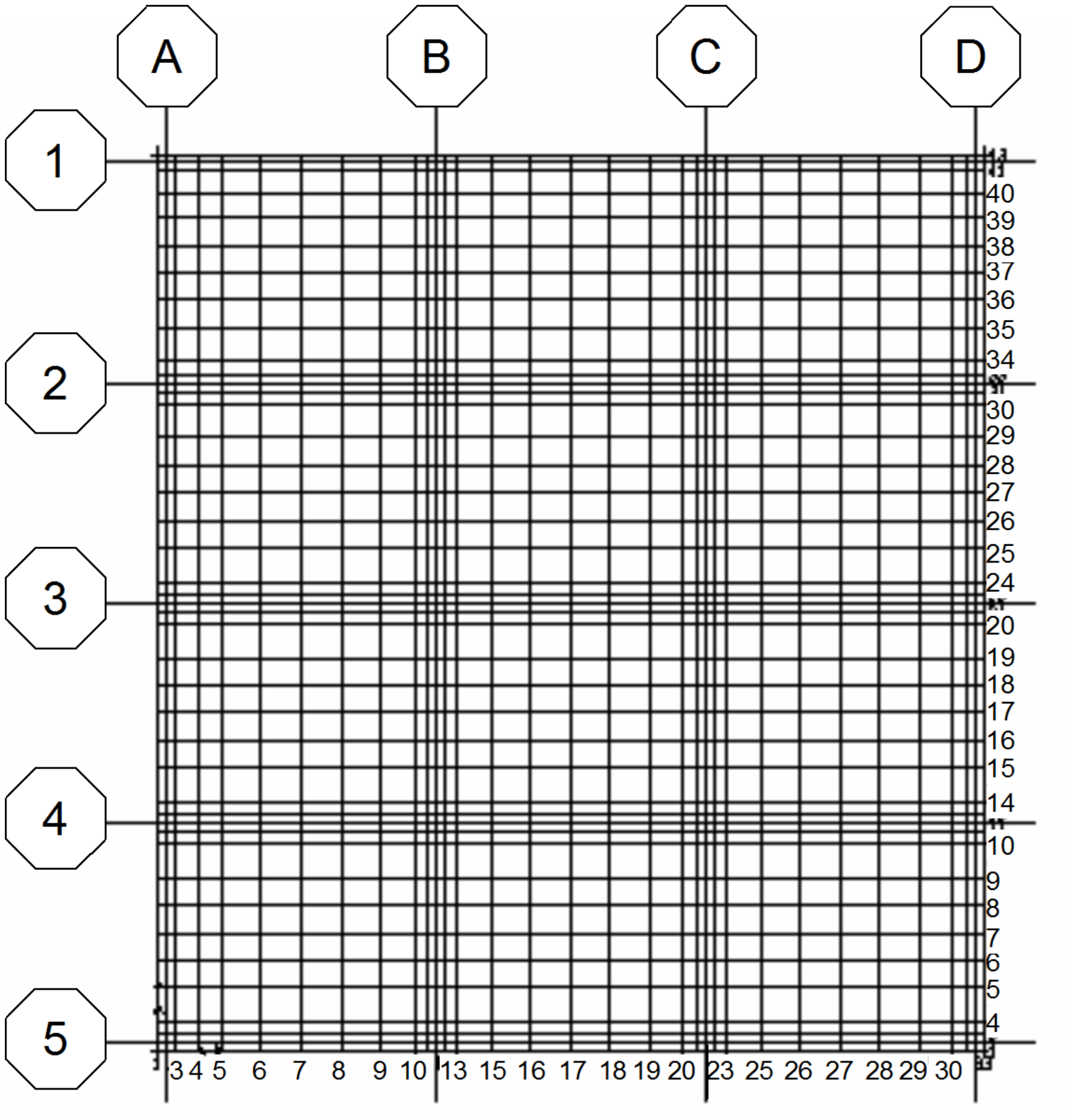


Figure 8-3 SAFE Mesh (10 × 10 per panel)

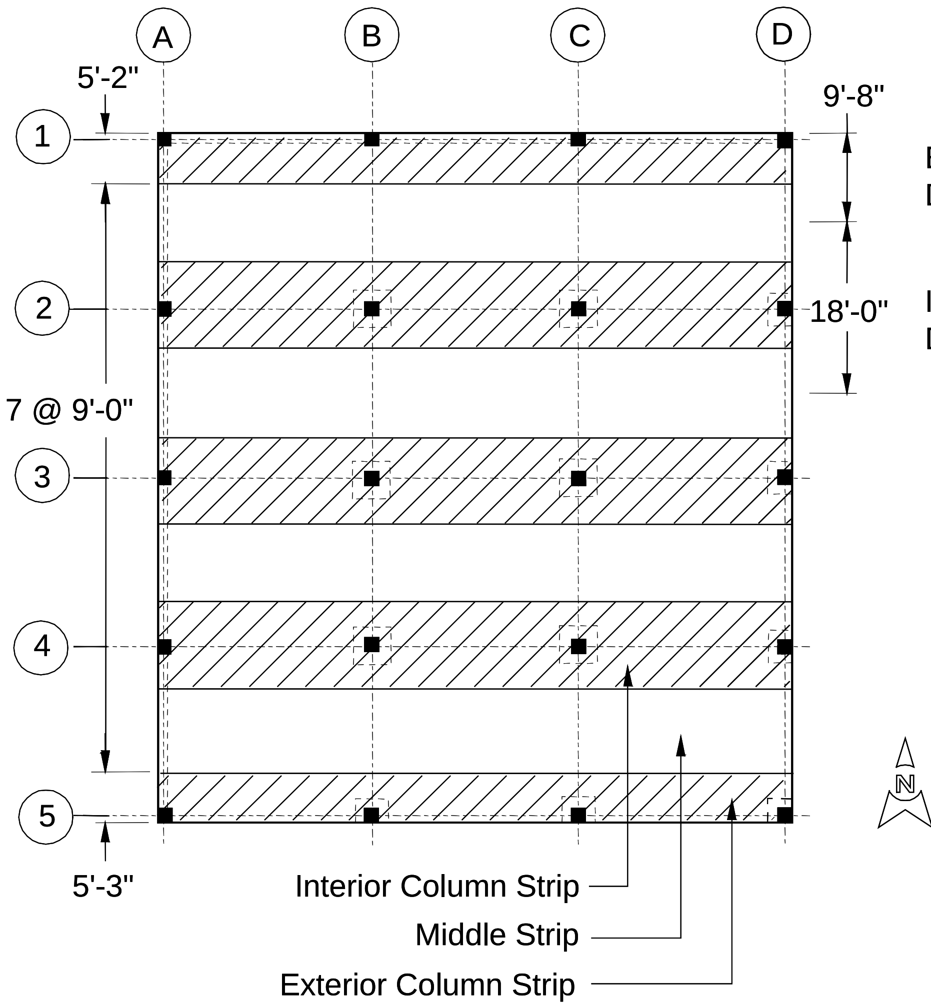


Figure 8-4 Definition of E-W Design Frames and Strips

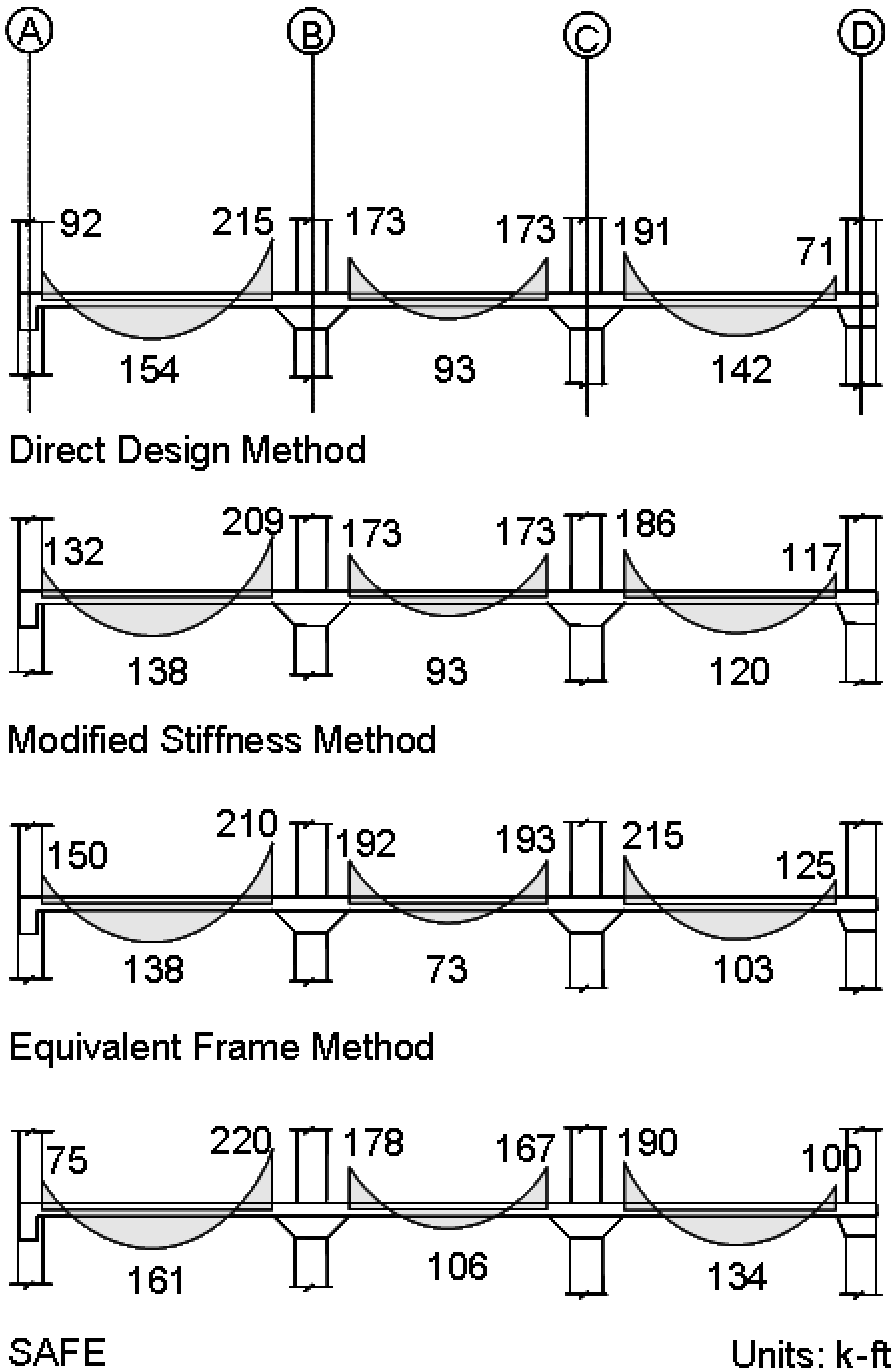


Figure 8-5 Comparison of Total Factored Moments (E-W Design Frame)

GEOMETRY, PROPERTIES AND LOADING

Materials:

Concrete strength	f_c'	=	3	ksi
Yield strength of steel	f_y	=	40	ksi
Concrete unit weight	γ_c	=	150	pcf
Modulus of elasticity	E_c	=	3320	ksi
Poisson's ratio	ν	=	0.2	

TECHNICAL FEATURES OF SAFE TESTED

- Comparison of factored moments in slab.

RESULTS COMPARISON

The SAFE results for the total factored moments in an interior E-W design frame are compared in Figure 8-5 with the results obtained by the Direct Design Method (DDM), the Modified Stiffness Method (MSM), and the Equivalent Frame Method (EFM). Only uniform loading with load factors of 1.4 and 1.7 has been considered. The DDM, MSM, and EFM results are all reproduced from Example 1 of ACI Committee 340 (1997), the Alternative Example 1 of ACI Committee 340 (1991), and from Example 3 of ACI Committee 340 (1991), respectively. Moments reported are calculated at the face of column capitals. Overall, they compare well. A noticeable discrepancy is observed in the negative column moment in the west side of the exterior bay (the edge beam side). In contrast to the EFM, the DDM appears to underestimate this moment. The SAFE result are between the two extreme values. The basic cause of this discrepancy is the way in which each method accounts for the combined flexural stiffness of columns framing into the joint. The DDM uses a stiffness coefficient k_c of 4 in the calculation of column and slab flexural stiffnesses. The EFM, on the other hand, uses higher value of k_c to allow for the added stiffness of the capital and the slab-column joint. The use of MSM affects mainly the exterior bay moments, which is not the case when the DDM is employed. In SAFE, member contributions to joint stiffness are dealt with more systematically than any of the preceding approaches. Hence, the possibility of over designing or under designing a section is greatly reduced.

The factored strip moments are compared in Table 8-1. There is a discrepancy in the end bays, particularly on the edge beam (west) side, where the SAFE and EFM results for exterior negative column strip moment show the greatest difference. This is expected because EFM simplifies a 3D structure to a 2D structure, thereby neglecting the transverse interaction between adjacent strips. Except for this localized difference, the comparison is good.

Table 8-1 Comparison of Total Factored Strip Moments (k-ft) (Interior E-W Design Frame)

Strip	Method	Factored Strip Moment (k-ft)								
		Span AB			Span BC			Span CD		
		^-M	^+M	^-M	^-M	^+M	^-M	^-M	^+M	^-M
Column Strip	DDM	86	92	161	130	56	130	143	85	71
	MSM	122	83	157	130	56	130	140	72	117
	EFM	140	83	157	144	44	145	161	62	125
	SAFE	69	85	159	128	58	121	138	72	88
Middle Strip	DDM	6	62	54	43	37	43	48	57	0
	MSM	10	55	52	43	37	43	46	48	0
	EFM	10	55	53	48	29	48	54	41	0
	SAFE	7	78	62	51	48	46	52	62	13

COMPUTER FILE:

S08.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

EXAMPLE 9

ACI Handbook Two-Way Slab Example 2

PROBLEM DESCRIPTION

The two-way slab system arranged three-by-three is shown in Figure 9-1. The slab consists of nine 6.5-inch-thick 20-foot × 24-foot panels. Beams extend 12 inches below the slab soffit. Details are shown in Figure 9-2. Sixteen inch × 16 inch columns are used throughout the system. Floor to floor height is 15 feet. A full description of this problem is given in Example 2 of ACI 340.R-91 (ACI Committee 340, 1991). The total factored moments in an interior design frame obtained from SAFE are compared with the Direct Design Method, the Modified Stiffness Method, and the Equivalent Frame Method.

The computational model uses a 10 × 10 mesh of elements per panel, as shown in Figure 9-3. The mesh contains grid lines at both column centerlines and column faces. The grid lines are extended to the slab edges. The slab is modeled using plate elements. The columns are modeled as point supports with vertical and rotational stiffnesses. A stiffness coefficient of 4 EI/L is used in the calculation of support flexural stiffness. Torsional constants of 4790 in^4 and 5478 in^4 are defined for the edge and interior beams respectively, in accordance with Section 13.7.5 of ACI 318-89 and Section 13.0 of ACI 318-95 code. The model is analyzed for uniform factored total load of 0.347 ksf ($w_u = 1.4w_d + 1.7w_1$), including self weight. To obtain factored moments in an interior design frame, the slab is divided into strips in the X-direction (E-W direction), as shown in Figure 9-4. An interior design frame consists of one column strip and two halves of adjacent middle strips.

GEOMETRY, PROPERTIES AND LOADING

Concrete strength	f_c'	=	3	ksi
Yield strength of steel	f_y	=	40	ksi
Concrete unit weight	w_c	=	150	psf
Modulus of elasticity	E_c	=	3120	ksi
Poisson's ratio	v	=	0.2	
Live load	w_1	=	125	psf
Mechanical load	w_d	=	15	psf
Exterior wall load	w_{wall}	=	400	plf

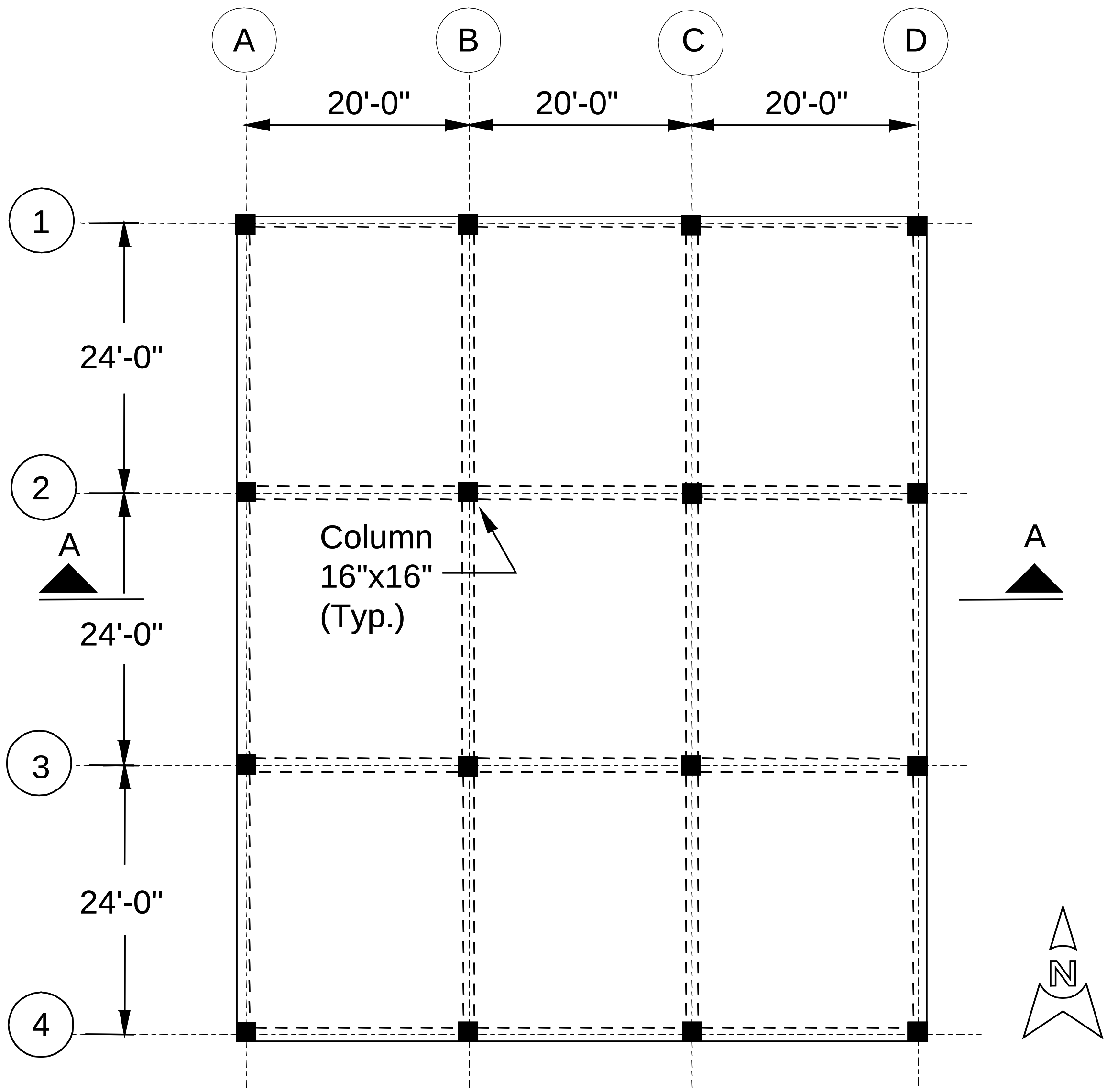


Figure 9-1 ACI Handbook Two-Way Slab Example

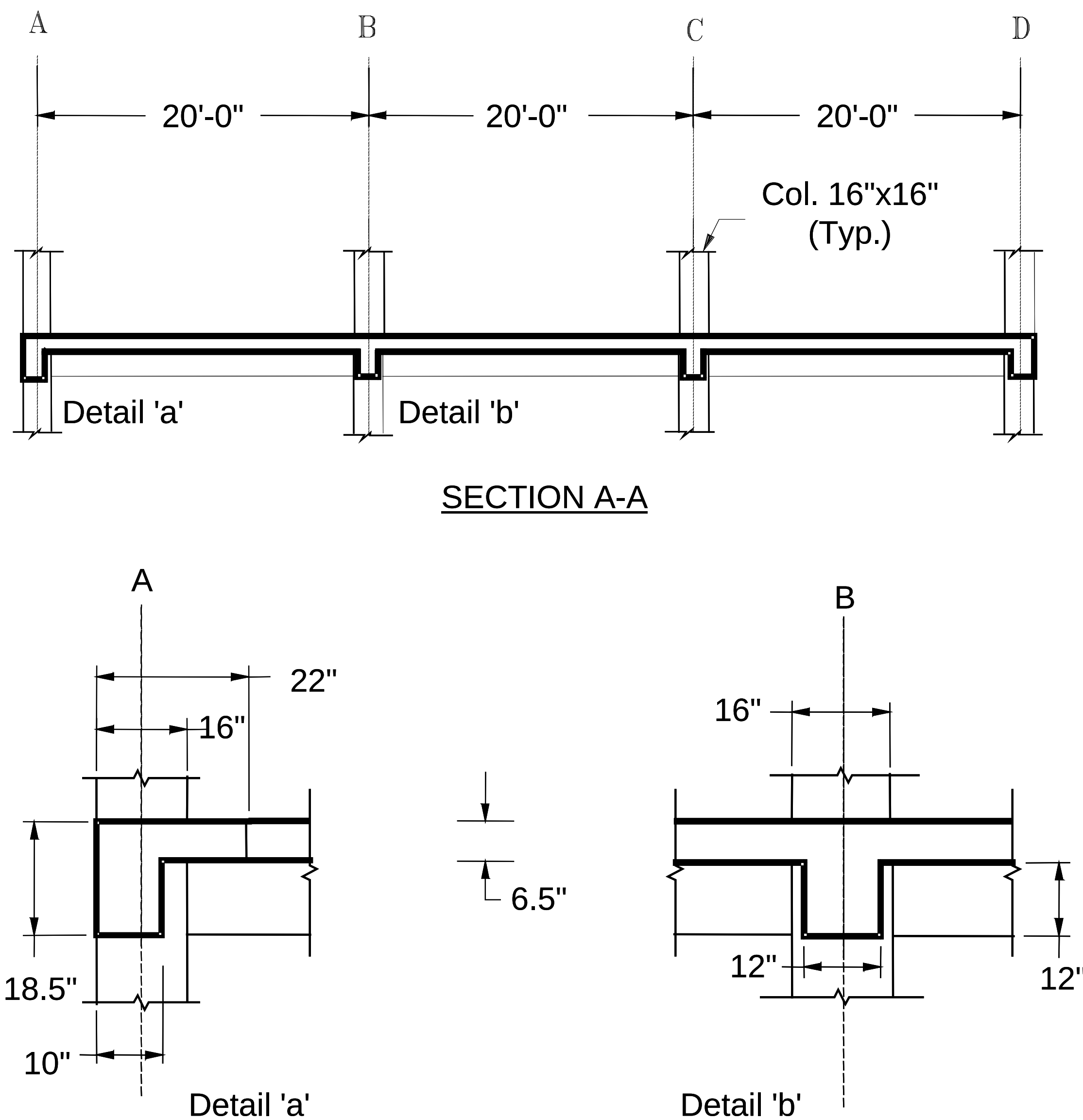


Figure 9-2 Details of Two-Way Slab Example from ACI Handbook

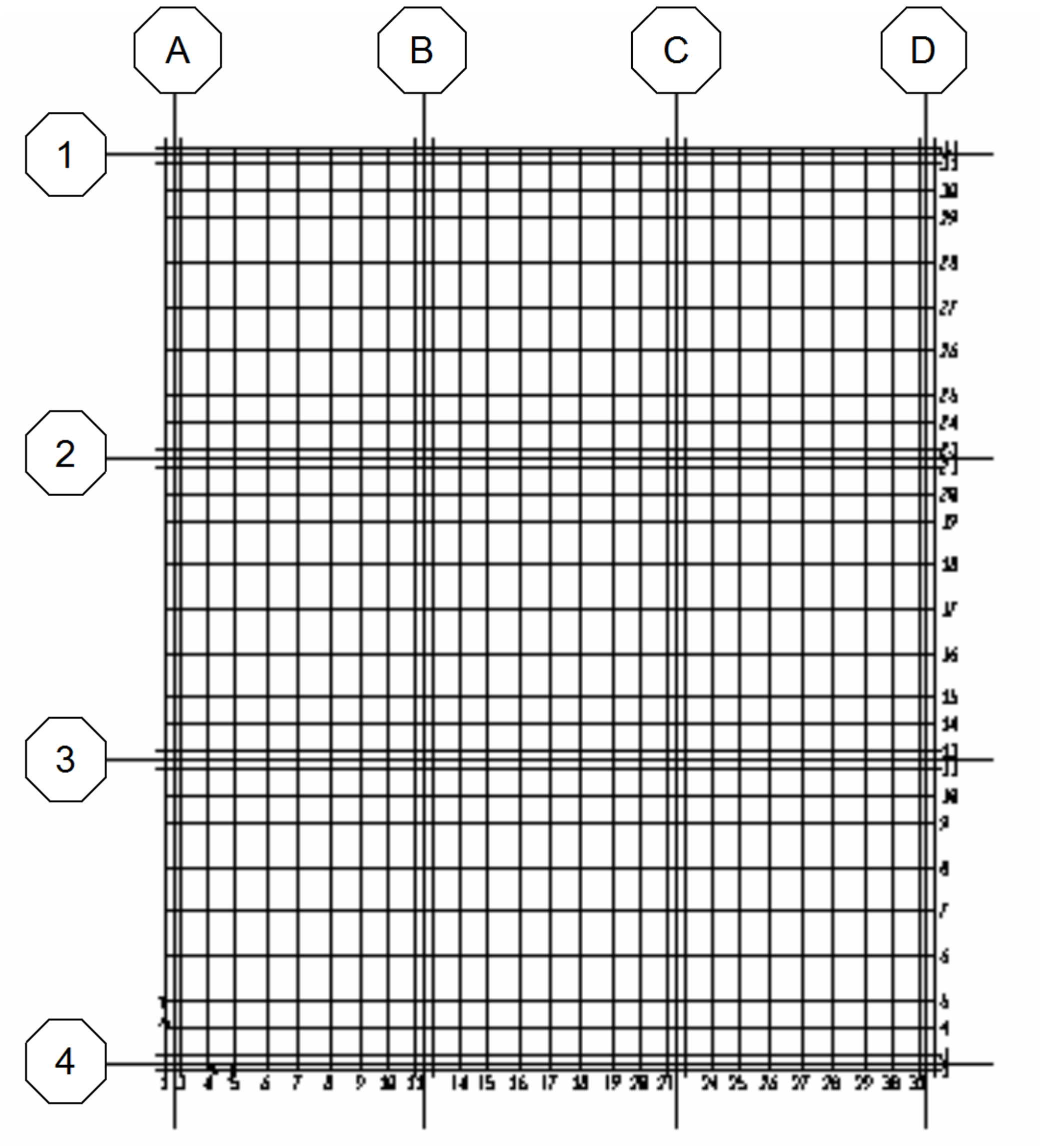


Figure 9-3 SAFE Mesh (10 × 10 per panel)

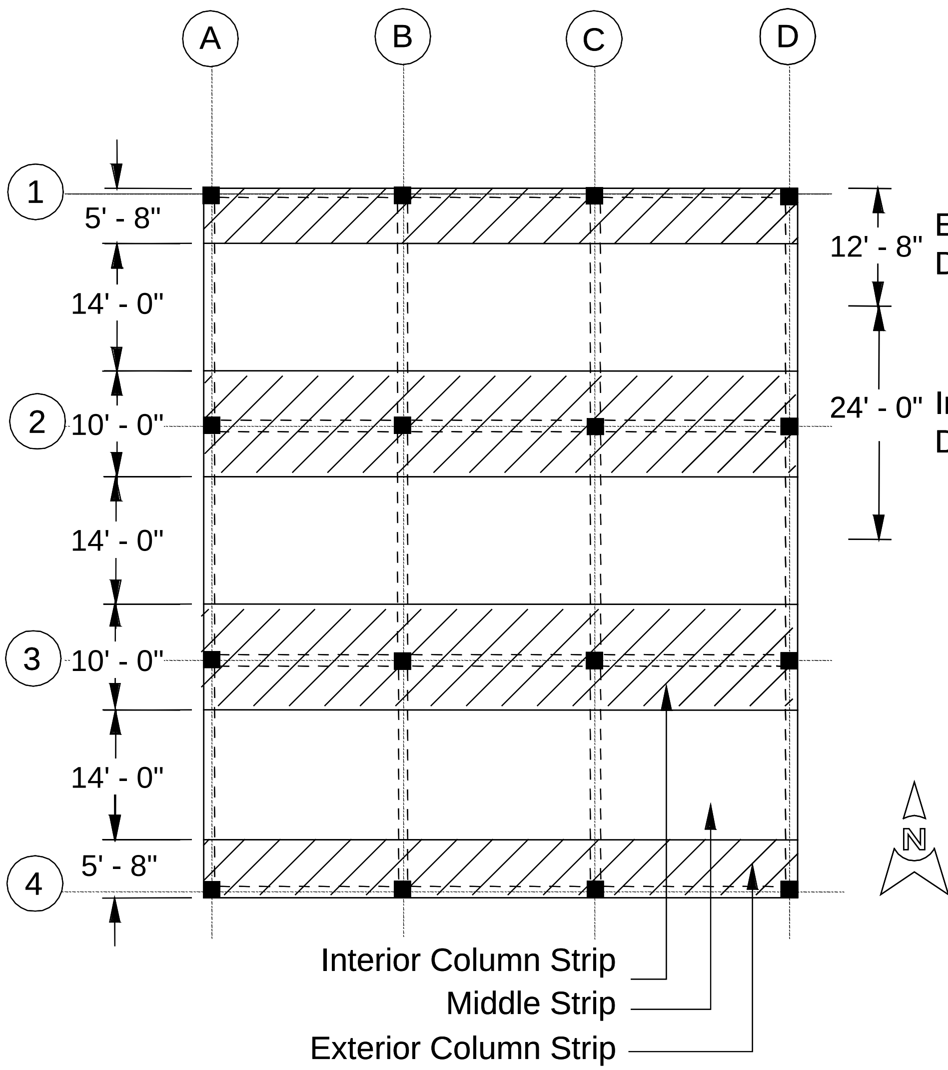


Figure 9-4 Definition of E-W Design Frames and Strips

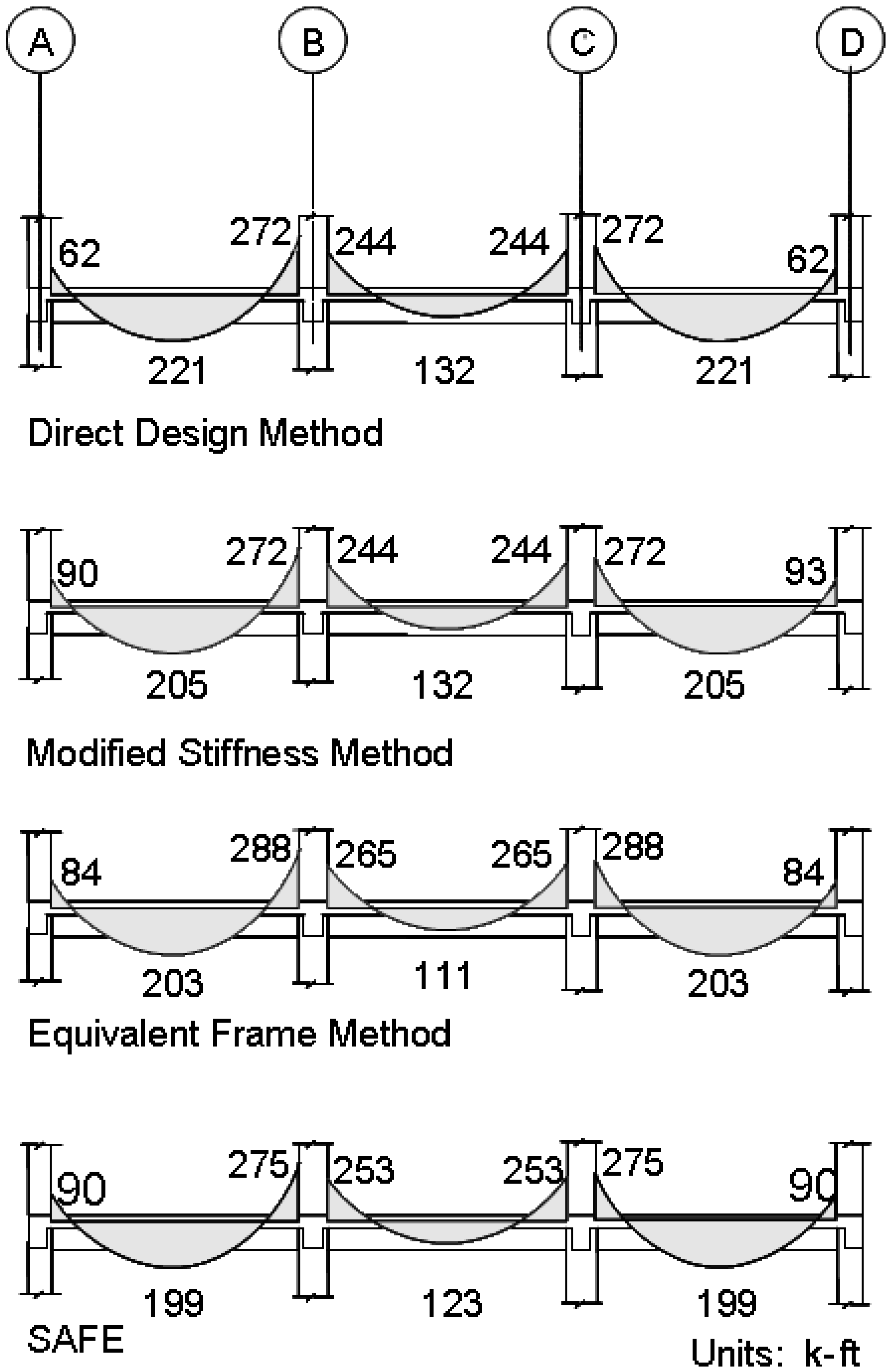


Figure 9-5 Comparison of Total Factored Moments (k-ft) in an Interior E-W Design Frame

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of factored moments in slab.

RESULTS COMPARISON

The SAFE results for the total factored moments in an interior E-W design frame are compared with the results obtained by the Direct Design Method (DDM), the Modified Stiffness Method (MSM), and the Equivalent Frame Method (EFM) as shown in Figure 9-5. The results are for uniform loading with load factors. The results are reproduced from ACI Committee 340 (1991). Moments reported are calculated at the column face. For all practical purposes they compare well. At the end bays, the MSM appears to overestimate the exterior column negative moments with the consequent reduction in the mid-span moments.

The distribution of total factored moments to the beam, column strip, and middle strip is shown in Table 9-1. The middle strip moments compare well. The total column strip moments also compare well. The distribution of the column strip moments between the slab and the beam has a larger scatter.

Table 9-1 Comparison of Total Factored Moments (kip-ft)

Strip	Method	Exterior Span ^-M	Exterior Span ^+M	Exterior Span ^-M	Interior Span ^-M	Interior Span ^+M	Interior Span ^-M
Slab Column Strip	DDM	9	23	28	25	14	25
	MSM	13	21	28	25	14	25
	EFM	12	21	30	27	11	27
	SAFE	22	27	62	58	14	58
Slab Middle Strip	DDM	3	69	84	76	41	76
	MSM	5	63	84	76	41	76
	EFM	4	63	89	82	34	82
	SAFE	6	71	73	73	49	73
Beam	DDM	50	129	160	143	77	143
	MSM	72	121	160	143	77	143
	EFM	68	119	169	156	66	156
	SAFE	62	102	141	122	60	122

Total Factored Moments in an E-W Design Frame(kip-ft)

COMPUTER FILE:

S09.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

EXAMPLE 10

PCA Flat Plate Test

PROBLEM DESCRIPTION

This example models the flat plate structure tested by the Portland Cement Association (Guralnick and LaFraugh 1963). The structure consists of nine 5.25-inch-thick 15-foot × 15-foot panels arranged 3 × 3, as shown in Figure 10-1. Deep and shallow beams are used on the exterior edges. The structure is symmetric about the diagonal line through columns A1, B2, C3, and D4, except the columns themselves are not symmetric about this line. The corner columns are 12 inches × 12 inches and the interior columns are 18 inches × 18 inches. Columns along the edges are 12 inches × 18 inches, with the longer dimension parallel to the plate edge. A typical section of the plate and details of edge beams are given in Figure 10-2. The total moments in an interior frame obtained numerically from SAFE are compared with the test results and the numerical values obtained by the Equivalent Frame Method (EFM).

A finite element model, shown in Figure 10-3, with 6 × 6 mesh per panel is employed in the analysis. The slab is modeled using the plate elements in SAFE. The columns are modeled as point supports with vertical and rotational stiffnesses. The reduced-height columns in the test structure are fixed at the base. Hence, rotational stiffnesses of point supports are calculated using a stiffness coefficient of 4 and an effective height of 39.75 inches ($K_c = 4EI / l_c$). In order to account for rigidity of the slab-column joint, the portion of slab occupying the column area is modeled as rigid by using a special stiff area element. A total uniformly distributed design load of 156 psf (not factored) is applied to all the panels.

To obtain design moment coefficients, the plate is divided into column and middle strips. An interior design frame consists of one column strip and half of each adjacent middle strip. Normalized values of design moments are used in the comparison.

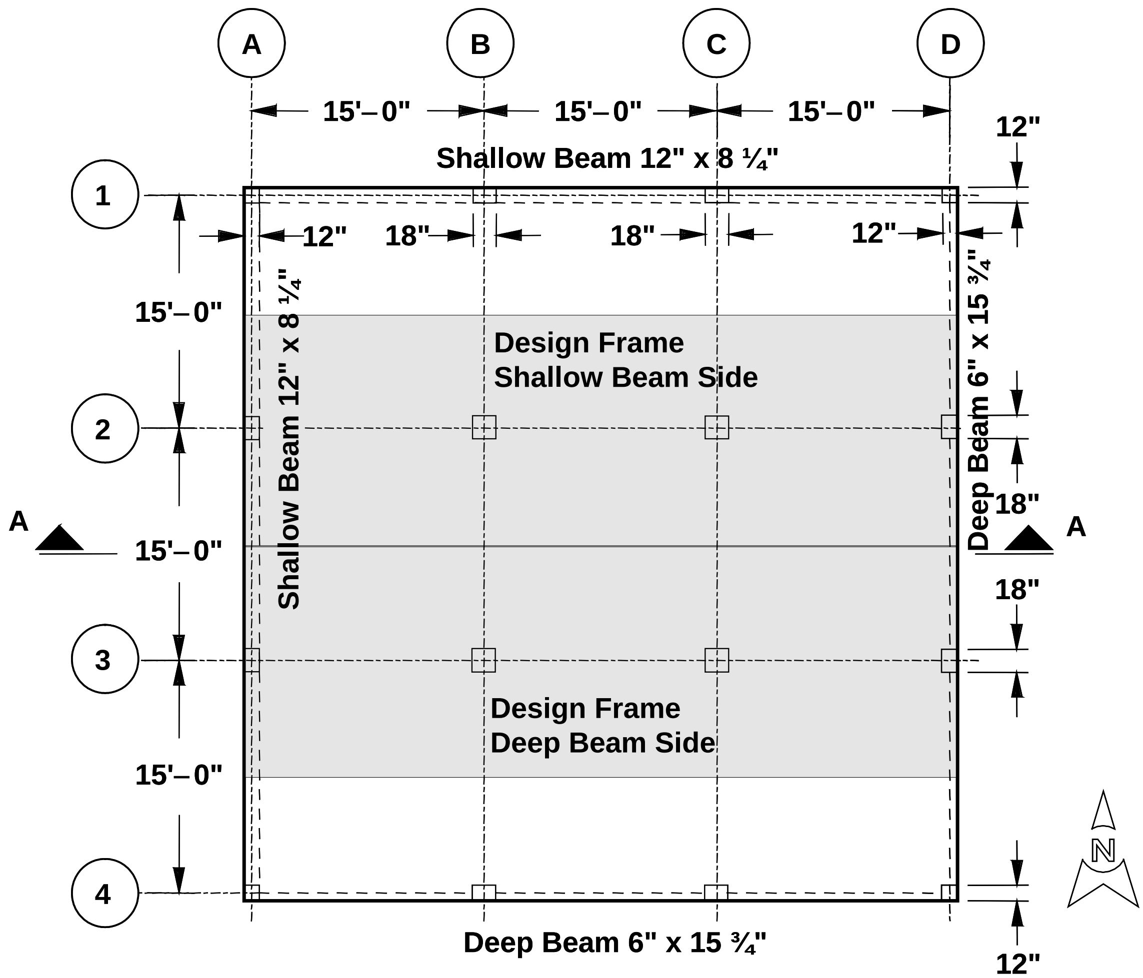
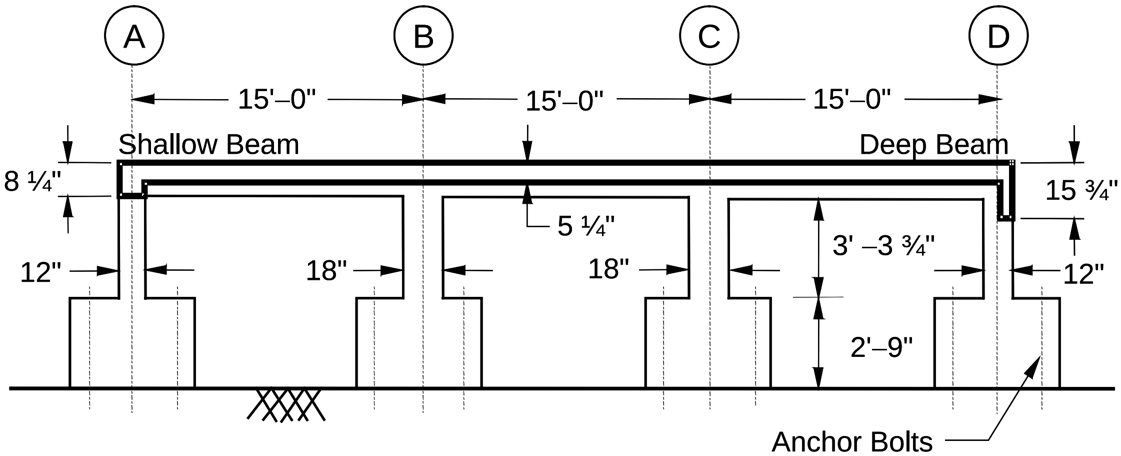


Figure 10-1 PCA Flat Plate Example



SECTION A-A

Figure 10-2 Section and Details of PCA Flat Plate Example

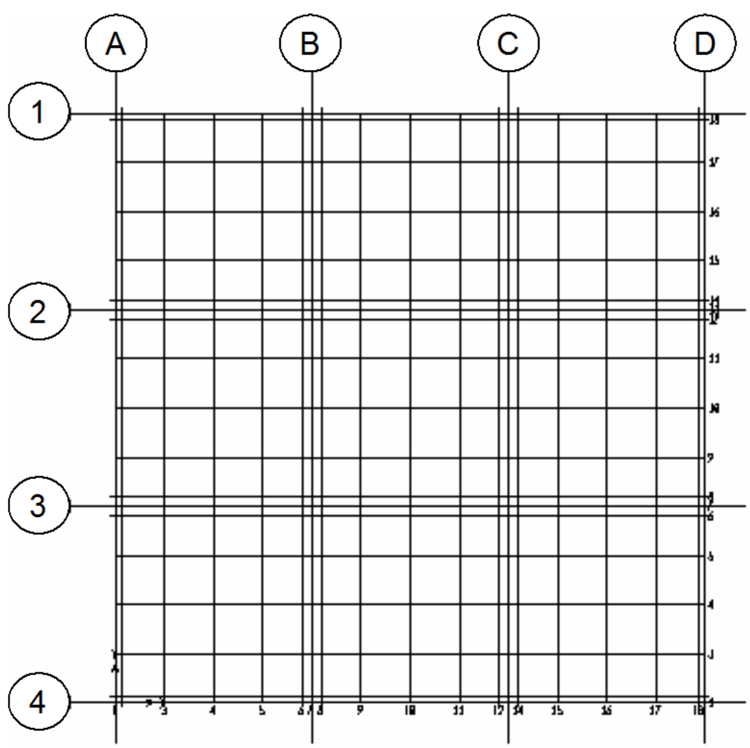


Figure 10-3 SAFE Mesh (6 × 6 per panel)

GEOMETRY, PROPERTIES AND LOADING

Concrete strength	f_c'	=	4.1	ksi
Yield strength of steel	f_y	=	40	ksi
Concrete unit weight	w_c	=	150	pcf
Modulus of elasticity	E_c	=	3670	ksi
Poisson's ratio	v	=	0.2	
Live load	w_1	=	70	psf
Dead load	w_d	=	86	psf

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of factored forces in slab.

RESULTS COMPARISON

The SAFE results for the total non-factored moments in an interior frame are compared with test results and the Equivalent Frame Method (EFM). The test and EFM results are all obtained from Corley and Jirsa (1970). The moments are compared in Table 10-1. The negative design moments reported are at the faces of the columns. Overall, the agreement between the SAFE and EFM results is good. The experimental negative moments at exterior sections, however, are comparatively lower. This may be partially the result of a general reduction of stiffness due to cracking in the beam and column connection at the exterior column, which is not accounted for in an elastic analysis. It is interesting to note that even with an approximate representation of the column flexural stiffness, the comparison of negative exterior moments between EFM and SAFE is excellent.

Table 10-1 Comparison of Measured and Computer Moments

Method	Moments in an Interior Design Frame (M / Wl_1 *)								
	End Span (Shallow Beam Side)			Middle Span			End Span (Deep Beam Side)		
	^-M	^+M	^-M	^-M	^+M	^-M	^-M	^+M	^-M
PCA Test	0.037	0.047	0.068	0.068	0.031	0.073	0.073	0.042	0.031
EFM	0.044	0.048	0.067	0.062	0.038	0.062	0.068	0.049	0.043
SAFE (Shallow Beam Slide)	0.040	0.051	0.069	0.062	0.041	0.062	0.068	0.052	0.039
SAFE (Deep Beam Slide)	0.040	0.051	0.068	0.062	0.041	0.062	0.068	0.052	0.039

* $Wl_1 = 526.5$ *kip-ft*

COMPUTER FILE:

S10.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

EXAMPLE 11

University of Illinois Flat Plate Test F1

PROBLEM DESCRIPTION

This example models the flat plate structure tested at the University of Illinois by Hatcher, Sozen, and Siess (1965). The structure consists of nine 1.75-inch-thick 5-foot × 5-foot panels arranged 3 × 3 as shown in Figure 11-1. Two adjacent edges are supported by 2.00-inch-wide × 5.25-inch-deep beams and the other two edges by shallow beams, 4 inches wide by 2.75 inches deep, producing a single diagonal line of symmetry through columns A1, B2, C3, and D4. A typical section and details of columns and edge beams are shown in Figure 11-2. The moments computed numerically using SAFE are compared with the test results and the EFM results.

The computational model uses a 6 × 6 mesh of elements per panel, as shown in Figure 11-3. The mesh contains grid lines at column centerlines as well as column faces. The slab is modeled using slab area elements and the columns are modeled as point supports with vertical and rotational stiffnesses. The reduced-height columns in the test structure are pinned at the base. Hence, an approximate value of $3(K_c = 3EI/l_c)$ is used to calculate flexural stiffness of the supports, taking the column height as 9.5 inches. In order to account for rigidity of the slab-column joint, the portion of slab occupying the column area is modeled as rigid by using a special stiff area element. Shallow and deep beams are defined on the edges with properties derived from cross-section geometry. The model is analyzed for uniform total load of 140 psf.

To obtain maximum factored moments in an interior design frame, the plate is divided into columns and middle strips. An interior design frame consists of one column strip and half of each adjacent middle strip.

GEOMETRY, PROPERTIES AND LOADING

Material:

Concrete strength	f'_c	=	2.5	ksi
Yield strength of steel	f_y	=	36.7	ksi
Modulus of elasticity	E_c	=	2400	ksi
Poisson's ratio	v	=	0.2	

Loading:

Total uniform load	w	=	140	psf

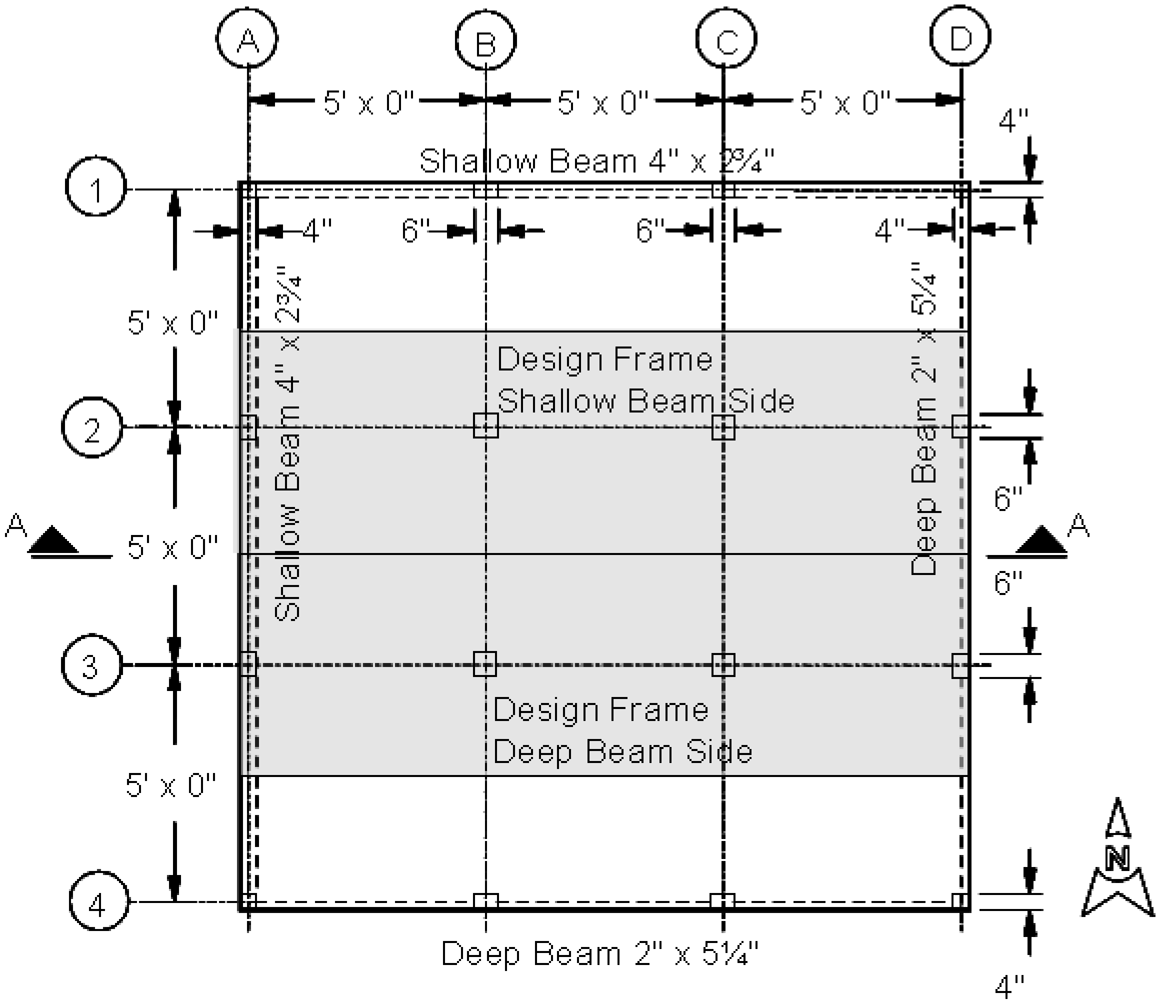


Figure 11-1 University of Illinois Flat Plate Test F1

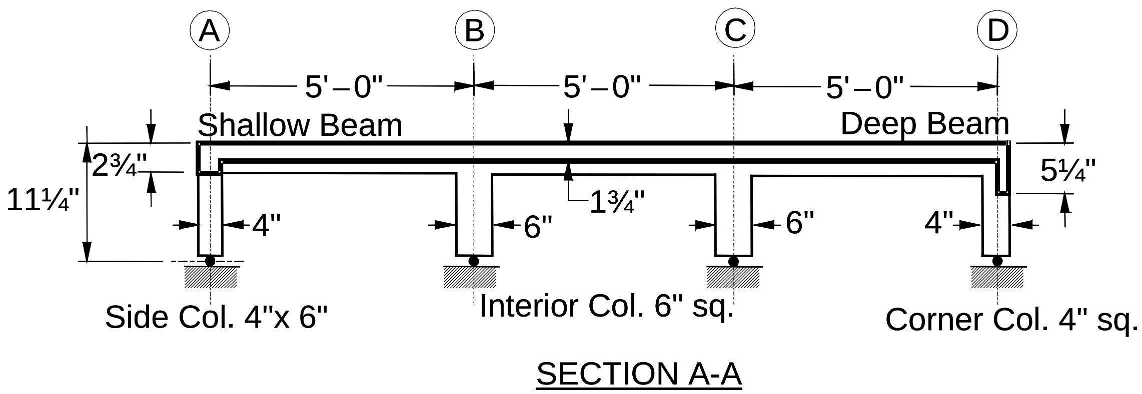


Figure 11-2 Sections and Details of University of Illinois Flat Plate Test F1

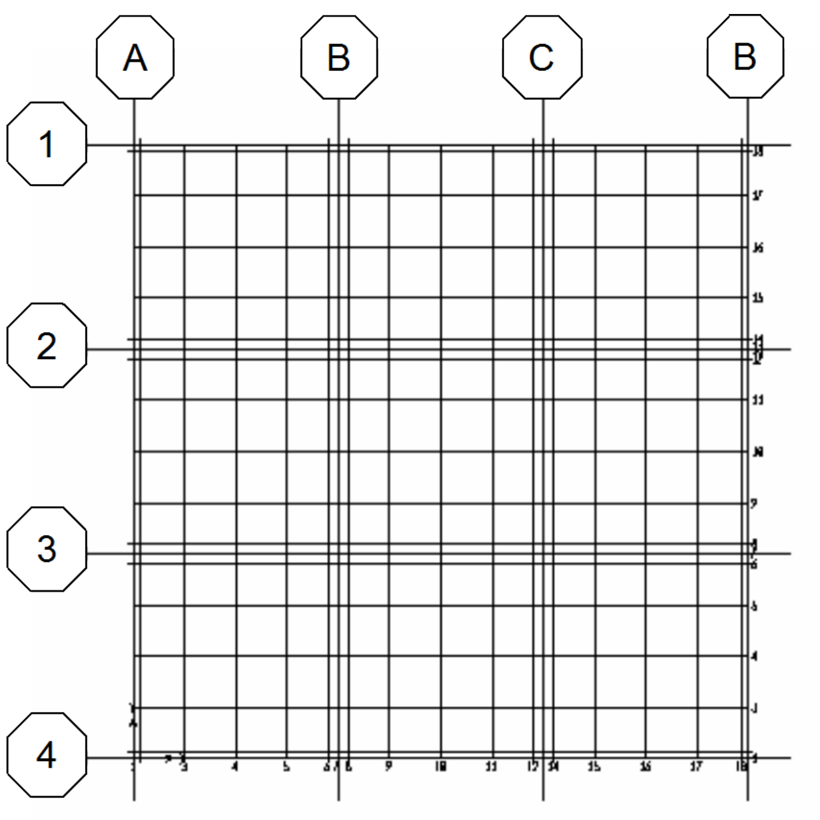


Figure 11-3 SAFE Mesh (6 $\times$ 6 per panel)

TECHNICAL FEATURES OF SAFE TESTED

- Calculation frame moments for uniform loading.

RESULTS COMPARISON

Table 11-1 shows the comparison of the SAFE results for uniform load moments for an interior frame with the experimental and EFM results. The experimental and EFM results are all obtained from Corley and Jirsa (1970).

Table 11-1 Comparison of Measured and Computed Moments

	Moments in an Interior Design Frame (M/Wl_1 *)								
	End Span (Shallow Beam Side)			Middle Span			End Span (Deep Beam Side)		
Method	-M	+M	-M	-M	+M	-M	-M	+M	-M
TEST F1	0.027	0.049	0.065	0.064	0.040	0.058	0.058	0.047	0.034
EFM	0.047	0.044	0.072	0.066	0.034	0.067	0.073	0.044	0.046
SAFE (Shallow Beam Side)	0.044	0.047	0.066	0.060	0.039	0.059	0.065	0.048	0.043
SAFE (Deep Beam Side)	0.043	0.047	0.064	0.059	0.039	0.058	0.064	0.047	0.042

* Wl_1 = 17.5 kip-ft

The negative design moments reported are at the faces of the columns. From a practical standpoint, even with a coarse mesh, the agreement between the SAFE and EFM results is good. In general the experimentally obtained moments at exterior sections are low, implying a loss of stiffness in the beam-column joint area.

In comparing absolute moments at a section, the sum of positive and average negative moments in the bay should add up to the total static moment. The SAFE and EFM results comply with this requirement within an acceptable margin of accuracy. The experimental results are expected to show greater discrepancy because of the difficulty in taking accurate strain measurements.

COMPUTER FILE: S11.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

EXAMPLE 12

University of Illinois Flat Slab Tests F2 and F3

PROBLEM DESCRIPTION

This example models F2 and F3, the flat slab structures tested at the University of Illinois by Hatcher, Sozen, and Siess (1969) and Jirsa, Sozen, and Siess (1966) respectively. A typical structure used in tests F2 and F3 is shown in Figure 12-1. The fundamental difference between these two test structures is in the type of reinforcement used. In test F2, the slab is reinforced with medium grade reinforcement, whereas in test F3, welded wire fabrics are used. The structure consists of nine 5-foot × 5-foot panels arranged 3 × 3. Two adjacent edges are supported by deep beams, 2 inches wide by 6 inches deep, and the other two edges by shallow beams, 4.5 inches wide by 2.5 inches deep, producing a single diagonal line of symmetry through columns A1, B2, C3, and D4. A typical section and details of columns, drop panels, and column capitals are shown in Figure 12-2. For both structures, the numerical results obtained for an interior frame by SAFE are compared with the experimental results and the EFM results due to uniformly distributed load.

The computational model uses an 8 × 8 mesh of elements per panel, as shown in Figure 12-3. The mesh contains grid lines at the column centerlines as well as the edges of drop panels and interior column capitals. The slab thickness is increased to 2.5 inches over the drop panels. A thickness of 4.5 inches is used to approximately model the interior capitals. Short deep beams are used to model the edge column capitals. In this model, the slab is modeled using plate elements and the columns are modeled as point supports with vertical and rotational stiffnesses. A stiffness coefficient of 4.91 ($K_c = 4.91EI_c / l_c$) is used in the calculation of the support flexural stiffness based on a column height of 21.375 inches, measured from the mid-depth of the slab to the support center. Due to the presence of capitals, columns are treated as non-prismatic. Shallow and deep beams are defined on the edges with properties derived from their cross-section geometry.

The test problems use two different concrete moduli of elasticity, E_c = 2100ksi and E_c = 3700 ksi for the beams and slab. However, both test problems are modeled in SAFE with concrete modulus of elasticity of 2100 ksi. This affects the slab, beam, and column stiffness since the distribution of moment depends on the relative stiffness.

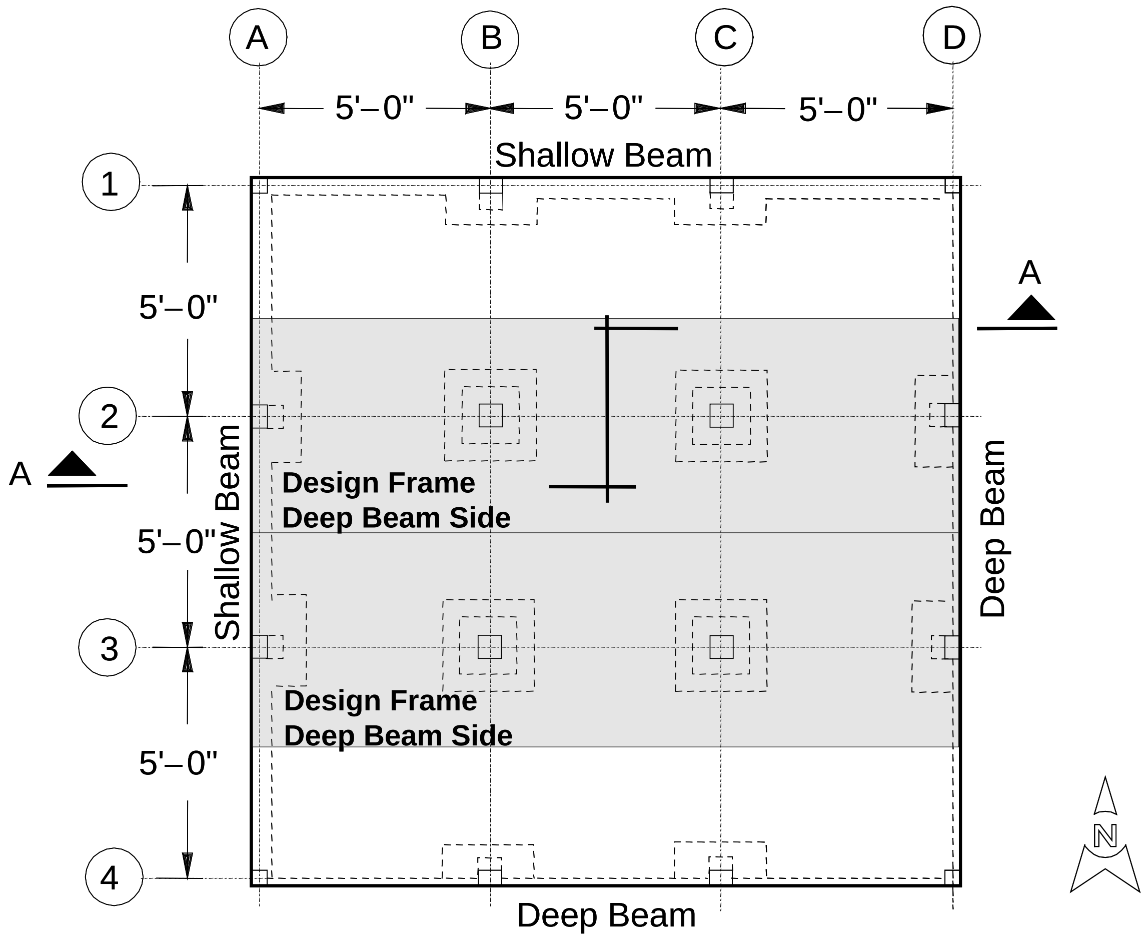
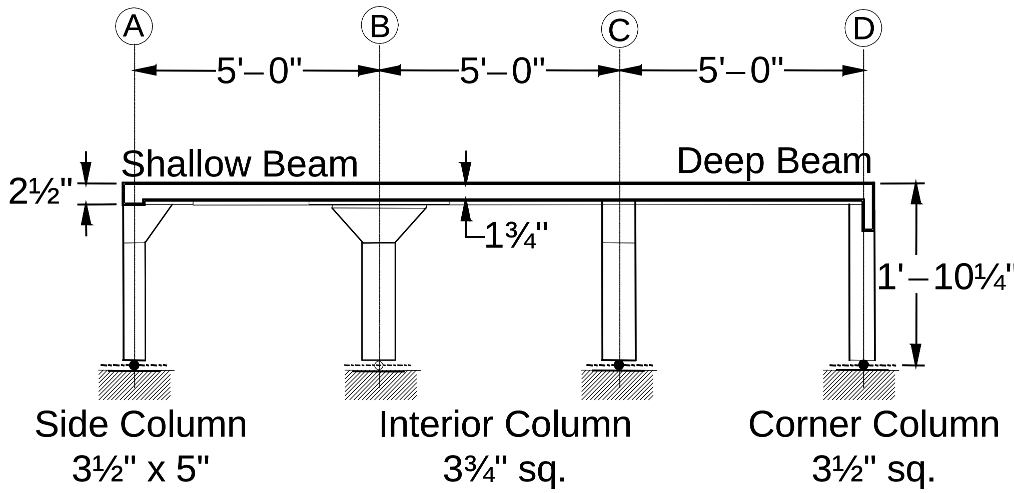
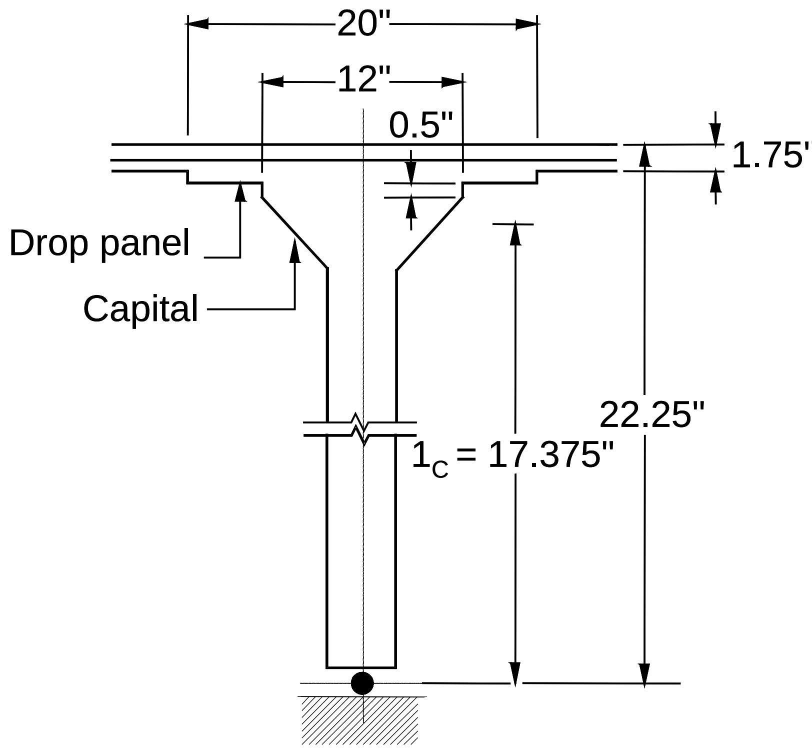


Figure 12-1 University of Illinois Flat Slab Tests F2 and F3

The model is analyzed for uniform load. To obtain maximum factored moments in an interior design frame, the slab is divided into two interior and two exterior design frames spanning in the X direction (E-W direction). Because of symmetry, results are shown for X strips only. An interior design frame consists of one column strip and half of each adjacent middle strip.



SECTION A–A



INTERIOR COLUMNS

Figure 12-2 Sections and Details of Flat Slabs F2 and F3

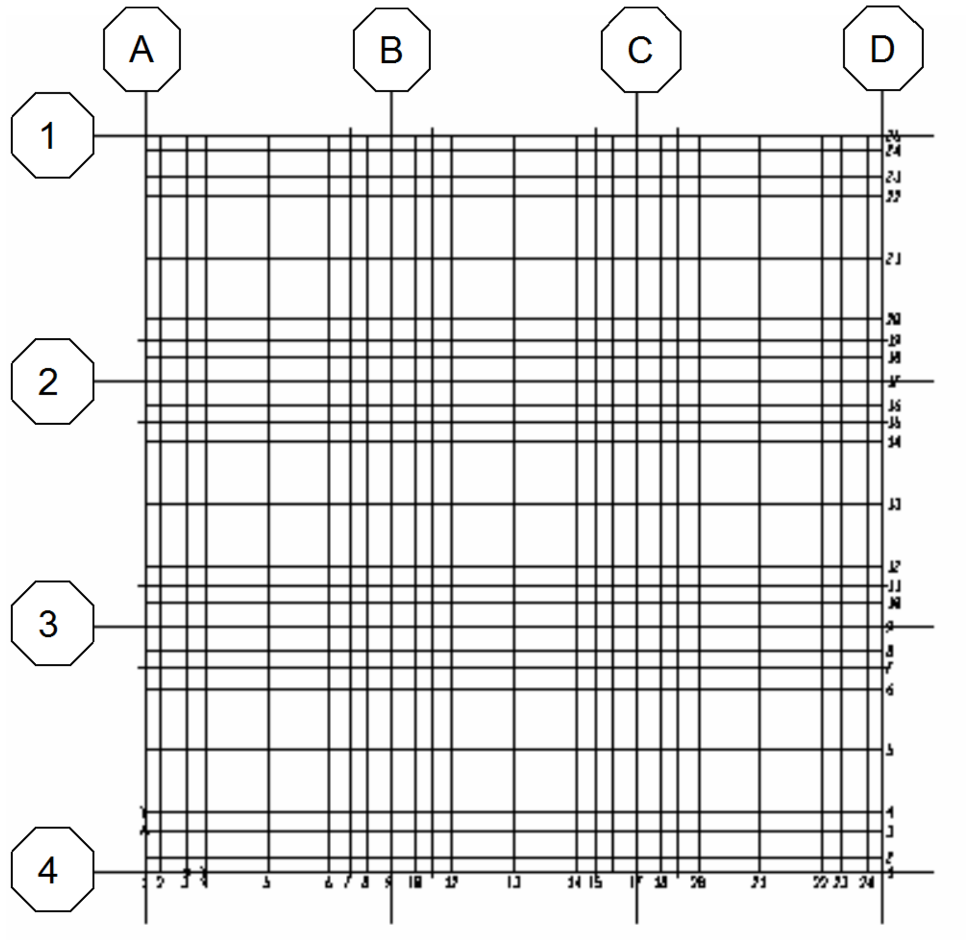


Figure 12-3 SAFE Mesh (8 × 8 per mesh)

GEOMETRY, PROPERTIES AND LOADING

Concrete strength:

$f_c' =$ 2.76 ksi (Test F2)
$f_c' =$ 3.76 ksi (Test F3)

Yield strength of slab reinforcement:

$f_y =$ 49 ksi (Test F2)
$f_y =$ 54 ksi (Test F3)

Modulus of elasticity:

E_c = 2100 ksi (Test F2)
E_c = 3700 ksi (Test F3)

Poisson's ratio:

ν = 0.2

Loading:

Total uniform design load, w = 280 psf

TECHNICAL FEATURES OF SAFE TESTED

- Calculation frame moments.

RESULTS COMPARISON

Table 12-1 shows the comparison of the SAFE results for moments in an interior frame with the experimental and EFM results for both structures F2 and F3. The experimental and EFM results are all obtained from Corley and Jirsa (1970).

Table 12-1 Comparison of Measured and Computer Moments

	Moments in an Interior Design Frame (M / Wl_1 *)								
	End Span (Shallow Beam Side)			Middle Span			End Span (Deep Beam Side)		
Method	^-M	^+M	^-M	^-M	^+M	^-M	^-M	^+M	^-M
TEST F2	0.025	0.042	0.068	0.062	0.029	0.061	0.065	0.038	0.025
TEST F3	0.029	0.038	0.057	0.055	0.023	0.058	0.060	0.034	0.024
EFM	0.021	0.044	0.057	0.050	0.026	0.049	0.057	0.044	0.021
SAFE (Shallow Beam Side)	0.026	0.042	0.067	0.058	0.025	0.057	0.066	0.042	0.024
SAFE (Deep Beam Side)	0.026	0.041	0.066	0.057	0.024	0.057	0.066	0.042	0.024

** Wl_1 = 35.0 k-ft*

Moments are compared at the edge of column capitals. Table 12-1 shows that the SAFE and the EFM results are in excellent agreement. In general, the measured positive moments appear to be lower than the SAFE and EFM values.

COMPUTER FILE: S12.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

EXAMPLE 13

University of Illinois Two-Way Slab Test T1

PROBLEM DESCRIPTION

This example models the slab structure tested at the University of Illinois by Gamble, Sozen, and Siess (1969). The structure is a two-way slab, 1.5 inches thick, in which each panel is supported along all four edges by beams, as shown in Figure 13-1. The structure consists of nine 5-foot × 5-foot panels arranged 3 × 3. The edge beams extend 2.75 inches below the soffit of the slab and the interior beams have an overall depth of 5 inches. The corner columns are 4 inches × 4 inches and the interior columns are 6 inches × 6 inches. Edge columns are 4 inches × 6 inches with the longer dimension parallel to the slab edge. A typical section of the slab and details are shown in Figure 13-2. The moments in an interior design frame due to uniform loads obtained from SAFE are compared with the corresponding experimental results and the numerical values obtained from the EFM.

The computational model uses a 6 × 6 mesh of elements per panel, as shown in Figure 13-3. Grid lines are defined at column faces as well as the column centerlines. The slab is modeled using the plate elements available in SAFE. The columns are modeled as supports with both vertical and rotational stiffnesses. A stiffness coefficient of 8.0 is used in the calculation of support flexural stiffnesses based on a column height of 15.875 inches, measured from the mid-depth of the slab to the support center. The column is assumed to be infinitely rigid over the full depth of the beams framing into it. The value of 8.0 is 75% of the figure obtained from Table 6.2 of ACI Committee 340 (1997) to approximately account for the pinned end condition at the column base. In order to account for rigidity of the slab-column joint, the portion of slab occupying the column area is modeled as rigid by using a special stiff area element. Edge beam properties are derived from their cross-section geometries.

To obtain maximum factored moments in an interior design frame, the slab is divided into two interior and two exterior design frames spanning in the X direction (E-W direction). Because of double symmetry, comparison is confined to X strips only. An interior design frame consists of one column strip and half of each adjacent middle strip.

