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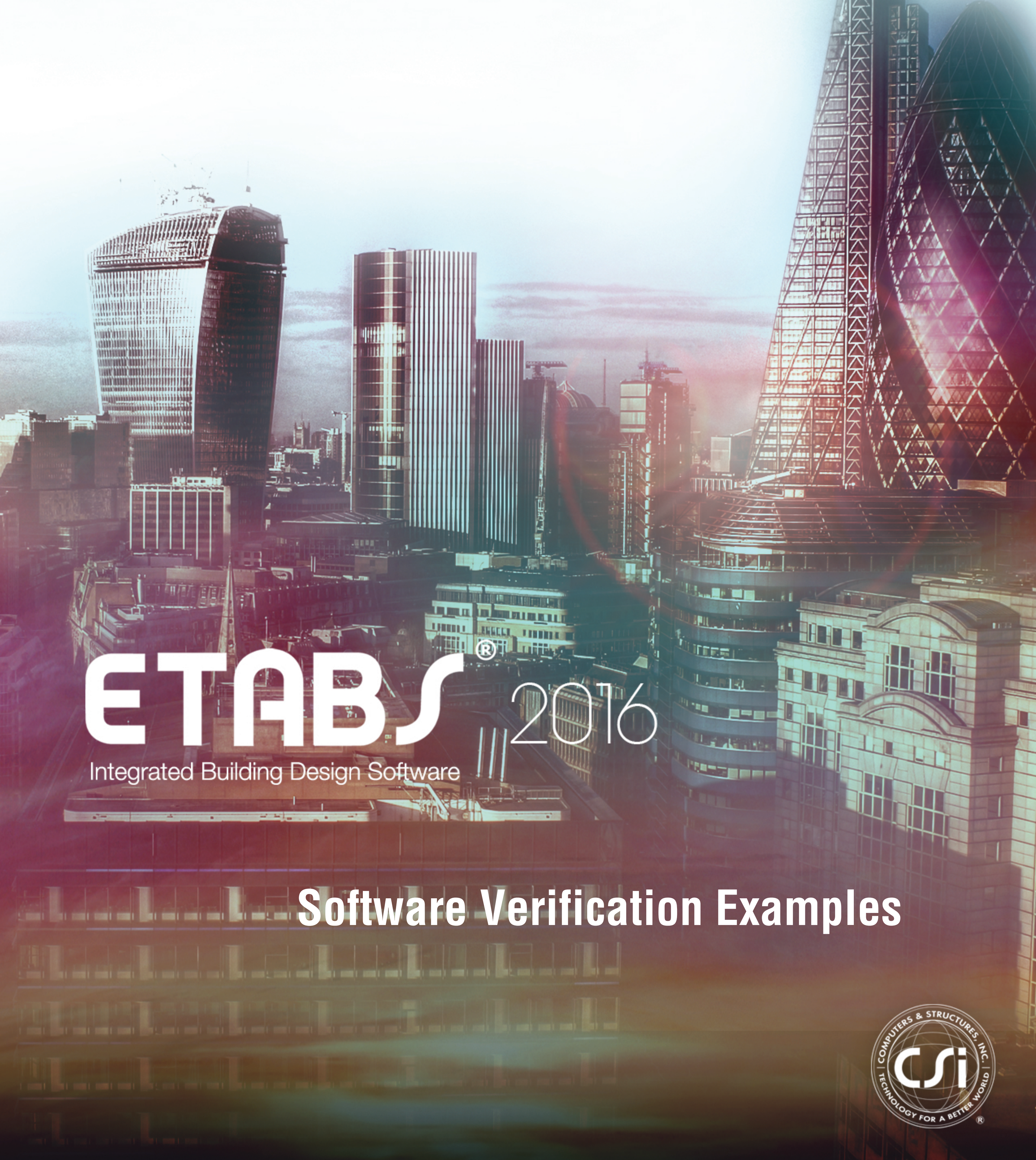

ETABS® 2016

Integrated Building Design Software

Software Verification Examples

Software Verification Examples

For ETABS® 2016

ISO ETA122815M6 Rev. 1
Proudly developed in the United States of America

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Computers & Structures, Inc.
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References

ETABS Software Verification Log		
Revision Number	**Date**	**Description**
0	19 Apr 2013	Initial release of ETABS, Version 13.0.0
1	9 July 2013	• Minor documentation errors in the Verification manuals have been corrected • Minor improvements have been made to some of the examples, and some example file names have been changed for consistency. The design results produced and reported by ETABS are correct. The reported results are not changed except where the model has been changed. • Three new examples have been added for steel frame design.
2	11 Apr 2014	• Analysis model EX8.EDB - The response-spectrum function damping was incorrect and did not match the response-spectrum load case damping, hence the results produced did not match the documented value. After correction, the example produces the expected and documented results. No change was made to the Verification manual. • Analysis Example 03 - The name of code IBC2000 was changed to ASCE 7-02, as actually used in ETABS (IBC2000 was used in v9.7.4). In addition, the Verification manual was corrected for the actual values produced by ETABS. These values have not changed since v13.0.0. The documented values were for ETABS v9.7.4 and some changed in v13.0.0 due to the use of a different solver. The change has no engineering significance. • Analysis Example 06 and Example 07 - The Verification manual was corrected for the actual values produced by ETABS. These values have not changed since v13.0.0. The documented values were for ETABS v9.7.4 and some changed in v13.0.0 due to the use of a different solver. The change has no engineering significance. • Analysis Example 15 - The Verification manual was corrected for the actual values produced by ETABS. These values have not changed since v13.0.0. The documented values were for ETABS v9.7.4 and some changed in v13.0.0 due to the use of a different solver, and due to the difference in how wall elements are connected to beams. The change due to the solver has no engineering significance. The change for wall elements was an enhancement.

ETABS Software Verification Log		
Revision Number	**Date**	**Description**
		• Concrete Frame Design EN 2-2004 Example 001, Concrete Frame Design NTC 2008 Example 002 - The values produced by ETABS 2014 were updated in the Verification manual for a change in v13.1.3 under Incident 59154 (Ticket 23901) where the coefficients Alpha_CC and Alpha_LCC were not taken into account in certain cases. • Concrete Frame Design AS 3600-2009 Example 002, Shear Wall Design AS 3600-2009 WALL-002 - The values produced by ETABS were updated in the Verification manual for a change in v13.1.4 under Incident 59973 where the phi factor was incorrectly computed.
3	3 Nov 2014	• Analysis Example 14 – Minor changes have been made to the results as the result of an enhancement made under Incident 67283 to improve the convergence behavior of nonlinear static and nonlinear direct-integration time history analysis. • Composite Beam Design AISC-360-05 Example 001 was updated to reflect the fact that, under Incident 59912 it is now possible to specify that the shear stud strength is to be computed assuming the weak stud position. A typo in the version number of the referenced Design Guide example was corrected. A slight error in the hand-calculation for the partial composite action ΦM_n was corrected, resulting in perfect agreement with the value produced by ETABS. • Composite Beam Design AISC-360-10 Example 001 was updated to reflect the fact that, under Incident 59912 it is possible to specify that the shear stud strength is to be computed assuming the weak stud position. The hand-calculation for the partial composite action ΦM_n was revised to account for a lower percentage of composite action caused by an increase in the number of shear studs per deck rib in places, and a corresponding decrease in shear stud strength. • Composite Beam Design BS-5950-90 Example 001- The hand-calculations in the Verification manual were updated to reflect the actual section area of a UKB457x191x167, which differs from the value in the reference example, and to reflect that the maximum number of shear studs that can be placed on the beam is 78 studs and not the 80 the reference example calls for. Also the value of the live load deflection produced by

ETABS Software Verification Log		
Revision Number	**Date**	**Description**
		ETABS was updated for a change in v13.2.0 under Incident 56782. • Composite Beam Design CSA-S16-09 Example 001. The values produced by ETABS for the shear stud capacity were updated in the Verification manual for a change in v13.2.0 under Incident 71303. This change in turn affects the value of the partial composite moment capacity M_c but has no engineering significance. A typo affecting the value of pre-composite deflection in the Results Comparison table was corrected. • Composite Beam Design EC-4-2004 Example 001. The hand-calculation of the construction moment capacity, $M_{a,pl,Rd}$ was updated to reflect a more accurate value of the section W_{pl} and typos affecting the pre-composite deflection and beam camber were corrected. None of the values computed by ETABS changed.
4	7 Jan 2015	Initial release of ETABS 2015, Version 15.0.0 • Shear Wall Design example Eurocode 2-2004 Wall-002 has been updated due to changes previously reported under Incident #56569. • Shear Wall Design example AS 3600-09 Wall-001 has been updated due to changes previously reported under Incident #56113. • Shear Wall Design example CSA A23.3-04 Example 001 has been updated due to changes previously reported under Incident #71922. • Concrete Frame Design example CSA A23.3-04 Example 002 has been updated due to changes previously reported under Incident #71922. • New steel frame design examples have been added for CSA S16-14 and KBC 2009. • New concrete frame design examples have been added for ACI 318-14, CSA A23.3-14, and KBC 2009. • New shear wall design examples have been added for ACI 318-14, CSA A23.3-14, and KBC 2009.

ETABS Software Verification Log		
Revision Number	**Date**	**Description**
5	7 July 2016	Initial release of ETABS 2016, Version 16.0.0 • Added SAFE design verification examples for slab design, punching shear design, and post-tension design for all codes supported in both SAFE and ETABS.

INTRODUCTION

This manual provides example problems used to test various features and capabilities of the ETABS program. Users should supplement these examples as necessary for verifying their particular application of the software.

METHODOLOGY

A series of test problems, or examples, designed to test the various elements and analysis features of the program were created. For each example, this manual contains a short description of the problem; a list of significant ETABS options tested; and a comparison of key results with theoretical results or results from other computer programs. The comparison of the ETABS results with results obtained from independent sources is provided in tabular form as part of each example.

To validate and verify ETABS results, the test problems were run on a PC platform that was a Dell machine with a Pentium III processor and 512 MB of RAM operating on a Windows XP operating system.

Acceptance Criteria

The comparison of the ETABS validation and verification example results with independent results is typically characterized in one of the following three ways.

Exact: There is no difference between the ETABS results and the independent results within the larger of the accuracy of the typical ETABS output and the accuracy of the independent result.

Acceptable: For force, moment and displacement values, the difference between the ETABS results and the independent results does not exceed five percent (5%). For internal force and stress values, the difference between the ETABS results and the independent results does not exceed ten percent (10%). For experimental values, the difference between the ETABS results and the independent results does not exceed twenty five percent (25%).

Unacceptable: For force, moment and displacement values, the difference between the ETABS results and the independent results exceeds five percent (5%). For internal force and stress values, the difference between the ETABS results and the independent results exceeds ten percent (10%). For experimental values, the difference between the ETABS results and the independent results exceeds twenty five percent (25%).

The percentage difference between results is typically calculated using the following formula:

$$\text{Percent Difference} = 100\left[\frac{\text{ETABS Result}}{\text{Independent Result}} - 1\right]$$

For examples with multiple versions of meshing density of area elements, only the models with the finest meshing density are expected to fall within Exact or Acceptable limits.

Summary of Examples

The example problems addressed plane frame, three-dimensional, and wall structures as well as shear wall and floor objects. The analyses completed included dynamic response spectrum, eigenvalue, nonlinear time history, and static gravity and lateral load.

Other program features tested include treatment of automatic generation of seismic and wind loads, automatic story mass calculation, biaxial friction pendulum and biaxial hysteretic elements, brace and column members with no bending stiffness, column pinned end connections, multiple diaphragms, non-rigid joint offsets on beams and columns, panel zones, point assignments, rigid joint offsets, section properties automatically recovered from the database, uniaxial damper element, uniaxial gap elements, vertical beam span loading and user specified lateral loads and section properties.

Slab design examples verify the design algorithms used in ETABS for flexural, shear design of beam; flexural and punching shear of reinforced concrete slab; and flexural design and serviceability stress checks of post-tensioned slab by comparing ETABS results with hand calculations.

Analysis: Of the fifteen Analysis problems, eight showed exact agreement while the remaining seven showed acceptable agreement between ETABS and the cited independent sources.

Design – Steel Frame: All 30 Steel Frame Design problems showed acceptable agreement between ETABS and the cited independent sources.

Design – Concrete Frame: All 34 Concrete Frame Design problems showed acceptable agreement between ETABS and the cited independent sources.

Design – Shear Wall: All 32 of the Shear Wall Design problems showed acceptable agreement between ETABS and the cited independent sources.

Design – Composite Beam: The 6 Composite Beam Design problems showed acceptable agreement between ETABS and the cited independent sources.

Design – Composite Column: The 3 Composite Column Design problems showed acceptable agreement between ETABS and cited independent sources.

Design – Slab: The 48 Slab Design problems showed acceptable agreement between ETABS and cited independent sources.

CONCLUSIONS

ETABS is the latest release of the ETABS series of computer programs. Since development, ETABS has been used widely for structural analysis. The ongoing usage of the program coupled with continuing program upgrades are strong indicators that most program bugs have been identified and corrected.

Additionally, the verification process conducted as described in this document demonstrates that the program features tested are operating reliably and with accuracy consistent with current computer technology capabilities.

MESHING OF AREA ELEMENTS

It is important to adequately mesh area elements to obtain satisfactory results. The art of creating area element models includes determining what constitutes an adequate mesh. In general, meshes should always be two or more elements wide. Rectangular elements give the best results and the aspect ratio should not be excessive. A tighter mesh may be needed in areas where the stress is high or the stress is changing quickly.

When reviewing results, the following process can help determine if the mesh is adequate. Pick a joint in a high stress area that has several different area elements connected to it. Review the stress reported for that joint for each of the area elements. If the stresses are similar, the mesh likely is adequate. Otherwise, additional meshing is required. If you choose to view the stresses graphically when using this process, be sure to turn off the stress averaging feature when displaying the stresses.

EXAMPLE 1

Plane Frame with Beam Span Loads - Static Gravity Load Analysis

Problem Description

This is a one-story, two-dimensional frame subjected to vertical static loading.

To be able to compare ETABS results with theoretical results using prismatic members and elementary beam theory, rigid joint offsets on columns and beams are not modeled, and axial and shear deformations are neglected. Thus, the automatic property generation feature of ETABS is not used; instead, the axial area and moment of inertia for each member are explicitly input.

Geometry, Properties and Loading

The frame is a three-column line, two-bay system. Kip-inch-second units are used. The modulus of elasticity is 3000 ksi. All columns are 12"x24"; all beams are 12"x30".

The frame geometry and loading patterns are shown in Figure 1-1.

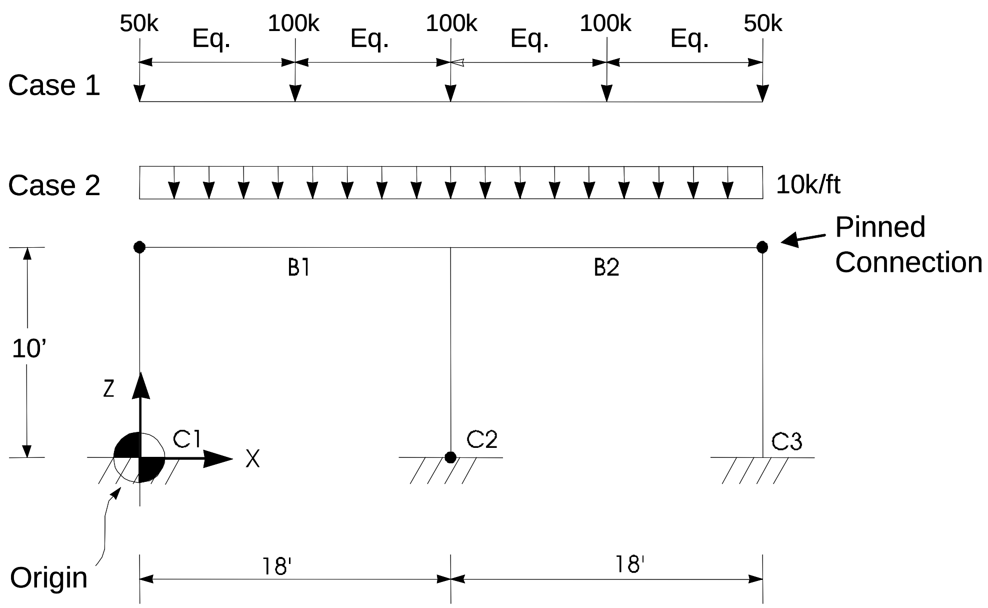


Figure 1-1 Plane Frame with Beam Span Loads

Technical Features of ETABS Tested

- Two-dimensional frame analysis
- Vertical beam span loading
- No rigid joint offsets on beams and columns
- Column pinned end connections

Results Comparison

The theoretical results for bending moments and shear forces on beams B1 and B2 are easily obtained from tabulated values for propped cantilevers (American Institute of Steel Construction 1989). These values for beam B1 are compared with ETABS results in Table 1-1.

Table 1-1 Comparison of Results for Beam B1 – Case 1

Quantity	Location	Load Case I (Concentrated Load)	
		ETABS	**Theoretical**
Bending Moments	End I	0.00	0.00
	¼ Point	1,687.50	1,687.50
	½ point	3,375.00	3,375.00
	¾ point	-337.50	-337.50
	End J	-4,050.00	-4,050.00
Shear Forces	End I	-31.25	-31.25
	¼ Point	-31.25	-31.25
	½ point	68.75	68.75
	¾ point	68.75	68.75
	End J	68.75	68.75

Table 1-1 Comparison of Results for Beam B1 – Case II

Quantity	Location	Load Case II (Uniformly Distributed Load)	
		ETABS	**Theoretical**
Bending Moments	End I	0.00	0.00
	¼ Point	2,430.00	2,430.00
	½ point	2,430.00	2,430.00
	¾ point	0.00	0.00

Table 1-1 Comparison of Results for Beam B1 – Case II

Quantity	Location	Load Case II (Uniformly Distributed Load)	
		ETABS	**Theoretical**
	End J	-4,860.00	-4,860.00
Shear Forces	End I	-67.50	-67.50
	¼ Point	-22.50	-22.50
	½ point	22.50	22.50
	¾ point	67.50	67.50
	End J	112.50	112.50

Computer File

The input data file for this example is Example 01.EDB. This file is provided as part of the ETABS installation.

Conclusion

The comparison of results shows an exact match between the ETABS results and the theoretical data.

EXAMPLE 2

Three-Story Plane Frame - Dynamic Response Spectrum Analysis

Problem Description

This is a three-story plane frame subjected to the El Centro 1940 seismic response spectra, N-S component, 5 percent damping.

Assuming the beams to be rigid and a rigid offset at the column top ends of 24 inches (i.e., equal to the depth of the beams), and neglecting both shear deformations and axial deformations, the story lateral stiffness for this example can be calculated (Przemieniecki 1968).

The example then reduces to a three-spring, three-mass system with equal stiffnesses and masses. This can be analyzed using any exact method (Paz 1985) to obtain the three natural periods and mass normalized mode shapes of the system.

The spectral accelerations at the three natural periods can then be linearly interpolated from the response spectrum used.

The spectral accelerations can in turn be used with the mode shapes and story mass information to obtain the modal responses (Paz 1985). The modal responses for story displacements and column moments can then be combined using the complete quadratic combination procedure (Wilson, et al. 1981).

Geometry, Properties and Loading

The frame is modeled as a two-column line, single bay system. Kip-inch-second units are used. Other parameters associated with the structure are as follows:

All columns are W14X90

All beams are infinitely rigid and 24" deep

Modulus of elasticity = 29500 ksi

Typical story mass = 0.4 kip-sec^2/in

The column is modeled to have infinite axial area, so that axial deformation is neglected. Also, zero column shear area is input to trigger the ETABS option of neglecting shear deformations. These deformations are neglected to be consistent with the hand-calculated model with which the results are compared.

The frame geometry is shown in Figure 2-1.

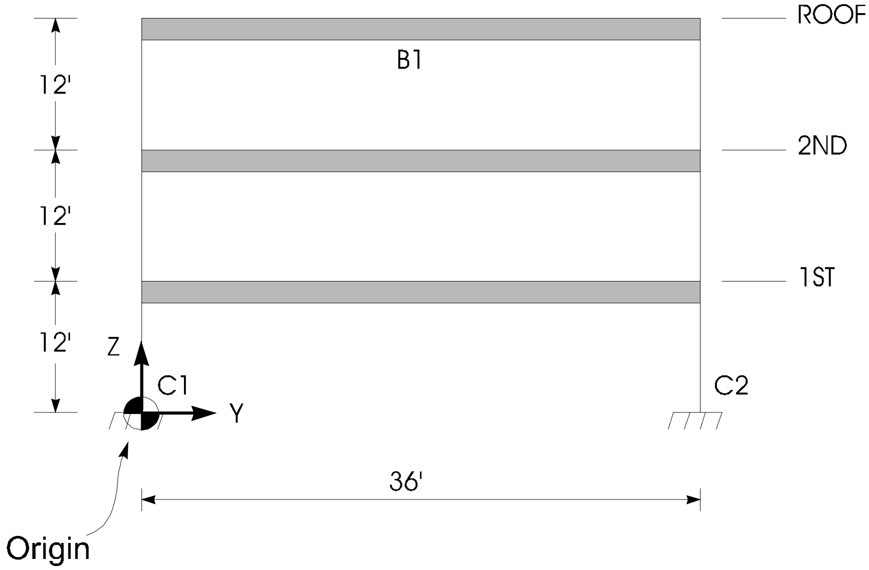


Figure 2-1 Three-Story Plane Frame

Technical Features in ETABS Tested

- Two-dimensional frame analysis
- Rigid joint offsets on beams and columns automatically calculated
- Dynamic response spectrum analysis

Results Comparison

The three theoretical natural periods and mass normalized mode shapes are compared in Table 2-1 with ETABS results.

Table 2-1 Comparison of Results for Periods and Mode Shapes

Mode	Period, secs.	Mode Shape	ETABS	Theoretical
1	0.4414	Roof Level	1.165	1.165
		2nd Level	0.934	0.934
		1st Level	0.519	0.519
2	0.1575	Roof Level	0.934	0.934
		2nd Level	-0.519	-0.519
		1st Level	-1.165	-1.165
3	0.1090	Roof Level	0.519	0.519

Table 2-1 Comparison of Results for Periods and Mode Shapes

Mode	Period, secs.	Mode Shape	ETABS	Theoretical
		2nd Level	-1.165	-1.165
		1st Level	0.934	0.934

The story displacements and column moments thus obtained are compared in Table 2-2 with ETABS results. The results are identical.

Table 2-2 Comparison of Displacements and Column Moments

Quantity	ETABS	Theoretical
Displacement at		
Roof	2.139	2.139
2nd	1.716	1.716
1st	0.955	0.955
Moment, Column C1, at Base	11,730	11,730

Computer Files

The input data file for this example is Example 02.EDB. The response spectrum file is ELCN-RS1. These files are provided as part of the ETABS installation.

Conclusion

The result comparison shows an exact match between the ETABS results and the theoretical data.

EXAMPLE 3

Three-Story Plane Frame, Code-Specified Static Lateral Load Analysis

Problem Description

The frame is modeled as a two-column line, single bay system. This three-story plane frame is subjected to the following three code-specified lateral load cases:

- UBC 1997 specified seismic loads (International Conference of Building Officials 1997)
- ASCE 7-02 specified seismic loads (American Society of Civil Engineers 2002)
- UBC 1997 specified wind loads (International Conference of Building Officials 1997)

Geometry, Properties and Loads

Kip-inch-second units are used. Other parameters associated with the structure are as follows:

All columns are W14X90
All beams are infinitely rigid and 24" deep
Modulus of elasticity = 29500 ksi
Poisson's ratio = 0.3
Typical story mass = 0.4 kip-sec^2/in

The frame geometry is shown in Figure 3-1.

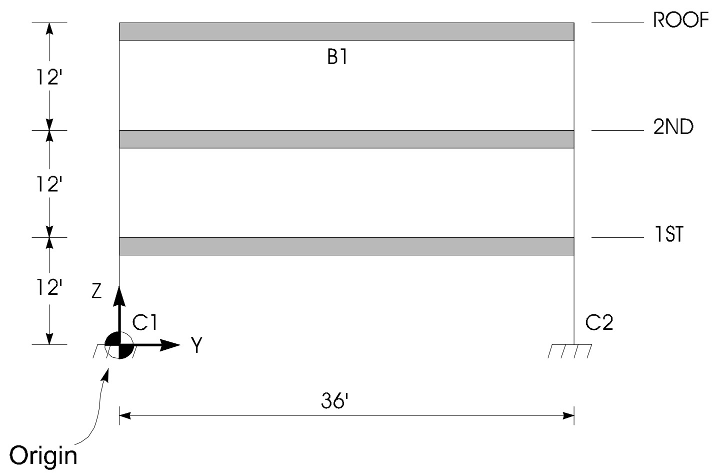


Figure 3-1 Three-Story Plane Frame

For the UBC97 seismic load analysis, the code parameters associated with the analysis are as follows:

UBC Seismic zone factor, Z	= 0.40
UBC Soil Profile Type	= SC
UBC Importance factor, I	= 1.25
UBC Overstrength Factor	= 8.5
UBC coefficient C_t	= 0.035
UBC Seismic Source Type	= B
Distance to Source	= 15 km

For the ASCE 7-02 seismic load analysis, the code parameters associated with the analysis are as follows:

Site Class	= C
Response Accel, S_s	= 1
Response Accel, S_1	= 0.4
Response Modification, R	= 8
Coefficient C_t	= 0.035
Seismic Group	= I

For the UBC97 wind load analysis, the exposure and code parameters associated with the analysis are as follows:

Width of structure supported by frame	= 20 ft
UBC Basic wind speed	= 100 mph
UBC Exposure type	= B
UBC Importance factor, I	= 1
UBC Windward coefficient, C_q	= 0.8
UBC Leeward coefficient, C_q	= 0.5

Technical Features in ETABS Tested

- Two-dimensional frame analysis
- Section properties automatically recovered from AISC database
- Automatic generation of UBC 1997 seismic loads
- Automatic generation of ASCE 7-02 seismic loads
- Automatic generation of UBC 1997 wind loads

Results Comparison

For each of the static lateral load analyses, the story shears can be computed using the formulae given in the applicable references. For the seismic loads, the fundamental period computed by ETABS can be used in the formulae. From ETABS results, this fundamental period is 0.5204 second. (Note the difference between the calculated fundamental period for this example and Example 2, which neglects shear and axial deformations.)

Hand-calculated story shears are compared with story shears produced by the ETABS program in Table 3-1 for UBC seismic loads, Table 3-2 for ASCE 7-02 seismic loads and Table 3-3 for UBC wind loads.

Table 3-1 Comparison of Results for Story Shears - UBC 1997 Seismic

Level	**ETABS** (kips)	**Theoretical** (kips)
Roof	34.07	34.09
2nd	56.78	56.82
1st	68.13	68.19

Table 3-2 Comparison of Results for Story Shears - ASCE 7-02 Seismic

Level	**ETABS** (kips)	**Theoretical** (kips)
Roof	19.37	19.38
2nd	32.23	32.25
1st	38.61	38.64

Table 3-3 Comparison of Results for Story Shears - UBC 1997 Wind

Level	**ETABS** (kips)	**Theoretical** (kips)
Roof	3.30	3.30
2nd	9.49	9.49
1st	15.21	15.21

Computer File

The input data file for this example is Example 03.EDB. This file is provided as part of the ETABS installation.

Conclusion

The results comparison shows an exact match between the ETABS results and the theoretical data.

EXAMPLE 4

Single-Story, Three-Dimensional Frame - Dynamic Response Spectrum Analysis

Problem Description

This is a one-story, four-bay, three-dimensional frame. The frame is subjected to the El Centro 1940 N-S component seismic response spectrum, for 5 percent damping, in two orthogonal directions. The columns are modeled to neglect shear and axial deformations to be consistent with the assumptions of hand calculations with which the results are compared.

The example is a three-degree-of-freedom system. From the individual column lateral stiffnesses, assuming rigid beams and rigid offsets at column top ends equal to 36 inches (i.e., the depth of the beams) and neglecting both shear deformations and column axial deformations, the structural stiffness matrix can be assembled (Przemieniecki 1968).

Geometry, Properties and Loads

The frame geometry is shown in Figure 4-1.

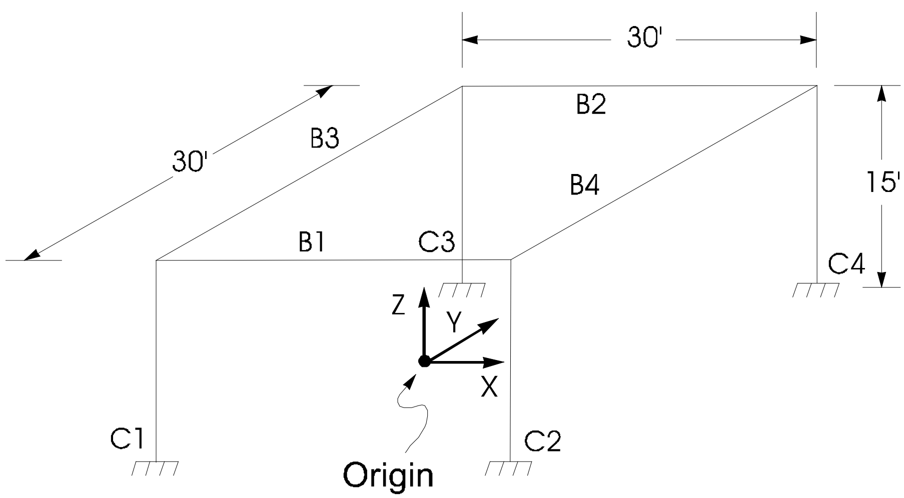


Figure 4-1 Single-Story Three-Dimensional Frame

The structure is modeled as a single frame with four column lines and four bays. Kip-inch-second units are used. Other parameters associated with the structure are as follows:

Columns on lines C1 and C2: 24" x 24"

Columns on lines C3 and C4: 18" x 18"
All beams infinitely rigid and 36" deep

Modulus of elasticity = 3000 ksi
Story weight = 150 psf

Technical Features of ETABS Tested

- Three-dimensional frame analysis
- Automatic story mass calculation
- Dynamic response spectrum analysis

Results Comparison

From the stiffness and mass matrices of the system, the three natural periods and mass normalized mode shapes of the system can be obtained (Paz 1985). These are compared in Table 4-1 with ETABS results.

Table 4-1 Comparison of Results for Periods and Mode Shapes

Mode	Quantity	ETABS	Theoretical
1	Period, sec.	0.1389	0.1389
	Mode Shape		
	X-translation	-1.6244	-1.6244
	Y-translation	0.0000	0.000
	Z-rotation	0.0032	0.0032
2	Period, sec.	0.1254	0.1254
	Mode Shape		
	X-translation	0.000	0.000
	Y-translation	1.6918	1.6918
	Z-rotation	0.000	0.000
3	Period,sec.	0.0702	0.070
	Mode Shape		
	X-translation	0.4728	0.4728
	Y-translation	0.000	0.000
	Z-rotation	0.0111	0.0111

Computer File

The input data file for this example is Example 04.EDB. This file is provided as part of the ETABS installation.

Conclusion

The results comparison shows an exact match between the ETABS results and the theoretical data.

EXAMPLE 5

Three-Story, Three-Dimensional Braced Frame - Dynamic Response Spectrum Analysis

Problem Description

This is an L-shaped building structure with four identical braced frames. All members (columns and braces) carry only axial loads.

The structure is subject to the El Centro 1940 N-S component seismic response spectrum in the X-direction. The structural damping is 5 percent. The structure is modeled by appropriately placing four identical planar frames. Each frame is modeled using three column lines. Kip-inch-second units are used.

Geometry, Properties and Loading

The modulus of elasticity is taken as 29500 ksi and the typical member axial area as 6 in^2. A story mass of 1.242 kip-sec^2/in and a mass moment of inertia of 174,907.4 kip-sec^2-in are used.

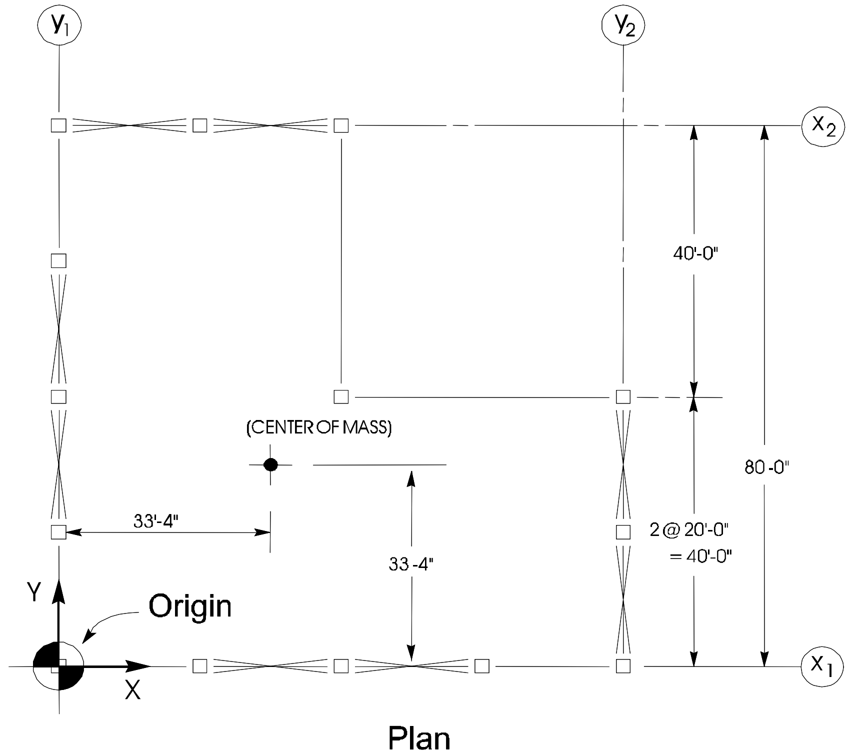
The geometry of the structure and a typical frame are shown in Figure 5-1.

Technical Features of ETABS Tested

- Three-dimensional structure analysis using planar frames
- Brace (diagonal) and column members with no bending stiffness
- Dynamic response spectrum analysis

Results Comparison

This example has been solved in Wilson and Habibullah (1992) and Peterson (1981). A comparison of ETABS results for natural periods and key member forces for one frame with these references is given in Table 5-1.



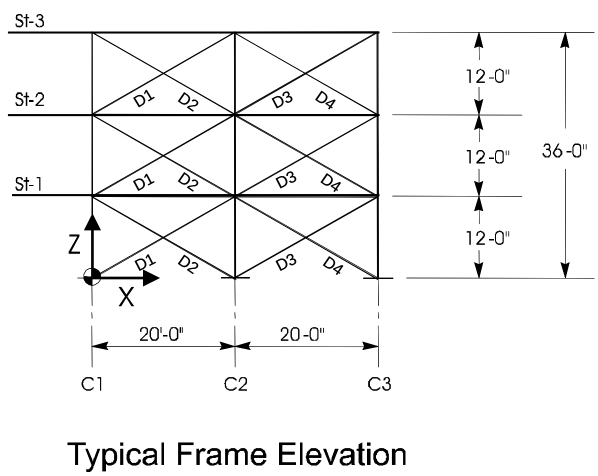


Figure 5-1 Three-Story, Three-Dimensional Braced Frame Building

Table 5-1 Comparison of Results

Quantity	ETABS	Wilson and Habibullah	Peterson
Period, Mode 1	0.32686	0.32689	0.32689
Period, Mode 2	0.32061	0.32064	0.32064
Axial Force Column C1, Story 1	279.39	279.47	279.48
Axial Force Brace D1, Story 1	194.44	194.51	194.50
Axial Force Brace D3, Story 1	120.49	120.53	120.52

Computer File

The input data file is Example 05.EDB. This file is provided as part of the ETABS installation.

Conclusions

The results comparison reflects acceptable agreement between the ETABS results and reference data.

EXAMPLE 6

Nine-Story, Ten-Bay Plane Frame - Eigenvalue Analysis

Problem Description

An eigenvalue analysis is completed.

Geometry, Properties and Loads

The frame is modeled with eleven column lines and ten bays. Kip-ft-second units are used. A modulus of elasticity of 432,000 ksf is used. A typical member axial area of 3ft^2 and moment of inertia of 1ft^4 are used. A mass of 3kip-sec^2/ft/ft of member length is converted to story mass using tributary lengths and used for the analysis.

This is a nine-story, ten-bay plane frame, as shown in Figure 6-1.

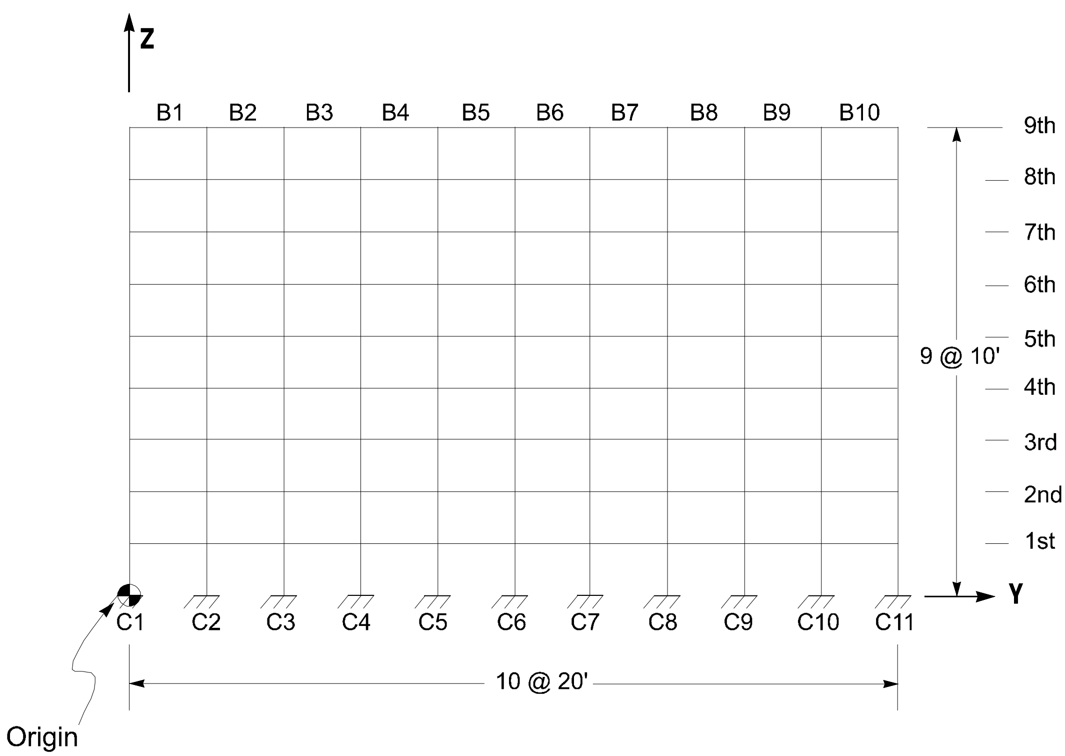


Figure 6-1 Nine-Story, Ten-Bay Plane Frame

Technical Features of ETABS Tested

- Two-dimensional frame analysis
- Eigenvalue analysis

Results Comparison

This example is also analyzed in Wilson and Habibullah (1992) and Bathe and Wilson (1972). There are two differences between the ETABS analysis and the analyses of the references. The models of the references assign vertical and horizontal mass degrees of freedom to each joint in the structure. However, the ETABS model only assigns horizontal masses and additionally, only one horizontal mass is assigned for all the joints associated with any one floor level.

The eigenvalues obtained from ETABS are compared in Table 6-1 with results from Wilson and Habibullah (1992) and Bathe and Wilson (1972).

Table 6-1 Comparison of Results for Eigenvalues

Quantity	ETABS	Wilson and Habibullah	Bathe and Wilson
1	0.58965	0.58954	0.58954
2	5.53196	5.52696	5.52695
3	16.5962	16.5879	16.5878

Computer File

The input data filename for this example is Example 06.EDB. This file is provided as part of the ETABS installation.

Conclusions

Considering the differences in modeling enumerated herein, the results comparison between ETABS and the references is acceptable.

EXAMPLE 7

Seven-Story, Plane Frame - Gravity and Lateral Loads Analysis

Problem Description

This is a seven-story plane frame. The frame is modeled with three column lines and two bays. Kip-inch-second units are used. Because the wide flange members used in the frame are older sections, their properties are not available in the AISC section property database included with the ETABS program, and the required properties therefore need to be explicitly provided in the input data.

The example frame is analyzed in Wilson and Habibullah (1992) for gravity loads, static lateral loads and dynamic response spectrum loads. DYNAMIC/EASE2 analyzes the example frame under static lateral loads and dynamic response spectrum and time history loads. A comparison of key ETABS results with Wilson and Habibullah (1992) and DYNAMIC/EASE2 results is presented in Tables 7-1, 7-2, 7-3 and 7-4. Note the difference in modal combination techniques between ETABS and Wilson and Habibullah, which uses complete quadratic combination (CQC), and DYNAMIC/EASE2, which uses square root of the sum of the squares combination (SRSS).

Geometry, Properties and Loads

The gravity loads and the geometry of the frame are shown in Figure 7-1.

The frame is subjected to the following lateral loads:

- Static lateral loads, shown in Figure 7-1
- Lateral loads resulting from the El Centro 1940 N-S component seismic response spectra, 5 percent damping
- Lateral loads resulting from the El Centro 1940 N-S component acceleration time history

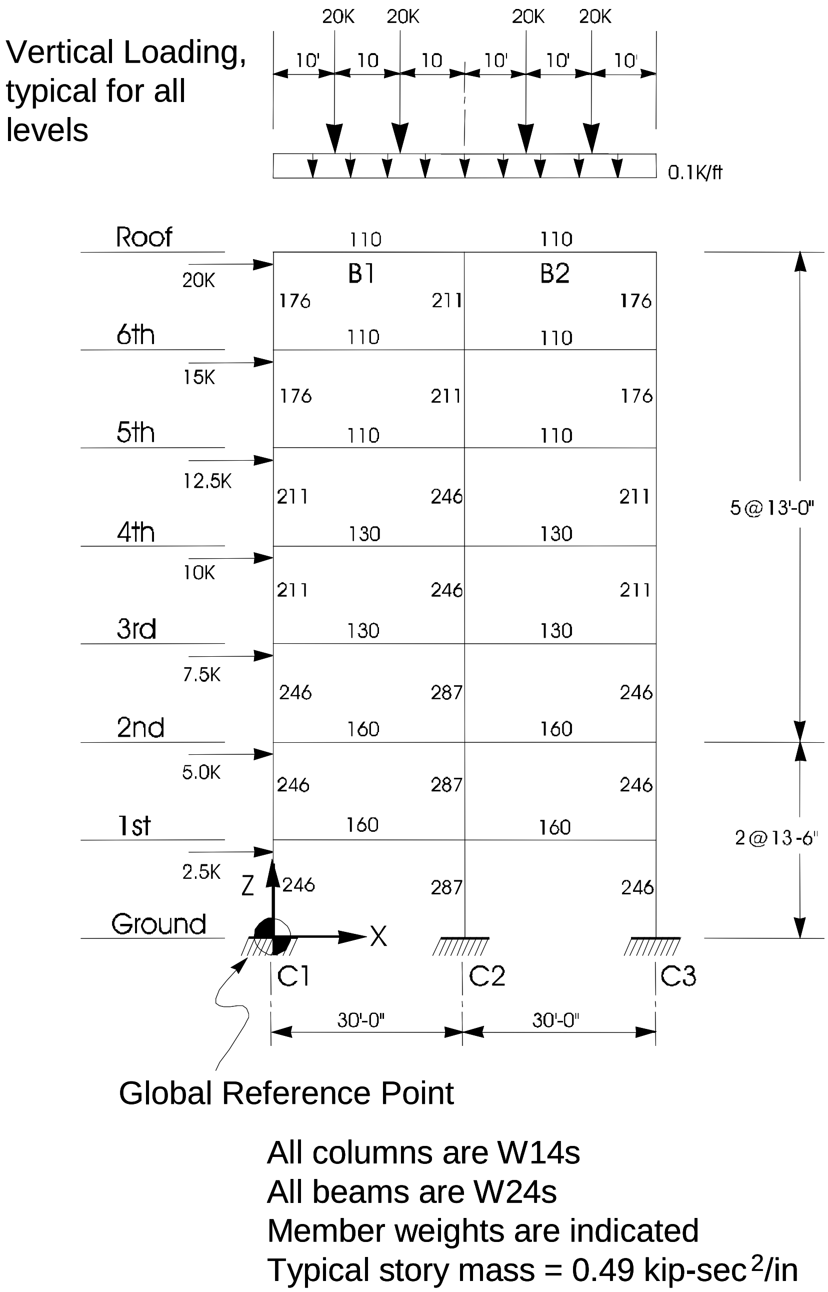


Figure 7-1 Seven-Story Plane Frame

Technical Features of ETABS Tested

- Two-dimensional frame analysis
- User-specified section properties
- User-specified lateral loads
- Dynamic response spectrum analysis
- Dynamic time history analysis

Results Comparison

The comparison of the results for all three analyses is excellent.

Table 7-1 Comparison of Results for Static Lateral Loads

Quantity	ETABS	Wilson and Habibullah	DYNAMIC/EASE2
Lateral Displacement at Roof	1.4508	1.4508	1.4508
Axial Force Column C1, at ground	69.99	69.99	69.99
Moment Column C1, at ground	2324.68	2324.68	2324.68

Table 7-2 Comparison of Results for Periods of Vibration

Mode	ETABS	Wilson and Habibullah	DYNAMIC/EASE2
1	1.27321	1.27321	1.27321
2	0.43128	0.43128	0.43128
3	0.24205	0.24204	0.24204
4	0.16018	0.16018	0.16018
5	0.11899	0.11899	0.11899
6	0.09506	0.09506	0.09506
7	0.07952	0.07951	0.07951

Table 7-3 Comparison of Results for Response Spectrum Analysis

Quantity	ETABS CQC Combination	Wilson and Habibullah CQC Combination	DYNAMIC/EASE2 SRSS Combination
Lateral Displacement at Roof	5.4314	5.4314	5.4378
Axial Force Column C1 at ground	261.52	261.50	261.76
Moment Column C1 at ground	9916.12	9916.11	9868.25

Table 7-4 Comparison of Results for Time History Analysis

Quantity	ETABS	Wilson and Habibullah
Maximum Roof Displacement	5.49	5.48
Maximum Base Shear	285	284
Maximum Axial Force, Column C1 at ground	263	258
Maximum Moment, Column C1 at ground	9104	8740

Computer Files

The input data file is Example 07.EDB. The input history is ELCN-THU. Time history results are obtained for the first eight seconds of the excitation. This is consistent with DYNAMIC/EASE2, with which the results are compared. These computer files are provided as part of the ETABS installation.

Conclusions

Noting the difference in modal combination techniques between ETABS and Wilson and Habibullah, which uses complete quadratic combination (CQC), and DYNAMIC/EASE2, which uses square root of the sum of the squares combination (SRSS), the results of the testing are acceptable.

EXAMPLE 8

Two-Story, Three-Dimensional Frame - Dynamic Response Spectrum Analysis

Problem Description

This is a two-story, three-dimensional building frame subjected to a response spectrum of constant amplitude. The three-dimensional structure is modeled as a single frame with nine column lines and twelve bays. Kip-foot-second units are used.

For consistency with the models documented in other computer programs with which the ETABS results are compared (see Table 8-1), no story mass moments of inertia are assigned in the ETABS model.

Geometry, Properties and Loads

The geometry of the structure is shown in Figure 8-1.

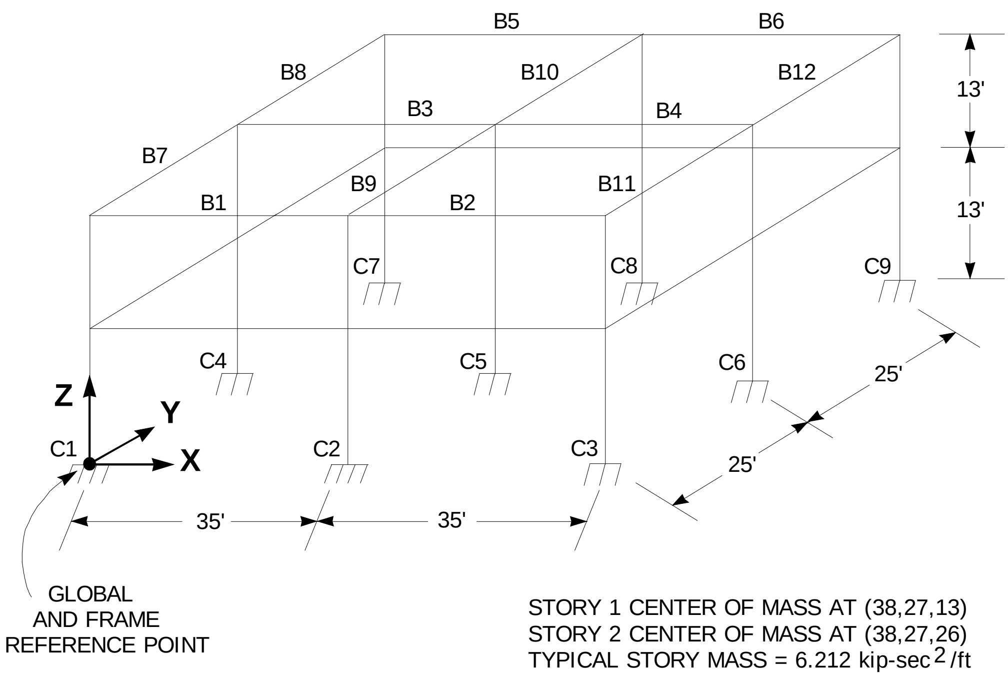


Figure 8-1 Two-Story Three-Dimensional Frame

A response spectrum with a constant value of 0.4g is used. Other parameters associated with the structure are as follows:

	Columns	Beams
Axial area	4 ft^2	5 ft^2
Minor moment of inertia	1.25 ft^4	1.67 ft^4
Major moment of inertia	1.25 ft^4	2.61 ft^4
Modulus of elasticity	350000 ksf	500000 ksf

Technical Features of ETABS Tested

- Three-dimensional frame analysis
- User-specified section properties
- Dynamic response spectrum analysis

Comparison of Results

This example is also analyzed in Wilson and Habibullah (1992) and Peterson (1981). A comparison of the key ETABS results with Wilson and Habibullah (Reference 1) and Peterson (Reference 2) is shown in Table 8-1.

Table 8-1 Comparison of Results

Quantity	ETABS	Reference 1	Reference 2
Period, Mode 1	0.22708	0.22706	0.22706
Period, Mode 2	0.21565	0.21563	0.21563
Period, Mode 3	0.07335	0.07335	0.07335
Period, Mode 4	0.07201	0.07201	0.07201
X-Displacement Center of mass, 2^{nd} Story	0.0201	0.0201	0.0201

Computer File

The input data file is Example 08.EDB. This file is provided as part of the ETABS installation.

Conclusion

The results comparison shows acceptable agreement between ETABS and the references.

EXAMPLE 9

Two-Story, 3D Unsymmetrical Building Frame - Dynamic Response Spectrum Analysis

Problem Description

This is a two-story three-dimensional unsymmetrical building frame. The structure is subjected to a seismic response spectrum along two horizontal axes that are at a 30-degree angle to the building axes. The seismic excitation is identical to the one used in Wilson and Habibullah (1992).

Geometry, Properties and Loads

The geometry of the structure is shown in Figure 9-1. The three-dimensional structure is modeled as a single frame with six column lines and five bays. Kip-foot-second units are used. Typical columns are 18"x18" and beams are 12"x24". The modulus of elasticity is taken as 432,000 ksf.

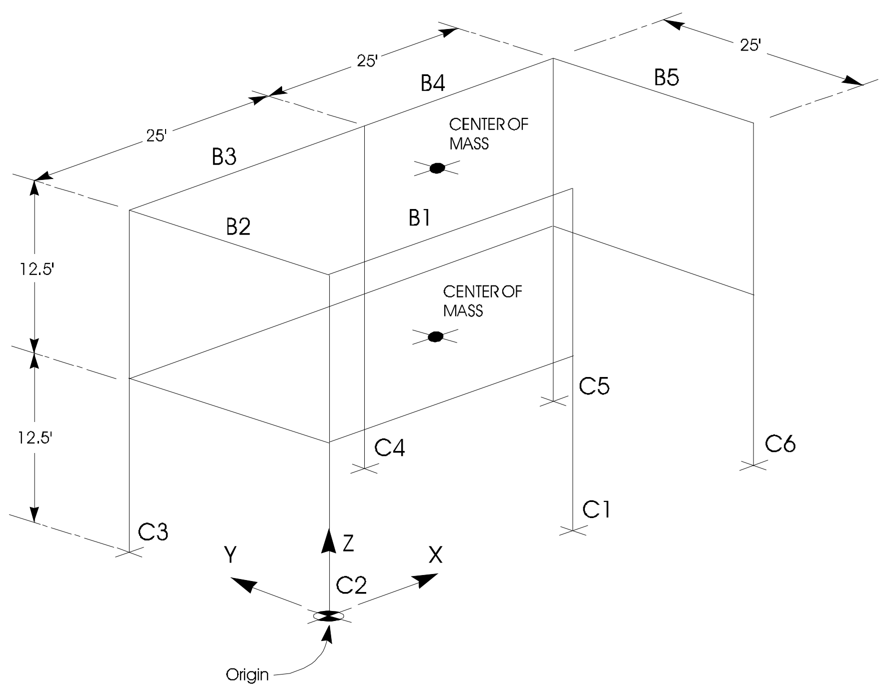


Figure 9-1 Two-Story Three-Dimensional Unsymmetrical Building Frame

Technical Features of ETABS Tested

- Three-dimensional frame analysis
- Dynamic response spectrum analysis

Results Comparison

The structure is also analyzed in Wilson and Habibullah (1992). Key ETABS results are compared in Table 9-1.

Table 9-1 Comparison of Results

Quantity	ETABS	Wilson and Habibullah
Period, Mode 1	0.4146	0.4146
Period, Mode 2	0.3753	0.3753
Period, Mode 3	0.2436	0.2436
Period, Mode 4	0.1148	0.1148
Period, Mode 5	0.1103	0.1103
Period, Mode 6	0.0729	0.0729
X- Displacement Center of Mass at 2nd Story for:		
Seismic at 30° to X	0.1062	0.1062
Seismic at 120° to X	0.0617	0.0617

Computer File

The input data file is Example 09.EDB. This file is provided as part of the ETABS installation.

Conclusions

The results comparison shows exact agreement between ETABS and the reference material.

EXAMPLE 10

Three-Story Plane Frame with ADAS Elements - Nonlinear Time History Analysis

Problem Description

This is a single bay three-story plane frame subjected to ground motion, as shown in Figure 10-1. The El Centro 1940 (N-S) record is used in the nonlinear time history analysis. Three elements that absorb energy through hysteresis (ADAS elements as described in Scholl 1993 and Tsai, et al. 1993) are used to connect the chevron braces to the frame. Two models are investigated. In the first model, the ADAS elements are intended to produce about 5% damping in the fundamental mode. In the second model, damping is increased to 25%. The manufacturer supplied the properties of the ADAS elements.

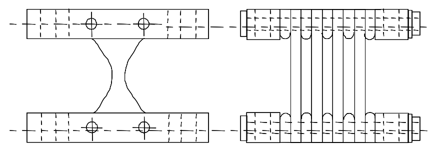
The ADAS elements are modeled in ETABS by assigning a panel zone with a nonlinear link property to the mid-span point object where the chevrons intersect the beams at each story. The link properties use the uniaxial hysteretic spring property (PLASTIC1) and provide beam-brace connectivity with nonlinear behavior in the U2 (shear in the 1-2 plane) direction. Under this arrangement, displacements are transferred between the chevrons and the frame via the link elements undergoing shear deformation.

Geometry, Properties and Loads

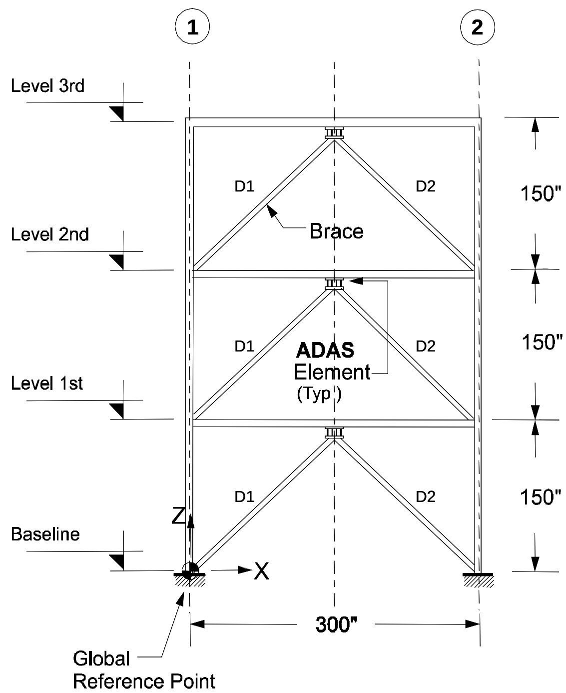
The frame is modeled as a two-column line, one-bay system. Kip-inch-second units are used. The modulus of elasticity is taken as 29000 ksi. Column, beam and brace section properties are user-defined.

A single rigid diaphragm is allocated to each story level and connects all three point objects (two column points and one mid-span point) at each story. Because of the rigid diaphragms, no axial force will occur in the beam members. All members are assigned a rigid zone factor of 1.

In both models the value of post yield stiffness ratio is taken as 5% and the time increment for output sampling is specified as 0.02 second.



Typical ADAS Element



Frame Elevation

Figure 10-1 Planar Frame with ADAS Elements

Technical Features of ETABS Tested

- Two-dimensional frame analysis
- Panel zones
- Point assignments
- Nonlinear time history analysis
- Ritz vectors

Results Comparison

Sample results are compared in Table 10-1 with results from the nonlinear analysis program DRAIN-2DX (Prakash, et al. 1993) for both 5% and 25% damping cases.

Table 10-1 Results Comparison

Level	5% Damping		25% Damping	
	ETABS	**DRAIN-2DX**	**ETABS**	**DRAIN-2DX**
Comparison of Maximum Story Deflections				
3rd	4.57	4.57	2.10	1.92
2nd	3.48	3.51	1.68	1.55
1st	1.82	1.82	0.92	0.86
Comparison of Maximum Link Shear Force				
3rd	7.29	7.31	17.75	17.40
2nd	13.97	13.92	36.70	36.20
1st	17.98	18.00	47.79	47.10
Comparison of Maximum Brace Axial Force				
3rd	5.16	5.17	12.55	12.30
2nd	9.88	9.84	25.95	25.60
1st	12.71	12.70	33.79	33.28

Computer Files

The input data files for this example are Example 10A.EDB (5% damping) and Example 10B.EDB (25% damping). The time history file is ELCN-THE. These files are provided as part of the ETABS installation.

Conclusions

The results comparison show acceptable to exact agreement between ETABS and DRAIN-2DX.

Example 11

Three-Story Plane Frame with Viscous Damper Elements - Nonlinear Time History Analysis

Problem Description

The El Centro 1940 (N-S) record is used in the nonlinear time history analysis. Three viscous damper elements of the type described in Hanson (1993) are used to connect the chevron braces to the frame. Two models are investigated. In the first model, the damper elements are intended to produce about 5% damping in the fundamental mode. In the second model, damping is increased to 25%.

The ETABS viscous damper element (DAMPER) is a uniaxial damping device with a linear or nonlinear force-velocity relationship given by $F = CV^{\alpha}$.

The damper elements are modeled in ETABS by assigning a panel zone with a nonlinear link property to the mid-span point object where the chevrons intersect the beams at each story. The link properties use the uniaxial damper property (DAMPER) and provide beam-brace connectivity with nonlinear behavior in the U2 (shear in the 1-2 plane) direction. Under this arrangement, displacements are transferred between the chevrons and the frame via the link elements (dampers) undergoing shear deformation.

The time increment for output sampling is specified as 0.02 second.

Geometry, Properties and Loads

This is a single-bay, three-story plane frame subjected to ground motion, as shown in Figure 11-1. The frame is modeled as a two-column line, one-bay system. Kip-inch-second units are used. The modulus of elasticity is taken as 29000 ksi. Column, beam and brace section properties are user defined.

A single rigid diaphragm is allocated to each story level and connects all three point objects (two column points and one mid-span point) at each story. Because of the rigid diaphragms, no axial force will occur in the beam members. All members are assigned a rigid zone factor of 1.

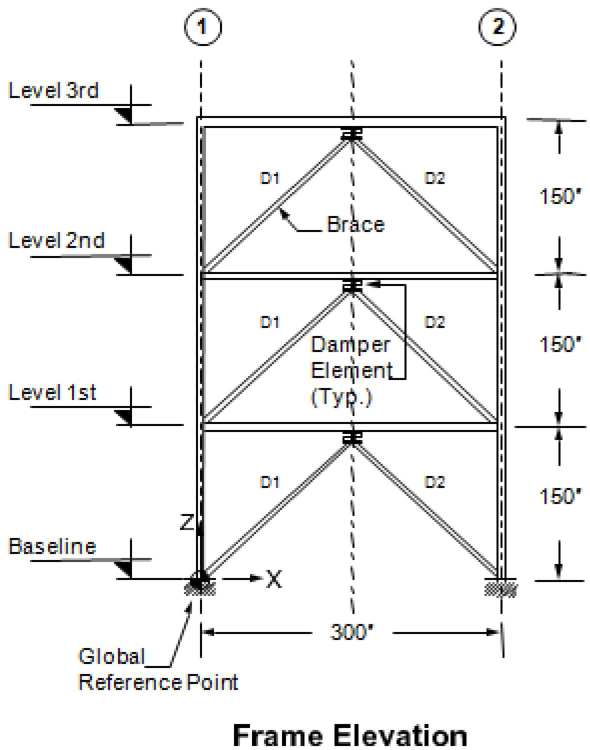


Figure 11-1 Planar Frame with Damper Elements

Technical Features of ETABS Tested

- Two-dimensional frame analysis
- Use of panel zones
- Use of uniaxial damper elements
- Point assignments
- Nonlinear time history analysis
- Ritz vectors

Results Comparison

Sample results for $\alpha = 1$ are compared in Table 11-1 with results from the nonlinear analysis program DRAIN-2DX (Prakash, et al. 1993) for both 5% and 25% damping cases.

Table 11-1 Results Comparison

Level	5% Damping		25% Damping	
	ETABS	DRAIN-2DX	ETABS	DRAIN-2DX
Comparison of Maximum Story Deflections				
3rd	4.09	4.11	2.26	2.24
2nd	3.13	3.14	1.75	1.71
1st	1.63	1.63	0.89	0.87
Comparison of Maximum Link Shear Force				
3rd	6.16	5.98	14.75	14.75
2nd	10.79	10.80	32.82	32.84
1st	15.15	15.02	44.90	44.97
Comparison of Maximum Brace Axial Force				
3rd	4.36	4.23	10.43	10.43
2nd	7.63	7.63	23.21	23.22
1st	10.71	10.62	31.75	31.80

Computer File

The input data files for this example are Example 11A.EDB (5% damping) and Example 11B.EDB (25% damping). The time history file is ELCN-THE. These files are provided as part of the ETABS installation.

Conclusions

The comparison of results shows acceptable agreement between ETABS and DRAIN-2DX.

EXAMPLE 12

Pounding of Two Planar Frames, Nonlinear Time History Analysis

Problem Description

A two-bay, seven-story plane frame is linked to a one-bay four-story plane frame using ETABS GAP elements. The structure experiences pounding because of ground motion. The El Centro 1940 (N-S) record is used in the nonlinear time history analysis.

This example illustrates the use of gap elements to model pounding between buildings.

Geometry, Properties and Loads

The geometry of the structure is shown in Figure 12-1.

The combined structure is modeled as a single frame with five column lines and three beam bays. Kip-inch-second units are used. The modulus of elasticity is taken as 29500 ksi. Column and beam section properties are user defined.

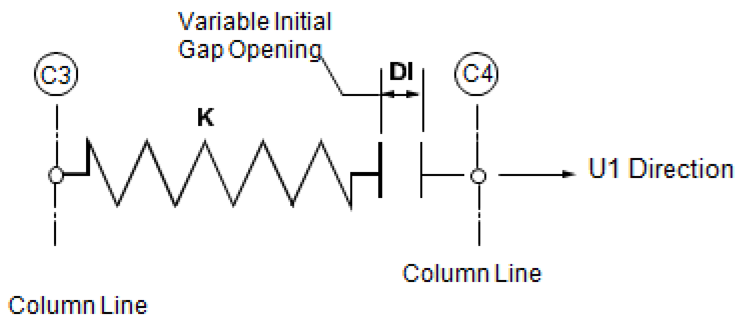
Through the joint assignment option, Column lines 4 and 5 are connected to Diaphragm 2. Column lines 1 to 3 remain connected to Diaphragm 1 by default. This arrangement physically divides the structure into two parts. The interaction is provided via the gap elements, which are used as links spanning Column lines 3 and 4. The local axis 1 of the links is in the global X-direction.

Technical Features of ETABS Tested

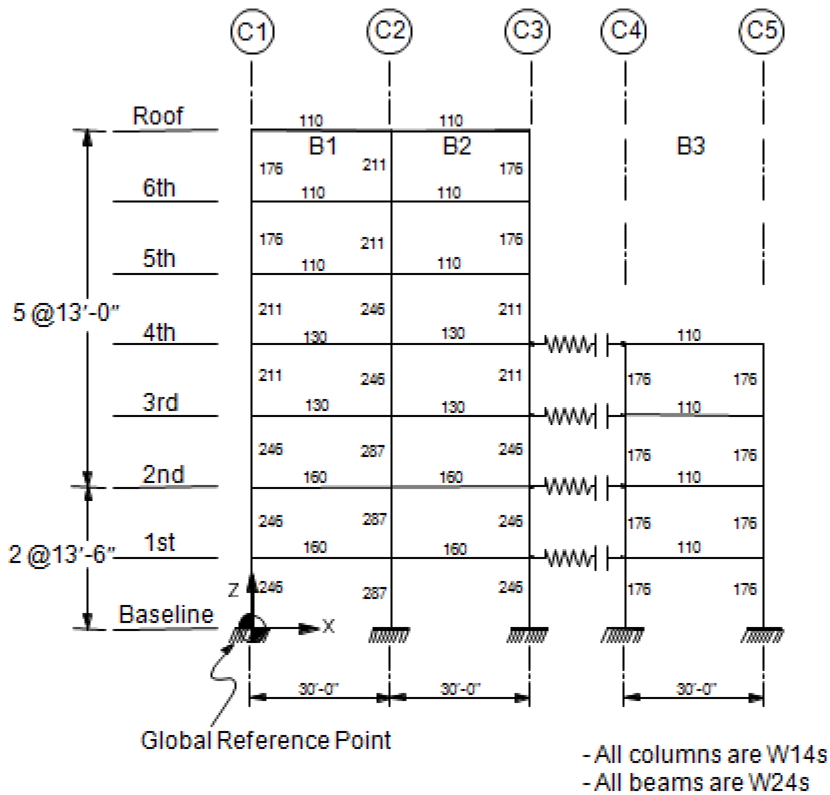
- Two-dimensional frame analysis
- Use of uniaxial gap elements
- Point assignments
- Nonlinear time history analysis
- Use of multiple diaphragms

Results Comparison

The example frame analyzed using ETABS is also analyzed using SAP2000 (Computers and Structures 2002) for time history loads (SAP2000 has been verified independently). A comparison of key ETABS results with SAP2000 is presented in Table 12-1.



Gap Element Used as a Link



Frame Elevation

Figure 12-1 Planar Frame with Gap Elements

Table 12-1 Comparison of Results for Time History Analysis

Quantity	ETABS	SAP2000
Maximum Lateral Displacement at Roof	5.5521	5.5521
Maximum Axial Force, Column C1 at ground	266.89	266.88

A typical output produced by the program is shown in Figure 12-2. It shows the variations of the displacement of Column lines 3 and 4 and the link force at Story 4. It is clearly evident that the link force is generated whenever the two column lines move in phase and their separation is less than the specified initial opening or if they move towards each other out of phase. For display purposes, the link forces are scaled down by a factor of 0.01.

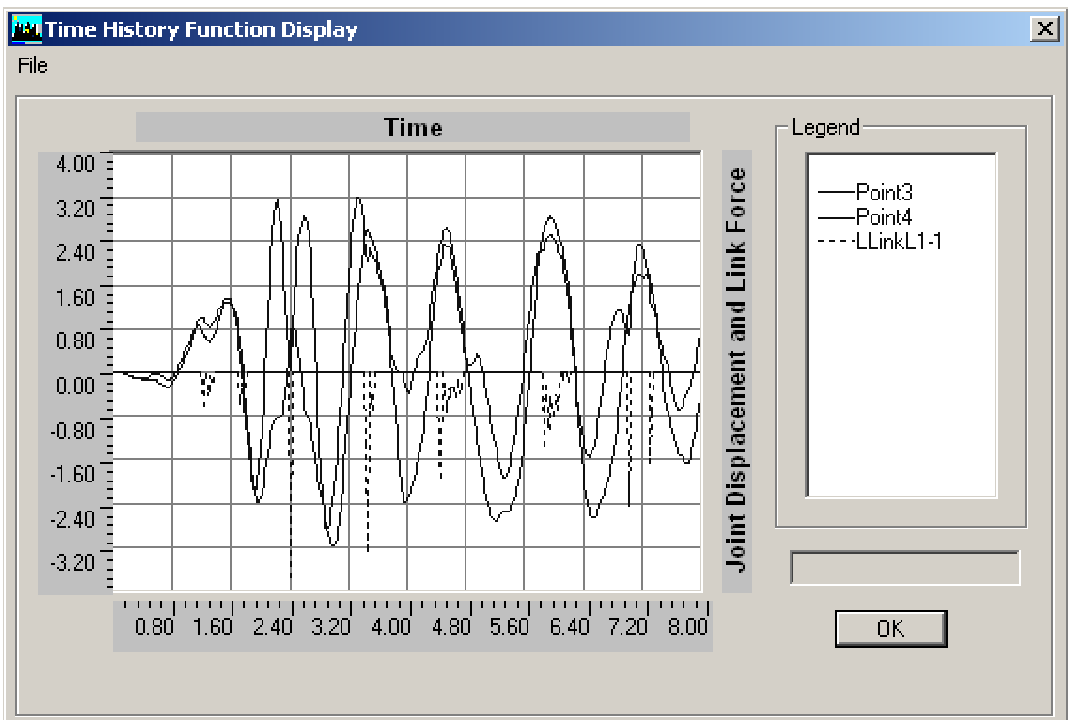


Figure 12-2 Variations of Displacement of Column Lines 3 and 4 and Link Force at Story 4

Computer Files

The input data for this example is Example 12.EDB. The time history file is ELCN-THU. Both of the files are provided as part of the ETABS installation.

Conclusions

The results comparison shows essentially exact agreement between ETABS and SAP2000.

EXAMPLE 13

Base-Isolated, Two-Story, 3D Frame - Nonlinear Time History Analysis

Problem Description

This is a two-story, three-dimensional frame with base isolation. The structure is subjected to earthquake motion in two perpendicular directions using the Loma Prieta acceleration records.

Hysteretic base isolators of the type described in Nagarajaiah et al. (1991) are modeled using the ETABS ISOLATOR1 elements, which show biaxial hysteretic characteristics.

Geometry, Properties and Loads

The structure is modeled as a single reinforced concrete frame with nine column lines and twelve bays. The floor slab is taken to be 8 inches thick, covering all of the specified floor bays at the base and the 1st story level. At the second story level the corner column as well as the two edge beams are eliminated, together with the floor slab, to render this particular level unsymmetric, as depicted in Figure 13-1.

A modulus of elasticity of 3000 ksi is used. The self-weight of concrete is taken as 150 pcf. Kip-inch-second units are used.

The geometry of the structure is shown in Figure 13-1.

Technical Features of ETABS Tested

- Three-dimensional frame analysis
- Use of area (floor) objects
- Use of biaxial hysteretic elements
- Point assignments
- Nonlinear time history analysis using ritz vectors

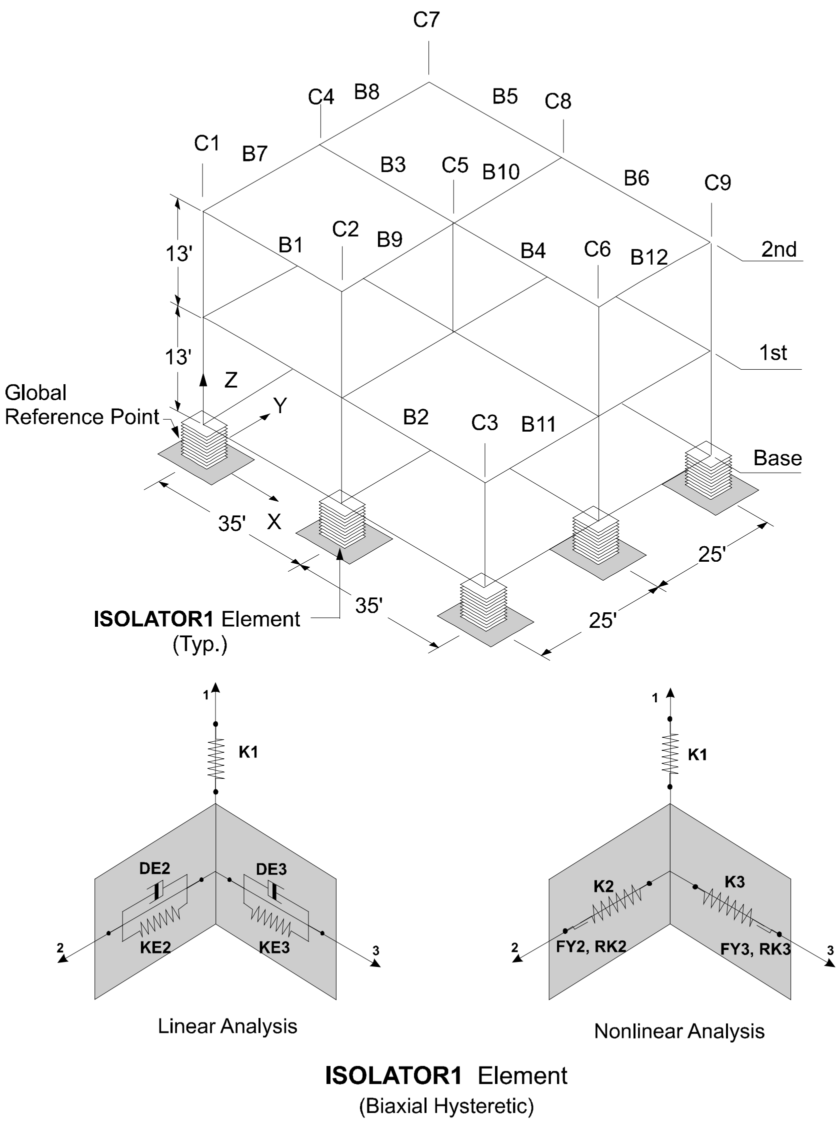


Figure 13-1 Base-Isolated Three-Dimensional Frame

Results Comparison

The example frame analyzed using ETABS is also analyzed using SAP2000 (Computers and Structures 2002) for time history loads (SAP2000 has been verified independently). A comparison of key ETABS results with SAP200 is presented in Table 13-1.

Table 13-1 Comparison of Results for Time History Analysis

Quantity	ETABS	SAP2000
Maximum Uy Displacement, Column C9 at 2nd Floor	3.4735	3.4736
Maximum Axial Force, Column C1 at base	13.56	13.55

A typical output produced by the program is shown in Figure 13-2. It shows the load-deformation relationship in the major direction for a typical isolator member.

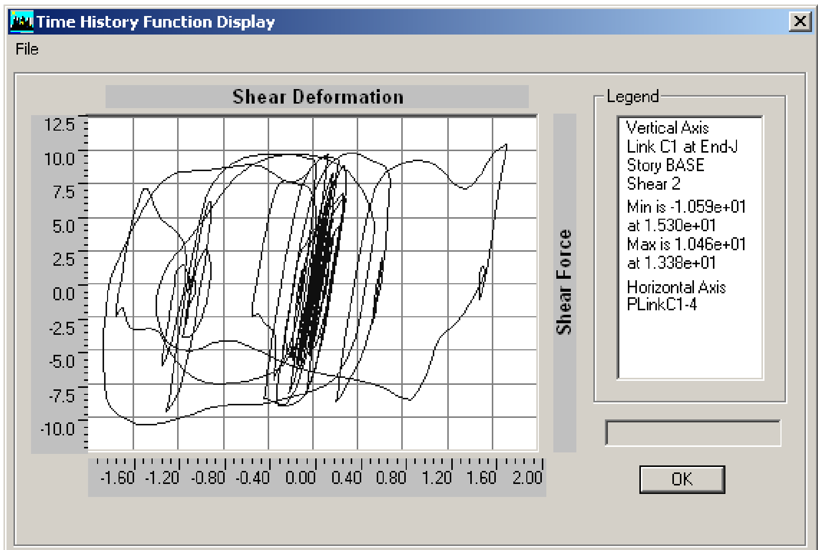


Figure 13-2 Load Deformation Diagram

Computer Files

The input data file for this example is Example 13.EDB. The time history files are LP-TH0 and LP-TH90. All of these files are provided as part of the ETABS installation.

Conclusion

The results comparison shows essentially exact agreement between ETABS and SAP2000.

EXAMPLE 14

Friction Pendulum Base-Isolated 3D Frame - Nonlinear Time History Analysis

Problem Description

This is a two-story, three-dimensional frame with base isolation using friction pendulum base isolators. The structure is subjected to earthquake motion in two perpendicular directions using the Loma Prieta acceleration records.

Friction pendulum type base isolators of the type described in Zayas and Low (1990) are modeled using the ETABS ISOLATOR2 elements.

It is important for these isolator elements that the axial load from other loads be modeled before starting the nonlinear analysis. This is achieved by using a factor of unity on the dead load (self weight) on the structure in the nonlinear analysis initial conditions data.

Geometry, Properties and Loads

The structure is modeled as a single reinforced concrete frame with nine column lines and twelve bays. The floor slab is taken to be 8 inches thick, covering all of the specified floor bays at the base and the 1st story level. At the second story level, the corner column and the two edge beams are eliminated, together with the floor slab, to render this particular level anti-symmetric, as depicted in Figure 14-1.

The isolator properties are defined as follows:

Stiffness in direction 1	1E3
Stiffness in directions 2 and 3	1E2
Coefficient of friction at fast speed	.04
Coefficient of friction at slow speed	.03
Parameter determining the variation of the coefficient of friction with velocity	20
Radius of contact surface in directions 2 and 3	60

A modulus of elasticity of 3000 ksi is used. The self-weight of concrete is taken as 150 pcf. Kip-inch-second units are used.

The geometry of the structure is shown in Figure 14-1.

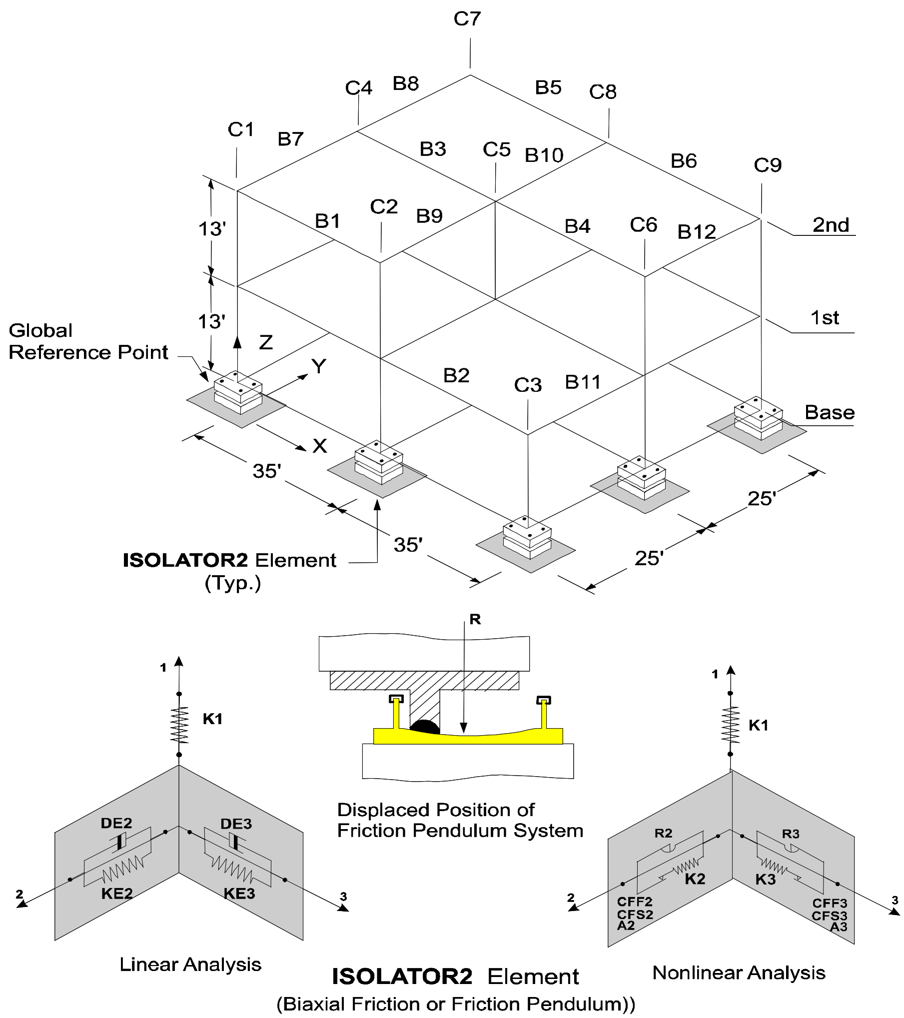


Figure 14-1 Base-Isolated Three-Dimensional Frame

Technical Features of ETABS Tested

- Three-dimensional frame analysis
- Use of area (floor) objects
- Use of biaxial friction pendulum elements
- Point assignments
- Nonlinear time history analysis using ritz vectors

Results Comparison

The example frame analyzed using ETABS is also analyzed using SAP2000 (Computers and Structures 2002) for time history loads (SAP2000 has been verified independently). A comparison of key ETABS results with SAP2000 is presented in Table 14-1.

Table 14-1 Comparison of Result for Time History Analysis

Quantity	ETABS	SAP2000
Maximum Uy Displacement, Column C9 at 2nd Floor	4.2039	4.2069
Maximum Axial Force, Column C1 at base	37.54	38.25

A typical output produced by the program is shown in Figure 14-2. It shows the variation of the displacement of the second story at column line 1.

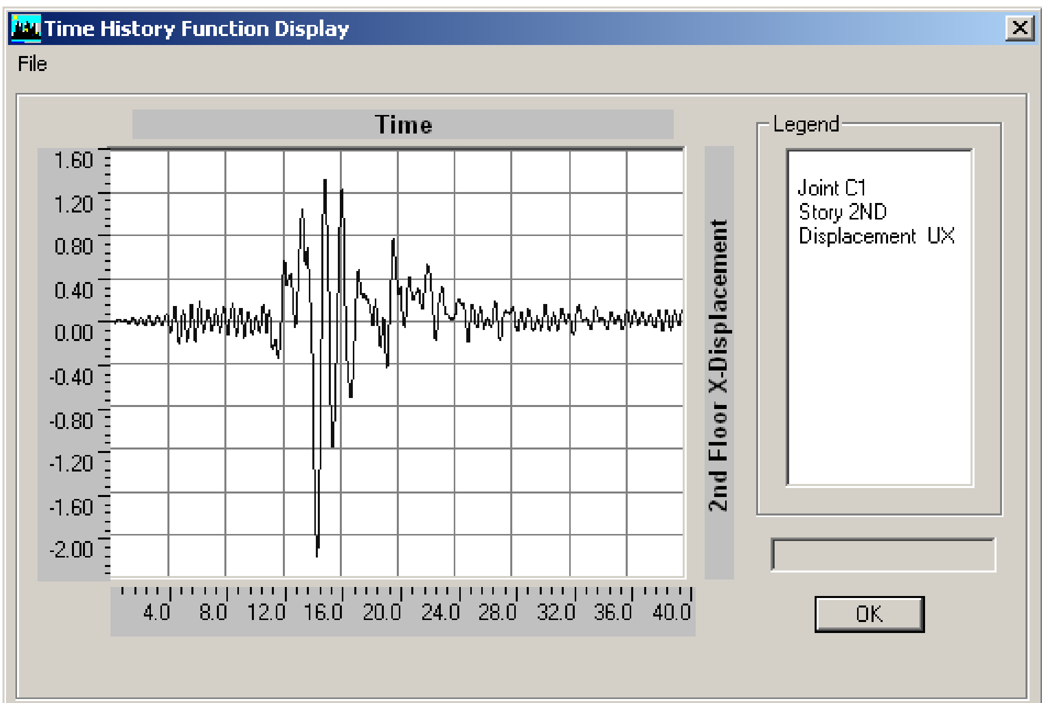


Figure 14-2 Variation of Displacement

Computer Files

The input data file for this example is Example 14.EDB. The time history files are LP-TH0 and LP-TH90. All of the files are provided as part of the ETABS installation.

Conclusion

The results comparison shows acceptable agreement between ETABS and SAP2000.

EXAMPLE 15

Wall Object Behavior - Static Lateral Loads Analysis

Problem Description

This example analyzes a series of wall configurations to evaluate the behavior of the ETABS shell object with wall section assignments. All walls are subjected to a static lateral load applied at the top of the wall.

The following walls are included:

- Planar shear wall, shown in Figure 15-1
- Wall supported on columns, shown in Figure 15-2
- Wall-spandrel system, shown in Figure 15-3
- C-shaped wall section, shown in Figure 15-4
- Wall with edges thickened, shown in Figure 15-5
- E-shaped wall section, shown in Figure 15-6

Geometry, Properties and Loads

A modulus of elasticity of 3000 ksi and a Poisson's ratio of 0.2 are used for all walls. Kip-inch-second units are used throughout. The following sections describe the models for the different walls.

Planar Shear Wall , Example 15a
This shear wall is modeled with one panel per story. Three different wall lengths of 120", 360" and 720" are analyzed. Also, one-story and three-story walls are analyzed, together with the six-story wall shown in Figure 15-1. A wall thickness of 12" is used.

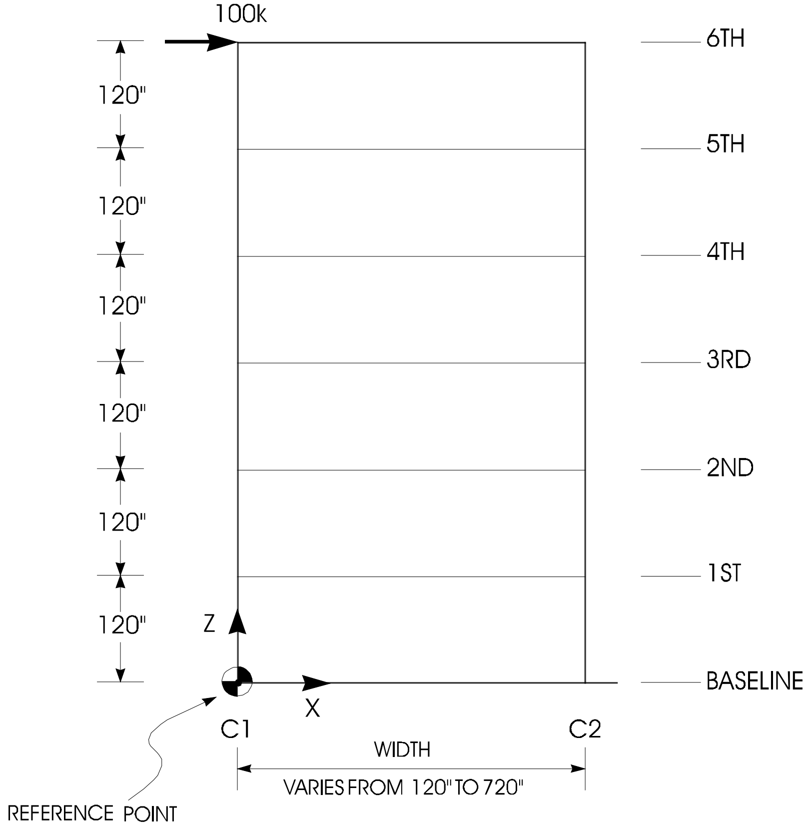


Figure 15-1 Planar Shear Wall, Example 15a

Wall Supported on Columns, Example 15b

This wall is modeled with two column lines. Columns are used for the first story, and the top two stories have a single shell object with end piers, as shown in Figure 15-2. End piers are 40" by 12" in cross section and panels are 12" thick. Columns are 40" by 20" in cross section.

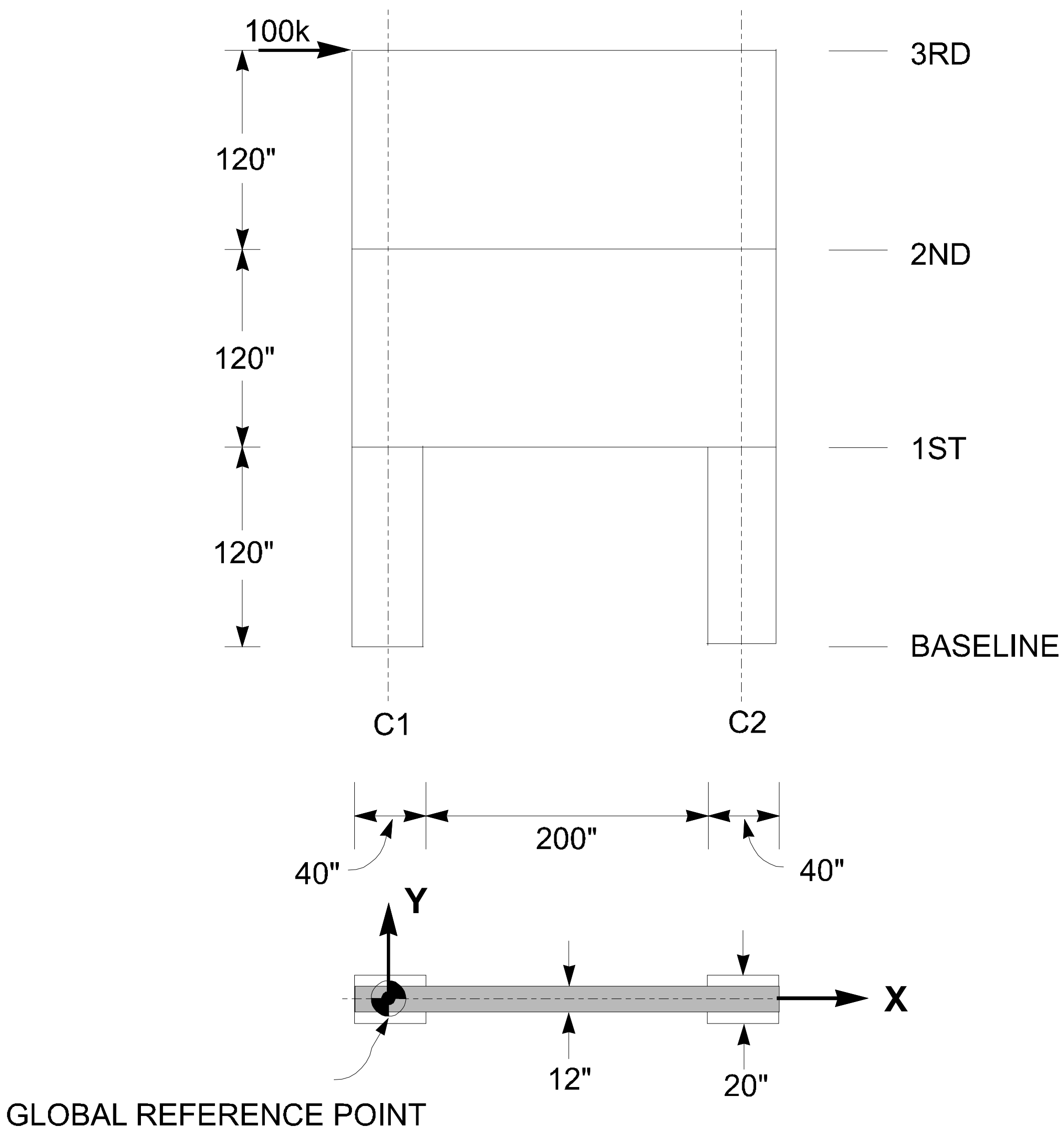


Figure 15-2 Wall Supported on Columns, Example 15b

Wall-Spandrel System, Example 15c

This wall is modeled with four column lines. The spandrels are modeled as beams. Two different spandrel lengths of 60" and 240" are analyzed. Each wall is modeled with two shell objects per story. Three-story walls are also analyzed together with the six-story wall shown in Figure 15-3. A wall and spandrel thickness of 12" is used.

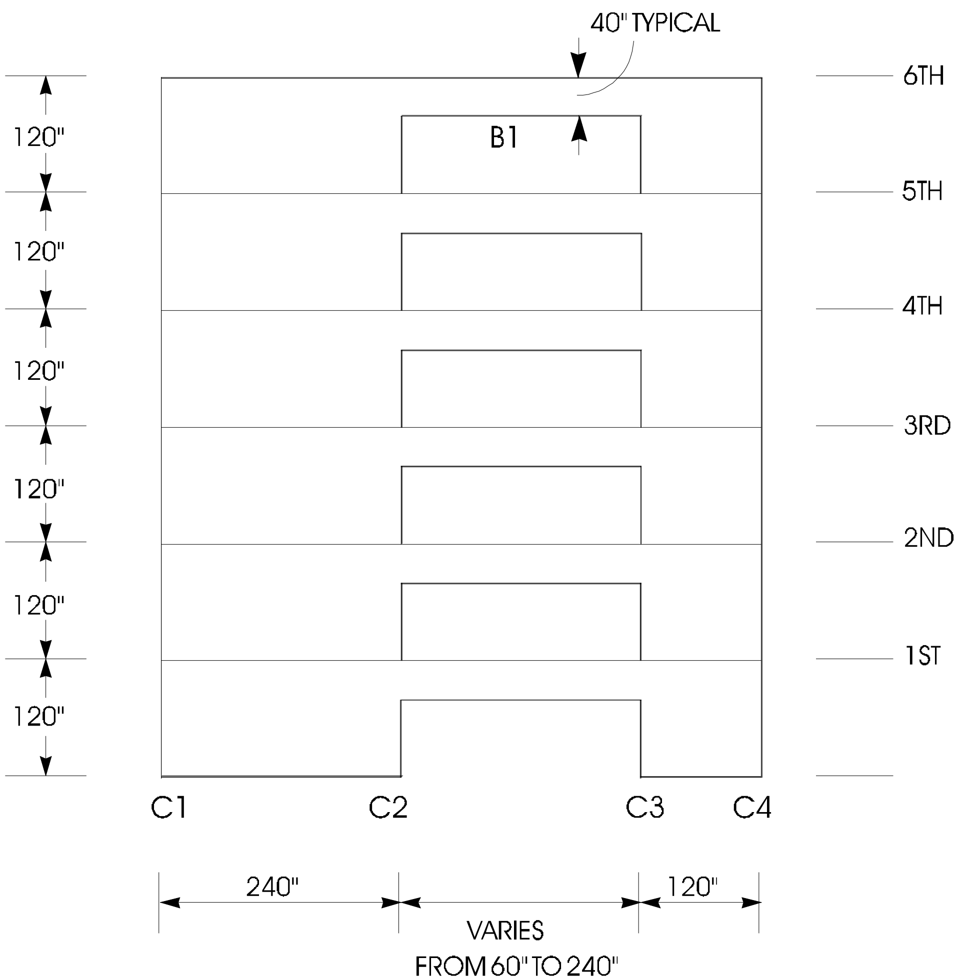


Figure 15-3 Wall-Spandrel System, Example 15c

Shaped Wall Section, Example 15d

This wall is modeled with six column lines and five shell objects per story, to model the shape of the wall. A three-story wall was also analyzed together with the six-story wall, as shown in Figure 15-4. A wall thickness of 6" is used.

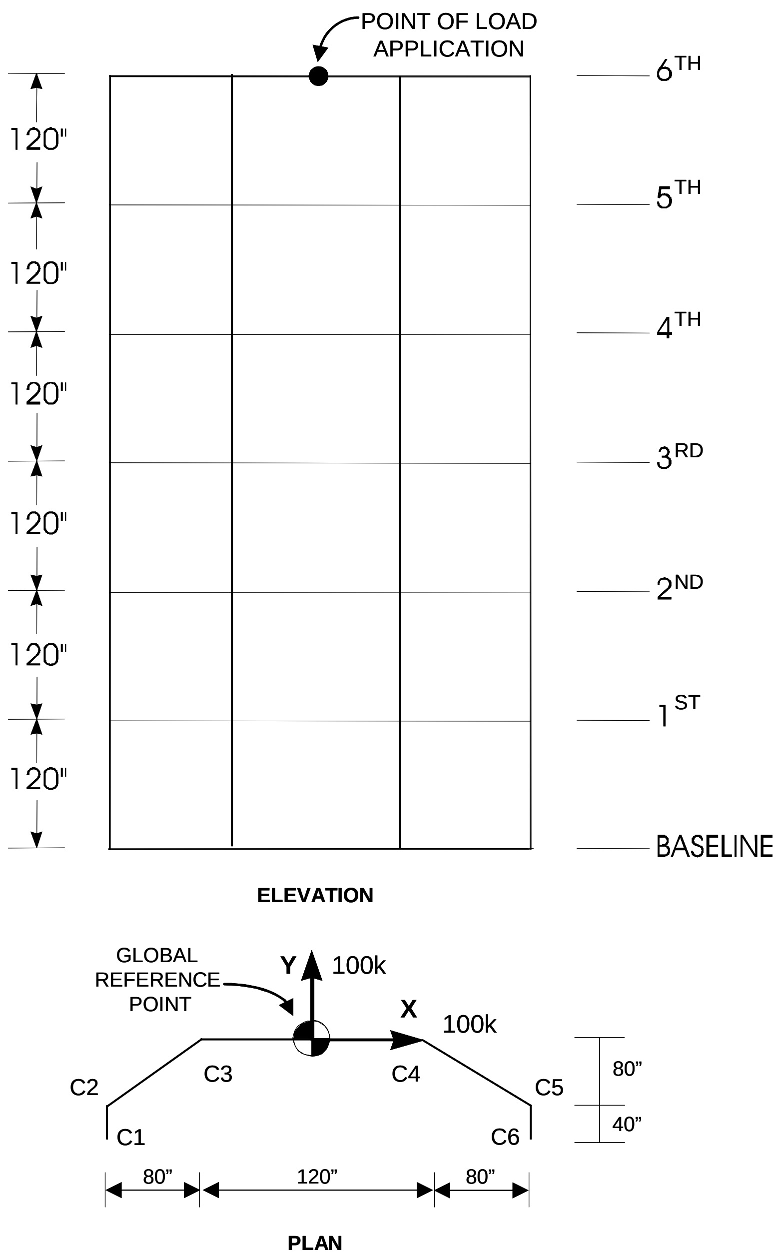


Figure 15-4 C-Shaped Wall Section, Example 15d

Wall with Edges Thickened, Example 15e

This wall is modeled with two column lines and one shell object, with end piers, per story as shown in Figure 15-5. A three-story wall was also analyzed together with the six-story wall shown in Figure 15-5.

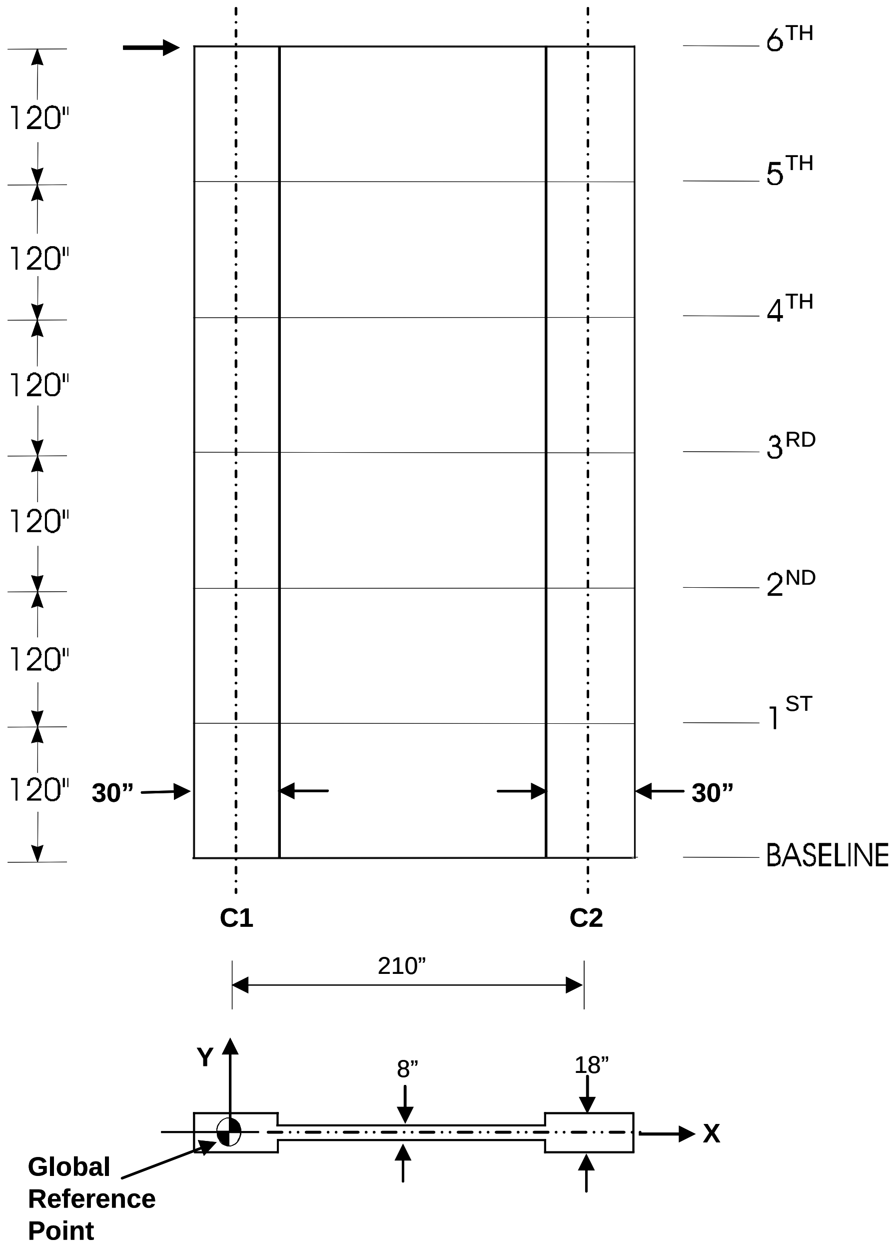


Figure 15-5 Wall with Thickened Edges, Example 15e

E-Shaped Wall Section, Example 15f

This wall is modeled with six column lines and five shell objects per story to model the shape of the wall. A three-story wall was also analyzed together with the six-story wall, as shown in Figure 15-6. A wall thickness of 6" is used.

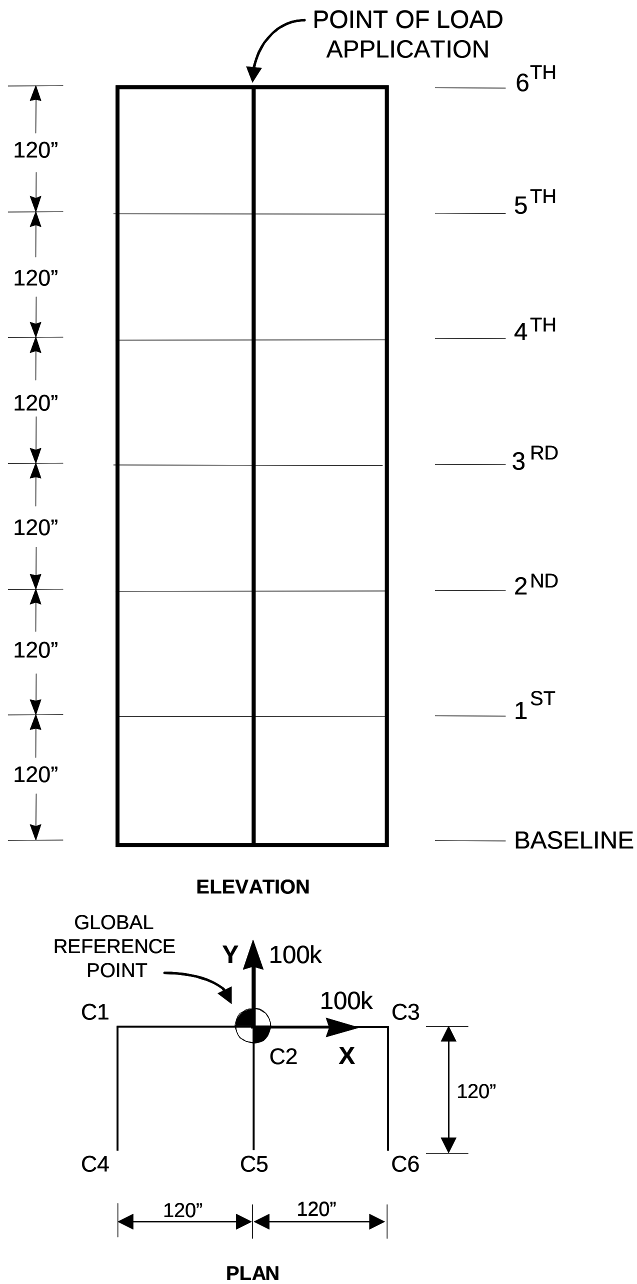


Figure 15-6 E-Shaped Wall Section, Example 15f

Technical Features of ETABS Tested

- ❑ Use of area objects
- ❑ Two-dimensional and three-dimensional shear wall systems
- ❑ Static lateral loads analysis

Results Comparison

All walls analyzed in this example using ETABS were also analyzed using the general structural analysis program SAP2000 (Computers and Structure 2002), using refined meshes of the membrane/shell element of that program. The SAP2000 meshes used are shown in Figures 15-7, 15-8, 15-9, 15-10, 15-11 and 15-12. For the SAP2000 analysis, the rigid diaphragms at the floor levels were modeled by constraining all wall nodes at the floor to have the same lateral displacement for planar walls, or by adding rigid members in the plane of the floor for three-dimensional walls.

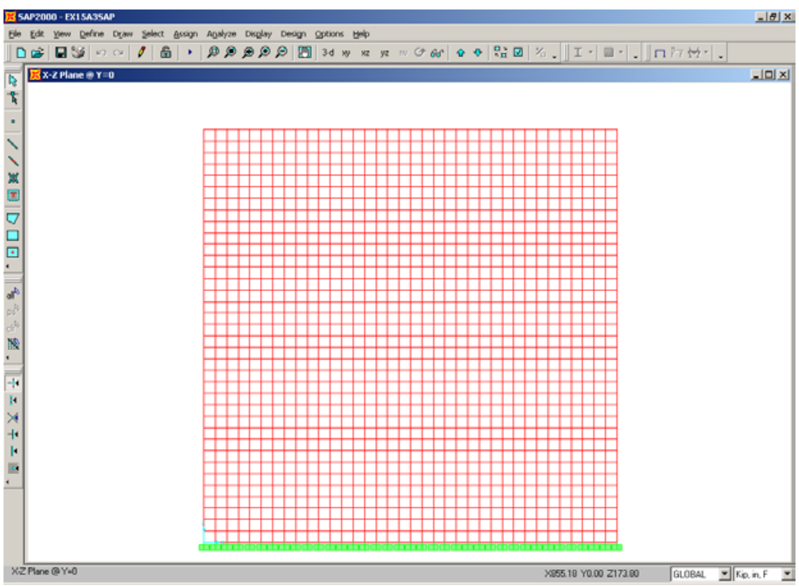


Figure 15-7 SAP2000 Mesh, Example 15a

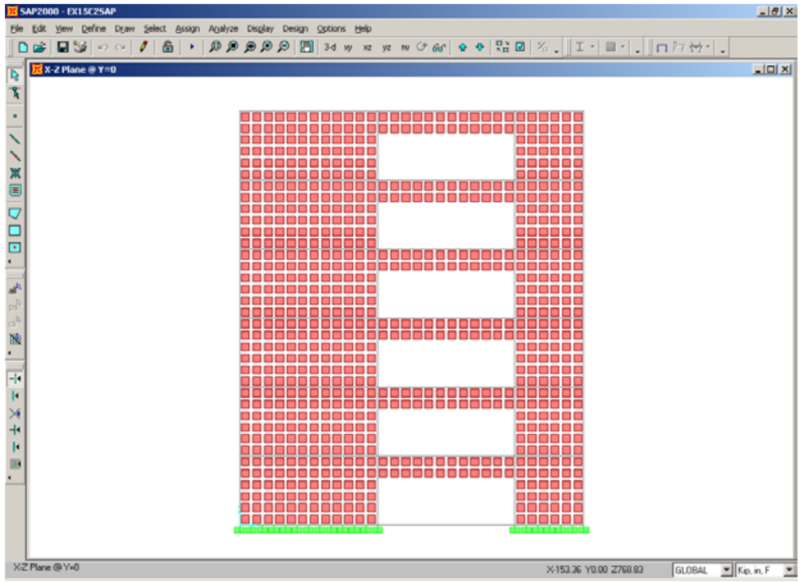


Figure 15-8 SAP2000 Mesh, Example 15b

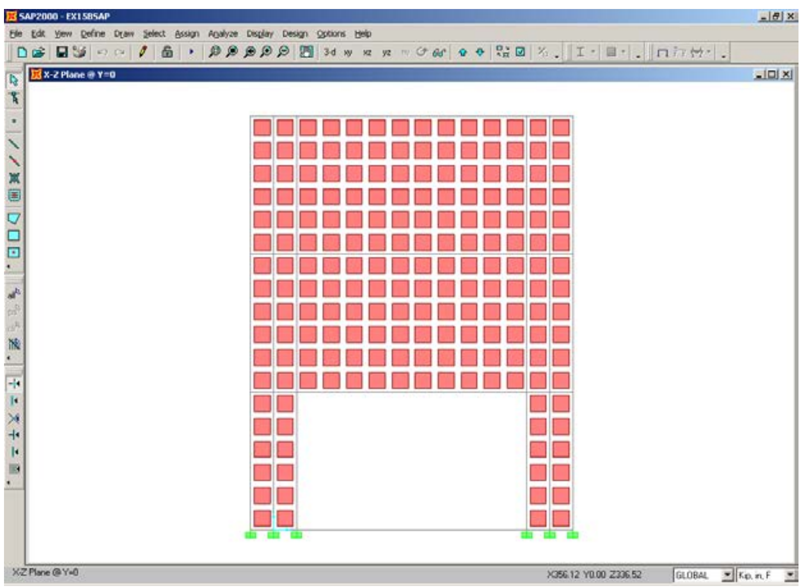


Figure 15-9 SAP2000 Mesh, Example 15c

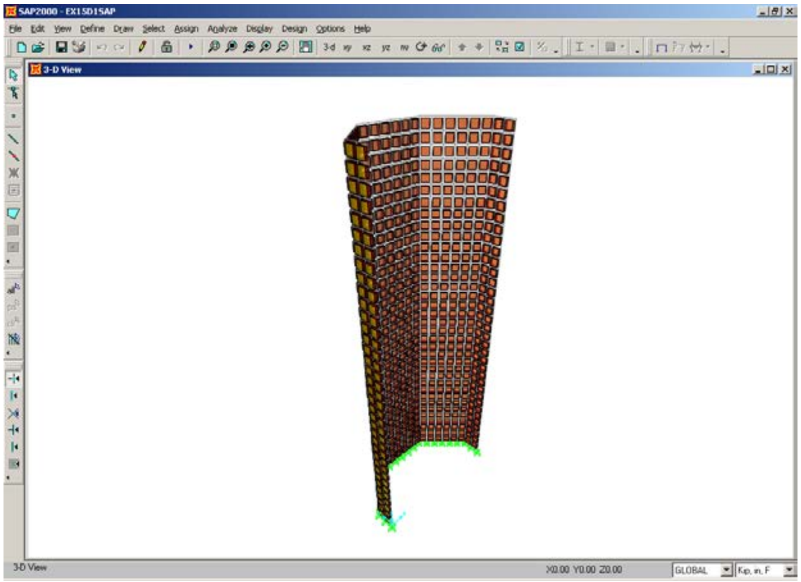


Figure 15-10 SAP2000 Mesh, Example 15d

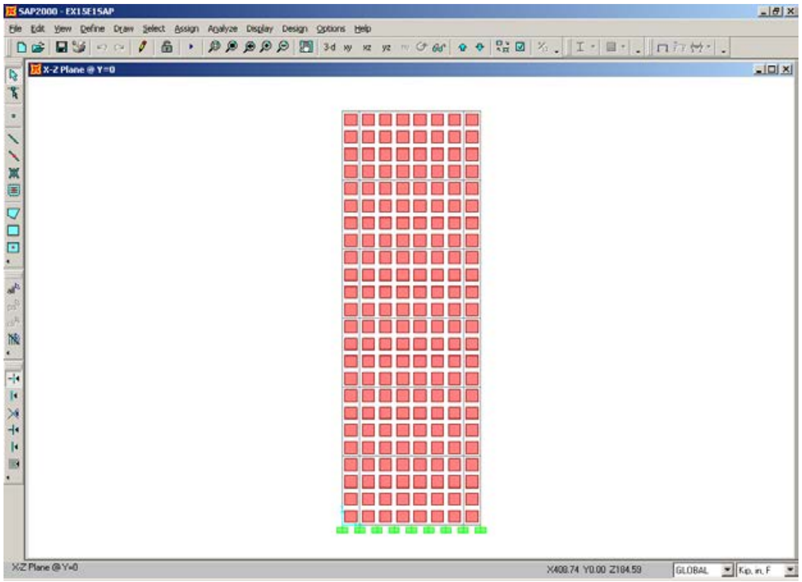


Figure 15-11 SAP2000 Mesh, Example 15e

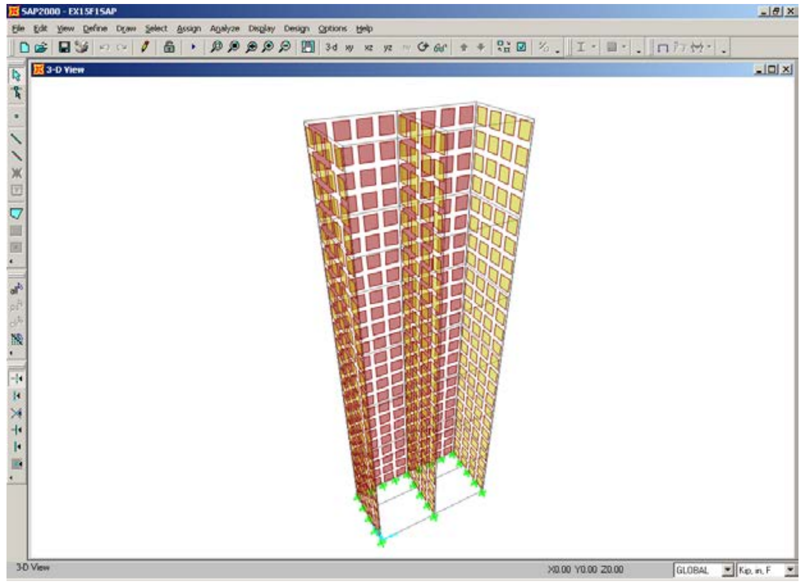


Figure 15-12 SAP2000 Mesh, Example 15f

The lateral displacements from the ETABS and SAP2000 analyses are compared in Tables 15-1, 15-2, 15-3, 15-4, 15-5 and 15-6 for the various walls.

Table 15-1 Results Comparison for Top Displacements (Inches), Example 15a

Number of Stories	Wall Height (inches)	Wall Length (inches)	ETABS	SAP2000
6	720	120	2.3921	2.4287
		360	0.0986	0.1031
		720	0.0172	0.0186
3	360	120	0.3071	0.3205
		360	0.0170	0.0187
		720	0.0046	0.0052
1	120	120	0.0145	0.0185
		360	0.0025	0.0029
		720	0.0011	0.0013

Table 15-2 Results Comparison for Displacements (Inches), Example 15b

Location	ETABS	SAP2000
Story 3	0.0691	0.0671
Story 2	0.0524	0.0530
Story 1	0.0390	0.0412

Table 15-3 Results Comparison for Top Displacements (inches) Example 15c (1-4)

Number of Stories	Beam Length (inches)	ETABS	SAP2000
6	60	0.0844	0.0869
	240	0.1456	0.1505
3	60	0.0188	0.0200
	240	0.0313	0.0332

Table 15-4 Results Comparison for Top Displacements (Inches) at Load Application Point, Example 15d (1-2)

Number of Stories	**Load Direction**	**Displacement Direction**	**ETABS**	**SAP2000**
6	X	X	0.8637	0.8936
	X	Z-Rotation	0.0185	0.0191
	Y	Y	1.1447	1.1882
3	X	X	0.1249	0.1337
	X	Z-Rotation	0.0024	0.0025
	Y	Y	0.1623	0.1733

Table 15-5 Results Comparison for Top Displacements (Inches), Example 15e(1-2)

Number of Stories	ETABS	SAP2000
6	0.2822	0.2899
3	0.0464	0.0480

Table 15-6 Results Comparison for Displacements at Load Application, Example 15f (1-2)

Number of Stories	**Load Direction**	**Displacement Direction**	**ETABS**	**SAP2000**
6	X	X	0.3707	0.3655
	X	Z-Rotation	0.0042	0.0039
	Y	Y	0.7295	0.7490
3	X	X	0.0602	0.0628
	X	Z-Rotation	0.0005	0.0005
	Y	Y	0.0993	0.1058

Computer Files

The input data files for the planar shear walls are included as files Example 15A1.EDB through Example 15A9.EDB. These and the following input data files are provided as part of the ETABS installation.

The input data for the wall supported on columns is Example 15B.EDB.

The input data files for the wall-spandrel system are Example C1.EDB through Example C4.EDB.

The input data files for the shaped wall section are included as files Example 15D1.EDB and Example 15D2.EDB.

The input data for the wall with thickened edges are included as files Example 15E1.EDB and Example 15E2.EDB.

The input data for the E-shaped wall section are included as files Example 15F1.EDB and Example 15F2.EDB.

Conclusion

The results comparison show acceptable agreement between ETABS and SAP2000. In general, the comparisons become better as the number of stories increases.

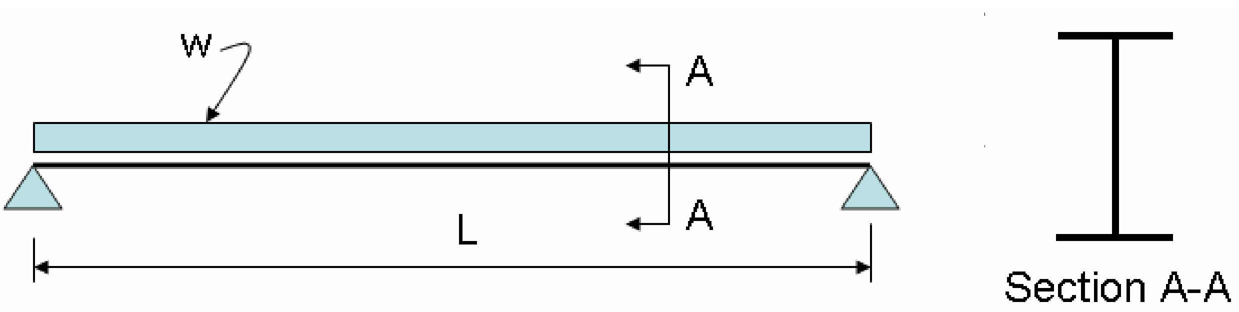
AISC 360-05 Example 001

WIDE FLANGE MEMBER UNDER BENDING

EXAMPLE DESCRIPTION

The design flexural strengths are checked for the beam shown below. The beam is loaded with a uniform load of 0.45 klf (D) and 0.75 klf (L). The flexural moment capacity is checked for three unsupported lengths in the weak direction, L_b = 5 ft, 11.667 ft and 35 ft.

GEOMETRY, PROPERTIES AND LOADING



Member Properties
W18X50
E = 29000 ksi
F_y = 50 ksi

Loading
w = 0.45 klf (D)
w = 0.75 klf (L)

Geometry
Span, L = 35 ft

TECHNICAL FEATURES TESTED

- Section Compactness Check (Bending)
- Member Bending Capacities
- Unsupported length factors

RESULTS COMPARISON

Independent results are comparing with the results of Example F.1-2a from the AISC Design Examples, Volume 13 on the application of the 2005 AISC Specification for Structural Steel Buildings (ANSI/AISC 360-05).

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Compact	Compact	0.00%
C_b (L_b =5ft)	1.004	1.002	0.20%
$\phi_b M_n$ (L_b =5ft) (k-ft)	378.750	378.750	0.00%
C_b (L_b =11.67ft)	1.015	1.014	0.10%
$\phi_b M_n$ (L_b =11.67ft) (k-ft)	307.124	306.657	0.15%
C_b (L_b =35ft)	1.138	1.136	0.18%
$\phi_b M_n$ (L_b =35ft) (k-ft)	94.377	94.218	0.17%

COMPUTER FILE: AISC 360-05 Ex001

CONCLUSION

The results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Material: ASTM A572 Grade 50 Steel

E = 29,000 ksi, F_y = 50 ksi

Section: W18x50

b_f = 7.5 in, t_f = 0.57 in, d = 18 in, t_w = 0.355 in

$$h = d - 2t_f = 18 - 2 \bullet 0.57 = 16.86 \ in$$

$$h_0 = d - t_f = 18 - 0.57 = 17.43 \ in$$

S_{33} = 88.9 in^3, Z_{33} = 101 in^3

I_y =40.1 in^4, r_y = 1.652 in, C_w = 3045.644 in^6, J = 1.240 in^4

$$r_{ts} = \sqrt{\frac{\sqrt{I_y C_w}}{S_{33}}} = \sqrt{\frac{\sqrt{40.1 \bullet 3045.644}}{88.889}} = 1.98 in$$

$R_m = 1.0$ *for doubly-symmetric sections*

Other:

c = 1.0

L = 35 ft

Loadings:

$w_u = (1.2w_d + 1.6w_l) = 1.2(0.45) + 1.6(0.75) = 1.74$ k/ft

$$M_u = \frac{w_u L^2}{8} = 1.74 \bullet 35^2/8 = 266.4375 \text{ k-ft}$$

Section Compactness:

Localized Buckling for Flange:

$$\lambda = \frac{b_f}{2t_f} = \frac{7.50}{2 \bullet 0.57} = 6.579$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29000}{50}} = 9.152$$

$\lambda < \lambda_p$, *No localized flange buckling*

Flange is Compact.

Localized Buckling for Web:

$$\lambda = \frac{h}{t_w} = \frac{16.86}{0.355} = 47.49$$

$$\lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29000}{50}} = 90.553$$

$\lambda < \lambda_p$, *No localized web buckling*

Web is Compact.

Section is Compact.

Section Bending Capacity:

$$M_p = F_y Z_{33} = 50 \bullet 101 = 5050\, k - in$$

Lateral-Torsional Buckling Parameters:

Critical Lengths:

$$L_p = 1.76\, r_y \sqrt{\frac{E}{F_y}} = 1.76 \bullet 1.652\sqrt{\frac{29000}{50}} = 70.022\ in = 5.835\ ft$$

$$L_r = 1.95 r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_{33}h_o}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7F_y}{E}\frac{S_{33}h_o}{Jc}\right)^2}}$$

$$L_r = 1.95 \bullet 1.98\frac{29000}{0.7 \bullet 50}\sqrt{\frac{1.240 \bullet 1.0}{88.9 \bullet 17.43}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7 \bullet 50}{29000}\frac{88.9 \bullet 17.43}{1.240 \bullet 1.0}\right)^2}}$$

$L_r = 16.966\ ft$

Non-Uniform Moment Magnification Factor:

For the lateral-torsional buckling limit state, the non-uniform moment magnification factor is calculated using the following equation:

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} R_m \leq 3.0 \qquad \textit{Eqn. 1}$$

Where M_A = first quarter-span moment, M_B = mid-span moment, M_C = second quarter-span moment.

The required moments for Eqn. 1 can be calculated as a percentage of the maximum mid-span moment. Since the loading is uniform and the resulting moment is symmetric:

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2$$

Member Bending Capacity for L_b = 5 ft:

$$M_{\max} = M_B = 1.00$$

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2 = 1 - \frac{1}{4}\left(\frac{5}{35}\right)^2 = 0.995$$

$$C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.995) + 4(1.00) + 3(0.995)}$$

$$C_b = 1.002$$

$L_b < L_p$, Lateral-Torsional buckling capacity is as follows:

$$M_n = M_p = 5050\,k - in$$

$$\varphi_b M_n = 0.9 \bullet 5050/12$$

$$\varphi_b M_n = 378.75\ \ k - ft$$

Member Bending Capacity for L_b = 11.667 ft:

$$M_{\max} = M_B = 1.00$$

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2 = 1 - \frac{1}{4}\left(\frac{11.667}{35}\right)^2 = 0.972$$

$$C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.972) + 4(1.00) + 3(0.972)}$$

$$C_b = 1.014$$

$L_p < L_b < L_r$, Lateral-Torsional buckling capacity is as follows:

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_{33}\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

$$M_n = 1.014\left[5050 - \left(5050 - 0.7 \bullet 50 \bullet 88.889\right)\left(\frac{11.667 - 5.835}{16.966 - 5.835}\right)\right] = 4088.733\ k - in$$

$$\varphi_b M_n = 0.9 \bullet 4088.733 / 12$$

$$\varphi_b M_n = 306.657\ \ k - ft$$

Member Bending Capacity for L_b = 35 ft:

$$M_{\max} = M_B = 1.00$$

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2 = 1 - \frac{1}{4}\left(\frac{35}{35}\right)^2 = 0.750\,.$$

$$C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.750) + 4(1.00) + 3(0.750)}(1.00)$$

$$C_b = 1.136$$

$L_b > L_r$, Lateral-Torsional buckling capacity is as follows:

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_{33} h_o} \left(\frac{L_b}{r_{ts}}\right)^2}$$

$$F_{cr} = \frac{1.136 \bullet \pi^2 \bullet 29000}{\left(\frac{420}{1.983}\right)^2} \sqrt{1 + 0.078 \frac{1.24 \bullet 1}{88.889 \bullet 17.4} \left(\frac{420}{1.983}\right)^2} = 14.133 ksi$$

$$M_n = F_{cr} S_{33} \leq M_p$$

$$M_n = 14.133 \bullet 88.9 = 1256.245 \ k - in$$

$$\varphi_b M_n = 0.9 \bullet 1256.245 / 12$$

$$\varphi_b M_n = 94.218 \ \ k - ft$$

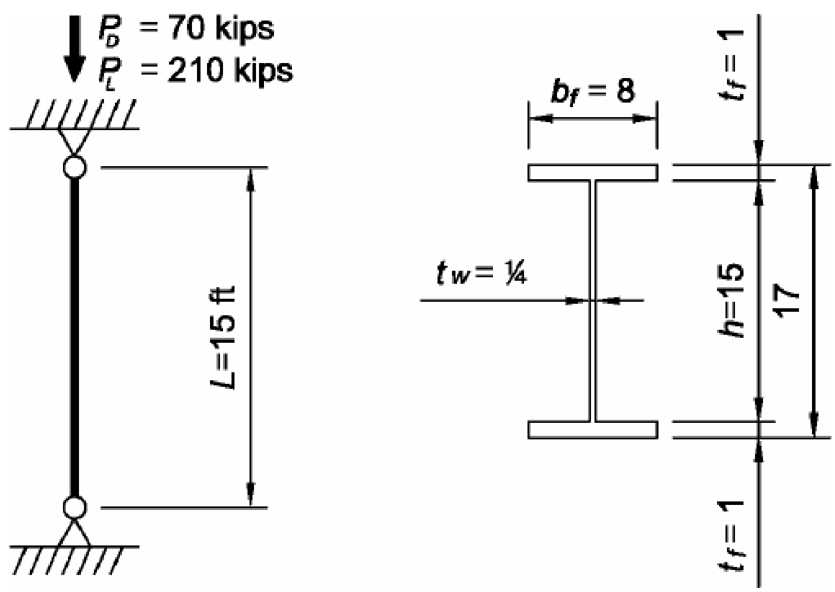
AISC 360-05 Example 002

BUILT UP WIDE FLANGE MEMBER UNDER COMPRESSION

EXAMPLE DESCRIPTION

A demand capacity ratio is calculated for the built-up, ASTM A572 grade 50, column shown below. An axial load of 70 kips (D) and 210 kips (L) is applied to a simply supported column with a height of 15 ft.