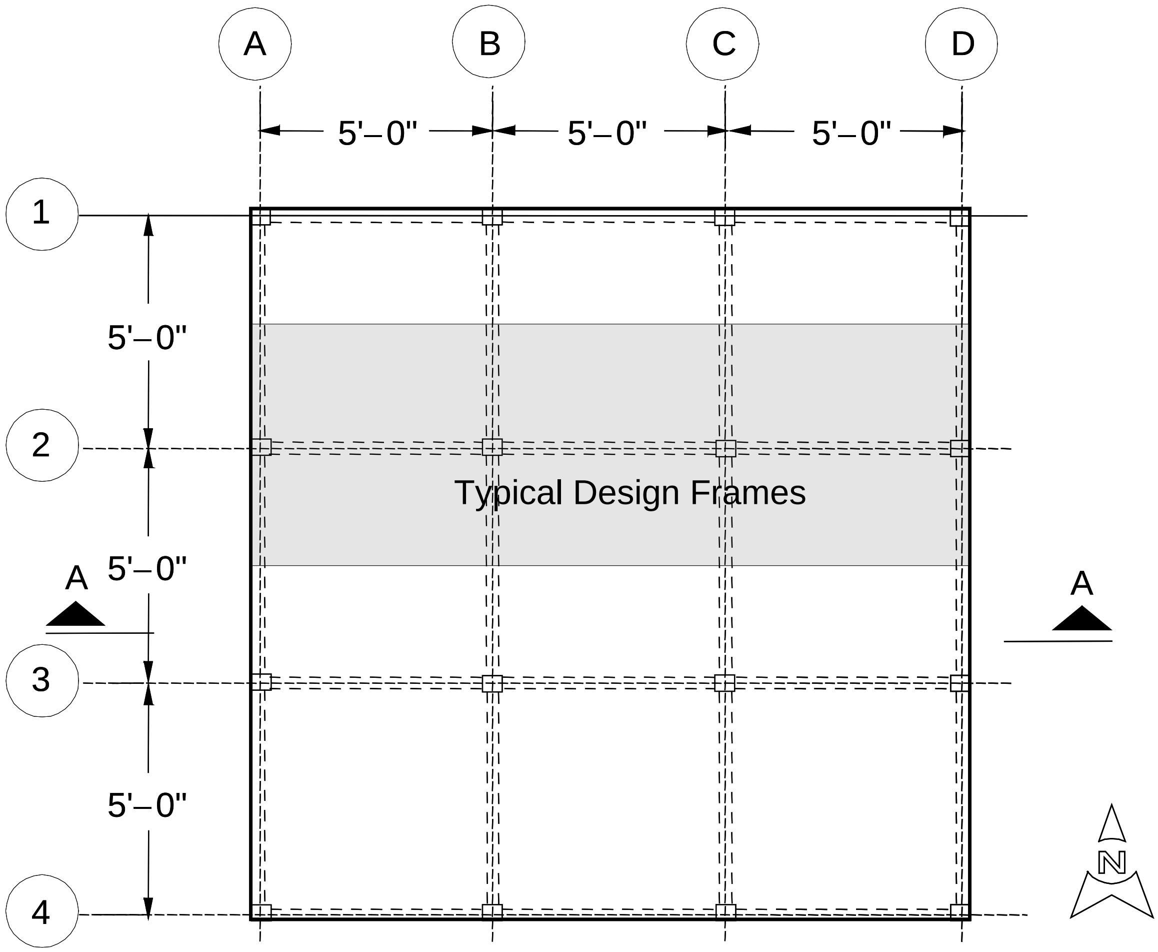


Figure 13-1 University of Illinois Two-Way Slab Example T1

GEOMETRY, PROPERTIES AND LOADING

Concrete strength	f_c'	=	3	ksi
Yield strength of reinforcements	f_y	=	42	ksi
Modulus of elasticity	E_c	=	3000	ksi
Poisson's ratio	ν	=	0.2	
Loading: Total uniform load	w	=	150	psf

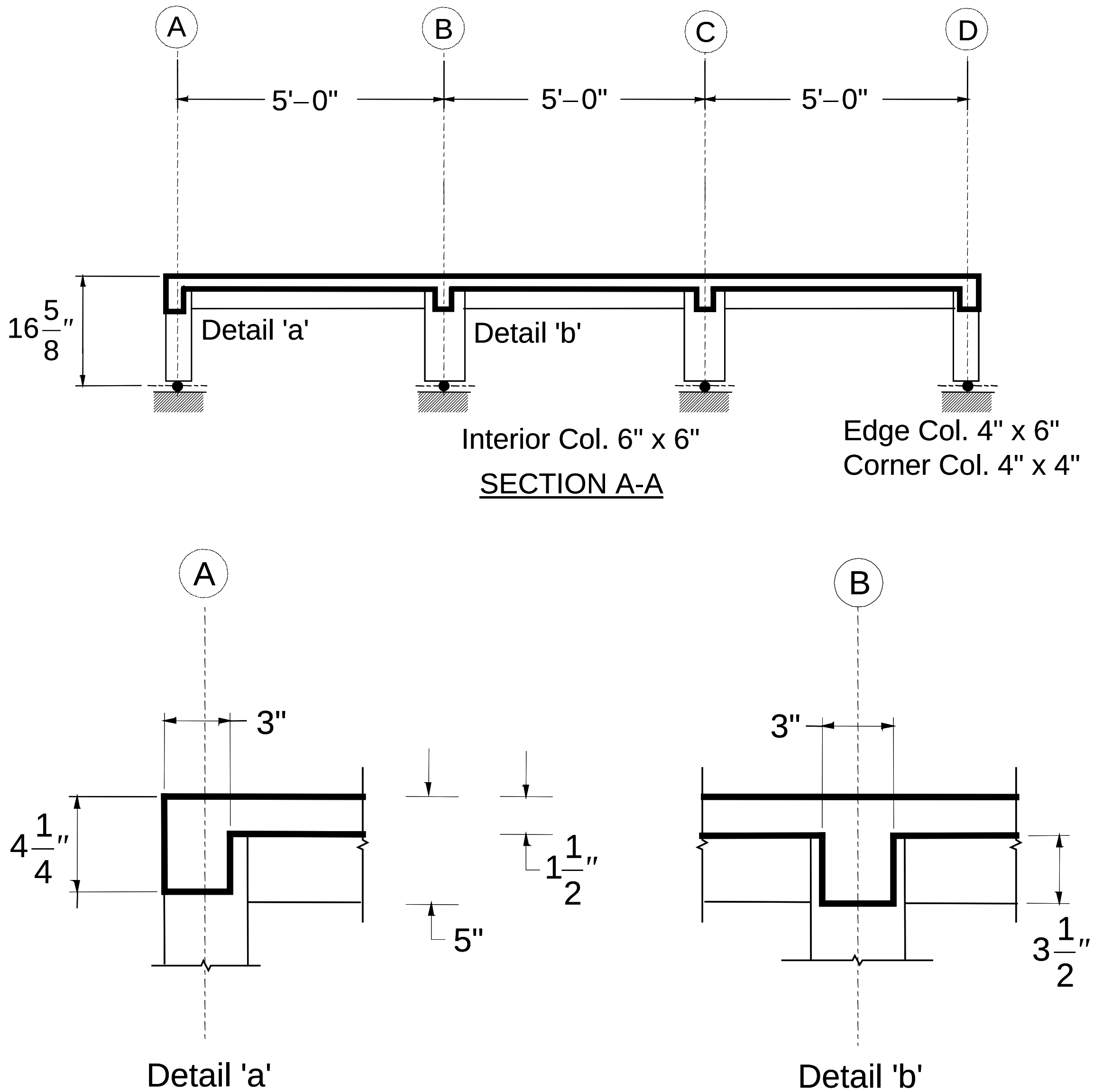


Figure 13-2 Sections and Details of Slab T1

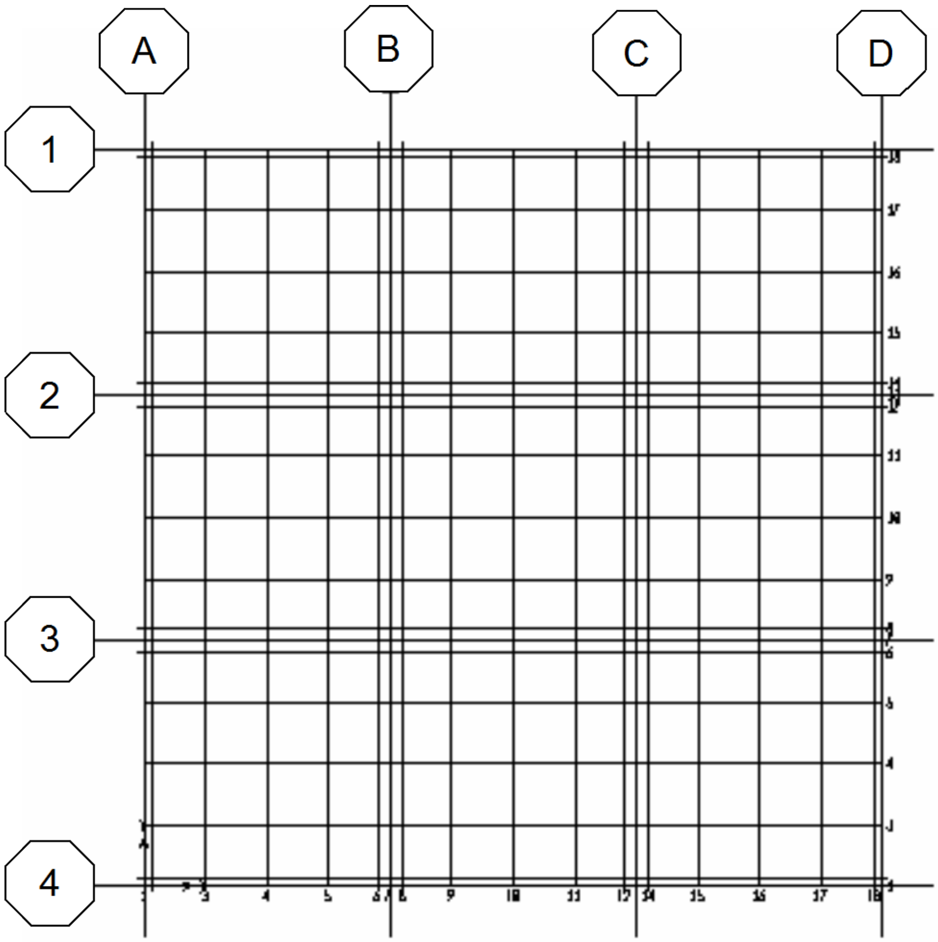


Figure 13-3 SAFE Mesh of Slab T1 (6 × 6 per panel)

TECHNICAL FEATURES OF SAFE TESTED

- Calculation frame moments and comparison with experimental and FEM results.

RESULTS COMPARISON

Table 13.-1 shows the comparison of the moments in an interior design frame obtained numerically from SAFE with the experimental results and the EFM results. The experimental and EFM results are all obtained from Corley and Jirsa (1970).

Table 13-1 Comparison of Measured and Computer Moments

Method	Moments in an Interior Design Frame (*M / Wl₁* *)					
	Exterior Span			Middle Span		
	$^{-}$M	$^{+}$M	$^{-}$M	$^{-}$M	$^{+}$M	$^{-}$M
Test T1	0.043	0.046	0.079	0.071	0.036	0.071
EFM	0.035	0.047	0.079	0.066	0.034	0.066
SAFE	0.044	0.049	0.071	0.061	0.041	0.061

* $Wl_1 = 18.75$ *k-ft*

The negative design moments reported are at the face of columns. The comparison is excellent. The minor discrepancy is attributed to the loss of stiffness due to the development of cracks and the difficulty in measuring strains accurately at desired locations.

COMPUTER FILE: S13.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

EXAMPLE 14

University of Illinois Two-Way Slab Test T2

PROBLEM DESCRIPTION

This example models the slab structure tested at the University of Illinois by Vanderbilt, Sozen, and Siess (1969). The structure is a two-way slab arranged in 3 × 3 panels in which each panel is supported along all four edges by beams, as shown in Figure 14-1. The structure consists of nine 1.5-inch thick 5-foot × 5-foot panels. The edge beams and the interior beams extend 1.5 inches below the soffit of the slab. The corner columns are 4 inches × 4 inches and the interior columns are 6 inches × 6 inches. Edge columns are 4 inches × 6 inches with the longer dimension parallel to the slab edge. A typical section of the slab and details is shown in Figure 14-2.

The computational model uses a 6 × 6 mesh of elements per panel, as shown in Figure 14-3. Grid lines are defined at column faces as well as the column centerlines. The slab is modeled using plate elements and the columns are modeled as supports with both vertical and rotational stiffnesses. A stiffness coefficient of 6.33 is used in the calculation of support flexural stiffnesses based on a column height of 13.125 inches, measured from the mid-depth of the slab to the support center. The column stiffness is assumed to be infinitely rigid over the full depth of the beams framing into it. The value of 6.33 is 75% of the figure obtained from Table A7 of Portland Cement Association (1990) to approximately account for the pinned end condition at the column base. In order to account for rigidity of the slab-column joint, the portion of slab occupying the column area is modeled as rigid by using a special stiff area element. Edge beam properties are derived from their cross-section geometries.

To obtain maximum factored moments in an interior design frame, the slab is divided into two interior and two exterior design frames spanning in the X direction (E-W direction). An interior design frame consists of one column strip and half of each adjacent middle strip.

GEOMETRY, PROPERTIES AND LOADING

Concrete strength	f_c'	=	3	ksi
Yield strength of reinforcement	f_y	=	47.6	ksi
Modulus of elasticity	E_c	=	3000	ksi
Poisson's ratio	ν	=	0.2	
Loading: Total uniform load	w	=	139	psf

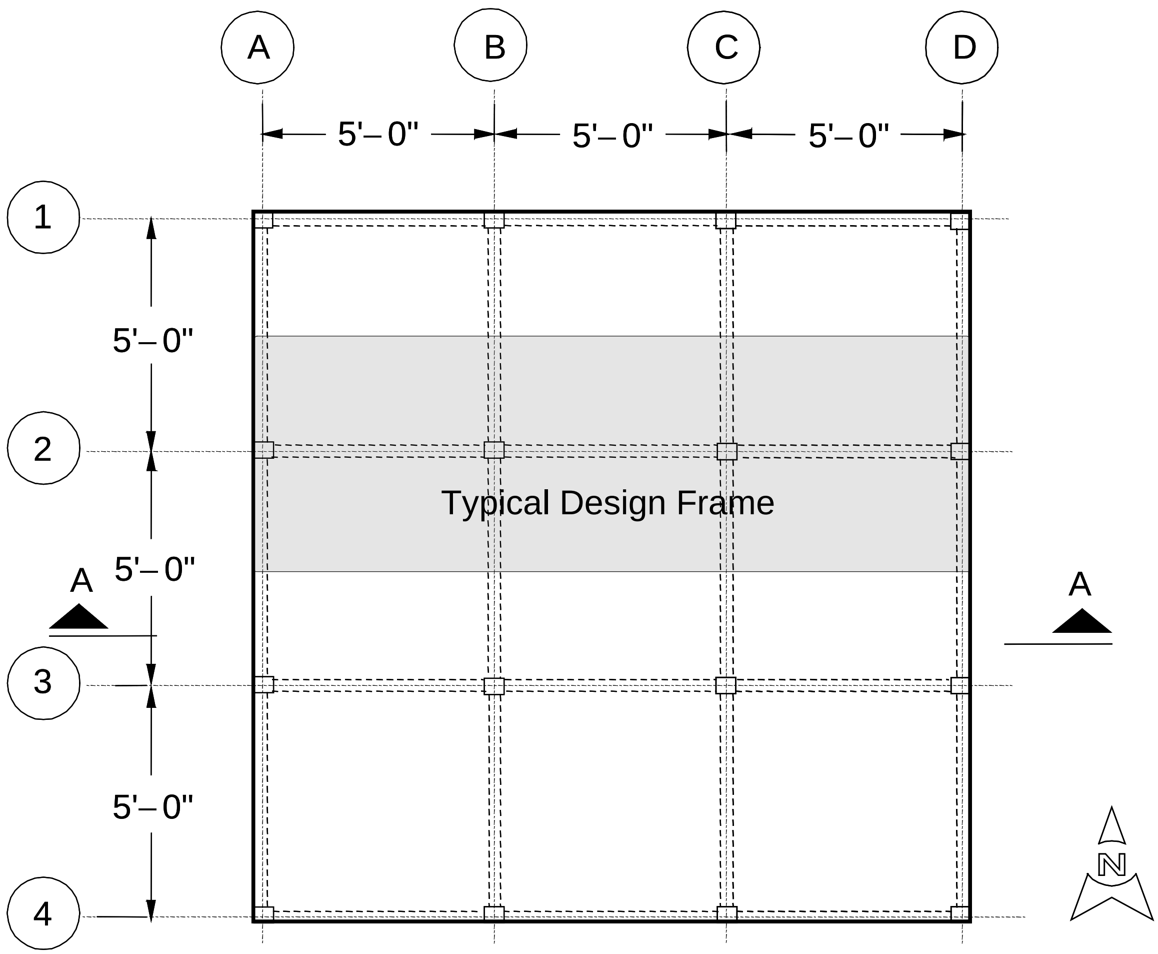
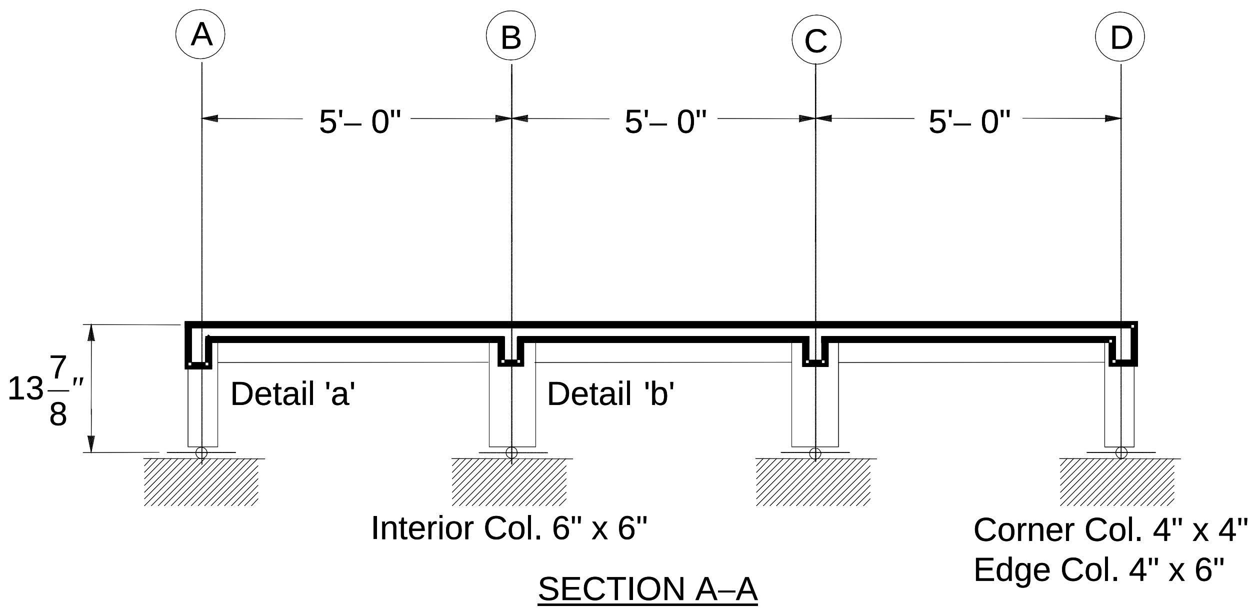


Figure 14-1 University of Illinois Two-Way Floor Slab T2



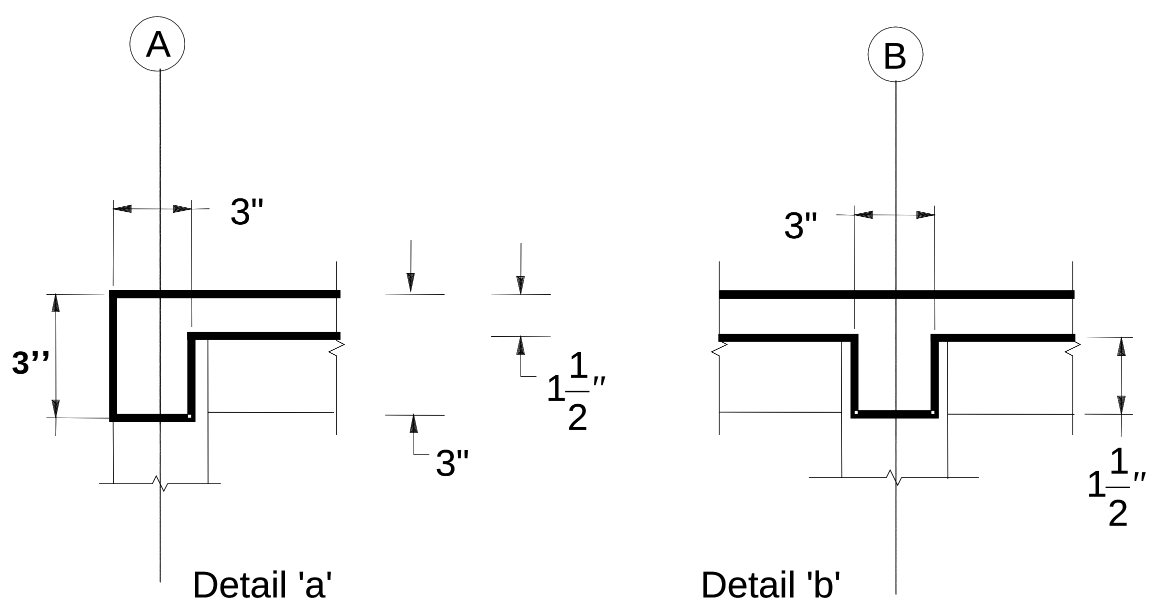


Figure 14-2 Sections and Details of Floor Slab T2

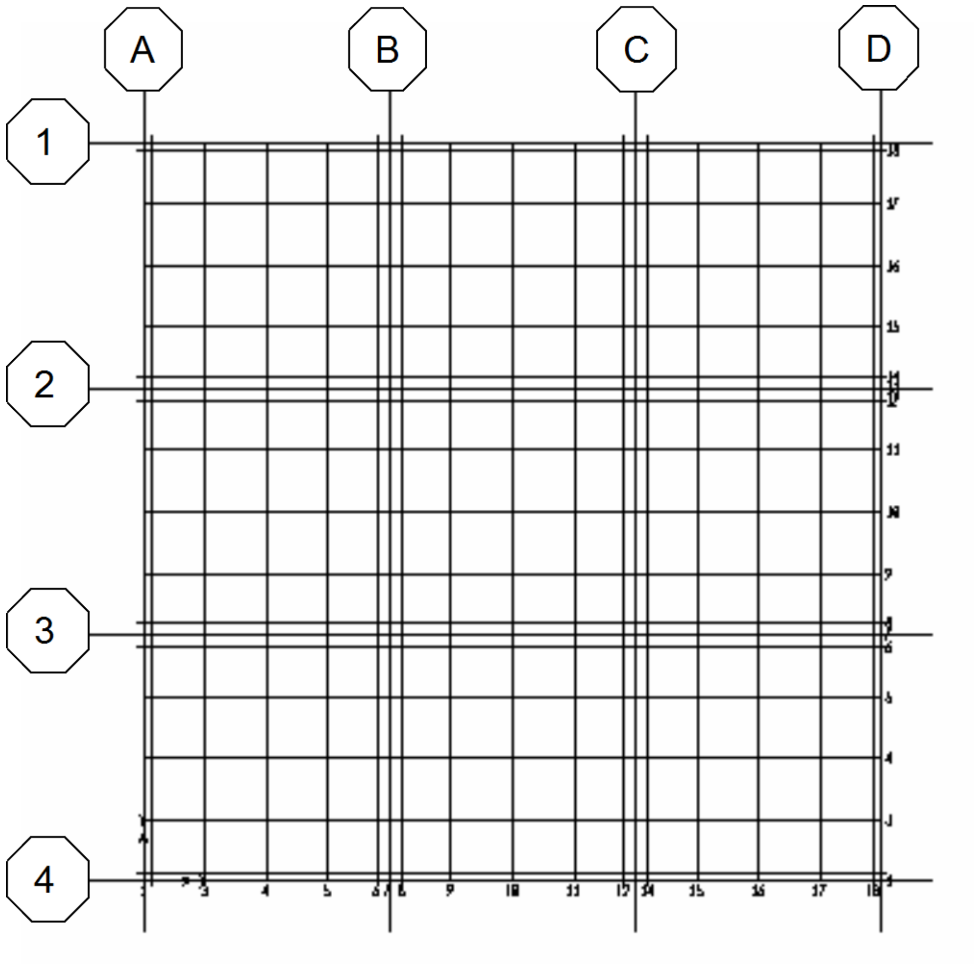


Figure 14-3 SAFE Mesh of Slab T2 (6 × 6 per panel)

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of frame forces and comparison with experimental and FEM results.

RESULTS COMPARISON

Table 14-1 shows the comparison of the moments in an interior design frame obtained numerically from SAFE with the experimental results and the EFM results. The experimental and EFM results are all obtained from Corley and Jirsa (1970).

Table 14-1 Comparison of Measured and Computer Moments

Method	Moments in an Interior Design Frame (M / Wl_1 *)					
	Exterior Span			Middle Span		
	^{-}M	^{+}M	^{-}M	^{-}M	^{+}M	^{-}M
TEST T1	0.036	0.056	0.069	0.061	0.045	0.061
EFM	0.046	0.044	0.074	0.066	0.034	0.066
SAFE	0.046	0.047	0.067	0.060	0.039	0.060

* Wl_1 = *17.375 kip-ft*

The negative design moments reported are at the face of columns. The comparison is excellent except for the negative exterior moments where the experimental results are lower than both the SAFE and the EFM results. The discrepancy is attributed not only to the loss of stiffness due to the development of cracks, but also to the difficulty in taking accurate strain measurements at desired locations.

COMPUTER FILE: S14.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

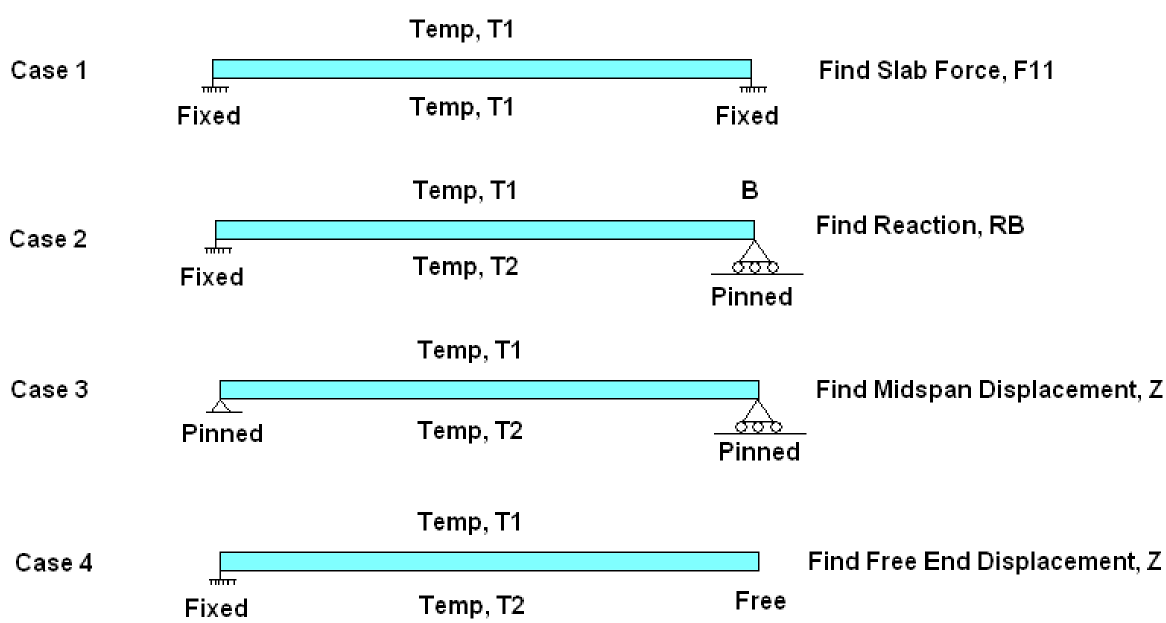
EXAMPLE 15

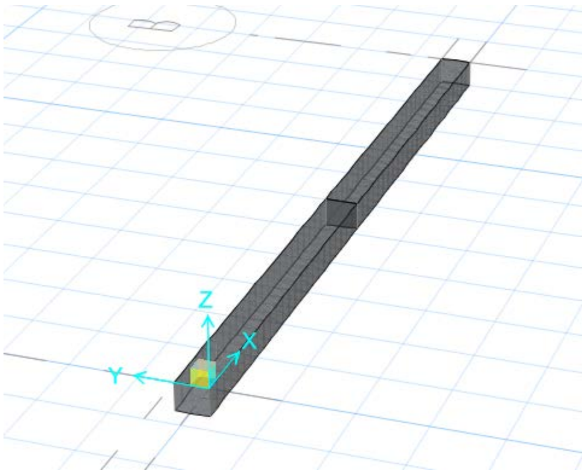
Temperature Loading

PROBLEM DESCRIPTION

In SAFE, two types of temperature loads can be applied to slab elements: an overall change in temperature or a temperature gradient across the slab thickness. This example tests each of these temperature loading methods using a 10-inch-deep x 12-inch-wide concrete slab. The slab is restrained in four different ways, and different temperature loads are applied and analyzed using SAFE. The results are compared to hand calculations and summarized in Table 15-1.

Temp, T1 = 100 degrees, F, Temp, T2 = 0 degrees, F, Span = 24 ft





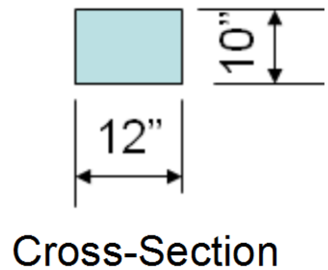


Figure 1 One-Way Slab

GEOMETRY, PROPERTIES AND LOADING

Slab thickness	h	=	10	in
Slab width	b	=	12	in
Clear span	L	=	288	in
Concrete strength	f'_c	=	4,000	psi
Modulus of elasticity	E_c	=	3,605	ksi
Poisson's ratio	ν	=	0.001	
Temp, T1	$T1$	=	100	degrees, F
Temp, T2	$T2$	=	0	degrees, F

TECHNICAL FEATURES OF SAFE TESTED

- Temperature and Temperature Gradient Loading

RESULTS COMPARISON

The force, reaction, or displacements are found using the SAFE program for the cases described previously. The SAFE values were then compared to the independent hand calculations and summarized in Table 15-1.

Table 15-1 Comparison of Results

CASE AND FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Case 1, Force, F11 (k/ft)	237.93	237.95	0.01%
Case 2, Reaction, RB (k)	1.033	1.032	0.09%
Case 3, Mid-Span Deflection, (in)	0.570	0.570	0.00%
Case 4, Free-End Displacement, (in)	–2.281	–2.281	0.00%

COMPUTER FILES: S15a.FDB, S15b.FDB, S15c.FDB, S15d.FDB

CONCLUSION

The SAFE results show an exact or nearly exact comparison with the independent hand-calculated results.

COMMENT

In Case 4, a stiffness modifier of 100 for V13 and V23 is used to avoid shear deformation in plate.

The vertical offset of a slab can have a significant effect on the thermal loading results. Therefore, it is recommended that users turn off the option to ignore the vertical offsets when temperature loading is considered in a model (see the **Run menu > Ignore Vertical Offsets in Non P/T Models** command).

CALCULATIONS:

Design Parameters: $T1$ = 100, $T2$ = 0, h= 10, L = 24 ft (288 in), b = 36, ε = 5.5E-06

Case 1:

Slab Force, $F11 = \varepsilon t AE = 0.0000055(100)(10 \times 12)(3605) = 237.93\text{ k/ft}$

Case 2:

Reaction, $RB = \frac{3EI\varepsilon}{2hL^3}\left(T2 - T1\right)L^2 = \frac{3EI\varepsilon}{2hL}\left(T2 - T1\right)$ From Roark and Young, p. 107

$$= \frac{3(3605)(1000)(0.0000055)}{2(10)(288)}(100) = 1.033\text{ kips}$$

Case 3:

Deflection, $Z = \frac{-\varepsilon}{8h}\left(T2 - T1\right)L^2 = \frac{-0.0000055}{8(10)}\left(-100\right)(288)^2 = 0.5702\text{ in}$ Roark…, p. 108

Case 4:

Deflection, $Z = \frac{-\varepsilon}{2h}\left(T2 - T1\right)L^2 = \frac{-0.0000055}{2(10)}\left(-100\right)(288)^2 = 2.281\text{ in}$ Roark…, p. 108

EXAMPLE 16

Cracked Slab Analysis

CRACKED ANALYSIS METHOD

The moment curvature diagram shown in Figure 16-1 depicts a plot of the uncracked and cracked conditions, Ψ_1 State 1, and, Ψ_2 State 2, for a reinforced beam or slab. Plot A-B-C-D shows the theoretical moment versus curvature of a slab or beam. The slope of the moment curvature between points A and B remains linearly elastic until the cracking moment, *Mr*, is reached. The increase in moment curvature between B-C at the cracking moment, *Mr*, accounts for the introduction of cracks to the member cross-section. The slope of the moment curvature between point C-D approaches that of the fully cracked condition, Ψ_2 State 2, as the moment increases.

Since the moments vary along the span of a slab or beam, it is generally not accurate to assign the same cracked section effective moment of inertia along the entire length of a span. A better approach and the one recently added to the SAFE program is to account for the proper amount of curvature for each distinct finite element of the slab or beam that corresponds to the amount of moment being applied to that element. After the moment curvatures are known for each element, the deflections can be calculated accordingly.

This verification example will compare the results from Example 8.4, *Concrete Structures, Stresses and Deformations, Third Edition, A Ghali, R Favre and M Elbadry, pages 285-289*, with the results obtained from SAFE. Both the calculations and the SAFE analysis use the cracked analysis methodology described in the preceding paragraphs.

PROBLEM DESCRIPTION

The slab used in this example has dimensions b = 0.3 m and h = 0.6 m. The slab spans 8.0 m and has an applied load of 17.1 KN/m.

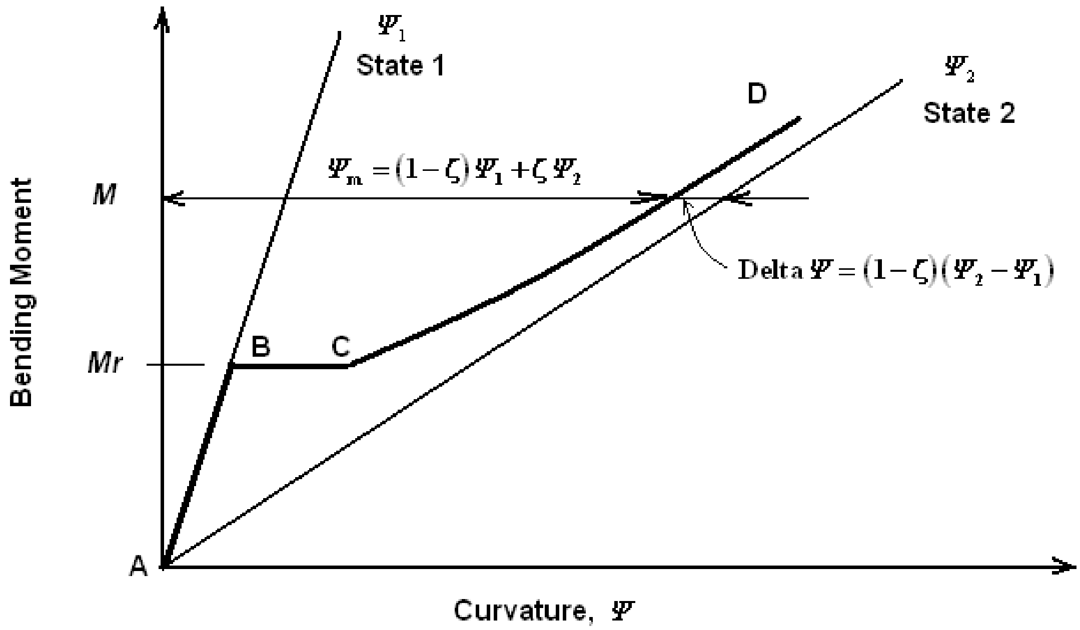


Figure 16-1 Moment versus curvature for a reinforced slab member

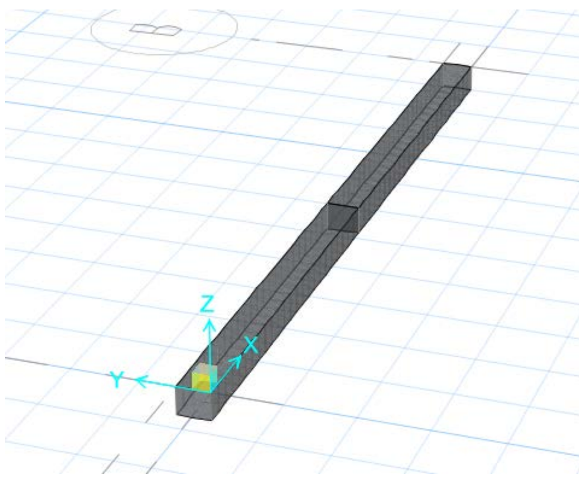


Figure 16-2 One-Way Slab

GEOMETRY, PROPERTIES AND LOADING

Slab thickness	h	=	0.65	m
Slab width	b	=	0.3	m
Clear span	L	=	8.0	m
Concrete Ultimate Strength h	f'_c	=	30	MPa
Concrete cracking strength	f_{cr}	=	2.5	MPa
Modulus of elasticity, Conc.	E_s	=	30	GPa
Modulus of elasticity, Steel	E_c	=	200	GPa
Poisson's ratio	ν	=	0.2	
Uniform load	w	=	17.1	KN/m
Creep coefficient	$\varphi(t,t_0)$	=	2.5	
Free shrinkage	$\varepsilon_{CS}(t,t_0)$	=	–250E-6	

Note: The concrete cracking strength of f_{cr} = 2.5 MPa was used in this example using the **Run menu** > **Cracking Analysis Option** command.

TECHNICAL FEATURES OF SAFE TESTED

- Cracked Slab Analysis

RESULTS COMPARISON

SAFE calculated the displacements using a Nonlinear Cracked Load Case (see Figure 16-1). The first nonlinear load case was calculated without creep and shrinkage effects and the second nonlinear load case included creep and shrinkage effects. Table 16-1 shows the results obtained from SAFE compared with the referenced example.

Table 16-1 Comparison of Results

CASE AND FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Mid-Span Displacement No Creep / Shrinkage (m)	14.4 mm	13.55 mm	5.90%
Mid-Span Displacement with Creep / Shrinkage (m)	23.9 mm	24.51 mm	2.51%

COMPUTER FILES: S16.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

CALCULATIONS:

Design Parameters: E_s = 200 GPa, E_c = 30 GPa, h = 0.65 m, b = 0.3m,
A_s = 1080 mm^2, A_s' = 270 mm^2, Center of reinf. at 0.05 m
Span = 8.0 m, Uniform Load = 17.1 KN/m

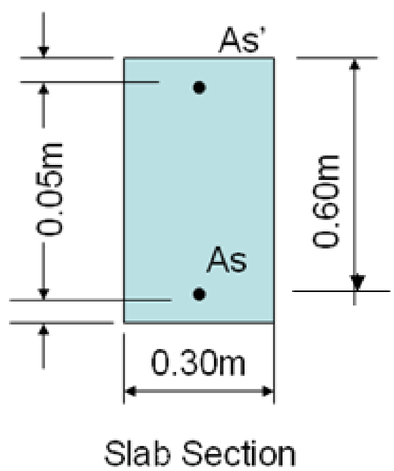


Figure 16-3 Slab Cross-Section

Case 1 – Nonlinear cracked slab analysis without creep and shrinkage

1.1 Transformed Uncracked Section Properties:
Area, A = 0.2027m^2
Y = 0.319m
I, transformed = 7.436E-03 m^4

1.2 Transformed Cracked Section Properties:
Area, A = 0.2027 m^2
C = 0.145 m
I, cracked = 1.809E-03 m^4

1.3 Cracked Bending Moment, *Mr* = 23.3E-03 × 2.5 × 10E6 = 58.3 KN-m

1.4 Interpolation coefficient, $\zeta = 1 - \beta_1 \beta_2 \left(\frac{M_r}{M}\right)^2 = 1 - 1.0\left(\frac{58.3}{136}\right)^2 = 0.82$

where $\beta_1 = 1.0$ and $\beta_2 = 1.0$

1.5 Curvature:

State1: Uncracked

$$\Psi_1 = \frac{136\text{E-06}}{30 \times 10^9 \times 7.436\text{E-03}} = 610\text{E-06}/\text{m}$$

State2: Fully Cracked

$$\Psi_2 = \frac{136\text{E-06}}{30 \times 10^9 \times 1.809\text{E-03}} = 2506\text{E-06}/\text{m}$$

Interpolated curvature:

$$\Psi_m = (1-\zeta)\Psi_1 + \zeta(\Psi_2) = (1-0.82)(610\text{E-06}/\text{m}) + 0.82(2506\text{E-06}) = 2157\text{E-06}/\text{m}$$

1.6 Slab Curvature:

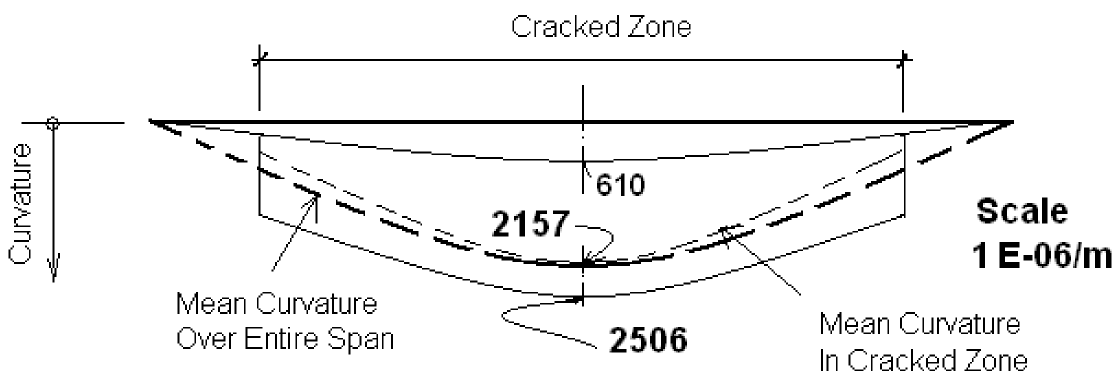


Figure 16-4 Span-Curvature Diagram

1.7 Deflection:

By assuming a parabolic distribution of curvature across the entire span (see the Mean Curvature over Entire Span plot in Figure 16-4), the deflection can be calculated as,

$$\text{Deflection} = 0.002157\frac{8^2}{96}\times 10\times 1000 = 14.4 \text{ mm (See Table 16-1)}$$

Case 2 – Nonlinear cracked slab analysis with creep and shrinkage

2.1 Aged adjusted concrete modulus,

$$\overline{E}_C\left(t,t_0\right) = \frac{E_C\left(t_0\right)}{1+X\varphi\left(t,t_0\right)} = \frac{30e9}{1+0.8(2.5)} = 10\text{GPa}$$

Where $X\left(t,t_0\right) = 0.8$ (SAFE Program Default), $\varphi\left(t,t_0\right) = 2.5$ (aging coefficient, see Figure 16-5 below)

$$n = \frac{E_S}{\overline{E}_C\left(t,t_0\right)} = \frac{200}{10} = 20$$

2.2 Age-adjusted transformed section in State1:

$\overline{A}_1 = 0.2207\,\text{m}^2$

$NA = 0.344\,\text{m}$ from top of slab

$\overline{I}_1 = 8.724\times 10^{-3}\,\text{m}^4$

$y_C = -0.020\,\text{m},$ distance from top of slab to the centroid of the concrete area

$A_C = 0.1937\,\text{m}^2,$ area of concrete

$I_C = 6.937\times 10^{-3}\,\text{m}^4,$ moment of inertia of A_C about NA

$$r_C^2 = \frac{I_C}{A_C} = 35.34\times 10^{-3}\,\text{m}^2$$

$$\kappa_1 = \frac{I_C}{\overline{I}} = \frac{6.937\times 10^{-3}}{8.724\times 10^{-3}} = 0.795, \text{ curvature reduction factor}$$

2.3 Age adjusted transformed section in State2:

$\overline{A}_2 = 0.0701\,\text{m}^2$

$NA = 0.233\,\text{m}$ from top of slab

$\overline{I}_2 = 4.277\times 10^{-3}\,\text{m}^4$

$y_C = -0.161\,\text{m},$ distance from top of slab to the centroid of the concrete area

$A_C = 0.0431\text{m}^2$, area of concrete

$I_C = 1.190 \times 10^{-3}\ \text{m}^4$, moment of inertia of A_C about NA

$$r_C^2 = \frac{I_C}{A_C} = 27.62 \times 10^{-3}\ \text{m}^2$$

$$\kappa_2 = \frac{I_C}{\bar{I}} = \frac{1.190 \times 10^{-3}}{4.277 \times 10^{-3}} = 0.278$$

2.4 Changes in curvature due to creep and shrinkage:

State 1, Change in curvature between period t_0 to t,

$$Delta\psi = \kappa\left[\varphi(t,t_0)\left(\psi(t_0) + \varepsilon_0(t_0)\frac{y_C}{r_C^2}\right) + \varepsilon_{CS}(t,t_0)\,\varepsilon\frac{y_C}{r_C^2}\right]$$

$$= 0.795\left[2.5\left(610 \times 10^{-6} + 8 \times 10^{-6}\frac{-0.020}{35.34x10^{-3}}\right) + \left(-250 \times 10^{-6}\right)\frac{-0.020}{35.34 \times 10^{-3}}\right]$$

$$= 1299 \times 10^{-6}\ /\ \text{m}$$

The curvature at time *t* (State 1)

$$\boldsymbol{\Psi}_1(\text{t}) = (610 + 1299)\text{x}10^{-6}\ /\ \text{m} = 1909\text{x}10^{-6}\ /\ \text{m}$$

State 2, Change in curvature between period t_0 to t,

$$Delta\psi = \kappa\left[\varphi(t,t_0)\left(\psi(t_0) + \varepsilon_0(t_0)\frac{y_C}{r_C^2}\right) + \varepsilon_{CS}(t,t_0)\,\varepsilon\frac{y_C}{r_C^2}\right]$$

$$= 0.278\left[2.5\left(2506 \times 10^{-6} + 222 \times 10^{-6}\frac{-0.161}{27.62x10^{-3}}\right) + \left(-250 \times 10^{-6}\right)\frac{-0.161}{27.62 \times 10^{-3}}\right]$$

$$= 1248 \times 10^{-6}\ /\ \text{m}$$

The curvature at time *t* (State 2)

$$\boldsymbol{\Psi}_2(t) = (2506 + 1248) \times 10^{-6}/\ \text{m} = 3754 \times 10^{-6}/\ \text{m}$$

Interpolated curvature:

$$\boldsymbol{\Psi}_t = (1 - \boldsymbol{\zeta})\boldsymbol{\Psi}_1(t) + \boldsymbol{\zeta}\left(\boldsymbol{\Psi}_2(t)\right) = (1 - 0.91)(1909 \times 10^{-6}) + 0.91\left(3754 \times 10^{-6}\right) = 3584 \times 10^{-6}\ /\ \text{m}$$

2.5 Deflection at center at time, *t*:

By assuming a parabolic distribution of curvature across the entire span, the deflection can be calculated as,

$$\text{Deflection} = 0.003584\frac{8^2}{96}\times 10\times 1000 = 23.90\,\text{mm (See Table 16-1)}$$

2.6 The *Load Case Data* form for Nonlinear Long-Term Cracked Analysis:

The Creep Coefficient and Shrinkage Strain values must be user defined. For this example, a shrinkage strain value of –250E-6 was used. Note that the value is input as a positive value.

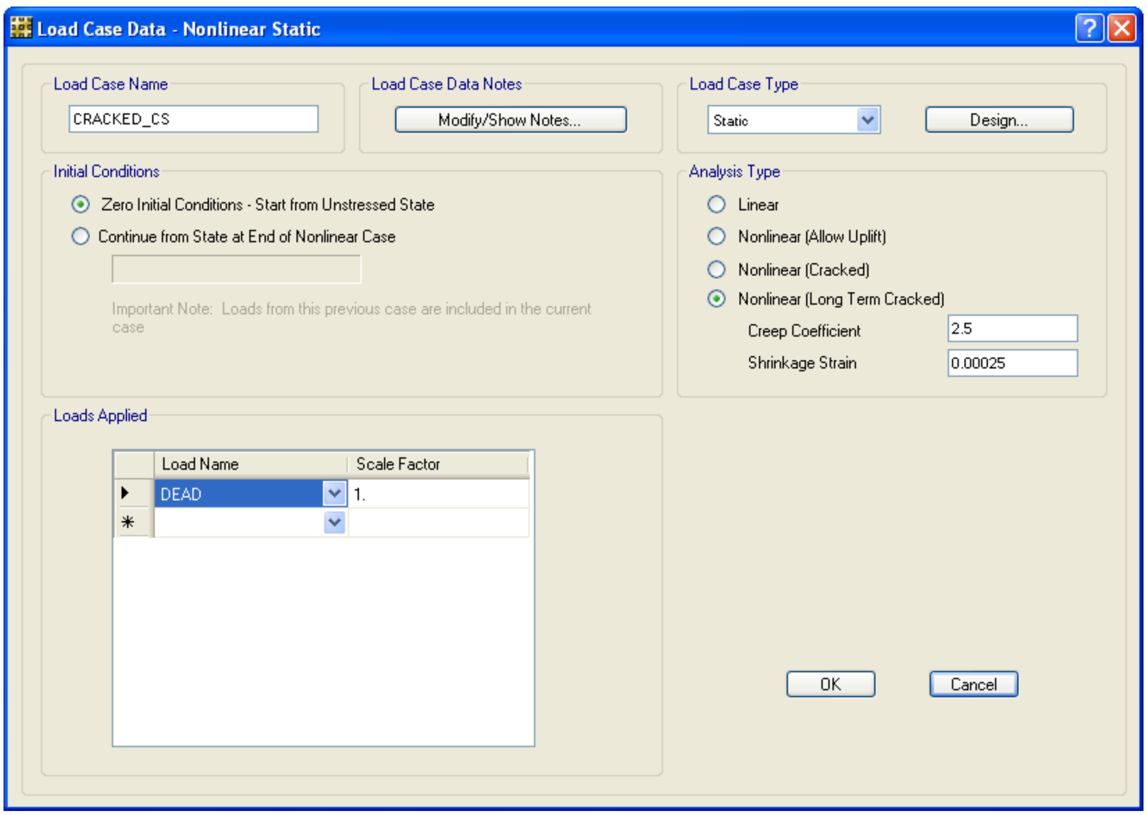


Figure 16-5 Load Case Data form for Nonlinear Long-Term Cracked Analysis

EXAMPLE 17

Crack Width Analysis

The crack width, w_k, is calculated using the methodology described in the Eurocode EN 1992-1-1:2004, Section 7.3.4, which makes use of the following expressions:

(1) $w_k = s_{r,\max}(\varepsilon_{sm} - \varepsilon_{cm})$ (eq. 7.8)

where

$s_{r,\max}$ is the maximum crack spacing

ε_{sm} is the mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening. Only the additional tensile strain beyond the state of zero strain of the concrete at the same level is considered.

ε_{cm} is the mean strain in the concrete between cracks

(2) $\varepsilon_{sm} - \varepsilon_{cm}$ may be calculated from the expression

$$\varepsilon_{sm} - \varepsilon_{cm} = E\frac{\sigma_s - k_t \dfrac{f_{ct,\text{eff}}}{\rho_{p,\text{eff}}}\left(1 + \alpha_e \rho_{p.\text{eff}}\right)}{E_s} \geq 0.6\frac{\sigma_s}{E_s} \quad \text{(eq. 7.9)}$$

where

σ_s is the stress in the tension reinforcement assuming a cracked section. For pretensioned members, σ_s may be replaced by $\Delta\sigma_s$, the stress variation in prestressing tendons from the state of zero strain of the concrete at the same level.

α_e is the ratio E_c/E_{cm}

$\rho_{p,\text{eff}}$ is A_s / $A_{c,\text{eff}}$

A_p' and $A_{c,\text{eff}}$; A_p' is the area of tendons within $A_{c,\text{eff}}$, and $A_{c,\text{eff}}$ is the area of tension concrete surrounding the reinforcing.

k_t is a factor dependent on the duration of the load
k_t = 0.6 for short term loading
k_t = 0.4 for long-term loading

(3) In situations where bonded reinforcement is fixed at reasonably close centers within the tension zone [spacing ≤ 5(c + ϕ / 2)], the maximum final crack spacing may be calculated from

$$s_{r,\max} = k_3 c + k_1 k_2 k_4 \phi / \rho_{p,\text{eff}} \quad \text{(eq. 7.11)}$$

where

ϕ is the bar diameter. Where a mixture of bar diameters is used in a section, an equivalent diameter, ϕ_{eq}, should be used. For a section with n_1 bars of diameter ϕ_1 and n_2 bars of diameter ϕ_2, the following equation should be used:

$$\phi_{eq} = \frac{n_1\phi_1^2 + n_2\phi_2^2}{n_1\phi_1 + n_2\phi_2} \quad \text{(eq. 7.12)}$$

where

c is the cover to the longitudinal reinforcement

k_1 is a coefficient that takes into account the bond properties of the bonded reinforcement:
= 0.8 for high bond bars
= 1.6 for bars with an effectively plain surface (e.g., prestressing tendons)

k_2 is a coefficient that takes into account the distribution of strain:
= 0.5 for bending
= 1.0 for pure tension

k_3 and k_4 are recommended as 3.4 and 0.425 respectively. See the National Annex for more information.

For cases of eccentric tension or for local areas, intermediate values of k_2 should be used that may be calculated from the relation:

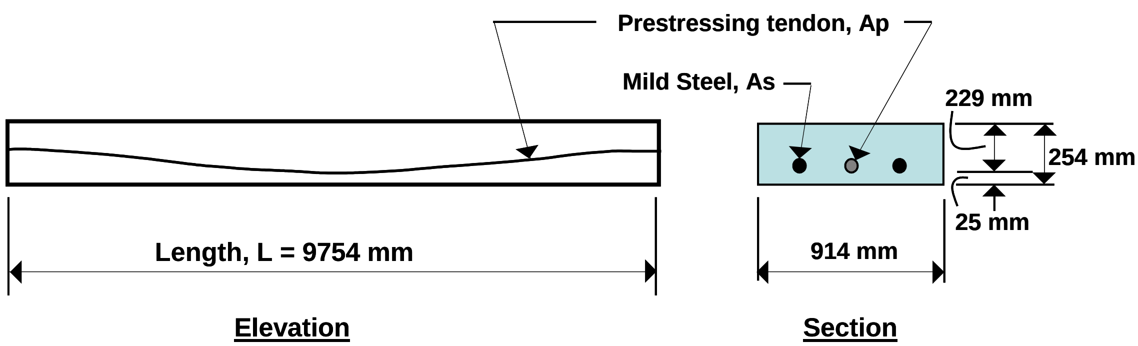
$$k_2 = (\varepsilon_1 + \varepsilon_2) / 2\varepsilon_1 \quad \text{(eq. 7.13)}$$

where ε_1 is the greater and ε_2 is the lesser tensile strain at the boundaries of the section considered, assessed on the basis of a cracked section.

PROBLEM DESCRIPTION

The purpose of this example is to verify that the crack width calculation performed by SAFE is consistent with the methodology described above. Hand calculations using the Eurocode EN 1992-1-1:2004, Section 7.3.4 are shown below as well as a comparison of the SAFE and hand calculated results.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 914 mm wide and spans 9,754 mm, as shown in Figure 17-1, and is the same slab used to validate the Eurocode PT design (see design verification example *Eurocode 2-04 PT-SL-001*). To test the crack width calculation, seven #5 longitudinal bars have been added to the slab. The total area of mild steel reinforcement is 1,400mm^2. Currently, SAFE will account for some of the PT effects. SAFE accounts for the PT effects on the moments and reinforcing stresses but the tendon areas are not considered effective to resist cracking.



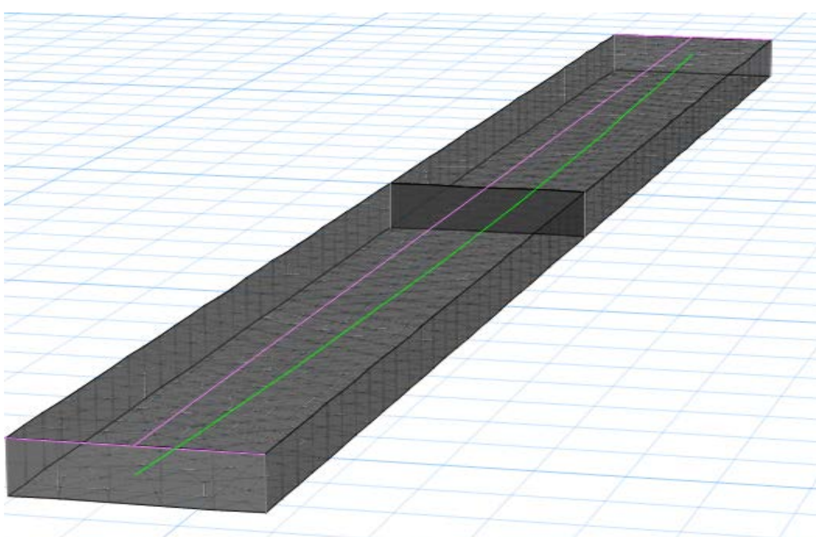


Figure 17-1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, was added to the A-Strip. The loads are as follows:

Loads: Dead = self weight

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	KN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	KN/m^2

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the reported crack widths.

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE crack widths to those calculated by hand.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Crack Widths (mm)	0.151mm	0.161mm	6.62%

COMPUTER FILE: S17.FDB

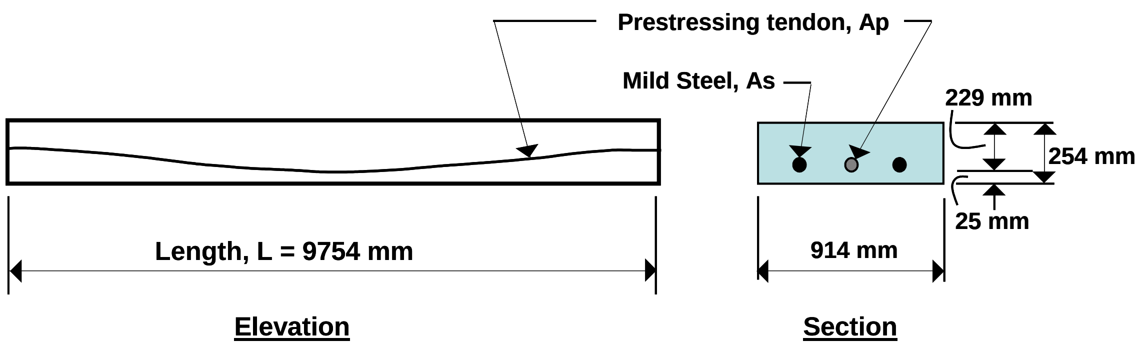
CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
$f'c$ = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa



Loads:

Dead, self-wt = 0.254 m x 23.56 kN/m^3 = 5.984 kN/m^2 (D)

ω =5.984 kN/m^2 x 0.914 m = 5.469 kN/m

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8}$ = 5.469 x (9.754)^2/8 = 65.0 kN-m

Reinforcing steel stress, $\sigma = 207 N/mm^2$ (calculated but not reported by SAFE)

Check of Concrete Stresses at Midspan:

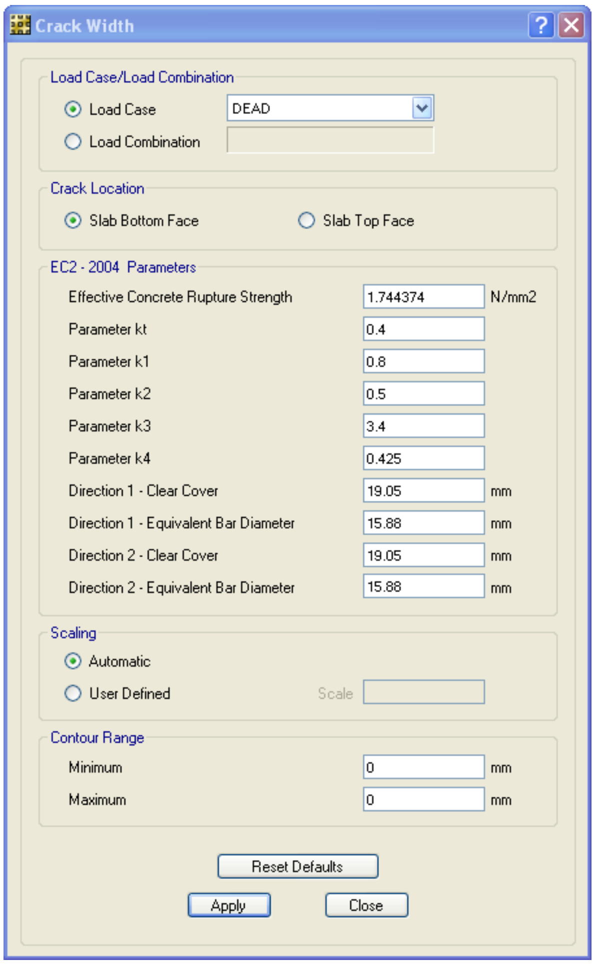


Figure 17-1 Settings used for this example

Calculation of Crack Width:

$$w_k = s_{r,\max}(\varepsilon_{sm} - \varepsilon_{cm})$$

where

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,\text{eff}}}{\rho_{p,\text{eff}}}\left(1 + \alpha_e \rho_{p,\text{eff}}\right)}{E_s} \geq 0.6\frac{\sigma_s}{E_s}, \text{ where}$$

$$\rho_{p,\text{eff}} = As / A_{c.\text{eff}} = 1.53\text{mm}^2/\text{mm} / \left(60\text{mm}^2/\text{mm}\right)$$

$$\rho_{p,eff} = 0.026$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{206N/\text{mm}^2 - 0.4\frac{1.744N/\text{mm}^2}{0.026}\left(1 + 8\left(0.026\right)\right)}{199948} \geq 0.6\frac{206}{199948}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = 0.0009 \geq 0.0006$$

$$s_{r,\max} = k_3 c + k_1 k_2 k_4 \phi / \rho_{p,\text{eff}} = 3.4\left(19.0\text{mm}\right) + 0.8\left(0.5\right)\left(0.425\right)15.8\text{mm} / 0.026$$

$$= 168\text{mm}$$

Total crack width, $w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) = 168\text{mm}\left(0.0009\right) = 0.151\text{mm}$

EXAMPLE ACI 318-14 PT-SL 001

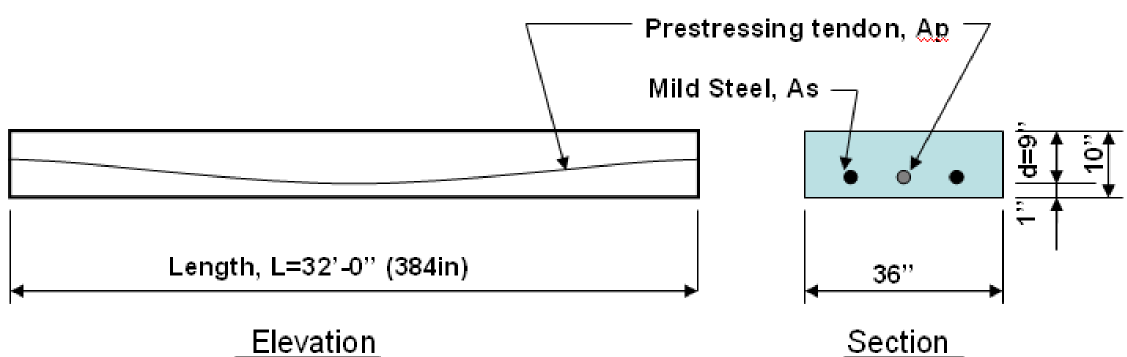
Design Verification of Post-Tensioned Slab using the ACI 318-14 code

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 10 inches thick by 36 inches wide and spans 32 feet, as shown in shown in Figure 1. A 36-inch-wide design strip was centered along the length of the slab and was defined as an A-Strip. B-strips were placed at each end of the span perpendicular to the Strip-A (the B-Strips are necessary to define the tendon profile). A tendon, with two strands having an area of 0.153 square inches each, was added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are shown below. The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the SAFE results and summarized for verification and validation of the SAFE results.

Loads: Dead = self weight, Live = 100psf



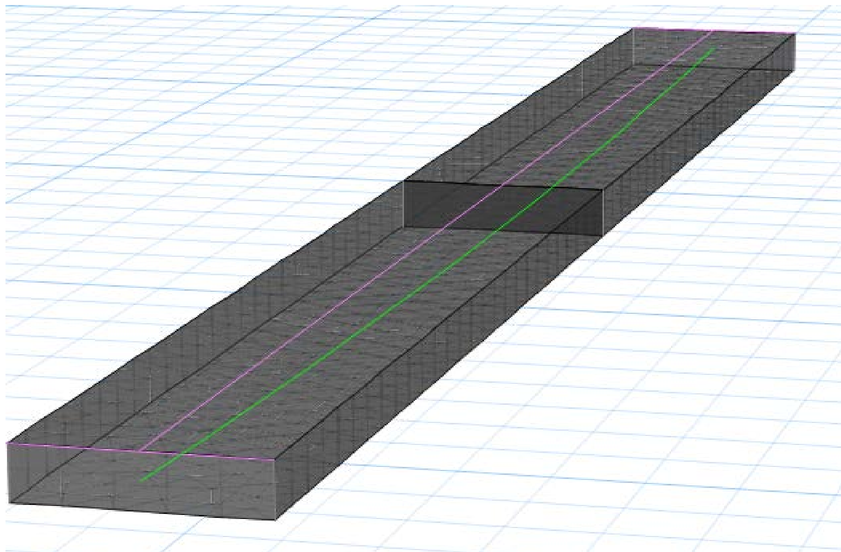


Figure 1 One-Way Slab

GEOMETRY, PROPERTIES AND LOADING

Thickness,	T, h	=	10	in
Effective depth,	d	=	9	in
Clear span,	L	=	384	in
Concrete strength,	f'_c	=	4,000	psi
Yield strength of steel,	f_y	=	60,000	psi
Prestressing, ultimate	f_{pu}	=	270,000	psi
Prestressing, effective	f_e	=	175,500	psi
Area of Prestress (single strand),	A_P	=	0.153	sq in
Concrete unit weight,	w_c	=	0.150	pcf
Modulus of elasticity,	E_c	=	3,600	ksi
Modulus of elasticity,	E_s	=	29,000	ksi
Poisson's ratio,	ν	=	0	
Dead load,	w_d	=	self	psf
Live load,	w_l	=	100	psf

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live and post-tensioning loads.

RESULTS COMPARISON

The SAFE total factored moments, required mild steel reinforcing and slab stresses are compared to the independent hand calculations in Table 1.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (k-in)	1429.0	1428.3	0.05%
Area of Mild Steel req'd, As (sq-in)	2.20	2.20	0.00%
Transfer Conc. Stress, top $(D+PT_I)$, ksi	–0.734	–0.735	0.14%
Transfer Conc. Stress, bot $(D+PT_I)$, ksi	0.414	0.414	0.00%
Normal Conc. Stress, top $(D+L+PT_F)$, ksi	–1.518	–1.519	0.07%
Normal Conc. Stress, bot $(D+L+PT_F)$, ksi	1.220	1.221	0.08%
Long-Term Conc. Stress, top $(D+0.5L+PT_{F(L)})$, ksi	–1.134	–1.135	0.09%
Long-Term Conc. Stress, bot $(D+0.5L+PT_{F(L)})$, ksi	0.836	0.837	0.12%

COMPUTER FILE: ACI 318-14 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

CALCULATIONS:

Design Parameters:

$\phi = 0.9$

Mild Steel Reinforcing

$f'c$ = 4000 psi

fy = 60,000 psi

Post-Tensioning

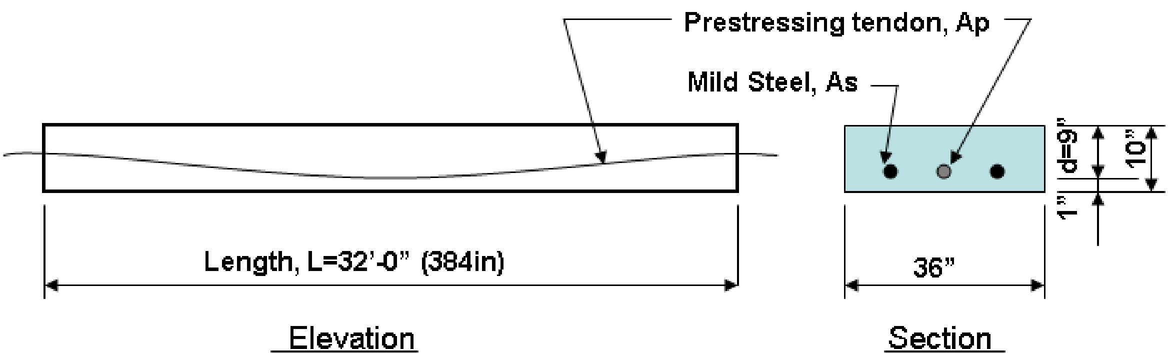
f_j = 216.0 ksi

Stressing Loss = 27.0 ksi

Long-Term Loss = 13.5 ksi

f_i = 189.0 ksi

f_e = 175.5 ksi



Loads:

Dead, self-wt = 10 / 12 ft × 0.150 kcf = 0.125 ksf (D) × 1.2 = 0.150 ksf (D_u)

Live, 0.100 ksf (L) × 1.6 = 0.160 ksf (L_u)

Total =0.225 ksf (D+L) 0.310 ksf (D+L)ult

ω =0.225 ksf × 3 ft = 0.675 klf, ω_u = 0.310 ksf × 3ft = 0.930 klf

Ultimate Moment, $M_U = \frac{wl_1^2}{8}$ = 0.310 klf × 32^2/8 = 119.0 k-ft = 1429.0 k-in

Ultimate Stress in strand, $f_{PS} = f_{SE} + 10000 + \frac{f'c}{300\rho_P}$ (span-to-depth ratio > 35)

$$= 175,500 + 10,000 + \frac{4,000}{300(0.000944)}$$

$$= 199,624 \text{ psi} \leq 205,500 \text{ psi}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 2(0.153)(199.62) = 61.08 \text{ kips}$

Ultimate force in RC, $F_{ult,RC} = A_s(f)_y = 2.00\text{(assumed)}(60.0) = 120.0 \text{ kips}$

Total Ultimate force, $F_{ult,Total} = 61.08 + 120.0 = 181.08 \text{ kips}$

Stress block depth, $a = \frac{F_{ult,Total}}{0.85f'cb} = \frac{181.08}{0.85(4)(36)} = 1.48 \text{ in}$

Ultimate moment due to PT, $M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right)\phi = 61.08\left(9 - \frac{1.48}{2}\right)(0.9) = 454.1 \text{ k-in}$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 1429.0 - 454.1 = 974.9 \text{ k-in}$

Required area of mild steel reinforcing, $A_S = \frac{M_{net}}{\phi f_y\left(d - \frac{a}{2}\right)} = \frac{974.9}{0.9(60)\left(9 - \frac{1.48}{2}\right)} = 2.18 \text{ in}^2$

Note: The required area of mild steel reinforcing was calculated from an assumed amount of steel. Since the assumed value and the calculated value are not the same a second iteration can be performed. The second iteration changes the depth of the stress block and the calculated area of steel value. Upon completion of the second iteration the area of steel was found to be 2.21in^2

Check of Concrete Stresses at Mid-Span:

Initial Condition (Transfer), load combination (D + L + PT_i) = 1.0D + $1.0PT_I$

The stress in the tendon at transfer = jacking stress − stressing losses = 216.0 − 27.0 = 189.0 ksi

The force in the tendon at transfer, = $189.0(2)(0.153) = 57.83\,\text{kips}$

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0\text{ k-ft} = 576\text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 57.83(4\text{ in}) = 231.3\text{ k-in}$

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-57.83}{10(36)} \pm \dfrac{576.0 - 231.3}{600}$, where S = 600 in^3

$$\boldsymbol{f} = -0.161 \pm 0.5745$$

$$f = -0.735(\text{Comp})\text{max}, 0.414(\text{Tension})\text{max}$$

Normal Condition, load combinations: (D + L + PT_F) = 1.0D + 1.0L + $1.0PT_F$

Tendon stress at normal = jacking − stressing − long-term = 216.0 − 27.0 − 13.5 = 175.5 ksi
The force in tendon at Normal, = $175.5(2)(0.153) = 53.70\text{ kips}$

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0\text{ k-ft} = 576\text{ k-in}$

Moment due to dead load, $M_L = 0.100(3)(32)^2/8 = 38.4\text{ k-ft} = 461\text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 53.70(4\text{ in}) = 214.8\text{ k-in}$

Stress in concrete for (D + L+ PT_F), $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_{D+L} - M_{PT}}{S} = \dfrac{-53.70}{10(36)} \pm \dfrac{1037.0 - 214.8}{600}$

$$\boldsymbol{f} = -0.149 \pm 1.727 \pm 0.358$$

$$f = -1.518(\text{Comp})\,\text{max}, 1.220(\text{Tension})\,\text{max}$$

Long-Term Condition, load combinations: (D + 0.5L + $PT_{F(L)}$) = 1.0D + 0.5L + $1.0PT_F$

Tendon stress at normal = jacking − stressing − long-term = 216.0 − 27.0 − 13.5 = 175.5 ksi
The force in tendon at Normal, = $175.5(2)(0.153) = 53.70\text{ kips}$

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0\text{ k-ft} = 576\text{ k-in}$

Moment due to dead load, $M_L = 0.100(3)(32)^2/8 = 38.4\text{ k-ft} = 460\text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 53.70(4\text{ in}) = 214.8\text{ k-in}$

Stress in concrete for (D + 0.5L + $PT_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-53.70}{10(36)} \pm \frac{806.0 - 214.8}{600}$$

$$f = -0.149 \pm 0.985$$

$$f = -1.134(\text{Comp})\max, 0.836(\text{Tension})\max$$

EXAMPLE ACI 318-14 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

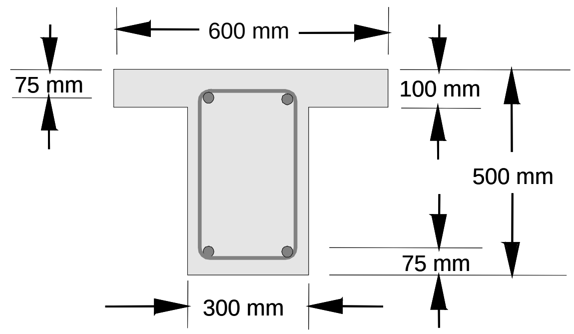
The purpose of this example is to verify the beam flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by ACI 318-14.
- The average shear stress in the beam falls below the maximum shear stress allowed by ACI 318-14, requiring design shear reinforcement.

A simple-span, 20-foot-long, 12-inch-wide, and 18-inch-deep T beam with a flange 4 inches thick and 24 inches wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size is specified as 6 inches. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kip/in).

The beam is loaded with symmetric third-point loading. One dead load (DL02) case and one live load (LL30) case, with only symmetric third-point loads of magnitudes 3, and 30 kips, respectively, are defined in the model. One load combination (COMB30) is defined using the ACI 318-14 load combination factors of 1.2 for dead load and 1.6 for live load. The model is analyzed for both of these load cases and the load combination.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results and found to be identical. After completing the analysis, the design is performed using the ACI 318-14 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcement. Table 2 shows the comparison of the design shear reinforcement.



Beam Section

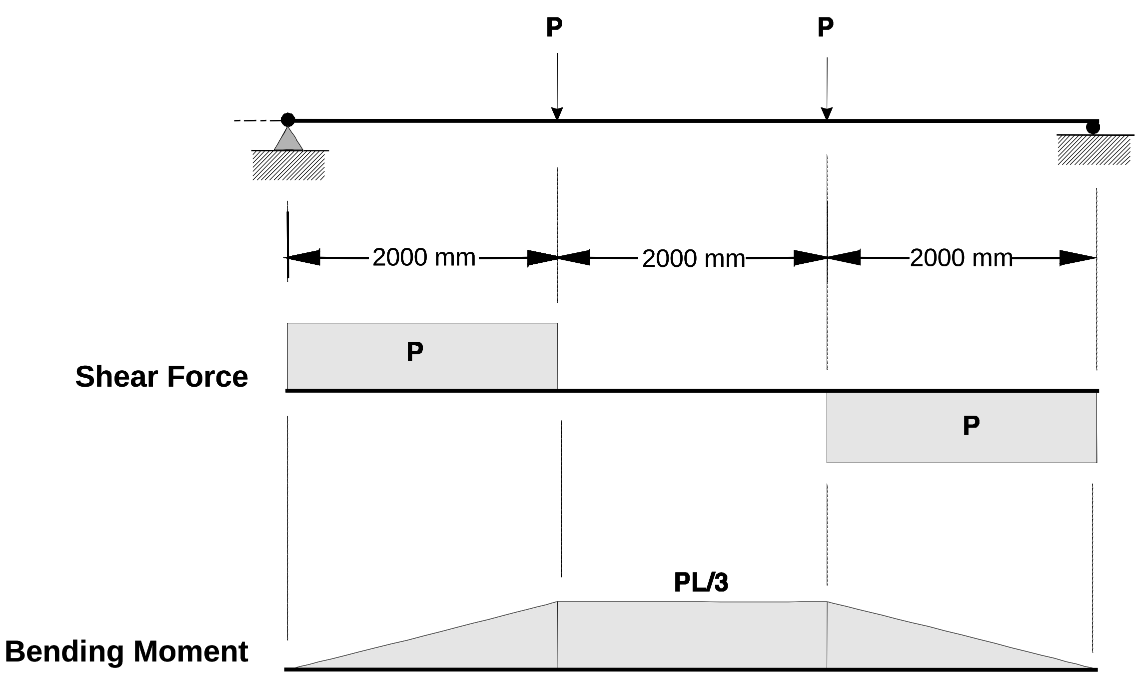


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span	l	=	240	in
Overall depth	h	=	18	in
Flange thickness	d_s	=	4	in
Width of web	b_w	=	12	in
Width of flange,	b_f	=	24	in
Depth of tensile reinf.	d_c	=	3	in
Effective depth	d	=	15	in
Depth of comp. reinf.	d'	=	3	in
Concrete strength	f'_c	=	4,000	psi
Yield strength of steel	f_y	=	60,000	psi
Concrete unit weight	w_c	=	0	pcf
Modulus of elasticity	E_c	=	3,600	ksi
Modulus of elasticity	E_s	=	29,000	ksi
Poisson's ratio	v	=	0.2	
Dead load	P_d	=	2	kips
Live load	P_l	=	30	kips

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the beam with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the comparison of the design reinforcement.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (k-in)	Reinforcement Area (sq-in)
		A_s^+
SAFE	4032	5.808
Calculated	4032	5.808

$A^+_{s,min}$ = 0.4752 sq-in

Table 2 Comparison of Shear Reinforcements

Shear Force (kip)	Reinforcement Area, $\frac{A_v}{s}$ (sq-in/ft)	
	SAFE	Calculated
50.40	0.592	0.592

COMPUTER FILE: ACI 318-14 RC-BM-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\varphi = 0.9$

$A_g = 264$ sq-in

$A_{s,\min} = 0.0018A_g = 0.4752$ sq-in

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) = 0.85$$

$$c_{\max} = \frac{0.003}{0.003 + 0.005} d = 5.625 \text{ in}$$

$$a_{\max} = \beta_1 c_{\max} = 4.78125 \text{ in}$$

$A_s = \min[A_{s,\min}, (4/3)\, A_{s,\text{required}}] = \min[0.4752, (4/3)5.804] = 0.4752$ sq-in

COMB30

$P_u = (1.2P_d + 1.6P_t) = 50.4$ k

$$M_u = \frac{P_u l}{3} = 4032 \text{ k-in}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f'_c \varphi\, b_f}} = 4.2671 \text{ in } (a > d_s)$$

Calculation for A_s is performed in two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w. C_f is given by:

$$C_f = 0.85 f'_c\,(b_f - b_w)\, d_s = 163.2 \text{ k}$$

The portion of M_u that is resisted by the flange is given by:

$$M_{uf} = C_f\left(d - \frac{d_s}{2}\right)\varphi = 1909.44 \text{ k-in}$$

Therefore, the area of tensile steel reinforcement to balance flange compression is:

$$A_{s1} = \frac{M_{uf}}{f_y\left(d - d_s/2\right)\varphi} = 2.7200 \text{ sq-in}$$

The balance of the moment to be carried by the web is given by:

$$M_{uw} = M_u - M_{uf} = 2122.56 \text{ k-in}$$

The web is a rectangular section with dimensions b_w and d, for which the design depth of the compression block is recalculated as

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85\, f'_c\, \varphi\, b_w}} = 4.5409 \text{ in} \quad (a_1 \le a_{\max})$$

The area of tensile steel reinforcement to balance the web compression is then given by:

$$A_{s2} = \frac{M_{uw}}{\varphi\, f_y\left(d - \dfrac{a_1}{2}\right)\varphi} = 3.0878 \text{ sq-in}$$

The area of total tensile steel reinforcement is then given by:

$$A_s = A_{s1} + A_{s2} = 5.808 \text{ sq-in}$$

Shear Design

The following quantities are computed for all of the load combinations:

$$\varphi = 0.75$$

Check the limit of $\sqrt{f'_c}$:

$$\sqrt{f'_c} = 63.246 \text{ psi} < 100 \text{ psi}$$

The concrete shear capacity is given by:

$$\varphi V_c = \varphi\, 2\sqrt{f'_c}\, b_w d = 17.076 \text{ k}$$

The maximum shear that can be carried by reinforcement is given by:

$\varphi V_s = \varphi\, 8\sqrt{f'_c}\, b_w d = 68.305$ k

The following limits are required in the determination of the reinforcement:

$(\varphi V_c/2) = 8.538$ k

$(\varphi V_c + \varphi\, 50\, b_w d) = 23.826$ k

$V_{max} = \varphi V_c + \varphi V_s = 85.381$ k

Given V_u, V_c and V_{max}, the required shear reinforcement in area/unit length for any load combination is calculated as follows:

If $V_u \le (V_c/2)\,\varphi$,

$$\frac{A_v}{s} = 0,$$

else if $(V_c/2)\,\varphi < V_u \le (\varphi V_c + \varphi\, 50\, b_w d)$,

$$\frac{A_v}{s} = \frac{50\, b_w}{f_y},$$

else if $(\varphi V_c + \varphi\, 50\, b_w d) < V_u \le \varphi V_{max}$

$$\frac{A_v}{s} = \frac{(V_u - \varphi V_c)}{\varphi\, f_{ys}\, d}$$

else if $V_u > \varphi V_{max}$,

a failure condition is declared.