GEOMETRY, PROPERTIES AND LOADING



TECHNICAL FEATURES TESTED

- Section compactness check (compression)
- Warping constant calculation, C_w
- Member compression capacity with slenderness reduction

RESULTS COMPARISON

Independent results are hand calculated and compared with the results from Example E.2 AISC *Design Examples, Volume 13.0* on the application of the *2005 AISC Specification for Structural Steel Buildings* (ANSI/AISC 360-05).

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Slender	Slender	0.00%
$\phi_c P_n$ (kips)	506.1	506.1	0.00 %

COMPUTER FILE: AISC 360-05 EX002

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material: ASTM A572 Grade 50
E = 29,000 ksi, F_y = 50 ksi

Section: Built-Up Wide Flange

d = 17.0 in, b_f = 8.00 in, t_f = 1.00 in, h = 15.0 in, t_w = 0.250 in.

Ignoring fillet welds:

$$A = 2(8.00)(1.00) + (15.0)(0.250) = 19.75 \text{ in}^2$$

$$I_y = \frac{2(1.0)(8.0)^3}{12} + \frac{(15.0)(0.25)^3}{12} = 85.35 \text{ in}^3$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{85.4}{19.8}} = 2.08 \text{ in.}$$

$$I_x = \sum Ad^2 + \sum I_x$$

$$I_x = 2(8.0)(8.0)^2 + \frac{(0.250)(15.0)^3}{12} + \frac{2(8.0)(1.0)^3}{12} = 1095.65 \text{ in}^4$$

$$d' = d - \frac{t_1 + t_2}{2} = 17 - \frac{1+1}{2} = 16 \text{ in}$$

$$C_w = \frac{I_y \bullet d'^2}{4} = \frac{(85.35)(16.0)^2}{4} = 5462.583 \text{ in}^4$$

$$J = \sum \frac{bt^3}{3} = \frac{2(8.0)(1.0)^3 + (15.0)(0.250)^3}{3} = 5.41 \text{ in}^4$$

Member:
K = 1.0 for a pinned-pinned condition
L = 15 ft

Loadings:

P_u = 1.2(70.0) + 1.6(210) = 420 kips

Section Compactness:

Check for slender elements using Specification Section E7

Localized Buckling for Flange:

$$\lambda = \frac{b}{t} = \frac{4.0}{1.0} = 4.0$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29000}{50}} = 9.152$$

$\lambda < \lambda_p$, No localized flange buckling

Flange is Compact.

Localized Buckling for Web:

$$\lambda = \frac{h}{t} = \frac{15.0}{0.250} = 60.0,$$

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29000}{50}} = 35.9$$

$\lambda > \lambda_r$, Localized web buckling

Web is Slender.

Section is Slender

Member Compression Capacity:

Elastic Flexural Buckling Stress

Since the unbraced length is the same for both axes, the *y*-*y* axis will govern by inspection.

$$\frac{KL_y}{r_y} = \frac{1.0(15 \bullet 12)}{2.08} = 86.6$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \bullet 29000}{(86.6)^2} = 38.18 \text{ ksi}$$

Elastic Critical Torsional Buckling Stress

Note: Torsional buckling will not govern if $KL_y > KL_z$, however, the check is included here to illustrate the calculation.

$$F_e = \left[\frac{\pi^2 E C_w}{(K_z L)^2} + GJ\right]\frac{1}{I_x + I_y}$$

$$F_e = \left[\frac{\pi^2 \bullet 29000 \bullet 5462.4}{(180)^2} + 11200 \bullet 5.41\right]\frac{1}{1100 + 85.4} = 91.8 \text{ ksi} > 38.18 \text{ ksi}$$

Therefore, the flexural buckling limit state controls.

$F_e = 38.18$ ksi

Section Reduction Factors

Since the flange is not slender,

$Q_s = 1.0$

Since the web is slender,

For equation E7-17, take f as F_{cr} with $Q = 1.0$

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29000}{1.0(50)}} = 113 > \frac{KL_y}{r_y} = 86.6$$

So

$$f = F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y = 1.0\left[0.658^{\frac{1.0(50)}{38.2}}\right] \bullet 50 = 28.9 \text{ ksi}$$

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{(b/t)}\sqrt{\frac{E}{f}}\right] \le b, \text{ where } b = h$$

$$b_e = 1.92(0.250)\sqrt{\frac{29000}{28.9}}\left[1 - \frac{0.34}{(15.0/0.250)}\sqrt{\frac{29000}{28.9}}\right] \le 15.0 \text{ in}$$

$$b_e = 12.5 \text{ in} \le 15.0 \text{ in}$$

therefore compute A_{eff} with reduced effective web width.

$$A_{eff} = b_e t_w + 2b_f t_f = (12.5)(0.250) + 2(8.0)(1.0) = 19.1 \text{ in}^2$$

where A_{eff} is effective area based on the reduced effective width of the web, b_e.

$$Q_a = \frac{A_{eff}}{A} = \frac{19.1}{19.75} = 0.968$$

$$Q = Q_s Q_a = (1.00)(0.968) = 0.968$$

Critical Buckling Stress

Determine whether Specification Equation E7-2 or E7-3 applies

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29000}{0.966(50)}} = 115.4 > \frac{KL_y}{r_y} = 86.6$$

Therefore, Specification Equation E7-2 applies.

When $4.71\sqrt{\frac{E}{QF_y}} \geq \frac{KL}{r}$

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y = 0.966\left[0.658^{\frac{1.0(50)}{38.18}}\right] \bullet 50 = 28.47 \text{ ksi}$$

Nominal Compressive Strength

$$P_n = F_{cr} A_g = 28.5 \bullet 19.75 = 562.3 \text{ kips}$$

$$\phi_c = 0.90$$

$$\phi_c P_n = F_{cr} A_g = 0.90(562.3) = 506.1 \text{ kips} > 420 \text{ kips}$$

$$\phi_c P_n = 506.1 \text{ kips}$$

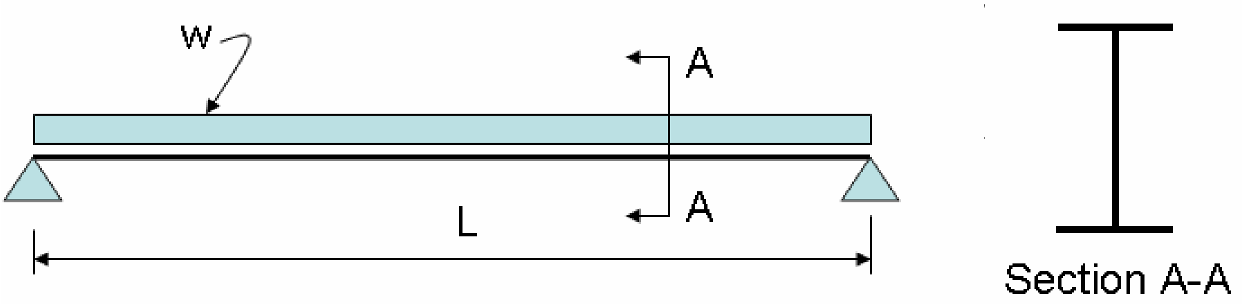
AISC 360-10 Example 001

WIDE FLANGE MEMBER UNDER BENDING

EXAMPLE DESCRIPTION

The design flexural strengths are checked for the beam shown below. The beam is loaded with a uniform load of 0.45 klf (D) and 0.75 klf (L). The flexural moment capacity is checked for three unsupported lengths in the weak direction, L_b = 5 ft, 11.667 ft and 35 ft.

GEOMETRY, PROPERTIES AND LOADING



Member Properties
W18X50
E = 29000 ksi
F_y = 50 ksi

Loading
w = 0.45 klf (D)
w = 0.75 klf (L)

Geometry
Span, L = 35 ft

TECHNICAL FEATURES TESTED

- Section compactness check (bending)
- Member bending capacities
- Unsupported length factors

RESULTS COMPARISON

Independent results are comparing with the results of Example F.1-2a from the AISC Design Examples, Volume 13 on the application of the 2005 AISC Specification for Structural Steel Buildings (ANSI/AISC 360-10).

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Compact	Compact	0.00%
C_b (L_b =5ft)	1.004	1.002	0.20%
$\phi_b M_n$ (L_b =5ft) (k-ft)	378.750	378.750	0.00%
C_b (L_b =11.67ft)	1.015	1.014	0.10%
$\phi_b M_n$ (L_b =11.67ft) (k-ft)	307.124	306.657	0.15%
C_b (L_b =35ft)	1.138	1.136	0.18%
$\phi_b M_n$ (L_b =35ft) (k-ft)	94.377	94.218	0.17%

COMPUTER FILE: AISC 360-10 EX001

CONCLUSION

The results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Material: ASTM A572 Grade 50 Steel

E = 29,000 ksi, F_y = 50 ksi

Section: W18x50

b_f = 7.5 in, t_f = 0.57 in, d = 18 in, t_w = 0.355 in

$$h = d - 2t_f = 18 - 2 \bullet 0.57 = 16.86 \text{ in}$$

$$h_0 = d - t_f = 18 - 0.57 = 17.43 \text{ in}$$

S_{33} = 88.9 in^3, Z_{33} = 101 in^3

I_y =40.1 in^4, r_y = 1.652 in, C_w = 3045.644 in^6, J = 1.240 in^4

$$r_{ts} = \sqrt{\frac{\sqrt{I_y C_w}}{S_{33}}} = \sqrt{\frac{\sqrt{40.1 \bullet 3045.644}}{88.889}} = 1.98 \text{in}$$

$R_m = 1.0$ for doubly-symmetric sections

Other:

c = 1.0

L = 35 ft

Loadings:

w_u = (1.2w_d + 1.6w_l) = 1.2(0.45) + 1.6(0.75) = 1.74 k/ft

$$M_u = \frac{w_u L^2}{8} = 1.74 \bullet 35^2/8 = 266.4375 \text{ k-ft}$$

Section Compactness:

Localized Buckling for Flange:

$$\lambda = \frac{b_f}{2t_f} = \frac{7.50}{2 \bullet 0.57} = 6.579$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29000}{50}} = 9.152$$

$\lambda < \lambda_p$, No localized flange buckling

Flange is Compact.

Localized Buckling for Web:

$$\lambda = \frac{h}{t_w} = \frac{16.86}{0.355} = 47.49$$

$$\lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29000}{50}} = 90.553$$

$\lambda < \lambda_p$, No localized web buckling

Web is Compact.

Section is Compact.

Section Bending Capacity:

$$M_p = F_y Z_{33} = 50 \bullet 101 = 5050 \text{ k-in}$$

Lateral-Torsional Buckling Parameters:

Critical Lengths:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76 \bullet 1.652 \sqrt{\frac{29000}{50}} = 70.022 \text{ in} = 5.835 \text{ ft}$$

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_{33} h_o}} \sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7F_y}{E}\frac{S_{33} h_o}{Jc}\right)^2}}$$

$$L_r = 1.95 \bullet 1.98 \frac{29000}{0.7 \bullet 50} \sqrt{\frac{1.240 \bullet 1.0}{88.9 \bullet 17.43}} \sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7 \bullet 50}{29000}\frac{88.9 \bullet 17.43}{1.240 \bullet 1.0}\right)^2}}$$

$$L_r = 16.966 \text{ ft}$$

Non-Uniform Moment Magnification Factor:

For the lateral-torsional buckling limit state, the non-uniform moment magnification factor is calculated using the following equation:

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} R_m \le 3.0 \qquad \textit{Eqn. 1}$$

where M_A = first quarter-span moment, M_B = mid-span moment, M_C = second quarter-span moment.

The required moments for Eqn. 1 can be calculated as a percentage of the maximum mid-span moment. Since the loading is uniform and the resulting moment is symmetric:

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2$$

Member Bending Capacity for L_b = 5 ft:

$$M_{\max} = M_B = 1.00$$

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2 = 1 - \frac{1}{4}\left(\frac{5}{35}\right)^2 = 0.995$$

$$C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.995) + 4(1.00) + 3(0.995)}$$

$$\boxed{C_b = 1.002}$$

$L_b < L_p$, Lateral-Torsional buckling capacity is as follows:

$$M_n = M_p = 5050 \text{ k-in}$$

$$\phi_b M_n = 0.9 \bullet 5050 / 12$$

$$\boxed{\phi_b M_n = 378.75 \text{ k-ft}}$$

Member Bending Capacity for L_b = 11.667 ft:

$$M_{\max} = M_B = 1.00$$

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2 = 1 - \frac{1}{4}\left(\frac{11.667}{35}\right)^2 = 0.972$$

$$C_b = \frac{12.5(1.00)}{2.5(1.00)+3(0.972)+4(1.00)+3(0.972)}$$

$$\boxed{C_b = 1.014}$$

$L_p < L_b < L_r$, Lateral-Torsional buckling capacity is as follows:

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_y S_{33}\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

$$M_n = 1.014\left[5050 - \left(5050 - 0.7 \bullet 50 \bullet 88.889\right)\left(\frac{11.667 - 5.835}{16.966 - 5.835}\right)\right] = 4088.733 \text{ k-in}$$

$$\phi_b M_n = 0.9 \bullet 4088.733/12$$

$$\boxed{\phi_b M_n = 306.657 \text{ k-ft}}$$

Member Bending Capacity for L_b = 35 ft:

$$M_{\max} = M_B = 1.00$$

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2 = 1 - \frac{1}{4}\left(\frac{35}{35}\right)^2 = 0.750 .$$

$$C_b = \frac{12.5(1.00)}{2.5(1.00)+3(0.750)+4(1.00)+3(0.750)}(1.00)$$

$$\boxed{C_b = 1.136}$$

$L_b > L_r$, Lateral-Torsional buckling capacity is as follows:

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_{33}h_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$

$$F_{cr} = \frac{1.136 \bullet \pi^2 \bullet 29000}{\left(\frac{420}{1.983}\right)^2}\sqrt{1 + 0.078\frac{1.24 \bullet 1}{88.889 \bullet 17.4}\left(\frac{420}{1.983}\right)^2} = 14.133 \text{ksi}$$

$$M_n = F_{cr} S_{33} \le M_p$$

$$M_n = 14.133 \bullet 88.9 = 1256.245 \text{ k-in}$$

$$\phi_b M_n = 0.9 \bullet 1256.245 / 12$$

$$\boxed{\phi_b M_n = 94.218 \text{ k-ft}}$$

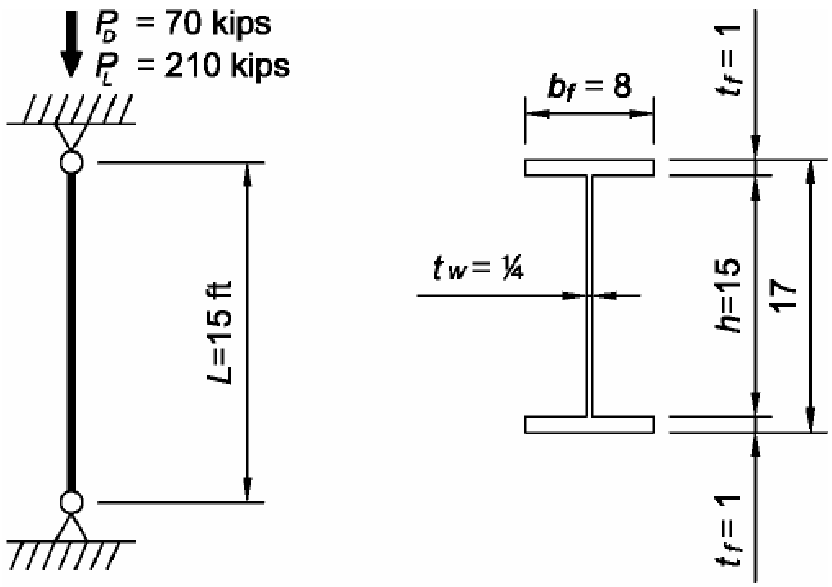
AISC 360-10 Example 002

BUILT UP WIDE FLANGE MEMBER UNDER COMPRESSION

EXAMPLE DESCRIPTION

A demand capacity ratio is calculated for the built-up, ASTM A572 grade 50, column shown below. An axial load of 70 kips (D) and 210 kips (L) is applied to a simply supported column with a height of 15 ft.

GEOMETRY, PROPERTIES AND LOADING



TECHNICAL FEATURES TESTED

- Section compactness check (compression)
- Warping constant calculation, C_w
- Member compression capacity with slenderness reduction

RESULTS COMPARISON

Independent results are hand calculated and compared with the results from Example E.2 AISC *Design Examples, Volume 13.0* on the application of the *2005 AISC Specification for Structural Steel Buildings* (ANSI/AISC 360-10).

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Slender	Slender	0.00%
$\phi_c P_n$ (kips)	506.1	506.1	0.00 %

COMPUTER FILE: AISC 360-10 EX002

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:
Material: ASTM A572 Grade 50
E = 29,000 ksi, F_y = 50 ksi

Section: Built-Up Wide Flange
d = 17.0 in, b_f = 8.00 in, t_f = 1.00 in, h = 15.0 in, t_w = 0.250 in.

Ignoring fillet welds:

A = 2(8.00)(1.00) + (15.0)(0.250) = 19.75 in^2

$$I_y = \frac{2(1.0)(8.0)^3}{12} + \frac{(15.0)(0.25)^3}{12} = 85.35\ in^3$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{85.4}{19.8}} = 2.08\ in.$$

$$I_x = \sum Ad^2 + \sum I_x$$

$$I_x = 2(8.0)(8.0)^2 + \frac{(0.250)(15.0)^3}{12} + \frac{2(8.0)(1.0)^3}{12} = 1095.65\ in^4$$

$$d' = d - \frac{t_1 + t_2}{2} = 17 - \frac{1+1}{2} = 16\ in$$

$$C_w = \frac{I_y \bullet d'^2}{4} = \frac{(85.35)(16.0)^2}{4} = 5462.583\ in^4$$

$$J = \sum \frac{bt^3}{3} = \frac{2(8.0)(1.0)^3 + (15.0)(0.250)^3}{3} = 5.41\ in^4$$

Member:
K = 1.0 for a pinned-pinned condition
L = 15 ft

Loadings:
P_u = 1.2(70.0) + 1.6(210) = 420 kips

Section Compactness:

Check for slender elements using Specification Section E7

Localized Buckling for Flange:

$$\lambda = \frac{b}{t} = \frac{4.0}{1.0} = 4.0$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29000}{50}} = 9.152$$

$\lambda < \lambda_p$, No localized flange buckling

Flange is Compact.

Localized Buckling for Web:

$$\lambda = \frac{h}{t} = \frac{15.0}{0.250} = 60.0,$$

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29000}{50}} = 35.9$$

$\lambda > \lambda_r$, Localized web buckling

Web is Slender.

Section is Slender

Member Compression Capacity:

Elastic Flexural Buckling Stress

Since the unbraced length is the same for both axes, the y-y axis will govern by inspection.

$$\frac{KL_y}{r_y} = \frac{1.0(15 \bullet 12)}{2.08} = 86.6$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \bullet 29000}{(86.6)^2} = 38.18 \text{ ksi}$$

Elastic Critical Torsional Buckling Stress

Note: Torsional buckling will not govern if $KL_y > KL_z$, however, the check is included here to illustrate the calculation.

$$F_e = \left[\frac{\pi^2 E C_w}{(K_z L)^2} + GJ\right]\frac{1}{I_x + I_y}$$

$$F_e = \left[\frac{\pi^2 \bullet 29000 \bullet 5462.4}{(180)^2} + 11200 \bullet 5.41\right]\frac{1}{1100 + 85.4} = 91.8 \text{ ksi} > 38.18 \text{ ksi}$$

Therefore, the flexural buckling limit state controls.

F_e = 38.18 ksi

Section Reduction Factors

Since the flange is not slender,
Q_s = 1.0

Since the web is slender,
For equation E7-17, take f as F_{cr} with Q = 1.0

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29000}{1.0(50)}} = 113 > \frac{KL_y}{r_y} = 86.6$$

So

$$f = F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y = 1.0\left[0.658^{\frac{1.0(50)}{38.2}}\right]\bullet 50 = 28.9\,\text{ksi}$$

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{(b/t)}\sqrt{\frac{E}{f}}\right] \le b, \text{ where } b = h$$

$$b_e = 1.92(0.250)\sqrt{\frac{29000}{28.9}}\left[1 - \frac{0.34}{(15.0/0.250)}\sqrt{\frac{29000}{28.9}}\right] \le 15.0\,\text{in}$$

$$b_e = 12.5\,\text{in} \le 15.0\,\text{in}$$

therefore compute A_{eff} with reduced effective web width.

$$A_{eff} = b_e t_w + 2b_f t_f = (12.5)(0.250) + 2(8.0)(1.0) = 19.1 \text{ in}^2$$

where A_{eff} is effective area based on the reduced effective width of the web, b_e.

$$Q_a = \frac{A_{eff}}{A} = \frac{19.1}{19.75} = 0.968$$

$$Q = Q_s Q_a = (1.00)(0.968) = 0.968$$

Critical Buckling Stress

Determine whether Specification Equation E7-2 or E7-3 applies

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29000}{0.966(50)}} = 115.4 > \frac{KL_y}{r_y} = 86.6$$

Therefore, Specification Equation E7-2 applies.

When $4.71\sqrt{\frac{E}{QF_y}} \geq \frac{KL}{r}$

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y = 0.966\left[0.658^{\frac{1.0(50)}{38.18}}\right] \bullet 50 = 28.47\,\text{ksi}$$

Nominal Compressive Strength

$$P_n = F_{cr} A_g = 28.5 \bullet 19.75 = 562.3\,\text{kips}$$

$$\phi_c = 0.90$$

$$\phi_c P_n = F_{cr} A_g = 0.90(562.3) = 506.1\,\text{kips} > 420\text{ kips}$$

$$\boxed{\phi_c P_n = 506.1\,\text{kips}}$$

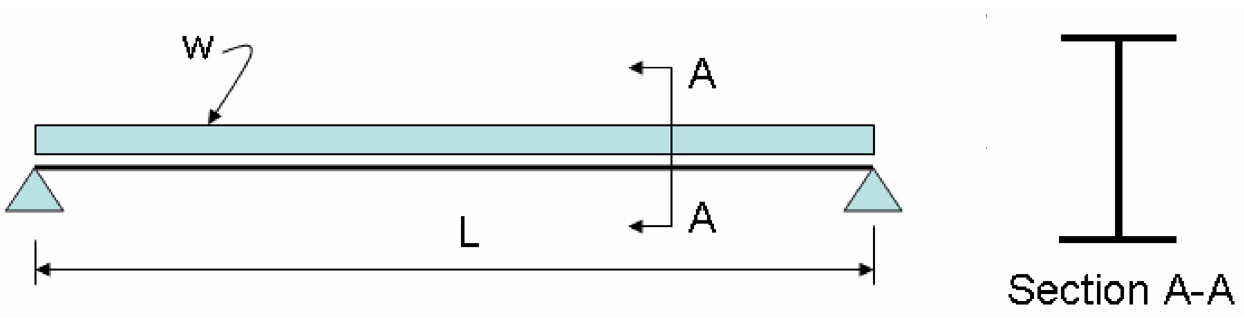
AISC ASD-89 Example 001

WIDE FLANGE MEMBER UNDER BENDING

EXAMPLE DESCRIPTION

The beam below is subjected to a bending moment of 20 kip-ft. The compression flange is braced at 3.0 ft intervals. The selected member is non-compact due to flange criteria.

GEOMETRY, PROPERTIES AND LOADING



Member Properties	Loading	Geometry
W6X12, M10X9, W8X10	w = 1.0 klf	Span, L = 12.65 ft
E = 29000 ksi		

TECHNICAL FEATURES TESTED

- Section compactness check (bending)
- Member bending capacity

RESULTS COMPARISON

Independent results are taken from Allowable Stress Design Manual of Steel Construction, Ninth Edition, 1989, Example 3, Page 2-6.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Non-Compact	Non-Compact	0.00%
Design Bending Stress, f_b (ksi)	30.74	30.74	0.00%
Allowable Bending Stress, F_b (ksi)	32.70	32.70	0.00 %

COMPUTER FILE: AISC ASD-89 EX001

CONCLUSION

The results show an exact match with the independent results.

HAND CALCULATION

Properties:

Material: ASTM A572 Grade 50 Steel

E = 29,000 ksi, F_y = 50 ksi

Section: W8x10

b_f = 3.94 in, t_f = 0.205 in, d = 7.98 in, t_w = 0.17 in

$$h = h - 2t_f = 7.89 - 2 \bullet 0.205 = 7.48 \text{ in}$$

Member:

L = 12.65 ft

l_b = 3 ft

Loadings:

w = 1.0 k/ft

$$M = \frac{wL^2}{8} = 1.0 \bullet 12.65^2/8 = 20.0 \text{ k-ft}$$

Design Bending Stress

$$f_b = M / S_{33} = 20 \bullet 12 / 7.8074$$

$$\boxed{f_b = 30.74 \text{ ksi}}$$

Section Compactness:

Localized Buckling for Flange:

$$\lambda = \frac{b_f}{2t_f} = \frac{3.94}{2 \bullet 0.205} = 9.610$$

$$\lambda_p = \frac{65}{\sqrt{F_y}} = \frac{65}{\sqrt{50}} = 9.192$$

$$\lambda_r = \frac{95}{\sqrt{F_y}} = \frac{95}{\sqrt{50}} = 13.435$$

$\lambda > \lambda_p$, Localized flange buckling is present.

$\lambda < \lambda_r$,

Flange is Non-Compact.

Localized Buckling for Web:

$$\lambda = \frac{d}{t_w} = \frac{7.89}{0.17} = 46.412$$

No axial force is present, so $f_a = \frac{P}{A} = 0$ and $\frac{f_a}{F_y} = 0 \leq 0.16,$ so

$$\lambda_p = \frac{640}{\sqrt{F_y}}\left(1 - 3.74\frac{f_a}{F_y}\right) = \frac{640}{\sqrt{50}}\left(1 - 3.74 \bullet \frac{0}{50}\right) = 90.510$$

$\lambda < \lambda_p$, No localized web buckling

Web is Compact.

Section is Non-Compact.

Section Bending Capacity

Allowable Bending Stress

Since section is Non-Compact

$$F_{b33} = \left(0.79 - 0.002\frac{b_f}{2t_f}\sqrt{F_y}\right)F_y$$

$$F_{b33} = \left(0.79 - 0.002 \bullet 9.61 \bullet \sqrt{50}\right)50$$

$$\boxed{F_{b33} = 32.70 \text{ ksi}}$$

Member Bending Capacity for L_b = 3.0 ft:

Critical Length, l_c:

$$l_c = \min\left\{\frac{76 b_f}{\sqrt{F_y}}, \frac{20{,}000 A_f}{d F_y}\right\}$$

$$l_c = \min\left\{\frac{76 \bullet 3.94}{\sqrt{50}}, \frac{20{,}000 \bullet 3.94 \bullet 0.205}{7.89 \bullet 50}\right\}$$

$$l_c = \min\left\{42.347, 40.948\right\}$$

$$l_c = 40.948 \text{ in}$$

$$l_{22} = l_b = 3 \bullet 12 = 36 \text{ in}$$

$l_{22} < l_c$, section capacity is as follows:

$$\boxed{F_{b33} = 32.70 \text{ ksi}}$$

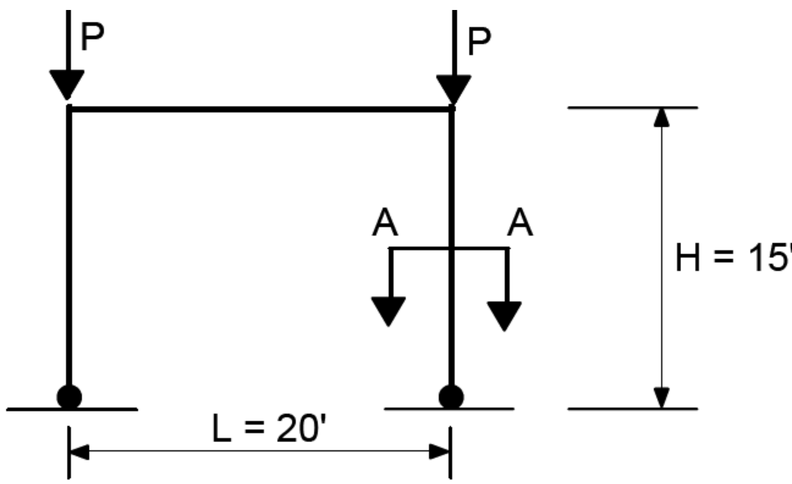
AISC ASD-89 Example 002

WIDE FLANGE MEMBER UNDER COMPRESSION

EXAMPLE DESCRIPTION

The column design features for the AISC ASD-89 code are checked for the frame shown below. This frame is presented in the *Allowable Stress Design Manual of Steel Construction*, Ninth Edition, 1989, Example 3, Pages 3-6 and 3-7. The column *K* factors were overwritten to a value of 2.13 to match the example. The transverse direction was assumed to be continuously supported. Two point loads of 560 kips are applied at the tops of each column. The ratio of allow axial stress, F_a, to the actual, f_a, was checked and compared to the referenced design code.

GEOMETRY, PROPERTIES AND LOADING



Member Properties	Loading	Geometry
W12X120	P = 560 kips (D)+(L)	Span, L = 20 ft
F_y = 50 ksi		Height, H = 15 ft
E = 29000 ksi		

TECHNICAL FEATURES TESTED

- Section compactness check (compression)
- Member compression capacity

RESULTS COMPARISON

Independent results are taken from *Allowable Stress Design Manual of Steel Construction*, Ninth Edition, 1989, Example 3, Pages 3-6 and 3-7.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Compact	Compact	0.00%
Design Axial Stress, f_a (ksi)	15.86	15.86	0.00%
Allowable Axial Stress, F_a (ksi)	16.47	16.47	0.00%

COMPUTER FILE: AISC ASD-89 EX002

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material: A36 Steel

E = 29,000 ksi, F_y = 36 ksi

Section: W12x120:

b_f = 12.32 in, t_f = 1.105 in, d =13.12 in, t_w=0.71 in

A = 35.3 in^2

r_x=5.5056 in

Member:

K = 2.13

L = 15 ft

Loadings:

P = 560 kips

Design Axial Stress:

$$f_a = \frac{P}{A} = \frac{560}{35.3}$$

$$\boxed{f_a = 15.86\,\text{ksi}}$$

Compactness:

Localized Buckling for Flange:

$$\lambda = \frac{b_f}{2t_f} = \frac{12.32}{2 \bullet 1.105} = 5.575$$

$$\lambda_p = \frac{65}{\sqrt{F_y}} = \frac{65}{\sqrt{36}} = 10.83$$

$\lambda < \lambda_p$, No localized flange buckling

Flange is Compact.

Localized Buckling for Web:

$$\frac{f_a}{F_y} = \frac{15.86}{36} = 0.44$$

$$\lambda = \frac{d}{t_w} = \frac{13.12}{0.71} = 18.48$$

Since $\frac{f_a}{F_y} = 0.44 > 0.16$

$$\lambda_p = \frac{257}{\sqrt{F_y}} = \frac{257}{\sqrt{36}} = 42.83$$

$\lambda < \lambda_p$, No localized web buckling

Web is Compact.

Section is Compact.

Member Compression Capacity

$$\frac{KL_x}{r_x} = \frac{2.13 \bullet (15 \bullet 12)}{5.5056} = 69.638$$

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}} = \sqrt{\frac{2\pi^2 \bullet 29000}{36}} = 126.099$$

$$\frac{\frac{KL_x}{r_x}}{C_c} = \frac{69.638}{126.099} = 0.552$$

$$\frac{KL_x}{r_x} < C_c$$

$$F_a = \frac{\left\{1.0 - \frac{1}{2}\left(\frac{KL_x/r_x}{C_c}\right)^2\right\}F_y}{\frac{5}{3} + \frac{3}{8}\left(\frac{KL_x/r_x}{C_c}\right) - \frac{1}{8}\left(\frac{KL_x/r_x}{C_c}\right)^3}$$

$$F_a = \frac{\left\{1.0 - \frac{1}{2}(0.552)^2\right\} \bullet 36}{\frac{5}{3} + \frac{3}{8}(0.552) - \frac{1}{8}(0.552)^3}$$

$$\boxed{F_a = 16.47 \text{ ksi}}$$

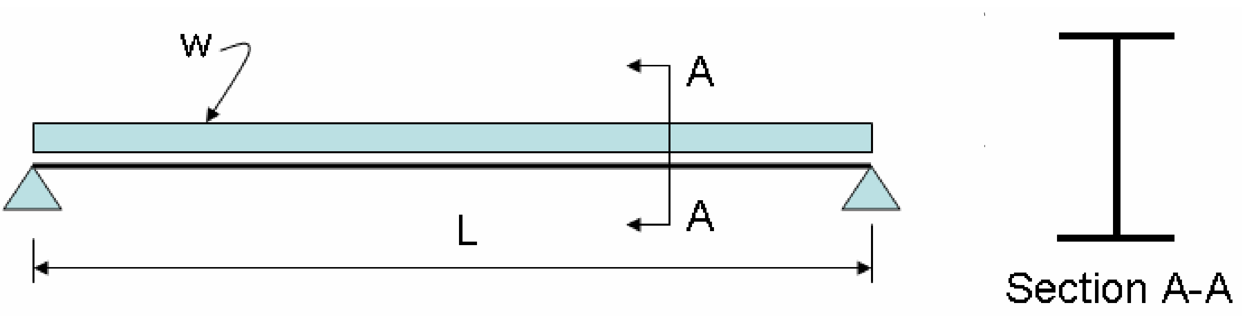
AISC LRFD-93 Example 001

WIDE FLANGE MEMBER UNDER BENDING

EXAMPLE DESCRIPTION

The design flexural strengths are checked for the beam shown below. The beam is loaded with an ultimate uniform load of 1.6 klf. The flexural moment capacity is checked for three unsupported lengths in the weak direction, L_b = 4.375 ft, 11.667 ft and 35 ft.

GEOMETRY, PROPERTIES AND LOADING



Member Properties
W18X40
E = 29000 ksi
F_y = 50 ksi

Loading
w_u = 1.6 klf

Geometry
Span, L = 35 ft

TECHNICAL FEATURES TESTED

- Section compactness check (bending)
- Member bending capacity
- Unsupported length factors

RESULTS COMPARISON

Independent results are comparing with the results of Example 5.1 in the 2nd Edition, LRFD Manual of Steel Construction, pages 5-12 to 5-15.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Compact	Compact	0.00%
C_b (L_b =4.375ft)	1.003	1.002	0.10%
$\phi_b M_n$ (L_b =4.375 ft) (k-ft)	294.000	294.000	0.00%
C_b (L_b =11.67 ft)	1.015	1.014	0.10%
$\phi_b M_n$ (L_b = 11.67ft) (k-ft)	213.0319	212.703	0.15%
C_b (L_b = 35ft)	1.138	1.136	0.18%
$\phi_b M_n$ (L_b = 35ft) (k-ft)	50.6845	50.599	0.17%

COMPUTER FILE: AISC LRFD-93 EX001

CONCLUSION

The results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Material: ASTM A572 Grade 50 Steel

E = 29,000 ksi, F_y = 50 ksi

F_r = 10 ksi (for rolled shapes)

$$F_L = F_y - F_r = 50 - 10 = 40 \text{ ksi}$$

Section: W18x40

b_f = 6.02 in, t_f = 0.525 in, d = 17.9 in, t_w = 0.315 in

$$h_c = d - 2t_f = 17.9 - 2 \bullet 0.525 = 16.85 \text{ in}$$

A = 11.8 in^2

S_{33} = 68.3799 in^3, Z_{33} = 78.4 in^3

I_y = 19.1 in^4, r_y = 1.2723 in

C_w = 1441.528 in^6, J = 0.81 in^4

Other:

L = 35 ft

ϕ_b = 0.9

Loadings:

w_u = 1.6 k/ft

$$M_u = \frac{w_u L^2}{8} = 1.6 \bullet 35^2/8 = 245.0 \text{ k-ft}$$

Section Compactness:

Localized Buckling for Flange:

$$\lambda = \frac{b_f}{2t_f} = \frac{6.02}{2 \bullet 0.525} = 5.733$$

$$\lambda_p = \frac{65}{\sqrt{F_y}} = \frac{65}{\sqrt{50}} = 9.192$$

$\lambda < \lambda_p$, No localized flange buckling

Flange is Compact.

Localized Buckling for Web:

$$\lambda = \frac{h_c}{t_w} = \frac{16.85}{0.315} = 53.492$$

$$\lambda_p = \frac{640}{\sqrt{F_y}} = \frac{640}{\sqrt{50}} = 90.510$$

$\lambda < \lambda_p$, No localized web buckling

Web is Compact.

Section is Compact.

Section Bending Capacity

$$M_p = F_y Z_{33} = 50 \bullet 78.4 = 3920 \text{ k-in}$$

Lateral-Torsional Buckling Parameters:

Critical Lengths*:*

$$X_1 = \frac{\pi}{S_{33}}\sqrt{\frac{EGJA}{2}} = \frac{\pi}{68.3799}\sqrt{\frac{29000 \bullet 11153.85 \bullet 0.81 \bullet 11.8}{2}} = 1806 \text{ ksi}$$

$$X_2 = 4\frac{C_w}{I_{22}}\left(\frac{S_{33}}{GJ}\right)^2 = 4\frac{1441.528}{19.1}\left(\frac{68.3799}{11153.85 \bullet 0.81}\right)^2 = 0.0173 \text{ in}^4$$

$$L_p = \frac{300\, r_{22}}{\sqrt{F_y}} = \frac{300 \bullet 1.2723}{\sqrt{50}} = 53.979 \text{ in} = 4.498 \text{ ft}$$

$$L_r = r_{22}\frac{X_1}{F_L}\sqrt{1 + \sqrt{1 + X_2 F_L^{\,2}}}$$

$$L_r = \frac{1.27 \bullet 1810}{40}\sqrt{1 + \sqrt{1 + 0.0172 \bullet 40^2}} = 144.8 \text{ in} = 12.069 \text{ ft}$$

Non-Uniform Moment Magnification Factor:

For the lateral-torsional buckling limit state, the non-uniform moment magnification factor is calculated using the following equation:

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} R_m \le 3.0 \qquad \text{Eqn. 1}$$

where M_A = first quarter-span moment, M_B = mid-span moment, M_C = second quarter-span moment.

The required moments for Eqn. 1 can be calculated as a percentage of the maximum mid-span moment. Since the loading is uniform and the resulting moment is symmetric:

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2$$

Member Bending Capacity for L_b = 4.375 ft:

$$M_{\max} = M_B = 1.00$$

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2 = 1 - \frac{1}{4}\left(\frac{4.375}{35}\right)^2 = 0.996$$

$$C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.996) + 4(1.00) + 3(0.996)}$$

$$\boxed{C_b = 1.002}$$

$L_b < L_p$, Lateral-Torsional buckling capacity is as follows:

$$M_n = M_p = F_y Z_{33} = 50 \bullet 78.4 = 3920 < 1.5 S_{33} F_y = 1.5 \bullet 68.3799 \bullet 50 = 5128.493 \text{ k-in}$$

$$\phi_b M_n = 0.9 \bullet 3920 / 12$$

$$\boxed{\phi_b M_n = 294.0 \text{ k-ft}}$$

Member Bending Capacity for L_b = 11.667 ft:

$$M_{\max} = M_B = 1.00$$

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2 = 1 - \frac{1}{4}\left(\frac{11.667}{35}\right)^2 = 0.972$$

$$C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.972) + 4(1.00) + 3(0.972)}$$

$$\boxed{C_b = 1.014}$$

$L_p < L_b < L_r$, Lateral-Torsional buckling capacity is as follows:

$$M_n = C_b\left[M_p - \left(M_p - F_L S_{33}\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

$$M_n = 1.01\left[3920 - \left(3920 - 40 \bullet 68.4\right)\left(\frac{11.667 - 4.486}{12.06 - 4.486}\right)\right] = 2836.042 \text{ k-in}$$

$$\phi_b M_n = 0.9 \bullet 2836.042/12$$

$$\boxed{\phi_b M_n = 212.7031 \text{ k-ft}}$$

Member Bending Capacity for L_b = 35 ft:

$$M_{\max} = M_B = 1.00$$

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2 = 1 - \frac{1}{4}\left(\frac{35}{35}\right)^2 = 0.750.$$

$$C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.750) + 4(1.00) + 3(0.750)}(1.00)$$

$$\boxed{C_b = 1.136}$$

$L_b > L_r$, Lateral-Torsional buckling capacity is as follows:

$$M_n = F_{cr} S_{33} \le M_p$$

$$M_{cr} = \frac{C_b \pi}{L_b} \sqrt{E I_{22} G J + \left(\frac{\pi E}{L_b}\right)^2 I_{22} C_W}$$

$$M_{cr} = \frac{1.136 \bullet \pi}{35 \bullet 12} \sqrt{29000 \bullet 19.1 \bullet 11153.85 \bullet 0.81 + \left(\frac{\pi \bullet 29000}{35 \bullet 12}\right)^2 19.1 \bullet 1441.528}$$

$$M_n = M_{cr} = 674.655 \text{ k-in}$$

$$\phi_b M_n = 0.9 \bullet 674.655 / 12$$

$$\boxed{\phi_b M_n = 50.599 \text{ k-ft}}$$

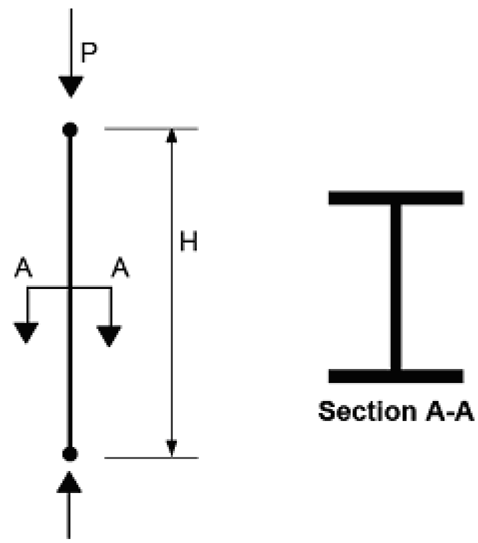
AISC LRFD-93 Example 002

WIDE FLANGE MEMBER UNDER COMBINED COMPRESSION & BIAXIAL BENDING

EXAMPLE DESCRIPTION

A check of the column adequacy is checked for combined axial compression and flexural loads. The column is 14 feet tall and loaded with an axial load, $\boldsymbol{P_u} = 1400$ kips and bending, $\boldsymbol{M_{ux}}, \boldsymbol{M_{uy}}$ = 200k-ft and 70k-ft, respectively. It is assumed that there is reverse-curvature bending with equal end moments about both axes and no loads along the member. The column demand/capacity ratio is checked against the results of Example 6.2 in the 3rd Edition, *LRFD Manual of Steel Construction*, pages 6-6 to 6-8.

GEOMETRY, PROPERTIES AND LOADING



Member Properties	Loading	Geometry
W14X176	P_u = 1,400 kips	H = 14.0 ft
E = 29000 ksi	M_{ux} = 200 kip-ft	
F_y = 50 ksi	M_{uy} = 70 kip-ft	

TECHNICAL FEATURES TESTED

- Section compactness check (compression)
- Member compression capacity
- Member bending capacity
- Demand/capacity ratio, D/C

RESULTS COMPARISON

Independent results are hand calculated and compared with the results from Example 6.2 in the 3rd Edition, *LRFD Manual of Steel Construction*, pages 6-6 to 6-8.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Compact	Compact	0.00%
$\phi_c P_n$ (kips)	1937.84	1937.84	0.00%
$\phi_b M_{nx}$ (k-ft)	1200	1200	0.00%
$\phi_b M_{ny}$ (k-ft)	600.478	600.478	0.00%
D/C	0.974	0.974	0.00%

COMPUTER FILE: AISC LRFD-93 EX002

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material: ASTM A992 Grade 50 Steel

F_y = 50 ksi, E = 29,000 ksi

Section: W14x176

A = 51.8 in^2,

b_f = 15.7 in, t_f = 1.31 in, d = 15.2 in, t_w = 0.83 in

$h_c = d - 2t_f = 15.2 - 2 \bullet 1.31 = 12.58\, in$

I_x = 2,140 in^4, I_y = 838 in^4, r_x = 6.4275 in, r_y = 4.0221 in

S_x = 281.579 in^3, S_y = 106.7516 in^3, Z_x = 320.0 in^3, Z_y = 163.0 in^3.

Member:

$K_x = K_y = 1.0$

$L = L_b$ = 14 ft

Other

$\phi_c = 0.85$

$\phi_b = 0.9$

Loadings:

P_u = 1400 kips

M_{ux} = 200 k-ft

M_{uy} = 70 k-ft

Section Compactness:

Localized Buckling for Flange:

$$\lambda = \frac{(b_f / 2)}{t_f} = \frac{(15.7 / 2)}{1.31} = 5.99$$

$$\lambda_p = \frac{65}{\sqrt{F_y}} = \frac{65}{\sqrt{50}} = 9.19$$

$\lambda < \lambda_p$, No localized flange buckling

Flange is Compact.

Localized Buckling for Web:

$$\lambda = \frac{h_c}{t_w} = \frac{12.58}{0.83} = 15.16$$

$$\phi_b P_y = \phi_b A_g F_y = 0.9 \bullet 51.8 \bullet 50 = 2331 \text{ kips}$$

$$\frac{P_u}{\phi_b P_y} = \frac{1400}{2331} = 0.601$$

Since $\frac{P_u}{\phi_b P_y} = 0.601 > 0.125$

$$\lambda_p = \frac{191}{\sqrt{F_y}}\left(2.33 - \frac{P_u}{\phi_b P_y}\right) \geq \frac{253}{\sqrt{F_y}}$$

$$\lambda_p = \frac{191}{\sqrt{50}}(2.33 - 0.601) = 46.714 \geq \frac{253}{\sqrt{50}} = 35.780$$

$\lambda < \lambda_p$, No localized web buckling

Web is Compact.

Section is Compact.

Member Compression Capacity:

For braced frames, K = 1.0 and $K_xL_x = K_yL_y$ = 14.0 ft, From AISC Table 4-2,

$$\phi_c P_n = 1940 \text{ kips}$$

Or by hand,

$$\lambda_c = \frac{K_y L}{r_y \pi}\sqrt{\frac{F_y}{E}} = \frac{1.0 \bullet 14 \bullet 12}{4.022 \bullet \pi}\sqrt{\frac{50}{29000}} = 0.552$$

Since $\lambda_c < 1.5$,

$$F_{cr} = F_y\left(0.658^{\lambda_c^2}\right) = 50 \bullet 0.658^{0.552^2} = 44.012 \text{ ksi}$$

$$\phi_c P_n = \phi_c F_{cr} A_g = 0.85 \bullet 44.012 \bullet 51.8$$

$$\boxed{\phi_c P_n = 1937.84 \text{ kips}}$$

From LRFD Specification Section H1.2,

$$\frac{P_u}{\phi_c P_n} = \frac{1400}{1937.84} = 0.722 > 0.2$$

Therefore, LRFD Specification Equation H1-1a governs.

Section Bending Capacity

$$M_{px} = F_y Z_x = \frac{50 \bullet 310}{12} = 1333.333 \text{ k-ft}$$

$$M_{py} = F_y Z_y$$

However, $\frac{Z_y}{S_y} = \frac{163}{106.7516} = 1.527 > 1.5$,

So

$$Z_y = 1.5 S_y = 1.5 \bullet 106.7516 = 160.1274 \text{in}^3$$

$$M_{py} = \frac{50 \bullet 160.1274}{12} = 667.198 \text{k-ft}$$

Member Bending Capacity

From LRFD Specification Equation F1-4,

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_{yf}}}$$

$$L_p = 1.76 \bullet 4.02 \sqrt{\frac{29000}{50}} \bullet 1/12 = 14.2 \text{ft} > L_b = 14 \text{ft}$$

$$\phi_b M_{nx} = \phi_b M_{px}$$

$$\phi_b M_{nx} = 0.9 \bullet 1333.333$$

$$\boxed{\phi_b M_{nx} = 1200 \text{k-ft}}$$

$$\phi_b M_{ny} = \phi_b M_{py}$$

$$\phi_b M_{ny} = 0.9 \bullet 667.198$$

$$\boxed{\phi_b M_{ny} = 600.478 \text{ k-ft}}$$

Interaction Capacity: Compression & Bending

From LRFD Specification section C1.2, for a braced frame, $M_{lt} = 0$.

$$M_{ux} = B_{1x} M_{ntx}, \text{ where } M_{ntx} = 200 \text{ kip-ft; and}$$
$$M_{uy} = B_{1y} M_{nty}, \text{ where } M_{nty} = 70 \text{ kip-ft}$$

$$B_1 = \frac{C_m}{\left(1 - \frac{P_u}{P_{e1}}\right)} \geq 1$$

For reverse curvature bending and equal end moments:

$$\frac{M_1}{M_2} = +1.0$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right)$$
$$C_m = 0.6 - 0.4(1.0) = 0.2$$
$$p_{e1} = \frac{\pi^2 EI}{(KL)^2}$$

$$p_{e1x} = \frac{\pi^2 \bullet 29000 \bullet 2140}{(14.0 \bullet 12)^2} = 21,702\, kips$$
$$p_{e1y} = \frac{\pi^2 \bullet 29000 \bullet 838}{(14.0 \bullet 12)^2} = 8,498$$

$$B_{1x} = \frac{C_{mx}}{\left(1 - \frac{P_u}{P_{e1x}}\right)} \geq 1$$
$$B_{1x} = \frac{0.2}{\left(1 - \frac{1400}{21702}\right)} = 0.214 \geq 1$$

$$B_{1x} = 1$$

$$B_{1y} = \frac{C_{my}}{\left(1 - \frac{P_u}{P_{e1y}}\right)} \geq 1$$

$$B_{1y} = \frac{0.2}{\left(1 - \frac{1400}{8498}\right)} = 0.239 \geq 1$$

$$B_{1y} = 1$$

$$M_{ux} = 1.0 \bullet 200 = 200 \text{ kip-ft;}$$

and

$$M_{uy} = 1.0 \bullet 70 = 70 \text{ kip-ft}$$

From LRFD Specification Equation H1-1a,

$$\frac{1400}{1940} + \frac{8}{9}\left(\frac{200}{1200} + \frac{70}{600.478}\right) = 0.974 < 1.0 \text{, OK}$$

$$\boxed{\frac{D}{C} = 0.974}$$

AS 4100-1998 Example 001

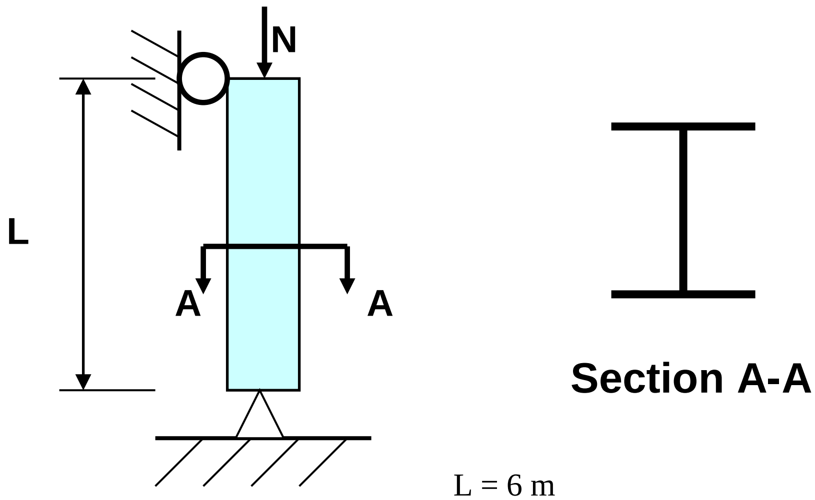
WIDE FLANGE MEMBER UNDER COMPRESSION

EXAMPLE DESCRIPTION

The frame object axial strengths are tested in this example.

A continuous column is subjected to factored load N = 200 kN. This example was tested using the AS4100-1998 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



L = 6 m

Material Properties	Loading	Design Properties
E = 200x10^3 MPa	N = 200 kN	f_y = 250 MPa
v = 0.3		Section: 350WC197
G = 76923.08 MPa		

TECHNICAL FEATURES TESTED

- Section compactness check (compression)
- Section compression capacity
- Member compression capacity

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file “SFD-AS-4100-1998.pdf”, which is also available through the program “Help” menu.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Compact	Compact	0.00%
Section Axial Capacity, N_s (kN)	6275	6275	0.00%
Member Axial Capacity, N_c (kN)	4385	4385	0.00%

COMPUTER FILE: AS 4100-1998 EX001

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material:

f_y = 250 MPa

Section: 350WC197

$A_g = A_n$ = 25100 mm^2

b_f = 350 mm, t_f = 28 mm, h = 331 mm, t_w = 20 mm

r_{33} = 139.15 mm, r_{22} = 89.264 mm

Member:

$l_{e33} = l_{e22}$ = 6000 mm (unbraced length)

Considered to be a braced frame

Loadings:

$N^* = 200\ kN$

Section Compactness:

Localized Buckling for Flange:

$$\lambda_e = \frac{(b_f - t_w)}{2 \bullet t_f}\sqrt{\frac{f_y}{250}} = \frac{350-20}{2 \bullet 28}\sqrt{\frac{250}{250}} = 5.89$$

Flange is under uniform compression, so:

$\lambda_{ep} = 9,\ \lambda_{ey} = 16,\ \lambda_{ew} = 90$

$\lambda_e = 5.89 < \lambda_{ep} = 9$, No localized flange buckling

Flange is compact

Localized Buckling for Web:

$$\lambda_e = \frac{h}{t_w}\sqrt{\frac{f_y}{250}} = \frac{331}{20}\sqrt{\frac{250}{250}} = 16.55$$

Web is under uniform compression, so:

$\lambda_{ep} = 30, \lambda_{ey} = 45, \lambda_{ew} = 180$

$\lambda_e = 16.55 < \lambda_{ep} = 30$, No localized web buckling

Web is compact.

Section is Compact.

Section Compression Capacity:

Section is not Slender, so K_f = 1.0

$N_s = K_f A_n f_y = 1 \bullet 25{,}100 \bullet 250 / 10^3$

$N_s = 6275\text{kN}$

Member Weak-Axis Compression Capacity:

Frame is considered a braced frame in both directions, so $k_{e22} = k_{e33} = 1$

$\frac{l_{e22}}{r_{22}} = \frac{6000}{89.264} = 67.216$ and $\frac{l_{e33}}{r_{33}} = \frac{6000}{139.15} = 43.119$

Buckling will occur on the 22-axis.

$$\lambda_{n22} = \frac{l_{e22}}{r_{22}} \sqrt{\frac{K_f f_y}{250}} = \frac{6000}{89.264} \bullet \sqrt{\frac{(1 \bullet 250)}{250}} = 67.216$$

$$\alpha_{a22} = \frac{2100(\lambda_{n22} - 13.5)}{\lambda_{n22}{}^2 - 15.3\lambda_{n22} + 2050} = 20.363$$

$\alpha_{b22} = 0.5$ since cross-section is not a UB or UC section

$$\lambda_{22} = \lambda_{n22} + \alpha_{a22}\alpha_{b22} = 67.216 + 20.363 \bullet 0.5 = 77.398$$

$$\eta_{22} = 0.00326(\lambda_{22} - 13.5) = 0.2083 \geq 0$$

$$\xi_{22} = \frac{\left(\frac{\lambda_{22}}{90}\right)^2 + 1 + \eta_{22}}{2\left(\frac{\lambda_{22}}{90}\right)^2} = \frac{\left(\frac{77.398}{90}\right)^2 + 1 + 0.2083}{2\left(\frac{77.398}{90}\right)^2} = 1.317$$

$$\alpha_{c22} = \xi_{22}\left(1 - \sqrt{\left(1 - \left(\frac{90}{\xi_{22}\lambda_{22}}\right)^2\right)}\right)$$

$$\alpha_{c22} = 1.317\left(1 - \sqrt{\left(1 - \left(\frac{90}{1.317 \bullet 77.398}\right)^2\right)}\right) = 0.6988$$

$$N_{c22} = \alpha_{c22} N_s \leq N_s$$

$$\boxed{N_{c22} = 0.6988 \bullet 6275 = 4385 \text{ kN}}$$

AS 4100-1998 Example 002

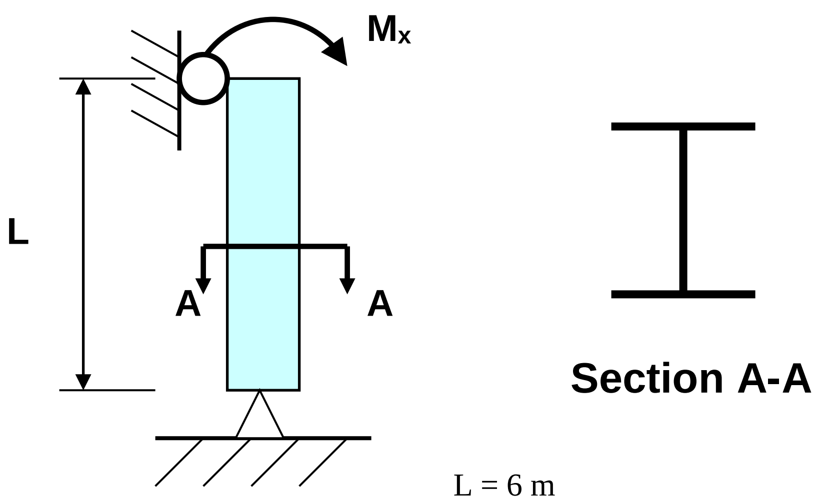
WIDE FLANGE MEMBER UNDER BENDING

EXAMPLE DESCRIPTION

The frame object bending strengths are tested in this example.

A continuous column is subjected to factored moment M_x = 1000 kN-m. This example was tested using the AS 4100-1998 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



L = 6 m

Material Properties	Loading	Design Properties
E = 200x10^3 MPa	M_x = 1000 kN-m	f_y = 250 MPa
v = 0.3		Section: 350WC197
G = 76923.08 MPa		

TECHNICAL FEATURES TESTED

- Section compactness check (bending)
- Section bending capacity
- Member bending capacity

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file "SFD-AS-4100-1998.pdf," which is also available through the program "Help" menu.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Compact	Compact	0.00%
Section Bending Capacity, $M_{s,\text{major}}$ (kN-m)	837.5	837.5	0.00%
Member Bending Capacity, M_b (kN-m)	837.5	837.5	0.00%

COMPUTER FILE: AS 4100-1998 EX002

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material:

f_y = 250 MPa

Section: 350WC197

b_f = 350 mm, t_f = 28 mm, h = 331 mm, t_w = 20 mm

I_{22} = 200,000,000 mm^4

Z_{33} = 2,936,555.891 mm^2

S_{33} = 3,350,000 mm^2

J = 5,750,000 mm^4

I_w = 4,590,000,000,000 mm^6

Member:

l_{e22} = 6000 mm (unbraced length)

Considered to be a braced frame

Loadings:

$M_m* = 1000$ kN-m

This leads to:

$M_2* = 250$ kN-m

$M_3* = 500$ kN-m

$M_4* = 750$ kN-m

Section Compactness:

Localized Buckling for Flange:

$$\lambda_e = \frac{(b_f - t_w)}{2 \bullet t_f}\sqrt{\frac{f_y}{250}} = \frac{350-20}{2 \bullet 28}\sqrt{\frac{250}{250}} = 5.89$$

Flange is under uniform compression, so:

$\lambda_{ep} = 9, \lambda_{ey} = 16, \lambda_{ew} = 90$

$\lambda_e = 5.89 < \lambda_{ep} = 9$, No localized flange buckling

Flange is compact

Localized Buckling for Web:

$$\lambda_e = \frac{h}{t_w}\sqrt{\frac{f_y}{250}} = \frac{331}{20}\sqrt{\frac{250}{250}} = 16.55$$

Web is under bending, so:

$\lambda_{ep} = 82, \lambda_{ey} = 115, \lambda_{ew} = 180$

$\lambda_e = 16.55 < \lambda_{ep} = 30$, No localized web buckling

Web is compact.

Section is Compact.

Section Bending Capacity:

$Z_e = Z_c = \min(S, 1.5Z)$ for compact sections

$$Z_{e33} = Z_{c33} = 3{,}350{,}000 \text{ mm}^2$$

$$M_{s33} = M_{s,\text{major}} = f_y Z_{e33} = 250 \bullet 3{,}350{,}000 / 1000^2$$

$$M_{s33} = M_{s,\text{major}} = 837.5 \text{ kN-m}$$

Member Bending Capacity:

k_t = 1 (Program default)

k_l = 1.4 (Program default)

k_r = 1 (Program default)

$l_{LTB} = l_{e22}$ = 6000 mm

$$l_e = k_t k_l k_r l_{LTB} = 1 \bullet 1.4 \bullet 1 \bullet 6000 = 8400 \text{ mm}^2$$

$$M_{oa} = M_o = \sqrt{\left(\left(\frac{\pi^2 EI_{22}}{l_e^{\,2}}\right)\left(GJ + \frac{\pi^2 EI_w}{l_e^{\,2}}\right)\right)}$$

$$M_{oa} = M_o = \sqrt{\left(\left(\frac{\pi^2 \bullet 2 \bullet 10^5 \bullet 2 \bullet 10^8}{8,400^2}\right)\left(76,923.08 \bullet 5,750,000 + \frac{\pi^2 \bullet 2 \bullet 10^5 \bullet 4.59 \bullet 10^{12}}{8,400^2}\right)\right)}$$

$$M_{oa} = M_o = 1786.938 \text{ kN-m}$$

$$\alpha_s = 0.6\left(\sqrt{\left(\left(\frac{M_s}{M_{oa}}\right)^2 + 3\right)} - \frac{M_s}{M_{oa}}\right) = 0.6\left(\sqrt{\left(\left(\frac{837.5}{1786.938}\right)^2 + 3\right)} - \frac{837.5}{1786.938}\right)$$

$$\alpha_s = 0.7954$$

$$\alpha_m = \frac{1.7 M_m *}{\sqrt{\left(M_2 *\right)^2 + \left(M_3 *\right)^2 + \left(M_4 *\right)^2}} \le 2.5$$

$$\alpha_m = \frac{1.7 \bullet 1000}{\sqrt{\left(250\right)^2 + \left(500\right)^2 + \left(750\right)^2}} = 1.817 \le 2.5$$

$$M_b = \alpha_m \alpha_s M_s = 0.7954 \bullet 1.817 \bullet 837.5 \le M_s$$

$$\boxed{M_b = 1210.64 \text{ kN-m} \le 837.5 \text{ kN-m}}$$

AS 4100-1998 Example 003

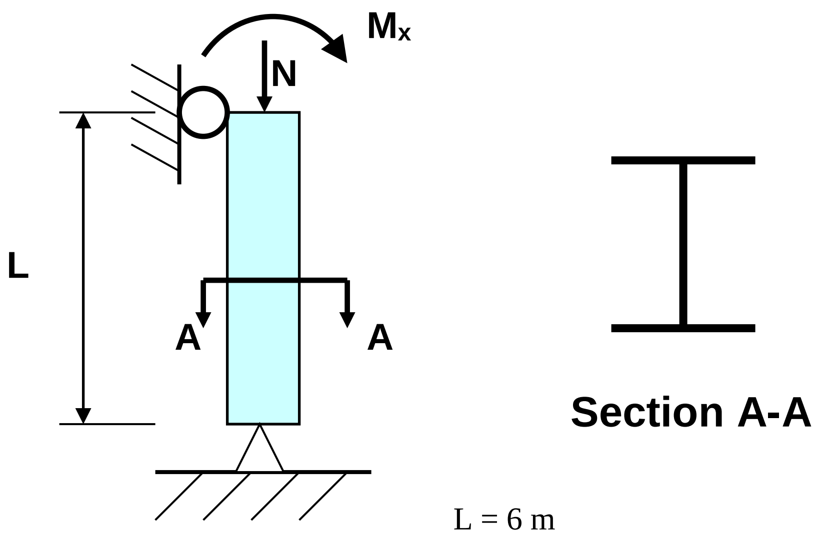
WIDE FLANGE MEMBER UNDER COMBINED COMPRESSION & BENDING

EXAMPLE DESCRIPTION

The frame object interacting axial and bending strengths are tested in this example.

A continuous column is subjected to factored loads and moments N = 200 kN; M_x = 1000 kN-m. This example was tested using the AS4100-1998 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



L = 6 m

Material Properties
E = 200x10³ MPa
v = 0.3
G = 76923.08 MPa

Loading
N = 200 kN
M_x = 1000 kN-m

Design Properties
f_y = 250 MPa
Section: 350WC197

TECHNICAL FEATURES TESTED

- Section compactness check (bending, compression)
- Section bending capacity with compression reduction
- Member bending capacity with in-plane compression reduction
- Member bending capacity with out-of-plane compression reduction

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file "SFD-AS-4100-1998.pdf," which is available through the program "Help" menu.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Compact	Compact	0.00%
Reduced Section Bending Capacity, M_{r33} (kN-m)	837.5	837.5	0.00%
Reduced In-Plane Member Bending Capacity, M_{i33} (kN-m)	823.1	823.1	0.00%
Reduced Out-of-Plane Member Bending Capacity, M_o (kN-m)	837.5	837.5	0.00%

COMPUTER FILE: AS 4100-1998 EX003

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Section: 350WC197

A_g = A_n = 25100 mm^2

I_{22} = 200,000,000 mm^4

I_{33} = 486,000,000 mm^4

J = 5,750,000 mm^4

I_w = 4,590,000,000,000 mm^6

Member:

$l_z = l_{e33} = l_{e22}$ = 6000 mm (unbraced length)

Considered to be a braced frame

$\phi = 0.9$

Loadings:

$N^* = 200\ kN$

$M_m^* = 1000\ kN - m$

Section Compactness:

From example SFD – IN-01-1, section is **Compact in Compression**

From example SFD – IN-01-2, section is **Compact in Bending**

Section Compression Capacity:

From example SFD – IN-01-1, $N_s = 6275\text{kN}$

Member Compression Capacity:

From example SFD – IN-01-1, $N_{c22} = 4385\text{ kN}$

Section Bending Capacity:

From example SFD – IN-01-2, $M_{s33} = M_{s,\text{major}} = 837.5\text{ kN-m}$

Section Interaction: Bending & Compression Capacity:

$$M_{r33} = 1.18 M_{s33}\left(1 - \frac{N*}{\phi N_s}\right) = 1.18 \bullet 837.5\left(1 - \frac{200}{0.9 \bullet 6275}\right) \leq M_{s33} = 837.5$$

$$M_{r33} = 953.252 \leq 837.5$$

$$M_{r33} = 837.5\text{kN-m}$$

Member Strong-Axis Compression Capacity:

Strong-axis buckling strength needs to be calculated:

Frame is considered a braced frame in both directions, so $k_{e33} = 1$

$$\frac{l_{e33}}{r_{33}} = \frac{6000}{139.15} = 43.119$$

$$\lambda_{n33} = \frac{l_{e33}}{r_{33}}\sqrt{\frac{K_f f_y}{250}} = \frac{6000}{139.15} \bullet \sqrt{\frac{(1 \bullet 250)}{250}} = 43.119$$

$$\alpha_{a33} = \frac{2100(\lambda_{n33} - 13.5)}{\lambda_{n33}{}^2 - 15.3\lambda_{n33} + 2050} = 19.141$$

$\alpha_{b33} = 0.5$ since cross-section is not a UB or UC section

$$\lambda_{33} = \lambda_{n33} + \alpha_{a33}\alpha_{b33} = 43.119 + 19.141 \bullet 0.5 = 52.690$$

$$\eta_{33} = 0.00326(\lambda_{33} - 13.5) = 0.1278 \geq 0$$

$$\xi_{33} = \frac{\left(\frac{\lambda_{33}}{90}\right)^2 + 1 + \eta_{33}}{2\left(\frac{\lambda_{33}}{90}\right)^2} = \frac{\left(\frac{52.690}{90}\right)^2 + 1 + 0.1278}{2\left(\frac{52.690}{90}\right)^2} = 2.145$$

$$\alpha_{c33} = \xi_{33}\left(1-\sqrt{\left(1-\left(\frac{90}{\xi_{33}\lambda_{33}}\right)^2\right)}\right)$$

$$\alpha_{c33} = 2.145\left(1-\sqrt{\left(1-\left(\frac{90}{2.145\bullet 50.690}\right)^2\right)}\right) = 0.8474$$

$$N_{c33} = \alpha_{c33}N_s \le N_s$$

$$N_{c33} = 0.8474\bullet 6275$$
$$N_{c33} = 5318\ kN$$

Member Interaction: In-Plane Bending and Compression Capacity:

$$\beta_m = \frac{M_{\min}}{M_{\max}} = \frac{0}{1000} = 0$$

Since the section is compact,

$$M_i = M_{s33}\left(\left(1-\left(\frac{1+\beta_m}{2}\right)^3\right)\left(1-\frac{N^*}{\phi N_{c33}}\right)+1.18\left(\frac{1+\beta_m}{2}\right)^3\sqrt{1-\frac{N^*}{\phi N_{c33}}}\right)$$

$$M_i = 837.5\left(\left(1-\left(\frac{1+0}{2}\right)^3\right)\left(1-\frac{200}{0.9\bullet 5318}\right)+1.18\left(\frac{1+0}{2}\right)^3\sqrt{1-\frac{200}{0.9\bullet 5318}}\right)$$

$$\boxed{M_i = 823.11\ \text{kN-m}}$$

Member Interaction: Out-of-Plane Bending and Compression Capacity:

$$\alpha_{bc} = \frac{1}{\left(\frac{1-\beta_m}{2}+\left(\frac{1-\beta_m}{2}\right)^3\left(0.4-0.23\frac{N^*}{\phi N_{c22}}\right)\right)}$$

$$\alpha_{bc} = \frac{1}{\left(\frac{1-0}{2} + \left(\frac{1-0}{2}\right)^3 \left(0.4 - 0.23\frac{200}{0.9 \bullet 4385}\right)\right)}$$

$$\alpha_{bc} = 4.120$$

$$N_{oz} = GJ + \frac{\frac{\pi^2 EI_w}{l_z^{\ 2}}}{\frac{I_{33}+I_{22}}{A_g}} = 76923.08 \bullet 5.75 \bullet 10^6 + \frac{\frac{\pi^2 \bullet 2 \bullet 10^6 \bullet 4.59 \bullet 10^{12}}{6000^2}}{\frac{(4.86+2)\bullet 10^8}{25100}}$$

$$N_{oz} = 4.423 \bullet 10^{11} \text{ kN}$$

$M_{b33o} = \alpha_m \alpha_s M_{sx}$ w/ an assumed uniform moment such that $\alpha_m = 1.0$

$$M_{b33o} = 1.0 \bullet 0.7954 \bullet 837.5 = 666.145 \text{ kN-m}$$

$$M_{o33} = \alpha_{bc} M_{b33o} \sqrt{\left(1 - \frac{N*}{\phi N_{c22}}\right)\left(1 - \frac{N*}{\phi N_{oz}}\right)} \le M_{r33}$$

$$M_{o33} = 4.12 \bullet 666.145 \sqrt{\left(1 - \frac{200}{0.9 \bullet 4385}\right)\left(1 - \frac{200}{0.9 \bullet 4.423 \bullet 10^{11}}\right)} = 2674 \le 837.5$$

$$\boxed{M_{o33} = 837.5 \text{ kN-m}}$$

BS 5950-2000 Example 001

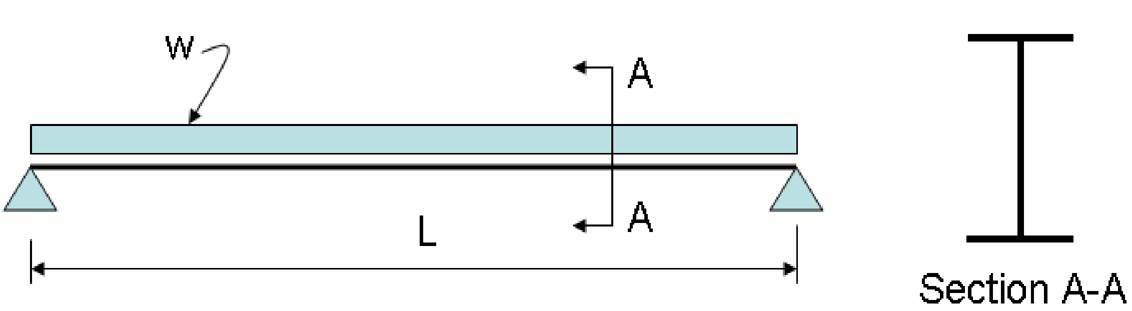
WIDE FLANGE SECTION UNDER BENDING

EXAMPLE DESCRIPTION

The frame object moment and shear strength is tested in this example.

A simply supported beam is laterally restrained along its full length and is subjected to a uniform factored load of 69 kN/m and a factored point load at the mid-span of 136 kN. This example was tested using the BS 5950-2000 steel frame design code. The moment and shear strengths are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



L=6.5 m

Material Properties
E = 205000 MPa
v = 0.3
G = 78846.15 MPa

Loading
W = 69 kN/m
P = 136 kN

Design Properties
Y_s = 275 MPa
Section: UB533x210x92

TECHNICAL FEATURES TESTED

- Section compactness check (bending)
- Section bending capacity
- Section shear capacity

RESULTS COMPARISON

Independent results are hand calculated based on the methods in Example 2 on page 5 of the SCI Publication P326, Steelwork Design Guide to BS5950-1:2000 Volume 2: Worked Examples by M.D. Heywood & J.B. Lim.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Class 1	Class 1	0.00%
Design Moment, M_{33} (kN-m)	585.4	585.4	0.00%
Design Shear, F_v (kN)	292.25	292.25	0.00%
Moment Capacity, M_c (kN-m)	649.0	649	0.00%
Shear Capacity, P_v (kN)	888.4	888.4	0.00%

COMPUTER FILE: BS 5950-2000 EX001

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material:

E = 205000 MPa

Y_s = 275 MPa

$\rho_y = 1.0 \bullet Y_s = 275\,\text{MPa}$

Section: UB533x210x92

A_g = 11,700 mm^2

D = 533.1 mm, b = 104.65 mm

t = 10.1 mm, T = 15.6 mm

$d = D - 2t = 533.1 - 2 \bullet 10.1 = 501.9\,\text{mm}$

Z_{33} = 2,072,031.5 mm^3

S_{33} = 2,360,000 mm^3

Loadings:

$P_{axial} = 0$

$w_u = (1.4w_d + 1.6w_l) = 1.4(15) + 1.6(30) = 69$ kN/m

$P_u = (1.4P_d + 1.6P_l) = 1.4(40) + 1.6(50) = 136$ kN

$$M_u = \frac{w_u l^2}{8} + \frac{P_u l}{4} = \frac{69 \bullet 6.5^2}{8} + \frac{136 \bullet 6.5}{4}$$

$$\boxed{M_u = 585.4\,\text{kN-m}}$$

$$F_v = \frac{w_u l + P_u}{2} = \frac{69 \bullet 6.5 + 136}{2}$$

$$\boxed{F_v = 292.25\,\text{kN}}$$

Section Compactness:

$$r_1 = \frac{P}{dt\rho_y} = 0 \text{ (since there is no axial force)}$$

$$r_2 = \frac{P}{A_g\rho_y} = 0 \text{ (since there is no axial force)}$$

$$\varepsilon = \sqrt{\frac{275}{\rho_y}} = \sqrt{\frac{275}{275}} = 1$$

Localized Buckling for Flange:

$$\lambda = \frac{b}{T} = \frac{104.65}{15.6} = 6.71$$

$$\lambda_{ep} = 9\varepsilon = 9$$

$\lambda = 6.71 < \lambda_p = 9$, No localized flange buckling

Flange is Class 1.

Localized Buckling for Web:

$$\lambda = \frac{d}{t} = \frac{501.9}{10.1} = 49.69$$

Since $r_1 = r_2 = 0$ and there is no axial compression:

$$\lambda_p = 80\varepsilon = 80$$

$\lambda = 49.69 < \lambda_p = 80$, No localized web buckling

Web is Class 1.

Section is Class 1.

Section Shear Capacity:

$$A_{v2} = Dt = 533.1 \bullet 10.1 = 5,384.31 \text{mm}^2$$

$$P_{v2} = 0.6 \rho_y A_{v2} = 0.6 \bullet 275 \bullet 5384.31$$

$$\boxed{P_{v2} = 888.4 \text{ kN}}$$

Section Bending Capacity:

With Shear Reduction:

$$0.6 \bullet P_{v2} = 533 kN > F_v = 292.3 \text{ kN}$$

So no shear reduction is needed in calculating the bending capacity.

$$M_c = \rho_y S_{33} \le 1.2 \rho_y Z_{33} = 275 \bullet 2,360,000 \le 1.2 \bullet 275 \bullet 2,072,031.5$$

$$M_c = 649 \text{ kN-m} \le 683.77 \text{ kN-m}$$

$$\boxed{M_c = 649 \text{ kN-m}}$$

BS 5950-2000 Example 002

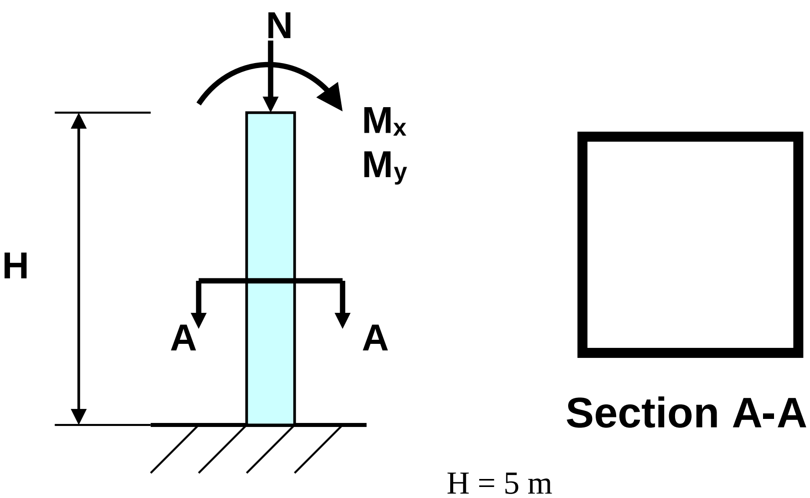
SQUARE TUBE MEMBER UNDER COMPRESSION & BENDING

EXAMPLE DESCRIPTION

The frame object axial and moment strengths are tested in this example.

A continuous column is subjected to factored loads and moments N = 640 kN; M_x = 10.5 kN-m; M_y = 0.66 kN-m. The moment on the column is caused by eccentric beam connections. This example was tested using the BS 5950-2000 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



H = 5 m

Material Properties
E = 205000 MPa
v = 0.3
G = 78846.15 MPa

Loading
N = 640 kN
M_x = 10.5 kN-m
M_y = 0.66 kN-m

Design Properties
Y_s = 355 MPa

TECHNICAL FEATURES TESTED

- Section compactness check (compression & bending)
- Member compression capacity
- Section bending capacity

RESULTS COMPARISON

Independent results are taken from Example 15 on page 83 of the SCI Publication P326, Steelwork Design Guide to BS5950-1:2000 Volume 2: Worked Examples by M.D. Heywood & J.B. Lim.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Class 1	Class 1	0.00%
Axial Capacity, N_c (kN)	773.2	773.2	0.00%
Bending Capacity, M_c (kN-m)	68.3	68.3	0.00%

COMPUTER FILE: BS 5950-2000 EX002

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material:

E = 205000 MPa

G = 78846.15 MPa

Y_s = 355 MPa

$$\rho_y = 1.0 \bullet Y_s = 355\,\text{MPa}$$

Section: RHS 150x150x6.3:

A_g = 3580 mm^2

D = B = *150* mm, T=t = 6.3 mm

$$b = B - 3 \bullet t = d = D - 3 \bullet T = 150 - 2 \bullet 6.3 = 131.1\text{mm}$$

r_{33} = 58.4483 mm

Z_{33} = 163,066.7 mm^3

S_{33} = 192,301.5 mm^3

Loadings:

N = 640 kN

M_x = 10.5 kN-m

M_y = 0.66 kN-m

F_{v33} = M_x/H = 10.5 / 5 = *2.1 kN*

Section Compactness:

$$r_1 = \frac{P}{dt\rho_y} = \frac{640}{131 \bullet 6.3 \bullet 355} = 0.002183$$

$$\varepsilon = \sqrt{\frac{275}{\rho_y}} = \sqrt{\frac{275}{355}} = 0.880$$

<u>Localized Buckling for Flange:</u>

$$\lambda = \frac{b}{T} = \frac{131.1}{6.3} = 20.81$$

$$\lambda_p = 28\varepsilon < 80\varepsilon - \frac{d}{t} = 28 \bullet 0.880 < 80 \bullet 0.880 - \frac{131.1}{6.3}$$

$$\lambda_p = 24.6 < 49.6$$

$\lambda = 20.81 < \lambda_p = 24.6$, No localized flange buckling

Flange is Compact.

<u>Localized Buckling for Web:</u>

$$\lambda = \frac{d}{t} = \frac{131.1}{6.3} = 20.81:$$

$$\lambda_p = \frac{64\varepsilon}{1+0.6r_1} < 40\varepsilon = \frac{64 \bullet 0.88}{1+0.6 \bullet 0.002183} < 40 \bullet 0.88 = 56.3 > 35.2$$

So $\lambda_p = 35.2$

$\lambda = 20.81 < \lambda_p = 35.2$, No localized web buckling

Web is compact.

Section is Compact.

Member Compression Capacity:

$$\lambda_{22} = \lambda_{33} = \frac{l_{e33}}{r_{33}} = \frac{K_{33} l_{33}}{r_{33}} = \frac{1.0 \bullet 5000}{58.4483} = 85.546$$

$$\lambda = \max\left\{\lambda_{22}, \lambda_{33}\right\} = 85.546$$

$$\lambda_o = 0.2\sqrt{\frac{\pi^2 E}{\rho_y}} = 0.2\sqrt{\frac{\pi^2 \bullet 205000}{355}} = 15.1$$

Robertson Constant: a = 2.0 (from Table VIII-3 for Rolled Box Section in CSI code documentation)

Perry Factor: $\eta = 0.001a\left(\lambda - \lambda_0\right) = 0.001 \bullet 2\left(85.546 - 15.1\right) = 0.141$

Euler Strength: $\rho_E = \frac{\pi^2 E}{\lambda^2} = \frac{\pi^2 \bullet 205000}{85.546^2} = 276.5\,MPa$

$$\phi = \frac{\rho_y + (\eta + 1)\rho_E}{2} = \frac{355 + (0.141 + 1) \bullet 276.5}{2} = 355.215\,\text{MPa}$$

$$\rho_c = \frac{\rho_E \rho_y}{\phi + \sqrt{\phi^2 - \rho_E \rho_y}} = \frac{276.5 \bullet 355}{335.215 + \sqrt{335.215^2 - 276.5 \bullet 355}} = 215.967\,\text{MPa}$$

$$N_c = A_g \rho_c = 3580 \bullet 215.967$$

$$\boxed{N_c = 773.2\,\text{kN}}$$

Section Shear Capacity:

$$\rho_y = 1.0 \bullet Y_s = 275\,\text{MPa}$$

$$A_v = A_g \left(\frac{D}{D + B} \right) = 3580 \bullet \left(\frac{150}{150 + 150} \right) = 1790\,\text{mm}^2$$

$$P_v = 0.6 \rho_y A_{v2} = 0.6 \bullet 355 \bullet 1790$$

$$P_v = 381.3\,\text{kN}$$

Section Bending Capacity:

<u>With Shear Reduction</u>

$$0.6 \bullet P_v = 228.8\,\text{kN} > F_v = 2.1\,\text{kN}$$

So no shear reduction is needed in calculating the bending capacity.

$$M_c = \rho_y S_{33} \leq 1.2 \rho_y Z_{33} = 355 \bullet 192{,}301.5 \leq 1.2 \bullet 355 \bullet 163{,}066.7$$

$$M_c = 68.3\,\text{kN-m} \leq 69.5\,\text{kN-m}$$

$$\boxed{M_c = 68.3\,\text{kN-m}}$$

<u>With LTB Reduction</u>

Not considered since the section is square.

CSA S16-09 Example 001

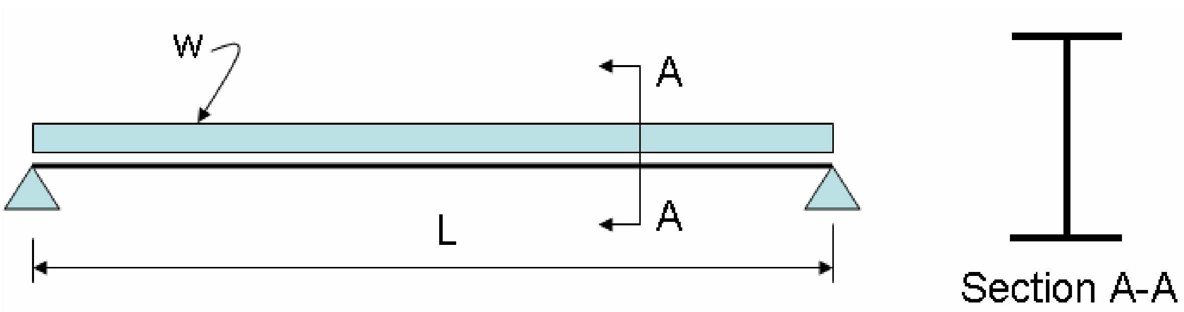
WIDE FLANGE MEMBER UNDER COMPRESSION & BENDING

EXAMPLE DESCRIPTION

The frame object moment and shear strength is tested in this example.

A simply supported beam is (a) laterally restrained along its full length, (b) laterally restrained along its quarter points, at mid-span, and at the ends (c) laterally restrained along mid-span, and is subjected to a uniform factored load of DL = 7 kN/m and LL = 15 kN/m. This example was tested using the CSA S16-09 steel frame design code. The moment and shear strengths are compared with Handbook of Steel construction (9th Edition) results.

GEOMETRY, PROPERTIES AND LOADING



L = 8.0 m

Material Properties	Loading	Design Properties
E = 2×10^8 kN/m^2	W_D = 7 kN/m	ASTM A992
F_y = 350 kN/m^2	W_L = 15 kN/m	CSA G40.21 350W
		W410X46
		W410X60

TECHNICAL FEATURES TESTED

- Section compactness check (bending)
- Member bending capacity, M_r (fully restrained)
- Member bending capacity, M_r (buckling)
- Member bending capacity, M_r (LTB)

RESULT COMPARISON

Independent results are taken from Examples 1, 2 and 3 on pages 5-84 and 5-85 of the *Hand Book of Steel Construction to CSA S16-01* published by Canadian Institute of Steel Construction.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Class 1	Class 1	0.00%
Design Moment, M_f (kN-m)	250.0	250.0	0.00%
(a) Moment Capacity, M_{r33} of W410X46 (kN-m) w/ l_b = 0 m	278.775	278.775	0.00%
(b) Moment Capacity, M_{r33} of W410X46 (kN-m) w/ l_b = 2 m	268.97	268.83	0.05%
(c) Moment Capacity, M_{r33} of W410X60 (kN-m) w/ l_b = 4 m	292.10	292.05	0.02%

COMPUTER FILE: CSA S16-09 EX001

CONCLUSION

The results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Material: CSA G40.21 Grade 350W

f_y = 350 MPa

E = 200,000 MPa

G = 76923 MPa

Section: W410x46

b_f = 140 mm, t_f = 11.2 mm, d = 404 mm, t_w = 7 mm

$h = d - 2t_f = 404 - 2 \bullet 11.2 = 381.6 \text{ mm}$

A_g = 5890 mm^2

I_{22} = 5,140,000 mm^4

Z_{33} = 885,000 mm^3

J = 192,000 mm^4

$C_w = 1.976 \bullet 10^{11} \text{mm}^6$

Section: W410x60

b_f = 178 mm, t_f = 12.8 mm, d = 408 mm, t_w = 7.7 mm

$h = d - 2t_f = 408 - 2 \bullet 12.8 = 382.4 \text{ mm}$

A_g = 7580 mm^2

I_{22} = 12,000,000 mm^4

Z_{33} = 1,190,000 mm^3

J = 328,000 mm^4

$C_w = 4.698 \bullet 10^{11} \text{mm}^6$

Member:

L = 8 m

Φ = 0.9

Loadings:

$w_f = (1.25w_d + 1.5w_l) = 1.25(7) + 1.5(15) = 31.25$ kN/m

$$M_f = \frac{w_f L^2}{8} = \frac{31.25 \bullet 8^2}{8}$$

$$\boxed{M_f = 250\,\text{kN-m}}$$

Section Compactness:

Localized Buckling for Flange:

$$\lambda_{Cl.1} = \frac{145}{\sqrt{F_y}} = \frac{145}{\sqrt{350}} = 7.75$$

W410x46

$$\lambda = \frac{b_f}{2t_f} = \frac{140}{2 \bullet 11.2} = 6.25$$

$\lambda < \lambda_{Cl.1}$, No localized flange buckling

Flange is Class 1.

W410x60

$$\lambda = \frac{b_f}{2t_f} = \frac{178}{2 \bullet 12.8} = 6.95$$

$\lambda < \lambda_{Cl.1}$, No localized flange buckling

Flange is Class 1.

Localized Buckling for Web:

$$\lambda_{Cl.1} = \frac{1100}{\sqrt{F_y}}\left(1 - 0.39\frac{C_f}{C_y}\right) = \frac{1100}{\sqrt{350}}\left(1 - 0.39\frac{0}{5890 \bullet 350}\right) = 58.8$$

W410x46

$$\lambda = \frac{h}{t_w} = \frac{381.6}{7} = 54.51$$

$\lambda < \lambda_{Cl.1}$, No localized web buckling

Web is Class 1.

Section is Class 1

W410x60

$$\lambda = \frac{h}{t_w} = \frac{382.4}{7.7} = 49.66$$

$\lambda < \lambda_{Cl.1}$, No localized web buckling

Web is Class 1.

Section is Class 1

Calculation of ω_2:

ω_2 is calculated from the moment profile so is independent of cross section and is calculated as:

$$\omega_2 = \frac{4 \bullet M_{\max}}{\sqrt{M_{\max}{}^2 + 4M_a{}^2 + 7M_b{}^2 + 4M_c{}^2}}$$

where: $M_{\max}$ = maximum moment

M_a = moment at ¼ unrestrained span

M_b = moment at ½ unrestrained span

M_c = moment at ¾ unrestrained span

Section Bending Capacity for W410x46:

$$M_p = F_y Z_{33} = 350 \bullet 885{,}000 / 10^6 = 309.75 \text{ kN-m}$$

$$\phi M_p = 0.9 \bullet 309.75 = 278.775 \text{ kN-m}$$

Member Bending Capacity for L_b = 0 mm (Fully Restrained):

L_b = 0, so $M_{\max} = M_a = M_b = M_c = M_u$ = 250 kN-m and ω_2 = 1.000

$$M_u = \frac{\omega_2 \pi}{L}\sqrt{EI_{22}GJ + \left(\frac{\pi E}{L}\right)^2 I_{22}C_w} \rightarrow \infty \; as \; L \rightarrow 0$$

$$M_{r33} = 1.15\phi M_{p33}\left[1 - 0.28\frac{M_{p33}}{M_u}\right] \le \phi M_{p33}$$

$$0.28\frac{M_{p33}}{M_u} \rightarrow 0 \; as \; M_u \rightarrow \infty$$

leading to $M_{r33} = 1.15\,\phi\, M_{p33} > \phi\, M_{p33}$

So

$$\boxed{M_{r33} = \phi\, M_{p33} = 278.775 \text{ kN-m}}$$

Member Bending Capacity for L_b = 2000 mm:

$$M_a \;@\; x_a = \frac{L - L_b}{2} + \frac{L_b}{4} = \frac{8-2}{2} + \frac{2}{4} = 3.5 \text{ m}$$

$$M_a = \frac{\omega_f L x_a}{2} - \frac{\omega_f x_a^{\,2}}{2} = \frac{31.25 \bullet 8 \bullet 3.5}{2} - \frac{31.25 \bullet 3.5^2}{2} = 246.094 \text{ kN-m}$$

$M_a = M_c$ = 246.094 kN-m @ 3500 mm and 4500 mm

$M_{\max} = M_b$ = 250 kN-m @ 4000 mm

$$\omega_2 = \frac{4 \bullet 250}{\sqrt{250^2 + 4 \bullet 246.094^2 + 7 \bullet 250^2 + 4 \bullet 246.094^2}} = 1.008$$

ω_2 = 1.008

$$M_u = \frac{\omega_2 \pi}{L}\sqrt{EI_{22}GJ + \left(\frac{\pi E}{L}\right)^2 I_{22}C_w}$$

$$M_u = \frac{1.008 \bullet \pi}{2000}\sqrt{\left(2 \bullet 10^5\right) \bullet \left(5.14 \bullet 10^6\right) \bullet 76923 \bullet \left(192 \bullet 10^3\right) + \left(\frac{\pi\left(2 \bullet 10^5\right)}{2000}\right)^2 \left(5.14 \bullet 10^6\right)\left(197.6 \bullet 10^9\right)}$$

$$M_u = 537.82 \bullet 10^6 \text{ N-mm} = 537.82 \text{ kN-m}$$

$$0.67 M_p = 0.67 \bullet 309.75 = 208 < M_u = 537.82 \text{ kN-m, so}$$

$$M_{r33} = 1.15\phi M_{p33}\left[1 - 0.28\frac{M_{p33}}{M_u}\right] \le \phi M_{p33}$$

$$M_{r33} = 1.15 \bullet 0.9 \bullet 309.75\left[1 - 0.28\frac{309.75}{537.82}\right] = 268.89 \text{ kN-m} < 278.775 \text{ kN-m}$$

$$\boxed{M_{r33} = 268.89 \text{ kN-m}}$$

Section Capacity for W410x60:

$$M_p = F_y Z_{33} = 350 \bullet 1190{,}000 / 10^6 = 416.5 \text{ kN-m}$$

$$\phi M_p = 0.9 \bullet 416.5 = 374.85 \text{ kN-m}$$

Member Bending Capacity for L_b = 4000 m:

$$M_a \text{ @ } x_a = \frac{L - L_b}{2} + \frac{L_b}{4} = \frac{8-4}{2} + \frac{4}{4} = 3 \text{m}$$

$$M_a = \frac{\omega_f L x_a}{2} - \frac{\omega_f {x_a}^2}{2} = \frac{31.25 \bullet 8 \bullet 3}{2} - \frac{31.25 \bullet 3^2}{2} = 234.375 \text{ kN-m}$$

$M_a = M_c$ = 234.375 kN-m @ 3500 mm and 4500 mm

$M_{\max} = M_b$ = 250 kN-m @ 4000 mm

$$\omega_2 = \frac{4 \bullet 250}{\sqrt{250^2 + 4 \bullet 234.375^2 + 7 \bullet 250^2 + 4 \bullet 234.375^2}} = 1.032$$

ω_2 = 1.032

$$M_u = \frac{\omega_2 \pi}{L_y}\sqrt{EI_{22}GJ + \left(\frac{\pi E}{L}\right)^2 I_{22}C_w}$$

$$M_u = \frac{1.032 \bullet \pi}{4000}\sqrt{\left(2 \bullet 10^5\right) \bullet \left(12 \bullet 10^6\right) \bullet 76923 \bullet \left(328 \bullet 10^3\right) + \left(\frac{\pi\left(2 \bullet 10^5\right)}{4000}\right)^2 \left(12 \bullet 10^6\right)\left(469.8 \bullet 10^9\right)}$$

$$M_u = 362.06 \bullet 10^6 \text{ N-mm} = 362.06 \text{ kN-m}$$

$$0.67\,M_p = 0.67 \bullet 309.75 = 279 < M_u = 362.06 \text{ kN-m, so}$$

$$M_{r33} = 1.15\phi M_{p33}\left[1 - 0.28\frac{M_{p33}}{M_u}\right] \le \phi M_{p33}$$

$$M_{r33} = 1.15 \bullet 0.9 \bullet 416.5\left[1 - 0.28\frac{416.5}{362.06}\right]$$

$$\boxed{M_{r33} = 292.23 \text{ kN-m}}$$

CSA S16-09 Example 002

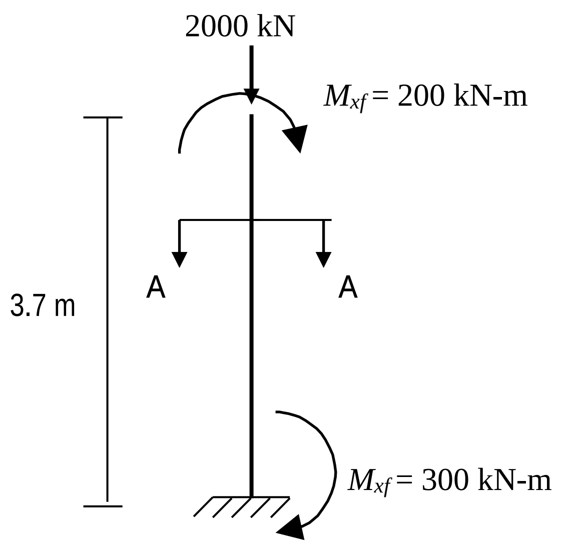
WIDE FLANGE MEMBER UNDER COMPRESSION & BENDING

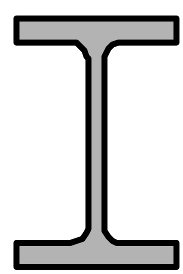
EXAMPLE DESCRIPTION

The frame object axial and moment strengths are tested in this example.

A continuous column is subjected to factored loads and moments C_f = 2000 kN; $M_{fx\text{-top}}$= 200 kN-m; $M_{fx\text{-bottom}}$= 300 kN-m. This example was tested using the CSA S16-09 steel frame design code. The design capacities are compared with Handbook of Steel Construction (9th Edition) results.

GEOMETRY, PROPERTIES AND LOADING





W310x118

Section A-A

Material Properties		Loading		Design Properties	
E =	200,000 MPa	C_f	= 2000 kN	f_y	= 345 MPa
ν =	0.3	$M_{fx\text{-top}}$	= 200 kN-m		
G=	76,923.08 MPa	$M_{fx\text{-bottom}}$	= 300 kN-m		

TECHNICAL FEATURES TESTED

- Section compactness check (compression & bending)
- Member compression capacity
- Member bending capacity with no mid-span loading

RESULTS COMPARISON

Independent results are taken from Example 1 on page 4-114 of the Hand Book of Steel Construction to CSA S16-01 published by the Canadian Institute of Steel Construction.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Class 2	Class 2	0.00%
Axial Capacity, C_r (kN)	3849.5	3849.5	0.00%
Bending Capacity, M_{r33} (kN-m)	605.5	605.5	0.00%

COMPUTER FILE: CSA S16-09 EX002

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material:

f_y = 345 MPa

E = 200,000 MPa

G = 76923.08 MPa

Section: W310x118

A_g = 15000 mm^2

r_{33} = 135.4006 mm, r_{22} = 77.5457 mm

I_{22} = 90,200,000 mm^4

Z_{33} = 1,950,000 mm^3

J = 1,600,000 mm^4

$$C_w = 1.966 \bullet 10^{12} \text{ mm}^6$$

$$\bar{r}_o^{\,2} = \bar{x}_o^{\,2} + \bar{y}_o^{\,2} + r_{22}^{\,2} + r_{33}^{\,2} = 0^2 + 0^2 + 77.5457^2 + 135.4006^2$$

$$\bar{r}_o^{\,2} = 24346.658 \text{ mm}^2$$

Member:

$l_z = l_{e33} = l_{e22}$ = 3700 mm (unbraced length)

$k_z = k_{33} = k_{22}$ =1.0

$$\phi = 0.9$$

Loadings:

$$C_f = 2000 \text{ kN}$$

$$M_a = M_{xf,\text{top}} = 200 \text{ kN-m}$$

$$M_b = M_{xf,\text{bottom}} = 300 \text{ kN-m}$$

Section Compactness:

Localized Buckling for Flange:

$$\lambda_{Cl.1} = \frac{145}{\sqrt{F_y}} = \frac{145}{\sqrt{345}} = 7.81$$

$$\lambda_{Cl.2} = \frac{170}{\sqrt{F_y}} = \frac{170}{\sqrt{345}} = 9.15$$

$$\lambda = \frac{b_f}{2t_f} = \frac{307}{2 \bullet 18.7} = 8.21$$

$\lambda_{Cl.1} < \lambda < \lambda_{Cl.2}$,

Flange is Class 2.

Localized Buckling for Web:

$$C_y = f_y A_g = \frac{345 \bullet 15000}{1000} = 5175 \text{kN}$$

$$\lambda_{Cl.1} = \frac{1100}{\sqrt{F_y}}\left(1 - 0.39\frac{C_f}{C_y}\right) = \frac{1100}{\sqrt{345}}\left(1 - 0.39\frac{2000}{5175}\right) = 50.30$$

$$\lambda = \frac{h}{t_w} = \frac{276.6}{11.9} = 23.24$$

$\lambda < \lambda_{Cl.1}$,

Web is Class 1.

Section is Class 2

Member Compression Capacity:

Flexural Buckling

n = 1.34 (wide flange section)

$$\lambda = \max(\lambda_{22}, \lambda_{33}) = \lambda_{22} = \frac{k_{22} l_{22}}{r_{22} \pi} \sqrt{\frac{f_y}{E}} = \frac{1.0 \bullet 3700}{77.5457} \sqrt{\frac{345}{200000}} = 0.6308$$

$$C_r = \phi A_g F_y \left(1 + \lambda^{2n}\right)^{-\frac{1}{n}} = 0.9 \bullet 15000 \bullet 345 \bullet \left(1 + 0.6308^{2 \bullet 1.34}\right)^{-\frac{1}{1.34}}$$

$$\boxed{C_r = 3489.5 \, \text{kN}}$$

Torsional & Lateral-Torsional Buckling

$$F_{ex} = \frac{\pi^2 E}{\left(\dfrac{k_{33} l_{33}}{r_{33}}\right)^2} = \frac{\pi^2 2 \bullet 10^5}{\left(\dfrac{1 \bullet 3700}{135.4006}\right)^2} = 2643 \, \text{MPa}$$

$$F_{ey} = \frac{\pi^2 E}{\left(\dfrac{k_{22} l_{22}}{r_{22}}\right)^2} = \frac{\pi^2 2 \bullet 10^5}{\left(\dfrac{1 \bullet 3700}{77.5457}\right)^2} = 867 \, \text{MPa}$$

$$F_{ez} = \left(\frac{\pi^2 E C_w}{\left(k_z l_z\right)^2} + GJ\right) \frac{1}{A_g \bar{r}_o^2}$$

$$F_{ez} = \left(\frac{\pi^2 \bullet 2 \bullet 10^5 \bullet 1.966 \bullet 10^{12}}{\left(1 \bullet 3700\right)^2} + 76923.08 \bullet 1.6 \bullet 10^6\right) \frac{1}{15000 \bullet 24347}$$

$$F_{eZ} = 1113.222 \, \text{MPa}$$

$$F_e = \min\left(F_{ex}, F_{ey}, F_{ez}\right) = F_{ey} = 867 \, \text{MPa}$$

$$C_r = \phi A_g F_e \left(1 + \lambda^{2n}\right)^{-\frac{1}{n}} = 0.9 \bullet 15000 \bullet 867 \bullet \left(1 + 0.6308^{2 \bullet 1.34}\right)^{-\frac{1}{1.34}}$$

$$C_r = 9674.5 \, \text{kN} \ \text{(does not govern)}$$

Section Bending Capacity:

$$M_{p33} = Z_{33} F_y = 1{,}950{,}000 \bullet 345 = 672.75 \, \text{kN-m}$$

Member Bending Capacity:

$$\omega_2 = 1.75 + 1.05\left(\frac{M_a}{M_b}\right) + 0.3\left(\frac{M_a}{M_b}\right)^2 \leq 2.5$$

$$\omega_2 = 1.75 + 1.05\left(\frac{200}{300}\right) + 0.3\left(\frac{200}{300}\right)^2 = 2.583 \leq 2.5$$

So $\omega_2 = 2.5$

$$M_u = \frac{\omega_2 \pi}{l_{22}}\sqrt{EI_{22}GJ + \left(\frac{\pi E}{l_{22}}\right)^2 I_{22}C_w}$$

$$M_u = \frac{2.5 \bullet \pi}{3700}\sqrt{2 \bullet 10^5 \bullet 9.02 \bullet 10^7 \bullet 76923.08 \bullet 1.6 \bullet 10^6 + \left(\frac{\pi \bullet 2 \bullet 10^5}{3700}\right)^2 9.02 \bullet 10^7 \bullet 1.966 \bullet 10^{12}}$$

$$M_u = 3163.117 \text{ kN-m}$$

Since $M_u > 0.67 \bullet M_{p33}$

$$M_{r33} = 1.15\phi M_{p33}\left[1 - 0.28\frac{M_{p33}}{M_u}\right] \leq \phi M_{p33}$$

$$M_{r33} = 1.15 \bullet 0.9 \bullet 672.75\left[1 - 0.28\frac{672.75}{3163.117}\right] \leq 0.9 \bullet 672.75$$

$$M_{r33} = 654.830 \leq 605.475$$

$$\boxed{M_{r33} = 605.5 \text{ kN-m}}$$

CSA S16-14 Example 001

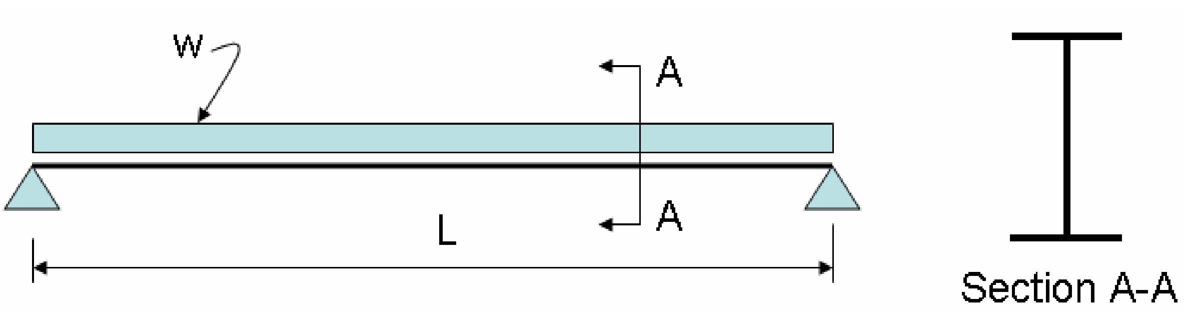
WIDE FLANGE MEMBER UNDER COMPRESSION & BENDING

EXAMPLE DESCRIPTION

The frame object moment and shear strength is tested in this example.

A simply supported beam is (a) laterally restrained along its full length, (b) laterally restrained along its quarter points, at mid-span, and at the ends (c) laterally restrained along mid-span, and is subjected to a uniform factored load of DL = 7 kN/m and LL = 15 kN/m. This example was tested using the CSA S16-14 steel frame design code. The moment and shear strengths are compared with Handbook of Steel construction (9th Edition) results.

GEOMETRY, PROPERTIES AND LOADING



L = 8.0 m

Material Properties

$E = 2x10^8$ kN/m^2
$F_y = 350$ kN/m^2

Loading

$W_D = 7$ kN/m
$W_L = 15$ kN/m

Design Properties

ASTM A992
CSA G40.21 350W
W410X46
W410X60

TECHNICAL FEATURES TESTED

- Section compactness check (bending)
- Member bending capacity, M_r (fully restrained)
- Member bending capacity, M_r (buckling)
- Member bending capacity, M_r (LTB)

RESULT COMPARISON

Independent results are taken from Examples 1, 2 and 3 on pages 5-84 and 5-85 of the *Hand Book of Steel Construction to CSA S16-01* published by Canadian Institute of Steel Construction.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Class 1	Class 1	0.00%
Design Moment, M_f (kN-m)	250.0	250.0	0.00%
(a) Moment Capacity, M_{r33} of W410X46 (kN-m) w/ l_b = 0 m	278.775	278.775	0.00%
(b) Moment Capacity, M_{r33} of W410X46 (kN-m) w/ l_b = 2 m	268.97	268.83	0.05%
(c) Moment Capacity, M_{r33} of W410X60 (kN-m) w/ l_b = 4 m	292.10	292.05	0.02%

COMPUTER FILE: CSA S16-14 EX001

CONCLUSION

The results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Material: CSA G40.21 Grade 350W

f_y = 350 MPa

E = 200,000 MPa

G = 76923 MPa

Section: W410x46

b_f = 140 mm, t_f = 11.2 mm, d = 404 mm, t_w = 7 mm

$$h = d - 2t_f = 404 - 2 \bullet 11.2 = 381.6 \text{ mm}$$

A_g = 5890 mm^2

I_{22} = 5,140,000 mm^4

Z_{33} = 885,000 mm^3

J = 192,000 mm^4

$$C_w = 1.976 \bullet 10^{11} \text{mm}^6$$

Section: W410x60

b_f = 178 mm, t_f = 12.8 mm, d = 408 mm, t_w = 7.7 mm

$$h = d - 2t_f = 408 - 2 \bullet 12.8 = 382.4 \text{ mm}$$

A_g = 7580 mm^2

I_{22} = 12,000,000 mm^4

Z_{33} = 1,190,000 mm^3

J = 328,000 mm^4

$$C_w = 4.698 \bullet 10^{11} \text{mm}^6$$

Member:

L = 8 m

Φ = 0.9

Loadings:

$w_f = (1.25w_d + 1.5w_l) = 1.25(7) + 1.5(15) = 31.25$ kN/m

$$M_f = \frac{w_f L^2}{8} = \frac{31.25 \bullet 8^2}{8}$$

$$\boxed{M_f = 250 \text{ kN-m}}$$

Section Compactness:

Localized Buckling for Flange:

$$\lambda_{Cl.1} = \frac{145}{\sqrt{F_y}} = \frac{145}{\sqrt{350}} = 7.75$$

W410x46

$$\lambda = \frac{b_f}{2t_f} = \frac{140}{2 \bullet 11.2} = 6.25$$

$\lambda < \lambda_{Cl.1}$, No localized flange buckling

Flange is Class 1.

W410x60

$$\lambda = \frac{b_f}{2t_f} = \frac{178}{2 \bullet 12.8} = 6.95$$

$\lambda < \lambda_{Cl.1}$, No localized flange buckling

Flange is Class 1.

Localized Buckling for Web:

$$\lambda_{Cl.1} = \frac{1100}{\sqrt{F_y}}\left(1 - 0.39\frac{C_f}{C_y}\right) = \frac{1100}{\sqrt{350}}\left(1 - 0.39\frac{0}{5890 \bullet 350}\right) = 58.8$$

W410x46

$$\lambda = \frac{h}{t_w} = \frac{381.6}{7} = 54.51$$

$\lambda < \lambda_{Cl.1}$, No localized web buckling

Web is Class 1.

Section is Class 1

W410x60

$$\lambda = \frac{h}{t_w} = \frac{382.4}{7.7} = 49.66$$

$\lambda < \lambda_{Cl.1}$, No localized web buckling

Web is Class 1.

Section is Class 1

Calculation of ω_2:

ω_2 is calculated from the moment profile so is independent of cross section and is calculated as:

$$\omega_2 = \frac{4 \bullet M_{\max}}{\sqrt{M_{\max}{}^2 + 4M_a{}^2 + 7M_b{}^2 + 4M_c{}^2}}$$

where: $M_{\max}$ = maximum moment

M_a = moment at ¼ unrestrained span

M_b = moment at ½ unrestrained span

M_c = moment at ¾ unrestrained span

Section Bending Capacity for W410x46:

$$M_p = F_y Z_{33} = 350 \bullet 885{,}000 / 10^6 = 309.75 \text{ kN-m}$$

$$\phi M_p = 0.9 \bullet 309.75 = 278.775 \text{ kN-m}$$

Member Bending Capacity for L_b = 0 mm (Fully Restrained):

L_b = 0, so $M_{\max} = M_a = M_b = M_c = M_u$ = 250 kN-m and ω_2 = 1.000

$$M_u = \frac{\omega_2 \pi}{L}\sqrt{EI_{22}GJ + \left(\frac{\pi E}{L}\right)^2 I_{22}C_w} \rightarrow \infty \, as \, L \rightarrow 0$$

$$M_{r33} = 1.15\phi M_{p33}\left[1 - 0.28\frac{M_{p33}}{M_u}\right] \le \phi M_{p33}$$

$$0.28\frac{M_{p33}}{M_u} \rightarrow 0 \, as \, M_u \rightarrow \infty$$

leading to $M_{r33} = 1.15\,\phi\, M_{p33} > \phi\, M_{p33}$

So

$$\boxed{M_{r33} = \phi\, M_{p33} = 278.775 \, \text{kN-m}}$$

Member Bending Capacity for L_b = 2000 mm:

$$M_a \; @ \; x_a = \frac{L - L_b}{2} + \frac{L_b}{4} = \frac{8-2}{2} + \frac{2}{4} = 3.5 \, \text{m}$$

$$M_a = \frac{\omega_f L x_a}{2} - \frac{\omega_f x_a^{\;2}}{2} = \frac{31.25 \bullet 8 \bullet 3.5}{2} - \frac{31.25 \bullet 3.5^2}{2} = 246.094 \, \text{kN-m}$$

$M_a = M_c$ = 246.094 kN-m @ 3500 mm and 4500 mm

$M_{\max} = M_b$ = 250 kN-m @ 4000 mm

$$\omega_2 = \frac{4 \bullet 250}{\sqrt{250^2 + 4 \bullet 246.094^2 + 7 \bullet 250^2 + 4 \bullet 246.094^2}} = 1.008$$

ω_2 = 1.008

$$M_u = \frac{\omega_2 \pi}{L}\sqrt{EI_{22}GJ + \left(\frac{\pi E}{L}\right)^2 I_{22}C_w}$$

$$M_u = \frac{1.008 \bullet \pi}{2000}\sqrt{\left(2 \bullet 10^5\right) \bullet \left(5.14 \bullet 10^6\right) \bullet 76923 \bullet \left(192 \bullet 10^3\right) + \left(\frac{\pi\left(2 \bullet 10^5\right)}{2000}\right)^2 \left(5.14 \bullet 10^6\right)\left(197.6 \bullet 10^9\right)}$$

$$M_u = 537.82 \bullet 10^6 \text{ N-mm} = 537.82 \text{ kN-m}$$

$$0.67 M_p = 0.67 \bullet 309.75 = 208 < M_u = 537.82 \text{ kN-m, so}$$

$$M_{r33} = 1.15\phi M_{p33}\left[1 - 0.28\frac{M_{p33}}{M_u}\right] \leq \phi M_{p33}$$

$$M_{r33} = 1.15 \bullet 0.9 \bullet 309.75\left[1 - 0.28\frac{309.75}{537.82}\right] = 268.89 \text{ kN-m} < 278.775 \text{ kN-m}$$

$$\boxed{M_{r33} = 268.89 \text{ kN-m}}$$

Section Capacity for W410x60:

$$M_p = F_y Z_{33} = 350 \bullet 1190{,}000 / 10^6 = 416.5 \text{ kN-m}$$

$$\phi M_p = 0.9 \bullet 416.5 = 374.85 \text{ kN-m}$$

Member Bending Capacity for L_b = 4000 m:

$$M_a \ @ \ x_a = \frac{L - L_b}{2} + \frac{L_b}{4} = \frac{8-4}{2} + \frac{4}{4} = 3 \text{m}$$

$$M_a = \frac{\omega_f L x_a}{2} - \frac{\omega_f x_a^{\ 2}}{2} = \frac{31.25 \bullet 8 \bullet 3}{2} - \frac{31.25 \bullet 3^2}{2} = 234.375 \text{ kN-m}$$

$M_a = M_c$ = 234.375 kN-m @ 3500 mm and 4500 mm

$M_{\max} = M_b$ = 250 kN-m @ 4000 mm

$$\omega_2 = \frac{4 \bullet 250}{\sqrt{250^2 + 4 \bullet 234.375^2 + 7 \bullet 250^2 + 4 \bullet 234.375^2}} = 1.032$$

ω_2 = 1.032

$$M_u = \frac{\omega_2 \pi}{L_y}\sqrt{EI_{22}GJ + \left(\frac{\pi E}{L}\right)^2 I_{22}C_w}$$

$$M_u = \frac{1.032 \bullet \pi}{4000}\sqrt{\begin{gathered}\left(2 \bullet 10^5\right) \bullet \left(12 \bullet 10^6\right) \bullet 76923 \bullet \left(328 \bullet 10^3\right) \\ + \left(\frac{\pi\left(2 \bullet 10^5\right)}{4000}\right)^2 \left(12 \bullet 10^6\right)\left(469.8 \bullet 10^9\right)\end{gathered}}$$

$$M_u = 362.06 \bullet 10^6 \text{ N-mm} = 362.06 \text{ kN-m}$$

$$0.67\,M_p = 0.67 \bullet 309.75 = 279 < M_u = 362.06 \text{ kN-m, so}$$

$$M_{r33} = 1.15\phi M_{p33}\left[1 - 0.28\frac{M_{p33}}{M_u}\right] \le \phi M_{p33}$$

$$M_{r33} = 1.15 \bullet 0.9 \bullet 416.5\left[1 - 0.28\frac{416.5}{362.06}\right]$$

$$\boxed{M_{r33} = 292.23 \text{ kN-m}}$$

CSA S16-14 Example 002

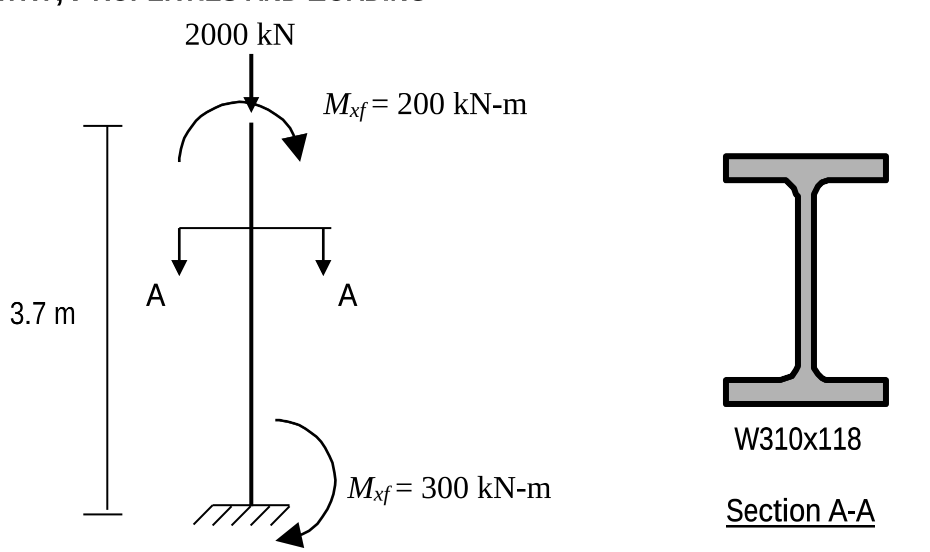
WIDE FLANGE MEMBER UNDER COMPRESSION & BENDING

EXAMPLE DESCRIPTION

The frame object axial and moment strengths are tested in this example.

A continuous column is subjected to factored loads and moments C_f = 2000 kN; $M_{fx\text{-top}}$= 200 kN-m; $M_{fx\text{-bottom}}$= 300 kN-m. This example was tested using the CSA S16-14 steel frame design code. The design capacities are compared with Handbook of Steel Construction (9th Edition) results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 200,000 MPa
v = 0.3
G= 76,923.08 MPa

Loading

C_f = 2000 kN
$M_{fx\text{-top}}$ = 200 kN-m
$M_{fx\text{-bottom}}$ = 300 kN-m

Design Properties

f_y = 345 MPa

TECHNICAL FEATURES TESTED

- Section compactness check (compression & bending)
- Member compression capacity
- Member bending capacity with no mid-span loading

RESULTS COMPARISON

Independent results are taken from Example 1 on page 4-114 of the Hand Book of Steel Construction to CSA S16-01 published by the Canadian Institute of Steel Construction.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Class 2	Class 2	0.00%
Axial Capacity, C_r (kN)	3849.5	3849.5	0.00%
Bending Capacity, M_{r33} (kN-m)	605.5	605.5	0.00%

COMPUTER FILE: CSA S16-14 EX002

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material:

f_y = 345 MPa

E = 200,000 MPa

G = 76923.08 MPa

Section: W310x118

A_g = 15000 mm^2

r_{33} = 135.4006 mm, r_{22} = 77.5457 mm

I_{22} = 90,200,000 mm^4

Z_{33} = 1,950,000 mm^3

J = 1,600,000 mm^4

$$C_w = 1.966 \bullet 10^{12} \text{ mm}^6$$

$$\overline{r}_o^{\,2} = \overline{x}_o^{\,2} + \overline{y}_o^{\,2} + r_{22}^{\,2} + r_{33}^{\,2} = 0^2 + 0^2 + 77.5457^2 + 135.4006^2$$

$$\overline{r}_o^{\,2} = 24346.658 \text{ mm}^2$$

Member:

$l_z = l_{e33} = l_{e22}$ = 3700 mm (unbraced length)

$k_z = k_{33} = k_{22} = 1.0$

$$\phi = 0.9$$

Loadings:

$$C_f = 2000 \text{ kN}$$

$$M_a = M_{xf,\text{top}} = 200 \text{ kN-m}$$

$$M_b = M_{xf,\text{bottom}} = 300 \text{ kN-m}$$

Section Compactness:

Localized Buckling for Flange:

$$\lambda_{Cl.1} = \frac{145}{\sqrt{F_y}} = \frac{145}{\sqrt{345}} = 7.81$$

$$\lambda_{Cl.2} = \frac{170}{\sqrt{F_y}} = \frac{170}{\sqrt{345}} = 9.15$$

$$\lambda = \frac{b_f}{2t_f} = \frac{307}{2 \bullet 18.7} = 8.21$$

$$\lambda_{Cl.1} < \lambda < \lambda_{Cl.2},$$

Flange is Class 2.

Localized Buckling for Web:

$$C_y = f_y A_g = \frac{345 \bullet 15000}{1000} = 5175\,\text{kN}$$

$$\lambda_{Cl.1} = \frac{1100}{\sqrt{F_y}}\left(1 - 0.39\frac{C_f}{C_y}\right) = \frac{1100}{\sqrt{345}}\left(1 - 0.39\frac{2000}{5175}\right) = 50.30$$

$$\lambda = \frac{h}{t_w} = \frac{276.6}{11.9} = 23.24$$

$$\lambda < \lambda_{Cl.1},$$

Web is Class 1.

Section is Class 2

Member Compression Capacity:

Flexural Buckling

n = 1.34 (wide flange section)

$$\lambda = \max(\lambda_{22}, \lambda_{33}) = \lambda_{22} = \frac{k_{22} l_{22}}{r_{22} \pi} \sqrt{\frac{f_y}{E}} = \frac{1.0 \bullet 3700}{77.5457} \sqrt{\frac{345}{200000}} = 0.6308$$

$$C_r = \phi A_g F_y \left(1 + \lambda^{2n}\right)^{-\frac{1}{n}} = 0.9 \bullet 15000 \bullet 345 \bullet \left(1 + 0.6308^{2 \bullet 1.34}\right)^{-\frac{1}{1.34}}$$

$$\boxed{C_r = 3489.5 \text{ kN}}$$

Torsional & Lateral-Torsional Buckling

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{k_{33} l_{33}}{r_{33}}\right)^2} = \frac{\pi^2 2 \bullet 10^5}{\left(\frac{1 \bullet 3700}{135.4006}\right)^2} = 2643 \text{ MPa}$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{k_{22} l_{22}}{r_{22}}\right)^2} = \frac{\pi^2 2 \bullet 10^5}{\left(\frac{1 \bullet 3700}{77.5457}\right)^2} = 867 \text{ MPa}$$

$$F_{ez} = \left(\frac{\pi^2 E C_w}{\left(k_z l_z\right)^2} + GJ\right) \frac{1}{A_g \bar{r}_o^2}$$

$$F_{ez} = \left(\frac{\pi^2 \bullet 2 \bullet 10^5 \bullet 1.966 \bullet 10^{12}}{\left(1 \bullet 3700\right)^2} + 76923.08 \bullet 1.6 \bullet 10^6\right) \frac{1}{15000 \bullet 24347}$$

$$F_{eZ} = 1113.222 \text{ MPa}$$

$$F_e = \min\left(F_{ex}, F_{ey}, F_{ez}\right) = F_{ey} = 867 \text{ MPa}$$

$$C_r = \phi A_g F_e \left(1 + \lambda^{2n}\right)^{-\frac{1}{n}} = 0.9 \bullet 15000 \bullet 867 \bullet \left(1 + 0.6308^{2 \bullet 1.34}\right)^{-\frac{1}{1.34}}$$

$$C_r = 9674.5 \text{ kN} \text{ (does not govern)}$$

Section Bending Capacity:

$$M_{p33} = Z_{33} F_y = 1{,}950{,}000 \bullet 345 = 672.75 \text{ kN-m}$$

Member Bending Capacity:

$$\omega_2 = 1.75 + 1.05\left(\frac{M_a}{M_b}\right) + 0.3\left(\frac{M_a}{M_b}\right)^2 \leq 2.5$$

$$\omega_2 = 1.75 + 1.05\left(\frac{200}{300}\right) + 0.3\left(\frac{200}{300}\right)^2 = 2.583 \leq 2.5$$

So $\omega_2 = 2.5$

$$M_u = \frac{\omega_2 \pi}{l_{22}}\sqrt{EI_{22}GJ + \left(\frac{\pi E}{l_{22}}\right)^2 I_{22}C_w}$$

$$M_u = \frac{2.5 \bullet \pi}{3700}\sqrt{2 \bullet 10^5 \bullet 9.02 \bullet 10^7 \bullet 76923.08 \bullet 1.6 \bullet 10^6 + \left(\frac{\pi \bullet 2 \bullet 10^5}{3700}\right)^2 9.02 \bullet 10^7 \bullet 1.966 \bullet 10^{12}}$$

$$M_u = 3163.117 \text{ kN-m}$$

Since $M_u > 0.67 \bullet M_{p33}$

$$M_{r33} = 1.15\phi M_{p33}\left[1 - 0.28\frac{M_{p33}}{M_u}\right] \leq \phi M_{p33}$$

$$M_{r33} = 1.15 \bullet 0.9 \bullet 672.75\left[1 - 0.28\frac{672.75}{3163.117}\right] \leq 0.9 \bullet 672.75$$

$$M_{r33} = 654.830 \leq 605.475$$

$$\boxed{M_{r33} = 605.5 \text{ kN-m}}$$

EN 3-2005 Example 001

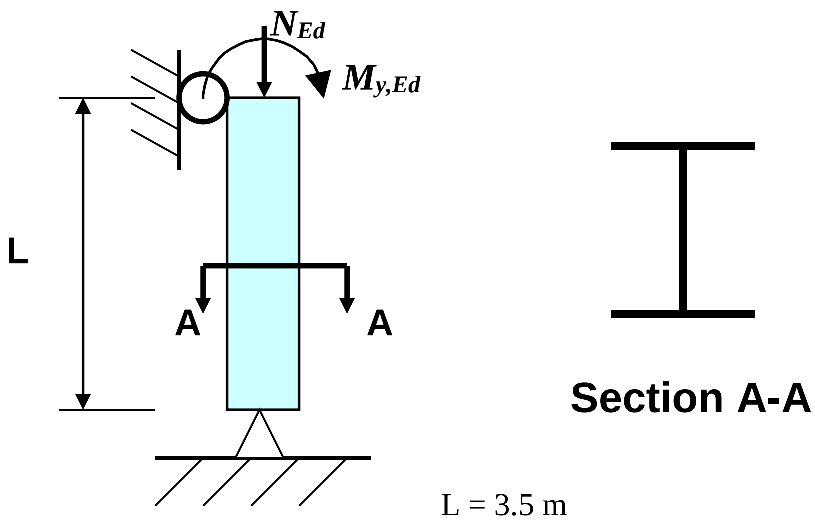
WIDE FLANGE MEMBER UNDER COMBINED COMPRESSION & BENDING

EXAMPLE DESCRIPTION

The frame object axial strengths are tested in this example considering in-plane behavior only.

A continuous column is subjected to factored load N = 210 kN and $M_{y,Ed}$ = 43 kN-m. This example was tested using the Eurocode 3-2005 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



L = 3.5 m

Material Properties

E = 210x10^3 MPa

v = 0.3

G = 80770 MPa

Loading

N = 210 kN

$M_{y,Ed}$ = 43 kN-m

Design Properties

f_y = 235 MPa

Section: IPE 200

TECHNICAL FEATURES TESTED

- Section compactness check (beam-column)
- Member interaction capacities, D/C, Method 1

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file “SFD-EC-3-2005.pdf,” which is available through the program “Help” menu. This example was taken from "New design rules in EN 1993-1-1 for member stability," Worked example 1 in section 5.2.1, page 151.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Class 1	Class 1	0.00%
D/C_{Axial}	0.334	0.334	0.00%
$D/C_{Bending}$	0.649	0.646	0.46%

COMPUTER FILE: EN 3-2005 EX001

CONCLUSION

The results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Material: S235

f_y = 235 MPa

E = 210,000 MPa

G = 80,770 MPa

Section: IPE 200

A = 2848 mm^2

h = 200 mm, b_f = 100 mm, t_f = 8.5 mm, t_w = 5.6 mm, r = 12 mm

$$h_w = h - 2t_f = 200 - 2 \bullet 85 = 183 \text{mm}$$

$$c = \frac{b_f - t_w - 2r}{2} = \frac{100 - 5.6 - 2 \bullet 12}{2} = 35.2 \text{mm}$$

I_{yy} = 19,430,000 mm^4

$W_{el,y}$ = 194,300 mm^3

$W_{pl,y}$ = 220,600 mm^3

Member:

L_{yy} = L_{zz} = 3,500 mm (unbraced length)

$\gamma_{M0} = 1$

$\gamma_{M1} = 1$

α_y = 0.21

Loadings:

$$N_{Ed} = 210,000 \text{ N}$$

$$M_{Ed,y,\text{Left}} = 0 \text{ N-m}$$

$$M_{Ed,y,\text{Right}} = 43000 \text{ N-m}$$

Section Compactness:

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{235}} = 1$$

$$-1 \le \alpha = \frac{1}{2}\left(1 - \frac{N_{Ed}}{2ht_w f_y}\right) \le 1$$

$$\alpha = \frac{1}{2}\left(1 - \frac{210,000}{2 \bullet 200 \bullet 5.6 \bullet 235}\right) = 0.6737$$

Localized Buckling for Flange:

For the tip in compression under combined bending and compression

$$\lambda_{cl.1} = \frac{9\varepsilon}{\alpha} = \frac{9 \bullet 1}{0.6737} = 13.36$$

$$\lambda_e = \frac{c}{t_f} = \frac{35.2}{8.5} = 4.14$$

$$\lambda_e = 4.14 < \lambda_{cl.1} = 13.36$$

So Flange is Class 1 in combined bending and compression

Localized Buckling for Web:

$\alpha > 0.5$, so

$$\lambda_{cl.1} = \frac{396\varepsilon}{13\alpha - 1} = \frac{396 \bullet 1}{13 \bullet 0.6737 - 1} = 51.05 \text{ for combined bending \& compression}$$

$$\lambda_e = \frac{d}{t_w} = \frac{183}{5.6} = 28.39$$

$$\lambda_e = 32.68 < \lambda_{cl.1} = 51.05$$

So Web is Class 1 in combined bending and compression

Since Flange and Web are Class 1, Section is Class 1.

Section Compression Capacity:

$$N_{c,Rk} = Af_y = 2.848 \bullet 10^{-3} \bullet 235 \bullet 10^6 = 669\,\text{kN}$$

Member Compression Capacity:

$$N_{cr,22} = \frac{\pi^2 EI_{22}}{L^2} = \frac{\pi^2 \bullet 210000 \bullet 10^6 \bullet 19.43 \bullet 10^{-6}}{3.5^2} = 3287\,\text{kN}$$

Section Bending Capacity:

$$M_{pl,y,Rk} = W_{pl,y} f_y = 220.6 \bullet 10^{-6} \bullet 235 \bullet 10^6 = 51.8\,\text{kN-m}$$

Interaction Capacities - Method 1:

Member Bending & Compression Capacity with Buckling

Compression Buckling Factors

$$\bar{\lambda}_y = \sqrt{\frac{Af_y}{N_{cr,y}}} = \sqrt{\frac{2.858 \bullet 10^{-3} \bullet 235 \bullet 10^6}{3287 \bullet 10^3}} = 0.451$$

$$\phi_y = 0.5\left[1 + \alpha_y\left(\bar{\lambda}_y - 0.2\right) + \bar{\lambda}_y^{\,2}\right] = 0.5\left[1 + 0.21 \bullet \left(0.451 - 0.2\right) + 0.451^2\right] = 0.628$$

$$\chi_y = \frac{1}{\left[\phi_y + \sqrt{\left(\phi_y^{\,2} - \bar{\lambda}_y^{\,2}\right)}\right]} = \frac{1}{\left[0.628 + \sqrt{\left(0.628^2 - 0.451^2\right)}\right]} = 0.939 \le 1$$

Auxiliary Terms

$$\mu_y = \frac{1 - \dfrac{N_{Ed}}{N_{cr,y}}}{1 - \chi_y \dfrac{N_{Ed}}{N_{cr,y}}} = \frac{1 - \dfrac{210}{3287}}{1 - 0.939\dfrac{210}{3287}} = 0.996$$

$$w_y = \frac{W_{pl,y}}{W_{el,y}} = \frac{220.6 \bullet 10^{-6}}{194.3 \bullet 10^{-6}} = 1.135 \le 1.5$$

C_{mo} Factor

$$\psi_y = \frac{M_{Ed,y,\text{right}}}{M_{Ed,y,\text{left}}} = \frac{0}{43 \bullet 10^3} = 0$$

$$C_{my,0} = 0.79 + 0.21\psi_y + 0.36\left(\psi_y - 0.33\right)\frac{N_{Ed}}{N_{cr,y}}$$

$$C_{my,0} = 0.79 + 0.21 \bullet 0 + 0.36\left(0 - 0.33\right)\frac{210}{3287} = 0.782$$

$C_{my} = C_{my,0} = 0.782$ because no LTB is likely to occur.

Elastic-Plastic Bending Resistance

Because LTB is prevented, $b_{LT} = 0$ so $a_{LT} = 0$

$$C_{yy} = 1 + \left(w_y - 1\right)\left[\left(2 - \frac{1.6}{w_y}C_{my}{}^2\bar{\lambda}_{22} - \frac{1.6}{w_y}C_{my}{}^2\bar{\lambda}_y{}^2\right)\frac{N_{Ed}}{\frac{N_{c,Rk}}{\gamma_{M1}}} - b_{LT}\right]$$

$$C_{yy} = 1 + \left(1.135 - 1\right)\left[\left(2 - \frac{1.6}{1.135} \bullet 0.782^2 \bullet 0.451 - \frac{1.6}{1.135} \bullet 0.782^2 \bullet 0.451^2\right)\frac{210 \bullet 10^3}{\frac{669 \bullet 10^3}{1.0}} - 0\right]$$

$$C_{yy} = 1.061 \geq \frac{W_{el,y}}{W_{pl,y}} = \frac{194.3 \bullet 10^{-6}}{220.6 \bullet 10^{-6}} = 0.881$$

$$D/C_{\text{Axial}} = \frac{N_{Ed}}{\chi_y \frac{N_{c,Rk}}{\gamma_{M1}}} = \frac{210 \bullet 10^3}{0.939\frac{669 \bullet 10^3}{1}}$$

$$\boxed{D/C_{\text{Axial}} = 0.334}$$

$$D/C_{\text{Bending}} = \mu_y \left[\frac{C_{my} M_{Ed,y,\text{right}}}{\left(1 - \frac{N_{Ed}}{N_{cr,y}}\right) C_{yy} \frac{M_{pl,y,Rk}}{\gamma_{M1}}} \right] = 0.996 \left[\frac{0.782 \bullet 43 \bullet 10^3}{\left(1 - \frac{210 \bullet 10^3}{3287 \bullet 10^3}\right) 1.061 \frac{51.8 \bullet 10^3}{1}} \right]$$

$$\boxed{D/C_{\text{Bending}} = 0.646}$$

$$D/C_{\text{Total}} = 0.980$$

EN 3-2005 Example 002

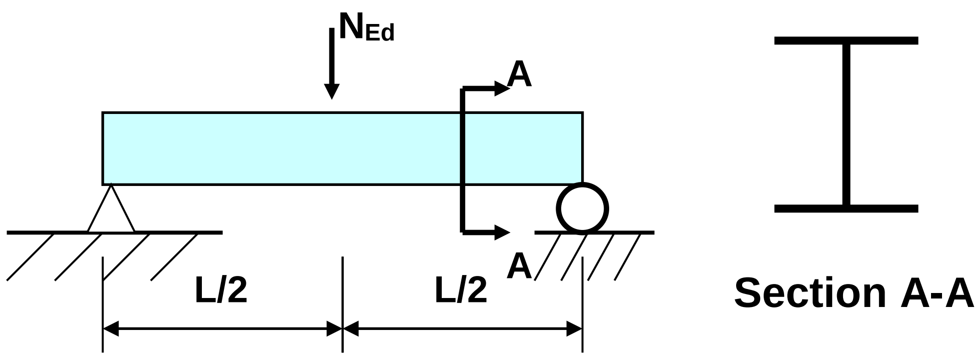
WIDE FLANGE SECTION UNDER BENDING

EXAMPLE DESCRIPTION

The frame object axial strengths are tested in this example.

A beam is subjected to factored load N = 1050 kN. This example was tested using the Eurocode 3-2005 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



L = 1.4 m

Material Properties
E = 210×10^3 MPa
v = 0.3
G = 80770 MPa

Loading
N = 1050 kN

Design Properties
f_y = 275 MPa
Section: 406x178x74 UB

TECHNICAL FEATURES TESTED

- Section compactness (beam)
- Section shear capacity
- Section bending capacity with shear reduction

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file "SFD-EC-3-2005.pdf," which is available through the program "Help" menu. Examples were taken from Example 6.5 on pp. 53-55 from the book "Designers' Guide to EN1993-1-1" by R.S. Narayanan & A. Beeby.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Class 1	Class 1	0.00%
Section Shear Resistance, $V_{pl,Rd}$ (kN)	689.2	689.2	0.00%
Section Bending Resistance, $M_{c,y,Rd}$ (kN-m)	412.8	412.8	0.00%
Section Shear-Reduced Bending Resistance, $M_{v,y,Rd}$ (kN-m)	386.8	386.8	0.00%

COMPUTER FILE: EN 3-2005 EX002

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material: S275 Steel

f_y = 275 MPa

E = 210000 MPa

Section: 406x178x74 UB

A = 9450 mm^2

b = 179.5 mm, t_f = 16 mm, h = 412.8 mm, t_w = 9.5 mm, r = 10.2 mm

$$h_w = h - 2t_f = 412.8 - 2 \bullet 16 = 380.8 \text{ mm}$$

$$d = h - 2\left(t_f + r\right) = 412.8 - 2 \bullet \left(16 + 10.2\right) = 360.4 \text{ mm}$$

$$c = \frac{b - t_w - 2r}{2} = \frac{179.5 - 9.5 - 2 \bullet 10.2}{2} = 74.8 \text{ mm}$$

$W_{pl,y}$ = 501,000 mm^3

Other:

$$\gamma_{M0} = 1.0$$

$$\eta = 1.2$$

Loadings:

$$N_{Ed} = 0 \text{ kN}$$

$N = 1050$ kN @ mid-span

Results in the following internal forces:

$$V_{Ed} = 525 \text{ kN}$$

$$M_{Ed} = 367.5 \text{ kN-m}$$

Section Compactness:

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0.924$$

<u>Localized Buckling for Flange:</u>

$$\lambda_{cl.1} = 9\varepsilon = 9 \bullet 0.924 = 8.32 \text{ for pure compression}$$

$$\lambda_e = \frac{c}{t_f} = \frac{74.8}{16} = 4.68$$

$$\lambda_e = 4.68 < \lambda_{cl.1} = 8.32$$

So Flange is Class 1 in pure compression

<u>Localized Buckling for Web:</u>

$$\lambda_{cl.1} = 72\varepsilon = 72 \bullet 0.924 = 66.56 \text{ for pure bending}$$

$$\lambda_e = \frac{d}{t_w} = \frac{360.4}{9.5} = 37.94$$

$$\lambda_e = 37.94 < \lambda_{cl.1} = 66.56$$

So Web is Class 1 in pure bending

Since Flange & Web are Class 1, Section is Class 1.

Section Shear Capacity

$$A_{v-\min} = \eta h_w t_w = 1.2 \bullet 380.8 \bullet 9.5 = 4341 \,\text{mm}^2$$

$$A_v = A - 2bt_f + (t_w + 2r)t_f = 9450 - 2 \bullet 179.5 \bullet 16 + \left(9.5 + 2 \bullet 10.2\right) \bullet 16$$

$$A_v = 4021.2 \,\text{mm}^2 < A_{v-\min}$$

$$So \;\; A_v = 4341 \,\text{mm}^2$$

$$V_{pl,Rd} = \frac{A_v}{\gamma_{M0}}\left(\frac{f_y}{\sqrt{3}}\right) = \frac{4341}{1.0}\left(\frac{275}{\sqrt{3}}\right) = 689,245\,N$$

$$\boxed{V_{pl,Rd} = 689.2\,\text{kN}}$$

Section Bending Capacity

$$M_{c,y,Rd} = \frac{W_{pl,y}\,f_y}{\gamma_{M0}} = \frac{1501,000 \bullet 275}{1} = 412,775,000\,\text{N-mm}$$

$$\boxed{M_{c,y,Rd} = 412.8\,\text{kN-m}}$$

With Shear Reduction:

$$\rho = \left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1\right)^2 = \left(\frac{2 \bullet 525}{689.2} - 1\right)^2 = 0.27$$

$$A_w = h_w t_w = 380.8 \bullet 9.5 = 3617.6\,\text{mm}^2$$

$$M_{v,y,Rd} = \frac{f_y}{\gamma_{M0}}\left(W_{pl,y} - \frac{\rho A_w{}^2}{4t_w}\right) = \frac{275}{1.0}\left(1,501,000 - \frac{0.27 \bullet 3617.6^2}{4 \bullet 9.5}\right)$$

$$M_{v,y,Rd} = 386,829,246\,\text{N-mm}$$

$$\boxed{M_{v,y,Rd} = 386.8\,\text{kN-m}}$$

EN 3-2005 Example 003

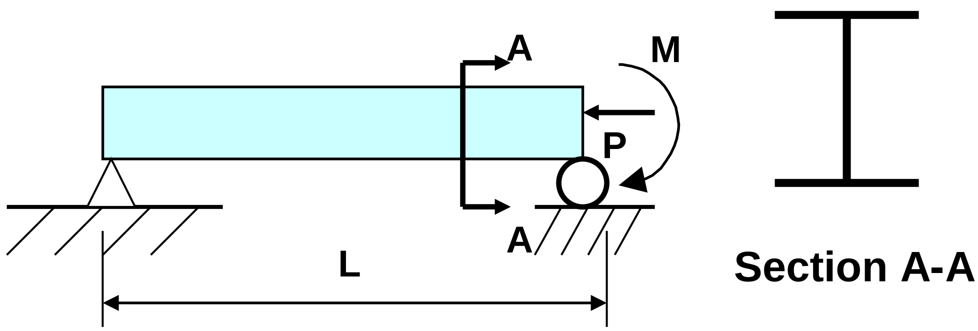
WIDE FLANGE SECTION UNDER COMBINED COMPRESSION & BENDING

EXAMPLE DESCRIPTION

The frame object axial strengths are tested in this example.

A continuous beam-column is subjected to factored axial load P = 1400 kN and major-axis bending moment M = 200 kN-m. This example was tested using the Eurocode 3-2005 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



L = 0.4 m

Material Properties
- E = 210x10[3] MPa
- v = 0.3
- G = 80769 MPa

Loading
- P = 1400 kN
- M = 200 kN-m

Design Properties
- f_y = 235 MPa
- Section: 457x191x98 UB

TECHNICAL FEATURES TESTED

- Section compactness check (beam-column)
- Section compression capacity
- Section bending capacity with compression reduction

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file "SFD-EC-3-2005.pdf", which is also available through the program "Help" menu. Examples were taken from Example 6.6 on pp. 57-59 from the book "Designers' Guide to EN1993-1-1" by R.S. Narayanan & A. Beeby.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Class 2	Class 2	0.00%
Section Compression Resistance, $N_{pl,Rd}$ (kN)	2937.5	2937.5	0.00%
Section Plastic Bending Resistance, $M_{pl,y,Rd}$ (kN-m)	524.1	524.5	-0.08%
Section Reduced Bending Resistance, $M_{n,y,Rd}$ (kN-m)	341.9	342.2	-0.09%

COMPUTER FILE: EN 3-2005 EX003

CONCLUSION

The results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Material: S275 Steel

E = 210000 MPa

f_y = 235 MPa

Section: 457x191x98 UB

A = 12,500 mm^2

b = 192.8 mm, t_f = 19.6 mm, h = 467.2 mm, t_w = 11.4 mm, r = 10.2 mm

$$h_w = h - 2t_f = 467.2 - 2 \bullet 19.6 = 428 \text{ mm}$$

$$d = h - 2\left(t_f + r\right) = 467.2 - 2 \bullet \left(19.6 + 10.2\right) = 407.6 \text{ mm}$$

$$c = \frac{b - t_w - 2r}{2} = \frac{192.8 - 11.4 - 2 \bullet 10.2}{2} = 80.5 \text{ mm}$$

$W_{pl,y}$ = 2,232,000 mm^3

Other:

$$\gamma_{M0} = 1.0$$

Loadings:

$P = 1400$ kN axial load

Results in the following internal forces:

$N_{Ed} = 1400$ kN

M = 200 kN-m

Section Compactness:

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{235}} = 1$$

$$-1 \le \alpha = \frac{1}{2}\left(1 - \frac{N_{Ed}}{2ht_w f_y}\right) \le 1$$

$$\alpha = \frac{1}{2}\left(1 - \frac{1,400,000}{2 \bullet 467.2 \bullet 11.4 \bullet 235}\right) = 2.7818 > 1, \text{ so}$$

$$\alpha = 1.0$$

Localized Buckling for Flange:

For the tip in compression under combined bending & compression

$$\lambda_{cl.1} = \frac{9\varepsilon}{\alpha} = \frac{9 \bullet 1}{1} = 9$$

$$\lambda_e = \frac{c}{t_f} = \frac{80.5}{19.6} = 4.11$$

$$\lambda_e = 4.11 < \lambda_{cl.1} = 9$$

So Flange is Class 1 in combined bending and compression

Localized Buckling for Web:

$$\alpha > 0.5, \text{ so}$$

$$\lambda_{cl.1} = \frac{396\varepsilon}{13\alpha - 1} = \frac{396 \bullet 1}{13 \bullet 1 - 1} = 33.00 \text{ for combined bending \& compression}$$

$$\lambda_e = \frac{d}{t_w} = \frac{407.6}{11.4} = 35.75$$

$$\lambda_e = 35.75 > \lambda_{cl.1} = 33.00$$

$$\lambda_{cl.2} = \frac{456\varepsilon}{13\alpha - 1} = \frac{456 \bullet 1}{13 \bullet 1 - 1} = 38.00$$

$$\lambda_e = 35.75 < \lambda_{cl.2} = 38.00$$

So Web is Class 2 in combined bending & compression.

Since Web is Class 2, Section is Class 2 in combined bending & compression.

Section Compression Capacity

$$N_{pl,Rd} = \frac{Af_y}{\gamma_{M0}} = \frac{12,500 \bullet 235}{1}$$

$$\boxed{N_{pl,Rd} = 2937.5 \text{ kN}}$$

Section Bending Capacity

$$M_{pl,y,Rd} = \frac{W_{pl,y} f_y}{\gamma_{M0}} = \frac{2,232,000 \bullet 235}{1}$$

$$\boxed{M_{pl,y,Rd} = 524.5 \text{ kN-m}}$$

<u>Axial Reduction</u>

$$N_{Ed} = 1400\text{kN} > 0.25 N_{pl,Rd} = 0.25 \bullet 2937.5 = 734.4 \text{ kN}$$

$$N_{Ed} = 1400\text{kN} > 0.5 \frac{h_w t_w f_y}{\gamma_{M0}} = 0.5 \bullet \frac{428 \bullet 11.4 \bullet 235}{1} = 573.3 \text{ kN}$$

So moment resistance must be reduced.

$$n = \frac{N_{Ed}}{N_{pl,Rd}} = \frac{1400}{2937.5} = 0.48$$

$$a = \frac{A - 2bt_f}{A} = \frac{12,500 - 2 \bullet 192.8 \bullet 19.6}{12,500} = 0.40$$

$$M_{N,y,Rd} = M_{pl,y,Rd} \frac{1-n}{1-0.5a} = 524.5 \bullet \frac{1-0.48}{1-0.5 \bullet 0.4}$$

$$\boxed{M_{N,y,Rd} = 342.2 \text{ kN-m}}$$

IS 800-2007 Example 001

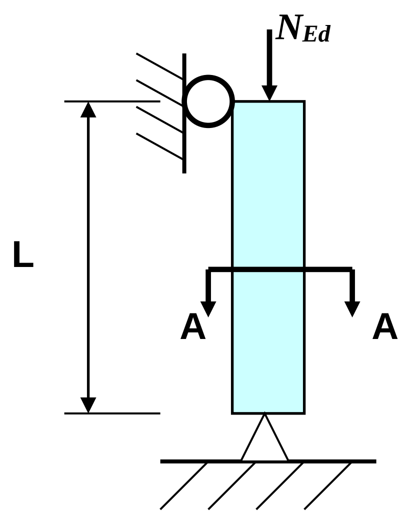
WIDE FLANGE MEMBER UNDER COMPRESSION

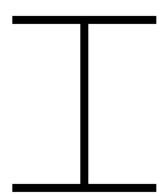
EXAMPLE DESCRIPTION

The frame object axial strengths are tested in this example.

A continuous column is subjected to factored load N = 1 kN. This example was tested using the Indian IS 800:2007 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING





Section A-A

L = 3m

Material Properties
E = 200x10^3 MPa
ν = 0.3
G = 76923 MPa

Loading
N = 1 kN

Design Properties
f_y = 250 MPa
f_u = 410 MPa
Section: ISMB 350

TECHNICAL FEATURES TESTED

- Section compactness check (column)
- Member compression capacity

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file "SFD-IS-800-2007.pdf," which is available through the program "Help" menu. The example was taken from Example 9.2 on pp. 765-766 in "Design of Steel Structures" by N. Subramanian.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Plastic	Plastic	0.00%
Design Axial Strength, N_{crd}	733.85	734.07	-0.03%

COMPUTER FILE: IS 800-2007 EX001

CONCLUSION

The results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Material: Fe 250

E = 200,000 MPa

f_y = 250 MPa

Section: ISMB 350

A = 6670 mm^2

b = 140 mm, t_f = 14.2 mm, d = 350 mm, t_w = 8.1 mm, r = 1.8 mm

$$h = d - 2\left(t_f + r\right) = 350 - 2\left(14.2 + 1.8\right) = 318 \text{ mm}$$

r_y = *28.4* mm, r_z = *143* mm

Member:

$KL_y = KL_z$ = 3,000 mm (unbraced length)

$$\gamma_{M0} = 1.1$$

Loadings:

$$N_{Ed} = 1 \text{ kN}$$

Section Compactness:

$$\varepsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

Localized Buckling for Flange:

$$\lambda_p = 8.4\varepsilon = 8.4 \bullet 1 = 8.4$$

$$\lambda_e = \frac{b}{t_f} = \frac{70}{14.2} = 4.93$$

$$\lambda_e = 4.93 < \lambda_p = 8.40$$

So Flange is Plastic in compression

Localized Buckling for Web:

$$\lambda_p = N/A \;\; \& \;\; \lambda_s = 42\varepsilon = 42 \text{ for compression}$$

$$\lambda_e = \frac{d}{t_w} = \frac{318}{8.1} = 39.26$$

$$\lambda_e = 39.26 < \lambda_s = 42$$

So Web is Plastic in compression

Since Flange & Web are Plastic, Section is Plastic.

Member Compression Capacity:

Non-Dimensional Slenderness Ratio:

$$\frac{h}{b_f} = \frac{350}{140} = 2.5 > 1.2$$

and

$$t_f = 14.2\,\text{mm} < 40\,\text{mm}$$

So we should use the Buckling Curve ‘a’ for the z-z axis and Buckling Curve ‘b’ for the y-y axis (IS 7.1.1, 7.1.2.1, Table 7).

Z-Z Axis Parameters:

For buckling curve a, $\alpha = 0.21$ (IS 7.1.1, 7.1.2.1, Table 7)

Euler Buckling Stress: $$f_{cc} = \frac{\pi^2 E}{\left(\frac{K_z L_z}{r_z}\right)^2} = \frac{\pi^2 200,000}{\left(\frac{3,000}{143}\right)^2} = 4485\,\text{MPa}$$

$$\lambda_z = \sqrt{\frac{f_y}{f_{cc}}} = \sqrt{\frac{250}{4485}} = 0.2361$$

$$\phi = 0.5\left[1+\alpha\left(\lambda-0.2\right)+\lambda^2\right] = 0.5\left[1+0.21\left(0.2361-0.2\right)+0.2361^2\right]$$

$$\phi = 0.532$$

Stress Reduction Factor: $\chi = \dfrac{1}{\phi+\sqrt{\phi^2-\lambda^2}} = \dfrac{1}{0.532+\sqrt{0.532^2-0.2361^2}} = 0.9920$

$$f_{cd,z} = \chi\frac{f_y}{\gamma_{M0}} = 0.992 \bullet \frac{250}{1.1} = 255.5\,\text{MPa}$$

Y-Y Axis Parameters:

For buckling curve b, $\alpha = 0.34$ (IS 7.1.1, 7.1.2.1, Table 7)

Euler Buckling Stress: $f_{cc} = \dfrac{\pi^2 E}{\left(\dfrac{K_z L_z}{r_z}\right)^2} = \dfrac{\pi^2\,200{,}000}{\left(\dfrac{3{,}000}{28.4}\right)^2} = 177\,\text{MPa}$

$$\lambda_y = \sqrt{\frac{f_y}{f_{cc}}} = \sqrt{\frac{250}{177}} = 1.189$$

$$\phi = 0.5\left[1+\alpha\left(\lambda-0.2\right)+\lambda^2\right] = 0.5\left[1+0.34\left(1.189-0.2\right)+1.189^2\right]$$

$$\phi = 1.375$$

Stress Reduction Factor: $\chi = \dfrac{1}{\phi+\sqrt{\phi^2-\lambda^2}} = \dfrac{1}{1.375+\sqrt{1.375^2-1.189^2}} = 0.4842$

$$f_{cd,y} = \chi\frac{f_y}{\gamma_{M0}} = 0.4842 \bullet \frac{250}{1.1} = 110.1\,\text{MPa} \quad \text{Governs}$$

$$P_d = Af_{cd,y} = 6670 \bullet 110.1$$

$$\boxed{P_d = 734.07\,\text{kN}}$$

IS 800-2007 Example 002

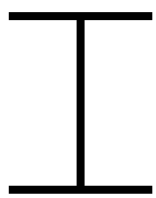
WIDE FLANGE MEMBER UNDER BENDING

EXAMPLE DESCRIPTION

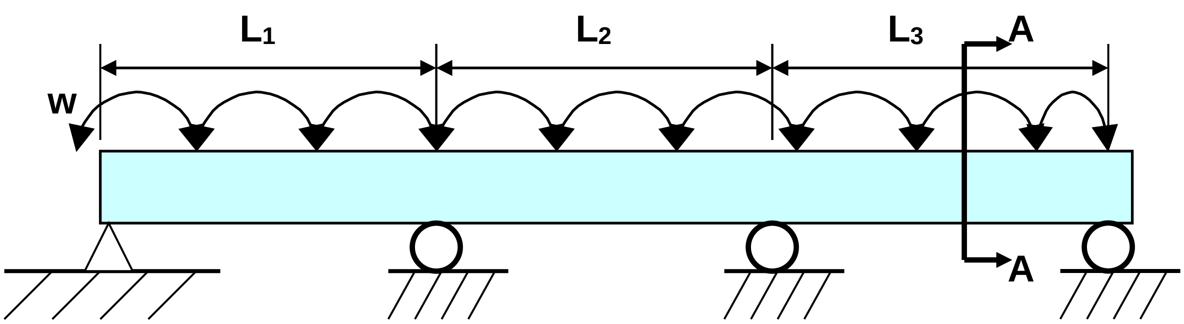
The frame object axial strengths are tested in this example.

A continuous beam is subjected to factored distributed load w = 48.74 kN/m. This example was tested using the Indian IS 800:2007 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



Section A-A



L_1 = 4.9 m **L_2 = 6 m** **L_3 = 4.9 m**

<u>Material Properties</u>

E = 200x10^3 MPa

v = 0.3

G = 76923 MPa

<u>Loading</u>

w = 48.74 kN/m

<u>Design Properties</u>

f_y = 250 MPa

Section: ISLB 500

TECHNICAL FEATURES TESTED

- Section compactness check (beam)
- Section shear capacity
- Member bending capacity

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file "SFD-IS-800-2007.pdf," which is available through the program "Help" menu. The example was taken from Example 10.8 on pp. 897-901 in "Design of Steel Structures" by N. Subramanian. The torsional constant, I_t, is calculated by the program as a slightly different value, which accounts for the percent different in section bending resistance.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Plastic	Plastic	0.00%
Section Bending Resistance, $M_{d(LTB)}$ (kN-m)	157.70	157.93	0.14%
Section Shear Resistance, V_d (kN)	603.59	603.59	0.00%

COMPUTER FILE: IS 800-2007 EX002

CONCLUSION

The results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Material: Fe 250

E = 200,000 MPa

G = 76,923 MPa

f_y = 250 MPa

Section: ISLB 500

(Note: In ETABS, the section is not available with original example properties, including fillet properties. A similar cross-section with fillet r = 0 was used, with similar results, shown below.)

A = 9550 mm^2

h = 500 mm, b_f = 180 mm, t_f = 14.1 mm, t_w = 9.2 mm

$$b = \frac{b_f}{2} = \frac{180}{2} = 90\,\text{mm}$$

$$d = h - 2\left(t_f + r\right) = 500 - 2\left(14.1 + 0\right) = 471.8\,\text{mm}$$

I_z = 385,790,000 mm^4, I_y = 10,639,000.2 mm^4

Z_{ez} = 1,543,160 mm^3, Z_{pz} = 1,543,200 mm^3

r_y = 33.4 mm

Member:

L_{left} = 4.9 m

L_{center} = 6 m (governs)

L_{right} = 4.9 m

KL_y = KL_z = 6,000 mm (unbraced length)

$\gamma_{M0} = 1.1$

Loadings:

$$N_{Ed} = 0 \text{ kN}$$

$$\omega = 48.75 \text{ kN/m}$$

Section Compactness:

$$\varepsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

r_1 = 0 since there is no axial force

Localized Buckling for Flange:

$$\lambda_p = 9.4\varepsilon = 9.4 \bullet 1 = 9.4$$

$$\lambda_e = \frac{b}{t_f} = \frac{90}{14.1} = 6.38$$

$$\lambda_e = 6.38 < \lambda_p = 9.40$$

So Flange is Plastic in pure bending

Localized Buckling for Web:

$$\lambda_p = \frac{84\varepsilon}{(1+r_1)} = \frac{84 \bullet 1}{(1+0)} = 84$$

$$\lambda_e = \frac{d}{t_w} = \frac{471.8}{9.2} = 51.28$$

$$\lambda_e = 51.28 < \lambda_p = 84.00$$

So Web is Plastic in pure bending

Since Flange & Web are Class 1, Section is Plastic.

Section Shear Capacity:

$$V_d = \frac{f_y}{\gamma_{M0}\sqrt{3}} h t_w = \frac{250}{1.1\sqrt{3}} \bullet 500 \bullet 9.2$$

$$\boxed{V_d = 603.59\,\text{kN}}$$

Member Bending Capacity

$$h_f = h - t_f = 500 - 14.1 = 485.9$$

$$I_t = \sum \frac{b_i t_i^3}{3} = \frac{2 b_f t_f^{\,3}}{3} + \frac{d_i t_w^{\,3}}{3} = \frac{2 \bullet 180 \bullet 14.1^3}{3} + \frac{485.9 \bullet 9.2^3}{3} = 4.63 \bullet 10^5\ \text{mm}^4$$

From Roark & Young, 5th Ed., 1975, Table 21, Item 7, pg.302

$t_1 = t_2 = t_f$ and $b_1 = b_2 = b_f$ for symmetric sections

$$I_w = \frac{h_f^{\,2} t_1 t_2 b_1^3 b_2^3}{12\left(t_1 b_1^3 + t_2 b_2^3\right)} = \frac{485.9^2 \bullet 14.1 \bullet 14.1 \bullet 180^3 \bullet 180^3}{12 \bullet \left(14.2 \bullet 180^3 + 14.2 \bullet 180^3\right)} = 8.089 \bullet 10^{11}\text{mm}^6$$

C_1 = 1.0 (Assumed in example and specified in ETABS)

$$M_{cr} = C_1 \sqrt{\frac{\pi^2 E I_y}{(KL)^2}\left(G I_t + \frac{\pi^2 E I_w}{(KL)^2}\right)}$$

$$M_{cr} = 1.0\sqrt{\frac{\pi^2 \bullet 200{,}000 \bullet 10{,}639{,}000.2}{(6{,}000)^2}\left(76{,}923 \bullet 462{,}508 + \frac{\pi^2 \bullet 200{,}000 \bullet 8.089 \bullet 10^{11}}{(6{,}000)^2}\right)}$$

$$M_{cr} = 215{,}936{,}919.3\,\text{N-mm}$$

$$\alpha_{LT} = 0.21$$

$$\beta_b = 1.0$$

$$\lambda_{LT} = \sqrt{\frac{\beta_b Z_{pz} f_y}{M_{cr}}} = \sqrt{\frac{1 \bullet 1{,}543{,}200 \bullet 250}{215{,}936{,}919.3}} = 1.337$$

$$\phi_{LT} = 0.5\left[1+\alpha_{LT}\left(\lambda_{LT}-0.2\right)+\lambda_{LT}{}^{2}\right] = 0.5\left[1+0.21\bullet\left(1.337-0.2\right)+1.337^{2}\right]$$

$$\phi_{LT} = 1.5127$$

$$\chi_{LT} = \frac{1}{\phi_{LT}+\sqrt{\phi_{LT}{}^{2}-\lambda_{LT}{}^{2}}} \leq 1.0$$

$$\chi_{LT} = \frac{1}{1.5127+\sqrt{1.5127^{2}-1.337^{2}}} = 0.450 \leq 1.0$$

$$f_{bd} = \frac{\chi_{LT} f_y}{\gamma_{M0}} = \frac{0.450\bullet 250}{1.1} = 102.3\,\text{MPa}$$

$$M_{d,LTB} = Z_{pz} f_{bd} = 1543.2\bullet 10^{3}\bullet 102.3 = 157,925,037.7\ \text{N-mm}$$

$$\boxed{M_{d,LTB} = 157.93\text{kN-m}}$$

IS 800-2007 Example 003

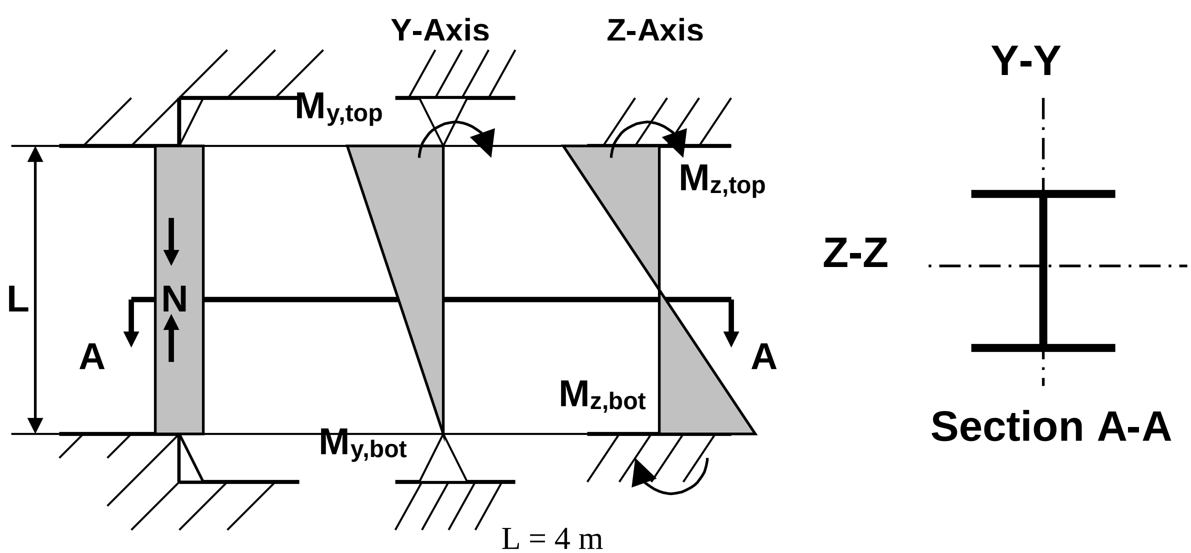
WIDE FLANGE MEMBER UNDER COMBINED COMPRESSION & BIAXIAL BENDING

EXAMPLE DESCRIPTION

The frame object axial strengths are tested in this example.

In this example a beam-column is subjected factored distributed load N = 2500 kN, M_z = 350 kN-m, and M_y = 100 kN-m. The element is moment-resisting in the z-direction and pinned in the y-direction. This example was tested using the Indian IS 800:2007 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties
E = 200x10^3 MPa
v = 0.3
G = 76923.08 MPa

Loading
N = 2500 kN
$M_{z,top}$ = 350 kN-m
$M_{z,bot}$ = -350 kN-m
$M_{y,top}$ = 100 kN-m
$M_{y,bot}$ = 0

Design Properties
f_y = 250 MPa
Section: *W310x310x226*

TECHNICAL FEATURES TESTED

- Section Compactness Check (Beam-Column)
- Section Compression Capacity
- Section Shear Capacity for Major & Minor Axes
- Section Bending Capacity for Major & Minor Axes
- Member Compression Capacity for Major & Minor Axes
- Member Bending Capacity for Major & Minor Axes
- Interaction Capacity, D/C, for Major & Minor Axes

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file "SFD-IS-800-2007.pdf", which is also available through the program "Help" menu. The example was taken from Example 13.2 on pp. 1101-1106 in "Design of Steel Structures" by N. Subramanian.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Plastic	Plastic	0.00%
Plastic Compression Resistance, N_d (kN)	6520	6520	0.00%
Buckling Resistance in Compression, P_{dz} (kN)	6511	6511	0.00%
Buckling Resistance in Compression, P_{dy} (kN)	5295	5295	0.00%
Section Bending Resistance, M_{dz} (kN-m)	897.46	897.46	0.00%
Section Bending Resistance, M_{dy} (kN-m)	325.65	325.65	0.00%
Buckling Resistance in Bending, M_{dLTB} (kN-m)	886.84	886.84	0.00%
Section Shear Resistance, V_{dy} (kN)	1009.2	1009.2	0.00%
Section Shear Resistance, V_{dz} (kN)	2961.6	2961.6	0.00%
Interaction Capacity, D/C	1.050	1.050	0.00%

COMPUTER FILE: IS 800-2007 EX003

CONCLUSION

The results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Material: *Fe 410*

$E = 200{,}000\ MPa$

$G = 76{,}923.08\ MPa$

$f_y = 250\ MPa$

Section: *W310x310x226*

$A = 28{,}687.7\ mm^2$

$b_f = 317\ mm,\ t_f = 35.6\ mm,\ h = 348\ mm,\ t_w = 22.1\ mm,\ r = 0\ mm$

$$b = \frac{b_f}{2} = \frac{317}{2} = 158.5\,mm,$$

$$d = h - 2\left(t_f + r\right) = 348 - 2\left(35.6 + 0\right) = 276.8\,mm$$

$I_z = 592{,}124{,}221\ mm^4,\ I_y = 189{,}255{,}388.9\ mm^4$

$r_z = 143.668\ mm,\ r_y = 81.222\ mm$

$Z_{ez} = 3{,}403{,}012.8\ mm^3,\ Z_{ey} = 1{,}194{,}040.3\ mm^3$

$Z_{pz} = 3{,}948{,}812.3\ mm^3,\ Z_{py} = 1{,}822{,}502.2\ mm^3$

$I_t = 10{,}658{,}941.4\ mm^6,\ I_w = 4.611 \bullet 10^{12}\ mm^6$

Member:

$L_y = L_z = 4{,}000\ mm$ *(unbraced length)*

$\gamma_{M0} = 1.1$

Loadings:

$P = 2500\ kN$

$V_z = 25\,kN$

$V_y = 175\,kN$

$$M_{z-1} = 350\, kN - m$$

$$M_{z-2} = -350\, kN - m$$

$$M_{y-1} = 100\, kN - m$$

$$M_{y-2} = 0\, kN - m$$

Section Compactness:

$$\varepsilon = \sqrt{\frac{f_y}{250}} = \sqrt{\frac{250}{250}} = 1$$

$$r_1 = \frac{P}{dt_w \frac{f_y}{\gamma_{mo}}} = \frac{2,500,000}{246.8 \bullet 22.1 \bullet \frac{2.5}{1.1}} = 2.01676$$

Localized Buckling for Flange:

$$\lambda_p = 9.4\varepsilon = 9.4 \bullet 1 = 9.4$$

$$\lambda_e = \frac{b}{t_f} = \frac{158.5}{35.6} = 4.45$$

$$\lambda_e = 4.45 < \lambda_p = 9.40$$

So Flange is Plastic in pure bending

Localized Buckling for Web:

$$\lambda_p = \frac{84\varepsilon}{(1 + r_1)} = \frac{84 \bullet 1}{(1 + 2.01676)} = 27.84$$

$$\lambda_e = \frac{d}{t_w} = \frac{246.8}{22.1} = 11.20$$

$$\lambda_e = 11.20 < \lambda_p = 27.84$$

So Web is Plastic in bending & compression

Section is Plastic.

Section Compression Capacity:

$$N_d = \frac{A_g f_y}{\gamma_{M0}} = \frac{28687.7 \bullet 250}{1.1}$$

$$N_d = 6520\, kN$$

Section Shear Capacity:

For major z-z axis

$$A_{vz} = ht_w = 348 \bullet 22.1 = 7690.8\, mm^2$$

$$V_{Pz} = \frac{f_y}{\gamma_{M0}\sqrt{3}} A_{vz} = \frac{250}{1.1\sqrt{3}} \bullet 7690.8$$

$$V_{Pz} = 1009.2\, kN$$

For minor y-y axis

$$A_{vy} = 2b_f t_f = 2 \bullet 317 \bullet 35.6 = 22,570.4\, mm^2$$

$$V_{Py} = \frac{f_y}{\gamma_{M0}\sqrt{3}} A_{vy} = \frac{250}{1.1\sqrt{3}} \bullet 22570.4$$

$$V_{Py} = 2961.6\, kN$$

Section Bending Capacity:

For major z-z axis

$$M_{dz} = \frac{\beta_b Z_{pz} f_y}{\gamma_{M0}} = \frac{1 \bullet 3,948,812.3 \bullet 250}{1.1} \le \frac{1.2 Z_{ez} f_y}{\gamma_{M0}} = \frac{1.2 \bullet 3,403,012.8 \bullet 250}{1.1}$$

$$M_{dz} = 897.46\, kN-m \le 933.54\, kN-m$$

$$M_{dz} = 897.46\ kN-m$$

For minor y-y axis

$$M_{dy} = \frac{\beta_b Z_{py} f_y}{\gamma_{M0}} = \frac{1 \bullet 1,822,502.2 \bullet 250}{1.1} \leq \frac{1.2 Z_{ey} f_y}{\gamma_{M0}} = \frac{1.2 \bullet 1,194,040.3 \bullet 250}{1.1}$$

$$M_{dy} = 414.21\, kN-m \leq 325.65\, kN-m$$

$$M_{dy} = 325.65\ kN-m$$

With Shear Reduction:

For major z-z axis

$V_z = 25\, kN < 0.6 V_{Pz} = 0.6 \bullet 1009.2 = 605.5\, kN$ *No shear reduction is needed.*

For minor y-y axis

$V_y = 175\, kN < 0.6 V_{Py} = 0.6 \bullet 2961.6 = 1777\, kN$ *No shear reduction is needed.*

With Compression Reduction:

$$n = \frac{P}{N_d} = \frac{2500}{6520} = 0.383$$

For major z-z axis

$$M_{ndz} = 1.11 M_{dz} \left(1-n\right) = 1.11 \bullet 897.46 \left(1-0.383\right) \leq M_{dz}$$

$$M_{ndz} = 614.2\, kN-m < 897.46\, kN-m$$

For minor y-y axis, since $n > 0.2$

$$M_{ndy} = 1.56 M_{dy} \left(1-n\right)\left(n+0.6\right) = 1.56 \bullet 325.65 \left(1-0.383\right)\left(0.383+0.6\right)$$

$$M_{ndy} = 308.0\, kN-m$$

Member Compression Capacity:

Non-Dimensional Slenderness Ratio:

$$\frac{h}{b_f} = \frac{348}{317} = 1.1 < 1.2$$

and

$$t_f = 35.6\,mm < 40\,mm$$

So we should use the Buckling Curve 'b' for the z-z axis and Buckling Curve 'c' for the y-y axis (IS 7.1.1, 7.1.2.1, Table 7).

Z-Z Axis Parameters:

For buckling curve b, $\alpha = 0.34$ *(IS 7.1.1, 7.1.2.1, Table 7)*

$$K_z = 0.65$$

$$K_z L_z = 0.65 \bullet 4000 = 2600\,mm, \ \frac{K_z L_z}{r_z} = \frac{2600}{143.668} = 18.097$$

Euler Buckling Stress: $f_{cr,z} = \dfrac{\pi^2 E}{\left(\dfrac{K_z L_z}{r_z}\right)^2} = \dfrac{\pi^2 \bullet 200,000}{(18.097)^2} = 6027\,MPa$

$$\lambda_z = \sqrt{\frac{f_y}{f_{cr,z}}} = \sqrt{\frac{250}{6022}} = 0.2037$$

$$\phi_z = 0.5\left[1 + \alpha_z\left(\lambda_z - 0.2\right) + \lambda_z^{\,2}\right] = 0.5\left[1 + 0.34\left(0.2037 - 0.2\right) + 0.2037^2\right]$$

$$\phi_z = 0.5214$$

Stress Reduction Factor: $\chi_z = \dfrac{1}{\phi_z + \sqrt{\phi_z^{\,2} - \lambda_z^{\,2}}} = \dfrac{1}{0.5214 + \sqrt{0.5214^2 - 0.2037^2}} = 0.9987$

$$f_{cd,z} = \chi \frac{f_y}{\gamma_{M0}} = 0.9987 \bullet \frac{250}{1.1} = 226.978\,MPa$$

$$P_{dz} = f_{cd,z} A_g = 226.978 \bullet 28,687.7$$

$P_{dz} = 6511 kN$

Y-Y Axis Parameters:

For buckling curve c, $\alpha = 0.49$ *(IS 7.1.1, 7.1.2.1, Table 7)*

$K_y = 1.00$

$K_y L_y = 1 \bullet 4000 = 4000\,mm, \quad \frac{K_y L_y}{r_y} = \frac{4000}{81.222} = 49.25$

Euler Buckling Stress: $f_{cr,y} = \frac{\pi^2 E}{\left(\frac{K_y L_y}{r_y}\right)^2} = \frac{\pi^2 \bullet 200,000}{(49.25)^2} = 813.88\,MPa$

$\lambda_y = \sqrt{\frac{f_y}{f_{cr,y}}} = \sqrt{\frac{250}{813.88}} = 0.5542$

$\phi_y = 0.5\left[1 + \alpha_y\left(\lambda_y - 0.2\right) + {\lambda_y}^2\right] = 0.5\left[1 + 0.49\left(0.5542 - 0.2\right) + 0.5542^2\right]$

$\phi_y = 0.7404$

Stress Reduction Factor: $\chi_y = \frac{1}{\phi_y + \sqrt{{\phi_y}^2 - {\lambda_y}^2}} = \frac{1}{0.7404 + \sqrt{0.7404^2 - 0.5542^2}} = 0.8122$

$f_{cd,y} = \chi \frac{f_y}{\gamma_{M0}} = 0.8122 \bullet \frac{250}{1.1} = 184.584\,MPa$

$P_{dy} = f_{cd,y} A_g = 184.584 \bullet 28,687.7$

$P_{dy} = 5295\,kN$

Member Bending Capacity:

C_1 = 2.7 (Program Calculation from AISC equation, where $C_1 \leq 2.7$)

$$M_{cr} = C_1 \sqrt{\frac{\pi^2 EI_y}{(KL)^2}\left(GI_t + \frac{\pi^2 EI_w}{(KL)^2}\right)}$$

$$M_{cr} = 2.7\sqrt{\frac{\pi^2 \bullet 200,000 \bullet 189,300,000}{(4,000)^2}\left(76,923.08 \bullet 10,658,941.4 + \frac{\pi^2 \bullet 200,000 \bullet 4.611 \bullet 10^{12}}{(4,000)^2}\right)}$$

$$M_{cr} = 15,374,789,309\ N-mm$$

$$\alpha_{LT} = 0.21$$

$$\beta_b = 1.0$$

$$\lambda_{LT} = \sqrt{\frac{\beta_b Z_{pz} f_y}{M_{cr}}} = \sqrt{\frac{1 \bullet 3,948,812.3 \bullet 250}{15,374,789,309}} = 0.2534$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\lambda_{LT} - 0.2\right) + \lambda_{LT}{}^2\right] = 0.5\left[1 + 0.21 \bullet \left(0.2534 - 0.2\right) + 0.2534^2\right]$$

$$\phi_{LT} = 0.5377$$

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}{}^2 + \lambda_{LT}{}^2}} \leq 1.0$$

$$\chi_{LT} = \frac{1}{0.5377 + \sqrt{0.5377^2 + 0.2534^2}} = 0.9882 \leq 1.0$$

$$f_{bd} = \frac{\chi_{LT} f_y}{\gamma_{M0}} = \frac{0.9882 \bullet 250}{1.1} = 224.58\,MPa$$

$$M_{dLTB} = Z_{pz} f_{bd} = 3,948,812.3 \bullet 224.58 = 886,839,489\ N-mm$$

$$M_{dLTB} = 886.84\,kN-m$$

Interaction Capacity: Compression & Bending

Member Bending & Compression Capacity with Buckling

Z-Z Axis

$$n_z = \frac{P}{P_{dz}} = \frac{2500}{6511} = 0.3839$$

$$K_z = 1 + \left(\lambda_z - 0.2\right) n_z = 1 + \left(0.2037 - 0.2\right) \bullet 0.3839 \le 1 + 0.8 n_z = 1 + 0.8 \bullet \left(0.3839\right)$$
$$K_z = 1.0014 \le 1.3072 \;\; so \;\; K_z = 1.0014$$

$$\psi_z = \frac{M_2}{M_1} = \frac{-350}{350} = -1$$

$$C_{mz} = 0.6 + 0.4\psi = 0.6 + 0.4 \bullet -1 = 0.2 > 0.4 \;\; so \;\; C_{mz} = 0.4$$

Y-Y Axis

$$n_y = \frac{P}{P_{dy}} = \frac{2500}{5295} = 0.4721$$

$$K_y = 1 + \left(\lambda_y - 0.2\right) n_y = 1 + \left(0.554 - 0.2\right) \bullet 0.4721 \le 1 + 0.8 n_y = 1 + 0.8 \bullet \left(0.4721\right)$$

$$K_y = 1.167 \le 1.378 \, so \;\; K_y = 1.167$$

$$\psi_y = \frac{M_2}{M_1} = \frac{0}{100} = 0$$

$$C_{my} = 0.6 + 0.4\psi = 0.6 + 0.4 \bullet 0 = 0.6 > 0.4 \;\; so \;\; C_{my} = 0.6$$

Lateral-Torsional Buckling

$$C_{mLT} = 0.4$$

$$K_{LT} = 1 - \frac{0.1 \lambda_{LT} n_y}{C_{mLT} - 0.25} \ge 1 - \frac{0.1 n_y}{C_{mLT} - 0.25}$$

$$K_{LT}=1-\frac{0.1\bullet 0.2534\bullet 0.4721}{0.4-0.25}=0.920\geq 1-\frac{0.1\bullet 0.4721}{0.4-0.25}=0.831$$

$$K_{LT}=0.920$$

Formula IS 9.3.2.2 (a)

$$\frac{D}{C}=\frac{P}{P_{dy}}+\frac{K_y C_{my} M_y}{M_{dy}}+\frac{K_{LT} M_z}{M_{dLTB}}=\frac{2500}{5295}+\frac{1.167\bullet 0.6\bullet 100}{325.65}+\frac{0.920\bullet 350}{886.84}$$

$$\frac{D}{C}=0.472+0.215+0.363$$

$$\frac{D}{C}=1.050\ \text{(Governs)}$$

Formula IS 9.3.2.2 (b)

$$\frac{D}{C}=\frac{P}{P_{dz}}+\frac{0.6 K_y C_{my} M_y}{M_{dy}}+\frac{K_z C_{mz} M_z}{M_{dLTB}}=\frac{2500}{6511}+\frac{0.6\bullet 1.167\bullet 0.6\bullet 100}{325.65}+\frac{1.0014\bullet 0.4\bullet 350}{886.84}$$

$$\frac{D}{C}=0.384+0.129+0.158$$

$$\frac{D}{C}=0.671$$

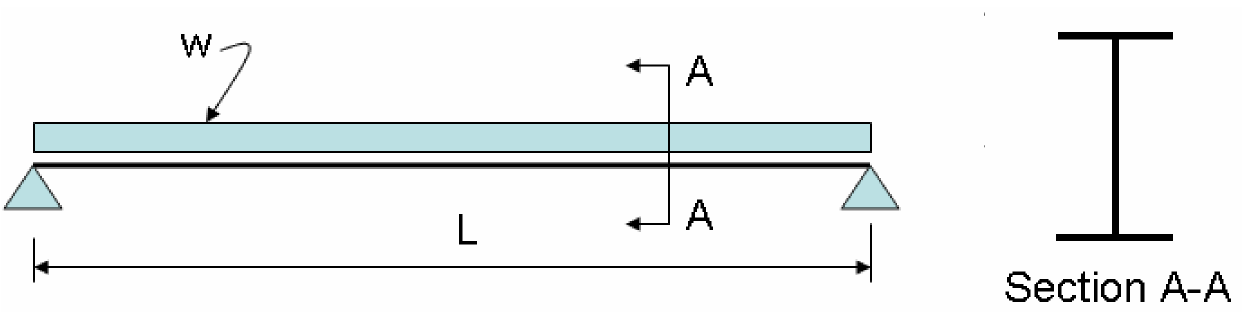
KBC 2009 Example 001

WIDE FLANGE MEMBER UNDER BENDING

EXAMPLE DESCRIPTION

The design flexural strengths are checked for the beam shown below. The beam is loaded with a uniform load of 6.5 kN/m (D) and 11 kN/m (L). The flexural moment capacity is checked for three unsupported lengths in the weak direction, L_b = 1.75 m, 4 m and 12 m.

GEOMETRY, PROPERTIES AND LOADING



Member Properties
W460x74
E = 205,000 MPa
F_y = 345 MPa

Loading
w = 6.5 kN/m (D)
w = 11.0 kN/m (L)

Geometry
Span, L = 12m

TECHNICAL FEATURES TESTED

- Section Compactness Check (Bending)
- Member Bending Capacities
- Unsupported length factors

RESULTS COMPARISON

Independent results are comparing with the results of ETABS.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Compact	Compact	0.00%
C_b (L_b =1.75m)	1.004	1.002	0.20%
$\phi_b M_n$ (L_b =1.75m) (kN-m)	515.43	515.43	0.00%
C_b (L_b =4m)	1.015	1.014	0.10%
$\phi_b M_n$ (L_b =4m) (kN-m)	394.8	394.2	0.15%
C_b (L_b =12m)	1.136	1.136	0.00%
$\phi_b M_n$ (L_b =12m) (kN-m)	113.48	113.45	0.03%

COMPUTER FILE: KBC 2009 Ex001

CONCLUSION

The results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Material:

E = 205,000 MPa, F_y = 345 MPa

Section: W460x74

b_f = 191 mm, t_f = 14.5 mm, d = 457 mm, t_w = 9 mm

$$h = d - 2t_f = 457 - 2 \bullet 14.5 = 428 \text{ mm}$$

$$h_0 = d - t_f = 457 - 14.5 = 442.5 \text{ mm}$$

S_{33} = 1457.3 cm^3, Z_{33} = 1660 cm^3

I_y =1670 cm^4, r_y = 42 mm, C_w = 824296.4 cm^6, J = 51.6 cm^4

$$r_{ts} = \sqrt{\frac{\sqrt{I_y C_w}}{S_{33}}} = \sqrt{\frac{\sqrt{1670 \bullet 824296.4}}{1457.3}} = 50.45 \text{ mm}$$

$R_m = 1.0$ *for doubly-symmetric sections*

Other:

c = 1.0

L = 12 m

Loadings:

w_u = (1.2w_d + 1.6w_l) = 1.2(6.5) + 1.6(11) = 25.4 kN/m

$$M_u = \frac{w_u L^2}{8} = 25.4 \bullet 12^2/8 = 457.2 \text{ kN-m}$$

Section Compactness:

Localized Buckling for Flange:

$$\lambda = \frac{b_f}{2t_f} = \frac{191}{2 \bullet 14.5} = 6.586$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{205,000}{345}} = 9.263$$

$\lambda < \lambda_p$, *No localized flange buckling*

Flange is Compact.

<u>*Localized Buckling for Web:*</u>

$$\lambda = \frac{h}{t_w} = \frac{428}{9} = 47.56$$

$$\lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{205,000}{345}} = 91.654$$

$\lambda < \lambda_p$, *No localized web buckling*

Web is Compact.

Section is Compact.

Section Bending Capacity:

$$M_p = F_y Z_{33} = 345 \bullet 1660 = 572.7 \text{ kN-m}$$

Lateral-Torsional Buckling Parameters:

<u>*Critical Lengths:*</u>

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76 \bullet 42\sqrt{\frac{205,000}{345}} = 1801.9 \text{ mm} = 1.8 \text{ m}$$

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_{33}h_o}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7F_y}{E}\frac{S_{33}h_o}{Jc}\right)^2}}$$

$$L_r = 1.95 \bullet 50.45 \frac{205,000}{0.7 \bullet 345}\sqrt{\frac{51.6 \bullet 1}{1457.3 \bullet 44.25}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7 \bullet 345}{205,000}\frac{1457.3 \bullet 44.8}{51.6 \bullet 1}\right)^2}}$$

$L_r = 5.25\,\text{m}$

Non-Uniform Moment Magnification Factor:

For the lateral-torsional buckling limit state, the non-uniform moment magnification factor is calculated using the following equation:

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} R_m \le 3.0 \qquad \textit{Eqn. 1}$$

Where M_A = first quarter-span moment, M_B = mid-span moment, M_C = second quarter-span moment.

The required moments for Eqn. 1 can be calculated as a percentage of the maximum mid-span moment. Since the loading is uniform and the resulting moment is symmetric:

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2$$

Member Bending Capacity for L_b = 1.75 m:

$$M_{\max} = M_B = 1.00$$

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2 = 1 - \frac{1}{4}\left(\frac{1.75}{12}\right)^2 = 0.995$$

$$C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.995) + 4(1.00) + 3(0.995)}$$

$$C_b = 1.002$$

$L_b < L_p$, Lateral-Torsional buckling capacity is as follows:

$$M_n = M_p = 572.7\,\text{kN-m}$$

$$\phi_b M_n = 0.9 \bullet 572.7$$

$$\phi_b M_n = 515.43\ \text{kN-m}$$

Member Bending Capacity for L_b = 4 m:

$$M_{\max} = M_B = 1.00$$

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2 = 1 - \frac{1}{4}\left(\frac{4}{12}\right)^2 = 0.972$$

$$C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.972) + 4(1.00) + 3(0.972)}$$

$$C_b = 1.014$$

$L_p < L_b < L_r$, Lateral-Torsional buckling capacity is as follows:

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_{33}\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

$$M_n = 1.014\left[572.7 - (572.7 - 0.7 \bullet 0.345 \bullet 1457.3)\left(\frac{4.00 - 1.80}{5.25 - 1.80}\right)\right] = 437.97 \text{ kN-m}$$

$$\phi_b M_n = 0.9 \bullet 437.97$$

$$\phi_b M_n = 394.2 \text{ kN-m}$$

Member Bending Capacity for L_b = 12 m:

$$M_{\max} = M_B = 1.00$$

$$M_A = M_C = 1 - \frac{1}{4}\left(\frac{L_b}{L}\right)^2 = 1 - \frac{1}{4}\left(\frac{12}{12}\right)^2 = 0.750.$$

$$C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.750) + 4(1.00) + 3(0.750)}(1.00)$$

$$C_b = 1.136$$

$L_b > L_r$, Lateral-Torsional buckling capacity is as follows:

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_{33} h_o} \left(\frac{L_b}{r_{ts}}\right)^2}$$

$$F_{cr} = \frac{1.136 \bullet \pi^2 \bullet 205,000}{\left(\frac{12000}{50.45}\right)^2} \sqrt{1 + 0.078 \frac{51.6 \bullet 1}{1457.3 \bullet 44.25} \left(\frac{12000}{50.45}\right)^2} = 86.5\,\text{MPa}$$

$$M_n = F_{cr} S_{33} \le M_p$$

$$M_n = 86.5 \bullet 1457.3 = 126.056\text{kN-m}$$

$$\phi_b M_n = 0.9 \bullet 126.056$$

$$\phi_b M_n = 113.45 \text{ kN-m}$$

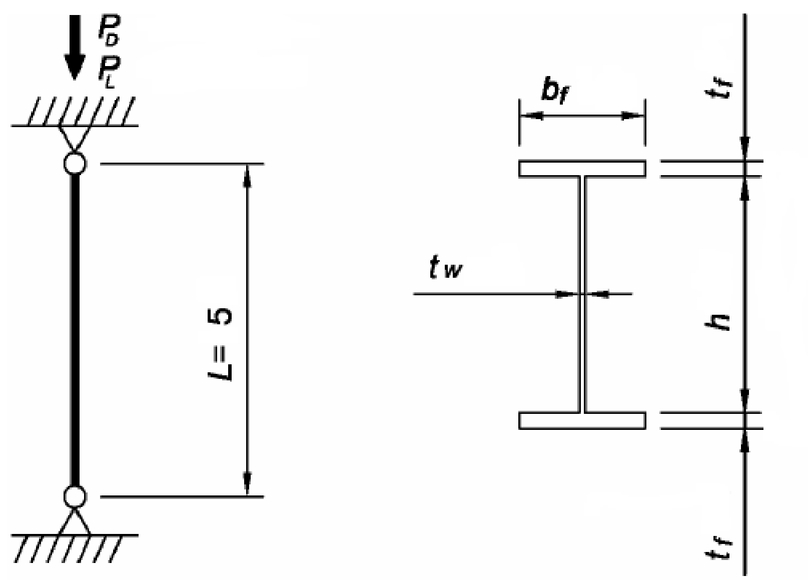
KBC 2009 Example 002

BUILT UP WIDE FLANGE MEMBER UNDER COMPRESSION

EXAMPLE DESCRIPTION

A demand capacity ratio is calculated for the built-up, ASTM A572 grade 50, column shown below. An axial load of 300 kips (D) and 900 kips (L) is applied to a simply supported column with a height of 5m.

GEOMETRY, PROPERTIES AND LOADING



TECHNICAL FEATURES TESTED

- Section compactness check (compression)
- Warping constant calculation, C_w
- Member compression capacity with slenderness reduction

RESULTS COMPARISON

Independent results are hand calculated and compared with the results from ETABS.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Slender	Slender	0.00%
$\phi_c P_n$ (kN)	2056.7	2056.7	0.00 %

COMPUTER FILE: KBC 2009 EX002

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material:

E = 205,000 MPa, F_y =*345* MPa

Section: Built-Up Wide Flange

d = 432 mm, b_f = 203 mm, t_f = 25 mm, h = 382 mm, t_w = 7 in.

Ignoring fillet welds:

$$A = 2(203)(25) + (382)(7) = 128.24 \text{ cm}^2$$

$$I_y = \frac{2(25)(203)^3}{12} + \frac{(382)(7)^3}{12} = 34.867E06 \text{ mm}^3$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{34.867E06}{12824}} = 52.1 \text{ mm.}$$

$$I_x = \sum Ad^2 + \sum I_x$$

$$C_w = 1443463.1 \text{ cm}^6$$

$$J = \sum \frac{bt^3}{3} = 216.1\ cm^4$$

Member:

K = 1.0 for a pinned-pinned condition
L = 5 m

Loadings:

P_u = 1.2(300) + 1.6(700) = 1800 kN

Section Compactness:

Check for slender elements using Specification KBC 2009:

Localized Buckling for Flange:

$$\lambda = \frac{b}{t} = \frac{101.5}{25} = 4.06$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{205,000}{345}} = 9.263$$

$\lambda < \lambda_p$, No localized flange buckling

Flange is Compact.

Localized Buckling for Web:

$$\lambda = \frac{h}{t} = \frac{382}{7} = 54.57,$$

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{205,000}{345}} = 36.32$$

$\lambda > \lambda_r$, Localized web buckling

Web is Slender.

Section is Slender

Member Compression Capacity:

Elastic Flexural Buckling Stress

Since the unbraced length is the same for both axes, the *y*-*y* axis will govern by inspection.

$$\frac{KL_y}{r_y} = \frac{1.0 \bullet 5000}{52.1} = 95.97$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \bullet 205,000}{(95.97)^2} = 219.68 \text{ MPa}$$

Elastic Critical Torsional Buckling Stress

Note: Torsional buckling will not govern if $KL_y > KL_z$, however, the check is included here to illustrate the calculation.

$$F_e = \left[\frac{\pi^2 EC_w}{(K_z L)^2} + GJ\right]\frac{1}{I_x + I_y}$$

$$F_e = \left[\frac{\pi^2 \bullet 205{,}000 \bullet 1443463.1E06}{(5000)^2} + 78846.15 \bullet 216.1E04\right]\frac{1}{(45338 + 3486.7)E04}$$

= 588 MPa > 288.84 MPa

Therefore, the flexural buckling limit state controls.

F_e = 220 MPa

Section Reduction Factors

Since the flange is not slender,

Q_s = 1.0

Since the web is slender,

Take f as F_{cr} with Q = 1.0

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{205{,}000}{1.0(345)}} = 114.8 > \frac{KL_y}{r_y} = 95.97$$

So

$$f = F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y = 1.0\left[0.658^{\frac{1.0(345)}{220}}\right] \bullet 345 = 178.98\,\text{MPa}$$

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{(b/t)}\sqrt{\frac{E}{f}}\right] \le b, \text{ where } b = h$$

$$b_e = 1.92(7)\sqrt{\frac{205{,}000}{178.98}}\left[1 - \frac{0.34}{(382/7)}\sqrt{\frac{205{,}000}{178.98}}\right] \le 359.12\,\text{mm}$$

$$b_e = 359.12\,\text{mm} \le 382\,\text{mm}$$

therefore compute A_{eff} with reduced effective web width.

$$A_{eff} = b_e t_w + 2b_f t_f = (359.12)(7) + 2(203)(25) = 12663.84\ \text{mm}^2$$

where A_{eff} is effective area based on the reduced effective width of the web, b_e.

$$Q_a = \frac{A_{eff}}{A} = \frac{12663.84}{12824} = 0.9875$$
$$Q = Q_s Q_a = (1.00)(0.9875) = 0.9875$$

Critical Buckling Stress
Determine whether Specification Equation E7-2 or E7-3 applies

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{205,000}{0.9875(345)}} = 138.47 > \frac{KL_y}{r_y} = 95.97$$

Therefore, Specification Equation E7-2 applies.

When $4.71\sqrt{\frac{E}{QF_y}} \geq \frac{KL}{r}$

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y = 0.9875\left[0.658^{\frac{0.9875(345)}{220}}\right] \bullet 345 = 178.2\,\text{MPa}$$

Nominal Compressive Strength

$$P_n = F_{cr}A_g = 12824 \bullet 178.2 = 2285236.8\,\text{N}$$
$$\phi_c = 0.90$$
$$\phi_c P_n = F_{cr}A_g = 0.90(2285.24) = 2056.7\,\text{kN} > 1800\text{ kN}$$

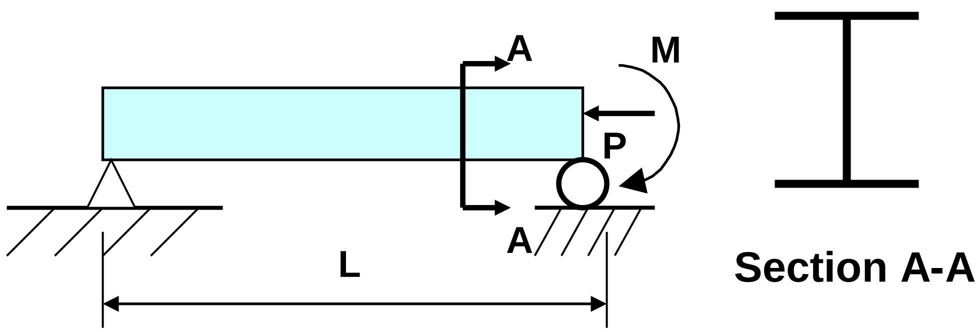
NTC 2008 Example 001

WIDE FLANGE SECTION UNDER COMBINED COMPRESSION & BENDING

EXAMPLE DESCRIPTION

In this example a continuous beam-column is subjected to factored axial load P = 1400 kN and major-axis bending moment M = 200 kN-m. The beam is continuously braced to avoid any buckling effects. This example was tested using the Italian NTC-2008 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



L = 0.4 m

Material Properties
E = $210\text{x}10^3$ MPa
v = 0.3
G = 80769 MPa

Loading
P = 1400 kN
M = 200 kN-m

Design Properties
f_y = 235 MPa
Section: *457x191x98 UB*

TECHNICAL FEATURES TESTED

- Section compactness check (beam-column)
- Section compression capacity
- Section shear capacity
- Section bending capacity with compression & shear reductions
- Interaction capacity, D/C

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file "SFD-NTC-2008.pdf," which is available through the program "Help" menu. Examples were taken from Example 6.6 on pp. 57-59 from the book "Designers' Guide to EN1993-1-1" by R.S. Narayanan & A. Beeby.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Class 2	Class 2	0.00%
Section Compression Resistance, $N_{c,Rd}$ (kN)	2797.6	2797.6	0.00%
Section Shear Resistance, $V_{c,Rd,y}$ (kN)	667.5	667.5	0.00%
Section Plastic Bending Resistance, $M_{c,y,Rd}$ (kN-m)	499.1	499.1	0.00%
Section Bending Resistance Axially Reduced, $M_{N,y,Rd}$ (kN-m)	310.8	310.8	0.00%
Section Bending Resistance Shear Reduced, $M_{V,y,Rd}$ (kN-m)	470.1	470.1	0.00%
Interaction Capacity, D/C	0.644	0.644	0.00%

COMPUTER FILE: NTC 2008 EX001

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material: S275 Steel

E = 210000 MPa

f_y = 235 MPa

Section: 457x191x98 UB

A = 12,500 mm^2

b = 192.8 mm, t_f = 19.6 mm, h = 467.2 mm, t_w = 11.4 mm, r = 0 mm

$$h_w = h - 2t_f = 467.2 - 2 \bullet 19.6 = 428 \text{ mm}$$

$$d = h - 2\left(t_f + r\right) = 467.2 - 2 \bullet \left(19.6 + 0\right) = 428 \text{ mm}$$

$$c = \frac{b - t_w - 2r}{2} = \frac{192.8 - 11.4 - 2 \bullet 0}{2} = 90.7 \text{ mm}$$

$W_{pl,y}$ = 2,230,000 mm^3

Other:

$$\gamma_{M0} = 1.05$$

Loadings:

$P = 1400$ kN axial load

$M_y = 200$ kN-m bending load at one end

Results in the following internal forces:

$$N_{Ed} = 1400 \text{ kN}$$

$$V_{Ed} = 500 \text{ kN}$$

$$M_{y,Ed} = 200 \text{ kN-m}$$

Section Compactness:

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{235}} = 1$$

$$-1 \le \alpha = \frac{1}{2}\left(1 - \frac{N_{Ed}}{2ht_w f_y}\right) \le 1$$

$$\alpha = \frac{1}{2}\left(1 - \frac{1,400,000}{2 \bullet 467.2 \bullet 11.4 \bullet 235}\right) = 2.7818 > 1, \text{ so}$$

$$\alpha = 1.0$$

Localized Buckling for Flange:

For the tip in compression under combined bending & compression

$$\lambda_{cl.1} = \frac{9\varepsilon}{\alpha} = \frac{9 \bullet 1}{1} = 9$$

$$\lambda_e = \frac{c}{t_f} = \frac{90.7}{19.6} = 4.63$$

$$\lambda_e = 4.63 < \lambda_{cl.1} = 9$$

So Flange is Class 1 in combined bending and compression

Localized Buckling for Web:

$$\alpha > 0.5, \ so$$

$$\lambda_{cl.1} = \frac{396\varepsilon}{13\alpha - 1} = \frac{396 \bullet 1}{13 \bullet 1 - 1} = 33.00$$ for combined bending & compression

$$\lambda_e = \frac{d}{t_w} = \frac{428}{11.4} = 37.54$$

$$\lambda_e = 37.54 > \lambda_{cl.1} = 33.00$$

$$\lambda_{cl.2} = \frac{456\varepsilon}{13\alpha - 1} = \frac{456 \bullet 1}{13 \bullet 1 - 1} = 38.00$$

$$\lambda_e = 37.54 < \lambda_{cl.2} = 38.00$$

So Web is Class 2 in combined bending & compression.

Since Web is Class 2, Section is Class 2 in combined bending & compression.

Section Compression Capacity

$$N_{c,Rd} = N_{pl,Rd} = \frac{Af_y}{\gamma_{M0}} = \frac{12,500 \bullet 235}{1.05}$$

$$\boxed{N_{c,Rd} = 2797.6\,\text{kN}}$$

Section Shear Capacity

$$A_{V,y} = A - 2bt_f + t_f\left(t_w + 2r\right) = 12,500 - 2 \bullet 192.8 \bullet 19.6 + 19.6\left(11.4 + 2 \bullet 0\right)$$

$$A_{V,y} = 5,165.7\,\text{mm}^2$$

$$V_{c,Rd,y} = \frac{f_y}{\gamma_{M0}\sqrt{3}} A_{vy} = \frac{235}{1.05\sqrt{3}} \bullet 5,165.7$$

$$\boxed{V_{c,Rd,y} = 667.5\,\text{kN}}$$

$$\eta = 1.0$$

$$\frac{h_w}{t_w} = \frac{428}{11.4} = 37.5 < \frac{72}{\eta}\sqrt{\frac{235}{f_y}} = \frac{72}{1.0}\sqrt{\frac{235}{235}} = 72$$

So no shear buckling needs to be checked.

Section Bending Capacity

$$M_{c,y,Rd} = M_{pl,y,Rd} = \frac{W_{pl,y} f_y}{\gamma_{M0}} = \frac{2,230,000 \bullet 235}{1.05}$$

$$\boxed{M_{c,y,Rd} = 499.1\,\text{kN-m}}$$

with Shear Reduction

$V_{Ed} = 500\,\text{kN} > 0.5 \bullet V_{c,Rd} = 333.7\,\text{kN}$ Shear Reduction is needed

$A_v = ht_w = 467.2 \bullet 11.4 = 4{,}879.2\,\text{mm}^2$

$$\rho = \left(\frac{2V_{Ed}}{V_{c,Rd}} - 1\right)^2 = \left(\frac{2 \bullet 500}{667.5} - 1\right)^2 = 0.2482$$

$$M_{y,V,Rd} = \frac{\left[W_{pl,y} - \frac{\rho A_v^{\,2}}{4t_w}\right] f_{yk}}{\gamma_{M0}} = \frac{\left[2{,}230{,}000 - \frac{0.2482 \bullet 4879.2^2}{4 \bullet 11.4}\right] \bullet 235}{1.05} \le M_{y,c,Rd}$$

$$\boxed{M_{V,r,Rd} = 470.1\,\text{kN-m}}$$

with Compression Reduction

$$n = \frac{N_{Ed}}{N_{pl,Rd}} = \frac{1400}{2797.6} = 0.50$$

$$a = \frac{A - 2bt_f}{A} = \frac{12{,}500 - 2 \bullet 192.8 \bullet 19.6}{12{,}500} = 0.40 \le 0.5$$

$$M_{N,y,Rd} = M_{pl,y,Rd} \frac{1-n}{1-0.5a} = 499.1 \bullet \frac{1-0.5}{1-0.5 \bullet 0.4}$$

$$\boxed{M_{N,y,Rd} = 310.8\,\text{kN-m}}$$

Interaction Capacity: Compression & Bending

Section Bending & Compression Capacity

Formula NTC 4.2.39

$$\frac{D}{C} = \left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^2 + \left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^{5n} = \left[\frac{200}{310.8}\right]^2 + 0 = 0.414 \le \frac{M_{y,Ed}}{M_{N,y,Rd}} = 0.644$$

$$\boxed{\frac{D}{C} = 0.644 \text{ (Governs)}}$$

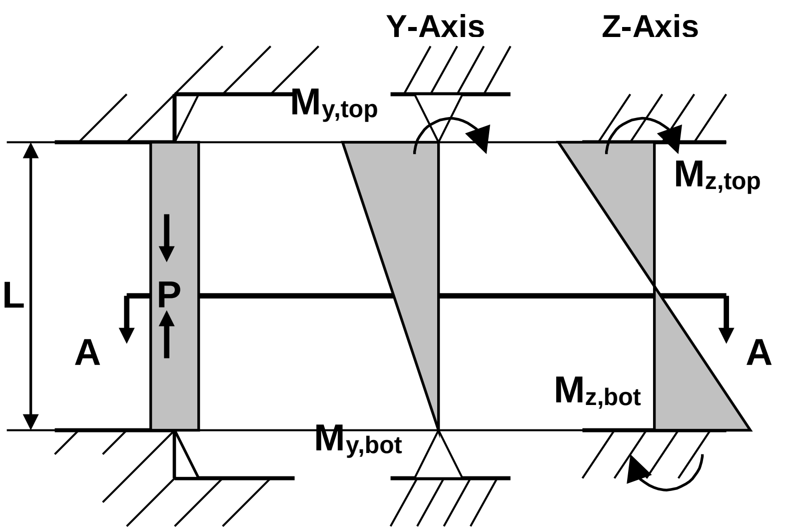
NTC 2008 Example 002

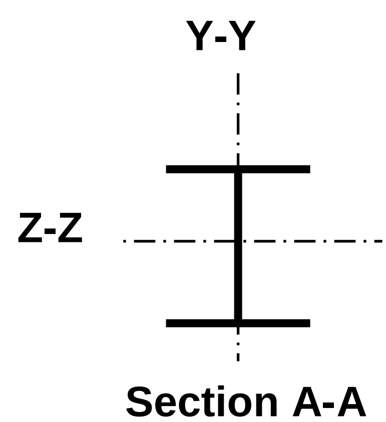
WIDE FLANGE MEMBER UNDER COMBINED COMPRESSION & BENDING

EXAMPLE DESCRIPTION

In this example a continuous beam-column is subjected to factored axial load P = 1400 kN, major-axis bending moment M_y = 200 kN-m, and a minor axis bending moment of M_z = 100 kN-m. This example was tested using the Italian NTC-2008 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING





L = 0.4 m

Material Properties

E = 210x10^3 MPa
v = 0.3
G = 80769 MPa

Loading

P = 1400 kN
$M_{z,top}$ = 100 kN-m
$M_{z,bot}$ = -100 kN-m
$M_{y,top}$ = 200 kN-m
$M_{y,bot}$ = 0

Design Properties

f_y = 235 MPa
Section: *457x191x98 UB*

TECHNICAL FEATURES TESTED

- Section compactness check (beam-column)
- Section compression capacity
- Section shear capacity for major & minor axes
- Section bending capacity for major & minor axes
- Member compression capacity for major & minor axes
- Member bending capacity
- Interaction capacity, D/C, for major & minor axes

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file "SFD-NTC-2008.pdf," which is available through the program "Help" menu. Examples were taken from Example 6.6 on pp. 57-59 from the book "Designers' Guide to EN1993-1-1" by R.S. Narayanan & A. Beeby.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Class 2	Class 2	0.00%
Section Compression Resistance, $N_{c,Rd}$ (kN)	2,797.6	2,797.6	0.00%
Buckling Resistance in Compression, $N_{byy,Rd}$ (kN)	2,797.6	2,797.6	0.00%
Buckling Resistance in Compression, $N_{bzz,Rd}$ (kN)	2,797.6	2,797.6	0.00%
Section Plastic Bending Resistance, $M_{c,y,Rd}$ (kN-m)	499.1	499.1	0.00%
Section Plastic Bending Resistance, $M_{c,z,Rd}$ (kN-m)	84.8	84.8	0.00%
Section Bending Resistance Shear Reduced, $M_{V,y,Rd}$ (kN-m)	470.1	470.1	0.00%
Section Bending Resistance Axially Reduced, $M_{N,y,Rd}$ (kN-m)	310.8	310.8	0.00%
Section Bending Resistance Axially Reduced, $M_{N,z,Rd}$ (kN-m)	82.26	82.26	0.00%
Member Bending Resistance, $M_{b,Rd}$ (kN-m)	499.095	499.095	0.00%
Section Shear Resistance, $V_{c,y,Rd}$ (kN)	667.5	667.5	0.00%
Section Shear Resistance, $V_{c,z,Rd}$ (kN)	984.7	984.7	0.00%
Interaction Capacity, D/C	2.044	2.044	0.00%

COMPUTER FILE: NTC 2008 EX002

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material: S275 Steel

E = 210,000 MPa

G = 80,769 MPa

f_y = 235 MPa

Section: 457x191x98 UB

A = 12,500 mm^2

b = 192.8 mm, t_f = 19.6 mm, h = 467.2 mm, t_w = 11.4 mm, r = 0 mm

$$h_w = h - 2t_f = 467.2 - 2 \bullet 19.6 = 428 \text{ mm}$$

$$d = h - 2\left(t_f + r\right) = 467.2 - 2 \bullet \left(19.6 + 0\right) = 428 \text{ mm}$$

$$c = \frac{b - t_w - 2r}{2} = \frac{192.8 - 11.4 - 2 \bullet 0}{2} = 90.7 \text{ mm}$$

$W_{pl,y}$ = 2,230,000 mm^3

$W_{pl,z}$ = 379,000 mm^3

r_{yy} = 191.3 mm

r_{zz} = 43.3331 mm

I_{zz} = 23,469,998 mm^4

$$I_w = 1.176 \bullet 10^{12} \text{ mm}^6$$

I_T = 1,210,000 mm^4

Member:

$L = L_b = L_{\text{unbraced}}$ = 400 mm

K_{yy} = 1.0, K_{zz} = 1.0

Other:

$$\gamma_{M0} = 1.05$$

$$\gamma_{M1} = 1.05$$

Loadings:

$P = 1400\,\text{kN}$ axial load

$M_{z-1} = 100\,\text{kN-m}$

$M_{z-2} = -100\,\text{kN-m}$

$M_{y-1} = 200\,\text{kN-m}$

$M_{y-2} = 0\,\text{kN-m}$

Results in the following internal forces:

$N_{Ed} = 1400\ \text{kN}$

$M_{y,Ed} = 200\ \text{kN-m}$

$M_{z,Ed} = 100\ \text{kN-m}$

$V_{y,Ed} = 500\ \text{kN-m}$

$V_{z,Ed} = 0\ \text{kN-m}$

Section Compactness:

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{235}} = 1$$

$$-1 \le \alpha = \frac{1}{2}\left(1 - \frac{N_{Ed}}{2ht_w f_y}\right) \le 1$$

$$\alpha = \frac{1}{2}\left(1 - \frac{1,400,000}{2 \bullet 467.2 \bullet 11.4 \bullet 235}\right) = 2.7818 > 1,\ so$$

$$\alpha = 1.0$$

Localized Buckling for Flange:

For the tip in compression under combined bending & compression

$$\lambda_{cl.1} = \frac{9\varepsilon}{\alpha} = \frac{9 \bullet 1}{1} = 9$$

$$\lambda_e = \frac{c}{t_f} = \frac{90.7}{19.6} = 4.63$$

$$\lambda_e = 4.63 < \lambda_{cl.1} = 9$$

So Flange is Class 1 in combined bending and compression

Localized Buckling for Web:

$\alpha > 0.5$, so

$$\lambda_{cl.1} = \frac{396\varepsilon}{13\alpha - 1} = \frac{396 \bullet 1}{13 \bullet 1 - 1} = 33.00 \text{ for combined bending \& compression}$$

$$\lambda_e = \frac{d}{t_w} = \frac{428}{11.4} = 37.54$$

$$\lambda_e = 37.54 > \lambda_{cl.1} = 33.00$$

$$\lambda_{cl.2} = \frac{456\varepsilon}{13\alpha - 1} = \frac{456 \bullet 1}{13 \bullet 1 - 1} = 38.00$$

$$\lambda_e = 37.54 < \lambda_{cl.2} = 38.00$$

So Web is Class 2 in combined bending & compression.

Since Web is Class 2, Section is Class 2 in combined bending & compression.

Section Compression Capacity

$$N_{c,Rd} = N_{pl,Rd} = \frac{Af_y}{\gamma_{M0}} = \frac{12,500 \bullet 235}{1.05}$$

$$\boxed{N_{c,Rd} = 2,797.6\,\text{kN}}$$

Section Shear Capacity

For major y-y axis

$$A_{V,y} = A - 2bt_f + t_f\left(t_w + 2r\right) = 12,500 - 2 \bullet 192.8 \bullet 19.6 + 19.6\left(11.4 + 2 \bullet 0\right)$$

$A_{V,y} = 5{,}165.7\,\text{mm}^2$

$$V_{c,y,Rd} = \frac{f_y}{\gamma_{M0}\sqrt{3}} A_{vy} = \frac{235}{1.05\sqrt{3}} \bullet 5{,}165.7$$

$$\boxed{V_{c,y,Rd} = 667.5\,\text{kN}}$$

For minor *z-z* axis

$$A_{V,z} = A - h_w t_w = 12{,}500 - 428 \bullet 11.4 = 7{,}620.8\ \text{mm}^2$$

$$V_{c,z,Rd} = \frac{f_y}{\gamma_{M0}\sqrt{3}} A_{vy} = \frac{235}{1.05\sqrt{3}} \bullet 7{,}620.8$$

$$\boxed{V_{c,z,Rd} = 984.7\,\text{kN}}$$

$\eta = 1.0$

$$\frac{h_w}{t_w} = \frac{428}{11.4} = 37.5 < \frac{72}{\eta}\sqrt{\frac{235}{f_y}} = \frac{72}{1.0}\sqrt{\frac{235}{235}} = 72$$

So no shear buckling needs to be checked.

Section Bending Capacity

For major *y-y* axis

$$M_{c,y,Rd} = M_{pl,y,Rd} = \frac{W_{pl,y} f_y}{\gamma_{M0}} = \frac{2{,}230{,}000 \bullet 235}{1.05}$$

$$\boxed{M_{c,y,Rd} = 499.1\,\text{kN-m}}$$

For minor *z-z* axis

$$M_{c,z,Rd} = M_{pl,z,Rd} = \frac{W_{pl,z} f_y}{\gamma_{M0}} = \frac{379{,}000 \bullet 235}{1.05}$$

$$\boxed{M_{c,z,Rd} = 84.8\,\text{kN-m}}$$

With Shear Reduction

For major *y-y* axis

$V_{y,Ed} = 500\,\text{kN} > 0.5 \bullet V_{c,y,Rd} = 333.7\,\text{kN}$ Shear Reduction is needed

$A_v = ht_w = 467.2 \bullet 11.4 = 4,879.2\,\text{mm}^2$

$$\rho = \left(\frac{2V_{Ed}}{V_{c,Rd}} - 1\right)^2 = \left(\frac{2 \bullet 500}{667.5} - 1\right)^2 = 0.2482$$

$$M_{y,V,Rd} = \frac{\left[W_{pl,y} - \frac{\rho A_v^{\,2}}{4t_w}\right] f_{yk}}{\gamma_{M0}} = \frac{\left[2,230,000 - \frac{0.1525 \bullet 4879.2^2}{4 \bullet 11.4}\right] \bullet 235}{1.05} \le M_{y,c,Rd}$$

$\boxed{M_{V,r,Rd} = 470.1\text{kN-m}}$

For minor *z-z* axis

$V_{z,Ed} = 0\,\text{kN} < 0.5 \bullet V_{c,z,Rd}$ No shear Reduction is needed

With Compression Reduction

$$n = \frac{N_{Ed}}{N_{pl,Rd}} = \frac{1400}{2797.6} = 0.50$$

$$a = \frac{A - 2bt_f}{A} = \frac{12,500 - 2 \bullet 192.8 \bullet 19.6}{12,500} = 0.40 \le 0.5$$

For major y-y axis

$$M_{N,y,Rd} = M_{pl,y,Rd} \frac{1-n}{1-0.5a} = 499.1 \bullet \frac{1-0.5}{1-0.5 \bullet 0.4}$$

$\boxed{M_{N,y,Rd} = 310.8\,\text{kN-m}}$

For minor *z-z* axis

$n < a$

$$M_{N,z,Rd} = M_{pl,z,Rd}\left[1-\left(\frac{n-a}{1-a}\right)^2\right] = 84.8 \bullet \left[1-\left(\frac{0.5-0.4}{1-0.4}\right)^2\right]$$

$$\boxed{M_{N,z,Rd} = 82.26\,\text{kN-m}}$$

Member Compression Capacity:

Non-Dimensional Slenderness Ratio:

Steel is S235

$$\frac{h}{b_f} = \frac{467.2}{192.8} = 2.4 > 1.2$$

and

$$t_f = 19.6\,\text{mm} < 40\,\text{mm}$$

So we should use the Buckling Curve 'a' for the *z-z* axis and Buckling Curve 'b' for the *y-y* axis (NTC 2008, Table 4.2.VI).

Y-Y Axis Parameters:

For buckling curve a, $\alpha = 0.21$ (NTC 2008, Table 4.2 VI)

$$K_y = 1.00$$

$$L_{cr,y} = K_y L_y = 1 \bullet 400 = 400\,\text{mm}, \quad \frac{L_{cr,y}}{r_y} = \frac{400}{191.3} = 2.091$$

$$N_{cr,y} = \frac{\pi^2 E}{A\left(\dfrac{K_y L_y}{r_y}\right)^2} = \frac{\pi^2 \bullet 210{,}000}{12{,}500 \bullet (2.091)^2} = 5{,}925{,}691\,\text{kN}$$

$$\bar{\lambda}_y = \sqrt{\frac{A f_y}{N_{cr,y}}} = \sqrt{\frac{12{,}500 \bullet 235}{5{,}925{,}691}} = 0.022$$

$$\phi_y = 0.5\left[1+\alpha_y\left(\bar{\lambda}_y - 0.2\right)+\bar{\lambda}_y^{\,2}\right] = 0.5\left[1+0.21\left(0.022-0.2\right)+0.022^2\right]$$

$$\phi_y = 0.482$$

Stress Reduction Factor: $\chi_y = \dfrac{1}{\phi_y + \sqrt{\phi_y^{\,2} - \bar{\lambda}_y^{\,2}}} = \dfrac{1}{0.482+\sqrt{0.482^2 - 0.022^2}} = 1.0388$

$$\chi_y = 1.0388 > 1.0,\ \ so\ \ \chi_y = 1.0$$

$$N_{byy,Rd} = \frac{\chi_y A f_y}{\gamma_{M1}} = \frac{1.0 \bullet 12{,}500 \bullet 235}{1.05}$$

$$\boxed{N_{byy,Rd} = 2{,}797.6\,\text{kN}}$$

Z-Z Axis Parameters:

For buckling curve b, $\alpha = 0.34$ (NTC 2008, Table 4.2 VI)

$$K_z = 1.00$$

$$L_{cr,z} = K_z L_z = 1 \bullet 400 = 400\,\text{mm},\ \ \frac{L_{cr,z}}{r_z} = \frac{400}{43.33} = 9.231$$

$$N_{cr,z} = \frac{\pi^2 E}{A\left(\dfrac{K_z L_z}{r_z}\right)^2} = \frac{\pi^2 \bullet 210{,}000}{12{,}500 \bullet \left(9.231\right)^2} = 304{,}052\,\text{kN}$$

$$\bar{\lambda}_z = \sqrt{\frac{A f_y}{N_{cr,z}}} = \sqrt{\frac{12{,}500 \bullet 235}{304{,}052}} = 0.098$$

$$\phi_z = 0.5\left[1+\alpha_z\left(\bar{\lambda}_z - 0.2\right)+\bar{\lambda}_z^{\,2}\right] = 0.5\left[1+0.34\left(0.098-0.2\right)+0.098^2\right]$$

$$\phi_z = 0.488$$

Stress Reduction Factor: $\chi_z = \dfrac{1}{\phi_z + \sqrt{\phi_z^{\,2} - \bar{\lambda}_z^{\,2}}} = \dfrac{1}{0.488+\sqrt{0.488^2 - 0.098^2}} = 1.0362$

$\chi_z = 1.0362 > 1.0,\ so\ \ \chi_z = 1.0$

$$N_{bzz,Rd} = \frac{\chi_z A f_y}{\gamma_{M1}} = \frac{1.0 \bullet 12,500 \bullet 235}{1.05}$$

$$\boxed{N_{bzz,Rd} = 2,797.6\,\text{kN}}$$

Member Bending Capacity:

$$\frac{h}{b_f} = \frac{467.2}{192.8} = 2.4 > 2$$

So we should use the Buckling Curve ‘c’ for lateral-torsional buckling (NTC 2008, Table 4.2.VII).

$\alpha_{LT} = 0.49$

$\overline{\lambda}_{LT,0} = 0.4$ (default for rolled section)

$\beta = 0.75$ (default for rolled section)

$M_B = M_{y-2} = 0,\ \ M_A = M_{y-1} = 200\,\text{kN-m}$

$$\psi = 1.75 - 1.05\frac{M_B}{M_A} + 0.3\left(\frac{M_B}{M_A}\right)^2 = 1.75 - 1.05\frac{0}{200} + 0.3\left(\frac{0}{200}\right)^2 = 1.75$$

Corrective Factor is determined from NTC 2008, Table 4.2 VIII

$$k_c = \frac{1}{1.33 - 0.33\psi} = \frac{1}{1.33 - 0.33 \bullet 1.75} = 1.329$$

$$M_{cr} = \psi \frac{\pi^2 E I_z}{\left(L_{cr,z}\right)^2} \sqrt{\left(\frac{I_w}{I_z} + \frac{\left(L_{cr,z}\right)^2 G I_T}{\pi^2 E I_z}\right)}$$

$$M_{cr} = 1.75 \bullet \frac{\pi^2 \bullet 210,000 \bullet 23,469,998}{400^2} \sqrt{\left(\frac{1.176 \bullet 10^{12}}{23,469,998} + \frac{400^2 \bullet 80,769 \bullet 1,210,000}{\pi^2 \bullet 210,000 \bullet 23,469,998} \right)}$$

$$M_{cr} = 119,477,445,900 \text{ N-mm}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{pl,y} f_y}{M_{cr}}} = \sqrt{\frac{2,230,000 \bullet 235}{119,477,445,900}} = 0.066$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}\right) + \beta \bar{\lambda}_{LT}^{\,2}\right] = 0.5\left[1 + 0.49 \bullet \left(0.066 - 0.4\right) + 0.75 \bullet 0.066^2\right]$$

$$\phi_{LT} = 0.420$$

$$f = 1 - 0.5(1 - k_c)\left[1 - 2\left(\bar{\lambda}_{LT} - 0.8\right)^2\right] = 1 - 0.5(1 - 1.329)\left[1 - 2\left(0.066 - 0.8\right)^2\right] = 0.987$$

$$\chi_{LT} = \frac{1}{f} \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^{\,2} + \beta \lambda_{LT}^{\,2}}} \le \left(1.0 \text{ or } \frac{1}{\bar{\lambda}_{LT}^{\,2}} \frac{1}{f}\right)$$

$$\chi_{LT} = \left(\frac{1}{0.987}\right) \frac{1}{0.420 + \sqrt{0.420^2 + 0.75 \bullet 0.066^2}} \le \left(1.0 \text{ or } \frac{1}{0.066^2} \frac{1}{0.987}\right)$$

$$\chi_{LT} = 1.2118 \le \left(1.0 \text{ or } 230.9\right)$$

so

$$\chi_{LT} = 1.0$$

$$M_{b,Rd} = \chi_{LT} W_{pl,y} \frac{f_y}{\gamma_{M1}} = 1.0 \bullet 2,230,000 \bullet \frac{235}{1.05}$$

$$\boxed{M_{b,Rd} = 499.095 \text{ kN-m}}$$

Interaction Capacity: Compression & Bending

Section Bending & Compression Capacity

Formula NTC 4.2.39

$$\frac{D}{C} = \left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^2 + \left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^{5n} = \left[\frac{200}{310.8}\right]^2 + \left[\frac{100}{82.3}\right]^{5 \bullet 0.5} = 0.414 + 1.630$$

$$\boxed{\frac{D}{C} = 2.044 \text{ (Governs)}}$$

Member Bending & Compression Capacity: Method B

k factors used are taken from the software, and determined from Method 2 in Annex B of Eurocode 3.

$k_{yy} = 0.547$

$k_{yz} = 0.479$

$k_{zy} = 0.698$

$k_{zz} = 0.798$

Formula NTC 4.2.37

$$\frac{D}{C} = \frac{N_{Ed}}{\dfrac{\chi_y A f_{yk}}{\gamma_{M1}}} + k_{yy} \frac{M_{y,Ed}}{\chi_{LT} \dfrac{W_{pl,y} f_{yk}}{\gamma_{M1}}} + k_{yz} \frac{M_{z,Ed}}{\dfrac{W_{pl,z} f_{yk}}{\gamma_{M1}}}$$

$$\frac{D}{C} = \frac{1,400}{\dfrac{1 \times 12,500 \times 235}{1.05}} + 0.547 \times \frac{200}{1 \times \dfrac{2,230,000 \times 235}{1.05}} + 0.479 \times \frac{100}{\dfrac{379,000 \times 235}{1.05}}$$

$$\frac{D}{C} = 0.5 + 0.22 + 0.56$$

$$\frac{D}{C} = 1.284$$

Formula NTC 4.2.38

$$\frac{D}{C}=\frac{N_{Ed}}{\dfrac{\chi_z A f_{yk}}{\gamma_{M1}}}+k_{zy}\frac{M_{y,Ed}}{\chi_{LT}\dfrac{W_{pl,y}f_{yk}}{\gamma_{M1}}}+k_{zz}\frac{M_{z,Ed}}{\dfrac{W_{pl,z}f_{yk}}{\gamma_{M1}}}$$

$$\frac{D}{C}=\frac{1,400}{\dfrac{1\times 12,500\times 235}{1.05}}+0.698\times\frac{200}{1\times\dfrac{2,230,000\times 235}{1.05}}+0.798\times\frac{100}{\dfrac{379,000\times 235}{1.05}}$$

$$\frac{D}{C}=0.5+0.28+0.941$$

$$\frac{D}{C}=1.721$$

NZS 3404-1997 Example 001

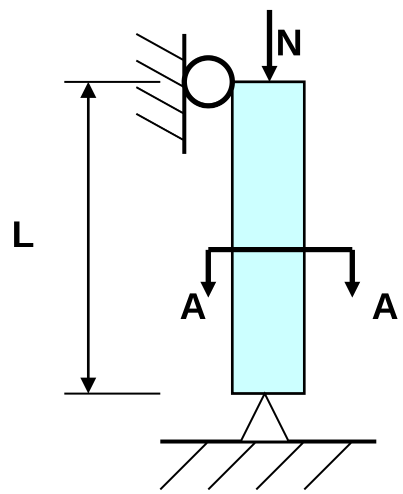
WIDE FLANGE MEMBER UNDER COMPRESSION

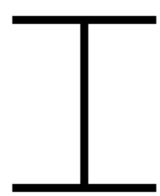
EXAMPLE DESCRIPTION

The frame object axial strengths are tested in this example.

A continuous column is subjected to factored load N = 200 kN. This example was tested using the NZS 3404-1997 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING





Section A-A

L = 6 m

Material Properties
E = 200x10^3 MPa
v = 0.3
G = 76923.08 MPa

Loading
N = 200 kN

Design Properties
f_y = 250 MPa
Section: 350WC197

TECHNICAL FEATURES TESTED

- Section compactness check (compression)
- Section compression capacity
- Member compression capacity

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file "SFD-NZS-3404-1997.pdf," which is available through the program "Help" menu.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Compact	Compact	0.00%
Section Axial Capacity, N_s (kN)	6275	6275	0.00%
Member Axial Capacity, N_c (kN)	4385	4385	0.00%

COMPUTER FILE: NZS 3404-1997 EX001

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material:

f_y = 250 MPa

Section: 350WC197

$A_g = A_n$ = 25100 mm^2

b_f = 350 mm, t_f = 28 mm, h = 331 mm, t_w = 20 mm

r_{33} = 139.15 mm, r_{22} = 89.264 mm

Member:

$l_{e33} = l_{e22}$ = 6000 mm (unbraced length)

Considered to be a braced frame

Loadings:

$N^* = 200$ kN

Section Compactness:

Localized Buckling for Flange:

$$\lambda_e = \frac{(b_f - t_w)}{2 \bullet t_f}\sqrt{\frac{f_y}{250}} = \frac{350-20}{2 \bullet 28}\sqrt{\frac{250}{250}} = 5.89$$

Flange is under uniform compression, so:

$\lambda_{ep} = 9, \lambda_{ey} = 16, \lambda_{ew} = 90$

$\lambda_e = 5.89 < \lambda_{ep} = 9$, No localized flange buckling

Flange is compact

Localized Buckling for Web:

$$\lambda_e = \frac{h}{t_w}\sqrt{\frac{f_y}{250}} = \frac{331}{20}\sqrt{\frac{250}{250}} = 16.55$$

Web is under uniform compression, so:

$\lambda_{ep} = 30, \lambda_{ey} = 45, \lambda_{ew} = 180$

$\lambda_e = 16.55 < \lambda_{ep} = 30$, No localized web buckling

Web is compact.

Section is Compact.

Section Compression Capacity:

Section is not Slender, so $K_f = 1.0$

$N_s = K_f A_n f_y = 1 \bullet 25{,}100 \bullet 250 / 10^3$

$N_s = 6275\text{kN}$

Member Weak-Axis Compression Capacity:

Frame is considered a braced frame in both directions, so $k_{e22} = k_{e33} = 1$

$\frac{l_{e22}}{r_{22}} = \frac{6000}{89.264} = 67.216$ and $\frac{l_{e33}}{r_{33}} = \frac{6000}{139.15} = 43.119$

Buckling will occur on the 22-axis.

$$\lambda_{n22} = \frac{l_{e22}}{r_{22}}\sqrt{\frac{K_f f_y}{250}} = \frac{6000}{89.264} \bullet \sqrt{\frac{(1 \bullet 250)}{250}} = 67.216$$

$$\alpha_{a22} = \frac{2100(\lambda_{n22} - 13.5)}{\lambda_{n22}^2 - 15.3\lambda_{n22} + 2050} = 20.363$$

$\alpha_{b22} = 0.5$ since cross-section is not a UB or UC section

$$\lambda_{22} = \lambda_{n22} + \alpha_{a22}\alpha_{b22} = 67.216 + 20.363 \bullet 0.5 = 77.398$$

$$\eta_{22} = 0.00326(\lambda_{22} - 13.5) = 0.2083 \geq 0$$

$$\xi_{22} = \frac{\left(\frac{\lambda_{22}}{90}\right)^2 + 1 + \eta_{22}}{2\left(\frac{\lambda_{22}}{90}\right)^2} = \frac{\left(\frac{77.398}{90}\right)^2 + 1 + 0.2083}{2\left(\frac{77.398}{90}\right)^2} = 1.317$$

$$\alpha_{c22} = \xi_{22}\left(1 - \sqrt{\left(1 - \left(\frac{90}{\xi_{22}\lambda_{22}}\right)^2\right)}\right)$$

$$\alpha_{c22} = 1.317\left(1 - \sqrt{\left(1 - \left(\frac{90}{1.317 \bullet 77.398}\right)^2\right)}\right) = 0.6988$$

$$N_{c22} = \alpha_{c22} N_s \leq N_s$$

$$\boxed{N_{c22} = 0.6988 \bullet 6275 = 4385\ \text{kN}}$$

NZS 3404-1997 Example 002

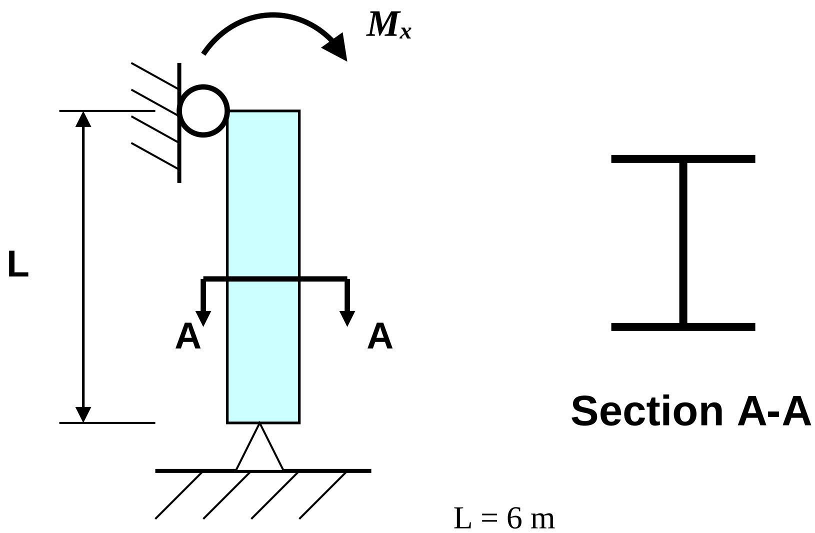
WIDE FLANGE MEMBER UNDER BENDING

EXAMPLE DESCRIPTION

The frame object bending strengths are tested in this example.

A continuous column is subjected to factored moment M_x = 1000 kN-m. This example was tested using the NZS 3404-1997 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



L = 6 m

Material Properties	Loading	Design Properties
E = 200x10^3 MPa	M_x = 1000 kN-m	f_y = 250 MPa
v = 0.3		Section: 350WC197
G = 76923.08 MPa		

TECHNICAL FEATURES TESTED

- Section compactness check (bending)
- Section bending capacity
- Member bending capacity

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file “SFD-NZS-3404-1997.pdf, ” which is available through the program “Help” menu.

Output Parameter	ETABS	Independent	Percent Difference
Compactness	Compact	Compact	0%
Section Bending Capacity, $M_{s,\mathrm{major}}$ (kN-m)	837.5	837.5	0%
Member Bending Capacity, M_b (kN-m)	837.5	837.5	0%

COMPUTER FILE: NZS 3404-1997 EX002

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Material:

f_y = 250 MPa

Section: 350WC197

b_f = 350 mm, t_f = 28 mm, h = 331 mm, t_w = 20 mm

I_{22} = 200,000,000 mm^4

Z_{33} = 2,936,555.891 mm^2

S_{33} = 3,350,000 mm^2

J = 5,750,000 mm^4

I_w = 4,590,000,000,000 mm^6

Member:

l_{e22} = 6000 mm (unbraced length)

Considered to be a braced frame

Loadings:

$M_m* = 1000$ kN-m

This leads to:

$M_2* = 250$ kN-m

$M_3* = 500$ kN-m

$M_4* = 750$ kN-m

Section Compactness:

Localized Buckling for Flange:

$$\lambda_e = \frac{(b_f - t_w)}{2 \bullet t_f}\sqrt{\frac{f_y}{250}} = \frac{350-20}{2 \bullet 28}\sqrt{\frac{250}{250}} = 5.89$$

Flange is under uniform compression, so:

$\lambda_{ep} = 9, \lambda_{ey} = 16, \lambda_{ew} = 90$

$\lambda_e = 5.89 < \lambda_{ep} = 9$, No localized flange buckling

Flange is compact

Localized Buckling for Web:

$$\lambda_e = \frac{h}{t_w}\sqrt{\frac{f_y}{250}} = \frac{331}{20}\sqrt{\frac{250}{250}} = 16.55$$

Web is under bending, so:

$\lambda_{ep} = 82, \lambda_{ey} = 115, \lambda_{ew} = 180$

$\lambda_e = 16.55 < \lambda_{ep} = 30$, No localized web buckling

Web is compact.

Section is Compact.

Section Bending Capacity:

$Z_e = Z_c = \min(S, 1.5Z)$ for compact sections

$Z_{e33} = Z_{c33} = 3,350,000 \text{ mm}^2$

$M_{s33} = M_{s,major} = f_y Z_{e33} = 250 \bullet 3,350,000 / 1000^2$

$M_{s33} = M_{s,\text{major}} = 837.5 \text{ kN-m}$

Member Bending Capacity:

k_t = 1 (Program default)

k_l = 1.4 (Program default)

k_r = 1 (Program default)

$l_{LTB} = l_{e22}$ = 6000 mm

$$l_e = k_t k_l k_r l_{LTB} = 1 \bullet 1.4 \bullet 1 \bullet 6000 = 8400 \text{ mm}^2$$

$$M_{oa} = M_o = \sqrt{\left(\left(\frac{\pi^2 EI_{22}}{l_e^2}\right)\left(GJ + \frac{\pi^2 EI_w}{l_e^2}\right)\right)}$$

$$M_{oa} = M_o = \sqrt{\left(\left(\frac{\pi^2 \bullet 2 \bullet 10^5 \bullet 2 \bullet 10^8}{8,400^2}\right)\left(76,923.08 \bullet 5,750,000 + \frac{\pi^2 \bullet 2 \bullet 10^5 \bullet 4.59 \bullet 10^{12}}{8,400^2}\right)\right)}$$

$$M_{oa} = M_o = 1786.938 \text{ kN-m}$$

$$\alpha_s = 0.6\left(\sqrt{\left(\left(\frac{M_s}{M_{oa}}\right)^2 + 3\right)} - \frac{M_s}{M_{oa}}\right) = 0.6\left(\sqrt{\left(\left(\frac{837.5}{1786.938}\right)^2 + 3\right)} - \frac{837.5}{1786.938}\right)$$

$$\alpha_s = 0.7954$$

$$\alpha_m = \frac{1.7 M_m *}{\sqrt{(M_2 *)^2 + (M_3 *)^2 + (M_4 *)^2}} \le 2.5$$

$$\alpha_m = \frac{1.7 \bullet 1000}{\sqrt{(250)^2 + (500)^2 + (750)^2}} = 1.817 \le 2.5$$

$$M_b = \alpha_m \alpha_s M_s = 0.7954 \bullet 1.817 \bullet 837.5 \le \text{M}_\text{s}$$

$$\boxed{M_b = 1210.64 \text{ kN-m} \le 837.5 \text{ kN-m}}$$

NZS 3404-1997 Example 003

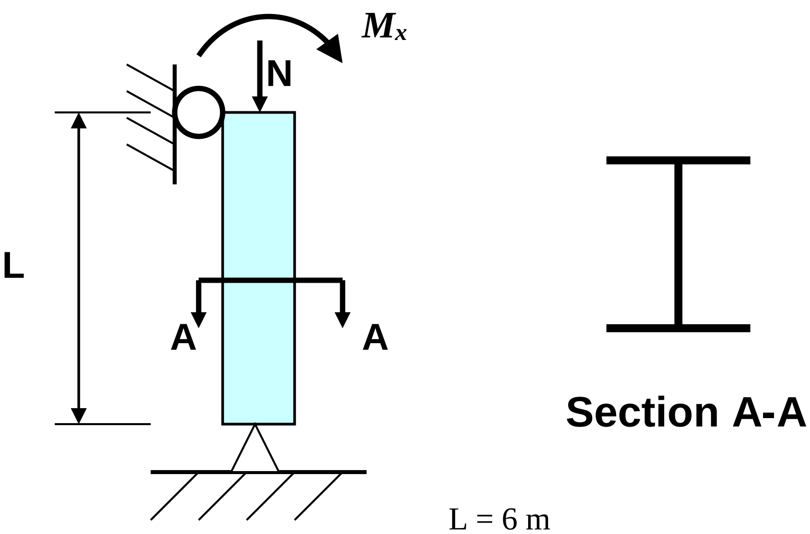
WIDE FLANGE MEMBER UNDER COMBINED COMPRESSION & BENDING

EXAMPLE DESCRIPTION

The frame object interacting axial and bending strengths are tested in this example.

A continuous column is subjected to factored loads and moments N= 200 kN; M_x= 1000 kN-m. This example was tested using the NZS 3404-1997 steel frame design code. The design capacities are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



L = 6 m

Material Properties
E = 200x10^3 MPa
v = 0.3
G = 76923.08 MPa

Loading
N = 200 kN
M_x = 1000 kN-m

Design Properties
f_y = 250 MPa
Section: 350WC197

TECHNICAL FEATURES TESTED

- Section compactness check (compression & bending)
- Section bending capacity with compression reduction
- Member in-plane bending capacity with compression reduction
- Member out-of-plane bending capacity with compression reduction

RESULTS COMPARISON

Independent results are taken from hand calculations based on the CSI steel design documentation contained in the file "SFD-NZS-3404-1997.pdf," which is available through the program "Help" menu.

Output Parameter	ETABS	Independent	Percent Difference
Compactness`	Compact	Compact	0.00%
Reduced Section Bending Capacity, M_{rx} (kN-m)	837.5	837.5	0.00%
Reduced In-Plane Member Bending Capacity, M_{ix} (kN-m)	823.1	823.1	0.00%
Reduced Out-of-Plane Member Bending Capacity, M_{o} (kN-m)	837.5	837.5	0.00%

COMPUTER FILE: NZS 3404-1997 EX003

CONCLUSION

The results show an exact comparison with the independent results.

HAND CALCULATION

Properties:

Section: 350WC197

A_g = A_n = 25100 mm^2

I_{22} = 200,000,000 mm^4

I_{33} = 486,000,000 mm^4

J = 5,750,000 mm^4

I_w = 4,590,000,000,000 mm^6

Member:

l_z=l_{e33} = l_{e22} = 6000 mm (unbraced length)

Considered to be a braced frame

ϕ =0.9

Loadings:

$N^* = 200$ kN

$M_m{}^* = 1000$ kN-m

Section Compactness:

From example SFD – IN-01-1, section is **Compact in Compression**

From example SFD – IN-01-2, section is **Compact in Bending**

Section Compression Capacity:

From example SFD – IN-01-1, $N_s = 6275\text{kN}$

Member Compression Capacity:

From example SFD – IN-01-1, $N_{c22} = 4385\text{ kN}$

Section Bending Capacity:

From example SFD – IN-01-2, $M_{s33} = M_{s,\text{major}} = 837.5\text{ kN-m}$

Section Interaction: Bending & Compression Capacity:

$$M_{r33} = 1.18M_{s33}\left(1 - \frac{N*}{\phi N_s}\right) = 1.18 \bullet 837.5\left(1 - \frac{200}{0.9 \bullet 6275}\right) \le M_{s33} = 837.5$$

$$M_{r33} = 953.252 \le 837.5$$

$$\boxed{M_{r33} = 837.5\text{kN-m}}$$

Member Strong-Axis Compression Capacity:

Strong-axis buckling strength needs to be calculated:

Frame is considered a braced frame in both directions, so $k_{e33} = 1$

$$\frac{l_{e33}}{r_{33}} = \frac{6000}{139.15} = 43.119$$

$$\lambda_{n33} = \frac{l_{e33}}{r_{33}}\sqrt{\frac{K_f f_y}{250}} = \frac{6000}{139.15} \bullet \sqrt{\frac{(1 \bullet 250)}{250}} = 43.119$$

$$\alpha_{a33} = \frac{2100(\lambda_{n33} - 13.5)}{\lambda_{n33}{}^2 - 15.3\lambda_{n33} + 2050} = 19.141$$

$\alpha_{b33} = 0.5$ since cross-section is not a UB or UC section

$$\lambda_{33} = \lambda_{n33} + \alpha_{a33}\alpha_{b33} = 43.119 + 19.141 \bullet 0.5 = 52.690$$

$$\eta_{33} = 0.00326(\lambda_{33} - 13.5) = 0.1278 \ge 0$$

$$\xi_{33}=\frac{\left(\frac{\lambda_{33}}{90}\right)^2+1+\eta_{33}}{2\left(\frac{\lambda_{33}}{90}\right)^2}=\frac{\left(\frac{52.690}{90}\right)^2+1+0.1278}{2\left(\frac{52.690}{90}\right)^2}=2.145$$

$$\alpha_{c33}=\xi_{33}\left(1-\sqrt{\left(1-\left(\frac{90}{\xi_{33}\lambda_{33}}\right)^2\right)}\right)$$

$$\alpha_{c33}=2.145\left(1-\sqrt{\left(1-\left(\frac{90}{2.145\bullet 50.690}\right)^2\right)}\right)=0.8474$$

$$N_{c33}=\alpha_{c33}N_s\le N_s$$

$$N_{c33}=0.8474\bullet 6275$$

$$N_{c33}=5318\text{ kN}$$

Member Interaction: In-Plane Bending & Compression Capacity:

$$\beta_m=\frac{M_{\min}}{M_{\max}}=\frac{0}{1000}=0$$

Since the section is compact,

$$M_i=M_{s33}\left(\left(1-\left(\frac{1+\beta_m}{2}\right)^3\right)\left(1-\frac{N^*}{\phi N_{c33}}\right)+1.18\left(\frac{1+\beta_m}{2}\right)^3\sqrt{1-\frac{N^*}{\phi N_{c33}}}\right)$$

$$M_i=837.5\left(\left(1-\left(\frac{1+0}{2}\right)^3\right)\left(1-\frac{200}{0.9\bullet 5318}\right)+1.18\left(\frac{1+0}{2}\right)^3\sqrt{1-\frac{200}{0.9\bullet 5318}}\right)$$

$$\boxed{M_i=823.11\text{ kN-m}}$$

Member Interaction: Out-of-Plane Bending & Compression Capacity:

$$\alpha_{bc} = \frac{1}{\left(\frac{1-\beta_m}{2} + \left(\frac{1-\beta_m}{2}\right)^3 \left(0.4 - 0.23\frac{N^*}{\phi N_{c22}}\right)\right)}$$

$$\alpha_{bc} = \frac{1}{\left(\frac{1-0}{2} + \left(\frac{1-0}{2}\right)^3 \left(0.4 - 0.23\frac{200}{0.9 \bullet 4385}\right)\right)}$$

$$\alpha_{bc} = 4.120$$

$$N_{oz} = GJ + \frac{\frac{\pi^2 EI_w}{l_z^{\,2}}}{\frac{I_{33} + I_{22}}{A_g}} = 76923.08 \bullet 5.75 \bullet 10^6 + \frac{\frac{\pi^2 \bullet 2 \bullet 10^6 \bullet 4.59 \bullet 10^{12}}{6000^2}}{\frac{\left(4.86+2\right) \bullet 10^8}{25100}}$$

$$N_{oz} = 4.423 \bullet 10^{11} \text{ kN}$$

$M_{b33o} = \alpha_m \alpha_s M_{sx}$ w/ an assumed uniform moment such that $\alpha_m = 1.0$

$$M_{b33o} = 1.0 \bullet 0.7954 \bullet 837.5 = 666.145 \text{ kN-m}$$

$$M_{o33} = \alpha_{bc} M_{b33o} \sqrt{\left(1 - \frac{N^*}{\phi N_{c22}}\right)\left(1 - \frac{N^*}{\phi N_{oz}}\right)} \le M_{r33}$$

$$M_{o33} = 4.12 \bullet 666.145 \sqrt{\left(1 - \frac{200}{0.9 \bullet 4385}\right)\left(1 - \frac{200}{0.9 \bullet 4.423 \bullet 10^{11}}\right)} = 2674 \le 837.5$$

$$\boxed{M_{o33} = 837.5 \text{ kN-m}}$$

ACI 318-08 Example 001

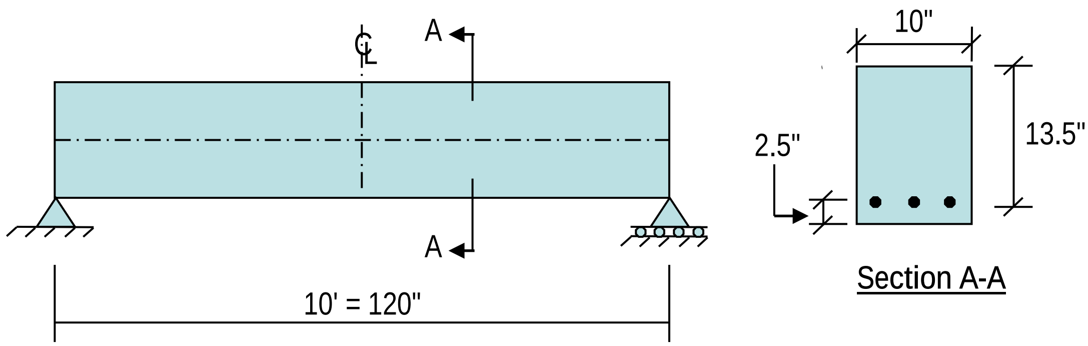
SHEAR AND FLEXURAL REINFORCEMENT DESIGN OF A SINGLY REINFORCED RECTANGLE

EXAMPLE DESCRIPTION

The flexural and shear design of a rectangular concrete beam is calculated in this example.

A simply supported beam is subjected to an ultimate uniform load of 9.736 k/ft. This example is tested using the ACI 318-08 concrete design code. The flexural and shear reinforcing computed is compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 3600 k/in^2
v = 0.2
G = 1500 k/in^2

Section Properties

d = 13.5 in
b = 10.0 in
I = 3,413 in^4

Design Properties

f'_c = 4 k/in^2
f_y = 60 k/in^2

TECHNICAL FEATURES TESTED

- Calculation of Flexural reinforcement, A_s
- Enforcement of Minimum tension reinforcement, $A_{s,min}$
- Calculation of Shear reinforcement, A_v
- Enforcement of Minimum shear reinforcing, $A_{v,min}$

RESULTS COMPARISON

Independent results are hand calculated based on the equivalent rectangular stress distribution described in Example 6.1 in Notes on ACI 318-08 Building Code.

Output Parameter	ETABS	Independent	Percent Difference
Design Moment, M_u (k-in)	1460.4	1460.4	0.00%
Tension Reinf, A_s (in^2)	2.37	2.37	0.00%
Design Shear Force, V_u	37.73	37.73	0.00%
Shear Reinf, A_v/s (in^2/in)	0.041	0.041	0.00%

COMPUTER FILE: ACI 318-08 Ex001

CONCLUSION

The computed results show an exact match for the flexural and the shear reinforcing.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\varphi = 0.9$, $A_g = 160$ sq-in

$$A_{s,min} = \frac{200}{f_y} b_w d = 0.450 \text{ sq-in (Govern)}$$

$$= \frac{3\sqrt{f_c'}}{f_y} b_w d = 0.427 \text{ sq-in}$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right) = 0.85$$

$$c_{max} = \frac{0.003}{0.003 + 0.005} d = 5.0625 \text{ in}$$

$$a_{max} = \beta_1 c_{max} = 4.303 \text{ in}$$

Combo1

$$w_u = (1.2w_d + 1.6w_l) = 9.736 \text{ k/ft}$$

$$M_u = \frac{w_u l^2}{8} = 9.736 \bullet 10^2/8 = 121.7 \text{ k-ft} = 1460.4 \text{ k-in}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2|M_u|}{0.85 f_c' \varphi b}} = 4.183 \text{ in } (a < a_{max})$$

The area of tensile steel reinforcement is given by:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{1460.4}{0.9 \bullet 60 \bullet (13.5 - 4.183/2)}$$

$$A_s = 2.37 \text{ sq-in}$$

Shear Design

The following quantities are computed for all of the load combinations:

$\varphi = 0.75$

Check the limit of $\sqrt{f'_c}$:

$\sqrt{f'_c}$ = 63.246 psi < 100 psi

The concrete shear capacity is given by:

$\varphi V_c = \varphi 2 \sqrt{f'_c}\, bd = 12.807$ k

The maximum shear that can be carried by reinforcement is given by:

$\varphi V_s = \varphi 8 \sqrt{f'_c}\, bd = 51.229$ k

The following limits are required in the determination of the reinforcement:

$(\varphi V_c/2) = 6.4035$ k

$\varphi V_{max} = \varphi V_c + \varphi V_s = 64.036$ k

Given V_u, V_c and V_{max}, the required shear reinforcement in area/unit length for any load combination is calculated as follows:

If $V_u \le \varphi (V_c/2)$,

$$\frac{A_v}{s} = 0,$$

else if $\varphi (V_c/2) < V_u \le \varphi V_{max}$

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d} \ge \left(\frac{A_v}{s}\right)_{min}$$

where:

$$\left(\frac{A_v}{s}\right)_{min} = \max\left\{50\left(\frac{b_w}{f_{yt}}\right), \left(\frac{b_w}{f_{yt}}\right) \bullet \frac{3}{4}\sqrt{f'_c}\right\}$$

else if $V_u > \varphi V_{max}$,

a failure condition is declared.

Combo1

$$V_u = 9.736 \bullet (5-13.5/12) = 37.727 \text{ k}$$

$$\phi\left(V_c / 2\right) = 6.4035\,k \le V_u = 37.727\,k \le \phi V_{\max} = 64.036\,k$$

$$\left(\frac{A_v}{s}\right)_{\min} = \max\left\{50\left(\frac{10}{60,000}\right), \left(\frac{10}{60,000}\right)\bullet\frac{3}{4}\sqrt{4,000}\right\}$$

$$\left(\frac{A_v}{s}\right)_{\min} = \max\left\{0.0083,\ 0.0079\right\} = 0.0083\frac{in^2}{in}$$

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d} = 0.041\frac{in^2}{in} = 0.492\frac{in^2}{ft}$$

ACI 318-08 Example 002

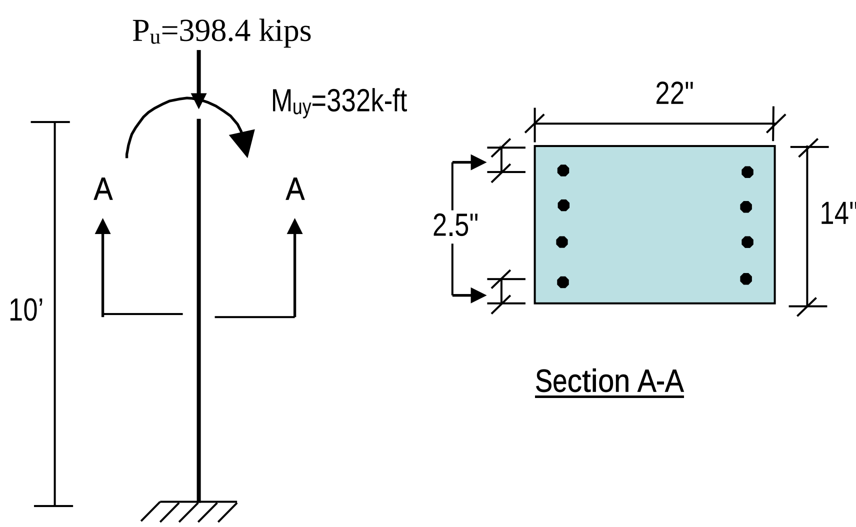
P-M INTERACTION CHECK FOR COMPRESSION-CONTROLLED RECTANGULAR COLUMN

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

A reinforced concrete column is subjected factored axial load P_u = 398.4 k and moments M_{uy} = 332 k-ft. This column is reinforced with 4 #9 bars. The total area of reinforcement is 8.00 in^2. The design capacity ratio is checked by hand calculations and result is compared.