For each load combination, the P_u and V_u are calculated as follows:

$P_u = 1.2P_d + 1.6P_l$

$V_u = P_u$

(COMB30)

$P_d = 2$ k

$P_l = 30$ k

$P_u = 50.4$ k

$V_u = 50.4$ k, $(\varphi V_c + \varphi\, 50\, b_w d) < V_u \le \varphi V_{max}$

$$\frac{A_v}{s} = \frac{(V_u - \varphi V_c)}{\varphi f_{ys} d} = 0.04937 \text{ sq-in/in or } 0.592 \text{ sq-in/ft}$$

EXAMPLE ACI 318-14 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE.

The numerical example is a flat slab that has three 24-foot-long spans in each direction, as shown in Figure 1.

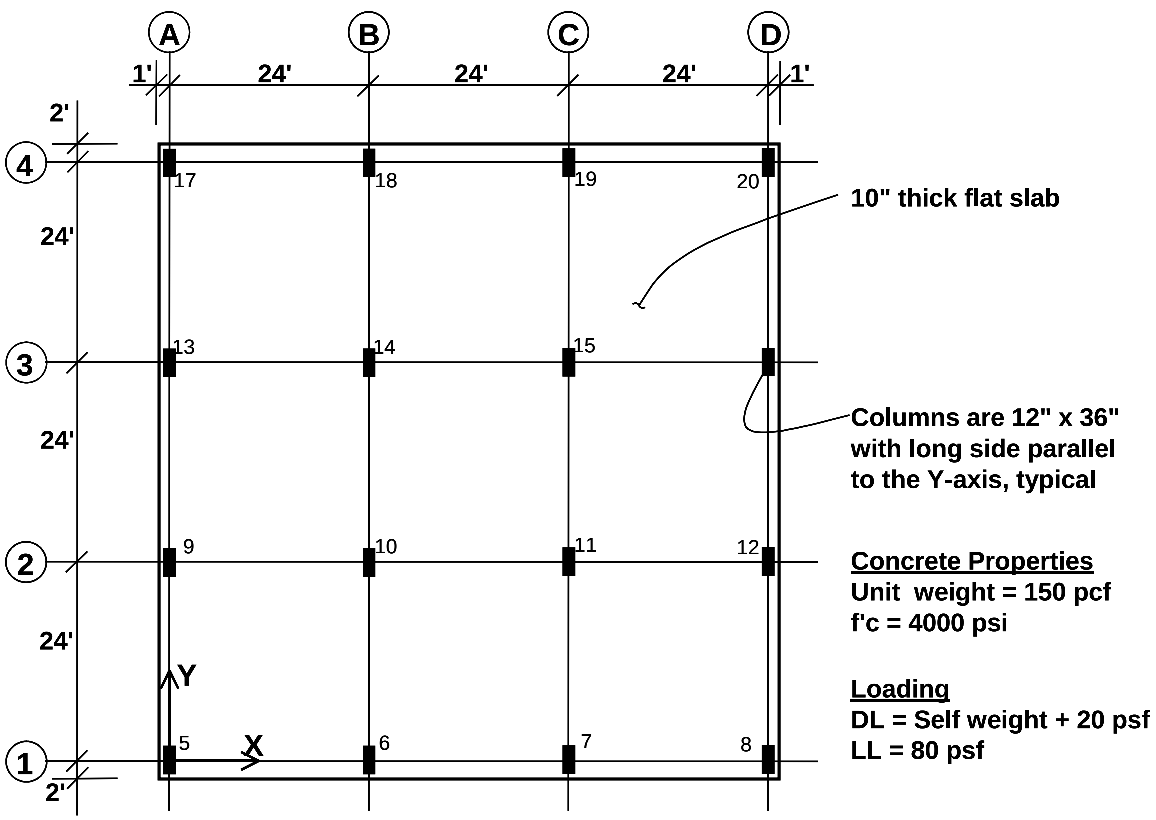


Figure 1: Flat Slab For Numerical Example

The slab overhangs the face of the column by 6 inches along each side of the structure. The columns are typically 12 inches wide by 36 inches long, with the long side parallel to the Y-axis. The slab is typically 10 inches thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 150 pcf and an f'c of 4000 psi. The dead load consists of the self weight of the structure plus an additional 20 psf. The live load is 80 psf.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress, and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE punching shear capacity, shear stress ratio, and D/C ratio with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this example.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (ksi)	Shear Capacity (ksi)	D/C ratio
SAFE	0.192	0.158	1.21
Calculated	0.193	0.158	1.22

COMPUTER FILE: ACI 318-14 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation for Interior Column Using SAFE Method

d = [(10 - 1) + (10 - 2)] / 2 = 8.5"

Refer to Figure 2.

b_0 = 44.5 + 20.5 + 44.5 + 20.5 = 130"

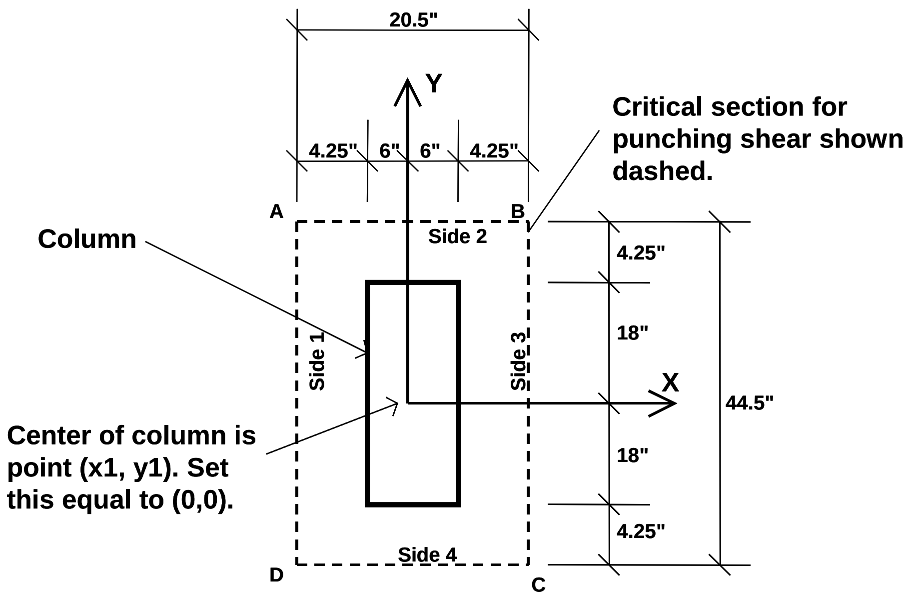


Figure 2: Interior Column, Grid B-2 in SAFE Model

$$\gamma_{V2} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{44.5}{20.5}}} = 0.4955$$

$$\gamma_{V3} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{20.5}{44.5}}} = 0.3115$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear, as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–10.25	0	10.25	0	N.A.
y_2	0	22.25	0	–22.25	N.A.
L	44.5	20.5	44.5	20.5	b_0 = 130
d	8.5	8.5	8.5	8.5	N.A.
Ld	378.25	174.25	378.25	174.25	1105
Ldx_2	–3877.06	0	3877.06	0	0
Ldy_2	0	3877.06	0	–3877.06	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{1105} = 0"$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{1105} = 0"$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the “Sum” column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	44.5	20.5	44.5	20.5	N.A.
d	8.5	8.5	8.5	8.5	N.A.
$x_2 - x_3$	–10.25	0	10.25	0	N.A.
$y_2 - y_3$	0	22.25	0	–22.25	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	64696.5	86264.6	64696.5	86264.6	301922.3
I_{YY}	39739.9	7151.5	39739.9	7151.5	93782.8
I_{XY}	0	0	0	0	0

From the SAFE output at Grid B-2:

V_U = 189.45 k

$\gamma_{V2} M_{U2}$ = –156.39 k-in

$\gamma_{V3} M_{U3}$ = 91.538 k-in

At the point labeled A in Figure 2, x_4 = –10.25 and y_4 = 22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(22.25-0\right)-\left(0\right)\left(-10.25-0\right)\right]}{(301922.3)(93782.8)-(0)^2} - \frac{91.538\left[301922.3\left(-10.25-0\right)-\left(0\right)\left(22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 – 0.0115 – 0.0100 = **0.1499 ksi** at point A

At the point labeled B in Figure 2, x_4 = 10.25 and y_4 = 22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(22.25-0\right)-\left(0\right)\left(10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(10.25-0\right)-\left(0\right)\left(22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 – 0.0115 + 0.0100 = **0.1699 ksi** at point B

At the point labeled C in Figure 2, x_4 = 10.25 and y_4 = –22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(-22.25-0\right)-\left(0\right)\left(10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(10.25-0\right)-\left(0\right)\left(-22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 + 0.0115 + 0.0100 = **0.1930 ksi** at point C

At the point labeled D in Figure 2, x_4 = –10.25 and y_4 = –22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(-22.25-0\right)-\left(0\right)\left(-10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(-10.25-0\right)-\left(0\right)\left(-22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 + 0.0115 - 0.0100 = **0.1729 ksi** at point D

Point C has the largest absolute value of v_u, thus v_{max} = 0.1930 ksi

The shear capacity is calculated based on the smallest of ACI 318-14 equations 11-34, 11-35 and 11-36 with the b_0 and d terms removed to convert force to stress.

$$\varphi v_C = \frac{0.75\left(2+\frac{4}{36/12}\right)\sqrt{4000}}{1000} = 0.158$$ ksi in accordance with equation 11-34

$$\varphi v_C = \frac{0.75\left(\frac{40 \bullet 8.5}{130}+2\right)\sqrt{4000}}{1000} = 0.219$$ ksi in accordance with equation 11-35

$$\varphi v_C = \frac{0.75 \bullet 4 \bullet \sqrt{4000}}{1000} = 0.190$$ ksi in accordance with equation 11-36

Equation 11-34 yields the smallest value of ϕv_C = 0.158 ksi and thus this is the shear capacity.

$$Shear\ Ratio = \frac{v_U}{\varphi v_C} = \frac{0.193}{0.158} = 1.22$$

EXAMPLE ACI 318-14 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 6 inches thick and spans 12 feet between walls. The slab is modeled using thin plate elements. The walls are modeled as line supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified to be 36 inches. To obtain factored moments and flexural reinforcement in a design strip, one one-foot-wide strip is defined in the X-direction on the slab, as shown in Figure 1.

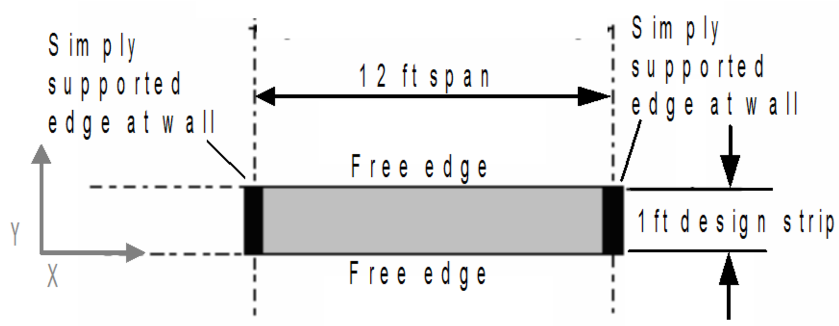


Figure 1 Plan View of One-Way Slab

One dead load case (DL80) and one live load case (LL100) with uniformly distributed surface loads of magnitudes 80 and 100 psf, respectively, are defined in the model. A load combination (COMB100) is defined using the ACI 318-14 load combination factors, 1.2 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing the analysis, design is performed in accordance with ACI 318-14 using SAFE and also by hand computation. Table 1 shows the comparison of the moments and design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness T, h = 6 in

Depth of tensile reinf.	d_c	=	1	in
Effective depth	d	=	5	in
Clear span	l_n, l_1	=	144	in
Concrete strength	f'_c	=	4,000	psi
Yield strength of steel	f_y	=	60,000	psi
Concrete unit weight	w_c	=	0	pcf
Modulus of elasticity	E_c	=	3,600	ksi
Modulus of elasticity	E_s	=	29,000	ksi
Poisson's ratio	ν	=	0	
Dead load	w_d	=	80	psf
Live load	w_l	=	100	psf

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (k-in)	Reinforcement Area (sq-in)
			A_s^+
Medium	SAFE	55.22	0.213
	Calculated	55.22	0.213

$A^+_{s,min}$ = 0.1296 sq-in

COMPUTER FILE: ACI 318-14 RC-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\varphi = 0.9$

$b = 12$ in

$A_{s,min} = 0.0018bh = 0.1296$ sq-in

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) = 0.85$$

$$c_{\max} = \frac{0.003}{0.003 + 0.005} d = 1.875 \text{ in}$$

$a_{max} = \beta_1 c_{max} = 1.59375$ in

For the load combination, w and M_u are calculated as follows:

$w = (1.2w_d + 1.6w_t)\, b \,/\, 144$

$$M_u = \frac{wl_1^2}{8}$$

$A_s = \min[A_{s,\min}, (4/3)\, A_{s,\text{required}}] = \min[0.1296, (4/3)2.11] = 0.1296$ sq-in

COMB100

w_d = 80 psf

w_t = 100 psf

w = 21.33 lb/in

$M_{u\text{-}strip}$ = 55.22 k-in

$M_{u\text{-}design}$ = 55.629 k-in

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f'_c \varphi\, b}} = 0.3128 \text{ in} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = 0.213 \text{ sq-in} > A_{s,\min}$$

A_s = 0.2114 sq-in

EXAMPLE ACI 318-11 PT-SL 001

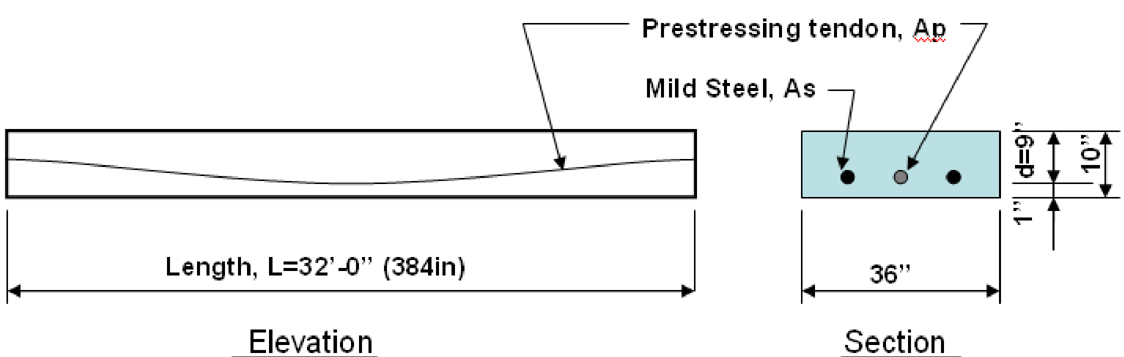
Design Verification of Post-Tensioned Slab using the ACI 318-11 code

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 10 inches thick by 36 inches wide and spans 32 feet, as shown in shown in Figure 1. A 36-inch-wide design strip was centered along the length of the slab and was defined as an A-Strip. B-strips were placed at each end of the span perpendicular to the Strip-A (the B-Strips are necessary to define the tendon profile). A tendon, with two strands having an area of 0.153 square inches each, was added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are shown below. The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the SAFE results and summarized for verification and validation of the SAFE results.

Loads: Dead = self weight , Live = 100psf



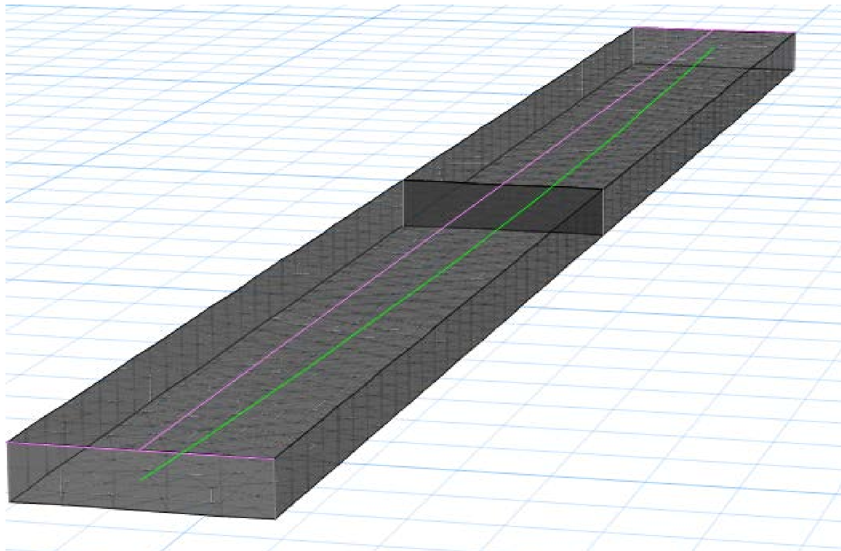


Figure 1 One-Way Slab

GEOMETRY, PROPERTIES AND LOADING

Thickness,	T, h	=	10	in
Effective depth,	d	=	9	in
Clear span,	L	=	384	in
Concrete strength,	f'_c	=	4,000	psi
Yield strength of steel,	f_y	=	60,000	psi
Prestressing, ultimate	f_{pu}	=	270,000	psi
Prestressing, effective	f_e	=	175,500	psi
Area of Prestress (single strand),	A_P	=	0.153	sq in
Concrete unit weight,	w_c	=	0.150	pcf
Modulus of elasticity,	E_c	=	3,600	ksi
Modulus of elasticity,	E_s	=	29,000	ksi
Poisson's ratio,	ν	=	0	
Dead load,	w_d	=	self	psf
Live load,	w_l	=	100	psf

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live and post-tensioning loads.

RESULTS COMPARISON

The SAFE total factored moments, required mild steel reinforcing and slab stresses are compared to the independent hand calculations in Table 1.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (k-in)	1429.0	1428.3	0.05%
Area of Mild Steel req'd, As (sq-in)	2.20	2.20	0.00%
Transfer Conc. Stress, top ($D+PT_I$), ksi	–0.734	–0.735	0.14%
Transfer Conc. Stress, bot ($D+PT_I$), ksi	0.414	0.414	0.00%
Normal Conc. Stress, top ($D+L+PT_F$), ksi	–1.518	–1.519	0.07%
Normal Conc. Stress, bot ($D+L+PT_F$), ksi	1.220	1.221	0.08%
Long-Term Conc. Stress, top ($D+0.5L+PT_{F(L)}$), ksi	–1.134	–1.135	0.09%
Long-Term Conc. Stress, bot ($D+0.5L+PT_{F(L)}$), ksi	0.836	0.837	0.12%

COMPUTER FILE: ACI 318-11 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

CALCULATIONS:

Design Parameters:

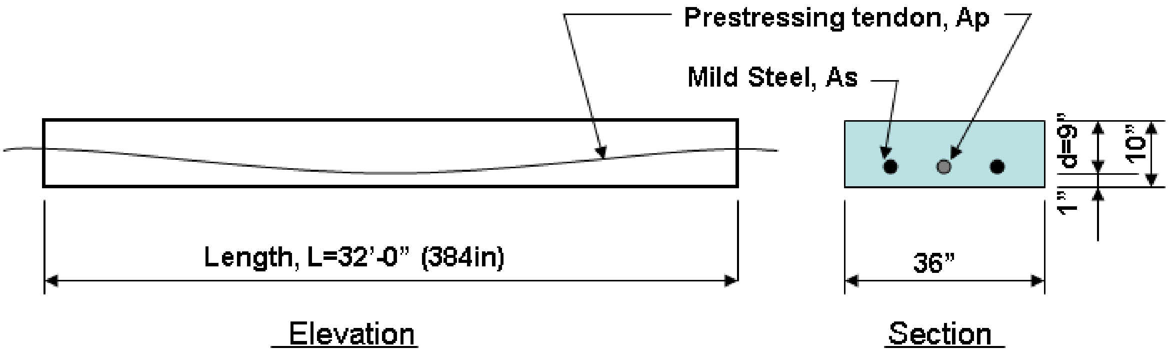
$\phi = 0.9$

Mild Steel Reinforcing

$f'c$ = 4000 psi
fy = 60,000 psi

Post-Tensioning

f_j = 216.0 ksi
Stressing Loss = 27.0 ksi
Long-Term Loss = 13.5 ksi
f_i = 189.0 ksi
f_e = 175.5 ksi



Loads:

Dead, self-wt = 10 / 12 ft × 0.150 kcf = 0.125 ksf (D) × 1.2 = 0.150 ksf (D_u)
Live, 0.100 ksf (L) × 1.6 = 0.160 ksf (L_u)
Total =0.225 ksf (D+L) 0.310 ksf (D+L)ult

ω =0.225 ksf × 3 ft = 0.675 klf, ω_u = 0.310 ksf × 3ft = 0.930 klf

Ultimate Moment, $M_U = \frac{wl_1^2}{8}$ = 0.310 klf × 32^2/8 = 119.0 k-ft = 1429.0 k-in

Ultimate Stress in strand, $f_{PS} = f_{SE} + 10000 + \frac{f'c}{300\rho_P}$ (span-to-depth ratio > 35)

$$= 175,500 + 10,000 + \frac{4,000}{300(0.000944)}$$

$$= 199,624 \text{ psi} \leq 205,500 \text{ psi}$$

Ultimate force in PT, $F_{ult,PT} = A_P\left(f_{PS}\right) = 2(0.153)(199.62) = 61.08 \text{ kips}$

Ultimate force in RC, $F_{ult,RC} = A_s\left(f\right)_y = 2.00\text{(assumed)}(60.0) = 120.0 \text{ kips}$

Total Ultimate force, $F_{ult,Total} = 61.08 + 120.0 = 181.08 \text{ kips}$

Stress block depth, $a = \frac{F_{ult,Total}}{0.85 f'cb} = \frac{181.08}{0.85(4)(36)} = 1.48 \text{ in}$

Ultimate moment due to PT, $M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right)\phi = 61.08\left(9 - \frac{1.48}{2}\right)(0.9) = 454.1 \text{ k-in}$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 1429.0 - 454.1 = 974.9 \text{ k-in}$

Required area of mild steel reinforcing, $A_S = \frac{M_{net}}{\phi f_y\left(d - \frac{a}{2}\right)} = \frac{974.9}{0.9(60)\left(9 - \frac{1.48}{2}\right)} = 2.18 \text{ in}^2$

Note: The required area of mild steel reinforcing was calculated from an assumed amount of steel. Since the assumed value and the calculated value are not the same a second iteration can be performed. The second iteration changes the depth of the stress block and the calculated area of steel value. Upon completion of the second iteration the area of steel was found to be 2.21in^2

Check of Concrete Stresses at Mid-Span:

Initial Condition (Transfer), load combination (D + L + PT_i) = 1.0D + 1.0PT_I

The stress in the tendon at transfer = jacking stress − stressing losses = 216.0 − 27.0 = 189.0 ksi

The force in the tendon at transfer, = $189.0(2)(0.153) = 57.83\,\text{kips}$

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0 \text{ k-ft} = 576 \text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 57.83(4 \text{ in}) = 231.3 \text{ k-in}$

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-57.83}{10(36)} \pm \dfrac{576.0 - 231.3}{600}$, where S = 600 in^3

$$\boldsymbol{f} = -0.161 \pm 0.5745$$

$$f = -0.735(\text{Comp})\max, 0.414(\text{Tension})\max$$

Normal Condition, load combinations: (D + L + PT_F) = 1.0D + 1.0L + 1.0PT_F

Tendon stress at normal = jacking − stressing − long-term = 216.0 − 27.0 − 13.5 = 175.5 ksi
The force in tendon at Normal, = $175.5(2)(0.153) = 53.70\,\text{kips}$

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0 \text{ k-ft} = 576 \text{ k-in}$

Moment due to dead load, $M_L = 0.100(3)(32)^2/8 = 38.4 \text{ k-ft} = 461 \text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 53.70(4 \text{ in}) = 214.8 \text{ k-in}$

Stress in concrete for (D + L+ PT_F), $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_{D+L} - M_{PT}}{S} = \dfrac{-53.70}{10(36)} \pm \dfrac{1037.0 - 214.8}{600}$

$$\boldsymbol{f} = -0.149 \pm 1.727 \pm 0.358$$

$$f = -1.518(\text{Comp})\max, 1.220(\text{Tension})\max$$

Long-Term Condition, load combinations: (D + 0.5L + $PT_{F(L)}$) = 1.0D + 0.5L + 1.0PT_F

Tendon stress at normal = jacking − stressing − long-term = 216.0 − 27.0 − 13.5 = 175.5 ksi
The force in tendon at Normal, = $175.5(2)(0.153) = 53.70\,\text{kips}$

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0 \text{ k-ft} = 576 \text{ k-in}$

Moment due to dead load, $M_L = 0.100(3)(32)^2/8 = 38.4 \text{ k-ft} = 460 \text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 53.70(4 \text{ in}) = 214.8 \text{ k-in}$

Stress in concrete for (D + 0.5L + $PT_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-53.70}{10(36)} \pm \frac{806.0 - 214.8}{600}$$

$$f = -0.149 \pm 0.985$$

$$f = -1.134(\text{Comp})\,\text{max}, 0.836(\text{Tension})\,\text{max}$$

EXAMPLE ACI 318-11 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

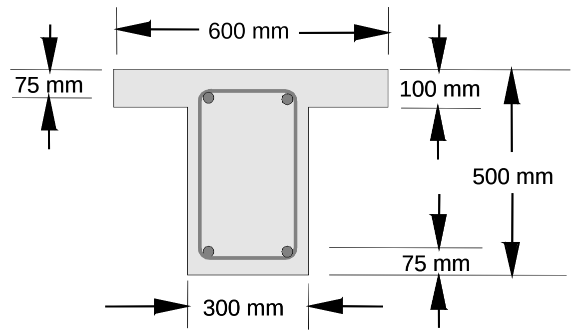
The purpose of this example is to verify the beam flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by ACI 318-11.
- The average shear stress in the beam falls below the maximum shear stress allowed by ACI 318-11, requiring design shear reinforcement.

A simple-span, 20-foot-long, 12-inch-wide, and 18-inch-deep T beam with a flange 4 inches thick and 24 inches wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size is specified as 6 inches. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kip/in).

The beam is loaded with symmetric third-point loading. One dead load (DL02) case and one live load (LL30) case, with only symmetric third-point loads of magnitudes 3, and 30 kips, respectively, are defined in the model. One load combination (COMB30) is defined using the ACI 318-11 load combination factors of 1.2 for dead load and 1.6 for live load. The model is analyzed for both of these load cases and the load combination.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results and found to be identical. After completing the analysis, the design is performed using the ACI 318-11 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcement. Table 2 shows the comparison of the design shear reinforcement.



Beam Section

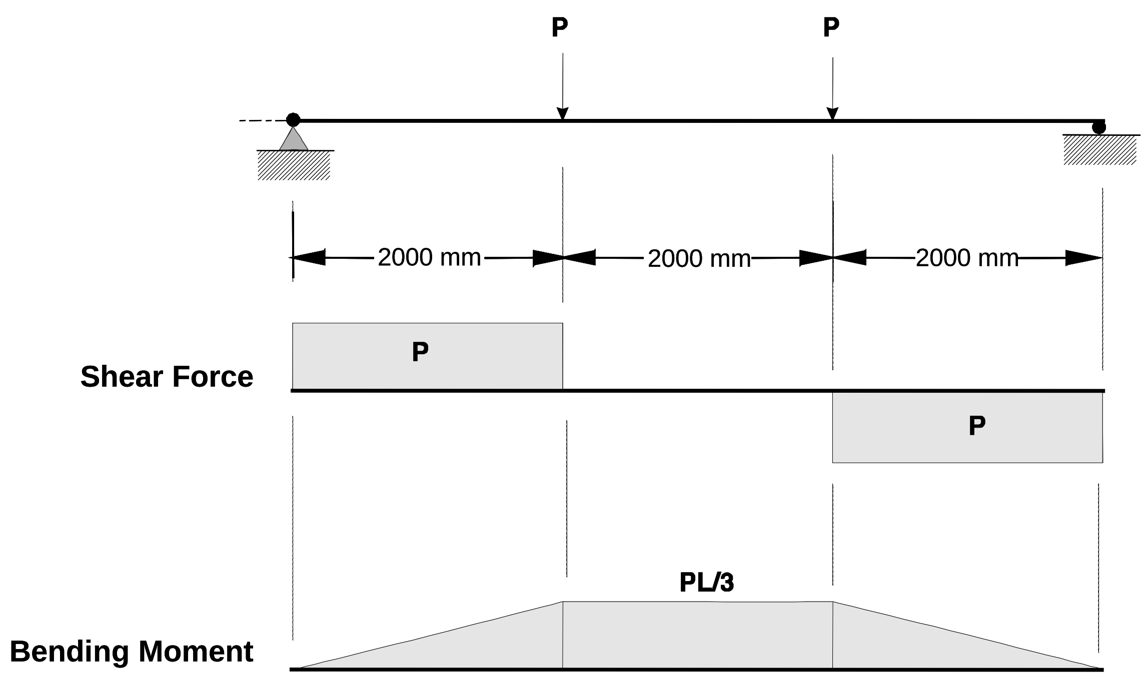


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span	l	=	240	in
Overall depth	h	=	18	in
Flange thickness	d_s	=	4	in
Width of web	b_w	=	12	in
Width of flange,	b_f	=	24	in
Depth of tensile reinf.	d_c	=	3	in
Effective depth	d	=	15	in
Depth of comp. reinf.	d'	=	3	in
Concrete strength	f'_c	=	4,000	psi
Yield strength of steel	f_y	=	60,000	psi
Concrete unit weight	w_c	=	0	pcf
Modulus of elasticity	E_c	=	3,600	ksi
Modulus of elasticity	E_s	=	29,000	ksi
Poisson's ratio	v	=	0.2	
Dead load	P_d	=	2	kips
Live load	P_l	=	30	kips

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the beam with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the comparison of the design reinforcement.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (k-in)	Reinforcement Area (sq-in)
		A_s^+
SAFE	4032	5.808
Calculated	4032	5.808

$A^+_{s,min}$ = 0.4752 sq-in

Table 2 Comparison of Shear Reinforcements

Shear Force (kip)	Reinforcement Area, $\frac{A_v}{s}$ (sq-in/ft)	
	SAFE	Calculated
50.40	0.592	0.592

COMPUTER FILE: ACI 318-11 RC-BM-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\varphi = 0.9$

$A_g = 264$ sq-in

$A_{s,min} = 0.0018A_g = 0.4752$ sq-in

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) = 0.85$$

$$c_{max} = \frac{0.003}{0.003 + 0.005} d = 5.625 \text{ in}$$

$$a_{max} = \beta_1 c_{max} = 4.78125 \text{ in}$$

$A_s = \min[A_{s,min}, (4/3)\, A_{s,required}] = \min[0.4752, (4/3)5.804] = 0.4752$ sq-in

COMB30

$P_u = (1.2P_d + 1.6P_t) = 50.4$ k

$$M_u = \frac{P_u l}{3} = 4032 \text{ k-in}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f'_c \varphi\, b_f}} = 4.2671 \text{ in } (a > d_s)$$

Calculation for A_s is performed in two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w. C_f is given by:

$$C_f = 0.85 f'_c (b_f - b_w)\, d_s = 163.2 \text{ k}$$

The portion of M_u that is resisted by the flange is given by:

$$M_{uf} = C_f \left(d - \frac{d_s}{2}\right)\varphi = 1909.44 \text{ k-in}$$

Therefore, the area of tensile steel reinforcement to balance flange compression is:

$$A_{s1} = \frac{M_{uf}}{f_y\left(d - d_s/2\right)\varphi} = 2.7200 \text{ sq-in}$$

The balance of the moment to be carried by the web is given by:

$$M_{uw} = M_u - M_{uf} = 2122.56 \text{ k-in}$$

The web is a rectangular section with dimensions b_w and d, for which the design depth of the compression block is recalculated as

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85\, f_c'\, \varphi\, b_w}} = 4.5409 \text{ in} \quad (a_1 \le a_{max})$$

The area of tensile steel reinforcement to balance the web compression is then given by:

$$A_{s2} = \frac{M_{uw}}{\varphi\, f_y\left(d - \frac{a_1}{2}\right)\varphi} = 3.0878 \text{ sq-in}$$

The area of total tensile steel reinforcement is then given by:

$$A_s = A_{s1} + A_{s2} = 5.808 \text{ sq-in}$$

Shear Design

The following quantities are computed for all of the load combinations:

$$\varphi = 0.75$$

Check the limit of $\sqrt{f_c'}$:

$$\sqrt{f_c'} = 63.246 \text{ psi} < 100 \text{ psi}$$

The concrete shear capacity is given by:

$$\varphi V_c = \varphi\, 2\sqrt{f_c'}\, b_w d = 17.076 \text{ k}$$

The maximum shear that can be carried by reinforcement is given by:

$\varphi V_s = \varphi 8 \sqrt{f'_c}\, b_w d = 68.305$ k

The following limits are required in the determination of the reinforcement:

$(\varphi V_c/2) = 8.538$ k

$(\varphi V_c + \varphi 50\, b_w d) = 23.826$ k

$V_{\max} = \varphi V_c + \varphi V_s = 85.381$ k

Given V_u, V_c and $V_{\max}$, the required shear reinforcement in area/unit length for any load combination is calculated as follows:

If $V_u \le (V_c/2)\, \varphi$,

$$\frac{A_v}{s} = 0,$$

else if $(V_c/2)\, \varphi < V_u \le (\varphi V_c + \varphi\, 50\, b_w d)$,

$$\frac{A_v}{s} = \frac{50\, b_w}{f_y},$$

else if $(\varphi V_c + \varphi\, 50\, b_w d) < V_u \le \varphi\, V_{\max}$

$$\frac{A_v}{s} = \frac{(V_u - \varphi V_c)}{\varphi\, f_{ys}\, d}$$

else if $V_u > \varphi\, V_{\max}$,

a failure condition is declared.

For each load combination, the P_u and V_u are calculated as follows:

$P_u = 1.2P_d + 1.6P_l$

$V_u = P_u$

(COMB30)

$P_d = 2$ k

$P_l = 30$ k

$P_u = 50.4$ k

$V_u = 50.4$ k, $(\varphi V_c + \varphi\, 50\, b_w d) < V_u \le \varphi\, V_{\max}$

$$\frac{A_v}{s} = \frac{(V_u - \varphi V_c)}{\varphi f_{ys} d} = 0.04937 \text{ sq-in/in or } 0.592 \text{ sq-in/ft}$$

EXAMPLE ACI 318-11 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE.

The numerical example is a flat slab that has three 24-foot-long spans in each direction, as shown in Figure 1.

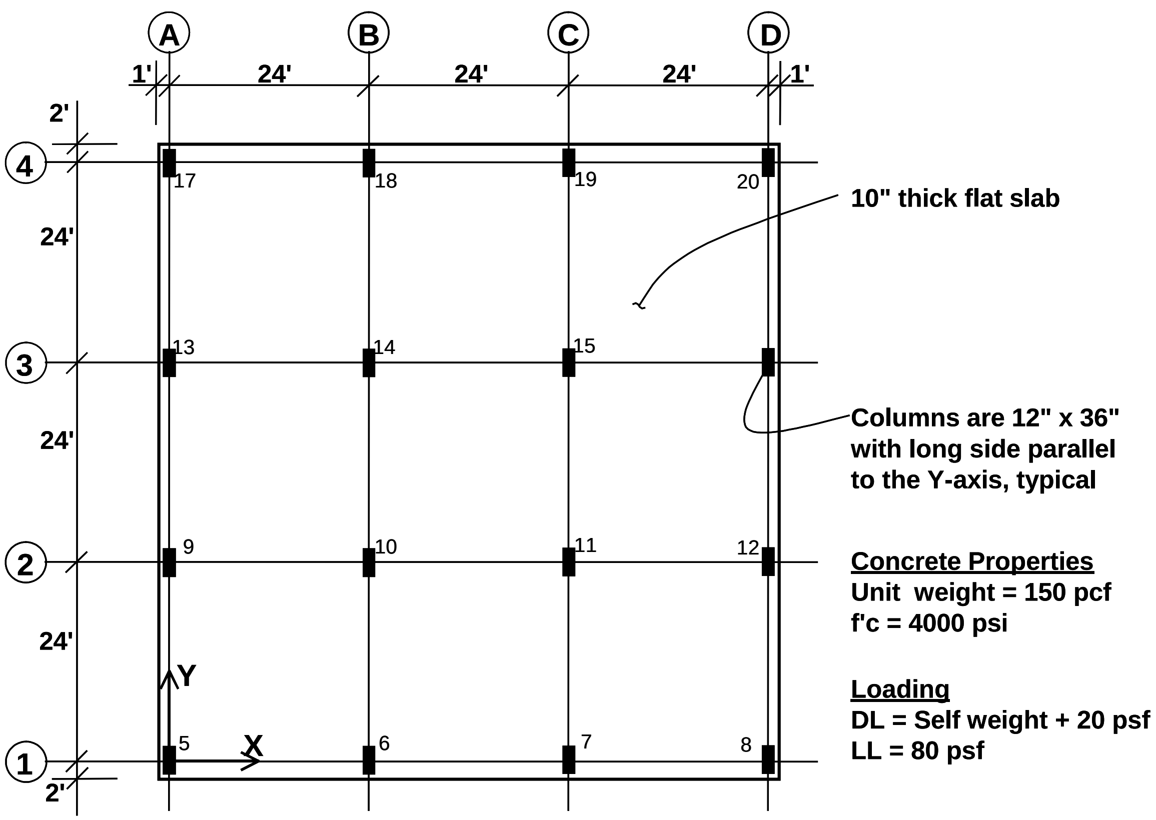


Figure 1: Flat Slab For Numerical Example

The slab overhangs the face of the column by 6 inches along each side of the structure. The columns are typically 12 inches wide by 36 inches long, with the long side parallel to the Y-axis. The slab is typically 10 inches thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 150 pcf and an f'c of 4000 psi. The dead load consists of the self weight of the structure plus an additional 20 psf. The live load is 80 psf.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress, and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE punching shear capacity, shear stress ratio, and D/C ratio with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this example.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (ksi)	Shear Capacity (ksi)	D/C ratio
SAFE	0.192	0.158	1.21
Calculated	0.193	0.158	1.22

COMPUTER FILE: ACI 318-11 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation for Interior Column Using SAFE Method

d = [(10 - 1) + (10 - 2)] / 2 = 8.5"

Refer to Figure 2.

b_0 = 44.5 + 20.5 + 44.5 + 20.5 = 130"

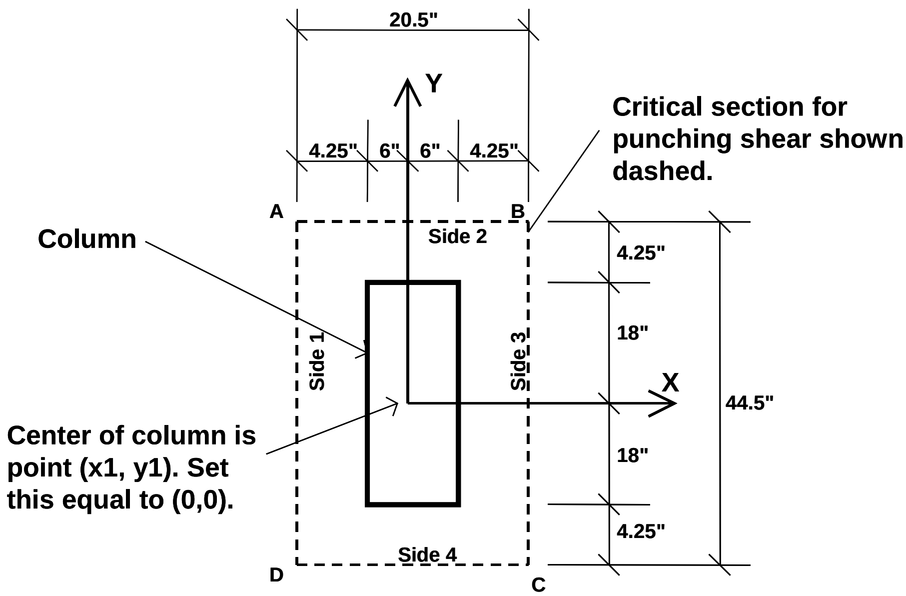


Figure 2: Interior Column, Grid B-2 in SAFE Model

$$\gamma_{V2} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{44.5}{20.5}}} = 0.4955$$

$$\gamma_{V3} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{20.5}{44.5}}} = 0.3115$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear, as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–10.25	0	10.25	0	N.A.
y_2	0	22.25	0	–22.25	N.A.
L	44.5	20.5	44.5	20.5	b_0 = 130
d	8.5	8.5	8.5	8.5	N.A.
Ld	378.25	174.25	378.25	174.25	1105
Ldx_2	–3877.06	0	3877.06	0	0
Ldy_2	0	3877.06	0	–3877.06	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{1105} = 0"$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{1105} = 0"$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	44.5	20.5	44.5	20.5	N.A.
d	8.5	8.5	8.5	8.5	N.A.
x_2 - x_3	–10.25	0	10.25	0	N.A.
y_2 - y_3	0	22.25	0	–22.25	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	64696.5	86264.6	64696.5	86264.6	301922.3
I_{YY}	39739.9	7151.5	39739.9	7151.5	93782.8
I_{XY}	0	0	0	0	0

From the SAFE output at Grid B-2:

V_U = 189.45 k

$\gamma_{V2} M_{U2}$ = –156.39 k-in

$\gamma_{V3} M_{U3}$ = 91.538 k-in

At the point labeled A in Figure 2, $x_4 = -10.25$ and $y_4 = 22.25$, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(22.25-0\right)-\left(0\right)\left(-10.25-0\right)\right]}{(301922.3)(93782.8)-(0)^2} - \frac{91.538\left[301922.3\left(-10.25-0\right)-\left(0\right)\left(22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 – 0.0115 – 0.0100 = **0.1499 ksi** at point A

At the point labeled B in Figure 2, $x_4 = 10.25$ and $y_4 = 22.25$, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(22.25-0\right)-\left(0\right)\left(10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(10.25-0\right)-\left(0\right)\left(22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 – 0.0115 + 0.0100 = **0.1699 ksi** at point B

At the point labeled C in Figure 2, $x_4 = 10.25$ and $y_4 = -22.25$, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(-22.25-0\right)-\left(0\right)\left(10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(10.25-0\right)-\left(0\right)\left(-22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 + 0.0115 + 0.0100 = **0.1930 ksi** at point C

At the point labeled D in Figure 2, $x_4 = -10.25$ and $y_4 = -22.25$, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(-22.25-0\right)-\left(0\right)\left(-10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(-10.25-0\right)-\left(0\right)\left(-22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 + 0.0115 - 0.0100 = **0.1729 ksi** at point D

Point C has the largest absolute value of v_u, thus v_{max} = 0.1930 ksi

The shear capacity is calculated based on the smallest of ACI 318-11 equations 11-34, 11-35 and 11-36 with the b_0 and d terms removed to convert force to stress.

$$\varphi v_C = \frac{0.75\left(2+\frac{4}{36/12}\right)\sqrt{4000}}{1000} = 0.158$$ ksi in accordance with equation 11-34

$$\varphi v_C = \frac{0.75\left(\frac{40 \bullet 8.5}{130}+2\right)\sqrt{4000}}{1000} = 0.219$$ ksi in accordance with equation 11-35

$$\varphi v_C = \frac{0.75 \bullet 4 \bullet \sqrt{4000}}{1000} = 0.190$$ ksi in accordance with equation 11-36

Equation 11-34 yields the smallest value of $\phi v_C = 0.158$ ksi and thus this is the shear capacity.

$$Shear\ Ratio = \frac{v_U}{\varphi v_C} = \frac{0.193}{0.158} = 1.22$$

EXAMPLE ACI 318-11 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 6 inches thick and spans 12 feet between walls. The slab is modeled using thin plate elements. The walls are modeled as line supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified to be 36 inches. To obtain factored moments and flexural reinforcement in a design strip, one one-foot-wide strip is defined in the X-direction on the slab, as shown in Figure 1.

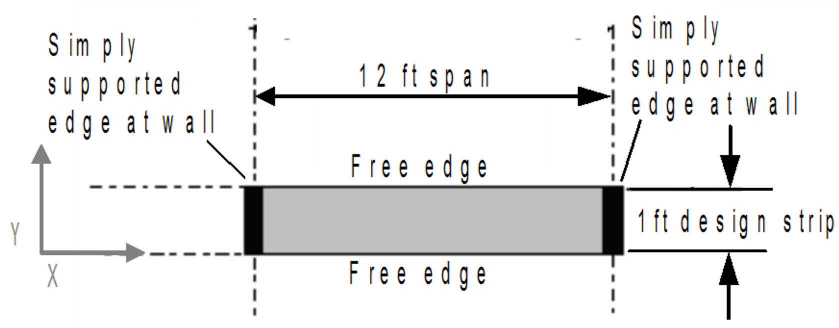


Figure 1 Plan View of One-Way Slab

One dead load case (DL80) and one live load case (LL100) with uniformly distributed surface loads of magnitudes 80 and 100 psf, respectively, are defined in the model. A load combination (COMB100) is defined using the ACI 318-11 load combination factors, 1.2 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing the analysis, design is performed in accordance with ACI 318-11 using SAFE and also by hand computation. Table 1 shows the comparison of the moments and design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness T, h = 6 in

Depth of tensile reinf.	d_c	=	1	in
Effective depth	d	=	5	in
Clear span	l_n, l_1	=	144	in
Concrete strength	f'_c	=	4,000	psi
Yield strength of steel	f_y	=	60,000	psi
Concrete unit weight	w_c	=	0	pcf
Modulus of elasticity	E_c	=	3,600	ksi
Modulus of elasticity	E_s	=	29,000	ksi
Poisson's ratio	ν	=	0	
Dead load	w_d	=	80	psf
Live load	w_l	=	100	psf

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (k-in)	Reinforcement Area (sq-in)
			A_s^+
Medium	SAFE	55.22	0.213
	Calculated	55.22	0.213

$A^+_{s,min}$ = 0.1296 sq-in

COMPUTER FILE: ACI 318-11 RC-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\varphi = 0.9$

$b = 12$ in

$A_{s,\min} = 0.0018bh = 0.1296$ sq-in

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right) = 0.85$$

$$c_{\max} = \frac{0.003}{0.003 + 0.005} d = 1.875 \text{ in}$$

$a_{max} = \beta_1 c_{max} = 1.59375$ in

For the load combination, w and M_u are calculated as follows:

$w = (1.2w_d + 1.6w_t)\, b / 144$

$$M_u = \frac{wl_1^2}{8}$$

$A_s = \min[A_{s,\min}, (4/3)\, A_{s,\text{required}}] = \min[0.1296, (4/3)2.11] = 0.1296$ sq-in

COMB100

w_d = 80 psf

w_t = 100 psf

w = 21.33 lb/in

$M_{u\text{-}strip}$ = 55.22 k-in

$M_{u\text{-}design}$ = 55.629 k-in

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f_c' \varphi\, b}} = 0.3128 \text{ in} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \dfrac{a}{2}\right)} = 0.213 \text{ sq-in} > A_{s,\min}$$

A_s = 0.2114 sq-in

EXAMPLE ACI 318-08 PT-SL 001

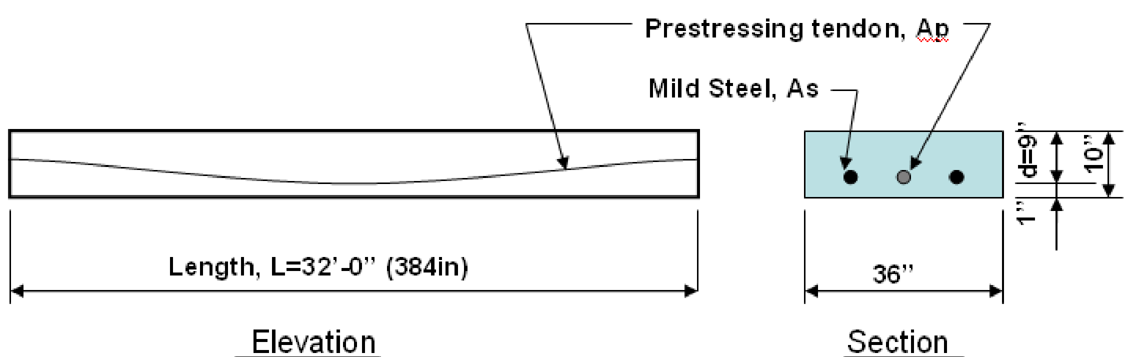
Design Verification of Post-Tensioned Slab using the ACI 318-08 code

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 10 inches thick by 36 inches wide and spans 32 feet, as shown in shown in Figure 1. A 36-inch-wide design strip was centered along the length of the slab and was defined as an A-Strip. B-strips were placed at each end of the span perpendicular to the Strip-A (the B-Strips are necessary to define the tendon profile). A tendon, with two strands having an area of 0.153 square inches each, was added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are shown below. The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the SAFE results and summarized for verification and validation of the SAFE results.

Loads: Dead = self weight , Live = 100psf



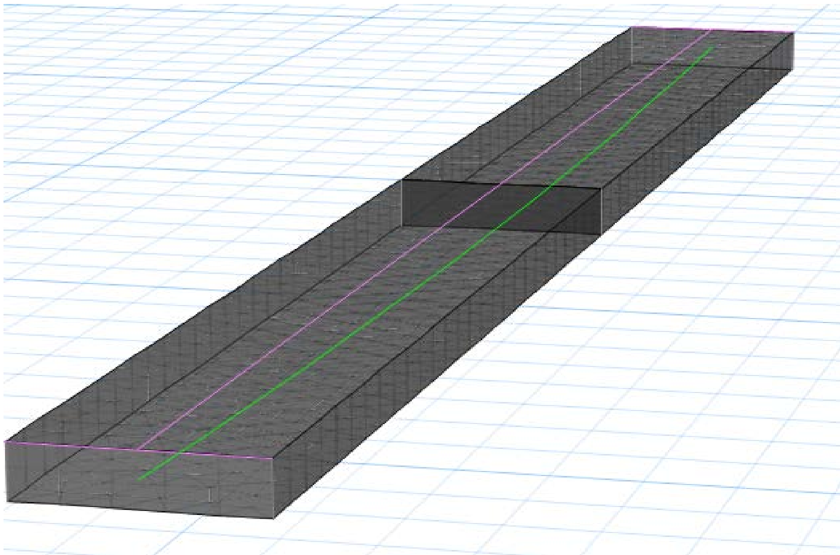


Figure 1 One-Way Slab

GEOMETRY, PROPERTIES AND LOADING

Thickness,	T, h	=	10	in
Effective depth,	d	=	9	in
Clear span,	L	=	384	in
Concrete strength,	f'_c	=	4,000	psi
Yield strength of steel,	f_y	=	60,000	psi
Prestressing, ultimate	f_{pu}	=	270,000	psi
Prestressing, effective	f_e	=	175,500	psi
Area of Prestress (single strand),	A_P	=	0.153	sq in
Concrete unit weight,	w_c	=	0.150	pcf
Modulus of elasticity,	E_c	=	3,600	ksi
Modulus of elasticity,	E_s	=	29,000	ksi
Poisson's ratio,	ν	=	0	
Dead load,	w_d	=	self	psf
Live load,	w_l	=	100	psf

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live and post-tensioning loads.

RESULTS COMPARISON

The SAFE total factored moments, required mild steel reinforcing and slab stresses are compared to the independent hand calculations in Table 1.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (k-in)	1429.0	1428.3	0.05%
Area of Mild Steel req'd, As (sq-in)	2.20	2.20	0.00%
Transfer Conc. Stress, top (D+PT_I), ksi	–0.734	–0.735	0.14%
Transfer Conc. Stress, bot (D+PT_I), ksi	0.414	0.414	0.00%
Normal Conc. Stress, top (D+L+PT_F), ksi	–1.518	–1.519	0.07%
Normal Conc. Stress, bot (D+L+PT_F), ksi	1.220	1.221	0.08%
Long-Term Conc. Stress, top (D+0.5L+$PT_{F(L)}$), ksi	–1.134	–1.135	0.09%
Long-Term Conc. Stress, bot (D+0.5L+$PT_{F(L)}$), ksi	0.836	0.837	0.12%

COMPUTER FILE: ACI 318-05 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

CALCULATIONS:

Design Parameters:

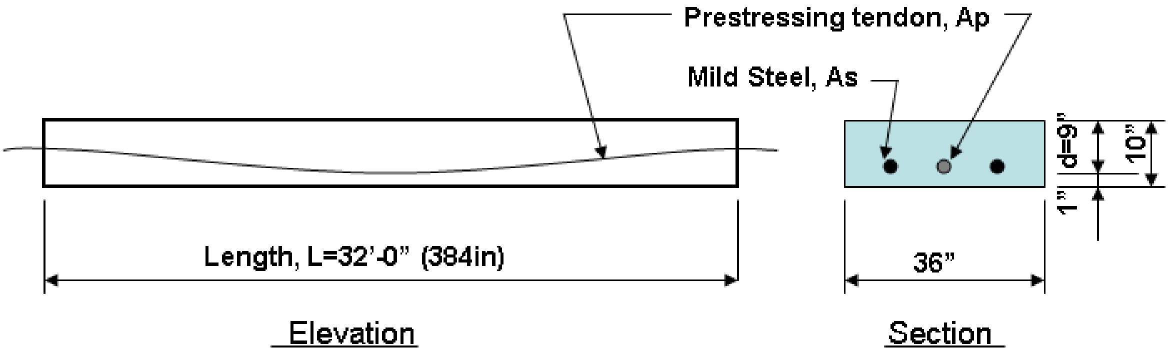
$\phi = 0.9$

Mild Steel Reinforcing

$f'c$ = 4000 psi
fy = 60,000 psi

Post-Tensioning

f_j = 216.0 ksi
Stressing Loss = 27.0 ksi
Long-Term Loss = 13.5 ksi
f_i = 189.0 ksi
f_e = 175.5 ksi



Loads:

Dead, self-wt = 10 / 12 ft × 0.150 kcf = 0.125 ksf (D) × 1.2 = 0.150 ksf (D_u)
Live, 0.100 ksf (L) × 1.6 = 0.160 ksf (L_u)
Total = 0.225 ksf (D+L)   0.310 ksf (D+L)ult

ω = 0.225 ksf × 3 ft = 0.675 klf, ω_u = 0.310 ksf × 3ft = 0.930 klf

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8}$ = 0.310 klf × $32^2/8$ = 119.0 k-ft = 1429.0 k-in

Ultimate Stress in strand, $f_{PS} = f_{SE} + 10000 + \dfrac{f'c}{300\rho_P}$ (span-to-depth ratio > 35)

$$= 175{,}500 + 10{,}000 + \frac{4{,}000}{300\left(0.000944\right)}$$

$$= 199{,}624 \text{ psi} \le 205{,}500 \text{ psi}$$

Ultimate force in PT, $F_{ult,PT} = A_P\left(f_{PS}\right) = 2\left(0.153\right)\left(199.62\right) = 61.08 \text{ kips}$

Ultimate force in RC, $F_{ult,RC} = A_s\left(f\right)_y = 2.00\text{(assumed)}\left(60.0\right) = 120.0 \text{ kips}$

Total Ultimate force, $F_{ult,Total} = 61.08 + 120.0 = 181.08 \text{ kips}$

Stress block depth, $a = \dfrac{F_{ult,Total}}{0.85 f'cb} = \dfrac{181.08}{0.85\left(4\right)\left(36\right)} = 1.48 \text{ in}$

Ultimate moment due to PT, $M_{ult,PT} = F_{ult,PT}\left(d - \dfrac{a}{2}\right)\phi = 61.08\left(9 - \dfrac{1.48}{2}\right)\left(0.9\right) = 454.1 \text{ k-in}$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 1429.0 - 454.1 = 974.9 \text{ k-in}$

Required area of mild steel reinforcing, $A_S = \dfrac{M_{net}}{\phi f_y\left(d - \dfrac{a}{2}\right)} = \dfrac{974.9}{0.9\left(60\right)\left(9 - \dfrac{1.48}{2}\right)} = 2.18 \text{ in}^2$

Note: The required area of mild steel reinforcing was calculated from an assumed amount of steel. Since the assumed value and the calculated value are not the same a second iteration can be performed. The second iteration changes the depth of the stress block and the calculated area of steel value. Upon completion of the second iteration the area of steel was found to be 2.21in^2

Check of Concrete Stresses at Mid-Span:

Initial Condition (Transfer), load combination (D + L + PT$_i$) = 1.0D + 1.0PT$_I$

The stress in the tendon at transfer = jacking stress − stressing losses = 216.0 − 27.0 = 189.0 ksi

The force in the tendon at transfer, = $189.0(2)(0.153) = 57.83\,\text{kips}$

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0\text{ k-ft} = 576\text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 57.83(4\text{ in}) = 231.3\text{ k-in}$

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-57.83}{10(36)} \pm \dfrac{576.0 - 231.3}{600}$, where S = 600 in^3

$$\boldsymbol{f} = -0.161 \pm 0.5745$$

$$f = -0.735(\text{Comp})\max, 0.414(\text{Tension})\max$$

Normal Condition, load combinations: (D + L + PT$_F$) = 1.0D + 1.0L + 1.0PT$_F$

Tendon stress at normal = jacking − stressing − long-term = 216.0 − 27.0 − 13.5 = 175.5 ksi
The force in tendon at Normal, = $175.5(2)(0.153) = 53.70\text{ kips}$

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0\text{ k-ft} = 576\text{ k-in}$

Moment due to dead load, $M_L = 0.100(3)(32)^2/8 = 38.4\text{ k-ft} = 461\text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 53.70(4\text{ in}) = 214.8\text{ k-in}$

Stress in concrete for (D + L+ PT$_F$), $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_{D+L} - M_{PT}}{S} = \dfrac{-53.70}{10(36)} \pm \dfrac{1037.0 - 214.8}{600}$

$$\boldsymbol{f} = -0.149 \pm 1.727 \pm 0.358$$

$$f = -1.518(\text{Comp})\max, 1.220(\text{Tension})\max$$

Long-Term Condition, load combinations: (D + 0.5L + PT$_{F(L)}$) = 1.0D + 0.5L + 1.0PT$_F$

Tendon stress at normal = jacking − stressing − long-term = 216.0 − 27.0 − 13.5 = 175.5 ksi
The force in tendon at Normal, = $175.5(2)(0.153) = 53.70\text{ kips}$

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0\text{ k-ft} = 576\text{ k-in}$

Moment due to dead load, $M_L = 0.100(3)(32)^2/8 = 38.4\text{ k-ft} = 460\text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 53.70(4\text{ in}) = 214.8\text{ k-in}$

Stress in concrete for (D + 0.5L + $PT_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-53.70}{10(36)} \pm \frac{806.0 - 214.8}{600}$$

$$f = -0.149 \pm 0.985$$

$$f = -1.134(\text{Comp})\,\text{max}, 0.836(\text{Tension})\,\text{max}$$

EXAMPLE ACI 318-08 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

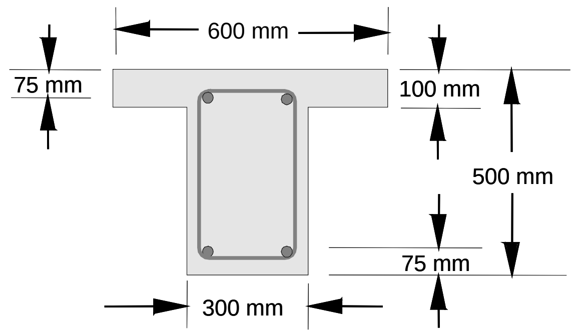
The purpose of this example is to verify the beam flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by ACI 318-08.
- The average shear stress in the beam falls below the maximum shear stress allowed by ACI 318-08, requiring design shear reinforcement.

A simple-span, 20-foot-long, 12-inch-wide, and 18-inch-deep T beam with a flange 4 inches thick and 24 inches wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size is specified as 6 inches. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kip/in).

The beam is loaded with symmetric third-point loading. One dead load (DL02) case and one live load (LL30) case, with only symmetric third-point loads of magnitudes 3, and 30 kips, respectively, are defined in the model. One load combination (COMB30) is defined using the ACI 318-08 load combination factors of 1.2 for dead load and 1.6 for live load. The model is analyzed for both of these load cases and the load combination.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results and found to be identical. After completing the analysis, the design is performed using the ACI 318-08 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcement. Table 2 shows the comparison of the design shear reinforcement.



Beam Section

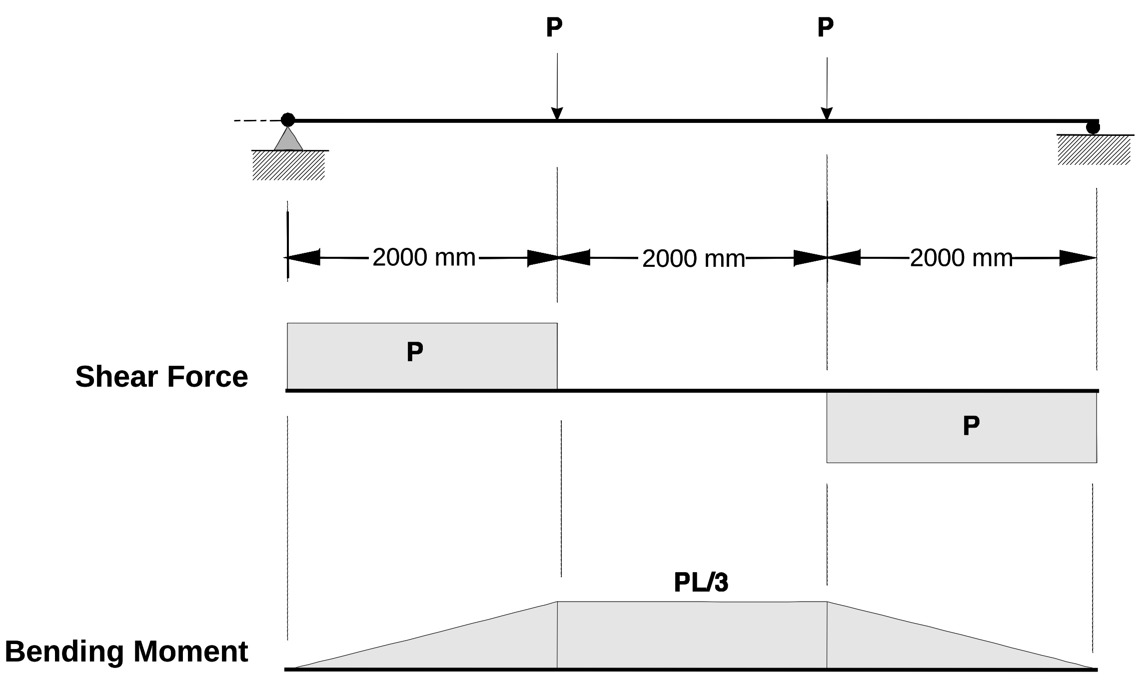


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span	l	=	240	in
Overall depth	h	=	18	in
Flange thickness	d_s	=	4	in
Width of web	b_w	=	12	in
Width of flange,	b_f	=	24	in
Depth of tensile reinf.	d_c	=	3	in
Effective depth	d	=	15	in
Depth of comp. reinf.	d'	=	3	in
Concrete strength	f'_c	=	4,000	psi
Yield strength of steel	f_y	=	60,000	psi
Concrete unit weight	w_c	=	0	pcf
Modulus of elasticity	E_c	=	3,600	ksi
Modulus of elasticity	E_s	=	29,000	ksi
Poisson's ratio	v	=	0.2	
Dead load	P_d	=	2	kips
Live load	P_l	=	30	kips

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the beam with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the comparison of the design reinforcement.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (k-in)	Reinforcement Area (sq-in)
		A_s^+
SAFE	4032	5.808
Calculated	4032	5.808

$A^+_{s,min}$ = 0.4752 sq-in

Table 2 Comparison of Shear Reinforcements

Shear Force (kip)	Reinforcement Area, $\frac{A_v}{s}$ (sq-in/ft)	
	SAFE	Calculated
50.40	0.592	0.592

COMPUTER FILE: ACI 318-08 RC-BM-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\varphi = 0.9$

$A_g = 264$ sq-in

$A_{s,min} = 0.0018 A_g = 0.4752$ sq-in

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right) = 0.85$$

$$c_{max} = \frac{0.003}{0.003 + 0.005} d = 5.625 \text{ in}$$

$$a_{max} = \beta_1 c_{max} = 4.78125 \text{ in}$$

$A_s = \min[A_{s,min}, (4/3)\, A_{s,required}] = \min[0.4752, (4/3)5.804] = 0.4752$ sq-in

COMB30

$P_u = (1.2P_d + 1.6P_t) = 50.4$ k

$$M_u = \frac{P_u l}{3} = 4032 \text{ k-in}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f_c' \varphi\, b_f}} = 4.2671 \text{ in } (a > d_s)$$

Calculation for A_s is performed in two parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w. C_f is given by:

$$C_f = 0.85 f_c' (b_f - b_w)\, d_s = 163.2 \text{ k}$$

The portion of M_u that is resisted by the flange is given by:

$$M_{uf} = C_f \left(d - \frac{d_s}{2}\right)\varphi = 1909.44 \text{ k-in}$$

Therefore, the area of tensile steel reinforcement to balance flange compression is:

$$A_{s1} = \frac{M_{uf}}{f_y\left(d - d_s/2\right)\varphi} = 2.7200 \text{ sq-in}$$

The balance of the moment to be carried by the web is given by:

$$M_{uw} = M_u - M_{uf} = 2122.56 \text{ k-in}$$

The web is a rectangular section with dimensions b_w and d, for which the design depth of the compression block is recalculated as

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85\, f_c'\, \varphi\, b_w}} = 4.5409 \text{ in} \quad (a_1 \le a_{\max})$$

The area of tensile steel reinforcement to balance the web compression is then given by:

$$A_{s2} = \frac{M_{uw}}{\varphi\, f_y\left(d - \frac{a_1}{2}\right)\varphi} = 3.0878 \text{ sq-in}$$

The area of total tensile steel reinforcement is then given by:

$$A_s = A_{s1} + A_{s2} = 5.808 \text{ sq-in}$$

Shear Design

The following quantities are computed for all of the load combinations:

$$\varphi = 0.75$$

Check the limit of $\sqrt{f_c'}$:

$$\sqrt{f_c'} = 63.246 \text{ psi} < 100 \text{ psi}$$

The concrete shear capacity is given by:

$$\varphi V_c = \varphi\, 2\sqrt{f_c'}\, b_w d = 17.076 \text{ k}$$

The maximum shear that can be carried by reinforcement is given by:

$\varphi V_s = \varphi\, 8\sqrt{f'_c}\, b_w d = 68.305$ k

The following limits are required in the determination of the reinforcement:

$(\varphi V_c/2) = 8.538$ k

$(\varphi V_c + \varphi\, 50\, b_w d) = 23.826$ k

$V_{max} = \varphi V_c + \varphi V_s = 85.381$ k

Given V_u, V_c and V_{max}, the required shear reinforcement in area/unit length for any load combination is calculated as follows:

If $V_u \le (V_c/2)\,\varphi$,

$$\frac{A_v}{s} = 0,$$

else if $(V_c/2)\,\varphi < V_u \le (\varphi V_c + \varphi\, 50\, b_w d)$,

$$\frac{A_v}{s} = \frac{50\, b_w}{f_y},$$

else if $(\varphi V_c + \varphi\, 50\, b_w d) < V_u \le \varphi V_{max}$

$$\frac{A_v}{s} = \frac{(V_u - \varphi V_c)}{\varphi\, f_{ys}\, d}$$

else if $V_u > \varphi V_{max}$,

a failure condition is declared.

For each load combination, the P_u and V_u are calculated as follows:

$P_u = 1.2P_d + 1.6P_l$

$V_u = P_u$

(COMB30)

$P_d = 2$ k

$P_l = 30$ k

$P_u = 50.4$ k

$V_u = 50.4$ k, $(\varphi V_c + \varphi\, 50\, b_w d) < V_u \le \varphi V_{max}$

$$\frac{A_v}{s} = \frac{(V_u - \varphi V_c)}{\varphi f_{ys} d} = 0.04937 \text{ sq-in/in or } 0.592 \text{ sq-in/ft}$$

EXAMPLE ACI 318-08 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE.

The numerical example is a flat slab that has three 24-foot-long spans in each direction, as shown in Figure 1.

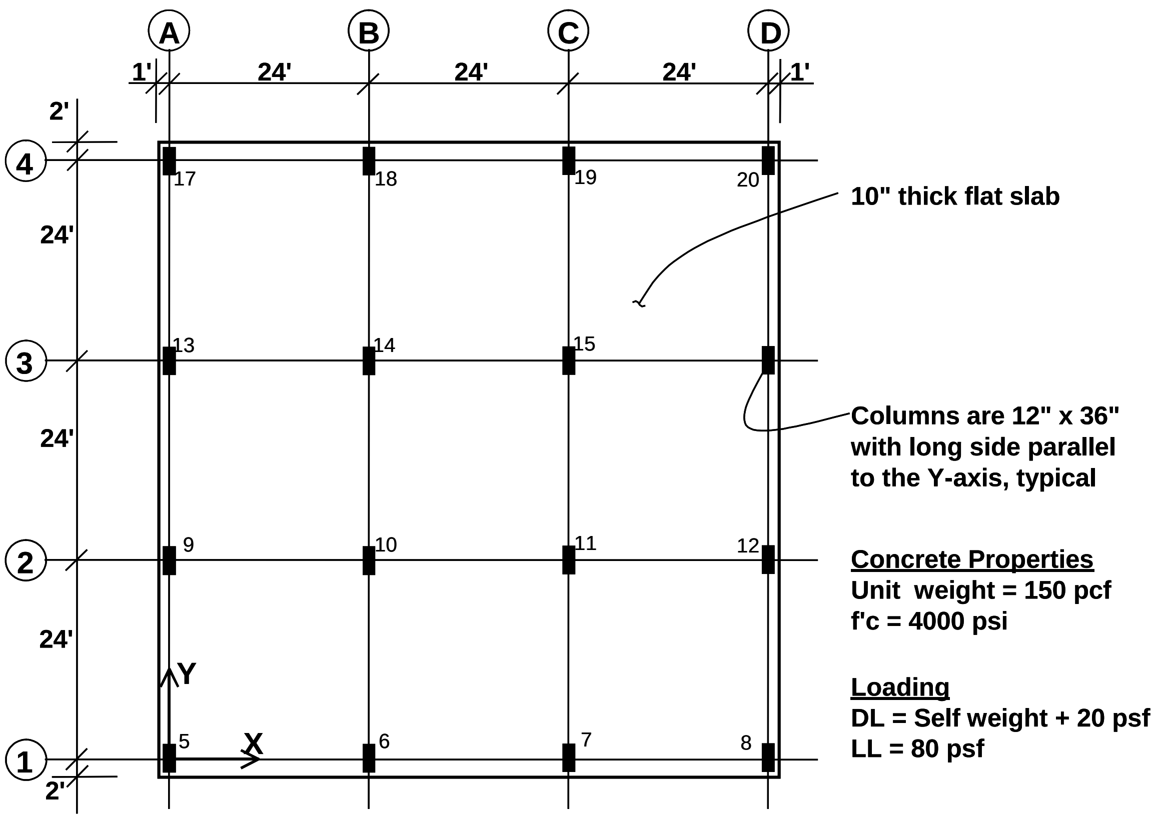


Figure 1: Flat Slab For Numerical Example

The slab overhangs the face of the column by 6 inches along each side of the structure. The columns are typically 12 inches wide by 36 inches long, with the long side parallel to the Y-axis. The slab is typically 10 inches thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 150 pcf and an f'c of 4000 psi. The dead load consists of the self weight of the structure plus an additional 20 psf. The live load is 80 psf.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress, and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE punching shear capacity, shear stress ratio, and D/C ratio with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this example.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (ksi)	Shear Capacity (ksi)	D/C ratio
SAFE	0.192	0.158	1.21
Calculated	0.193	0.158	1.22

COMPUTER FILE: ACI 318-08 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation for Interior Column Using SAFE Method

d = [(10 - 1) + (10 - 2)] / 2 = 8.5"

Refer to Figure 2.

b_0 = 44.5 + 20.5 + 44.5 + 20.5 = 130"

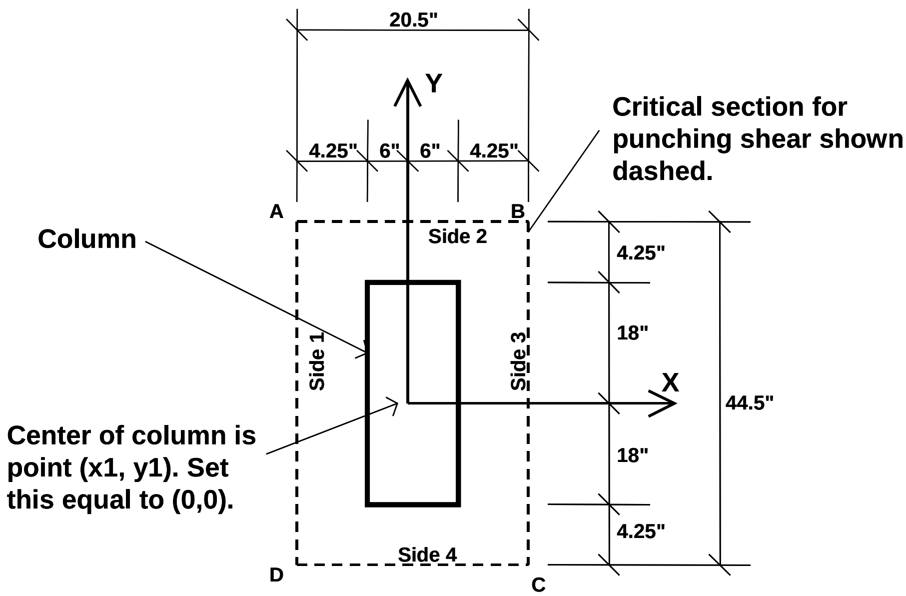


Figure 2: Interior Column, Grid B-2 in SAFE Model

$$\gamma_{V2} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{44.5}{20.5}}} = 0.4955$$

$$\gamma_{V3} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{20.5}{44.5}}} = 0.3115$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear, as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–10.25	0	10.25	0	N.A.
y_2	0	22.25	0	–22.25	N.A.
L	44.5	20.5	44.5	20.5	b_0 = 130
d	8.5	8.5	8.5	8.5	N.A.
Ld	378.25	174.25	378.25	174.25	1105
Ldx_2	–3877.06	0	3877.06	0	0
Ldy_2	0	3877.06	0	–3877.06	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{1105} = 0"$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{1105} = 0"$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	44.5	20.5	44.5	20.5	N.A.
d	8.5	8.5	8.5	8.5	N.A.
x_2 - x_3	–10.25	0	10.25	0	N.A.
y_2 - y_3	0	22.25	0	–22.25	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	64696.5	86264.6	64696.5	86264.6	301922.3
I_{YY}	39739.9	7151.5	39739.9	7151.5	93782.8
I_{XY}	0	0	0	0	0

From the SAFE output at Grid B-2:

V_U = 189.45 k

$\gamma_{V2} M_{U2}$ = –156.39 k-in

$\gamma_{V3} M_{U3}$ = 91.538 k-in

At the point labeled A in Figure 2, x_4 = –10.25 and y_4 = 22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(22.25-0\right)-\left(0\right)\left(-10.25-0\right)\right]}{(301922.3)(93782.8)-(0)^2} - \frac{91.538\left[301922.3\left(-10.25-0\right)-\left(0\right)\left(22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 – 0.0115 – 0.0100 = **0.1499 ksi** at point A

At the point labeled B in Figure 2, x_4 = 10.25 and y_4 = 22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(22.25-0\right)-\left(0\right)\left(10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(10.25-0\right)-\left(0\right)\left(22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 – 0.0115 + 0.0100 = **0.1699 ksi** at point B

At the point labeled C in Figure 2, x_4 = 10.25 and y_4 = –22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(-22.25-0\right)-\left(0\right)\left(10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(10.25-0\right)-\left(0\right)\left(-22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 + 0.0115 + 0.0100 = **0.1930 ksi** at point C

At the point labeled D in Figure 2, x_4 = –10.25 and y_4 = –22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(-22.25-0\right)-\left(0\right)\left(-10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(-10.25-0\right)-\left(0\right)\left(-22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 + 0.0115 - 0.0100 = **0.1729 ksi** at point D

Point C has the largest absolute value of v_u, thus v_{max} = 0.1930 ksi

The shear capacity is calculated based on the smallest of ACI 318-08 equations 11-34, 11-35 and 11-36 with the b_0 and d terms removed to convert force to stress.

$$\varphi v_C = \frac{0.75\left(2+\frac{4}{36/12}\right)\sqrt{4000}}{1000} = 0.158$$ ksi in accordance with equation 11-34

$$\varphi v_C = \frac{0.75\left(\frac{40\bullet 8.5}{130}+2\right)\sqrt{4000}}{1000} = 0.219$$ ksi in accordance with equation 11-35

$$\varphi v_C = \frac{0.75\bullet 4\bullet \sqrt{4000}}{1000} = 0.190$$ ksi in accordance with equation 11-36

Equation 11-34 yields the smallest value of $\phi v_C = 0.158$ ksi and thus this is the shear capacity.

$$Shear\ Ratio = \frac{v_U}{\varphi v_C} = \frac{0.193}{0.158} = 1.22$$

EXAMPLE ACI 318-08 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 6 inches thick and spans 12 feet between walls. The slab is modeled using thin plate elements. The walls are modeled as line supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified to be 36 inches. To obtain factored moments and flexural reinforcement in a design strip, one one-foot-wide strip is defined in the X-direction on the slab, as shown in Figure 1.

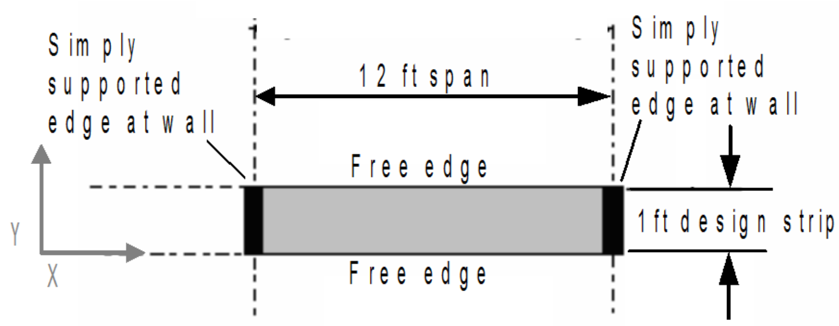


Figure 1 Plan View of One-Way Slab

One dead load case (DL80) and one live load case (LL100) with uniformly distributed surface loads of magnitudes 80 and 100 psf, respectively, are defined in the model. A load combination (COMB100) is defined using the ACI 318-08 load combination factors, 1.2 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing the analysis, design is performed in accordance with ACI 318-08 using SAFE and also by hand computation. Table 1 shows the comparison of the moments and design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness T, h = 6 in

Depth of tensile reinf.	d_c	=	1	in
Effective depth	d	=	5	in
Clear span	l_n, l_1	=	144	in
Concrete strength	f'_c	=	4,000	psi
Yield strength of steel	f_y	=	60,000	psi
Concrete unit weight	w_c	=	0	pcf
Modulus of elasticity	E_c	=	3,600	ksi
Modulus of elasticity	E_s	=	29,000	ksi
Poisson's ratio	ν	=	0	
Dead load	w_d	=	80	psf
Live load	w_l	=	100	psf

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (k-in)	Reinforcement Area (sq-in)
			A_s^+
Medium	SAFE	55.22	0.213
	Calculated	55.22	0.213

$A^+_{s,min}$ = 0.1296 sq-in

COMPUTER FILE: ACI 318-08 RC-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\varphi = 0.9$

$b = 12$ in

$A_{s,min} = 0.0018bh = 0.1296$ sq-in

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right) = 0.85$$

$$c_{\max} = \frac{0.003}{0.003 + 0.005} d = 1.875 \text{ in}$$

$a_{max} = \beta_1 c_{max} = 1.59375$ in

For the load combination, w and M_u are calculated as follows:

$w = (1.2w_d + 1.6w_t)\ b\ /\ 144$

$$M_u = \frac{wl_1^2}{8}$$

$A_s = \min[A_{s,\min}, (4/3)\ A_{s,\text{required}}] = \min[0.1296, (4/3)2.11] = 0.1296$ sq-in

COMB100

w_d = 80 psf

w_t = 100 psf

w = 21.33 lb/in

$M_{u\text{-}strip}$ = 55.22 k-in

$M_{u\text{-}design}$ = 55.629 k-in

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f_c' \varphi\ b}} = 0.3128 \text{ in} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = 0.213 \text{ sq-in} > A_{s,\min}$$

A_s = 0.2114 sq-in

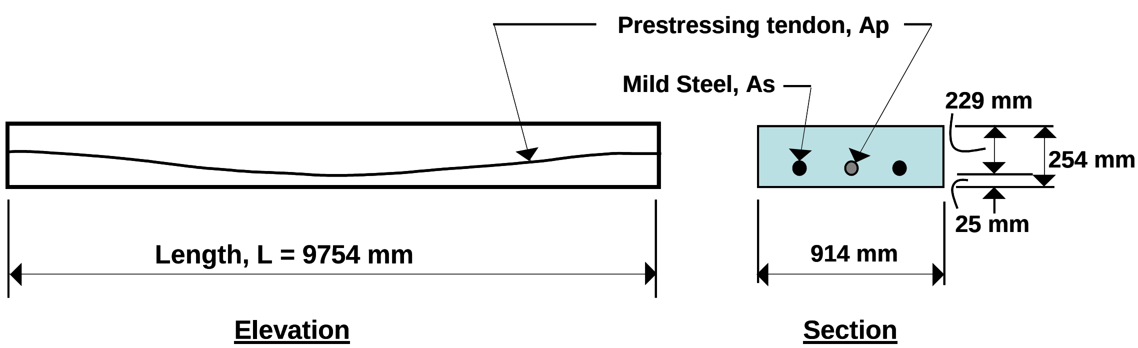
EXAMPLE AS 3600-09 PT-SL-001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel reinforcing strength for a post-tensioned slab.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.



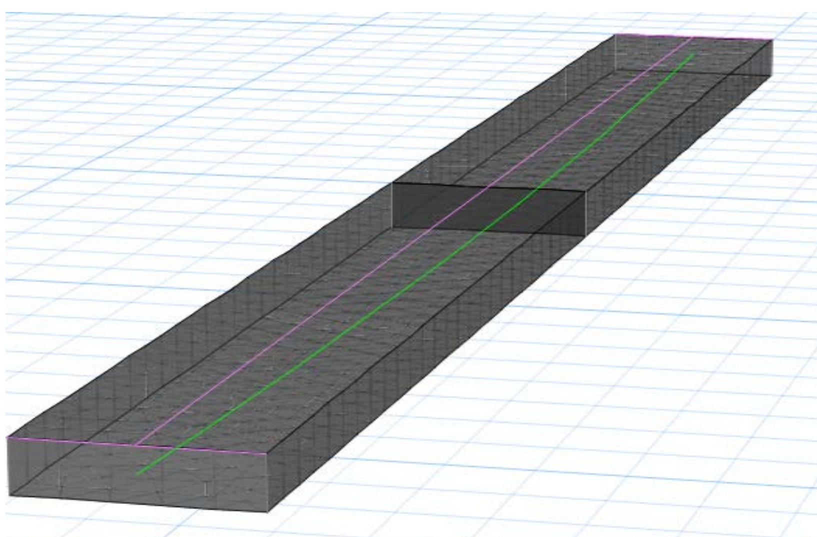


Figure 1 One-Way Slab

A 914-mm-wide design strip is centered along the length of the slab and is defined as an A-Strip. B-Strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the SAFE results and summarized for verification and validation of the SAFE results.

GEOMETRY, PROPERTIES AND LOADING

Thickness,	T, h	=	254	mm
Effective depth,	d	=	229	mm
Clear span,	L	=	9754	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of prestress (single tendon),	A_p	=	198	mm^2
Concrete unit weight,	w_c	=	23.56	KN/m^3
Concrete modulus of elasticity,	E_c	=	25000	N/mm^3
Rebar modulus of elasticity,	E_s	=	200,000	N/mm^3
Poisson's ratio,	ν	=	0	
Dead load,	w_d	=	self	KN/m^2
Live load,	w_l	=	4.788	KN/m^2

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments, required mild steel reinforcing and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	156.12	156.14	0.01%
Area of Mild Steel req'd, As (sq-cm)	16.55	16.59	0.24%
Transfer Conc. Stress, top (0.8D+1.15PT_I), MPa	–3.500	–3.498	0.06%
Transfer Conc. Stress, bot (0.8D+1.15PT_I), MPa	0.950	0.948	0.21%
Normal Conc. Stress, top (D+L+PT_F), MPa	–10.460	–10.465	0.10%
Normal Conc. Stress, bot (D+L+PT_F), MPa	8.402	8.407	0.05%
Long-Term Conc. Stress, top (D+0.5L+$PT_{F(L)}$), MPa	–7.817	–7.817	0.00%
Long-Term Conc. Stress, bot (D+0.5L+$PT_{F(L)}$), MPa	5.759	5.759	0.00%

COMPUTER FILE: AS 3600-09 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

Design Parameters:

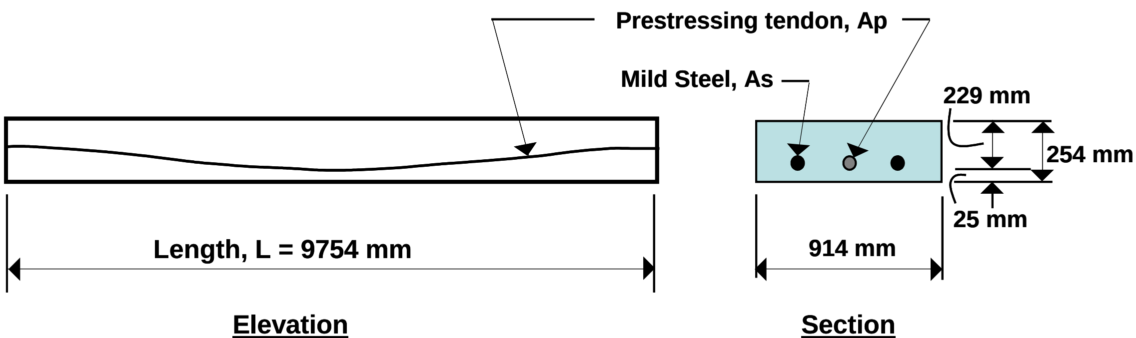
Mild Steel Reinforcing	Post-Tensioning
f'_c = 30MPai	f_{pu} = 1862 MPa
f_y = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\phi = 0.80$

$\alpha_2 = 1.0 - 0.003 f'_c$ = 0.91 > 0.85, Use $\alpha_2 = 0.85$

$\gamma = 1.0 - 0.003 f'_c$ = 0.91 > 0.85, Use $\gamma = 0.85$

$a_{max} = \gamma k_u d$ = 0.85*0.36*229 = 70.07 mm



Loads:

Dead, self-wt = 0.254 m x 23.56 kN/m^3 = 5.984 kN/m^2 (D) x 1.2 = 7.181 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) x 1.5 = 7.182 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 14.363 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 x 0.914m = 9.846 kN/m, ω_u = 14.363 kN/m^2 x 0.914m = 13.128 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8}$ = 13.128 x (9.754)^2/8 = 156.12 kN-m

Ultimate Stress in strand, $f_{PS} = f_{SE} + 70 + \dfrac{f'_C b_{ef} d_P}{300 A_P}$

$$= 1210 + 70 + \frac{30(914)(229)}{300(198)}$$

$$= 1386 \text{ MPa} \leq f_{SE} + 200 = 1410 \text{ MPa}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197.4(1386)/1000 = 273.60$ kN

Total Ultimate force, $F_{ult,Total} = 273.60 + 560.0 = 833.60$ kN

Stress block depth, $a = d - \sqrt{d^2 - \dfrac{2M^*}{0.85 f'_c \phi b}}$

$$= 0.229 - \sqrt{0.229^2 - \frac{2(159.12)}{0.85(30000)(0.80)(0.914)}} = 40.90$$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right)\phi = 273.60\left(229 - \frac{40.90}{2}\right)(0.80)\Big/1000 = 45.65 \text{ kN-m}$$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 156.1 - 45.65 = 110.45$ kN-m

Required area of mild steel reinforcing,

$$A_S = \frac{M_{net}}{\phi f_y\left(d - \frac{a}{2}\right)} = \frac{110.45}{0.80(400000)\left(0.229 - \frac{0.04090}{2}\right)}(1e6) = 1655 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (0.8D+1.15PT$_i$) = 0.80D+0.0L+1.15PT$_I$

Tendon stress at transfer = jacking stress – stressing losses =1490 – 186 = 1304 MPa

The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{(1.15)(-257.4)}{0.254(0.914)} \pm \dfrac{(0.80)65.04 - (1.15)26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.275 \pm 2.225 \text{ MPa}$$

$$f = -3.500(\text{Comp})\max, 0.950(\text{Tension})\max$$

Normal Condition, load combinations: $(D+L+PT_F) = 1.0D+1.0L+1.0PT_F$

Tendon stress at Normal = jacking – stressing – long-term = 1490 – 186 – 94= 1210 MPa

The force in tendon at Normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for $(D+L+PT_F)$,

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08-24.37}{0.00983}$$

$$f = -1.029 \pm 9.431$$

$$f = -10.460(\text{Comp})\max, 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: $(D+0.5L+PT_{F(L)}) = 1.0D+0.5L+1.0PT_F$

Tendon stress at Normal = jacking – stressing – long-term =1490 – 186 – 94 = 1210 MPa

The force in tendon at Normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to dead load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for $(D+0.5L+PT_{F(L)})$,

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06-24.33}{0.00983}$$

$$f = -1.029 \pm 6.788$$

$$f = -7.817(\text{Comp})\max, 5.759(\text{Tension})\max$$

EXAMPLE AS 3600-09 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

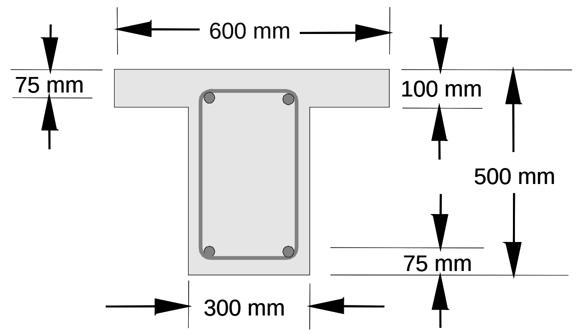
- The stress-block extends below the flange but remains within the balanced condition permitted by AS 3600-09.

- The average shear stress in the beam is below the maximum shear stress allowed by AS 3600-09, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T-beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading. One dead load case (DL30) and one live load case (LL130), with only symmetric third-point loads of magnitudes 30, and 130 kN, respectively, are defined in the model. One load combinations (COMB130) is defined using the AS 3600-09 load combination factors of 1.2 for dead load and 1.5 for live load. The model is analyzed for both of these load cases and the load combination.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results and found to be identical. After completing the analysis, the design is performed using the AS 3600-09 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.



Beam Section

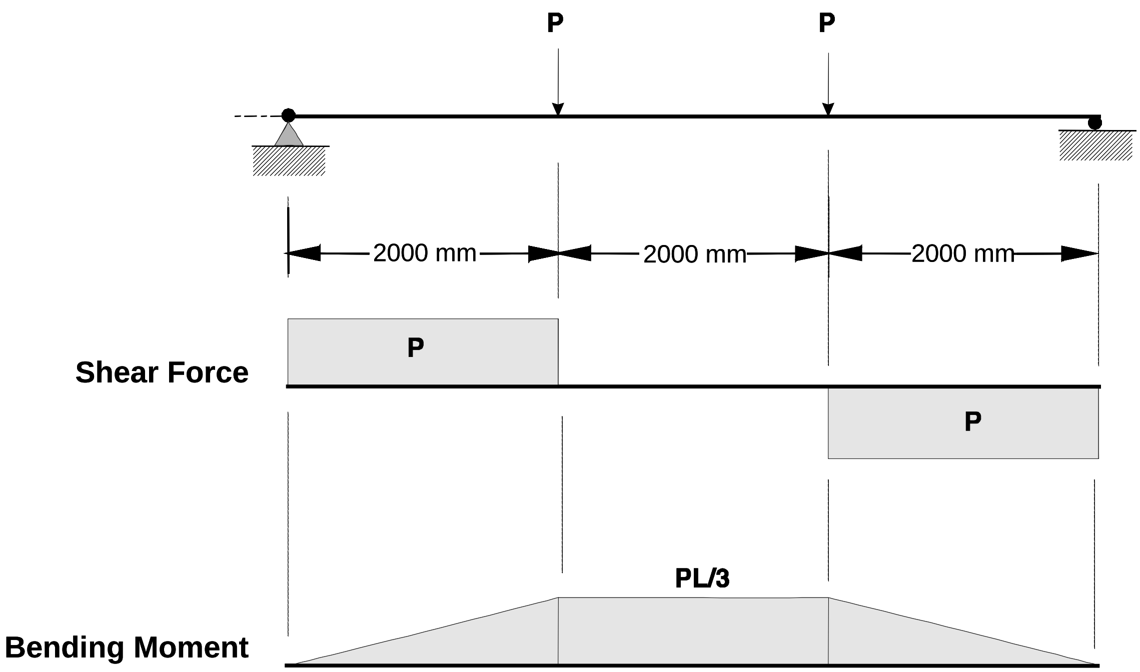


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span,	l	=	6000	mm
Overall depth,	h	=	500	mm
Flange thickness,	d_s	=	100	mm
Width of web,	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.,	d_c	=	75	mm
Effective depth,	d	=	425	mm
Depth of comp. reinf.,	d'	=	75	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	$25x10^5$	MPa
Modulus of elasticity,	E_s	=	$2x10^8$	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	30	kN
Live load,	P_l	=	130	kN

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the design reinforcement comparison.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-cm)
		A_s^+
SAFE	462	33.512
Calculated	462	33.512

$A^+_{s,min}$ = 3.00 sq-cm

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-cm/m)	
	SAFE	Calculated
231	12.05	12.05

COMPUTER FILE: AS 3600-09 RC-BM-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\varphi = 0.8$

$\alpha_2 = 1.0 - 0.003 f'_c$ = 0.91 > 0.85, Use $\alpha_2 = 0.85$

$\gamma = 1.05 - 0.007 f'_c$ = 0.84 < 0.85, Use $\gamma = 0.84$

$a_{\max} = \gamma k_u d$ = 0.84 • 0.36 • 425 = 128.52 mm

$$A_{st.\min} = \alpha_b \left(\frac{D}{d}\right)^2 \frac{f'_{ct,f}}{f_{sy}} b_w d, \text{ where}$$

for L- and T-Sections with the web in tension:

$$\alpha_b = 0.20 + \left(\frac{b_f}{b_w} - 1\right)\left(0.4\frac{D_s}{D} - 0.18\right) \geq 0.20\left(\frac{b_f}{b_w}\right)^{1/4} = 0.2378$$

$$A_{st.\min} = 0.2378\left(\frac{D}{d}\right)^2 \frac{f'_{ct,f}}{f_{sy}} bd$$

= 0.2378 • (500/425)^2 • 0.6 • SQRT(30)/460 • 300*425

= 299.8 sq-mm

COMB130

$N^* = (1.2N_d + 1.5N_t)$ = 231kN

$$M^* = \frac{N^* l}{3} = 462 \text{ kN-m}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M^*}{0.85 f'_c \phi b_{ef}}} = 100.755 \text{ mm } (a > D_s)$$

The compressive force developed in the concrete alone is given by:

The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, 2. C_f is given by:

$$C_f = 0.85 f'_c \left(b_{ef} - b_w\right) \times \min\left(D_s, a_{\max}\right) = 765 \text{ kN}$$

Therefore, $A_{s1} = \dfrac{C_f}{f_{sy}}$ and the portion of M^* that is resisted by the flange is given by:

$$M_{uf} = \phi C_f \left(d - \frac{\min\left(D_s, a_{\max}\right)}{2}\right) = 229.5 \text{ kN-m}$$

$$A_{s1} = \frac{C_f}{f_{sy}} = 1663.043 \text{ sq-mm}$$

Again, the value for ϕ is 0.80 by default. Therefore, the balance of the moment, M^* to be carried by the web is:

$$M_{uw} = M^* - M_{uf} = 462 - 229.5 = 232.5$$

The web is a rectangular section of dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f'_c \, \phi \, b_w}} = 101.5118 \text{ mm}$$

If $a_1 \le a_{\max}$, the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{uw}}{\phi \, f_{sy} \left(d - \dfrac{a_1}{2}\right)} = 1688.186 \text{ sq-mm}$$

$$A_{st} = A_{s1} + A_{s2} = 3351.23 \text{ sq-mm} = 33.512 \text{ sq-cm}$$

Shear Design

The shear force carried by the concrete, V_{uc}, is calculated as:

$$V_{uc} = \beta_1 \beta_2 \beta_3 b_w d_o f'_{cv} \left[\frac{A_{st}}{b_w d_o}\right]^{1/3} = 0 \text{ kN}$$

where,

$$f'_{cv} = \left(f'_c\right)^{1/3} = 3.107 \text{ N/mm}^2 \le 4\text{MPa}$$

$\beta_1 = 1.1\left(1.6 - \frac{d_o}{1000}\right) \geq 1.1$ =1.2925, β_2 = 1 and β_3 = 1

The shear force is limited to a maximum of:

$$V_{u.\max} = 0.2 f'_c b d_o = 765 \text{ kN}$$

Given V^*, V_{uc}, and $V_{u.\max}$, the required shear reinforcement is calculated as follows, where, ϕ, the strength reduction factor, is 0.7.

If $V^* \leq \phi V_{uc} / 2$,

$\frac{A_{sv}}{s} = 0$, if $D \leq 750$ mm, otherwise $A_{sv.\min}$ shall be provided.

If $\phi V_{u.\min} < V^* \leq \phi V_{u.\max}$,

$$\frac{A_{sv}}{s} = \frac{\left(V^* - \phi V_{uc}\right)}{\phi f_{sy.f} d_o \cot \theta_v},$$

and greater than $A_{sv.\min}$, defined as:

$$\frac{A_{sv.\min}}{s} = \left(0.35 \frac{b_w}{f_{sy.f}}\right) = 0.22826 \text{ sq-mm/mm} = 228.26 \text{ sq-mm/m}$$

θ_v = the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when $V^*=\phi V_{u.\min}$ to 45 degrees when $V^*=\phi V_{u,\max}$ = 35.52 degrees

If $V^* > \phi V_{\max}$, a failure condition is declared.

For load combination, the N^* and V^* are calculated as follows:

$N^* = 1.2N_d + 1.5N_l$

$V^* = N^*$

(COMB130)

N_d = 30 kips

N_l = 130 kips

N^* = 231 kN

$N^* = 231$ kN, $(\phi V_{u.\min} < V^* \le \phi V_{u.\max},)$

$$\frac{A_{sv}}{s} = \frac{\left(V^* - \phi V_{uc}\right)}{\phi f_{sy.f} d_o \cot\theta_v}, \quad = 1.205 \text{ sq-mm/mm or } 12.05 \text{ sq-cm/m}$$

EXAMPLE AS 3600-09 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

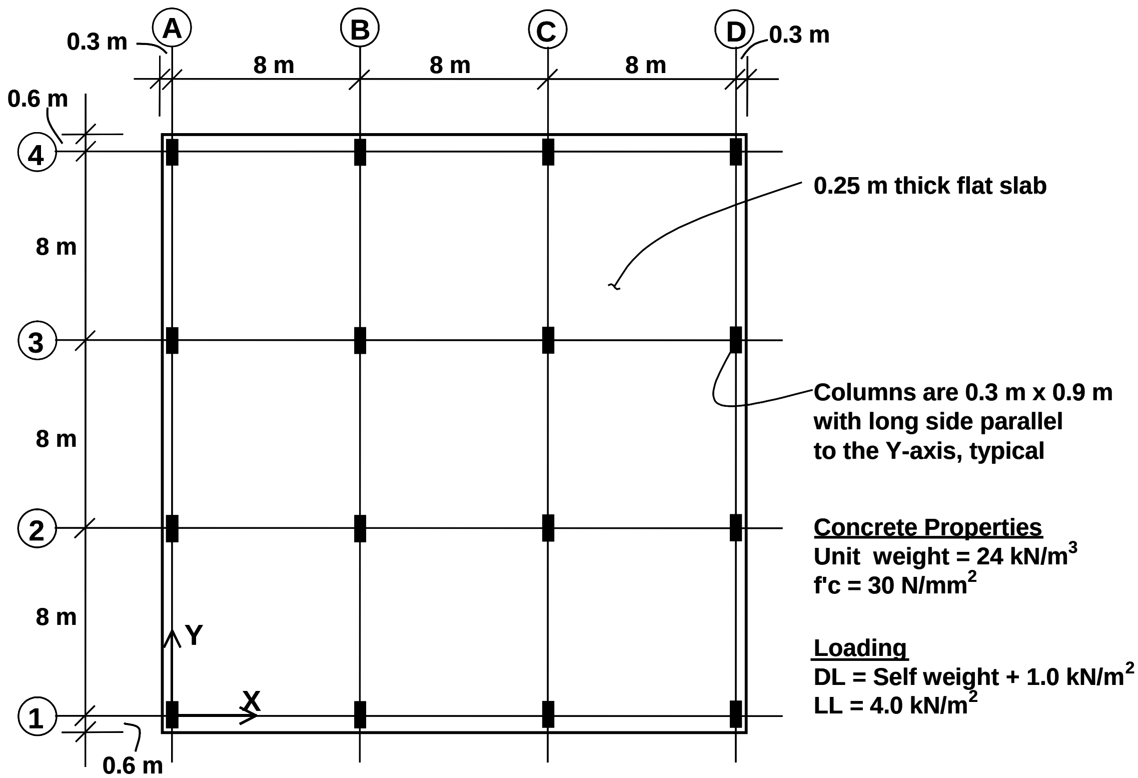


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m, with the long side parallel to the Y-axis. The slab is typically 0.25-m thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f'_c of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress, and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio, and D/C ratio obtained from SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid Point B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
SAFE	1.811	1.086	1.67
Calculated	1.811	1.086	1.67

COMPUTER FILE: AS 3600-09 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using SAFE Method

d_{om} = [(250 – 26) + (250 – 38)] / 2 = 218 mm

Refer to Figure 2.

U = 518+ 1118 + 1118 + 518 = 3272 mm

a_x = 518 mm

a_y = 1118 mm

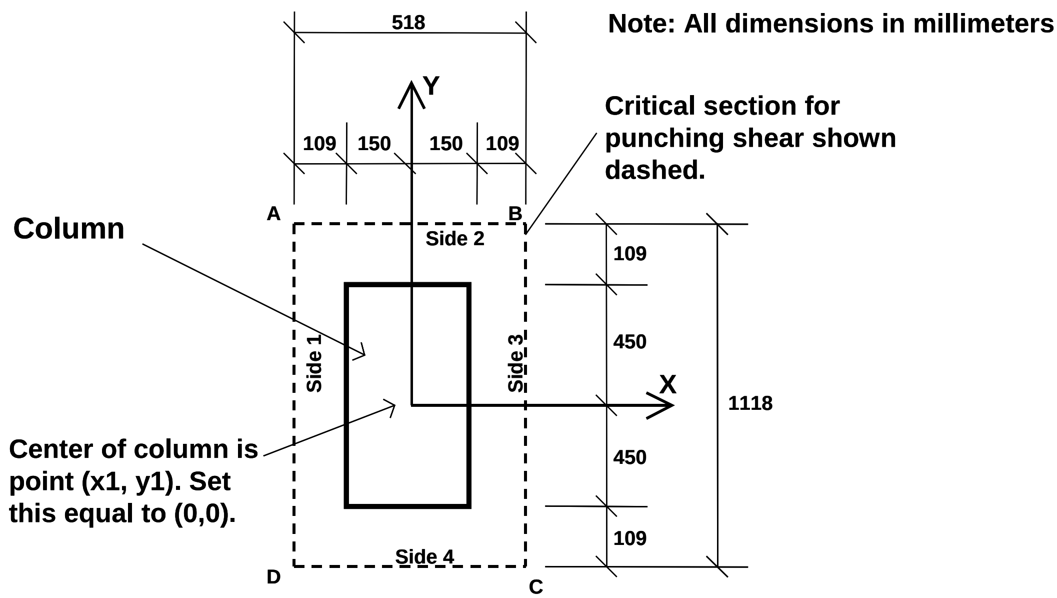


Figure 2: Interior Column, Grid B-2 in SAFE Model

From the SAFE output at grid line B-2:

$V^* = 1126.498$ kN

$M_{v2} = -51.991$ kN-m

$M_{v3} = 45.723$ kN-m

The maximum design shear stress is computed along the major and minor axis of column separately:

$$v_{\max} = \frac{V^*}{ud_{om}}\left[1.0 + \frac{uM_v}{8V^*ad_{om}}\right]$$

$$v_{\max,X} = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} \bullet \left(1 + \frac{3272 \bullet 51.991 \bullet 10^6}{8 \bullet 1126.498 \bullet 10^3 \bullet 1118 \bullet 218}\right)$$

$v_{\max,X}$ = 1.579 $\bullet$ 1.0774 = **1.7013 N/mm²**

$$v_{\max,Y} = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} \bullet \left(1 + \frac{3272 \bullet 45.723 \bullet 10^6}{8 \bullet 1126.498 \bullet 10^3 \bullet 518 \bullet 218}\right)$$

$v_{\max,Y}$ = 1.579 $\bullet$ 1.1470 = **1.811 N/mm²** (Govern)

The largest absolute value of $v_{\max}$= **1.811 N/mm²**

The shear capacity is calculated based on the smallest of AS 3600-09 equation 11-35, with the d_{om} and u terms removed to convert force to stress.

$$\varphi f_{cv} = \min\begin{cases} 0.17\left(1 + \frac{2}{\beta_h}\right)\varphi\sqrt{f'_c} \\ 0.34\varphi\sqrt{f'_c} \end{cases}$$ = **1.803N/mm²** in accordance with AS 9.2.3(a)

AS 9.2.3(a) yields the smallest value of φf_{cv} = 1.086 N/mm², and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{\varphi f_{cv}} = \frac{1.811}{1.086} = 1.67$$

EXAMPLE AS 3600-2009 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

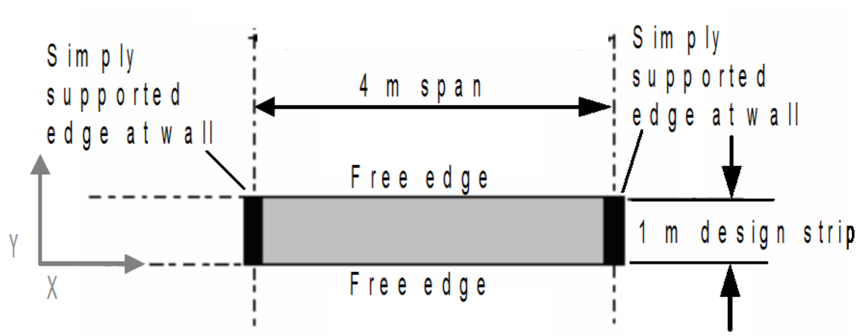


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa), with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the AS 3600-2009 load combination factors, 1.2 for dead loads and 1.5 for live loads. The model is analyzed for both load cases and the load combinations.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing analysis, design is performed using the AS 3600-2009 code using SAFE and also by hand computation. Table 1 shows the comparison of the design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	$2x10^6$	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	SAFE	24.597	5.58
	Calculated	24.600	5.58

$A^+_{s,min}$ = 370.356 sq-mm

COMPUTER FILE: AS 3600-2009 RC-SL-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\varphi = 0.8$

$b = 1000$ mm

$\alpha_2 = 1.0 - 0.003 f'_c = 0.91 > 0.85$, Use $\alpha_2 = 0.85$

$\gamma = 1.05 - 0.007 f'_c = 0.84 < 0.85$, Use $\gamma = 0.84$

$a_{\max} = \gamma k_u d = 0.84 \cdot 0.36 \cdot 125 = 37.80$ mm

For the load combination, w and M^* are calculated as follows:

$w = (1.2w_d + 1.5w_t)\, b$

$$M_u = \frac{wl_1^2}{8}$$

$$A_s = 0.24\left(\frac{h}{d}\right)^2 \frac{f'_{ct,f}}{f_{sy,f}} bh \text{ for flat slabs}$$

$$A_{st.\min} = 0.24\left(\frac{h}{d}\right)^2 \frac{f'_{ct,f}}{f_{sy,f}} bd$$

$= 0.24 \cdot (150/125)^2 \cdot 0.6 \cdot \text{SQRT}(30)/460 \cdot 1000 \cdot 150$

$= 370.356$ sq-mm

COMB100

$w_d = 4.0$ kPa

$w_t = 5.0$ kPa

$w = 12.3$ kN/m

$M_{\text{-strip}}^* = 24.6$ kN-m

$M_{\text{-design}}^* = 24.633$ kN-m

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M^*}{0.85 f'_c \phi b}} = 10.065 \text{ mm} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_{st} = \frac{M^*}{\phi f_{sy}\left(d - \frac{a}{2}\right)} = 557.966 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.57966 \text{ sq-cm}$$

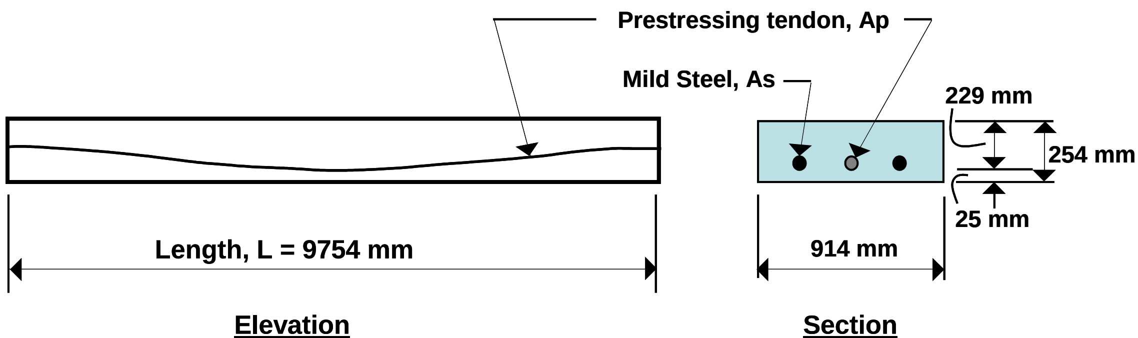
EXAMPLE AS 3600-01 PT-SL-001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel reinforcing strength for a post-tensioned slab.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm as shown in shown in Figure 1.



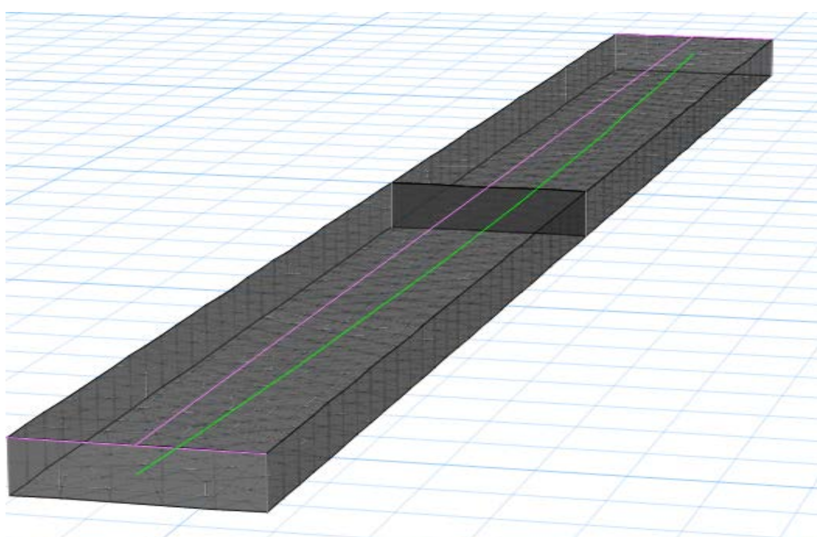


Figure 1 One-Way Slab

A 914-mm-wide design strip is centered along the length of the slab and is defined as an A-Strip. B-Strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self-weight and live loads were added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the midspan of the slab. Independent hand calculations were compared with the SAFE results and summarized for verification and validation of the SAFE results.

GEOMETRY, PROPERTIES AND LOADING

Thickness,	T, h	=	254	mm
Effective depth,	d	=	229	mm
Clear span,	L	=	9754	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of prestress (single tendon),	A_p	=	198	mm^2
Concrete unit weight,	w_c	=	23.56	KN/m^3
Concrete modulus of elasticity,	E_c	=	25000	N/mm^3
Rebar modulus of elasticity,	E_s	=	200,000	N/mm^3
Poisson's ratio,	ν	=	0	
Dead load,	w_d	=	self	KN/m^2
Live load,	w_l	=	4.788	KN/m^2

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments, required mild steel reinforcing and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	156.12	156.14	0.01%
Area of Mild Steel req'd, As (sq-cm)	16.55	16.59	0.24%
Transfer Conc. Stress, top ($0.8D+1.15PT_I$), MPa	–3.500	–3.498	0.06%
Transfer Conc. Stress, bot ($0.8D+1.15PT_I$), MPa	0.950	0.948	0.21%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	–10.460	–10.465	0.10%
Normal Conc. Stress, bot ($D+L+PT_F$), MPa	8.402	8.407	0.05%
Long-Term Conc. Stress, top ($D+0.5L+PT_{F(L)}$), MPa	–7.817	–7.817	0.00%
Long-Term Conc. Stress, bot ($D+0.5L+PT_{F(L)}$), MPa	5.759	5.759	0.00%

COMPUTER FILE: AS 3600-01 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

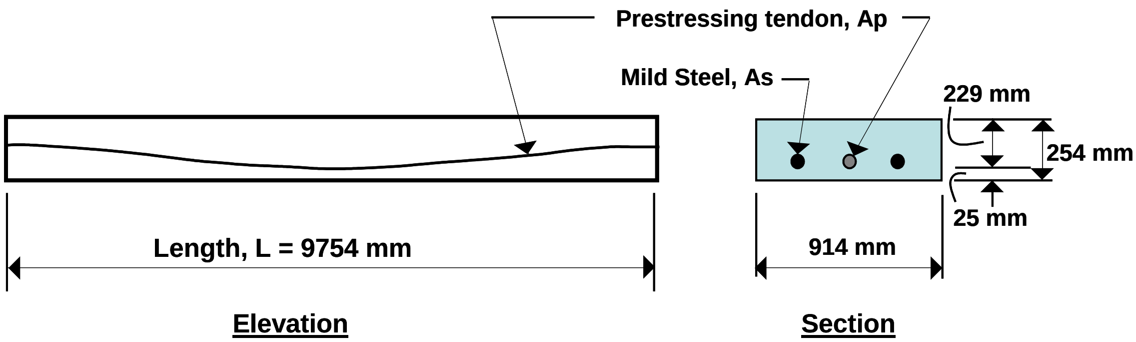
Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
f'_c = 30MPai	f_{pu} = 1862 MPa
f_y = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\phi = 0.80$

$\gamma = \left[0.85 - 0.007\left(f'_c - 28\right)\right] = 0.836$

$a_{max} = \gamma k_u d$ = 0.836*0.4*229 = 76.5 mm



Loads:

Dead, self-wt = 0.254 m x 23.56 kN/m^3 = 5.984 kN/m^2 (D) x 1.2 = 7.181 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) x 1.5 = 7.182 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 14.363 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 x 0.914m = 9.846 kN/m, ω_u = 14.363 kN/m^2 x 0.914m = 13.128 kN/m

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8}$ = 13.128 x (9.754)^2/8 = 156.12 kN-m

Ultimate Stress in strand, $f_{PS} = f_{SE} + 70 + \dfrac{f'_C b_{ef} d_P}{300 A_P}$

$$= 1210 + 70 + \frac{30(914)(229)}{300(198)}$$

$$= 1386 \text{ MPa} \le f_{SE} + 200 = 1410 \text{ MPa}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197.4(1386)/1000 = 273.60$ kN

Total Ultimate force, $F_{ult,Total} = 273.60 + 560.0 = 833.60$ kN

Stress block depth, $a = d - \sqrt{d^2 - \dfrac{2M^*}{0.85 f'_c \phi b}}$

$$= 0.229 - \sqrt{0.229^2 - \frac{2(159.12)}{0.85(30000)(0.80)(0.914)}} = 40.90$$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right)\phi = 273.60\left(229 - \frac{40.90}{2}\right)(0.80)\Big/1000 = 45.65 \text{ kN-m}$$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 156.1 - 45.65 = 110.45$ kN-m

Required area of mild steel reinforcing,

$$A_S = \frac{M_{net}}{\phi f_y\left(d - \frac{a}{2}\right)} = \frac{110.45}{0.80(400000)\left(0.229 - \frac{0.04090}{2}\right)}(1e6) = 1655 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (0.8D+1.15PT$_i$) = 0.80D+0.0L+1.15PT$_I$

Tendon stress at transfer = jacking stress − stressing losses =1490 − 186 = 1304 MPa

The force in the tendon at transfer, $= 1304(197.4)/1000 = 257.4$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{(1.15)(-257.4)}{0.254(0.914)} \pm \dfrac{(0.80)65.04 - (1.15)26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.275 \pm 2.225 \text{ MPa}$$

$$f = -3.500(\text{Comp})\max, 0.950(\text{Tension})\max$$

Normal Condition, load combinations: ($D+L+PT_F$) = $1.0D+1.0L+1.0PT_F$

Tendon stress at Normal = jacking – stressing – long-term = 1490 – 186 – 94= 1210 MPa
The force in tendon at Normal, = $1210(197.4)/1000 = 238.9$ kN
Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m
Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m
Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for ($D+L+PT_F$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$
$$f = -1.029 \pm 9.431$$
$$f = -10.460(\text{Comp})\max, 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: ($D+0.5L+PT_{F(L)}$) = $1.0D+0.5L+1.0PT_F$

Tendon stress at Normal = jacking – stressing – long-term =1490 – 186 – 94 = 1210 MPa
The force in tendon at Normal, = $1210(197.4)/1000 = 238.9$ kN
Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m
Moment due to dead load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m
Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for ($D+0.5L+PT_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$
$$f = -1.029 \pm 6.788$$
$$f = -7.817(\text{Comp})\max, 5.759(\text{Tension})\max$$

EXAMPLE AS 3600-01 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

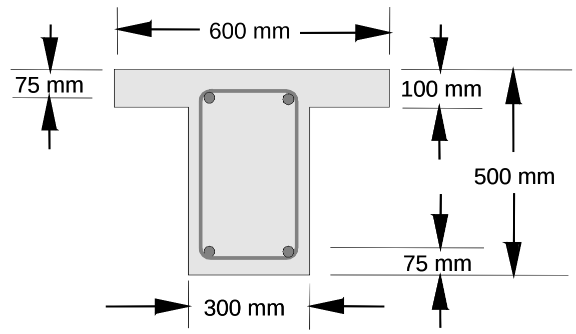
The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by AS 3600-01.
- The average shear stress in the beam is below the maximum shear stress allowed by AS 3600-01, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading. One dead load case (DL30) and one live load case (LL130), with only symmetric third-point loads of magnitudes 30, and 130 kN, respectively, are defined in the model. One load combinations (COMB130) is defined using the AS 3600-01 load combination factors of 1.2 for dead load and 1.5 for live load. The model is analyzed for both of these load cases and the load combination.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results and found to be identical. After completing the analysis, the design is performed using the AS 3600-01 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.



Beam Section

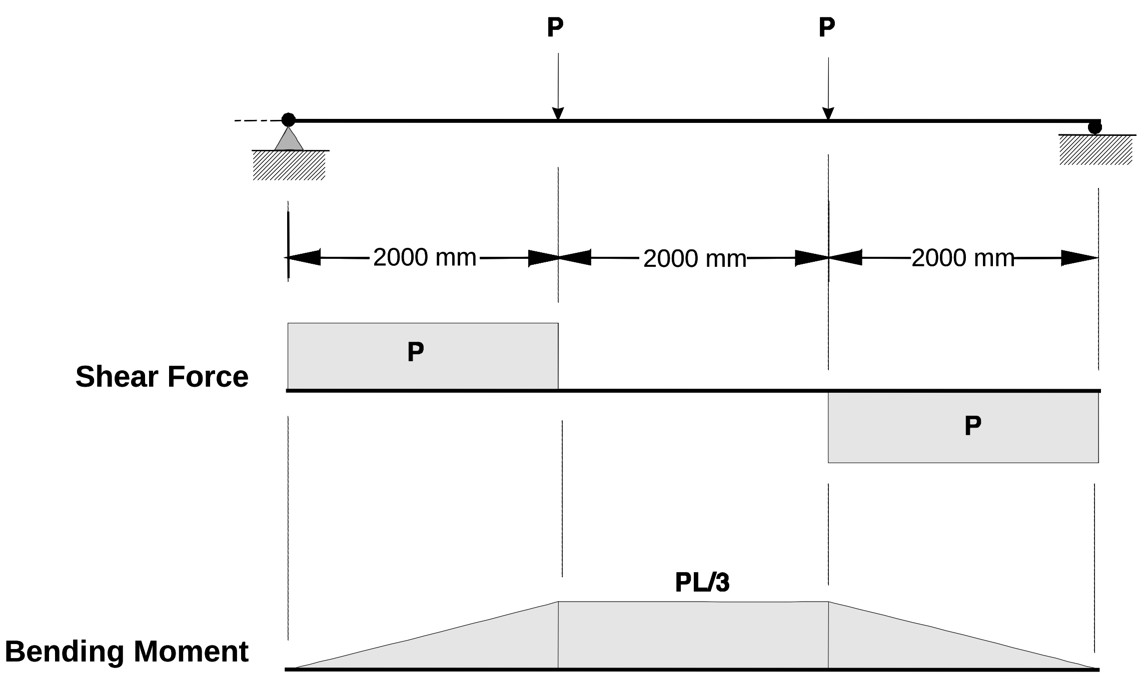


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span,	l	=	6000	mm
Overall depth,	h	=	500	mm
Flange thickness,	d_s	=	100	mm
Width of web,	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.,	d_c	=	75	mm
Effective depth,	d	=	425	mm
Depth of comp. reinf.,	d'	=	75	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	25x10^5	MPa
Modulus of elasticity,	E_s	=	2x10^8	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	30	kN
Live load,	P_l	=	130	kN

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the design reinforcement comparison.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
SAFE	462	33.512
Calculated	462	33.512

$A^{+}_{s,min}$ = 3.92 sq-cm

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-cm/m)	
	SAFE	Calculated
231	12.05	12.05

COMPUTER FILE: AS 3100-01 RC-BM-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$$\varphi = 0.8$$

$$\gamma = \left[0.85 - 0.007\left(f'_c - 28\right)\right] = 0.836$$

$$a_{\max} = \gamma k_u d = 0.836 \bullet 0.4 \bullet 425 = 142.12 \text{ mm}$$

$$A_{st.\min} = 0.22\left(\frac{D}{d}\right)^2 \frac{f'_{cf}}{f_{sy}} A_c$$

$$= 0.22 \bullet (500/425)^2 \bullet 0.6 \bullet \text{SQRT}(30)/460 \bullet 180{,}000$$

$$= 391.572 \text{ sq-mm}$$

COMB130

$$N^* = (1.2N_d + 1.5N_t) = 231\text{kN}$$

$$M^* = \frac{N^* l}{3} = 462 \text{ kN-m}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M^*}{0.85\, f'_c\, \phi\, b_{ef}}} = 100.755 \text{ mm } (a > D_s)$$

The compressive force developed in the concrete alone is given by:

The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, 2. C_f is given by:

$$C_f = 0.85 f'_c \left(b_{ef} - b_w\right) \times \min\left(D_s, a_{\max}\right) = 765 \text{ kN}$$

Therefore, $A_{s1} = \dfrac{C_f}{f_{sy}}$ and the portion of M^* that is resisted by the flange is given by:

$$M_{uf} = \phi C_f \left(d - \frac{\min\left(D_s, a_{\max}\right)}{2}\right) = 229.5 \text{ kN-m}$$

$$A_{s1} = \frac{C_f}{f_{sy}} = 1663.043 \text{ sq-mm}$$

Again, the value for ϕ is 0.80 by default. Therefore, the balance of the moment, M^* to be carried by the web is:

$$M_{uw} = M^* - M_{uf} = 462 - 229.5 = 232.5$$

The web is a rectangular section of dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f'_c \phi\, b_w}} = 101.5118 \text{ mm}$$

If $a_1 \le a_{\max}$, the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{uw}}{\phi\, f_{sy}\left(d - \frac{a_1}{2}\right)} = 1688.186 \text{ sq-mm}$$

$$A_{st} = A_{s1} + A_{s2} = 3351.23 \text{ sq-mm} = 33.512 \text{ sq-cm}$$

Shear Design

The shear force carried by the concrete, V_{uc}, is calculated as:

$$V_{uc} = \beta_1\beta_2\beta_3 b_w d_o\left[\frac{A_{st} f'_c}{b_w d_o}\right]^{1/3} = 0 \text{ kN}$$

where, $\beta_1 = 1.1\left(1.6 - \frac{d_o}{1000}\right) \ge 1.1 = 1.2925$, $\beta_2 = 1$ and $\beta_3 = 1$

The shear force is limited to a maximum of:

$$V_{u.\max} = 0.2 f'_c\, b d_o = 765 \text{ kN}$$

Given V^*, V_{uc}, and $V_{u.\max}$, the required shear reinforcement is calculated as follows, where, ϕ, the strength reduction factor, is 0.7.

If $V^* \le \phi V_{uc}/2$,

$\frac{A_{sv}}{s} = 0$, if $D \le 750$ mm, otherwise $A_{sv.\min}$ shall be provided.

If $\phi V_{u.\min} < V^* \le \phi V_{u.\max}$,

$$\frac{A_{sv}}{s} = \frac{\left(V^* - \phi V_{uc}\right)}{\phi f_{sy.f} d_o \cot\theta_v},$$

and greater than $A_{sv.\min,}$ defined as:

$$\frac{A_{sv.\min}}{s} = \left(0.35 \frac{b_w}{f_{sy.f}}\right) = 0.22826 \text{ sq-mm/mm} = 228.26 \text{ sq-mm/m}$$

θ_v = the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when $V^*=\phi V_{u.\min}$ to 45 degrees when $V^*=\phi V_{u,\max}$ = 35.52 degrees

If $V^* > \phi V_{\max}$, a failure condition is declared.

For load combination, the N^* and V^* are calculated as follows:

$N^* = 1.2N_d + 1.5N_1$

$V^* = N^*$

(COMB130)

N_d = 30 kips

N_l = 130 kips

N^* = 231 kN

N^* = 231 kN, ($\phi V_{u.\min} < V^* \le \phi V_{u.\max}$,)

$$\frac{A_{sv}}{s} = \frac{\left(V^* - \phi V_{uc}\right)}{\phi f_{sy.f} d_o \cot\theta_v}, \quad = 1.205 \text{ sq-mm/mm or } 12.05 \text{ sq-cm/m}$$

EXAMPLE AS 3600-01 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

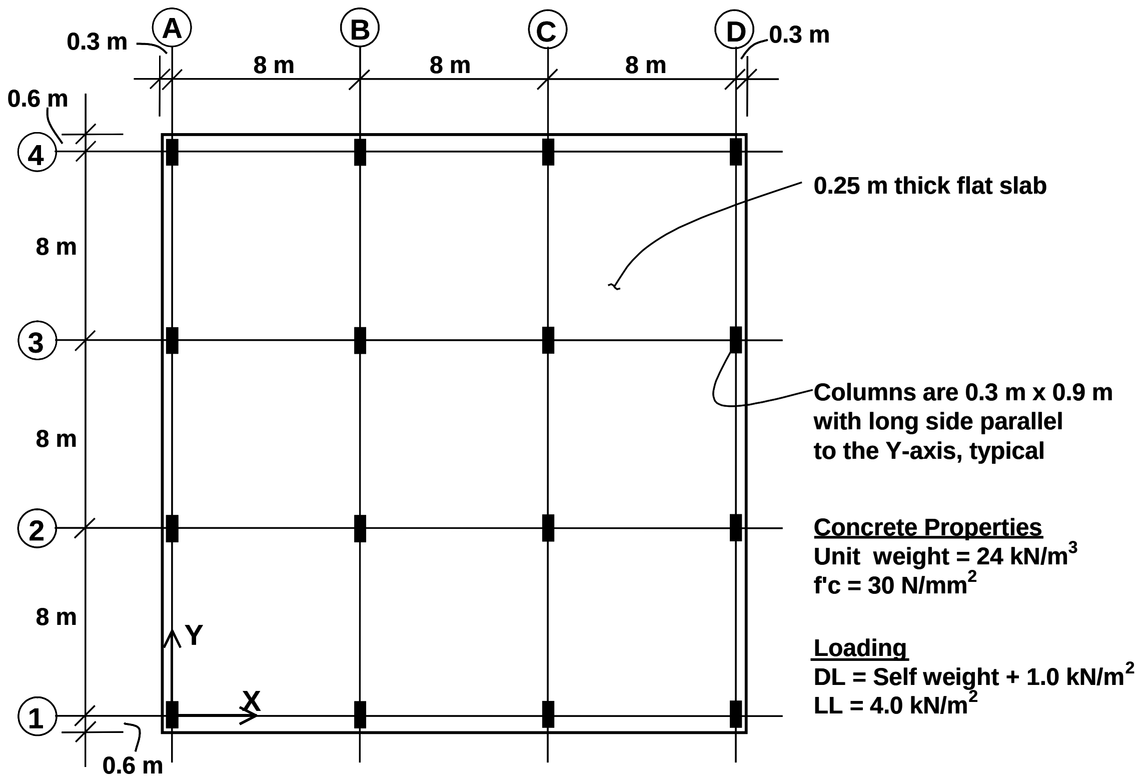


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m, with the long side parallel to the Y-axis. The slab is typically 0.25-m thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a $f'c$ of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress, and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio, and D/C ratio obtained from SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid Point B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
SAFE	1.811	1.086	1.67
Calculated	1.811	1.086	1.67

COMPUTER FILE: AS 3600-01 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using SAFE Method

d_{om} = [(250 – 26) + (250 – 38)] / 2 = 218 mm

Refer to Figure 2.

U = 518+ 1118 + 1118 + 518 = 3272 mm

a_x = 518 mm

a_y = 1118 mm

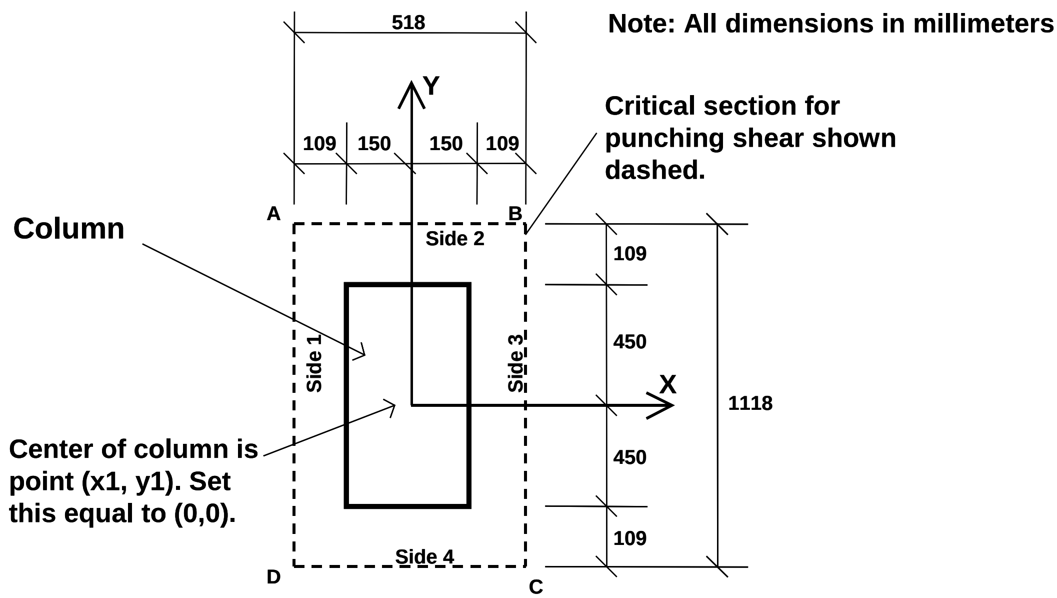


Figure 2: Interior Column, Grid B-2 in SAFE Model

From the SAFE output at grid line B-2:

V^* = 1126.498 kN

M_{v2} = –51.991 kN-m

M_{v3} = 45.723 kN-m

The maximum design shear stress is computed along the major and minor axis of column separately:

$$v_{\max} = \frac{V^*}{ud_{om}}\left[1.0 + \frac{uM_v}{8V^* ad_{om}}\right]$$

$$v_{\max,X} = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} \bullet \left(1 + \frac{3272 \bullet 51.991 \bullet 10^6}{8 \bullet 1126.498 \bullet 10^3 \bullet 1118 \bullet 218}\right)$$

$v_{\max,X}$ = 1.579 $\bullet$ 1.0774 = **1.7013 N/mm²**

$$v_{\max,Y} = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} \bullet \left(1 + \frac{3272 \bullet 45.723 \bullet 10^6}{8 \bullet 1126.498 \bullet 10^3 \bullet 518 \bullet 218}\right)$$

$v_{\max,Y}$ = 1.579 $\bullet$ 1.1470 = **1.811 N/mm²** (Govern)

The largest absolute value of $v_{\max}$= **1.811 N/mm²**

The shear capacity is calculated based on the smallest of AS 3600-01 equation 11-35, with the d_{om} and u terms removed to convert force to stress.

$$\varphi f_{cv} = \min\begin{cases} 0.17\left(1 + \frac{2}{\beta_h}\right)\varphi\sqrt{f'_c} \\ 0.34\varphi\sqrt{f'_c} \end{cases}$$ = **1.803N/mm²** in accordance with AS 9.2.3(a)

AS 9.2.3(a) yields the smallest value of φf_{cv} = 1.086 N/mm², and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{\varphi f_{cv}} = \frac{1.811}{1.086} = 1.67$$

EXAMPLE AS 3600-2001 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

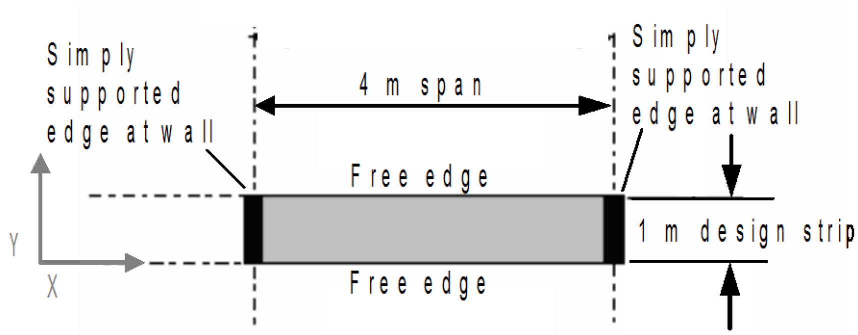


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa), with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the AS 3600-2001 load combination factors, 1.2 for dead loads and 1.5 for live loads. The model is analyzed for both load cases and the load combinations.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing analysis, design is performed using the AS 3600-2001 code using SAFE and also by hand computation. Table 1 shows the comparison of the design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	$2x10^6$	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	SAFE	24.597	5.58
	Calculated	24.600	5.58

$A_{s,min}^{+}$ = 282.9 sq-mm

COMPUTER FILE: AS 3600-2001 RC-SL-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\varphi = 0.8$

$b = 1000$ mm

$\gamma = \left[0.85 - 0.007\left(f'_c - 28\right)\right] = 0.836$

$a_{\max} = \gamma k_u d = 0.836 \cdot 0.4 \cdot 125 = 41.8$ mm

For the load combination, w and M^* are calculated as follows:

$w = (1.2w_d + 1.5w_t)\, b$

$$M_u = \frac{wl_1^2}{8}$$

$$A_{st.\min} = 0.22\left(\frac{D}{d}\right)^2 \frac{f'_{cf}}{f_{sy}} bd$$

$= 0.22 \cdot (150/125)^2 \cdot 0.6 \cdot \text{SQRT}(30)/460 \cdot 100 \cdot 125$

$= 282.9$ sq-mm

COMB100

$w_d = 4.0$ kPa

$w_t = 5.0$ kPa

$w = 12.3$ kN/m

$M_{\text{-}strip}^* = 24.6$ kN-m

$M_{\text{-}design}^* = 24.633$ kN-m

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M^*}{0.85 f'_c \phi b}} = 10.065 \text{ mm} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_{st} = \frac{M^*}{\phi f_{sy}\left(d - \frac{a}{2}\right)} = 557.966 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.57966 sq-cm

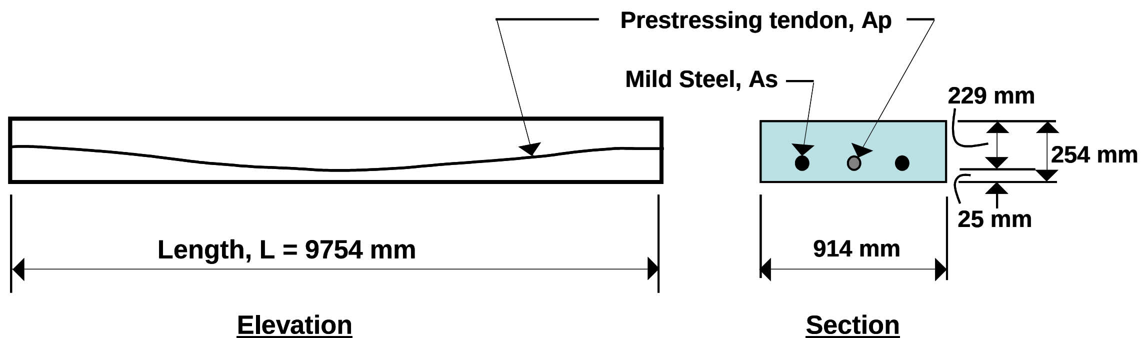
EXAMPLE BS 8110-97 PT-SL-001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.



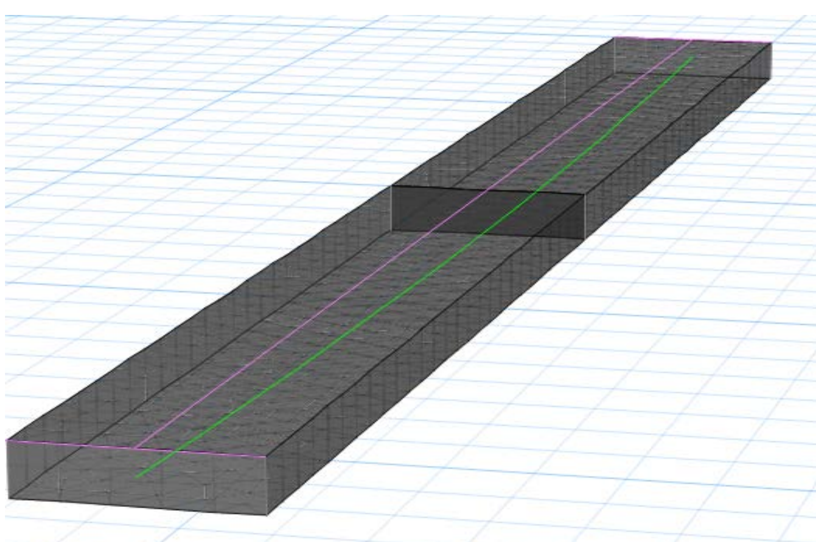


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, was added to the A-Strip. The self-weight and live loads were added to the slab. The loads and post-tensioning forces are as follows.

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the midspan of the slab. Independent hand calculations have been compared with the SAFE results and summarized for verification and validation of the SAFE results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	kN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	kN/m^2
Live load	w_l	=	4.788	kN/m^2

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	174.4	174.4	0.00%
Area of Mild Steel req'd, As (sq-cm)	19.65	19.79	0.71%
Transfer Conc. Stress, top $(D+PT_I)$, MPa	–5.058	–5.057	0.02%
Transfer Conc. Stress, bot $(D+PT_I)$, MPa	2.839	2.839	0.00%
Normal Conc. Stress, top $(D+L+PT_F)$, MPa	–10.460	–10.465	0. 50%
Normal Conc. Stress, bot $(D+L+PT_F)$, MPa	8.402	8.407	0.06%

COMPUTER FILE: BS 8110-97 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

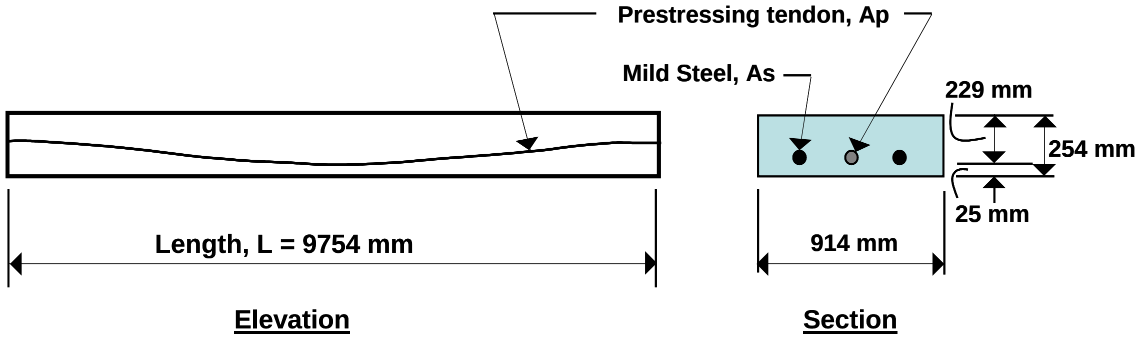
HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
f_{cu} = 30 MPa	f_{pu} = 1862 MPa
f_y = 400 MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\gamma_{m,\,steel}$ = 1.15

$\gamma_{m,\,concrete}$ = 1.50



Loads:

Dead, self-wt = 0.254 m x 23.56 kN/m^3 = 5.984 kN/m^2 (D) x 1.4 = 8.378 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) x 1.6 = 7.661 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 16.039 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 x 0.914m = 9.846 kN/m, ω_u = 16.039 kN/m^2 x 0.914m = 14.659 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8}$ = 14.659 x (9.754)^2/8 = 174.4 kN-m

Ultimate Stress in strand, $f_{pb} = f_{pe} + \frac{7000}{l/d}\left(1 - 1.7\frac{f_{pu}A_p}{f_{cu}bd}\right)$

$$= 1210 + \frac{7000}{9.754/0.229}\left(1 - 1.7\frac{1862(198)}{30(914)(229)}\right)$$

$$= 1358 \text{ MPa} \le 0.7 f_{pu} = 1303 \text{ MPa}$$

K factor used to determine the effective depth is given as:

$$K = \frac{M}{f_{cu} b d^2} = \frac{174.4}{30000(0.914)(0.229)^2} = 0.1213 < 0.156$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d = 192.2 \text{ mm}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197.4(1303)/1000 = 257.2KN$

Ultimate moment due to PT, $M_{ult,PT} = F_{ult,PT}(z)/\gamma = 257.2(0.192)/1.15 = 43.00$ kN-m

Net Moment to be resisted by As, $M_{NET} = M_U - M_{PT}$

$$= 174.4 - 43.00 = 131.40 \text{ kN-m}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_{NET}}{0.87 f_y z} = \frac{131.4}{0.87(400)(192)}(1e6) = 1965 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination ($D+PT_i$) = $1.0D+0.0L+1.0PT_I$

Tendon stress at transfer = jacking stress – stressing losses = 1490 – 186 = 1304 MPa

The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102\text{mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \frac{F_{PTI}}{A} \pm \frac{M_D - M_{PT}}{S} = \frac{-257.4}{0.254(0.914)} \pm \frac{65.04 - 26.23}{0.00983}$

where $S=0.00983\text{m}^3$

$$f = -1.109 \pm 3.948 \text{ MPa}$$

$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: ($D+L+PT_F$) = $1.0D+1.0L+1.0PT_F$

Tendon stress at Normal = jacking – stressing – long-term = 1490 – 186 – 94= 1210 MPa

The force in tendon at Normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT_F),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$

$$f = -1.029 \pm 9.431$$

$$f = -10.460(\text{Comp})\max, 8.402(\text{Tension})\max$$

EXAMPLE BS 8110-97 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by BS 8110-97.
- The average shear stress in the beam is below the maximum shear stress allowed by BS 8110-97, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T-beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading. One dead load case (DL20) and one live load case (LL80) with only symmetric third-point loads of magnitudes 20 and 80 kN, respectively, are defined in the model. One load combinations (COMB80) is defined using the BS 8110-97 load combination factors of 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both of these load cases and the load combinations.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results. These moment and shear force are identical. After completing the analysis, design is performed using the BS 8110-97 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.

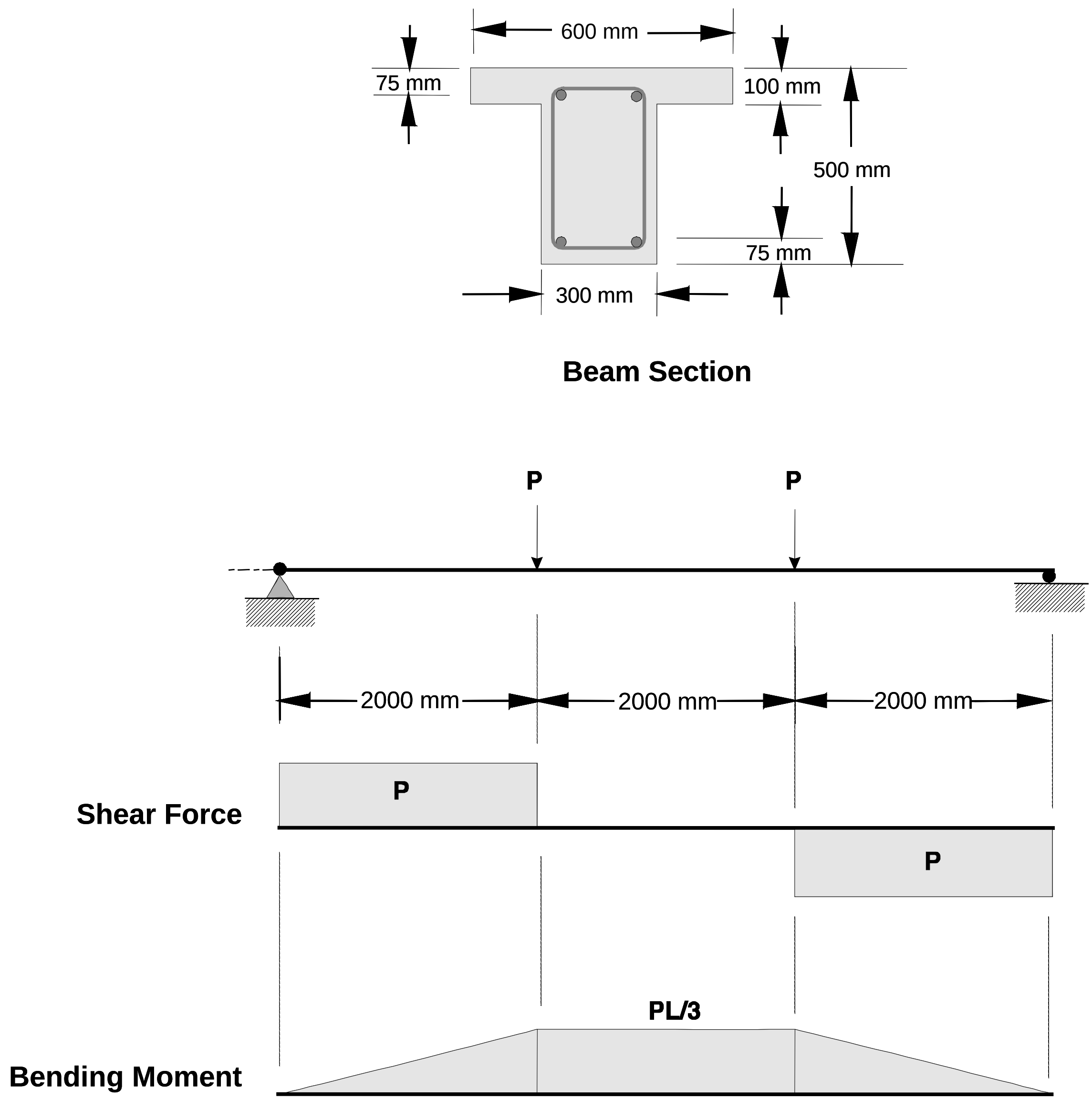


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span	l	=	6000	mm
Overall depth	h	=	500	mm
Flange thickness	d_s	=	100	mm
Width of web	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.	d_c	=	75	mm
Effective depth	d	=	425	mm
Depth of comp. reinf.	d'	=	75	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	460	MPa
Concrete unit weight	w_c	=	0	kN/m^3
Modulus of elasticity	E_c	=	$25x10^5$	MPa
Modulus of elasticity	E_s	=	$2x10^8$	MPa
Poisson's ratio	v	=	0.2	
Dead load	P_d	=	20	kN
Live load	P_l	=	80	kN

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 Also shows the design reinforcement comparison.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-cm)
		A_s^+
SAFE	312	20.90
Calculated	312	20.90

$A^+_{s,min}$ = 195.00 sq-mm

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-cm/m)	
	SAFE	Calculated
156	6.50	6.50

COMPUTER FILE: BS 8110-97 RCBM-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\gamma_{m,\text{ steel}} = 1.15$

$\gamma_{m,\text{ concrete}} = 1.50$

$$A_{s,\min} = 0.0013 b_w h$$

$$= 195.00 \text{ sq-mm}$$

COMB80

$P = (1.4P_d + 1.6P_t) = 156$ kN

$$M^* = \frac{N^* l}{3} = 312 \text{ kN-m}$$

The depth of the compression block is given by:

$$K = \frac{M}{f_{cu} b_f d^2} = 0.095963 < 0.156$$

Then the moment arm is computed as:

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \leq 0.95d = 373.4254 \text{ mm}$$

The depth of the neutral axis is computed as:

$$x = \frac{1}{0.45}(d - z) = 114.6102 \text{ mm}$$

And the depth of the compression block is given by:

$$a = 0.9x = 103.1492 \text{ mm} > h_f$$

The ultimate resistance moment of the flange is given by:

$$M_f = \frac{0.67}{\gamma_c} f_{cu}\left(b_f - b_w\right)h_f\left(d - 0.5h_f\right) = 150.75 \text{ kN-m}$$

The moment taken by the web is computed as:

$$M_w = M - M_f = 161.25 \text{ kN-m}$$

and the normalized moment resisted by the web is given by:

$$K_w = \frac{M_w}{f_{cu} b_w d^2} = 0.0991926 < 0.156$$

If $K_w \leq 0.156$ (BS 3.4.4.4), the beam is designed as a singly reinforced concrete beam. The reinforcement is calculated as the sum of two parts: one to balance compression in the flange and one to balance compression in the web.