GEOMETRY, PROPERTIES AND LOADING



Material Properties		Section Properties		Design Properties
E =	3600 k/in^2	b =	14 in	f'_c = 4 k/in^2
v =	0.2	d =	19.5 in	f_y = 60 k/in^2
G =	1500 k/in^2			

TECHNICAL FEATURES TESTED

- Tied Reinforced Concrete Column Demand/Capacity Ratio

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	1.000	1.00	0.00%

COMPUTER FILE: ACI 318-08 Ex002

CONCLUSION

The computed results show an exact match with the independent results.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

f'_c = 4 ksi	f_y = 60 ksi
b = 14 inch	d = 19.5 inch
P_u = 398.4 kips	M_u = 332 k-ft

1) Because *e* = 10 inch < (2/3)*d* = 13 inch., assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis for a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{87}{87 + f_y} d_t = \frac{87}{87 + 60}(19.5) = 11.54 \text{ inch}$$

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = 0.85 f'_c ab = 0.85 \bullet 4 \bullet 14a = 47.6a$$

$$C_s = A'_s \left(f_y - 0.85 f'_c \right) = 4\left(60 - 0.85 \bullet 4\right) = 226.4 \text{ kips}$$

Assume compression steels yields, (this assumption will be checked later).

$$T = A_s f_s = 4 f_s \left(f_s < f_y \right)$$

$$P_n = 47.6a + 226.4 - 4 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$P_n = \frac{1}{e'}\left[C_c \left(d - \frac{a}{2} \right) + C_s \left(d - d' \right) \right]$$

The plastic centroid is at the center of the section and d'' = 8.5 inch

$$e' = e + d'' = 10 + 8.5 = 18.5 \text{ inch.}$$

$$P_n = \frac{1}{18.5}\left[47.6a\left(19.5 - \frac{a}{2} \right) + 226.4\left(19.5 - 2.5\right) \right]$$

$$P_n = 50.17a - 1.29a^2 + 208 \qquad \text{(Eqn. 2)}$$

4) Assume c = 13.45 inch, which exceed c_b (11.54 inch).

$$a = 0.85 \bullet 13.45 = 11.43 \text{ inch}$$

Substitute in Eqn. 2:

$$P_n = 50.17 \bullet 11.43 - 1.29 \bullet (11.43)^2 + 208 = 612.9 \text{ kips}$$

5) Calculate f_s from the strain diagram when c = 13.45 inch.

$$f_s = \left(\frac{19.5 - 13.45}{13.45}\right)87 = 39.13 \text{ ksi}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.00135$$

6) Substitute a = 13.45 inch and f_s = 39.13 ksi in Eqn. 1 to calculate P_{n2}:

$$P_{n2} = 47.6(11.43) + 226.4 - 4(39.13) = 613.9 \text{ kips}$$

Which is very close to the calculated P_{n2} of 612.9 kips (less than 1% difference)

$$M_n = P_n e = 612.9\left(\frac{10}{12}\right) = 510.8 \text{ kips-ft}$$

7) Check if compression steels yield. From strain diagram,

$$\varepsilon_s' = \left(\frac{13.45 - 2.5}{13.45}\right)(0.003) = 0.00244 > \varepsilon_y = 0.00207 \text{ ksi}$$

Compression steels yields, as assumed.

8) Calculate ϕ,

$d_t = d = 19.5$ inch, $\qquad c$ = 13.45 inch

$$\varepsilon_t \text{ (at the tension reinforcement level)} = 0.003\left(\frac{19.45 - 13.45}{13.45}\right) = 0.00135$$

Since $\varepsilon_t < 0.002$, then $\phi = 0.65$

$$\phi P_n = 0.65(612.9) = 398.4 \text{ kips}$$

$$\phi M_n = 0.65(510.8) = 332 \text{ k-ft.}$$

ACI 318-11 Example 001

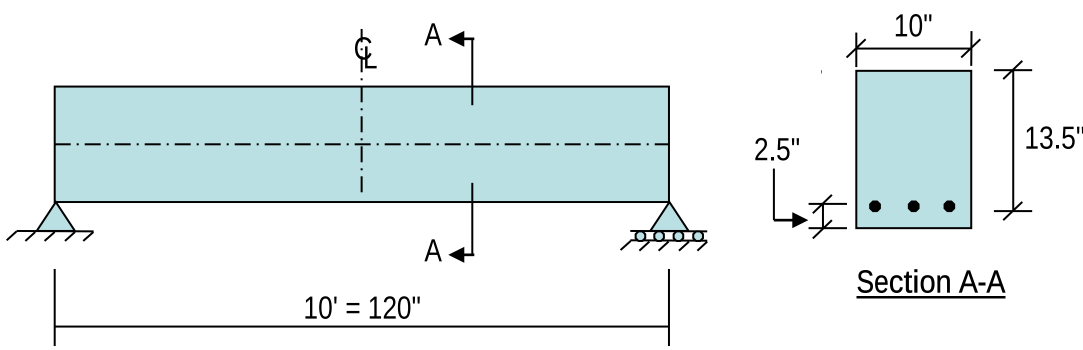
SHEAR AND FLEXURAL REINFORCEMENT DESIGN OF A SINGLY REINFORCED RECTANGLE

EXAMPLE DESCRIPTION

The flexural and shear design of a rectangular concrete beam is calculated in this example.

A simply supported beam is subjected to an ultimate uniform load of 9.736 k/ft. This example is tested using the ACI 318-11 concrete design code. The flexural and shear reinforcing computed is compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 3600 k/in^2
v = 0.2
G = 1500 k/in^2

Section Properties

d = 13.5 in
b = 10.0 in
I = 3,413 in^4

Design Properties

f'_c = 4 k/in^2
f_y = 60 k/in^2

TECHNICAL FEATURES TESTED

- Calculation of Flexural reinforcement, A_s
- Enforcement of Minimum tension reinforcement, $A_{s,min}$
- Calculation of Shear reinforcement, A_v
- Enforcement of Minimum shear reinforcing, $A_{v,min}$

RESULTS COMPARISON

Independent results are hand calculated based on the equivalent rectangular stress distribution described in Example 6.1 in Notes on ACI 318-11 Building Code.

Output Parameter	ETABS	Independent	Percent Difference
Design Moment, M_u (k-in)	1460.4	1460.4	0.00%
Tension Reinf, A_s (in^2)	2.37	2.37	0.00%
Design Shear Force, V_u	37.73	37.73	0.00%
Shear Reinf, A_v/s (in^2/in)	0.041	0.041	0.00%

COMPUTER FILE: ACI 318-11 Ex001

CONCLUSION

The computed results show an exact match for the flexural and the shear reinforcing.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\varphi = 0.9$, $A_g = 160$ sq-in

$$A_{s,min} = \frac{200}{f_y} b_w d = 0.450 \text{ sq-in (Govern)}$$

$$= \frac{3\sqrt{f_c^{'}}}{f_y} b_w d = 0.427 \text{ sq-in}$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right) = 0.85$$

$$c_{\max} = \frac{0.003}{0.003 + 0.005} d = 5.0625 \text{ in}$$

$$a_{\max} = \beta_1 c_{\max} = 4.303 \text{ in}$$

Combo1

$$w_u = (1.2 w_d + 1.6 w_l) = 9.736 \text{ k/ft}$$

$$M_u = \frac{w_u l^2}{8} = 9.736 \bullet 10^2/8 = 121.7 \text{ k-ft} = 1460.4 \text{ k-in}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f_c^{'} \varphi b}} = 4.183 \text{ in } (a < a_{max})$$

The area of tensile steel reinforcement is given by:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{1460.4}{0.9 \bullet 60 \bullet (13.5 - 4.183/2)}$$

$$A_s = 2.37 \text{ sq-in}$$

Shear Design

The following quantities are computed for all of the load combinations:

$\varphi = 0.75$

Check the limit of $\sqrt{f'_c}$:

$\sqrt{f'_c}$ = 63.246 psi < 100 psi

The concrete shear capacity is given by:

$\varphi V_c = \varphi 2 \sqrt{f'_c}\, bd = 12.807$ k

The maximum shear that can be carried by reinforcement is given by:

$\varphi V_s = \varphi 8 \sqrt{f'_c}\, bd = 51.229$ k

The following limits are required in the determination of the reinforcement:

$(\varphi V_c/2) = 6.4035$ k

$(\varphi V_c + \varphi 50\, bd) = 11.466$ k

$\varphi V_{max} = \varphi V_c + \varphi V_s = 64.036$ k

Given V_u, V_c and V_{max}, the required shear reinforcement in area/unit length for any load combination is calculated as follows:

If $V_u \le \varphi (V_c/2)$,

$$\frac{A_v}{s} = 0,$$

else if $\varphi (V_c/2) < V_u \le \varphi V_{max}$

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d} \ge \left(\frac{A_v}{s}\right)_{min}$$

where:

$$\left(\frac{A_v}{s}\right)_{min} = \max\left\{50\left(\frac{b_w}{f_{yt}}\right), \left(\frac{b_w}{f_{yt}}\right) \bullet \frac{3}{4}\sqrt{f'_c}\right\}$$

else if $V_u > \varphi V_{max}$,

a failure condition is declared.

Combo1

$$V_u = 9.736 \bullet (5-13.5/12) = 37.727 \text{ k}$$

$$\phi\left(V_c / 2\right) = 6.4035\,k \le V_u = 37.727\,k \le \phi V_{\max} = 64.036\,k$$

$$\left(\frac{A_v}{s}\right)_{\min} = \max\left\{50\left(\frac{10}{60,000}\right), \left(\frac{10}{60,000}\right) \bullet \frac{3}{4}\sqrt{4,000}\right\}$$

$$\left(\frac{A_v}{s}\right)_{\min} = \max\left\{0.0083,\ 0.0079\right\} = 0.0083\frac{in^2}{in}$$

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d} = 0.041\frac{in^2}{in} = 0.492\frac{in^2}{ft}$$

ACI 318-11 Example 002

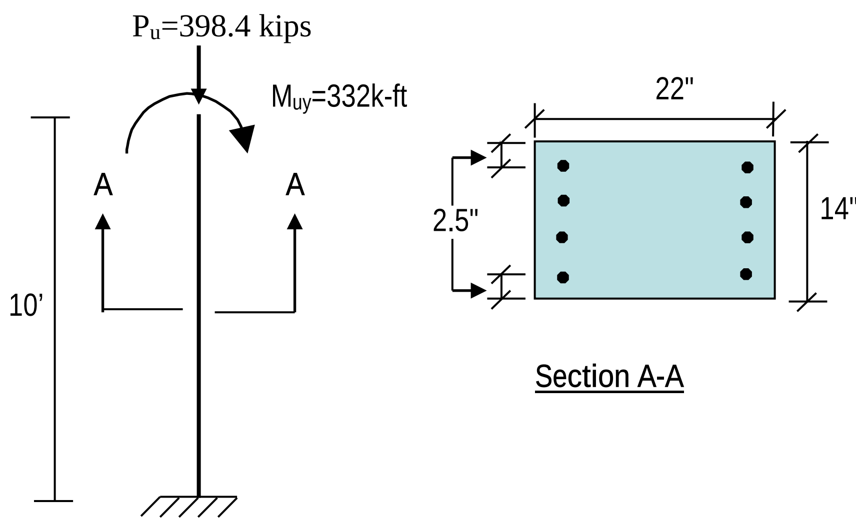
P-M INTERACTION CHECK FOR COMPRESSION-CONTROLLED RECTANGULAR COLUMN

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

A reinforced concrete column is subjected factored axial load P_u = 398.4 k and moments M_{uy} = 332 k-ft. This column is reinforced with 4 #9 bars. The total area of reinforcement is 8.00 in^2. The design capacity ratio is checked by hand calculations and result is compared.

GEOMETRY, PROPERTIES AND LOADING



Material Properties		Section Properties		Design Properties
E =	3600 k/in^2	b =	14 in	f'_c = 4 k/in^2
v =	0.2	d =	19.5 in	f_y = 60 k/in^2
G =	1500 k/in^2			

TECHNICAL FEATURES TESTED

- Tied Reinforced Concrete Column Demand/Capacity Ratio

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	1.000	1.00	0.00%

COMPUTER FILE: ACI 318-11 Ex002

CONCLUSION

The computed results show an exact match with the independent results.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

f'_c = 4 ksi	f_y = 60 ksi
b = 14 inch	d = 19.5 inch
P_u = 398.4 kips	M_u = 332 k-ft

1) Because e = 10 inch < (2/3)d = 13 inch., assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis fro a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{87}{87 + f_y} d_t = \frac{87}{87+60}(19.5) = 11.54 \text{ inch}$$

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = 0.85 f_c' ab = 0.85 \bullet 4 \bullet 14a = 47.6a$$

$$C_s = A_s' \left(f_y - 0.85 f_c' \right) = 4(60 - 0.85 \bullet 4) = 226.4 \text{ kips}$$

Assume compression steels yields, (this assumption will be checked later).

$$T = A_s f_s = 4 f_s \left(f_s < f_y \right)$$

$$P_n = 47.6a + 226.4 - 4 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$P_n = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2} \right) + C_s\left(d - d' \right) \right]$$

The plastic centroid is at the center of the section and d'' = 8.5 inch

$$e' = e + d'' = 10 + 8.5 = 18.5 \text{ inch.}$$

$$P_n = \frac{1}{18.5}\left[47.6a\left(19.5 - \frac{a}{2} \right) + 226.4(19.5 - 2.5) \right]$$

$$P_n = 50.17a - 1.29a^2 + 208 \quad \text{(Eqn. 2)}$$

4) Assume c = 13.45 inch, which exceed c_b (11.54 inch).

$$a = 0.85 \bullet 13.45 = 11.43 \text{ inch}$$

Substitute in Eqn. 2:

$$P_n = 50.17 \bullet 11.43 - 1.29 \bullet (11.43)^2 + 208 = 612.9 \text{ kips}$$

5) Calculate f_s from the strain diagram when c = 13.45 inch.

$$f_s = \left(\frac{19.5 - 13.45}{13.45}\right) 87 = 39.13 \text{ ksi}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.00135$$

6) Substitute a = 13.45 inch and f_s = 39.13 ksi in Eqn. 1 to calculate P_{n2}:

$$P_{n2} = 47.6(11.43) + 226.4 - 4(39.13) = 613.9 \text{ kips}$$

Which is very close to the calculated P_{n2} of 612.9 kips (less than 1% difference)

$$M_n = P_n e = 612.9\left(\frac{10}{12}\right) = 510.8 \text{ kips-ft}$$

7) Check if compression steels yield. From strain diagram,

$$\varepsilon_s' = \left(\frac{13.45 - 2.5}{13.45}\right)(0.003) = 0.00244 > \varepsilon_y = 0.00207 \text{ ksi}$$

Compression steels yields, as assumed.

8) Calculate ϕ,

$d_t = d = 19.5$ inch, $\quad c$ = 13.45 inch

$$\varepsilon_t \text{ (at the tension reinforcement level)} = 0.003\left(\frac{19.45 - 13.45}{13.45}\right) = 0.00135$$

Since $\varepsilon_t < 0.002$, then $\phi = 0.65$

$$\phi P_n = 0.65(612.9) = 398.4 \text{ kips}$$

$$\phi M_n = 0.65(510.8) = 332 \text{ k-ft.}$$

ACI 318-14 Example 001

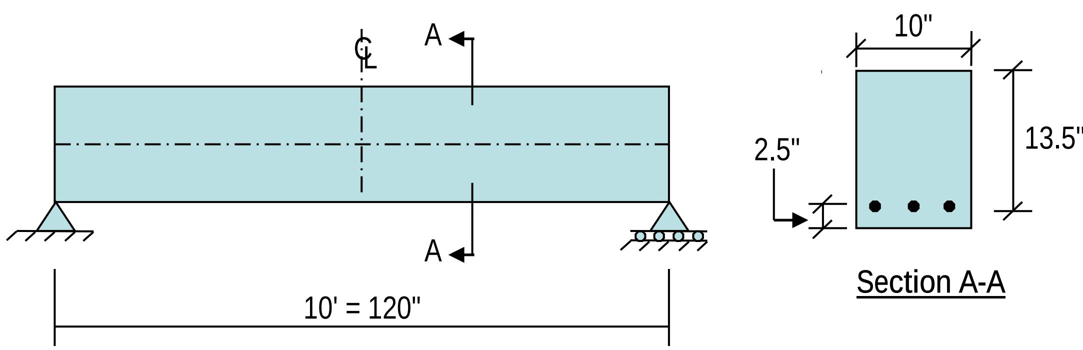
SHEAR AND FLEXURAL REINFORCEMENT DESIGN OF A SINGLY REINFORCED RECTANGLE

EXAMPLE DESCRIPTION

The flexural and shear design of a rectangular concrete beam is calculated in this example.

A simply supported beam is subjected to an ultimate uniform load of 9.736 k/ft. This example is tested using the ACI 318-14 concrete design code. The flexural and shear reinforcing computed is compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 3600 k/in^2
v = 0.2
G = 1500 k/in^2

Section Properties

d = 13.5 in
b = 10.0 in
I = 3,413 in^4

Design Properties

f'_c = 4 k/in^2
f_y = 60 k/in^2

TECHNICAL FEATURES TESTED

- Calculation of Flexural reinforcement, A_s
- Enforcement of Minimum tension reinforcement, $A_{s,min}$
- Calculation of Shear reinforcement, A_v
- Enforcement of Minimum shear reinforcing, $A_{v,min}$

RESULTS COMPARISON

Independent results are hand calculated based on the equivalent rectangular stress distribution described in Example 6.1 in Notes on ACI 318-14 Building Code.

Output Parameter	ETABS	Independent	Percent Difference
Design Moment, M_u (k-in)	1460.4	1460.4	0.00%
Tension Reinf, A_s (in^2)	2.37	2.37	0.00%
Design Shear Force, V_u	37.73	37.73	0.00%
Shear Reinf, A_v/s (in^2/in)	0.041	0.041	0.00%

COMPUTER FILE: ACI 318-14 Ex001

CONCLUSION

The computed results show an exact match for the flexural and the shear reinforcing.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\varphi = 0.9$, $A_g = 160$ sq-in

$$A_{s,min} = \frac{200}{f_y} b_w d = 0.450 \text{ sq-in (Govern)}$$

$$= \frac{3\sqrt{f_c'}}{f_y} b_w d = 0.427 \text{ sq-in}$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right) = 0.85$$

$$c_{\max} = \frac{0.003}{0.003 + 0.005} d = 5.0625 \text{ in}$$

$$a_{\max} = \beta_1 c_{\max} = 4.303 \text{ in}$$

Combo1

$w_u = (1.2w_d + 1.6w_l) = 9.736$ k/ft

$$M_u = \frac{w_u l^2}{8} = 9.736 \bullet 10^2/8 = 121.7 \text{ k-ft} = 1460.4 \text{ k-in}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f_c' \varphi b}} = 4.183 \text{ in } (a < a_{max})$$

The area of tensile steel reinforcement is given by:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{1460.4}{0.9 \bullet 60 \bullet (13.5 - 4.183/2)}$$

$$A_s = 2.37 \text{ sq-in}$$

Shear Design

The following quantities are computed for all of the load combinations:

$\varphi = 0.75$

Check the limit of $\sqrt{f'_c}$:

$\sqrt{f'_c}$ = 63.246 psi < 100 psi

The concrete shear capacity is given by:

$\varphi V_c = \varphi 2 \sqrt{f'_c}\, bd = 12.807$ k

The maximum shear that can be carried by reinforcement is given by:

$\varphi V_s = \varphi 8 \sqrt{f'_c}\, bd = 51.229$ k

The following limits are required in the determination of the reinforcement:

$(\varphi V_c/2) = 6.4035$ k

$(\varphi V_c + \varphi 50\, bd) = 11.466$ k

$\varphi V_{\max} = \varphi V_c + \varphi V_s = 64.036$ k

Given V_u, V_c and $V_{\max}$, the required shear reinforcement in area/unit length for any load combination is calculated as follows:

If $V_u \le \varphi (V_c/2)$,

$$\frac{A_v}{s} = 0,$$

else if $\varphi (V_c/2) < V_u \le \varphi V_{\max}$

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d} \ge \left(\frac{A_v}{s}\right)_{\min}$$

where:

$$\left(\frac{A_v}{s}\right)_{\min} = \max\left\{50\left(\frac{b_w}{f_{yt}}\right), \left(\frac{b_w}{f_{yt}}\right) \bullet \frac{3}{4}\sqrt{f'_c}\right\}$$

else if $V_u > \varphi V_{\max}$,

a failure condition is declared.

Combo1

V_u = 9.736•(5-13.5/12) = 37.727 k

$$\phi\left(V_c / 2\right) = 6.4035\,k \le V_u = 37.727\,k \le \phi V_{\max} = 64.036\,k$$

$$\left(\frac{A_v}{s}\right)_{\min} = \max\left\{50\left(\frac{10}{60,000}\right), \left(\frac{10}{60,000}\right)\bullet\frac{3}{4}\sqrt{4,000}\right\}$$

$$\left(\frac{A_v}{s}\right)_{\min} = \max\left\{0.0083,\ 0.0079\right\} = 0.0083\frac{in^2}{in}$$

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_{ys} d} = 0.041\frac{in^2}{in} = 0.492\frac{in^2}{ft}$$

ACI 318-14 Example 002

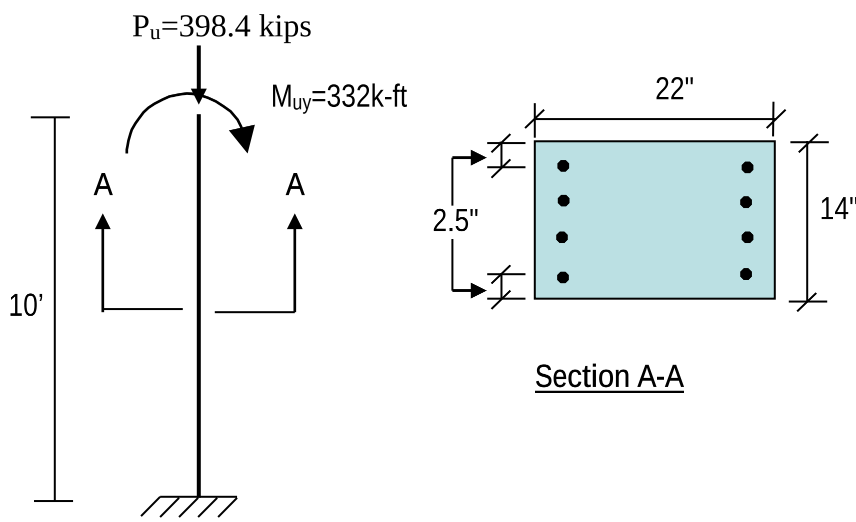
P-M Interaction Check for Compression-Controlled Rectangular Column

Example Description

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

A reinforced concrete column is subjected factored axial load P_u = 398.4 k and moments M_{uy} = 332 k-ft. This column is reinforced with 4 #9 bars. The total area of reinforcement is 8.00 in^2. The design capacity ratio is checked by hand calculations and result is compared.

Geometry, Properties and Loading



Material Properties		Section Properties		Design Properties
E =	3600 k/in^2	b =	14 in	f'_c = 4 k/in^2
v =	0.2	d =	19.5 in	f_y = 60 k/in^2
G =	1500 k/in^2			

TECHNICAL FEATURES TESTED

- Tied Reinforced Concrete Column Demand/Capacity Ratio

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	1.000	1.00	0.00%

COMPUTER FILE: ACI 318-14 Ex002

CONCLUSION

The computed results show an exact match with the independent results.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

f'_c = 4 ksi f_y = 60 ksi
b = 14 inch   d = 19.5 inch
P_u = 398.4 kips M_u = 332 k-ft

1) Because e = 10 inch < (2/3)d = 13 inch., assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis fro a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{87}{87 + f_y} d_t = \frac{87}{87 + 60}(19.5) = 11.54 \text{ inch}$$

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = 0.85 f_c' ab = 0.85 \bullet 4 \bullet 14a = 47.6a$$

$$C_s = A_s' \left(f_y - 0.85 f_c' \right) = 4(60 - 0.85 \bullet 4) = 226.4 \text{ kips}$$

Assume compression steels yields, (this assumption will be checked later).

$$T = A_s f_s = 4 f_s \left(f_s < f_y \right)$$

$$P_n = 47.6a + 226.4 - 4 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$P_n = \frac{1}{e'}\left[C_c \left(d - \frac{a}{2} \right) + C_s \left(d - d' \right) \right]$$

The plastic centroid is at the center of the section and d'' = 8.5 inch

$$e' = e + d'' = 10 + 8.5 = 18.5 \text{ inch.}$$

$$P_n = \frac{1}{18.5}\left[47.6a\left(19.5 - \frac{a}{2} \right) + 226.4(19.5 - 2.5) \right]$$

$$P_n = 50.17a - 1.29a^2 + 208 \qquad \text{(Eqn. 2)}$$

4) Assume c = 13.45 inch, which exceed c_b (11.54 inch).

$$a = 0.85 \bullet 13.45 = 11.43 \text{ inch}$$

Substitute in Eqn. 2:

$$P_n = 50.17 \bullet 11.43 - 1.29 \bullet (11.43)^2 + 208 = 612.9 \text{ kips}$$

5) Calculate f_s from the strain diagram when c = 13.45 inch.

$$f_s = \left(\frac{19.5 - 13.45}{13.45}\right)87 = 39.13 \text{ ksi}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.00135$$

6) Substitute a = 13.45 inch and f_s = 39.13 ksi in Eqn. 1 to calculate P_{n2}:

$$P_{n2} = 47.6(11.43) + 226.4 - 4(39.13) = 613.9 \text{ kips}$$

Which is very close to the calculated P_{n2} of 612.9 kips (less than 1% difference)

$$M_n = P_n e = 612.9\left(\frac{10}{12}\right) = 510.8 \text{ kips-ft}$$

7) Check if compression steels yield. From strain diagram,

$$\varepsilon_s' = \left(\frac{13.45 - 2.5}{13.45}\right)(0.003) = 0.00244 > \varepsilon_y = 0.00207 \text{ ksi}$$

Compression steels yields, as assumed.

8) Calculate ϕ,

$$d_t = d = 19.5 \text{ inch}, \qquad c = 13.45 \text{ inch}$$

$$\varepsilon_t \text{ (at the tension reinforcement level)} = 0.003\left(\frac{19.45 - 13.45}{13.45}\right) = 0.00135$$

Since $\varepsilon_t < 0.002$, then $\phi = 0.65$

$$\phi P_n = 0.65(612.9) = 398.4 \text{ kips}$$

$$\phi M_n = 0.65(510.8) = 332 \text{ k-ft.}$$

AS 3600-2009 Example 001

Shear and Flexural Reinforcement Design of a Singly Reinforced T-Beam

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block extends below the flange but remains within the balanced condition permitted by AS 3600-09.
- The average shear stress in the beam is below the maximum shear stress allowed by AS 3600-09, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 500-mm-deep T-beam with a flange 100 mm thick and 600 mm wide is considered. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements automatically generated. The maximum element size has been specified to be 500 mm. The beam is supported by columns without rotational stiffnesses and with very large vertical stiffness (1×10^{20} kN/m).

The beam is loaded with symmetric third-point loading. One dead load case (DL30) and one live load case (LL130), with only symmetric third-point loads of magnitudes 30, and 130 kN, respectively, are defined in the model. One load combinations (COMB130) is defined using the AS 3600-09 load combination factors of 1.2 for dead load and 1.5 for live load. The model is analyzed for both of these load cases and the load combination.

The beam moment and shear force are computed analytically. Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.

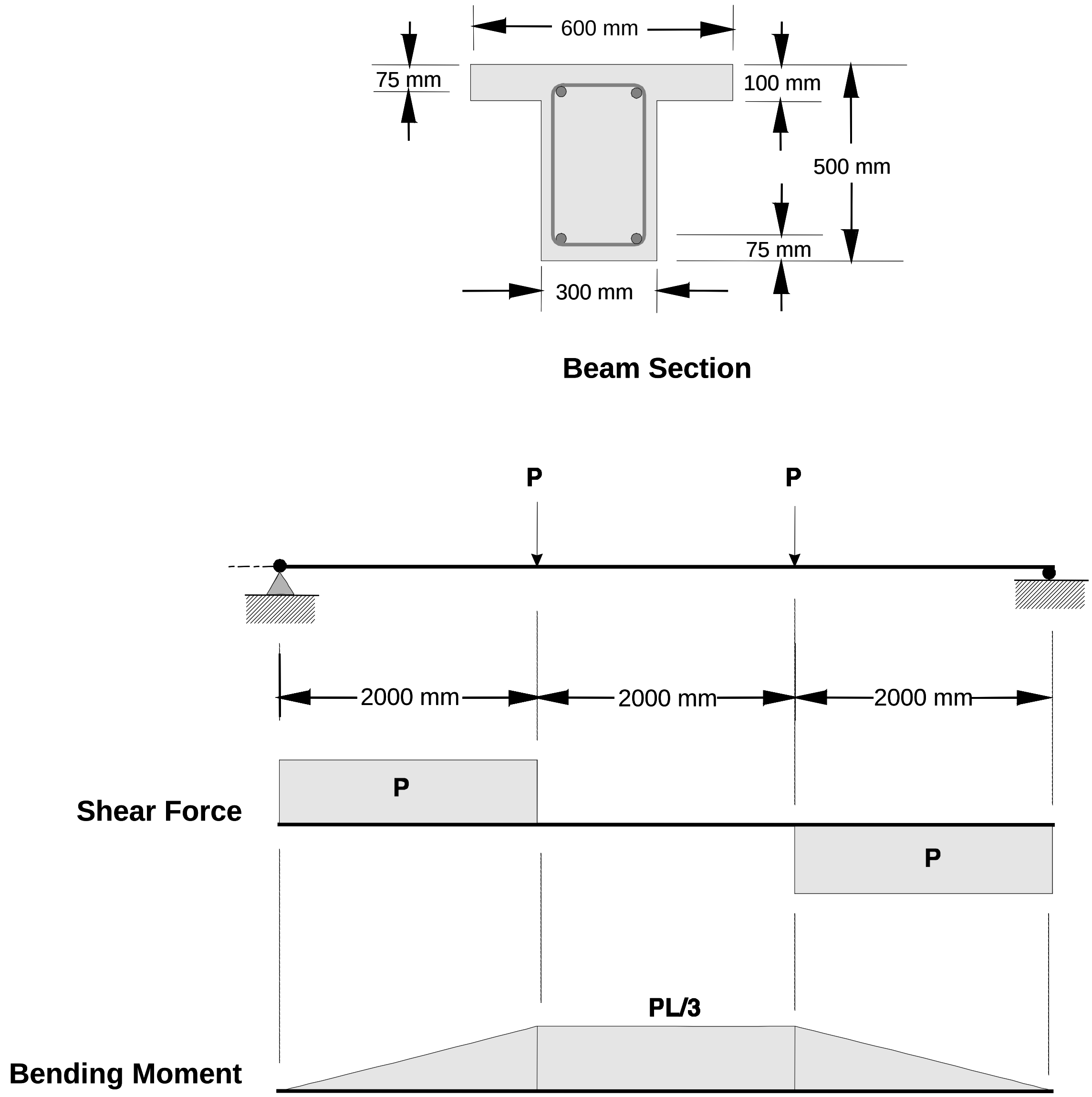


Figure 1 The Model Beam for Flexural and Shear Design

GEOMETRY, PROPERTIES AND LOADING

Clear span,	L	=	6000	mm
Overall depth,	h	=	500	mm
Flange thickness,	D_s	=	100	mm
Width of web,	b_w	=	300	mm
Width of flange,	b_f	=	600	mm
Depth of tensile reinf.,	d_{sc}	=	75	mm

Effective depth,	d	=	425	mm
Depth of comp. reinf.,	d'	=	75	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	$25x10^5$	MPa
Modulus of elasticity,	E_s	=	$2x10^8$	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	30	kN
Live load,	P_l	=	130	kN

TECHNICAL FEATURES TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the total factored moments in the design strip. They match exactly for this problem. Table 1 also shows the design reinforcement comparison.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-cm)
		A_s^+
ETABS	462	33.512
Calculated	462	33.512

$A^+_{s,min}$ = 3.00 sq-cm

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-cm/m)	
	ETABS	**Calculated**
231	12.05	12.05

COMPUTER FILE: AS 3600-2009 Ex001

CONCLUSION

The computed results show an exact match for the flexural and the shear reinforcing.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

ϕ = 0.8 for bending

$0.67 \le \alpha_2 \le 0.85,\ where\ \alpha_2 = 1.0 - 0.003 \bullet f_c' = 0.91$, use $\alpha_2 = 0.85$

$0.67 \le \gamma \le 0.85,\ where\ \alpha_2 = 1.05 - 0.007 \bullet f_c' = 0.84$, use $\gamma = 0.84$

$k_u \le 0.36$

$a_{\max} = \gamma k_u d = 0.84 \bullet 0.36 \bullet 425 = 128.52\,mm$

$$A_{st.\min} = \alpha_b \left(\frac{D}{d}\right)^2 \frac{f'_{ct,f}}{f_{sy}} b_w d$$

where for L- and T-Sections with the web in tension:

$D = h = 500\,mm$

$$\alpha_b = 0.20 + \left(\frac{b_f}{b_w} - 1\right)\left(0.4\frac{D_s}{D} - 0.18\right) \ge 0.20\left(\frac{b_f}{b_w}\right)^{1/4} = 0.2378$$

$f'_{ct,f} = 0.5\sqrt{f'_c} = 0.5\sqrt{30} = 3.3\,MPa$

$f_{sy} = f_y = 460\,MPa \le 500\,MPa$

$$A_{st.\min} = 0.2378\left(\frac{D}{d}\right)^2 \frac{f'_{ct,f}}{f_{sy}} bd$$

$= 0.2378 \bullet (500/425)^2 \bullet 3.3/460 \bullet 300 \bullet 425$

$= 299.9\ mm^2$

COMB130

$V^* = (1.2P_d + 1.5P_l) = 231$kN

$$M^* = \frac{V^* L}{3} = 462 \text{ kN-m}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M^*}{\alpha_2 f'_c \phi b_f}}$$ = 100.755 mm ($a > D_s$), so design as a T-beam.

The compressive force developed in the concrete alone is given by the following methodology:

The first part of the calculation is for balancing the compressive force from the flange, C_f, and the second part of the calculation is for balancing the compressive force from the web, C_w. C_f is given by:

$$C_f = \alpha_2 f'_c \left(b_f - b_w\right) \bullet \min\left(D_s, a_{\max}\right) = 765\,kN$$

Therefore,

$$A_{s1} = \frac{C_f}{f_{sy}} = \frac{765}{460} = 1663.043\,mm^2$$

and the portion of M^* that is resisted by the flange is given by:

$$M_{uf} = \phi C_f \left(d - \frac{\min\left(D_s, a_{\max}\right)}{2}\right) = 229.5 \text{ kN-m}$$

Again, the value for ϕ is 0.80 by default. Therefore, the balance of the moment, M^* to be carried by the web is:

$$M_{uw} = M^* - M_{uf} = 462 - 229.5 = 232.5$$

The web is a rectangular section of dimensions b_w and d, for which the design depth of the compression block is recalculated as:

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{\alpha_2 f'_c \phi b_w}} = 101.5118 \text{ mm}$$

$a_1 \le a_{\max}$, so no compression reinforcement is needed, and the area of tension reinforcement is then given by:

$$A_{s2} = \frac{M_{uw}}{\phi f_{sy}\left(d - \frac{a_1}{2}\right)} = 1688.186 \text{ mm}^2$$

$$A_{st} = A_{s1} + A_{s2} = 3351.23 \text{ sq-mm} = 33.512 \text{ sq-cm}$$

Shear Design

$\phi = 0.7$ for shear

Calculated at the end of the beam, so M=0 and $A_{st} = 0$.

The shear force carried by the concrete, V_{uc}, is calculated as:

$$V_{uc} = \beta_1 \beta_2 \beta_3 b_v d_o f'_{cv} \left[\frac{A_{st}}{b_v d_o} \right]^{1/3} = 0\, kN$$

where,

$f'_{cv} = \left(f'_c \right)^{1/3} = 3.107$ N/mm^2 ≤ 4MPa

$\beta_1 = 1.1\left(1.6 - \frac{d_o}{1000}\right) \geq 1.1 = 1.2925,$

$\beta_2 = 1$ since no significant axial load is present

$\beta_3 = 1$

$b_v = b_w = 300$mm as there are no grouted ducts

$d_o = d = 425$ mm

The shear force is limited to a maximum of:

$$V_{u.\max} = 0.2 f'_c b d_o = 765 \text{ kN}$$

And the beam must have a minimum shear force capacity of:

$$V_{u.\min} = V_{uc} + 0.6 \bullet b_w \bullet d_o = 0 + 0.6 \bullet 300 \bullet 425 = 77\, kN$$

$V^* = 231 kN > \phi V_{uc} / 2 = 0$, so reinforcement is needed.

$V^* = 231 kN \leq \phi V_{u.\max} = 535.5 kN$, so concrete crushing does not occur.

$$\left(\frac{A_{sv}}{s} \right)_{\min} = \max \left\{ 0.35 \frac{b_w}{f_{sy}}, 0.06 \frac{\sqrt{f'_c} b_v}{f_{sy}} \right\} = \max \left\{ 228.26, 214.33 \right\} \frac{mm^2}{m}$$

$$\left(\frac{A_{sv}}{s} \right)_{\min} = 228.26 \frac{mm^2}{m}$$

COMB130

Since $\phi V_{u.\min} = 53.55 kN < V^* = 231 kN \le \phi V_{u.\max} = 535.5 kN$

$$\frac{A_{sv}}{s} = \frac{\left(V^* - \phi V_{uc}\right)}{\phi f_{sy} d_o \cot \theta_v} \ge \left(\frac{A_{sv}}{s}\right)_{\min}$$

θ_v = the angle between the axis of the concrete compression strut and the longitudinal axis of the member, which varies linearly from 30 degrees when $V^*=\phi V_{u,\min}$ to 45 degrees when $V^*=\phi V_{u,\max}$ = 35.52 degrees

θ_v = 35.52 degrees

$$\frac{A_{sv}}{s} = \frac{\left(213 - 0\right)}{0.7 \bullet 460 \bullet 425 \bullet \cot\left(35.52^o\right)} = 1205.04 \frac{mm^2}{m} \ge \left(\frac{A_{sv}}{s}\right)_{\min} = 228.26 \frac{mm^2}{m}$$

$$\frac{A_{sv}}{s} = 12.05 \frac{cm^2}{m}$$

AS 3600-2009 Example 002

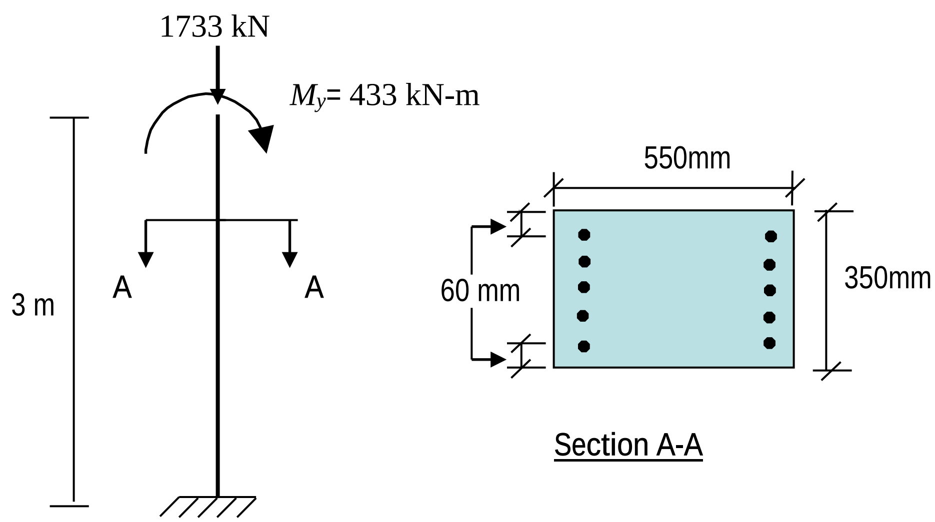
P-M INTERACTION CHECK FOR COMPRESSION-CONTROLLED RECTANGULAR COLUMN

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

In this example, a reinforced concrete column is subjected to factored axial load N = 1733 kN and moment M_y = 433 kN-m. This column is reinforced with five 25M bars. The design capacity ratio is checked by hand calculations and the result is compared with computed results. The column is designed as a short, non-sway member.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E_c = 25x10^6 kN/m^2	b = 350 m	f_{cu} = 30 MPa
v = 0.2	d = 490 mm	f_y = 460 MPa
G = 10416666.7kN/m^2		

TECHNICAL FEATURES TESTED

- Tied reinforced concrete column design

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	1.089	1.00	8.9%

COMPUTER FILE: AS 3600-2009 EX002

CONCLUSION

The computed results show an acceptable comparison with the independent results.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

f_{cu} = 30 MPa f_y = 460 MPa
b = 350 mm d = 490 mm

1) Because e = 250 mm < (2/3)d = 327 mm, assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis for a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{600}{600 + f_y} d_t = \frac{600}{600 + 460}(490) = 277.4 \text{ mm}$$

2) From the equation of equilibrium:

$$N = C_c + C_s - T$$

where

$$C_c = \alpha_2 f_c' ab = 0.85 \bullet 30 \bullet 350a = 8925a$$

$$C_s = A_s \left(f_y - \alpha_2 f_c' \right) = 2500\left(460 - 0.85 \bullet 30\right) = 1{,}086{,}250 \text{ N}$$

Assume compression steel yields, (this assumption will be checked later).

$$T = A_s f = 2500 f_s = 2500 f_s \left(f_s < f_y \right)$$

$$N_1 = 8925a + 1.086{,}250 - 2500 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$N_2 = \frac{1}{e'}\left[C_c \left(d - \frac{a}{2} \right) + C_s \left(d - d' \right) \right]$$

The plastic centroid is at the center of the section and d'' = 215 mm

$$e' = e + d'' = 250 + 215 = 465 \text{ mm}$$

$$N = \frac{1}{465}\left[8925a\left(490 - \frac{a}{2} \right) + 1{,}086{,}250\left(490 - 60\right) \right]$$

$$N = 9404.8a - 9.597a^2 + 1{,}004{,}489 \qquad \text{(Eqn. 2)}$$

4) Assume c = 333.9 mm, which exceeds c_b (296 mm).

$$a = 0.84 \bullet 333.9 = 280.5 \text{ mm}$$

Substitute in Eqn. 2:

$$N_2 = 8925 \bullet 280.5 - 9.597(280.5)^2 + 1,004,489 = 2,888,240 \text{ N}$$

5) Calculate f_s from the strain diagram when c = 365 mm.

$$f_s = \left(\frac{490 - 333.9}{333.9}\right)600 = 280.5 \text{ MPa}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.0014$$

6) Substitute a = 280.5 mm and f_s = 280.5 MPa in Eqn. 1 to calculate N_1:

$$N_1 = 8925(280.5) + 1,086,250 - 2500(280.5) = 2,887,373 \text{ N}$$

which is very close to the calculated N_2 of 2,888,240 (less than 1% difference)

$$M = Ne = 2888\left(\frac{250}{1000}\right) = 722 \text{ kN-m}$$

7) Check if compression steel yields. From strain diagram,

$$\varepsilon_s' = \left(\frac{333.9 - 60}{333.9}\right)(0.003) = 0.0025 > \varepsilon_y = 0.0023$$

Compression steel yields, as assumed.

8) Therefore, section capacity is

$$N = \phi \bullet 2888 = 1733 \text{ kN}$$

$$M = \phi \bullet 2888 \bullet \frac{e}{1000} = 0.60 \bullet 2888 \bullet \frac{250}{1000} = 433 \text{ kN-m}$$

BS 8110-1997 Example 001

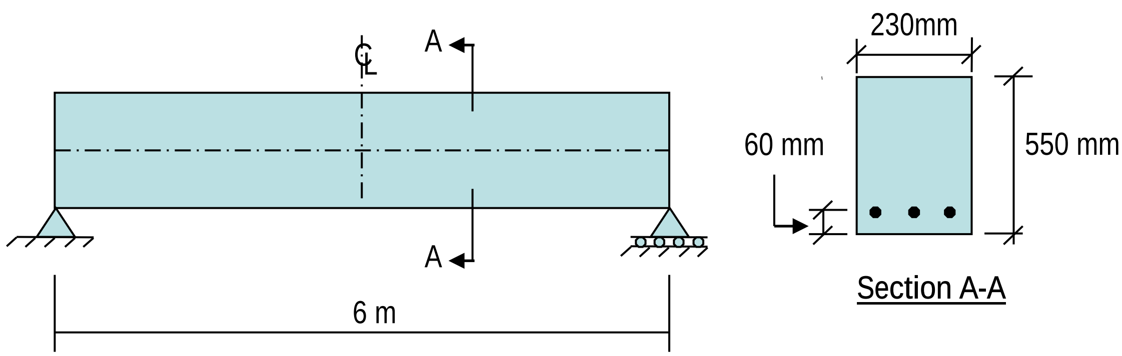
SHEAR AND FLEXURAL REINFORCEMENT DESIGN OF A SINGLY REINFORCED RECTANGLE

Example Description

The flexural and shear strength of a rectangular concrete beam is tested in this example.

In the example a simple supported beam is subjected to a uniform factored load of 36.67 kN/m. This example was tested using the BS 8110-97 concrete design code. The flexural and shear reinforcing computed is compared with independent results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 25×10^6 kN/m^2

v = 0.2

G = 10416666.7kN/m^2

Section Properties

d = 490 mm

W = 36.67 kN/m

Design Properties

f_{cu} = 30 MPa

f_y = 460 MPa

TECHNICAL FEATURES TESTED

- Calculation of Flexural reinforcement, A_s
- Enforcement of Minimum tension reinforcement, $A_{s,min}$
- Calculation of Shear reinforcement, A_v
- Enforcement of Minimum shear reinforcing, $A_{v,min}$

RESULTS COMPARISON

Independent results are hand calculated based on the equivalent rectangular stress distribution described in Example 7.2 on page 149 of Reinforced Concrete Design by W. H. Mosley, J. H. Bungey & R. Hulse.

Output Parameter	ETABS	Independent	Percent Difference
Design Moment, M_u (kN-m)	165.02	165.02	0.00%
Tension Reinf, A_s (mm^2)	964.1	964.1	0.00%
Design Shear, V_u (kN)	92.04	92.04	0.00%
Shear Reinf, A_{sv}/s_v (mm^2/mm)	0.231	0.231	0.00%

COMPUTER FILE: BS 8110-1997 Ex001

CONCLUSION

The computed results show an exact match with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\gamma_{m,\ steel}$ = 1.15

$$A_{s,\min} = 0.0013 b_w h = 0.0013 \bullet 230 \bullet 550 = 164.45\, mm^2$$

Design Combo COMB1

w_u = =36.67 kN/m

$$M_u = \frac{w_u l^2}{8} = 165 \text{ kN-m}$$

The depth of the compression block is given by:

$$K = \frac{M}{f_{cu} b d^2} = 0.0996 < 0.156$$

If $K \leq 0.156$ (BS 3.4.4.4), the beam is designed as a singly reinforced concrete beam.

Then the moment arm is computed as:

$$z = d \left\{ 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \right\} \leq 0.95d = 427.90 \text{ mm}$$

The ultimate resistance moment is given by:

$$A_s = \frac{M}{\left(f_y / 1.15 \right) z} = 964.1 \text{ sq-mm}$$

Shear Design

$$V_U = \omega_U \left(\frac{L^2}{2} - d \right) = 92.04 \text{ kN at distance, d, from support}$$

$$v = \frac{V_U}{b\,d} = 0.8167 \text{ MPa}$$

$$v_{\max} = \min(0.8\sqrt{f_{cu}}\,,\ 5 \text{ MPa}) = 4.38178 \text{ MPa}$$

$v \le v_{\max}$, so no concrete crushing

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m} \left(\frac{100 A_s}{bd} \right)^{1/3} \left(\frac{400}{d} \right)^{1/4} = 0.415 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{25} \right)^{1/3} = 1.06266,\ 1 \le k_2 \le \left(\frac{40}{25} \right)^{1/3}$$

$$\gamma_{m,\ concrete} = 1.25$$

$$0.15 \le \frac{100\ A_s}{bd} \le 3$$

$$\frac{100\ A_s}{bd} = \frac{100 \bullet 266}{230 \bullet 490} = 0.2359$$

$$\left(\frac{400}{d} \right)^{1/4} = 0.95 \ge 1,\ so \left(\frac{400}{d} \right)^{1/4} \text{ is taken as 1.}$$

$f_{cu} \le 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

If $(v_c + 0.4) < v \le v_{\max}$

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c) b_w}{0.87 f_{yv}}$$

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c)b_w}{0.87 f_{yv}} = \frac{(0.8167 - 0.4150)}{0.87 \bullet 460} = 0.231 \text{ sq-mm/mm}$$

BS 8110-1997 Example 002

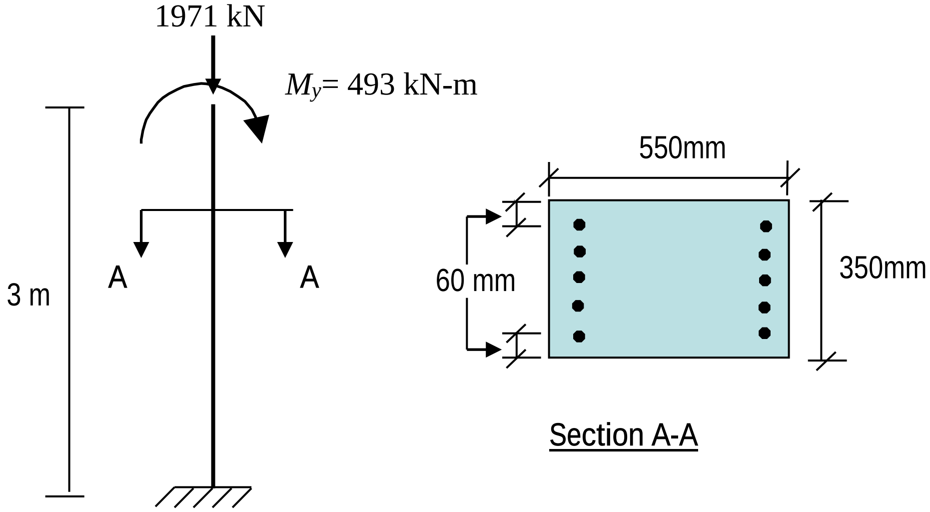
P-M Interaction Check for Compression-Controlled Rectangular Column

Example Description

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

In this example, a reinforced concrete column is subjected to factored axial load N = 1971 kN and moment M_y = 493 kN-m. This column is reinforced with five 25M bars. The design capacity ratio is checked by hand calculations and the result is compared with computed results. The column is designed as a short, non-sway member.

Geometry, Properties and Loading



Material Properties	Section Properties	Design Properties
E_c = 25x10^6 kN/m^2	b = 350 m	f_{cu} = 30 MPa
v = 0.2	d = 490 mm	f_y = 460 MPa
G = 10416666.7kN/m^2		

Technical Features Tested

- Tied reinforced concrete column design

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	0.994	1.00	0.40%

COMPUTER FILE: BS 8110-1997 EX002

CONCLUSION

The computed result shows an acceptable comparison with the independent result.

HAND CALCULATION

Column Strength under compression control

f_{cu} = 30 MPa f_y = 460 MPa
b = 350 mm d = 490 mm

1) Because e = 250 mm < (2/3)d = 327 mm, assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis for a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{700}{700 + f_y / \gamma_s} d_t = \frac{700}{700 + 460/1.15}(490) = 312 \text{ mm}$$

2) From the equation of equilibrium:

$$N = C_c + C_s - T$$

where

$$C_c = \frac{0.67}{\gamma_M} f_{cu} ab = 0.67/1.5 \bullet 30 \bullet 350a = 4667a$$

$$C_s = \frac{A'_s}{\gamma_s}\left(f_y - 0.4467 f_{cu}\right) = \frac{2500}{1.15}\left(460 - 0.4467 \bullet 30\right) = 971{,}014 \text{ N}$$

Assume compression steel yields (this assumption will be checked later).

$$T = \frac{A_s f_s}{\gamma_s} = \frac{2500 f_s}{1.15} = 2174 f_s \left(f_s < f_y\right)$$

$$N_1 = 4{,}667a + 971{,}014 - 2174 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$N = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_s\left(d - d'\right)\right]$$

The plastic centroid is at the center of the section and d'' = 215 mm

$$e' = e + d'' = 250 + 215 = 465 \text{ mm}$$

$$N = \frac{1}{465}\left[4{,}667a\left(490 - \frac{a}{2}\right) + 971{,}014\left(490 - 60\right)\right]$$

$$N = 4917.9a - 5.018a^2 + 897{,}926 \qquad \text{(Eqn. 2)}$$

4) Assume c = 364 mm, which exceed c_b (296 mm).

$$a = 0.9 \bullet 364 = 327.6 \text{ mm}$$

Substitute in Eqn. 2:

$$N_2 = 4917.9 \bullet 327.6 - 5.018(327.6)^2 + 897,926 = 1,970,500 \text{ N}$$

5) Calculate f_s from the strain diagram when c = 365 mm.

$$f_s = \left(\frac{490 - 364}{364}\right) 700 = 242.3 \text{ MPa}$$
$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.0012$$

6) Substitute a = 327.6 mm and f_s = 242.3 MPa in Eqn. 1 to calculate N_1:

$$N_1 = 4,667(327.6) + 971,014 - 2174(242.3) = 1,973,163 \text{ N}$$

which is very close to the calculated N_2 of 1,970,500 (less than 1% difference)

$$M = Ne = 1971\left(\frac{250}{1000}\right) = 493 \text{ kN-m}$$

7) Check if compression steels yield. From strain diagram,

$$\varepsilon'_s = \left(\frac{365 - 60}{365}\right)(0.0035) = 0.00292 > \varepsilon_y = 0.0023$$

Compression steel yields, as assumed.

8) Therefore, the section capacity is

$$N = 1971 \text{ kN}$$
$$M = 493 \text{ kN-m}$$

CSA A23.3-04 Example 001

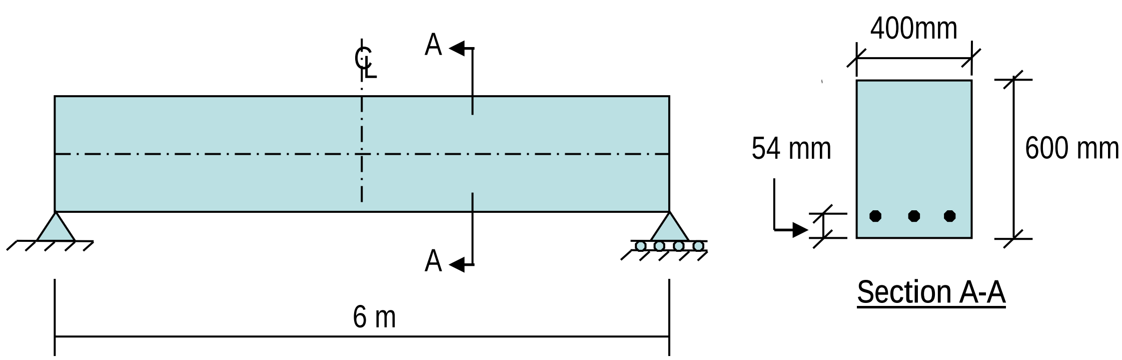
SHEAR AND FLEXURAL REINFORCEMENT DESIGN OF A SINGLY REINFORCED RECTANGLE

EXAMPLE DESCRIPTION

The flexural and shear strength of a rectangular concrete beam is tested in this example.

In the example a simply supported beam is subjected to a uniform factored load of 92.222 kN/m. This example is tested using the CSA A23.3-04 concrete design code. The flexural and shear reinforcing computed is compared with independent results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 25×10^6 kN/m^2

v = 0.2

G = 10416666.7kN/m^2

Section Properties

d = 546 mm

W = 92.222 kN/m

Design Properties

f'_c = 40 MPa

f_y = 400 MPa

TECHNICAL FEATURES TESTED

- Calculation of Flexural reinforcement, A_s
- Enforcement of Minimum tension reinforcement, $A_{s,min}$
- Calculation of Shear reinforcement, A_v
- Enforcement of Minimum shear reinforcing, $A_{v,min}$

RESULTS COMPARISON

Independent results are hand calculated based on the equivalent rectangular stress distribution described in Example 2.2 on page 2-12 in Part II on Concrete Design Handbook of Cement Association of Canada.

Output Parameter	ETABS	Independent	Percent Difference
Design Moment, M_f (kN-m)	415.00	415.00	0.00%
Tension Reinf, A_s (mm^2)	2466	2466	0.00%
Design Shear, V_f (kN)	226.31	226.31	0.00%
Shear Reinf, A_v/s (mm^2/mm)	0.379	0.379	0.00%

COMPUTER FILE: CSA A23.3-04 Ex001

CONCLUSION

The computed results show an exact match with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

ϕ_c = 0.65 for concrete

ϕ_s = 0.85 for reinforcement

$$A_{s,\min} = \frac{0.2\sqrt{f'_c}}{f_y}\; b\,h = 758.95 \text{ mm}^2$$

$$\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67 = 0.79$$

$$\beta_1 = 0.97 - 0.0025 f'_c \geq 0.67 = 0.87$$

$$c_b = \frac{700}{700 + f_y} d = 347.45 \text{ mm}$$

$$a_b = \beta_1 c_b = 302.285 \text{ mm}$$

COMB1

$$M_f = \frac{w_u l^2}{8} = 415 \text{ kN-m}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M_f}{\alpha_1 f'_c \phi_c b}} = 102.048 \text{ mm}$$

If $a \leq a_b$, the area of tension reinforcement is then given by:

$$A_s = \frac{M_f}{\phi_s f_y \left(d - \frac{a}{2}\right)} = 2466 \text{ mm}^2$$

$$A_{s,\min} = \min\left\{A_{s,\min}, \left(\frac{4}{3}\right)A_{s,required}\right\} = \min\left\{758.95, \left(\frac{4}{3}\right)2466\right\} = 758.95\,mm^2$$

Shear Design

The basic shear strength for rectangular section is computed as,

$\phi_c =$ 0.65 for shear

$\lambda = \{1.00,$ for normal density concrete

d_v is the effective shear depth. It is taken as the greater of 0.72h = 432 mm or 0.9d = 491.4 mm (governing).

$\beta =$ 0.18 since minimum transverse reinforcement is provided

V_f = 92.222•(3 - 0.546) = 226.31 kN

$V_c = \phi_c \lambda \beta \sqrt{f'_c} b_w d_v$ = 145.45 kN

$V_{r,\max} = 0.25\phi_c f'_c b_w d$ = 1419.60 kN

θ = 35º since $f_y \leq 400\,MPa$ and $f'_c \leq 60\,MPa$

$$\frac{A_v}{s} = \frac{(V_f - V_c)\tan\theta}{\phi_s f_{yt} d_v} = 0.339 \text{ mm}^2\text{/mm}$$

$$\left(\frac{A_v}{s}\right)_{\min} = 0.06\frac{\sqrt{f'_c}}{f_y}b = 0.379 \text{ mm}^2\text{/mm (Govern)}$$

CSA A23.3-04 Example 002

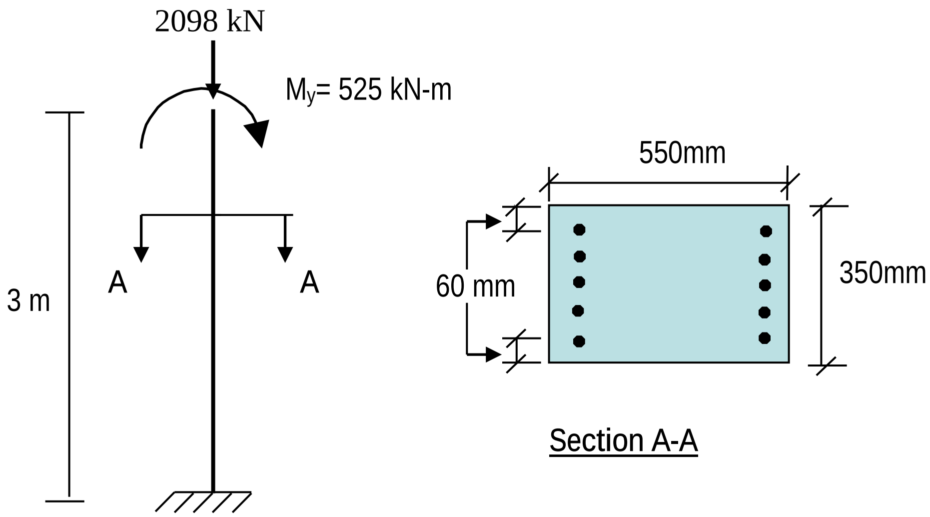
P-M Interaction Check for Compression-Controlled Rectangular Column

Example Description

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

In this example, a reinforced concrete column is subjected factored axial load N = 2098 kN and moment M_y = 525 kN-m. This column is reinforced with 5 T25 bars. The design capacity ratio is checked by hand calculations and results are compared. The column is designed as a short, non-sway member.

Geometry, Properties and Loading



Material Properties	Section Properties	Design Properties
E_c = 25x10^6 kN/m^2	b = 350 m	f'_c = 30 MPa
v = 0.2	d = 490 mm	f_y = 460 MPa
G = 10416666.7kN/m^2		

Technical Features Tested

- Tied Reinforced Concrete Column Design

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	0.9869	1.00	-1.31%

COMPUTER FILE: CSA A23.3-04 Ex002

CONCLUSION

The computed results show an acceptable comparison with the independent results.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

f_{cu} = 30 MPa f_y = 460 MPa
b = 350 mm   d = 490 mm

1) Because e = 250 mm < (2/3)d = 327 mm, assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis fro a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{700}{700 + f_y} d_t = \frac{700}{700 + 460}(490) = 296 \text{ mm}$$

2) From the equation of equilibrium:

$$P_r = C_c + C_s - T$$

where

$$C_c = \phi_c \bullet \alpha_1 f_c^{'} ab = 0.65 \bullet 0.805 \bullet 30 \bullet 350a = 5494.1a$$

$$C_s = \phi_s A_s^{'} \left(f_y - 0.805 f_c^{'} \right) = 0.85 \bullet 2500 \left(460 - 0.805 \bullet 30 \right) = 926{,}181 \text{ N}$$

Assume compression steels yields, (this assumption will be checked later).

$$T = \phi_s A_s f_s = 0.85 \bullet 2500 f_s = 2125 f_s \left(f_s < f_y \right)$$

$$P_r = 5{,}494.1a + 926{,}181 - 2125 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$P_r = \frac{1}{e^{'}} \left[C_c \left(d - \frac{a}{2} \right) + C_s \left(d - d^{'} \right) \right]$$

The plastic centroid is at the center of the section and $d^{''}$ = 215 mm

$$e^{'} = e + d^{''} = 250 + 215 = 465 \text{ mm}$$

$$P_r = \frac{1}{465} \left[5{,}494.1a \left(490 - \frac{a}{2} \right) + 926{,}181 \left(490 - 60 \right) \right]$$

$$P_r = 5789.5a - 5.91a^2 + 856{,}468.5 \qquad \text{(Eqn. 2)}$$

4) Assume c = 355 mm, which exceed c_b (296 mm).

$$a = 0.895 \bullet 355 = 317.7 \text{ mm}$$

Substitute in Eqn. 2:

$$P_r = 5789.5 \bullet 317.7 - 5.91(317.7)^2 + 856,468.5 = 2,099,327.8 \text{ N}$$

5) Calculate f_s from the strain diagram when c = 350 mm.

$$f_s = \left(\frac{490 - 355}{355}\right)700 = 266.2 \text{ MPa}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.0013$$

6) Substitute a = 317.7 mm and f_s = 266.2 MPa in Eqn. 1 to calculate P_{r2}:

$$P_{r2} = 5,494.1(317.7) + 926,181 - 2125(266.2) = 2,106,124.9 \text{ N}$$

Which is very close to the calculated P_{r1} of 2,012,589.8 (less than 1% difference)

$$M_r = P_r e = 2100\left(\frac{250}{1000}\right) = 525 \text{ kN-m}$$

7) Check if compression steels yield. From strain diagram,

$$\varepsilon_s' = \left(\frac{355 - 60}{355}\right)(0.0035) = 0.00291 > \varepsilon_y = 0.0023$$

Compression steels yields, as assumed.

8) Therefore, section capacity is

$$P_r = 2098 \text{ kN}$$

$$M_r = 525 \text{ kN-m}$$

CSA A23.3-14 Example 001

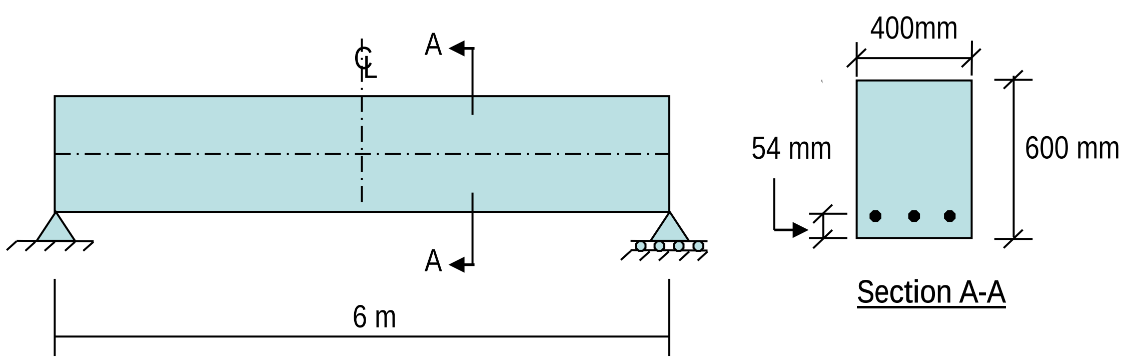
SHEAR AND FLEXURAL REINFORCEMENT DESIGN OF A SINGLY REINFORCED RECTANGLE

EXAMPLE DESCRIPTION

The flexural and shear strength of a rectangular concrete beam is tested in this example.

In the example a simply supported beam is subjected to a uniform factored load of 92.222 kN/m. This example is tested using the CSA A23.3-14 concrete design code. The flexural and shear reinforcing computed is compared with independent results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = $25x10^6$ kN/m^2
v = 0.2
G = 10416666.7kN/m^2

Section Properties

d = 546 mm
W = 92.222 kN/m

Design Properties

f'_c = 40 MPa
f_y = 400 MPa

TECHNICAL FEATURES TESTED

- Calculation of Flexural reinforcement, A_s
- Enforcement of Minimum tension reinforcement, $A_{s,min}$
- Calculation of Shear reinforcement, A_v
- Enforcement of Minimum shear reinforcing, $A_{v,min}$

RESULTS COMPARISON

Independent results are hand calculated based on the equivalent rectangular stress distribution described in Example 2.2 on page 2-12 in Part II on Concrete Design Handbook of Cement Association of Canada.

Output Parameter	ETABS	Independent	Percent Difference
Design Moment, M_f (kN-m)	415.00	415.00	0.00%
Tension Reinf, A_s (mm^2)	2466	2466	0.00%
Design Shear, V_f (kN)	226.31	226.31	0.00%
Shear Reinf, A_v/s (mm^2/mm)	0.379	0.379	0.00%

COMPUTER FILE: CSA A23.3-14 Ex001

CONCLUSION

The computed results show an exact match with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

ϕ_c = 0.65 for concrete

ϕ_s = 0.85 for reinforcement

$$A_{s,\min} = \frac{0.2\sqrt{f'_c}}{f_y}\, b\, h = 758.95 \text{ mm}^2$$

$$\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67 = 0.79$$

$$\beta_1 = 0.97 - 0.0025 f'_c \geq 0.67 = 0.87$$

$$c_b = \frac{700}{700 + f_y} d = 347.45 \text{ mm}$$

$$a_b = \beta_1 c_b = 302.285 \text{ mm}$$

COMB1

$$M_f = \frac{w_u l^2}{8} = 415 \text{ kN-m}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M_f}{\alpha_1 f'_c \phi_c b}} = 102.048 \text{ mm}$$

If $a \leq a_b$, the area of tension reinforcement is then given by:

$$A_s = \frac{M_f}{\phi_s f_y \left(d - \frac{a}{2}\right)} = 2466 \text{ mm}^2$$

$$A_{s,\min} = \min\left\{A_{s,\min}, \left(\frac{4}{3}\right)A_{s,required}\right\} = \min\left\{758.95, \left(\frac{4}{3}\right)2466\right\} = 758.95\,mm^2$$

Shear Design

The basic shear strength for rectangular section is computed as,

$\phi_c =$ 0.65 for shear

$\lambda = \{1.00,$ for normal density concrete

d_v is the effective shear depth. It is taken as the greater of
0.72h = 432 mm or 0.9d = 491.4 mm (governing).

$\beta =$ 0.18 since minimum transverse reinforcement is provided

V_f = 92.222•(3 - 0.546) = 226.31 kN

$V_c = \phi_c \lambda \beta \sqrt{f'_c}\, b_w d_v$ = 145.45 kN

$V_{r,\max} = 0.25\phi_c f'_c b_w d$ = 1419.60 kN

θ = 35º since $f_y \leq 400\,MPa$ and $f'_c \leq 60\,MPa$

$$\frac{A_v}{s} = \frac{\left(V_f - V_c\right)\tan\theta}{\phi_s f_{yt} d_v} = 0.339 \text{ mm}^2/\text{mm}$$

$$\left(\frac{A_v}{s}\right)_{\min} = 0.06\frac{\sqrt{f'_c}}{f_y}b = 0.379 \text{ mm}^2/\text{mm (Govern)}$$

CSA A23.3-14 Example 002

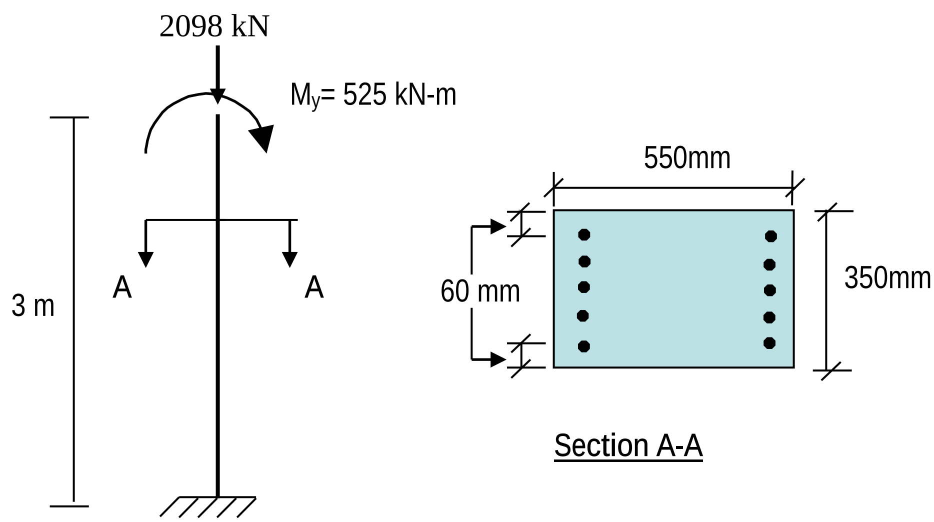
P-M INTERACTION CHECK FOR COMPRESSION-CONTROLLED RECTANGULAR COLUMN

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

In this example, a reinforced concrete column is subjected factored axial load N = 2098 kN and moment M_y = 525 kN-m. This column is reinforced with 5 T25 bars. The design capacity ratio is checked by hand calculations and results are compared. The column is designed as a short, non-sway member.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E_c = 25x10^6 kN/m^2	b = 350 m	f'_c = 30 MPa
v = 0.2	d = 490 mm	f_y = 460 MPa
G = 10416666.7kN/m^2		

TECHNICAL FEATURES TESTED

- Tied Reinforced Concrete Column Design

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	0.9869	1.00	-1.31%

COMPUTER FILE: CSA A23.3-14 Ex002

CONCLUSION

The computed results show an acceptable comparison with the independent results.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

f_{cu} = 30 MPa f_y = 460 MPa
b = 350 mm d = 490 mm

1) Because *e* = 250 mm < (2/3)*d* = 327 mm, assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis fro a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{700}{700+f_y} d_t = \frac{700}{700+460}(490) = 296 \text{ mm}$$

2) From the equation of equilibrium:

$$P_r = C_c + C_s - T$$

where

$$C_c = \phi_c \bullet \alpha_1 f_c^{'} ab = 0.65 \bullet 0.805 \bullet 30 \bullet 350a = 5494.1a$$

$$C_s = \phi_s A_s^{'}\left(f_y - 0.805 f_c^{'}\right) = 0.85 \bullet 2500\left(460 - 0.805 \bullet 30\right) = 926{,}181 \text{ N}$$

Assume compression steels yields, (this assumption will be checked later).

$$T = \phi_s A_s f_s = 0.85 \bullet 2500 f_s = 2125 f_s \left(f_s < f_y\right)$$

$$P_r = 5{,}494.1a + 926{,}181 - 2125 f_s \quad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$P_r = \frac{1}{e^{'}}\left[C_c\left(d - \frac{a}{2}\right) + C_s\left(d - d^{'}\right)\right]$$

The plastic centroid is at the center of the section and $d^{''}$ = 215 mm

$$e^{'} = e + d^{''} = 250 + 215 = 465 \text{ mm}$$

$$P_r = \frac{1}{465}\left[5{,}494.1a\left(490 - \frac{a}{2}\right) + 926{,}181\left(490 - 60\right)\right]$$

$$P_r = 5789.5a - 5.91a^2 + 856{,}468.5 \quad \text{(Eqn. 2)}$$

4) Assume c = 355 mm, which exceed c_b (296 mm).

$$a = 0.895 \bullet 355 = 317.7 \text{ mm}$$

Substitute in Eqn. 2:

$$P_r = 5789.5 \bullet 317.7 - 5.91(317.7)^2 + 856{,}468.5 = 2{,}099{,}327.8 \text{ N}$$

5) Calculate f_s from the strain diagram when c = 350 mm.

$$f_s = \left(\frac{490 - 355}{355}\right)700 = 266.2 \text{ MPa}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.0013$$

6) Substitute a = 317.7 mm and f_s = 266.2 MPa in Eqn. 1 to calculate P_{r2}:

$$P_{r2} = 5{,}494.1(317.7) + 926{,}181 - 2125(266.2) = 2{,}106{,}124.9 \text{ N}$$

Which is very close to the calculated P_{r1} of 2,012,589.8 (less than 1% difference)

$$M_r = P_r e = 2100\left(\frac{250}{1000}\right) = 525 \text{ kN-m}$$

7) Check if compression steels yield. From strain diagram,

$$\varepsilon_s^{'} = \left(\frac{355 - 60}{355}\right)(0.0035) = 0.00291 > \varepsilon_y = 0.0023$$

Compression steels yields, as assumed.

8) Therefore, section capacity is

$$P_r = 2098 \text{ kN}$$

$$M_r = 525 \text{ kN-m}$$

EN 2-2004 Example 001

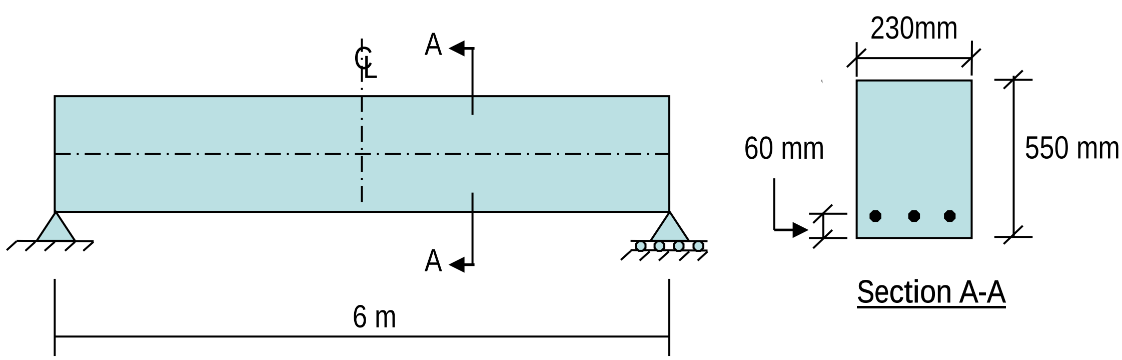
SHEAR AND FLEXURAL REINFORCEMENT DESIGN OF A SINGLY REINFORCED RECTANGLE

EXAMPLE DESCRIPTION

The flexural and shear strength of a rectangular concrete beam is tested in this example.

In the example a simple supported beam is subjected to a uniform factored load of 36.67 kN/m. This example is tested using the Eurocode concrete design code. The flexural and shear reinforcing computed is compared with independent results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 25×10^6 kN/m^2
v = 0.2
G = 10416666.7kN/m^2

Section Properties

d = 490 mm
b = 230 mm

Design Properties

f_{ck} = 30 MPa
f_{yk} = 460 MPa

TECHNICAL FEATURES TESTED

- Calculation of Flexural reinforcement, A_s
- Enforcement of Minimum tension reinforcement, $A_{s,min}$
- Calculation of Shear reinforcement, A_v
- Enforcement of Minimum shear reinforcing, $A_{v,min}$

RESULTS COMPARISON

Independent results are hand calculated based on the equivalent rectangular stress distribution.

Country	γ_c	γ_s	α_{cc}	k_1	k_2	k_3	k_4
CEN Default, Slovenia, Sweden, Portugal	1.5	1.15	1.0	0.44	1.25	0.54	1.25
UK	1.5	1.15	0.85	0.40	1.25	0.40	1.25
Norway	1.5	1.15	0.85	0.44	1.25	0.54	1.25
Singapore	1.5	1.15	0.85	0.40	1.25	0.54	1.25
Finland	1.5	1.15	0.85	0.44	1.10	0.54	1.25
Denmark	1.45	1.2	1.0	0.44	1.25	0.54	1.25
Germany	1.5	1.15	0.85	0.64	0.80	0.72	0.80
Poland	1.4	1.15	1.0	0.44	1.25	0.54	1.25

Country	Design Moment, M_{Ed} (kN-m)		Tension Reinforcing, As+ (sq-mm)		Design Shear, V_{Ed} (kN)		Shear Reinforcing, A_{sw}/s (sq-mm/m)		% diff.
Method	ETABS	Hand	ETABS	Hand	ETABS	Hand	ETABS	Hand	0.00%
CEN Default, Slovenia, Sweden, Portugal	165	165	916	916	110	110	249.5	249.5	0.00%
UK	165	165	933	933	110	110	249.5	249.5	0.00%
Norway	165	165	933	933	110	110	249.5	249.5	0.00%
Singapore	165	165	933	933	110	110	249.5	249.5	0.00%
Finland	165	165	933	933	110	110	249.5	249.5	0.00%
Denmark	165	165	950	950	110	110	249.5	249.5	0.00%
Germany	165	165	933	933	110	110	249.5	249.5	0.00%
Poland	165	165	925	925	110	110	249.5	249.5	0.00%

COMPUTER FILE: EN 2-2004 Ex001

CONCLUSION

The computed results show an exact match with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for both of the load combinations:

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$

$\alpha_{cc} = 1.0$

$k_1 = 0.44 \quad k_2 = k_4 = 1.25\left(0.6 + 0.0014/\varepsilon_{cu2}\right) = 1.25 \quad k_3 = 0.54$

$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c$ = 1.0(30)/1.5 = 20 MPa

$f_{yd} = f_{yk} / \gamma_s$ = 460/1.15 = 400 Mpa

$f_{ywd} = f_{yk} / \gamma_s$ = 460/1.15 = 400 Mpa

$\eta = 1.0$ for $f_{ck} \leq 50$ MPa

$\lambda = 0.8$ for $f_{ck} \leq 50$ MPa

$A_{s,\min} = 0.26 \dfrac{f_{ctm}}{f_{yk}} bd$ = 184.5 sq-mm,

where $f_{ctm} = 0.3 f_{cwk}^{2/3} = 0.3(30)^{2/3}$ = 2.896 N/sq-mm

$A_{s,\min} = 0.0013bh$ = 164.5 sq-mm

COMB1

The factored design load and moment are given as,

w_u = 36.67 kN/m

$M = \dfrac{w_u l^2}{8} = 36.67 \bullet 6^2/8$ = 165.0 kN-m

The limiting value of the ratio of the neutral axis depth at the ultimate limit state to the effective depth, $(x/d)_{\lim}$, is given as,

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa},$$

where $\delta = 1$, assuming no moment redistribution

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} = \frac{(1-0.44)}{1.25} = 0.448$$

The normalized section capacity as a singly reinforced beam is given as,

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.29417$$

The limiting normalized steel ratio is given as,

$$\omega_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim} = 1 - \sqrt{1-2m_{\lim}} = 0.3584$$

The normalized moment, *m*, is given as,

$$m = \frac{M}{bd^2\eta f_{cd}} = \frac{165 \bullet 10^6}{230 \bullet 490^2 \bullet 1.0 \bullet 20} = 0.1494 < m_{\lim}$$ so a singly reinforced beam will be adequate.

$$\omega = 1 - \sqrt{1-2m} = 0.16263 < \omega_{\lim}$$

$$A_s = \omega\left[\frac{\eta f_{cd} bd}{f_{yd}}\right] = 0.1626\left[\frac{1.0 \bullet 20 \bullet 230 \bullet 490}{400}\right] = 916 \text{ sq-mm}$$

Shear Design

The shear force demand is given as,

$$V_{Ed} = \omega L/2 = 110.01 \text{ kN}$$

The shear force that can be carried without requiring design shear reinforcement,

$$V_{Rd,c} = \left[C_{Rd,c} k \left(100 \rho_1 f_{ck}\right)^{1/3} + k_1 \sigma_{cp} \right] \bullet b_w \bullet d$$

$$V_{Rd,c} = \left[0.12 \bullet 1.6389 \left(100 \bullet 0.0 \bullet 30\right)^{1/3} + 0.0 \right] 230 \bullet 490 = 0\, kN$$

with a minimum of:

$$V_{Rd,c} = \left[v_{\min} + k_1 \sigma_{cp} \right] bd \ = \left[0.4022 + 0.0\right] 230 \bullet 490 = 45.3 \text{ kN}$$

where,

$$k = 1 + \sqrt{\frac{200}{d}} \le 2.0 \ = 1.6389$$

$$\rho_1 = \frac{A_S}{bd} = \frac{0}{230 \bullet 490} = 0.0 \le 0.02$$

As = 0 for ρ_l at the end of a simply-supported beam as it taken as the tensile reinforcement at the location offset by $d+l_{db}$ beyond the point considered.

(EN 1992-1-1:2004 6.2.2(1) Figure 6.3)

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} = 0.0$$

$$C_{Rd,c} = 0.18 / \gamma_c = 0.12$$

$$v_{\min} = 0.035 k^{3/2} fck^{1/2} \ = 0.4022$$

The maximum design shear force that can be carried without crushing of the notional concrete compressive struts,

$$V_{Rd,\max} = \alpha_{cw} b z v_1 f_{cd} / \left(\cot\theta + \tan\theta\right)$$

where,

$$\alpha_{cw} = 1.0$$

$$z = 0.9d = 441.0 \text{ mm}$$

$$v_1 = 0.6\left[1 - \frac{f_{ck}}{250}\right] = 0.528$$

$$\theta = 0.5\sin^{-1}\frac{v_{Ed}}{0.2\,f_{ck}\left(1-f_{ck}/250\right)} = 5.33^{\circ}$$

where,

$$v_{Ed} = \frac{V_{Ed}}{b_w \bullet d} = 0.9761$$

$21.8^{\circ} \le \theta \le 45^{\circ}$, therefore use $\theta = 21.8^{\circ}$

$$V_{Rd,\max} = \alpha_{cw} b z v_1 f_{cd} / \left(\cot\theta + \tan\theta\right) = 369\text{ kN}$$

$V_{Rd,\max} > V_{Ed}$, so there is no concrete crushing.

The required shear reinforcing is,

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{z f_{ywd}\cot\theta} = \frac{110.01 \bullet 1e6}{441 \bullet \frac{460}{1.15} \bullet 2.5} = 249.5\text{ sq-mm/m}$$

EN 2-2004 Example 002

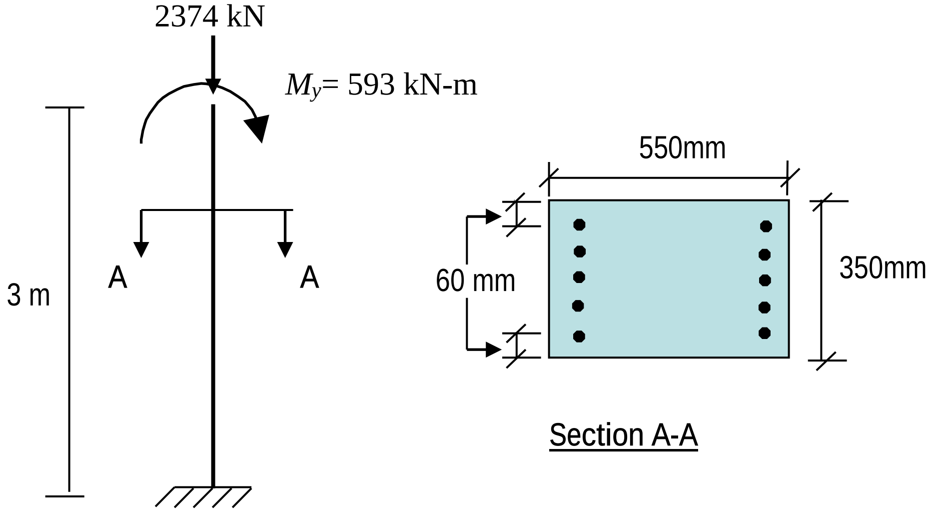
P-M INTERACTION CHECK FOR COMPRESSION-CONTROLLED RECTANGULAR COLUMN

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

In this example, a reinforced concrete column is subjected to factored axial load N = 2374 kN and moment M_y = 593 kN-m. This column is reinforced with five 25 bars. The design capacity ratio is checked by hand calculations and the result is compared with computed results. The column is designed as a short, non-sway member.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E_c = 25x10^6 kN/m^2	b = 350 m	f_{ck} = 30 MPa
v = 0.2	d = 490 mm	f_y = 460 MPa
G = 10416666.7kN/m^2		

TECHNICAL FEATURES TESTED

- Tied reinforced concrete column design

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	1.009	1.00	0.90%

COMPUTER FILE: EN 2-2004 EX002

CONCLUSION

The computed results show an acceptable comparison with the independent results.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

f_{ck} = 30 MPa f_y = 460 MPa
b = 350 mm   d = 490 mm

1) Because e = 250 mm < (2/3)d = 327 mm, assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis for a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{700}{700 + f_y/\gamma_s} d_t = \frac{700}{700 + 460/1.15}(490) = 312 \text{ mm}$$

2) From the equation of equilibrium:

$$N = C_c + C_s - T$$

where

$$C_c = \alpha_{cc}\frac{f_{ck}}{\gamma_c}ab = 1.0\frac{30}{1.5}\bullet 350a = 7000a$$

$$C_s = \frac{A_s'}{\gamma_s}\left(f_y - \alpha_{cc}\frac{f_{ck}}{\gamma_c}\right) = \frac{2500}{1.15}\left(460 - 1.0\bullet\frac{30}{1.5}\right) = 956,521.7 \text{ N}$$

Assume compression steel yields (this assumption will be checked later).

$$T = \frac{A_s f_s}{\gamma_s} = \frac{2500 f_s}{1.15} = 2174 f_s \left(f_s < f_y\right)$$

$$N_1 = 7,000a + 956,521.7 - 2174 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$N_2 = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_s\left(d - d'\right)\right]$$

The plastic centroid is at the center of the section and d'' = 215 mm

$$e' = e + d'' = 250 + 215 = 465 \text{ mm}$$

$$N_2 = \frac{1}{465}\left[7,000a\left(490 - \frac{a}{2}\right) + 956,521.7\left(490 - 60\right)\right]$$

$$N_2 = 7376.3a - 7.527a^2 + 884,525.5 \qquad \text{(Eqn. 2)}$$

4) Assume c = 356 mm, which exceed c_b (312 mm).

$$a = 0.8 \bullet 356 = 284.8 \text{ mm}$$

Substitute in Eqn. 2:

$$N_2 = 7376.3 \bullet 284.8 - 7.527(284.8)^2 + 884,525.5 = 2,374,173 \text{ N}$$

5) Calculate f_s from the strain diagram when c = 356 mm.

$$f_s = \left(\frac{490 - 356}{356}\right)700 = 263.4 \text{ MPa}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.00114$$

6) Substitute a = 284.8 mm and f_s = 263.4 MPa in Eqn. 1 to calculate N_1:

$$N_1 = 7,000(284.8) + 956,522 - 2174(263.5) = 2,377,273 \text{ N}$$

which is very close to the calculated N_2 of $2,374,173$ (less than 1% difference)

$$M = Ne = 2374\left(\frac{250}{1000}\right) = 593.5 \text{ kN-m}$$

7) Check if compression steels yield. From strain diagram,

$$\varepsilon'_s = \left(\frac{356 - 60}{356}\right)(0.0035) = 0.0029 > \varepsilon_y = 0.0023$$

Compression steel yields, as assumed.

8) Therefore, section capacity is

N = 2,374 kN
M = 593 kN-m

HK CP-2004 Example 002

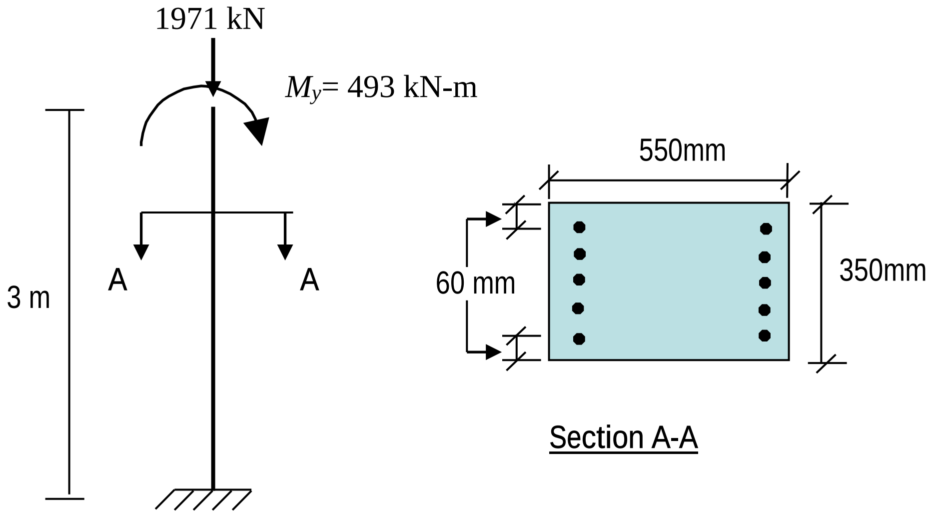
P-M INTERACTION CHECK FOR COMPRESSION-CONTROLLED RECTANGULAR COLUMN

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

In this example, a reinforced concrete column is subjected to factored axial load N = 1971 kN and moment M_y = 493 kN-m. This column is reinforced with five 25M bars. The design capacity ratio is checked by hand calculations and the result is compared with the computed results. The column is designed as a short, non-sway member.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E_c = 25x10^6 kN/m^2	b = 350 m	f_{cu} = 30 MPa
v = 0.2	d = 490 mm	f_y = 460 MPa
G = 10416666.7kN/m^2		

TECHNICAL FEATURES TESTED

- Tied reinforced concrete column design

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	0.994	1.00	0.60%

COMPUTER FILE: HK CP-2004 EX002

CONCLUSION

The computed result shows an acceptable comparison with the independent result.

HAND CALCULATION

Column Strength under compression control

f_{cu} = 30 MPa f_y = 460 MPa
b = 350 mm   d = 490 mm

1) Because e = 250 mm < (2/3)d = 327 mm, assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis for a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{700}{700 + f_y / \gamma_s} d_t = \frac{700}{700 + 460/1.15}(490) = 312 \text{ mm}$$

2) From the equation of equilibrium:

$$N = C_c + C_s - T$$

where

$$C_c = \frac{0.67}{\gamma_M} f_{cu} ab = 0.67/1.5 \bullet 30 \bullet 350a = 4667a$$

$$C_s = \frac{A_s'}{\gamma_s}\left(f_y - 0.4467 f_{cu}\right) = \frac{2500}{1.15}\left(460 - 0.4467 \bullet 30\right) = 971{,}014 \text{ N}$$

Assume compression steels yields, (this assumption will be checked later).

$$T = \frac{A_s f_s}{\gamma_s} = \frac{2500 f_s}{1.15} = 2174 f_s \left(f_s < f_y\right)$$

$$N_1 = 4{,}667a + 971{,}014 - 2174 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$N = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_s\left(d - d'\right)\right]$$

The plastic centroid is at the center of the section and d'' = 215 mm

$$e' = e + d'' = 250 + 215 = 465 \text{ mm}$$

$$N = \frac{1}{465}\left[4{,}667a\left(490 - \frac{a}{2}\right) + 971{,}014\left(490 - 60\right)\right]$$

$$N = 4917.9a - 5.018a^2 + 897{,}926 \qquad \text{(Eqn. 2)}$$

4) Assume c = 364 mm, which exceed c_b (312 mm).

$$a = 0.9 \bullet 364 = 327.6 \text{ mm}$$

Substitute in Eqn. 2:

$$N_2 = 4917.9 \bullet 327.6 - 5.018(327.6)^2 + 897,926 = 1,970,500 \text{ N}$$

5) Calculate f_s from the strain diagram when c = 365 mm.

$$f_s = \left(\frac{490 - 364}{364}\right)700 = 242.3 \text{ MPa}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.0012$$

6) Substitute a = 327.6 mm and f_s = 242.3 MPa in Eqn. 1 to calculate N_1:

$$N_1 = 4,667(327.6) + 971,014 - 2174(242.3) = 1,973,163 \text{ N}$$

which is very close to the calculated N_2 of 1,970,500 (less than 1% difference)

$$M = Ne = 1971\left(\frac{250}{1000}\right) = 493 \text{ kN-m}$$

7) Check if compression steel yields. From strain diagram,

$$\varepsilon_s' = \left(\frac{365 - 60}{365}\right)(0.0035) = 0.00292 > \varepsilon_y = 0.0023$$

Compression steel yields, as assumed.

8) Therefore, the section capacity is

N = 1971 kN
M = 493 kN-m

IS 456-2000 Example 001

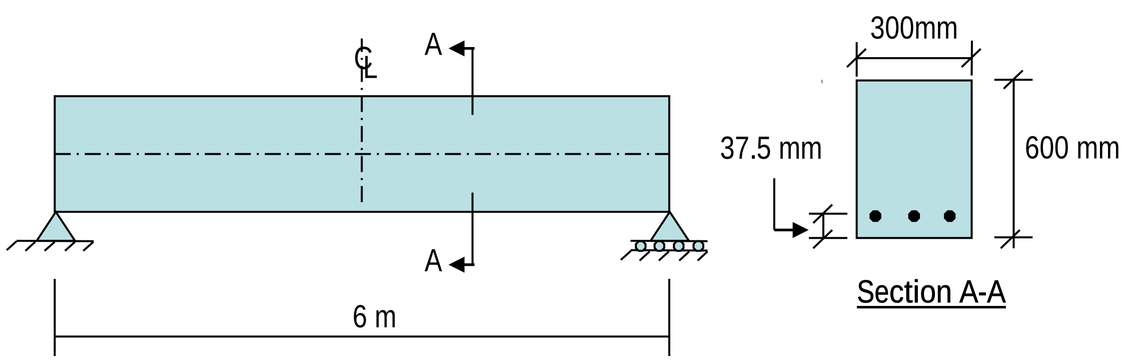
SHEAR AND FLEXURAL REINFORCEMENT DESIGN OF A SINGLY REINFORCED RECTANGLE

EXAMPLE DESCRIPTION

The flexural and shear strength of a rectangular concrete beam is tested in this example.

In the example a simply supported beam is subjected to a uniform factored load of 37.778 kN/m. This example is tested using the IS 456-2000 concrete design code. The flexural and shear reinforcing computed is compared with independent results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

$E =$ 19.365x10^6 kN/m^2

$v =$ 0.2

$G =$ 8068715.3kN/m^2

Section Properties

$d =$ 562.5 mm

$w =$ 37.778 kN/m

Design Properties

f_{ck} = 15 MPa

f_y = 415 MPa

TECHNICAL FEATURES TESTED

- Calculation of Flexural reinforcement, A_s
- Enforcement of Minimum tension reinforcement, $A_{s,min}$
- Calculation of Shear reinforcement, A_v
- Enforcement of Minimum shear reinforcing, $A_{v,min}$

RESULTS COMPARISON

The example problem is same as Example-1 given in SP-16 Design Aids for Reinforced Concrete published by Bureau of Indian Standards. For this example a direct comparison for flexural steel only is possible as corresponding data for shear steel reinforcement is not available in the reference for this problem.

Output Parameter	ETABS	Independent	Percent Difference
Design Moment, M_u (kN-m)	170.00	170.00	0.00%
Tension Reinf, A_s (mm^2)	1006	1006	0.00%
Design Shear, V_u (kN)	113.33	113.33	0.00%
Shear Reinf, A_{sv}/s (mm^2/mm)	0.333	0.333	0.00%

COMPUTER FILE: IS 456-2000 Ex001

CONCLUSION

The computed results show an exact match with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\gamma_{m,\ steel}$ = 1.15

$\gamma_{m,\ concrete}$ = 1.50

α = 0.36

β = 0.42

$$A_{s,\min} \geq \frac{0.85}{f_y} bd \ = 345.63 \text{ sq-mm}$$

COMB1

M_u = 170 kN-m

V_u = 113.33 kN-m

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \leq 250 \text{ MPa} \\ 0.53 - 0.05\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \leq 415 \text{ MPa} \\ 0.48 - 0.02\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \leq 500 \text{ MPa} \\ 0.46 & \text{if} \quad f_y \geq 500 \text{ MPa} \end{cases}$$

$$\frac{Xu_{,\max}}{d} = 0.48$$

The normalized design moment, *m*, is given by

$$m = \frac{M_u}{b_w d^2 \alpha f_{ck}} = 0.33166$$

$$M_{w,\text{single}} = \alpha f_{ck} b_w d^2 \ \frac{x_{u,\max}}{d}\left[1 - \beta\frac{x_{u,\max}}{d}\right] = 196.436 \text{ kN-m} > M_u$$

So no compression reinforcement is needed

$$\frac{x_u}{d} = \frac{1-\sqrt{1-4\beta \bullet m}}{2\beta} = 0.3983$$

$$z = d\left\{1-\beta\frac{x_u}{d}\right\} = 562.5\{1-0.42 \bullet 0.3983\} = 468.406$$

$$As = \left(\frac{M_u}{\left(f_y/\gamma_s\right)z}\right) = 1006 \text{ sq-mm}$$

Shear Design

$$\tau_v = \frac{V_u}{bd} = 0.67161$$

τ_{max} = 2.5 for M15 concrete

k = 1.0

$\delta = 1$ if $P_u \le 0$, Under Tension

$$0.15 \le \frac{100\ A_s}{bd} \le 3$$

$$\frac{100\ A_s}{bd} = 0.596$$

$\tau_c = 0.49$ From Table 19 of IS 456:2000 code, interpolating between rows.

$\tau_{cd} = k\delta\tau_c = 0.49$

The required shear reinforcement is calculated as follows:

Since $\tau_v > \tau_{cd}$

$$\frac{A_{sv}}{s} = \max\left\{\frac{0.4b}{\left(f_y/\gamma\right)}, \frac{\left(\tau_v-\tau_{cd}\right)b}{\left(f_y/\gamma\right)_y}\right\} = \max\left\{\frac{0.4 \bullet 300}{\left(415/1.15\right)}, \frac{\left(0.67161-0.49\right) \bullet 300}{\left(415/1.15\right)}\right\}$$

$$\frac{A_{sv}}{s} = \max\{0.333, 0.150\} = 0.333\frac{mm^2}{mm}$$

IS 456-2000 Example 002

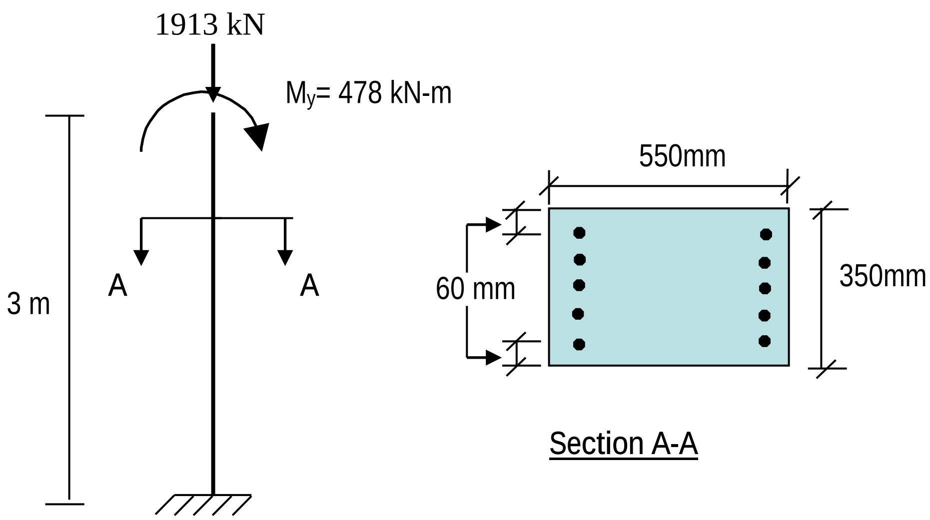
P-M INTERACTION CHECK FOR COMPRESSION-CONTROLLED RECTANGULAR COLUMN

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

In this example, a reinforced concrete column is subjected factored axial load N = 1913 kN and moment M_y= 478 kN-m. This column is reinforced with 5 25M bars. The design capacity ratio is checked by hand calculations and computed result is compared. The column is designed as a short, non-sway member.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E_c = 25x10^6 kN/m^2	b = 350 m	f'_c = 30 MPa
v = 0.2	d = 490 mm	f_y = 460 MPa
G = 10416666.7kN/m^2		

TECHNICAL FEATURES TESTED

- Tied Reinforced Concrete Column Design

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	0.997	1.00	0.30%

COMPUTER FILE: IS 456-2000 Ex002

CONCLUSION

The computed results show an acceptable comparison with the independent results. The larger variation is due to equivalent rectangular compression block assumption.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

f_{cu} = 30 MPa $\quad$ f_y = 460 MPa
b = 350 mm $\quad$ d = 490 mm

1) Because e = 250 mm < (2/3)d = 327 mm, assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis fro a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{700}{700 + f_y} d_t = \frac{700}{700 + 460}(490) = 296 \text{ mm}$$

2) From the equation of equilibrium:

$$N = C_c + C_s - T$$

where

$$C_c = \frac{0.36}{0.84} f_{ck} ab = 0.4286 \bullet 30 \bullet 350a = 4500a$$

$$C_s = \frac{A_s'}{\gamma_s}\left(f_y - 0.4286 f_{ck}\right) = \frac{2500}{1.15}\left(460 - 0.4286 \bullet 30\right) = 972{,}048 \text{ N}$$

Assume compression steels yields, (this assumption will be checked later).

$$T = \frac{A_s f_s}{\gamma_s} = \frac{2500 f_s}{1.15} = 2174 f_s \left(f_s < f_y\right)$$

$$N_1 = 4500a + 972{,}048 - 2174 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$N_2 = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_s\left(d - d'\right)\right]$$

The plastic centroid is at the center of the section and $d'' = 215$ mm

$$e' = e + d'' = 250 + 215 = 465 \text{mm}$$

$$N_2 = \frac{1}{465}\left[4500a\left(490 - \frac{a}{2}\right) + 972{,}048\left(490 - 60\right)\right]$$

$$N_2 = 4742a - 4.839a^2 + 898{,}883 \qquad \text{(Eqn. 2)}$$

4) Assume c = 374 mm, which exceed c_b (296 mm).

$$a = 0.84 \bullet 374 = 314.2 \text{ mm}$$

Substitute in Eqn. 2:

$$N_2 = 4742 \bullet 314.2 - 4.039(314.2)^2 + 898,883 = 1,911,037 \text{ N}$$

5) Calculate f_s from the strain diagram when c = 365 mm.

$$f_s = \left(\frac{490 - 374}{374}\right)700 = 217.11 \text{ MPa}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.0011$$

6) Substitute a = 314.2 mm and f_s = 217.11 MPa in Eqn. 1 to calculate N_1:

$$N_1 = 4500(314.2) + 972,048 - 2174(217.4) = 1,913,765 \text{ N}$$

Which is very close to the calculated N_2 of 1,911,037 (less than 1% difference)

$$M = Ne = 1913\left(\frac{250}{1000}\right) = 478 \text{ kN-m}$$

7) Check if compression steels yield. From strain diagram,

$$\varepsilon_s' = \left(\frac{374 - 60}{374}\right)(0.0035) = 0.0029 > \varepsilon_y = 0.0023$$

Compression steels yields, as assumed.

8) Therefore, section capacity is

$$N = 1913 \text{ kN}$$
$$M = 478 \text{ kN-m}$$

NTC 2008 Example 001

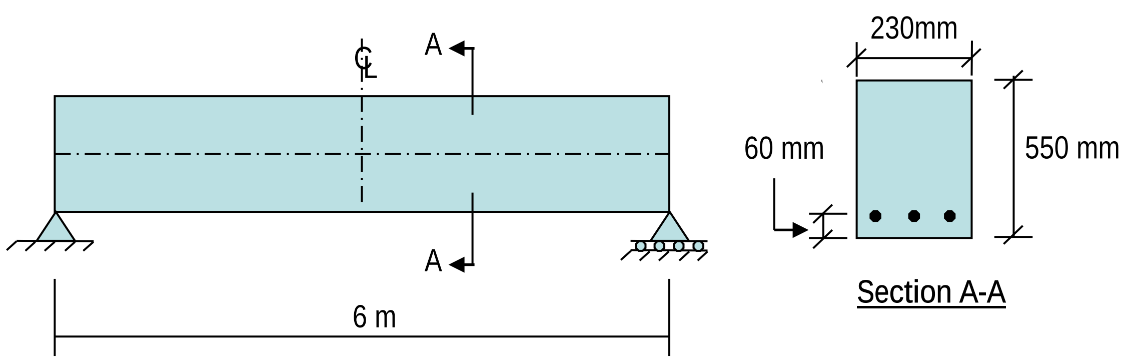
SHEAR AND FLEXURAL REINFORCEMENT DESIGN OF A SINGLY REINFORCED RECTANGLE

EXAMPLE DESCRIPTION

The flexural and shear strength of a rectangular concrete beam is tested in this example.

In the example a simple supported beam is subjected to a uniform factored load of 36.67 kN/m. This example is tested using the Italian NTC 2008 concrete design code. The flexural and shear reinforcing computed is compared with independent results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 25×10^6 kN/m^2
v = 0.2
G = 10416666.7kN/m^2

Section Properties

d = 490 mm
b = 230 mm

Design Properties

f_{ck} = 30 MPa
f_{yk} = 460 MPa

TECHNICAL FEATURES TESTED

- Calculation of Flexural reinforcement, A_s
- Enforcement of Minimum tension reinforcement, $A_{s,min}$
- Calculation of Shear reinforcement, A_v
- Enforcement of Minimum shear reinforcing, $A_{v,min}$

RESULTS COMPARISON

Independent results are hand calculated based on the equivalent rectangular stress distribution.

Output Parameter	ETABS	Independent	Percent Difference
Design Moment, M_{Ed} (kN-m)	165.00	165.00	0.00%
Tension Reinf, A_s (mm^2)	933	933	0.00%
Design Shear, V_{Ed} (kN)	110.0	110.0	0.00%
Shear Reinf, A_{sw}/s (mm^2/m)	345.0	345.0	0.00%

COMPUTER FILE: NTC 2008 Ex001

CONCLUSION

The computed results show an exact match with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for both of the load combinations:

$\gamma_{c,\ concrete}$ = 1.50

$\alpha_{cc} = 0.85$

$k_1 = 0.44 \quad k_2 = k_4 = 1.25\left(0.6 + 0.0014 / \varepsilon_{cu2}\right) = 1.25 \quad k_3 = 0.54$

$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c$ = 0.85(30)/1.5 = 17 MPa

$f_{yd} = \dfrac{f_y}{\gamma_s} \; \dfrac{460}{1.15}$ = 400 Mpa

$\eta = 1.0$ for $f_{ck} \leq 50$ MPa

$\lambda = 0.8$ for $f_{ck} \leq 50$ MPa

$A_{s,\min} = 0.26 \dfrac{f_{ctm}}{f_{yk}} bd$ = 184.5 sq-mm,

where $f_{ctm} = 0.3 f_{cwk}^{2/3}$ = 0.3(30)$^{2/3}$ = 2.896 N/sq-mm

$A_{s,\min} = 0.0013bh$ = 164.5 sq-mm

COMB1

The factored design load and moment are given as,

w_u = 36.67 kN/m

$M = \dfrac{w_u l^2}{8}$ = 36.67•6^2/8 = 165.0 kN-m

The limiting value of the ratio of the neutral axis depth at the ultimate limit state to the effective depth, $(x/d)_{\lim}$, is given as,

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa},$$

where $\delta = 1$, assuming no moment redistribution

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} = \frac{(1-0.44)}{1.25} = 0.448$$

The normalized section capacity as a singly reinforced beam is given as,

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.29417$$

The limiting normalized steel ratio is given as,

$$\omega_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim} = 1 - \sqrt{1 - 2m_{\lim}} = 0.3584$$

The normalized moment, *m*, is given as,

$$m = \frac{M}{bd^2 f_{cd}} = \frac{165 \bullet 10^6}{230 \bullet 490^2 \bullet 17} = 0.1758 < m_{\lim}$$ so a singly reinforced beam will be adequate.

$$\omega = 1 - \sqrt{1 - 2m} = 0.1947 < \omega_{\lim}$$

$$A_s = \omega\left[\frac{f_{cd}bd}{f_{yd}}\right] = 0.1947\left[\frac{17 \bullet 230 \bullet 490}{400}\right] = 933 \text{ sq-mm}$$

Shear Design

The shear force demand is given as,

$$V_{Ed} = \omega L / 2 = 110.0 \text{ kN}$$

The shear force that can be carried without requiring design shear reinforcement,

$$V_{Rd,c} = \left[C_{Rd,c} k \left(100 \rho_1 f_{ck}\right)^{1/3} + k_1 \sigma_{cp}\right] \bullet b_w \bullet d$$

$$V_{Rd,c} = \left[0.12 \bullet 1.6389 \left(100 \bullet 0.0 \bullet 30\right)^{1/3} + 0.0\right] 230 \bullet 490 = 0\, kN$$

with a minimum of:

$$V_{Rd,c} = \left[v_{\min} + k_1 \sigma_{cp} \right] bd = \left[0.4022 + 0.0 \right] 230x490 = 45.3\,\text{kN}$$

where,

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 = 1.6389$$

$$\rho_1 = \frac{A_S}{bd} = \frac{0}{230 \bullet 490} = 0.0 \leq 0.02$$

As = 0 for ρ_l at the end of a simply-supported beam as it taken as the tensile reinforcement at the location offset by $d+l_{db}$ beyond the point considered.

(EN 1992-1-1:2004 6.2.2(1) Figure 6.3)

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} = 0.0$$

$$C_{Rd,c} = 0.18 / \gamma_c = 0.12$$

$$v_{\min} = 0.035 k^{3/2} fck^{1/2} = 0.4022$$

The maximum design shear force that can be carried without crushing of the notional concrete compressive struts,

$$V_{Rd,\max} = zb\alpha_c f'_{cd} \bullet \left(\frac{\cot\alpha + \cot\vartheta}{1 + \cot^2\vartheta} \right) = 297\,kN$$

where,

$$z = 0.9d = 441.0\,mm$$

$\alpha_c = 1.0$ since there is no axial compression

$$f'_{cd} = 0.5 f_{cd}$$

$\alpha = 90^0$ for vertical stirrups

$$\vartheta = 0.5 \sin^{-1} \frac{v_{Ed}}{0.2 f_{ck} \left(1 - f_{ck} / 250 \right)} = 5.33^{\circ}$$

where,

$$v_{Ed} = \frac{V_{Ed}}{b_w \bullet d} = 0.9761$$

$21.8^\circ \le \vartheta \le 45^\circ$, therefore use $\vartheta = 21.8^\circ$

The required shear reinforcing is,

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{zf_{ywd}} \frac{1}{\left(\cot\alpha + \cot\vartheta\right)\sin\alpha} = \frac{110.0 \bullet 10^6}{441 \bullet \frac{460}{1.15} \bullet 2.5} = 249.4 \frac{mm^2}{m}$$

The minimum required shear reinforcing is,

$$\left(\frac{A_{sw}}{s}\right)_{\min} = 1.5b = 1.5 \bullet 230 = 345.0 \frac{mm^2}{m} \text{ (controls)}$$

NTC 2008 Example 002

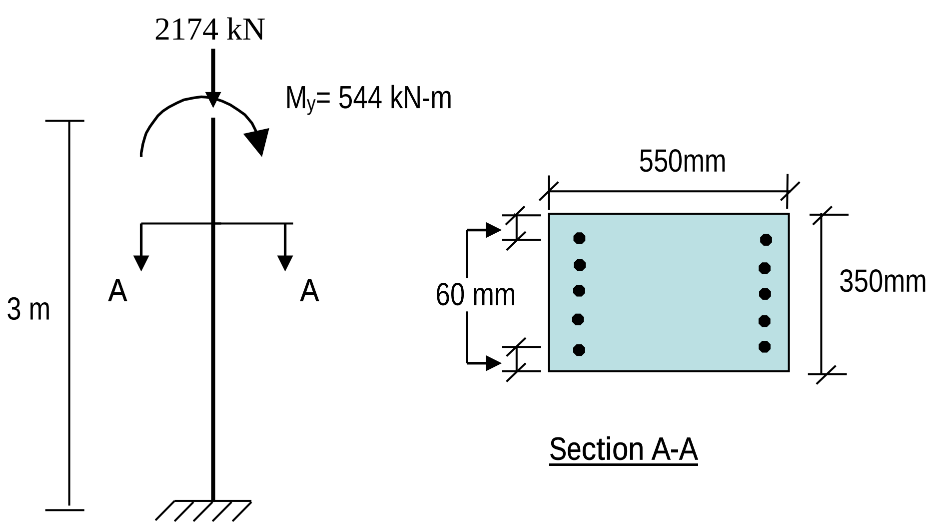
P-M Interaction Check for Compression-Controlled Rectangular Column

Example Description

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

In this example, a reinforced concrete column is subjected factored axial load N = 2174 kN and moment M_y = 544 kN-m. This column is reinforced with 5-25 bars. The design capacity ratio is checked by hand calculations and computed result is compared. The column is designed as a short, non-sway member.

Geometry, Properties and Loading



Material Properties	Section Properties	Design Properties
E_c = 25x10^6 kN/m^2	b = 350 m	f_{ck} = 25 MPa
v = 0.2	d = 490 mm	f_y = 460 MPa
G = 10416666.7kN/m^2		

TECHNICAL FEATURES TESTED

- Tied Reinforced Concrete Column Design

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	1.092	1.00	9.20%

COMPUTER FILE: EN 2-2004 Ex002

CONCLUSION

The computed results show an acceptable comparison with the independent results.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

f_{cu} = 25 MPa f_y = 460 MPa
b = 350 mm d = 490 mm

1) Because e = 250 mm < (2/3) d = 327 mm, assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis for a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{700}{700+f_y} d_t = \frac{700}{700+460}(490) = 296 \text{ mm}$$

2) From the equation of equilibrium:

$$N = C_c + C_s - T$$

where

$$C_c = \frac{\alpha f_{ck}}{\gamma_c} ab = \frac{0.85 \bullet 30}{1.5} \bullet 350a = 5950a$$

$$C_s = \frac{A_s^{'}}{\gamma_s}\left(f_y - \frac{\alpha f_{ck}}{\gamma_c}\right) = \frac{2500}{1.15}\left(460 - \frac{0.85 \bullet 30}{1.5}\right) = 963,043 \text{N}$$

Assume compression steels yields, (this assumption will be checked later).

$$T = \frac{A_s f_s}{\gamma_s} = \frac{2500 f_s}{1.15} = 2174 f_s \left(f_s < f_y\right)$$

$$N_1 = 5,950a + 963,043 - 2174 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$N_2 = \frac{1}{e^{'}}\left[C_c\left(d - \frac{a}{2}\right) + C_s\left(d - d^{'}\right)\right]$$

The plastic centroid is at the center of the section and $d^{''}$ = 215 mm

$$e^{'} = e + d^{''} = 250 + 215 = 465 \text{mm}$$

$$N_2 = \frac{1}{465}\left[5950a\left(490 - \frac{a}{2}\right) + 963,043\left(490 - 60\right)\right]$$

$$N_2 = 6270a - 6.3978a^2 + 890,556 \qquad \text{(Eqn. 2)}$$

4) Assume c = 365 mm, which exceed c_b (296 mm).

$$a = 0.8 \bullet 365 = 292 \text{ mm}$$

Substitute in Eqn. 2:

$$N_2 = 6270 \bullet 292 - 6.3978(292)^2 + 890,556 = 2,175,893 \text{ N}$$

5) Calculate f_s from the strain diagram when c = 356 mm.

$$f_s = \left(\frac{490 - 365}{365}\right)700 = 240.0 \text{ MPa}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.0012$$

6) Substitute a = 284.8 mm and f_s = 263.4 MPa in Eqn. 1 to calculate N_1:

$$N_1 = 5950(292) + 963,043 - 2174(240.0) = 2,178,683 \text{ N}$$

Which is very close to the calculated N_2 of 2,175,893 (less than 1% difference)

$$M = Ne = 2175\left(\frac{250}{1000}\right) = 544 \text{ kN-m}$$

7) Check if compression steels yield. From strain diagram,

$$\varepsilon_s' = \left(\frac{365 - 60}{365}\right)(0.0035) = 0.0029 > \varepsilon_y = 0.0023$$

Compression steels yields, as assumed.

8) Therefore, section capacity is

N = 2,174 kN
M = 544 kN-m

KBC 2009 Example 001

Shear and Flexural Reinforcement Design of a Singly Reinforced Rectangle

PROBLEM DESCRIPTION

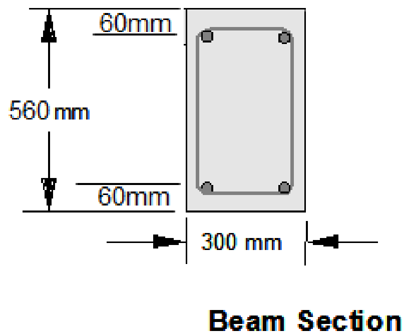
The purpose of this example is to verify the flexural and shear design. A simple-span, 6-m-long, 300-mm-wide, and 560-mm-deep beam is modeled. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated. The maximum element size has been specified to be 200 mm. The beam is supported by joint restraints that have no rotational stiffness. One end of the beam has no longitudinal stiffness.

The beam is loaded with symmetric third-point loading. One dead load case (DL50) and one live load case (LL130) with only symmetric third-point loads of magnitudes 50, and 130 kN, respectively, are defined in the model. One load combination (COMB130) is defined using the KBC 2009 load combination factors of 1.2 for dead loads and 1.5 for live loads. The model is analyzed for both of those load cases and the load combinations.

Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.

GEOMETRY, PROPERTIES AND LOADING

Clear span,	L	=	6000	mm
Overall depth,	h	=	560	mm
Width of beam,	b	=	300	mm
Effective depth,	d	=	500	mm
Depth of comp. reinf.,	d'	=	60	mm
Concrete strength,	f_{ck}	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	$25x10^5$	MPa
Modulus of elasticity,	E_s	=	$2x10^5$	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	50	kN
Live load,	P_l	=	130	kN



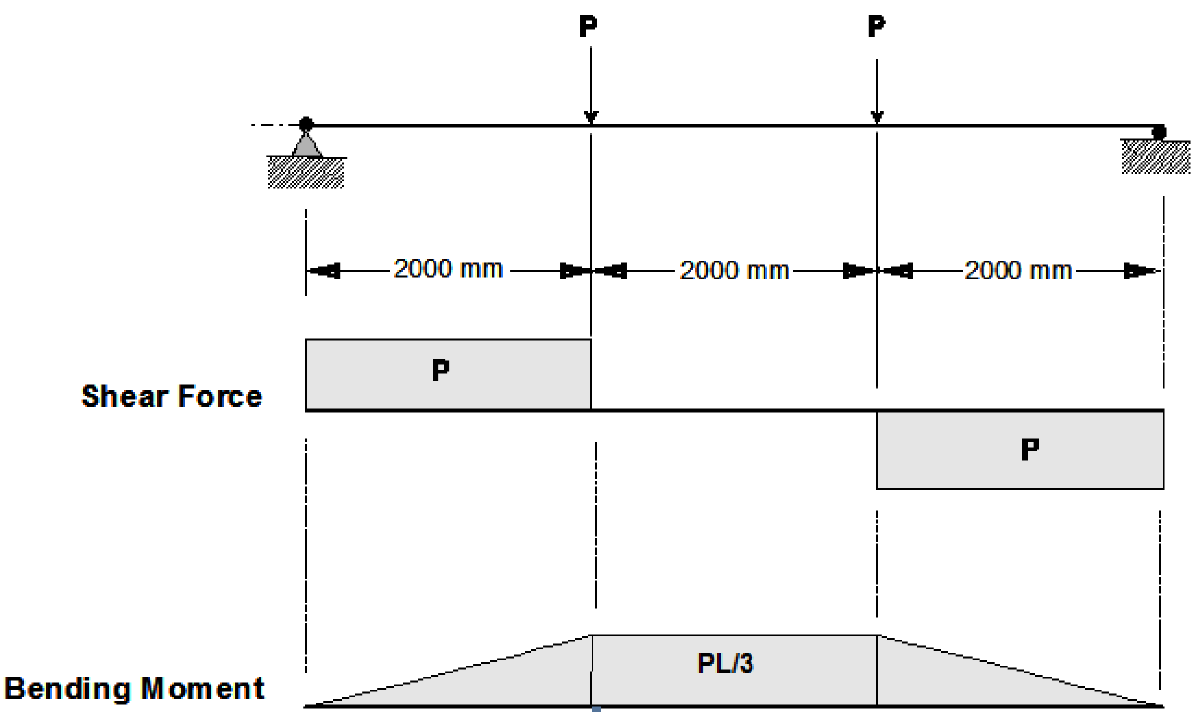


Figure 1 The Model Beam for Flexural and Shear Design

TECHNICAL FEATURES TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the total factored moments in the design strip with the moments obtained using the analytical method. They match exactly for this problem. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-mm)	
		A_s^+	A_s^-
ETABS	360	2109	0
Calculated	360	2109	0

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-mm/m)	
	ETABS	Calculated
180	515.3	515.4

COMPUTER FILE: KBC 2009 EX001

CONCLUSION

The computed results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for the load combination:

$\phi_b = 0.85$

$\beta_1 = 0.85 \;-.007(30-28) = 0.836 \text{ for } \; f_{ck} = 30\text{MPa},$

$$c_{\max} = \frac{\varepsilon_c}{\varepsilon_c + f_y / E_s} d \;= 187.5 \text{ mm}$$

$a_{\max} = \beta_1 c_{max} = 156.75$ mm

$A_c = b \bullet d = 150{,}000\, mm^2$

$$A_{s,\min} = \max \begin{cases} \dfrac{0.25\sqrt{f_{ck}}}{f_y} A_c = 446.5 \\ 1.4 \dfrac{A_c}{f_y} = 456.5 \end{cases} \text{mm}^2$$

$$= 456.5 \text{ mm}^2$$

COMB130

$V_u = (1.0P_d + 1.0P_l) = 180$ kN – Loads were Ultimate

$$M_u = \frac{V_u\, L}{3} = 360 \text{ kN-m}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f_{ck} \phi_b b}} = 26.81 \text{ mm} \; ; \text{a} < a_{\max}$$

Since $a \; < a_{\max}$, compression reinforcing is NOT required.

The required tension reinforcing is:

$$A_s = \frac{M_u}{f_y\left(d - \frac{a}{2}\right)\phi_b} = 2108.9\,\text{mm}^2$$

Shear Design

The following quantities are computed for all of the load combinations:

$\phi \quad = \quad 0.75$

The concrete limit is:

$\sqrt{f_{ck}}$ = 5.48 MPa < 8.4 MPa

The concrete shear capacity is given by:

$\phi V_c \quad = \quad 1/6\phi\sqrt{f_{ck}}\,bd \quad = \quad 102.69$ kN

The maximum shear that can be carried by reinforcement is given by:

$\phi V_s = 0.25\phi\sqrt{f_{ck}}\;bd \quad = 154.05$ kN

The following limits are required in the determination of the reinforcement:

$\phi V_c/2 \quad = \quad 51.35$ kN

$\phi V_{\max} = \phi V_c + \phi V_s \quad = \quad 256.75$ kN

Given V_u, V_c and $V_{\max}$, the required shear reinforcement in area/unit length for any load combination is calculated as follows:

If $V_u \leq \phi(V_c/2)$,

$$\frac{A_v}{s} = 0,$$

else if $\phi(V_c/2) < V_u \leq \phi V_{\max}$

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi\, f_{ys}\, d} \geq \left(\frac{A_v}{s}\right)_{\min}$$

where:

$$\left(\frac{A_v}{s}\right)_{\min} = \max\left\{3.5\left(\frac{b_w}{f_y}\right), \left(\frac{b_w}{f_y}\right) \bullet 0.2\sqrt{f_{ck}}\right\}$$

else if $V_u > \phi V_{max}$,

a failure condition is declared.