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K_w}{0.9}}\right) \leq 0.95d = 371.3988 \text{ mm}$$

$$A_s = \frac{M_f}{\frac{f_y}{\gamma_s}\left(d - 0.5h_f\right)} + \frac{M_w}{\frac{f_y}{\gamma_s} z} = 2090.4 \text{ sq-mm}$$

Shear Design

$$v = \frac{V}{b_w d} \leq v_{\max} = 1.2235 \text{ MPa}$$

$$v_{\max} = \min(0.8\sqrt{f_{cu}}, 5 \text{ MPa}) = 4.38178 \text{ MPa}$$

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} = 0.3568 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3} = 1.06266,\ 1 \leq k_2 \leq \left(\frac{40}{25}\right)^{1/3}$$

$$\gamma_m = 1.25$$

$$\frac{100\, A_s}{bd} = 0.15$$

$$\left(\frac{400}{d}\right)^{1/4} = 1$$

However, the following limitations also apply:

$$0.15 \le \frac{100\,A_s}{bd} \le 3$$

$$\left(\frac{400}{d}\right)^{1/4} \ge 1$$

$f_{cu} \le 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

Given v, v_c, and $v_{\max}$, the required shear reinforcement is calculated as follows:

If $v \le (v_c + 0.4)$

$$\frac{A_{sv}}{s_v} = \frac{0.4 b_w}{0.87\,f_{yv}}$$

If $(v_c + 0.4) < v \le v_{\max}$

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c) b_w}{0.87\,f_{yv}}$$

If $v > v_{\max}$, a failure condition is declared.

(COMB80)

P_d = 20 kN

P_l = 80 kN

V = 156 kN

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c) b_w}{0.87\,f_{yv}} = 0.64967 \text{ sq-mm/mm} = 649.67 \text{ sq-mm/m}$$

EXAMPLE BS 8110-97 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

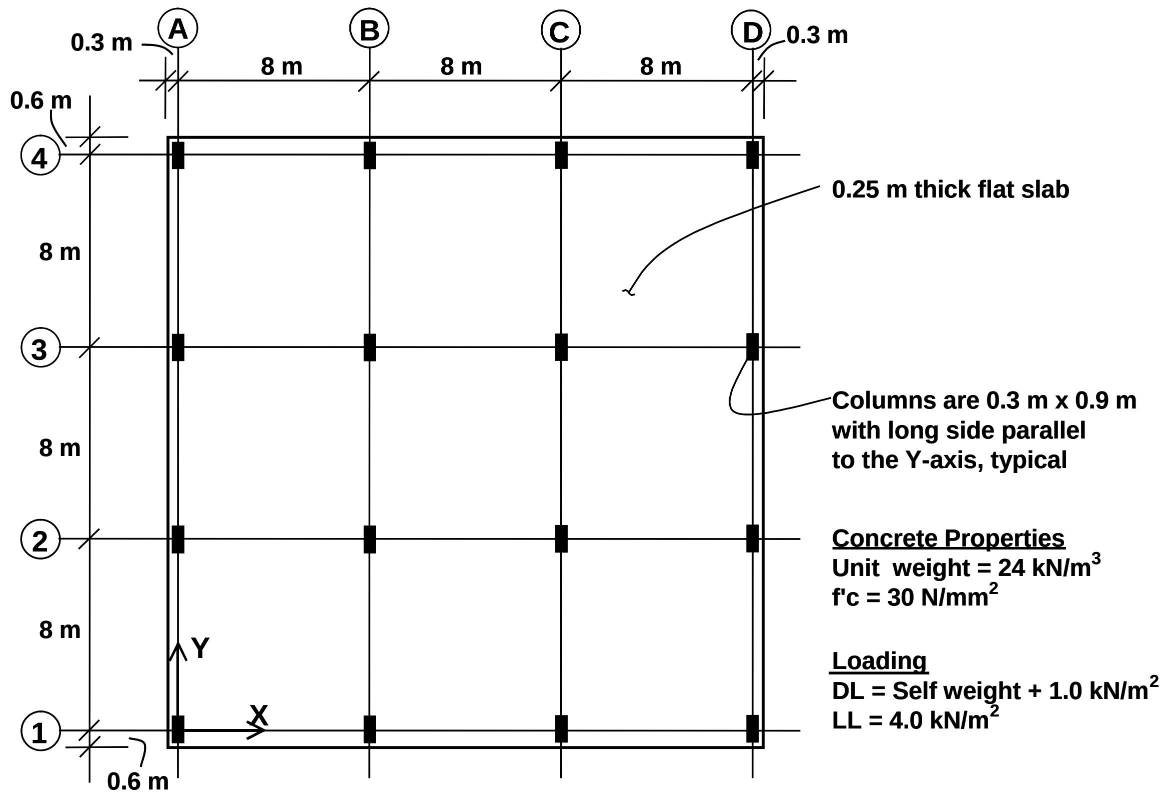


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25-m thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f_{cu} of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
SAFE	1.105	0.625	1.77
Calculated	1.105	0.625	1.77

COMPUTER FILE: BS 8110-97 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using SAFE Method

$$d = [(250-26)+(250-38)]/2 = 218 \text{ mm}$$

Refer to Figure 2.

u = 954+ 1554 + 954 + 1554 = 5016 mm

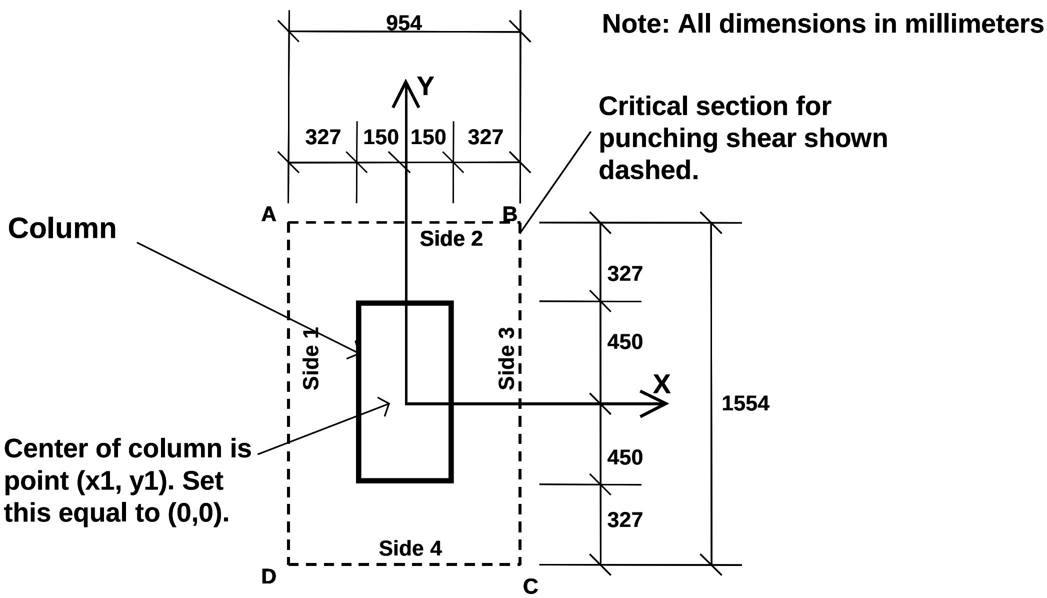


Figure 2: Interior Column, Grid B-2 in SAFE Model

From the SAFE output at Grid B-2:

V = 1126.498 kN

M_2 = 51.9908 kN-m

M_3 = 45.7234 kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{eff,x} = \frac{V}{ud}\left(f + \frac{1.5M_x}{Vy} \right)$$ (BS 3.7.7.3)

$$v_{eff,x} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 51.9908 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 954} \right) = 1.1049 \text{ (Govern)}$$

$$v_{eff,y} = \frac{V}{ud}\left(f + \frac{1.5M_y}{Vx} \right)$$

$$v_{eff,y} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 45.7234 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 1554} \right) = 1.0705$$

The largest absolute value of v = **1.1049 N/mm²**

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd} \right)^{1/3}\left(\frac{400}{d} \right)^{1/4} = 0.3568 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{25} \right)^{1/3} = \left(\frac{30}{25} \right)^{1/3} = 1.0627 > 1.0 \text{ OK}$$

$\gamma_m = 1.25$

$$\left(\frac{400}{d} \right)^{1/4} = 1.16386 > 1 \text{ OK.}$$

$f_{cu} \leq 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

Areas of reinforcement at the face of column for the design strips are as follows:

A_s in Strip Layer A = 9494.296 mm^2

A_s in Strip Layer B = 8314.486 mm^2

Average A_s = (9494.296+8314.486)/2 = 8904.391 mm^2

$$\frac{100\ A_s}{bd} = 100 \bullet 8904.391/(8000 \bullet 218) = 0.51057$$

$$v_c = \frac{0.79 \bullet 1.0 \bullet 1.0627}{1.25} \bullet \left(0.51057\right)^{1/3} \bullet 1.16386 = 0.6247 \text{ MPa}$$

BS 3.7.7.3 yields the value of v = **0.625 N/mm²**, and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{v} = \frac{1.1049}{0.6247} = 1.77$$

EXAMPLE BS 8110-97 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

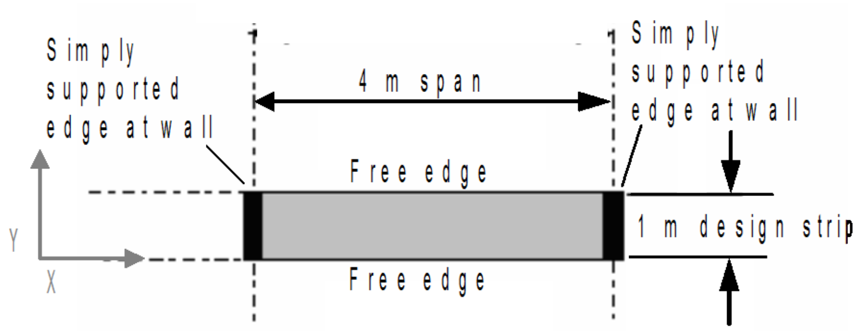


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the BS 8110-97 load combination factors, 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing the analysis, design was performed using the BS 8110-97 code by SAFE and also by hand computation. Table 1 shows the comparison of the design reinforcements computed by the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	$2x10^6$	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	SAFE	27.197	5.853
	Calculated	27.200	5.850

$A^+_{s,min}$ = 162.5 sq-mm

COMPUTER FILE: BS 8110-97 RC-SL-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\gamma_{m, steel}$ = 1.15

$\gamma_{m, concrete}$ = 1.50

b = 1000 mm

For the load combination, w and M are calculated as follows:

$$w = (1.4w_d + 1.6w_t)\, b$$

$$M = \frac{wl_1^2}{8}$$

$$A_{s,\min} = 0.0013 b_w d$$

$$= 162.5 \text{ sq-mm}$$

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 13.6 kN/m

$M_{\text{-strip}}$ = 27.2 kN-m

$M_{\text{-design}}$ = 27.2366 kN-m

The depth of the compression block is given by:

$$K = \frac{M}{f_{cu} b d^2} = 0.05810 < 0.156$$

The area of tensile steel reinforcement is then given by:

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d = 116.3283$$

$$A_s = \frac{M}{0.87 f_y z} = 585.046 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.850 \text{ sq-cm}$$

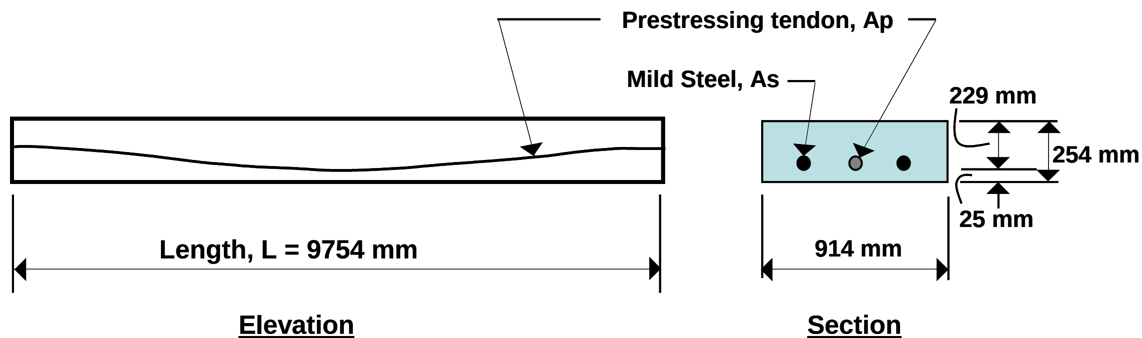
EXAMPLE CSA A23.3-14 PT-SL-001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm as shown in shown in Figure 1.



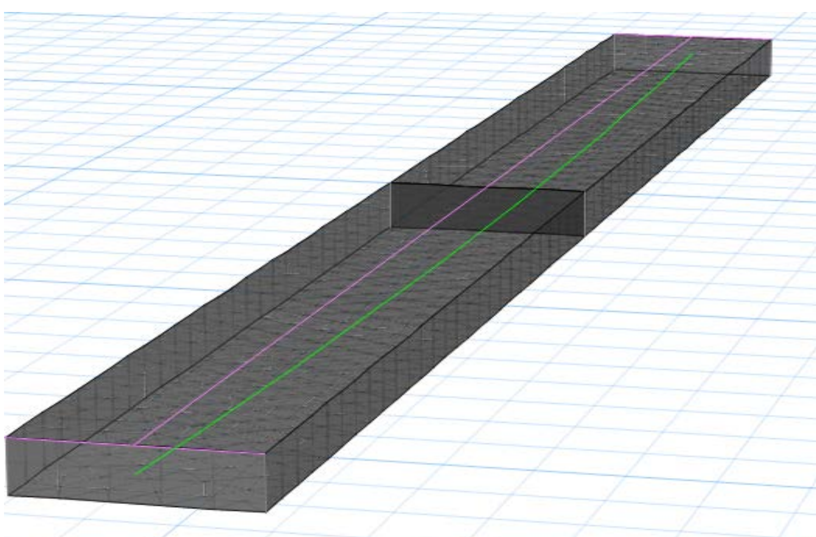


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 KN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the midspan of the slab. Independent hand calculations have been compared with the SAFE results and summarized for verification and validation of the SAFE results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	KN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	KN/m^2
Live load	w_l	=	4.788	KN/m^2

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	159.4	159.4	0.00%
Area of Mild Steel req'd, As (sq-cm)	16.25	16.32	0.43%
Transfer Conc. Stress, top ($D+PT_I$), MPa	–5.058	–5.057	-0.02%
Transfer Conc. Stress, bot ($D+PT_I$), MPa	2.839	2.839	0.00%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	–10.460	–10.465	0.05%
Normal Conc. Stress, bot ($D+L+PT_F$), MPa	8.402	8.407	0.06%
Long-Term Conc. Stress, top ($D+0.5L+PT_{F(L)}$), MPa	–7.817	–7.817	0.00%
Long-Term Conc. Stress, bot ($D+0.5L+PT_{F(L)}$), MPa	5.759	5.759	0.00%

COMPUTER FILE: CSA A23.3-14 PT-SL-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATIONS:

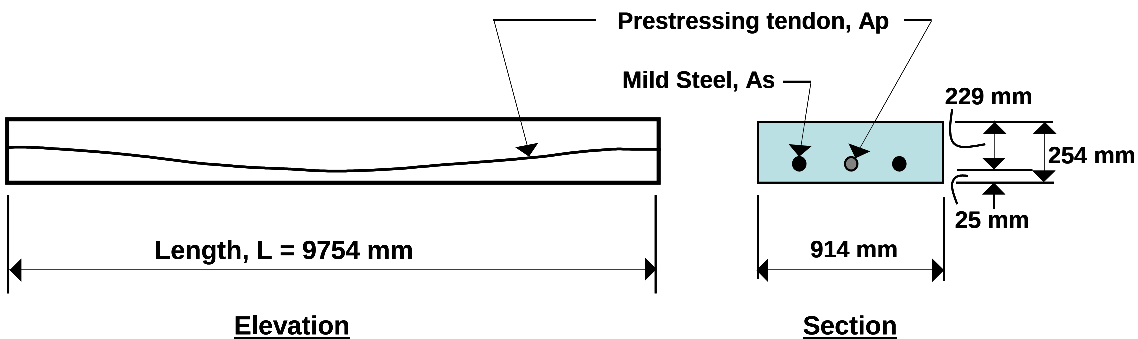
Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
f_{cu} = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\phi_c = 0.65$, $\phi_S = 0.85$

$\alpha_1 = 0.85 - 0.0015f'_c \geq 0.67 = 0.805$

$\beta_1 = 0.97 - 0.0025f'_c \geq 0.67 = 0.895$



Loads:

Dead, self-wt = 0.254 m x 23.56 kN/m^3 = 5.984 kN/m^2 (D) x 1.25 = 7.480 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) x 1.50 = 7.182 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 14.662 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 x 0.914m = 9.846 kN/m, ω_u = 16.039 kN/m^2 x 0.914m = 13.401 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8}$ = 13.401 x (9.754)^2/8 = 159.42 kN-m

Ultimate Stress in strand, $f_{pb} = f_{pe} + \frac{8000}{l_o}\left(d_p - c_y\right)$

$$c_y = \frac{\phi_p A_p f_{pr} + \phi_s A_s f_y}{\alpha_1 \phi_c f'_c \beta_1 b} = \frac{0.9(197)(1347) + 0.85(1625)(400)}{0.805(0.65)(30.0)(0.895)(914)} = 61.66 \text{ mm}$$

$$f_{pb} = 1210 + \frac{8000}{9754}\left(229 - 61.66\right) = 1347 \text{ MPa}$$

Depth of the compression block, *a*, is given as:

Stress block depth, $a = d - \sqrt{d^2 - \frac{2M^*}{\alpha_1 f'_c \phi_c b}}$

$$= 0.229 - \sqrt{0.229^2 - \frac{2(159.42)}{0.805(30000)(0.65)(0.914)}} = 55.18$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197(1347)/1000 = 265.9 \text{ kN}$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right)\phi = 265.9\left(0.229 - \frac{55.18}{2}\right)(0.85) = 45.52 \text{ kN-m}$$

Net Moment to be resisted by As, $M_{NET} = M_U - M_{PT}$

$$= 159.42 - 45.52 = 113.90 \text{ kN-m}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_{NET}}{0.87 f_y z} = \frac{113.90}{0.87(400)\left(229 - \frac{55.18}{2}\right)}(1e6) = 1625 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (D+PT$_i$) = 1.0D+0.0L+1.0PT$_I$

Tendon stress at transfer = jacking stress − stressing losses = 1490 − 186 = 1304 MPa
The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4 \text{ kN}$

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102\text{ mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-257.4}{0.254(0.914)} \pm \dfrac{65.04 - 26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.109 \pm 3.948 \text{ MPa}$$
$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT$_F$) = 1.0D+1.0L+1.0PT$_F$

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT$_F$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$
$$f = -1.029 \pm 9.431$$
$$f = -10.460(\text{Comp})\max,\ 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: (D+0.5L+PT$_{F(L)}$) = 1.0D+0.5L+1.0PT$_F$

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+0.5L+PT$_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$
$$f = -1.029 \pm 6.788$$
$$f = -7.817(\text{Comp})\max,\ 5.759(\text{Tension})\max$$

EXAMPLE CSA A23.3-14 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

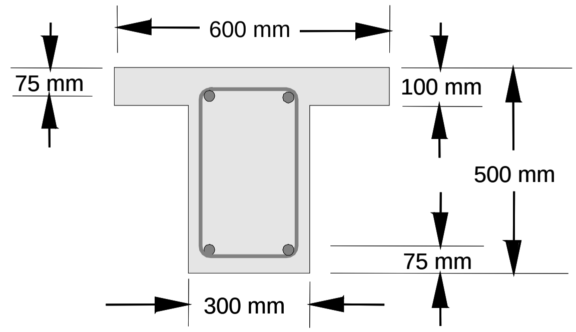
The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by CSA A23.3-14.
- The average shear stress in the beam is below the maximum shear stress allowed by CSA A23.3-14, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T-beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading. One dead load case (DL30) and one live load case (LL100) with only symmetric third-point loads of magnitudes 30, and 100 kN, respectively, are defined in the model. One load combinations (COMB100) is defined using the CSA A23.3-14 load combination factors of 1.25 for dead loads and 1.5 for live loads. The model is analyzed for both of these load cases and the load combinations.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results. These moment and shear force are identical. After completing the analysis, design is performed using the CSA A23.3-14 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.



Beam Section

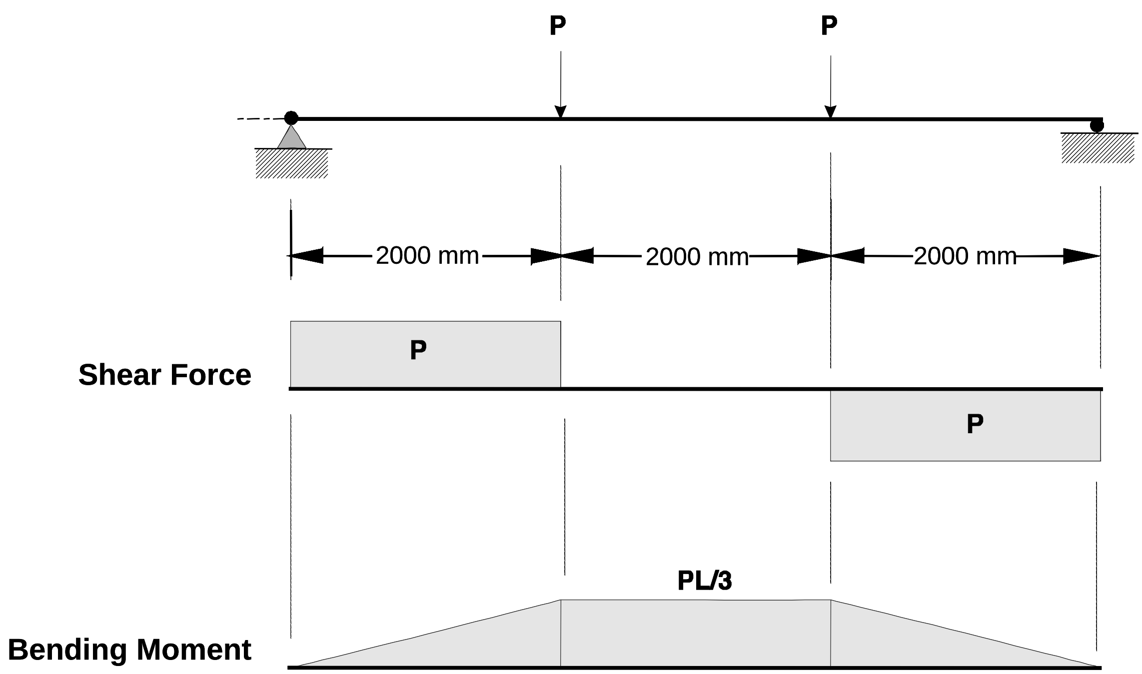


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span,	l	=	6000	mm
Overall depth,	h	=	500	mm
Flange thickness,	d_s	=	100	mm
Width of web,	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.,	d_c	=	75	mm
Effective depth,	d	=	425	mm
Depth of comp. reinf.,	d'	=	75	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	$25x10^5$	MPa
Modulus of elasticity,	E_s	=	$2x10^8$	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	30	kN
Live load,	P_l	=	100	kN

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the design reinforcement comparison.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-cm)
		A_s^+
SAFE	375	25.844
Calculated	375	25.844

$A^+_{s,min}$ = 535.82 sq-m

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-cm/m)	
	SAFE	**Calculated**
187.5	12.573	12.573

COMPUTER FILE: CSA A23.3-14 RC-BM-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

ϕ_c = 0.65 for concrete

ϕ_s = 0.85 for reinforcement

$$A_{s,\min} = \frac{0.2\sqrt{f'_c}}{f_y}\, b_w\, h = 357.2 \text{ sq-mm}$$

$$\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67 = 0.805$$

$$\beta_1 = 0.97 - 0.0025 f'_c \geq 0.67 = 0.895$$

$$c_b = \frac{700}{700 + f_y}\, d = 256.46 \text{ mm}$$

$$a_b = \beta_1 c_b = 229.5366 \text{ mm}$$

$$A_s = \min[A_{s,\min}, (4/3)\, A_{s,\text{required}}] = \min[357.2, (4/3)2445] = 357.2 \text{ sq-mm}$$

COMB100

$$P = (1.25P_d + 1.5P_t) = 187.5\text{kN}$$

$$M^* = \frac{Pl}{3} = 375 \text{ kN-m}$$

$$M_f = 375 \text{ kN-m}$$

The depth of the compression block is given by:

$$C_f = \alpha_1 f'_c \left(b_f - b_w\right) \min\left(h_s, a_b\right) = 724.5 \text{ kN}$$

Therefore, $A_{s1} = \dfrac{C_f \phi_c}{f_y \phi_s}$ and the portion of M_f that is resisted by the flange is given by:

$$A_{s1} = \frac{C_f \phi_c}{f_y \phi_s} = 1204.411 \text{ sq-mm}$$

$$M_{ff} = C_f \left(d - \frac{\min(h_s, a_b)}{2} \right) \phi_c = 176.596 \text{ kN-m}$$

Therefore, the balance of the moment, M_f to be carried by the web is:

$$M_{fw} = M_f - M_{ff} = 198.403 \text{ kN-m}$$

The web is a rectangular section with dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{fw}}{\alpha_1 f'_c \phi_c b_w}} = 114.5745 \text{ mm}$$

If $a_1 \le a_b$, the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{fw}}{\phi_s f_y \left(d - \frac{a_1}{2} \right)} = 1379.94 \text{ sq-mm}$$

$$A_s = A_{s1} + A_{s2} = 2584.351 \text{ sq-mm}$$

Shear Design

The basic shear strength for rectangular section is computed as,

$\phi_c =$ 0.65 for shear

$\lambda = \{1.00,$ for normal density concrete

d_v is the effective shear depth. It is taken as the greater of $0.9d$ or $0.72h$ = 382.5 mm (governing) or 360 mm.

$S_{ze} = 300$ if minimum transverse reinforcement

$$\varepsilon_x = \frac{M_f / d_v + V_f + 0.5N_f}{2(E_s A_s)} \text{ and } \varepsilon_x \le 0.003$$

$$\beta = \frac{0.40}{(1+1500\varepsilon_x)} \bullet \frac{1300}{(1000+S_{ze})} = 0.07272$$

$$V_c = \phi_c \lambda \beta \sqrt{f'_c} b_w d_v = 29.708 \text{ kN}$$

$$V_{r,\max} = 0.25\phi_c f'_c b_w d = 621.56 \text{ kN}$$

θ = 50

$$\frac{A_v}{s} = \frac{(V_f - V_c)\tan\theta}{\phi_s f_{yt} d_v} = 1.2573 \text{ mm}^2/\text{mm} = 12.573 \text{ cm}^2/\text{m}.$$

EXAMPLE CSA A23.3-14 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8 m spans in each direction, as shown in Figure 1.

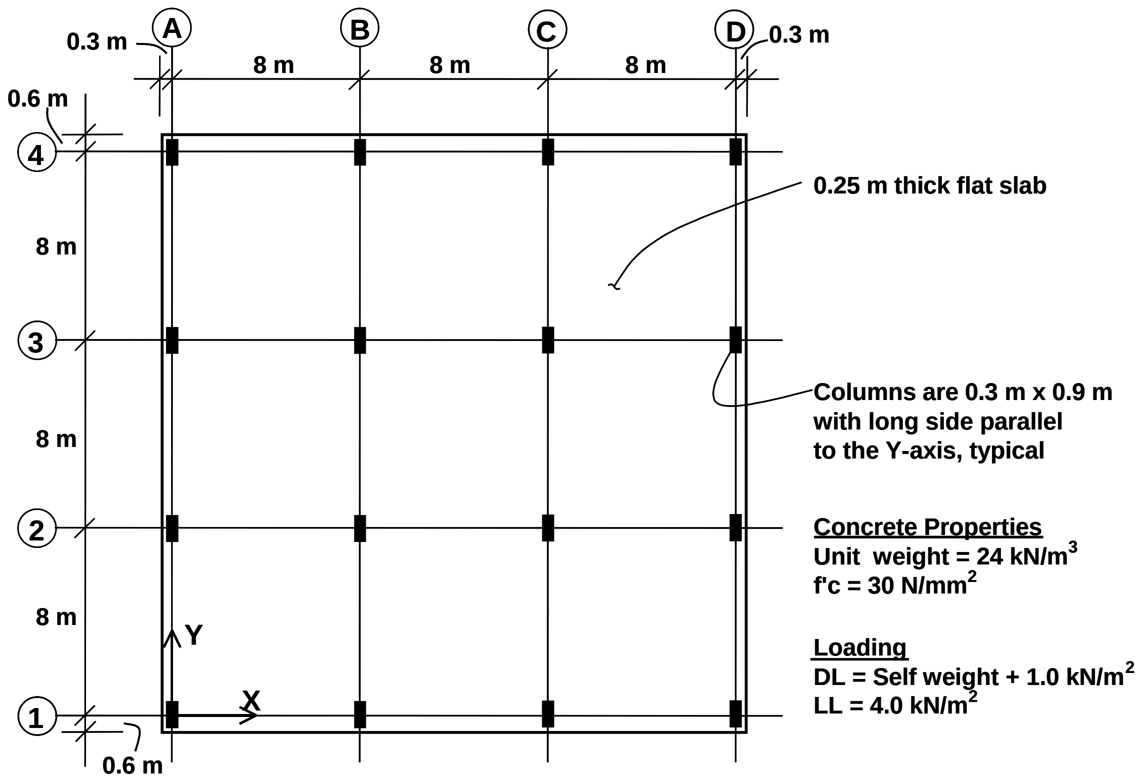


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f'_c of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
SAFE	1.792	1.127	1.59
Calculated	1.792	1.127	1.59

COMPUTER FILE: CSA A23.3-14 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using SAFE Method

$d = [(250-26)+(250-38)]/2 = 218$ mm

Refer to Figure 2.

b_0 = 518+ 1118 + 1118 + 518 = 3272 mm

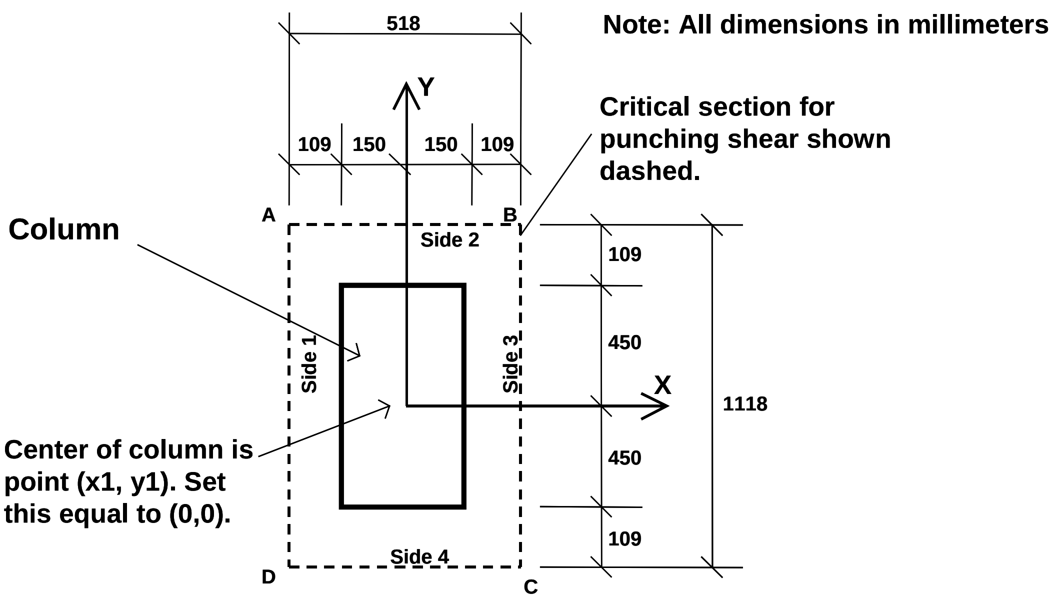


Figure 2: Interior Column, Grid B-2 in SAFE Model

$$\gamma_{V2} = 1 - \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{1118}{518}}} = 0.495$$

$$\gamma_{V3} = 1 - \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{518}{1118}}} = 0.312$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–259	0	259	0	N.A.
y_2	0	559	0	–559	N.A.
L	1118	518	1118	518	b_0 = 3272
d	218	218	218	218	N.A.
Ld	243724	112924	243724	112924	713296
Ldx_2	–63124516	0	63124516	0	0
Ldy_2	0	63124516	0	–63124516	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{713296} = 0\,mm$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{713296} = 0\,mm$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	1118	518	1118	518	N.A.
d	218	218	218	218	N.A.
$x_2 - x_3$	–259	0	259	0	N.A.
$y_2 - y_3$	0	559	0	–559	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	2.64E+10	3.53E+10	2.64E+10	3.53E+10	1.23E+11
I_{YY}	1.63E+10	2.97E+09	1.63E+10	2.97E+09	3.86E+10
I_{XY}	0	0	0	0	0

From the SAFE output at Grid B-2:

V_f = 1126.498 kN

$\gamma_{V2} M_{f,2}$ = –25.725 kN-m

$\gamma_{V3} M_{f,3}$ = 14.272 kN-m

At the point labeled A in Figure 2, x_4 = –259 and y_4 = 559, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(559-0) - (0)(-259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(-259-0) - (0)(559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

v_f = 1.5793 – 0.1169 – 0.0958 = **1.3666 N/mm²** at point A

At the point labeled B in Figure 2, x_4 = 259 and y_4 = 559, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(559-0) - (0)(259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(259-0) - (0)(559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

v_f = 1.5793 – 0.1169 + 0.0958 =**1.5582 N/mm²** at point B

At the point labeled C in Figure 2, x_4 = 259 and y_4 = –559, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(-559-0) - (0)(259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(259-0) - (0)(-559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

v_f = 1.5793 + 0.1169 + 0.0958 = **1.792 N/mm²** at point C

At the point labeled D in Figure 2, x_4 = –259 and y_4 = –559, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(-559-0) - (0)(-259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(-259-0) - (0)(-559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

v_f = 1.5793 + 0.1169 – 0.0958 = **1.6004 N/mm²** at point D

Point C has the largest absolute value of v_u, thus v_{max} = **1.792 N/mm²**

The shear capacity is calculated based on the minimum of the following three limits:

$$v_v = \min \begin{cases} \phi_c \left(1 + \frac{2}{\beta_c}\right) 0.19 \lambda \sqrt{f'_c} \\ \phi_c \left(0.19 + \frac{\alpha_s d}{b_0}\right) \lambda \sqrt{f'_c} \\ \phi_c 0.38 \lambda \sqrt{f'_c} \end{cases} \qquad 1.127 \text{ N/mm}^2 \text{ in accordance with CSA 13.3.4.1}$$

CSA 13.3.4.1 yields the smallest value of $\boxed{v_v = 1.127 \text{ N/mm}^2}$, and thus this is the shear capacity.

$$\boxed{\text{Shear Ratio} = \frac{v_U}{\varphi v_v} = \frac{1.792}{1.127} = 1.59}$$

EXAMPLE CSA A23.3-14 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

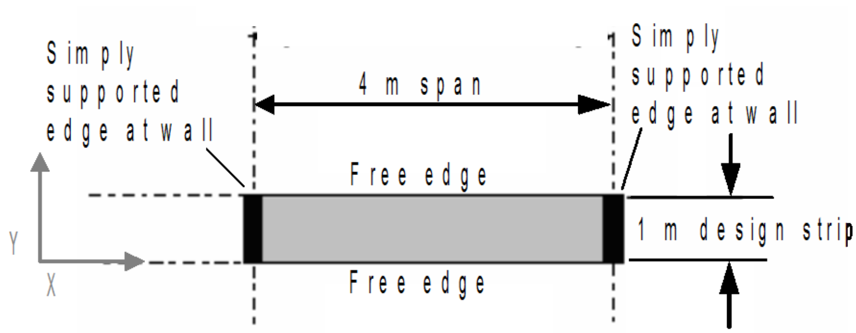


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the CSA A23.3-14 load combination factors, 1.25 for dead loads and 1.5 for live loads. The model is analyzed for these load cases and load combinations.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing the analysis, design is performed using the CSA A23.3-14 code by SAFE and also by hand computation. Table 1 show the comparison of the design reinforcements computed using the two methods.

PROGRAM NAME: SAFE
REVISION NO.: 0

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2×10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm)
			A_s^+
Medium	SAFE	25.00	5.414
	Calculated	25.00	5.528

$A^+_{s,min}$ = 357.2 sq-mm

COMPUTER FILE: CSA A23.3-14 RC-SL-001.FDB

CONCLUSION

The SAFE results show a very close comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

ϕ_c = 0.65 for concrete

ϕ_s = 0.85 for reinforcement

$$A_{s,\min} = \frac{0.2\sqrt{f'_c}}{f_y} b_w h = 357.2 \text{ sq-mm}$$

b = 1000 mm

$\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67 = 0.805$

$\beta_1 = 0.97 - 0.0025 f'_c \geq 0.67 = 0.895$

$$c_b = \frac{700}{700 + f_y} d = 75.43 \text{ mm}$$

$a_b = \beta_1 c_b = 67.5$ mm

For the load combination, w and M^* are calculated as follows:

$w = (1.25 w_d + 1.5 w_t)\, b$

$$M_u = \frac{w l_1^2}{8}$$

$A_s = \min[A_{s,\min}, (4/3)\, A_{s,\text{required}}] = \min[357.2, (4/3)540.63] = 357.2$ sq-mm

$= 0.22 \bullet (150/125)^2 \bullet 0.6 \bullet \text{SQRT}(30)/460 \bullet 100 \bullet 125$

$=$ 282.9 sq-mm

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 12.5 kN/m

$M_{f\text{-}strip}$ = 25.0 kN-m

$M_{f\text{-}design}$ = 25.529 kN-m

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_f\right|}{\alpha_1 f'_c \phi_c b}} = 13.769 \text{ mm} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_f}{\phi_s f_y \left(d - \frac{a}{2}\right)} = 552.77 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.528 sq-cm

EXAMPLE CSA A23.3-04 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8 m spans in each direction, as shown in Figure 1.

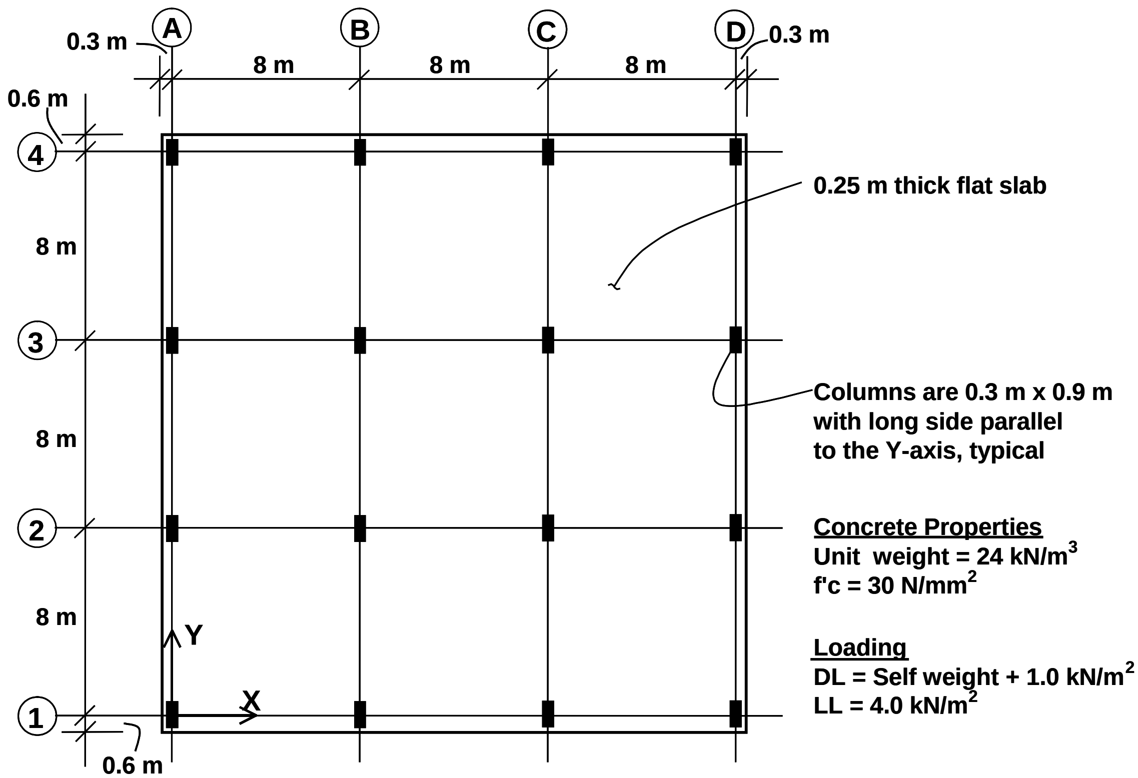


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f'_c of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
SAFE	1.792	1.127	1.59
Calculated	1.792	1.127	1.59

COMPUTER FILE: CSA A23.3-04 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using SAFE Method

$d = [(250-26)+(250-38)]/2 = 218$ mm

Refer to Figure 2.

b_0 = 518+ 1118 + 1118 + 518 = 3272 mm

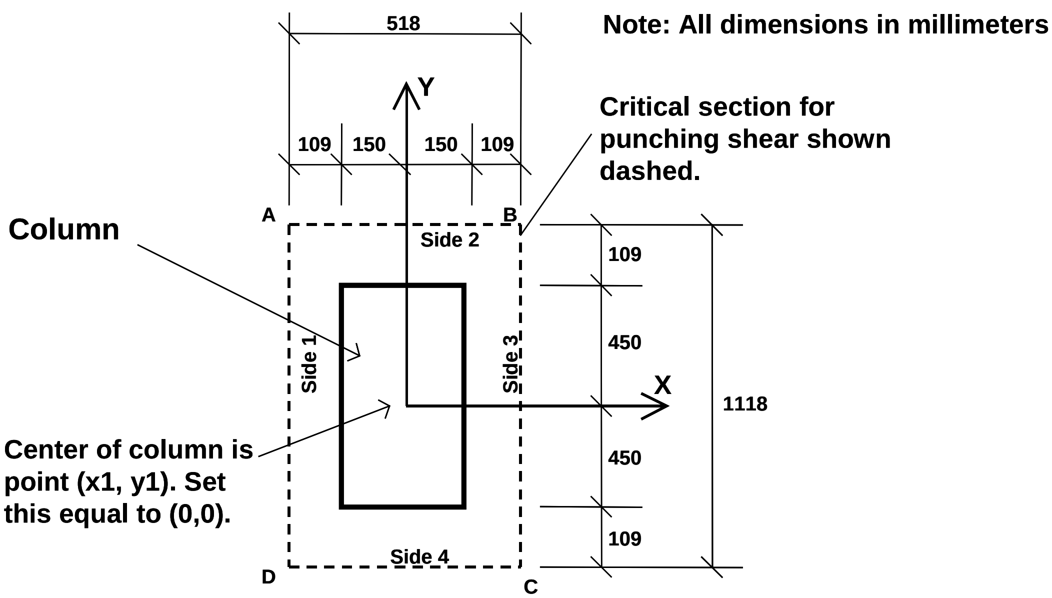


Figure 2: Interior Column, Grid B-2 in SAFE Model

$$\gamma_{V2} = 1 - \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{1118}{518}}} = 0.495$$

$$\gamma_{V3} = 1 - \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{518}{1118}}} = 0.312$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–259	0	259	0	N.A.
y_2	0	559	0	–559	N.A.
L	1118	518	1118	518	b_0 = 3272
d	218	218	218	218	N.A.
Ld	243724	112924	243724	112924	713296
Ldx_2	–63124516	0	63124516	0	0
Ldy_2	0	63124516	0	–63124516	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{713296} = 0\,mm$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{713296} = 0\,mm$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	1118	518	1118	518	N.A.
d	218	218	218	218	N.A.
$x_2 - x_3$	–259	0	259	0	N.A.
$y_2 - y_3$	0	559	0	–559	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	2.64E+10	3.53E+10	2.64E+10	3.53E+10	1.23E+11
I_{YY}	1.63E+10	2.97E+09	1.63E+10	2.97E+09	3.86E+10
I_{XY}	0	0	0	0	0

From the SAFE output at Grid B-2:

V_f = 1126.498 kN

$\gamma_{V2} M_{f,2}$ = –25.725 kN-m

$\gamma_{V3} M_{f,3}$ = 14.272 kN-m

At the point labeled A in Figure 2, $x_4 = -259$ and $y_4 = 559$, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(559-0) - (0)(-259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(-259-0) - (0)(559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

v_f = 1.5793 – 0.1169 – 0.0958 = **1.3666 N/mm^2** at point A

At the point labeled B in Figure 2, $x_4 = 259$ and $y_4 = 559$, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(559-0) - (0)(259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(259-0) - (0)(559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

v_f = 1.5793 – 0.1169 + 0.0958 =**1.5582 N/mm^2** at point B

At the point labeled C in Figure 2, $x_4 = 259$ and $y_4 = -559$, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(-559-0) - (0)(259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(259-0) - (0)(-559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

v_f = 1.5793 + 0.1169 + 0.0958 = **1.792 N/mm^2** at point C

At the point labeled D in Figure 2, $x_4 = -259$ and $y_4 = -559$, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(-559-0) - (0)(-259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(-259-0) - (0)(-559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

v_f = 1.5793 + 0.1169 – 0.0958 = **1.6004 N/mm^2** at point D

Point C has the largest absolute value of v_u, thus v_{max} = **1.792 N/mm^2**

The shear capacity is calculated based on the minimum of the following three limits:

$$v_v = \min \begin{cases} \phi_c \left(1 + \dfrac{2}{\beta_c}\right) 0.19 \lambda \sqrt{f'_c} \\ \phi_c \left(0.19 + \dfrac{\alpha_s d}{b_0}\right) \lambda \sqrt{f'_c} \\ \phi_c 0.38 \lambda \sqrt{f'_c} \end{cases} \qquad 1.127 \text{ N/mm}^2 \text{ in accordance with CSA 13.3.4.1}$$

CSA 13.3.4.1 yields the smallest value of $\boxed{v_v = 1.127 \text{ N/mm}^2}$, and thus this is the shear capacity.

$$\boxed{\text{Shear Ratio} = \frac{v_U}{\varphi v_v} = \frac{1.792}{1.127} = 1.59}$$

EXAMPLE CSA 23.3-04 PT-SL-001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm as shown in shown in Figure 1.

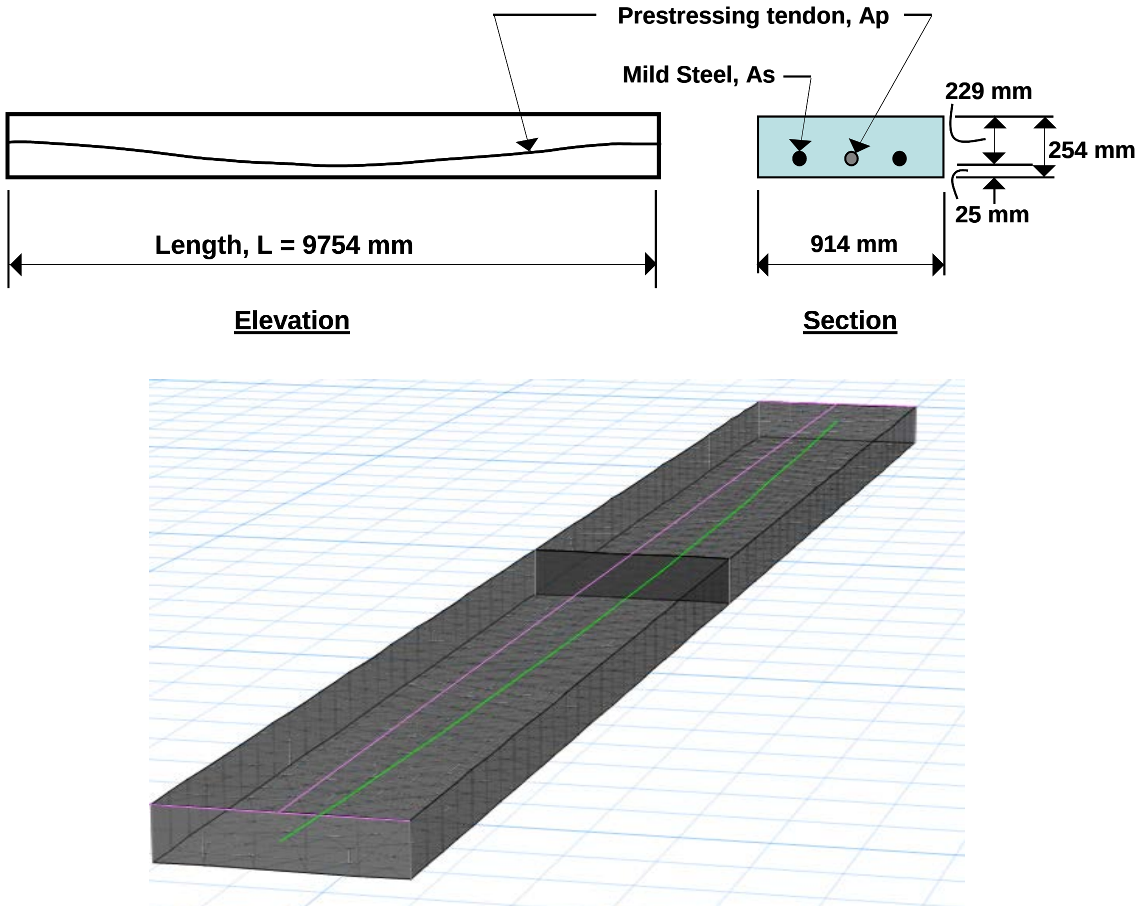


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 KN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the midspan of the slab. Independent hand calculations have been compared with the SAFE results and summarized for verification and validation of the SAFE results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	KN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	KN/m^2
Live load	w_l	=	4.788	KN/m^2

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	159.4	159.4	0.00%
Area of Mild Steel req'd, As (sq-cm)	16.25	16.32	0.43%
Transfer Conc. Stress, top ($D+PT_I$), MPa	–5.058	–5.057	0.02%
Transfer Conc. Stress, bot ($D+PT_I$), MPa	2.839	2.839	0.00%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	–10.460	–10.465	0.05%
Normal Conc. Stress, bot ($D+L+PT_F$), MPa	8.402	8.407	0.06%
Long-Term Conc. Stress, top ($D+0.5L+PT_{F(L)}$), MPa	–7.817	–7.817	0.00%
Long-Term Conc. Stress, bot ($D+0.5L+PT_{F(L)}$), MPa	5.759	5.759	0.00%

COMPUTER FILE: CSA A23.3-04 PT-SL-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATIONS:

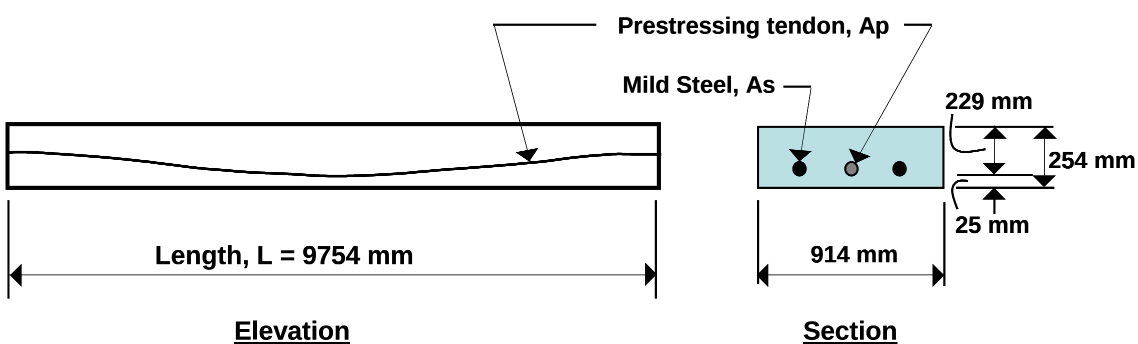
Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
f_{cu} = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\phi_c = 0.65,\ \ \phi_S = 0.85$

$\alpha_1 = 0.85 - 0.0015f'_c \geq 0.67 = 0.805$

$\beta_1 = 0.97 - 0.0025f'_c \geq 0.67 = 0.895$



Loads:

Dead, self-wt = 0.254 m x 23.56 kN/m^3 = 5.984 kN/m^2 (D) x 1.25 = 7.480 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) x 1.50 = 7.182 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 14.662 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 x 0.914m = 9.846 kN/m, ω_u = 16.039 kN/m^2 x 0.914m = 13.401 kN/m

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8}$ = 13.401 x (9.754)^2/8 = 159.42 kN-m

Ultimate Stress in strand, $f_{pb} = f_{pe} + \frac{8000}{l_o}\left(d_p - c_y\right)$

$$c_y = \frac{\phi_p A_p f_{pr} + \phi_s A_s f_y}{\alpha_1 \phi_c f'_c \beta_1 b} = \frac{0.9(197)(1347) + 0.85(1625)(400)}{0.805(0.65)(30.0)(0.895)(914)} = 61.66 \text{ mm}$$

$$f_{pb} = 1210 + \frac{8000}{9754}(229 - 61.66) = 1347 \text{ MPa}$$

Depth of the compression block, *a*, is given as:

Stress block depth, $a = d - \sqrt{d^2 - \frac{2M^*}{\alpha_1 f'_c \phi_c b}}$

$$= 0.229 - \sqrt{0.229^2 - \frac{2(159.42)}{0.805(30000)(0.65)(0.914)}} = 55.18$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197(1347)/1000 = 265.9 \text{ kN}$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right)\phi = 265.9\left(0.229 - \frac{55.18}{2}\right)(0.85) = 45.52 \text{ kN-m}$$

Net Moment to be resisted by As, $M_{NET} = M_U - M_{PT}$

$$= 159.42 - 45.52 = 113.90 \text{ kN-m}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_{NET}}{0.87 f_y z} = \frac{113.90}{0.87(400)\left(229 - \frac{55.18}{2}\right)}(1e6) = 1625 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (D+PT$_i$) = 1.0D+0.0L+1.0PT$_I$

Tendon stress at transfer = jacking stress − stressing losses = 1490 − 186 = 1304 MPa
The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4 \text{ kN}$

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102\text{ mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-257.4}{0.254(0.914)} \pm \dfrac{65.04 - 26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.109 \pm 3.948 \text{ MPa}$$
$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT_F) = 1.0D+1.0L+1.0PT_F

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT_F),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$
$$f = -1.029 \pm 9.431$$
$$f = -10.460(\text{Comp})\max,\ 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: (D+0.5L+$PT_{F(L)}$) = 1.0D+0.5L+1.0PT_F

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+0.5L+$PT_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$
$$f = -1.029 \pm 6.788$$
$$f = -7.817(\text{Comp})\max,\ 5.759(\text{Tension})\max$$

EXAMPLE CSA A23.3-04 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

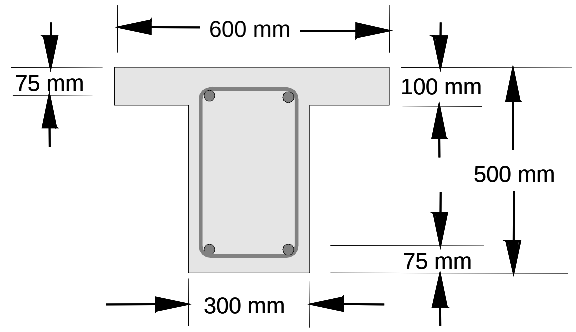
The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by CSA A23.3-04.
- The average shear stress in the beam is below the maximum shear stress allowed by CSA A23.3-04, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T-beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading. One dead load case (DL30) and one live load case (LL100) with only symmetric third-point loads of magnitudes 30, and 100 kN, respectively, are defined in the model. One load combinations (COMB100) is defined using the CSA A23.3-04 load combination factors of 1.25 for dead loads and 1.5 for live loads. The model is analyzed for both of these load cases and the load combinations.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results. These moment and shear force are identical. After completing the analysis, design is performed using the CSA A23.3-04 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.



Beam Section

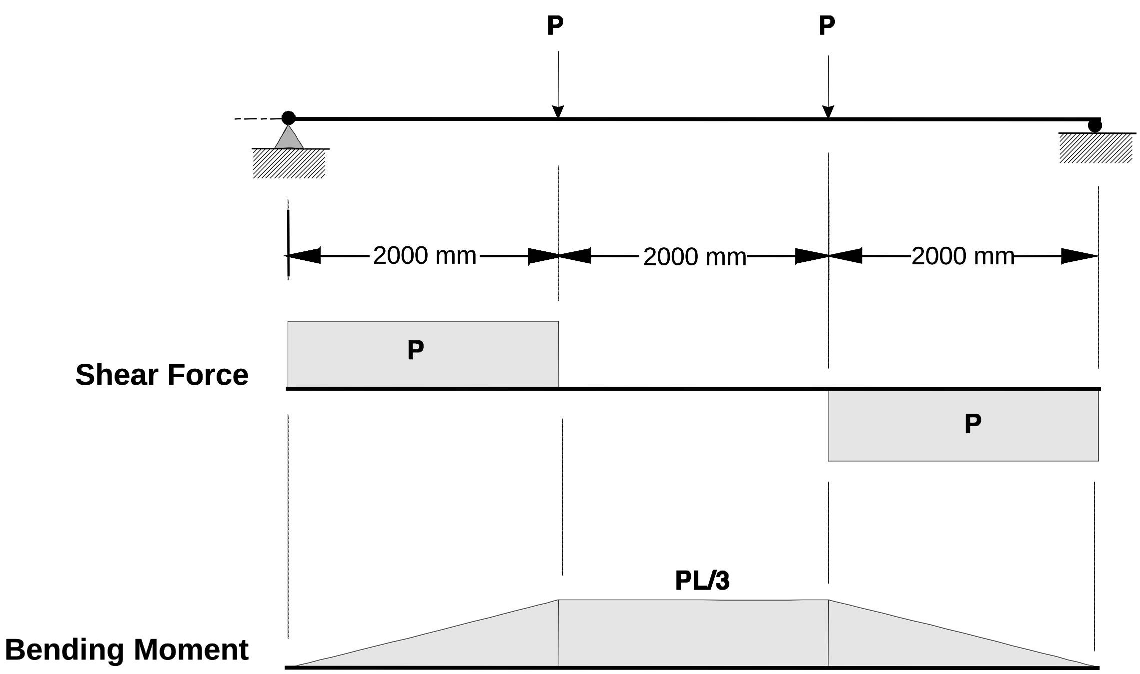


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span,	l	=	6000	mm
Overall depth,	h	=	500	mm
Flange thickness,	d_s	=	100	mm
Width of web,	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.,	d_c	=	75	mm
Effective depth,	d	=	425	mm
Depth of comp. reinf.,	d'	=	75	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	$25x10^5$	MPa
Modulus of elasticity,	E_s	=	$2x10^8$	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	30	kN
Live load,	P_l	=	100	kN

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the design reinforcement comparison.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
SAFE	375	25.844
Calculated	375	25.844

$A^+_{s,min}$ = 535.82 sq-m

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-cm/m)	
	SAFE	**Calculated**
187.5	12.573	12.573

COMPUTER FILE: CSA A23.3-04 RC-BM-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

ϕ_c = 0.65 for concrete

ϕ_s = 0.85 for reinforcement

$$A_{s,\min} = \frac{0.2\sqrt{f'_c}}{f_y}\, b_w\, h = 357.2 \text{ sq-mm}$$

$$\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67 = 0.805$$

$$\beta_1 = 0.97 - 0.0025 f'_c \geq 0.67 = 0.895$$

$$c_b = \frac{700}{700 + f_y} d = 256.46 \text{ mm}$$

$$a_b = \beta_1 c_b = 229.5366 \text{ mm}$$

$$A_s = \min[A_{s,\min}, (4/3)\, A_{s,\text{required}}] = \min[357.2, (4/3)2445] = 357.2 \text{ sq-mm}$$

COMB100

$$P = (1.25 P_d + 1.5 P_t) = 187.5\text{kN}$$

$$M^* = \frac{Pl}{3} = 375 \text{ kN-m}$$

$$M_f = 375 \text{ kN-m}$$

The depth of the compression block is given by:

$$C_f = \alpha_1 f'_c \left(b_f - b_w\right) \min\left(h_s, a_b\right) = 724.5 \text{ kN}$$

Therefore, $A_{s1} = \dfrac{C_f \phi_c}{f_y \phi_s}$ and the portion of M_f that is resisted by the flange is given by:

$$A_{s1} = \frac{C_f \phi_c}{f_y \phi_s} = 1204.411 \text{ sq-mm}$$

$$M_{ff} = C_f \left(d - \frac{\min(h_s, a_b)}{2} \right) \phi_c = 176.596 \text{ kN-m}$$

Therefore, the balance of the moment, M_f to be carried by the web is:

$$M_{fw} = M_f - M_{ff} = 198.403 \text{ kN-m}$$

The web is a rectangular section with dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{fw}}{\alpha_1 f'_c \phi_c b_w}} = 114.5745 \text{ mm}$$

If $a_1 \le a_b$, the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{fw}}{\phi_s f_y \left(d - \frac{a_1}{2} \right)} = 1379.94 \text{ sq-mm}$$

$A_s = A_{s1} + A_{s2} = 2584.351$ sq-mm

Shear Design

The basic shear strength for rectangular section is computed as,

$\phi_c =$ 0.65 for shear

$\lambda = \{1.00,$ for normal density concrete

d_v is the effective shear depth. It is taken as the greater of $0.9d$ or $0.72h$ = 382.5 mm (governing) or 360 mm.

$S_{ze} = 300$ if minimum transverse reinforcement

$$\varepsilon_x = \frac{M_f / d_v + V_f + 0.5N_f}{2(E_s A_s)} \text{ and } \varepsilon_x \le 0.003$$

$$\beta = \frac{0.40}{(1+1500\varepsilon_x)} \bullet \frac{1300}{(1000+S_{ze})} = 0.07272$$

$$V_c = \phi_c \lambda \beta \sqrt{f'_c} b_w d_v = 29.708 \text{ kN}$$

$$V_{r,\max} = 0.25\phi_c f'_c b_w d = 621.56 \text{ kN}$$

θ = 50

$$\frac{A_v}{s} = \frac{(V_f - V_c)\tan\theta}{\phi_s f_{yt} d_v} = 1.2573 \text{ mm}^2\text{/mm} = 12.573 \text{ cm}^2\text{/m.}$$

EXAMPLE CSA A23.3-04 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

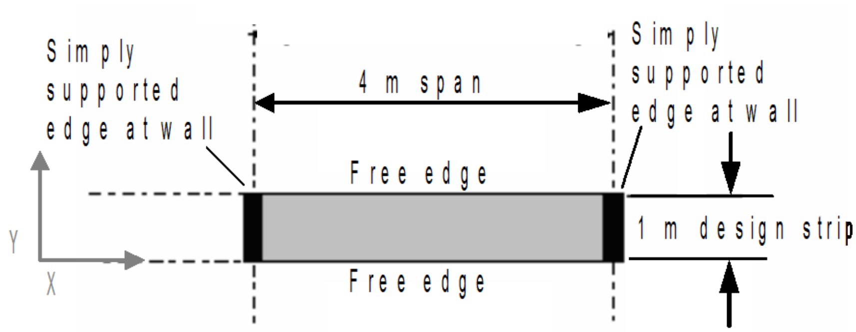


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the CSA A23.3-04 load combination factors, 1.25 for dead loads and 1.5 for live loads. The model is analyzed for these load cases and load combinations.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing the analysis, design is performed using the CSA A23.3-04 code by SAFE and also by hand computation. Table 1 show the comparison of the design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	$2x10^6$	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	SAFE	25.00	5.414
	Calculated	25.00	5.528

$A^+_{s,min}$ = 357.2 sq-mm

COMPUTER FILE: CSA A23.3-04 RC-SL-001.FDB

CONCLUSION

The SAFE results show a very close comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

ϕ_c = 0.65 for concrete

ϕ_s = 0.85 for reinforcement

$$A_{s,\min} = \frac{0.2\sqrt{f'_c}}{f_y} b_w h = 357.2 \text{ sq-mm}$$

b = 1000 mm

$\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67 = 0.805$

$\beta_1 = 0.97 - 0.0025 f'_c \geq 0.67 = 0.895$

$$c_b = \frac{700}{700 + f_y} d = 75.43 \text{ mm}$$

$a_b = \beta_1 c_b$ = 67.5 mm

For the load combination, w and M^* are calculated as follows:

$w = (1.25 w_d + 1.5 w_t)\, b$

$$M_u = \frac{w l_1^2}{8}$$

$A_s = \min[A_{s,\min}, (4/3)\, A_{s,\text{required}}] = \min[357.2, (4/3)540.63]$ = 357.2 sq-mm

= 0.22•(150/125)^2•0.6•SQRT(30)/460•100•125

= 282.9 sq-mm

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 12.5 kN/m

$M_{f\text{-}strip}$ = 25.0 kN-m

$M_{f\text{-}design}$ = 25.529 kN-m

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_f\right|}{\alpha_1 f'_c \phi_c b}} = 13.769 \text{ mm} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_f}{\phi_s f_y \left(d - \frac{a}{2}\right)} = 552.77 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.528 sq-cm

EXAMPLE Eurocode 2-04 PT-SL-001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm as shown in shown in Figure 1.

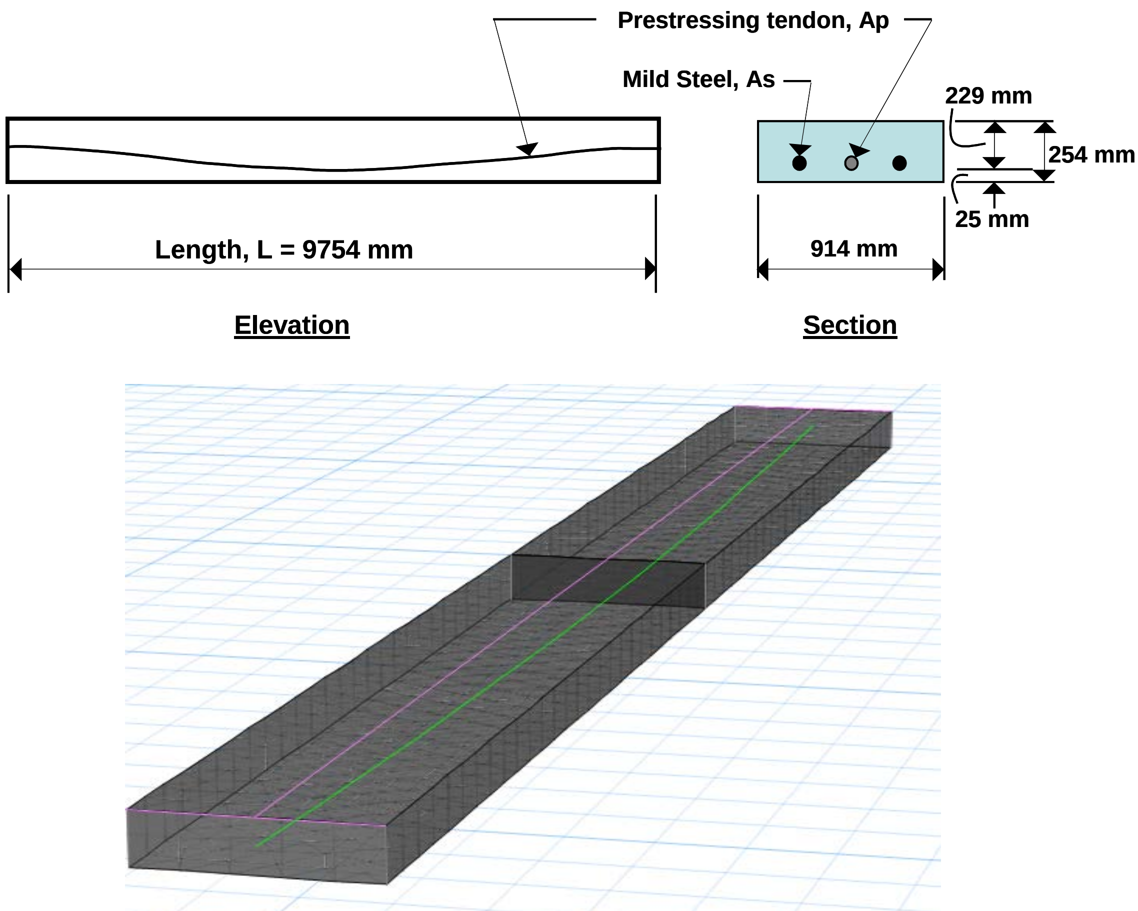


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, was added to the A-Strip. The self weight and live loads have been added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the SAFE results and summarized for verification and validation of the SAFE results.

Geometry, Properties and Loading

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	KN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	KN/m^2
Live load	w_l	=	4.788	KN/m^2

Technical Features of SAFE Tested

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

Results Comparison

Table 1 shows the comparison of the SAFE total factored moments, required mild steel reinforcing, and slab stresses with independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	166.41	166.41	0.00%
Transfer Conc. Stress, top ($D+PT_I$), MPa	–5.057	–5.057	0.00%
Transfer Conc. Stress, bot ($D+PT_I$), MPa	2.839	2.839	0.00%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	–10.460	–10.465	0.05%
Normal Conc. Stress, bot ($D+L+PT_F$), MPa	8.402	8.407	0.06%
Long-Term Conc. Stress, top ($D+0.5L+PT_{F(L)}$), MPa	–7.817	–7.817	0.00%
Long-Term Conc. Stress, bot ($D+0.5L+PT_{F(L)}$), MPa	5.759	5.759	0.00%

Table 2 Comparison of Design Moments and Reinforcements

National Annex	Method	Design Moment (kN-m)	Reinforcement Area (sq-cm)
			A_s^+
CEN Default, Norway, Slovenia and Sweden	SAFE	166.41	15.39
	Calculated	166.41	15.36
Finland , Singapore and UK	SAFE	166.41	15.89
	Calculated	166.41	15.87
Denmark	SAFE	166.41	15.96
	Calculated	166.41	15.94

COMPUTER FILE: EUROCODE 2-04 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

Design Parameters:

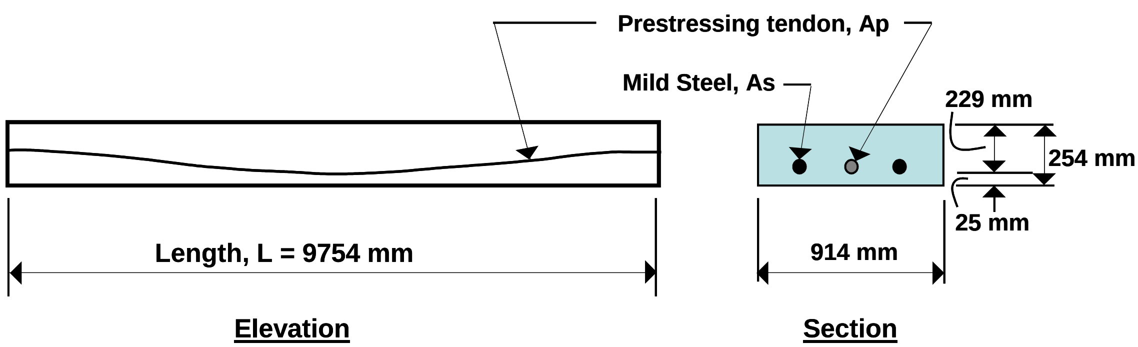
Mild Steel Reinforcing	Post-Tensioning
$f'c$ = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\gamma_{m,\,steel} = 1.15$

$\gamma_{m,\,concrete} = 1.50$

$\eta = 1.0$ for $f_{ck} \leq 50$ MPa

$\lambda = 0.8$ for $f_{ck} \leq 50$ MPa



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.35 = 8.078 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) × 1.50 = 7.182 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 15.260 kN/m^2 (D+L)ult

ω = 10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 15.260 kN/m^2 × 0.914 m = 13.948 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8} = 13.948 \times (9.754)^2 / 8 = 165.9$ kN-m

Ultimate Stress in strand, $f_{PS} = f_{SE} + 7000d\left(1 - 1.36\frac{f_{PU}A_P}{f_{CK}bd}\right)\Big/ l$

$$= 1210 + 7000(229)\left(1 - 1.36\frac{1862(198)}{30(914)(229)}\right)\Big/(9754)$$

$$= 1361\,\text{MPa}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 2(99)(1361)/1000 = 269.5\,\text{kN}$

CEN Default, Norway, Slovenia and Sweden:
Design moment M = 166.4122 kN-m

Compression block depth ratio: $m = \dfrac{M}{bd^2\eta f_{cd}}$

$$= \frac{166.4122}{(0.914)(0.229)^2(1)(30000/1.50)} = 0.1736$$

Required area of mild steel reinforcing,

$$\omega = 1 - \sqrt{1 - 2m} = 1 - \sqrt{1 - 2(0.1736)} = 0.1920$$

$$A_{EquivTotal} = \omega\left(\frac{\eta f_{cd}bd}{f_{yd}}\right) = 0.1920\left(\frac{1(30/1.5)(914)(229)}{400/1.15}\right) = 2311\,\text{mm}^2$$

$$A_{EquivTotal} = A_P\left(\frac{1361}{400/1.15}\right) + A_S = 2311\,\text{mm}^2$$

$$A_S = 2311 - 198\left(\frac{1361}{400/1.15}\right) = 1536\,\text{mm}^2$$

Finland, Singapore and UK:
Design moment M = 166.4122 kN-m

Compression block depth ratio: $m = \dfrac{M}{bd^2\eta f_{cd}}$

$$= \frac{166.4122}{(0.914)(0.229)^2(0.85)(30000/1.50)} = 0.2042$$

Required area of mild steel reinforcing,

$$\omega = 1 - \sqrt{1 - 2m} = 1 - \sqrt{1 - 2(0.2042)} = 0.23088$$

$$A_{EquivTotal} = \omega\left(\frac{\eta\, f_{cd} bd}{f_{yd}}\right) = 0.23088\left(\frac{0.85(30/1.5)(914)(229)}{400/1.15}\right) = 2362 \text{ mm}^2$$

$$A_{EquivTotal} = A_P\left(\frac{1361}{400/1.15}\right) + A_S = 2362 \text{ mm}^2$$

$$A_S = 2362 - 198\left(\frac{1361}{400/1.15}\right) = 1587 \text{ mm}^2$$

Denmark:

Design moment M = 166.4122 kN-m

Compression block depth ratio: $m = \dfrac{M}{bd^2 \eta f_{cd}}$

$$= \frac{166.4122}{(0.914)(0.229)^2(1.0)(30000/1.45)} = 0.1678$$

Required area of mild steel reinforcing,

$$\omega = 1 - \sqrt{1-2m} = 1 - \sqrt{1-2(0.1678)} = 0.1849$$

$$A_{EquivTotal} = \omega\left(\frac{\eta\, f_{cd} bd}{f_{yd}}\right) = 0.1849\left(\frac{1.0(30/1.45)(914)(229)}{400/1.20}\right) = 2402 \text{ mm}^2$$

$$A_{EquivTotal} = A_P\left(\frac{1361}{400/1.2}\right) + A_S = 2402 \text{ mm}^2$$

$$A_S = 2402 - 198\left(\frac{1361}{400/1.2}\right) = 1594 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination ($D+PT_i$) = $1.0D+0.0L+1.0PT_I$

Tendon stress at transfer = jacking stress − stressing losses =1490 − 186 = 1304 MPa

The force in the tendon at transfer = $1304(197.4)/1000 = 257.4$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-257.4}{0.254(0.914)} \pm \dfrac{65.04 - 26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.109 \pm 3.948 \text{ MPa}$$
$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT$_F$) = 1.0D+1.0L+1.0PT$_F$

Tendon stress at normal = jacking – stressing – long-term=1490 – 186 – 94 = 1210 MPa
The force in tendon at normal = $1210(197.4)/1000 = 238.9$ kN
Moment due to dead load $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m
Moment due to live load $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m
Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT$_F$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$
$$f = -1.029 \pm 9.431$$
$$f = -10.460(\text{Comp})\max,\ 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: (D+0.5L+PT$_{F(L)}$) = 1.0D+0.5L+1.0PT$_F$

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa
The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN
Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m
Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m
Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+0.5L+PT$_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$
$$f = -1.029 \pm 6.788$$
$$f = -7.817(\text{Comp})\max,\ 5.759(\text{Tension})\max$$

EXAMPLE Eurocode 2-04 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

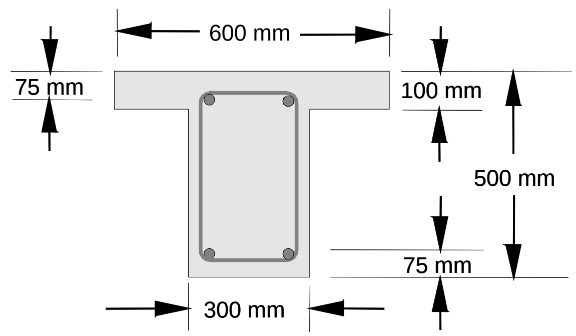
The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by Eurocode 2-04.
- The average shear stress in the beam is below the maximum shear stress allowed by Eurocode 2-04, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T-beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading One dead load case (DL30) and one live load case (LL130) with only symmetric third-point loads of magnitudes 30, and 130 kN, respectively, are defined in the model. One load combinations (COMB130) is defined using the Eurocode 2-04 load combination factors of 1.35 for dead loads and 1.5 for live loads. The model is analyzed for both of these load cases and the load combinations.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results. These moment and shear force are identical. After completing the analysis, design is performed using the Eurocode 2-04 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.



Beam Section

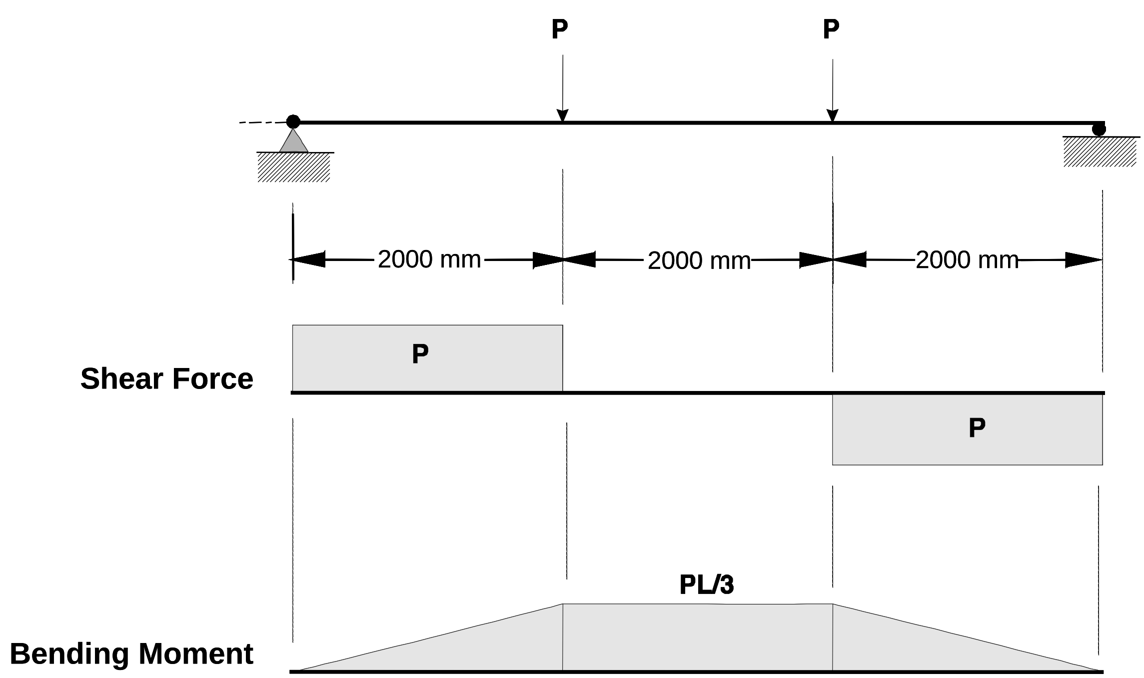


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span,	l	=	6000	mm
Overall depth,	h	=	500	mm
Flange thickness,	d_s	=	100	mm
Width of web,	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.,	d_c	=	75	mm
Effective depth,	d	=	425	mm
Depth of comp. reinf.,	d'	=	75	mm
Concrete strength,	f'_{ck}	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	$25x10^5$	MPa
Modulus of elasticity,	E_s	=	$2x10^8$	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	30	kN
Live load,	P_l	=	130	kN

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Moments and Flexural Reinforcements

National Annex	Method	Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
CEN Default, Norway, Slovenia and Sweden	SAFE	471	31.643
	Calculated	471	31.643
Finland , Singapore and UK	SAFE	471	32.98
	Calculated	471	32.98
Denmark	SAFE	471	32.83
	Calculated	471	32.83

$A^{+}_{s,\min}$ = 2.09 sq-cm

Table 2 Comparison of Shear Reinforcements

National Annex	Method	Shear Force (kN)	Reinforcement Area , $\frac{A_v}{s}$ (sq-cm/m) A_s^+
CEN Default, Norway, Slovenia and Sweden	SAFE	235.5	6.16
	Calculated	235.5	6.16
Finland , Singapore and UK	SAFE	235.5	6.16
	Calculated	235.5	6.16
Denmark	SAFE	235.5	6.42
	Calculated	235.5	6.42

COMPUTER FILE: Eurocode 2-04 RC-BM-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for both of the load combinations:

$\gamma_s = 1.15$

$\gamma_c = 1.50$

$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c$

$f_{yd} = f_{yk} / \gamma_s$

$\eta = 1.0$ for $f_{ck} \leq 50$ MPa

$\lambda = 0.8$ for $f_{ck} \leq 50$ MPa

$$A_{s,\min} = 0.26 \frac{f_{ctm}}{f_{yk}} bd = 208.73 \text{ sq-mm}$$

$$A_{s,\min} = 0.0013 b_w h = 195.00 \text{ sq-mm}$$

For CEN Default, Norway, Slovenia and Sweden—COMB130:

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$

$\alpha_{cc} = 1.0$

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta f_{cd}} = \frac{471 \bullet 10^6}{600 \bullet 425^2 \bullet 1.0 \bullet 1.0 \bullet 30 / 1.5} = 0.217301$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

$k_1 = 0.44$

$k_2 = k_4 = 1.25(0.6 + 0.0014/\varepsilon_{cu2}) = 1.25$

δ is assumed to be 1

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa} = 0.448$$

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1-\frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.29417$$

$$\omega_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim} = 1-\sqrt{1-2m_{\lim}} = 0.3584$$

$$a_{max} = \omega_{\lim} d = 152.32 \text{ mm}$$

$$\omega = 1 - \sqrt{1-2m} = 0.24807$$

$$a = \omega d = 105.4299 \text{ mm} \leq a_{max}$$

$$A_{s2} = \frac{\left(b_f - b_w\right) h_f \eta\, f_{cd}}{f_{yd}} = 1500 \text{ sq-mm}$$

$$M_2 = A_{s2} f_{yd}\left(d - \frac{h_f}{2}\right) = 225 \text{ kN-m}$$

$$M_1 = M - M_2 = 246 \text{ kN-m}$$

$$m_1 = \frac{M_1}{b_w d^2 \eta\, f_{cd}} = 0.2269896 \leq m_{\lim}$$

$$\omega_1 = 1-\sqrt{1-2m_1} = 0.2610678$$

$$A_{s1} = \omega_1\left[\frac{\eta\, f_{cd} b_w d}{f_{yd}}\right] = 1664.304 \text{ sq-mm}$$

$$A_s = A_{s1} + A_{s2} = 3164.307 \text{ sq-mm}$$

For Singapore and UK—COMB130:

$$\gamma_{m,\,steel} = 1.15$$

$$\gamma_{m,\,concrete} = 1.50$$

$$\alpha_{cc} = 0.85$$

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta\, f_{cd}} = \frac{471 \bullet 10^6}{600 \bullet 425^2 \bullet 1.0 \bullet 0.85 \bullet 30/1.5} = 0.255648$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

$$k_1 = 0.40$$

$k_2 = (0.6 + 0.0014/\varepsilon_{cu2}) = 1.00$

δ is assumed to be 1

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa} = 0.60$$

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.3648$$

$$\omega_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim} = 1 - \sqrt{1 - 2m_{\lim}} = 0.48$$

$a_{max} = \omega_{\lim} d = 204$ mm

$\omega = 1 - \sqrt{1 - 2m} = 0.300923$

$a = \omega d = 127.8939 \text{ mm} \leq a_{max}$

$$A_{s2} = \frac{\left(b_f - b_w\right) h_f \eta\, f_{cd}}{f_{yd}} = 1275 \text{ sq-mm}$$

$$M_2 = A_{s2} f_{yd}\left(d - \frac{h_f}{2}\right) = 191.25 \text{ kN-m}$$

$M_1 = M - M_2 = 279.75$ kN-m

$$m_1 = \frac{M_1}{b_w d^2 \eta\, f_{cd}} = 0.30368 \leq m_{\lim}$$

$$\omega_1 = 1 - \sqrt{1 - 2m_1} = 0.37339$$

$$A_{s1} = \omega_1\left[\frac{\eta f_{cd} b_w d}{f_{yd}}\right]$$

$$= 0.37339\left[\frac{1.0 \bullet \frac{0.85 \bullet 30}{1.5} \bullet 300 \bullet 425}{400}\right] = 2023.307 \text{ sq-mm}$$

$A_s = A_{s1} + A_{s2} = 3298.31$ sq-mm

For Finland—COMB130:

$\gamma_{m,\, steel} = 1.15$

$\gamma_{m,\ concrete}$ = 1.50

α_{cc} = 0.85

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta f_{cd}} = \frac{471 \bullet 10^6}{600 \bullet 425^2 \bullet 1.0 \bullet 0.85 \bullet 30/1.5} = 0.255648$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

$k_1 = 0.44$

$k_2 = 1.10$

δ is assumed to be 1

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa} = 0.5091$$

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.3243$$

$$\omega_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim} = 1 - \sqrt{1 - 2m_{\lim}} = 0.40728$$

$a_{\max} = \omega_{\lim} d = 173.094$ mm

$\omega = 1 - \sqrt{1 - 2m} = 0.300923$

$a = \omega d = 127.8939 \text{ mm} \leq a_{\max}$

$$A_{s2} = \frac{\left(b_f - b_w\right) h_f \eta\, f_{cd}}{f_{yd}} = 1275 \text{ sq-mm}$$

$$M_2 = A_{s2} f_{yd}\left(d - \frac{h_f}{2}\right) = 191.25 \text{ kN-m}$$

$M_1 = M - M_2 = 279.75$ kN-m

$$m_1 = \frac{M_1}{b_w d^2 \eta\, f_{cd}} = 0.30368 \leq m_{\lim}$$

$$\omega_1 = 1 - \sqrt{1 - 2m_1} = 0.37339$$

$$A_{s1} = \omega_1\left[\frac{\eta f_{cd} b_w d}{f_{yd}}\right]$$

$$= 0.37339\left[\frac{1.0 \bullet \frac{0.85 \bullet 30}{1.5} \bullet 300 \bullet 425}{400}\right] = 2023.307 \text{ sq-mm}$$

$A_s = A_{s1} + A_{s2} = 3298.31$ sq-mm

For Denmark—COMB130:

$\gamma_{m,\,steel} = 1.20$

$\gamma_{m,\,concrete} = 1.45$

$\alpha_{cc} = 1.0$

The depth of the compression block is given by:

$$m = \frac{M}{bd^2\eta f_{cd}} = \frac{471 \bullet 10^6}{600 \bullet 425^2 \bullet 1.0 \bullet 1.0 \bullet 30/1.45} = 0.210058$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

$k_1 = 0.44$

$k_2 = k_4 = 1.25(0.6 + 0.0014/\varepsilon_{cu2}) = 1.25$

δ is assumed to be 1

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa} = 0.448$$

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.29417$$

$$\omega_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim} = 1 - \sqrt{1 - 2m_{\lim}} = 0.3584$$

$a_{\max} = \omega_{\lim} d = 152.32$ mm

$\omega = 1 - \sqrt{1 - 2m} = 0.238499$

$a = \omega d = 101.3620 \text{ mm} \leq a_{\max}$

$$A_{s2} = \frac{\left(b_f - b_w\right)h_f \eta\, f_{cd}}{f_{yd}} = 1619.19 \text{ sq-mm}$$

$$M_2 = A_{s2} f_{yd}\left(d - \frac{h_f}{2}\right) = 232.76 \text{ kN-m}$$

$$M_1 = M - M_2 = 238.24 \text{ kN-m}$$

$$m_1 = \frac{M_1}{b_w d^2 \eta\, f_{cd}} = 0.21250 \le m_{\lim}$$

$$\omega_1 = 1 - \sqrt{1 - 2m_1} = 0.241715$$

$$A_{s1} = \omega_1 \left[\frac{\eta\, f_{cd} b_w d}{f_{yd}}\right] = 1663.37 \text{ sq-mm}$$

$$A_s = A_{s1} + A_{s2} = 3282.56 \text{ sq-mm}$$

Shear Design

For CEN Default, Finland, Singapore, Slovenia and UK

$$C_{Rd,c} = 0.18/\gamma_c = 0.18/1.5 = 0.12$$

For Denmark

$$C_{Rd,c} = 0.18/\gamma_c = 0.18/1.45 = 0.124$$

For Sweden and Norway

$$C_{Rd,c} = 0.15/\gamma_c = 0.15/1.5 = 0.10$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1.686 \le 2.0 \text{ with } d \text{ in mm}$$

$$\rho_1 = 0.0$$

$$\sigma_{cp} = N_{Ed}/A_c < 0.2\, f_{cd} = 0.0 \text{ MPa}$$

For CEN Default, Denmark, Norway, Singapore, Slovenia, Sweden and UK:

$$v_{\min} = 0.035 k^{3/2} f_{ck}^{\,1/2} = 0.419677$$

For Finland:

$$v_{\min} = 0.035 k^{2/3} f_{ck}^{\,1/2} = 0.271561$$

$$V_{Rd,c} = \left[C_{Rd,c} k \left(100 \rho_1 f_{ck}\right)^{1/3} + k_1 \sigma_{cp}\right] b_w d = 34.62 \text{ kN for Finland}$$

$V_{Rd,c} = \left[C_{Rd,c}k\left(100\rho_1 f_{ck}\right)^{1/3} + k_1\sigma_{cp}\right]b_w d$ = 53.5 kN for all other NA

$\alpha_{cw} = 1$

$$\nu_1 = 0.6\left(1 - \frac{f_{ck}}{250}\right) = 0.528$$

$z = 0.9d = 382.5$ mm

θ is taken as 1.

$$V_{Rd,\max} = \frac{\alpha_{cw} b_w z \nu_1 f_{cd}}{\cot\theta + \tan\theta} = 1253.54 \text{ kN for Denmark}$$

$$V_{Rd,\max} = \frac{\alpha_{cw} b_w z \nu_1 f_{cd}}{\cot\theta + \tan\theta} = 1211.76 \text{ kN for all other NA}$$

$V_{R,dc} < V_{Ed} \le V_{Rd,\max}$ (govern)

Computing the angle using v_{Ed} :

$$v_{Ed} = \frac{235.5 \bullet 10^3}{0.9 \bullet 425 \bullet 300} = 2.0522$$

$$\theta = 0.5\sin^{-1}\frac{v_{Ed}}{0.2 f_{ck}\left(1 - f_{ck}/250\right)}$$

$$\theta = 0.5\sin^{-1}\frac{2.0522}{0.2 \bullet 30\left(1 - 30/250\right)} = 11.43°$$

$21.8° \le \theta \le 45°$, therefore use $\theta = 21.8°$

$$\frac{A_{sw}}{s} = \frac{v_{Ed} b_w}{f_{ywd}\cot\theta}$$

$$\frac{A_{sw}}{s} = \frac{2.0522 \bullet 300}{460/1.20 \bullet 2.5} = 0.64243 \text{ sq-mm/m} = 6.42 \text{ sq-cm/m for Denmark}$$

$$\frac{A_{sw}}{s} = \frac{2.0522 \bullet 300}{460/1.15 \bullet 2.5} = 0.61566 \text{ sq-mm/m} = 6.16 \text{ sq-cm/m for all other NA}$$

EXAMPLE EUROCODE 2-04 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

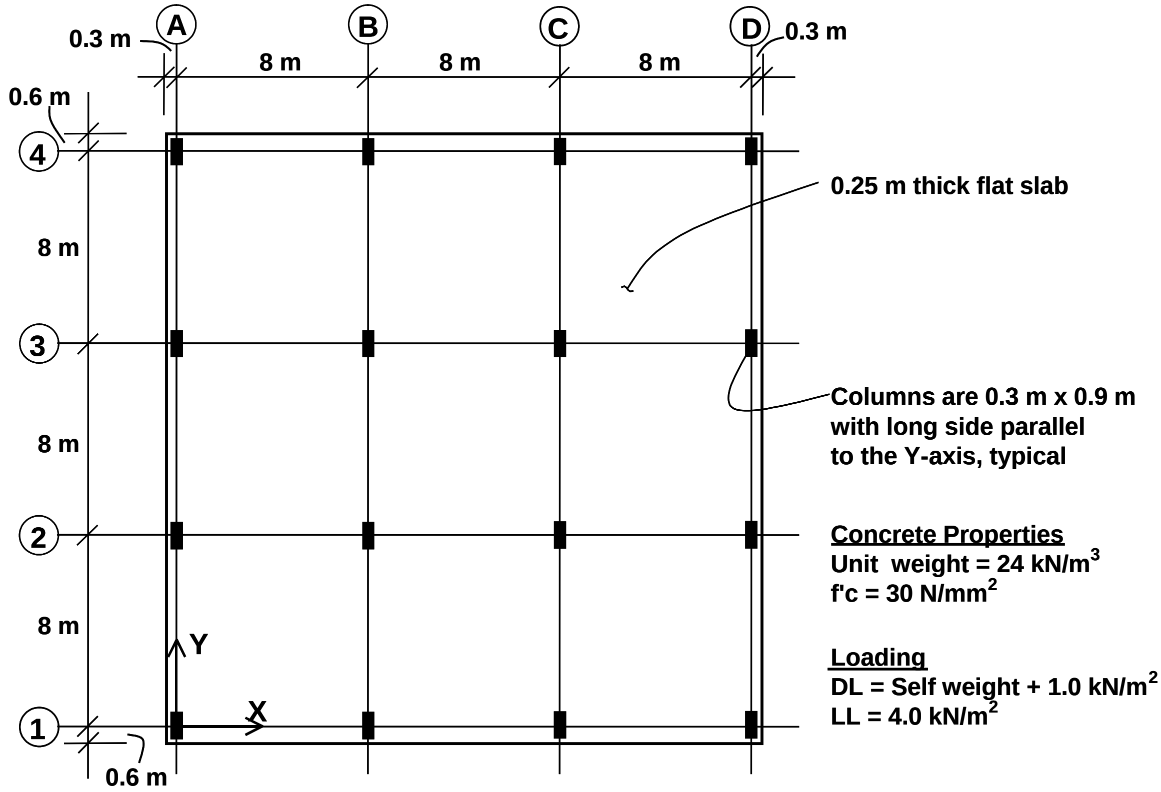


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a *f'c* of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

National Annex	Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
CEN Default, Norway, Slovenia and Sweden	SAFE	1.100	0.578	1.90
	Calculated	1.099	0.578	1.90
Finland, Singapore and UK	SAFE	1.100	0.5796	1.90
	Calculated	1.099	0.5796	1.90
Denmark	SAFE	1.100	0.606	1.82
	Calculated	1.099	0.606	1.81

COMPUTER FILE: EUROCODE 2-04 RC-PN-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATION

Hand Calculation for Interior Column using SAFE Method

$d = [(250-26)+(250-38)]/2$ = 218 mm

Refer to Figure 2.

$u_1 = u = 2 \bullet 300 + 2 \bullet 900 + 2 \bullet \pi \bullet 436 = 5139.468$ mm

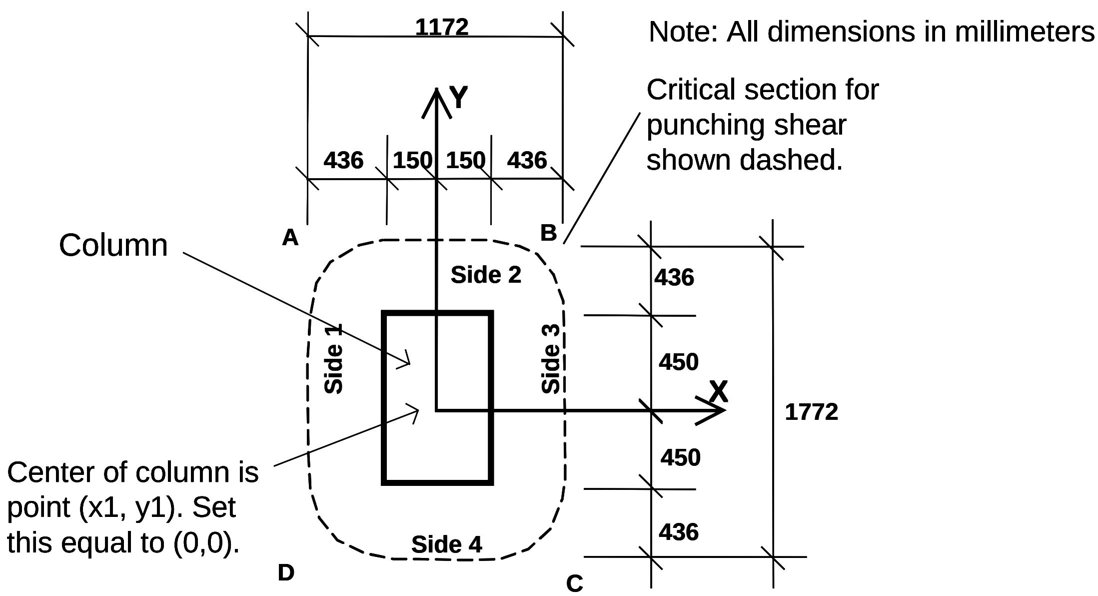


Figure 2: Interior Column, Grid B-2 in SAFE Model

From the SAFE output at Grid B-2:

V_{Ed} = 1112.197 kN

$k_2 M_{Ed2}$ = 41.593 kN-m

$k_3 M_{Ed3}$ = 20.576 kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{Ed} = \frac{V_{Ed}}{ud}\left[1 + \frac{k_2 M_{Ed,2} u_1}{V_{Ed} W_{1,2}} + \frac{k_3 M_{Ed,3} u_1}{V_{Ed} W_{1,3}}\right] \quad \text{(EC2 6.4.4(2))}$$

$$W_1 = \frac{c_1^2}{2} + c_1 c_2 + 4c_2 d + 16d^2 + 2\pi d c_1$$

$$W_{1,2} = \frac{900^2}{2} + 300 \bullet 900 + 4 \bullet 300 \bullet 218 + 16 \bullet 218^2 + 2\pi \bullet 218 \bullet 900$$

$$W_{1,2} = 2{,}929{,}744.957 \text{ mm}^2$$

$$W_{1,3} = 3\frac{900^2}{2} + 900 \bullet 300 + 4 \bullet 900 \bullet 218 + 16 \bullet 218^2 + 2\pi \bullet 218 \bullet 300$$

$$W_{1,2} = 2{,}271{,}104.319 \text{ mm}^2$$

$$v_{Ed} = \frac{V_{Ed}}{u_1 d}\left[1 + \frac{k_2 M_{Ed,2} u_1}{V_{Ed} W_{1,2}} + \frac{k_3 M_{Ed,3} u_1}{V_{Ed} W_{1,3}}\right]$$

$$v_{Ed} = \frac{1112.197 \bullet 10^3}{5139.468 \bullet 218}\left[1 + \frac{41.593 \bullet 10^6 \bullet 5139.468}{1112.197 \bullet 10^3 \bullet 2929744.957} + \frac{20.576 \bullet 10^6 \bullet 5139.468}{1112.197 \bullet 10^3 \bullet 2271104.319}\right]$$

$$v_{Ed} = 1.099 \text{ N/mm}^2$$

Thus v_{max} = **1.099 N/mm²**

For CEN Default, Finland, Norway, Singapore, Slovenia, Sweden and UK:

$$C_{Rd,c} = 0.18/\gamma_c = 0.18/1.5 = 0.12 \quad \text{(EC2 6.4.4)}$$

For Denmark:

$$C_{Rd,c} = 0.18/\gamma_c = 0.18/1.45 = 0.124 \quad \text{(EC2 6.4.4)}$$

The shear stress carried by the concrete, $V_{Rd,c}$, is calculated as:

$$V_{Rd,c} = \left[C_{Rd,c} k \left(100\rho_1 f_{ck}\right)^{1/3} + k_1 \sigma_{cp}\right] \quad \text{(EC2 6.4.4)}$$

with a minimum of:

$$v_{Rd,c} = \left(v_{\min} + k_1 \sigma_{cp}\right) \quad \text{(EC2 6.4.4)}$$

$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 = 1.9578$ (EC2 6.4.4(1))

k_1 = 0.15. (EC2 6.2.2(1))

$\rho_1 = \frac{A_{s1}}{b_w d} \leq 0.02$

Area of reinforcement at the face of column for design strip are as follows:

For CEN Default, Norway, Slovenia and Sweden:

A_s in Strip Layer A = 9204.985 mm^2

A_s in Strip Layer B = 8078.337 mm^2

Average A_s = $(9204.985 + 8078.337)/2$ = 8641.661 mm^2

ρ_1 = $8641.661/(8000 \bullet 218)$ = 0.004955 ≤ 0.02

For Finland, Singapore and UK:

A_s in Strip Layer A = 9319.248 mm^2

A_s in Strip Layer B = 8174.104 mm^2

Average A_s = $(9319.248 + 8174.104)/2$ = 8746.676 mm^2

ρ_1 = $8746.676/(8000 \bullet 218)$ = 0.005015 ≤ 0.02

For Denmark:

A_s in Strip Layer A = 9606.651 mm^2

A_s in Strip Layer B = 8434.444 mm^2

Average A_s = $(9606.651 + 8434.444)/2$ = 9020.548 mm^2

ρ_1 = $9020.548/(8000 \bullet 218)$ = 0.005172 ≤ 0.02

For CEN Default, Denmark, Norway, Singapore, Slovenia, Sweden and UK:

$$v_{\min} = 0.035k^{3/2} f_{ck}^{\;1/2} = 0.035(1.9578)^{3/2}(30)^{1/2} = 0.525 \text{ N/mm}^2$$

For Finland:

$$v_{\min} = 0.035k^{2/3} f_{ck}^{\;1/2} = 0.035(1.9578)^{2/3}(30)^{1/2} = 0.3000 \text{ N/mm}^2$$

For CEN Default, Norway, Slovenia and Sweden:

$$v_{Rd,c} = \left[0.12 \bullet 1.9578(100 \bullet 0.004955 \bullet 30)^{1/3} + 0\right] = 0.5777 \text{ N/mm}^2$$

For Finland, Singapore, and UK:

$$v_{Rd,c} = \left[0.12 \bullet 1.9578(100 \bullet 0.005015 \bullet 30)^{1/3} + 0\right] = 0.5796 \text{ N/mm}^2$$

For Denmark:

$$v_{Rd,c} = \left[0.124 \bullet 1.9578(100 \bullet 0.005015 \bullet 30)^{1/3} + 0\right] = 0.606 \text{ N/mm}^2$$

For CEN Default, Norway, Slovenia and Sweden:

$$\text{Shear Ratio} = \frac{v_{\max}}{v_{Rd,c}} = \frac{1.092}{0.5777} = 1.90$$

For Finland, Singapore and UK:

$$\text{Shear Ratio} = \frac{v_{\max}}{v_{Rd,c}} = \frac{1.092}{0.5796} = 1.90$$

For Denmark:

$$\text{Shear Ratio} = \frac{v_{\max}}{v_{Rd,c}} = \frac{1.092}{0.606} = 1.81$$

EXAMPLE Eurocode 2-04 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

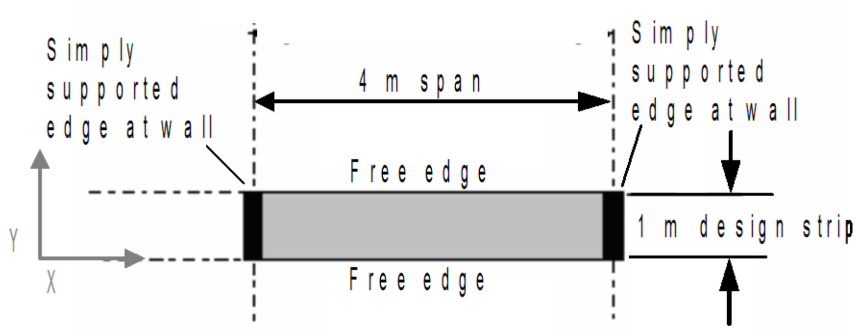


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the Eurocode 2-04 load combination factors, 1.35 for dead loads and 1.5 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. These moments are identical. After completing the analysis, design is performed using the Eurocode 2-04 code by SAFE and also by hand computation. Table 1 shows the comparison of the design reinforcements computed by the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_{ck}	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	$2x10^6$	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

National Annex	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
CEN Default, Norway, Slovenia and Sweden	SAFE	25.797	5.400
	Calculated	25.800	5.400
Finland , Singapore and UK	SAFE	25.797	5.446
	Calculated	25.800	5.446
Denmark	SAFE	25.797	5.626

	Calculated	25.800	5.626

$A^{+}_{s,\min}$ = 204.642 sq-mm

COMPUTER FILE: Eurocode 2-04 RC-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$$\eta = 1.0 \text{ for } f_{ck} \leq 50 \text{ MPa}$$

$$\lambda = 0.8 \text{ for } f_{ck} \leq 50 \text{ MPa}$$

b = 1000 mm

For the load combination, w and M are calculated as follows:

$$w = (1.35w_d + 1.5w_t)\, b$$

$$M = \frac{wl_1^2}{8}$$

$$A_{s,\min} = \max \begin{cases} 0.0013 b_w d \\ 0.26 \dfrac{f_{ctm}}{f_{yk}} bd \end{cases}$$

= 204.642 sq-mm

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 12.9 kN/m

$M_{\text{-strip}}$ = 25.8 kN-m

$M_{\text{-design}}$ = 25.8347 kN-m

For CEN Default, Norway, Slovenia and Sweden:

$\gamma_{m,\ steel}$ = 1.15

$\gamma_{m,\ concrete}$ = 1.50

α_{cc} = 1.0

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta f_{cd}} = \frac{25.8347 \bullet 10^6}{1000 \bullet 125^2 \bullet 1.0 \bullet 1.0 \bullet 30/1.5} = 0.08267$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

$k_1 = 0.44$

$k_2 = k_4 = 1.25(0.6 + 0.0014/\varepsilon_{cu2}) = 1.25$

δ is assumed to be 1

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa} = 0.448$$

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.294$$

$$\omega = 1 - \sqrt{1 - 2m} = 0.08640$$

$$A_s = \omega\left(\frac{\eta f_{cd} bd}{f_{yd}}\right) = 540.024 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.400 sq-cm

For Singapore and UK:

$\gamma_{m,\ steel}$ = 1.15

$\gamma_{m,\ concrete}$ = 1.50

α_{cc} = 0.85:

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta f_{cd}} = \frac{25.8347 \bullet 10^6}{1000 \bullet 125^2 \bullet 1.0 \bullet 0.85 \bullet 30/1.5} = 0.097260$$

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.48$$

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa} = 0.60$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

$k_1 = 0.40$

$k_2 = (0.6 + 0.0014/\varepsilon_{cu2}) = 1.00$

δ is assumed to be 1

$$\omega = 1 - \sqrt{1-2m} = 0.10251$$

$$A_s = \omega\left(\frac{\eta f_{cd} bd}{f_{yd}}\right) = 544.61 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.446 sq-cm

For Finland:

$\gamma_{m,\ steel}$ = 1.15

$\gamma_{m,\ concrete}$ = 1.50

α_{cc} = 0.85:

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta f_{cd}} = \frac{25.8347 \bullet 10^6}{1000 \bullet 125^2 \bullet 1.0 \bullet 0.85 \bullet 30/1.5} = 0.097260$$

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 032433$$

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \le 50 \text{ MPa} = 0.5091$$

For reinforcement with $f_{yk} \le 500$ MPa, the following values are used:

k_1 = 0.44

k_2 = 1.1

$k_4 = 1.25(0.6 + 0.0014/\varepsilon_{cu2}) = 1.25$

δ is assumed to be 1

$$\omega = 1 - \sqrt{1-2m} = 0.10251$$

$$A_s = \omega\left(\frac{\eta f_{cd} bd}{f_{yd}}\right) = 544.61 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.446 sq-cm

For Denmark:

$\gamma_{m,\ steel} = 1.20$

$\gamma_{m,\ concrete} = 1.45$

$\alpha_{cc} = 1.0$

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta f_{cd}} = \frac{25.8347 \bullet 10^6}{1000 \bullet 125^2 \bullet 1.0 \bullet 1.0 \bullet 30/1.5} = 0.0799153$$

$$m_{\lim} = \lambda \left(\frac{x}{d}\right)_{\lim} \left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.294$$

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa} = 0.448$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

$k_1 = 0.44$

$k_2 = k_4 = 1.25(0.6 + 0.0014/\varepsilon_{cu2}) = 1.25$

δ is assumed to be 1

$$\omega = 1 - \sqrt{1 - 2m} = 0.08339$$

$$A_s = \omega \left(\frac{\eta f_{cd} bd}{f_{yd}}\right) = 562.62 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.626 sq-cm

EXAMPLE Hong Kong CP-04 PT-SL-001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.

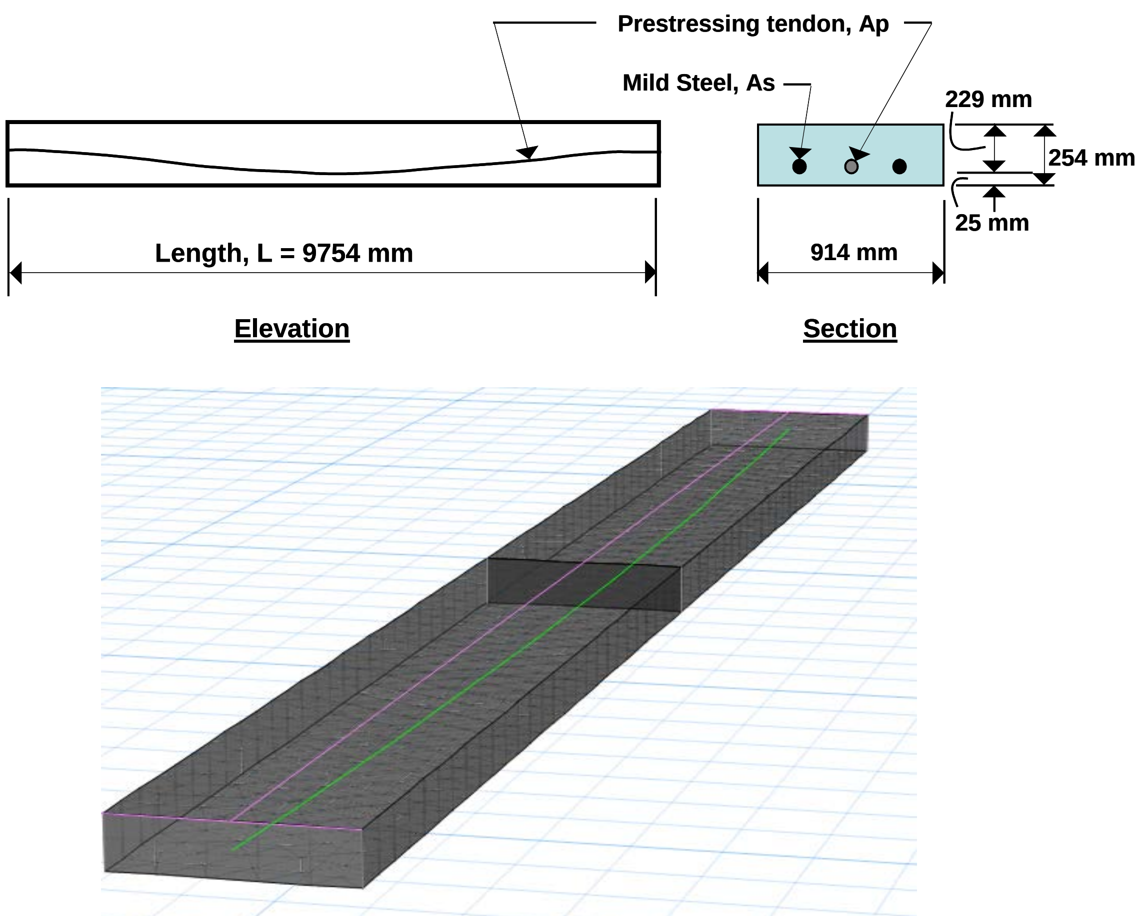


Figure 1 One-Way Slab

To ensure one-way action Poisson's ratio is taken to be zero. A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, was added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement and slab stresses are reported at the midspan of the slab. Independent hand calculations were compared with the SAFE results and summarized for verification and validation of the SAFE results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	KN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	KN/m^2
Live load	w_l	=	4.788	KN/m^2

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments, required mild steel reinforcing, and slab stresses with independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (KN-m)	174.4	174.4	0.00%
Area of Mild Steel req'd, As (cm^2)	19.65	19.79	0.35%
Transfer Conc. Stress, top ($D+PT_I$), MPa	–5.056	–5.056	0.00%
Transfer Conc. Stress, bot ($D+PT_I$), MPa	2.836	2.839	0.11%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	–10.547	–10.465	0.77%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	8.323	8.407	1.01%

COMPUTER FILE: HONG KONG CP-04 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

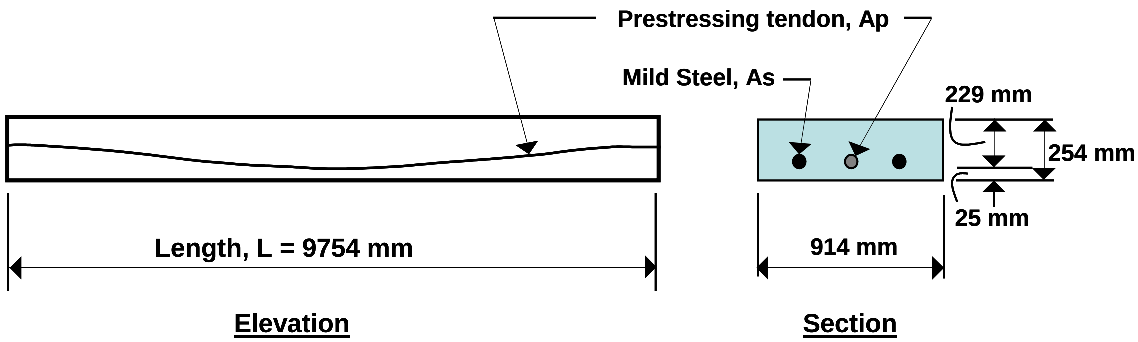
HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
fc = 30 MPa	f_{pu} = 1862 MPa
fy = 400 MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.4 = 8.378 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) × 1.6 = 7.661 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 16.039 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 16.039 kN/m^2 × 0.914 m = 14.659 kN/m

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8} = 14.659 \times (9.754)^2 / 8$ = 174.4 kN-m

Ultimate Stress in strand, $f_{pb} = f_{pe} + \frac{7000}{l/d}\left(1 - 1.7\frac{f_{pu}A_p}{f_{cu}bd}\right)$

$$= 1210 + \frac{7000}{9.754/0.229}\left(1 - 1.7\frac{1862(198)}{30(914)(229)}\right)$$

$$= 1358 \text{ MPa} \le 0.7 f_{pu} = 1303 \text{ MPa}$$

K factor used to determine the effective depth is given as:

$$K = \frac{M}{f_{cu}bd^2} = \frac{174.4}{30000(0.914)(0.229)^2} = 0.1213 < 0.156$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d = 192.2 \text{ mm}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197.4(1303)/1000 = 257.2$ KN

Ultimate moment due to PT, $M_{ult,PT} = F_{ult,PT}(z)/\gamma = 257.2(0.192)/1.15 = 43.00$ KN-m

Net Moment to be resisted by As, $M_{NET} = M_U - M_{PT}$

$$= 174.4 - 43.00 = 131.40 \text{ kN-m}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M}{0.87 f_y z} = \frac{131.40}{0.87(400)(192)}(1e6) = 1965 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (D+PT$_i$) = 1.0D+0.0L+1.0PT$_I$

Tendon stress at transfer = jacking stress − stressing losses = 1490 − 186 = 1304 MPa

The force in the tendon at transfer, = $1304(2)(99)/1000 = 258.2$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 258.2(101.6 \text{ mm})/1000 = 26.23$ kN-m

Stress in concrete, $f = \frac{F_{PTI}}{A} \pm \frac{M_D}{S} \pm \frac{M_{PT}}{S} = \frac{-258.2}{0.254(0.914)} \pm \frac{65.04}{0.00983} \pm \frac{26.23}{0.00983}$

where S = 0.00983 m^3

$$f = -1.112 \pm 6.6166 \pm 2.668 \text{ MPa}$$

$$f = -5.060(\text{Comp})\max,\ 2.836(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT_F) = 1.0D+1.0L+1.0PT_F

Tendon stress at normal = jacking − stressing − long-term = 1490 − 186 − 94 = 1210 MPa

The force in tendon at normal, = $1210(2)(99)/1000 = 239.5$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 239.5(101.6\text{ mm})/1000 = 24.33$ kN-m

Stress in concrete for (D+L+PT_F),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_D}{S} \pm \frac{M_{PT}}{S} = \frac{-258.2}{0.254(0.914)} \pm \frac{117.08}{0.00983} \pm \frac{24.33}{0.00983}$$

$$f = -1.112 \pm 11.910 \pm 2.475$$

$$f = -10.547(\text{Comp})\,\text{max},\ 8.323(\text{Tension})\,\text{max}$$

EXAMPLE HONG KONG CP-04 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

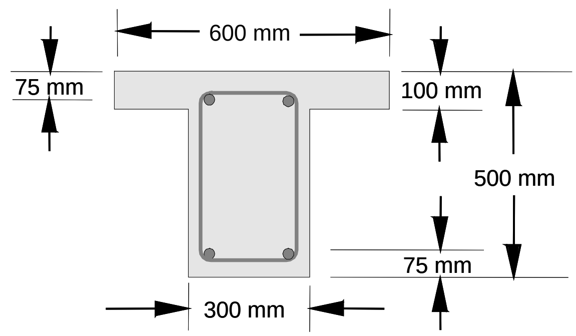
The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by Hong Kong CP 2004.
- The average shear stress in the beam is below the maximum shear stress allowed by Hong Kong CP 2004, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading. One dead load case (DL20) and one live load case (LL80) with only symmetric third-point loads of magnitudes 20, and 80 kN, respectively, are defined in the model. One load combinations (COMB80) is defined using the Hong Kong CP 2004 load combination factors of 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both of these load cases and the load combinations.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results. These moment and shear force are identical. After completing the analysis, design is performed using the Hong Kong CP 2004 code in SAFE and also by hand computation. The design longitudinal reinforcements are compared in Table 1. The design shear reinforcements are compared in Table 2.



Beam Section

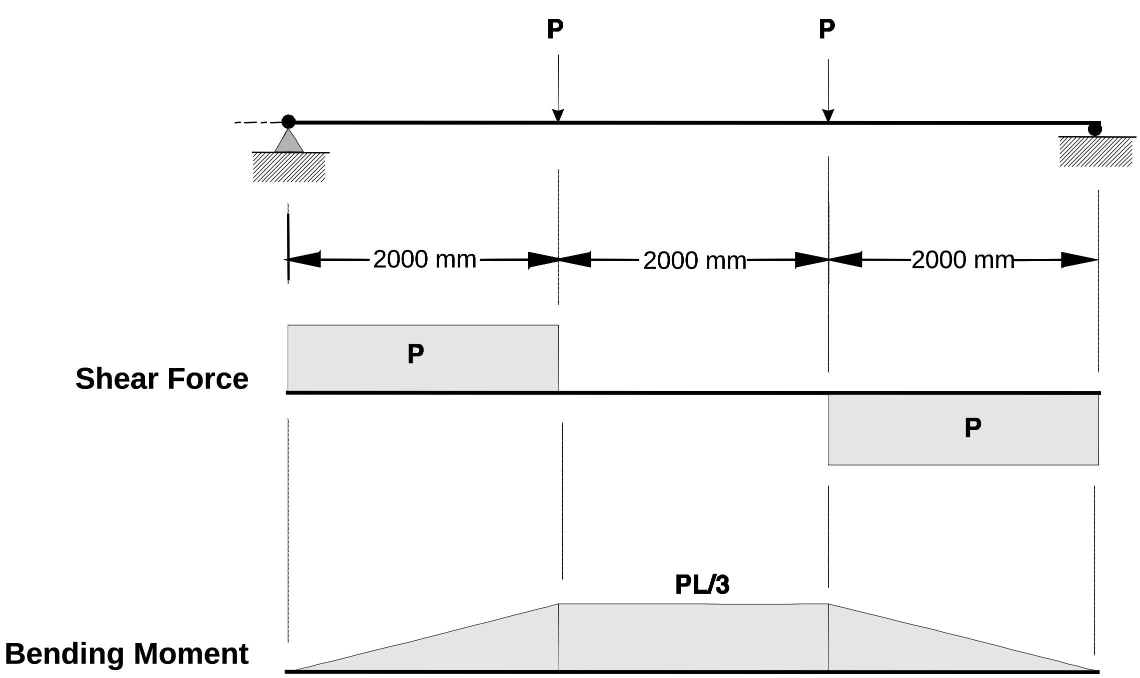


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span,	l	=	6000	mm
Overall depth,	h	=	500	mm
Flange Thickness,	d_s	=	100	mm
Width of web,	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.,	d_c	=	75	mm
Effective depth,	d	=	425	mm
Depth of comp. reinf.,	d'	=	75	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	25×10^5	MPa
Modulus of elasticity,	E_s	=	2×10^8	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	20	kN
Live load,	P_l	=	80	kN

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-cm)
		A_s^+
SAFE	312	20.904
Calculated	312	20.904

$A^+_{s,min}$ = 195.00 sq-mm

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-cm/m)	
	SAFE	**Calculated**
156	6.50	6.50

COMPUTER FILE: Hong Kong CP-04 RC-BM-001.FDB

CONCLUSION

The SAFE results show an approximate comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$

$$A_{s,\min} = 0.0013 b_w h$$

$$= 195.00 \text{ sq-mm}$$

COMB80

$P = (1.4P_d + 1.6P_t) = 156$ kN

$$M^* = \frac{N^* l}{3} = 312 \text{ kN-m}$$

The depth of the compression block is given by:

$$K = \frac{M}{f_{cu}\, b_f\, d^2} = 0.095963 < 0.156$$

Then the moment arm is computed as:

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \leq 0.95d = 373.4254 \text{ mm}$$

The depth of the neutral axis is computed as:

$$x = \frac{1}{0.45}(d - z) = 114.6102 \text{ mm}$$

And the depth of the compression block is given by:

$a = 0.9x = 103.1492 \text{ mm} > h_f$

The ultimate resistance moment of the flange is given by:

$$M_f = \frac{0.67}{\gamma_c} f_{cu}\left(b_f - b_w\right) h_f \left(d - 0.5h_f\right) = 150.75 \text{ kN-m}$$

The moment taken by the web is computed as:

$$M_w = M - M_f = 161.25 \text{ kN-m}$$

and the normalized moment resisted by the web is given by:

$$K_w = \frac{M_w}{f_{cu} b_w d^2} = 0.0991926 < 0.156$$

If $K_w \leq 0.156$ (BS 3.4.4.4), the beam is designed as a singly reinforced concrete beam. The reinforcement is calculated as the sum of two parts: one to balance compression in the flange and one to balance compression in the web.

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K_w}{0.9}}\right) \leq 0.95d = 371.3988 \text{ mm}$$

$$A_s = \frac{M_f}{\frac{f_y}{\gamma_s}\left(d - 0.5h_f\right)} + \frac{M_w}{\frac{f_y}{\gamma_s} z} = 2090.4 \text{ sq-mm}$$

Shear Design

$$v = \frac{V}{b_w d} \leq v_{\max} = 1.2235 \text{ MPa}$$

$v_{max} = \min(0.8\sqrt{f_{cu}}, 5 \text{ MPa}) = 4.38178$ MPa

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} = 0.3568 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3} = 1.06266,\ 1 \leq k_2 \leq \left(\frac{40}{25}\right)^{1/3}$$

$\gamma_m = 1.25$

$$\frac{100 A_s}{bd} = 0.15$$

$$\left(\frac{400}{d}\right)^{1/4} = 1$$

However, the following limitations also apply:

$$0.15 \le \frac{100\,A_s}{bd} \le 3$$

$$\left(\frac{400}{d}\right)^{1/4} \ge 1$$

$f_{cu} \le 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

Given v, v_c, and $v_{\max}$, the required shear reinforcement is calculated as follows:

If $v \le (v_c + 0.4)$,

$$\frac{A_{sv}}{s_v} = \frac{0.4 b_w}{0.87\, f_{yv}}$$

If $(v_c + 0.4) < v \le v_{\max}$,

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c) b_w}{0.87\, f_{yv}}$$

If $v > v_{\max}$, a failure condition is declared.

(COMB80)

P_d = 20 kN

P_l = 80 kN

V = 156 kN

$$v^* = \frac{V^*}{b_w d} = 2.0 \text{ MPa} \quad (\phi_s v_c < v^* \le \phi_s v_{\max})$$

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c) b_w}{0.87\, f_{yv}} = 0.64967\text{sq-mm/mm} = 6.50 \text{ sq-cm/m}$$

EXAMPLE Hong Kong CP-04 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

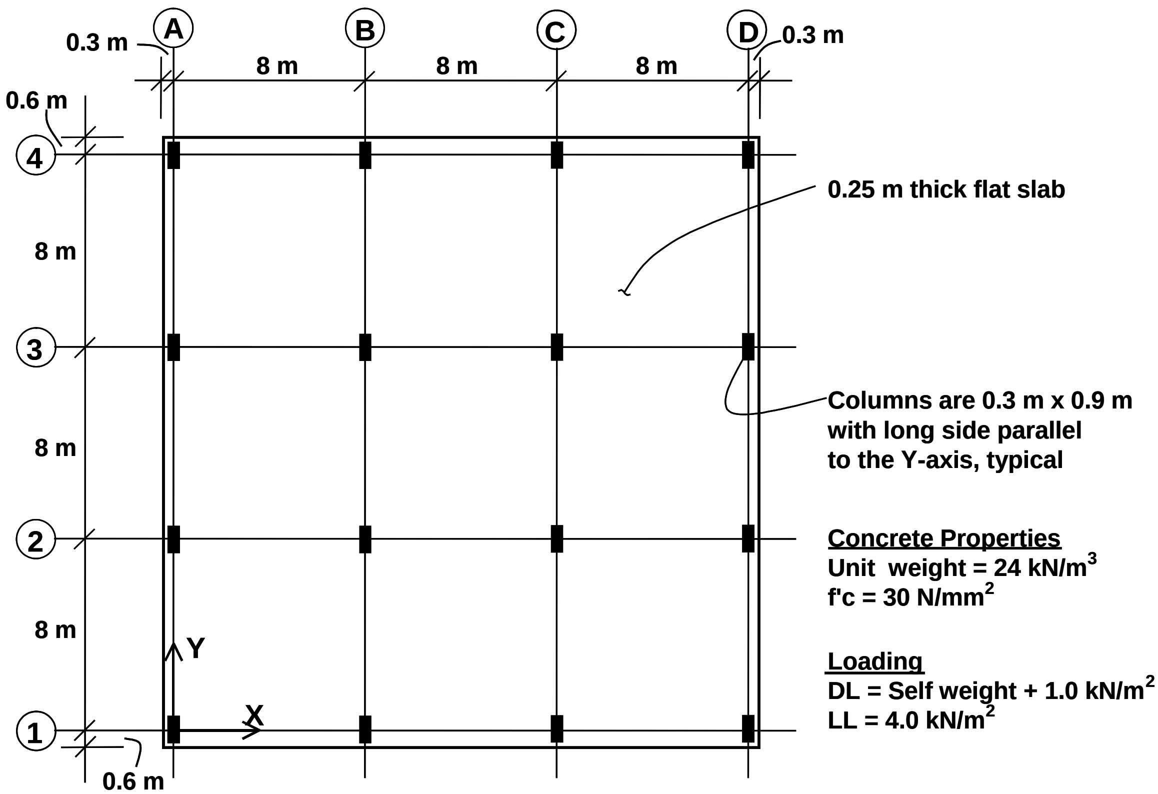


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f_{cu} of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
SAFE	1.105	0.625	1.77
Calculated	1.105	0.625	1.77

COMPUTER FILE: HONG KONG CP-04 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using SAFE Method

$d = [(250 - 26) + (250 - 38)]/2$ = 218 mm

Refer to Figure 1.

u = 954+ 1554 + 954 + 1554 = 5016 mm

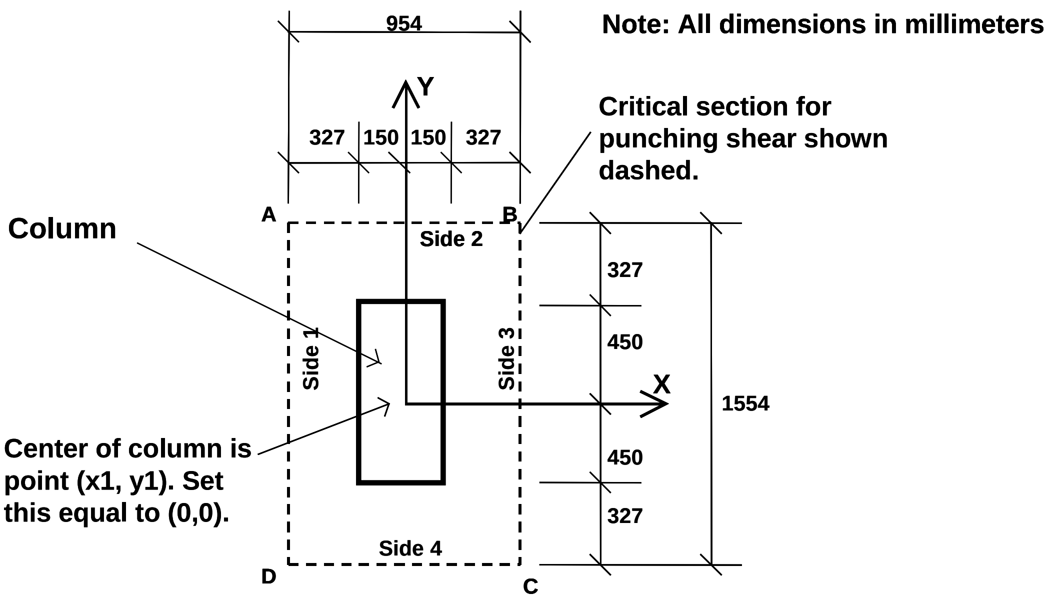


Figure 2: Interior Column, Grid B-2 in SAFE Model

From the SAFE output at Grid B-2:

V= 1126.498 kN

M_2 = 51.9908 kN-m

M_3 = 45.7234 kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{eff,x} = \frac{V}{ud}\left(f + \frac{1.5M_x}{Vy} \right)$$

$$v_{eff,x} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 51.9908 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 954} \right) = 1.1049 \text{ (Govern)}$$

$$v_{eff,y} = \frac{V}{ud}\left(f + \frac{1.5M_y}{Vx} \right)$$

$$v_{eff,y} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 45.7234 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 1554} \right) = 1.0705$$

The largest absolute value of v = **1.1049 N/mm²**

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd} \right)^{1/3}\left(\frac{400}{d} \right)^{1/4} = 0.3568 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{25} \right)^{1/3} = \left(\frac{30}{25} \right)^{1/3} = 1.0627 > 1.0 \text{ OK}$$

γ_m = 1.25

$$\left(\frac{400}{d} \right)^{1/4} = 1.16386 > 1 \text{ OK.}$$

$f_{cu} \le 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

Area of reinforcement at the face of column for design strip are as follows:

A_s in Strip Layer A = 9494.296 mm^2

A_s in Strip Layer B = 8314.486 mm^2

Average $A_s = (9494.296 + 8314.486)/2 = 8904.391$ mm^2

$$\frac{100\ A_s}{bd} = 100 \bullet 8904.391/(8000 \bullet 218) = 0.51057$$

$$v_c = \frac{0.79 \bullet 1.0 \bullet 1.0627}{1.25} \bullet (0.51057)^{1/3} \bullet 1.16386 = 0.6247 \text{ MPa}$$

BS 3.7.7.3 yields the value of v = **0.625 N/mm²**, and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{v} = \frac{1.1049}{0.6247} = 1.77$$

EXAMPLE Hong Kong CP-04 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

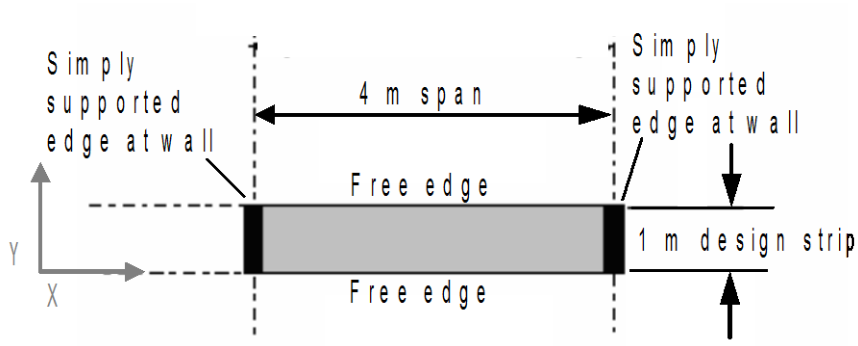


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the Hong Kong CP-04 load combination factors, 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing analysis, design is performed using the Hong Kong CP-04 code by SAFE and also by hand computation. Table 1 shows the comparison of the design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2×10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	SAFE	27.197	5.853
	Calculated	27.200	5.842

$A^+_{s,min}$ = 162.5 sq-mm

COMPUTER FILE: Hong Kong CP-04 RC-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\gamma_{m,\ steel}$ = 1.15

$\gamma_{m,\ concrete}$ = 1.50

b = 1000 mm

For the load combination, the w and M are calculated as follows:

$$w = (1.4w_d + 1.6w_t)\ b$$

$$M = \frac{wl_1^2}{8}$$

$$A_{s,\min} = 0.0013b_w d$$

$$= 162.5 \text{ sq-mm}$$

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 13.6 kN/m

$M_{\text{-strip}}$ = 27.2 kN-m

$M_{\text{-design}}$ = 27.2366 kN-m

The depth of the compression block is given by:

$$K = \frac{M}{f_{cu} b d^2} = 0.05810 < 0.156$$

The area of tensile steel reinforcement is then given by:

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d\ = 116.3283$$

$$A_s = \frac{M}{0.87 f_y z} = 585.046 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.850 \text{ sq-cm}$$

EXAMPLE Hong Kong CP-2013 PT-SL-001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.

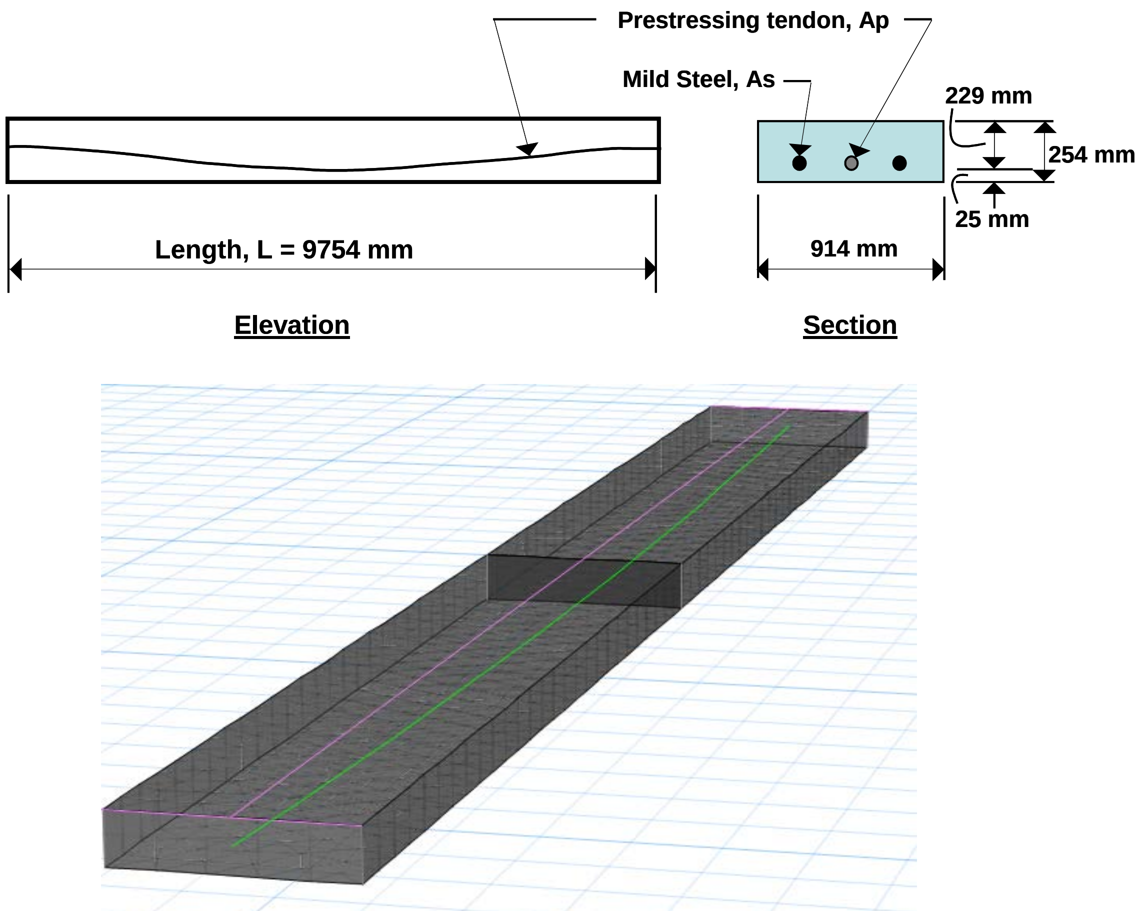


Figure 1 One-Way Slab

To ensure one-way action Poisson's ratio is taken to be zero. A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, was added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement and slab stresses are reported at the midspan of the slab. Independent hand calculations were compared with the SAFE results and summarized for verification and validation of the SAFE results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	KN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	KN/m^2
Live load	w_l	=	4.788	KN/m^2

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments, required mild steel reinforcing, and slab stresses with independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (KN-m)	174.4	174.4	0.00%
Area of Mild Steel req'd, As (cm^2)	19.65	19.79	0.35%
Transfer Conc. Stress, top ($D+PT_I$), MPa	–5.056	–5.056	0.00%
Transfer Conc. Stress, bot ($D+PT_I$), MPa	2.836	2.839	0.11%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	–10.547	–10.465	0.77%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	8.323	8.407	1.01%

COMPUTER FILE: HONG KONG CP-13 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing

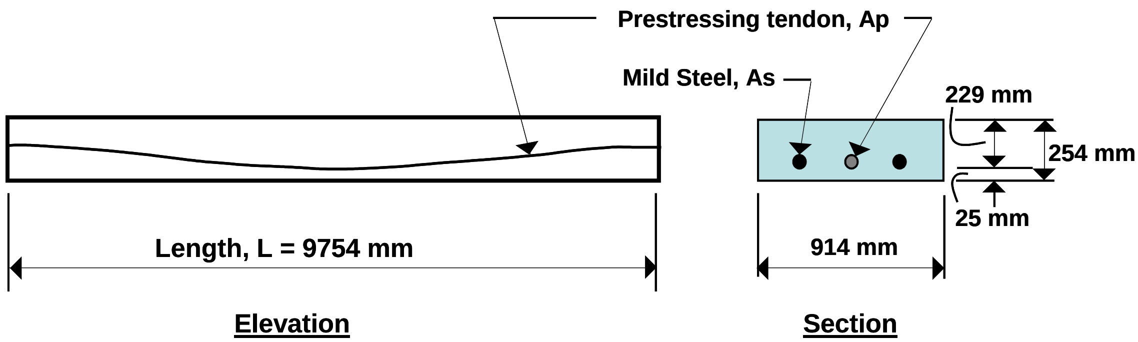
$f_c = 30$ MPa
$f_y = 400$ MPa

Post-Tensioning

$f_{pu} = 1862$ MPa
$f_{py} = 1675$ MPa
Stressing Loss = 186 MPa
Long-Term Loss = 94 MPa
$f_i = 1490$ MPa
$f_e = 1210$ MPa

$\gamma_{m,\, steel} = 1.15$

$\gamma_{m,\, concrete} = 1.50$



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.4 = 8.378 kN/m^2 (D_u)
Live, = 4.788 kN/m^2 (L) × 1.6 = 7.661 kN/m^2 (L_u)
Total = 10.772 kN/m^2 (D+L) = 16.039 kN/m^2 (D+L)ult

$\omega = 10.772$ kN/m^2 × 0.914 m = 9.846 kN/m, $\omega_u = 16.039$ kN/m^2 × 0.914 m = 14.659 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8} = 14.659 \times (9.754)^2 / 8 = 174.4$ kN-m

Ultimate Stress in strand, $f_{pb} = f_{pe} + \frac{7000}{l/d}\left(1 - 1.7\frac{f_{pu}A_p}{f_{cu}bd}\right)$

$$= 1210 + \frac{7000}{9.754/0.229}\left(1 - 1.7\frac{1862(198)}{30(914)(229)}\right)$$

$$= 1358 \text{ MPa} \le 0.7 f_{pu} = 1303 \text{ MPa}$$

K factor used to determine the effective depth is given as:

$$K = \frac{M}{f_{cu}bd^2} = \frac{174.4}{30000(0.914)(0.229)^2} = 0.1213 < 0.156$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d = 192.2 \text{ mm}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197.4(1303)/1000 = 257.2 \text{ KN}$

Ultimate moment due to PT, $M_{ult,PT} = F_{ult,PT}(z)/\gamma = 257.2(0.192)/1.15 = 43.00 \text{ KN-m}$

Net Moment to be resisted by As, $M_{NET} = M_U - M_{PT}$

$$= 174.4 - 43.00 = 131.40 \text{ kN-m}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M}{0.87 f_y z} = \frac{131.40}{0.87(400)(192)}(1e6) = 1965 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (D+PT$_i$) = 1.0D+0.0L+1.0PT$_I$

Tendon stress at transfer = jacking stress − stressing losses = 1490 − 186 = 1304 MPa

The force in the tendon at transfer, = $1304(2)(99)/1000 = 258.2 \text{ kN}$

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04 \text{ kN-m}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 258.2(101.6 \text{ mm})/1000 = 26.23 \text{ kN-m}$

Stress in concrete, $f = \frac{F_{PTI}}{A} \pm \frac{M_D}{S} \pm \frac{M_{PT}}{S} = \frac{-258.2}{0.254(0.914)} \pm \frac{65.04}{0.00983} \pm \frac{26.23}{0.00983}$

where S = 0.00983 m^3

$$f = -1.112 \pm 6.6166 \pm 2.668 \text{ MPa}$$

$$f = -5.060(\text{Comp})\max,\ 2.836(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT_F) = 1.0D+1.0L+1.0PT_F

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, = $1210(2)(99)/1000 = 239.5$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 239.5(101.6\text{ mm})/1000 = 24.33$ kN-m

Stress in concrete for (D+L+PT_F),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_D}{S} \pm \frac{M_{PT}}{S} = \frac{-258.2}{0.254(0.914)} \pm \frac{117.08}{0.00983} \pm \frac{24.33}{0.00983}$$

$$f = -1.112 \pm 11.910 \pm 2.475$$

$$f = -10.547(\text{Comp})\max,\ 8.323(\text{Tension})\max$$

EXAMPLE HONG KONG CP-2013 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by Hong Kong CP 2013.
- The average shear stress in the beam is below the maximum shear stress allowed by Hong Kong CP 2013, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading. One dead load case (DL20) and one live load case (LL80) with only symmetric third-point loads of magnitudes 20, and 80 kN, respectively, are defined in the model. One load combinations (COMB80) is defined using the Hong Kong CP 2013 load combination factors of 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both of these load cases and the load combinations.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results. These moment and shear force are identical. After completing the analysis, design is performed using the Hong Kong CP 2013 code in SAFE and also by hand computation. The design longitudinal reinforcements are compared in Table 1. The design shear reinforcements are compared in Table 2.

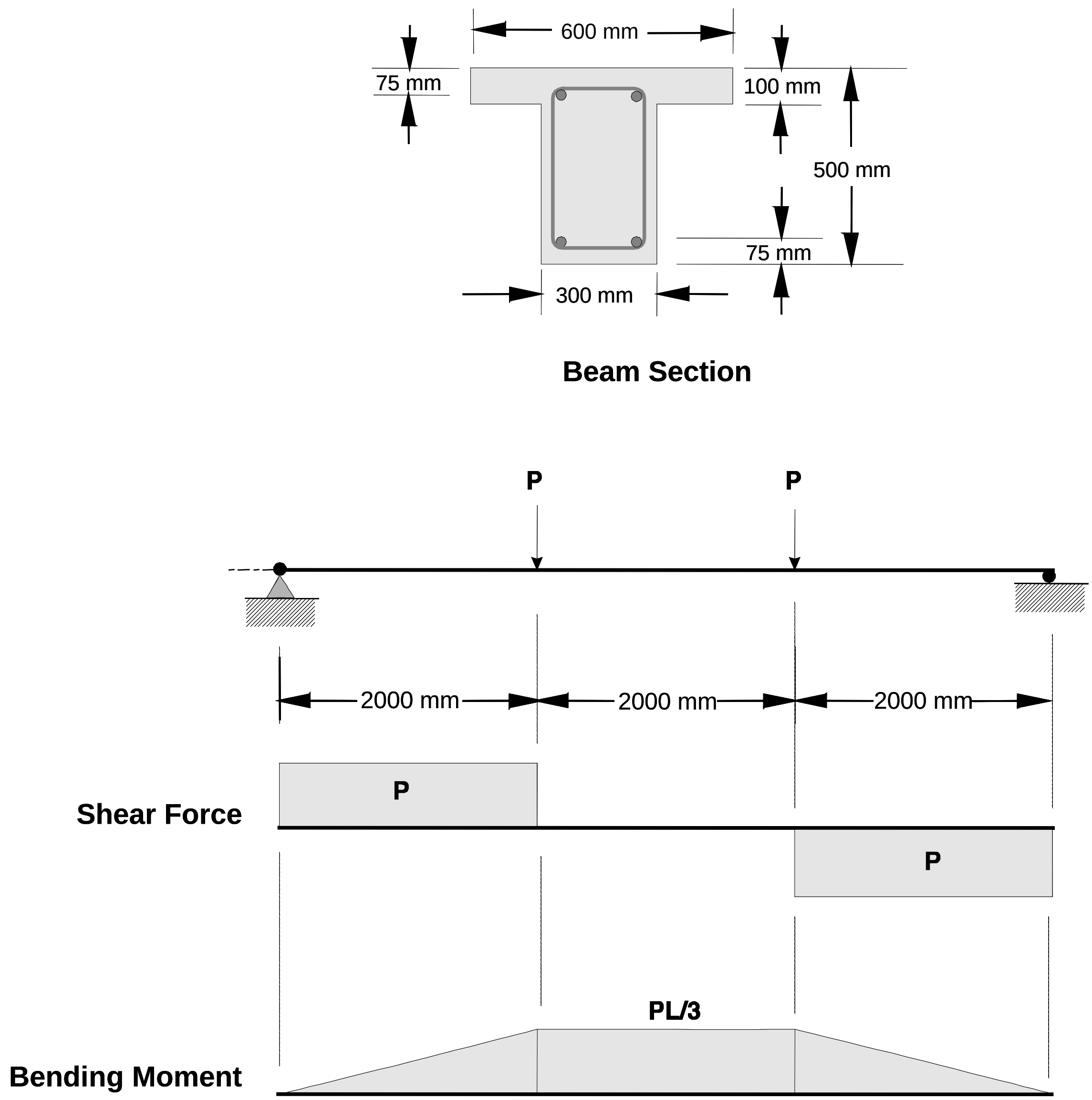


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span,	l	=	6000	mm
Overall depth,	h	=	500	mm
Flange Thickness,	d_s	=	100	mm
Width of web,	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.,	d_c	=	75	mm
Effective depth,	d	=	425	mm
Depth of comp. reinf.,	d'	=	75	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	25×10^5	MPa
Modulus of elasticity,	E_s	=	2×10^8	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	20	kN
Live load,	P_l	=	80	kN

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-cm)
		A_s^+
SAFE	312	20.904
Calculated	312	20.904

$A^+_{s,min}$ = 195.00 sq-mm

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-cm/m)	
	SAFE	Calculated
156	6.50	6.50

COMPUTER FILE: Hong Kong CP-13 RC-BM-001.FDB

CONCLUSION

The SAFE results show an approximate comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\gamma_{m,\,steel}$ = 1.15

$\gamma_{m,\,concrete}$ = 1.50

$$A_{s,\min} = 0.0013 b_w h$$

= 195.00 sq-mm

COMB80

P = (1.4P_d + 1.6P_t) =156 kN

$$M^* = \frac{N^* l}{3} = 312 \text{ kN-m}$$

The depth of the compression block is given by:

$$K = \frac{M}{f_{cu}\, b_f\, d^2} = 0.095963 < 0.156$$

Then the moment arm is computed as:

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \le 0.95d = 373.4254 \text{ mm}$$

The depth of the neutral axis is computed as:

$$x = \frac{1}{0.45}(d - z) = 114.6102 \text{ mm}$$

And the depth of the compression block is given by:

$a = 0.9x = 103.1492 \text{ mm} > h_f$

The ultimate resistance moment of the flange is given by:

$$M_f = \frac{0.67}{\gamma_c} f_{cu}\left(b_f - b_w\right) h_f \left(d - 0.5h_f\right) = 150.75 \text{ kN-m}$$

The moment taken by the web is computed as:

$$M_w = M - M_f = 161.25 \text{ kN-m}$$

and the normalized moment resisted by the web is given by:

$$K_w = \frac{M_w}{f_{cu} b_w d^2} = 0.0991926 < 0.156$$

If $K_w \le 0.156$ (BS 3.4.4.4), the beam is designed as a singly reinforced concrete beam. The reinforcement is calculated as the sum of two parts: one to balance compression in the flange and one to balance compression in the web.

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K_w}{0.9}}\right) \le 0.95d = 371.3988 \text{ mm}$$

$$A_s = \frac{M_f}{\frac{f_y}{\gamma_s}\left(d - 0.5h_f\right)} + \frac{M_w}{\frac{f_y}{\gamma_s} z} = 2090.4 \text{ sq-mm}$$

Shear Design

$$v = \frac{V}{b_w d} \le v_{max} = 1.2235 \text{ MPa}$$

v_{max} = min(0.8$\sqrt{f_{cu}}$, 5 MPa) = 4.38178 MPa

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} = 0.3568 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3} = 1.06266,\ 1 \le k_2 \le \left(\frac{40}{25}\right)^{1/3}$$

γ_m = 1.25

$$\frac{100\, A_s}{bd} = 0.15$$

$$\left(\frac{400}{d}\right)^{1/4} = 1$$

However, the following limitations also apply:

$$0.15 \le \frac{100\,A_s}{bd} \le 3$$

$$\left(\frac{400}{d}\right)^{1/4} \ge 1$$

$f_{cu} \le 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

Given v, v_c, and v_{max}, the required shear reinforcement is calculated as follows:

If $v \le (v_c + 0.4)$,

$$\frac{A_{sv}}{s_v} = \frac{0.4 b_w}{0.87 f_{yv}}$$

If $(v_c + 0.4) < v \le v_{max}$,

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c) b_w}{0.87 f_{yv}}$$

If $v > v_{max}$, a failure condition is declared.