Combo1

$$V_u = 180 \text{ kN}$$

$$\phi\left(V_c / 2\right) = 51.35 \text{ kN} \le V_u = 180 \text{ kN} \le \phi V_{max} = 256.75 \text{ kN}$$

$$\left(\frac{A_v}{s}\right)_{min} = \max\left\{3.5\left(\frac{300}{420}\right), \left(\frac{0.2\sqrt{30}}{420}\right) \bullet 300\right\}$$

$$\left(\frac{A_v}{s}\right)_{min} = \max\left\{2.5,\ 0.78\right\} = 0.0083 \frac{\text{mm}^2}{\text{mm}}$$

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_y d} = 0.5154 \frac{\text{mm}^2}{\text{mm}} = 515.4 \frac{\text{mm}^2}{\text{m}}$$

KBC 2009 Example 002

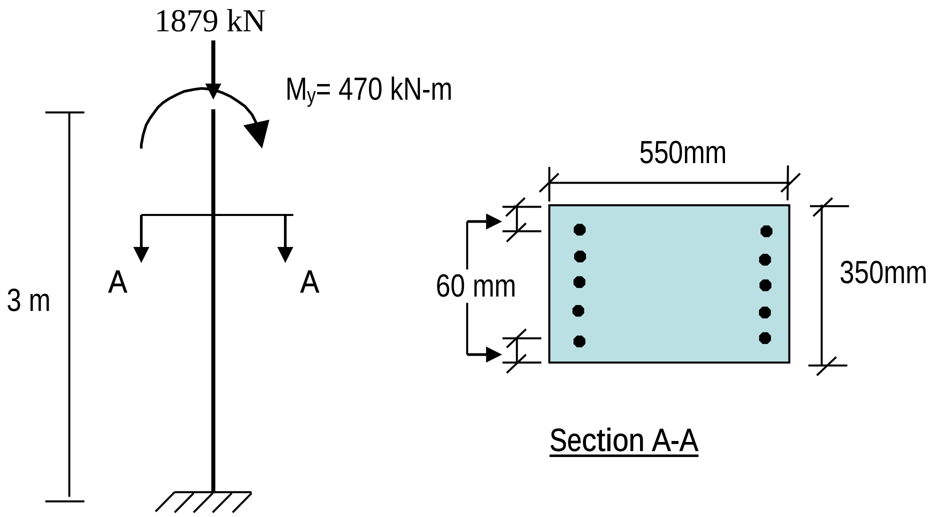
P-M INTERACTION CHECK FOR COMPRESSION-CONTROLLED RECTANGULAR COLUMN

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

In this example, a reinforced concrete column is subjected factored axial load P_u = 1879 kN and moment M_u = 470 kN-m. This column is reinforced with 5 T25 bars. The design capacity ratio is checked by hand calculations and computed result is compared. The column is designed as a short, non-sway member.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E_c = 25x10^6 kN/m^2	b = 350 m	f_{ck} = 30 MPa
v = 0.2	d = 490 mm	f_y = 460 MPa
G = 10416666.7kN/m^2		

TECHNICAL FEATURES TESTED

- Tied Reinforced Concrete Column Design

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	1.003	1.00	0.30%

COMPUTER FILE: KBC 2009 Ex002

CONCLUSION

The computed results show an acceptable comparison with the independent results.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

f_{ck} = 30 MPa $\quad$ f_y = 460 MPa
b = 350 mm $\quad$ d = 490 mm

1) Because e = 250 mm < (2/3)d = 327 mm, assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis fro a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_{\max} = \frac{0.003}{0.003 + 0.005} d = \frac{0.003}{0.003 + 0.005}(490) = 183.75 \text{ mm}$$

2) From the equation of equilibrium:

$$P_u = C_c + C_s - T$$

where

$$C_c = 0.85 f_{ck} ab = 0.85 \bullet 30 \bullet 350a = 8925a$$

$$C_s = A_s^{'}\left(f_y - 0.85 f_{ck}\right) = 2500\left(460 - 0.85 \bullet 30\right) = 1,086,250 \text{ N}$$

Assume compression steels yields, (this assumption will be checked later).

$$T = A_s f_s = 2500 f_s \left(f_s < f_y\right)$$

$$P_u = 8,925a + 1,086,250 - 2500 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$P_u = \frac{1}{e^{'}}\left[C_c\left(d - \frac{a}{2}\right) + C_s\left(d - d^{'}\right)\right]$$

The plastic centroid is at the center of the section and $d^{''}$ = 215 mm

$$e^{'} = e + d^{''} = 250 + 215 = 465 \text{ mm}$$

$$P_u = \frac{1}{465}\left[8,925a\left(490 - \frac{a}{2}\right) + 1,086,250\left(490 - 60\right)\right]$$

$$P_u = 9,404.8a - 9.6a^2 + 1,004,489.2 \qquad \text{(Eqn. 2)}$$

4) Assume c = 335 mm, which exceed c_{max} (183.75 mm).

$$\beta_1 = 0.85 \ -.007(30-28) = 0.836 \text{ for } \ f_{ck} = 30\text{MPa},$$
$$a = 0.836 \bullet 335 = 280 \text{ mm}$$

Substitute in Eqn. 2:

$$P_u = 9,404.8 \bullet 280 - 9.6\left(280\right)^2 + 1,004,489.2 = 2,885,193.2 \text{ N}$$

5) Calculate f_s from the strain diagram when c = 335 mm.

$$f_s = \left(\frac{490 - 335}{335}\right)600 = 277.8 \text{ MPa}$$
$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.00138$$

6) Substitute a = 280 mm and f_s = 277.7 MPa in Eqn. 1 to calculate P_{u2}:

$$P_{u2} = 8,925\left(280\right) + 1,086,250 - 2500\left(277.8\right) = 2,890,750 \text{ N}$$

Which is very close to the calculated P_{u1} of 2,885,193.2 (less than 1% difference)

$$M_u = P_u e = 2890\left(\frac{250}{1000}\right) = 722.5 \text{ kN-m}$$

7) Check if compression steels yield. From strain diagram,

$$\varepsilon_s^{'} = \left(\frac{335 - 60}{335}\right)\left(0.003\right) = 0.00263 > \varepsilon_y = 0.0023$$

Compression steels yields, as assumed.

8) Therefore, section capacity is

$$P_u = 0.65 \bullet 2890 = 21879 \text{ kN}$$
$$M_u = 0.65 \bullet 722.5 = 470 \text{ kN-m}$$

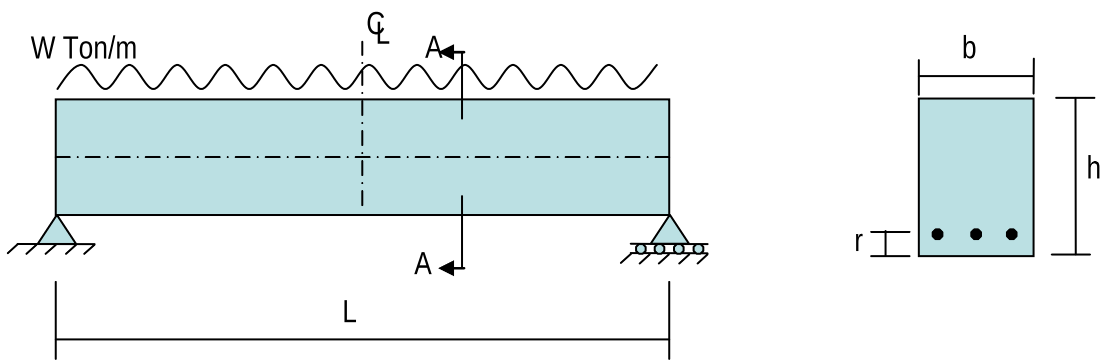
RCDF 2004 Example 001

SHEAR AND FLEXURAL REINFORCEMENT DESIGN OF A SINGLY REINFORCED RECTANGLE

EXAMPLE DESCRIPTION

In the example a simple supported beam is subjected to a uniform factored load of 6.58 Ton/m (64.528 kN/m). This example was tested using the Mexican RCDF 2004 concrete design code. The computed moment and shear strengths are compared with independent hand calculated results.

GEOMETRY, PROPERTIES AND LOADING



L = 6 m

Material Properties

E = 1979899 kg/cm^2
v = 0.2
G = 824958 kg/cm^2

Section Properties

h = 0.65 m
r = 0.05 m
b = 0.30 m
W = 6.58 Ton/m (64.528 kN/m)

Design Properties

f'c = 200 kg/cm^2 (19.6133 MPa)
f_y = 4200 kg/cm^2 (411.88 MPa)

TECHNICAL FEATURES TESTED

- Design moment calculation, M and factored moment resistance, $M_{u.}$
- Minimum reinforcement calculation, A_s
- Design Shear Strength, V, and factored shear strength, Vu

RESULTS COMPARISON

Independent results are hand calculated based on the equivalent rectangular stress distribution described in Example 5.2 on page 92 of "Aspectos Fundamentales del Concreto Reforzado" Fourth Edition by Óscar M. González Cuevas and Francisco Robles Fernández-Villegas.

Output Parameter	ETABS	Independent	Percent Difference
Design Moment (kN-m)	290.38	290.38	0%
A_s (mm^2)	1498	1498	0%
Design Shear (kN)	154.9	154.9	0%
A_v/s (mm^2/m)	563	563	0%

COMPUTER FILE: RCDF 2004 Ex001

CONCLUSION

The computed results show an acceptable comparison with the independent results for bending and an acceptable-conservative comparison for shear.

GEOMETRY AND PROPERTIES

Clear span,	L	=	6	m
Overall depth,	h	=	650	mm
Width of beam,	b	=	300	mm
Effective depth,	d	=	600	mm
Concrete strength,	f'_c	=	19.61	N/mm^2
Yield strength of steel,	f_y	=	411.88	N/mm^2
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	$20.6x10^3$	N/mm^2
Modulus of elasticity,	E_s	=	$20.0x10^4$	N/mm^2
Poisson's ratio,	v	=	0.2	

HAND CALCULATION

Flexural Design

The following quantities are computed for the load combination:

$$f_c^* = \frac{f'_c}{1.25} = \frac{19.61}{1.25} = 15.69 \text{ MPa}$$

$$c_b = \frac{\varepsilon_c E_s}{\varepsilon_c E_s + f_{yd}} d = 355.8 \text{ mm}$$

$$a_{\max} = \beta_1 c_b = 302.4 \text{ mm}$$

where, $\beta_1 = 1.05 - \left(\frac{f_c^*}{140}\right),\ 0.65 \le \beta_1 \le 0.85$

$$A_{s,\min} = \frac{0.22\sqrt{f'_c}}{f_y} bd = 425.8\, mm^2$$

COMB1

ω_u = 6.58 ton/cm (64.528kN/m)

$$M_u = \frac{\omega_u l^2}{8} = 64.528 \bullet 6.0^2/8 = 290.376 \text{ kN-m}$$

The depth of the compression block is given by: (RCDF-NTC 2.1, 1.5.1.2)

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f_c^* F_R b}} = 154.2 \text{ mm}$$

where $F_R = 0.9$

Compression steel not required since a < a_{max}.

The area of tensile steel reinforcement is given by:

$$A_s = \frac{M_u}{F_R f_y \left(d - \frac{a}{2}\right)} = \frac{290376000}{0.9(411.88)\left(600 - 154.2/2\right)} = 1498\, mm^2$$

Shear Design

The shear demand is computed as:

$V_u = \omega\left(L/2 - d\right)$ =15.79 ton (154.9 kN) at distance, d, from support for this example

The shear force is limited to a maximum of,

$$V_{\max} = V_{cR} + \left(0.8\sqrt{f_c^*}\right) A_{cv}$$

The nominal shear strength provided by concrete is computed as:

$$V_{cR} = 0.3 F_{Rv}\left(0.2 + 20\rho\right)\sqrt{f_c^*} A_{cv} = 0.3 \bullet 0.8\left(0.3665\right)\sqrt{15.69} \bullet 300 \bullet 600$$

=43.553 kN where $F_{Rv} = 0.8$

The shear reinforcement is computed as follows:

$$\left(\frac{A_v}{s}\right)_{\min} = \frac{0.1\sqrt{f_c'}}{f_y} b = 289 \frac{mm^2}{m}$$ (RCDF-NTC 2.5.2.3, Eqn 2.22)

$$\frac{A_v}{s} = \frac{\left(V_u - F_{Rv} V_{cR}\right)}{F_{Rv} f_{ys} d} = \frac{154.9 - 0.8 \bullet 43.553}{0.8 \bullet 411.88 \bullet 600} = 563 \frac{mm^2}{m}$$

(RCDF-NTC 2.5.2.3, Eqn 2.23)

RCDF 2004 Example 002

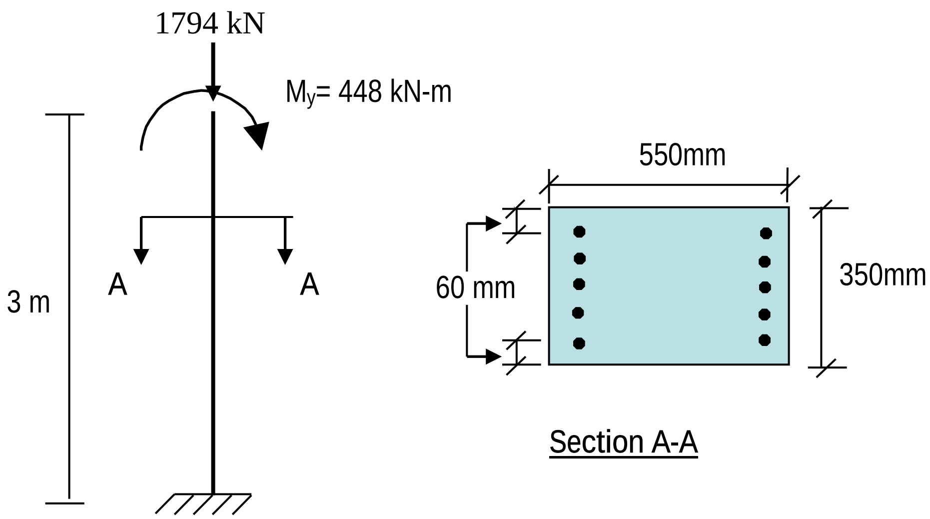
P-M INTERACTION CHECK FOR COMPRESSION-CONTROLLED RECTANGULAR COLUMN

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

A reinforced concrete column is subjected to factored axial load N = 1794 kN and moment M_y = 448 kN-m. This column is reinforced with five 25M bars. The design capacity ratio is checked by hand calculations and the result is compared with a computed result. The column is designed as a short, non-sway member.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E_c = 25x10^6 kN/m^2	b = 350 m	f_{cu} = 30 MPa
v = 0.2	d = 490 mm	f_y = 460 MPa
G = 10416666.7kN/m^2		

TECHNICAL FEATURES TESTED

- Tied reinforced concrete column design

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	0.999	1.00	0.10%

COMPUTER FILE: RCDF 2004 EX002

CONCLUSION

The computed results show an acceptable comparison with the independent results.

HAND CALCULATION

Column Strength under compression control

f_{cu} = 30 MPa $\quad$ f_y = 460 MPa
b = 350 mm $\quad$ d = 490 mm

1) Because e = 250 mm < (2/3)d = 327 mm, assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis for a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{600}{600 + f_y} d_t = \frac{600}{600 + 460}(490) = 277 \text{ mm}$$

2) From the equation of equilibrium:

$$N = C_c + C_s - T$$

where

$$C_c = 0.85 f^*{}_c ab = 0.85 \bullet 0.8 \bullet 30 \bullet 350a = 7140a$$

$$C_s = A'_s \left(f_y - 0.85 f^*{}_c \right) = 2500\left(460 - 0.85 \bullet 0.8 \bullet 30\right) = 1{,}099{,}000 \text{ N}$$

Assume compression steels yields, (this assumption will be checked later).

$$T = A_s f_s = 2500 f_s \left(f_s < f_y \right)$$

$$N_1 = 7140a + 1{,}099{,}000 - 2500 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$N = \frac{1}{e'}\left[C_c \left(d - \frac{a}{2} \right) + C_s \left(d - d' \right) \right]$$

The plastic centroid is at the center of the section and d'' = 215 mm

$$e' = e + d'' = 250 + 215 = 465 \text{ mm}$$

$$N = \frac{1}{465}\left[7140a\left(490 - \frac{a}{2} \right) + 1{,}099{,}000\left(490 - 60\right) \right]$$

$$N_2 = 7542a - 7.677a^2 + 1{,}016{,}280 \qquad \text{(Eqn. 2)}$$

4) Assume c = 347 mm, which exceeds c_b (277 mm).

$$a = \beta_1 a = 0.836 \bullet 347 = 290 \text{ mm}$$

Substitute in Eqn. 2:

$$N_2 = 7542 \bullet 290 - 7.677(290)^2 + 1{,}016{,}280 = 2{,}557{,}824 \text{ N}$$

5) Calculate f_s from the strain diagram when c = 365 mm.

$$f_s = \left(\frac{490 - 347}{347}\right) 600 = 247.3 \text{ MPa}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.0012$$

6) Substitute a = 290 mm and f_s = 247.3 MPa in Eqn. 1 to calculate N_1:

$$N_1 = 7140(290) + 1{,}099{,}000 - 2500(247.3) = 2{,}551{,}350 \text{ N}$$

which is very close to the calculated N_2 of 2,557,824 (less than 1% difference)

$$M = Ne = 2552\left(\frac{250}{1000}\right) = 638 \text{ kN-m}$$

7) Check if compression steel yields. From strain diagram,

$$\varepsilon_s' = \left(\frac{347 - 60}{347}\right)(0.003) = 0.0025 > \varepsilon_y = 0.0023$$

Compression steel yields, as assumed.

8) Therefore, section capacity is

$$N = F_R(2551) = 1794 \text{ kN}$$

$$M = F_R(638) = 448 \text{ kN-m}$$

NZS 3101-2006 Example 001

Shear and Flexural Reinforcement Design of a Singly Reinforced Rectangle

PROBLEM DESCRIPTION

The purpose of this example is to verify the flexural and shear design. The load level is adjusted for the case corresponding to the following conditions:

- The stress-block dimension, a, extends below, $a_{\max}$, which requires that compression reinforcement be provided as permitted by NZS 3101-06.
- The average shear stress in the beam is below the maximum shear stress allowed by NZS 3101-06, requiring design shear reinforcement.

A simple-span, 6-m-long, 300-mm-wide, and 560-mm-deep beam is modeled. The beam is shown in Figure 1. The computational model uses a finite element mesh of frame elements, automatically generated. The maximum element size has been specified to be 200 mm. The beam is supported by joint restraints that have no rotational stiffness. One end of the beam has no longitudinal stiffness.

The beam is loaded with symmetric third-point loading. One dead load case (DL50) and one live load case (LL130) with only symmetric third-point loads of magnitudes 50, and 130 kN, respectively, are defined in the model. One load combination (COMB130) is defined using the NZS 3101-06 load combination factors of 1.2 for dead loads and 1.5 for live loads. The model is analyzed for both of those load cases and the load combinations.

Table 1 shows the comparison of the design longitudinal reinforcements. Table 2 shows the comparison of the design shear reinforcements.

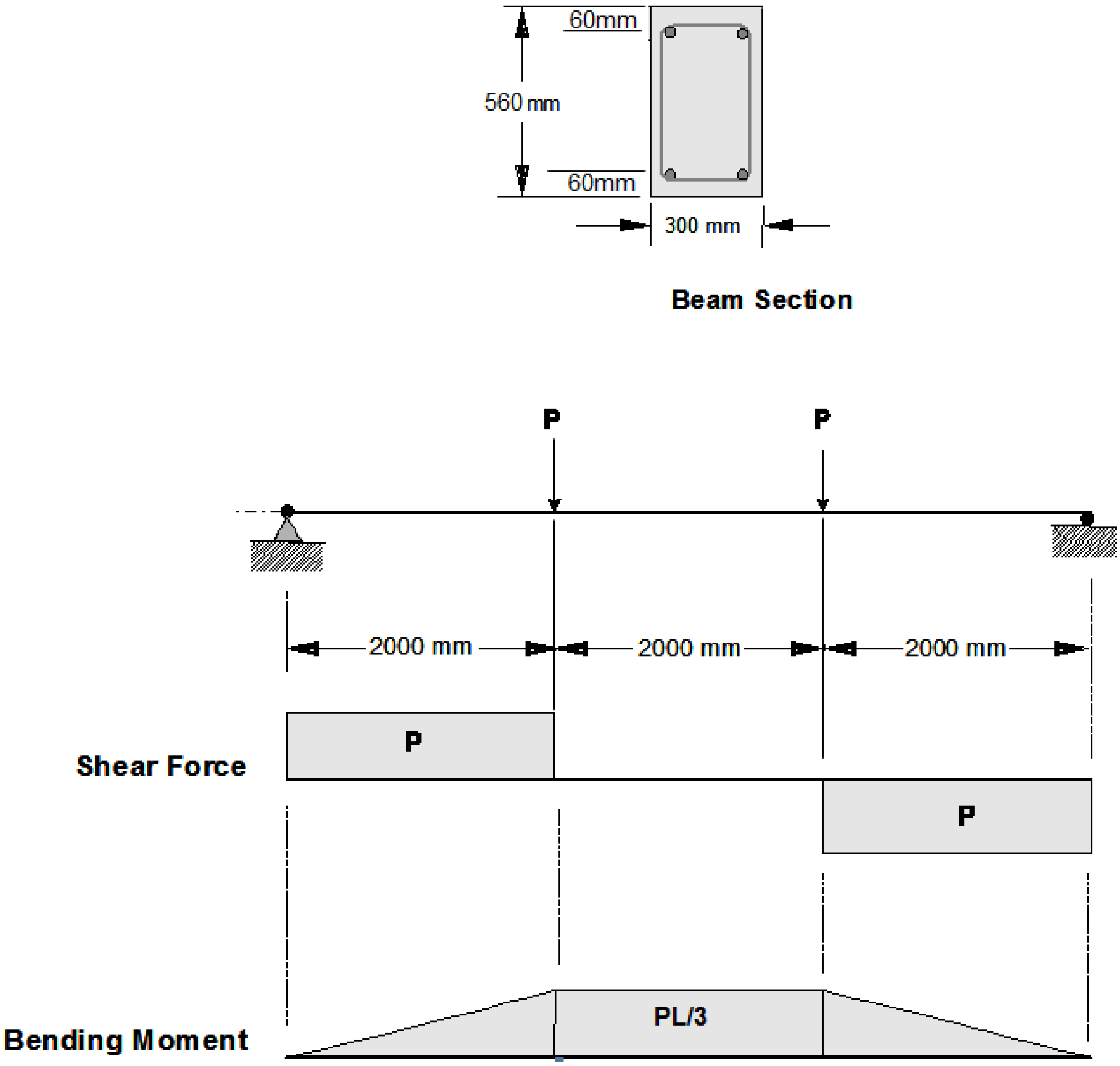


Figure 1 The Model Beam for Flexural and Shear Design

Geometry, Properties and Loading

Clear span,	L	=	6000	mm
Overall depth,	h	=	560	mm
Width of beam,	b	=	300	mm
Effective depth,	d	=	500	mm
Depth of comp. reinf.,	d'	=	60	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	460	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	$25x10^5$	MPa
Modulus of elasticity,	E_s	=	$2x10^5$	MPa
Poisson's ratio,	v	=	0.2	
Dead load,	P_d	=	50	kN
Live load,	P_l	=	130	kN

Technical Features Tested

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

Results Comparison

Table 1 shows the comparison of the total factored moments in the design strip with the moments obtained using the analytical method. They match exactly for this problem. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Moments and Flexural Reinforcements

Method	Moment (kN-m)	Reinforcement Area (sq-mm)	
		A_s^+	A_s^-
ETABS	510	3170	193
Calculated	510	3170	193

Table 2 Comparison of Shear Reinforcements

Shear Force (kN)	Reinforcement Area, $\frac{A_v}{s}$ (sq-mm/m)	
	ETABS	Calculated
255	1192.5	1192.5

COMPUTER FILE: NZS 3101-2006 EX001

CONCLUSION

The computed results show an exact comparison with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for the load combination:

$\phi_b = 0.85$

$\alpha_1 = 0.85$ for $f'_c \leq 55\text{MPa}$

$\beta_1 = 0.85$ for $f'_c \leq 30,$

$$c_b = \frac{\varepsilon_c}{\varepsilon_c + f_y / E_s} d = 283.02 \text{ mm}$$

$a_{max} = 0.75\beta_1 c_b = 180.42$ mm

$A_c = b \bullet d = 150{,}000\, mm^2$

$$A_{s,\min} = \max \begin{cases} \dfrac{\sqrt{f'_c}}{4 f_y} A_c = 446.5 \\ 1.4 \dfrac{A_c}{f_y} = 456.5 \end{cases} \text{mm}^2$$

$= 456.5 \text{ mm}^2$

COMB130

$V^* = (1.2P_d + 1.5P_l) = 255$ kN

$$M^* = \frac{V^* L}{3} = 510 \text{ kN-m}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M^*\right|}{\alpha_1 f'_c \phi_b b_f}} = 194.82 \text{ mm} ;\ a > a_{\max}$$

Since $a \geq a_{\max}$, compression reinforcing is required.

The compressive force, C, developed in the concrete alone is given by:

$$C = \alpha_1 f'_c b a_{\max} = 1{,}380.2\,kN$$

The resisting moment by the concrete compression and tension reinforcement is:

$$M_c^* = C\left(d - \frac{a_{\max}}{2}\right)\phi_b = 480.8 \text{ kN-m}$$

Therefore the moment required by concrete compression and tension reinforcement is:

$$M_s^* = M^* - M_c^* = 29.2\,kN - m$$

The required compression reinforcing is given by:

$$A'_s = \frac{M_s^*}{\left(f'_s - \alpha_1 f'_c\right)\left(d - d'\right)\phi_b} = 193\,mm^2, \text{ where}$$

$$c_{b,\max} = \frac{a_{\max}}{\beta_1} = 0.75 \bullet c_b = 0.75 \bullet 283.02 = 212.26\,mm$$

$$f'_s = \varepsilon_{c,\max} E_s \left[\frac{c_{b,\max} - d'}{c_{b,\max}}\right] \le f_y \quad ;$$

$$f'_s = 0.003 \bullet 200{,}000 \left[\frac{212.26 - 60}{212.26}\right] = 430\,MPa \le f_y = 460\,MPa$$

$$f'_s = 430\,MPa$$

The required tension reinforcing for balancing the compression in the concrete is:

$$A_{s1} = \frac{M_c^*}{f_y\left(d - \frac{a_{\max}}{2}\right)\phi_b} = 3{,}001\,mm^2$$

And the tension required for balancing the compression reinforcement is given by:

$$A_{s2} = \frac{M_s^*}{f_y\left(d - d'\right)\phi_b} = 169.9\,mm^2$$

Therefore, the total tension reinforcement, $A_s = A_{s1} + A_{s2}$ is given by:

$$A_s = A_{s1} + A_{s2} = 3001 + 169.9 = 3170.5\,mm^2$$

Shear Design

The nominal shear strength provided by concrete is computed as:

$V_C = v_C A_{CV}$, where

$v_C = k_d k_a v_b$, and

$k_d = 1.0$ since shear reinforcement provided will be equal to or greater than the nominal amount required.

k_a = 1.0 (Program default)

$v_b = \left(0.07 + 10\dfrac{A_s}{bd}\right)\sqrt{f'_C}$, except v_b is neither less than $0.08\sqrt{f'_C}$ nor greater than $0.2\sqrt{f'_C}$ and $f'_C \le 50\,MPa$

$v_C = 0.4382$

The average shear stress is limited to a maximum limit of,

$$v_{max} = \min\left\{0.2 f'_c, 8 \text{ MPa}\right\} = \min\{6, 8\} = 6 \text{ MPa}$$

For this example, the nominal shear strength provided by concrete is:

$$V_C = v_C A_{CV} = 0.4382 \bullet 300 \bullet 500 = 65.727\,kN$$

$$v^* = \frac{V^*}{b_w d} = 1.7\,MPa < v_{max}$$, so there is no concrete crushing.

If $v^* > v_{max}$, a failure condition is declared.

For this example the required shear reinforcing strength is:

$\phi_s = 0.75$

$$V_S = \frac{V^*}{\phi_S} - V_C = \frac{255}{0.75} - 65.727 = 274.3 \text{ kN}$$

The shear reinforcement is computed as follows:

Since $h = 560\,mm > \max\left\{300\,mm, 0.5b_w = 0.5 \bullet 300 = 150\,mm\right\}$

$\phi_s v_c = 0.328$ MPa

$\phi_s v_{\max}$ = 4.5 MPa

So $\phi_s v_c < v^* \le \phi_s v_{\max}$, and shear reinforcement is required and calculate as:

$$\frac{A_v}{s} = \frac{V_S}{f_{yt} d} = \frac{274.27 \bullet 1E6}{460 \bullet 500} = 1192.5\, mm^2$$

NZS 3101-2006 Example 002

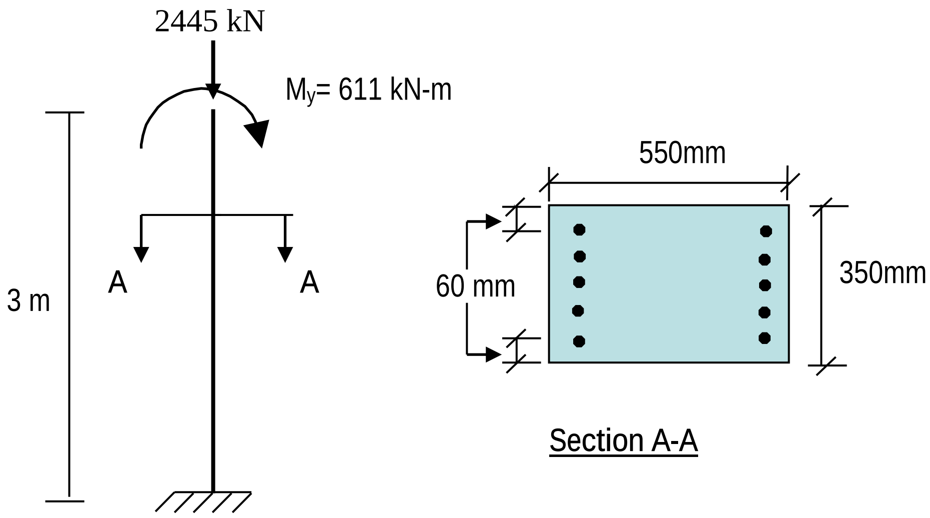
P-M INTERACTION CHECK FOR COMPRESSION-CONTROLLED RECTANGULAR COLUMN

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

In this example, a reinforced concrete column is subjected factored axial load N^* = 2445 kN and moment M_y = 611 kN-m. This column is reinforced with 5 T25 bars. The design capacity ratio is checked by hand calculations and computed result is compared. The column is designed as a short, non-sway member.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E_c = 25x10^6 kN/m^2	b = 350 m	f'_c = 30 MPa
v = 0.2	d = 490 mm	f_y = 460 MPa
G = 10416666.7kN/m^2		

TECHNICAL FEATURES TESTED

- Tied Reinforced Concrete Column Design

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	0.994	1.00	0.60%

COMPUTER FILE: NZS 3101-2006 Ex002

CONCLUSION

The computed results show an acceptable comparison with the independent results.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

f_{cu} = 30 MPa f_y = 460 MPa
b = 350 mm   d = 490 mm

1) Because e = 250 mm < (2/3)d = 327 mm, assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis fro a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{600}{600 + f_y} d_t = \frac{600}{600 + 460}\left(490\right) = 277 \text{ mm}$$

2) From the equation of equilibrium:

$$N^* = C_c + C_s - T$$

where

$$C_c = 0.85 f_c^{'} ab = 0.85 \bullet 30 \bullet 350a = 8925a$$

$$C_s = A_s^{'}\left(f_y - 0.85 f_c^{'}\right) = 2500\left(460 - 0.85 \bullet 30\right) = 1,086,250 \text{ N}$$

Assume compression steels yields, (this assumption will be checked later).

$$T = A_s f_s = 2500 f_s \left(f_s < f_y\right)$$

$$N^* = 8,925a + 1,086,250 - 2500 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$N^* = \frac{1}{e^{'}}\left[C_c\left(d - \frac{a}{2}\right) + C_s\left(d - d^{'}\right)\right]$$

The plastic centroid is at the center of the section and $d^{''}$ = 215 mm

$$e^{'} = e + d^{''} = 250 + 215 = 465 \text{ mm}$$

$$N^* = \frac{1}{465}\left[8,925a\left(490 - \frac{a}{2}\right) + 1,086,250\left(490 - 60\right)\right]$$

$$N^* = 9,404.8a - 9.6a^2 + 1,004,489.2 \qquad \text{(Eqn. 2)}$$

4) Assume c = 330 mm, which exceed c_b (296 mm).

$$a = 0.85 \bullet 330 = 280.5 \text{ mm}$$

Substitute in Eqn. 2:

$$N^* = 9,404.8 \bullet 280.5 - 9.6(280.5)^2 + 1,004,489.2 = 2,887,205.2 \text{ N}$$

5) Calculate f_s from the strain diagram when c = 330 mm.

$$f_s = \left(\frac{490 - 330}{330}\right)600 = 290.9 \text{ MPa}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.00145$$

6) Substitute a = 280.5 mm and f_s = 290.9 MPa in Eqn. 1 to calculate N^*_2:

$$N_2^* = 8,925(280.5) + 1,086,250 - 2500(290.9) = 2,862,462.5 \text{ N}$$

Which is very close to the calculated P_{r1} of 2,887,205.2 (less than 1% difference)

$$M^* = N^* e = 2877\left(\frac{250}{1000}\right) = 719 \text{ kN-m}$$

7) Check if compression steels yield. From strain diagram,

$$\varepsilon_s' = \left(\frac{330 - 60}{330}\right)(0.003) = 0.00245 > \varepsilon_y = 0.0023$$

Compression steels yields, as assumed.

8) Therefore, section capacity is

$$N^* = 0.85 \bullet 2877 = 2445 \text{ kN}$$

$$M^* = 0.85 \bullet 719 = 611 \text{ kN-m}$$

SS CP 65-1999 Example 001

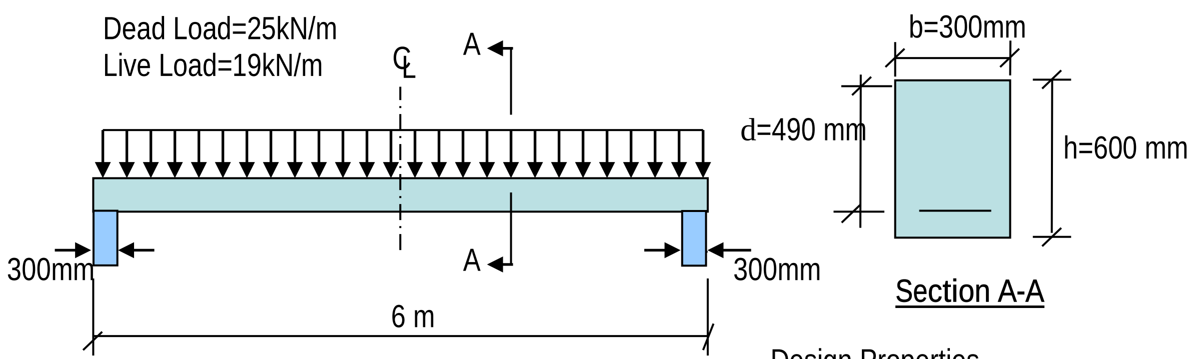
SHEAR AND FLEXURAL REINFORCEMENT DESIGN OF A SINGLY REINFORCED RECTANGLE

EXAMPLE DESCRIPTION

The flexural and shear strength of a rectangular concrete beam is tested in this example.

A simply supported beam is subjected to a uniform unfactored dead load and imposed load of 25 and 19 kN/m respectively spanning 6m. This example is tested using the Singapore CP65-99 concrete design code. The flexural and shear reinforcing computed is compared with independent results.

GEOMETRY, PROPERTIES AND LOADING



Design Properties

f_{cu} = 30 MPa

f_y = 460 MPa

f_{yv} = 250 MPa

TECHNICAL FEATURES OF TESTED

- Calculation of Flexural reinforcement, A_s
- Enforcement of Minimum tension reinforcement, $A_{s,min}$
- Calculation of Shear reinforcement, A_v
- Enforcement of Minimum shear reinforcing, $A_{v,min}$

RESULTS COMPARISON

The detailed work-out of the example above can be obtained from Example 3.4 of Chanakya Arya (1994). "Design of Structural Elements." *E & FN Spon*, 54-55

Output Parameter	ETABS	Independent	Percent Difference
Design Moment, M_u (kN-m)	294.30	294.30	0.00%
Tension Reinf, A_s (mm^2)	1555	1555	0.00%
Design Shear, V_u (kN)	160.23	160.23	0.00%
Shear Reinf, A_{sv}/s_v (mm^2/mm)	0.730	0.730	0.00 %

COMPUTER FILE: SS CP 65-1999 Ex001

CONCLUSION

The computed flexural results show an exact match with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for all the load combinations:

$\gamma_{m,\,steel}$ = 1.15

$A_{s,\min} = 0.0013bh$, where b=300mm, h=600mm

= 234.00 sq-mm

Design Combo COMB1

w_u = =65.4 kN/m

$M_u = \dfrac{w_u l^2}{8}$ = 294.3 kN-m

$V_u = \dfrac{w_u l}{2} - w_u d$ = 160.23 kN

The depth of the compression block is given by:

$K = \dfrac{M}{f_{cu} b d^2}$ = 0.108 < 0.156

If $K \leq 0.156$ (BS 3.4.4.4), the beam is designed as a singly reinforced concrete beam.

Then the moment arm is computed as:

$z = d \left\{ 0.5 + \sqrt{0.25 - \dfrac{K}{0.9}} \right\} \leq 0.95d$ = 473.221 mm, where d=550 mm

The ultimate resistance moment is given by:

$A_s = \dfrac{M}{\left(f_y/1.15\right)z}$ = 1555 sq-mm

Shear Design

V_u = =160.23 kN at distance, d, from support

$$v = \frac{V}{b_w d} = 0.9711 \text{ MPa}$$

$$v_{\max} = \min(0.8\sqrt{f_{cu}}, 5 \text{ MPa}) = 4.38178 \text{ MPa}$$

$v \le v_{\max}$, so no concrete crushing

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.84 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} = 0.4418 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{30}\right)^{1/3} = 1.0,\ 1 \le k_2 \le \left(\frac{80}{30}\right)^{1/3}$$

$\gamma_m = 1.25$

$$0.15 \le \frac{100\ A_s}{bd} \le 3$$

$$\frac{100\ A_s}{bd} = \frac{100 \bullet 469}{300 \bullet 550} = 0.2842$$

$$\left(\frac{400}{d}\right)^{1/4} = 0.95 \ge 1,\ so\left(\frac{400}{d}\right)^{1/4} \text{ is taken as 1.}$$

$f_{cu} \le 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

If $(v_c + 0.4) < v \le v_{\max}$

$$\frac{A_{sv}}{s_v} = \frac{(v - v_c) b_w}{0.87 f_{yv}} = \frac{(0.9711 - 0.4418)}{0.87 \bullet 250} = 0.730 \text{ sq-mm/mm}$$

SS CP 65-1999 Example 002

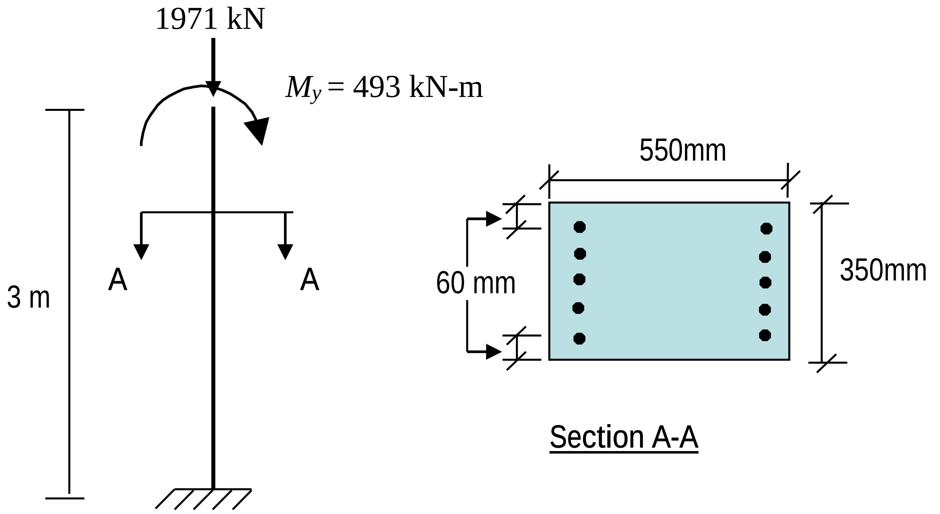
P-M INTERACTION CHECK FOR COMPRESSION-CONTROLLED RECTANGULAR COLUMN

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

A reinforced concrete column is subjected to factored axial load N = 1971 kN and moment M_y = 493 kN-m. This column is reinforced with five 25M bars. The design capacity ratio is checked by hand calculations and the result is compared with the calculated result. The column is designed as a short, non-sway member.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E_c = 25x10^6 kN/m^2	b = 350 m	f_{cu} = 30 MPa
v = 0.2	d = 490 mm	f_y = 460 MPa
G = 10416666.7kN/m^2		

TECHNICAL FEATURES TESTED

- Tied reinforced concrete column design

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	0.994	1.00	0.60%

COMPUTER FILE: SS CP 65-1999 EX002

CONCLUSION

The computed results show an acceptable comparison with the independent results.

HAND CALCULATION

Column Strength under compression control

f_{cu} = 30 MPa f_y = 460 MPa
b = 350 mm   d = 490 mm

1) Because e = 250 mm < (2/3)d = 327 mm, assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis for a balanced condition, c_b:

Position of neutral axis at balanced condition:

$$c_b = \frac{700}{700 + f_y} d_t = \frac{700}{700+460}(490) = 296 \text{ mm}$$

2) From the equation of equilibrium:

$$N = C_c + C_s - T$$

where

$$C_c = \frac{0.67}{\gamma_M} f_{cu} ab = 0.67/1.5 \cdot 30 \cdot 350a = 4667a$$

$$C_s = \frac{A'_s}{\gamma_s}\left(f_y - 0.4467 f_{cu}\right) = \frac{2500}{1.15}\left(460 - 0.4467 \cdot 30\right) = 971,014 \text{ N}$$

Assume compression steel yields (this assumption will be checked later).

$$T = \frac{A_s f_s}{\gamma_s} = \frac{2500 f_s}{1.15} = 2174 f_s \left(f_s < f_y\right)$$

$$N_1 = 4,667a + 971,014 - 2174 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$N = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_s\left(d - d'\right)\right]$$

The plastic centroid is at the center of the section and d'' = 215 mm

$$e' = e + d'' = 250 + 215 = 465 \text{ mm}$$

$$N = \frac{1}{465}\left[4,667a\left(490 - \frac{a}{2}\right) + 971,014\left(490 - 60\right)\right]$$

$$N = 4917.9a - 5.018a^2 + 897,926 \qquad \text{(Eqn. 2)}$$

4) Assume c = 364 mm, which exceeds c_b (296 mm).

$a = 0.9 \bullet 364 = 327.6 \text{ mm}$

Substitute in Eqn. 2:

$$N_2 = 4917.9 \bullet 327.6 - 5.018(327.6)^2 + 897,926 = 1,970,500 \text{ N}$$

5) Calculate f_s from the strain diagram when c = 365 mm.

$$f_s = \left(\frac{490-364}{364}\right)700 = 242.3 \text{ MPa}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.0012$$

6) Substitute a = 327.6 mm and f_s = 242.3 MPa in Eqn. 1 to calculate N_1:

$$N_1 = 4,667(327.6) + 971,014 - 2174(242.3) = 1,973,163 \text{ N}$$

which is very close to the calculated N_2 of 1,970,500 (less than 1% difference)

$$M = Ne = 1971\left(\frac{250}{1000}\right) = 493 \text{ kN-m}$$

7) Check if compression steel yields. From strain diagram,

$$\varepsilon_s' = \left(\frac{364-60}{364}\right)(0.0035) = 0.0029 > \varepsilon_y = 0.0023$$

Compression steel yields, as assumed.

8) Therefore, section capacity is

N = 1971 kN
M = 493 kN-m

TS 500-2000 Example 001

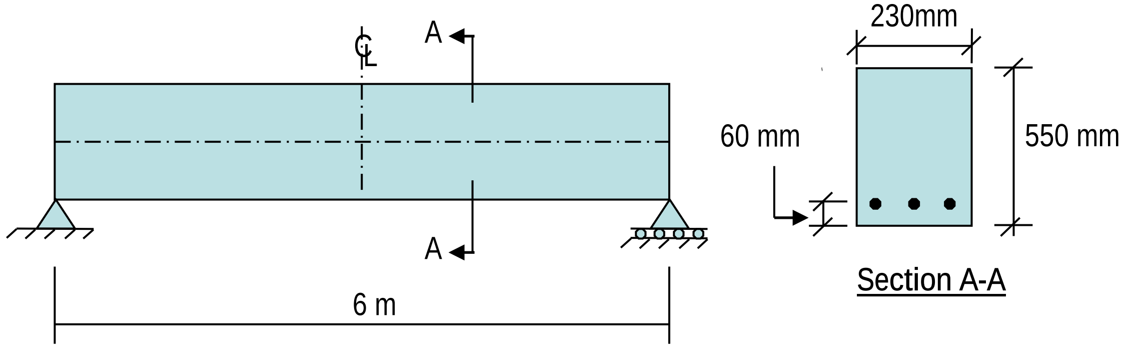
Shear and Flexural Reinforcement Design of a Singly Reinforced Rectangle

EXAMPLE DESCRIPTION

The flexural and shear strength of a rectangular concrete beam is tested in this example.

A simply supported beam is subjected to a uniform factored load of 36.67 kN/m. This example is tested using the Turkish TS 500-2000 concrete design code. The flexural and shear reinforcing computed is compared with independent results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 25.000×10^6 kN/m^2

v = 0.2

Section Properties

d = 543.75 mm

Design Properties

f_{ck} = 30 MPa

f_y = 420 MPa

Clear span,	L	=	6000	mm
Overall depth,	h	=	550	mm
Width of beam,	b	=	230	mm
Effective depth,	d	=	490	mm
Concrete strength,	f_{ck}	=	30	MPa
Yield strength of steel,	f_{yk}	=	420	MPa
Concrete unit weight,	w_c	=	0	kN/m^3
Modulus of elasticity,	E_c	=	25×10^3	MPa
Modulus of elasticity,	E_s	=	2×10^5	MPa
Poisson's ratio,	v	=	0.2	

TECHNICAL FEATURES TESTED

- Calculation of flexural and shear reinforcement
- Application of minimum flexural and shear reinforcement

RESULTS COMPARISON

Output Parameter	ETABS	Independent	Percent Difference
Design Moment, M_d (kN-m)	165.02	165.02	0.00%
Tension Reinf, A_s (mm^2)	1022	1022	0.00%
Design Shear, V_d (kN)	110.0	110.0	0.00%
Shear Reinf, A_{sw}/s (mm^2/mm)	0.2415	0.2415	0.00%

COMPUTER FILE: TS 500-2000 EX001

CONCLUSION

The computed results show an exact match with the independent results.

HAND CALCULATION

Flexural Design

The following quantities are computed for the load combination:

$$f_{cd} = \frac{f_{ck}}{\gamma_{mc}} = \frac{30}{1.5} = 20$$

$$f_{yd} = \frac{f_{yk}}{\gamma_{ms}} = \frac{420}{1.15} = 365$$

$$c_b = \frac{\varepsilon_{cu} E_s}{\varepsilon_{cu} E_s + f_{yd}} d \ = 304.6 \text{ mm}$$

$$a_{\max} = 0.85 k_1 c_b \ = 212.3 \text{ mm}$$

where, $k_1 = 0.85 - 0.006\left(f_{ck} - 25\right) = 0.82$, $0.70 \le k_1 \le 0.85$

$$A_{s,\min} = \frac{0.8 f_{ctd}}{f_{yd}} bd \ = 315.5\, mm^2$$

Where $f_{ctd} = \dfrac{0.35\sqrt{f_{cu}}}{\gamma_{mc}} = \dfrac{0.35\sqrt{30}}{1.5} = 1.278$

COMB1

$$\omega_d \ = 36.67 \text{ kN/m}$$

$$M_d = \frac{\omega_d L^2}{8} \ = 36.67 \bullet 6^2/8 = 165.02 \text{ kN-m}$$

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_d\right|}{0.85 f_{cd} b}} \ = 95.42 \text{ mm}$$

Compression steel not required since a < a_{max}.

The area of tensile steel reinforcement is given by:

$$A_s = \frac{M_d}{f_{yd}\left(d - \frac{a}{2}\right)} = \frac{165\text{E}6}{365 \bullet \left(490 - 95.41/2\right)}$$

$$A_s = \ 1022 \text{ mm}^2$$

Shear Design

The shear demand is computed as:

$$V_d = \frac{\omega L}{2} = 110.0 \text{ kN at face of support for this example}$$

The shear force is limited to a maximum of,

$$V_{\max} = 0.22 f_{cd} A_w = 496 \text{ kN}$$

The nominal shear strength provided by concrete is computed as:

$$V_{cr} = 0.65 f_{ctd} bd \left(1 + \frac{\gamma N_d}{A_g}\right) = 93.6 \text{ kN, where } N_d = 0$$

$$V_c = 0.8 V_{cr} = 74.9 \text{ kN}$$

The shear reinforcement is computed as follows:

If $V_d \le V_{cr}$

$$\left(\frac{A_{sw}}{s}\right)_{\min} = 0.3 \frac{f_{ctd}}{f_{ywd}} b = 0.2415 \frac{mm^2}{mm} \text{ (min. controls)} \qquad \text{(TS 8.1.5, Eqn 8.6)}$$

If $V_{cr} \le V_d \le V_{\max}$

$$\frac{A_{sw}}{s} = \frac{(V_d - V_c)}{f_{ywd} d} = 0.1962 \frac{mm^2}{mm} \qquad \text{(TS 8.1.4, Eqn 8.5)}$$

TS 500-2000 Example 002

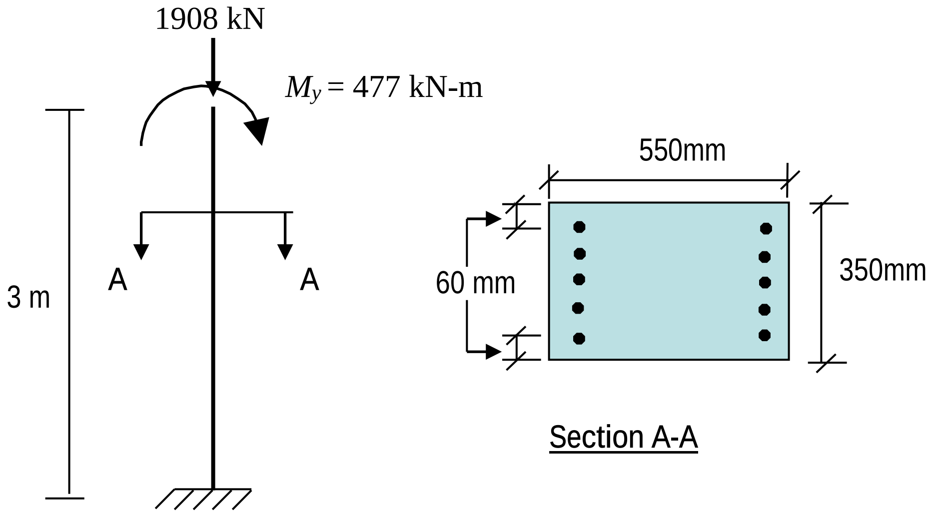
P-M INTERACTION CHECK FOR COMPRESSION-CONTROLLED RECTANGULAR COLUMN

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

A reinforced concrete column is subjected to factored axial load N = 1908 kN and moment M_y = 477 kN-m. This column is reinforced with five 25M bars. The design capacity ratio is checked by hand calculations and the result is compared with the computed result. The column is designed as a short, non-sway member.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E_c = 25x10^6 kN/m^2	b = 350 mm	f_{ck} = 25 MPa
v = 0.2	d = 550 mm	f_{yk} = 420 MPa
G = 10416666.7kN/m^2		

TECHNICAL FEATURES TESTED

- Tied reinforced concrete column design

RESULTS COMPARISON

Independent results are hand calculated and compared.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	0.992	1.00	0.80%

COMPUTER FILE: TS 500-2000 EX002

CONCLUSION

The computed result shows an acceptable comparison with the independent result.

HAND CALCULATION

Column Strength under compression control

f_{ck} = 25 MPa $\quad$ f_{yk} = 420 MPa
b = 350 mm $\quad$ d = 490 mm

1) Because e = 167.46 mm < (2/3)d = 326.67 mm, assume compression failure. This assumption will be checked later. Calculate the distance to the neutral axis for a balanced condition, c_b:

Position of neutral axis at balance condition:

$$c_b = \frac{0.003 \bullet 2\text{x}10^5}{0.003 \bullet 2\text{x}10^5 + f_{yk}} d_t = \frac{600}{600 + 420/1.15}(490) = 305 \text{ mm}$$

2) From the equation of equilibrium:

$$N = C_c + C_s - T$$

where

$$C_c = 0.85 f_{ck} ab = 0.85 \bullet 25/1.5 \bullet 350a = 4,958a$$

$$C_s = \frac{A'_s}{\gamma_s}\left(f_{yk} - 0.85\frac{f_{ck}}{\gamma_c}\right) = \frac{2500}{1.15}\left(420 - 0.85 \bullet 25/1.5\right) = 882,246 \text{ N}$$

Assume compression steels yields, (this assumption will be checked later).

$$T = \frac{A_s f_s}{\gamma_s} = \frac{2500 f_s}{1.15} = 2174 f_s \left(f_s < f_y\right)$$

$$N_1 = 4,958a + 882,246 - 2,174 f_s \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_s:

$$N = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_s\left(d - d'\right)\right]$$

The plastic centroid is at the center of the section and d'' = 215 mm

$$e' = e + d'' = 250 + 215 = 465 \text{ mm}$$

$$N = \frac{1}{465}\left[4,958a\left(490 - \frac{a}{2}\right) + 882,246\left(490 - 60\right)\right]$$

$$N_2 = 5525a - 5.3312a^2 + 815,840 \qquad \text{(Eqn. 2)}$$

4) Assume c = 358.3 mm, which exceed c_b (305 mm).

$$a = 0.85 \bullet 358 = 304.6 \text{ mm}$$

Substitute in Eqn. 2:

$$N_2 = 5525 \bullet 304.6 - 5.3312(304.6)^2 + 815,840 = 1,907,643 \text{ N}$$

5) Calculate f_s from the strain diagram when c = 359 mm.

$$f_s = \left(\frac{490 - 358.3}{358.3}\right)600 = 220.2 > 420 \text{ MPa}$$

$$\varepsilon_s = \varepsilon_t = f_s / E_s = 0.0011$$

6) Substitute a = 304.6m and f_s 221.2 MPa in Eqn. 1 to calculate N_1:

$$N_1 = 4,958(304.6) + 882,246 - 2174(220.2) = 1,907,601 \text{ N}$$

which is very close to the calculated N_2 of 2,002,751 (less than 1% difference)

$$M = Ne = 1908\left(\frac{250}{1000}\right) = 477 \text{ kN-m}$$

7) Check if compression steel yields. From strain diagram,

$$\varepsilon_s' = \left(\frac{358 - 60}{358}\right)(0.003) = 0.0025 > \varepsilon_y = 0.0021$$

Compression steel yields, as assumed.

8) Therefore, section capacity is

N = 1908 kN
M = 477 kN-m

EXAMPLE ACI 318-08 Wall-001

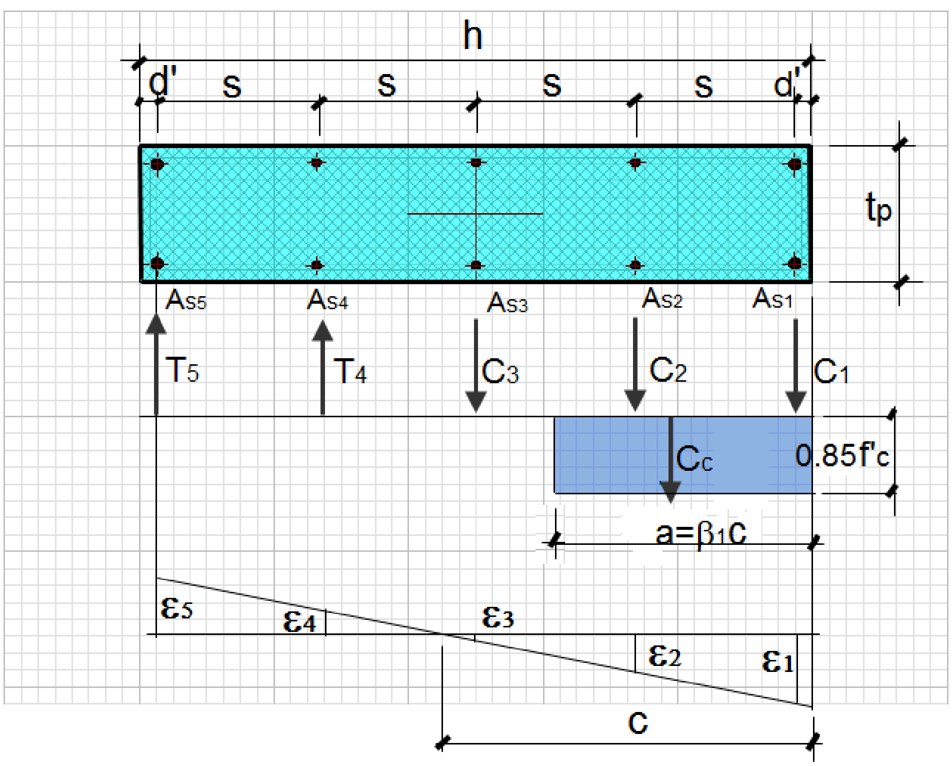
P-M INTERACTION CHECK FOR WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

A reinforced concrete wall is subjected to factored axial load P_u = 735 k and moments M_{uy} = 1504 k-ft. This wall is reinforced with two #9 bars at each end and #4 bars at 14.00 inches on center of each face. The total area of reinforcement is 5.20 in^2. The design capacity ratio is checked by hand calculations and results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties		Section Properties	Design Properties
E =	3600 k/in^2	tb = 12 in	f'_c = 4 k/in^2
v =	0.2	h = 60 in	f_y = 60 k/in^2
G =	1500 k/in^2	As1= As5 = 2-#9 (2.00 in^2)	
		As2, As3, As4 = 2-#4 (0.40 in^2)	

TECHNICAL FEATURES OF ETABS TESTED

Concrete wall flexural Demand/Capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.007	1.00	0.70%

COMPUTER FILE: ACI 318-08 WALL-001

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

1) A value of e = 24.58 inch was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model. The values of M_u and P_u were large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = 0.85 f_c' ab = 0.85 \bullet 4 \bullet 12a = 40.8a$$

$$C_s = A_1'\left(f_{s1} - 0.85 f_c'\right) + A_2'\left(f_{s2} - 0.85 f_c'\right) + A_3'\left(f_{s3} - 0.85 f_c'\right)$$

$$T = A_{s4} f_{s4} + A_{s5} f_{s5}$$

$$P_{n1} = 40.8a + A_1'\left(f_{s1} - 0.85 f_C'\right) + A_2'\left(f_{s2} - 0.85 f_c'\right) A_1'\left(f_{s1} - 0.85 f_c'\right) + A_3'\left(f_{s3} - 0.85 f_c'\right) - A_{s4} f_{s4} - A_{s5} f_{s5} \quad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(3s\right) - C_{s3}\left(2s\right) - T_{s4}\left(s\right)\right] \quad \text{(Eqn. 2)}$$

where $C_{s1} = A_1'\left(f_{s1} - 0.85 f_c'\right)$; $C_{sn} = A_n'\left(f_{sn} - 0.85 f_c'\right)$; $T_{sn} = f_{sn} A_{sn}$; and the bar strains are determined below. The plastic centroid is at the center of the section and d'' = 28 inch

$$e' = e + d'' = 24.54 + 28 = 52.55 \text{ inch.}$$

4) Using c = 30.1 inch (from iteration),

$$a = 0.85 \bullet 30.1 = 25.58 \text{ inch}$$

5) Assuming the extreme fiber strain equals 0.003 and c= 30.1 inch, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.003 \quad = 0.0028; \; f_s = \varepsilon_s E \le F_y; \; f_{s1} = 60.00 \text{ ksi}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.003 = 0.0014 \qquad f_{s2} = 40.75 \text{ ksi}$$

$$\varepsilon_{s3} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s5} \quad = 0.0000 \qquad f_{s3} = 00.29 \text{ ksi}$$

$$\varepsilon_{s4} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s5} \quad = 0.0014 \qquad f_{s4} = 40.20 \text{ ksi}$$

$$\varepsilon_{s5} = \left(\frac{d-c}{c}\right)0.003 \quad = 0.0028 \qquad f_{s5} = 60.00 \text{ ksi}$$

Substitute in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal gives,

$$P_{n1} = 1035 \text{ k}$$
$$P_{n2} = 1035 \text{ k}$$

$$M_n = P_n e = 1035(24.54)/12 = 2116 \text{ k-ft}$$

6) Determine if ϕ is tension controlled or compression controlled.

$$\varepsilon_t = 0.00244, \; \varepsilon_y = 0.0021$$

$$\text{for } \varepsilon_y < \varepsilon_t < 0.005; \; \phi = (\phi_t - \phi_c)\left(\frac{0.005 - \varepsilon_t}{0.005 - \varepsilon_y}\right) = 0.712$$

7) Calculate ϕ,

$$\phi P_n = 0.711(1035) = 735 \text{ kips}$$
$$\phi M_n = 0.711(2115) = 1504 \text{ k-ft.}$$

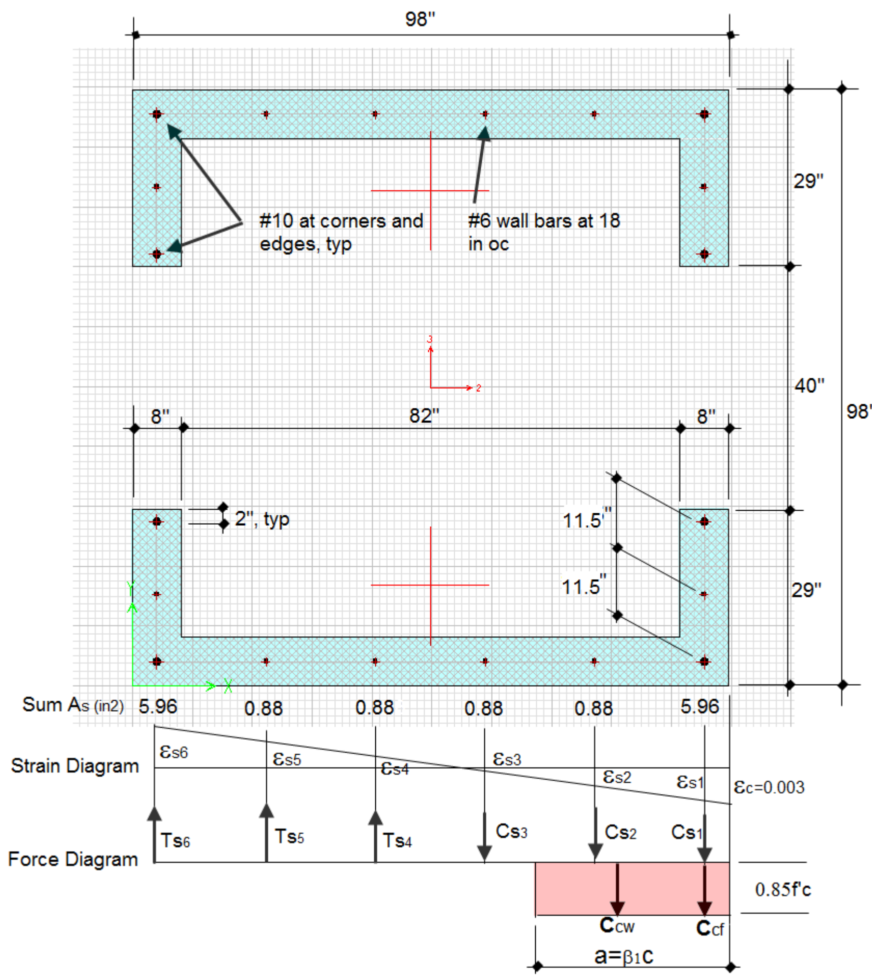
EXAMPLE ACI 318-08 Wall-002

P-M INTERACTION CHECK FOR WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example. The wall is reinforced as shown below. The concrete core wall is loaded with a factored axial load P_u = 2384 k and moments M_{u3} = 9293k-ft. The design capacity ratio is checked by hand calculations and results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties		Section Properties	Design Properties
E =	3600 k/in^2	tb = 8 in	f'_c = 4 k/in^2
v =	0.2	h = 98 in	f_y = 60 k/in^2
G =	1500 k/in^2	As1= As6 = 2-#10,2#6 (5.96 in^2)	
		As2, As3, As4 and As5 = 2-#6 (0.88 in^2)	

TECHNICAL FEATURES OF ETABS TESTED

- Concrete wall flexural Demand/Capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	0.999	1.00	0.10%

COMPUTER FILE: ACI 318-08 WALL-002

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength under compression and bending

1) A value of e = 46.78 inches was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model interaction diagram. The values of M_u and P_u were large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_{n1} = C_c + C_s - T$$

where

$C_c = C_{cw} + C_{cf}$, where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = 0.85 f'_c \bullet 8 \bullet (a - 8)$$

$$C_{cf} = 0.85 f'_c \left(8 \bullet (98 - 40)\right)$$

$$C_s = A'_1 \left(f_{s1} - 0.85 f'_c \right) + A'_2 \left(f_{s2} - 0.85 f'_c \right) + A'_3 \left(f_{s3} - 0.85 f'_c \right)$$

$$T = A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6}$$

$$P_{n1} = 0.85 f'_c \bullet 8 \bullet (a - 8) + 0.85 f'_c \left(8 \bullet (98 - 40)\right) + A'_1 \left(f_{s1} - 0.85 f'_c \right) + A'_2 \left(f_{s2} - 0.85 f'_c \right) + A'_3 \left(f_{s3} - 0.85 f'_c \right) + A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6} \quad \text{(Eqn. 1)}$$

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'} \left[\begin{array}{l} C_{cf} \left(d - d' \right) + C_{cw} \left(d - \frac{a - t_f}{2} - t_f \right) + C_{s1} \left(d - d' \right) + \\ C_{s2} \left(4s \right) + C_{s3} \left(3s \right) - T_{s4} \left(2s \right) - T_{s5} \left(s \right) \end{array} \right] \quad \text{(Eqn. 2)}$$

where $C_{s1} = A_1'\left(f_{s1} - 0.85 f_c'\right)$, $C_{sn} = A_n'\left(f_{sn} - 0.85 f_c'\right)$, $T_{sn} = f_{sn} A_{sn}$, and the bar strains are determined below. The plastic centroid is at the center of the section, $d'' = \dfrac{98-8}{2}$ = 45 inches

$$e' = e + d'' = 46.78 + 45 = 91.78 \text{ inches}$$

4) Iterating on a value of *c* until equations 1 and 2 are equal *c* is found to be *c* = 44.58 inches.

$$a = 0.85 \bullet c = 0.85 \bullet 44.58 = 37.89 \text{ inches}$$

5) Assuming the extreme fiber strain equals 0.003 and *c* = 44.58 inches, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.003 = 0.00273;\; f_s = \varepsilon_s E \le F_y;\; f_{s1} = 60.00 \text{ ksi}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.003 = 0.00152 \qquad f_{s2} = 44.07 \text{ ksi}$$

$$\varepsilon_{s3} = \left(\frac{c-2s-d'}{c}\right)0.003 = 0.00310 \qquad f_{s3} = 8.94 \text{ ksi}$$

$$\varepsilon_{s4} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s6} = 0.00090 \qquad f_{s4} = 26.2 \text{ ksi}$$

$$\varepsilon_{s5} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s6} = 0.00211 \qquad f_{s5} = 60.00 \text{ ksi}$$

$$\varepsilon_{s6} = \left(\frac{d-c}{c}\right)0.003 = 0.00333 \qquad f_{s6} = 60.00 \text{ ksi}$$

Substituting the above values of the compression block depth, *a*, and the rebar stresses into equations Eqn. 1 and Eqn. 2 give

$$P_{n1} = 3148 \text{ k}$$
$$P_{n2} = 3148 \text{ k}$$

$$M_n = P_n e = 3148(46.78)/12 = 12{,}273 \text{ k-ft}$$

6) Determine if ϕ is tension controlled or compression controlled.

$$\varepsilon_t = 0.00332\,,\ \varepsilon_y = 0.0021$$

$$\text{for } \varepsilon_y < \varepsilon_t < 0.005\,;\ \phi = \left(\phi_t - \phi_c\right)\left(\frac{0.005 - \varepsilon_t}{0.005 - \varepsilon_y}\right) = 0.757$$

7) Calculate the capacity,

$$\phi P_n = 0.757\left(3148\right) = 2384 \text{ kips}$$

$$\phi M_n = 0.757\left(12,273\right) = 9293 \text{ k-ft.}$$

EXAMPLE ACI 318-11 Wall-001

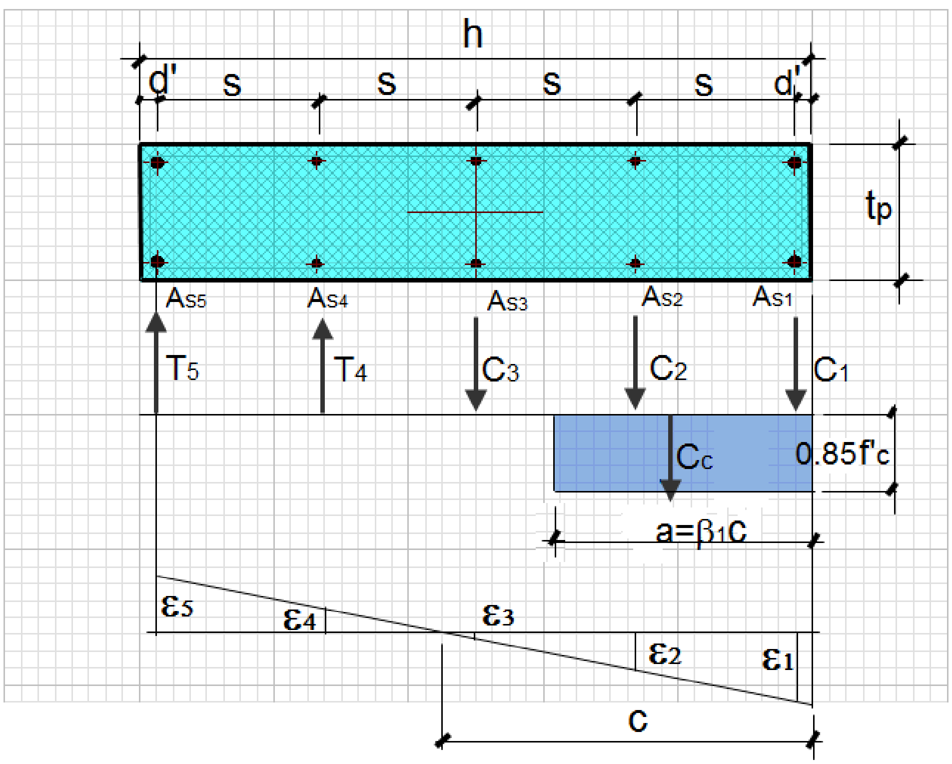
P-M INTERACTION CHECK FOR WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

A reinforced concrete wall is subjected to factored axial load P_u = 735 k and moments M_{uy} = 1,504 k-ft. This wall is reinforced with two #9 bars at each end and #4 bars at 14.00 inches on center of each face. The total area of reinforcement is 5.20 in^2. The design capacity ratio is checked by hand calculations and results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 3600 k/in^2
v = 0.2
G = 1500 k/in^2

Section Properties

tb = 12 in
h = 60 in
As1= As5 = 2-#9 (2.00 in^2)
As2, As3, As4 = 2-#4 (0.40 in^2)

Design Properties

f'_c = 4 k/in^2
f_y = 60 k/in^2

TECHNICAL FEATURES OF ETABS TESTED

- Concrete wall flexural Demand/Capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.007	1.00	0.70%

COMPUTER FILE: ACI 318-11 WALL-001

CONCLUSION

The ETABS results show an acceptable match with the independent results.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

1) A value of e = 24.58 inch was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model. The values of M_u and P_u were large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = 0.85 f_c' ab = 0.85 \bullet 4 \bullet 12a = 40.8a$$

$$C_s = A_1'\left(f_{s1} - 0.85 f_c'\right) + A_2'\left(f_{s2} - 0.85 f_c'\right) + A_3'\left(f_{s3} - 0.85 f_c'\right)$$

$$T = A_{s4} f_{s4} + A_{s5} f_{s5}$$

$$P_{n1} = 40.8a + A_1'\left(f_{s1} - 0.85 f_c'\right) + A_2'\left(f_{s2} - 0.85 f_c'\right) A_1'\left(f_{s1} - 0.85 f_c'\right) + A_3'\left(f_{s3} - 0.85 f_c'\right) - A_{s4} f_{s4} - A_{s5} f_{s5} \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(3s\right) - C_{s3}\left(2s\right) - T_{s4}\left(s\right)\right] \qquad \text{(Eqn. 2)}$$

where $C_{s1} = A'\left(f_{s1} - 0.85 f_c'\right)$; $C_{sn} = A_n'\left(f_{sn} - 0.85 f_c'\right)$; $T_{sn} = f_{sn} A_{sn}$; and the bar strains are determined below. The plastic centroid is at the center of the section and d'' = 28 inch

$$e' = e + d'' = 24.54 + 28 = 52.55 \text{ inch.}$$

4) Using c = 30.1 inch (from iteration),

$$a = 0.85 \bullet 30.1 = 25.58 \text{ inches}$$

5) Assuming the extreme fiber strain equals 0.003 and c= 30.1 inch, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.003 \quad = 0.0028; \; f_s = \varepsilon_s E \le F_y \; ; \; f_{s1} = \; 60.00 \text{ ksi}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.003 \quad = 0.0014 \qquad f_{s2} = \; 40.75 \text{ ksi}$$

$$\varepsilon_{s3} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s5} \quad = 0.0000 \qquad f_{s3} = \; 00.29 \text{ ksi}$$

$$\varepsilon_{s4} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s5} \quad = 0.0014 \qquad f_{s4} = \; 40.20 \text{ ksi}$$

$$\varepsilon_{s5} = \left(\frac{d-c}{c}\right)87 \quad = 0.0028 \qquad f_{s5} = \; 60.00 \text{ ksi}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

P_{n1} = 1035 k

P_{n2} = 1035 k

$$M_n = P_n e = 1035(24.54)/12 = 2116 \text{ k-ft}$$

6) Determine if ϕ is tension controlled or compression controlled.

$$\varepsilon_t = 0.00244, \; \varepsilon_y = 0.0021$$

$$\text{for } \varepsilon_y < \varepsilon_t < 0.005; \; \phi = (\phi_t - \phi_c)\left(\frac{0.005-\varepsilon_t}{0.005-\varepsilon_y}\right) = 0.712$$

7) Calculate ϕ,

$$\phi P_n = 0.711(1035) = 735 \text{ kips}$$

$$\phi M_n = 0.711(2115) = 1504 \text{ k-ft.}$$

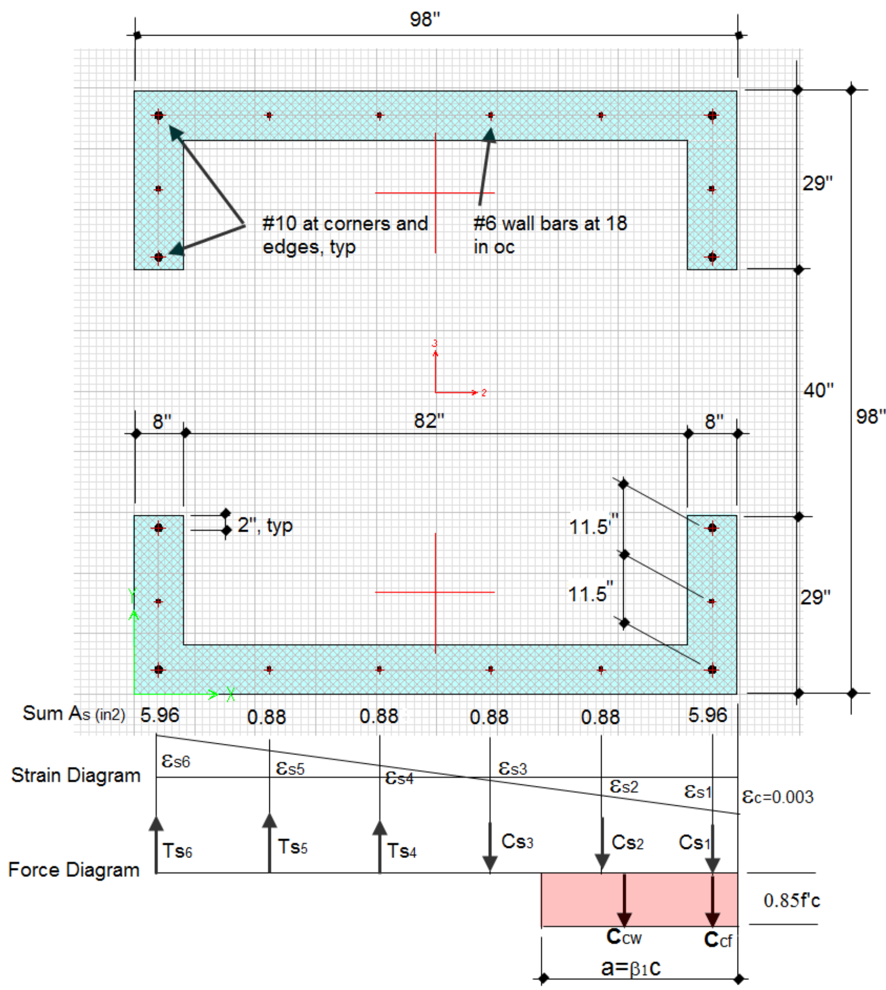
EXAMPLE ACI 318-11 Wall-002

P-M INTERACTION CHECK FOR WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example. The wall is reinforced as shown below. The concrete core wall is loaded with a factored axial load P_u = 2384 k and moments M_{u3} = 9293k-ft. The design capacity ratio is checked by hand calculations and results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E = 3600 k/in^2	tb = 8 in	f'_c = 4 k/in^2
v = 0.2	h = 98 in	f_y = 60 k/in^2
G = 1500 k/in^2	As1= As6 = 2-#10,2#6 (5.96 in^2)	
	As2, As3, As4 and As5 = 2-#6 (0.88 in^2)	

TECHNICAL FEATURES OF ETABS TESTED

- Concrete wall flexural Demand/Capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	0.999	1.00	0.10%

COMPUTER FILE: ACI 318-11 WALL-002

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength under compression and bending

1) A value of *e* = 46.78 inches was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model interaction diagram. The values of M_u and P_u were large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, *c*, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_{n1} = C_c + C_s - T$$

where

$C_c = C_{cw} + C_{cf}$ **,** where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = 0.85 f_c' \bullet 8 \bullet (a-8)$$

$$C_{cf} = 0.85 f_c' \left(8 \bullet (98-40)\right)$$

$$C_s = A_1'\left(f_{s1} - 0.85 f_c'\right) + A_2'\left(f_{s2} - 0.85 f_c'\right) + A_3'\left(f_{s3} - 0.85 f_c'\right)$$

$$T = A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6}$$

$$P_{n1} = 0.85 f_c' \bullet 8 \bullet (a-8) + 0.85 f_c'\left(8 \bullet (98-40)\right) + A_1'\left(f_{s1} - 0.85 f_c'\right) + A_2'\left(f_{s2} - 0.85 f_c'\right) + A_3'\left(f_{s3} - 0.85 f_c'\right) + A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6} \quad \text{(Eqn. 1)}$$

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'}\left[\begin{array}{l} C_{cf}\left(d-d'\right) + C_{cw}\left(d - \dfrac{a-t_f}{2} - t_f\right) + C_{s1}\left(d-d'\right) + \\ C_{s2}(4s) + C_{s3}(3s) - T_{s4}(2s) - T_{s5}(s) \end{array}\right] \quad \text{(Eqn. 2)}$$

where $C_{s1} = A_1'\left(f_{s1} - 0.85 f_c'\right)$, $C_{sn} = A_n'\left(f_{sn} - 0.85 f_c'\right)$, $T_{sn} = f_{sn} A_{sn}$, and the bar strains are determined below. The plastic centroid is at the center of the section, $d'' = \dfrac{98-8}{2}$ = 45 inches

$$e' = e + d'' = 46.78 + 45 = 91.78 \text{ inches}$$

4) Iterating on a value of c until equations 1 and 2 are equal *c* is found to be *c* = 44.58 inches.

$a = 0.85 \bullet c = 0.85 \bullet 44.58 = 37.89$ inches

5) Assuming the extreme fiber strain equals 0.003 and *c* = 44.58 inches, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain then, $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.003 = 0.00273; \; f_s = \varepsilon_s E \le F_y \; ; \; f_{s1} = 60.00 \text{ ksi}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.003 = 0.00152 \qquad f_{s2} = 44.07 \text{ ksi}$$

$$\varepsilon_{s3} = \left(\frac{c-2s-d'}{c}\right)0.003 = 0.00310 \qquad f_{s3} = 8.94 \text{ ksi}$$

$$\varepsilon_{s4} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s6} = 0.00090 \qquad f_{s4} = 26.2 \text{ ksi}$$

$$\varepsilon_{s5} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s6} = 0.00211 \qquad f_{s5} = 60.00 \text{ ksi}$$

$$\varepsilon_{s6} = \left(\frac{d-c}{c}\right)0.003 = 0.00333 \qquad f_{s6} = 60.00 \text{ ksi}$$

Substituting the above values of the compression block depth, a, and the rebar stresses into equations Eqn. 1 and Eqn. 2 give

$P_{n1} = 3148$ k
$P_{n2} = 3148$ k

$$M_n = P_n e = 3148(46.78)/12 = 12{,}273 \text{ k-ft}$$

6) Determine if ϕ is tension controlled or compression controlled.

$\varepsilon_t = 0.00332$, $\varepsilon_y = 0.0021$

for $\varepsilon_y < \varepsilon_t < 0.005$; $\phi = (\phi_t - \phi_c)\left(\frac{0.005 - \varepsilon_t}{0.005 - \varepsilon_y}\right) = 0.757$

7) Calculate the capacity,

$$\phi P_n = 0.757(3148) = 2384 \text{ kips}$$
$$\phi M_n = 0.757(12,273) = 9,293 \text{ k-ft.}$$

EXAMPLE ACI 318-14 Wall-001

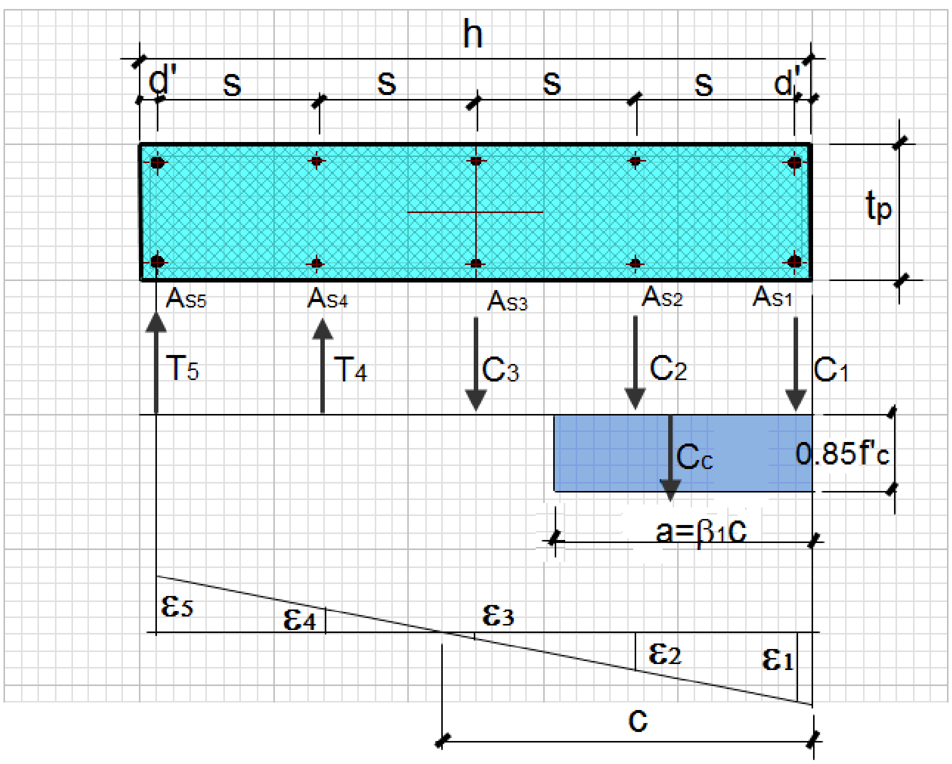
P-M INTERACTION CHECK FOR WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example.

A reinforced concrete wall is subjected to factored axial load P_u = 735 k and moments M_{uy} = 1,504 k-ft. This wall is reinforced with two #9 bars at each end and #4 bars at 14.00 inches on center of each face. The total area of reinforcement is 5.20 in^2. The design capacity ratio is checked by hand calculations and results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 3600 k/in²
v = 0.2
G = 1500 k/in²

Section Properties

tb = 12 in
h = 60 in
As1= As5 = 2-#9 (2.00 in^2)
As2, As3, As4 = 2-#4 (0.40 in^2)

Design Properties

f'_c = 4 k/in²
f_y = 60 k/in²

TECHNICAL FEATURES OF ETABS TESTED

- Concrete wall flexural Demand/Capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.007	1.00	0.70%

COMPUTER FILE: ACI 318-14 WALL-001

CONCLUSION

The ETABS results show an acceptable match with the independent results.

HAND CALCULATION

COLUMN STRENGTH UNDER COMPRESSION CONTROL

1) A value of e = 24.58 inch was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model. The values of M_u and P_u were large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = 0.85 f_c' ab = 0.85 \bullet 4 \bullet 12a = 40.8a$$

$$C_s = A_1'\left(f_{s1} - 0.85 f_c'\right) + A_2'\left(f_{s2} - 0.85 f_c'\right) + A_3'\left(f_{s3} - 0.85 f_c'\right)$$

$$T = A_{s4} f_{s4} + A_{s5} f_{s5}$$

$$P_{n1} = 40.8a + A_1'\left(f_{s1} - 0.85 f_c'\right) + A_2'\left(f_{s2} - 0.85 f_c'\right) A_1'\left(f_{s1} - 0.85 f_c'\right) + A_3'\left(f_{s3} - 0.85 f_c'\right) - A_{s4} f_{s4} - A_{s5} f_{s5} \quad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(3s\right) - C_{s3}\left(2s\right) - T_{s4}\left(s\right)\right] \quad \text{(Eqn. 2)}$$

where $C_{s1} = A'\left(f_{s1} - 0.85 f_c'\right)$; $C_{sn} = A_n'\left(f_{sn} - 0.85 f_c'\right)$; $T_{sn} = f_{sn} A_{sn}$; and the bar strains are determined below. The plastic centroid is at the center of the section and d'' = 28 inch

$$e' = e + d'' = 24.54 + 28 = 52.55 \text{ inch.}$$

4) Using c = 30.1 inch (from iteration),

$$a = 0.85 \bullet 30.1 = 25.58 \text{ inches}$$

5) Assuming the extreme fiber strain equals 0.003 and c= 30.1 inch, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.003 \quad = 0.0028;\ f_s = \varepsilon_s E \le F_y \ ; \ f_{s1} = 60.00 \text{ ksi}$

$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.003 \quad = 0.0014 \qquad f_{s2} = 40.75 \text{ ksi}$

$\varepsilon_{s3} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s5} \quad = 0.0000 \qquad f_{s3} = 00.29 \text{ ksi}$

$\varepsilon_{s4} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s5} \quad = 0.0014 \qquad f_{s4} = 40.20 \text{ ksi}$

$\varepsilon_{s5} = \left(\frac{d-c}{c}\right)87 \quad = 0.0028 \qquad f_{s5} = 60.00 \text{ ksi}$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$P_{n1} = 1035$ k
$P_{n2} = 1035$ k

$M_n = P_n e = 1035(24.54)/12 = 2116$ k-ft

6) Determine if ϕ is tension controlled or compression controlled.

$\varepsilon_t = 0.00244$, $\varepsilon_y = 0.0021$

for $\varepsilon_y < \varepsilon_t < 0.005$; $\phi = (\phi_t - \phi_c)\left(\frac{0.005-\varepsilon_t}{0.005-\varepsilon_y}\right) = 0.712$

7) Calculate ϕ,

$\phi P_n = 0.711(1035) = 735$ kips
$\phi M_n = 0.711(2115) = 1504$ k-ft.

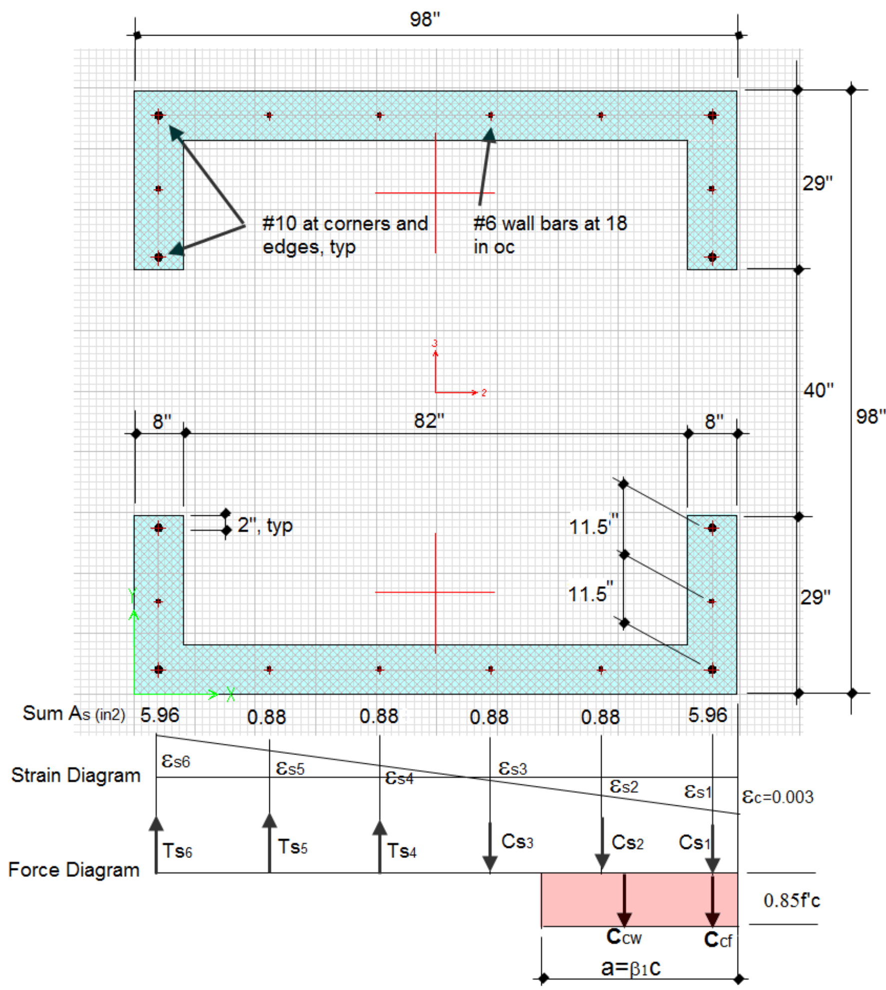
EXAMPLE ACI 318-14 Wall-002

P-M INTERACTION CHECK FOR WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example. The wall is reinforced as shown below. The concrete core wall is loaded with a factored axial load P_u = 2384 k and moments M_{u3} = 9293k-ft. The design capacity ratio is checked by hand calculations and results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties		Section Properties	Design Properties
E =	3600 k/in^2	tb = 8 in	f'_c = 4 k/in^2
v =	0.2	h = 98 in	f_y = 60 k/in^2
G =	1500 k/in^2	As1= As6 = 2-#10,2#6 (5.96 in^2)	
		As2, As3, As4 and As5 = 2-#6 (0.88 in^2)	

TECHNICAL FEATURES OF ETABS TESTED

- Concrete wall flexural Demand/Capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	0.999	1.00	0.10%

COMPUTER FILE: ACI 318-14 WALL-002

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength under compression and bending

1) A value of *e* = 46.78 inches was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model interaction diagram. The values of M_u and P_u were large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, *c*, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_{n1} = C_c + C_s - T$$

where

$C_c = C_{cw} + C_{cf}$, where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = 0.85 f_c' \bullet 8 \bullet (a-8)$$

$$C_{cf} = 0.85 f_c' \left(8 \bullet (98-40)\right)$$

$$C_s = A_1'\left(f_{s1} - 0.85 f_c'\right) + A_2'\left(f_{s2} - 0.85 f_c'\right) + A_3'\left(f_{s3} - 0.85 f_c'\right)$$

$$T = A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6}$$

$$P_{n1} = 0.85 f_c' \bullet 8 \bullet (a-8) + 0.85 f_c' \left(8 \bullet (98-40)\right) + A_1'\left(f_{s1} - 0.85 f_c'\right) + A_2'\left(f_{s2} - 0.85 f_c'\right) + A_3'\left(f_{s3} - 0.85 f_c'\right) + A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6} \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'}\left[\begin{array}{l} C_{cf}\left(d-d'\right) + C_{cw}\left(d - \dfrac{a-t_f}{2} - t_f\right) + C_{s1}\left(d-d'\right) + \\ C_{s2}(4s) + C_{s3}(3s) - T_{s4}(2s) - T_{s5}(s) \end{array}\right] \qquad \text{(Eqn. 2)}$$

where $C_{s1} = A_1'\left(f_{s1} - 0.85 f_c'\right)$, $C_{sn} = A_n'\left(f_{sn} - 0.85 f_c'\right)$, $T_{sn} = f_{sn} A_{sn}$, and the bar strains are determined below. The plastic centroid is at the center of the section, $d'' = \dfrac{98-8}{2}$ = 45 inches

$$e' = e + d'' = 46.78 + 45 = 91.78 \text{ inches}$$

4) Iterating on a value of c until equations 1 and 2 are equal c is found to be c = 44.58 inches.

$$a = 0.85 \bullet \text{c} = 0.85 \bullet 44.58 = 37.89 \text{ inches}$$

5) Assuming the extreme fiber strain equals 0.003 and c = 44.58 inches, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain then, $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.003 = 0.00273; \; f_s = \varepsilon_s E \le F_y \; ; \; f_{s1} = 60.00 \text{ ksi}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.003 = 0.00152 \qquad f_{s2} = 44.07 \text{ ksi}$$

$$\varepsilon_{s3} = \left(\frac{c-2s-d'}{c}\right)0.003 = 0.00310 \qquad f_{s3} = 8.94 \text{ ksi}$$

$$\varepsilon_{s4} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s6} = 0.00090 \qquad f_{s4} = 26.2 \text{ ksi}$$

$$\varepsilon_{s5} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s6} = 0.00211 \qquad f_{s5} = 60.00 \text{ ksi}$$

$$\varepsilon_{s6} = \left(\frac{d-c}{c}\right)0.003 = 0.00333 \qquad f_{s6} = 60.00 \text{ ksi}$$

Substituting the above values of the compression block depth, a, and the rebar stresses into equations Eqn. 1 and Eqn. 2 give

$$P_{n1} = 3148 \text{ k}$$
$$P_{n2} = 3148 \text{ k}$$

$$M_n = P_n e = 3148(46.78)/12 = 12{,}273 \text{ k-ft}$$

6) Determine if ϕ is tension controlled or compression controlled.

$$\varepsilon_t = 0.00332, \; \varepsilon_y = 0.0021$$

$$\text{for } \varepsilon_y < \varepsilon_t < 0.005; \; \phi = \left(\phi_t - \phi_c\right)\left(\frac{0.005-\varepsilon_t}{0.005-\varepsilon_y}\right) = 0.757$$

7) Calculate the capacity,

$$\phi P_n = 0.757(3148) = 2384 \text{ kips}$$
$$\phi M_n = 0.757(12,273) = 9,293 \text{ k-ft.}$$

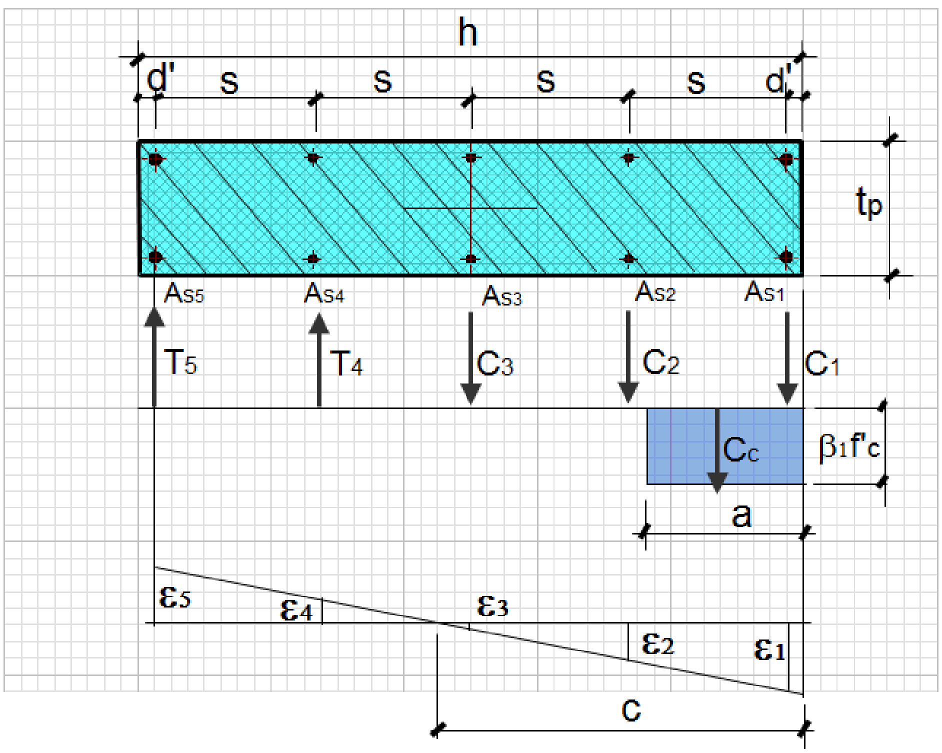
EXAMPLE ACI 530-11 Masonry Wall-001

P-M INTERACTION CHECK FOR WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example. A reinforced masonry wall is subjected to factored axial load P_u = 556 k and moments M_{uy} = 1331 k-ft. This wall is reinforced with two #9 bars at each end and #4 bars at 14.00 inches on center each of face module (The reinforcing is not aligned with the conventional masonry block spacing for calculation convenience. The same excel spreadsheet used in other concrete examples was used here). The total area of reinforcement is 5.20 in^2. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 2250 k/in^2
v = 0.2
G = 750 k/in^2

Section Properties

tb = 12 in
h = 60 in
As1= As5 = 2-#9 (2.00 in^2)
As2, As3, As4 = 2-#4 (0.40 in^2)

Design Properties

$f'm$ = 2.5 k/in^2
f_y = 60 k/in^2

TECHNICAL FEATURES OF ETABS TESTED

- Wall flexural Demand/Capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	0.939	1.00	-6.1%

COMPUTER FILE: ACI 530-11 MASONRY WALL-001

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Column Strength under compression control

1) A value of e = 28.722 inches was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model. The values of M_u and P_u were large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = \beta_1 f'_m ab = 0.8 \bullet 2.5 \bullet 12a = 24.0a$$
$$C_s = A'_1\left(f_{s1} - 0.8f'_m\right) + A'_2\left(f_{s2} - 0.8f'_m\right) + A'_3\left(f_{s3} - 0.8f'_m\right)$$
$$T = A_{s4} f_{s4} + A_{s5} f_{s5}$$
$$P_{n1} = 24a + A'_1\left(f_{s1} - 0.8f'_m\right) + A'_2\left(f_{s2} - 0.8f'_m\right) + A'_3\left(f_{s3} - 0.8f'_m\right) - A_{s4} f_{s4} - A_{s5} f_{s5}$$

(Eqn. 1)

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(3s\right) - T_{s3}\left(2s\right) - T_{s4}\left(s\right)\right]$$

(Eqn. 2)

where $C_{s1} = A'_1\left(f_{s1} - 0.8f'_m\right)$; $C_{sn} = A'_n\left(f_{sn} - 0.8f'_m\right)$; $T_{sn} = f_{sn} A_{sn}$; and the bar strains are determined below. The plastic centroid is at the center of the section and d'' = 28 inch

$$e' = e + d'' = 28.722 + 28 = 56.72 \text{ inch.}$$

4) Using c = 32.04 inch (from iteration),

$$a = 0.80 \bullet 332.04 = 25.64 \text{ inch}$$

5) Assuming the extreme fiber strain equals 0.0025 and c= 32.04 inch, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain then, $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c - d'}{c}\right)0.0025 = 0.00207;\ f_s = \varepsilon_s E \le F_y\ ;\ f_{s1} = 60.00 \text{ ksi}$$

$$\varepsilon_{s2} = \left(\frac{c - s - d'}{c}\right)0.0025 = 0.00125 \qquad f_{s2} = 36.30 \text{ ksi}$$

$$\varepsilon_{s3} = \left(\frac{c - 2s - d'}{c}\right)0.0025 = 0.00016 \qquad f_{s3} = 4.62 \text{ ksi}$$

$$\varepsilon_{s4} = \left(\frac{d - c - s}{c}\right)0.0025 = 0.00093 \qquad f_{s4} = 27.10 \text{ ksi}$$

$$\varepsilon_{s5} = \left(\frac{d - c}{c}\right)0.0025 = 0.00203 \qquad f_{s5} = 58.70 \text{ ksi}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$P_{n1} = 618$ k;
$P_{n2} = 618$ k

$$M_n = P_n e = 618(28.72)/12 = 1479 \text{ k-ft}$$

6) Calculate ϕ,

$$\phi P_n = 0.9(618) = 556 \text{ kips}$$
$$\phi M_n = 0.9(1479) = 1331 \text{ k-ft.}$$

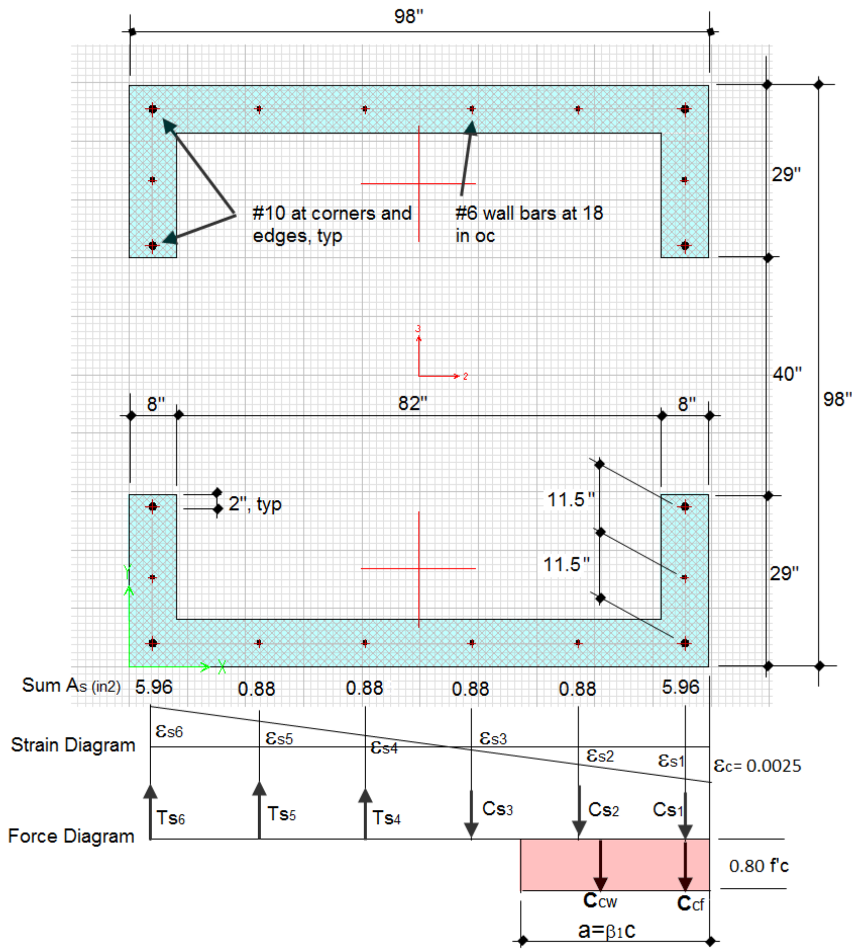
EXAMPLE ACI 530-11 Masonry Wall-002

P-M INTERACTION CHECK FOR WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example. The wall is reinforced as shown below. The concrete core wall is loaded with a factored axial load P_u = 1496 k and moments M_{u3} = 7387 k-ft. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties		Section Properties	Design Properties
E =	3600 k/in^2	tb = 8 in	f'_c = 4 k/in^2
v =	0.2	h = 98 in	f_y = 60 k/in^2
G =	1500 k/in^2	As1= As6 = 2-#10,2#6 (5.96 in^2)	
		As2, As3, As4 and As5 = 2-#6 (0.88 in^2)	

TECHNICAL FEATURES OF ETABS TESTED

- Concrete wall flexural Demand/Capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Column Demand/Capacity Ratio	0.998	1.00	-0.20%

COMPUTER FILE: ACI 530-11 MASONRY WALL-002

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength under compression and bending

1) A value of *e* = 59.24 inches was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model interaction diagram. The values of M_u and P_u were large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, *c*, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_{n1} = C_c + C_s - T$$

where

$$C_c = \beta_1 f'_m ab = 0.8 \bullet 2.5 \bullet 12a = 24.0a$$

$$C_s = A'_1\left(f_{s1} - 0.8 f'_m\right) + A'_2\left(f_{s2} - 0.8 f'_m\right) + A'_3\left(f_{s3} - 0.8 f'_m\right)$$

$$T = A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6}$$

$$\begin{aligned} P_{n1} = {} & 24a + A'_1\left(f_{s1} - 0.8 f'_m\right) + A'_2\left(f_{s2} - 0.8 f'_m\right) + \\ & A'_3\left(f_{s3} - 0.8 f'_m\right) - A_{s4} f_{s4} - A_{s5} f_{s5} - A_{s6} f_{s6} \end{aligned} \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'}\left[\begin{array}{l} C_{cf}\left(d - d'\right) + C_{cw}\left(d - \dfrac{a - t_f}{2}\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(4s\right) + \\ C_{s3}\left(3s\right) - T_{s4}\left(2s\right) - T_{s5}\left(s\right) \end{array}\right] \qquad \text{(Eqn. 2)}$$

where $C_{s1} = A'_1\left(f_{s1} - 0.8 f'_m\right)$; $C_{sn} = A'_n\left(f_{sn} - 0.8 f'_m\right)$; $T_{sn} = f_{sn} A_{sn}$; and the bar strains are determined below. The plastic centroid is at the center of the section and d'' = 45 inch

$$e' = e + d'' = 59.24 + 45 = 104.24 \text{ inch.}$$

4) Iterating on a value of *c* until equations 1 and 2 are equal *c* is found to be *c* = 41.15 inches.

$$a = 0.8 \bullet c = 0.8 \bullet 41.15 = 32.92 \text{ inches}$$

5) Assuming the extreme fiber strain equals 0.0025 and *c* = 41.15 inches, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.0025 = 0.00226; \; f_s = \varepsilon_s E \le F_y \; ; \; f_{s1} = 60.00 \text{ ksi}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.0025 = 0.00116 \qquad f_{s2} = 33.74 \text{ ksi}$$

$$\varepsilon_{s3} = \left(\frac{c-2s-d'}{c}\right)0.0025 = 0.00007 \qquad f_{s3} = 2.03 \text{ ksi}$$

$$\varepsilon_{s4} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s6} = 0.00102 \qquad f_{s4} = 29.7 \text{ ksi}$$

$$\varepsilon_{s5} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s6} = 0.00212 \qquad f_{s5} = 60.00 \text{ ksi}$$

$$\varepsilon_{s6} = \left(\frac{d-c}{c}\right)0.0025 = 0.00321 \qquad f_{s6} = 60.00 \text{ ksi}$$

Substituting the above values of the compression block depth, *a*, and the rebar stresses into equations Eqn. 1 and Eqn. 2 give

$$P_{n1} = 1662 \text{ k}$$
$$P_{n2} = 1662 \text{ k}$$

$$M_n = P_n e = 1662(41.15)/12 = 8208 \text{ k-ft}$$

6) Calculate the capacity,

$$\phi P_n = 0.9(1622) = 1496 \text{ kips}$$
$$\phi M_n = 0.9(8208) = 7387 \text{ k-ft.}$$

EXAMPLE AS 3600-09 Wall-001

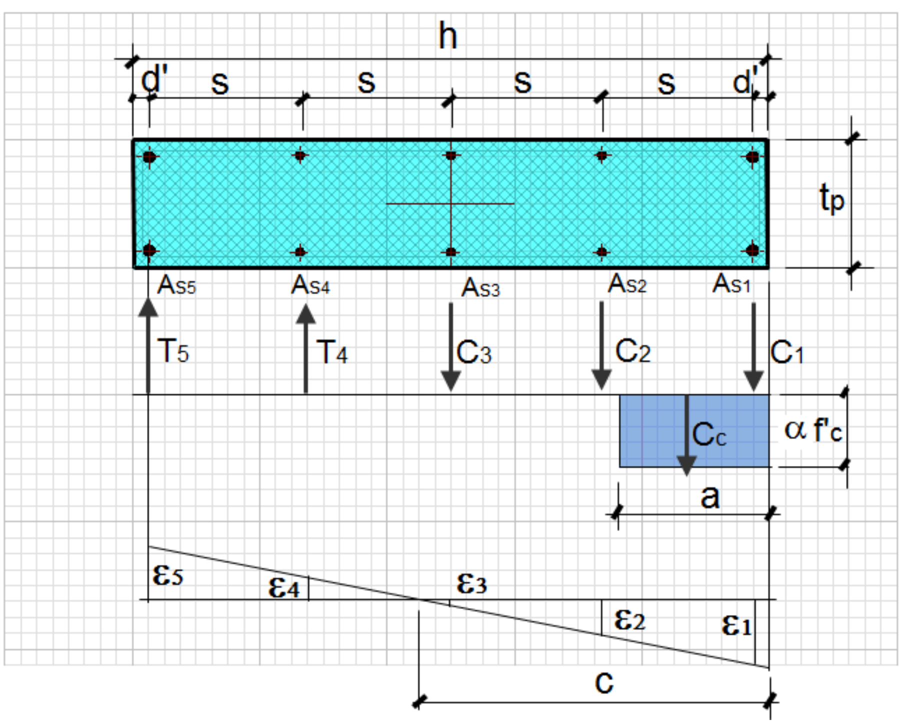
P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example.

A reinforced concrete wall is subjected to factored axial load P_u = 3438 kN and moments M_{uy} = 2003 kN-m. This wall is reinforced with two 30M bars at each end and 15M bars at 350 mm on center of each face. The total area of reinforcement is 4000 mm^2. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 25000 MPa
v = 0.2

Section Properties

tb = 300 mm
h = 1500 mm
d' = 50 mm
s = 350 mm
As1= As5 = 2-30M (1400 mm^2)
As2, As3, As4 = 2-15M (400 mm^2)

Design Properties

f'_c = 30 MPa
f_y = 460 MPa

TECHNICAL FEATURES OF ETABS TESTED

- Wall flexural Demand/Capacity ratio for a General Reinforcing pier section.

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.083	1.00	8.30%

COMPUTER FILE: AS 3600-09 WALL-001

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

1) A value of e = 582.6 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for pier P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = 0.85 f_c' ab = 0.85 \bullet 30 \bullet 300a = 7650a$$
$$C_s = A_1 \left(f_{s1} - 0.85 \bullet f_c' \right) + A_2 \left(f_{s2} - 0.85 \bullet f_c' \right) + A_3 \left(f_{s3} - 0.85 \bullet f_c' \right)$$
$$T = A_{s4} f_{s4} + A_{s5} f_{s5}$$
$$P_{n1} = 7650a + A_1 \left(f_{s1} - 0.85 \bullet f_c' \right) + A_2 \left(f_{s2} - 0.85 \bullet f_c' \right) + A_3 \left(f_{s3} - 0.85 \bullet f_c' \right) - A_{s4} f_{s4} - A_{s5} f_{s5} \quad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'} \left[C_c \left(d - \frac{a}{2} \right) + C_{s1} \left(d - d' \right) + C_{s2} \left(3s \right) + C_{s3} \left(2s \right) - T_{s4} \left(s \right) \right] \quad \text{(Eqn. 2)}$$

where $C_{s1} = A_1 \left(f_{s1} - 0.85 \bullet f_c' \right)$; $C_{s2} = A_2 \left(f_{s2} - 0.85 \bullet f_c' \right)$; $C_{s3} \left(f_{s3} - 0.85 \bullet f_c' \right)$; $T_{s4} = f_{s4} A_{s4}$ and the bar strains are determined below. The plastic centroid is at the center of the section and d'' = 700mm

$$e' = e + d'' = 582.6 + 700 = 1282.61 \text{ mm.}$$

4) Using c = 821.7 mm (from iteration),

$$a = \gamma c = 0.84 \bullet 821.7 = 690.2 \text{ mm, where } \gamma = 1.05 - 0.007(f_c') = 0.84$$

5) Assuming the extreme fiber strain equals 0.003 and c = 30 inch, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.003 = 0.0028; \; f_s = \varepsilon_s E \le F_y \; ; \; f_{s1} = 460.00 \text{ ksi}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.003 = 0.0015 \qquad f_{s2} = 307.9 \text{ ksi}$$

$$\varepsilon_{s3} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s5} = 0.0003 \qquad f_{s3} = 52.3 \text{ ksi}$$

$$\varepsilon_{s4} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s5} = 0.0010 \qquad f_{s4} = 203.2 \text{ ksi}$$

$$\varepsilon_{s5} = \left(\frac{d-c}{c}\right)0.003 = 0.0023 \qquad f_{s5} = 458.8 \text{ ksi}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$$P_{n1} = 5289 \text{ kN}$$
$$P_{n2} = 5289 \text{ kN}$$

$$M_n = P_n e = 5289(582.6)/1000000 = 3081 \text{ k-ft}$$

6) Calculate ϕ,

$$\phi P_n = 0.65(5289) = 3438 \text{ kN}$$
$$\phi M_n = 0.65(3081) = 2003 \text{ kN-m}$$

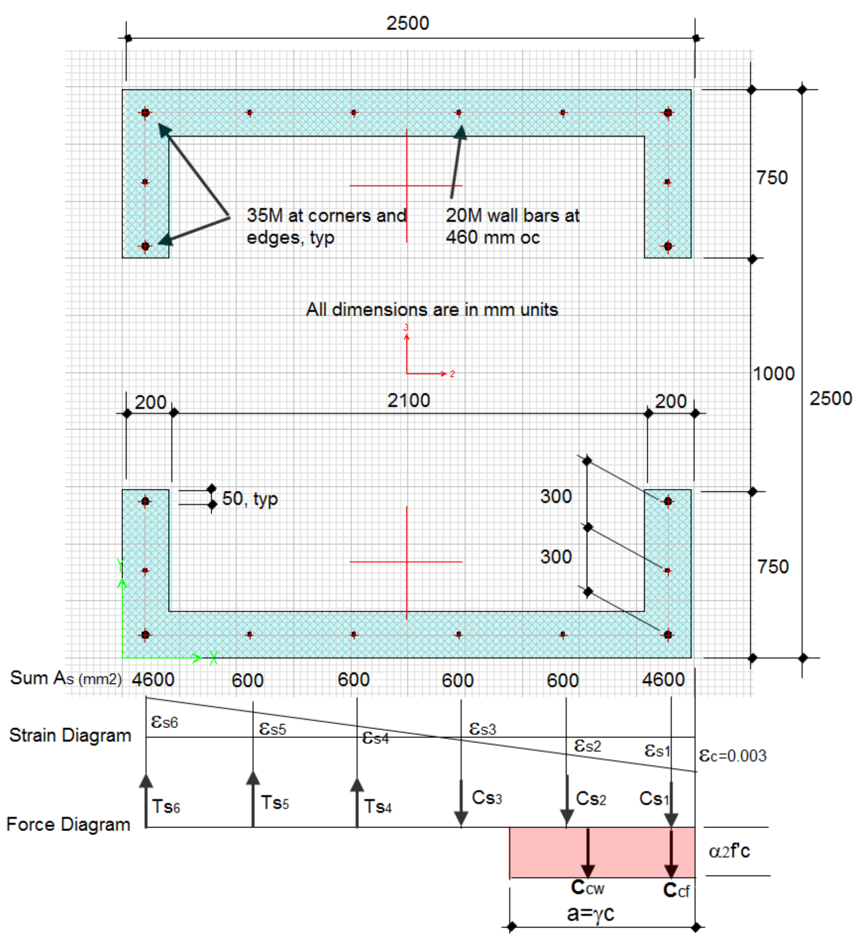
EXAMPLE AS 3600-09 Wall-002

P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example. A reinforced concrete wall is subjected to a factored axial load P_u = 11175 kN and moments M_{uy} = 12564 kN-m. This wall is reinforced as noted below. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E = 25000 MPa	tb = 200 mm	f'_c = 30 MPa
v = 0.2	H = 2500 mm	f_y = 460 MPa
	d = 2400 mm	
	s = 460 mm	
	As1= As5 = 4-35M+2-20M (4600 mm^2)	
	As2, As3, As4, As5 = 2-20M (600 mm^2)	

TECHNICAL FEATURES OF ETABS TESTED

- Demand/Capacity Ratio for a General Reinforcing pier section.

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.082	1.00	8.20%

COMPUTER FILE: AS 3600-09 WALL-002

CONCLUSION

The ETABS result shows an acceptable comparison with the independent result.

HAND CALCULATION

Wall Strength Determined as follows:

1) A value of *e* = 1124.3 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for the pier, P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, *c*, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = 0.85 f_c^{'} ab = 0.85 \bullet 30 \bullet 300a = 7650a$$

$C_c = C_{cw} + C_{cf}$, where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = 0.85 f_c' \bullet 200 \bullet (a - 200)$$

$$C_{cf} = 0.85 f_c' (200 \bullet 2500)$$

$$C_s = A_1' \left(f_{s1} - 0.85 f_c' \right) + A_2' \left(f_{s2} - 0.85 f_c' \right) + A_3' \left(f_{s3} - 0.85 f_c' \right)$$

$$T = A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6}$$

$$P_{n1} = 0.85 f_c' \bullet 8 \bullet (a - 8) + 0.85 f_c' (8 \bullet 98) + A_1' \left(f_{s1} - 0.85 f_c' \right) + A_2' \left(f_{s2} - 0.85 f_c' \right) + A_3' \left(f_{s3} - 0.85 f_c' \right) + A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6} \quad \text{(Eqn. 1)}$$

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'} \left[\begin{array}{l} C_{cf} \left(d - d' \right) + C_{cw} \left(d - \dfrac{a - t_f}{2} - t_f \right) + C_{s1} \left(d - d' \right) + \\ C_{s2} (4s) + C_{s3} (3s) - T_{s4} (2s) - T_{s5} (s) \end{array} \right] \quad \text{(Eqn. 2)}$$

where $C_{s1} = A_1'\left(f_{s1} - 0.85f_c'\right)$, $C_{sn} = A_n'\left(f_{sn} - 0.85f_c'\right)$, $T_{sn} = f_{sn}A_{sn}$, and the bar strains are determined below. The plastic centroid is at the center of the section, d'' $= \dfrac{2500-200}{2}$ = 1150 mm

$$e' = e + d'' = 1124.3 + 1150 = 2274.3\text{ mm}$$

(4) Using c = 1341.6 mm (from iteration)

$$a = \beta_1 c = 0.85 \bullet 1341.6 = 1140.4\ \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.003 and c = 1341.6 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.003 = 0.00278;\ f_s = \varepsilon_s E \le F_y;\ f_{s1} = 460.0\text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.003 = 0.00199 \qquad f_{s2} = 398.7\text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{c-2s-d'}{c}\right)0.003 = 0.00121 \qquad f_{s3} = 242.2\text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s6} = 0.00080 \qquad f_{s4} = 160.3\text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s6} = 0.00158 \qquad f_{s5} = 16.8\text{ MPa}$$

$$\varepsilon_{s6} = \left(\frac{d-c}{c}\right)0.003 = 0.00237 \qquad f_{s6} = 460.0\text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give,

$$P_{n1} = 17192\text{ kN}$$
$$P_{n2} = 17192\text{ kN}$$

$$M_n = P_n e = 17192(1124.3)/1000000 = 19329\text{ kN-m}$$

6) Calculate ϕ,

$$\phi P_n = 0.65(17192) = 11175 \text{ kN}$$

$$\phi M_n = 0.65(19329) = 12564 \text{ kN-m}$$

EXAMPLE BS 8110-97 Wall-001

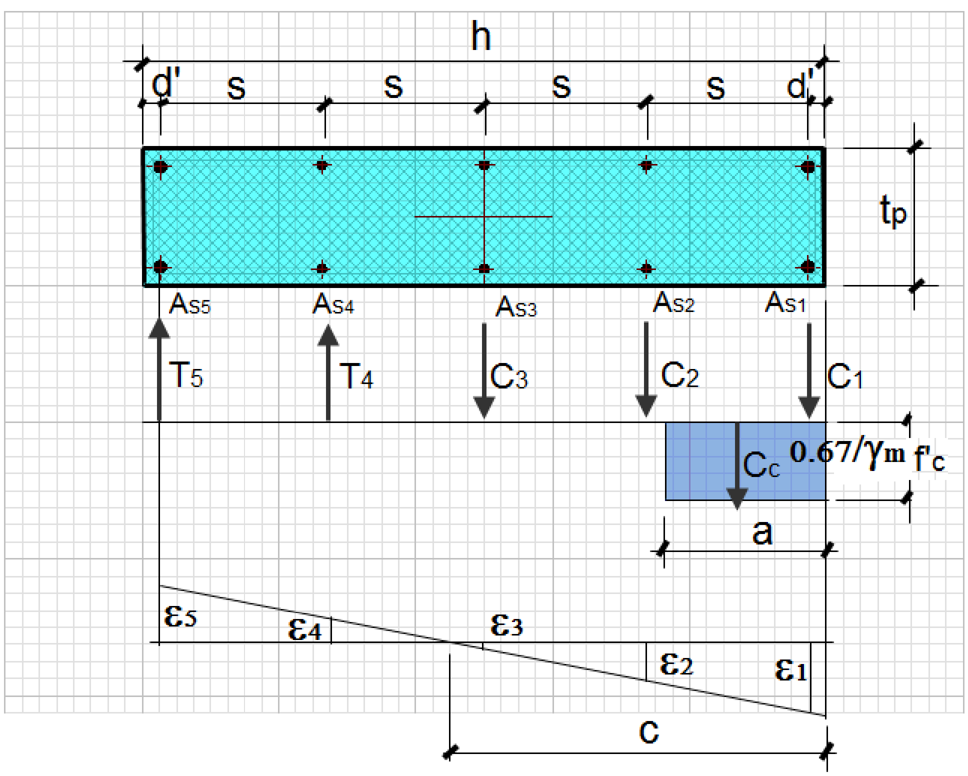
P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example.

A reinforced concrete wall is subjected to factored axial load P_u = 3246 kN and moments M_{uy} = 1969 kN-m. This wall is reinforced with two 30M bars at each end and 15M bars at 350 mm on center of each face. The total area of reinforcement is 4000 mm^2. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 25000 MPa
v = 0.2

Section Properties

tb = 300 mm
h = 1500 mm
d' = 50 mm
s = 350 mm
As1= As5 = 2-30M (1400 mm^2)
As2, As3, As4 = 2-15M (400 mm^2)

Design Properties

f'_c = 30 MPa
f_y = 460 MPa

TECHNICAL FEATURES OF ETABS TESTED

- Concrete wall flexural Demand/Capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	0.997	1.00	0.30%

COMPUTER FILE: BS 8110-97 WALL-001

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

f'_c = 30MPa f_y = 460 MPa
b = 300mm h = 1500 mm

1) A value of e = 606.5 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for pier P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

The distance to the neutral axis for a balanced condition, c_b:

$$c_b = \frac{700}{700 + f_y / \gamma_s} d_t = \frac{700}{700 + 460/1.15}(1450) = 922.7 \text{ mm}$$

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = \frac{0.67}{\gamma_m} f_{cu} ab = \frac{0.67}{1.5} \bullet 30 \bullet 300a = 4020a$$

$$C_s = \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.67}{\gamma_m} f'_c\right) + \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.67}{\gamma_m} f'_c\right) + \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.67}{\gamma_m} f'_c\right)$$

$$T = \frac{A_{s4}}{\gamma_s} f_{s4} + \frac{A_{s5}}{\gamma_s} f_{s5}$$

$$P_{n1} = 4709a + \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.67}{\gamma_m} f'_c\right) + \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.67}{\gamma_m} f'_c\right) + \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.67}{\gamma_m} f'_c\right) - \frac{A_{s4}}{\gamma_s} f_{s4} + \frac{A_{s5}}{\gamma_s} f_{s5} \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(d - d' - s\right) + C_{s3}\left(2s\right) - T_{s4}\left(s\right)\right] \qquad \text{(Eqn. 2)}$$

where $C_{s1} = \frac{A_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.67}{\gamma_m}f'_c\right)$; $C_{s2} = \frac{A_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.67}{\gamma_m}f'_c\right)$; $C_{s3} = \frac{A_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.67}{\gamma_m}f'_c\right)$;

$T_{s4} = \frac{A_{s4}}{\gamma_s}\left(f_{s4} - \frac{0.67}{\gamma_m}f'_c\right)$ and the bar strains and stresses are determined below.

The plastic centroid is at the center of the section and d'' = 700 mm

$$e' = e + d'' = 606.5 + 700 = 1306.5 \text{ mm.}$$

4) Using c = 875.2 mm (from iteration), which is more than c_b (722.7mm).

$$a = \beta_1 c = 0.9 \bullet 875.2 = 787.7 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.0035 and c = 643.6 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c - d'}{c}\right)0.003 = 0.00330;\ f_s = \varepsilon_s E \le F_y;\quad f_{s1} = 460 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c - s - d'}{c}\right)0.003 = 0.00190 \qquad f_{s2} = 380.1 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{d - c - 2s}{d - c}\right)\varepsilon_{s5} = 0.00050 \qquad f_{s3} = 100.1 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d - c - s}{d - c}\right)\varepsilon_{s5} = 0.00090 \qquad f_{s4} = 179.8 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d - c}{c}\right)0.003 = 0.00230 \qquad f_{s5} = 459.7 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$$P_{n1} = 3246 \text{ kN}$$
$$P_{n2} = 3246 \text{ kN}$$

$$M_n = P_n e = 3246(606.5)/1000 = 1969 \text{ kN-m}$$

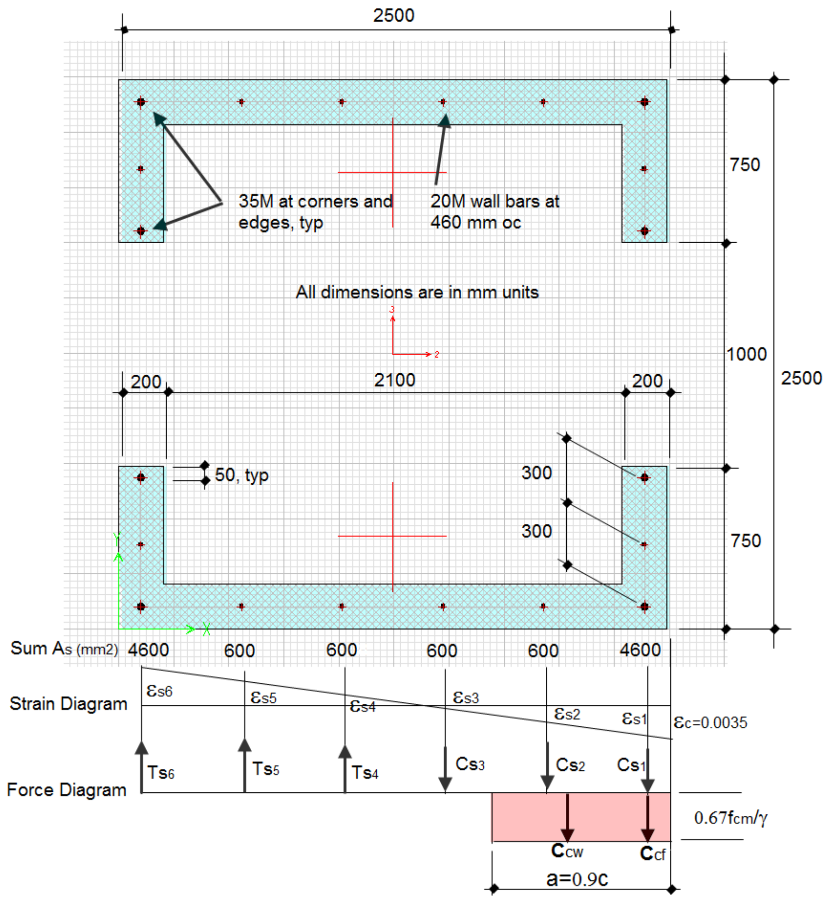
EXAMPLE BS 8110-97 Wall-002

P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example. A reinforced concrete wall is subjected to a factored axial load P_u = 8368 kN and moments M_{uy} = 11967 kN-m. This wall is reinforced as noted below. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties		Section Properties		Design Properties
E =	25000 MPa	tb =	200 mm	f'_c = 30 MPa
v =	0.2	H =	2500 mm	f_y = 460 MPa
		d =	2400 mm	
		s =	460 mm	
		As1= As5 = 4-35M+2-20M (4600 mm^2)		
		As2, As3, As4, As5 = 2-20M (600 mm^2)		

TECHNICAL FEATURES OF ETABS TESTED

- Wall flexural demand/capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.001	1.00	0.10%

COMPUTER FILE: BS 8110-97 WALL-002

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

1) A value of *e* = 1430 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for pier P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, *c*, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$C_c = C_{cw} + C_{cf}$, where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = \frac{0.67}{\gamma_m} f_{cu} \bullet 200 \bullet \left(a - 200\right)$$

$$C_{cf} = \frac{0.67}{\gamma_m} f_{cu} \left(200 \bullet 2500\right)$$

$$C_s = \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.67}{\gamma_m} f'_c\right) + \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.67}{\gamma_m} f'_c\right) + \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.67}{\gamma_m} f'_c\right)$$

$$T = \frac{A_{s4}}{\gamma_s} f_{s4} + \frac{A_{s5}}{\gamma_s} f_{s5} + \frac{A_{s6}}{\gamma_s} f_{s6}$$

$$P_{n1} = \frac{0.67}{\gamma_m} f_{cu} \bullet 200 \bullet \left(a - 200\right) + \frac{0.67}{\gamma_m} f_{cu} \left(200 \bullet 2500\right) + \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.67}{\gamma_m} f'_c\right)$$
$$+ \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.67}{\gamma_m} f'_c\right) + \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.67}{\gamma_m} f'_c\right) - \frac{A_{s4}}{\gamma_s} f_{s4} + \frac{A_{s5}}{\gamma_s} f_{s5} + \frac{A_{s6}}{\gamma_s} f_{s6} \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'}\left[\begin{array}{l} C_{cf}\left(d-d'\right)+C_{cw}\left(d-\frac{a-t_f}{2}-t_f\right)+C_{s1}\left(d-d'\right)+C_{s2}\left(4s\right) \\ +C_{s3}\left(3s\right)-T_{s4}\left(2s\right)-T_{s5}\left(s\right) \end{array}\right] \qquad \text{(Eqn. 2)}$$

where $C_{s1} = \frac{A_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.67}{\gamma_m}f'_c\right)$; $C_{sn} = \frac{A_{sn}}{\gamma_s}\left(f_{sn} - \frac{0.67}{\gamma_m}f'_c\right)$; $T_{sn} = \frac{A_{sn}}{\gamma_s}\left(f_{sn} - \frac{0.67}{\gamma_m}f'_c\right)$
and the bar strains and stresses are determined below.