(COMB80)

P_d = 20 kN

P_l = 80 kN

V = 156 kN

$$v^* = \frac{V^*}{b_w d} = 2.0 \text{ MPa} \quad (\phi_s v_c < v^* \le \phi_s v_{max})$$

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c) b_w}{0.87 f_{yv}} = 0.64967\text{sq-mm/mm} = 6.50 \text{ sq-cm/m}$$

EXAMPLE Hong Kong CP-2013 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

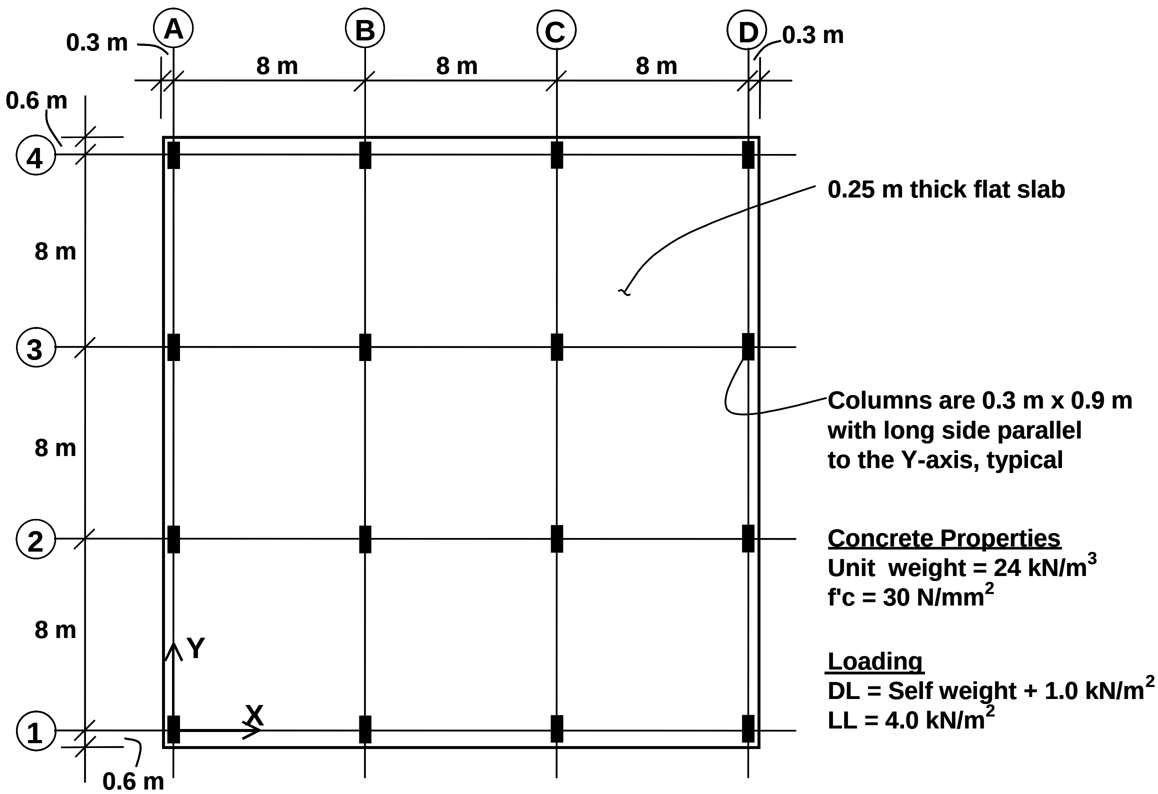


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f_{cu} of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
SAFE	1.105	0.625	1.77
Calculated	1.105	0.625	1.77

COMPUTER FILE: HONG KONG CP-13 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using SAFE Method

$d = [(250-26)+(250-38)]/2$ = 218 mm

Refer to Figure 1.

u = 954+ 1554 + 954 + 1554 = 5016 mm

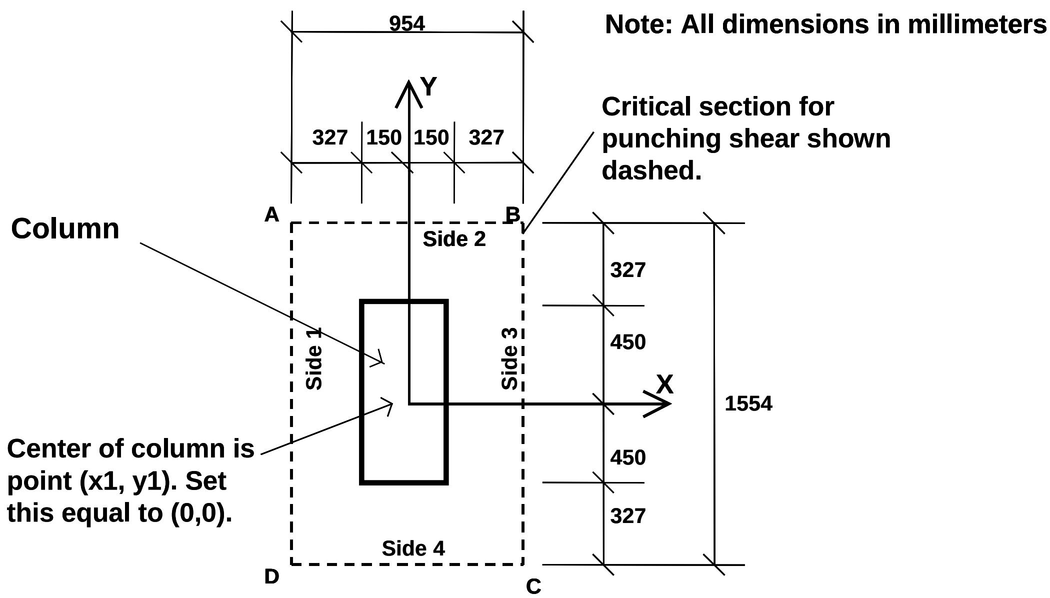


Figure 2: Interior Column, Grid B-2 in SAFE Model

From the SAFE output at Grid B-2:

V= 1126.498 kN

M_2 = 51.9908 kN-m

M_3 = 45.7234 kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{eff,x} = \frac{V}{ud}\left(f + \frac{1.5M_x}{Vy} \right)$$

$$v_{eff,x} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 51.9908 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 954} \right) = 1.1049 \text{ (Govern)}$$

$$v_{eff,y} = \frac{V}{ud}\left(f + \frac{1.5M_y}{Vx} \right)$$

$$v_{eff,y} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 45.7234 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 1554} \right) = 1.0705$$

The largest absolute value of v = **1.1049 N/mm²**

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd} \right)^{1/3}\left(\frac{400}{d} \right)^{1/4} = 0.3568 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{25} \right)^{1/3} = \left(\frac{30}{25} \right)^{1/3} = 1.0627 > 1.0 \text{ OK}$$

$\gamma_m = 1.25$

$$\left(\frac{400}{d} \right)^{1/4} = 1.16386 > 1 \text{ OK.}$$

$f_{cu} \leq 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

Area of reinforcement at the face of column for design strip are as follows:

A_s in Strip Layer A = 9494.296 mm²

A_s in Strip Layer B = 8314.486 mm²

$$\text{Average } A_s = (9494.296 + 8314.486)/2 = 8904.391 \text{ mm}^2$$

$$\frac{100\,A_s}{bd} = 100 \bullet 8904.391/(8000 \bullet 218) = 0.51057$$

$$v_c = \frac{0.79 \bullet 1.0 \bullet 1.0627}{1.25} \bullet (0.51057)^{1/3} \bullet 1.16386 = 0.6247 \text{ MPa}$$

BS 3.7.7.3 yields the value of v = **0.625 N/mm²**, and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{v} = \frac{1.1049}{0.6247} = 1.77$$

EXAMPLE Hong Kong CP-2013 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

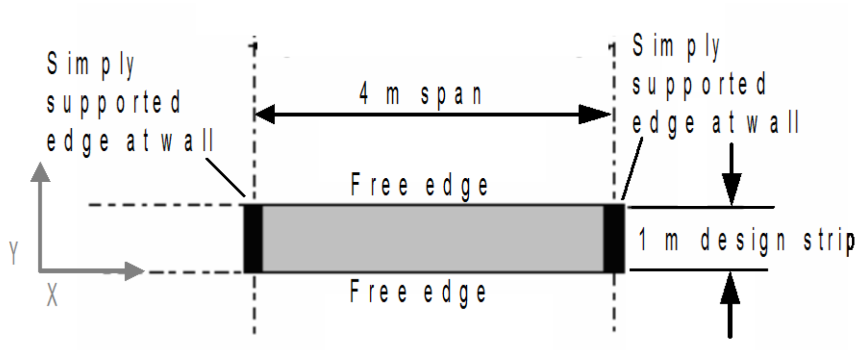


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the Hong Kong CP-04 load combination factors, 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing analysis, design is performed using the Hong Kong CP-04 code by SAFE and also by hand computation. Table 1 shows the comparison of the design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2×10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	SAFE	27.197	5.853
	Calculated	27.200	5.842

$A^{+}_{s,\min}$ = 162.5 sq-mm

COMPUTER FILE: Hong Kong CP-13 RC-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\gamma_{m,\ steel}$ = 1.15

$\gamma_{m,\ concrete}$ = 1.50

b = 1000 mm

For the load combination, the w and M are calculated as follows:

$$w = (1.4w_d + 1.6w_t)\ b$$

$$M = \frac{wl_1^2}{8}$$

$$A_{s,\min} = 0.0013b_w d$$

$$= 162.5 \text{ sq-mm}$$

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 13.6 kN/m

$M_{\text{-strip}}$ = 27.2 kN-m

$M_{\text{-design}}$ = 27.2366 kN-m

The depth of the compression block is given by:

$$K = \frac{M}{f_{cu}bd^2} = 0.05810 < 0.156$$

The area of tensile steel reinforcement is then given by:

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \leq 0.95d \ = 116.3283$$

$$A_s = \frac{M}{0.87 f_y z} = 585.046 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.850 \text{ sq-cm}$$

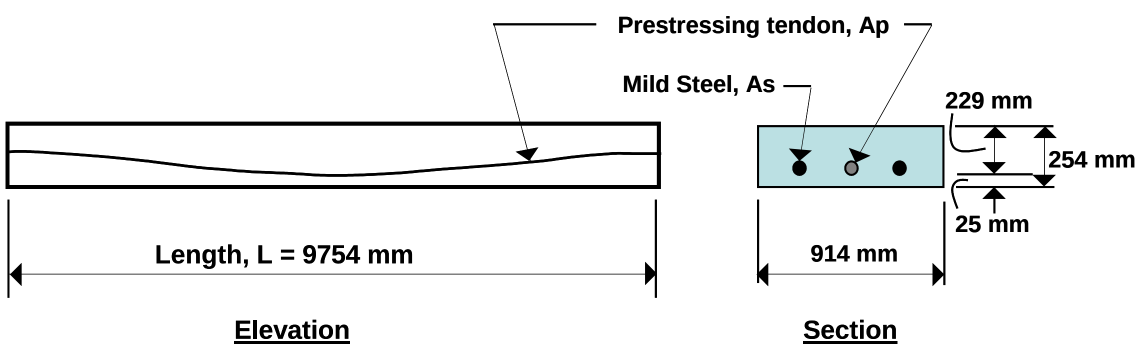
EXAMPLE IS 456-00 PT-SL-001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.



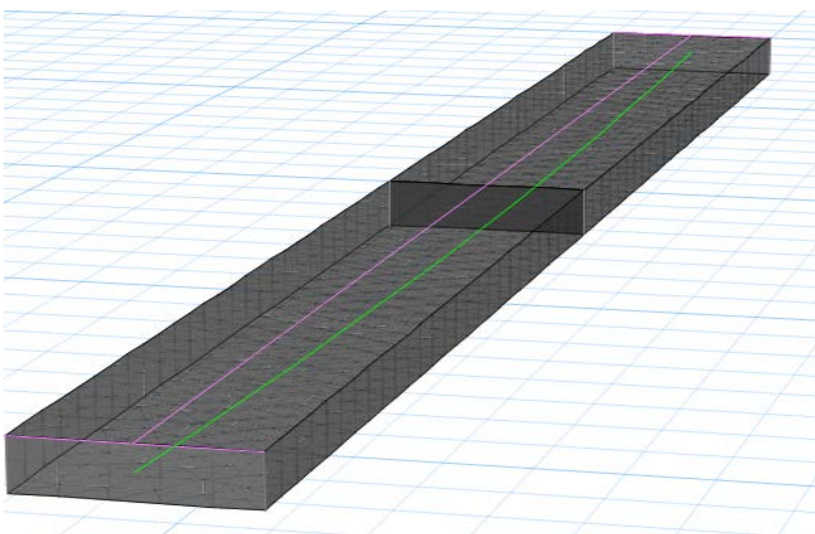


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads have been added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the SAFE results and summarized for verification and validation of the SAFE results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	kN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	kN/m^2
Live load	w_l	=	4.788	kN/m^2

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	175.6	175.65	0.03%
Area of Mild Steel req'd, As (sq-cm)	19.53	19.768	1.22%
Transfer Conc. Stress, top (D+PT_I), MPa	–5.058	–5.057	0.02%
Transfer Conc. Stress, bot (D+PT_I), MPa	2.839	2.839	0.00%
Normal Conc. Stress, top (D+L+PT_F), MPa	–10.460	–10.465	0.05%
Normal Conc. Stress, bot (D+L+PT_F), MPa	8.402	8.407	0.06%

COMPUTER FILE: IS 456-00 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
f_{ck} = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e =1210 MPa

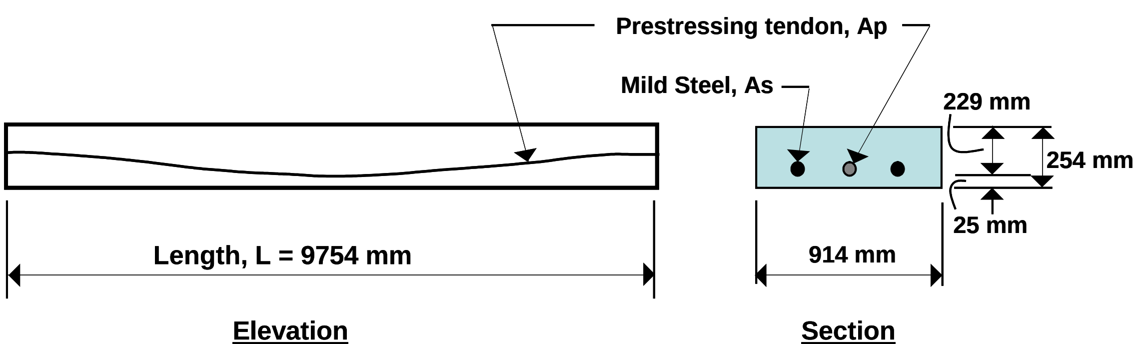
γ_s = 1.15

γ_c = 1.50

α = 0.36

β = 0.42 $\quad \dfrac{x_{\max}}{d} = 0.53 - 0.05\dfrac{f_y - 250}{165}$ if $250 < f_y \le 415$ MPa

$$\frac{x_{u,\max}}{d} = 0.484$$



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.50 = 8.976 kN/m^2 (D_u)
Live, = 4.788 kN/m^2 (L) × 1.50 = 7.182 kN/m^2 (L_u)
Total = 10.772 kN/m^2 (D+L) = 16.158 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 16.158 kN/m^2 × 0.914 m = 14.768 kN/m

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8} = 14.768 \times (9.754)^2 / 8$ = 175.6 kN-m

Ultimate Stress in strand, f_{PS} = from Table 11: f_P = 1435 MPa

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197.4(1435)/1000 = 283.3\text{ kN}$

Compression block depth ratio: $m = \dfrac{M}{bd^2\alpha f_{ck}}$

$$= \frac{175.6}{(0.914)(0.229)^2(0.36)(30000)} = 0.3392$$

Required area of mild steel reinforcing,

$$\frac{x_u}{d} = \frac{1-\sqrt{1-4\beta m}}{2\beta} = \frac{1-\sqrt{1-4(0.42)(0.3392)}}{2(0.42)} = 0.4094 \quad > \quad \frac{x_{u,\max}}{d} = 0.484$$

The area of tensile steel reinforcement is then given by:

$$z = d\left\{1-\beta\frac{x_u}{d}\right\} = 229\big(1-0.42(0.4094)\big) = 189.6\text{ mm}$$

$$A_{NET} = \frac{M_u}{(f_y/\gamma_s)\,z} = \frac{175.6}{(400/1.15)189.6}(1e6) = 2663\text{ mm}^2$$

$$A_s = A_{NET} - A_P\left(\frac{f_P}{f_y}\right) = 2663 - 198\left(\frac{1435}{400}\right) = 1953\text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination ($D+PT_i$) = $1.0D+0.0L+1.0PT_I$

Tendon stress at transfer = jacking stress − stressing losses =1490 − 186 = 1304 MPa

The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4\text{ kN}$

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04\text{ kN-m}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102\text{ mm})/1000 = 26.25\text{ kN-m}$

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-257.4}{0.254(0.914)} \pm \dfrac{65.04-26.23}{0.00983}$

where $S=0.00983\text{m}^3$

$f = -1.109 \pm 3.948\text{ MPa}$

$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$

Normal Condition, load combinations: $(D+L+PT_F) = 1.0D+1.0L+1.0PT_F$

Tendon stress at normal = jacking – stressing – long-term=1490 – 186 – 94 = 1210 MPa
The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN
Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m
Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m
Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for $(D+L+PT_F)$,

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$

$$f = -1.029 \pm 9.431$$

$$f = -10.460(\text{Comp})\max, 8.402(\text{Tension})\max$$

EXAMPLE IS 456-00 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress block extends below the flange but remains within the balanced condition permitted by IS 456-2000.
- The average shear stress in the beam is below the maximum shear stress allowed by IS 456-2000, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T-beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading. One dead load case (DL20) and one live load case (LL80) with only symmetric third-point loads of magnitudes 20, and 80 kN, respectively, are defined in the model. One load combinations (COMB80) is defined using the IS 456-2000 load combination factors of 1.5 for dead loads and 1.5 for live loads. The model is analyzed for both of these load cases and the load combinations.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results. The moment and shear force are identical. After completing the analysis, design is performed using the IS 456-2000 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.

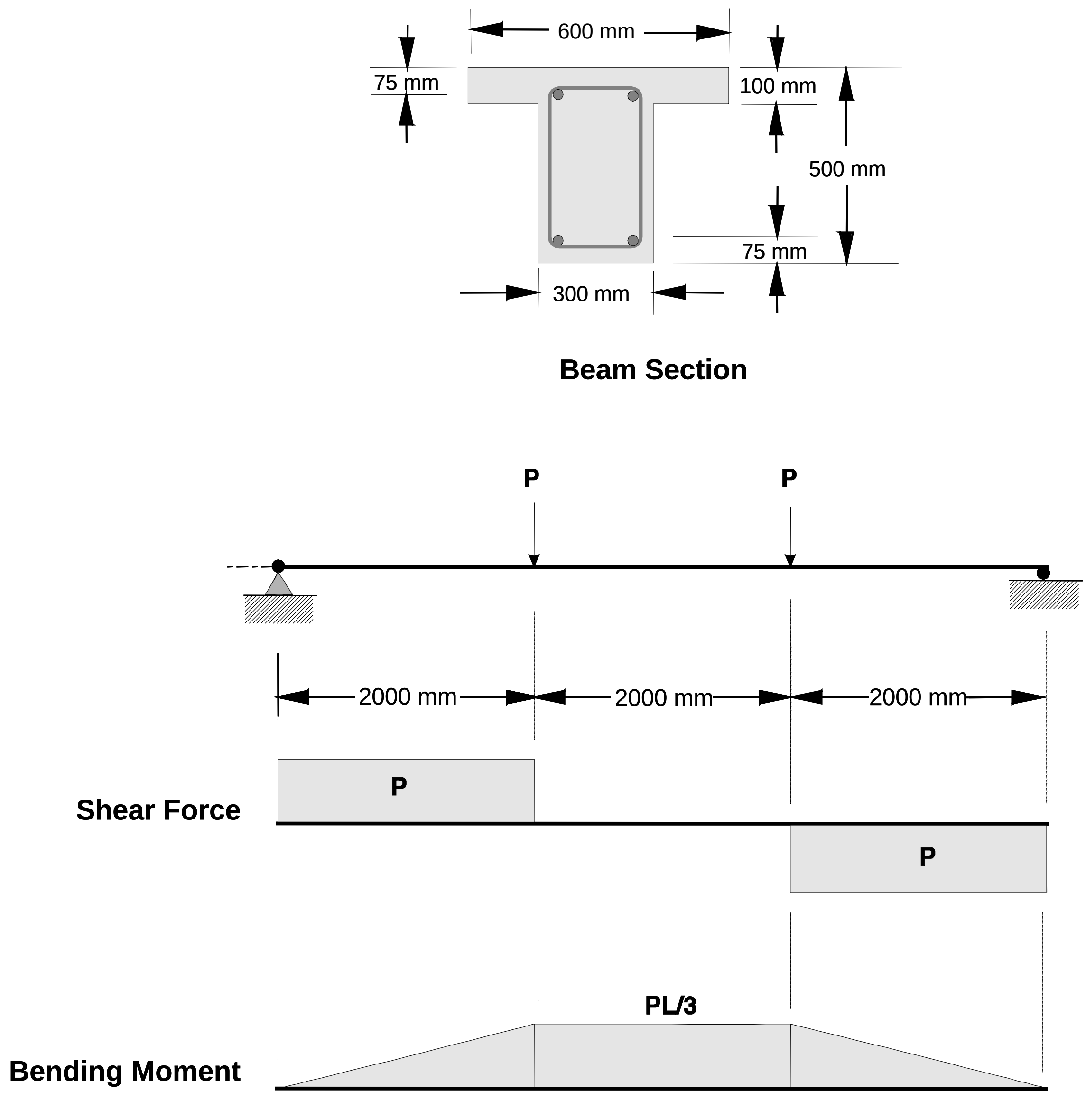


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span,	l	=	6000	mm
Overall depth,	h	=	500	mm
Flange Thickness,	d_s	=	100	mm
Width of web,	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.,	d_c	=	75	mm
Effective depth,	d	=	425	mm
Depth of comp. reinf.,	d'	=	75	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	25×10^5	MPa
Modulus of elasticity,	E_s	=	2×10^8	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	20	kN
Live load,	P_l	=	80	kN

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-cm)
		A_s^+
SAFE	312	21.13
Calculated	312	21.13

$A^+_{s,min}$ = 235.6 sq-mm

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-cm/m)	
	SAFE	Calculated
156	7.76	7.73

COMPUTER FILE: IS 456-00 RC-BM-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$

$\alpha = 0.36$

$\beta = 0.42$

$$A_{s,\min} \geq \frac{0.85}{f_y} bd = 235.6 \text{ sq-mm}$$

COMB80

$P = (1.4P_d + 1.6P_t) = 156$ kN

$$M^* = \frac{N^* l}{3} = 312 \text{ kN-m}$$

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \leq 250 \text{ MPa} \\ 0.53 - 0.05\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \leq 415 \text{ MPa} \\ 0.48 - 0.02\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \leq 500 \text{ MPa} \\ 0.46 & \text{if} \quad f_y \geq 500 \text{ MPa} \end{cases}$$

$$\frac{x_{u,\max}}{d} = 0.4666$$

The normalized design moment, m, is given by

$$m = \frac{M_u}{b_f d^2 \alpha f_{ck}}$$

$$\text{M} = 312\text{x}10^6/(600 \bullet 425^2 \bullet 0.36 \bullet 30) = 0.26656$$

$$\left(\frac{D_f}{d}\right) = 100/425 = 0.23529$$

$$\frac{x_u}{d} = \frac{1-\sqrt{1-4\beta m}}{2\beta} = 0.305848 > \left(\frac{D_f}{d}\right)$$

$$\gamma_f = 0.15 x_u + 0.65 D_f \quad \text{if} \qquad D_f > 0.2d = 84.49781$$

$$M_f = 0.45 f_{ck}\left(b_f - b_w\right)\gamma_f\left(d - \frac{\gamma_f}{2}\right) = 130.98359 \text{ kN-m}$$

$$M_w = M_u - M_f. = 181.0164 \text{ kN-m}$$

$$M_{w,\text{single}} = \alpha f_{ck} b_w d^2 \frac{x_{u,\max}}{d}\left[1-\beta\frac{x_{u,\max}}{d}\right] = 233.233 < M_w$$

$$m = \frac{M_w}{b_f d^2 \alpha f_{ck}} = 0.309310$$

$$\frac{x_u}{d} = \frac{1-\sqrt{1-4\beta m}}{2\beta} = 0.36538$$

$$A_s = \frac{M_f}{\left(f_y/\gamma_s\right)\left(d-0.5y_f\right)} + \frac{M_w}{\left(f_y/\gamma_s\right)z} = 2113 \text{ sq-mm}$$

Shear Design

$$\tau_v = \frac{V_u}{bd} = 1.2235$$

$\tau_{\max} = 3.5$ for M30 concrete

$k = 1.0$

$\delta = 1 \quad$ if $P_u \le 0$, Under Tension

$$\frac{100\,A_s}{bd} = 0.15 \text{ as } 0.15 \le \frac{100\,A_s}{bd} \le 3$$

$$\left(\frac{f_{ck}}{25}\right)^{1/4} = 1.0466$$

$\tau_c = 0.29$ From Table 19 of IS 456:2000 code

$\tau_{cd} = k\delta\tau_c = 0.29$

$\tau_{cd} + 0.4 = 0.69$

The required shear reinforcement is calculated as follows:

If $\tau_{cd} + 0.4 < \tau_v \le \tau_{c,\max}$,

$$\frac{A_{sv}}{s_v} \ge \frac{\left(\tau_v - \tau_{cd}\right) b}{0.87\ f_y} = 7.73\ \text{sq-cm/m}$$

EXAMPLE IS 456-00 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

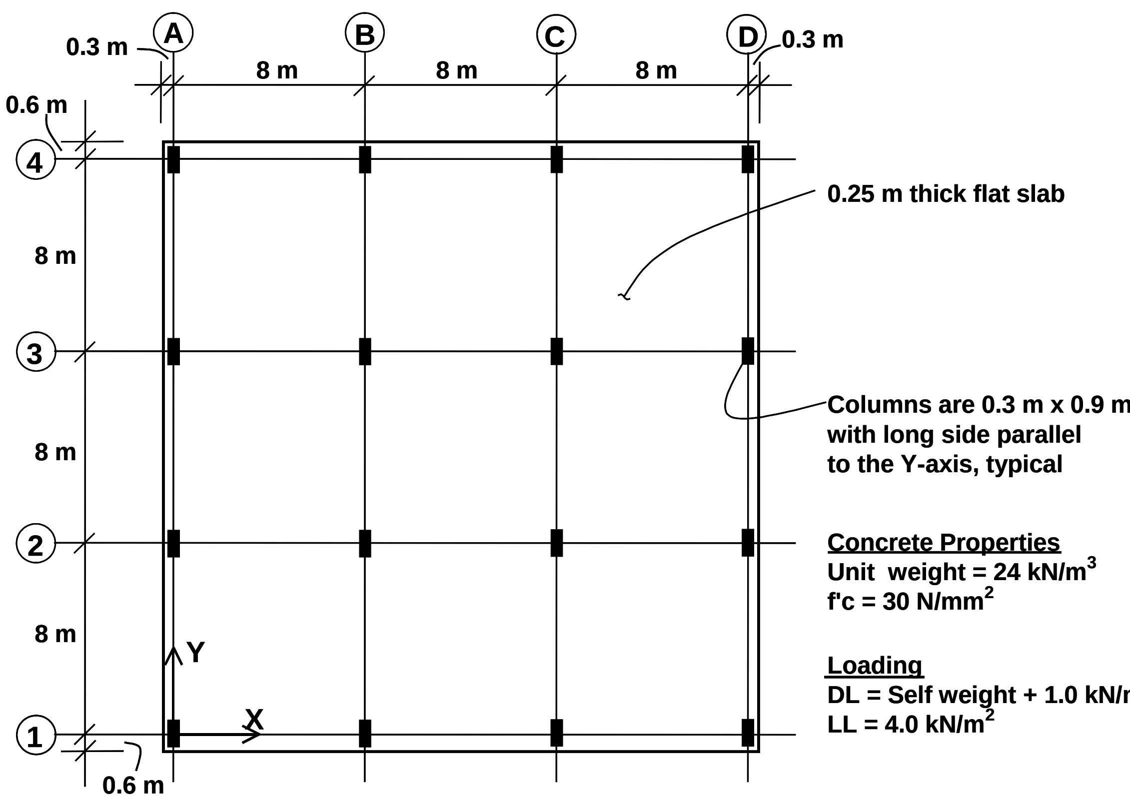


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a $f'c$ of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained in SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
SAFE	1.792	1.141	1.57
Calculated	1.792	1.141	1.57

COMPUTER FILE: IS 456-00 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using SAFE Method

$d = [(250 - 26) + (250 - 38)]/2 = 218$ mm

Refer to Figure 1.

$b_0 = 518 + 1118 + 1118 + 518 = 3272$ mm

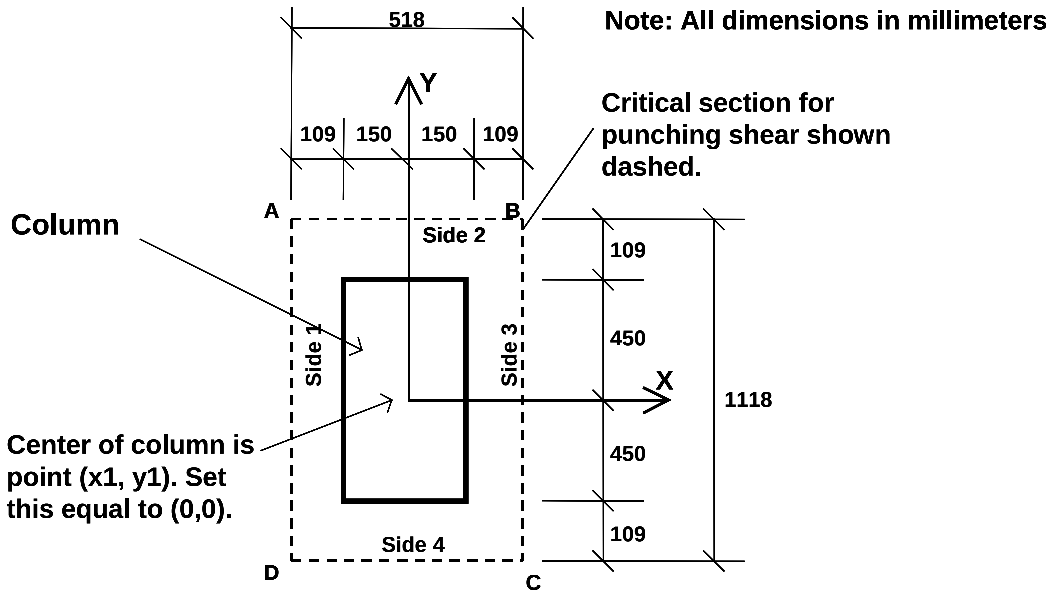


Figure 2: Interior Column, Grid B-2 in SAFE Model

$$\gamma_{V2} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{1118}{518}}} = 0.495$$

$$\gamma_{V3} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{518}{1118}}} = 0.312$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–259	0	259	0	N.A.
y_2	0	559	0	–559	N.A.
L	1118	518	1118	518	b_0 = 3272
d	218	218	218	218	N.A.
Ld	243724	112924	243724	112924	713296
Ldx_2	–63124516	0	63124516	0	0
Ldy_2	0	63124516	0	–63124516	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{713296} = 0\,mm$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{713296} = 0\,mm$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	1118	518	1118	518	N.A.
d	218	218	218	218	N.A.
$x_2 - x_3$	–259	0	259	0	N.A.
$y_2 - y_3$	0	559	0	–559	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	2.64E+10	3.53E+10	2.64E+10	3.53E+10	1.23E+11
I_{YY}	1.63E+10	2.97E+09	1.63E+10	2.97E+09	3.86E+10
I_{XY}	0	0	0	0	0

From the SAFE output at Grid B-2:

V_U = 1126.498 kN

$\gamma_{V2}\ M_{U2}$ = –25.725 kN-m

$\gamma_{V3}\ M_{U3}$ = 14.272 kN-m

At the point labeled A in Figure 2, x_4 = –259 and y_4 = 559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(559-0)-(0)(-259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(-259-0)-(0)(559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 – 0.1169 – 0.0958 = **1.3666 N/mm^2** at point A

At the point labeled B in Figure 2, x_4 = 259 and y_4 = 559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(559-0)-(0)(259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(259-0)-(0)(559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 – 0.1169 + 0.0958 =**1.5582 N/mm^2** at point B

At the point labeled C in Figure 2, x_4 = 259 and y_4 = –559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(-559-0)-(0)(259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(259-0)-(0)(-559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 + 0.1169 + 0.0958 = **1.792 N/mm^2** at point C

At the point labeled D in Figure 2, x_4 = –259 and y_4 = –559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(-559-0)-(0)(-259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(-259-0)-(0)(-559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 + 0.1169 – 0.0958 = **1.6004 N/mm^2** at point D

Point C has the largest absolute value of v_u, thus v_{max} = **1.792 N/mm^2**

The shear capacity is calculated based on the minimum of the following three limits:

$k_s = 0.5 + \beta_c \leq 1.0 = 0.833$ (IS 31.6.3.1)

$\tau_c = 0.25 = 1.127$ N/mm^2 (IS 31.6.3.1)

$v_c = k_s\ \tau_c = 1.141$ N/mm^2 (IS 31.6.3.1)

CSA 13.3.4.1 yields the smallest value of $v_c = 1.141$ N/mm^2, and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{v_c} = \frac{1.792}{1.141} = 1.57$$

EXAMPLE IS 456-00 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

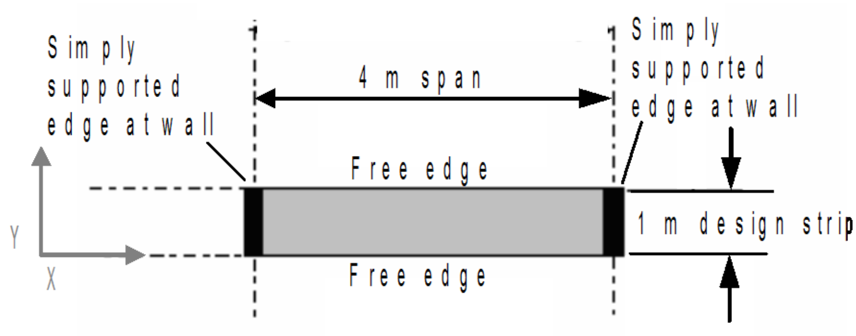


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the IS 456-00 load combination factors, 1.5 for dead loads and 1.5 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing analysis, design was performed using the IS 456-00 code by SAFE and also by hand computation. Table 1 shows the comparison of the design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2×10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm)	
			A_s^+	A_s^-
Medium	SAFE	26.997	5.830	--
	Calculated	27.000	5.830	--

$A^+_{s,min}$ = 230.978 sq-mm

COMPUTER FILE: IS 456-00 RC-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\gamma_s = 1.15$

$\gamma_c = 1.50$

$\alpha = 0.36$

$\beta = 0.42$

$b = 1000$ mm

For the load combination, w and M are calculated as follows:

$w = (1.5w_d + 1.5w_t)\, b$

$$M = \frac{wl_1^2}{8}$$

$$A_{s,\min} = \frac{0.85}{f_y} bd$$

$= 230.978$ sq-mm

COMB100

$w_d = 4.0$ kPa

$w_t = 5.0$ kPa

$w = 13.5$ kN/m

$M_{\text{-strip}} = 27.0$ kN-m

$M_{\text{-design}} = 27.0363$ kN-m

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \le 250 \text{ MPa} \\ 0.53 - 0.05\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \le 415 \text{ MPa} \\ 0.48 - 0.02\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \le 500 \text{ MPa} \\ 0.46 & \text{if} \quad f_y \ge 500 \text{ MPa} \end{cases}$$

$$\frac{x_{u,\max}}{d} = 0.466$$

The depth of the compression block is given by:

$$m = \frac{M_u}{bd^2 \alpha f_{ck}}$$

$$= 0.16$$

$$\frac{x_u}{d} = \frac{1 - \sqrt{1 - 4\beta m}}{2\beta} = 0.1727488 < \frac{x_{u,\max}}{d}$$

The area of tensile steel reinforcement is given by:

$$z = d\left\{1 - \beta \frac{x_u}{d}\right\}. = 115.9307 \text{ mm}$$

$$A_s = \frac{M_u}{\left(f_y / \gamma_s\right) z}, = 583.027 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.830 \text{ sq-cm}$$

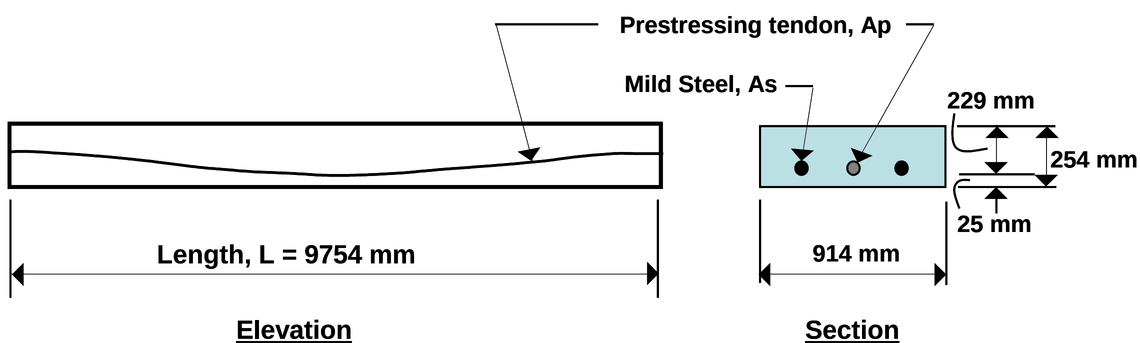
EXAMPLE Italian NTC 2008 PT-SL-001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm as shown in shown in Figure 1.



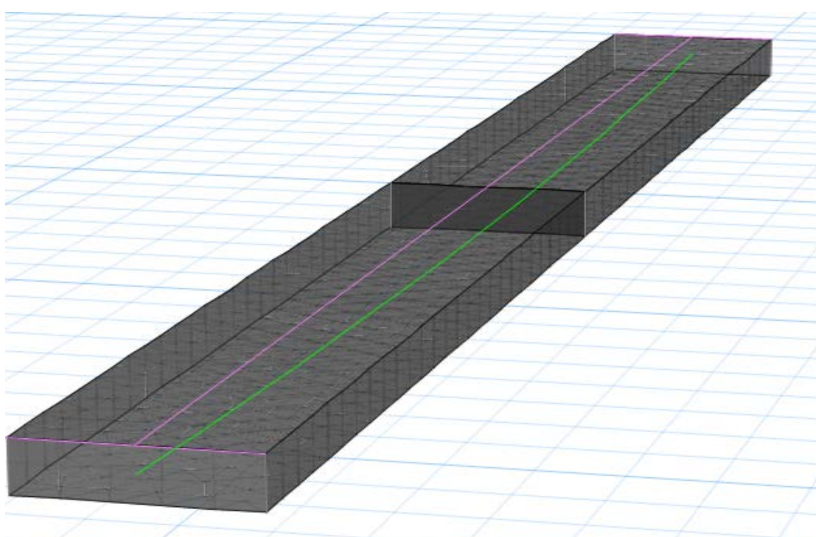


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, was added to the A-Strip. The self weight and live loads have been added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the SAFE results and summarized for verification and validation of the SAFE results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	KN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	KN/m^2
Live load	w_l	=	4.788	KN/m^2

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments, required mild steel reinforcing, and slab stresses with independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	165.9	165.9	0.00%
Transfer Conc. Stress, top ($D+PT_I$), MPa	–5.057	–5.057	0.00%
Transfer Conc. Stress, bot ($D+PT_I$), MPa	2.839	2.839	0.00%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	–10.460	–10.465	0.05%
Normal Conc. Stress, bot ($D+L+PT_F$), MPa	8.402	8.407	0.06%
Long-Term Conc. Stress, top ($D+0.5L+PT_{F(L)}$), MPa	–7.817	–7.817	0.00%
Long-Term Conc. Stress, bot ($D+0.5L+PT_{F(L)}$), MPa	5.759	5.759	0.00%

Table 2 Comparison of Design Moments and Reinforcements

Method	Design Moment (kN-m)	Reinforcement Area (sq-cm)
		A_s^+
SAFE	165.9	16.39
Calculated	165.9	16.29

COMPUTER FILE: ITALIAN NTC 2008 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

Design Parameters:

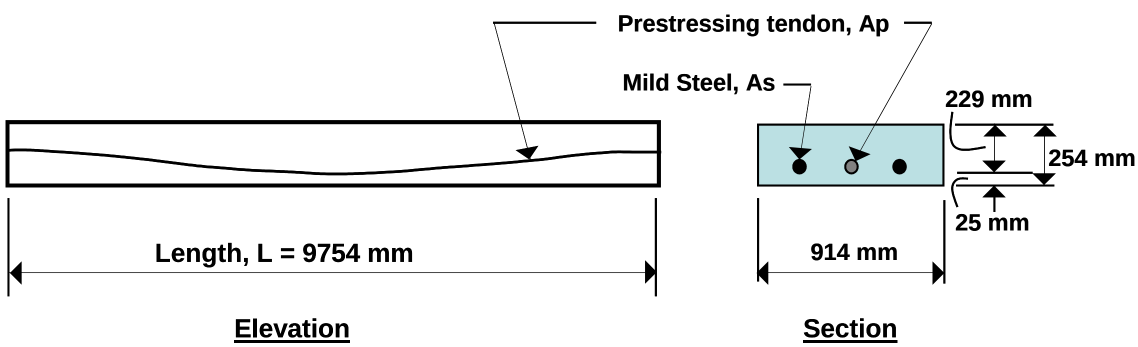
Mild Steel Reinforcing	Post-Tensioning
$f'c$ = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$

$\eta = 1.0$ for $f_{ck} \leq 50$ MPa

$\lambda = 0.8$ for $f_{ck} \leq 50$ MPa



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.35 = 8.078 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) × 1.50 = 7.182 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 15.260 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 15.260 kN/m^2 × 0.914 m = 13.948 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8} = 13.948 \times (9.754)^2 / 8 = 165.9$ kN-m

Ultimate Stress in strand, $f_{PS} = f_{SE} + 7000d\left(1 - 1.36\dfrac{f_{PU}A_P}{f_{CK}bd}\right)\Big/ l$

$$= 1210 + 7000(229)\left(1 - 1.36\frac{1862(198)}{30(914)(229)}\right)\Big/(9754)$$

$$= 1361\text{ MPa}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 2(99)(1361)/1000 = 269.5\text{ kN}$

Design moment M = 165.9 kN-m

Compression block depth ratio: $m = \dfrac{M}{bd^2\eta f_{cd}}$

$$= \frac{165.9}{(0.914)(0.229)^2(1)(30000/1.50)} = 0.1731$$

Required area of mild steel reinforcing,

$$\omega = 1 - \sqrt{1-2m} = 1 - \sqrt{1-2(0.1731)} = 0.1914$$

$$A_{EquivTotal} = \omega\left(\frac{\eta f_{cd}bd}{f_{yd}}\right) = 0.1914\left(\frac{1(30/1.5)(914)(229)}{400/1.15}\right) = 2303\text{ mm}^2$$

$$A_{EquivTotal} = A_P\left(\frac{1366}{400}\right) + A_S = 2311\text{ mm}^2$$

$$A_S = 2303 - 198\left(\frac{1361}{400}\right) = 1629\text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination ($D+PT_i$) = $1.0D+0.0L+1.0PT_I$

Tendon stress at transfer = jacking stress − stressing losses =1490 − 186 = 1304 MPa

The force in the tendon at transfer = $1304(197.4)/1000 = 257.4\text{ kN}$

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04\text{ kN-m}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102\text{ mm})/1000 = 26.25\text{ kN-m}$

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-257.4}{0.254(0.914)} \pm \dfrac{65.04 - 26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.109 \pm 3.948 \text{ MPa}$$
$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: $(D+L+PT_F) = 1.0D+1.0L+1.0PT_F$

Tendon stress at normal = jacking – stressing – long-term=1490 – 186 – 94 = 1210 MPa

The force in tendon at normal = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for $(D+L+PT_F)$,

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$
$$f = -1.029 \pm 9.431$$
$$f = -10.460(\text{Comp})\max,\ 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: $(D+0.5L+PT_{F(L)}) = 1.0D+0.5L+1.0PT_F$

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for $(D+0.5L+PT_{F(L)})$,

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$
$$f = -1.029 \pm 6.788$$
$$f = -7.817(\text{Comp})\max,\ 5.759(\text{Tension})\max$$

EXAMPLE Italian NTC 2008 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by Italian NTC 2008.
- The average shear stress in the beam is below the maximum shear stress allowed by Italian NTC 2008, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T-beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading One dead load case (DL30) and one live load case (LL130) with only symmetric third-point loads of magnitudes 30, and 130 kN, respectively, are defined in the model. One load combinations (COMB130) is defined using the Italian NTC 2008 load combination factors of 1.35 for dead loads and 1.5 for live loads. The model is analyzed for both of these load cases and the load combinations.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results. These moment and shear force are identical. After completing the analysis, design is performed using the Italian NTC 2008 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.

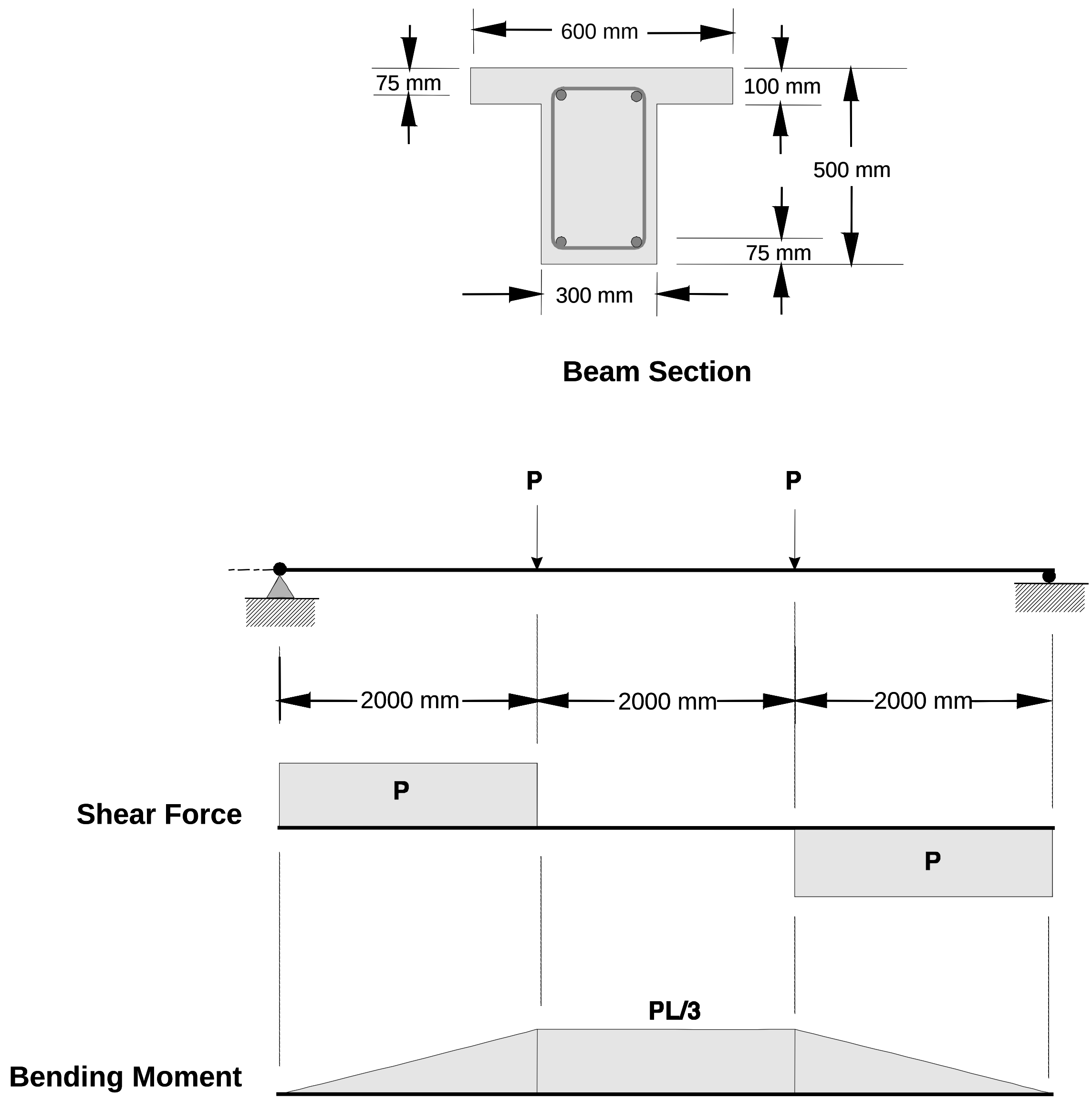


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span,	l	=	6000	mm
Overall depth,	h	=	500	mm
Flange thickness,	d_s	=	100	mm
Width of web,	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.,	d_c	=	75	mm
Effective depth,	d	=	425	mm
Depth of comp. reinf.,	d'	=	75	mm
Concrete strength,	f'_{ck}	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	$25x10^5$	MPa
Modulus of elasticity,	E_s	=	$2x10^8$	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	30	kN
Live load,	P_l	=	130	kN

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
SAFE	471	31.643
Calculated	471	31.643

$A^+_{s,min}$ = 2.09 sq-cm

Table 2 Comparison of Shear Reinforcements

Method	Shear Force (kN)	Reinforcement Area , $\frac{A_v}{s}$ (sq-cm/m) A_s^+
SAFE	235.5	6.16
Calculated	235.5	6.16

COMPUTER FILE: Italian NTC 2008 RC-BM-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for both of the load combinations:

γ_s = 1.15

γ_c = 1.50

$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c$

$f_{yd} = f_{yk} / \gamma_s$

$\eta = 1.0$ for $f_{ck} \leq 50$ MPa

$\lambda = 0.8$ for $f_{ck} \leq 50$ MPa

$A_{s,\min} = 0.26 \dfrac{f_{ctm}}{f_{yk}} bd$ = 208.73 sq-mm

$A_{s,\min} = 0.0013 b_w h$ = 195.00 sq-mm

$\gamma_{m,\ steel}$ = 1.15

$\gamma_{m,\ concrete}$ = 1.50

α_{cc} = 1.0

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta f_{cd}} = \frac{471 \bullet 10^6}{600 \bullet 425^2 \bullet 1.0 \bullet 1.0 \bullet 30/1.5} = 0.217301$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

k_1 = 0.44

$k_2 = k_4 = 1.25(0.6 + 0.0014/\varepsilon_{cu2}) = 1.25$

δ is assumed to be 1

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa} = 0.448$$

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1-\frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.29417$$

$$\omega_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim} = 1-\sqrt{1-2m_{\lim}} = 0.3584$$

$$a_{\max} = \omega_{\lim} d = 152.32 \text{ mm}$$

$$\omega = 1 - \sqrt{1-2m} = 0.24807$$

$$a = \omega d = 105.4299 \text{ mm} \le a_{\max}$$

$$A_{s2} = \frac{\left(b_f - b_w\right) h_f \eta\, f_{cd}}{f_{yd}} = 1500 \text{ sq-mm}$$

$$M_2 = A_{s2} f_{yd}\left(d - \frac{h_f}{2}\right) = 225 \text{ kN-m}$$

$$M_1 = M - M_2 = 246 \text{ kN-m}$$

$$m_1 = \frac{M_1}{b_w d^2 \eta\, f_{cd}} = 0.2269896 \le m_{\lim}$$

$$\omega_1 = 1-\sqrt{1-2m_1} = 0.2610678$$

$$A_{s1} = \omega_1\left[\frac{\eta\, f_{cd} b_w d}{f_{yd}}\right] = 1664.304 \text{ sq-mm}$$

$$A_s = A_{s1} + A_{s2} = 3164.307 \text{ sq-mm}$$

Shear Design

$$C_{Rd,c} = 0.18/\gamma_c = 0.18/1.5 = 0.12$$

$$k = 1+\sqrt{\frac{200}{d}} = 1.686 \le 2.0 \text{ with } d \text{ in mm}$$

$$\rho_1 = 0.0$$

$$\sigma_{cp} = N_{Ed}/A_c < 0.2\, f_{cd} = 0.0 \text{ MPa}$$

$$v_{\min} = 0.035 k^{3/2} f_{ck}^{\ 1/2} = 0.419677$$

$$V_{Rd,c} = \left[C_{Rd,c}k\left(100\rho_1 f_{ck}\right)^{1/3} + k_1\sigma_{cp}\right]b_w d = 53.5 \text{ kN}$$

$\alpha_{cw} = 1$

$$v_1 = 0.6\left(1 - \frac{f_{ck}}{250}\right) = 0.528$$

$z = 0.9d = 382.5$ mm

θ is taken as 1.

$$V_{Rd,\max} = \frac{\alpha_{cw} b_w z v_1 f_{cd}}{\cot\theta + \tan\theta} = 1211.76 \text{ kN}$$

$V_{R,dc} < V_{Ed} \leq V_{Rd,\max}$ (govern)

Computing the angle using v_{Ed} :

$$v_{Ed} = \frac{235.5 \bullet 10^3}{0.9 \bullet 425 \bullet 300} = 2.0522$$

$$\theta = 0.5\sin^{-1}\frac{v_{Ed}}{0.2 f_{ck}\left(1 - f_{ck}/250\right)}$$

$$\theta = 0.5\sin^{-1}\frac{2.0522}{0.2 \bullet 30\left(1 - 30/250\right)} = 11.43°$$

$21.8° \leq \theta \leq 45°$, therefore use $\theta = 21.8°$

$$\frac{A_{sw}}{s} = \frac{v_{Ed} b_w}{f_{ywd}\cot\theta}$$

$$\frac{A_{sw}}{s} = \frac{2.0522 \bullet 300}{460/1.15 \bullet 2.5} = 0.61566 \text{ sq-mm/m} = 6.16 \text{ sq-cm/m}$$

EXAMPLE Italian NTC 2008 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

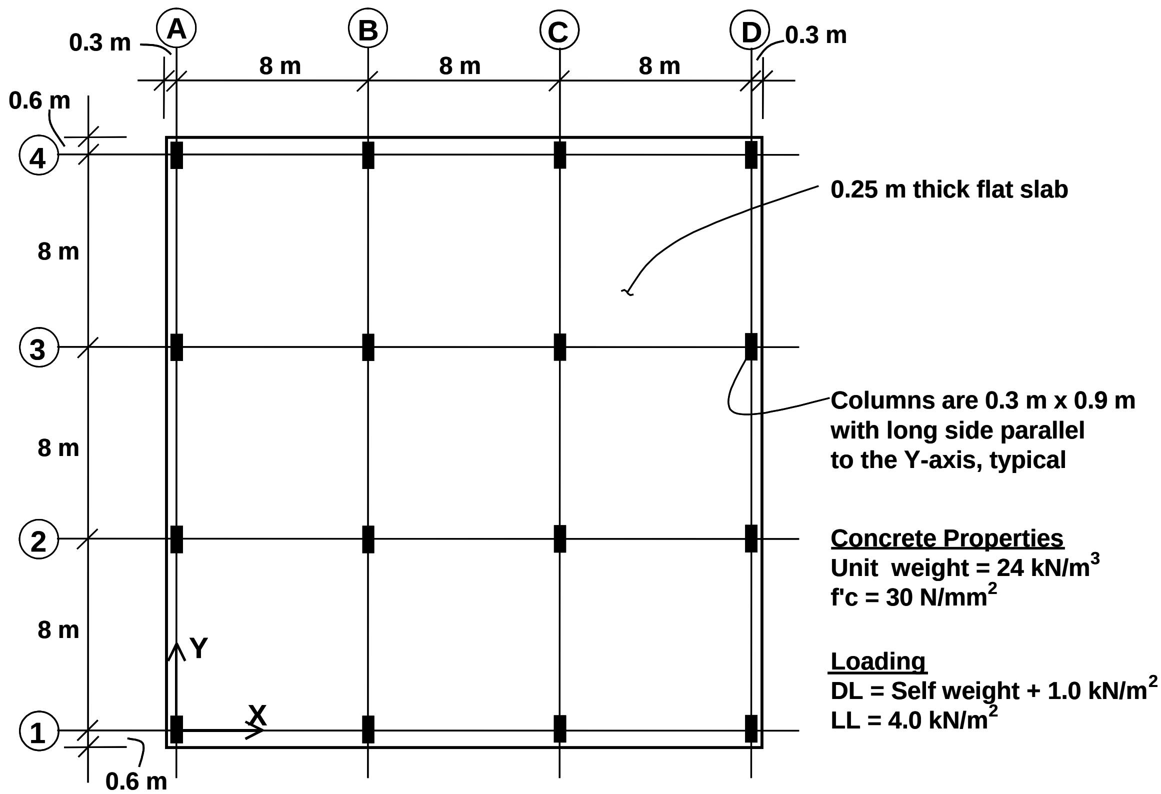


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a *f'c* of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
SAFE	1.100	0.578	1.90
Calculated	1.099	0.578	1.90

COMPUTER FILE: ITALIAN NTC 2008 RC-PN-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATION

Hand Calculation for Interior Column using SAFE Method

$d = [(250-26)+(250-38)]/2$ = 218 mm

Refer to Figure 2.

$u_1 = u = 2 \bullet 300 + 2 \bullet 900 + 2 \bullet \pi \bullet 436 = 5139.468$ mm

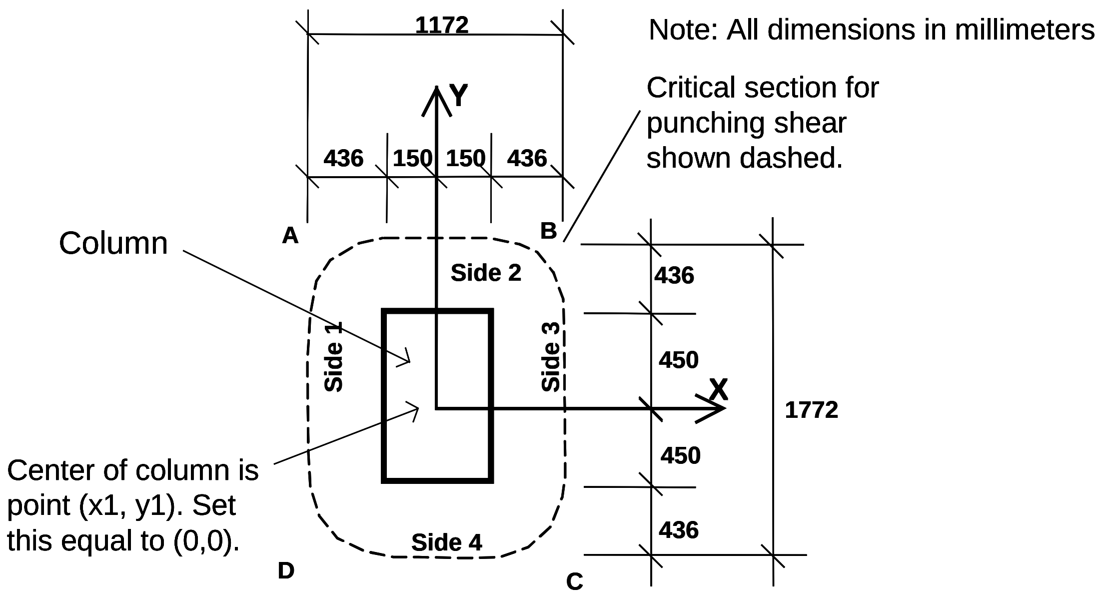


Figure 2: Interior Column, Grid B-2 in SAFE Model

From the SAFE output at Grid B-2:

V_{Ed} = 1112.197 kN

k_2M_{Ed2} = 41.593 kN-m

k_3M_{Ed3} = 20.576 kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{Ed} = \frac{V_{Ed}}{ud}\left[1 + \frac{k_2 M_{Ed,2} u_1}{V_{Ed} W_{1,2}} + \frac{k_3 M_{Ed,3} u_1}{V_{Ed} W_{1,3}}\right] \quad \text{(EC2 6.4.4(2))}$$

$$W_1 = \frac{c_1^2}{2} + c_1 c_2 + 4 c_2 d + 16 d^2 + 2\pi d c_1$$

$$W_{1,2} = \frac{900^2}{2} + 300 \bullet 900 + 4 \bullet 300 \bullet 218 + 16 \bullet 218^2 + 2\pi \bullet 218 \bullet 900$$

$$W_{1,2} = 2{,}929{,}744.957 \text{ mm}^2$$

$$W_{1,3} = 3\frac{900^2}{2} + 900 \bullet 300 + 4 \bullet 900 \bullet 218 + 16 \bullet 218^2 + 2\pi \bullet 218 \bullet 300$$

$$W_{1,2} = 2{,}271{,}104.319 \text{ mm}^2$$

$$v_{Ed} = \frac{V_{Ed}}{ud}\left[1 + \frac{k_2 M_{Ed,2} u_1}{V_{Ed} W_{1,2}} + \frac{k_3 M_{Ed,3} u_1}{V_{Ed} W_{1,3}}\right]$$

$$v_{Ed} = \frac{1112.197 \bullet 10^3}{5139.468 \bullet 218}\left[1 + \frac{41.593 \bullet 10^6 \bullet 5139.468}{1112.197 \bullet 10^3 \bullet 2929744.957} + \frac{20.576 \bullet 10^6 \bullet 5139.468}{1112.197 \bullet 10^3 \bullet 2271104.319}\right]$$

$$v_{Ed} = 1.099 \text{ N/mm}^2$$

Thus v_{max} = **1.099 N/mm²**

$$C_{Rd,c} = 0.18/\gamma_c = 0.18/1.5 = 0.12 \quad \text{(EC2 6.4.4)}$$

The shear stress carried by the concrete, $V_{Rd,c}$, is calculated as:

$$V_{Rd,c} = \left[C_{Rd,c} k \left(100 \rho_1 f_{ck}\right)^{1/3} + k_1 \sigma_{cp}\right] \quad \text{(EC2 6.4.4)}$$

with a minimum of:

$$v_{Rd,c} = \left(v_{\min} + k_1 \sigma_{cp}\right) \quad \text{(EC2 6.4.4)}$$

$$k = 1 + \sqrt{\frac{200}{d}} \le 2.0 = 1.9578 \quad \text{(EC2 6.4.4(1))}$$

k_1 = 0.15. (EC2 6.2.2(1))

$$\rho_1 = \frac{A_{s1}}{b_w d} \leq 0.02$$

Area of reinforcement at the face of column for design strip are as follows:

A_s in Strip Layer A = 9204.985 mm^2

A_s in Strip Layer B = 8078.337 mm^2

Average A_s = $(9204.985 + 8078.337)/2$ = 8641.661 mm^2

ρ_1 = $8641.661/(8000 \bullet 218)$ = 0.004955 ≤ 0.02

$$v_{\min} = 0.035k^{3/2} f_{ck}^{\ 1/2} = 0.035(1.9578)^{3/2}(30)^{1/2} = 0.525 \text{ N/mm}^2$$

$$v_{Rd,c} = \left[0.12 \bullet 1.9578(100 \bullet 0.004955 \bullet 30)^{1/3} + 0\right] = 0.5777 \text{ N/mm}^2$$

$$\text{Shear Ratio} = \frac{v_{\max}}{v_{Rd,c}} = \frac{1.099}{0.5777} = 1.90$$

EXAMPLE Italian NTC 2008 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

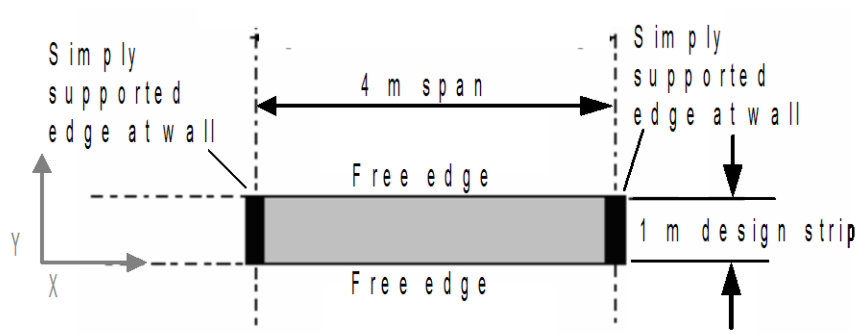


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the Italian NTC 2008 load combination factors, 1.35 for dead loads and 1.5 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. These moments are identical. After completing the analysis, design is performed using the Italian NTC 2008 code by SAFE and also by hand computation. Table 1 shows the comparison of the design reinforcements computed by the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_{ck}	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	$2x10^6$	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm)
		A_s^+
SAFE	25.797	5.400
Calculated	25.800	5.400

$A^+_{s,min}$ = 204.642 sq-mm

COMPUTER FILE: Italian NTC 2008 RC-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$$\eta = 1.0 \text{ for } f_{ck} \leq 50 \text{ MPa}$$

$$\lambda = 0.8 \text{ for } f_{ck} \leq 50 \text{ MPa}$$

b = 1000 mm

For the load combination, w and M are calculated as follows:

$w = (1.35w_d + 1.5w_t)\, b$

$$M = \frac{wl_1^2}{8}$$

$$A_{s,\min} = \max \begin{cases} 0.0013 b_w d \\ 0.26 \dfrac{f_{ctm}}{f_{yk}} bd \end{cases}$$

= 204.642 sq-mm

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 12.9 kN/m

$M_{\text{-strip}}$ = 25.8 kN-m

$M_{\text{-design}}$ = 25.8347 kN-m

$\gamma_{m,\ steel}$ = 1.15

$\gamma_{m,\ concrete}$ = 1.50

α_{cc} = 0.85:

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta f_{cd}} = \frac{25.8347 \bullet 10^6}{1000 \bullet 125^2 \bullet 1.0 \bullet 0.85 \bullet 30/1.5} = 0.097260$$

$$m_{\lim} = \lambda \left(\frac{x}{d}\right)_{\lim} \left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.48$$

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \le 50 \text{ MPa} = 0.60$$

For reinforcement with $f_{yk} \le 500$ MPa, the following values are used:

$k_1 = 0.40$

$k_2 = (0.6 + 0.0014/\varepsilon_{cu2}) = 1.00$

δ is assumed to be 1

$$\omega = 1 - \sqrt{1 - 2m} = 0.10251$$

$$A_s = \omega\left(\frac{\eta f_{cd} bd}{f_{yd}}\right) = 544.61 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.446 sq-cm

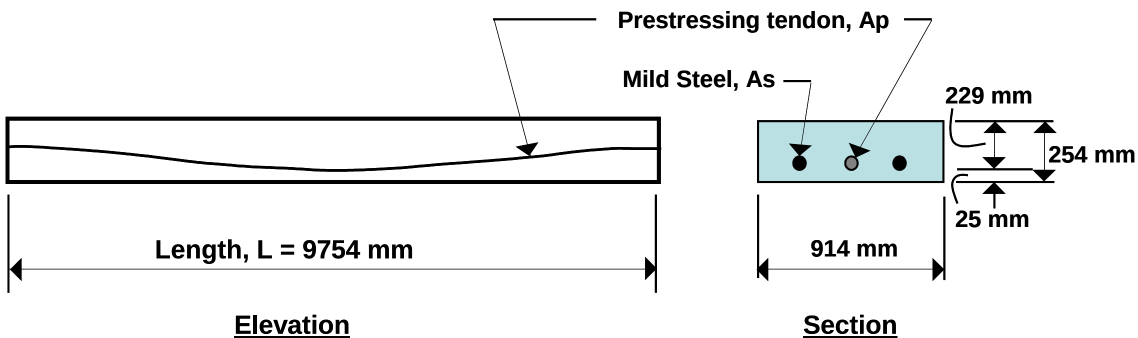
EXAMPLE NZS 3101-06 PT-SL-001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 915 mm wide and spans 9754 mm as, shown in shown in Figure 1.



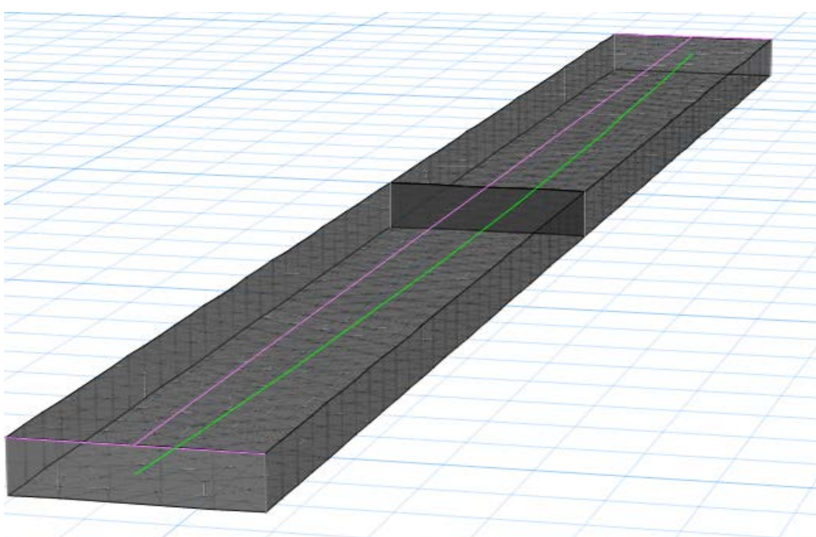


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads have been added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the midspan of the slab. Independent hand calculations were compared with the SAFE results and summarized for verification and validation of the SAFE results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	kN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	kN/m^2
Live load	w_l	=	4.788	kN/m^2

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	156.12	156.14	0.01%
Area of Mild Steel req'd, As (sq-cm)	14.96	15.08	0.74%
Transfer Conc. Stress, top ($D+PT_I$), MPa	–5.058	–5.057	0.02%
Transfer Conc. Stress, bot ($D+PT_I$), MPa	2.839	2.839	0.00%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	–10.460	–10.465	0.05%
Normal Conc. Stress, bot ($D+L+PT_F$), MPa	8.402	8.407	0.06%
Long-Term Conc. Stress, top ($D+0.5L+PT_{F(L)}$), MPa	–7.817	–7.817	0.00%
Long-Term Conc. Stress, bot ($D+0.5L+PT_{F(L)}$), MPa	5.759	5.759	0.00%

COMPUTER FILE: NZS 3101-06 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
$f'c$ = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

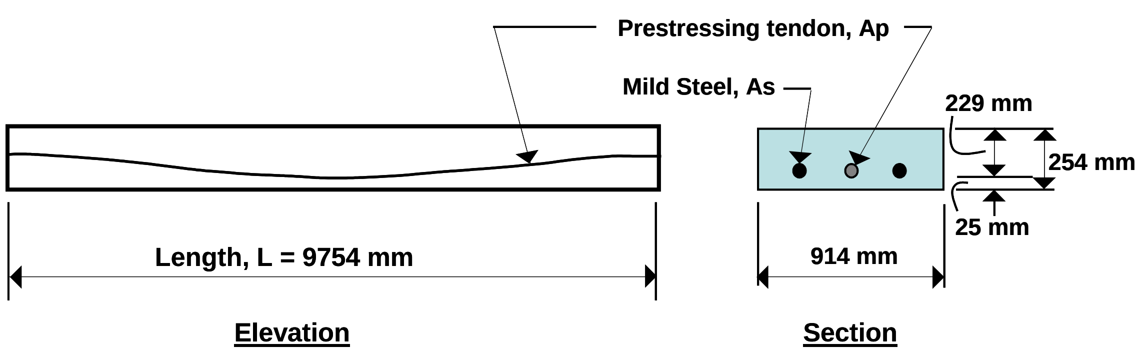
$\phi_b = 0.85$

$\alpha_1 = 0.85 \quad \text{for } f'_c \leq 55 \text{ MPa}$

$\beta_1 = 0.85 \quad \text{for } f'_c \leq 30,$

$$c_b = \frac{\varepsilon_c}{\varepsilon_c + f_y / E_s} d = 214.7$$

$a_{\max} = 0.75\beta_1 c_b = 136.8 \text{ mm}$



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.2 = 7.181kN/m^2 (D_u)
Live, = 4.788 kN/m^2 (L) × 1.5 = 7.182 kN/m^2 (L_u)
Total = 10.772 kN/m^2 (D+L) = 14.363 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 14.363 kN/m^2 × 0.914 m = 13.128 kN/m

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8}$ = 13.128 × (9.754)²/8 = 156.12 kN-m

Ultimate Stress in strand, $f_{PS} = f_{SE} + 70 + \dfrac{f'c}{300\rho_P}$

$$= 1210 + 70 + \frac{30}{300(0.00095)}$$

$$= 1385 \text{ MPa} \le f_{SE} + 200 = 1410 \text{ MPa}$$

Ultimate force in PT, $F_{ult,PT} = A_P\left(f_{PS}\right) = 2(99)(1385)/1000 = 274.23 \text{ kN}$

Stress block depth, $a = d - \sqrt{d^2 - \dfrac{2M^*}{\alpha f'_c \phi b}}$

$$= 0.229 - \sqrt{0.229^2 - \frac{2(156.12)}{0.85(30000)(0.85)(0.914)}}(1e3) = 37.48 \text{ mm}$$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right)\phi = 274.23\left(229 - \frac{37.48}{2}\right)(0.85)/1000 = 49.01 \text{ kN-m}$$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 156.1 - 49.10 = 107.0 \text{ kN-m}$

Required area of mild steel reinforcing,

$$A_S = \frac{M_{net}}{\phi f_y (d - \frac{a}{2})} = \frac{107.0}{0.85(400000)\left(0.229 - \frac{0.03748}{2}\right)}(1e6) = 1496 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (D+PT$_i$) = 1.0D+0.0L+1.0PT$_I$

Tendon stress at transfer = jacking stress − stressing losses =1490 − 186 = 1304 MPa

The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4 \text{ kN}$

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04 \text{ kN-m}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25 \text{ kN-m}$

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-257.4}{0.254(0.914)} \pm \dfrac{65.04 - 26.23}{0.00983}$

where S = 0.00983m³

$$f = -1.109 \pm 3.948 MPa$$
$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT_F) = 1.0D+1.0L+1.0PT_F

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT_F),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$
$$f = -1.029 \pm 9.431$$
$$f = -10.460(\text{Comp})\max,\ 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: (D+0.5L+$PT_{F(L)}$) = 1.0D+0.5L+1.0PT_F

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+0.5L+$PT_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$
$$f = -1.029 \pm 6.788$$
$$f = -7.817(\text{Comp})\max,\ 5.759(\text{Tension})\max$$

EXAMPLE NZS 3101-06 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by NZS 3101-06.
- The average shear stress in the beam is below the maximum shear stress allowed by NZS 3101-06, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading One dead load case (DL50) and one live load case (LL130) with only symmetric third-point loads of magnitudes 50, and 130 kN, respectively, are defined in the model. One load combinations (COMB130) is defined with the NZS 3101-06 load combination factors of 1.2 for dead loads and 1.5 for live loads. The model is analyzed for both of these load cases and the load combinations.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results. These moment and shear force are identical. After completing the analysis, design is performed using the NZS 3101-06 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.