The plastic centroid is at the center of the section and d'' = 1150 mm

$$e' = e + d'' = 1430 + 1150 = 2580 \text{ mm.}$$

4) Using c = 1160 mm (from iteration),

$$a = \beta_1 c = 0.9 \bullet 1160 = 1044 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.0035 and c = 1160 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.0035 = 0.00320; \; f_s = \varepsilon_s E \le F_y \; ; \; f_{s1} = 460 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.0035 = 0.00181 \qquad f_{s2} = 362.0 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{c-2s-d'}{c}\right)0.0035 = 0.00042 \qquad f_{s3} = 84.4 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s6} = 0.00097 \qquad f_{s4} = 193.2 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s6} = 0.00235 \qquad f_{s5} = 460.00 \text{ MPa}$$

$$\varepsilon_{s6} = \left(\frac{d-c}{c}\right)0.0035 = 0.00374 \qquad f_{s6} = 460.00 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

P_{n1} = 8368 kN

P_{n2} = 8368 kN

$M_n = P_n e = 8368(1430)/1000$ = 11,967 kN-m

EXAMPLE CSA A23.3-04 Wall-001

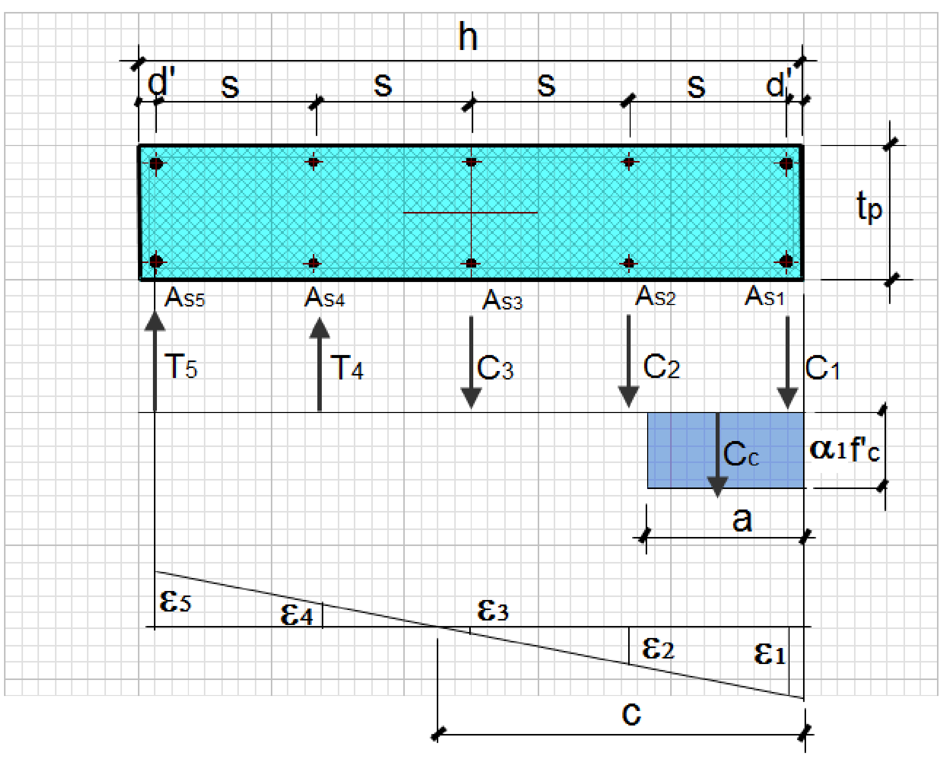
P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example.

A reinforced concrete column is subjected to factored axial load P_u = 3870 kN and moments M_{uy} = 2109 kN-m. This wall is reinforced with two 30M bars at each end and 15M bars at 350 mm on center of each face. The total area of reinforcement is 4000 mm^2. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E = 25000 MPa	tb = 300 mm	f'_c = 30 MPa
v = 0.2	h = 1500 mm	f_y = 460 MPa
	d' = 50 mm	
	s = 350 mm	
	As1= As5 = 2-30M (1400 mm^2)	
	As2, As3, As4 = 2-15M (400 mm^2)	

TECHNICAL FEATURES OF ETABS TESTED

- Flexural Demand/Capacity ratio for a General Reinforcing pier section.

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	0.986	1.00	-1.40%

COMPUTER FILE: CSA A23.3-04 WALL-001

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

f'_c = 30MPa f_y = 460 MPa
b = 300mm h = 1500 mm

1) A value of e = 545 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for pier P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

The distance to the neutral axis for a balanced condition, c_b:

$$c_b = \frac{700}{700 + f_y} d_t = \frac{700}{700 + 460}(1450) = 875 \text{ mm}$$

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = \phi_c \alpha_1 f'_c ab = 0.65 \bullet 0.805 \bullet 30 \bullet 300a = 4709a$$
$$C_s = \phi_s A'_{s1}\left(f_{s1} - \alpha_1 f'_c\right) + \phi_s A'_{s2}\left(f_{s2} - \alpha_1 f'_c\right) + \phi_s A'_{s3}\left(f_{s3} - \alpha_1 f'_c\right)$$
$$T = \phi_s A_{s4} f_{s4} + \phi A_{s5} f_{s5}$$

$$P_{n1} = 4709a + A'_1\left(f_{s1} - 0.805 f'_c\right) + A'_2\left(f_{s2} - 0.805 f'_c\right) - \phi A_{s3} f_{s3} - \phi A_{s4} f_{s4} - \phi A_{s5} f_{s5} \quad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(d - d' - s\right) + C_{s3}\left(2s\right) - T_{s4}\left(s\right)\right] \quad \text{(Eqn. 2)}$$

where $C_{s1}=\phi_s A'_{s1}\left(f_{s1}-\alpha_1 f'_c\right)$; $C_{s2}=\phi_s A'_{s2}\left(f_{s2}-\alpha_1 f'_c\right)$; $C_{s3}=\phi_s A'_{s3}\left(f_{s3}-\alpha_1 f'_c\right)$; $T_{s4}=\phi_s f_{s4} A_{s4}$ and the bar strains are determined below. The plastic centroid is at the center of the section and d'' = 700 mm

$$e'=e+d''=545+700=1245 \text{ inch.}$$

4) Using c = 894.5 mm (from iteration), which is more than c_b (875mm).

$$a=\beta_1 c=0.895\bullet 894.5=800.6 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.0035 and c = 643.6 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s=f_y$:

$$\varepsilon_{s1}=\left(\frac{c-d'}{c}\right)0.003 = 0.00330;\ f_s=\varepsilon_s E\le F_y;\quad f_{s1}=460.0 \text{ MPa}$$

$$\varepsilon_{s2}=\left(\frac{c-s-d'}{c}\right)0.003 = 0.00193 \quad f_{s2}=387.0 \text{ MPa}$$

$$\varepsilon_{s3}=\left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s5} = 0.00057 \quad f_{s3}=113.1 \text{ MPa}$$

$$\varepsilon_{s4}=\left(\frac{d-c-s}{d-c}\right)\varepsilon_{s5} = 0.00080 \quad f_{s4}=160.8 \text{ MPa}$$

$$\varepsilon_{s5}=\left(\frac{d-c}{c}\right)0.0035 = 0.00217 \quad f_{s5}= 434.7 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$$P_{n1}=3870 \text{ kN}$$
$$P_{n2}=3870 \text{ kN}$$

$$M_n=P_n e=3870(545)/1000 = 2109 \text{ kN-m}$$

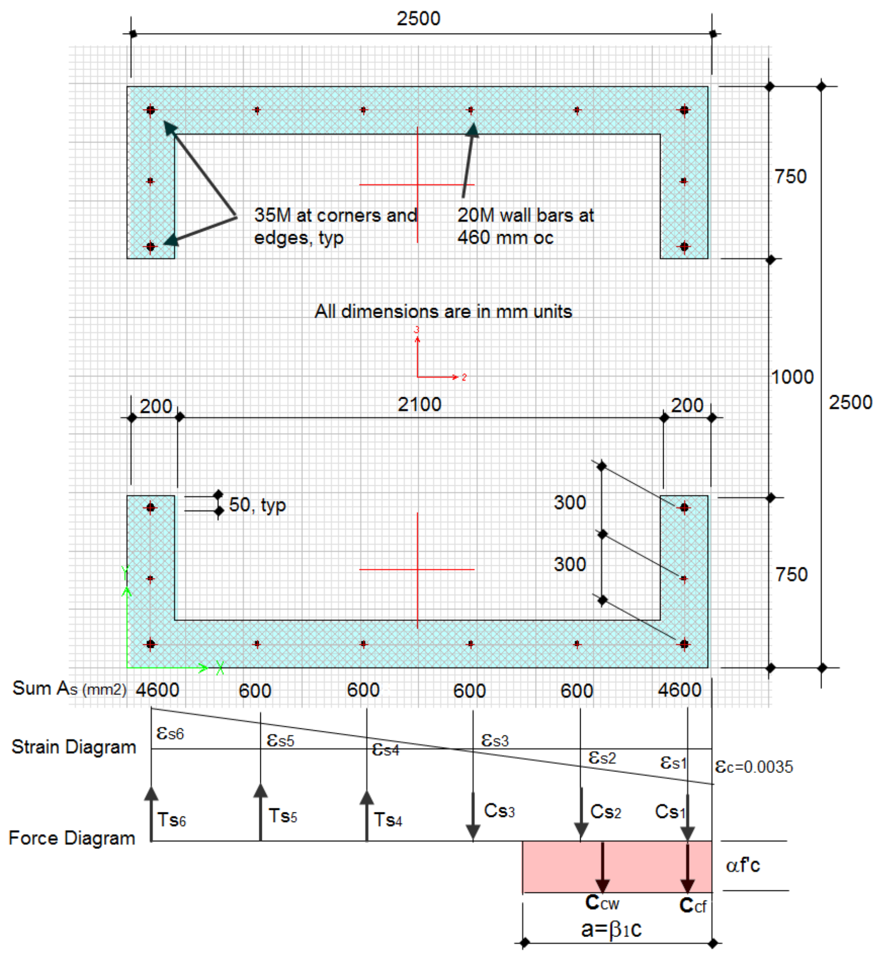
EXAMPLE CSA A23.3-04 Wall-002

P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example. A reinforced concrete wall is subjected to a factored axial load P_u = 10687 kN and moments M_{uy} = 13159 kN-m. This wall is reinforced as noted below. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E = 25000 MPa	tb = 200 mm	f'_c = 30 MPa
v = 0.2	H = 2500 mm	f_y = 460 MPa
	d = 2400 mm	
	s = 460 mm	
	As1= As5 = 4-35M+2-20M (4600 mm^2)	
	As2, As3, As4, As5 = 2-20M (600 mm^2)	

TECHNICAL FEATURES OF ETABS TESTED

- Demand/Capacity Ratio for a General Reinforcing pier section.

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	0.994	1.00	0.40%

COMPUTER FILE: CSA A23.3-04 WALL-002

CONCLUSION

The ETABS results show an acceptable match with the independent results.

HAND CALCULATION

WALL STRENGTH DETERMINED AS FOLLOWS:

f'_c = 30MPa f_y = 460 MPa
b = 300mm h = 1500 mm

1) A value of e = 1231.3 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for the pier, P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = \phi_c \alpha_1 f'_c ab = 0.65 \bullet 0.805 \bullet 30 \bullet 300a = 4709a$$

$C_c = C_{cw} + C_{cf}$, where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = \phi_c \alpha_1 f'_c \bullet 200 \bullet (a - 200)$$

$$C_{cf} = \phi_c \alpha_1 f'_c (200 \bullet 2500)$$

$$C_s = \phi_s A'_{s1}\left(f_{s1} - \alpha_1 \phi_c f'_c\right) + \phi_s A'_{s2}\left(f_{s2} - \alpha_1 \phi_c f'_c\right) + \phi_s A'_{s3}\left(f_{s3} - \alpha_1 \phi_c f'_c\right)$$

$$T = \phi_s A_{s4} f_{s4} + \phi_s A_{s5} f_{s5} + \phi_s A_{s6} f_{s6}$$

$$P_{n1} = \phi_c \alpha_1 f'_c \bullet 200 \bullet (a - 200) + \phi_c \alpha_1 f'_c (200 \bullet 2500) + \phi_s A'_{s1}\left(f_{s1} - \alpha_1 \phi_c f'_c\right) + \phi_s A'_{s2}\left(f_{s2} - \alpha_1 \phi_c f'_c\right) + \phi_s A'_{s3}\left(f_{s3} - \alpha_1 \phi_c f'_c\right) - \phi_s A_{s4} f_{s4} - \phi A_{s5} f_{s5} - \phi A_{s6} f_{s6}$$

(Eqn. 1)

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'}\begin{bmatrix} C_{cf}\left(d-d'\right)+C_{cw}\left(d-\dfrac{a-t_f}{2}-t_f\right)+C_{s1}\left(d-d'\right)+C_{s2}\left(4s\right)+ \\ C_{s3}\left(3s\right)-T_{s4}\left(2s\right)-T_{s5}\left(s\right) \end{bmatrix} \quad \text{(Eqn. 2)}$$

where $C_{s1} = \phi_s A'_{s1}\left(f_{s1}-\alpha_1\phi_c f'_c\right)$; $C_{sn} = \phi_s A'_{sn}\left(f_{sn}-\alpha_1\phi_c f'_c\right)$; $T_{s4} = \phi_s f_{sn} A_{sn}$ and the bar strains are determined below. The plastic centroid is at the center of the section and d'' = 700 mm

$$e' = e + d'' = 1231.3 + 1050 = 2381.3 \text{ inch.}$$

4) Using c = 1293.6 mm (from iteration),

$$a = \beta_1 c = 0.895 \bullet 1293.6 = 1157.8 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.0030 and c = 1293.6 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.0035 = 0.00323;\ f_s = \varepsilon_s E \le F_y;\quad f_{s1} = 460.0 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.0035 = 0.00198 \quad f_{s2} = 397.0 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{c-2s-d'}{c}\right)0.0035 = 0.00074 \quad f_{s3} = 148.1 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s6} = 0.00175 \quad f_{s4} = 100.9 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s6} = 0.00299 \quad f_{s5} = 349.8 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d-c}{c}\right)0.0035 = 0.00230 \quad f_{s6} = 460.0 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

P_{n1} = 10687 kN

P_{n2} = 10687 kN

$M_n = P_n e = 10687(1231.3)/1000000$ = 13159 kN-m

EXAMPLE CSA A23.3-14 Wall-001

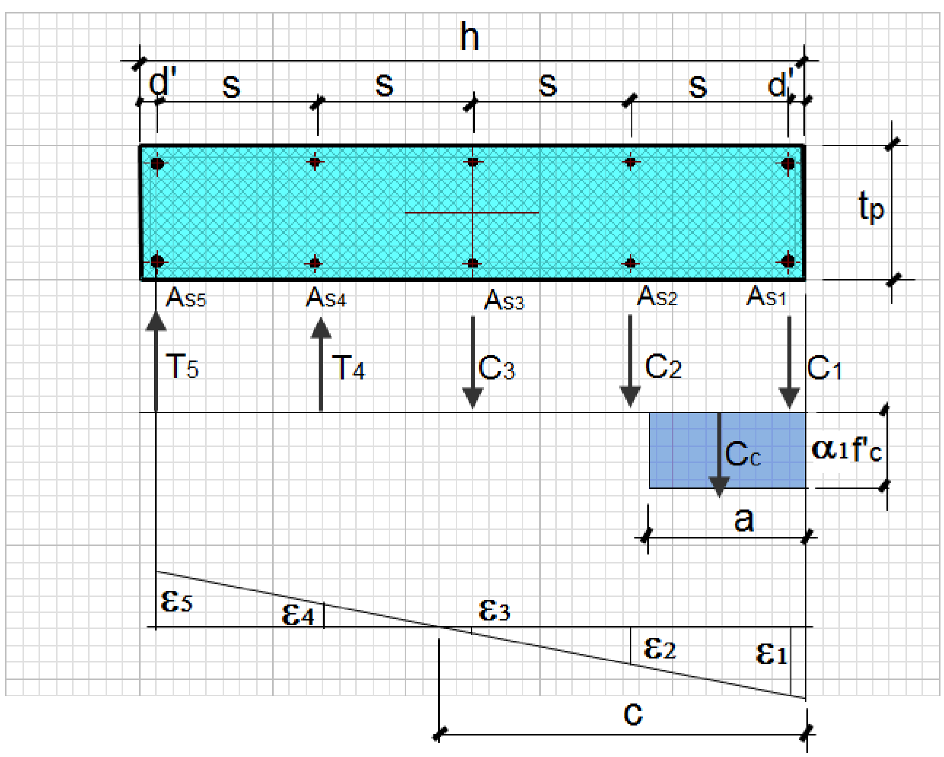
P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example.

A reinforced concrete column is subjected to factored axial load P_u = 3870 kN and moments M_{uy} = 2109 kN-m. This wall is reinforced with two 30M bars at each end and 15M bars at 350 mm on center of each face. The total area of reinforcement is 4000 mm^2. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 25000 MPa
v = 0.2

Section Properties

tb = 300 mm
h = 1500 mm
d' = 50 mm
s = 350 mm
As1= As5 = 2-30M (1400 mm^2)
As2, As3, As4 = 2-15M (400 mm^2)

Design Properties

f'_c = 30 MPa
f_y = 460 MPa

TECHNICAL FEATURES OF ETABS TESTED

- Flexural Demand/Capacity ratio for a General Reinforcing pier section.

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	0.986	1.00	-1.40%

COMPUTER FILE: CSA A23.3-14 WALL-001

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

f'_c = 30MPa f_y = 460 MPa
b = 300mm h = 1500 mm

1) A value of e = 545 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for pier P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

The distance to the neutral axis for a balanced condition, c_b:

$$c_b = \frac{700}{700 + f_y} d_t = \frac{700}{700 + 460}(1450) = 875 \text{ mm}$$

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = \phi_c \alpha_1 f'_c ab = 0.65 \bullet 0.805 \bullet 30 \bullet 300a = 4709a$$

$$C_s = \phi_s A'_{s1}\left(f_{s1} - \alpha_1 f'_c\right) + \phi_s A'_{s2}\left(f_{s2} - \alpha_1 f'_c\right) + \phi_s A'_{s3}\left(f_{s3} - \alpha_1 f'_c\right)$$

$$T = \phi_s A_{s4} f_{s4} + \phi A_{s5} f_{s5}$$

$$P_{n1} = 4709a + A'_1\left(f_{s1} - 0.805 f'_c\right) + A'_2\left(f_{s2} - 0.805 f'_c\right) - \phi A_{s3} f_{s3} - \phi A_{s4} f_{s4} - \phi A_{s5} f_{s5} \quad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(d - d' - s\right) + C_{s3}\left(2s\right) - T_{s4}\left(s\right)\right] \quad \text{(Eqn. 2)}$$

where $C_{s1} = \phi_s A'_{s1}\left(f_{s1} - \alpha_1 f'_c\right)$; $C_{s2} = \phi_s A'_{s2}\left(f_{s2} - \alpha_1 f'_c\right)$; $C_{s3} = \phi_s A'_{s3}\left(f_{s3} - \alpha_1 f'_c\right)$; $T_{s4} = \phi_s f_{s4} A_{s4}$ and the bar strains are determined below. The plastic centroid is at the center of the section and d'' = 700 mm

$$e' = e + d'' = 545 + 700 = 1245 \text{ inch.}$$

4) Using c = 894.5 mm (from iteration), which is more than c_b (875mm).

$$a = \beta_1 c = 0.895 \bullet 894.5 = 800.6 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.0035 and c = 643.6 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.003 = 0.00330; \; f_s = \varepsilon_s E \le F_y; \quad f_{s1} = 460.0 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.003 = 0.00193 \quad f_{s2} = 387.0 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s5} = 0.00057 \quad f_{s3} = 113.1 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s5} = 0.00080 \quad f_{s4} = 160.8 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d-c}{c}\right)0.0035 = 0.00217 \quad f_{s5} = 434.7 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$$P_{n1} = 3870 \text{ kN}$$
$$P_{n2} = 3870 \text{ kN}$$

$$M_n = P_n e = 3870(545)/1000 = 2109 \text{ kN-m}$$

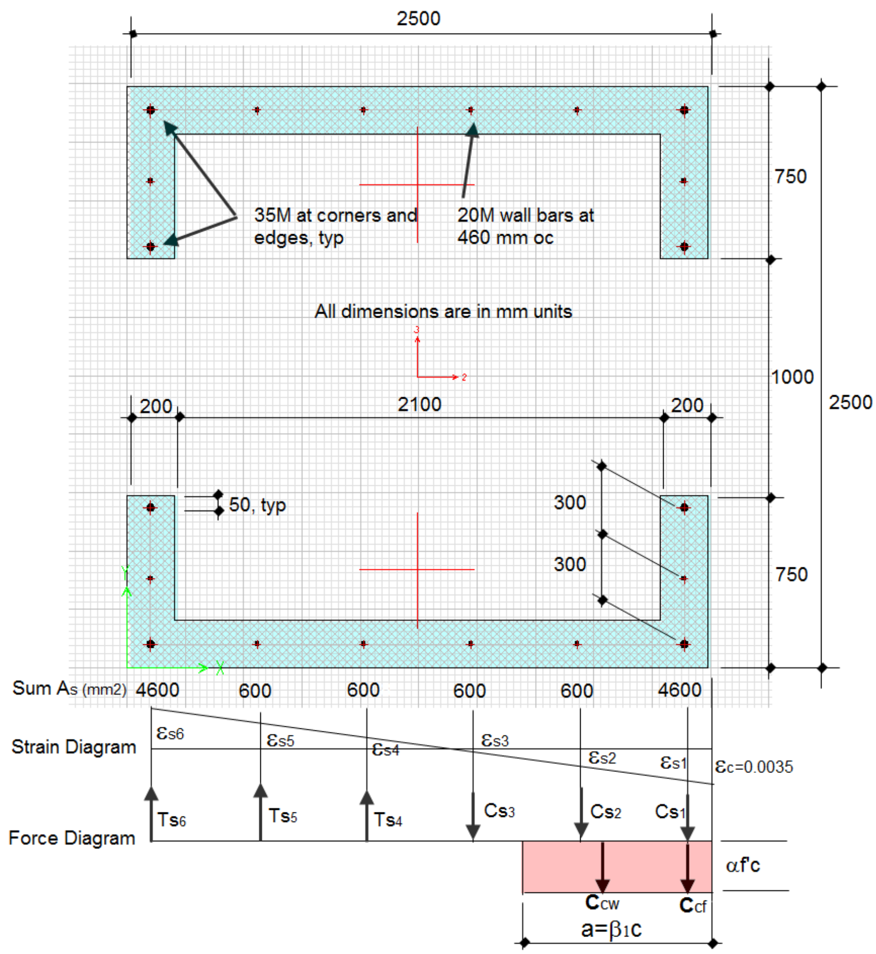
EXAMPLE CSA A23.3-14 Wall-002

P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example. A reinforced concrete wall is subjected to a factored axial load P_u = 10687 kN and moments M_{uy} = 13159 kN-m. This wall is reinforced as noted below. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties		Section Properties		Design Properties
E =	25000 MPa	tb =	200 mm	f'_c = 30 MPa
v =	0.2	H =	2500 mm	f_y = 460 MPa
		d =	2400 mm	
		s =	460 mm	
		As1= As5 = 4-35M+2-20M (4600 mm^2)		
		As2, As3, As4, As5 = 2-20M (600 mm^2)		

TECHNICAL FEATURES OF ETABS TESTED

- Demand/Capacity Ratio for a General Reinforcing pier section.

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	0.994	1.00	0.40%

COMPUTER FILE: CSA A23.3-14 WALL-002

CONCLUSION

The ETABS results show an acceptable match with the independent results.

HAND CALCULATION

WALL STRENGTH DETERMINED AS FOLLOWS:

f'_c = 30MPa f_y = 460 MPa
b = 300mm h = 1500 mm

1) A value of e = 1231.3 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for the pier, P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, *c*, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = \phi_c \alpha_1 f'_c ab = 0.65 \bullet 0.805 \bullet 30 \bullet 300a = 4709a$$

$C_c = C_{cw} + C_{cf}$, where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = \phi_c \alpha_1 f'_c \bullet 200 \bullet (a - 200)$$

$$C_{cf} = \phi_c \alpha_1 f'_c (200 \bullet 2500)$$

$$C_s = \phi_s A'_{s1} (f_{s1} - \alpha_1 \phi_c f'_c) + \phi_s A'_{s2} (f_{s2} - \alpha_1 \phi_c f'_c) + \phi_s A'_{s3} (f_{s3} - \alpha_1 \phi_c f'_c)$$

$$T = \phi_s A_{s4} f_{s4} + \phi_s A_{s5} f_{s5} + \phi_s A_{s6} f_{s6}$$

$$P_{n1} = \phi_c \alpha_1 f'_c \bullet 200 \bullet (a - 200) + \phi_c \alpha_1 f'_c (200 \bullet 2500) + \phi_s A'_{s1} (f_{s1} - \alpha_1 \phi_c f'_c) + \phi_s A'_{s2} (f_{s2} - \alpha_1 \phi_c f'_c) + \phi_s A'_{s3} (f_{s3} - \alpha_1 \phi_c f'_c) - \phi_s A_{s4} f_{s4} - \phi A_{s5} f_{s5} - \phi A_{s6} f_{s6}$$

(Eqn. 1)

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'}\left[\begin{array}{l} C_{cf}\left(d-d'\right)+C_{cw}\left(d-\dfrac{a-t_f}{2}-t_f\right)+C_{s1}\left(d-d'\right)+C_{s2}\left(4s\right)+ \\ C_{s3}\left(3s\right)-T_{s4}\left(2s\right)-T_{s5}\left(s\right) \end{array}\right] \quad \text{(Eqn. 2)}$$

where $C_{s1} = \phi_s A'_{s1}\left(f_{s1} - \alpha_1\phi_c f'_c\right)$; $C_{sn} = \phi_s A'_{sn}\left(f_{sn} - \alpha_1\phi_c f'_c\right)$; $T_{s4} = \phi_s f_{sn} A_{sn}$ and the bar strains are determined below. The plastic centroid is at the center of the section and d'' = 700 mm

$$e' = e + d'' = 1231.3 + 1050 = 2381.3 \text{ inch.}$$

4) Using c = 1293.6 mm (from iteration),

$$a = \beta_1 c = 0.895 \bullet 1293.6 = 1157.8 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.0030 and c = 1293.6 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.0035 = 0.00323;\ f_s = \varepsilon_s E \le F_y;\quad f_{s1} = 460.0 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.0035 = 0.00198 \qquad f_{s2} = 397.0 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{c-2s-d'}{c}\right)0.0035 = 0.00074 \qquad f_{s3} = 148.1 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s6} = 0.00175 \qquad f_{s4} = 100.9 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s6} = 0.00299 \qquad f_{s5} = 349.8 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d-c}{c}\right)0.0035 = 0.00230 \qquad f_{s6} = 460.0 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

P_{n1} = 10687 kN

P_{n2} = 10687 kN

$M_n = P_n e = 10687(1231.3)/1000000$ = 13159 kN-m

EXAMPLE Eurocode 2-2004 Wall-001

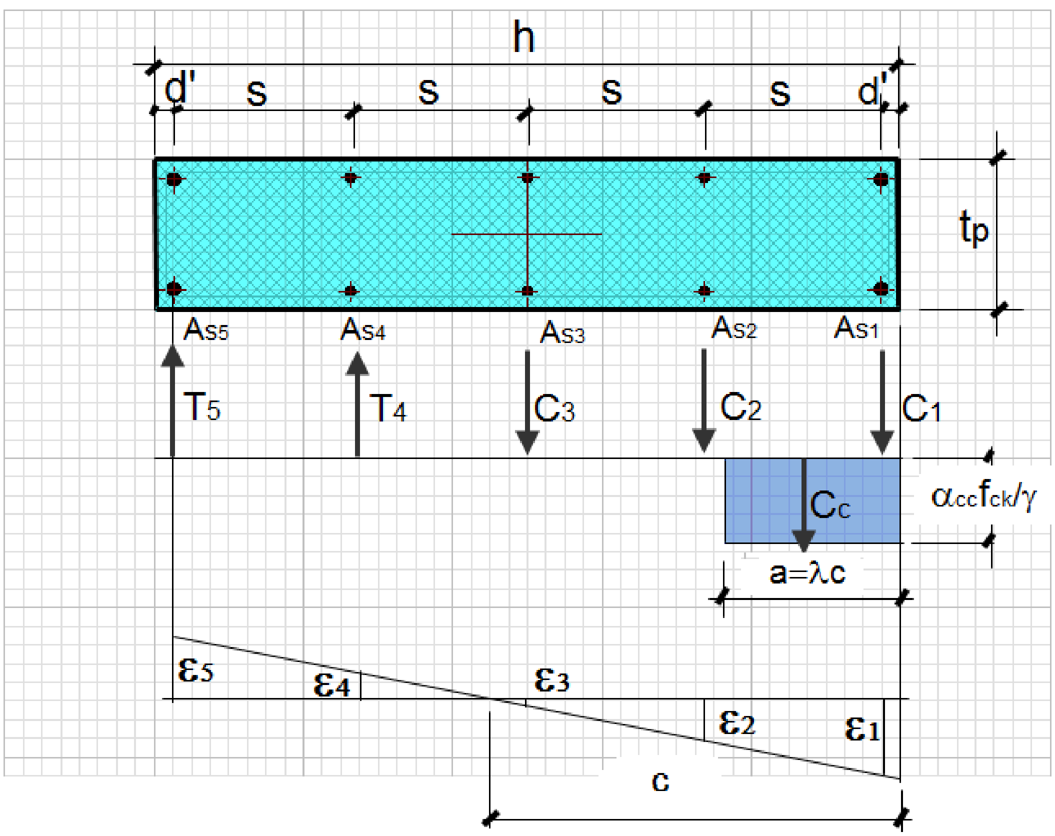
P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example.

A reinforced concrete wall is subjected to factored axial load P_u = 4340 kN and moments M_{uy} = 2503 kN-m. This wall is reinforced with two 30M bars at each end and 15M bars at 350 mm on center of each face. The total area of reinforcement is 4000 mm^2. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E = 25000 MPa	tb = 300 mm	f'_c = 30 MPa
v = 0.2	h = 1500 mm	f_y = 460 MPa
	d = 50 mm	
	s = 350 mm	
	As1= As5 = 2-30M (1400 mm^2)	
	As2, As3, As4 = 2-15M (400 mm^2)	

TECHNICAL FEATURES OF ETABS TESTED

- Concrete wall flexural demand/capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	0.993	1.00	0.70%

COMPUTER FILE: EUROCODE 2-2004 WALL-001

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

f'_c = 30MPa f_y = 460 MPa
b = 300mm h = 1500 mm

1) A value of e = 576.3 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for pier P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

The distance to the neutral axis for a balanced condition, c_b:

$$c_b = \frac{700}{700 + f_y / \gamma_s} d_t = \frac{700}{700 + 460/1.15}(1450) = 922.7 \text{ mm}$$

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = \frac{\alpha_{cc} f_{ck}}{\gamma_m} ab = \frac{1.0 \bullet 30}{1.5} \bullet 300a = 6000a$$

$$C_s = \frac{A_{s1}}{\gamma_s}\left(f_{s1} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right) + \frac{A_{s2}}{\gamma_s}\left(f_{s2} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right) + \frac{A_{s3}}{\gamma_s}\left(f_{s3} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right)$$

$$T = \frac{A_{s4}}{\gamma_s} f_{s4} + \frac{A_{s5}}{\gamma_s} f_{s5}$$

$$P_{n1} = 6000a + \frac{A_{s1}}{\gamma_s}\left(f_{s1} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right) + \frac{A_{s2}}{\gamma_s}\left(f_{s2} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right) + \frac{A_{s3}}{\gamma_s}\left(f_{s3} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right) - \frac{A_{s4}}{\gamma_s} f_{s4} - \frac{A_{s5}}{\gamma_s} f_{s5} \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(3s\right) + C_{s3}\left(2s\right) - T_{s4}\left(s\right)\right] \quad \text{(Eqn. 2)}$$

where $C_{s1} = \frac{A_{s1}}{\gamma_s}\left(f_{s1} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right)$; $C_{s2} = \frac{A_{s2}}{\gamma_s}\left(f_{s2} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right)$; $C_{s3} = \frac{A_{s3}}{\gamma_s}\left(f_{s3} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right)$;

$T_{s4} = \frac{A_{s4}}{\gamma_s}\left(f_{s4}\right)$ and the bar strains and stresses are determined below.

The plastic centroid is at the center of the section and d'' = 700 mm

$e' = e + d'' = 576.73 + 700 = 1276.73$ mm.

4) Using c = 885.33 mm (from iteration), which is more than c_b (922.7mm).

$a = \lambda_1 c = 0.80 \bullet 885.33 = 708.3$ mm

5) Assuming the extreme fiber strain equals 0.0035 and c = 885.33 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c - d'}{c}\right)0.0035 = 0.00330; \; f_s = \varepsilon_s E \le F_y \; ; \; f_{s1} = 460 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c - s - d'}{c}\right)0.0035 = 0.00192 \qquad f_{s2} = 383.7 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{d - c - 2s}{d - c}\right)\varepsilon_{s5} = 0.00054 \qquad f_{s3} = 107.0 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d - c - s}{d - c}\right)\varepsilon_{s5} = 0.00085 \qquad f_{s4} = 169.7 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d - c}{c}\right)0.0035 = 0.00223 \qquad f_{s5} = 446.5 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$P_{n1} = 4340$ kN

$P_{n2} = 4340$ kN

$M_n = P_n e = 4340(708.3)/1000 = 2503$ kN-m

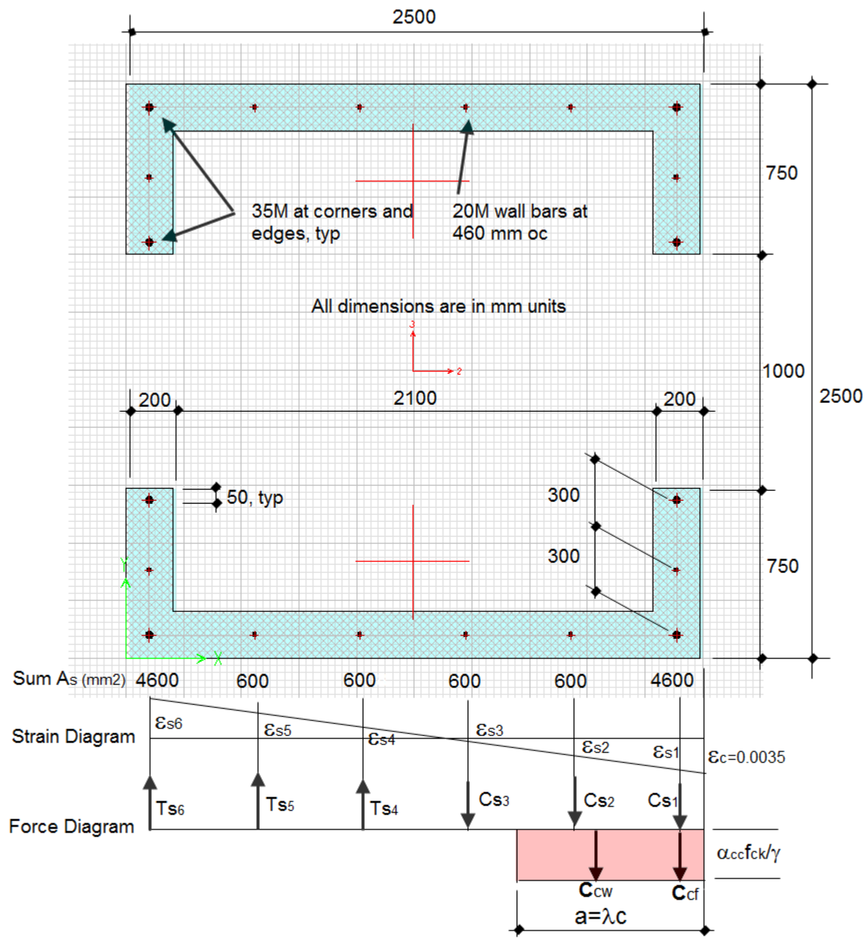
EXAMPLE Eurocode 2-2004 Wall-002

P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example. A reinforced concrete wall is subjected to factored axial load P_u = 11605 kN and moments M_{uy} = 15342 kN-m. This wall is reinforced as noted below. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties		Section Properties		Design Properties
E =	25000 MPa	tb =	200 mm	f'_c = 30 MPa
v =	0.2	H =	2500 mm	f_y = 460 MPa
		d =	2400 mm	
		s =	460 mm	
		As1= As5 = 4-35M+2-20M (4600 mm^2)		
		As2, As3, As4, As5 = 2-20M (600 mm^2)		

TECHNICAL FEATURES OF ETABS TESTED

- Concrete wall flexural demand/capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.011	1.00	1.10%

COMPUTER FILE: EUROCODE 2-2004 WALL-002

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

1) A value of *e* = 1322 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for pier P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, *c*, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

Where

$C_c = C_{cw} + C_{cf}$, where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = \frac{\alpha_{cc} f_{ck}}{\gamma_m} \bullet 200 \bullet (a - 200) = \frac{0.85 \bullet 30}{1.5} \bullet 200 \bullet (a - 200) = 3400(a - 200)$$

$$C_{cf} = \frac{\alpha_{cc} f_{ck}}{\gamma_m}\left(200 \bullet (2500 - 1000)\right) = \frac{0.85(30)}{1.5}\left(200 \bullet (2500 - 1000)\right) = 5{,}100{,}000$$

$$C_s = A_1'\left(f_{s1} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right) + A_2'\left(f_{s2} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right) + A_3'\left(f_{s3} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right)$$

$$T = A_{s4}\frac{f_{s4}}{\gamma_s} + A_{s5}\frac{f_{s4}}{\gamma_s} + A_{s6}\frac{f_{s4}}{\gamma_s}$$

$$P_{n1} = 3400(a - 200) + 5{,}100{,}000 + \frac{A_{s1}}{\gamma_s}\left(f_{s1} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right) + \frac{A_{s2}}{\gamma_s}\left(f_{s2} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right)$$

$$+ \frac{A_{s3}}{\gamma_s}\left(f_{s3} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right) - \frac{A_{s4}}{\gamma_s} f_{s4} - \frac{A_{s5}}{\gamma_s} f_{s5} - \frac{A_{s6}}{\gamma_s} f_{s6}$$

(Eqn. 1)

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'}\begin{bmatrix} C_{cf}\left(d - d'\right) + C_{cw}\left(d - \frac{a - t_f}{2} - t_f\right) + C_{s1}\left(d - d'\right) + \\ C_{s2}\left(4s\right) + C_{s3}\left(3s\right) - T_{s4}\left(2s\right) - T_{s5}\left(s\right) \end{bmatrix} \qquad \text{(Eqn. 2)}$$

where $C_{s1} = \frac{A_{s1}}{\gamma_s}\left(f_{s1} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right)$; $C_{s2} = \frac{A_{s2}}{\gamma_s}\left(f_{s2} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right)$; $C_{s3} = \frac{A_{s3}}{\gamma_s}\left(f_{s3} - \frac{\alpha_{cc} f_{ck}}{\gamma_m}\right)$;
$T_{s4} = \frac{A_{s4}}{\gamma_s}\left(f_{s4}\right)$ and the bar strains and stresses are determined below.

The plastic centroid is at the center of the section and d'' = 700 mm

$$e' = e + d'' = 1322 + 700 = 2472 \text{ mm.}$$

4) Using c = 1299 mm (from iteration),

$$a = \beta_1 c = 0.895 \bullet 1299 = 1163 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.0035 and c = 1299 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c - d'}{c}\right)0.0035 = 0.00323; \quad f_s = \varepsilon_s E \le F_y; \quad f_{s1} = 460 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c - s - d'}{c}\right)0.0035 = 0.00199 \qquad f_{s2} = 398.2 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{c - 2s - d'}{c}\right)0.0035 = 0.00075 \qquad f_{s3} = 150.3 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d - c - 2s}{d - c}\right)\varepsilon_{s6} = 0.00049 \qquad f_{s4} = 97.5 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d - c - s}{d - c}\right)\varepsilon_{s6} = 0.00173 \qquad f_{s5} = 345.4 \text{ MPa}$$

$$\varepsilon_{s6} = \left(\frac{d - c}{c}\right)0.0035 = 0.00297 \qquad f_{s6} = 460.00 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$P_{n1} = 11605$ kN

$P_{n2} = 11605$ kN

$M_n = P_n e = 11605(1322)/1000 = 15342$ kN-m

EXAMPLE Indian IS 456-2000 Wall-001

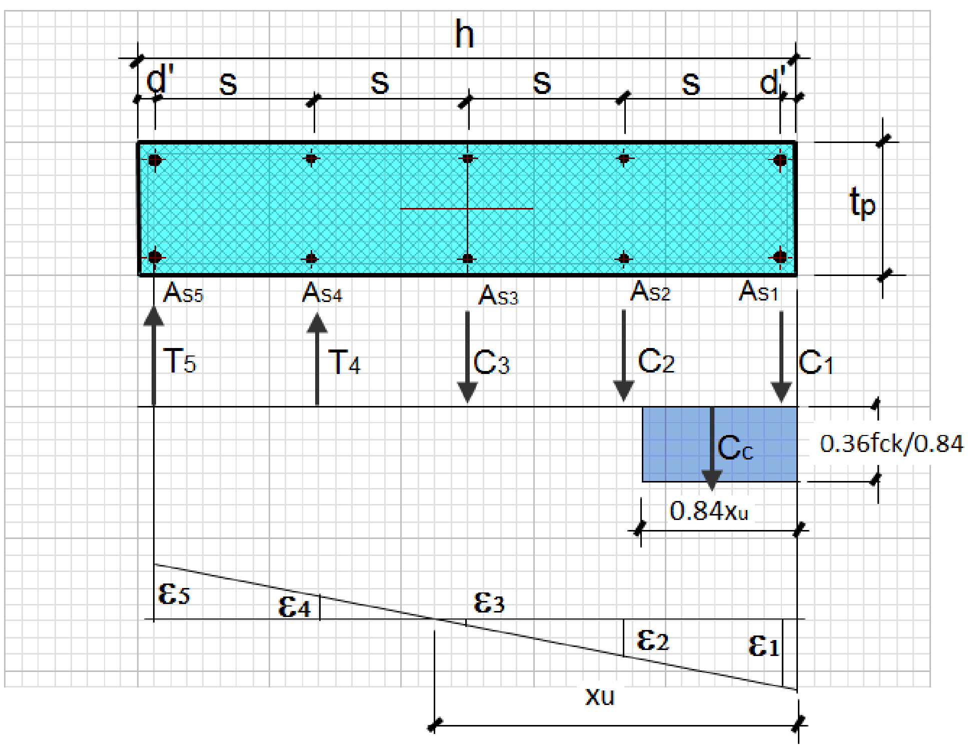
P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example.

A reinforced concrete wall is subjected factored axial load P_u = 3146 kN and moments M_{uy} = 1875 kN-m. This wall is reinforced with two 30M bars at each end and 15M bars at 350 mm on center of each face. The total area of reinforcement is 4000 mm^2. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 25000 MPa
v = 0.2

Section Properties

tb = 300 mm
h = 1500 mm
d' = 50 mm
s = 350 mm
As1= As5 = 2-30M (1400 mm^2)
As2, As3, As4 = 2-15M (400 mm^2)

Design Properties

f'_c = 30 MPa
f_y = 460 MPa

TECHNICAL FEATURES OF ETABS TESTED

- Concrete wall flexural demand/capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.035	1.00	3.50%

COMPUTER FILE: INDIAN IS 456-2000 WALL-001

CONCLUSION

The ETABS results show an acceptable match with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

F'_c = 30MPa $\quad f_y$ = 460 MPa

b = 300mm $\quad h$ = 1500 mm

1) A value of e = 596 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for the pier, P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = \frac{0.36}{0.84} f_{ck} ab = 0.4286 \bullet 30 \bullet 300a = 3857a\text{, where } a = 0.84x_u$$

$$C_s = \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - 0.4286 f'_c\right) + \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - 0.4286 f'_c\right) + \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - 0.4286 f'_c\right)$$

$$T = \frac{A_{s4}}{\gamma_s} f_{s4} + \frac{A_{s5}}{\gamma_s} f_{s5}$$

$$P_{n1} = 3857a + \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - 0.4286 f'_c\right) + \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - 0.4286 f'_c\right) + \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - 0.4286 f'_c\right) - \frac{A_{s4}}{\gamma_s} f_{s4} + \frac{A_{s5}}{\gamma_s} f_{s5} \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(d - d' - s\right) + C_{s3}\left(2s\right) - T_{s4}\left(s\right)\right] \qquad \text{(Eqn. 2)}$$

where $C_{s1}=\frac{A_{s1}}{\gamma_s}\left(f_{s1}-0.4286f_c'\right)$; $C_{s2}=\frac{A_{s2}}{\gamma_s}\left(f_{s2}-0.4286f_c'\right)$;
$C_{s3}=\frac{A_{s3}}{\gamma_s}\left(f_{s3}-0.4286f_c'\right)$; $T_{s4}=\frac{A_{s4}}{\gamma_s}\left(f_{s4}\right)$ and the bar strains and stresses are determined below.

The plastic centroid is at the center of the section and d'' = 700 mm

$$e'=e+d''=596+700=1296 \text{ mm.}$$

4) Using c = 917.3 mm (from iteration)

$$a=\beta_1 c=0.84\bullet 917.3=770.5 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.0035 and c = 917.3 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1}=\left(\frac{c-d'}{c}\right)0.0035 = 0.00331;\ f_s=\varepsilon_s E\le F_y;\quad f_{s1} = 460 \text{ MPa}$$

$$\varepsilon_{s2}=\left(\frac{c-s-d'}{c}\right)0.0035 = 0.00197 \qquad f_{s2} = 394.8 \text{ MPa}$$

$$\varepsilon_{s3}=\left(\frac{c-2s-d'}{c}\right)0.0035 = 0.00064 \qquad f_{s3} = 127.7 \text{ MPa}$$

$$\varepsilon_{s4}=\left(\frac{d-c-s}{d-c}\right)\varepsilon_{s5} = 0.00070 \qquad f_{s4} = 139.4 \text{ MPa}$$

$$\varepsilon_{s5}=\left(\frac{d-c}{c}\right)0.0035 = 0.00203 \qquad f_{s5} = 406.5 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$$P_{n1} = 3146 \text{ kN}$$
$$P_{n2} = 3146 \text{ kN}$$

$$M_n=P_n e=3146(596)/1000 = 1875 \text{ kN-m}$$

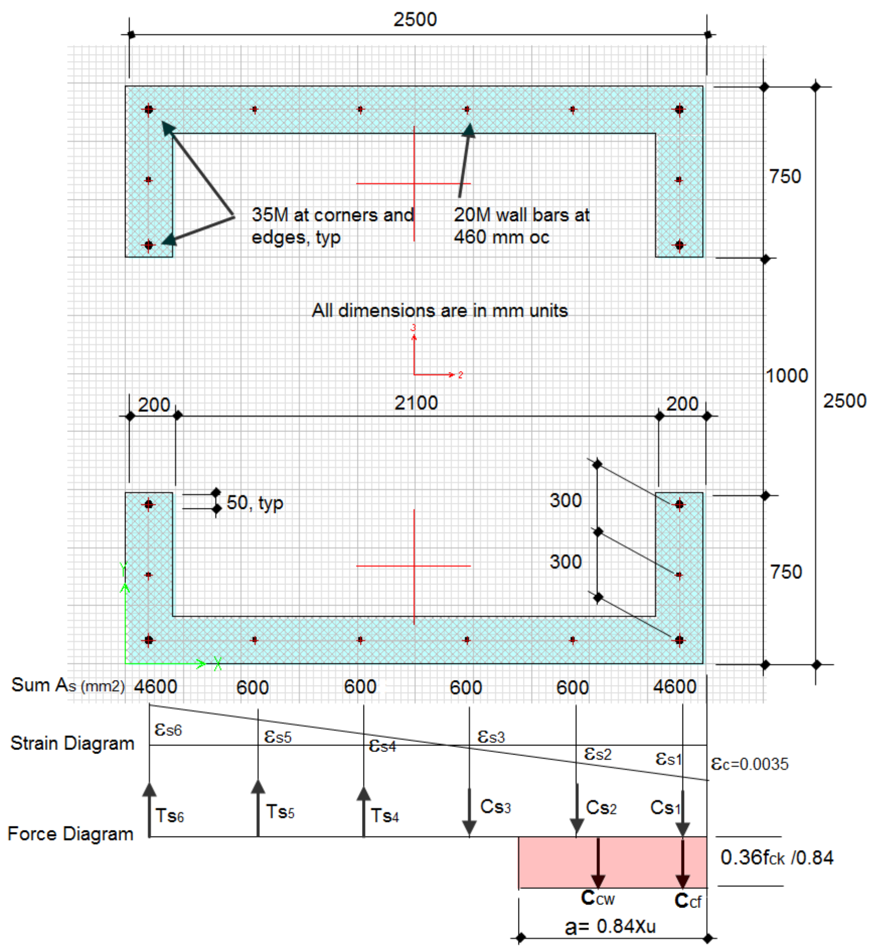
EXAMPLE Indian IS 456-2000 Wall-002

FRAME – P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example. A reinforced concrete wall is subjected to factored axial load P_u= 8426 kN and moments M_{uy}= 11670 kN-m. This wall is reinforced as noted below. The design capacity ratio is checked by hand calculations and results are compared with ETABS program.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 25000 MPa

v = 0.2

Section Properties

tb = 200 mm

H = 2500 mm

d = 2400 mm

s = 460 mm

As1= As5 = 4-35M+2-20M (4600 mm^2)

As2, As3, As4, As5 = 2-20M (600 mm^2)

Design Properties

f'_c = 30 MPa

f_y = 460 MPa

TECHNICAL FEATURES OF ETABS TESTED

- Concrete Wall Demand/Capacity Ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.003	1.00	0.30%

COMPUTER FILE: INDIAN IS 456-2000 WALL-002

CONCLUSION

The ETABS results show a very close match with the independent results.

HAND CALCULATION

WALL STRENGTH DETERMINED AS FOLLOWS:

1) A value of e = 1385 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for the pier, P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$C_c = C_{cw} + C_{cf}$, where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = \frac{0.36}{0.84} f_{ck} \bullet 200 \bullet (a - 200), \text{ where } a = 0.84 x_u$$

$$C_{cf} = \frac{0.36}{0.84} f_{ck} 200(2500\text{-}1000)$$

$$C_s = \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.36}{0.84} f_{ck}\right) + \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.36}{0.84} f_{ck}\right) + \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.36}{0.84} f_{ck}\right)$$

$$T = \frac{A_{s4}}{\gamma_s} f_{s4} + \frac{A_{s5}}{\gamma_s} f_{s5} + \frac{A_{s6}}{\gamma_s} f_{s6}$$

$$P_{n1} = \frac{0.36}{0.84} f_{ck} \bullet 200 \bullet (a - 200) + \frac{0.36}{0.84} f_{ck} 200(2500\text{-}1000) + \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.36}{0.84} f_{ck}\right) + \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.36}{0.84} f_{ck}\right)$$
$$+ \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.36}{0.84} f_{ck}\right) - \frac{A_{s4}}{\gamma_s} f_{s4} - \frac{A_{s5}}{\gamma_s} f_{s5} - \frac{A_{s6}}{\gamma_s} f_{s6}$$

(Eqn. 1)

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e^{'}}\left[C_{cf}\left(d - d'\right) + C_{cw}\left(d - \frac{a - t_f}{2} - t_f\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(4s\right) + C_{s3}\left(3s\right) - T_{s4}\left(2s\right) - T_{s5}\left(s\right)\right]$$

(Eqn. 2)

Where $C_{s1} = \frac{A_{s1}^{'}}{\gamma_s}\left(f_{s1} - \frac{0.36}{0.84}f_{ck}\right)$; $C_{s2} = \frac{A_{sn}^{'}}{\gamma_s}\left(f_{sn} - \frac{0.36}{0.84}f_{ck}\right)$; $T_{s4} = \frac{A_{sn}}{\gamma_s}\left(f_{sn}\right)$ and the bar strains and stresses are determined below.

The plastic centroid is at the center of the section and $d^{''}$ = 1150 mm

$e^{'} = e + d^{''} = 1138 + 1150 = 2535$ mm.

4) Using c = 1298.1 mm (from iteration)

$a = \beta_1 c = 0.84 \bullet 1298.1 = 1090.4$ mm

5) Assuming the extreme fiber strain equals 0.0035 and c= 1298.1 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain then, $f_s = f_y$:

$\varepsilon_{s1} = \left(\frac{c - d'}{c}\right)0.003$ =0.00323; $f_s = \varepsilon_s E \le F_y$; $f_{s1} =$ 460 MPa

$\varepsilon_{s2} = \left(\frac{c - s - d'}{c}\right)0.0035$ =0.00199 $f_{s2} =$ 398.0 MPa

$\varepsilon_{s3} = \left(\frac{c - 2s - d'}{c}\right)0.0035$ =0.00075 $f_{s3} =$ 150.0 MPa

$\varepsilon_{s4} = \left(\frac{d - c - 2s}{d - c}\right)\varepsilon_{s5}$ =0.00049 $f_{s4} =$ 98.1 MPa

$\varepsilon_{s5} = \left(\frac{d - c - s}{d - c}\right)\varepsilon_{s5}$ =0.00173 $f_{s5} =$ 346.1 MPa

$\varepsilon_{s6} = \left(\frac{d - c}{c}\right)0.0035$ =0.00297 $f_{s6} =$ 460.0 MPa

Substitute in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal gives,

P_{n1} = 8426 kN
P_{n2} = 8426 kN

$M_n = P_n e = 8426(1385)/1000$ = 11670 kN-m

EXAMPLE KBC 2009 Wall-001

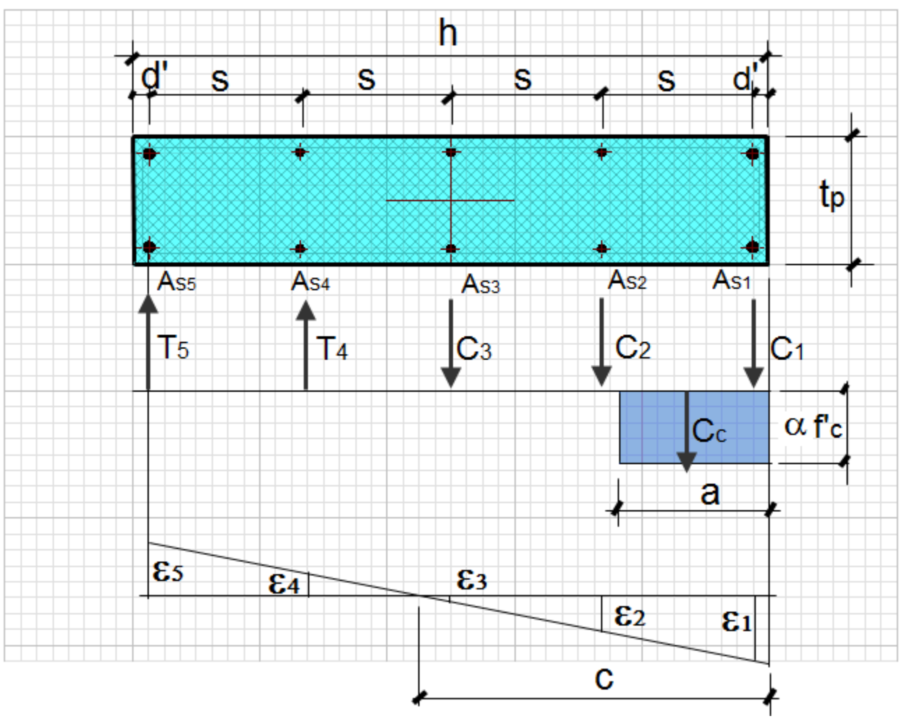
P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example.

A reinforced concrete wall is subjected to factored axial load P_u = 4549 kN and moments M_{uy} = 2622 kN-m. This wall is reinforced with two 30M bars at each end and 15M bars at 350 mm on center of each face. The total area of reinforcement is 4000 mm^2. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E = 25000 MPa	t = 300 mm	f_{ck} = 30 MPa
v = 0.2	h = 1500 mm	f_y = 460 MPa
	d = 50 mm	
	s = 350 mm	
	As_1= As_5 = 2-30M (1400 mm^2)	
	As_2, As_3, As_4 = 2-15M (400 mm^2)	

TECHNICAL FEATURES OF ETABS TESTED

- Flexural Demand/Capacity Ratio for a General Reinforcing pier section.

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.002	1.00	0.2%

COMPUTER FILE: KBC 2009 WALL-001

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

1) A value of *e* = 576.2 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for pier P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, *c*, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = 0.85 f_{ck} ab = 0.85 \bullet 30 \bullet 300a = 7650a$$
$$C_s = A_1 \left(f_{s1} - 0.85 \bullet f_{ck} \right) + A_2 \left(f_{s2} - 0.85 \bullet f_{ck} \right) + A_3 \left(f_{s3} - 0.85 \bullet f_{ck} \right)$$
$$T = A_{s4} f_{s4} + A_{s5} f_{s5}$$

$$P_{n1} = 7650a + A_1 \left(f_{s1} - 0.85 \bullet f_{ck} \right) + A_2 \left(f_{s2} - 0.85 \bullet f_{ck} \right) + A_3 \left(f_{s3} - 0.85 \bullet f_{ck} \right) - A_{s4} f_{s4} - A_{s5} f_{s5} \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'} \left[C_c \left(d - \frac{a}{2} \right) + C_{s1} \left(d - d' \right) + C_{s2} \left(3s \right) + C_{s3} \left(2s \right) - T_{s4} \left(s \right) \right] \qquad \text{(Eqn. 2)}$$

where $C_{s1} = A_1 \left(f_{s1} - 0.85 \bullet f_{ck} \right)$; $C_{s2} = A_2 \left(f_{s2} - 0.85 \bullet f_{ck} \right)$; $C_{s3} \left(f_{s3} - 0.85 \bullet f_{ck} \right)$; $T_{s4} = f_{s4} A_{s4}$ and the bar strains are determined below. The plastic centroid is at the center of the section and d'' = 700mm

$$e' = e + d'' = 576.2 + 700 = 1276.4 \text{ mm.}$$

4) Using c = 833.27 mm (from iteration),

$$a = \beta_1 c = 0.836 \bullet 833.27 = 696.61 \text{ mm},$$

5) Assuming the extreme fiber strain equals 0.003 and c= 833.27 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain then, $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c - d'}{c}\right)0.003 = 0.0028; \ f_s = \varepsilon_s E \le F_y \ ; \quad f_{s1} = 460.00 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c - s - d'}{c}\right)0.003 = 0.0016 \qquad f_{s2} = 312.0 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{c - 2s - d'}{c}\right)0.003 = 0.0003 \qquad f_{s3} = 60.0 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d - c - s}{d - c}\right)0.003 = 0.00103 \qquad f_{s4} = 259.5 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d - c}{c}\right)0.003 = 0.0022 \qquad f_{s5} = 444.1 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$$P_{n1} = 5340 \text{ kN}$$
$$P_{n2} = 5340 \text{ kN}$$

$$M_n = P_n e = 5340(576.4)/1000 = 3078 \text{ kN-m}$$

6) Calculate ϕ,

$$\phi P_n = 0.65(5340) = 3471 \text{ kN}$$
$$\phi M_n = 0.65(3078) = 2000.7 \text{ kN-m}$$

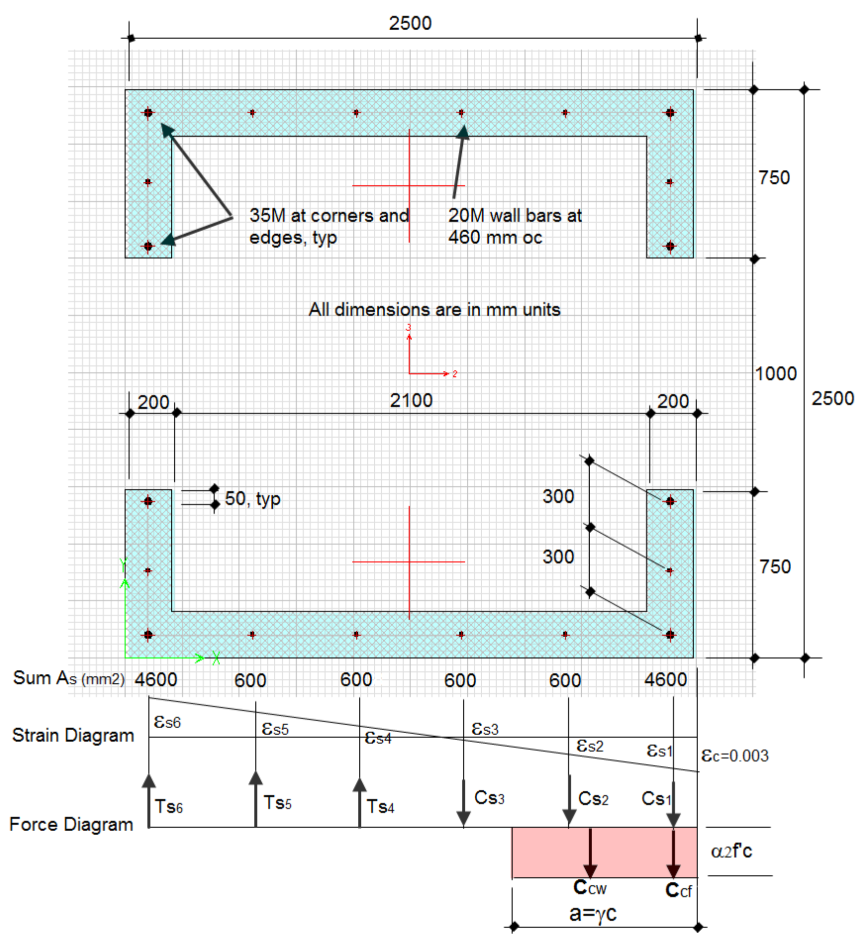
EXAMPLE KBC 2009 Wall-002

P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example. A reinforced concrete wall is subjected to a factored axial load P_u = 11256 kN and moments M_{uy} = 1498 kN-m. This wall is reinforced as noted below. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 25000 MPa
v = 0.2

Section Properties

t = 200 mm
H = 2500 mm
d = 2400 mm
s = 460 mm
$As_1 = As_5$ = 4-35M+2-20M (4600 mm^2)
As_2, As_3, As_4, As_5 = 2-20M (600 mm^2)

Design Properties

f_{ck} = 30 MPa
f_y = 420 MPa

TECHNICAL FEATURES OF ETABS TESTED

- Demand/Capacity Ratio for a General Reinforcing pier section.

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.007	1.00	0.7%

COMPUTER FILE: KBC 2009 WALL-002

CONCLUSION

The ETABS result shows a very close match with the independent result.

HAND CALCULATION

Wall Strength Determined as follows:

1) A value of e = 1199.2 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for the pier, P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$C_c = C_{cw} + C_{cf}$, where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = 0.85 f_{ck} \bullet 200 \bullet (a - 200)$$

$$C_{cf} = 0.85 f_{ck} (200 \bullet 1500)$$

$$C_s = A_1'(f_{s1} - 0.85 f_{ck}) + A_2'(f_{s2} - 0.85 f_{ck}) + A_3'(f_{s3} - 0.85 f_{ck})$$

$$T = A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6}$$

$$P_{n1} = 0.85 f_{ck} \bullet 200 \bullet (a - 200) + 0.85 f_{ck} (200 \bullet 1500) + A_1'(f_{s1} - 0.85 f_{ck}) + A_2'(f_{s2} - 0.85 f_{ck}) + A_3'(f_{s3} - 0.85 f_{ck}) + A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6} \quad \text{(Eqn. 1)}$$

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'} \left[\begin{array}{l} C_{cf}(d - d') + C_{cw}\left(d - \dfrac{a - t_f}{2} - t_f\right) + C_{s1}(d - d') + C_{s2}(4s) + \\ C_{s3}(3s) - T_{s4}(2s) - T_{s5}(s) \end{array} \right] \quad \text{(Eqn. 2)}$$

where $C_{s1} = A_1'\left(f_{s1} - 0.85 f_{ck}\right)$, $C_{sn} = A_n'\left(f_{sn} - 0.85 f_{ck}\right)$, $T_{sn} = f_{sn} A_{sn}$, and the bar strains are determined below. The plastic centroid is at the center of the section, d'' $= \dfrac{2500 - 200}{2}$ = 1150 mm

$$e' = e + d'' = 1199.2 + 1150 = 2349.2 \text{ mm}$$

4) Using c = 1480 mm (from iteration),

$$a = \beta_1 c = 0.836 \bullet 1480 = 1237.28 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.003 and c = 1480 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain then, $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c - d'}{c}\right)0.003 = 0.0028;\ f_s = \varepsilon_s E \le F_y;\qquad f_{s1} = 420.0 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c - s - d'}{c}\right)0.003 = 0.00186\qquad f_{s2} = 373.0 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{c - 2s - d'}{c}\right)0.003 = 0.00093\qquad f_{s3} = 186.5 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d - c - 2s}{d - c}\right)\varepsilon_{s6} = 0.0000\qquad f_{s4} = 0.0 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d - c - s}{d - c}\right)\varepsilon_{s6} = 0.00093\qquad f_{s5} = 186.5 \text{ MPa}$$

$$\varepsilon_{s6} = \left(\frac{d - c}{c}\right)0.003 = 0.00272\qquad f_{s6} = 373.0 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the axial force from two equations are less than 1%

P_{n1} = 13232 kN

P_{n2} = 13250 kN , use average P_n = 13242 kN

$M_n = P_n e = 13242(1199.2)/1000$ = 15879.8 kN-m

6) Calculate ϕ,

$$\phi P_n = 0.85(13242) = 11256 \text{ kN}$$

$$\phi M_n = 0.85(15879.8) = 13498 \text{ kN-m}$$

EXAMPLE Mexican RCDF-04 Wall-001

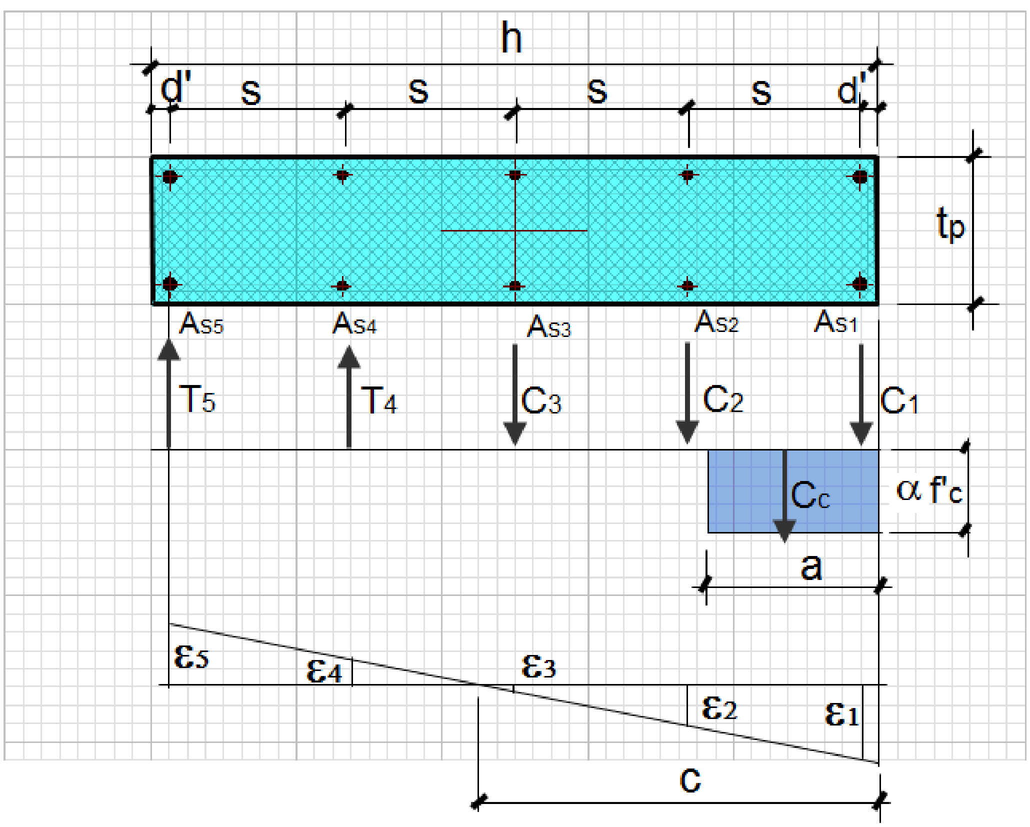
P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example.

A reinforced concrete column is subjected factored axial load P_u = 3545 kN and moments M_{uy} = 1817 kN-m. This wall is reinforced with two 30M bars at each end and 15M bars at 350 mm on center of each face. The total area of reinforcement is 4000 mm^2. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E = 25000 MPa	tb = 300 mm	f'_c = 30 MPa
v = 0.2	h = 1500 mm	f_y = 460 MPa
	d = 50 mm	
	s = 350 mm	
	As1= As5 = 2-30M (1400 mm^2)	
	As2, As3, As4 = 2-15M (400 mm^2)	

TECHNICAL FEATURES OF ETABS TESTED

- Flexural Demand/Capacity Ratio for a General Reinforcing pier section.

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.016	1.00	1.60%

COMPUTER FILE: MEXICAN RCDF-04 WALL-001

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

f'_c = 30MPa f_y = 460 MPa
b = 300mm h = 1500 mm

1) A value of e = 512.5 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for pier P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = 0.85 f_c^* ab = 0.85 \bullet 0.8 \bullet 30 \bullet 300a = 6120a$$

$$C_s = A_1 \left(f_{s1} - 0.85 \bullet 0.8 \bullet f_c^* \right) + A_2 \left(f_{s2} - 0.85 \bullet 0.8 \bullet f_c^* \right) + A_3 \left(f_{s3} - 0.85 \bullet 0.8 \bullet f_c^* \right)$$

$$T = A_{s4} f_{s4} + A_{s5} f_{s5}$$

$$P_{n1} = 6120a + A_1 \left(f_{s1} - 0.85 \bullet 0.8 \bullet f_c^* \right) + A_2 \left(f_{s2} - 0.85 \bullet 0.8 \bullet f_c^* \right) + A_3 \left(f_{s3} - 0.85 \bullet 0.8 \bullet f_c^* \right) - A_{s4} f_{s4} - A_{s5} f_{s5} \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'} \left[C_c \left(d - \frac{a}{2} \right) + C_{s1} \left(d - d' \right) + C_{s2} \left(3s \right) + C_{s3} \left(2s \right) - T_{s4} \left(s \right) \right] \qquad \text{(Eqn. 2)}$$

where $C_{s1} = A_1\left(f_{s1} - 0.85 \bullet 0.8 \bullet f_c^*\right)$; $C_{s2} = A_2\left(f_{s2} - 0.85 \bullet 0.8 \bullet f_c^*\right)$; $C_{s3}\left(f_{s3} - 0.85 \bullet 0.8 \bullet f_c^*\right)$; $T_{s4} = f_{s4}A_{s4}$ and the bar strains are determined below. The plastic centroid is at the center of the section and d'' = 700mm

$e' = e + d'' = 512.5 + 700 = 1212.5$ mm.

4) Using c = 936.2 mm (from iteration)

$$a = \beta c = 0.85 \bullet 916.2 = 805 \text{ mm},$$

5) Assuming the extreme fiber strain equals 0.003 and c = 936.2 inch, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain then, $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c - d'}{c}\right)0.003 = 0.0028; \quad f_s = \varepsilon_s E \le F_y; \quad f_{s1} = 460.00 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c - s - d'}{c}\right)0.003 = 0.0017 \qquad f_{s2} = 343.6 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{d - c - 2s}{d - c}\right)\varepsilon_{s5} = 0.0005 \qquad f_{s3} = 119.3 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d - c - s}{d - c}\right)\varepsilon_{s5} = 0.0060 \qquad f_{s4} = 105.4 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d - c}{c}\right)0.003 = 0.0175 \qquad f_{s5} = 329.3 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

P_{n1} = 5064 kN
P_{n2} = 5064 kN

$$M_n = P_n e = 5064(512.5)/1000000 = 2595 \text{ kN-m}$$

7) Calculate ϕP_n and, ϕM_n,

$$\phi P_n = 0.70(5064) = 3545 \text{ kips}$$
$$\phi M_n = 0.70(2595) = 1817 \text{ k-ft.}$$

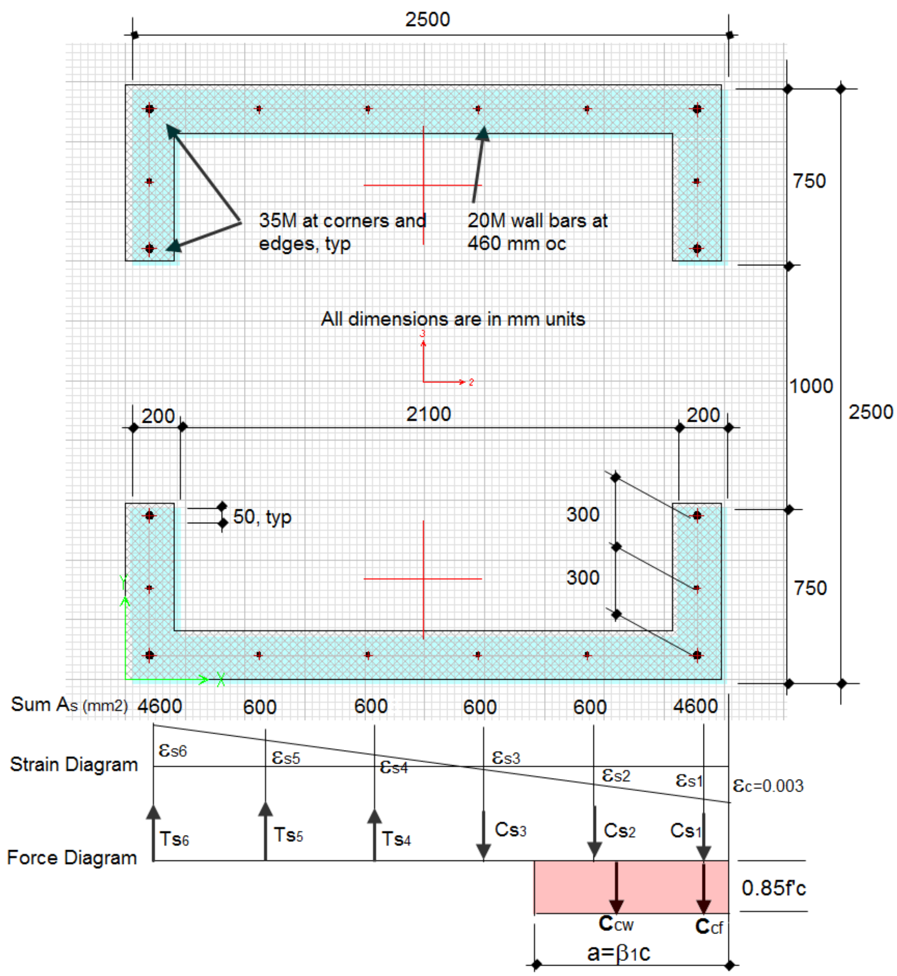
EXAMPLE Mexican RCDF-2004 Wall-002

P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial loading and moment is tested in this example. The wall is reinforced as shown below. The concrete core wall is loaded with a factored axial load P_u = 10165 kN and moments M_{u3} = 11430 kN-m. The design capacity ratio is checked by hand calculations and results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E = 25000 MPa	tb = 200 mm	f'_c = 30 MPa
v = 0.2	H = 2500 mm	f_y = 460 MPa
	d = 2400 mm	
	s = 460 mm	
	As1= As5 = 4-35M+2-20M (4600 mm^2)	
	As2, As3, As4, As5 = 2-20M (600 mm^2)	

TECHNICAL FEATURES OF ETABS TESTED

- Demand/Capacity Ratio for a General Reinforcing pier section.

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.000	1.00	0.000%

COMPUTER FILE: MEXICAN RCDF-04 WALL-002

CONCLUSION

The ETABS results show an acceptable match with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

1) A value of e = 1124.4 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for the pier, P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$C_c = C_{cw} + C_{cf}$, where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = 0.85 \bullet 0.8 f_c' \bullet 200 \bullet (a - 200)$$

$$C_{cf} = 0.85 \bullet 0.8 f_c' (200 \bullet 1500)$$

$$C_s = A_1 \left(f_{s1} - 0.85 \bullet 0.8 \bullet f_c' \right) + A_2 \left(f_{s2} - 0.85 \bullet 0.8 \bullet f_c' \right) + A_3 \left(f_{s3} - 0.85 \bullet 0.8 \bullet f_c' \right)$$

$$T = A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6}$$

$$P_{n1} = 0.85 \bullet 0.8 f_c' \bullet 200 \bullet (a - 200) + 0.85 \bullet 0.8 f_c' (200 \bullet 1500) + A_1 \left(f_{s1} - 0.85 \bullet 0.8 \bullet f_c' \right) + A_2 \left(f_{s2} - 0.85 \bullet 0.8 \bullet f_c' \right) + A_3 \left(f_{s3} - 0.85 \bullet 0.8 \bullet f_c' \right) - A_{s4} f_{s4} - A_{s5} f_{s5} - A_{s6} f_{s6}$$

(Eqn. 1)

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'} \left[\begin{array}{l} C_{cf} \left(d - d' \right) + C_{cw} \left(d - \dfrac{a - t_f}{2} - t_f \right) + C_{s1} \left(d - d' \right) + \\ C_{s2} (4s) + C_{s3} (3s) - T_{s4} (2s) - T_{s5} (s) \end{array} \right]$$

(Eqn. 2)

where $C_{s1} = A_1\left(f_{s1} - 0.85 \bullet 0.8 f_c'\right)$, $C_{sn} = A_n\left(f_{sn} - 0.85 \bullet 0.8 f_c'\right)$, $T_{sn} = f_{sn} A_{sn}$, and the bar strains are determined below. The plastic centroid is at the center of the section,

$$d'' = \frac{2500 - 200}{2} = 1150 \text{ mm}$$

$$e' = e + d'' = 1124.4 + 1150 = 2274.4 \text{ mm}$$

4) Using c = 1413 mm (from iteration)

$$a = 0.85c = 0.85 \bullet 1413 = 1201 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.003 and c = 1413 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain then, $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c - d'}{c}\right)0.003 = 0.00279; \quad f_s = \varepsilon_s E \le F_y; \quad f_{s1} = 460.0 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c - s - d'}{c}\right)0.003 = 0.00181 \qquad f_{s2} = 362.2 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{c - 2s - d'}{c}\right)0.003 = 0.00083 \qquad f_{s2} = 166.8 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d - c - 2s}{d - c}\right)\varepsilon_{s6} = 0.00014 \qquad f_{s3} = 28.6 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d - c - s}{d - c}\right)\varepsilon_{s6} = 0.00112 \qquad f_{s4} = 223.9 \text{ MPa}$$

$$\varepsilon_{s6} = \left(\frac{d - c}{c}\right)0.003 = 0.00210 \qquad f_{s5} = 419.3 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$$P_{n1} = 14522 \text{ kN}$$

$$P_{n2} = 14522 \text{ kN}$$

$$M_n = P_n e = 14522(1124.4)/1000000 = 16328 \text{ kN-m}$$

6) Calculate ϕP_n and ϕM_n,

$$\phi P_n = 0.70\left(14522\right) = 10165 \text{ kN}$$

$$\phi M_n = 0.70\left(16382\right) = 11430 \text{ kN-m}$$

EXAMPLE NZS-3101-2006 Wall-001

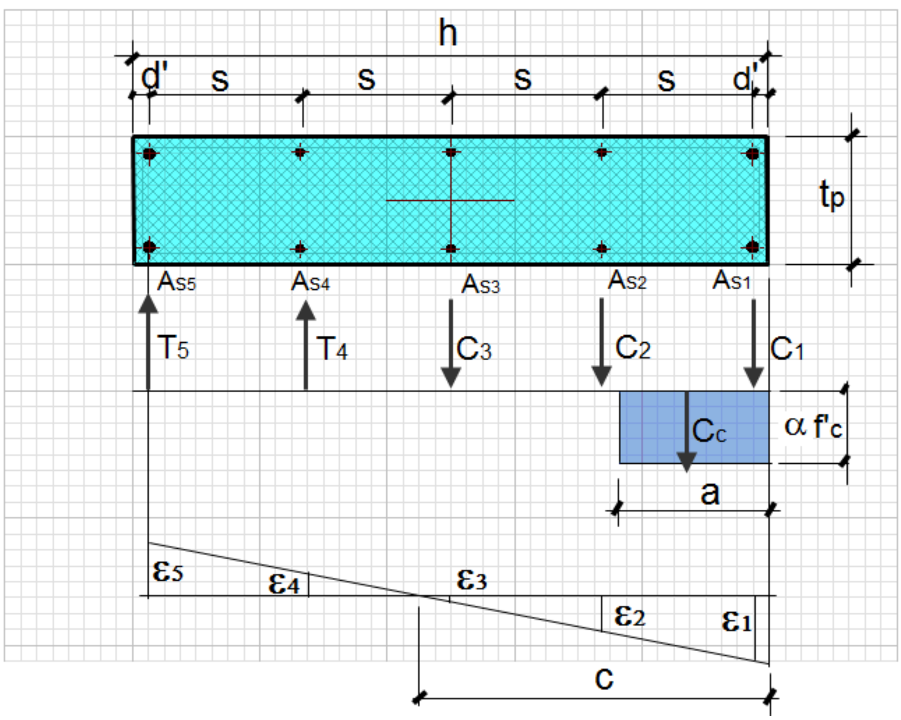
P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example.

A reinforced concrete wall is subjected to factored axial load P_u = 4549 kN and moments M_{uy} = 2622 kN-m. This wall is reinforced with two 30M bars at each end and 15M bars at 350 mm on center of each face. The total area of reinforcement is 4000 mm^2. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties		Section Properties		Design Properties
E =	25000 MPa	tb =	300 mm	f'_c = 30 MPa
v =	0.2	h =	1500 mm	f_y = 460 MPa
		d =	50 mm	
		s =	350 mm	
		As1= As5 = 2-30M (1400 mm^2)		
		As2, As3, As4 = 2-15M (400 mm^2)		

TECHNICAL FEATURES OF ETABS TESTED

- Flexural Demand/Capacity Ratio for a General Reinforcing pier section.

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.000	1.00	0.00%

COMPUTER FILE: NZS 3101-06 WALL-001

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

1) A value of e = 576.2 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for pier P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = 0.85 f_c' ab = 0.85 \bullet 30 \bullet 300a = 7650a$$
$$C_s = A_1\left(f_{s1} - 0.85 \bullet f_c'\right) + A_2\left(f_{s2} - 0.85 \bullet f_c'\right) + A_3\left(f_{s3} - 0.85 \bullet f_c'\right)$$
$$T = A_{s4} f_{s4} + A_{s5} f_{s5}$$

$$P_{n1} = 7650a + A_1\left(f_{s1} - 0.85 \bullet f_c'\right) + A_2\left(f_{s2} - 0.85 \bullet f_c'\right) + A_3\left(f_{s3} - 0.85 \bullet f_c'\right) - A_{s4} f_{s4} - A_{s5} f_{s5} \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(3s\right) + C_{s3}\left(2s\right) - T_{s4}\left(s\right)\right] \qquad \text{(Eqn. 2)}$$

where $C_{s1} = A_1\left(f_{s1} - 0.85 \bullet f_c'\right)$; $C_{s2} = A_2\left(f_{s2} - 0.85 \bullet f_c'\right)$; $C_{s3}\left(f_{s3} - 0.85 \bullet f_c'\right)$; $T_{s4} = f_{s4} A_{s4}$ and the bar strains are determined below. The plastic centroid is at the center of the section and d'' = 700mm

$$e' = e + d'' = 576.2 + 700 = 1276.4 \text{ mm.}$$

4) Using c = 821.7 mm (from iteration),

$$a = \gamma c = 0.85 \bullet 821.7 = 698.46 \text{ mm},$$

5) Assuming the extreme fiber strain equals 0.003 and c= 821.7 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain then, $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.003 = 0.0028; \; f_s = \varepsilon_s E \le F_y \;; \quad f_{s1} = 460.00 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.003 = 0.0015 \qquad f_{s2} = 307.9 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s5} = 0.0003 \qquad f_{s3} = 52.3 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s5} = 0.0010 \qquad f_{s4} = 203.2 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d-c}{c}\right)0.003 = 0.0023 \qquad f_{s5} = 458.8 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$$P_{n1} = 5352 \text{ kN}$$
$$P_{n2} = 5352 \text{ kN}$$

$$M_n = P_n e = 5352(576.4)/1000000 = 3085 \text{ kN-m}$$

6) Calculate ϕ,

$$\phi P_n = 0.85(5352) = 4549 \text{ kN}$$
$$\phi M_n = 0.85(3085) = 2622 \text{ kN-m}$$

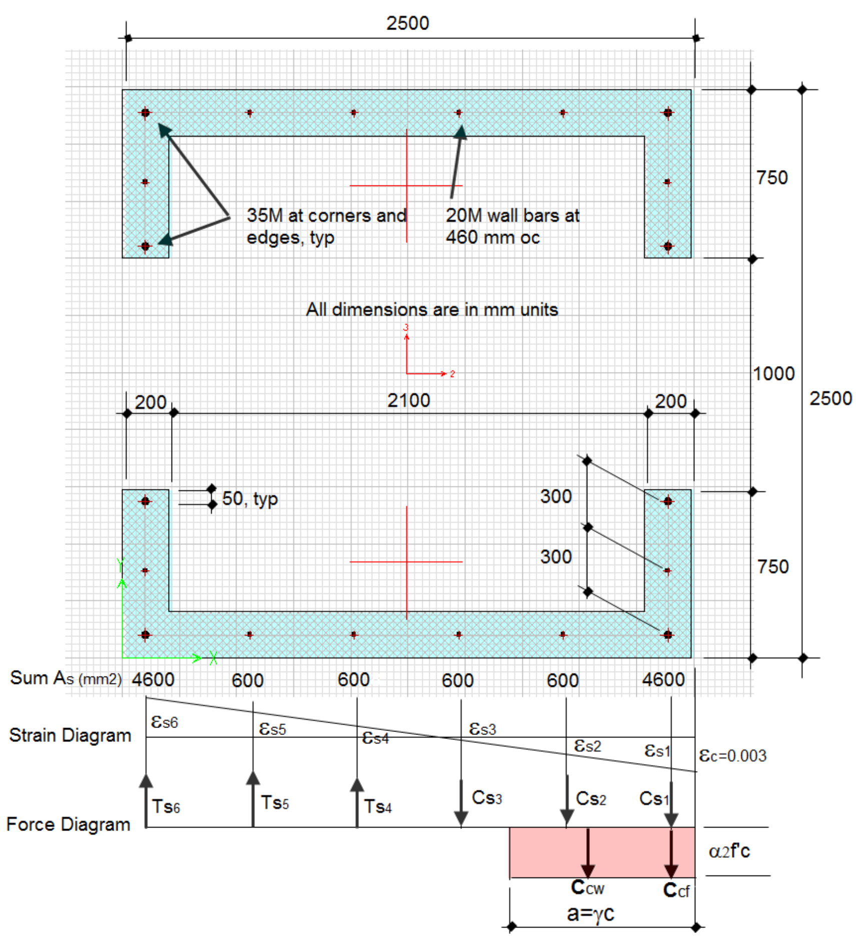
EXAMPLE NZS 3101-06 Wall-002

P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example. A reinforced concrete wall is subjected to a factored axial load P_u = 13625 kN and moments M_{uy} = 16339 kN-m. This wall is reinforced as noted below. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties		Section Properties		Design Properties
E =	25000 MPa	tb =	200 mm	f'_c = 30 MPa
v =	0.2	H =	2500 mm	f_y = 460 MPa
		d =	2400 mm	
		s =	460 mm	
		As1= As5 = 4-35M+2-20M (4600 mm^2)		
		As2, As3, As4, As5 = 2-20M (600 mm^2)		

TECHNICAL FEATURES OF ETABS TESTED

- Demand/Capacity Ratio for a General Reinforcing pier section.

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.000	1.00	0.00%

COMPUTER FILE: NZS 3101-06 WALL-002

CONCLUSION

The ETABS result shows a very close match with the independent result.

HAND CALCULATION

Wall Strength Determined as follows:

1) A value of e = 1199.2 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for the pier, P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = 0.85 f_c' ab = 0.85 \bullet 30 \bullet 300a = 7650a$$

$C_c = C_{cw} + C_{cf}$, where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = 0.85 f_c' \bullet 200 \bullet (a - 200)$$

$$C_{cf} = 0.85 f_c' (200 \bullet 2500)$$

$$C_s = A_1' (f_{s1} - 0.85 f_c') + A_2' (f_{s2} - 0.85 f_c') + A_3' (f_{s3} - 0.85 f_c')$$

$$T = A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6}$$

$$P_{n1} = 0.85 f_c' \bullet 8 \bullet (a - 8) + 0.85 f_c' (8 \bullet 98) + A_1' (f_{s1} - 0.85 f_c') + A_2' (f_{s2} - 0.85 f_c') + A_3' (f_{s3} - 0.85 f_c') + A_{s4} f_{s4} + A_{s5} f_{s5} + A_{s6} f_{s6} \quad \text{(Eqn. 1)}$$

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'} \left[C_{cf} (d - d') + C_{cw} \left(d - \frac{a - t_f}{2} - t_f \right) + C_{s1} (d - d') + C_{s2} (4s) + C_{s3} (3s) - T_{s4} (2s) - T_{s5} (s) \right] \quad \text{(Eqn. 2)}$$

where $C_{s1} = A_1'\left(f_{s1} - 0.85 f_c'\right)$, $C_{sn} = A_n'\left(f_{sn} - 0.85 f_c'\right)$, $T_{sn} = f_{sn} A_{sn}$, and the bar strains are determined below. The plastic centroid is at the center of the section, d'' $= \dfrac{2500 - 200}{2}$ = 1150 mm

$$e' = e + d'' = 1199.2 + 1150 = 2349.2 \text{ mm}$$

4) Using c = 1259.8 mm (from iteration),

$$a = \beta_1 c = 0.85 \bullet 1259.8 = 1070.83 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.003 and c = 1259.8 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain then, $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c - d'}{c}\right)0.003 = 0.00276; \; f_s = \varepsilon_s E \le F_y; \quad f_{s1} = 460.0 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c - s - d'}{c}\right)0.003 = 0.00167 \quad f_{s2} = 333.3 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{c - 2s - d'}{c}\right)0.003 = 0.00057 \quad f_{s3} = 114.2 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d - c - 2s}{d - c}\right)\varepsilon_{s6} = 0.00052 \quad f_{s4} = 104.9 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d - c - s}{d - c}\right)\varepsilon_{s6} = 0.00167 \quad f_{s5} = 324.0 \text{ MPa}$$

$$\varepsilon_{s6} = \left(\frac{d - c}{c}\right)0.003 = 0.00272 \quad f_{s6} = 460.0 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$$P_{n1} = 16029 \text{ kN}$$

$$P_{n2} = 16029 \text{ kN}$$

$$M_n = P_n e = 16029(1199.2)/1000000 = 19222 \text{ kN-m}$$

6) Calculate ϕ,

$$\phi P_n = 0.85(16029) = 13625 \text{ kN}$$

$$\phi M_n = 0.85(19222) = 16339 \text{ kN-m}$$

EXAMPLE Singapore CP65-99 Wall-001

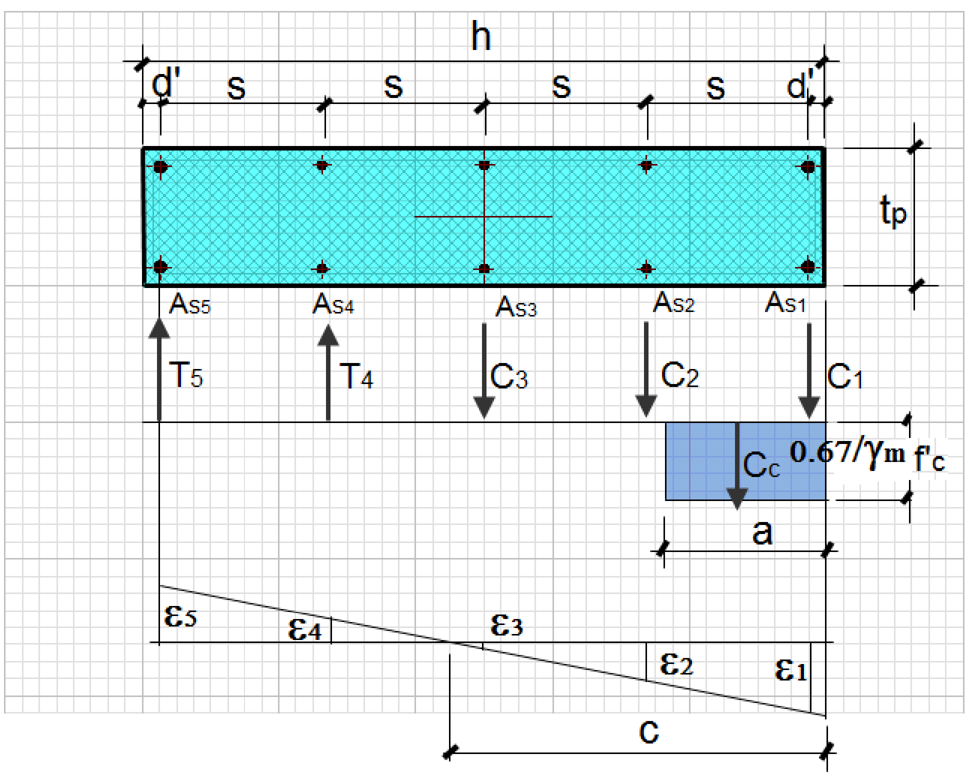
P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example.

A reinforced concrete wall is subjected factored axial load P_u = 3246 kN and moments M_{uy} = 1969 kN-m. This wall is reinforced with two 30M bars at each end and 15M bars at 350 mm on center of each face. The total area of reinforcement is 4000 mm^2. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 25000 MPa
v = 0.2

Section Properties

tb = 300 mm
h = 1500 mm
d' = 50 mm
s = 350 mm
As1= As5 = 2-30M (1400 mm^2)
As2, As3, As4 = 2-15M (400 mm^2)

Design Properties

f'_c = 30 MPa
f_y = 460 MPa

TECHNICAL FEATURES OF ETABS TESTED

- Concrete wall flexural demand/capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	0.997	1.00	0.30%

COMPUTER FILE: SINGAPORE CP65-99 WALL-001

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

f'_c = 30MPa f_y = 460 MPa
b = 300mm h = 1500 mm

1) A value of e = 606.5 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for pier P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

The distance to the neutral axis for a balanced condition, c_b:

$$c_b = \frac{700}{700 + f_y / \gamma_s} d_t = \frac{700}{700 + 460/1.15}(1450) = 922.7 \text{ mm}$$

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = \frac{0.67}{\gamma_m} f_{cu} ab = \frac{0.67}{1.5} \bullet 30 \bullet 300a = 4020a$$

$$C_s = \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.67}{\gamma_m} f'_c \right) + \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.67}{\gamma_m} f'_c \right) + \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.67}{\gamma_m} f'_c \right)$$

$$T = \frac{A_{s4}}{\gamma_s} f_{s4} + \frac{A_{s5}}{\gamma_s} f_{s5}$$

$$P_{n1} = 4709a + \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.67}{\gamma_m} f'_c \right) + \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.67}{\gamma_m} f'_c \right) + \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.67}{\gamma_m} f'_c \right) - \frac{A_{s4}}{\gamma_s} f_{s4} + \frac{A_{s5}}{\gamma_s} f_{s5} \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'}\begin{bmatrix} C_c\left(d - \frac{a}{2}\right) + C_{s1}(d - d') + C_{s2}(d - d' - s) + \\ C_{s3}(2s) - T_{s4}(s) \end{bmatrix} \qquad \text{(Eqn. 2)}$$

where $C_{s1} = \frac{A_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.67}{\gamma_m}f'_c\right)$; $C_{s2} = \frac{A_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.67}{\gamma_m}f'_c\right)$;
$C_{s3} = \frac{A_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.67}{\gamma_m}f'_c\right)$; $T_{s4} = \frac{A_{s4}}{\gamma_s}\left(f_{s4} - \frac{0.67}{\gamma_m}f'_c\right)$ and the bar strains and stresses are determined below.

The plastic centroid is at the center of the section and d'' = 700 mm

$$e' = e + d'' = 606.5 + 700 = 1306.5 \text{ mm.}$$

4) Using c = 887.5 mm (from iteration), which is slightly more than c_b (922.7mm).

$$a = \beta_1 c = 0.90 \bullet 875.2 = 787.6 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.0035 and c = 875.2 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.0035 = 0.00330; \; f_s = \varepsilon_s E \le F_y; \quad f_{s1} = 460 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.0035 = 0.00190 \qquad f_{s2} = 380.1 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s5} = 0.00050 \qquad f_{s3} = 100.1 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s5} = 0.00090 \qquad f_{s4} = 179.8 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d-c}{c}\right)0.0035 = 0.00230 \qquad f_{s5} = 459.7 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$$P_{n1} = 3246 \text{ kN}$$
$$P_{n2} = 3246 \text{ kN}$$

$$M_n = P_n e = 3246(606.5)/1000 = 1969 \text{ kN-m}$$

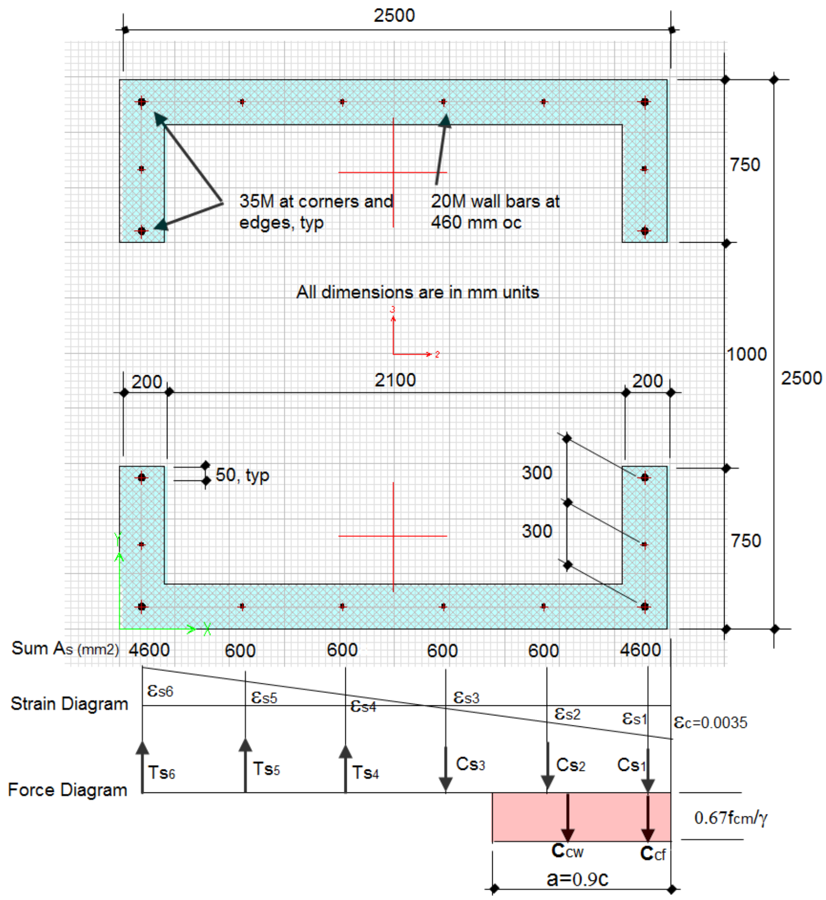
EXAMPLE Singapore CP65-99 Wall-002

P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example. A reinforced concrete wall is subjected to a factored axial load P_u = 8368 kN and moments M_{uy} = 11967 kN-m. This wall is reinforced as noted below. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties

E = 25000 MPa
v = 0.2

Section Properties

tb = 200 mm
H = 2500 mm
d = 2400 mm
s = 460 mm
As1= As5 = 4-35M+2-20M (4600 mm^2)
As2, As3, As4, As5 = 2-20M (600 mm^2)

Design Properties

f'_c = 30 MPa
f_y = 460 MPa

TECHNICAL FEATURES OF ETABS TESTED

- Concrete wall flexural demand/capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	1.001	1.00	0.10%

COMPUTER FILE: SINGAPORE CP65-99 WALL-002

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

1) A value of e = 1430 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for pier P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$C_c = C_{cw} + C_{cf}$, where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = \frac{0.67}{\gamma_m} f_{cu} \bullet 200 \bullet \left(a - 200\right)$$

$$C_{cf} = \frac{0.67}{\gamma_m} f_{cu} \left(200 \bullet 2500\right)$$

$$C_s = \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.67}{\gamma_m} f'_c\right) + \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.67}{\gamma_m} f'_c\right) + \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.67}{\gamma_m} f'_c\right)$$

$$T = \frac{A_{s4}}{\gamma_s} f_{s4} + \frac{A_{s5}}{\gamma_s} f_{s5} + \frac{A_{s6}}{\gamma_s} f_{s6}$$

$$P_{n1} = \frac{0.67}{\gamma_m} f_{cu} \bullet 200 \bullet \left(a - 200\right) + \frac{0.67}{\gamma_m} f_{cu}\left(200 \bullet 2500\right) + \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.67}{\gamma_m} f'_c\right) + \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.67}{\gamma_m} f'_c\right) + \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.67}{\gamma_m} f'_c\right) - \frac{A_{s4}}{\gamma_s} f_{s4} + \frac{A_{s5}}{\gamma_s} f_{s5} + \frac{A_{s6}}{\gamma_s} f_{s6} \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'}\left[C_{cf}\left(d - d'\right) + C_{cw}\left(d - \frac{a - t_f}{2} - t_f\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(4s\right) + C_{s3}\left(3s\right) - T_{s4}\left(2s\right) - T_{s5}\left(s\right)\right] \qquad \text{(Eqn. 2)}$$

where $C_{s1} = \frac{A_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.67}{\gamma_m} f_c'\right)$; $C_{sn} = \frac{A_{sn}}{\gamma_s}\left(f_{sn} - \frac{0.67}{\gamma_m} f_c'\right)$; $T_{sn} = \frac{A_{sn}}{\gamma_s}\left(f_{sn} - \frac{0.67}{\gamma_m} f_c'\right)$
and the bar strains and stresses are determined below.

The plastic centroid is at the center of the section and d'' = 1150 mm

$$e' = e + d'' = 1430 + 1150 = 2580 \text{ mm.}$$

4) Using c = 1160 mm (from iteration),

$$a = \beta_1 c = 0.9 \bullet 1160 = 1044 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.0035 and c= 1160 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain then, $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.0035 = 0.00320; \; f_s = \varepsilon_s E \le F_y; \quad f_{s1} = 460 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d}{c}\right)0.0035 = 0.00181 \qquad f_{s2} = 362.0 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{c-2s-d'}{c}\right)0.0035 = 0.00042 \qquad f_{s3} = 84.4 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s6} = 0.00097 \qquad f_{s4} = 193.2 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s6} = 0.00235 \qquad f_{s5} = 460.00 \text{ MPa}$$

$$\varepsilon_{s6} = \left(\frac{d-c}{c}\right)0.0035 = 0.00374 \qquad f_{s6} = 460.00 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

P_{n1} = 8368 kN
P_{n2} = 8368 kN

$$M_n = P_n e = 8368(1430)/1000 = 11{,}967 \text{ kN-m}$$

EXAMPLE Turkish TS 500-2000 Wall-001

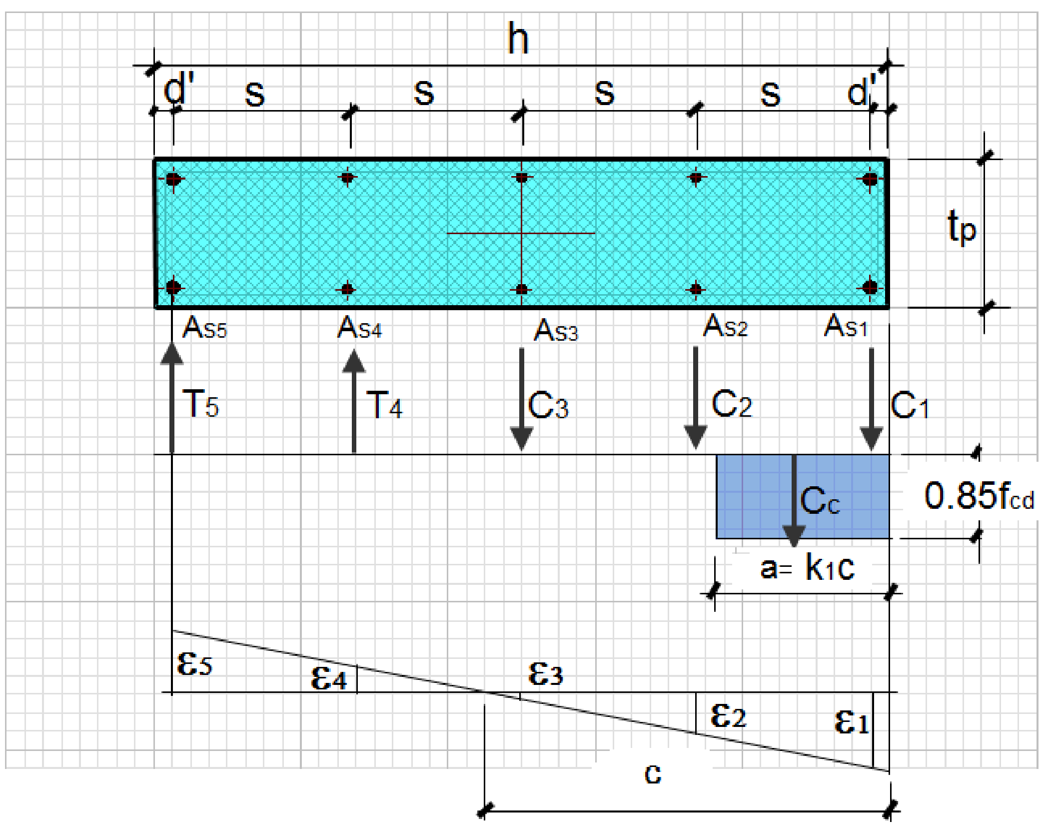
P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example.

A reinforced concrete wall is subjected factored axial load P_u = 3132 kN and moments M_{uy} = 1956 kN-m. This wall is reinforced with two 30M bars at each end and 15M bars at 350 mm on center of each face. The total area of reinforcement is 4000 mm^2. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties	Section Properties	Design Properties
E = 25000 MPa	tb = 300 mm	f'_c = 25 MPa
v = 0.2	h = 1500 mm	f_y = 420 MPa
	d = 50 mm	
	s = 350 mm	
	As1= As5 = 2-30M (1400 mm^2)	
	As2, As3, As4 = 2-15M (400 mm^2)	

TECHNICAL FEATURES OF ETABS TESTED

- Wall flexural demand/capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	0.997	1.00	0.30%

COMPUTER FILE: TURKISH TS 500-2000 WALL-001

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

f'_c = 25 MPa f_y = 420 MPa
b = 300mm h = 1500 mm

1) A value of e = 715 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for pier P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$$C_c = 0.85\frac{f_{ck}}{\gamma_c}ab = \frac{0.67}{1.5}\bullet 25\bullet 300a = 3350a$$

$$C_s = \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.85}{\gamma_c}f_{ck}\right) + \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.85}{\gamma_c}f_{ck}\right) + \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.85}{\gamma_c}f_{ck}\right)$$

$$T = \frac{A_{s4}}{\gamma_s}f_{s4} + \frac{A_{s5}}{\gamma_s}f_{s5}$$

$$P_{n1} = 3350a + \frac{A'_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.85}{\gamma_c}f_{ck}\right) + \frac{A'_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.85}{\gamma_c}f_{ck}\right) + \frac{A'_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.85}{\gamma_c}f_{ck}\right) - \frac{A_{s4}}{\gamma_s}f_{s4} + \frac{A_{s5}}{\gamma_s}f_{s5} \qquad \text{(Eqn. 1)}$$

3) Taking moments about A_{s5}:

$$P_{n2} = \frac{1}{e'}\left[C_c\left(d - \frac{a}{2}\right) + C_{s1}\left(d - d'\right) + C_{s2}\left(d - d' - s\right) + C_{s3}\left(2s\right) - T_{s4}\left(s\right)\right] \qquad \text{(Eqn. 2)}$$

where $C_{s1} = \frac{A_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.85}{\gamma_c} f_{ck} \right)$; $C_{s2} = \frac{A_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.85}{\gamma_c} f_{ck} \right)$;

$C_{s3} = \frac{A_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.85}{\gamma_c} f_{ck} \right)$; $T_{s4} = \frac{A_{s4}}{\gamma_s}\left(f_{s4} - \frac{0.85}{\gamma_c} f_{ck} \right)$ and the bar strains and stresses are determined below.

The plastic centroid is at the center of the section and d'' = 700 mm

$$e' = e + d'' = 715 + 700 = 1415 \text{ mm.}$$

4) Using c = 853.4 mm (from iteration),

$$a = k_1 c = 0.85 \bullet 853.4 = 725.4 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.0030 and c = 853.4 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain then, $f_s = f_y$:

$$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.003 = 0.00282; \; f_s = \varepsilon_s E \le F_y; \quad f_{s1} = 420.0 \text{ MPa}$$

$$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.003 = 0.00159 \qquad f_{s2} = 318.8 \text{ MPa}$$

$$\varepsilon_{s3} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s5} = 0.00036 \qquad f_{s3} = 72.7 \text{ MPa}$$

$$\varepsilon_{s4} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s5} = 0.00087 \qquad f_{s4} = 173.4 \text{ MPa}$$

$$\varepsilon_{s5} = \left(\frac{d-c}{c}\right)0.003 = 0.00210 \qquad f_{s5} = 419.5 \text{ MPa}$$

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$$P_{n1} = 3132 \text{ kN}$$
$$P_{n2} = 3132 \text{ kN}$$

$$M_n = P_n e = 3132(624.4)/1000 = 1956 \text{ kN-m}$$

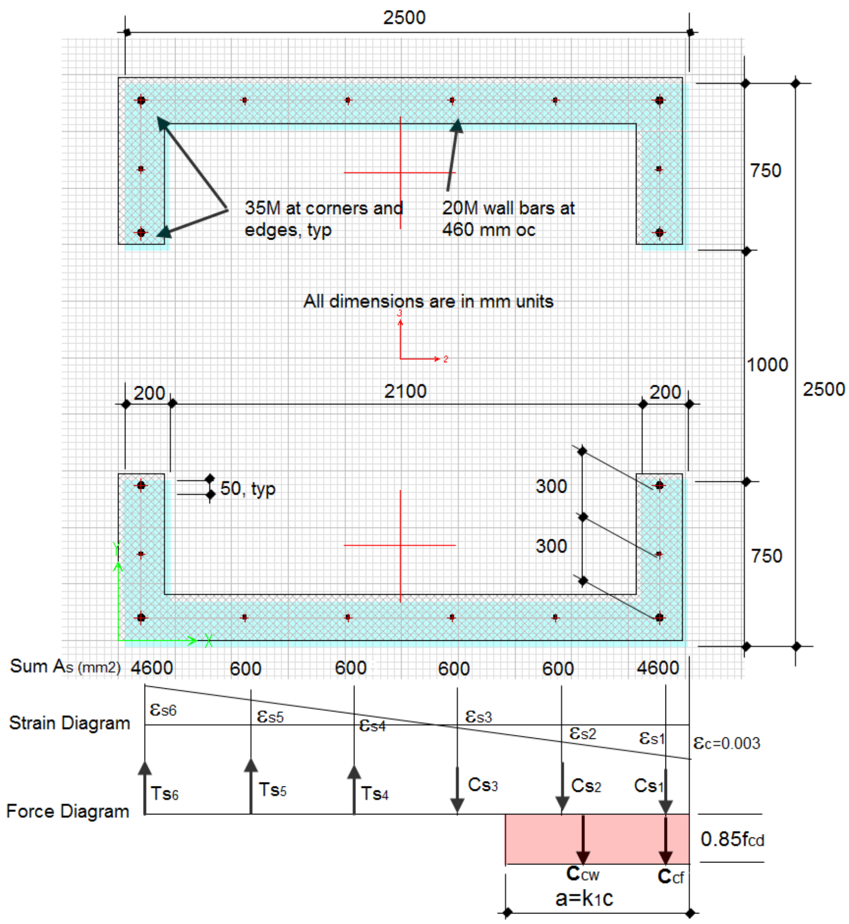
EXAMPLE Turkish TS 500-2000 Wall-002

P-M INTERACTION CHECK FOR A WALL

EXAMPLE DESCRIPTION

The Demand/Capacity ratio for a given axial load and moment are tested in this example. A reinforced concrete wall is subjected to factored axial load P_u = 9134 kN and moments M_{uy} = 11952 kN-m. This wall is reinforced as noted below. The design capacity ratio is checked by hand calculations and the results are compared with ETABS program results.

GEOMETRY, PROPERTIES AND LOADING



Material Properties		Section Properties		Design Properties
E =	25000 MPa	tb =	200 mm	f'_c = 25 MPa
v =	0.2	H =	2500 mm	f_y = 420 MPa
		d =	2400 mm	
		s =	460 mm	
		As1= As5 = 4-35M+2-20M (4600 mm^2)		
		As2, As3, As4, As5 = 2-20M (600 mm^2)		

TECHNICAL FEATURES OF ETABS TESTED

- Concrete wall demand/capacity ratio

RESULTS COMPARISON

Independent results are hand calculated and compared with ETABS design check.

Output Parameter	ETABS	Independent	Percent Difference
Wall Demand/Capacity Ratio	0.996	1.00	0.40%

COMPUTER FILE: TURKISH TS 500-2000 WALL-002

CONCLUSION

The ETABS results show an acceptable match with the independent results.

HAND CALCULATION

Wall Strength Determined as follows:

1) A value of e = 1308.6 mm was determined using $e = M_u / P_u$ where M_u and P_u were taken from the ETABS test model PMM interaction diagram for the pier, P1. Values for M_u and P_u were taken near the balanced condition and large enough to produce a flexural D/C ratio very close to or equal to one. The depth to the neutral axis, c, was determined by iteration using an excel spreadsheet so that equations 1 and 2 below were equal.

2) From the equation of equilibrium:

$$P_n = C_c + C_s - T$$

where

$C_c = C_{cw} + C_{cf}$, where C_{cw} and C_{cf} are the area of the concrete web and flange in compression

$$C_{cw} = \frac{f_{ck}}{\gamma_c} \bullet 200 \bullet (a - 200) = \frac{0.85 \bullet 30}{1.5} \bullet 200 \bullet (a - 200) = 3400(a - 200)$$

$$C_{cf} = 0.85 \bullet \frac{f_{ck}}{\gamma_c}\left(200 \bullet (2500 - 1000)\right) = \frac{0.85(30)}{1.5}\left(200 \bullet (2500 - 1000)\right) = 5{,}100{,}000$$

$$C_s = A_1'\left(f_{s1} - 0.85 \bullet \frac{f_{ck}}{\gamma_c}\right) + A_2'\left(f_{s2} - 0.85 \bullet \frac{f_{ck}}{\gamma_c}\right) + A_3'\left(f_{s3} - 0.85 \bullet \frac{f_{ck}}{\gamma_c}\right)$$

$$T = A_{s4}\frac{f_{s4}}{\gamma_s} + A_{s5}\frac{f_{s4}}{\gamma_s} + A_{s6}\frac{f_{s4}}{\gamma_s}$$

$$P_{n1} = 3400(a - 200) + 5{,}100{,}000 + \frac{A_{s1}}{\gamma_s}\left(f_{s1} - 0.85 \bullet \frac{f_{ck}}{\gamma_c}\right) + \frac{A_{s2}}{\gamma_s}\left(f_{s2} - 0.85 \bullet \frac{f_{ck}}{\gamma_c}\right)$$
$$+ \frac{A_{s3}}{\gamma_s}\left(f_{s3} - 0.85 \bullet \frac{f_{ck}}{\gamma_c}\right) - \frac{A_{s4}}{\gamma_s} f_{s4} - \frac{A_{s5}}{\gamma_s} f_{s5} - \frac{A_{s6}}{\gamma_s} f_{s6}$$

(Eqn. 1)

3) Taking moments about A_{s6}:

$$P_{n2} = \frac{1}{e'}\left[\begin{array}{l} C_{cf}\left(d-d'\right)+C_{cw}\left(d-\dfrac{a-t_f}{2}-t_f\right)+C_{s1}\left(d-d'\right)+C_{s2}\left(4s\right)+ \\ C_{s3}\left(3s\right)-T_{s4}\left(2s\right)-T_{s5}\left(s\right) \end{array}\right] \quad \text{(Eqn. 2)}$$

where $C_{s1} = \frac{A_{s1}}{\gamma_s}\left(f_{s1} - \frac{0.85}{\gamma_c}f_{ck}\right)$; $C_{s2} = \frac{A_{s2}}{\gamma_s}\left(f_{s2} - \frac{0.85}{\gamma_c}f_{ck}\right)$; $C_{s3} = \frac{A_{s3}}{\gamma_s}\left(f_{s3} - \frac{0.85}{\gamma_c}f_{ck}\right)$;

$T_{s4} = \frac{A_{s4}}{\gamma_s}$ and the bar strains and stresses are determined below.

The plastic centroid is at the center of the section and d'' = 1150 mm

$$e' = e + d'' = 1308.6 + 1150 = 2458.6 \text{ mm.}$$

4) Using c = 1327 mm (from iteration)

$$a = k_1 c = 0.85 \bullet 1327 = 1061.1 \text{ mm}$$

5) Assuming the extreme fiber strain equals 0.003 and c = 1327 mm, the steel stresses and strains can be calculated. When the bar strain exceeds the yield strain, then $f_s = f_y$:

$\varepsilon_{s1} = \left(\frac{c-d'}{c}\right)0.003$ = 0.00277; $f_s = \varepsilon_s E \le F_y$; f_{s1} = 420.0 MPa

$\varepsilon_{s2} = \left(\frac{c-s-d'}{c}\right)0.003$ = 0.00173 $\quad f_{s2}$ = 346.8 MPa

$\varepsilon_{s3} = \left(\frac{c-2s-d'}{c}\right)0.003$ = 0.00069 $\quad f_{s3}$ = 138.8 MPa

$\varepsilon_{s4} = \left(\frac{d-c-2s}{d-c}\right)\varepsilon_{s6}$ = 0.00035 $\quad f_{s4}$ = 69.2 MPa

$\varepsilon_{s5} = \left(\frac{d-c-s}{d-c}\right)\varepsilon_{s6}$ = 0.00139 $\quad f_{s5}$ = 277.2 MPa

$\varepsilon_{s6} = \left(\frac{d-c}{c}\right)0.003$ = 0.00243 $\quad f_{s6}$ = 420.0 MPa

Substituting in Eqn. 1 and 2 and iterating the value of the neutral axis depth until the two equations are equal give

$P_{n1} = 9134$ kN
$P_{n2} = 9134$ kN

$M_n = P_n e = 9134(1308.6)/1000 = 11952$ kN-m

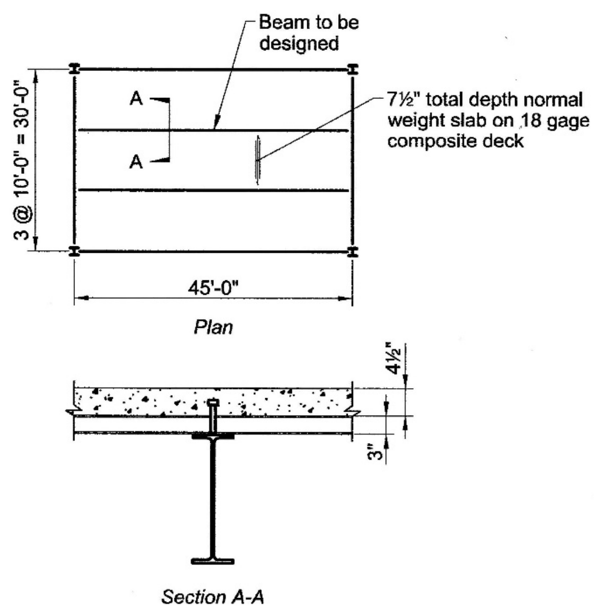
AISC-360-05 Example 001

COMPOSITE GIRDER DESIGN

EXAMPLE DESCRIPTION

A series of 45-ft. span composite beams at 10 ft. o/c carry the loads shown below. The beams are ASTM A992 and are unshored during construction. The concrete has a specified compressive strength, $f'_c = 4$ ksi. Design a typical floor beam with 3-in., 18-gage composite deck and 4 ½ in. normal weight concrete above the deck, for fire protection and mass. Select an appropriate beam and determine the required number of ¾ in.-diameter shear studs.

GEOMETRY, PROPERTIES AND LOADING



Member Properties	Loading	Geometry
W21x55	w = 830 plf (Dead Load)	Span, L = 45 ft
E = 29000 ksi	w = 200 plf (Construction)	
F_y = 50 ksi	w = 100 plf (SDL)	
	w = 1000 plf (Live Load)	

TECHNICAL FEATURES OF ETABS TESTED

Composite beam design, including:

- Selection of steel section, camber and shear stud distribution
- Member bending capacities, at construction and in service
- Member deflections, at construction and in service

RESULTS COMPARISON

Independent results are referenced from Example I.1 from the AISC Design Examples, Version 13.0.

Output Parameter	ETABS	Independent	Percent Difference
Pre-composite M_u (k-ft)	333.15	333.15	0.00%
Pre-composite $\Phi_b M_n$ (k-ft)	472.5	472.5	0.00%
Pre-composite Deflection (in.)	2.3	2.3	0.00%
Required Strength M_u (k-ft)	687.5	687.5	0.00%
Full Composite $\Phi_b M_n$ (k-ft)	1027.1	1027.1	0.00%
Partial Composite $\Phi_b M_n$ (k-ft)	770.3	770.3	0.00 %
Shear Stud Capacity Q_n	17.2	17.2	0.00 %
Shear Stud Distribution	35	34	2.9%
Live Load Deflection (in.)	1.35	1.30	3.70%
Required Strength V_u (kip)	61.1	61.1	0.00%
ΦV_n (k)	234	234	0.00%

COMPUTER FILE: AISC-360-05 EXAMPLE 001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results. The live load deflection differs due to a difference in methodology. In the AISC example, the live load deflection is computed based on a lower bound value of the beam moment of inertia, whereas in ETABS, it is computed based on the approximate value of the beam moment of inertia derived from Equation (C-I3-6) from the *Commentary on the AISC Load and Resistance Factor Design Specification – Second Edition*.

HAND CALCULATION

Properties:

Materials:

ASTM A572 Grade 50 Steel

E = 29,000 ksi, F_y = 50 ksi, w_{steel} = 490 pcf

4000 psi normal weight concrete

E_c = 3,644 ksi, $f_c' = 4$ ksi, $w_{concrete}$ = 145 pcf

Section:

W21x55

d = 20.8 in, b_f = 8.22 in, t_f = 0.522 in, t_w = 0.38 in, h = 18.75 in., r_{fillet} = 0.5 in.

A_{steel} = 16.2 in^2, S_{steel} = 109.6 in^3, Z_{steel} = 126 in^3, I_{steel} = 1140 in^4

Deck:

t_c =4 ½ in., h_r = 3 in., s_r =12 in., w_r = 6 in.

Shear Connectors:

d = ¾ in, h =4 ½ in, F_u = 65 ksi

Design for Pre-Composite Condition:

Construction Required Flexural Strength:

$$w_D = \left(10 \bullet 77.5 + 55.125\right) \bullet 10^{-3} = 0.830125 \text{ kip/ft}$$

$$w_L = 10 \bullet 20 \bullet 10^{-3} = 0.200 \text{ kip/ft}$$

$$w_u = 1.2 \bullet 0.830125 + 1.6 \bullet 0.200 = 1.31615 \text{ kip/ft}$$

$$M_u = \frac{w_u \bullet L^2}{8} = \frac{1.31615 \bullet 45^2}{8} = 333.15 \text{ kip-ft}$$

Moment Capacity:

$$\Phi_b M_n = \Phi_b \bullet Z_s \bullet F_y = \left(0.9 \bullet 126 \bullet 50\right) / 12 = 472.5 \text{ kip-ft}$$

Pre-Composite Deflection:

$$\Delta_{nc} = \frac{5w_D L^4}{384EI} = \frac{5 \bullet \frac{0.830}{12} \bullet (45 \bullet 12)^4}{384 \bullet 29,000 \bullet 1,140} = 2.31 \text{ in.}$$

Design for Composite Flexural Strength:

Required Flexural Strength:

$$w_u = 1.2 \bullet 0.830 + 1.2 \bullet 0.100 + 1.6 \bullet 1 = 2.71 \text{ kip/ft}$$

$$M_u = \frac{w_u \bullet L^2}{8} = \frac{2.68 \bullet 45^2}{8} = 687.5 \text{ kip-ft}$$

Full Composite Action Available Flexural Strength:

Effective width of slab:

$$b_{\text{eff}} = \frac{10.0}{2} \bullet 2 \text{ sides} = 10.0 \text{ ft} \leq \frac{45.0 \text{ f}t}{8} = 11.25 \text{ ft}$$

Resistance of steel in tension:

$$C = P_y = A_s \bullet F_y = 16.2 \bullet 50 = 810 \text{ kips} \quad \textbf{controls}$$

Resistance of slab in compression:

$$A_c = b_{\text{eff}} \bullet t_c = (10 \bullet 12) \bullet 4.5 = 540 \text{ in}^2$$

$$C = 0.85 \bullet f'_c A_c = 0.85 \bullet 4 \bullet 540 = 1836 \text{ kips}$$

Depth of compression block within slab:

$$a = \frac{C}{0.85 \bullet b_{\text{eff}} \bullet f'_c} = \frac{810}{0.85 \bullet (10 \bullet 12) \bullet 4} = 1.99 \text{ in.}$$

Moment resistance of composite beam for full composite action:

$$d_1 = (t_c + h_r) - \frac{a}{2} = (4.5 + 3) - \frac{2.00}{2} = 6.51 \text{ in.}$$

$$\Phi M_n = \Phi\left(P_y \bullet d_1 + P_y \bullet \frac{d}{2}\right) = 0.9\left(810 \bullet 6.51/12 + 810 \bullet \frac{20.8/12}{2}\right) = 1027.1 \text{ kip-ft}$$

Partial Composite Action Available Flexural Strength:

Assume 36.1% composite action:

$$C = 0.361 \bullet P_y = 0.361 \bullet 810 = 292.4 \text{ kips}$$

Depth of compression block within concrete slab:

$$a = \frac{C}{0.85 \bullet b_{\text{eff}} \bullet f'_c} = \frac{292.4}{0.85 \bullet (10 \bullet 12) \bullet 4} = 0.72 \text{ in.}$$

$$d_1 = (t_c + h_r) - \frac{a}{2} = (4.5 + 3) - \frac{0.72}{2} = 7.14 \text{ in.}$$

Compression force within steel section:

$$(P_y - C)/2 = (810 - 292.4)/2 = 258.8 \text{ kips}$$

Tensile resistance of one flange:

$$F_{\text{flange}} = b_f \bullet t_f \bullet F_{\text{y}} = 8.22 \bullet 0.522 \bullet 50 = 214.5 \text{ kip}$$

Tensile resistance of web:

$$F_{\text{web}} = T \bullet t_w \bullet F_{\text{y}} = 18.75 \bullet 0.375 \bullet 50 = 351.75 \text{ kips}$$

Tensile resistance of one fillet area:

$$F_{\text{fillet}} = (P_{\text{y}} - 2 \bullet F_{\text{flange}} - F_{\text{web}})/2 = (810 - 2 \bullet 214.5 - 351.2)/2 = 14.6 \text{ kips}$$

Compression force in web:

$$C_{\text{web}} = (P_y - C)/2 - F_{\text{flange}} - F_{\text{fillet}} = 258.8 - 214.5 - 14.6 = 29.7 \text{ kips}$$

Depth of compression block in web:

$$x = \frac{C_{\text{web}}}{F_{\text{web}}} \bullet T = \frac{29.7}{351.75} \bullet 18.76 = 1.584 \text{ in.}$$

Location of centroid of steel compression force measured from top of steel section:

$$d_2 = \frac{0.5 \bullet t_f \bullet F_{\text{flange}} + (t_f + 0.5 \bullet r_{\text{fillet}}) \bullet \text{F}_{\text{fillet}} + (t_f + r_{\text{fillet}} + 0.5 \bullet \text{x}) \bullet \text{C}_{\text{web}}}{(P_y - C)/2} =$$

$$= \frac{0.5 \bullet 0.522 \bullet 214.5 + (0.522 + 0.5 \bullet 0.5) \bullet 14.6 + (0.522 + 0.5 + 0.5 \bullet 1.58) \bullet 29.7}{258.8} = 0.467 \text{ in.}$$

Moment resistance of composite beam for partial composite action:

$$\Phi M_n = \Phi\left[C \bullet (d_1 + d_2) + P_y \bullet (d_3 - d_2)\right]$$

$$= 0.9\left[292.4 \bullet (7.14 + 0.467) + 810 \bullet \left(\frac{20.8}{2} - 0.467\right)\right] \Big/ 12 = 770.3 \text{ kip-ft}$$

<u>Shear Stud Strength:</u>

From AISC Manual Table 3.21 assuming one shear stud per rib placed in the weak position, the strength of ¾ in.-diameter shear studs in normal weight concrete with $f_c' = 4$ ksi and deck oriented perpendicular to the beam is:

$$Q_n = 17.2 \text{ kips}$$

<u>Shear Stud Distribution:</u>

$$n = \frac{\Sigma Q_n}{Q_n} = \frac{292.4}{17.2} = 17 \text{ from each end to mid-span, rounded up to 35 total}$$

<u>Live Load Deflection:</u>

Modulus of elasticity ratio:

$$n = E/E_c = 29{,}000/3{,}644 = 8.0$$

Transformed elastic moment of inertia assuming full composite action:

Element	Transformed Area A (in^2)	Moment Arm from Centroid y (in.)	Ay (in.^{3})	Ay^2 (in,^{4})	I_0 (in.^{4})
Slab	67.9	15.65	1,062	16,620	115
W21x50	16.2	0	0	0	1,140
	84.1		1,062	16,620	1,255

$$I_x = I_0 + Ay^2 = 1{,}255 + 16{,}620 = 17{,}874 \text{ in.}^4$$

$$\bar{y} = \frac{1{,}062}{84.1} = 12.6 \text{ in.}$$

$$I_{tr} = I_x - A \bullet \bar{y}^2 = 17{,}874 - 82.6 \bullet 12.6^2 = 4{,}458 \text{ in}^4$$

Effective moment inertia assuming partial composite action:

$$I_{\text{equiv}} = I_s + \sqrt{\Sigma Q_n / P_y}\left(I_{tr} - I_s\right) = 1{,}140 + \sqrt{0.361}\left(4{,}458 - 1{,}140\right) = 3{,}133 \text{ in}^4$$

$$I_{\text{eff}} = 0.75 \bullet I_{\text{equiv}} = 0.75 \bullet 3{,}133 = 2{,}350 \text{ in}^4$$

$$\Delta_{LL} = \frac{5 w_L L^4}{384 E I_{\text{eff}}} = \frac{5 \bullet \left(1/12\right) \bullet \left(30 \bullet 12\right)^4}{384 \bullet 29{,}000 \bullet 2{,}350} = 1.35 \text{ in.}$$

Design for Shear Strength:

Required Shear Strength:

$$w_u = 1.2 \bullet 0.830 + 1.2 \bullet 0.100 + 1.6 \bullet 1 = 2.71 \text{ kip/ft}$$

$$V_u = \frac{w_u \bullet L}{2} = \frac{2.71 \bullet 45}{2} = 61.1 \text{ kip-ft}$$

Available Shear Strength:

$$\Phi V_n = \Phi \bullet 0.6 \bullet d \bullet t_w \bullet F_y = 1.0 \bullet 0.6 \bullet 20.8 \bullet 0.375 \bullet 50 = 234 \text{ kips}$$

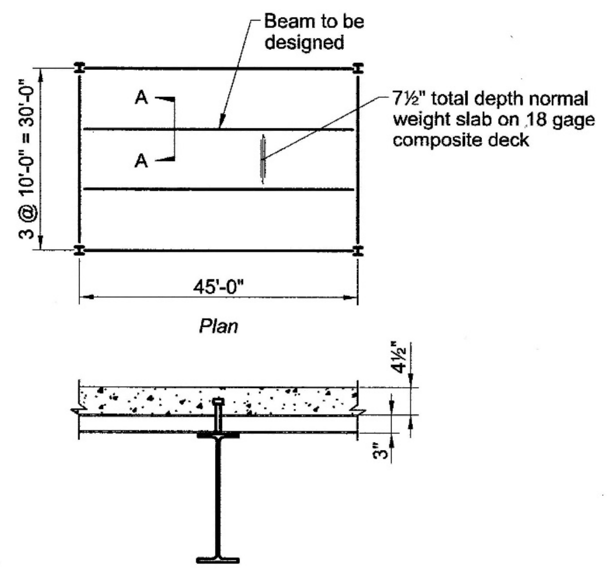
AISC-360-10 Example 001

COMPOSITE GIRDER DESIGN

EXAMPLE DESCRIPTION

A typical bay of a composite floor system is illustrated below. Select an appropriate ASTM A992 W-shaped beam and determine the required number of ¾ in.-diameter steel headed stud anchors. The beam will not be shored during construction. To achieve a two-hour fire rating without the application of spray applied fire protection material to the composite deck, 4 ½ in. of normal weight (145 lb/ft^3) concrete will be placed above the top of the deck. The concrete has a specified compressive strength, $f_c' = 4$ ksi.

GEOMETRY, PROPERTIES AND LOADING



Member Properties
W21x50
E = 29000 ksi
F_y = 50 ksi

Loading
w = 800 plf (Dead Load)
w = 250 plf (Construction)
w = 100 plf (SDL)
w = 1000 plf (Live Load)

Geometry
Span, L = 45 ft

TECHNICAL FEATURES OF ETABS TESTED

Composite beam design, including:

- Selection of steel section, camber and shear stud distribution
- Member bending capacities, at construction and in service
- Member deflections, at construction and in service

RESULTS COMPARISON

Independent results are referenced from Example I.1 from the AISC Design Examples, Version 14.0.

Output Parameter	ETABS	Independent	Percent Difference
Pre-composite M_u (k-ft)	344.2	344.2	0.00%
Pre-composite $\Phi_b M_n$ (k-ft)	412.5	412.5	0.00%
Pre-composite Deflection (in.)	2.6	2.6	0.00%
Required Strength M_u (k-ft)	678.3	678.4	0.01%
Full Composite $\Phi_b M_n$ (k-ft)	937.1	937.1	0.00%
Partial Composite $\Phi_b M_n$ (k-ft)	763.2	763.2	0.00%
Shear Stud Capacity Q_n	17.2; 14.6	17.2; 14.6	0.00%
Shear Stud Distribution	46	46	0.00%
Live Load Deflection (in.)	1.34	1.26	6.0%
Required Strength V_u (kip)	60.3	60.3	0.00%
ΦV_n (k)	237.1	237.1	0.00%

COMPUTER FILE: AISC-360-10 EXAMPLE 001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results. The live load deflection differs due to a difference in methodology. In the AISC example, the live load deflection is computed based on a lower bound value of the beam moment of inertia, whereas in ETABS, it is computed based on the approximate value of the beam moment of inertia derived from Equation (C-I3-6) from the *Commentary on the AISC Load and Resistance Factor Design Specification – Second Edition.*

HAND CALCULATION

Properties:

Materials:

ASTM A572 Grade 50 Steel

E = 29,000 ksi, F_y = 50 ksi, w_{steel} = 490 pcf

4000 psi normal weight concrete

E_c = 3,644 ksi, f_c' = 4 ksi, $w_{concrete}$ = 145 pcf

Section:

W21x50

d = 20.8 in, b_f = 6.53 in, t_f = 0.535 in, t_w = 0.38 in, k = 1.04 in

A_{steel} = 14.7 in^2, S_{steel} = 94.6 in^3, Z_{steel} = 110 in^3, I_{steel} = 984 in^4

Deck:

t_c =4 ½ in., h_r = 3 in., s_r =12 in., w_r = 6 in.

Shear Connectors:

d = ¾ in, h =4 ½ in, F_u = 65 ksi

Design for Pre-Composite Condition:

Construction Required Flexural Strength:

$$w_D = (10 \bullet 75 + 50) \bullet 10^{-3} = 0.800 \text{ kip/ft}$$

$$w_L = 10 \bullet 25 \bullet 10^{-3} = 0.250 \text{ kip/ft}$$

$$w_u = 1.2 \bullet 0.800 + 1.6 \bullet 0.250 = 1.36 \text{ kip/ft}$$

$$M_u = \frac{w_u \bullet L^2}{8} = \frac{1.36 \bullet 45^2}{8} = 344.25 \text{ kip-ft}$$

Moment Capacity:

$$\Phi_b M_n = \Phi_b \bullet Z_s \bullet F_y = (0.9 \bullet 110 \bullet 50)/12 = 412.5 \text{ kip-ft}$$

Pre-Composite Deflection:

$$\Delta_{nc} = \frac{5w_D L^4}{384EI} = \frac{5 \bullet \frac{0.800}{12} \bullet (45 \bullet 12)^4}{384 \bullet 29,000 \bullet 984} = 2.59 \text{ in.}$$

$\text{Camber} = 0.8 \bullet \Delta_{nc} = 0.8 \bullet 2.59 = 2.07$ in., which is rounded down to 2 in.

Design for Composite Flexural Strength:

Required Flexural Strength:

$$w_u = 1.2 \bullet 0.800 + 1.2 \bullet 0.100 + 1.6 \bullet 1 = 2.68 \text{ kip/ft}$$

$$M_u = \frac{w_u \bullet L^2}{8} = \frac{2.68 \bullet 45^2}{8} = 678.38 \text{ kip-ft}$$

Full Composite Action Available Flexural Strength:

Effective width of slab:

$$b_{\text{eff}} = \frac{10.0}{2} \bullet 2 \text{ sides} = 10.0 \text{ ft} \leq \frac{45.0 \text{ ft}}{8} = 11.25 \text{ ft}$$

Resistance of steel in tension:

$C = P_y = A_s \bullet F_y = 14.7 \bullet 50 = 735$ kips **controls**

Resistance of slab in compression:

$$A_c = b_{\text{eff}} \bullet t_c = (10 \bullet 12) \bullet 4.5 = 540 \text{ in}^2$$

$$C = 0.85 \bullet f'_c A_c = 0.85 \bullet 4 \bullet 540 = 1836 \text{ kips}$$

Depth of compression block within slab:

$$a = \frac{C}{0.85 \bullet b_{\text{eff}} \bullet f'_c} = \frac{735}{0.85 \bullet (10 \bullet 12) \bullet 4} = 1.80 \text{ in.}$$

Moment resistance of composite beam for full composite action:

$$d_1 = (t_c + h_r) - \frac{a}{2} = (4.5 + 3) - \frac{1.80}{2} = 6.60 \text{ in.}$$

$$\Phi M_n = \Phi \left(P_y \bullet d_1 + P_y \bullet \frac{d}{2} \right) = 0.9 \left(735 \bullet 6.60/12 + 735 \bullet \frac{20.8/12}{2} \right) = 937.1 \text{ kip-ft}$$

Partial Composite Action Available Flexural Strength:

Assume 50.9% composite action:

$$C = 0.509 \bullet P_y = 373.9 \text{ kips}$$

Depth of compression block within concrete slab:

$$a = \frac{C}{0.85 \bullet b_{\text{eff}} \bullet f'_c} = \frac{373.9}{0.85 \bullet (10 \bullet 12) \bullet 4} = 0.92 \text{ in.}$$

$$d_1 = (t_c + h_r) - \frac{a}{2} = (4.5 + 3) - \frac{0.92}{2} = 7.04 \text{ in.}$$

Compressive force in steel section:

$$\frac{P_y - C}{2} = \frac{735 - 373.9}{2} = 180.6 \text{ kips}$$

Steel section flange ultimate compressive force:

$$C_{flange} = b_f \bullet t_f \bullet F_y = 6.53 \bullet 0.535 \bullet 50 = 174.7 \text{ kips}$$

Steel section web (excluding fillet areas) ultimate compressive force:

$$C_{web} = (d - 2 \bullet k) \bullet t_w \bullet F_y = (20.8 - 2 \bullet 1.04) \bullet 0.38 \bullet 50 = 355.7 \text{ kips}$$

Steel section fillet ultimate compressive force:

$$C_{fillet} = \frac{P_y - (2 \bullet C_{flange} + C_{web})}{2} = \frac{735 - (2 \bullet 174.7 + 355.7)}{2} = 14.5 \text{ kips}$$

Assuming a rectangular fillet area, the distance from the bottom of the top flange to the neutral axis of the composite section is:

$$x = (\text{k} - \text{t}_\text{f}) \bullet \left[\frac{(P_y - C)/2 - C_{flange}}{C_{fillet}} \right]$$

$$= (1.04 - 0.535) \bullet \left[\frac{180.6 - 174.7}{14.98} \right] = 0.20 \text{ in.}$$

Distance from the centroid of the compressive force in the steel section to the top of the steel section:

$$d_2 = \frac{C_{flange} \bullet t_f / 2 + ((P_y - C)/2 - C_{flange}) \bullet (t_f + x/2)}{(P_y - C)/2}$$

$$= \frac{174.7 \bullet 0.535/2 + (180.6 - 174.7) \bullet (0.535 + 0.2/2)}{180.6} = 0.279 \text{ in.}$$

Moment resistance of composite beam for partial composite action:

$$\Phi M_n = \Phi\left[C \bullet \left(d_1 + d_2\right) + P_y \bullet \left(d_3 - d_2\right)\right]$$

$$= 0.9\left[373.9 \bullet \left(7.04 + 0.279\right) + 735 \bullet \left(\frac{20.8}{2} - 0.279\right)\right] \Big/ 12 = 763.2 \text{ kip-ft}$$

Shear Stud Strength:

From AISC Manual Table 3.21, assuming the shear studs are placed in the weak position, the strength of ¾ in.-diameter shear studs in normal weight concrete with $f_c' = 4$ ksi and deck oriented perpendicular to the beam is:

$Q_n = 17.2$ kips for one shear stud per deck flute

$Q_n = 14.6$ kips for two shear studs per deck flute

Shear Stud Distribution:

There are at most 22 deck flutes along each half of the clear span of the beam. ETABS only counts the studs in the first 21 deck flutes as the 22nd flute is potentially too close to the point of zero moment for any stud located in it to be effective. With two shear studs in the first flute, 20 in the next in the next twenty flutes, and one shear stud in the 22nd flute, in each half of the beam, there is a total of 46 shear studs on the beam, and the total force provided by the shear studs in each half span is:

$$\Sigma Q_n = 2 \bullet 14.6 + 20 \bullet 17.2 = 373.9 \text{ kip}$$

Live Load Deflection:

Modulus of elasticity ratio:

$$n = E/E_c = 29{,}000/3{,}644 = 8.0$$

Transformed elastic moment of inertia assuming full composite action:

Element	Transformed Area A (in^2)	Moment Arm from Centroid y (in.)	Ay (in.^{3})	Ay^2 (in,^{4})	I_0 (in.^{4})
Slab	67.9	15.65	1,062	16,620	115
W21x50	14.7	0	0	0	984
	82.6		1,062	16,620	1,099

$$I_x = I_0 + Ay^2 = 1{,}099 + 16{,}620 = 17{,}719 \text{ in.}^4$$

$$\bar{y} = \frac{1{,}062}{82.6} = 12.9 \text{ in.}$$

$$I_{tr} = I_x - A \bullet \bar{y}^2 = 17{,}719 - 82.6 \bullet 12.9^2 = 4{,}058 \text{ in}^4$$

Effective moment inertia assuming partial composite action:

$$I_{\text{equiv}} = I_s + \sqrt{\Sigma Q_n / P_y}\,(I_{tr} - I_s) = 984 + \sqrt{0.51}(4{,}058 - 984) = 3{,}176 \text{ in}^4$$

$$I_{\text{eff}} = 0.75 \bullet I_{\text{equiv}} = 0.75 \bullet 3{,}176 = 2{,}382 \text{ in}^4$$

$$\Delta_{LL} = \frac{5 w_L L^4}{384 E I_{\text{eff}}} = \frac{5 \bullet (1/12) \bullet (30 \bullet 12)^4}{384 \bullet 29{,}000 \bullet 2{,}382} = 1.34 \text{ in.}$$

Design for Shear Strength:

Required Shear Strength:

$$w_u = 1.2 \bullet 0.800 + 1.2 \bullet 0.100 + 1.6 \bullet 1 = 2.68 \text{ kip/ft}$$

$$V_u = \frac{w_u \bullet L}{2} = \frac{2.68 \bullet 45}{2} = 60.3 \text{ kip-ft}$$

Available Shear Strength:

$$\Phi V_n = \Phi \bullet 0.6 \bullet d \bullet t_w \bullet F_y = 1.0 \bullet 0.6 \bullet 20.8 \bullet 0.38 \bullet 50 = 237.1 \text{ kips}$$

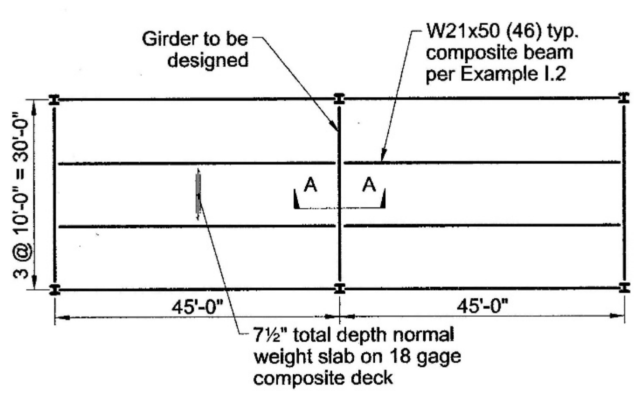
AISC-360-10 Example 002

COMPOSITE GIRDER DESIGN

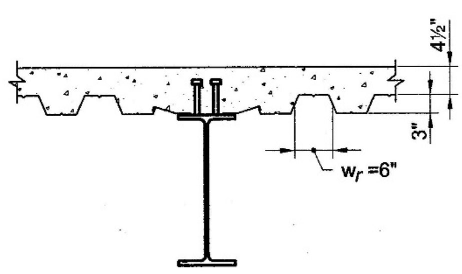
EXAMPLE DESCRIPTION

The design is checked for the composite girder shown below. The deck is 3 in. deep with 4 ½″ normal weight (145 pcf) concrete cover with a compressive strength of 4 ksi. The girder will not be shored during construction. The applied loads are the weight of the structure, a 25 psf construction live load, a 10 psf superimposed dead load and a 100 psf non-reducible service line load.

GEOMETRY, PROPERTIES AND LOADING



Plan



Section A-A

Member Properties	Loading	Geometry
W24x76	P = 36K (Dead Load)	Span, L = 45 ft
E = 29000 ksi	P = 4.5K (SDL)	
F_y = 50 ksi	P = 45K (Live Load)	

TECHNICAL FEATURES OF ETABS TESTED

Composite beam design, including:

- Selection of steel section, camber and shear stud distribution
- Member bending capacities, at construction and in service
- Member deflections, at construction and in service

RESULTS COMPARISON

Independent results are referenced from Example I.2 from the AISC Design Examples, Version 14.0.

Output Parameter	ETABS	Independent	Percent Difference
Pre-composite M_u (k-ft)	622.3	622.3	0.00%
Pre-composite $\Phi_b M_n$ (k-ft)	677.2	677.2	0.00%
Pre-composite Deflection (in.)	1.0	1.0	0.00%
Required Strength M_u (k-ft)	1216.3	1216.3	0.00%
Full Composite $\Phi_b M_n$ (k-ft)	1480.1	1480.1	0.00%
Partial Composite $\Phi_b M_n$ (k-ft)	1267.3	1267.3	0.00%
Shear Stud Capacity Q_n	21.54	21.54	0.00%
Shear Stud Distribution	26, 3, 26	26, 3, 26	0.00%
Live Load Deflection (in.)	0.63	0.55	12.7%
Required Strength V_u (kip)	122.0	122.0	0.00%
ΦV_n (k)	315.5	315.5	0.00%

COMPUTER FILE: AISC-360-10 EXAMPLE 002.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results. The live load deflection differs more markedly because of a difference in methodology. In the AISC example, the live load deflection is computed based on a lower bound value of the beam moment of inertia, whereas in ETABS, it is computed based on the approximate value of the beam moment of inertia derived from Equation (C-I3-6) from the *Commentary on the AISC Load and Resistance Factor Design Specification – Second Edition*.

HAND CALCULATION

Properties:

Materials:

ASTM A572 Grade 50 Steel

E = 29,000 ksi, F_y = 50 ksi, w_{steel} = 490 pcf

4000 psi normal weight concrete

E_c = 3,644 ksi, f_c' = 4 ksi, $w_{concrete}$ = 145 pcf

Section:

W24x76

d = 23.9 in, b_f = 8.99 in, t_f = 0.68 in, t_w = 0.44 in

A_{steel} = 22.4 in^2, I_{steel} = 2100 in^4

Deck:

t_c =4 ½ in., h_r = 3 in., s_r =12 in., w_r = 6 in.

Shear Connectors:

d = ¾ in, h =4 ½ in, F_u = 65 ksi

Design for Pre-Composite Condition:

Construction Required Flexural Strength:

$$w = A_{steel} \cdot w_{steel} = \left(\frac{22.4}{144}\text{sq.ft.}\right) \bullet 490 \text{ pcf} = 76.2 \text{ plf}$$

$$P_D = \left[(45 \text{ ft})(10 \text{ ft})(75 \text{ psf}) + (50 \text{ plf})(45 \text{ ft})\right](0.001 \text{ kip / lb}) = 36 \text{ kips}$$

$$P_L = \left[(45 \text{ ft})(10 \text{ ft})(25 \text{ psf})\right](0.001 \text{ kip/lb}) = 11.25 \text{ kips}$$

$$M_u = \frac{1.2wL^2}{8} + \left(1.2P_D + 1.6P_L\right)\frac{L}{3}$$

$$= 1.2\frac{76.2 \bullet 30^2}{8} + \left(1.2 \bullet 36 + 1.6 \bullet 11.25\right)\frac{30}{3} = 622.3 \text{ kip-ft}$$

Moment Capacity:

L_b = 10 ft

L_p = 6.78 ft

Lr = 19.5 ft

$\Phi_b BF$ = 22.6 kips

$\Phi_b M_{px}$ = 750 kip-ft

C_b = 1.0

$$\Phi_b M_n = C_b\left[\Phi_b M_{px} - \Phi_b BF(L_b - L_p)\right]$$
$$= 1.0\left[750 - 22.6 \bullet (10 - 6.78)\right] \qquad = 677.2 \text{ kip-ft}$$

Pre-Composite Deflection:

$$\Delta_{nc} = \frac{P_D L^3}{28EI} + \frac{5w_D L^4}{384EI} = \frac{36.0 \bullet 360^3}{28 \bullet 29,000 \bullet 2,100} + \frac{5 \bullet \frac{0.0762}{12} \bullet 360^4}{384 \bullet 29,000 \bullet 2,100} = 1.0$$

$\text{Camber} = 0.8 \bullet \Delta_{nc} = 0.8 \text{ in.}$ which is rounded down to ¾ in.

Design for Composite Flexural Strength:

Required Flexural Strength:

$$P_D = \left[(45 \text{ ft})(10 \text{ ft})(75 + 10\text{psf}) + (50 \text{ plf})(45 \text{ ft})\right](0.001 \text{ kip/lb}) = 40.5 \text{ kips}$$

$$P_L = \left[(45 \text{ ft})(10 \text{ ft})(100 \text{ psf})\right](0.001 \text{ kip/lb}) = 45 \text{ kips}$$

$$M_u = \frac{1.2wL^2}{8} + (1.2P_D + 1.6P_L)\frac{L}{3}$$
$$= \frac{1.2 \bullet 76.22 \bullet 30^2}{8} + (1.2 \bullet 40.5 + 1.6 \bullet 45)\frac{30}{3} = 1216.3 \text{ kip-ft}$$

Full Composite Action Available Flexural Strength:

Effective width of slab:

$$b_{\text{eff}} = \frac{30.0 \text{ ft}}{8} = 7.5 \text{ ft} = 90 \text{ in.}$$

Resistance of steel in tension:

$$C = P_y = A_s \bullet F_y = 22.4 \bullet 50 = 1{,}120 \text{ kips} \;\; \textbf{controls}$$

Resistance of slab in compression

$$A_c = b_{\text{eff}} \bullet t_c + (b_{\text{eff}}/2) \bullet h_r = (7.5 \bullet 12) \bullet 4.5 + \frac{7.5 \bullet 12}{2} \bullet 3 = 540 \text{ in}^2$$

$$C = 0.85 \bullet f'_c A_c = 0.85 \bullet 4 \bullet 540 = 1836 \text{ kips}$$

Depth of compression block within slab:

$$a = \frac{C}{0.85 \bullet b_{eff} \bullet f'_c} = \frac{1{,}120}{0.85 \bullet (7.5 \bullet 12) \bullet 4} = 3.66 \text{ in.}$$

Moment resistance of composite beam for full composite action:

$$d_1 = (t_c + h_r) - \frac{a}{2} = (4.5 + 3) - \frac{3.66}{2} = 5.67 \text{ in.}$$

$$\Phi M_n = \Phi\left(C \bullet d_1 + P_y \bullet \frac{d}{2}\right)$$

$$= 0.9 \bullet \left(1{,}120 \bullet 5.67 / 12 + 1{,}120 \bullet \frac{23.9/12}{2}\right) = 1480.1 \text{ kip-ft}$$

Partial Composite Action Available Flexural Strength:

Assume 50% composite action:

$$C = 0.5 \bullet P_y = 560 \text{ kips}$$

Depth of compression block within slab

$$a = \frac{C}{0.85 \bullet b_{eff} \bullet f'_c} = \frac{560}{0.85 \bullet (7.5 \bullet 12) \bullet 4} = 1.83 \text{ in.}$$

$$d_1 = (t_c + h_r) - \frac{a}{2} = (4.5 + 3) - \frac{1.83}{2} = 6.58 \text{ in.}$$

Depth of compression block within steel section flange

$$x = \frac{P_y - C}{2 \bullet b_f \bullet F_y} = \frac{1{,}120 - 560}{2 \bullet 8.99 \bullet 50} = 0.623 \text{ in.}$$

$$d_2 = x / 2 = 0.311 \text{ in.}$$

$$M_n = C \bullet (d_1 + d_2) + P_y \bullet (d_3 - d_2)$$

$$= \left[560 \bullet (6.58 + 0.312) + 1{,}120 \bullet \left(\frac{23.9}{2} - 0.312 \right) \right] \Big/ 12 = 1{,}408 \text{ kip-ft}$$

$$\Phi M_n = 0.9 M_n = 0.9 \bullet 1{,}408 = 1{,}267.3 \text{ kip-ft}$$

Shear Stud Strength:

$$Q_n = 0.5 A_{sa} \sqrt{f'_c E_c} \le R_g R_p A_{sa} F_u$$

$$A_{sa} = \pi d_{sa}{}^2 / 4 = \pi (0.75)^2 / 4 = 0.442 \text{ in}^2$$

$$f_c' = 4 \text{ ksi}$$

$$E = w_c^{1.5} \sqrt{f_c'} = 145^{1.5} \sqrt{4} = 3{,}490 \text{ ksi}$$

R_g = 1.0 Studs welded directly to the steel shape with the slab haunch

R_p = 0.75 Studs welded directly to the steel shape

F_u = 65 ksi

$$Q_n = 0.5 \bullet 0.442^2 \sqrt{4 \bullet 3{,}490} \le 1.0 \bullet 0.75 \bullet 0.442^2 \bullet 65$$

$$= 26.1 \text{ kips } \ge 21.54 \text{ kips controls}$$

Shear Stud Distribution:

$$n = \frac{\Sigma Q_n}{Q_n}$$

$$= \frac{560}{21.54} = 26 \text{ studs from each end to nearest concentrated load point}$$

Add 3 studs between load points to satisfy maximum stud spacing requirement.

Live Load Deflection:

Modulus of elasticity ratio:

$$n = E / E_c = 29{,}000 / 3{,}644 = 8.0$$

Transformed elastic moment of inertia assuming full composite action:

Element	Transformed Area A (in^2)	Moment Arm from Centroid y (in.)	Ay (in.^{3})	Ay^2 (in,^{4})	I_0 (in.^{4})
Slab	50.9	17.2	875	15,055	86
Deck ribs	17.0	13.45	228	3,069	13
W21x50	22.4	0	0	0	2,100
	89.5		1,103	18,124	2,199

$$I_x = I_0 + Ay^2 = 2{,}199 + 18{,}124 = 20{,}323 \text{ in.}^4$$

$$\bar{y} = \frac{1{,}092}{89.5} = 12.2 \text{ in.}$$

$$I_{tr} = I_x - A \bullet \bar{y}^2 = 20{,}323 - 90.3 \bullet 12.2^2 = 6{,}831 \text{ in}^4$$

Effective moment of inertia assuming partial composite action:

$$I_{\text{equiv}} = I_s + \sqrt{\Sigma Q_n / P_y}\left(I_{tr} - I_s\right) = 2{,}100 + \sqrt{0.5}\left(6{,}831 - 2{,}100\right) = 5{,}446 \text{ in}^4$$

$$I_{\text{eff}} = 0.75 \bullet I_{\text{equiv}} = 0.75 \bullet 5{,}446 = 4{,}084 \text{ in}^4$$

$$\Delta_{LL} = \frac{P_L L^3}{28 E I_{eff}} = \frac{45.0 \bullet (30 \bullet 12)^3}{28 \bullet 29{,}000 \bullet 4{,}084} = 0.633 \text{ in.}$$

Design for Shear Strength:

Required Shear Strength:

$$P_u = 1.2 \bullet P_D + 1.6 \bullet P_L = 1.2 \bullet 40.5 + 1.6 \bullet 45 = 120.6 \text{ kip}$$

$$V_u = \frac{1.2 \bullet w \bullet L}{2} + P_u = \frac{1.2 \bullet 0.076 \bullet 30}{2} + 120.6 = 121.2 \text{ kip-ft}$$

Available Shear Strength:

$$\Phi V_n = \Phi \bullet 0.6 \bullet d \bullet t_w \bullet F_y = 1.0 \bullet 0.6 \bullet 23.9 \bullet 0.44 \bullet 50 = 315.5 \text{ kips}$$

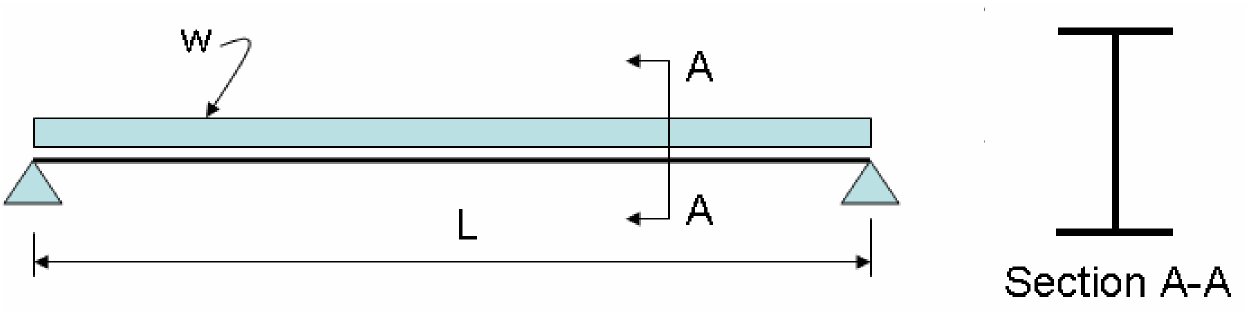
BS-5950-90 Example-001

STEEL DESIGNERS MANUAL SIXTH EDITION - DESIGN OF SIMPLY SUPPORTED COMPOSITE BEAM

EXAMPLE DESCRIPTION

Design a composite floor with beams at 3-m centers spanning 12 m. The composite slab is 130 mm deep. The floor is to resist an imposed load of 5.0 kN/m^2, partition loading of 1.0 kN/m^2 and a ceiling load of 0.5 kN/m^2. The floor is to be un-propped during construction.

GEOMETRY, PROPERTIES AND LOADING



Member Properties	Loading	Geometry
UKB457x191x67	w = 6.67kN/m (Dead Load)	Span, L = 12 m
E = 205,000 MPa	w = 1.5kN/m (Construction)	
F_y = 355 MPa	w = 1.5kN/m (Superimposed Load)	
	w = 18.00kN/m (Live Load)	

TECHNICAL FEATURES OF ETABS TESTED

Composite beam design, including:

- Selection of steel section, camber and shear stud distribution
- Member bending capacities, at construction and in service
- Member deflections, at construction and in service

RESULTS COMPARISON

Independent results are referenced from the first example, Design of Simply Supported Composite Beam, in Chapter 21 of the *Steel Construction Institute Steel Designer's Manual*, *Sixth Edition*.

Output Parameter	ETABS	Independent	Percent Difference
Construction Design Moment (kN-m)	211.2	211.3	0.05%
Construction M_s (kN-m)	522.2	522.2	0.00%
Construction Deflection (mm)	29.9	29.9	0.00%
Design Moment (kN-m)	724.2	724.3	0.01%
Full Composite M_{pc} (kN-m)	968.9	968.9	0.00%
Partial Composite M_c (kN-m)	910.8	910.9	0.01%
Shear Stud Capacity Q_n (kN)	57.6	57.6	0.00%
Live Load Deflection (mm)	33.2	33.2	0.00%
Applied Shear Force F_v (kN)	241.4	241.4	0.00%
Shear Resistance P_v (kN)	820.9	821.2	0.00%

COMPUTER FILE: BS-5950-90 EXAMPLE 001.EDB

CONCLUSION

The ETABS results show an excellent comparison with the independent results.

HAND CALCULATION

Properties:

Materials:

S355 Steel:

E = 205,000 MPa, p_y = 355 MPa, γ_s = 7850 kg/m^3

Light-weight concrete:

E = 24,855 MPa, f_{cu} = 30 MPa, γ_c = 1800 kg/m^3

Section:

UKB457x191x67

D = 453.6 mm, b_f = 189.9 mm, t_f = 12.7 mm, t_w = 8.5 mm

A_{steel} = 8,550 mm^2, I_{steel} = 29,380 cm^4

Deck:

D_s =130 mm, D_p = 50 mm, s_r = 300 mm, b_r = 150 mm

Shear Connectors:

d = 19 mm, h = 95 mm, F_u = 450 MPa

Loadings:

Self weight slab	= 2.0 kN/m^2
Self weight beam	= 0.67 kN/m
Construction load	= 0.5 kN/m^2
Ceiling	= 0.5 kN/m^2
Partitions (live load)	= 1.0 kN/m^2
Occupancy (live load)	= 5.0 kN/m^2

Design for Pre-Composite Condition:

Construction Required Flexural Strength:

$$w_{ult\ \text{construction}} = 1.4 \bullet 0.67 + \left(1.4 \bullet 2.0 + 1.6 \bullet 0.5\right) \bullet 3.0 = 11.74 \text{ kN/m}$$

$$M_{ult\ \text{construction}} = \frac{w_{ult\ \text{construction}} \bullet L^2}{8} = \frac{11.74 \bullet 12^2}{8} = 211.3\ \text{kN-m}$$

$$M_s = S_z \bullet P_y = 1,471 \bullet 10^3 \bullet 355 \bullet 10^{-6} = 522.2\ \text{kN-m}$$

Pre-Composite Deflection:

$$w_{\text{construction}} = 2.0 \bullet 3.0 + 0.67 = 6.67\ \text{kN/m}$$

$$\delta = \frac{5 \bullet w_{\text{construction}} \bullet L^4}{384 \bullet E \bullet I} = \frac{5 \bullet 6.67 \bullet 12,000^4}{384 \bullet 205,000 \bullet 29,380 \bullet 10^4} = 29.9\ \text{mm}$$

$\text{Camber} = 0.8 \bullet \delta = 24$ mm, which is rounded down to 20 mm

Design for Composite Flexural Strength:

Required Flexural Strength:

$$w_{ult} = 1.4 \bullet 0.67 + \left(1.4 \bullet 2.0 + 1.6 \bullet 1 + 1.6 \bullet 5\right) \bullet 3.0 = 40.24\ \text{kN/m}$$

$$M_{ult} = \frac{w_{ult} \bullet L^2}{8} = \frac{40.24 \bullet 12^2}{8} = 724.3\ \text{kN-m}$$

Full Composite Action Available Flexural Strength:

Effective width of slab:

$$B_e = \frac{\text{L}}{4} = \frac{12{,}000}{4} = 3,000\ \text{mm} \le 3{,}000\ \text{mmm}$$

Resistance of slab in compression:

$$R_c = 0.45 \bullet f_{cu} \bullet B_e \bullet (D_s - D_p) = 0.45 \bullet 30 \bullet 3,000 \bullet \left(130 - 50\right) \bullet 10^{-3} = 3,240\ \text{kN}$$

Resistance of steel in tension:

$$R_s = P_y = A_s \bullet p_y = 8,550 \bullet 355 \bullet 10^{-3} = 3,035\ \text{kN}\ \ \textbf{controls}$$

Moment resistance of composite beam for full composite action:

$$M_{pc} = R_s \left[\frac{D}{2} + D_s - \frac{R_s}{R_c} \frac{\left(\text{D}_\text{s} - \text{D}_\text{p}\right)}{2} \right] \text{ for } R_s \le R_c$$

$$= 3,035 \left[\frac{453.6}{2} + 130 - \frac{3,035}{3,240} \bullet \frac{80}{2} \right] \bullet 10^{-3} = 968.9\ \text{kN-m}$$

Partial Composite Action Available Flexural Strength:

Assume 72% composite action – the 75% assumed in the example requires more shear studs than can fit on the beam given its actual clear length.

$$R_q = 0.72 \bullet R_s = 2{,}189 \text{ kN}$$

Tensile Resistance of web:

$$R_w = t_w \bullet \left(D - 2 \bullet t_f\right) \bullet \mathrm{p_y} = 8.5 \bullet \left(453.6 - 2 \bullet 12.7\right) \bullet 355 \bullet 10^{-3} = 1{,}292 \text{ kN}$$

As $R_q > R_w$, the plastic axis is in the steel flange, and

$$M_c = R_s \frac{D}{2} + R_q \left[D_s - \frac{R_q}{R_c} \frac{\left(D_s - D_p\right)}{2} \right] - \frac{(R_s - R_q)^2}{R_f} \frac{t_f}{4}$$

$$= 3{,}035 \frac{453.6}{2} \bullet 10^{-3} + 2{,}1899 \left[130 - \frac{2{,}189}{3{,}240} \bullet \frac{80}{2} \right] \bullet 10^{-3} - \frac{\left(3{,}035 - 2{,}189\right)^2}{\left(3{,}035 - 1{,}292\right)} \frac{12.7}{4} \bullet 10^{-3}$$

$$= 910.9 \text{ kN-m}$$

Shear Stud Strength:

Characteristic resistance of 19 mm-diameter studs in normal weight 30 MPa concrete:

Q_k = 100 kN from BS 5950: Part 3 Table 5

Adjusting for light-weight concrete:

$$Q_k = 90 \text{ kN}$$

Reduction factor for profile shape with ribs perpendicular to the beam and two studs per rib:

$$k = 0.6 \bullet \frac{b_r}{D_p} \bullet \frac{\left(h - D_p\right)}{D_p} = 0.6 \bullet \frac{150}{50} \bullet \frac{\left(95 - 50\right)}{50} = 1.62 \text{ but } k \le 0.8$$

Design strength:

$$Q_p = k \bullet 0.8 \bullet Q_k = 0.8 \bullet 0.8 \bullet 90 = 57.6 \text{ kN}$$

Shear Stud Distribution:

The example places two rows of shear studs and computes the numbers of deck ribs available for placing shear studs based on the beam center to center span and the deck rib spacing: 12 m / 300 mm = 40

However, the number of deck ribs available for placing shear studs must be based on the beam clear span, and since the clear beam span is somewhat less than the 12 m center to center span there are only 39 deck ribs available.

ETABS selects 72% composite action, which is the highest achievable and sufficient to meet the live load deflection criteria. ETABS satisfies this 72% composite action by placing one stud per deck rib along the entire length of the beam, plus a second stud per rib in all the deck ribs except the mid-span rib since this is the location of the beam zero moment and a stud in that rib would not contribute anything to the total resistance of the shear connectors. The total resistance of the shear connectors is:

$$R_q = 2 \bullet 19 \bullet Q_p = 38 \bullet 57.6 = 2{,}189 \text{ kN}$$

<u>Live Load Deflection:</u>

The second moment of area of the composite section, based on elastic properties, I_c is given by:

$$I_c = \frac{A_{\text{steel}} \bullet \left(\text{D} + D_s + D_p\right)^2}{4 \bullet \left(1 + \alpha_e \bullet r\right)} + \frac{b_{\text{eff}} \bullet \left(D_s - D_p\right)^3}{12 \bullet \alpha_e} + I_{\text{steel}}$$

$$r = \frac{A_{\text{steel}}}{b_{\text{eff}} \bullet (D_s - D_p)} = \frac{8{,}550}{3{,}000 \bullet \left(130 - 50\right)} = 0.0356$$

For light-weight concrete:

$$\alpha_s = 10$$

$$\alpha_l = 25$$

Proportion of total loading which is long term:

$$\rho_l = \frac{w_{dl} + w_{sdl} + 0.33 \bullet w_{\text{live}}}{w_{dl} + w_{sdl} + w_{\text{live}}} = \frac{6.67 + 1.5 + 0.33 \bullet 18}{6.67 + 1.5 + 18} = 0.541$$

$$\alpha_e = \alpha_s + \rho_l \bullet \left(\alpha_l - \alpha_s\right) = 10 + 0.541 \bullet \left(25 - 10\right) = 18.1$$

$$I_c = \frac{8{,}550 \bullet \left(453.4 + 130 + 50\right)^2}{4 \bullet \left(1 + 18.1 \bullet 0.0356\right)} + \frac{3{,}000 \bullet 80^3}{12 \bullet 18.1} + 294 \bullet 10^6$$

$$= \left(521 + 7 + 294\right) \bullet 10^6 = 822 \bullet 10^6 \text{ mm}^4$$

Live load deflection assuming full composite action:

$$\delta_c = \frac{5 \bullet w_{\text{live}} \bullet L^4}{384 \bullet E \bullet I_c} = \frac{5 \bullet 18 \bullet (12,000)^4}{384 \bullet 205,000 \bullet 822 \bullet 10^6} = 28.8 \text{ mm}$$

Adjust for partial composite action:

$$\delta_s = \frac{5 \bullet w_{\text{live}} \bullet L^4}{384 \bullet E \bullet I_c} = \frac{5 \bullet 18 \bullet (12,000)^4}{384 \bullet 205,000 \bullet 294 \bullet 10^6}$$

$$= 80.7 \text{ mm non-composite reference deflection}$$

$$\delta_{\text{partial}} = \delta_c + 0.3 \bullet (1 - K) \bullet (\delta_s - \delta_c)$$

$$= 28.9 + 0.3 \bullet (1 - 0.72) \bullet (80.7 - 28.9) = 33.2 \text{ mm}$$

Design for Shear Strength:

Required Shear Strength:

$$F_v = \frac{w_{ult} \bullet L}{2} = \frac{40.24 \bullet 12}{2} = 241.4 \text{ kN}$$

Shear Resistance of Steel Section:

$$P_V = 0.6 \bullet p_y \bullet D_s \bullet t_w = 0.6 \bullet 355 \bullet 453.4 \bullet 8.5 \bullet 10^{-3} = 820.9 \text{ kN}$$

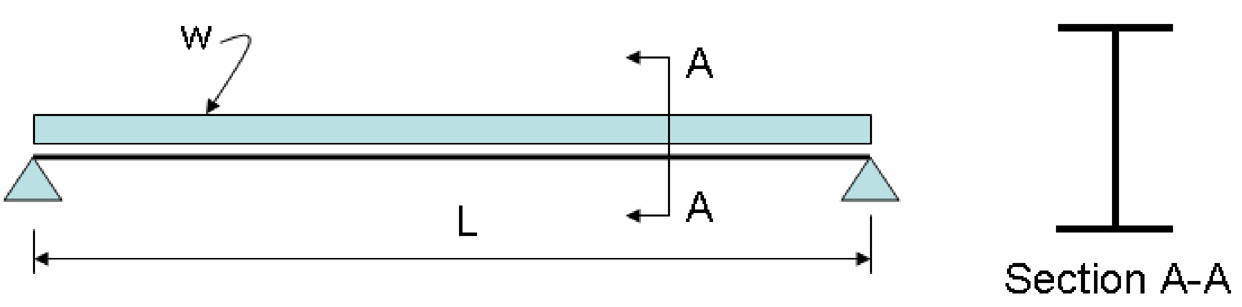
CSA-S16-09 Example-001

HANDBOOK OF STEEL CONSTRUCTION, TENTH EDITION - COMPOSITE BEAM

EXAMPLE DESCRIPTION

Design a simply supported composite beam to span 12 m and carry a uniformly distributed specified load of 18 kN/m live load and 12 kN/m dead load. Beams are spaced at 3 m on center and support a 75 mm steel deck (ribs perpendicular to the beam) with a 65 mm cover slab of 25 MPa normal density concrete. Calculations are based on F_y = 345 MPa. Live load deflections are limited to L/300.

GEOMETRY, PROPERTIES AND LOADING



Member Properties

W460x74
E = 205,000 MPa
F_y = 345 MPa

Loading

w = 8.0kN/m (Dead Load)
w = 2.5kN/m (Construction)
w = 4.0kN/m (Superimposed Load)
w = 18.00kN/m (Live Load)

Geometry

Span, L = 12 m

TECHNICAL FEATURES OF ETABS TESTED

Composite beam design, including:

- Selection of steel section, camber and shear stud distribution
- Member bending capacities, at construction and in service
- Member deflections, at construction and in service

RESULTS COMPARISON

Independent results are referenced from the design example on page 5-25 of the *Handbook of Steel Construction, Tenth Edition*.

Output Parameter	ETABS	Independent	Percent Difference
Construction Design Moment (kN-m)	247.4	247.5	0.04%
Construction M_s (kN-m)	512.3	512.3	0.00%
Construction Deflection (mm)	32.4	32.4	0.00%
Design Moment (kN-m)	755.8	756	0.02%
Full Composite M_{rc} (kN-m)	946.7	946.7	0.00%
Partial Composite M_{rc} (kN-m)	783.6	783.6	0.00%
Shear Stud Capacity Q_n (kN)	68.7	68.7	0.00%
Shear Stud Distribution	30	30	0.00%
Live Load Deflection (mm)	32.9	32.9	0.00%
Bottom Flange Tension (MPa)	267.2	267.1	0.04%
Design Shear Force V_f (kN)	251.9	251.9	0.00%
Shear Resistance V_r (kN)	842.9	842.9	0.00%

COMPUTER FILE: CSA-S16-09 EXAMPLE 001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Materials:

ASTM A992 Grade 50 Steel

E = 200,000 MPa, F_y = 345 MPa, γ_s = 7850 kg/m^3

Normal weight concrete

E = 23,400 MPa, f_{cu} = 20 MPa, γ_c = 2300 kg/m^3

Section:

W460x74

d = 457 mm, b_f = 190 mm, t_f = 14.5 mm, t_w = 9 mm, T = 395 mm, r_{fillet}=16.5 mm

A_s = 9,450 mm^2, $Z_s = 1,650 \bullet 10^3 \text{ mm}^3$, $I_s = 333 \bullet 10^6 \text{ mm}^4$

Deck:

t_c =65 mm, h_r = 75 mm, s_r = 300 mm, w_r = 150 mm

Shear Connectors:

d = 19 mm, h = 115 mm, F_u = 450 MPa

Loadings:

Self weight slab	= 2.42 kN/m^2
Self weight beam	= 0.73 kN/m
Construction load	= 0.83 kN/m^2
Superimposed dead load	= 1.33 kN/m^2
Live load	= 6.0 kN/m^2

Design for Pre-Composite Condition:

Construction Required Flexural Strength:

$$w_{f\,\text{construction}} = 1.25 \bullet 0.73 + \left(1.25 \bullet 2.42 + 1.5 \bullet 0.83\right) \bullet 3.0 = 13.75 \text{ kN/m}$$

$$M_{f\,\text{construction}} = \frac{w_{f\,\text{construction}} \bullet L^2}{8} = \frac{13.75 \bullet 12^2}{8} = 247.5 \text{ kN-m}$$

Moment Capacity:

$$M_s = Z_s \bullet 0.9 \bullet F_y = 1,650 \bullet 10^3 \bullet 0.9 \bullet 345 \bullet 10^{-6} = 512.3 \text{ kN-m}$$

Pre-Composite Deflection:

$$w_{\text{construction}} = 2.42 \bullet 3.0 + 0.73 = 8.0 \text{ kN/m}$$

$$\delta = \frac{5 \bullet w_{\text{construction}} \bullet L^4}{384 \bullet E \bullet I} = \frac{5 \bullet 8.0 \bullet 12,000^4}{384 \bullet 200,000 \bullet 33,300 \bullet 10^4} = 32.4 \; mm$$

Camber = 0.8 • δ = 25.9 mm, which is rounded down to 25 mm

Design for Composite Flexural Strength:

Required Flexural Strength:

$$w_f = 1.25 \bullet 0.73 + \left(1.25 \bullet 2.42 + 1.25 \bullet 1.33 + 1.5 \bullet 6\right) \bullet 3.0 = 42 \text{ kN/m}$$

$$M_f = \frac{w_f \bullet L^2}{8} = \frac{42 \bullet 12^2}{8} = 756.0 \text{ kN-m}$$

Full Composite Action Available Flexural Strength:

Effective width of slab:

$$b_l = \frac{\text{L}}{4} = \frac{12{,}000}{4} = 3,000 \text{ mm} \le 3,000 \text{ mmm}$$

Resistance of slab in compression:

$$\alpha_1 = 0.85 - 0.0015 \bullet f_c' = 0.8125$$

$$C'_r = \alpha_1 \bullet \Phi_c \bullet t \bullet b_f \bullet f_c' = 0.8125 \bullet 0.65 \bullet 65 \bullet 3,000 \bullet 25 \bullet 10^{-3} = 2,574 \text{ kN} \quad \textbf{controls}$$

Resistance of steel in tension:

$$\Phi \bullet A_s \bullet F_y = 0.9 \bullet 9,450 \bullet 345 \bullet 10^{-3} = 2,934 \text{ kN}$$

Depth of compression block within steel section top flange:

$$x = \frac{(\Phi \bullet A_s \bullet F_y - C'_r)/2}{\Phi \bullet F_y \bullet b_f} = \frac{\left(2,934 - 2,547\right) \bullet 10^3 / 2}{0.9 \bullet 345 \bullet 190} = 3.05 \text{ mm}$$

Moment resistance of composite beam for full composite action:

$$M_{rc} = C'_r \bullet \left(\mathrm{h_r} + \frac{t_c}{2} + \frac{x}{2} \right) + \Phi \bullet A_s \bullet F_y \bullet \left(\frac{d}{2} - \frac{x}{2} \right)$$

$$= 2,574 \bullet \left(75 + \frac{65}{2} + \frac{3}{2} \right) \bullet 10^{-3} + 2,934 \bullet \left(\frac{457}{2} - \frac{3}{2} \right) \bullet 10^{-3} = 946.7 \text{ kN-m}$$

Partial Composite Action Available Flexural Strength:

Assume 40.0% composite action:

$$Q_r = 0.4 \bullet R_c = 0.4 \bullet 2,574 = 1,031 \text{ kN}$$

Depth of compression block within concrete slab:

$$a = \frac{Q_r}{\alpha_1 \bullet \Phi c \bullet b_{\text{eff}} \bullet f'_c} = \frac{1,031 \bullet 10^3}{0.8125 \bullet 0.65 \bullet 3,000 \bullet 25} = 26 \text{ mm}$$

Compression force within steel section:

$$C_r = (P_y - Q_r)/2 = (2,934 - 1,031)/2 = 951.6 \text{ kN}$$

Tensile resistance of one flange:

$$F_{\text{flange}} = \Phi \bullet b_f \bullet t_f \bullet F_{\text{y}} = 0.9 \bullet 190 \bullet 14.5 \bullet 345 \bullet 10^{-3} = 855.4 \text{ kN}$$

Tensile resistance of web:

$$F_{\text{web}} = \Phi \bullet T \bullet t_w \bullet F_{\text{y}} = 0.9 \bullet 395 \bullet 9 \bullet 345 \bullet 10^{-3} = 1,103.8 \text{ kN}$$

Tensile resistance of one fillet area:

$$F_{\text{fillet}} = \left(P_{\text{y}} - 2 \bullet F_{\text{flange}} - F_{\text{web}} \right)/2 = \left(2,934 - 2 \bullet 855.4 - 1,103.8 \right)/2 = 59.8 \text{ kN}$$

Compression force in web:

$$C_{\text{web}} = C_r - F_{\text{flange}} - F_{\text{fillet}} = 951.6 - 855.4 - 59.7 = 36.4 \text{ kN}$$

Depth of compression block in web:

$$x = \frac{C_{\text{web}}}{F_{\text{web}}} \bullet T = \frac{36.4}{1,103.8} \bullet 395 = 13 \text{ mm}$$

Location of centroid of compressive force within steel section measured from top of steel section:

$$d_2 = \frac{0.5 \bullet t_f \bullet F_{\text{flange}} + \left(t_f + 0.5 \bullet r_{\text{fillet}}\right) \bullet \text{F}_{\text{fillet}} + \left(t_f + r_{\text{fillet}} + 0.5 \bullet \text{x}\right) \bullet \text{C}_{\text{web}}}{C_r} =$$

$$= \frac{0.5 \bullet 14 \bullet 855 + (14 + 0.5 \bullet 16.5) \bullet 60 + (14 + 16.5 + 0.5 \bullet 44) \bullet 36.4}{951.6} = 9.4 \text{ mm}$$

Moment resistance of composite beam for partial composite action:

$$M_{rc} = Q_r \bullet \left(\text{h}_\text{r} + t_c - \frac{a}{2} + d_2\right) + P_y \bullet \left(\frac{d}{2} - d_2\right)$$

$$= 1,031 \bullet \left(75 + 65 - \frac{26}{2} + 9.4\right) \bullet 10^{-3} + 2,934 \bullet \left(\frac{457}{2} - 9.4\right) \bullet 10^{-3} = 783.6 \text{ kN-m}$$

Shear Stud Strength:

From CISC Handbook of Steel Construction Tenth Edition for 19-mm-diameter studs,

h_d = 75 mm, w_d/h_d = 2.0, 25 MPa, 2,3000 kg/m^3 concrete:

q_{rr} = 68.7 kN

Total number of studs required = $\frac{2 \bullet Q_r}{q_{rr}} = \frac{2 \bullet 1,031}{68.7} = 30$

Live Load Deflection:

Modulus of elasticity ratio:

$$n = E/E_c = 200,000/23,400 = 8.55$$

Transformed elastic moment of inertia assuming full composite action:

Element	Transformed Area A (mn^2)	Moment Arm from Centroid y (mm)	Ay (10^3 mm^3)	Ay^2 (10^6 mm^4)	I_0 (10^6 mm^4)
Slab	22,815	336	7,666	2,576	8
W460x74	9,450	0	0	0	333
	32,265		7,666	2,576	341

$$I_x = I_0 + Ay^2 = 341 \bullet 10^6 + 2{,}576 \bullet 10^6 = 2{,}917 \bullet 10^6 \text{ mm}^4$$

$$\bar{y} = \frac{7{,}666 \bullet 10^6}{32{,}265} = 238 \text{ mm}$$

$$I_{tr} = I_x - A \bullet \bar{y}^2 = 2{,}917 \bullet 10^6 - 32{,}265 \bullet 238^2 = 1{,}095 \bullet 10^6 \text{ mm}^4$$

Effective moment of inertia assuming partial composite action:

$$\begin{aligned} I_{\text{eff}} &= I_s + 0.85 p^{0.25} \left(I_{tr} - I_s \right) \\ &= 333 + 0.85 \bullet 0.40^{0.25} \bullet \left(1{,}095 - 333 \right) \\ &= 848 \bullet 10^6 \text{ mm}^4 \end{aligned}$$

$$\Delta_{LL} = 1.15 \bullet \frac{5 w_L L^4}{384 E I_{\text{eff}}} = 1.15 \bullet \frac{5 \bullet 18 \bullet \left(12{,}000\right)^4}{384 \bullet 200{,}000 \bullet 848 \bullet 10^6} = 32.9 \text{ mm}$$

<u>Bottom Flange Tension:</u>

Stress in tension flange due to specified load acting on steel beam alone:

$$f_1 = \frac{M_1}{S_x} = \frac{8 \bullet 12000^2}{8 \bullet 1460 \bullet 10^3} = 98.6 \text{ MPa}$$

Bottom section modulus based on transformed elastic moment of inertia assuming, per the original example, <u>full</u> composite action:

$$S_t = \frac{I_{tr}}{(\frac{d}{2} + \bar{y})} = \frac{1{,}095 \bullet 10^6}{(228.5 + 237.6)} = 1350 \text{ mm}$$

Stress in tension flange due to specified live and superimposed dead loads acting on composite section:

$$f_2 = \frac{M_2}{S_t} = \frac{(18+4) \bullet 12000^2}{8 \bullet 2350 \bullet 10^3} = 168.5 \text{ MPa}$$

$$f_1 + f_2 = 98.6 + 168.5 = 267.1 \text{ MPa}$$

Design for Shear Strength:

Required Shear Strength:

$$V_f = \frac{w_{\text{factored}} \bullet L}{2} = \frac{42 \bullet 12}{2} = 252 \text{ kN}$$

Shear Resistance of Steel Section:

$$V_r = \Phi \bullet A_w \bullet F_s = 0.9 \bullet d \bullet t_w \bullet 0.66 \bullet F_y = 0.9 \bullet 457 \bullet 9 \bullet 0.66 \bullet 345 = 842.9 \text{ kN}$$

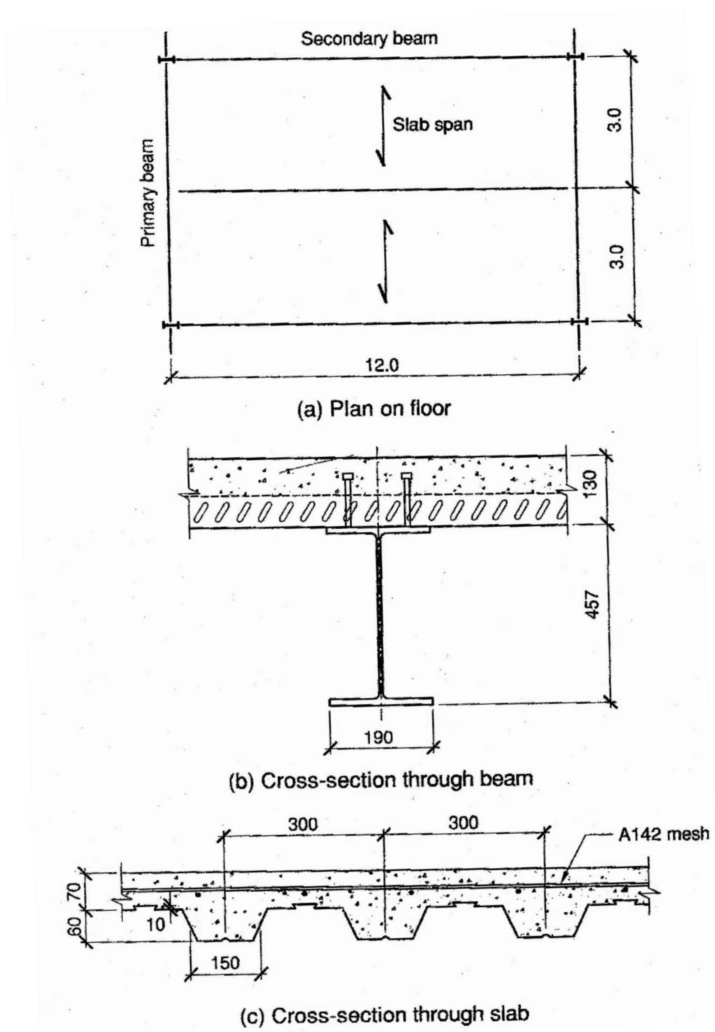
EC-4-2004 Example-001

STEEL DESIGNERS MANUAL SEVENTH EDITION - DESIGN OF SIMPLY SUPPORTED COMPOSITE BEAM

EXAMPLE DESCRIPTION

Consider an internal secondary composite beam of 12-m span between columns and subject to uniform loading. Choose a UKB457x191x74 in S 355 steel.

GEOMETRY, PROPERTIES AND LOADING



Member Properties

UKB457x191x74
E = 205,000 MPa
f_y = 355 MPa

Loading

w = 8.43kN/m (Dead Load)
w = 2.25kN/m (Construction)
w = 1.5kN/m (Superimposed Load)
w = 15.00kN/m (Live Load)

Geometry

Span, L = 12 m
Beam spacing, b =3 m

TECHNICAL FEATURES OF ETABS TESTED

Composite beam design, including:

- Selection of steel section, camber and shear stud distribution
- Member bending capacities, at construction and in service
- Member deflections, at construction and in service

RESULTS COMPARISON

Independent results are referenced from the first example, Design of Simply Supported Composite Beam, in Chapter 22 of the *Steel Construction Institute Steel Designer's Manual, Seventh Edition*.

Output Parameter	ETABS	Independent	Percent Difference
Construction M_{Ed} (kN-m)	250.4	250.4	0.00%
Construction $M_{a,pl,Rd}$ (kN-m)	587	587	0.00%
Construction Deflection (mm)	32.5	32.5	0.00%
Design Moment (kN-m)	628.4	628.4	0.01%
Full Composite M_{pc} (kN-m)	1020	1020	0.00%
Partial Composite M_c (kN-m)	971.2	971.2	0.00%
Shear Stud Capacity P_{Rd}	Input	52.0	NA
Shear Stud Distribution	77	76	1.3%
Live Load Deflection (mm)	19.3	19.1	1.03%

Output Parameter	ETABS	Independent	Percent Difference
Required Strength V_{Ed} (kN)	209.5	209.5	0.00%
$V_{pl,Rd}$ (kN)	843	843	0.00%

COMPUTER FILE: EC-4-2004 EXAMPLE 001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results. The shear stud capacity P_r was entered as an overwrite, since it is controlled by the deck profile geometry and the exact geometry of the example, which assumes a deck profile with a rib depth of 60 mm, a depth above profile of 60 mm and a total depth of 130 mm, cannot be modeled in ETABS, since in ETABS, only the rib depth and depth above profile can be specified.

HAND CALCULATION

Properties:

Materials:

S 355 Steel:

E = 210,000 MPa, f_y = 355 MPa, partial safety factor γ_a = 1.0

Normal weight concrete class C25/30:

E_{cm} = 30,500 MPa, f_{cu} = 30 MPa, density w_c = 24 kN/m^3

Section:

UKB457x191x74

h_a = 457 mm, b_f = 190.4 mm, t_f = 14.5 mm, t_w = 9 mm,

A_a = 9,460 mm^2, I_{ay} = 33,319 cm^4, W_{pl} = 1,653 cm^3

Deck:

Slab depth h_s =130 mm, depth above profile h_c = 60 mm,

Deck profile height h_p = 60 mm, $h_d = h_p$ + 10 mm for re-entrant stiffener, s_r = 300 mm, b_0 = 150 mm

Shear Connectors:

d = 19 mm, h = 95 mm, F_u = 450 MPa

Loadings:

Self weight slab, decking, reinforcement	= 2.567 kN/m^2
Self weight beam	= 0.73 kN/m
Construction load	= 0.75 kN/m^2
Ceiling	= 0.5 kN/m^2
Partitions (live load)	= 1.0 kN/m^2
Occupancy (live load)	= 4.0 kN/m^2

Design for Pre-Composite Condition:

Construction Required Flexural Strength:

$$w_{\text{factored construction}} = 1.25 \bullet (2.567 \bullet 3.0 + 0.73) + 1.5 \bullet 0.75 \bullet 3.0 = 13.91 \text{ kN/m}$$

$$M_{Ed} = \frac{w_{\text{factored construction}} \bullet L^2}{8} = \frac{13.91 \bullet 12^2}{8} = 250.4 \text{ kN-m}$$

Moment Capacity:

$$M_{a,pl,Rd} = W_{pl} \bullet f_d = 1,653 \bullet 10^3 \bullet 355 \bullet 10^{-6} = 587 \text{ kN-m}$$

Pre-Composite Deflection:

$$w_{\text{construction}} = 2.567 \bullet 3.0 + 0.73 = 8.43 \text{ kN/m}$$

$$\delta = \frac{5 \bullet w_{\text{construction}} \bullet L^4}{384 \bullet E \bullet I_{ay}} = \frac{5 \bullet 8.43 \bullet 12,000^4}{384 \bullet 210,000 \bullet 33,319 \bullet 10^4} = 32.5 \text{ mm}$$

Camber $= 0.8 \bullet \delta = 26$ mm, which is rounded down to 25 mm

Design for Composite Flexural Strength:

Required Flexural Strength:

$$w_{\text{factored}} = 1.25 \bullet 0.73 + (1.25 \bullet 2.567 + 1.25 \bullet 0.5 + 1.5 \bullet 1 + 1.5 \bullet 4.0) \bullet 3.0 = 34.91 \text{ kN/m}$$

$$M_{Ed} = \frac{w_{\text{factored}} \bullet L^2}{8} = \frac{34.91 \bullet 12^2}{8} = 628.4 \text{ kN-m}$$

Full Composite Action Available Flexural Strength:

Effective width of slab:

$$b_{\text{eff}} = \frac{2 \bullet \text{L}}{8} = \frac{2 \bullet 12}{8} = 3 \text{ m}$$

Resistance of slab in compression:

$$R_c = \frac{0.85 \bullet f_{ck}}{\gamma_c} \bullet b_{\text{eff}} \bullet h_c = 0.85 \bullet (25/1.5) \bullet 3,000 \bullet 60 \bullet 10^{-3} = 2,550 \text{ kN} \quad \textbf{controls}$$

Resistance of steel section in tension:

$$R_s = f_{yd} \bullet A_a = 355 \bullet 9,460 \bullet 10^{-3} = 3,358 \text{ kN}$$

Depth of compression block within steel section flange:

$$x = \frac{R_s - R_c}{2 \bullet b_f \bullet f_{yd}} = \frac{3,358 - 2,250}{2 \bullet 190.4 \bullet 355} = 6 \text{ mm}$$

$$d_2 = x/2 = 0.273 \text{ in.}$$

The plastic axis is in the steel flange and the moment resistance for full composite action is:

$$M_{a,pl,RD} = R_s\left[\frac{h}{2} - d_2\right]\frac{h}{2} + \mathrm{R_c}\left[h_s - \frac{h_c}{2}\right] - \frac{(R_s - R_c)^2}{R_f}\frac{t_f}{4}$$

$$= 3,358\frac{453.6}{2} \bullet 10^{-3} + 2,550\left[130 - \frac{60}{2}\right] \bullet 10^{-3} - \frac{(3,358 - 2,550)^2}{980}\frac{14.5}{4} \bullet 10^{-3}$$

$$= 1020.0 \text{ kN-m}$$

Partial Composite Action Available Flexural Strength:

Assume 77.5% composite action:

$$R_q = 0.775 \bullet R_s = 0.775 \bullet 3,358 = 1,976 \text{ kN}$$

Tensile Resistance of web:

$$R_w = t_w \bullet (D - 2 \bullet t_f) \bullet p_y = 8.5 \bullet (453.6 - 2 \bullet 12.7) \bullet 355 \bullet 10^{-3} = 1,292 \text{ kN}$$

As $R_q > R_w$, the plastic axis is in the steel flange, and

$$M_c = R_s\frac{h}{2} + R_q\left[h_s - \frac{R_q}{R_c}\frac{h_c}{2}\right] - \frac{(R_s - R_q)^2}{R_f}\frac{t_f}{4}$$

$$= 3,358\frac{453.6}{2} \bullet 10^{-3} + 1,976\left[130 - \frac{1,976}{2,250} \bullet \frac{60}{2}\right] \bullet 10^{-3} - \frac{(3,358 - 1,976)^2}{980}\frac{14.5}{4} \bullet 10^{-3}$$

$$= 971.2 \text{ kN-m}$$

Resistance of Shear Connector:

Resistance of shear connector in solid slab:

$$P_{Rd} = 0.29 \bullet \alpha \bullet d^2 \bullet \sqrt{f_{ck} \bullet E_{cm}}\Big/\gamma_v \le 0.8 \bullet f_u \bullet \left(\pi\frac{d^2}{4}\right)\Big/\gamma_v \text{ with } \alpha\text{=1.0 for } \frac{h}{d} = \frac{95}{19} > 4$$

$$0.29 \bullet \alpha \bullet d^2 \bullet \sqrt{f_{ck}E_{cm}}\Big/\gamma_v = 0.29 \bullet 1.0 \bullet 19^2 \bullet \sqrt{25 \bullet 30,500} \bullet 10^{-3}\Big/1.25 = 73 \text{ kN } \textbf{controls}$$

$$0.8 \bullet f_u \bullet \left(\pi \frac{d^2}{4}\right) \Big/ \gamma_v = 0.8 \bullet 450 \bullet \left(\pi \frac{19}{4}\right) \Big/ 1.25 = 81.7 \text{ kN}$$

Reduction factor for decking perpendicular to beam – assuming two studs per rib:

$$k_t = \frac{0.7}{\sqrt{n_r}} \left(b_0 / h_p\right) \left[\left(h_{sc} / h_p\right) - 1\right] \leq 0.75 \text{ per EN 1994-1-1 Table 6.2}$$

$$= \frac{0.7}{\sqrt{2}} \frac{150}{60} \left[\left(95/60\right) - 1\right] = 0.72 \leq 0.75$$

$$P_{Rd} = 0.72 \bullet 73 = 52 \text{ kN}$$

Total resistance with two studs per rib and 19 ribs from the support to the mid-span:

$$R_q = 2 \bullet 19 \bullet 52 = 1,976 \text{ kN}$$

Live Load Deflection:

The second moment of area of the composite section, based on elastic properties, I_c is given by:

$$I_c = \frac{A_a \bullet (h + 2 \bullet h_p + h_c)^2}{4 \bullet (1 + n \bullet r)} + \frac{b_{\text{eff}} \bullet h_c^3}{12 \bullet n} + I_{ay}$$

$$r = \frac{A_a}{b_{\text{eff}} \bullet h_c} = \frac{9,460}{3,000 \bullet 60} = 0.052$$

n = *modular ratio* = 10 for normal weight concrete subject to variable loads

$$I_c = \frac{9,460 \bullet (457 + 2 \bullet 70 + 60)^2}{4 \bullet (1 + 10 \bullet 0.052)} + \frac{3,000 \bullet 60^3}{12 \bullet 10} + 33,320 \bullet 10^4$$

$$= (6.69 + 0.05 + 3.33) \bullet 10^8 = 10.08 \bullet 10^8 \text{ mm}^4$$

$$\delta_{\text{live}} = \frac{5 \bullet w_{\text{live}} \bullet L^4}{384 \bullet E \bullet I_c} = \frac{5 \bullet 15 \bullet (12,000)^4}{384 \bullet 210,000 \bullet 10.08 \bullet 10^8} = 19.1 \text{ mm}$$

Design for Shear Strength:

Required Shear Strength:

$$V_{Ed} = \frac{w_{\text{factored}} \bullet L}{2} = \frac{34.91 \bullet 12}{2} = 209.5 \text{ kN}$$

Shear Resistance of Steel Section:

$$V_{pl,Rd} = \frac{457 \bullet 9.0 \bullet 355}{\sqrt{3} \bullet 10^{-3}} = 843 \text{ kN}$$

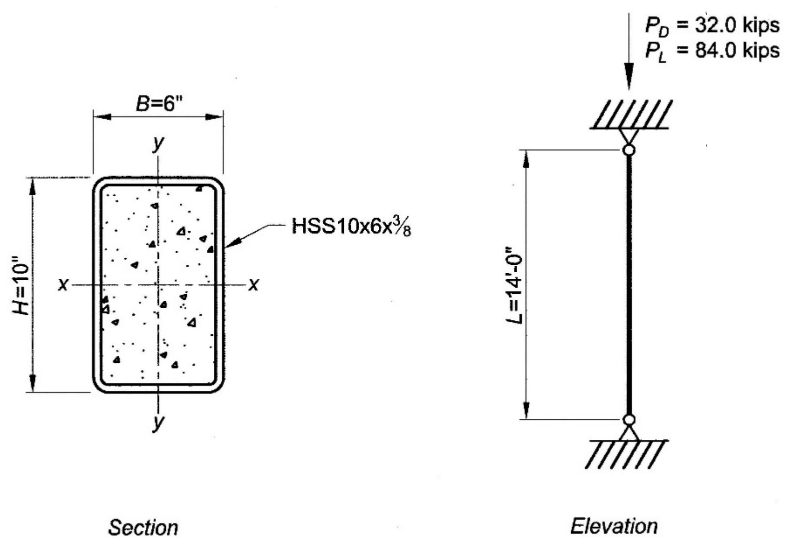
AISC-360-10 Example 001

COMPOSITE COLUMN DESIGN

EXAMPLE DESCRIPTION

Determine if the 14-ft.-long filled composite member illustrated below is adequate for the indicated dead and live loads. The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft^3) concrete fill having a specified compressive strength, $f'_c = 5$ ksi.

GEOMETRY, PROPERTIES AND LOADING



Member Properties	Loading	Geometry
HSS10x6 x3/8	P_D = 32.0 kips	Height, L = 14 ft
E = 29,000 ksi	P_L = 84.0 kips	
F_y = 46 ksi		

TECHNICAL FEATURE OF ETABS TESTED

Compression capacity of composite column design.

RESULTS COMPARISON

Independent results are referenced from Example I.4 from the AISC Design Examples, Version 14.0.

Output Parameter	ETABS	Independent	Percent Difference
Required Strength P_u (kip)	172.8	172.8	0.00%
Available Strength ΦP_n (kip)	342.93	354.78	3.34%

COMPUTER FILE: AISC-360-10 EXAMPLE 001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Materials:

ASTM A500 Grade B Steel

E = 29,000 ksi, F_y = 46 ksi, F_u = 58 ksi

5000 psi normal weight concrete

E_c = 3,900 ksi, $f_c' = 5$ ksi, $w_{concrete}$ = 145 pcf

Section dimensions and properties:

HSS10x6x⅜

H = 10.0 in, B= 6.00 in, t = 0.349 in

A_s = 10.4 in^2, I_{sx} = 137 in^4, I_{sy} = 61.8 in^4

Concrete area

$$h_i = H - 2 \bullet t = 10 - 2 \bullet 0.349 = 9.30 \text{ in.}$$

$$b_i = B - 2 \bullet t = 6 - 2 \bullet 0.349 = 5.30 \text{ in.}$$

$$A_c = b_i \bullet h_i - t^2 \bullet (4 - \pi) = 5.30 \bullet 9.30 - (0.349)^2 \bullet (4 - \pi) = 49.2 \text{ in.}^2$$

Moment of inertia for bending about the y-*y* axis:

$$I_{cy} = \frac{(H - 4 \bullet t) \bullet b_t^3}{12} + \frac{t \bullet (B - 4 \bullet t)^3}{6} + \frac{(9\pi^2 - 64) \bullet t^4}{36 \bullet \pi} + \pi \bullet t^2 \left(\frac{B - 4 \bullet t}{2} - \frac{4 \bullet t}{3 \bullet \pi} \right)^2$$

$$= \frac{(10 - 4 \bullet 0.349) \bullet 5.30^3}{12} + \frac{0.349 \bullet (6 - 4 \bullet 0.349)^3}{6} + \frac{(9\pi^2 - 64) \bullet 0.349^4}{36 \bullet \pi} +$$

$$\pi \bullet 0.349^2 (\frac{6 - 4 \bullet 0.349}{2} - \frac{4 \bullet 0.349}{3 \bullet \pi})^2$$

$$= 114.3 \text{ in.}^4$$

Design for Compression:

Required Compressive Strength:

$$P_u = 1.2 \bullet P_D + 1.6 \bullet P_L = 1.2 \bullet 32.0 + 1.6 \bullet 84.0 = 172.8 \text{ kips}$$

Nominal Compressive Strength:

$$P_{no} = P_p = F_y \bullet A_s + C_2 \bullet f_c' \left(A_c + A_{sr} \frac{E_s}{E_c} \right)$$

where

$C_2 = 0.85$ for rectangular sections

$A_{sr} = 0$ when no reinforcing is present within the HSS

$P_{no} = 46 \bullet 10.4 + 0.85 \bullet 5 \bullet (49.2 + 0.0) = 687.5$ kips

Weak-axis Elastic Buckling Force:

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_s} \right) \leq 0.9$$

$$= 0.6 + 2\left(\frac{10.4}{49.2 + 10.4} \right) \leq 0.9$$

$$= 0.949 > 0.9 \qquad \text{0.9 controls}$$

$$EI_{\text{eff}} = E_s \bullet I_{sy} + E_s \bullet I_{sr} + C_3 \bullet E_c \bullet I_{cy}$$

$$= 29{,}000 \bullet 62.1 + 0 + 0.9 \bullet 3{,}900 \bullet 114.3$$

$$= 2{,}201{,}000 \text{ kip-in}^2$$

$P_e = \pi^2 (EI_{\text{eff}}) / (KL)^2$ where K = 1.0 for a pin-ended member

$$P_e = \frac{\pi^2 \bullet 2{,}201{,}000}{1.0 \bullet (14.0 \bullet 12)^2} = 769.7 \text{ kips}$$

Available Compressive Strength:

$$\frac{P_{no}}{P_e} = \frac{688}{769.7} = 0.893 < 2.25$$

Therefore, use AISC Specification Equation I2-2:

$$\Phi P_n = \Phi P_{no} \left[0.658^{\frac{P_{no}}{P_e}} \right] = 0.75 \bullet 687.5 \bullet (0.658)^{0.893} = 354.8 \text{ kips}$$

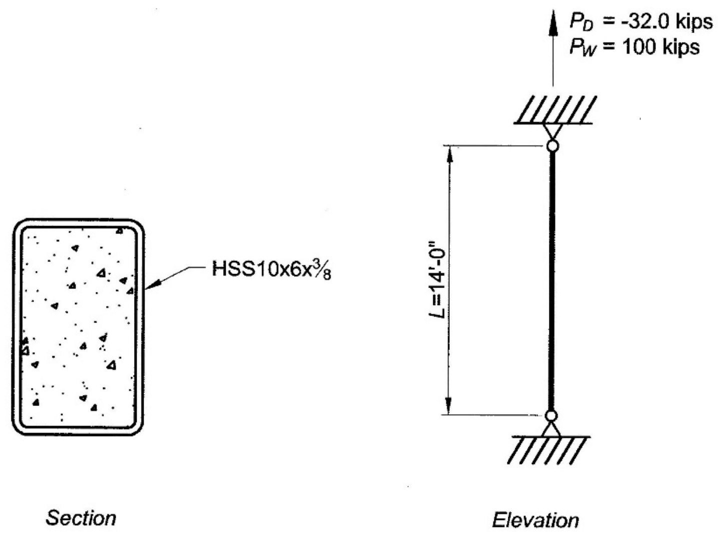
AISC-360-10 Example 002

COMPOSITE COLUMN DESIGN

EXAMPLE DESCRIPTION

Determine if the 14-ft.-long filled composite member illustrated below is adequate for the indicated dead load compression and wind load tension. The entire load is applied to the steel section.

GEOMETRY, PROPERTIES AND LOADING



Member Properties

HSS10x6 x3/8
E = 29,000 ksi
F_y = 46 ksi

Loading

P_D = -32.0 kips
P_W = 100.0 kips

Geometry

Height, L = 14 ft

TECHNICAL FEATURE OF ETABS TESTED

- Tension capacity of composite column design.

RESULTS COMPARISON

Independent results are referenced from Example I.5 from the AISC Design Examples, Version 14.0.

Output Parameter	ETABS	Independent	Percent Difference
Required Strength, P_u (kip)	71.2	71.2	0.00%
Available Strength, ΦP_n (kip)	430.5	430.0	0.12%

COMPUTER FILE: AISC-360-10 EXAMPLE 002.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Materials:

ASTM A500 Grade B Steel

E = 29,000 ksi, F_y = 46 ksi, F_u = 58 ksi

5000 psi normal weight concrete

E_c = 3,900 ksi, $f_c' = 5$ ksi, w_{concrete} = 145 pcf

Steel section dimensions:

HSS10x6x⅜

H = 10.0 in, B = 6.00 in, t = 0.349 in, A_s = 10.4 in^2

Design for Tension:

Required Compressive Strength:

The required compressive strength is (taking compression as negative and tension as positive):

$$P_u = 0.9 \bullet P_D + 1.0 \bullet P_W = 0.9 \bullet (-32.0) + 1.0 \bullet 100.0 = 71.2 \text{ kips}$$

Available Tensile Strength:

$$\Phi P_n = \Phi(A_s \bullet F_y + A_{sr} \bullet F_{ysr}) = 0.9(10.4 \bullet 46 + 0 \bullet 60) = 430 \text{ kips}$$

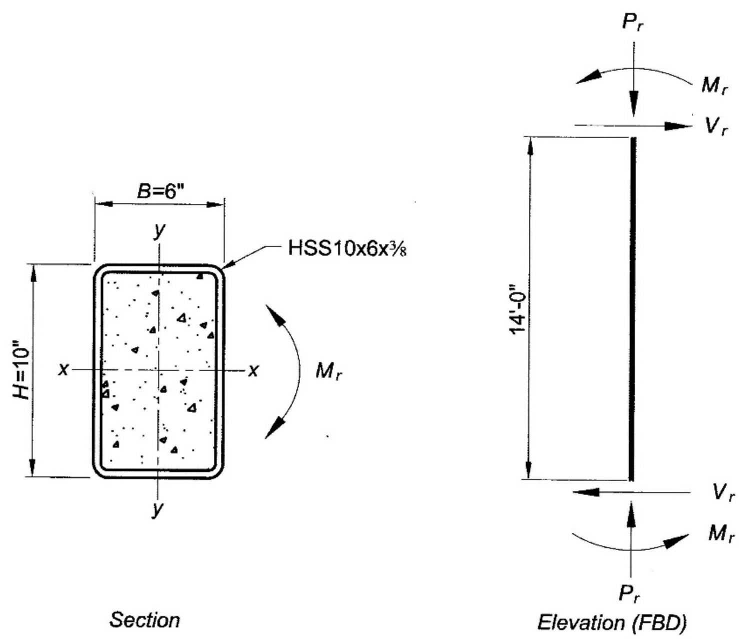
AISC-360-10 Example 003

COMPOSITE COLUMN DESIGN

EXAMPLE DESCRIPTION

Determine if the 14-ft.-long filled composite member illustrated below is adequate for the indicated axial forces, shears, and moments. The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft^3) concrete fill having a specified compressive strength, $f_c' = 5$ ksi.

GEOMETRY, PROPERTIES AND LOADING



Section

Elevation (FBD)

Member Properties	Loading	Geometry
HSS10x6 x⅜	P_r = 129.0 kips	Height, L = 14 ft
E = 29,000 ksi	M_r = 120.0 kip-ft	
F_y = 46 ksi	V_r = 17.1 kips	

TECHNICAL FEATURE OF ETABS TESTED

Tension capacity of composite column design.

RESULTS COMPARISON

Independent results are referenced from Example I.1 from the AISC Design Examples, Version 14.0.

Output Parameter	ETABS	Independent	Percent Difference
Required Strength, F_u (k)	129	129	0.00%
Available Strength, ΦP_n (kip)	342.9	354.78	-3.35%
Required Strength, M_u (k-ft)	120	120	0.00%
Available Strength, $\Phi_b M_n$ (k-ft)	130.58	130.5	0.06%
Interaction Equation H1-1a	1.19	1.18	0.85%

COMPUTER FILE: AISC-360-10 EXAMPLE 003.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Properties:

Materials:

ASTM A500 Grade B Steel

E = 29,000 ksi, F_y = 46 ksi, F_u = 58 ksi

5000 psi normal weight concrete

E_c = 3,900 ksi, $f_c' = 5$ ksi, w_{concrete} = 145 pcf

Section dimensions and properties:

HSS10x6x⅜

H = 10.0 in, B= 6.00 in, t = 0.349 in

A_s = 10.4 in^2, I_{sx} = 137 in^4, Z_{sx}=33.8 in^3, I_{sy} = 61.8 in^4

Concrete area

h_t = 9.30 in., b_t = 5.30 in., A_c = 49.2 in.2, I_{cx} = 353 in^4, I_{cy} = 115 in^4

Compression capacity:

Nominal Compressive Strength:

$\Phi_c P_n$= 354.78 kips as computed in Example I.4

Bending capacity:

Maximum Nominal Bending Strength:

Z_{sx} = 33.8 in^3

$$Z_c = \frac{b_i \bullet h_i^2}{4} - 0.192 \bullet r_i^3 \text{ where } r_i = t$$

$$= \frac{5.30 \bullet (9.30)^2}{4} - 0.192 \bullet (0.349)^3 = 114.7 \text{ in.}^3$$

$$M_D = F_y \bullet Z_{sx} + \frac{0.85 \bullet f_c' \bullet Zc}{2}$$

$$= 46 \bullet 33.8 + \frac{0.85 \bullet 5 \bullet 115}{2} = \frac{1{,}798.5 \text{ kip-in.}}{12 \text{ in./ft}} = 149.9 \text{ kip-ft}$$

Available Bending Strength:

$$h_n = \frac{0.85 \bullet f_c' \bullet A_c}{2(0.85 \bullet f_c' \bullet b_i + 4 \bullet t \bullet F_y)} \leq \frac{h_i}{2}$$

$$= \frac{0.85 \bullet 5 \bullet 49.2}{2(0.85 \bullet 5 \bullet 5.30 + 4 \bullet 0.349 \bullet 50)} \leq \frac{9.30}{2}$$

$$= 1.205 \leq 4.65$$

$$= 1.205 \text{ in.}$$

$$Z_{sn} = 2 \bullet t \bullet h_n^2 = 2 \bullet 0.349 \bullet (1.205)^2 = 1.01 \text{ in.}^3$$

$$Z_{cn} = b_i \bullet h_n^2 = 5.30 \bullet (1.205)^2 = 7.70 \text{ in.}^3$$

$$M_{nx} = M_D - F_y \bullet Z_{sn} - \frac{0.85 \bullet f_c' \bullet Z_{cn}}{2}$$

$$= 1{,}800 - 46 \bullet 1.02 - \frac{0.85 \bullet 5 \bullet 7.76}{2} = \frac{1{,}740 \text{ kip-in.}}{12 \text{ in./ft}} = 144.63 \text{ kip-ft}$$

$$\Phi_b M_{nx} = 0.9 \bullet 144.63 = 130.16 \text{ kip-ft}$$

Interaction Equation H1-1a:

$$\frac{P_u}{\Phi_c \bullet Pn} + \frac{8}{9}\left(\frac{M_u}{\Phi_b \bullet M_n}\right) \leq 1.0$$

$$\frac{129}{354.78} + \frac{8}{9}\left(\frac{120}{130.16}\right) \leq 1.0$$

1.18 > 1.0 **n.g.**

ACI 318-08 PT-SL Example 001

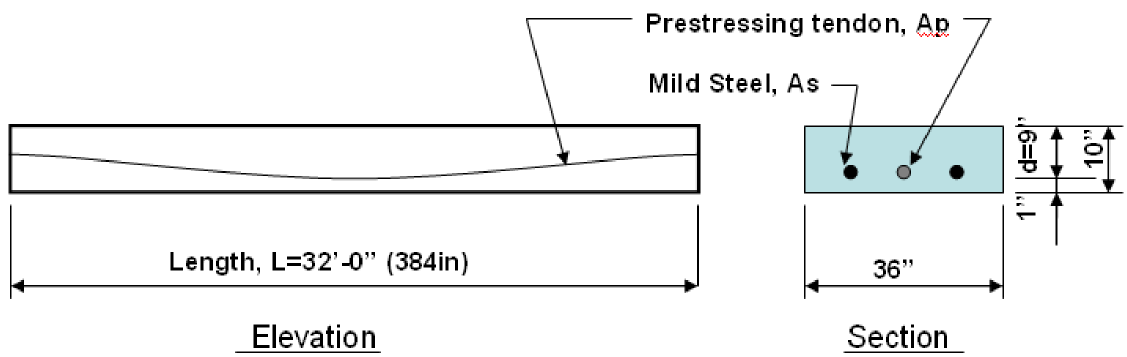
Design Verification of Post-Tensioned Slab using the ACI 318-08 code

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in ETABS. The modeled slab is 10 inches thick by 36 inches wide and spans 32 feet, as shown in shown in Figure 1. A 36-inch-wide design strip was centered along the length of the slab and was defined as an A-Strip. B-strips were placed at each end of the span perpendicular to the Strip-A (the B-Strips are necessary to define the tendon profile). A tendon, with two strands having an area of 0.153 square inches each, was added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are shown below. The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the ETABS results and summarized for verification and validation of the ETABS results.

Loads: Dead = self weight , Live = 100psf



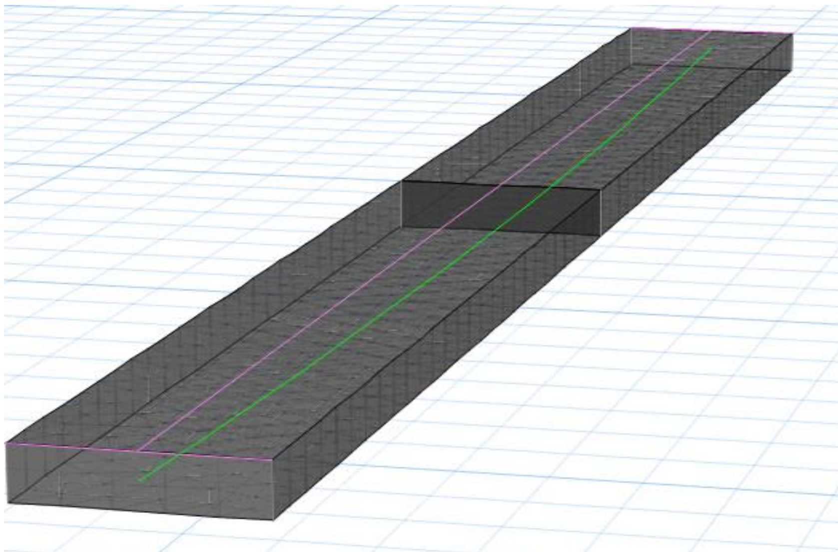


Figure 1 One-Way Slab

GEOMETRY, PROPERTIES AND LOADING

Thickness,	T, h	=	10	in
Effective depth,	d	=	9	in
Clear span,	L	=	384	in
Concrete strength,	f'_c	=	4,000	psi
Yield strength of steel,	f_y	=	60,000	psi
Prestressing, ultimate	f_{pu}	=	270,000	psi
Prestressing, effective	f_e	=	175,500	psi
Area of Prestress (single strand),	A_P	=	0.153	sq in
Concrete unit weight,	w_c	=	0.150	pcf
Modulus of elasticity,	E_c	=	3,600	ksi
Modulus of elasticity,	E_s	=	29,000	ksi
Poisson's ratio,	ν	=	0	
Dead load,	w_d	=	self	psf
Live load,	w_l	=	100	psf

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live and post-tensioning loads.

RESULTS COMPARISON

The ETABS total factored moments, required mild steel reinforcing and slab stresses are compared to the independent hand calculations in Table 1.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (k-in)	1429.0	1428.3	-0.05%
Area of Mild Steel req'd, As (sq-in)	2.21	2.21	0.00%
Transfer Conc. Stress, top (D+PT_I), ksi	–0.734	–0.735	0.14%
Transfer Conc. Stress, bot (D+PT_I), ksi	0.414	0.414	0.00%
Normal Conc. Stress, top (D+L+PT_F), ksi	–1.518	–1.519	0.07%
Normal Conc. Stress, bot (D+L+PT_F), ksi	1.220	1.221	0.08%
Long-Term Conc. Stress, top (D+0.5L+$PT_{F(L)}$), ksi	–1.134	–1.135	0.09%
Long-Term Conc. Stress, bot (D+0.5L+$PT_{F(L)}$), ksi	0.836	0.837	0.12%

COMPUTER FILE: ACI 318-08 PT-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

CALCULATIONS:

Design Parameters:

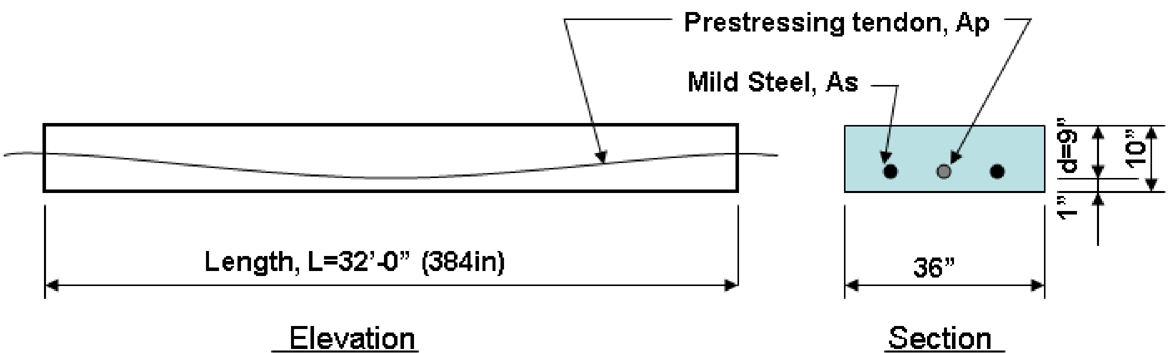
$\phi = 0.9$

Mild Steel Reinforcing

$f'c$ = 4000 psi
fy = 60,000 psi

Post-Tensioning

f_j = 216.0 ksi
Stressing Loss = 27.0 ksi
Long-Term Loss = 13.5 ksi
f_i = 189.0 ksi
f_e = 175.5 ksi



Loads:

Dead, self-wt = $10/12$ ft × 0.150 kcf = 0.125 ksf (D) × 1.2 = 0.150 ksf (D_u)
Live, 0.100 ksf (L) × 1.6 = 0.160 ksf (L_u)
Total =0.225 ksf (D+L) 0.310 ksf (D+L)ult

ω =0.225 ksf × 3 ft = 0.675 klf, ω_u = 0.310 ksf × 3ft = 0.930 klf

Ultimate Moment, $M_U = \dfrac{w\, l_1^2}{8}$ = 0.310 klf × $32^2/8$ = 119.0 k-ft = 1429.0 k-in

Ultimate Stress in strand, $f_{PS} = f_{SE} + 10000 + \dfrac{f'c}{300\rho_P}$ (span-to-depth ratio > 35)

$$= 175,500 + 10,000 + \frac{4,000}{300(0.000944)}$$

$$= 199,624 \text{ psi} \leq 205,500 \text{ psi}$$

Ultimate force in PT, $F_{ult,PT} = A_P\left(f_{PS}\right) = 2(0.153)(199.62) = 61.08 \text{ kips}$

Ultimate force in RC, $F_{ult,RC} = A_s\left(f\right)_y = 2.00(\text{assumed})(60.0) = 120.0 \text{ kips}$

Total Ultimate force, $F_{ult,Total} = 61.08 + 120.0 = 181.08 \text{ kips}$

Stress block depth, $a = \dfrac{F_{ult,Total}}{0.85 f'cb} = \dfrac{181.08}{0.85(4)(36)} = 1.48 \text{ in}$

Ultimate moment due to PT, $M_{ult,PT} = F_{ult,PT}\left(d - \dfrac{a}{2}\right)\phi = 61.08\left(9 - \dfrac{1.48}{2}\right)(0.9) = 454.1 \text{ k-in}$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 1429.0 - 454.1 = 974.9 \text{ k-in}$

Required area of mild steel reinforcing, $A_S = \dfrac{M_{net}}{\phi f_y\left(d - \dfrac{a}{2}\right)} = \dfrac{974.9}{0.9(60)\left(9 - \dfrac{1.48}{2}\right)} = 2.18 \text{ in}^2$

Note: The required area of mild steel reinforcing was calculated from an assumed amount of steel. Since the assumed value and the calculated value are not the same a second iteration can be performed. The second iteration changes the depth of the stress block and the calculated area of steel value. Upon completion of the second iteration the area of steel was found to be 2.21in^2

Check of Concrete Stresses at Mid-Span:

Initial Condition (Transfer), load combination $(D + L + PT_i) = 1.0D + 1.0PT_I$

The stress in the tendon at transfer = jacking stress − stressing losses = 216.0 − 27.0
= 189.0 ksi

The force in the tendon at transfer, = $189.0(2)(0.153) = 57.83$ kips

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0 \text{ k-ft} = 576 \text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 57.83(4 \text{ in}) = 231.3 \text{ k-in}$

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-57.83}{10(36)} \pm \dfrac{576.0 - 231.3}{600}$, where S = 600 in^3

$\boldsymbol{f} = -0.161 \pm 0.5745$

$f = -0.735(\text{Comp})\max, 0.414(\text{Tension})\max$

Normal Condition, load combinations: $(D + L + PT_F) = 1.0D + 1.0L + 1.0PT_F$

Tendon stress at normal = jacking − stressing − long-term = 216.0 − 27.0 − 13.5 = 175.5 ksi
The force in tendon at Normal, = $175.5(2)(0.153) = 53.70$ kips

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0 \text{ k-ft} = 576 \text{ k-in}$

Moment due to dead load, $M_L = 0.100(3)(32)^2/8 = 38.4 \text{ k-ft} = 461 \text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 53.70(4 \text{ in}) = 214.8 \text{ k-in}$

Stress in concrete for $(D + L + PT_F)$, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_{D+L} - M_{PT}}{S} = \dfrac{-53.70}{10(36)} \pm \dfrac{1037.0 - 214.8}{600}$

$\boldsymbol{f} = -0.149 \pm 1.727 \pm 0.358$

$f = -1.518(\text{Comp})\max, 1.220(\text{Tension})\max$

Long-Term Condition, load combinations: $(D + 0.5L + PT_{F(L)}) = 1.0D + 0.5L + 1.0PT_F$

Tendon stress at normal = jacking − stressing − long-term = 216.0 − 27.0 − 13.5 = 175.5 ksi
The force in tendon at Normal, = $175.5(2)(0.153) = 53.70$ kips

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0 \text{ k-ft} = 576 \text{ k-in}$

Moment due to dead load, $M_L = 0.100(3)(32)^2/8 = 38.4 \text{ k-ft} = 460 \text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 53.70(4 \text{ in}) = 214.8 \text{ k-in}$

Stress in concrete for (D + 0.5L + $PT_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-53.70}{10(36)} \pm \frac{806.0 - 214.8}{600}$$

$$f = -0.149 \pm 0.985$$

$$f = -1.134(\text{Comp})\max, 0.836(\text{Tension})\max$$

ACI 318-08 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 24-foot-long spans in each direction, as shown in Figure 1.

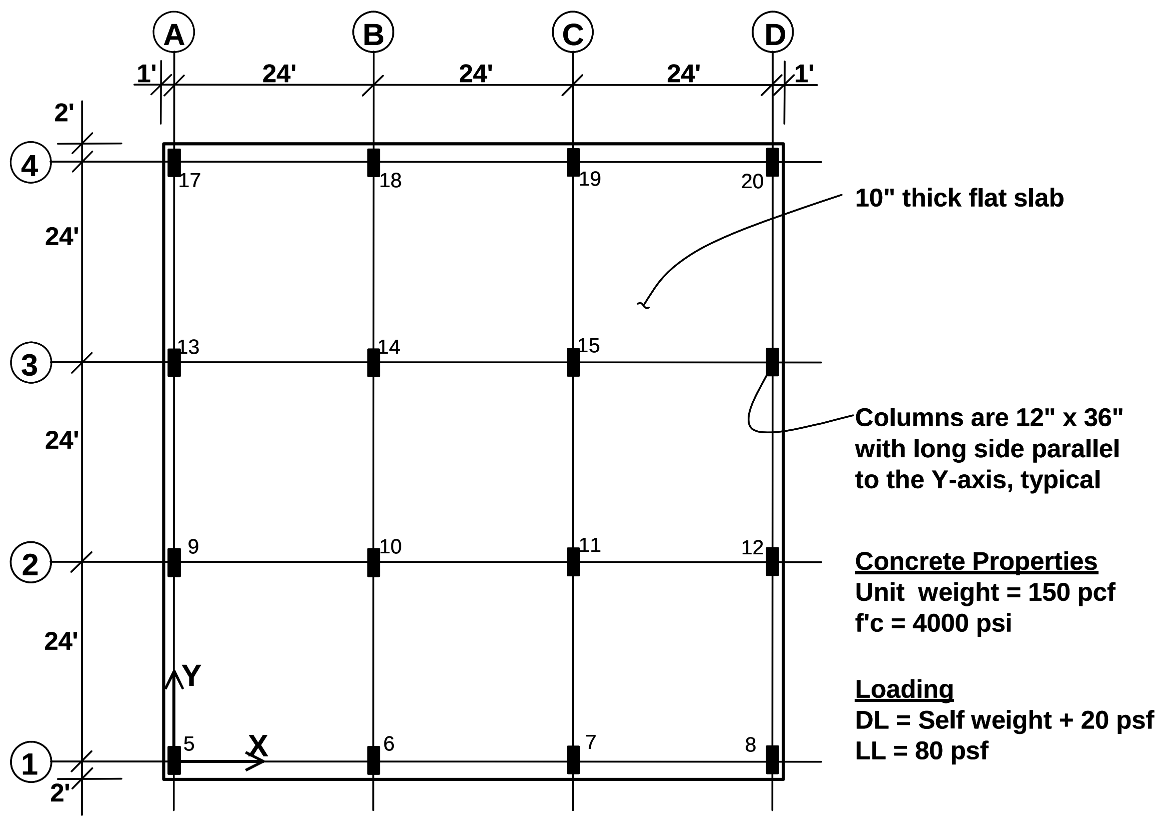


Figure 1: Flat Slab For Numerical Example

The slab overhangs the face of the column by 6 inches along each side of the structure. The columns are typically 12 inches wide by 36 inches long, with the long side parallel to the Y-axis. The slab is typically 10 inches thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 150 pcf and an f'c of 4000 psi. The dead load consists of the self weight of the structure plus an additional 20 psf. The live load is 80 psf.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress, and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS punching shear capacity, shear stress ratio, and D/C ratio with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this example.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (ksi)	Shear Capacity (ksi)	D/C ratio
ETABS	0.1930	0.158	1.22
Calculated	0.1930	0.158	1.22

COMPUTER FILE: ACI 318-08 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation for Interior Column Using ETABS Method

d = [(10 - 1) + (10 - 2)] / 2 = 8.5"

Refer to Figure 2.

b_0 = 44.5 + 20.5 + 44.5 + 20.5 = 130"

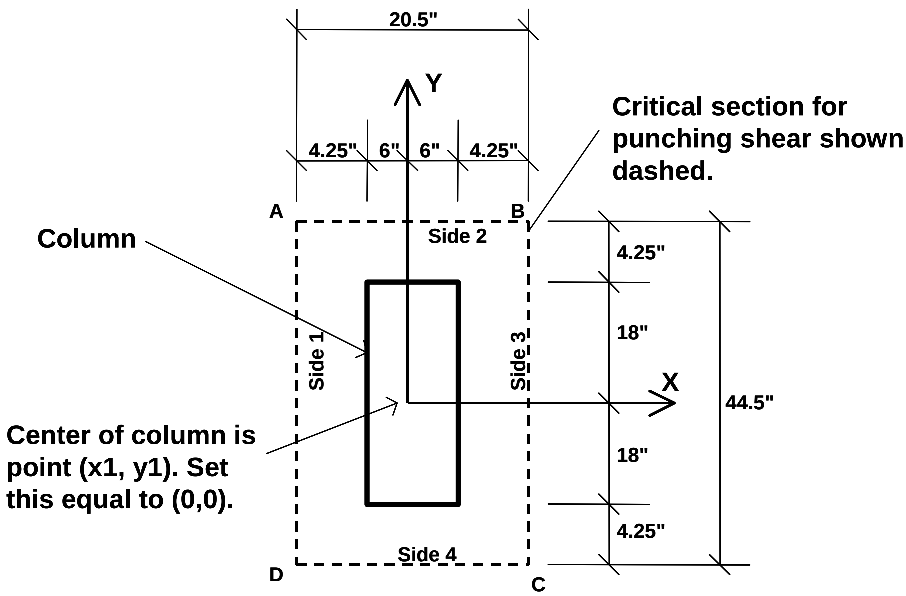


Figure 2: Interior Column, Grid B-2 in ETABS Model

$$\gamma_{V2} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{44.5}{20.5}}} = 0.4955$$

$$\gamma_{V3} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{20.5}{44.5}}} = 0.3115$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear, as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–10.25	0	10.25	0	N.A.
y_2	0	22.25	0	–22.25	N.A.
L	44.5	20.5	44.5	20.5	b_0 = 130
d	8.5	8.5	8.5	8.5	N.A.
Ld	378.25	174.25	378.25	174.25	1105
Ldx_2	–3877.06	0	3877.06	0	0
Ldy_2	0	3877.06	0	–3877.06	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{1105} = 0"$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{1105} = 0"$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	44.5	20.5	44.5	20.5	N.A.
d	8.5	8.5	8.5	8.5	N.A.
$x_2 - x_3$	–10.25	0	10.25	0	N.A.
$y_2 - y_3$	0	22.25	0	–22.25	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	64696.5	86264.6	64696.5	86264.6	301922.3
I_{YY}	39739.9	7151.5	39739.9	7151.5	93782.8
I_{XY}	0	0	0	0	0

From the ETABS output at Grid B-2:

V_U = 189.45 k

$\gamma_{V2} M_{U2}$ = –156.39 k-in

$\gamma_{V3} M_{U3}$ = 91.538 k-in

At the point labeled A in Figure 2, x_4 = –10.25 and y_4 = 22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(22.25-0\right)-\left(0\right)\left(-10.25-0\right)\right]}{(301922.3)(93782.8)-(0)^2} - \frac{91.538\left[301922.3\left(-10.25-0\right)-\left(0\right)\left(22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 – 0.0115 – 0.0100 = **0.1499 ksi** at point A

At the point labeled B in Figure 2, x_4 = 10.25 and y_4 = 22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(22.25-0\right)-\left(0\right)\left(10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(10.25-0\right)-\left(0\right)\left(22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 – 0.0115 + 0.0100 = **0.1699 ksi** at point B

At the point labeled C in Figure 2, x_4 = 10.25 and y_4 = –22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(-22.25-0\right)-\left(0\right)\left(10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(10.25-0\right)-\left(0\right)\left(-22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 + 0.0115 + 0.0100 = **0.1930 ksi** at point C

At the point labeled D in Figure 2, x_4 = –10.25 and y_4 = –22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(-22.25-0\right)-\left(0\right)\left(-10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(-10.25-0\right)-\left(0\right)\left(-22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 + 0.0115 - 0.0100 = **0.1729 ksi** at point D

Point C has the largest absolute value of v_u, thus v_{max} = 0.1930 ksi

The shear capacity is calculated based on the smallest of ACI 318-08 equations 11-34, 11-35 and 11-36 with the b_0 and d terms removed to convert force to stress.

$$\varphi v_C = \frac{0.75\left(2+\frac{4}{36/12}\right)\sqrt{4000}}{1000} = 0.158$$ ksi in accordance with equation 11-34

$$\varphi v_C = \frac{0.75\left(\frac{40 \bullet 8.5}{130}+2\right)\sqrt{4000}}{1000} = 0.219$$ ksi in accordance with equation 11-35

$$\varphi v_C = \frac{0.75 \bullet 4 \bullet \sqrt{4000}}{1000} = 0.190$$ ksi in accordance with equation 11-36

Equation 11-34 yields the smallest value of ϕv_C = 0.158 ksi and thus this is the shear capacity.

$$Shear\ Ratio = \frac{v_U}{\varphi v_C} = \frac{0.193}{0.158} = 1.22$$

ACI 318-08 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using ETABS. The slab is 6 inches thick and spans 12 feet between walls. The walls are modeled as line supports. The computational model uses a finite element mesh, automatically generated by ETABS. The maximum element size is specified to be 36 inches. To obtain factored moments and flexural reinforcement in a design strip, one one-foot-wide strip is defined in the X-direction on the slab, as shown in Figure 1.

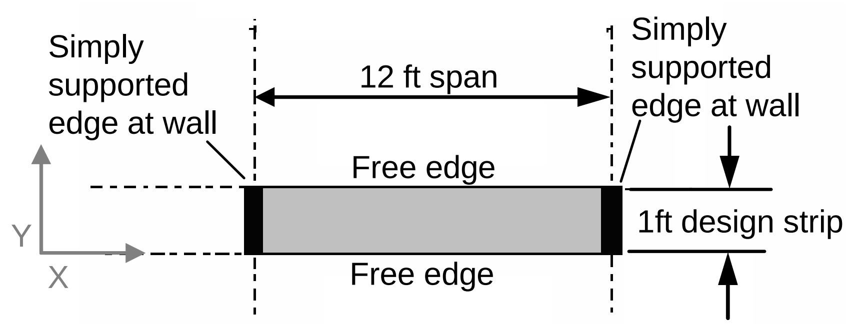


Figure 1 Plan View of One-Way Slab

One dead load case (DL80) and one live load case (LL100) with uniformly distributed surface loads of magnitudes 80 and 100 psf, respectively, are defined in the model. A load combination (COMB100) is defined using the ACI 318-08 load combination factors, 1.2 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing the analysis, design is performed in accordance with ACI 318-08 using ETABS and also by hand computation. Table 1 shows the comparison of the moments and design reinforcements computed using the two methods.

Geometry, Properties and Loading

Thickness	T, h	=	6	in
Depth of tensile reinf.	d_c	=	1	in
Effective depth	d	=	5	in
Clear span	l_n, l_1	=	144	in
Concrete strength	f_c	=	4,000	psi
Yield strength of steel	f_y	=	60,000	psi
Concrete unit weight	w_c	=	0	pcf
Modulus of elasticity	E_c	=	3,600	ksi
Modulus of elasticity	E_s	=	29,000	ksi
Poisson's ratio	ν	=	0	
Dead load	w_d	=	80	psf
Live load	w_l	=	100	psf

Technical Features of ETABS Tested

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

Results Comparison

Table 1 shows the comparison of the ETABS total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (k-in)	Reinforcement Area (sq-in) A_s^+
Medium	ETABS	55.22	0.213
	Calculated	55.22	0.213

$A^{+}_{s,min}$ = 0.1296 sq-in

Computer File: ACI 318-08 RC-SL Ex001.EDB

Conclusion

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\varphi = 0.9$

$b = 12$ in

$A_{s,min} = 0.0018bh = 0.1296$ sq-in

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right) = 0.85$$

$$c_{\max} = \frac{0.003}{0.003 + 0.005} d = 1.875 \text{ in}$$

$a_{max} = \beta_1 c_{max} = 1.59375$ in

For the load combination, w and M_u are calculated as follows:

$w = (1.2w_d + 1.6w_t)\ b / 144$

$$M_u = \frac{wl_1^2}{8}$$

$A_s = \min[A_{s,\min}, (4/3)\ A_{s,\text{required}}] = \min[0.1296, (4/3)2.11] = 0.1296$ sq-in

COMB100

w_d = 80 psf

w_t = 100 psf

w = 21.33 lb/in

$M_{u\text{-}strip}$ = 55.22 k-in

$M_{u\text{-}design}$ = 55.629 k-in

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f_c' \varphi\ b}} = 0.3128 \text{ in} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_u}{\varphi\, f_y \left(d - \frac{a}{2}\right)} = 0.213 \text{ sq-in} > A_{s,\min}$$

A_s = 0.2114 sq-in

ACI 318-11 PT-SL EXAMPLE 001

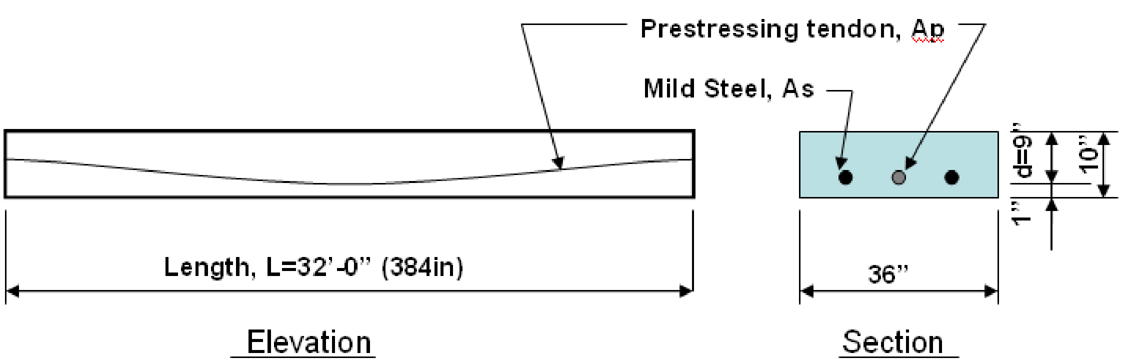
Design Verification of Post-Tensioned Slab using the ACI 318-11 code

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in ETABS. The modeled slab is 10 inches thick by 36 inches wide and spans 32 feet, as shown in shown in Figure 1. A 36-inch-wide design strip was centered along the length of the slab and was defined as an A-Strip. B-strips were placed at each end of the span perpendicular to the Strip-A (the B-Strips are necessary to define the tendon profile). A tendon, with two strands having an area of 0.153 square inches each, was added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are shown below. The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the ETABS results and summarized for verification and validation of the ETABS results.

Loads: Dead = self weight , Live = 100psf



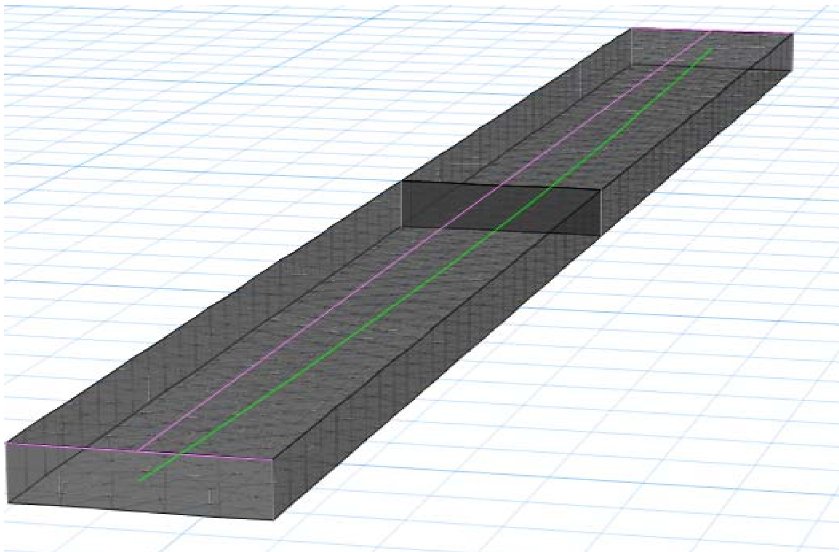


Figure 1 One-Way Slab

GEOMETRY, PROPERTIES AND LOADING

Thickness,	T, h	=	10	in
Effective depth,	d	=	9	in
Clear span,	L	=	384	in
Concrete strength,	f'_c	=	4,000	psi
Yield strength of steel,	f_y	=	60,000	psi
Prestressing, ultimate	f_{pu}	=	270,000	psi
Prestressing, effective	f_e	=	175,500	psi
Area of Prestress (single strand),	A_P	=	0.153	sq in
Concrete unit weight,	w_c	=	0.150	pcf
Modulus of elasticity,	E_c	=	3,600	ksi
Modulus of elasticity,	E_s	=	29,000	ksi
Poisson's ratio,	ν	=	0	
Dead load,	w_d	=	self	psf
Live load,	w_l	=	100	psf

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live and post-tensioning loads.

RESULTS COMPARISON

The ETABS total factored moments, required mild steel reinforcing and slab stresses are compared to the independent hand calculations in Table 1.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (k-in)	1429.0	1428.3	-0.05%
Area of Mild Steel req'd, As (sq-in)	2.21	2.21	0.00%
Transfer Conc. Stress, top (D+PT_I), ksi	–0.734	–0.735	0.14%
Transfer Conc. Stress, bot (D+PT_I), ksi	0.414	0.414	0.00%
Normal Conc. Stress, top (D+L+PT_F), ksi	–1.518	–1.519	0.07%
Normal Conc. Stress, bot (D+L+PT_F), ksi	1.220	1.221	0.08%
Long-Term Conc. Stress, top (D+0.5L+$PT_{F(L)}$), ksi	–1.134	–1.135	0.09%
Long-Term Conc. Stress, bot (D+0.5L+$PT_{F(L)}$), ksi	0.836	0.837	0.12%

COMPUTER FILE: ACI 318-11 PT-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

CALCULATIONS:

Design Parameters:

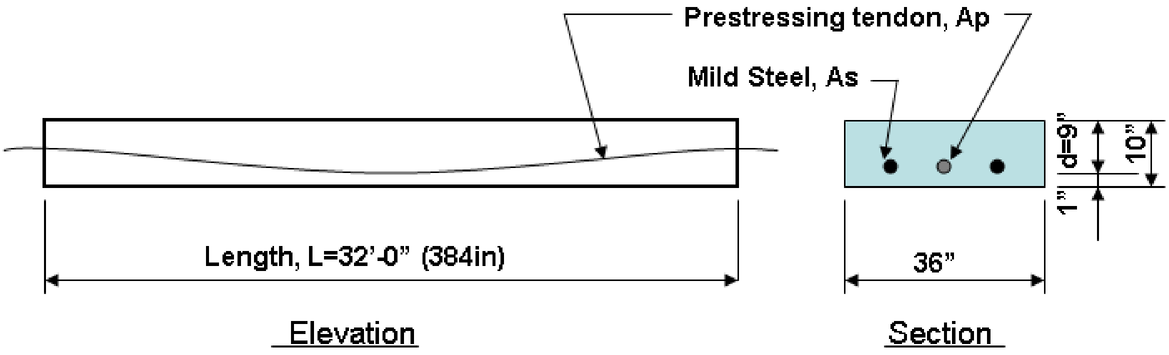
$\phi = 0.9$

Mild Steel Reinforcing

$f'c$ = 4000 psi
fy = 60,000 psi

Post-Tensioning

f_j = 216.0 ksi
Stressing Loss = 27.0 ksi
Long-Term Loss = 13.5 ksi
f_i = 189.0 ksi
f_e = 175.5 ksi



Loads:

Dead, self-wt = 10 / 12 ft × 0.150 kcf = 0.125 ksf (D) × 1.2 = 0.150 ksf (D_u)
Live, 0.100 ksf (L) × 1.6 = 0.160 ksf (L_u)
Total =0.225 ksf (D+L) 0.310 ksf (D+L)ult

ω =0.225 ksf × 3 ft = 0.675 klf, ω_u = 0.310 ksf × 3ft = 0.930 klf

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8}$ = 0.310 klf × 32^2/8 = 119.0 k-ft = 1429.0 k-in

Ultimate Stress in strand, $f_{PS} = f_{SE} + 10000 + \dfrac{f'c}{300\rho_P}$ (span-to-depth ratio > 35)

$$= 175,500 + 10,000 + \frac{4,000}{300(0.000944)}$$

$$= 199,624 \text{ psi} \le 205,500 \text{ psi}$$

Ultimate force in PT, $F_{ult,PT} = A_P\left(f_{PS}\right) = 2(0.153)(199.62) = 61.08 \text{ kips}$

Ultimate force in RC, $F_{ult,RC} = A_s\left(f\right)_y = 2.00\text{(assumed)}(60.0) = 120.0 \text{ kips}$

Total Ultimate force, $F_{ult,Total} = 61.08 + 120.0 = 181.08 \text{ kips}$

Stress block depth, $a = \dfrac{F_{ult,Total}}{0.85f'cb} = \dfrac{181.08}{0.85(4)(36)} = 1.48 \text{ in}$

Ultimate moment due to PT, $M_{ult,PT} = F_{ult,PT}\left(d - \dfrac{a}{2}\right)\phi = 61.08\left(9 - \dfrac{1.48}{2}\right)(0.9) = 454.1 \text{ k-in}$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 1429.0 - 454.1 = 974.9 \text{ k-in}$

Required area of mild steel reinforcing, $A_S = \dfrac{M_{net}}{\phi f_y\left(d - \dfrac{a}{2}\right)} = \dfrac{974.9}{0.9(60)\left(9 - \dfrac{1.48}{2}\right)} = 2.18 \text{ in}^2$

Note: The required area of mild steel reinforcing was calculated from an assumed amount of steel. Since the assumed value and the calculated value are not the same a second iteration can be performed. The second iteration changes the depth of the stress block and the calculated area of steel value. Upon completion of the second iteration the area of steel was found to be 2.21in^2

Check of Concrete Stresses at Mid-Span:

Initial Condition (Transfer), load combination (D + L + PT_i) = 1.0D + 1.0PT_I

The stress in the tendon at transfer = jacking stress − stressing losses = 216.0 − 27.0
= 189.0 ksi

The force in the tendon at transfer, = $189.0(2)(0.153) = 57.83\,\text{kips}$

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0\text{ k-ft} = 576\text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 57.83(4\text{ in}) = 231.3\text{ k-in}$

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-57.83}{10(36)} \pm \dfrac{576.0 - 231.3}{600}$, where S = 600 in^3

$\boldsymbol{f} = -0.161 \pm 0.5745$

$f = -0.735(\text{Comp})\max, 0.414(\text{Tension})\max$

Normal Condition, load combinations: (D + L + PT_F) = 1.0D + 1.0L + 1.0PT_F

Tendon stress at normal = jacking − stressing − long-term = 216.0 − 27.0 − 13.5 = 175.5 ksi
The force in tendon at Normal, = $175.5(2)(0.153) = 53.70\text{ kips}$

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0\text{ k-ft} = 576\text{ k-in}$

Moment due to dead load, $M_L = 0.100(3)(32)^2/8 = 38.4\text{ k-ft} = 461\text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 53.70(4\text{ in}) = 214.8\text{ k-in}$

Stress in concrete for (D + L+ PT_F), $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_{D+L} - M_{PT}}{S} = \dfrac{-53.70}{10(36)} \pm \dfrac{1037.0 - 214.8}{600}$

$\boldsymbol{f} = -0.149 \pm 1.727 \pm 0.358$

$f = -1.518(\text{Comp})\max, 1.220(\text{Tension})\max$

Long-Term Condition, load combinations: (D + 0.5L + $PT_{F(L)}$) = 1.0D + 0.5L + 1.0PT_F

Tendon stress at normal = jacking − stressing − long-term = 216.0 − 27.0 − 13.5 = 175.5 ksi
The force in tendon at Normal, = $175.5(2)(0.153) = 53.70\text{ kips}$

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0\text{ k-ft} = 576\text{ k-in}$

Moment due to dead load, $M_L = 0.100(3)(32)^2/8 = 38.4\text{ k-ft} = 460\text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 53.70(4\text{ in}) = 214.8\text{ k-in}$

Stress in concrete for (D + 0.5L + $PT_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-53.70}{10(36)} \pm \frac{806.0 - 214.8}{600}$$

$$f = -0.149 \pm 0.985$$

$$f = -1.134(\text{Comp})\,\text{max}, 0.836(\text{Tension})\,\text{max}$$

ACI 318-11 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 24-foot-long spans in each direction, as shown in Figure 1.