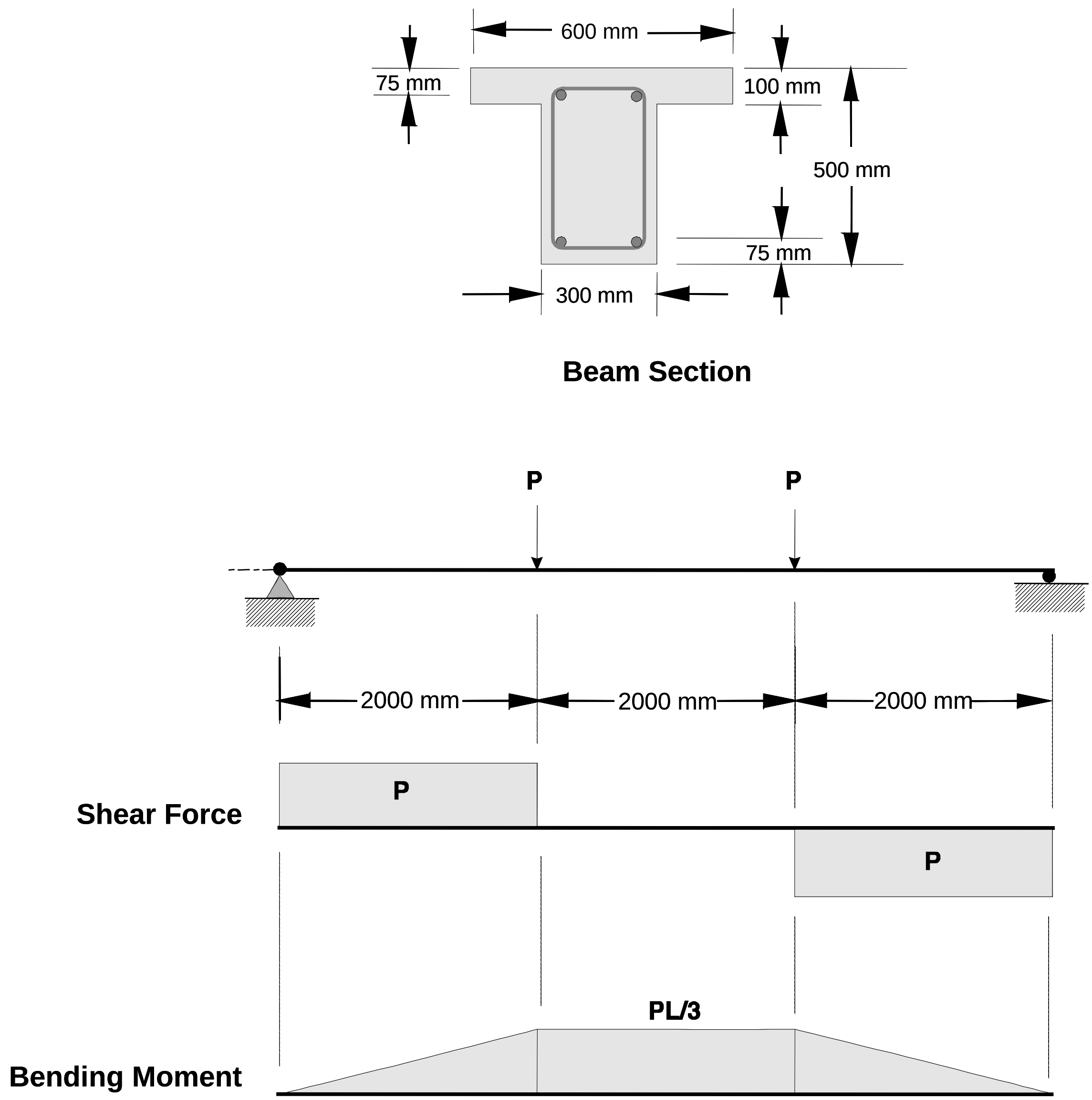


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span,	l	=	6000	mm
Overall depth,	h	=	500	mm
Flange Thickness,	d_s	=	100	mm
Width of web,	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.,	d_c	=	75	mm
Effective depth,	d	=	425	mm
Depth of comp. reinf.,	d'	=	75	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	$25x10^5$	MPa
Modulus of elasticity,	E_s	=	$2x10^8$	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	50	kN
Live load,	P_l	=	130	kN

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-cm)
		A_s^+
SAFE	510	35.046
Calculated	510	35.046

$A^+_{s,min}$ = 535.82 sq-m

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-cm/m)	
	SAFE	**Calculated**
255	14.962	14.89

COMPUTER FILE: NZS 3101-06 RC-BM-001.FDB

CONCLUSION

The SAFE results show an acceptable close comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for the load combination:

$\phi_b = 0.85$

$\alpha_1 = 0.85 \quad \text{for } f'_c \le 55MPa$

$\beta_1 = 0.85 \quad \text{for } f'_c \le 30,$

$$c_b = \frac{\varepsilon_c}{\varepsilon_c + f_y / E_s} d = 240.56 \text{ mm}$$

$a_{max} = 0.75\beta_1 c_b = 153.36$ mm

$$A_{s,\min} = \max \begin{cases} \dfrac{\sqrt{f'_c}}{4 f_y} A_c = 535.82 \\ 1.4 \dfrac{A_c}{f_y} = 136.96 \end{cases} \text{ sq-mm}$$

$= 535.82$ sq-mm

COMB130

$N^* = (1.2N_d + 1.5N_t) = 255$ kN

$$M^* = \frac{N^* l}{3} = 510 \text{ kN-m}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M^*\right|}{\alpha_1 f'_c \phi_b b_f}} = 105.322 \text{ mm } (a > D_s)$$

The compressive force developed in the concrete alone is given by:

C_f is given by:

$$C_f = \alpha_1 f'_c \left(b_f - b_w\right) h_f = 765 \text{ kN}$$

Therefore, $A_{s1} = \dfrac{C_f}{f_y}$ and the portion of M^* that is resisted by the flange is given by:

$$M_f^* = C_f\left(d - \frac{d_s}{2}\right)\phi_b = 243.84375 \text{ kN-m}$$

$$A_{s1} = \frac{C_f}{f_y} = 1663.043 \text{ sq-mm}$$

Therefore, the balance of the moment, M^*, to be carried by the web is:

$$M^*_w = M^* - M^*_f = 510 - 243.84375 = 266.15625 \text{ kN-m}$$

The web is a rectangular section with dimensions b_w and d, for which the depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_w^*}{\alpha_1 f'_c \phi_b b_w}} = 110.7354 \text{ mm} \le a_{\max}$$

If $a_1 \le a_{\max}$ (NZS 9.3.8.1), the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_w^*}{\phi_b f_y\left(d - \frac{a_1}{2}\right)} = 1841.577 \text{ sq-mm}$$

$$A_s = A_{s1} + A_{s2} = 3504.62 \text{ sq-mm}$$

Shear Design

The basic shear strength for rectangular section is computed as,

$$v_b = \left[0.07 + 10\frac{A_s}{b_w d}\right]\sqrt{f'_c} = 0.3834$$

$$f'_c \le 50 \text{ MPa, and}$$

$$0.08\sqrt{f'_c} = 0.438 \text{ MPa} \le v_b \le 0.2\sqrt{f'_c} = 1.095 \text{ MPa}$$

$$v_c = k_d\, k_a\, v_b = 0.438 \quad \text{where } (k_d = 1.0,\ k_a = 1.0)$$

The average shear stress is limited to a maximum limit of,

$$v_{\max} = \min\{0.2 f'_c, 8 \text{ MPa}\} = \min\{6, 8\} = 6 \text{ MPa}$$

The shear reinforcement is computed as follows:

If $v^* \le \phi_s\left(v_c/2\right)$ or $h \le \max(300 \text{ mm}, 0.5b_w)$

$$\frac{A_v}{s} = 0 \qquad \text{(NZS 9.3.9.4.13)}$$

If $\phi_s\left(v_c/2\right) < v^* \le \phi_s v_c$,

$$\frac{A_v}{s} = \frac{1}{16}\sqrt{f'_c}\,\frac{b_w}{f_{yt}} \qquad \text{(NZS 9.3.9.4.15)}$$

If $\phi_s v_c < v^* \le \phi_s v_{max}$, (NZS 9.3.9.4.2)

$$\frac{A_v}{s} = \frac{\left(v^* - \phi_s v_c\right)}{\phi_s f_{yt} d}$$

If $v^* > v_{max}$, a failure condition is declared.

For the load combination, the N^* and V^* are calculated as follows:

$$N^* = 1.2N_d + 1.5N_l$$

$$V^* = N^*$$

$$v^* = \frac{V^*}{b_w d}$$

(COMB130)

N_d = 50 kips

N_l = 130 kips

V^* = 255 kN

$$v^* = \frac{V^*}{b_w d} = 2.0 \text{ MPa} \quad (\phi_s v_c < v^* \le \phi_s v_{max})$$

$$\frac{A_v}{s} = \frac{\left(v^* - \phi_s v_c\right) b_w}{\phi_s f_{yt}} = 1.489 \text{ sq-mm/mm} = 1489 \text{ sq-mm/m}$$

EXAMPLE NZS 3101-06 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8 m spans in each direction, as shown in Figure 1.

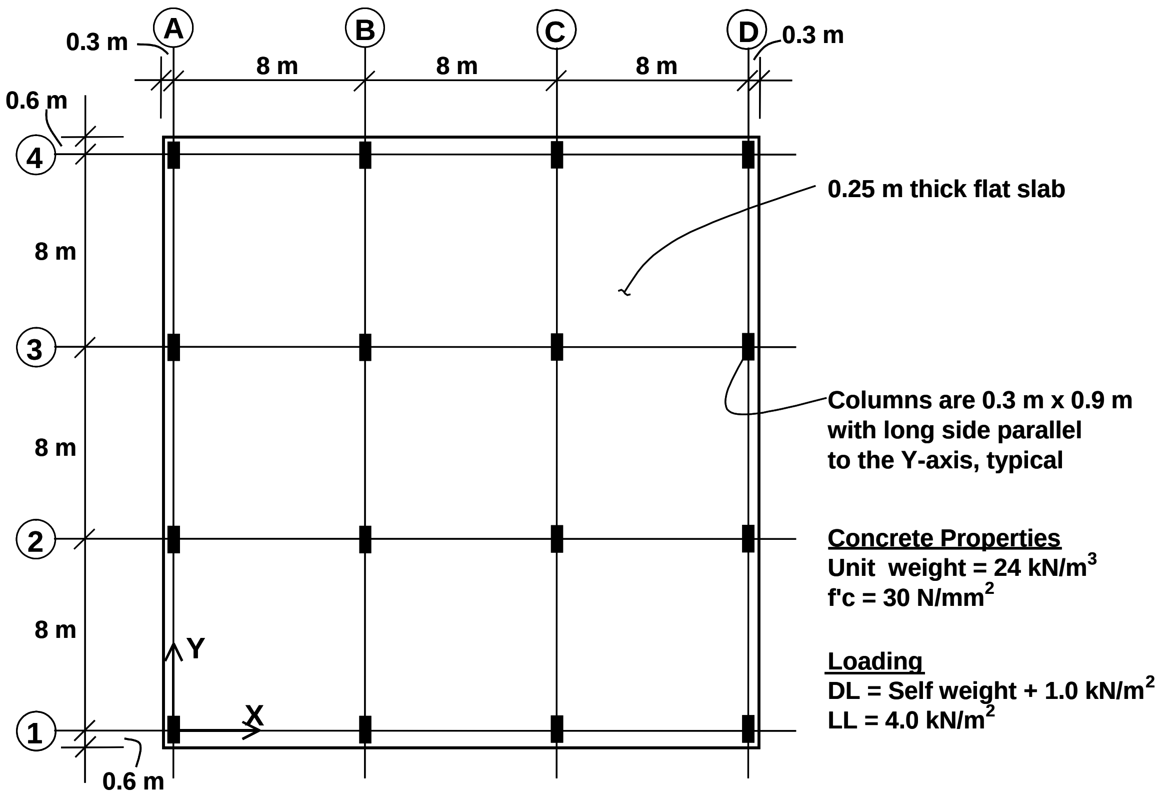


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a $f'c$ of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
SAFE	1.792	1.141	1.57
Calculated	1.792	1.141	1.57

COMPUTER FILE: NZS 3101-06 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using SAFE Method

$d = [(259 - 26) + (250 - 38)]/2 = 218$ mm

Refer to Figure 2.

b_0 = 518+ 1118 + 1118 + 518 = 3272 mm

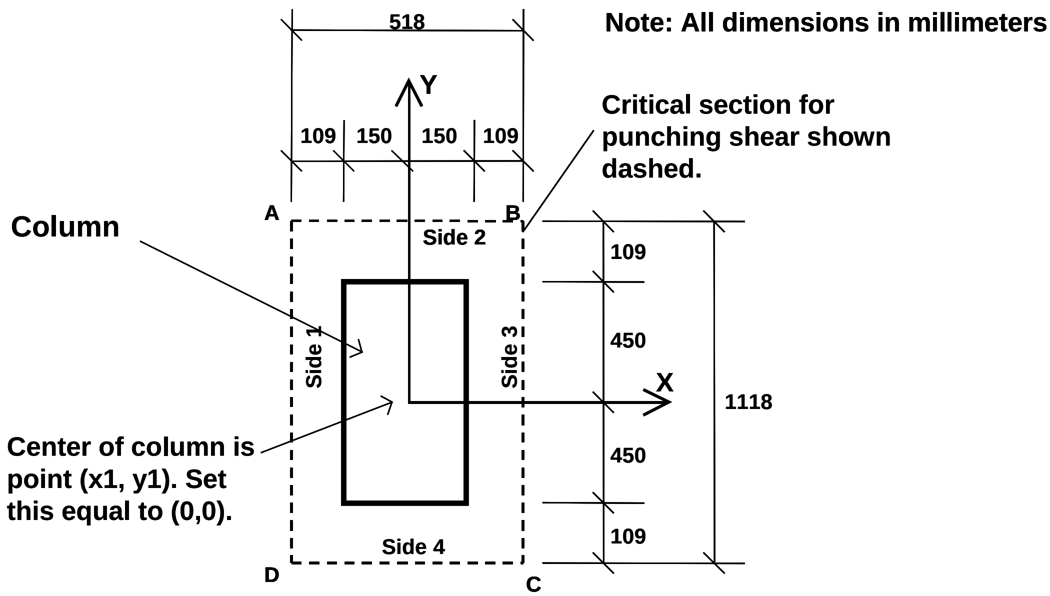


Figure 2: Interior Column, Grid B-2 in SAFE Model

$$\gamma_{V2} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{1118}{518}}} = 0.495$$

$$\gamma_{V3} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{518}{1118}}} = 0.312$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–259	0	259	0	N.A.
y_2	0	559	0	–559	N.A.
L	1118	518	1118	518	b_0 = 3272
d	218	218	218	218	N.A.
Ld	243724	112924	243724	112924	713296
Ldx_2	–63124516	0	63124516	0	0
Ldy_2	0	63124516	0	–63124516	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{713296} = 0\,mm$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{713296} = 0\,mm$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	1118	518	1118	518	N.A.
d	218	218	218	218	N.A.
$x_2 - x_3$	–259	0	259	0	N.A.
$y_2 - y_3$	0	559	0	–559	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	2.64E+10	3.53E+10	2.64E+10	3.53E+10	1.23E+11
I_{YY}	1.63E+10	2.97E+09	1.63E+10	2.97E+09	3.86E+10
I_{XY}	0	0	0	0	0

From the SAFE output at Grid B-2:

V_U = 1126.498 kN

$\gamma_{V2}\ M_{U2}$ = –25.725 kN-m

$\gamma_{V3}\ M_{U3}$ = 14.272 kN-m

At the point labeled A in Figure 2, x_4 = –259 and y_4 = 559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(559-0)-(0)(-259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(-259-0)-(0)(559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 – 0.1169 – 0.0958 = **1.3666 N/mm²** at point A

At the point labeled B in Figure 2, x_4 = 259 and y_4 = 559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(559-0)-(0)(259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(259-0)-(0)(559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 – 0.1169 + 0.0958 =**1.5582 N/mm²** at point B

At the point labeled C in Figure 2, x_4 = 259 and y_4 = –559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(-559-0)-(0)(259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(259-0)-(0)(-559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 + 0.1169 + 0.0958 = **1.792 N/mm²** at point C

At the point labeled D in Figure 2, x_4 = –259 and y_4 = –559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(-559-0)-(0)(-259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(-259-0)-(0)(-559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 + 0.1169 – 0.0958 = **1.6004 N/mm²** at point D

Point C has the largest absolute value of v_u, thus v_{max} = **1.792 N/mm²**

The shear capacity is calculated based on the smallest of NZS 3101-06, with the b_o and u terms removed to convert force to stress.

$$\varphi v_v = \min\begin{cases} \frac{1}{6}\left(1+\frac{2}{\beta_c}\right)\varphi\sqrt{f'_c} \\ \frac{1}{6}\left(1+\frac{\alpha_s d}{b_0}\right)\varphi\sqrt{f'_c} \\ \frac{1}{3}\varphi\sqrt{f'_c} \end{cases} = 1.141\text{N/mm}^2 \text{ per} \qquad \text{(NZS 12.7.3.2)}$$

NZS 12.7.3.2 yields the smallest value of $\boxed{\varphi v_v = 1.141 \text{ N/mm}^2}$, and thus this is the shear capacity.

$$\boxed{\text{Shear Ratio} = \frac{v_U}{\varphi v_v} = \frac{1.792}{1.141} = 1.57}$$

EXAMPLE NZS 3101-06 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

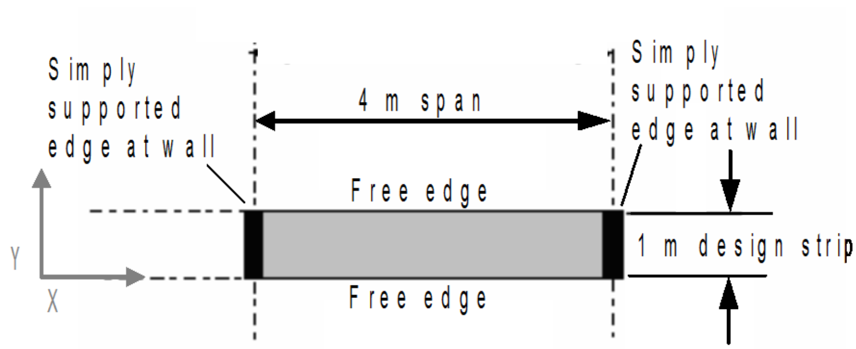


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the NZS 3101-06 load combination factors, 1.2 for dead loads and 1.5 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing analysis, design is performed using the NZS 3101-06 code by SAFE and also by hand computation. Table 1 shows the comparison of the design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2×10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	SAFE	24.597	5.238
	Calculated	24.6	5.238

$A^+_{s,min}$ = 380.43 sq-mm

COMPUTER FILE: NZS 3101-06 RC-SL-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\phi_b = 0.85$

$b = 1000$ mm

$\alpha_1 = 0.85 \quad \text{for } f'_c \leq 55\text{MPa}$

$\beta_1 = 0.85 \quad \text{for } f'_c \leq 30,$

$$c_b = \frac{\varepsilon_c}{\varepsilon_c + f_y / E_s} d = 70.7547$$

$a_{max} = 0.75\beta_1 c_b = 45.106$ mm

For the load combination, w and M^* are calculated as follows:

$w = (1.2w_d + 1.5w_t)\, b$

$$M_u = \frac{wl_1^2}{8}$$

$$A_{s,\min} = \max \begin{cases} \dfrac{\sqrt{f'_c}}{4 f_y} b_w d = 372.09 \text{ sq-mm} \\ 1.4 \dfrac{b_w d}{f_y} = 380.43 \text{ sq-mm} \end{cases}$$

$= 380.43$ sq-mm

COMB100

$w_d = 4.0$ kPa

$w_t = 5.0$ kPa

$w = 12.3$ kN/m

$M^*_{\text{-strip}} = 24.6$ kN-m

$M^*_{\text{-design}} = 24.6331$ kN-m

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M^*\right|}{\alpha_1 f'_c \phi_b b}} = 9.449 \text{ mm} < a_{max}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M^*}{\phi_b f_y \left(d - \frac{a}{2} \right)} = 523.799 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.238 sq-cm

EXAMPLE Singapore CP 65-99 PT-SL-001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.

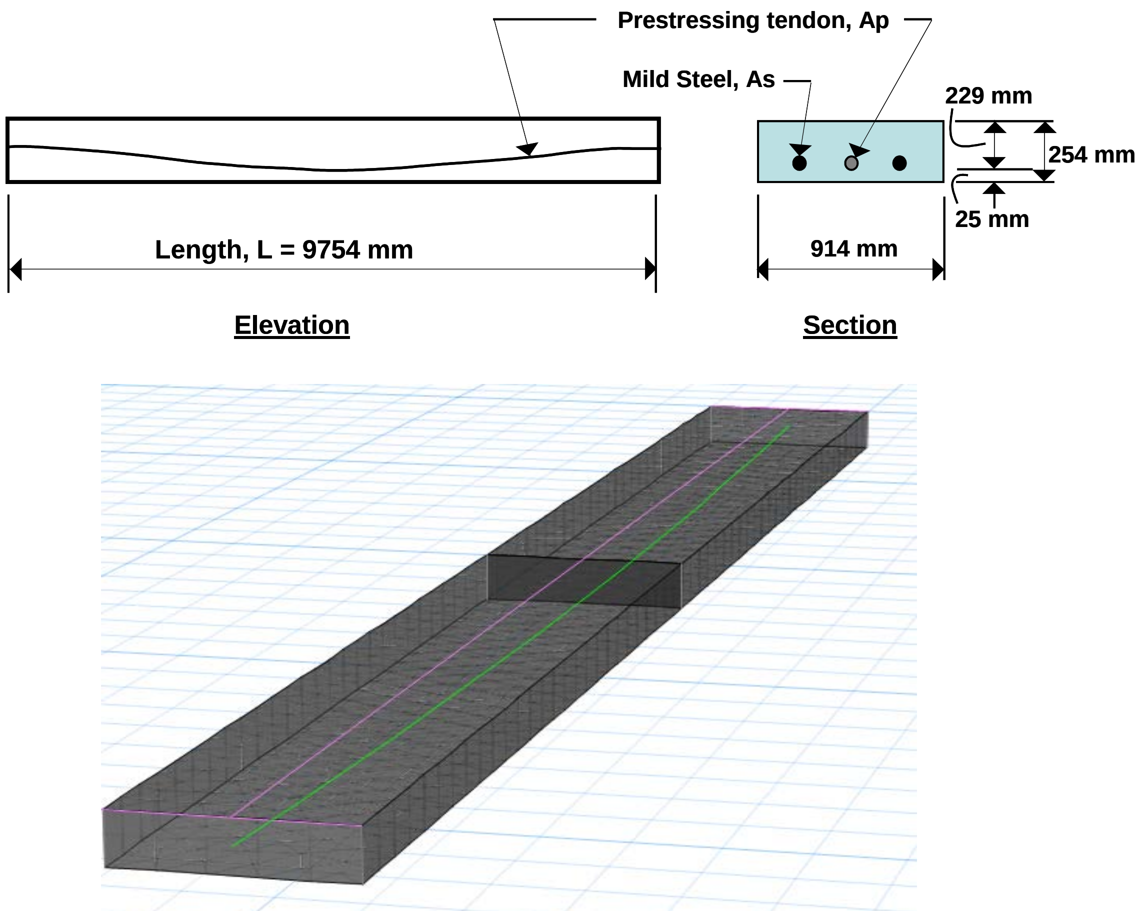


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads have been added to the slab. The loads and post-tensioning forces are as follows.

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations are compared with the SAFE results and summarized for verification and validation of the SAFE results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	kN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	kN/m^2
Live load	w_l	=	4.788	kN/m^2

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	174.4	174.4	0.00%
Area of Mild Steel req'd, As (sq-cm)	19.65	19.79	0.71%
Transfer Conc. Stress, top $(D+PT_I)$, MPa	–5.058	–5.057	0.02%
Transfer Conc. Stress, bot $(D+PT_I)$, MPa	2.839	2.839	0.00%
Normal Conc. Stress, top $(D+L+PT_F)$, MPa	–10.460	–10.465	0. 50%
Normal Conc. Stress, bot $(D+L+PT_F)$, MPa	8.402	8.407	0.06%

COMPUTER FILE: SINGAPORE CP 65-99 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

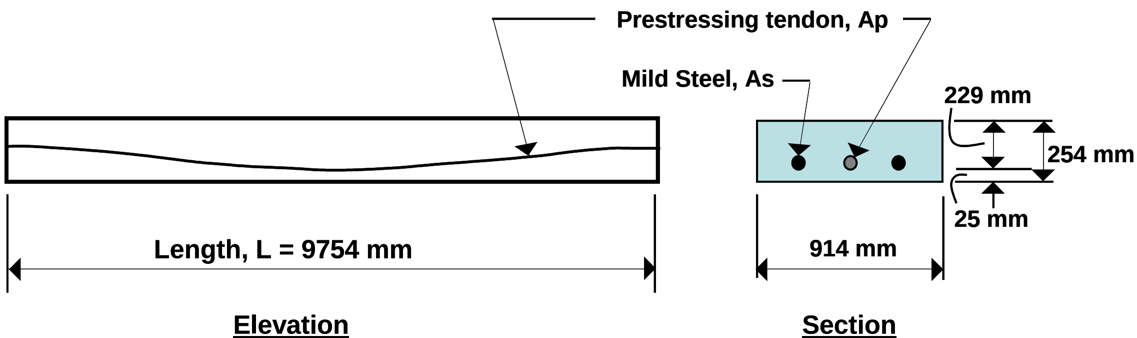
HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
$f'c$ = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.4 = 8.378 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) × 1.6 = 7.661 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 16.039 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 16.039 kN/m^2 × 0.914 m = 14.659 kN/m

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8} = 14.659 \times (9.754)^2/8 = 174.4$ kN-m

Ultimate Stress in strand, $f_{pb} = f_{pe} + \frac{7000}{l/d}\left(1 - 1.7\frac{f_{pu}A_p}{f_{cu}bd}\right)$

$$= 1210 + \frac{7000}{9754/229}\left(1 - 1.7\frac{1862(198)}{30(914)(229)}\right)$$

$$= 1358 \text{ MPa} \le 0.7 f_{pu} = 1303 \text{ MPa}$$

K factor used to determine the effective depth is given as:

$$K = \frac{M}{f_{cu}bd^2} = \frac{174.4}{30000(0.914)(0.229)^2} = 0.1213 < 0.156$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d = 192.2 \text{ mm}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 2(99)(1303)/1000 = 258.0 \text{ kN}$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}(z)/\gamma = 258.0(0.192)/1.15 = 43.12 \text{ kN-m}$$

Net Moment to be resisted by As,

$$M_{NET} = M_U - M_{PT}$$

$$= 174.4 - 43.12 = 131.28 \text{ kN-m}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_{NET}}{0.87 f_y z_X} = \frac{131.28}{0.87(400)(192)}(1e6) = 1965 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination ($D+PT_i$) = $1.0D+0.0L+1.0PT_I$

Tendon stress at transfer = jacking stress – stressing losses = 1490 – 186 = 1304 MPa

The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4 \text{ kN}$

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04 \text{ kN-m}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25 \text{ kN-m}$

Stress in concrete, $f = \frac{F_{PTI}}{A} \pm \frac{M_D - M_{PT}}{S} = \frac{-257.4}{0.254(0.914)} \pm \frac{65.04 - 26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.109 \pm 3.948 \text{ MPa}$$

$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT_F) = 1.0D+1.0L+1.0PT_F

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa
The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN
Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m
Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m
Moment due to PT, $M_{PT} = F_{PTI}\,(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT_F),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$

$$f = -1.029 \pm 9.431$$

$$f = -10.460(\text{Comp})\max,\ 8.402(\text{Tension})\max$$

EXAMPLE SINGAPORE CP 65-99 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

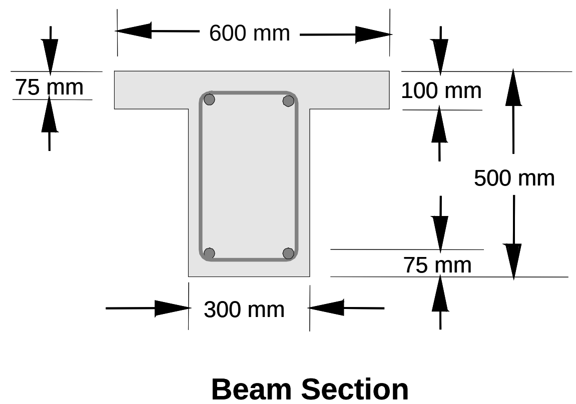
The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by Singapore CP 65-99.
- The average shear stress in the beam is below the maximum shear stress allowed by Singapore CP 65-99, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T-beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading. One dead load case (DL20) and one live load case (LL80) with only symmetric third-point loads of magnitudes 20, and 80 kN, respectively, are defined in the model. One load combinations (COMB80) is defined with the Singapore CP 65-99 load combination factors of 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both of these load cases and the load combinations.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results. These moment and shear force are identical. After completing the analysis, design is performed using the Singapore CP 65-99 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.



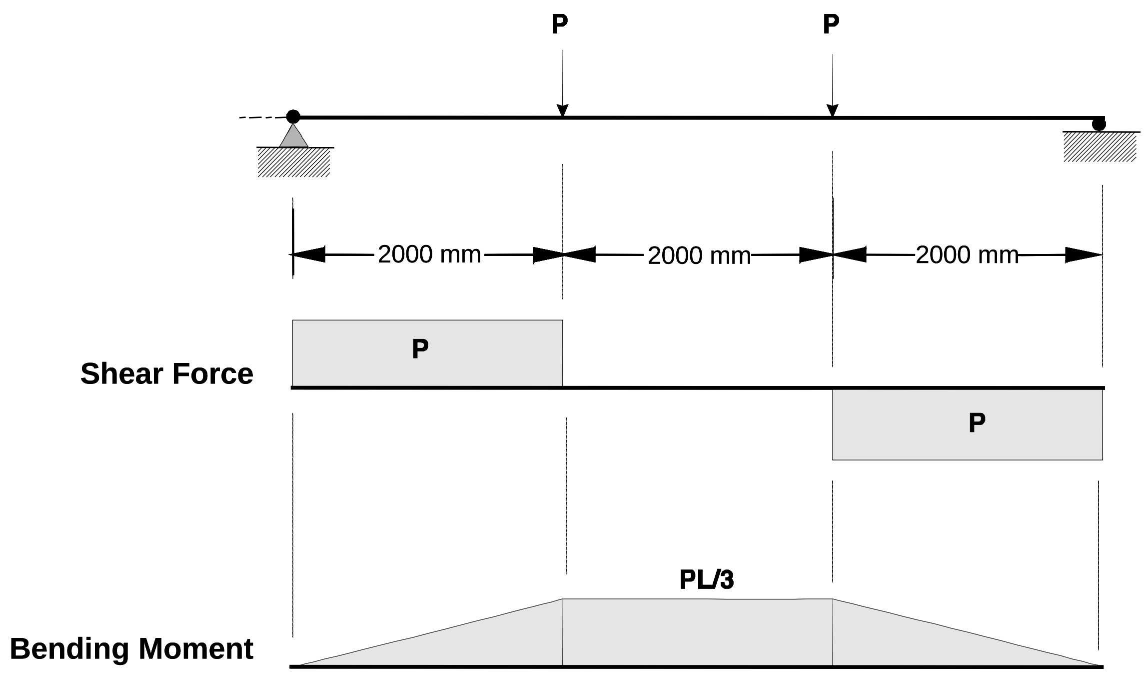


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span,	l	=	6000	mm
Overall depth,	h	=	500	mm
Flange Thickness,	d_s	=	100	mm
Width of web,	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.,	d_c	=	75	mm
Effective depth,	d	=	425	mm
Depth of comp. reinf.,	d'	=	75	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	25×10^5	MPa
Modulus of elasticity,	E_s	=	2×10^8	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	20	kN
Live load,	P_l	=	80	kN

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
SAFE	312	20.904
Calculated	312	20.904

$A^+_{s,min}$ = 195.00 sq-mm

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-cm/m)	
	SAFE	Calculated
156	6.50	6.50

COMPUTER FILE: SINGAPORE CP 65-99 RC-002.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\gamma_{m,\ steel}$ = 1.15

$\gamma_{m,\ concrete}$ = 1.50

$$A_{s,\min} = 0.0013 b_w h$$

= 195.00 sq-mm

COMB80

P = (1.4P_d + 1.6P_t) = 156 kN

$$M^* = \frac{N^* l}{3} = 312 \text{ kN-m}$$

The depth of the compression block is given by:

$$K = \frac{M}{f_{cu}\, b_f\, d^2} = 0.095963 < 0.156$$

Then the moment arm is computed as:

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \le 0.95d = 373.4254 \text{ mm}$$

The depth of the neutral axis is computed as:

$$x = \frac{1}{0.45}(d - z) = 114.6102 \text{ mm}$$

And the depth of the compression block is given by:

a = 0.9x = 103.1492 mm > h_f

The ultimate resistance moment of the flange is given by:

$$M_f = \frac{0.67}{\gamma_c} f_{cu}\left(b_f - b_w\right) h_f \left(d - 0.5 h_f\right) = 150.75 \text{ kN-m}$$

The moment taken by the web is computed as:

$$M_w = M - M_f = 161.25 \text{ kN-m}$$

And the normalized moment resisted by the web is given by:

$$K_w = \frac{M_w}{f_{cu} b_w d^2} = 0.0991926 < 0.156$$

If $K_w \le 0.156$ (BS 3.4.4.4), the beam is designed as a singly reinforced concrete beam. The reinforcement is calculated as the sum of two parts: one to balance compression in the flange and one to balance compression in the web.

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K_w}{0.9}}\right) \le 0.95d = 371.3988 \text{ mm}$$

$$A_s = \frac{M_f}{\frac{f_y}{\gamma_s}\left(d - 0.5h_f\right)} + \frac{M_w}{\frac{f_y}{\gamma_s} z} = 2090.4 \text{ sq-mm}$$

Shear Design

$$v = \frac{V}{b_w d} \le v_{\max} = 1.2235 \text{ MPa}$$

$$v_{\max} = \min(0.8\sqrt{f_{cu}}, 5 \text{ MPa}) = 4.38178 \text{ MPa}$$

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} = 0.3568 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3} = 1.06266, \; 1 \le k_2 \le \left(\frac{40}{25}\right)^{1/3}$$

$$\gamma_m = 1.25$$

$$\frac{100 A_s}{bd} = 0.15$$

$$\left(\frac{400}{d}\right)^{1/4} = 1$$

However, the following limitations also apply:

$$0.15 \le \frac{100\ A_s}{bd} \le 3$$

$$\left(\frac{400}{d}\right)^{1/4} \ge 1$$

$f_{cu} \le 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

Given v, v_c, and $v_{\max}$, the required shear reinforcement is calculated as follows:

If $v \le (v_c + 0.4)$,

$$\frac{A_{sv}}{s_v} = \frac{0.4 b_w}{0.87\, f_{yv}}$$

If $(v_c + 0.4) < v \le v_{\max}$,

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c) b_w}{0.87\, f_{yv}}$$

If $v > v_{\max}$, a failure condition is declared.

(COMB80)

P_d = 20 kN

P_l = 80 kN

V = 156 kN

$$v^* = \frac{V^*}{b_w d} = 2.0 \text{ MPa} \quad (\phi_s v_c < v^* \le \phi_s v_{\max})$$

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c) b_w}{0.87\, f_{yv}} = 0.64967\text{sq-mm/mm} = 6.50 \text{ sq-cm/m}$$

EXAMPLE Singapore CP 65-99 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

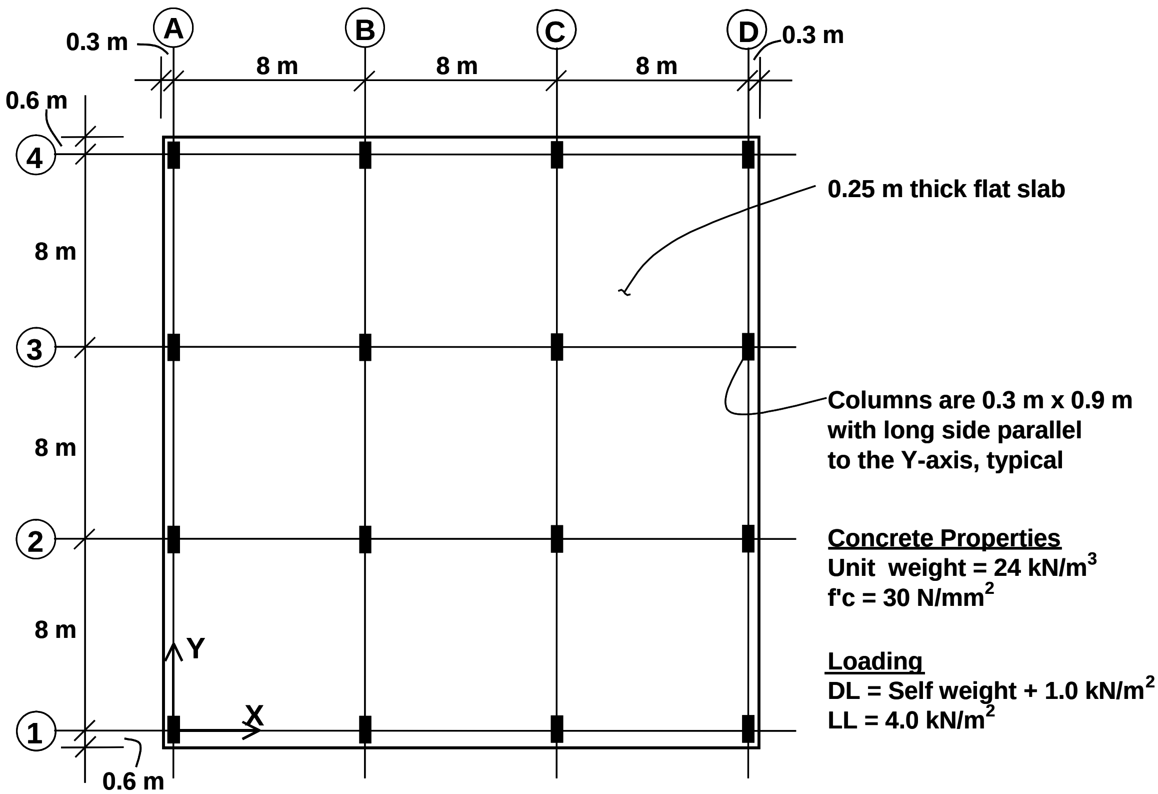


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f_{cu} of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
SAFE	1.105	0.625	1.77
Calculated	1.105	0.620	1.77

COMPUTER FILE: SINGAPORE CP 65-99 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using SAFE Method

$d = [(250 - 26) + (250 - 38)]/2$ = 218 mm

Refer to Figure 1.

u = 954+ 1554 + 954 + 1554 = 5016 mm

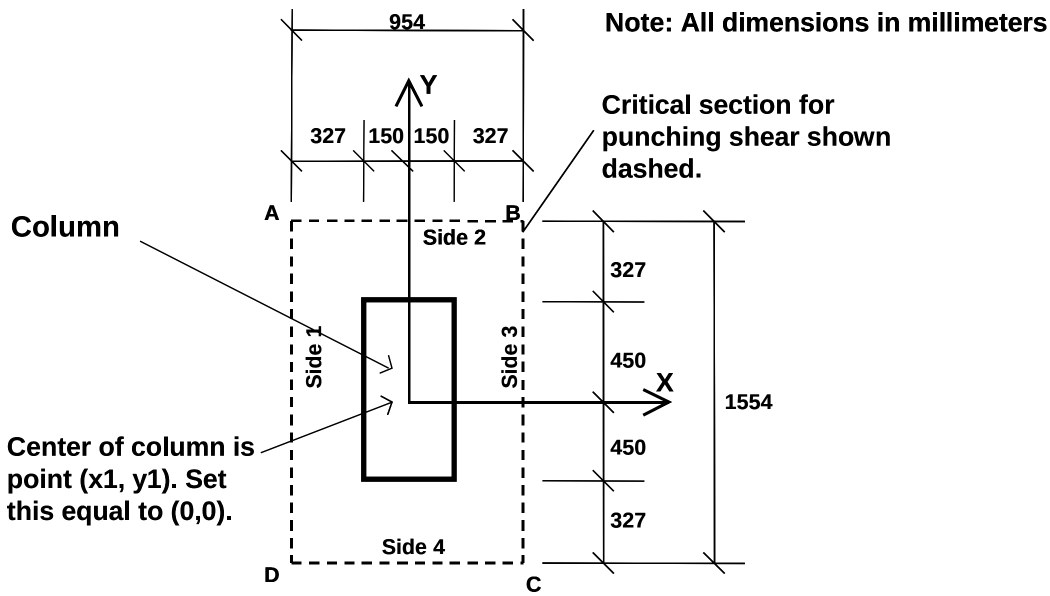


Figure 2: Interior Column, Grid B-2 in SAFE Model

From the SAFE output at Grid B-2:

V= 1126.498 kN

M_2 = 51.9908 kN-m

M_3 = 45.7234 kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{eff,x} = \frac{V}{ud}\left(f + \frac{1.5M_x}{Vy} \right) \qquad \text{(CP 3.7.7.3)}$$

$$v_{eff,x} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 51.9908 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 954} \right) = 1.1049 \ \text{(Govern)}$$

$$v_{eff,y} = \frac{V}{ud}\left(f + \frac{1.5M_y}{Vx} \right)$$

$$v_{eff,y} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 45.7234 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 1554} \right) = 1.0705$$

The largest absolute value of v = **1.1049 N/mm²**

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd} \right)^{1/3}\left(\frac{400}{d} \right)^{1/4} = 0.3568 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{25} \right)^{1/3} = \left(\frac{30}{25} \right)^{1/3} = 1.0627 > 1.0 \text{ OK}$$

$\gamma_m = 1.25$

$$\left(\frac{400}{d} \right)^{1/4} = 1.16386 > 1 \text{ OK.}$$

$f_{cu} \le 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

Area of reinforcement at the face of column for design strip are as follows:

A_s in Strip Layer A = 9494.296 mm^2

A_s in Strip Layer B = 8314.486 mm^2

Average A_s = (9494.296+8314.486)/2 = 8904.391 mm^2

$$\frac{100\ A_s}{bd} = 100 \bullet 8904.391/(8000 \bullet 218) = 0.51057$$

$$v_c = \frac{0.79 \bullet 1.0 \bullet 1.0627}{1.25} \bullet \left(0.51057\right)^{1/3} \bullet 1.16386 = 0.6247 \text{ MPa}$$

BS 3.7.7.3 yields the value of v = **0.625 N/mm²,** and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{v} = \frac{1.1049}{0.6247} = 1.77$$

EXAMPLE Singapore CP 65-99 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

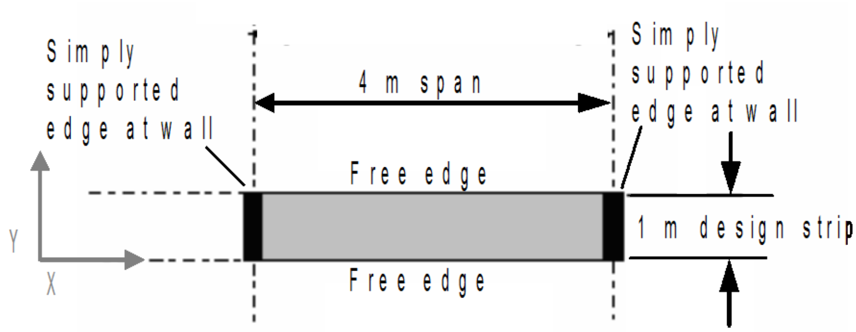


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 KN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the Singapore CP 65-99 load combination factors, 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing the analysis, design is performed using the Singapore CP 65-99 code by SAFE and also by hand computation. Table 1 shows the comparison of the design reinforcements computed by the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2×10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm)
			A_s^+
Medium	SAFE	27.197	5.853
	Calculated	27.200	5.850

$A_{s,\min}^{+}$ = 162.5 sq-mm

COMPUTER FILE: Singapore CP 65-99 RC-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for all the load combinations:

$\gamma_{m,\ steel}$ = 1.15

$\gamma_{m,\ concrete}$ = 1.50

b = 1000 mm

For each load combination, the w and M are calculated as follows:

$$w = (1.4w_d + 1.6w_t)\ b$$

$$M = \frac{wl_1^2}{8}$$

$$A_{s,min} = 0.0013 b_w d$$

$$= 162.5 \text{ sq-mm}$$

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 13.6 kN/m

$M_{\text{-}strip}$ = 27.2 kN-m

$M_{\text{-}design}$ = 27.2366 kN-m

The depth of the compression block is given by:

$$K = \frac{M}{f_{cu} b d^2} = 0.05810 < 0.156$$

The area of tensile steel reinforcement is then given by:

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d = 116.3283$$

$$A_s = \frac{M}{0.87 f_y z} = 585.046 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.850 sq-cm

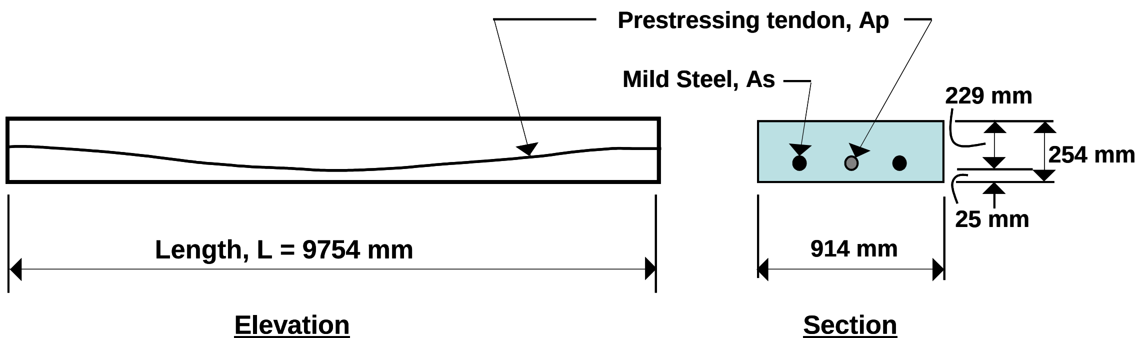
EXAMPLE Turkish TS 500-2000 PT-SL-001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in SAFE. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.



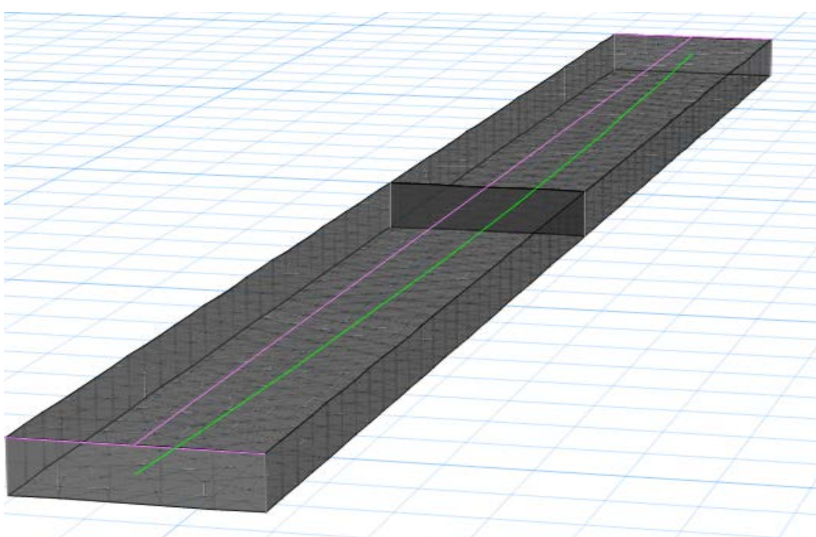


Figure 1 One-Way Slab

PROGRAM NAME: SAFE
REVISION NO.: 0

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads have been added to the slab. The loads and post-tensioning forces are as follows.

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations are compared with the SAFE results and summarized for verification and validation of the SAFE results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f_{ck}	=	30	MPa
Yield strength of steel	f_{yk}	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	kN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	kN/m^2
Live load	w_l	=	4.788	kN/m^2

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	SAFE RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	174.4	174.4	0.00%
Area of Mild Steel req'd, As (sq-cm)	14.88	14.90	0.13%
Transfer Conc. Stress, top $(D+PT_I)$, MPa	–5.058	–5.057	0.02%
Transfer Conc. Stress, bot $(D+PT_I)$, MPa	2.839	2.839	0.00%
Normal Conc. Stress, top $(D+L+PT_F)$, MPa	–10.460	–10.465	0. 50%
Normal Conc. Stress, bot $(D+L+PT_F)$, MPa	8.402	8.407	0.06%

COMPUTER FILE: TURKISH TS 500-2000 PT-SL-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

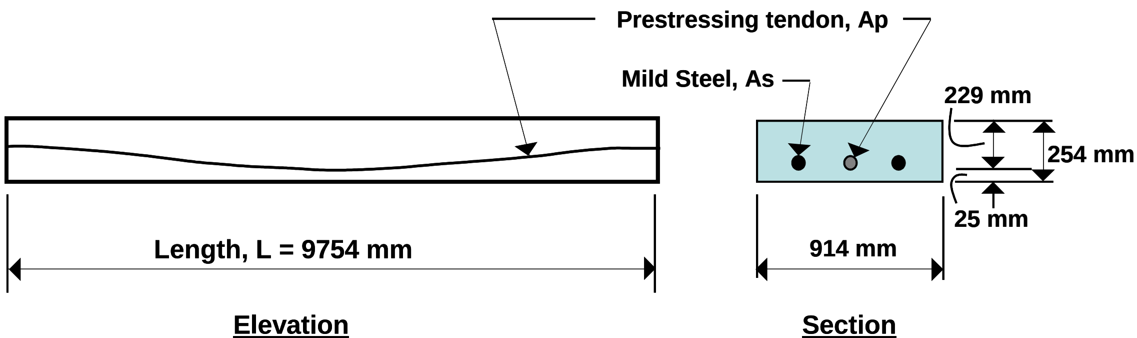
HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
f_{ck} = 30MPa	f_{pu} = 1862 MPa
f_{yk} = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\gamma_{m,\, steel} = 1.15$

$\gamma_{m,\, concrete} = 1.50$



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.4 = 8.378 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) × 1.6 = 7.661 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 16.039 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 16.039 kN/m^2 × 0.914 m = 14.659 kN/m

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8}$ = 14.659 × (9.754)^2/8 = 174.4 kN-m

Ultimate Stress in strand, $f_{Pd} = f_{pe} + 7000d\left(1 - 1.36\dfrac{f_{PU}A_P}{f_{CK}bd}\right)\Big/ l$

$$= 1210 + 7000(229)\left(1 - 1.36\frac{1862(198)}{30(914)(229)}\right)\Big/(9754)$$

$$= 1361\ \text{MPa}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 2(99)(1361)/1000 = 269.5$ kN

Stress block depth, $a = d - \sqrt{d^2 - \dfrac{2M_d}{0.85 f_{cd}\, b}}$

$$= 0.229 - \sqrt{0.229^2 - \frac{2(174.4)}{0.85(20000)(0.914)}}(1e3) = 55.816\ \text{mm}$$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right) = 269.5\left(229 - \frac{55.816}{2}\right)\Big/1000 = 54.194\ \text{kN-m}$$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 174.4 - 54.194 = 120.206$ kN-m

Required area of mild steel reinforcing,

$$A_S = \frac{M_{net}}{f_{yd}\left(d - \frac{a}{2}\right)} = \frac{120.206 \bullet 10^6}{(400)\left(229 - \frac{54.194}{2}\right)} = 1488.4\ \text{mm}^2$$

K factor used to determine the effective depth is given as:

$$K = \frac{M}{f_{cu}bd^2} = \frac{174.4}{30000/1.5(0.914)(0.229)^2} = 0.1819 < 0.156$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d = 192.2\ \text{mm}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 2(99)(1303)/1000 = 258.0$ kN

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}(z)/\gamma = 258.0(0.192)/1.15 = 43.12\ \text{kN-m}$$

Net Moment to be resisted by As,

$$M_{NET} = M_U - M_{PT}$$
$$= 174.4 - 43.12 = 131.28 \text{ kN-m}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_{NET}}{f_{yd} z}_X = \frac{131.28}{0.87(400)(192)}(1e6) = 1965 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (D+PT$_i$) = 1.0D+0.0L+1.0PT$_I$

Tendon stress at transfer = jacking stress – stressing losses = 1490 – 186 = 1304 MPa
The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-257.4}{0.254(0.914)} \pm \dfrac{65.04 - 26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.109 \pm 3.948 \text{ MPa}$$
$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT$_F$) = 1.0D+1.0L+1.0PT$_F$

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa
The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT$_F$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$
$$f = -1.029 \pm 9.431$$
$$f = -10.460(\text{Comp})\max, 8.402(\text{Tension})\max$$

EXAMPLE Turkish TS 500-2000 RC-BM-001

Flexural and Shear Beam Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by TS 500-2000.
- The average shear stress in the beam is below the maximum shear stress allowed by TS 500-2000, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T-beam with a flange 100 mm thick and 600 mm wide is modeled using SAFE. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated by SAFE. The maximum element size has been specified to be 200 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading. One dead load case (DL20) and one live load case (LL80) with only symmetric third-point loads of magnitudes 20, and 80 kN, respectively, are defined in the model. One load combinations (COMB80) is defined with the Turkish TS 500-2000 load combination factors of 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both of these load cases and the load combinations.

The beam moment and shear force are computed analytically. The total factored moment and shear force are compared with the SAFE results. These moment and shear force are identical. After completing the analysis, design is performed using the Turkish TS 500-2000 code in SAFE and also by hand computation. Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.

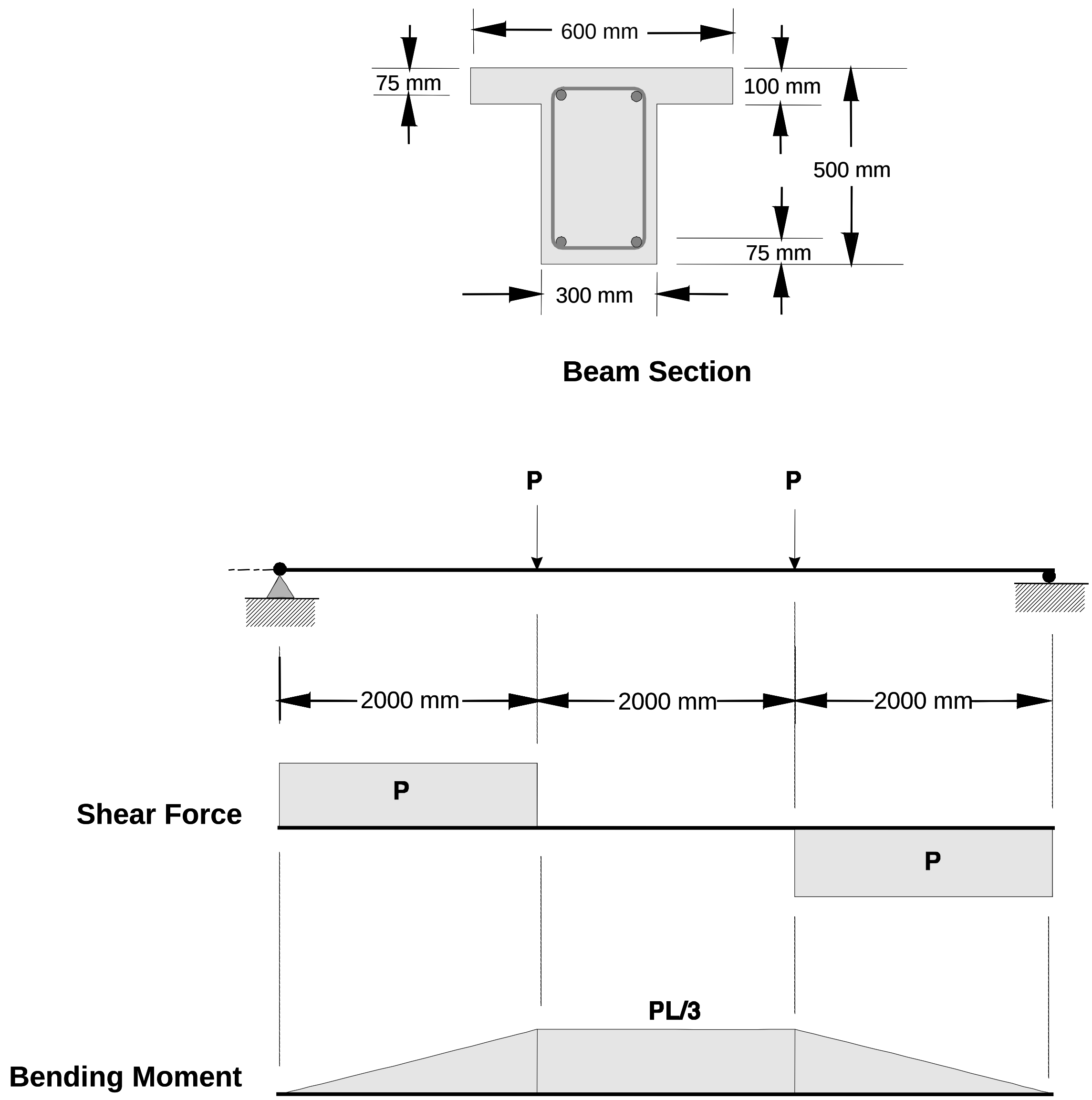


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span,	l	=	6000	mm
Overall depth,	h	=	500	mm
Flange Thickness,	d_s	=	100	mm
Width of web,	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.,	d_c	=	75	mm
Effective depth,	d	=	425	mm
Depth of comp. reinf.,	d'	=	75	mm
Concrete strength,	f'_{ck}	=	30	MPa
Yield strength of steel,	f_{yk}	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	25×10^5	MPa
Modulus of elasticity,	E_s	=	2×10^8	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	20	kN
Live load,	P_l	=	80	kN

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the analytical method. They match exactly for this problem. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-cm)
		A_s^+
SAFE	312	20.244
Calculated	312	20.244

$A^{+}_{s,min}$ = 325.9 sq-mm

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-cm/m)	
	SAFE	Calculated
156	4.19	4.19

COMPUTER FILE: TURKISH TS 500-2000 RC-002.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$

$$f_{cd} = \frac{f_{ck}}{\gamma_{mc}} = \frac{30}{1.5} = 20$$

$$f_{yd} = \frac{f_{yk}}{\gamma_{ms}} = \frac{460}{1.15} = 400$$

$$c_b = \frac{\varepsilon_{cu} E_s}{\varepsilon_{cu} E_s + f_{yd}} d \ = 255 \text{ mm}$$

$$a_{\max} = 0.85 k_1 c_b \ = 177.7 \text{ mm}$$

where, $k_1 = 0.85 - 0.006\left(f_{ck} - 25\right) = 0.82$, $0.70 \le k_1 \le 0.85$

$$A_{s,\min} = \frac{0.8 f_{ctd}}{f_{yd}} bd \ = 325.9\, mm^2$$

Where $f_{ctd} = \dfrac{0.35\sqrt{f_{cu}}}{\gamma_{mc}} = \dfrac{0.35\sqrt{30}}{1.5} = 1.278$

COMB80

$$P_d = (1.4P_G + 1.6P_Q) = 156 \text{ kN}$$

$$M_d = \frac{N_d l}{3} \ = \ 312 \text{ kN-m}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_d\right|}{0.85 f_{cd} b}} \ = 79.386 \text{ mm} < 100 \text{ mm}$$

since a < a_{max},

$$a = d - \sqrt{d^2 - \frac{2M_d}{0.85 f_{cd}\ b}} \qquad \text{(TS 7.1)}$$

$$a = 425 - \sqrt{425^2 - \frac{2 \bullet 312 \bullet 10^6}{0.85 \bullet 20 \bullet 600}} = 79.387 \text{ mm}$$

If $a \le a_{max}$ (TS 7.1), the area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_d}{f_{yd}\left(d - \frac{a}{2}\right)} = \frac{312 \bullet 10^6}{400\left(425 - \frac{79.387}{2}\right)} = 2024.36 \text{ mm}^2 \text{, and}$$

Shear Design

P_d = 20 kN

P_l = 80 kN

V = 156 kN

The shear force is limited to a maximum of,

$$V_{\max} = 0.22 f_{cd} A_w = 561\ kN$$

The nominal shear strength provided by concrete is computed as:

$$V_{cr} = 0.65 f_{ctd} b_w d\left(1 + \frac{\gamma N_d}{A_g}\right) = 105.9 \text{ kN, where } N_d = 0$$

$$V_c = 0.8 V_{cr} = 84.73 \text{ kN}$$

The shear reinforcement is computed as follows:

If $V_d \le V_{cr}$

$$\left(\frac{A_{sw}}{s}\right)_{\min} = 0.3 \frac{f_{ctd}}{f_{ywd}} b = 0.2876 \frac{mm^2}{mm} \qquad \text{(TS 8.1.5, Eqn 8.6)}$$

If $V_{cr} \le V_d \le V_{\max}$

$$\frac{A_{sw}}{s} = \frac{\left(V_d - V_c\right)}{f_{ywd}d} = 0.419\frac{mm^2}{mm} \qquad \text{(TS 8.1.4, Eqn 8.5)}$$

EXAMPLE Turkish TS 500-2000 RC-PN-001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in SAFE

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

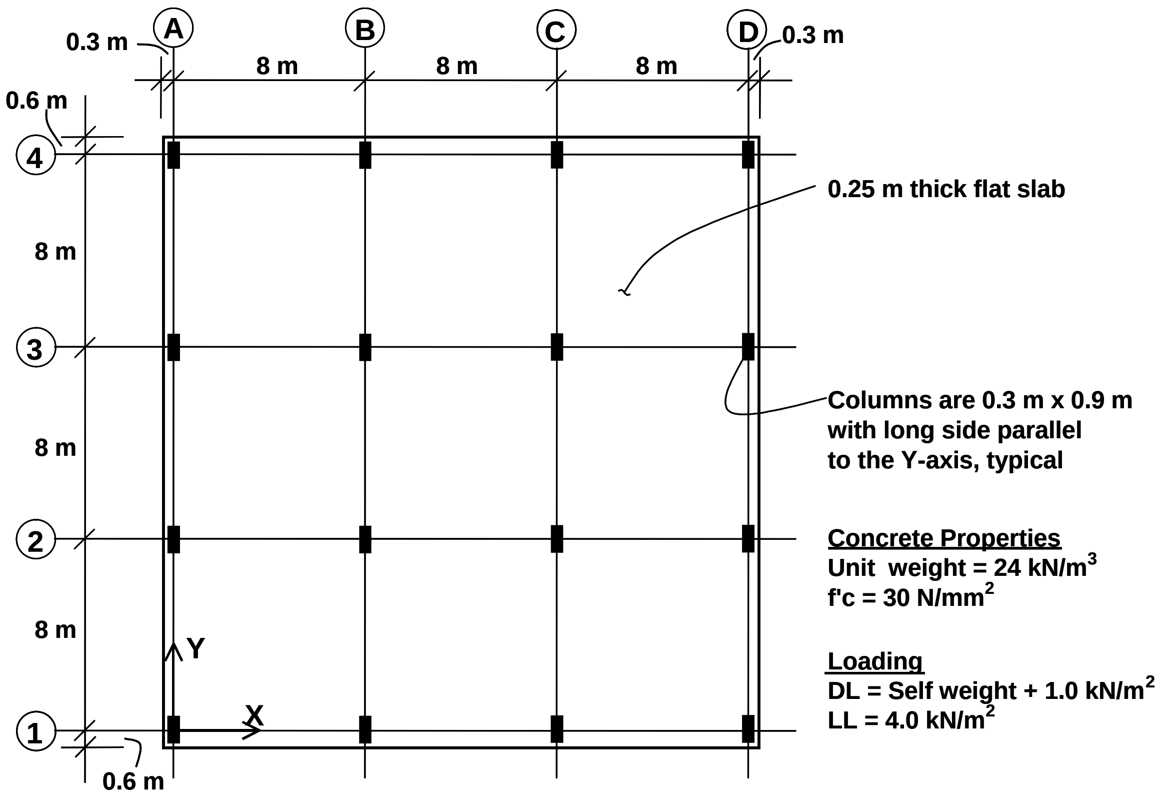


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick plate properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f_{ck} of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from SAFE with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
SAFE	1.690	1.278	1.32
Calculated	1.690	1.278	1.32

COMPUTER FILE: TURKISH TS 500-2000 RC-PN-001.FDB

CONCLUSION

The SAFE results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using SAFE Method

$$d = [(250 - 26) + (250 - 38)]/2 = 218 \text{ mm}$$

b_0 = 518+ 1118 + 1118 + 518 = 3272 mm

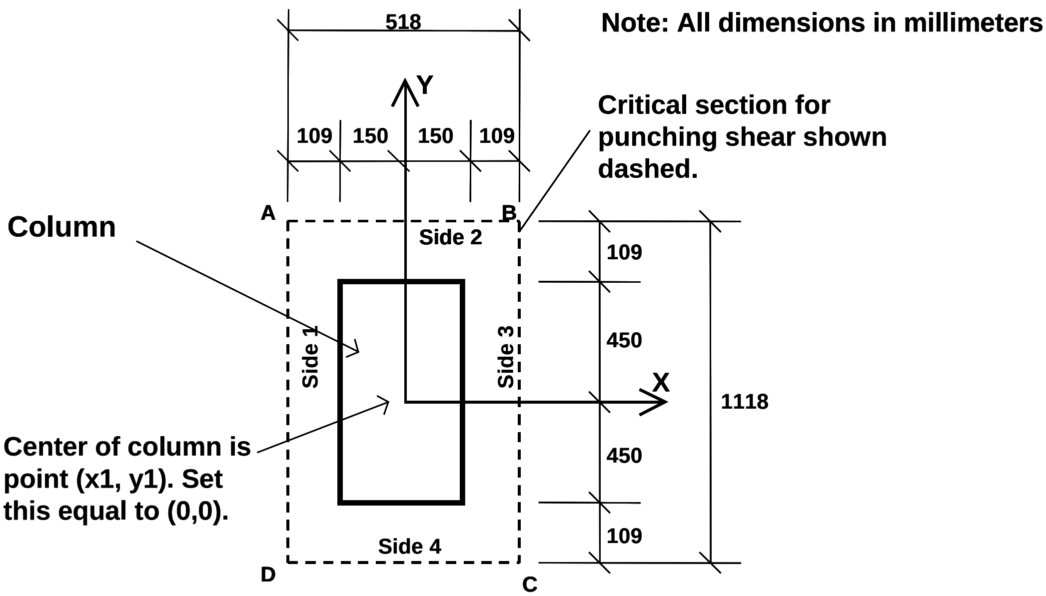


Figure 2: Interior Column, Grid B-2 in SAFE Model

$$\eta_2 = 1 - \frac{1}{1 + \sqrt{\frac{1118}{518}}} = 0.595$$

$$\eta_3 = 1 - \frac{1}{1 + \sqrt{\frac{518}{1118}}} = 0.405$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–259	0	259	0	N.A.
y_2	0	559	0	–559	N.A.
L	1118	518	1118	518	b_0 = 3272
d	218	218	218	218	N.A.
Ld	243724	112924	243724	112924	713296
Ldx_2	–63124516	0	63124516	0	0
Ldy_2	0	63124516	0	–63124516	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{713296} = 0\,mm$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{713296} = 0\,mm$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	1118	518	1118	518	N.A.
d	218	218	218	218	N.A.
$x_2 - x_3$	–259	0	259	0	N.A.
$y_2 - y_3$	0	559	0	–559	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b	5a, 6a	5b, 6b	5a, 6a	N.A.
I_{XX}	5.43E+07	6.31E+07	2.64E+10	3.53E+10	1.23E+11
I_{YY}	6.31E+07	1.39E07	1.63E+10	2.97E+09	3.86E+10

From the SAFE output at Grid B-2:

V_d= 1126.498 kN

$0.4\eta M_{d,2}$ = -8.4226 kN-m

$0.4\eta M_{d,3}$ = 10.8821 kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{pd} = \frac{V_{pd}}{u_p d}\left[1 + \eta\frac{0.4M_{pd,2}u_p d}{V_{pd}W_{m,2}} + \eta\frac{0.4M_{pd,3}u_p d}{V_{pd}W_{m,3}}\right], \qquad \text{(TS 8.3.1)}$$

At the point labeled A in Figure 2, x_4 = –259 and y_4 = 559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{8.423 \bullet 10^6 \left[3.86 \bullet 10^{10} \left(559 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)} + \frac{10.882 \bullet 10^6 \left[1.23 \bullet 10^{11} \left(-259 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)}$$

v_U = 1.5793 – 0.0383 – 0.0730 = **1.4680 N/mm²** at point A

At the point labeled B in Figure 2, x_4 = 259 and y_4 = 559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{8.423 \bullet 10^6 \left[3.86 \bullet 10^{10} \left(559 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)} + \frac{10.8821 \bullet 10^6 \left[1.23 \bullet 10^{11} \left(259 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)}$$

v_U = 1.5793 – 0.0383 + 0.0730 =**1.614 N/mm²** at point B

At the point labeled C in Figure 2, x_4 = 259 and y_4 = –559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{8.423 \bullet 10^6 \left[3.86 \bullet 10^{10} \left(-559 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)} + \frac{10.882 \bullet 10^6 \left[1.23 \bullet 10^{11} \left(259 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)}$$

v_U = 1.5793 + 0.0383 + 0.0730 = **1.690 N/mm²** at point C

At the point labeled D in Figure 2, x_4 = –259 and y_4 = –559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{8.423 \bullet 10^6 \left[3.86 \bullet 10^{10} \left(-559 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)} + \frac{10.8821 \bullet 10^6 \left[1.23 \bullet 10^{11} \left(-259 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)}$$

v_U = 1.5793 + 0.383 – 0.0730 = **1.5446 N/mm²** at point D

Point C has the largest absolute value of v_u, thus v_{max} = **1.690 N/mm²**

The concrete punching shear stress capacity of a section with punching shear reinforcement is limited to:

$$v_{pr} = f_{ctd} = 0.35\sqrt{f_{ck}}/\gamma_c \qquad \text{(TS 8.3.1)}$$

$$v_{pr} = f_{ctd} = 0.35\sqrt{30}/1.5 = 1.278\,\text{N/mm}^2$$

$$\text{Shear Ratio} = \frac{v_{pd}}{v_{pr}} = \frac{1.690}{1.278} = 1.32$$

EXAMPLE Turkish TS 500-2000 RC-SL-001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in SAFE.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

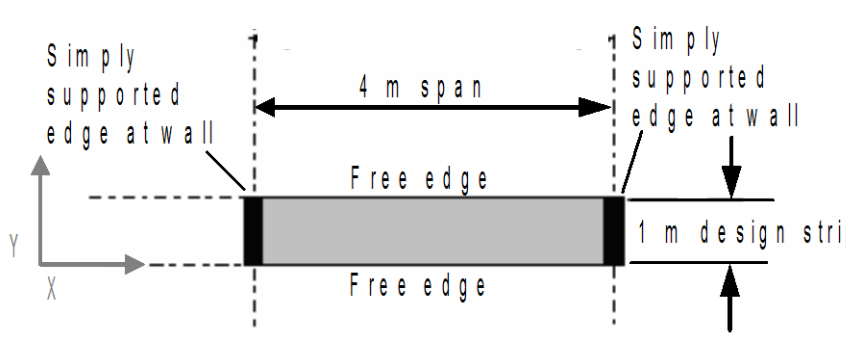


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 KN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the Turkish TS 500-2000 load combination factors, 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the SAFE results. After completing the analysis, design is performed using the Turkish TS 500-2000 code by SAFE and also by hand computation. Table 1 shows the comparison of the design reinforcements computed by the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_{ck}	=	30	MPa
Yield strength of steel	f_{yk}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2×10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF SAFE TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the SAFE total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	SAFE	27.197	5.760
	Calculated	27.200	5.760

$A^+_{s,\min}$ = 162.5 sq-mm

COMPUTER FILE: Turkish TS 500-2000 RC-001.FDB

CONCLUSION

The SAFE results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for all the load combinations:

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$

$$f_{cd} = \frac{f_{ck}}{\gamma_{mc}} = \frac{30}{1.5} = 20$$

$$f_{yd} = \frac{f_{yk}}{\gamma_{ms}} = \frac{460}{1.15} = 400$$

$$c_b = \frac{\varepsilon_{cu} E_s}{\varepsilon_{cu} E_s + f_{yd}} d \;\; = 75 \text{ mm}$$

$$a_{\max} = 0.85 k_1 c_b \;\; = 52.275 \text{ mm}$$

where, $k_1 = 0.85 - 0.006\left(f_{ck} - 25\right) = 0.82\,,\; 0.70 \le k_1 \le 0.85$

$$A_{s,\min} = \frac{0.8 f_{ctd}}{f_{yd}} bd \;\; = 325.9\, mm^2$$

Where $f_{ctd} = \dfrac{0.35\sqrt{f_{cu}}}{\gamma_{mc}} = \dfrac{0.35\sqrt{30}}{1.5} = 1.278$

For each load combination, the w and M are calculated as follows:

$$w = (1.4 w_d + 1.6 w_t)\, b$$

$$M = \frac{w l_1^2}{8}$$

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 13.6 kN/m

$M_{\text{-}strip}$ = 27.2 kN-m

$M_{\text{-}design}$ = 27.2366 kN-m

The depth of the compression block is given by:

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_d\right|}{0.85 f_{cd} b}} \quad \text{(TS 7.1)}$$

$$a = 125 - \sqrt{125^2 - \frac{2 \bullet 27.2366 \bullet 10^6}{0.85 \bullet 20 \bullet 1000}} = 13.5518 \text{ mm}$$

If $a \le a_{max}$ (TS 7.1), the area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_d}{f_{yd}\left(d - \frac{a}{2}\right)} = \frac{27.2366 \bullet 10^6}{400\left(125 - \frac{13.5518}{2}\right)} = 576 \text{ mm}^2$$

CONCLUSIONS

The conclusions are presented separately for analysis, reinforced concrete beam and slab design, and post-tensioned slab design in the following subsections.

ANALYSIS

The SAFE verification and validation example problems for analysis show Acceptable comparison with the independent solutions. The accuracy of the SAFE results for certain examples depends on the discretization of the area objects. For those examples, as the discretization is refined, the solution becomes more accurate.

DESIGN

The design results for flexural and shear design for reinforced concrete beams; flexural design for reinforced concrete and post-tensioned slab and stress checks for post-tensioned slabs show exact comparison with hand calculations.

MESHING OF AREA ELEMENTS

It is important to adequately mesh area elements to obtain satisfactory results. The art of creating area element models includes determining what constitutes an adequate mesh. In general, meshes should always be two or more elements wide. Rectangular elements give the best results and the aspect ratio should not be excessive. A tighter mesh may be needed in areas where the stress is high or the stress is changing quickly.

When reviewing results, the following process can help determine if the mesh is adequate. Pick a joint in a high stress area that has several different area elements connected to it. Review the stress reported for that joint for each of the area elements. If the stresses are similar, the mesh likely is adequate. Otherwise, additional meshing is required. If you choose to view the stresses graphically when using this process, be sure to turn off the stress averaging feature when displaying the stresses.

References

ACI Committee 435, 1984. Deflection of Two-way Reinforced Concrete Floor Systems: State-of-the-Art Report, (ACI 435-6R-74), (Reaffirmed 1984), American Concrete Institute, Detroit, Michigan.

ACI Committee 336, 1988. Suggested Analysis and Design Procedures for Combined Footings and Mats (ACI 336-2R-88), American Concrete Institute, Detroit, Michigan.

ACI Committee 340, 1991. Design Handbook In Accordance with the Strength Design Method of ACI 318-89, Volume 3, Two-way Slabs (ACI 340.4R-91), American Concrete Institute, Detroit, Michigan.

ACI Committee 340, 1997. ACI Design Handbook, Design of Structural Reinforced Concrete Elements in Accordance with the Strength Design Method of ACI 318-95 (ACI 340R-97), American Concrete Institute, Detroit, Michigan.

ACI Committee 318, 1995. Building Code Requirements for Reinforced Concrete (ACI 318-95) and Commentary (ACI 318R-95), American Concrete Institute, Detroit, Michigan.

Corley, W. G. and J. O. Jirsa, 1970. Equivalent Frame Analysis for Slab Design, ACI Journal, Vol. 67, No. 11, November.

Gamble, W. L., M. A. Sozen, and C. P. Siess, 1969. Tests of a Two-way Reinforced Concrete Floor Slab, Journal of the Structural Division, Proceedings of the ASCE, Vol. 95, ST6, June.

Guralnick, S. A. and R. W. LaFraugh, 1963. Laboratory Study of a 45-Foot Square Flat Plate Structure, ACI Journal, Vol. 60, No.9, September.

Hatcher, D. S., M. A. Sozen, and C. P. Siess, 1965. Test of a Reinforced Concrete Flat Plate, Journal of the Structural Division, Proceedings of the ASCE, Vol. 91, ST5, October.

Hatcher, D. S., M. A. Sozen, and C. P. Siess, 1969. Test of a Reinforced Concrete Flat Slab, Journal of the Structural Division, Proceedings of the ASCE, Vol. 95, ST6, June.

Jirsa, J. O., M. A. Sozen, and C. P. Siess, 1966. Test of a Flat Slab Reinforced with Welded Wire Fabric, Journal of the Structural Division, Proceedings of the ASCE, Vol. 92, ST3, June.

PCA, 1990. Notes on ACI 318-89 Building Code Requirements for Reinforced Concrete with Design Applications, Portland Cement Association, Skokie, Illinois.

PCA, 1996. Notes on ACI 318-95 Building Code Requirements for Reinforced Concrete with Design Applications, Portland Cement Association, Skokie, Illinois.

Roark, Raymond J., and Warren C. Young, 1975. Formulas for Stress and Strain, Fifth Edition, Table 3, p. 107-108. McGraw-Hill, 2 Penn Plaza, New York, NY 10121-0101.

Timoshenko, S. and S. Woinowsky-Krieger, 1959, Theory of Plates and Shells, McGraw-Hill, 2 Penn Plaza, New York, NY 10121-0101.

Ugural, A. C. 1981, Stresses in Plates and Shells, McGraw-Hill, 2 Penn Plaza, New York, NY 10121-0101.

Vanderbilt, M. D., M. A. Sozen, and C. P. Siess, 1969. Tests of a Modified Reinforced Concrete Two-Way Slab, Journal of the Structural Division, Proceedings of the ASCE, Vol. 95, ST6, June.