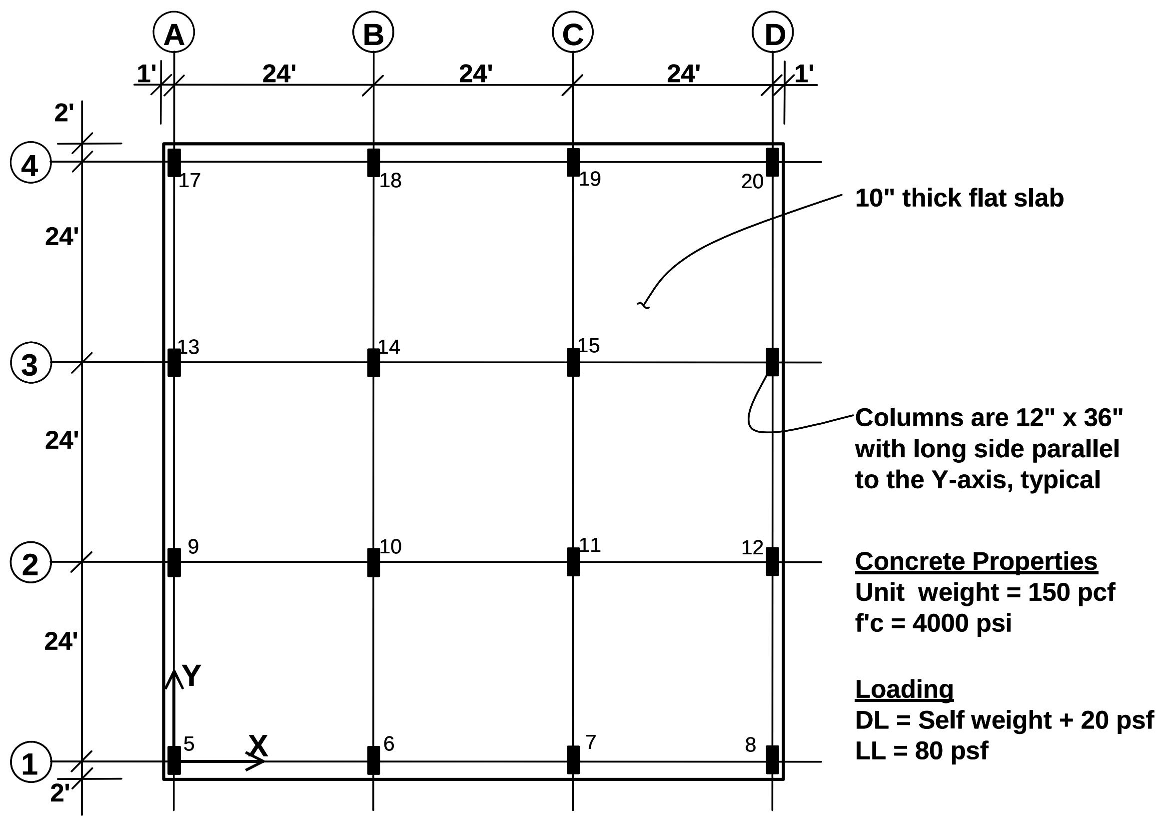


Figure 1: Flat Slab For Numerical Example

The slab overhangs the face of the column by 6 inches along each side of the structure. The columns are typically 12 inches wide by 36 inches long, with the long side parallel to the Y-axis. The slab is typically 10 inches thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 150 pcf and an f'c of 4000 psi. The dead load consists of the self weight of the structure plus an additional 20 psf. The live load is 80 psf.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress, and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS punching shear capacity, shear stress ratio, and D/C ratio with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this example.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (ksi)	Shear Capacity (ksi)	D/C ratio
ETABS	0.1930	0.158	1.22
Calculated	0.1930	0.158	1.22

COMPUTER FILE: ACI 318-11 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation for Interior Column Using ETABS Method

d = [(10 - 1) + (10 - 2)] / 2 = 8.5"

Refer to Figure 2.

b_0 = 44.5 + 20.5 + 44.5 + 20.5 = 130"

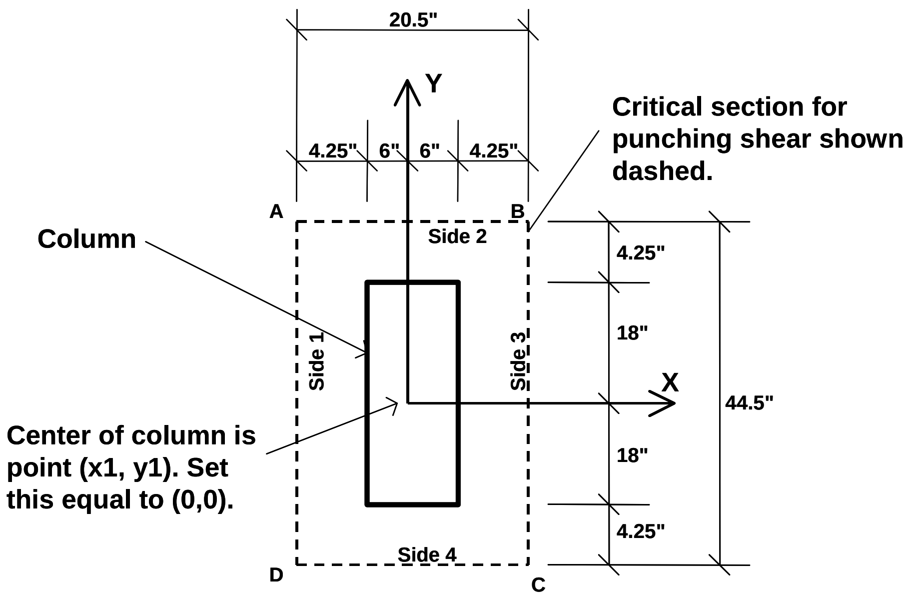


Figure 2: Interior Column, Grid B-2 in ETABS Model

$$\gamma_{V2} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{44.5}{20.5}}} = 0.4955$$

$$\gamma_{V3} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{20.5}{44.5}}} = 0.3115$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear, as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–10.25	0	10.25	0	N.A.
y_2	0	22.25	0	–22.25	N.A.
L	44.5	20.5	44.5	20.5	b_0 = 130
d	8.5	8.5	8.5	8.5	N.A.
Ld	378.25	174.25	378.25	174.25	1105
Ldx_2	–3877.06	0	3877.06	0	0
Ldy_2	0	3877.06	0	–3877.06	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{1105} = 0''$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{1105} = 0''$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the “Sum” column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	44.5	20.5	44.5	20.5	N.A.
d	8.5	8.5	8.5	8.5	N.A.
$x_2 - x_3$	–10.25	0	10.25	0	N.A.
$y_2 - y_3$	0	22.25	0	–22.25	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	64696.5	86264.6	64696.5	86264.6	301922.3
I_{YY}	39739.9	7151.5	39739.9	7151.5	93782.8
I_{XY}	0	0	0	0	0

From the ETABS output at Grid B-2:

V_U = 189.45 k

$\gamma_{V2} M_{U2}$ = –156.39 k-in

$\gamma_{V3} M_{U3}$ = 91.538 k-in

At the point labeled A in Figure 2, $x_4 = -10.25$ and $y_4 = 22.25$, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(22.25-0\right)-\left(0\right)\left(-10.25-0\right)\right]}{(301922.3)(93782.8)-(0)^2} - \frac{91.538\left[301922.3\left(-10.25-0\right)-\left(0\right)\left(22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 – 0.0115 – 0.0100 = **0.1499 ksi** at point A

At the point labeled B in Figure 2, $x_4 = 10.25$ and $y_4 = 22.25$, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(22.25-0\right)-\left(0\right)\left(10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(10.25-0\right)-\left(0\right)\left(22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 – 0.0115 + 0.0100 = **0.1699 ksi** at point B

At the point labeled C in Figure 2, $x_4 = 10.25$ and $y_4 = -22.25$, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(-22.25-0\right)-\left(0\right)\left(10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(10.25-0\right)-\left(0\right)\left(-22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 + 0.0115 + 0.0100 = **0.1930 ksi** at point C

At the point labeled D in Figure 2, $x_4 = -10.25$ and $y_4 = -22.25$, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(-22.25-0\right)-\left(0\right)\left(-10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(-10.25-0\right)-\left(0\right)\left(-22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 + 0.0115 - 0.0100 = **0.1729 ksi** at point D

Point C has the largest absolute value of v_u, thus v_{max} = 0.1930 ksi

The shear capacity is calculated based on the smallest of ACI 318-11 equations 11-34, 11-35 and 11-36 with the b_0 and d terms removed to convert force to stress.

$$\varphi v_C = \frac{0.75\left(2+\frac{4}{36/12}\right)\sqrt{4000}}{1000} = 0.158$$ ksi in accordance with equation 11-34

$$\varphi v_C = \frac{0.75\left(\frac{40 \bullet 8.5}{130}+2\right)\sqrt{4000}}{1000} = 0.219$$ ksi in accordance with equation 11-35

$$\varphi v_C = \frac{0.75 \bullet 4 \bullet \sqrt{4000}}{1000} = 0.190$$ ksi in accordance with equation 11-36

Equation 11-34 yields the smallest value of ϕv_C = 0.158 ksi and thus this is the shear capacity.

$$Shear\ Ratio = \frac{v_U}{\varphi v_C} = \frac{0.193}{0.158} = 1.22$$

ACI 318-11 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using ETABS. The slab is 6 inches thick and spans 12 feet between walls. The slab is modeled using thin plate elements. The walls are modeled as line supports. The computational model uses a finite element mesh, automatically generated by ETABS. The maximum element size is specified to be 36 inches. To obtain factored moments and flexural reinforcement in a design strip, one one-foot-wide strip is defined in the X-direction on the slab, as shown in Figure 1.

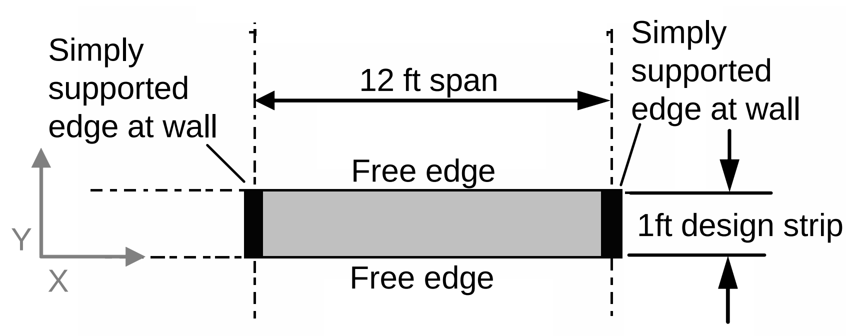


Figure 1 Plan View of One-Way Slab

One dead load case (DL80) and one live load case (LL100) with uniformly distributed surface loads of magnitudes 80 and 100 psf, respectively, are defined in the model. A load combination (COMB100) is defined using the ACI 318-11 load combination factors, 1.2 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing the analysis, design is performed in accordance with ACI 318-11 using ETABS and also by hand computation. Table 1 shows the comparison of the moments and design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	6	in
Depth of tensile reinf.	d_c	=	1	in
Effective depth	d	=	5	in
Clear span	l_n, l_1	=	144	in
Concrete strength	f_c	=	4,000	psi
Yield strength of steel	f_y	=	60,000	psi
Concrete unit weight	w_c	=	0	pcf
Modulus of elasticity	E_c	=	3,600	ksi
Modulus of elasticity	E_s	=	29,000	ksi
Poisson's ratio	ν	=	0	
Dead load	w_d	=	80	psf
Live load	w_l	=	100	psf

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (k-in)	Reinforcement Area (sq-in)
			A_s^+
Medium	ETABS	55.22	0.213
	Calculated	55.22	0.213

$A^+_{s,min}$ = 0.1296 sq-in

COMPUTER FILE: ACI 318-11 RC-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\varphi = 0.9$

$b = 12$ in

$A_{s,min} = 0.0018bh = 0.1296$ sq-in

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right) = 0.85$$

$$c_{max} = \frac{0.003}{0.003 + 0.005} d = 1.875 \text{ in}$$

$a_{max} = \beta_1 c_{max} = 1.59375$ in

For the load combination, w and M_u are calculated as follows:

$w = (1.2w_d + 1.6w_t)\, b\, /\, 144$

$$M_u = \frac{w l_1^2}{8}$$

$A_s = \min[A_{s,\min}, (4/3)\, A_{s,\text{required}}] = \min[0.1296, (4/3)2.11] = 0.1296$ sq-in

COMB100

w_d = 80 psf

w_t = 100 psf

w = 21.33 lb/in

$M_{u\text{-}strip}$ = 55.22 k-in

$M_{u\text{-}design}$ = 55.629 k-in

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_u\right|}{0.85 f_c' \varphi\, b}} = 0.3128 \text{ in} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = 0.213 \text{ sq-in} > A_{s,\min}$$

A_s = 0.2114 sq-in

ACI 318-14 PT-SL EXAMPLE 001

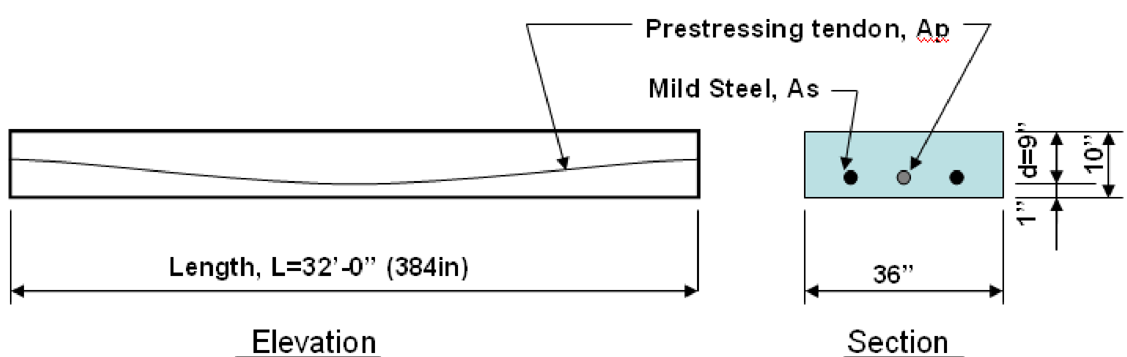
Design Verification of Post-Tensioned Slab using the ACI 318-14 code

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in ETABS. The modeled slab is 10 inches thick by 36 inches wide and spans 32 feet, as shown in shown in Figure 1. A 36-inch-wide design strip was centered along the length of the slab and was defined as an A-Strip. B-strips were placed at each end of the span perpendicular to the Strip-A (the B-Strips are necessary to define the tendon profile). A tendon, with two strands having an area of 0.153 square inches each, was added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are shown below. The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the ETABS results and summarized for verification and validation of the ETABS results.

Loads: Dead = self weight, Live = 100psf



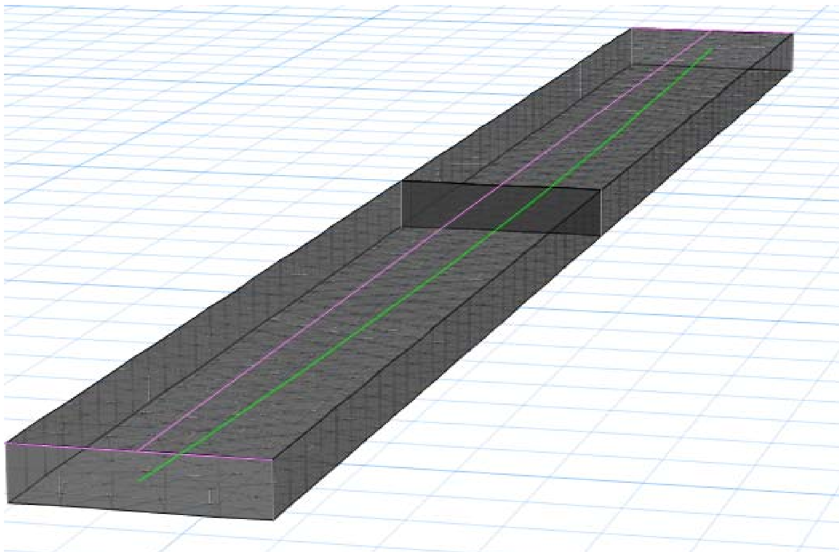


Figure 1 One-Way Slab

GEOMETRY, PROPERTIES AND LOADING

Thickness,	T, h	=	10	in
Effective depth,	d	=	9	in
Clear span,	L	=	384	in
Concrete strength,	f'_c	=	4,000	psi
Yield strength of steel,	f_y	=	60,000	psi
Prestressing, ultimate	f_{pu}	=	270,000	psi
Prestressing, effective	f_e	=	175,500	psi
Area of Prestress (single strand),	A_P	=	0.153	sq in
Concrete unit weight,	w_c	=	0.150	pcf
Modulus of elasticity,	E_c	=	3,600	ksi
Modulus of elasticity,	E_s	=	29,000	ksi
Poisson's ratio,	ν	=	0	
Dead load,	w_d	=	self	psf
Live load,	w_l	=	100	psf

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live and post-tensioning loads.

RESULTS COMPARISON

The ETABS total factored moments, required mild steel reinforcing and slab stresses are compared to the independent hand calculations in Table 1.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (k-in)	1429.0	1428.3	-0.05%
Area of Mild Steel req'd, As (sq-in)	2.21	2.21	0.00%
Transfer Conc. Stress, top ($D+PT_I$), ksi	–0.734	–0.735	0.14%
Transfer Conc. Stress, bot ($D+PT_I$), ksi	0.414	0.414	0.00%
Normal Conc. Stress, top ($D+L+PT_F$), ksi	–1.518	–1.519	0.07%
Normal Conc. Stress, bot ($D+L+PT_F$), ksi	1.220	1.221	0.08%
Long-Term Conc. Stress, top ($D+0.5L+PT_{F(L)}$), ksi	–1.134	–1.135	0.09%
Long-Term Conc. Stress, bot ($D+0.5L+PT_{F(L)}$), ksi	0.836	0.837	0.12%

COMPUTER FILE: ACI 318-14 PT-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

CALCULATIONS:

Design Parameters:

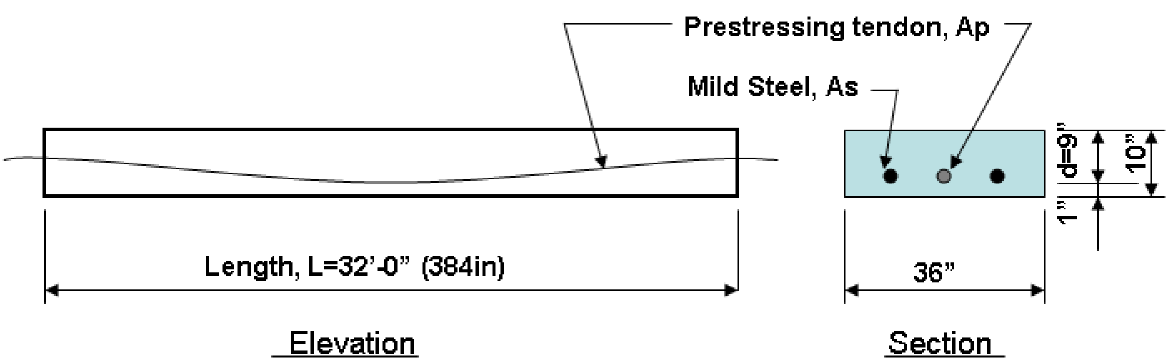
$\phi = 0.9$

Mild Steel Reinforcing

$f'c$ = 4000 psi
fy = 60,000 psi

Post-Tensioning

f_j = 216.0 ksi
Stressing Loss = 27.0 ksi
Long-Term Loss = 13.5 ksi
f_i = 189.0 ksi
f_e = 175.5 ksi



Loads:

Dead, self-wt = 10 / 12 ft × 0.150 kcf = 0.125 ksf (D) × 1.2 = 0.150 ksf (D_u)
Live, 0.100 ksf (L) × 1.6 = 0.160 ksf (L_u)
Total =0.225 ksf (D+L) 0.310 ksf (D+L)ult

ω =0.225 ksf × 3 ft = 0.675 klf, ω_u = 0.310 ksf × 3ft = 0.930 klf

Ultimate Moment, $M_U = \frac{wl_1^2}{8}$ = 0.310 klf × 32^2/8 = 119.0 k-ft = 1429.0 k-in

Ultimate Stress in strand, $f_{PS} = f_{SE} + 10000 + \dfrac{f'c}{300\rho_P}$ (span-to-depth ratio > 35)

$$= 175{,}500 + 10{,}000 + \frac{4{,}000}{300(0.000944)}$$

$$= 199{,}624 \text{ psi} \le 205{,}500 \text{ psi}$$

Ultimate force in PT, $F_{ult,PT} = A_P\left(f_{PS}\right) = 2(0.153)(199.62) = 61.08 \text{ kips}$

Ultimate force in RC, $F_{ult,RC} = A_s\left(f\right)_y = 2.00\text{(assumed)}(60.0) = 120.0 \text{ kips}$

Total Ultimate force, $F_{ult,Total} = 61.08 + 120.0 = 181.08 \text{ kips}$

Stress block depth, $a = \dfrac{F_{ult,Total}}{0.85 f'cb} = \dfrac{181.08}{0.85(4)(36)} = 1.48 \text{ in}$

Ultimate moment due to PT, $M_{ult,PT} = F_{ult,PT}\left(d - \dfrac{a}{2}\right)\phi = 61.08\left(9 - \dfrac{1.48}{2}\right)(0.9) = 454.1 \text{ k-in}$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 1429.0 - 454.1 = 974.9 \text{ k-in}$

Required area of mild steel reinforcing, $A_S = \dfrac{M_{net}}{\phi f_y\left(d - \dfrac{a}{2}\right)} = \dfrac{974.9}{0.9(60)\left(9 - \dfrac{1.48}{2}\right)} = 2.18 \text{ in}^2$

Note: The required area of mild steel reinforcing was calculated from an assumed amount of steel. Since the assumed value and the calculated value are not the same a second iteration can be performed. The second iteration changes the depth of the stress block and the calculated area of steel value. Upon completion of the second iteration the area of steel was found to be 2.21in^2

Check of Concrete Stresses at Mid-Span:

Initial Condition (Transfer), load combination (D + L + PT_i) = 1.0D + 1.0PT_I

The stress in the tendon at transfer = jacking stress − stressing losses = 216.0 − 27.0 = 189.0 ksi

The force in the tendon at transfer, = $189.0(2)(0.153) = 57.83$ kips

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0 \text{ k-ft} = 576 \text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}\,(\text{sag}) = 57.83(4 \text{ in}) = 231.3 \text{ k-in}$

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-57.83}{10(36)} \pm \dfrac{576.0 - 231.3}{600}$, where S = 600 in^3

$$\boldsymbol{f} = -0.161 \pm 0.5745$$

$$f = -0.735(\text{Comp})\max, 0.414(\text{Tension})\max$$

Normal Condition, load combinations: (D + L + PT_F) = 1.0D + 1.0L + 1.0PT_F

Tendon stress at normal = jacking − stressing − long-term = 216.0 − 27.0 − 13.5 = 175.5 ksi

The force in tendon at Normal, = $175.5(2)(0.153) = 53.70$ kips

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0 \text{ k-ft} = 576 \text{ k-in}$

Moment due to dead load, $M_L = 0.100(3)(32)^2/8 = 38.4 \text{ k-ft} = 461 \text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}\,(\text{sag}) = 53.70(4 \text{ in}) = 214.8 \text{ k-in}$

Stress in concrete for (D + L+ PT_F), $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_{D+L} - M_{PT}}{S} = \dfrac{-53.70}{10(36)} \pm \dfrac{1037.0 - 214.8}{600}$

$$\boldsymbol{f} = -0.149 \pm 1.727 \pm 0.358$$

$$f = -1.518(\text{Comp})\max, 1.220(\text{Tension})\max$$

Long-Term Condition, load combinations: (D + 0.5L + $PT_{F(L)}$) = 1.0D + 0.5L + 1.0PT_F

Tendon stress at normal = jacking − stressing − long-term = 216.0 − 27.0 − 13.5 = 175.5 ksi

The force in tendon at Normal, = $175.5(2)(0.153) = 53.70$ kips

Moment due to dead load, $M_D = 0.125(3)(32)^2/8 = 48.0 \text{ k-ft} = 576 \text{ k-in}$

Moment due to dead load, $M_L = 0.100(3)(32)^2/8 = 38.4 \text{ k-ft} = 460 \text{ k-in}$

Moment due to PT, $M_{PT} = F_{PTI}\,(\text{sag}) = 53.70(4 \text{ in}) = 214.8 \text{ k-in}$

Stress in concrete for (D + 0.5L + $PT_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-53.70}{10(36)} \pm \frac{806.0 - 214.8}{600}$$

$$f = -0.149 \pm 0.985$$

$$f = -1.134(\text{Comp})\,\text{max}, 0.836(\text{Tension})\,\text{max}$$

ACI 318-14 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 24-foot-long spans in each direction, as shown in Figure 1.

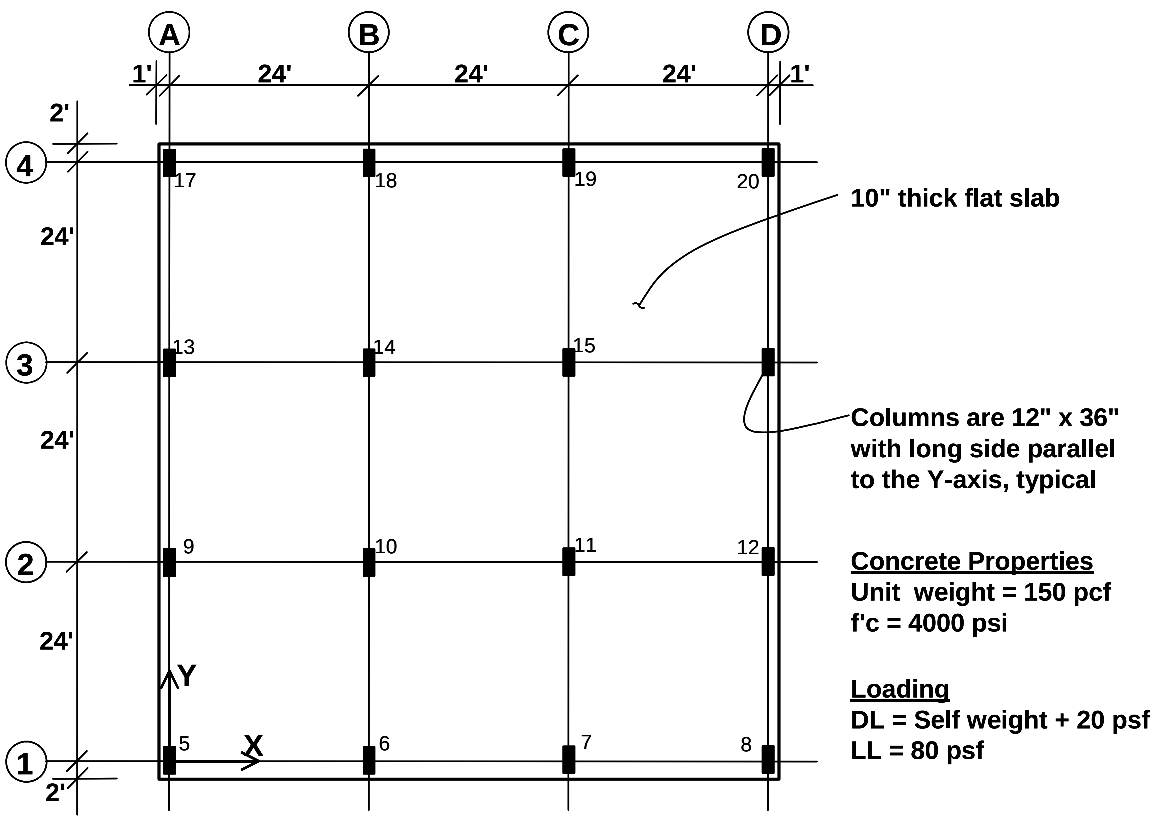


Figure 1: Flat Slab For Numerical Example

The slab overhangs the face of the column by 6 inches along each side of the structure. The columns are typically 12 inches wide by 36 inches long, with the long side parallel to the Y-axis. The slab is typically 10 inches thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 150 pcf and an f'c of 4000 psi. The dead load consists of the self weight of the structure plus an additional 20 psf. The live load is 80 psf.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress, and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS punching shear capacity, shear stress ratio, and D/C ratio with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this example.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (ksi)	Shear Capacity (ksi)	D/C ratio
ETABS	0.1930	0.158	1.22
Calculated	0.1930	0.158	1.22

COMPUTER FILE: ACI 318-14 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation for Interior Column Using ETABS Method

d = [(10 - 1) + (10 - 2)] / 2 = 8.5"

Refer to Figure 2.

b_0 = 44.5 + 20.5 + 44.5 + 20.5 = 130"

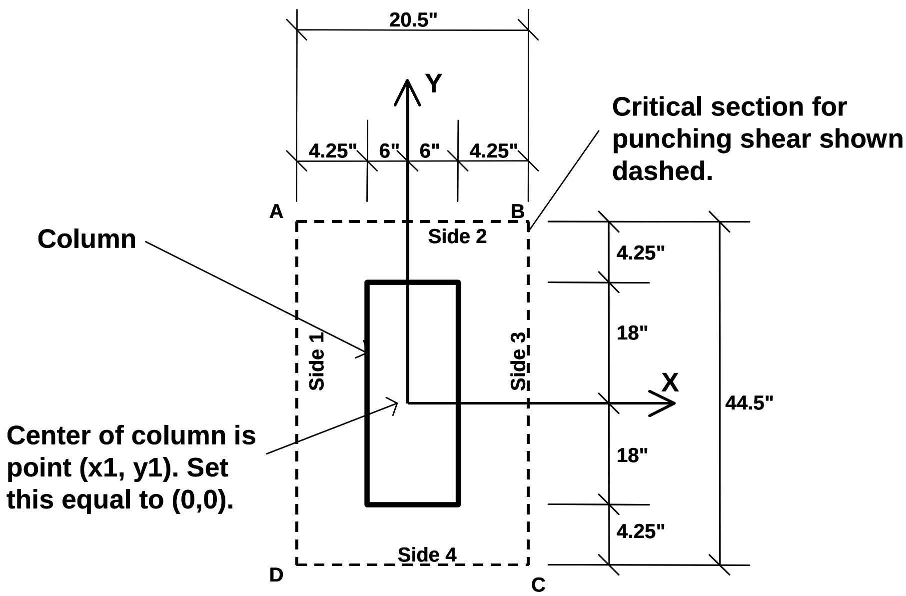


Figure 2: Interior Column, Grid B-2 in ETABS Model

$$\gamma_{V2} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{44.5}{20.5}}} = 0.4955$$

$$\gamma_{V3} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{20.5}{44.5}}} = 0.3115$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear, as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–10.25	0	10.25	0	N.A.
y_2	0	22.25	0	–22.25	N.A.
L	44.5	20.5	44.5	20.5	b_0 = 130
d	8.5	8.5	8.5	8.5	N.A.
Ld	378.25	174.25	378.25	174.25	1105
Ldx_2	–3877.06	0	3877.06	0	0
Ldy_2	0	3877.06	0	–3877.06	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{1105} = 0''$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{1105} = 0''$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	44.5	20.5	44.5	20.5	N.A.
d	8.5	8.5	8.5	8.5	N.A.
x_2 - x_3	–10.25	0	10.25	0	N.A.
y_2 - y_3	0	22.25	0	–22.25	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	64696.5	86264.6	64696.5	86264.6	301922.3
I_{YY}	39739.9	7151.5	39739.9	7151.5	93782.8
I_{XY}	0	0	0	0	0

From the ETABS output at Grid B-2:

V_U = 189.45 k

$\gamma_{V2} M_{U2}$ = –156.39 k-in

$\gamma_{V3} M_{U3}$ = 91.538 k-in

At the point labeled A in Figure 2, x_4 = –10.25 and y_4 = 22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(22.25-0\right)-\left(0\right)\left(-10.25-0\right)\right]}{(301922.3)(93782.8)-(0)^2} - \frac{91.538\left[301922.3\left(-10.25-0\right)-\left(0\right)\left(22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 – 0.0115 – 0.0100 = **0.1499 ksi** at point A

At the point labeled B in Figure 2, x_4 = 10.25 and y_4 = 22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(22.25-0\right)-\left(0\right)\left(10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(10.25-0\right)-\left(0\right)\left(22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 – 0.0115 + 0.0100 = **0.1699 ksi** at point B

At the point labeled C in Figure 2, x_4 = 10.25 and y_4 = –22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(-22.25-0\right)-\left(0\right)\left(10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(10.25-0\right)-\left(0\right)\left(-22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 + 0.0115 + 0.0100 = **0.1930 ksi** at point C

At the point labeled D in Figure 2, x_4 = –10.25 and y_4 = –22.25, thus:

$$v_U = \frac{189.45}{130 \bullet 8.5} - \frac{156.39\left[93782.8\left(-22.25-0\right)-\left(0\right)\left(-10.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2} - \frac{91.538\left[301922.3\left(-10.25-0\right)-\left(0\right)\left(-22.25-0\right)\right]}{\left(301922.3\right)\left(93782.8\right)-\left(0\right)^2}$$

v_U = 0.1714 + 0.0115 - 0.0100 = **0.1729 ksi** at point D

Point C has the largest absolute value of v_u, thus v_{max} = 0.1930 ksi

The shear capacity is calculated based on the smallest of ACI 318-14 equations 11-34, 11-35 and 11-36 with the b_0 and d terms removed to convert force to stress.

$$\varphi v_C = \frac{0.75\left(2+\frac{4}{36/12}\right)\sqrt{4000}}{1000} = 0.158$$ ksi in accordance with equation 11-34

$$\varphi v_C = \frac{0.75\left(\frac{40 \bullet 8.5}{130}+2\right)\sqrt{4000}}{1000} = 0.219$$ ksi in accordance with equation 11-35

$$\varphi v_C = \frac{0.75 \bullet 4 \bullet \sqrt{4000}}{1000} = 0.190$$ ksi in accordance with equation 11-36

Equation 11-34 yields the smallest value of ϕv_C = 0.158 ksi and thus this is the shear capacity.

$$Shear\ Ratio = \frac{v_U}{\varphi v_C} = \frac{0.193}{0.158} = 1.22$$

ACI 318-14 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using ETABS. The slab is 6 inches thick and spans 12 feet between walls. The walls are modeled as line supports. The computational model uses a finite element mesh, automatically generated by ETABS. The maximum element size is specified to be 36 inches. To obtain factored moments and flexural reinforcement in a design strip, one one-foot-wide strip is defined in the X-direction on the slab, as shown in Figure 1.

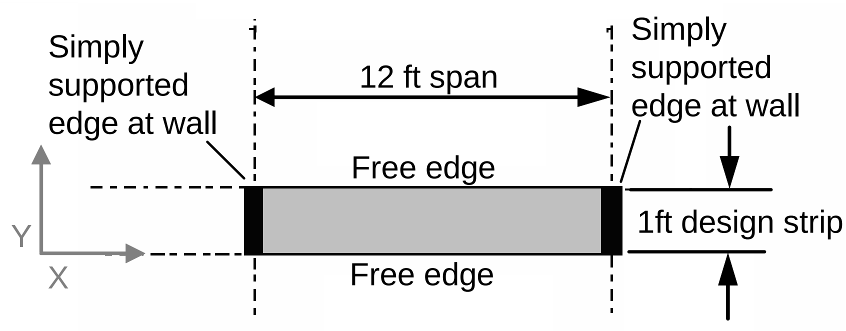


Figure 1 Plan View of One-Way Slab

One dead load case (DL80) and one live load case (LL100) with uniformly distributed surface loads of magnitudes 80 and 100 psf, respectively, are defined in the model. A load combination (COMB100) is defined using the ACI 318-14 load combination factors, 1.2 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing the analysis, design is performed in accordance with ACI 318-14 using ETABS and also by hand computation. Table 1 shows the comparison of the moments and design reinforcements computed using the two methods.

PROGRAM NAME: ETABS
REVISION NO.: 0

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	6	in
Depth of tensile reinf.	d_c	=	1	in
Effective depth	d	=	5	in
Clear span	l_n, l_1	=	144	in
Concrete strength	f_c	=	4,000	psi
Yield strength of steel	f_y	=	60,000	psi
Concrete unit weight	w_c	=	0	pcf
Modulus of elasticity	E_c	=	3,600	ksi
Modulus of elasticity	E_s	=	29,000	ksi
Poisson's ratio	ν	=	0	
Dead load	w_d	=	80	psf
Live load	w_l	=	100	psf

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (k-in)	Reinforcement Area (sq-in) A_s^+
Medium	ETABS	55.22	0.213
	Calculated	55.22	0.213

$A^+_{s,min}$ = 0.1296 sq-in

COMPUTER FILE: ACI 318-14 RC-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\varphi = 0.9$

$b = 12$ in

$A_{s,min} = 0.0018bh = 0.1296$ sq-in

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right) = 0.85$$

$$c_{max} = \frac{0.003}{0.003 + 0.005} d = 1.875 \text{ in}$$

$a_{max} = \beta_1 c_{max} = 1.59375$ in

For the load combination, w and M_u are calculated as follows:

$w = (1.2w_d + 1.6w_t)\, b / 144$

$$M_u = \frac{wl_1^2}{8}$$

$A_s = \min[A_{s,\min}, (4/3)\, A_{s,\text{required}}] = \min[0.1296, (4/3)2.11] = 0.1296$ sq-in

COMB100

w_d = 80 psf

w_t = 100 psf

w = 21.33 lb/in

$M_{u\text{-}strip}$ = 55.22 k-in

$M_{u\text{-}design}$ = 55.629 k-in

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2|M_u|}{0.85 f_c' \varphi\, b}} = 0.3128 \text{ in} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = 0.213 \text{ sq-in} > A_{s,\min}$$

A_s = 0.2114 sq-in

AS 3600-2001 PT-SL Example 001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel reinforcing strength for a post-tensioned slab.

A one-way, simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm as shown in shown in Figure 1.

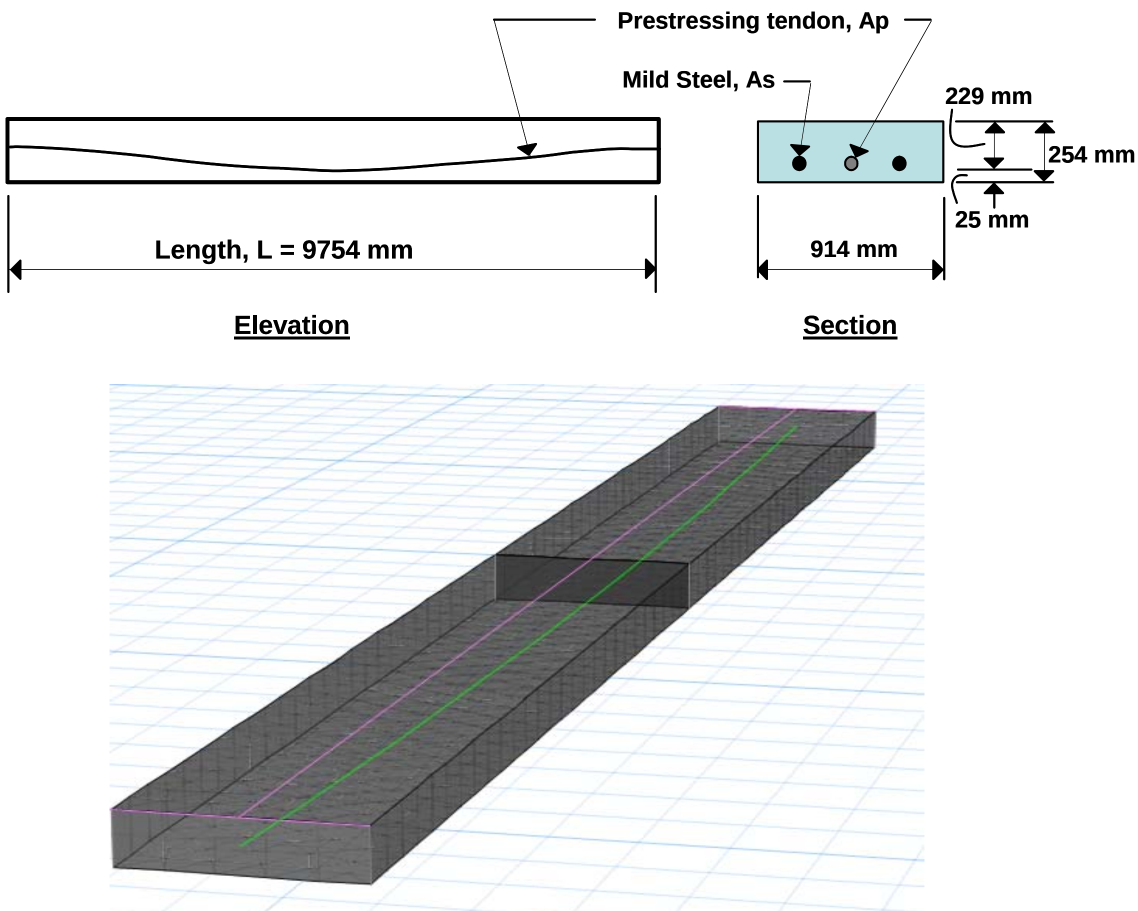


Figure 1 One-Way Slab

A 914-mm-wide design strip is centered along the length of the slab and is defined as an A-Strip. B-Strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self-weight and live loads were added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the midspan of the slab. Independent hand calculations were compared with the ETABS results and summarized for verification and validation of the ETABS results.

GEOMETRY, PROPERTIES AND LOADING

Thickness,	T, h	=	254	mm
Effective depth,	d	=	229	mm
Clear span,	L	=	9754	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of prestress (single tendon),	A_p	=	198	mm^2
Concrete unit weight,	w_c	=	23.56	KN/m^3
Concrete modulus of elasticity,	E_c	=	25000	N/mm^3
Rebar modulus of elasticity,	E_s	=	200,000	N/mm^3
Poisson's ratio,	ν	=	0	
Dead load,	w_d	=	self	KN/m^2
Live load,	w_l	=	4.788	KN/m^2

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	156.12	156.17	0.03%
Area of Mild Steel req'd, As (sq-cm)	16.55	16.60	0.30%
Transfer Conc. Stress, top (0.8D+1.15PT_I), MPa	–3.500	–3.498	-0.06%
Transfer Conc. Stress, bot (0.8D+1.15PT_I), MPa	0.950	0.948	-0.21%
Normal Conc. Stress, top (D+L+PT_F), MPa	–10.460	–10.467	0.07%
Normal Conc. Stress, bot (D+L+PT_F), MPa	8.402	8.409	0.08%
Long-Term Conc. Stress, top (D+0.5L+$PT_{F(L)}$), MPa	–7.817	–7.818	0.01%
Long-Term Conc. Stress, bot (D+0.5L+$PT_{F(L)}$), MPa	5.759	5.760	0.02%

COMPUTER FILE: AS 3600-2001 PT-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

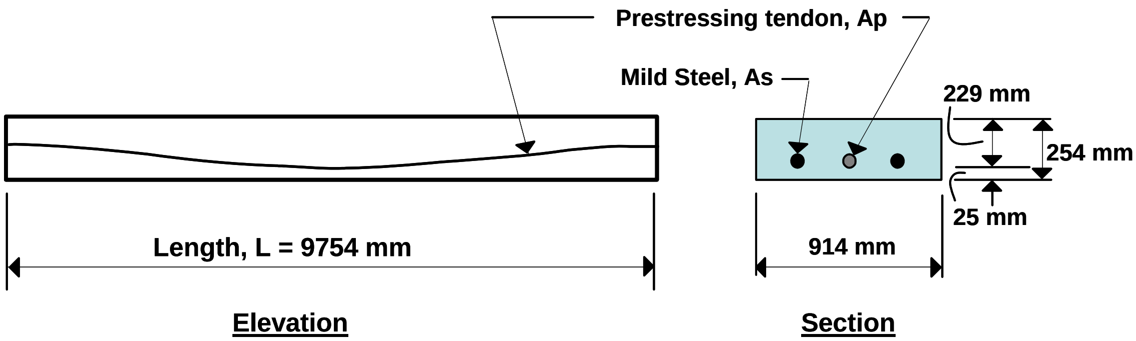
Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
f'_c = 30MPai	f_{pu} = 1862 MPa
f_y = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$$\phi = 0.80$$

$$\gamma = \left[0.85 - 0.007\left(f'_c - 28\right)\right] = 0.836$$

$$a_{\max} = \gamma k_u d = 0.836*0.4*229 = 76.5 \text{ mm}$$



Loads:

Dead, self-wt = 0.254 m x 23.56 kN/m^3 = 5.984 kN/m^2 (D) x 1.2 = 7.181 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) x 1.5 = 7.182 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 14.363 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 x 0.914m = 9.846 kN/m, ω_u = 14.363 kN/m^2 x 0.914m = 13.128 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8}$ = 13.128 x (9.754)^2/8 = 156.12 kN-m

Ultimate Stress in strand, $f_{PS} = f_{SE} + 70 + \frac{f'_C b_{ef} d_P}{300 A_P}$

$$= 1210 + 70 + \frac{30(914)(229)}{300(198)}$$

$$= 1386 \text{ MPa} \le f_{SE} + 200 = 1410 \text{ MPa}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197.4(1386)/1000 = 273.60$ kN

Total Ultimate force, $F_{ult,Total} = 273.60 + 560.0 = 833.60$ kN

Stress block depth, $a = d - \sqrt{d^2 - \frac{2M^*}{0.85 f'_c \phi b}}$

$$= 0.229 - \sqrt{0.229^2 - \frac{2(159.12)}{0.85(30000)(0.80)(0.914)}} = 40.90$$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right)\phi = 273.60\left(229 - \frac{40.90}{2}\right)(0.80)\Big/1000 = 45.65 \text{ kN-m}$$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 156.1 - 45.65 = 110.45$ kN-m

Required area of mild steel reinforcing,

$$A_S = \frac{M_{net}}{\phi f_y\left(d - \frac{a}{2}\right)} = \frac{110.45}{0.80(400000)\left(0.229 - \frac{0.04090}{2}\right)}(1e6) = 1655 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (0.8D+1.15PT_i) = 0.80D+0.0L+1.15PT_I

Tendon stress at transfer = jacking stress − stressing losses =1490 − 186 = 1304 MPa

The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \frac{F_{PTI}}{A} \pm \frac{M_D - M_{PT}}{S} = \frac{(1.15)(-257.4)}{0.254(0.914)} \pm \frac{(0.80)65.04 - (1.15)26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.275 \pm 2.225 \text{ MPa}$$

$$f = -3.500(\text{Comp})\max, 0.950(\text{Tension})\max$$

Normal Condition, load combinations: ($D+L+PT_F$) = $1.0D+1.0L+1.0PT_F$

Tendon stress at Normal = jacking − stressing − long-term = 1490 − 186 − 94= 1210 MPa
The force in tendon at Normal, = $1210(197.4)/1000 = 238.9$ kN
Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m
Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m
Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for ($D+L+PT_F$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$

$$f = -1.029 \pm 9.431$$

$$f = -10.460(\text{Comp})\max, 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: ($D+0.5L+PT_{F(L)}$) = $1.0D+0.5L+1.0PT_F$

Tendon stress at Normal = jacking − stressing − long-term =1490 − 186 − 94 = 1210 MPa
The force in tendon at Normal, = $1210(197.4)/1000 = 238.9$ kN
Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m
Moment due to dead load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m
Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for ($D+0.5L+PT_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$

$$f = -1.029 \pm 6.788$$

$$f = -7.817(\text{Comp})\max, 5.759(\text{Tension})\max$$

AS 3600-2001 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

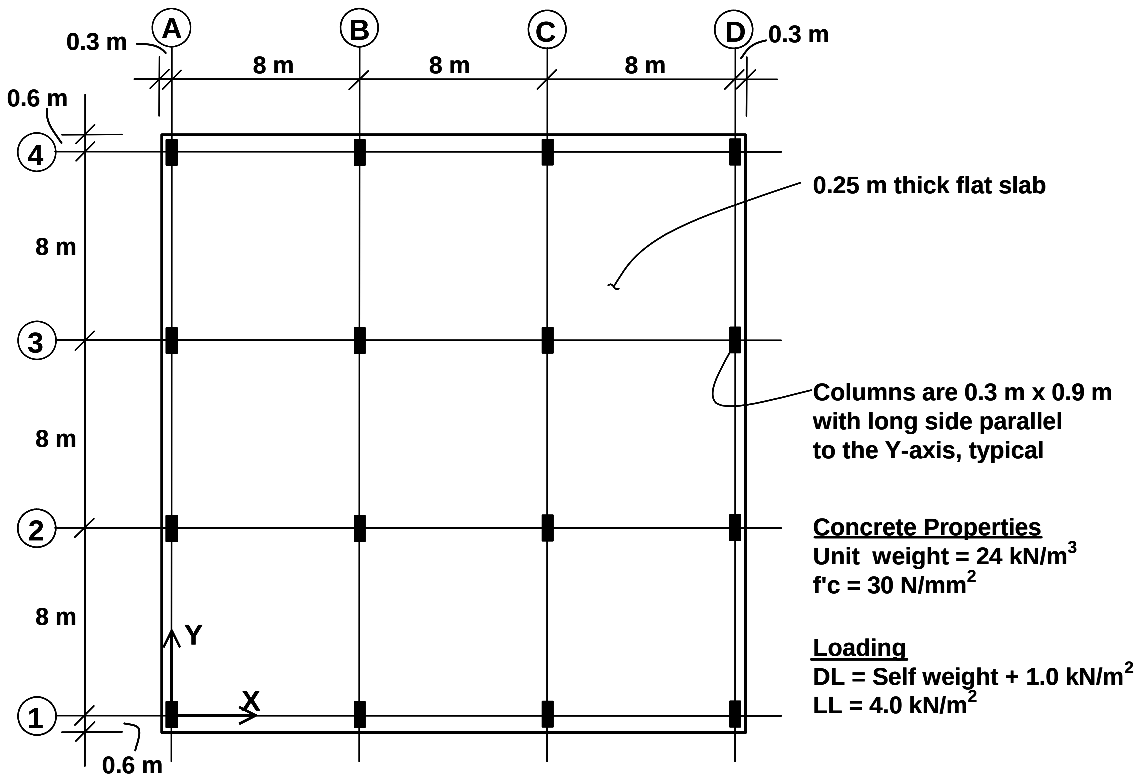


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m, with the long side parallel to the Y-axis. The slab is typically 0.25-m thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a $f'c$ of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress, and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio, and D/C ratio obtained from ETABS with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid Point B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
ETABS	1.799	1.086	1.66
Calculated	1.811	1.086	1.67

COMPUTER FILE: AS 3600-2001 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using ETABS Method

d_{om} = [(250 – 26) + (250 – 38)] / 2 = 218 mm

Refer to Figure 2.

U = 518+ 1118 + 1118 + 518 = 3272 mm

a_x = 518 mm

a_y = 1118 mm

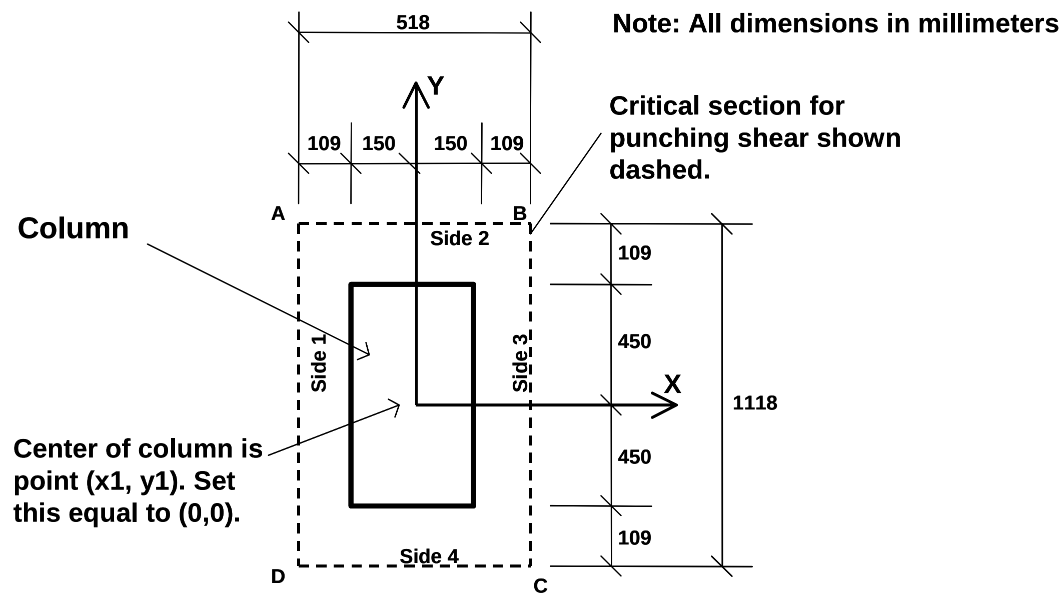


Figure 2: Interior Column, Grid B-2 in ETABS Model

From the ETABS output at grid line B-2:

V^* = 1126.498 kN

M_{v2} = –51.991 kN-m

M_{v3} = 45.723 kN-m

The maximum design shear stress is computed along the major and minor axis of column separately:

$$v_{\max} = \frac{V^*}{ud_{om}}\left[1.0 + \frac{uM_v}{8V^* ad_{om}}\right]$$

$$v_{\max,X} = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} \bullet \left(1 + \frac{3272 \bullet 51.991 \bullet 10^6}{8 \bullet 1126.498 \bullet 10^3 \bullet 1118 \bullet 218}\right)$$

$v_{\max,X}$ = 1.579 $\bullet$ 1.0774 = **1.7013 N/mm²**

$$v_{\max,Y} = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} \bullet \left(1 + \frac{3272 \bullet 45.723 \bullet 10^6}{8 \bullet 1126.498 \bullet 10^3 \bullet 518 \bullet 218}\right)$$

$v_{\max,Y}$ = 1.579 $\bullet$ 1.1470 = **1.811 N/mm²** (Govern)

The largest absolute value of $v_{\max}$= **1.811 N/mm²**

The shear capacity is calculated based on the smallest of AS 3600-01 equation 11-35, with the d_{om} and u terms removed to convert force to stress.

$$\varphi f_{cv} = \min\begin{cases} 0.17\left(1 + \frac{2}{\beta_h}\right)\varphi\sqrt{f'_c} \\ 0.34\varphi\sqrt{f'_c} \end{cases}$$ = **1.803N/mm²** in accordance with AS 9.2.3(a)

AS 9.2.3(a) yields the smallest value of φf_{cv} = 1.086 N/mm², and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{\varphi f_{cv}} = \frac{1.811}{1.086} = 1.67$$

AS 3600-2001 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

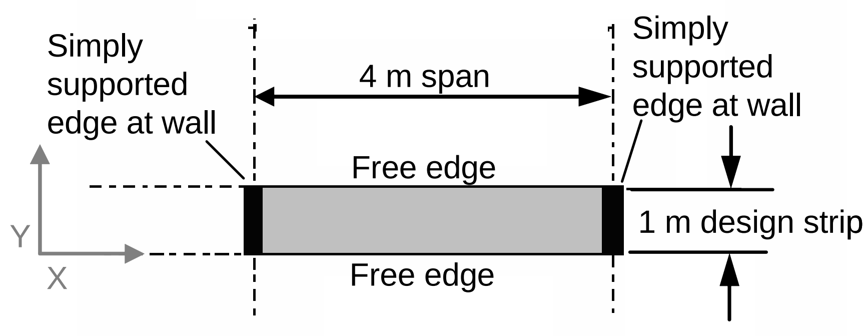


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa), with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the AS 3600-2001 load combination factors, 1.2 for dead loads and 1.5 for live loads. The model is analyzed for both load cases and the load combinations.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing analysis, design is performed using the AS 3600-2001 code using ETABS and also by hand computation. Table 1 shows the comparison of the design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	$2\text{x}10^6$	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	ETABS	24.597	5.58
	Calculated	24.600	5.58

$A^+_{s,min}$ = 282.9 sq-mm

COMPUTER FILE: AS 3600-2001 RC-SL EX001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\varphi = 0.8$

$b = 1000$ mm

$$\gamma = \left[0.85 - 0.007\left(f'_c - 28\right)\right] = 0.836$$

$$a_{\max} = \gamma k_u d = 0.836 \bullet 0.4 \bullet 125 = 41.8 \text{ mm}$$

For the load combination, w and M^* are calculated as follows:

$$w = (1.2w_d + 1.5w_t)\, b$$

$$M_u = \frac{w l_1^2}{8}$$

$$A_{st.\min} = 0.22\left(\frac{D}{d}\right)^2 \frac{f'_{cf}}{f_{sy}} bd$$

$$= 0.22 \bullet (150/125)^2 \bullet 0.6 \bullet \text{SQRT}(30)/460 \bullet 100 \bullet 125$$

$$= 282.9 \text{ sq-mm}$$

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 12.3 kN/m

$M_{\text{-strip}}^*$ = 24.6 kN-m

$M_{\text{-design}}^*$ = 24.633 kN-m

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M^*}{0.85\, f'_c\, \phi b}} = 10.065 \text{ mm} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_{st} = \frac{M^*}{\phi f_{sy}\left(d - \dfrac{a}{2}\right)} = 557.966 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.57966 \text{ sq-cm}$$

AS 3600-2009 PT-SL EXAMPLE 001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel reinforcing strength for a post-tensioned slab.

A one-way, simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.

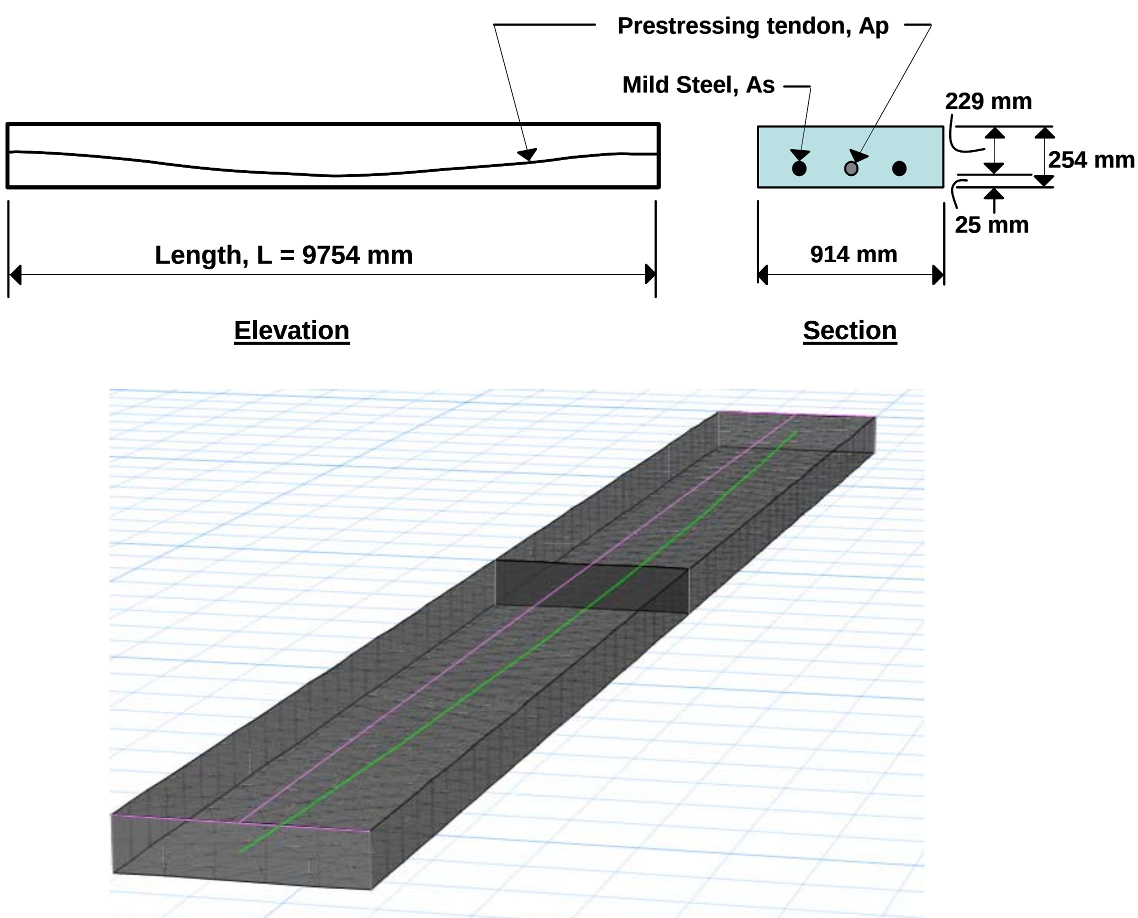


Figure 1 One-Way Slab

A 914-mm-wide design strip is centered along the length of the slab and is defined as an A-Strip. B-Strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the ETABS results and summarized for verification and validation of the ETABS results.

GEOMETRY, PROPERTIES AND LOADING

Thickness,	T, h	=	254	mm
Effective depth,	d	=	229	mm
Clear span,	L	=	9754	mm
Concrete strength,	f'_c	=	30	MPa
Yield strength of steel,	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of prestress (single tendon),	A_p	=	198	mm^2
Concrete unit weight,	w_c	=	23.56	KN/m^3
Concrete modulus of elasticity,	E_c	=	25000	N/mm^3
Rebar modulus of elasticity,	E_s	=	200,000	N/mm^3
Poisson's ratio,	ν	=	0	
Dead load,	w_d	=	self	KN/m^2
Live load,	w_l	=	4.788	KN/m^2

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	156.12	156.17	0.03%
Area of Mild Steel req'd, As (sq-cm)	16.55	16.60	0.30%
Transfer Conc. Stress, top ($0.8D+1.15PT_I$), MPa	–3.500	–3.498	-0.06%
Transfer Conc. Stress, bot ($0.8D+1.15PT_I$), MPa	0.950	0.948	-0.21%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	–10.460	–10.467	0.07%
Normal Conc. Stress, bot ($D+L+PT_F$), MPa	8.402	8.409	0.08%
Long-Term Conc. Stress, top ($D+0.5L+PT_{F(L)}$), MPa	–7.817	–7.818	0.01%
Long-Term Conc. Stress, bot ($D+0.5L+PT_{F(L)}$), MPa	5.759	5.760	0.02%

COMPUTER FILE: AS 3600-2009 PT-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

Design Parameters:

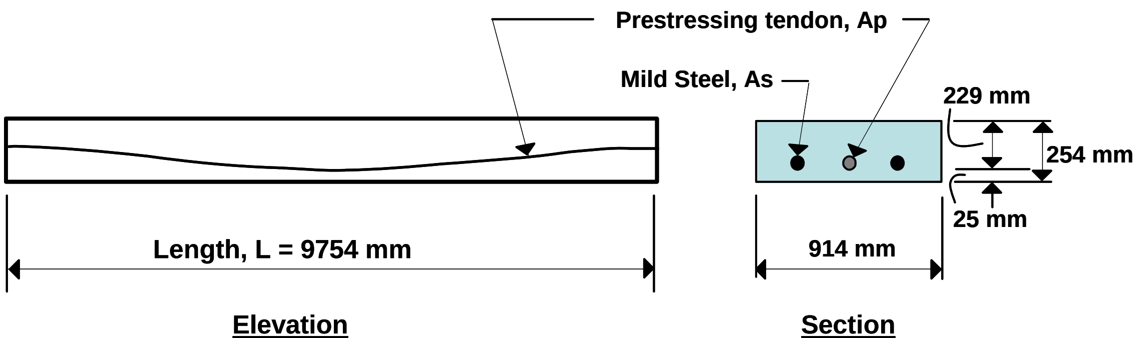
Mild Steel Reinforcing	Post-Tensioning
f'_c = 30MPai	f_{pu} = 1862 MPa
f_y = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\phi = 0.80$

$\alpha_2 = 1.0 - 0.003 f'_c$ = 0.91 > 0.85, Use $\alpha_2 = 0.85$

$\gamma = 1.0 - 0.003 f'_c$ = 0.91 > 0.85, Use $\gamma = 0.85$

$a_{max} = \gamma k_u d$ = 0.85*0.36*229 = 70.07 mm



Loads:

Dead, self-wt = 0.254 m x 23.56 kN/m^3 = 5.984 kN/m^2 (D) x 1.2 = 7.181 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) x 1.5 = 7.182 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 14.363 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 x 0.914m = 9.846 kN/m, ω_u = 14.363 kN/m^2 x 0.914m = 13.128 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8}$ = 13.128 x (9.754)^2/8 = 156.12 kN-m

Ultimate Stress in strand, $f_{PS} = f_{SE} + 70 + \dfrac{f'_C b_{ef} d_P}{300 A_P}$

$$= 1210 + 70 + \frac{30(914)(229)}{300(198)}$$

$$= 1386 \text{ MPa} \le f_{SE} + 200 = 1410 \text{ MPa}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197.4(1386)/1000 = 273.60 \text{ kN}$

Total Ultimate force, $F_{ult,Total} = 273.60 + 560.0 = 833.60 \text{ kN}$

Stress block depth, $a = d - \sqrt{d^2 - \dfrac{2M^*}{0.85 f'_c \phi b}}$

$$= 0.229 - \sqrt{0.229^2 - \frac{2(159.12)}{0.85(30000)(0.80)(0.914)}} = 40.90$$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right)\phi = 273.60\left(229 - \frac{40.90}{2}\right)(0.80)\Big/1000 = 45.65 \text{ kN-m}$$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 156.1 - 45.65 = 110.45 \text{ kN-m}$

Required area of mild steel reinforcing,

$$A_S = \frac{M_{net}}{\phi f_y\left(d - \frac{a}{2}\right)} = \frac{110.45}{0.80(400000)\left(0.229 - \frac{0.04090}{2}\right)}(1e6) = 1655 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (0.8D+1.15PT$_i$) = 0.80D+0.0L+1.15PT$_I$

Tendon stress at transfer = jacking stress − stressing losses =1490 − 186 = 1304 MPa

The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4 \text{ kN}$

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04 \text{ kN-m}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25 \text{ kN-m}$

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{(1.15)(-257.4)}{0.254(0.914)} \pm \dfrac{(0.80)65.04 - (1.15)26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.275 \pm 2.225 \text{ MPa}$$

$$f = -3.500(\text{Comp})\max, 0.950(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT_F) = 1.0D+1.0L+1.0PT_F

Tendon stress at Normal = jacking – stressing – long-term = 1490 – 186 – 94= 1210 MPa
The force in tendon at Normal, = $1210(197.4)/1000 = 238.9$ kN
Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m
Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m
Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT_F),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$

$$f = -1.029 \pm 9.431$$

$$f = -10.460(\text{Comp})\max, 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: (D+0.5L+$PT_{F(L)}$) = 1.0D+0.5L+1.0PT_F

Tendon stress at Normal = jacking – stressing – long-term =1490 – 186 – 94 = 1210 MPa
The force in tendon at Normal, = $1210(197.4)/1000 = 238.9$ kN
Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m
Moment due to dead load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m
Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+0.5L+$PT_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$

$$f = -1.029 \pm 6.788$$

$$f = -7.817(\text{Comp})\max, 5.759(\text{Tension})\max$$

AS 3600-2009 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

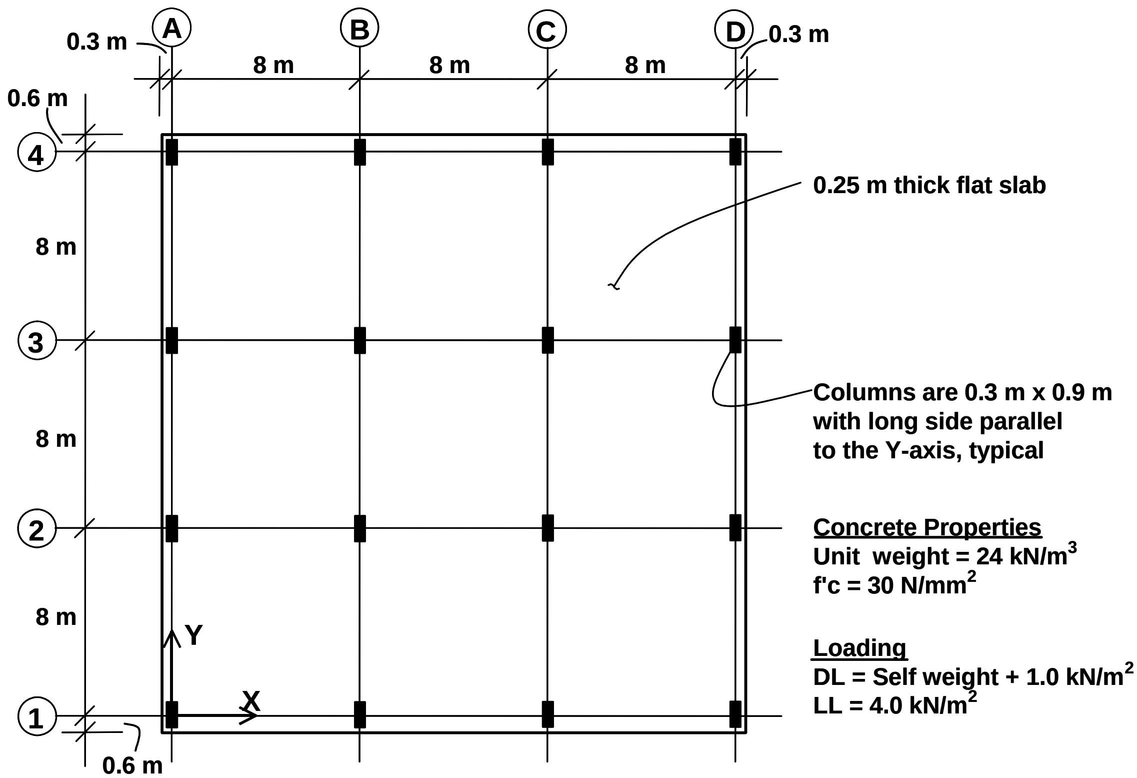


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m, with the long side parallel to the Y-axis. The slab is typically 0.25-m thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f'_c of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress, and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio, and D/C ratio obtained from ETABS with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid Point B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
ETABS	1.793	1.127	1.60
Calculated	1.811	1.086	1.67

COMPUTER FILE: AS 3600-2009 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using ETABS Method

d_{om} = [(250 – 26) + (250 – 38)] / 2 = 218 mm

Refer to Figure 2.

U = 518+ 1118 + 1118 + 518 = 3272 mm

a_x = 518 mm

a_y = 1118 mm

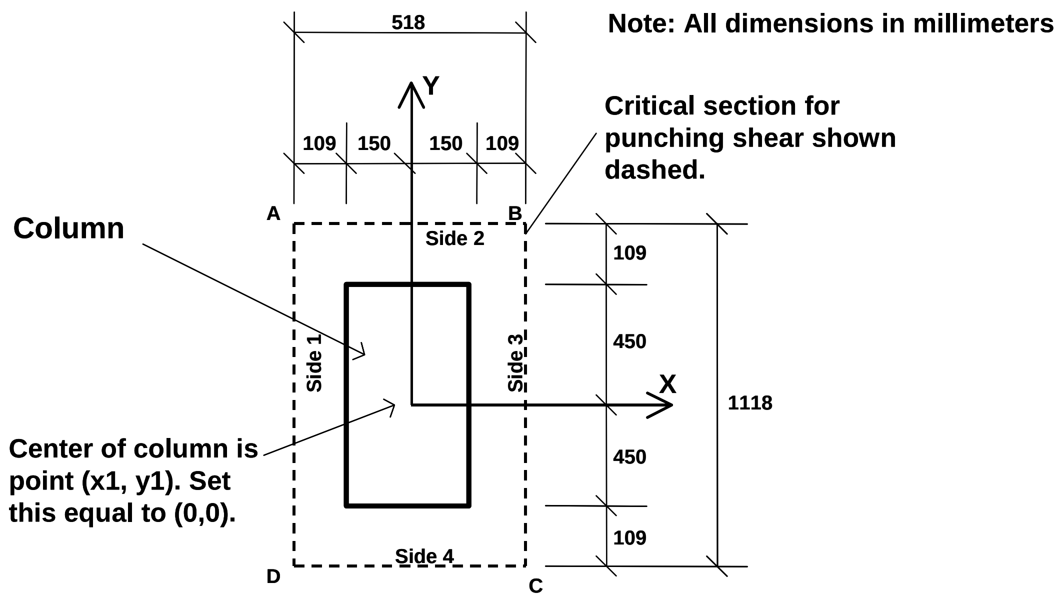


Figure 2: Interior Column, Grid B-2 in ETABS Model

From the ETABS output at grid line B-2:

V^* = 1126.498 kN

M_{v2} = –51.991 kN-m

M_{v3} = 45.723 kN-m

The maximum design shear stress is computed along the major and minor axis of column separately:

$$v_{max} = \frac{V^*}{ud_{om}}\left[1.0 + \frac{uM_v}{8V^* ad_{om}}\right]$$

$$v_{max,X} = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} \bullet \left(1 + \frac{3272 \bullet 51.991 \bullet 10^6}{8 \bullet 1126.498 \bullet 10^3 \bullet 1118 \bullet 218}\right)$$

$v_{max,X}$ = 1.579 $\bullet$ 1.0774 = **1.7013 N/mm²**

$$v_{max,Y} = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} \bullet \left(1 + \frac{3272 \bullet 45.723 \bullet 10^6}{8 \bullet 1126.498 \bullet 10^3 \bullet 518 \bullet 218}\right)$$

$v_{max,Y}$ = 1.579 $\bullet$ 1.1470 = **1.811 N/mm²** (Govern)

The largest absolute value of v_{max}= **1.811 N/mm²**

The shear capacity is calculated based on the smallest of AS 3600-09 equation 11-35, with the d_{om} and u terms removed to convert force to stress.

$$\varphi f_{cv} = \min\begin{cases} 0.17\left(1 + \frac{2}{\beta_h}\right)\varphi\sqrt{f'_c} \\ 0.34\varphi\sqrt{f'_c}\end{cases}$$ = **1.803N/mm²** in accordance with AS 9.2.3(a)

AS 9.2.3(a) yields the smallest value of φf_{cv} = 1.086 N/mm², and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{\varphi f_{cv}} = \frac{1.811}{1.086} = 1.67$$

AS 3600-2009 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

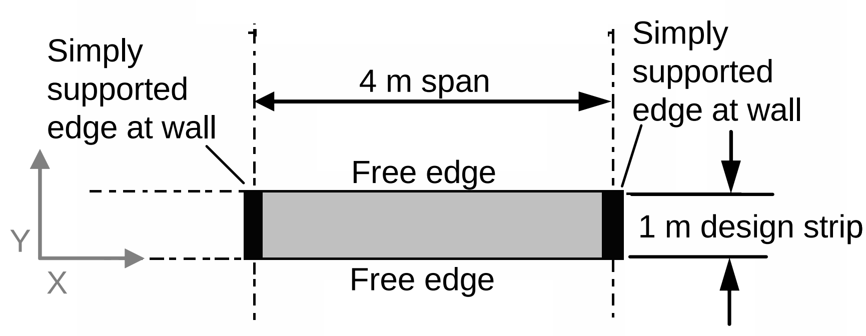


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa), with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the AS 3600-2009 load combination factors, 1.2 for dead loads and 1.5 for live loads. The model is analyzed for both load cases and the load combinations.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing analysis, design is performed using the AS 3600-2009 code using ETABS and also by hand computation. Table 1 shows the comparison of the design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	$2x10^6$	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm)
			A_s^+
Medium	ETABS	24.597	5.58
	Calculated	24.600	5.58

$A_{s,min}^{+}$ = 370.356 sq-mm

COMPUTER FILE: AS 3600-2009 RC-SL EX001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\varphi = 0.8$

$b = 1000$ mm

$\alpha_2 = 1.0 - 0.003 f'_c = 0.91 > 0.85$, Use $\alpha_2 = 0.85$

$\gamma = 1.05 - 0.007 f'_c = 0.84 < 0.85$, Use $\gamma = 0.84$

$a_{\max} = \gamma k_u d = 0.84 \bullet 0.36 \bullet 125 = 37.80$ mm

For the load combination, w and M^* are calculated as follows:

$w = (1.2w_d + 1.5w_t)\, b$

$$M_u = \frac{wl_1^2}{8}$$

$$A_s = 0.24\left(\frac{h}{d}\right)^2 \frac{f'_{ct,f}}{f_{sy,f}} bh \text{ for flat slabs}$$

$$A_{st.\min} = 0.24\left(\frac{h}{d}\right)^2 \frac{f'_{ct,f}}{f_{sy,f}} bd$$

$= 0.24 \bullet (150/125)^2 \bullet 0.6 \bullet \text{SQRT}(30)/460 \bullet 1000 \bullet 150$

$= 370.356$ sq-mm

COMB100

$w_d = 4.0$ kPa

$w_t = 5.0$ kPa

$w = 12.3$ kN/m

$M_{\text{-strip}}^* = 24.6$ kN-m

$M_{\text{-design}}^* = 24.633$ kN-m

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2M^*}{0.85 f'_c \phi b}} = 10.065 \text{ mm} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_{st} = \frac{M^*}{\phi f_{sy}\left(d - \frac{a}{2}\right)} = 557.966 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.57966 sq-cm

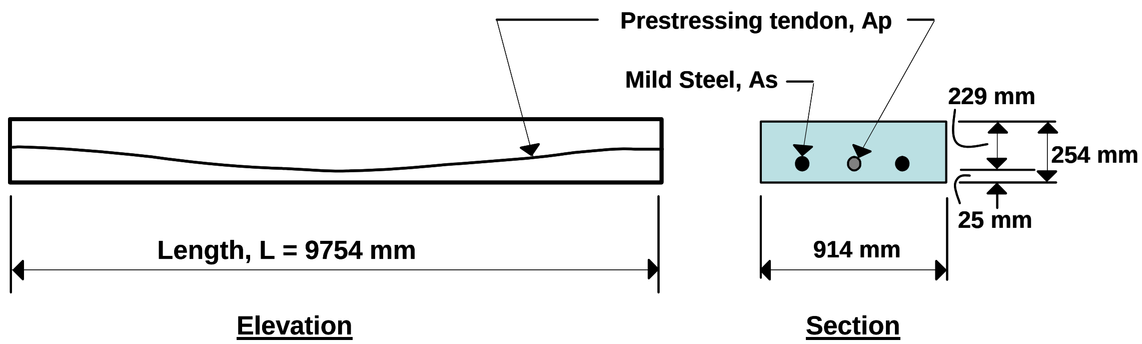
BS 8110-1997 PT-SL EXAMPLE 001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.



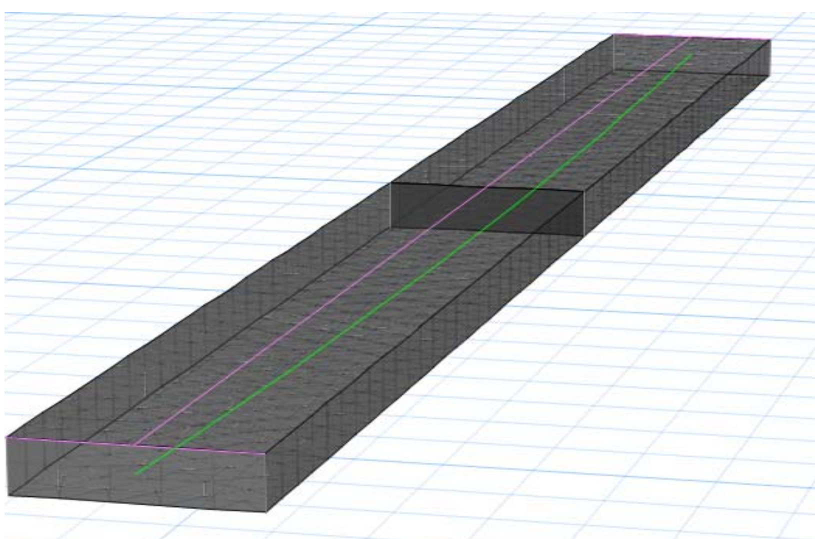


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, was added to the A-Strip. The self-weight and live loads were added to the slab. The loads and post-tensioning forces are as follows.

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the midspan of the slab. Independent hand calculations have been compared with the ETABS results and summarized for verification and validation of the ETABS results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	kN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	kN/m^2
Live load	w_l	=	4.788	kN/m^2

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	174.4	174.4	0.00%
Area of Mild Steel req'd, As (sq-cm)	19.65	19.80	0.76%
Transfer Conc. Stress, top $(D+PT_I)$, MPa	–5.058	–5.057	0.02%
Transfer Conc. Stress, bot $(D+PT_I)$, MPa	2.839	2.839	0.00%
Normal Conc. Stress, top $(D+L+PT_F)$, MPa	–10.460	–10.467	0.07%
Normal Conc. Stress, bot $(D+L+PT_F)$, MPa	8.402	8.409	0.08%

COMPUTER FILE: BS 8110-1997 PT-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

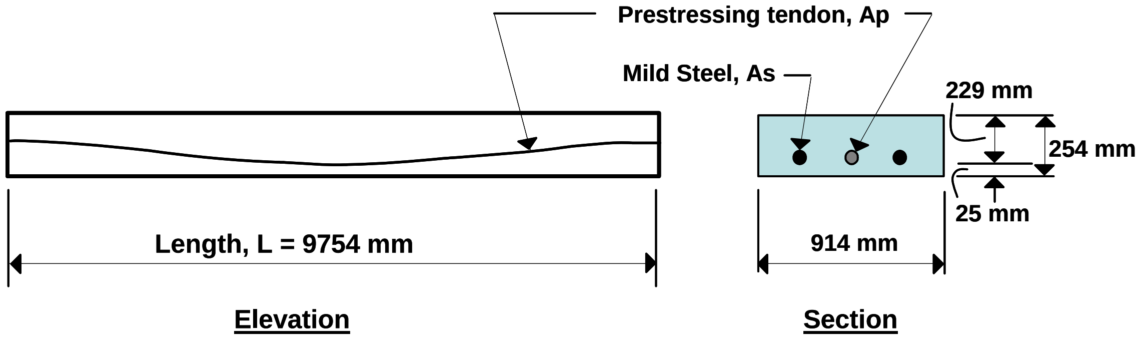
HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
f_{cu} = 30 MPa	f_{pu} = 1862 MPa
f_y = 400 MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$



Loads:

Dead, self-wt = 0.254 m x 23.56 kN/m^3 = 5.984 kN/m^2 (D) x 1.4 = 8.378 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) x 1.6 = 7.661 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 16.039 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 x 0.914m = 9.846 kN/m, ω_u = 16.039 kN/m^2 x 0.914m = 14.659 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8}$ = 14.659 x (9.754)^2/8 = 174.4 kN-m

Ultimate Stress in strand, $f_{pb} = f_{pe} + \frac{7000}{l/d}\left(1 - 1.7\frac{f_{pu}A_p}{f_{cu}bd}\right)$

$$= 1210 + \frac{7000}{9.754/0.229}\left(1 - 1.7\frac{1862(198)}{30(914)(229)}\right)$$

$$= 1358 \text{ MPa} \le 0.7 f_{pu} = 1303 \text{ MPa}$$

K factor used to determine the effective depth is given as:

$$K = \frac{M}{f_{cu}bd^2} = \frac{174.4}{30000(0.914)(0.229)^2} = 0.1213 < 0.156$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \leq 0.95d = 192.2 \text{ mm}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197.4(1303)/1000 = 257.2KN$

Ultimate moment due to PT, $M_{ult,PT} = F_{ult,PT}(z)/\gamma = 257.2(0.192)/1.15 = 43.00$ kN-m

Net Moment to be resisted by As, $M_{NET} = M_U - M_{PT}$

$= 174.4 - 43.00 = 131.40$ kN-m

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_{NET}}{0.87 f_y z} = \frac{131.4}{0.87(400)(192)}(1e6) = 1965 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination ($D+PT_i$) = $1.0D+0.0L+1.0PT_I$

Tendon stress at transfer = jacking stress – stressing losses = 1490 – 186 = 1304 MPa

The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102\text{mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \frac{F_{PTI}}{A} \pm \frac{M_D - M_{PT}}{S} = \frac{-257.4}{0.254(0.914)} \pm \frac{65.04 - 26.23}{0.00983}$

where $S=0.00983\text{m}^3$

$f = -1.109 \pm 3.948$ MPa

$f = -5.058(\text{Comp})\max, 2.839(\text{Tension})\max$

Normal Condition, load combinations: ($D+L+PT_F$) = $1.0D+1.0L+1.0PT_F$

Tendon stress at Normal = jacking – stressing – long-term = 1490 – 186 – 94= 1210 MPa

The force in tendon at Normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT_F),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$

$$f = -1.029 \pm 9.431$$

$$f = -10.460(\text{Comp})\,\text{max},\ 8.402(\text{Tension})\,\text{max}$$

BS 8110-1997 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

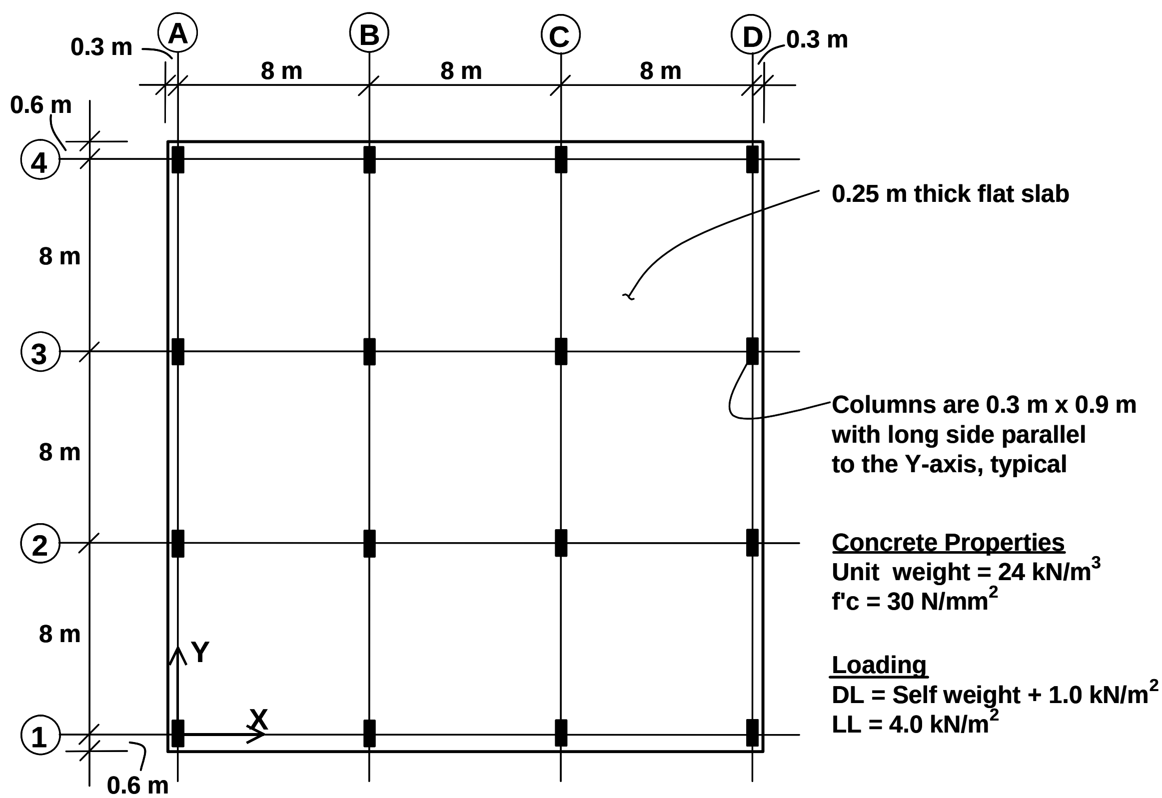


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25-m thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f_{cu} of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from ETABS with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
ETABS	1.119	0.660	1.70
Calculated	1.105	0.625	1.77

COMPUTER FILE: BS 8110-1997 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using ETABS Method

$d = [(250 - 26) + (250 - 38)]/2 = 218$ mm

Refer to Figure 2.

u = 954+ 1554 + 954 + 1554 = 5016 mm

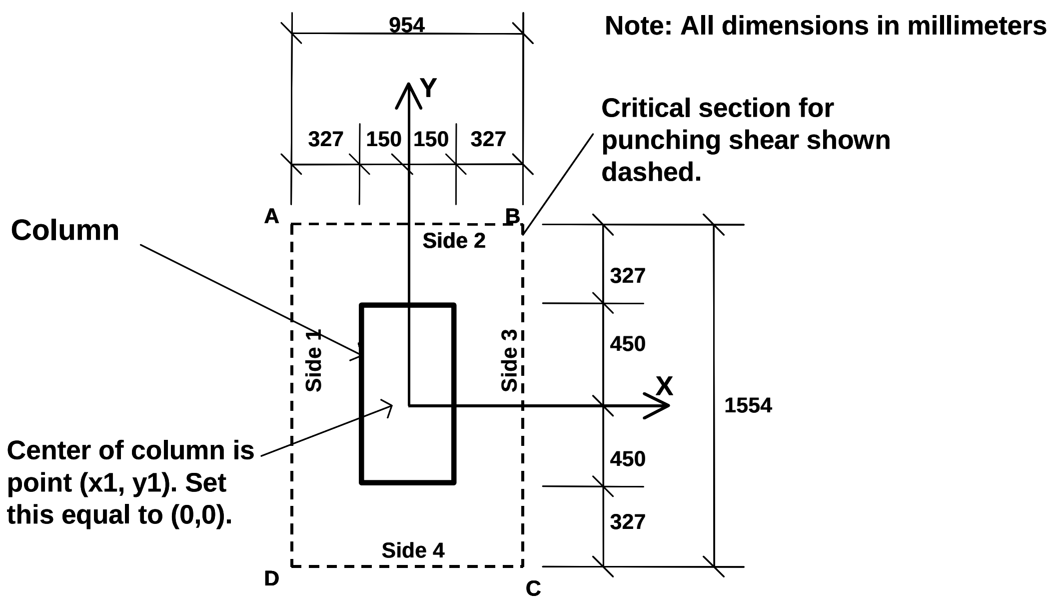


Figure 2: Interior Column, Grid B-2 in ETABS Model

From the ETABS output at Grid B-2:

V = 1126.498 kN

M_2 = 51.9908 kN-m

M_3 = 45.7234 kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{eff,x} = \frac{V}{ud}\left(f + \frac{1.5M_x}{Vy} \right) \qquad \text{(BS 3.7.7.3)}$$

$$v_{eff,x} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 51.9908 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 954} \right) = 1.1049 \text{ (Govern)}$$

$$v_{eff,y} = \frac{V}{ud}\left(f + \frac{1.5M_y}{Vx} \right)$$

$$v_{eff,y} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 45.7234 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 1554} \right) = 1.0705$$

The largest absolute value of v = **1.1049 N/mm²**

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100A_s}{bd} \right)^{1/3}\left(\frac{400}{d} \right)^{1/4} = 0.3568 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{25} \right)^{1/3} = \left(\frac{30}{25} \right)^{1/3} = 1.0627 > 1.0 \text{ OK}$$

γ_m = 1.25

$$\left(\frac{400}{d} \right)^{1/4} = 1.16386 > 1 \text{ OK.}$$

$f_{cu} \le 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

Areas of reinforcement at the face of column for the design strips are as follows:

A_s in Strip Layer A = 9494.296 mm^2

A_s in Strip Layer B = 8314.486 mm^2

Average A_s = (9494.296+8314.486)/2 = 8904.391 mm^2

$$\frac{100\ A_s}{bd} = 100 \bullet 8904.391/(8000 \bullet 218) = 0.51057$$

$$v_c = \frac{0.79 \bullet 1.0 \bullet 1.0627}{1.25} \bullet \left(0.51057\right)^{1/3} \bullet 1.16386 = 0.6247 \text{ MPa}$$

BS 3.7.7.3 yields the value of v = **0.625 N/mm^2**, and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{v} = \frac{1.1049}{0.6247} = 1.77$$

BS 8110-1997 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

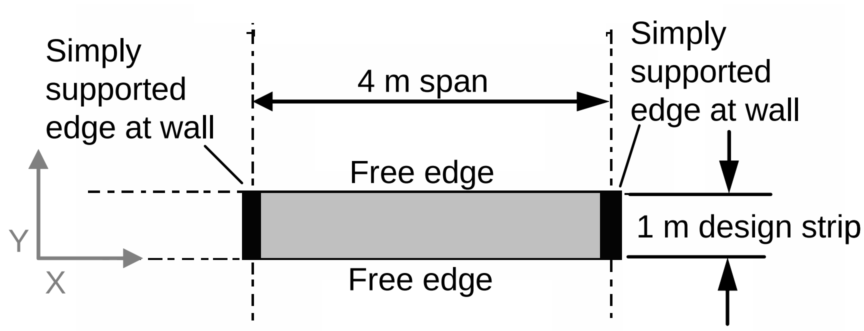


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the BS 8110-97 load combination factors, 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing the analysis, design was performed using the BS 8110-97 code by ETABS and also by hand computation. Table 1 shows the comparison of the design reinforcements computed by the two methods.

PROGRAM NAME: ETABS

REVISION NO.: 0

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	$2x10^6$	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	ETABS	27.197	5.853
	Calculated	27.200	5.850

$A^{+}_{s,min}$ = 162.5 sq-mm

COMPUTER FILE: BS 8110-1997 RC-SL Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\gamma_{m,\,steel}$ = 1.15

$\gamma_{m,\,concrete}$ = 1.50

b = 1000 mm

For the load combination, w and M are calculated as follows:

$$w = (1.4w_d + 1.6w_t)\,b$$

$$M = \frac{wl_1^2}{8}$$

$$A_{s,\min} = 0.0013 b_w d$$
$$= 162.5 \text{ sq-mm}$$

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 13.6 kN/m

$M_{\text{-}strip}$ = 27.2 kN-m

$M_{\text{-}design}$ = 27.2366 kN-m

The depth of the compression block is given by:

$$K = \frac{M}{f_{cu} b d^2} = 0.05810 < 0.156$$

The area of tensile steel reinforcement is then given by:

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d = 116.3283$$

$$A_s = \frac{M}{0.87 f_y z} = 585.046 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.850 sq-cm

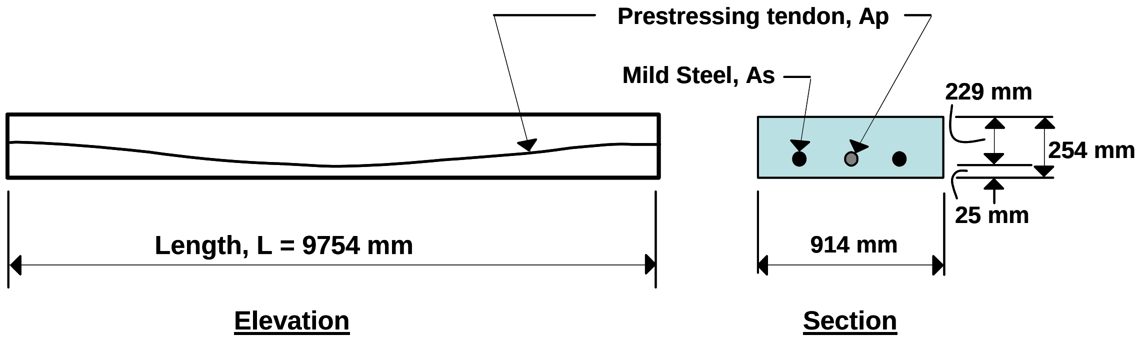
CSA 23.3-04 PT-SL EXAMPLE 001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm as shown in shown in Figure 1.



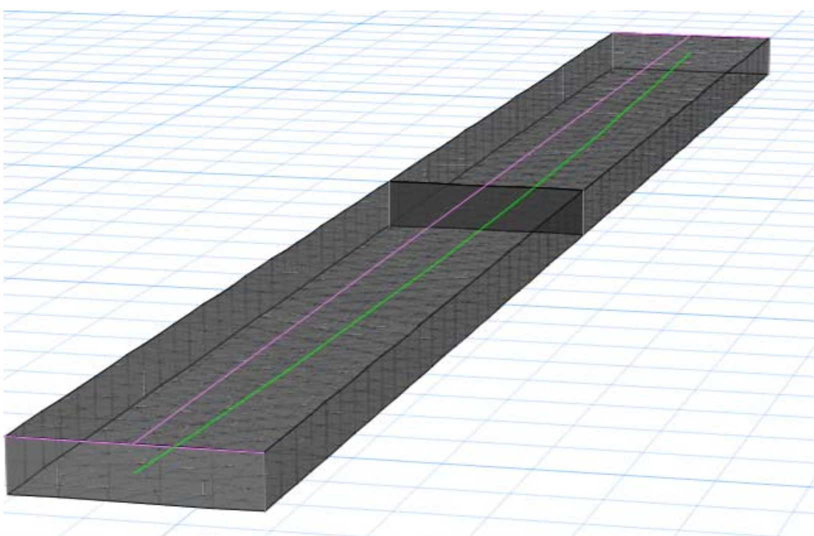


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 KN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the midspan of the slab. Independent hand calculations have been compared with the ETABS results and summarized for verification and validation of the ETABS results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	KN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	KN/m^2
Live load	w_l	=	4.788	KN/m^2

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	159.4	159.4	0.00%
Area of Mild Steel req'd, As (sq-cm)	16.25	16.33	0.49%
Transfer Conc. Stress, top $(D+PT_I)$, MPa	–5.058	–5.057	-0.02%
Transfer Conc. Stress, bot $(D+PT_I)$, MPa	2.839	2.839	0.00%
Normal Conc. Stress, top $(D+L+PT_F)$, MPa	–10.460	–10.467	0.07%
Normal Conc. Stress, bot $(D+L+PT_F)$, MPa	8.402	8.409	0.08%
Long-Term Conc. Stress, top $(D+0.5L+PT_{F(L)})$, MPa	–7.817	–7.818	0.01%
Long-Term Conc. Stress, bot $(D+0.5L+PT_{F(L)})$, MPa	5.759	5.760	0.02%

COMPUTER FILE: CSA A23.3-04 PT-SL EX001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATIONS:

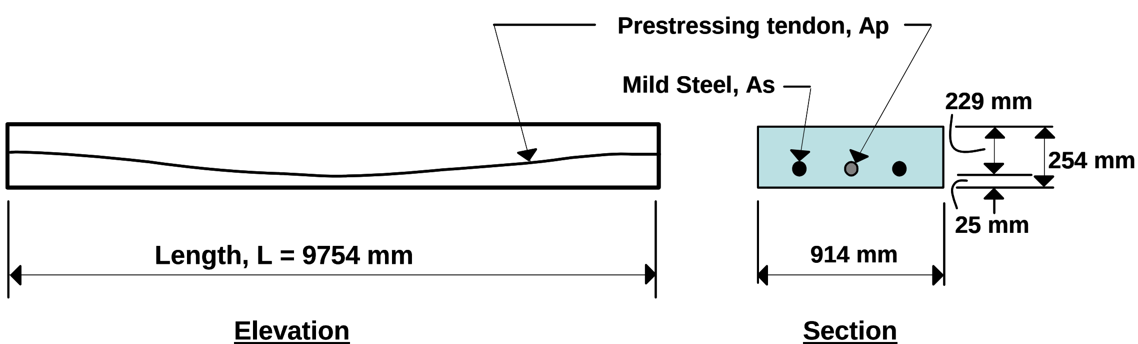
Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
f_{cu} = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\phi_c = 0.65$, $\phi_S = 0.85$

$\alpha_1 = 0.85 - 0.0015f'_c \geq 0.67 = 0.805$

$\beta_1 = 0.97 - 0.0025f'_c \geq 0.67 = 0.895$



Loads:

Dead, self-wt = 0.254 m x 23.56 kN/m^3 = 5.984 kN/m^2 (D) x 1.25 = 7.480 kN/m^2 (D_u)
Live, = 4.788 kN/m^2 (L) x 1.50 = 7.182 kN/m^2 (L_u)
Total = 10.772 kN/m^2 (D+L) = 14.662 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 x 0.914m = 9.846 kN/m, ω_u = 16.039 kN/m^2 x 0.914m = 13.401 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8}$ = 13.401 x (9.754)^2/8 = 159.42 kN-m

Ultimate Stress in strand, $f_{pb} = f_{pe} + \frac{8000}{l_o}\left(d_p - c_y\right)$

$$c_y = \frac{\phi_p A_p f_{pr} + \phi_s A_s f_y}{\alpha_1 \phi_c f'_c \beta_1 b} = \frac{0.9(197)(1347) + 0.85(1625)(400)}{0.805(0.65)(30.0)(0.895)(914)} = 61.66 \text{ mm}$$

$$f_{pb} = 1210 + \frac{8000}{9754}(229 - 61.66) = 1347 \text{ MPa}$$

Depth of the compression block, *a*, is given as:

Stress block depth, $a = d - \sqrt{d^2 - \frac{2M^*}{\alpha_1 f'_c \phi_c b}}$

$$= 0.229 - \sqrt{0.229^2 - \frac{2(159.42)}{0.805(30000)(0.65)(0.914)}} = 55.18$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197(1347)/1000 = 265.9 \text{ kN}$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right)\phi = 265.9\left(0.229 - \frac{55.18}{2}\right)(0.85) = 45.52 \text{ kN-m}$$

Net Moment to be resisted by As, $M_{NET} = M_U - M_{PT}$

$$= 159.42 - 45.52 = 113.90 \text{ kN-m}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_{NET}}{0.87 f_y z} = \frac{113.90}{0.87(400)\left(229 - \frac{55.18}{2}\right)}(1e6) = 1625 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination ($D+PT_i$) = $1.0D+0.0L+1.0PT_I$

Tendon stress at transfer = jacking stress − stressing losses = 1490 − 186 = 1304 MPa
The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4 \text{ kN}$

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-257.4}{0.254(0.914)} \pm \dfrac{65.04 - 26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.109 \pm 3.948 \text{ MPa}$$
$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT$_F$) = 1.0D+1.0L+1.0PT$_F$

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT$_F$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$
$$f = -1.029 \pm 9.431$$
$$f = -10.460(\text{Comp})\max,\ 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: (D+0.5L+PT$_{F(L)}$) = 1.0D+0.5L+1.0PT$_F$

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+0.5L+PT$_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$
$$f = -1.029 \pm 6.788$$
$$f = -7.817(\text{Comp})\max,\ 5.759(\text{Tension})\max$$

CSA A23.3-04 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 8 m spans in each direction, as shown in Figure 1.

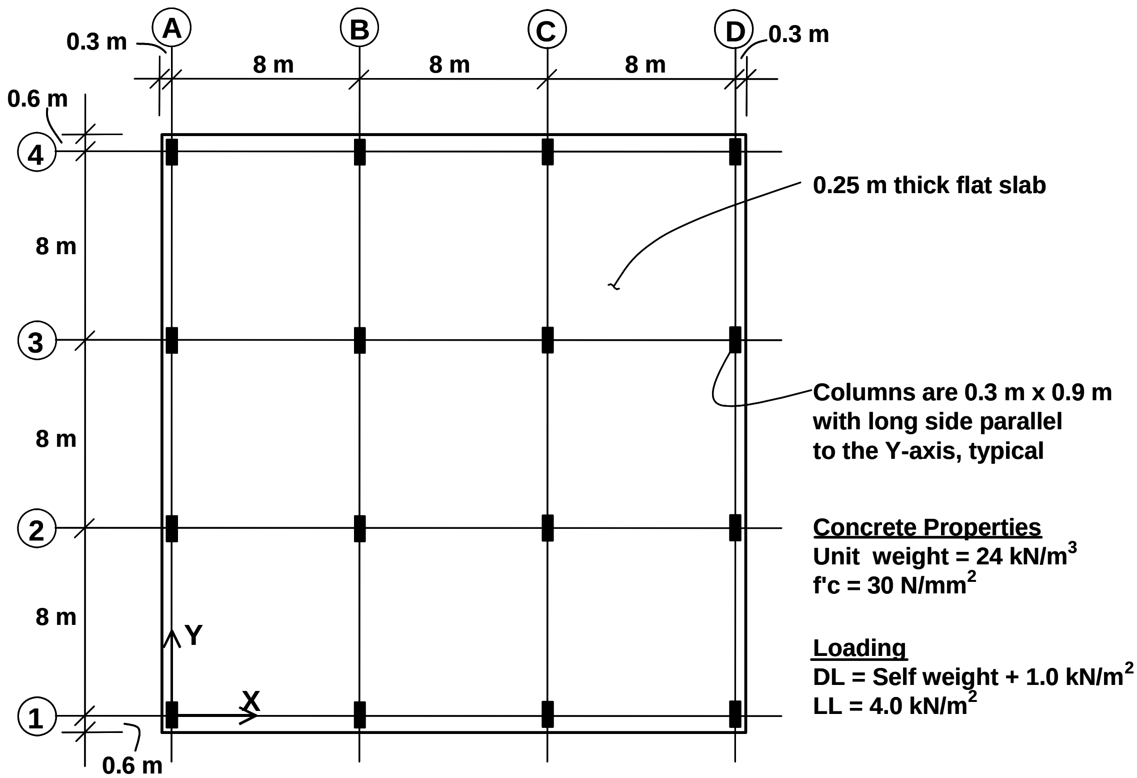


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f'_c of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from ETABS with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
ETABS	1.793	1.127	1.59
Calculated	1.792	1.127	1.59

COMPUTER FILE: CSA A23.3-04 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using ETABS Method

$d = [(250-26)+(250-38)]/2 = 218$ mm

Refer to Figure 2.

b_0 = 518+ 1118 + 1118 + 518 = 3272 mm

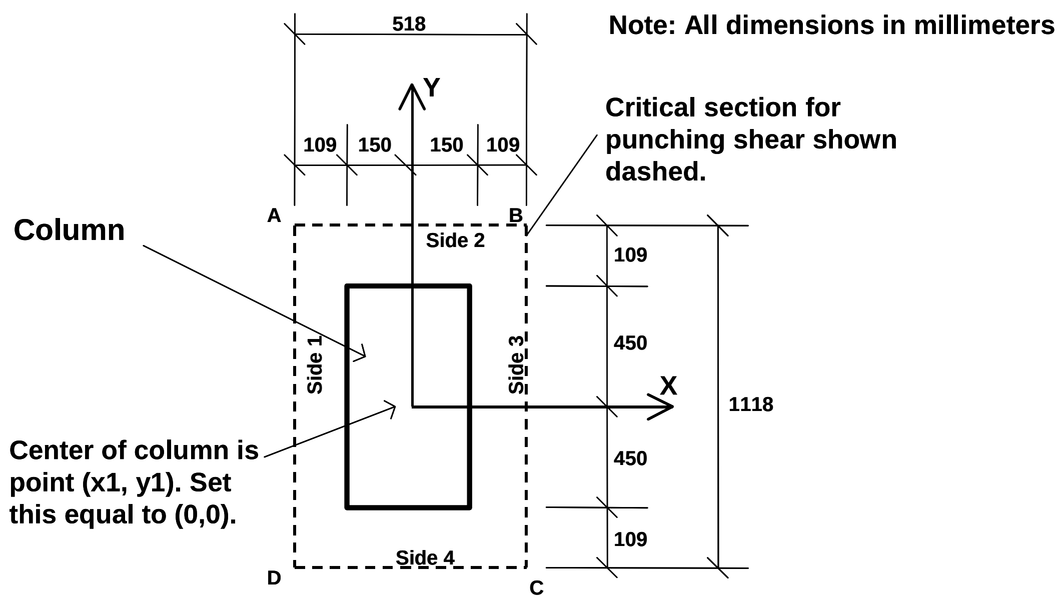


Figure 2: Interior Column, Grid B-2 in ETABS Model

$$\gamma_{V2} = 1 - \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{1118}{518}}} = 0.495$$

$$\gamma_{V3} = 1 - \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{518}{1118}}} = 0.312$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–259	0	259	0	N.A.
y_2	0	559	0	–559	N.A.
L	1118	518	1118	518	b_0 = 3272
d	218	218	218	218	N.A.
Ld	243724	112924	243724	112924	713296
Ldx_2	–63124516	0	63124516	0	0
Ldy_2	0	63124516	0	–63124516	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{713296} = 0\,mm$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{713296} = 0\,mm$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	1118	518	1118	518	N.A.
d	218	218	218	218	N.A.
$x_2 - x_3$	–259	0	259	0	N.A.
$y_2 - y_3$	0	559	0	–559	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	2.64E+10	3.53E+10	2.64E+10	3.53E+10	1.23E+11
I_{YY}	1.63E+10	2.97E+09	1.63E+10	2.97E+09	3.86E+10
I_{XY}	0	0	0	0	0

From the ETABS output at Grid B-2:

V_f = 1126.498 kN

$\gamma_{V2} M_{f,2}$ = –25.725 kN-m

$\gamma_{V3} M_{f,3}$ = 14.272 kN-m

At the point labeled A in Figure 2, $x_4 = -259$ and $y_4 = 559$, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(559-0) - (0)(-259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(-259-0) - (0)(559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

$v_f = 1.5793 - 0.1169 - 0.0958 =$ **1.3666 N/mm²** at point A

At the point labeled B in Figure 2, $x_4 = 259$ and $y_4 = 559$, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(559-0) - (0)(259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(259-0) - (0)(559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

$v_f = 1.5793 - 0.1169 + 0.0958 =$ **1.5582 N/mm²** at point B

At the point labeled C in Figure 2, $x_4 = 259$ and $y_4 = -559$, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(-559-0) - (0)(259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(259-0) - (0)(-559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

$v_f = 1.5793 + 0.1169 + 0.0958 =$ **1.792 N/mm²** at point C

At the point labeled D in Figure 2, $x_4 = -259$ and $y_4 = -559$, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(-559-0) - (0)(-259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(-259-0) - (0)(-559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

$v_f = 1.5793 + 0.1169 - 0.0958 =$ **1.6004 N/mm²** at point D

Point C has the largest absolute value of v_u, thus v_{max} = **1.792 N/mm²**

The shear capacity is calculated based on the minimum of the following three limits:

$$v_v = \min \begin{cases} \phi_c \left(1 + \dfrac{2}{\beta_c}\right) 0.19 \lambda \sqrt{f'_c} \\ \phi_c \left(0.19 + \dfrac{\alpha_s d}{b_0}\right) \lambda \sqrt{f'_c} \\ \phi_c 0.38 \lambda \sqrt{f'_c} \end{cases} \quad 1.127 \text{ N/mm}^2 \text{ in accordance with CSA 13.3.4.1}$$

CSA 13.3.4.1 yields the smallest value of v_v = 1.127 N/mm^2, and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{\varphi v_v} = \frac{1.792}{1.127} = 1.59$$

CSA A23.3-04 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

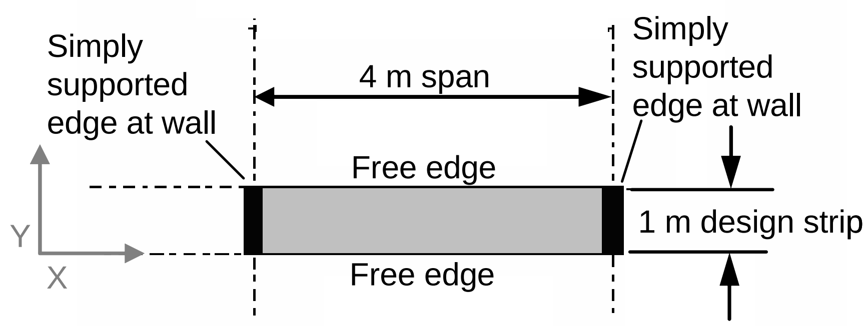


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the CSA A23.3-04 load combination factors, 1.25 for dead loads and 1.5 for live loads. The model is analyzed for these load cases and load combinations.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing the analysis, design is performed using the CSA A23.3-04 code by ETABS and also by hand computation. Table 1 show the comparison of the design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	$2x10^6$	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	ETABS	25.00	5.414
	Calculated	25.00	5.528

$A^+_{s,min}$ = 357.2 sq-mm

COMPUTER FILE: CSA A23.3-04 RC-SL Ex001.EDB

CONCLUSION

The ETABS results show a very close comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

ϕ_c = 0.65 for concrete

ϕ_s = 0.85 for reinforcement

$$A_{s,\min} = \frac{0.2\sqrt{f'_c}}{f_y}\ b_w\, h = 357.2 \text{ sq-mm}$$

b = 1000 mm

$\alpha_1 = 0.85 - 0.0015f'_c \geq 0.67 = 0.805$

$\beta_1 = 0.97 - 0.0025f'_c \geq 0.67 = 0.895$

$$c_b = \frac{700}{700 + f_y} d = 75.43 \text{ mm}$$

$a_b = \beta_1 c_b = 67.5$ mm

For the load combination, w and M^* are calculated as follows:

$w = (1.25w_d + 1.5w_t)\, b$

$$M_u = \frac{wl_1^2}{8}$$

$A_s = \min[A_{s,\min}, (4/3)\, A_{s,\text{required}}] = \min[357.2, (4/3)540.63] = 357.2$ sq-mm

$= 0.22\bullet(150/125)^2\bullet0.6\bullet\text{SQRT}(30)/460\bullet100\bullet125$

$=$ 282.9 sq-mm

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 12.5 kN/m

$M_{f\text{-}strip}$ = 25.0 kN-m

$M_{f\text{-}design}$ = 25.529 kN-m

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_f\right|}{\alpha_1 f'_c \phi_c b}} = 13.769 \text{ mm} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_f}{\phi_s f_y \left(d - \frac{a}{2}\right)} = 552.77 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.528 \text{ sq-cm}$$

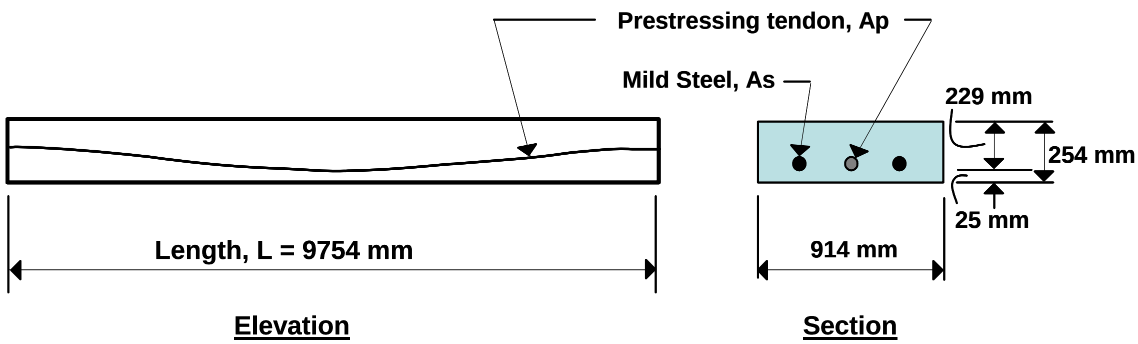
CSA A23.3-14 PT-SL EXAMPLE 001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm as shown in shown in Figure 1.



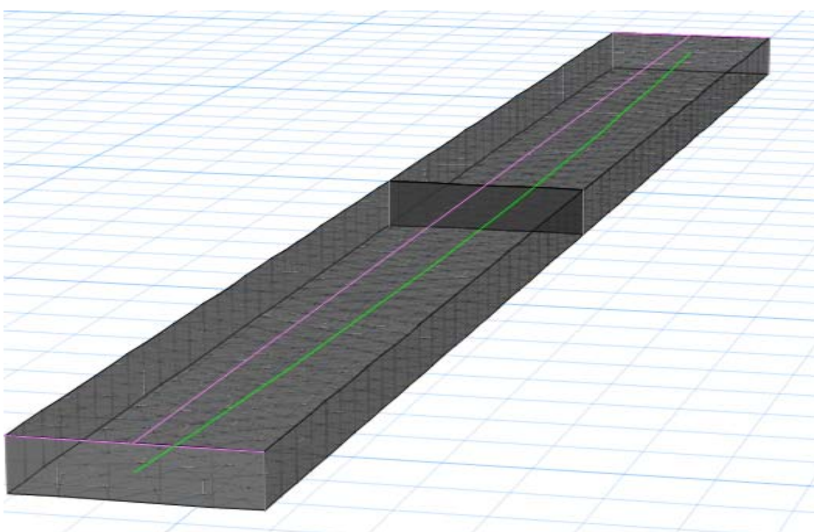


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 KN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the midspan of the slab. Independent hand calculations have been compared with the ETABS results and summarized for verification and validation of the ETABS results.

Geometry, Properties and Loading

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	KN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	KN/m^2
Live load	w_l	=	4.788	KN/m^2

Technical Features of ETABS Tested

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	159.4	159.4	0.00%
Area of Mild Steel req'd, As (sq-cm)	16.25	16.33	0.49%
Transfer Conc. Stress, top (D+PT_I), MPa	–5.058	–5.057	-0.02%
Transfer Conc. Stress, bot (D+PT_I), MPa	2.839	2.839	0.00%
Normal Conc. Stress, top (D+L+PT_F), MPa	–10.460	–10.467	0.07%
Normal Conc. Stress, bot (D+L+PT_F), MPa	8.402	8.409	0.08%
Long-Term Conc. Stress, top (D+0.5L+$PT_{F(L)}$), MPa	–7.817	–7.818	0.01%
Long-Term Conc. Stress, bot (D+0.5L+$PT_{F(L)}$), MPa	5.759	5.760	0.02%

COMPUTER FILE: CSA A23.3-14 PT-SL EX001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing

f_{cu} = 30MPa
fy = 400MPa

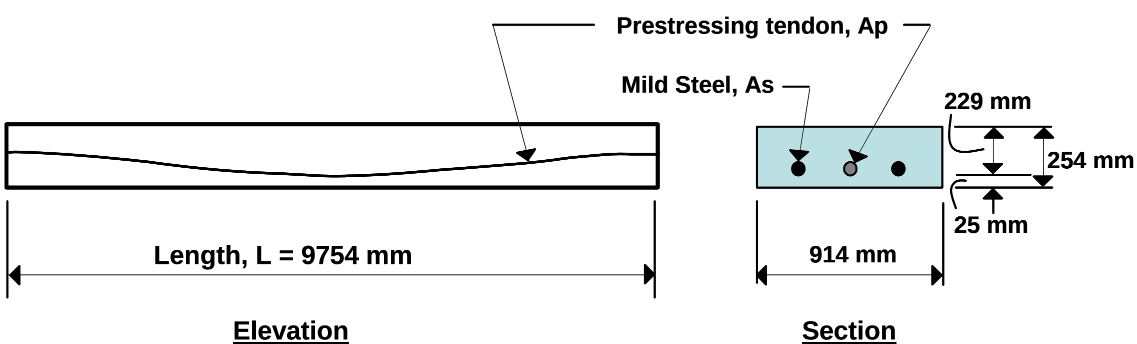
Post-Tensioning

f_{pu} = 1862 MPa
f_{py} = 1675 MPa
Stressing Loss = 186 MPa
Long-Term Loss = 94 MPa
f_i = 1490 MPa
f_e = 1210 MPa

$\phi_c = 0.65$, $\phi_S = 0.85$

$\alpha_1 = 0.85 - 0.0015f'_c \geq 0.67 = 0.805$

$\beta_1 = 0.97 - 0.0025f'_c \geq 0.67 = 0.895$



Loads:

Dead, self-wt = 0.254 m x 23.56 kN/m^3 = 5.984 kN/m^2 (D) x 1.25 = 7.480 kN/m^2 (D_u)
Live, = 4.788 kN/m^2 (L) x 1.50 = 7.182 kN/m^2 (L_u)
Total = 10.772 kN/m^2 (D+L) = 14.662 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 x 0.914m = 9.846 kN/m, ω_u = 16.039 kN/m^2 x 0.914m = 13.401 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8}$ = 13.401 x (9.754)^2/8 = 159.42 kN-m

Ultimate Stress in strand, $f_{pb} = f_{pe} + \frac{8000}{l_o}\left(d_p - c_y\right)$

$$c_y = \frac{\phi_p A_p f_{pr} + \phi_s A_s f_y}{\alpha_1 \phi_c f'_c \beta_1 b} = \frac{0.9(197)(1347) + 0.85(1625)(400)}{0.805(0.65)(30.0)(0.895)(914)} = 61.66 \text{ mm}$$

$$f_{pb} = 1210 + \frac{8000}{9754}\left(229 - 61.66\right) = 1347 \text{ MPa}$$

Depth of the compression block, *a*, is given as:

Stress block depth, $a = d - \sqrt{d^2 - \frac{2M^*}{\alpha_1 f'_c \phi_c b}}$

$$= 0.229 - \sqrt{0.229^2 - \frac{2(159.42)}{0.805(30000)(0.65)(0.914)}} = 55.18$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197(1347)/1000 = 265.9 \text{ kN}$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right)\phi = 265.9\left(0.229 - \frac{55.18}{2}\right)(0.85) = 45.52 \text{ kN-m}$$

Net Moment to be resisted by As, $M_{NET} = M_U - M_{PT}$

$$= 159.42 - 45.52 = 113.90 \text{ kN-m}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_{NET}}{0.87 f_y z} = \frac{113.90}{0.87(400)\left(229 - \frac{55.18}{2}\right)}(1e6) = 1625 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (D+PT$_i$) = 1.0D+0.0L+1.0PT$_I$

Tendon stress at transfer = jacking stress − stressing losses = 1490 − 186 = 1304 MPa
The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4 \text{ kN}$

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-257.4}{0.254(0.914)} \pm \dfrac{65.04 - 26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.109 \pm 3.948 \text{ MPa}$$
$$f = -5.058(\text{Comp})\max, 2.839(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT$_F$) = 1.0D+1.0L+1.0PT$_F$

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa
The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT$_F$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$
$$f = -1.029 \pm 9.431$$
$$f = -10.460(\text{Comp})\max, 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: (D+0.5L+PT$_{F(L)}$) = 1.0D+0.5L+1.0PT$_F$

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa
The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+0.5L+PT$_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$
$$f = -1.029 \pm 6.788$$
$$f = -7.817(\text{Comp})\max, 5.759(\text{Tension})\max$$

CSA A23.3-14 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 8 m spans in each direction, as shown in Figure 1.

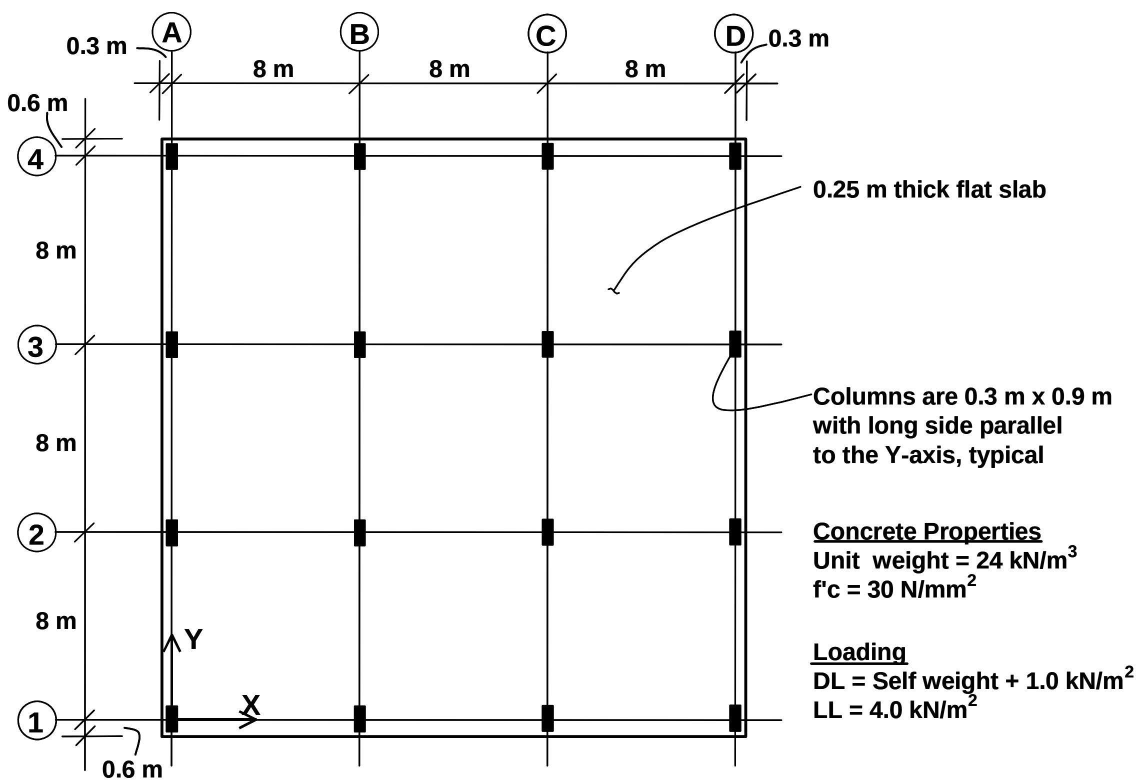


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f'_c of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from ETABS with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
ETABS	1.793	1.127	1.59
Calculated	1.792	1.127	1.59

COMPUTER FILE: CSA A23.3-14 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using ETABS Method

$$d = [(250-26)+(250-38)]/2 = 218 \text{ mm}$$

Refer to Figure 2.

b_0 = 518+ 1118 + 1118 + 518 = 3272 mm

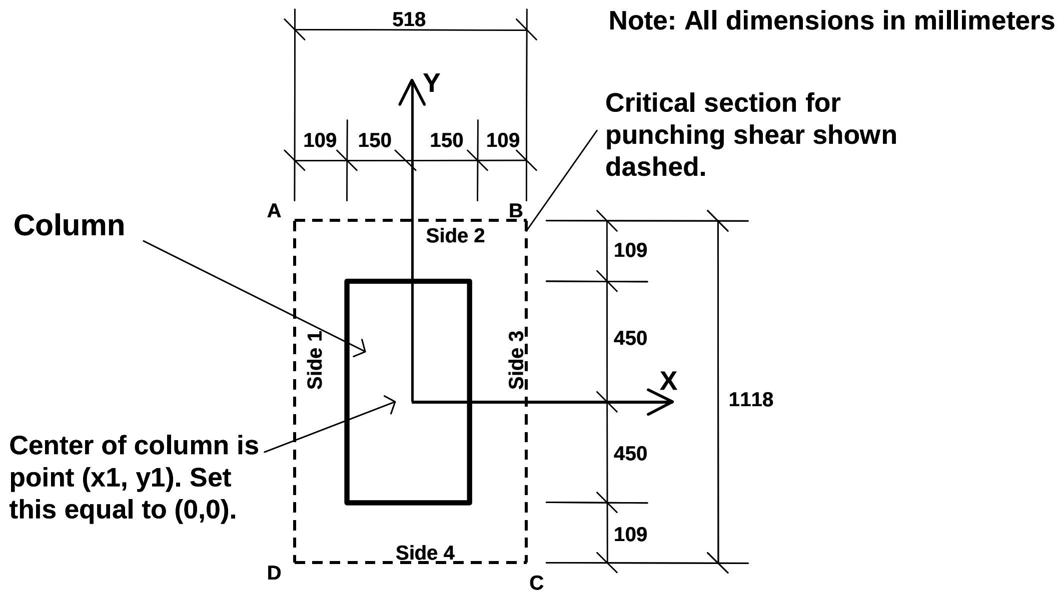


Figure 2: Interior Column, Grid B-2 in ETABS Model

$$\gamma_{V2} = 1 - \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{1118}{518}}} = 0.495$$

$$\gamma_{V3} = 1 - \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{518}{1118}}} = 0.312$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–259	0	259	0	N.A.
y_2	0	559	0	–559	N.A.
L	1118	518	1118	518	b_0 = 3272
d	218	218	218	218	N.A.
Ld	243724	112924	243724	112924	713296
Ldx_2	–63124516	0	63124516	0	0
Ldy_2	0	63124516	0	–63124516	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{713296} = 0\,mm$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{713296} = 0\,mm$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	1118	518	1118	518	N.A.
d	218	218	218	218	N.A.
$x_2 - x_3$	–259	0	259	0	N.A.
$y_2 - y_3$	0	559	0	–559	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	2.64E+10	3.53E+10	2.64E+10	3.53E+10	1.23E+11
I_{YY}	1.63E+10	2.97E+09	1.63E+10	2.97E+09	3.86E+10
I_{XY}	0	0	0	0	0

From the ETABS output at Grid B-2:

V_f = 1126.498 kN

$\gamma_{V2} M_{f,2}$ = –25.725 kN-m

$\gamma_{V3} M_{f,3}$ = 14.272 kN-m

At the point labeled A in Figure 2, $x_4 = -259$ and $y_4 = 559$, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(559-0) - (0)(-259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(-259-0) - (0)(559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

$v_f = 1.5793 - 0.1169 - 0.0958 =$ **1.3666 N/mm²** at point A

At the point labeled B in Figure 2, $x_4 = 259$ and $y_4 = 559$, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(559-0) - (0)(259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(259-0) - (0)(559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

$v_f = 1.5793 - 0.1169 + 0.0958 =$ **1.5582 N/mm²** at point B

At the point labeled C in Figure 2, $x_4 = 259$ and $y_4 = -559$, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(-559-0) - (0)(259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(259-0) - (0)(-559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

$v_f = 1.5793 + 0.1169 + 0.0958 =$ **1.792 N/mm²** at point C

At the point labeled D in Figure 2, $x_4 = -259$ and $y_4 = -559$, thus:

$$v_f = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 [3.86 \bullet 10^{10}(-559-0) - (0)(-259-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2} + \frac{14.272 \bullet 10^6 [1.23 \bullet 10^{11}(-259-0) - (0)(-559-0)]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10}) - (0)^2}$$

$v_f = 1.5793 + 0.1169 - 0.0958 =$ **1.6004 N/mm²** at point D

Point C has the largest absolute value of v_u, thus v_{max} = **1.792 N/mm²**

The shear capacity is calculated based on the minimum of the following three limits:

$$v_v = \min\begin{cases} \phi_c\left(1+\dfrac{2}{\beta_c}\right)0.19\lambda\sqrt{f'_c} \\ \phi_c\left(0.19+\dfrac{\alpha_s d}{b_0}\right)\lambda\sqrt{f'_c} \\ \phi_c 0.38\lambda\sqrt{f'_c} \end{cases} \quad 1.127\ \text{N/mm}^2 \text{ in accordance with CSA 13.3.4.1}$$

CSA 13.3.4.1 yields the smallest value of $\boxed{v_v = 1.127\ \text{N/mm}^2}$, and thus this is the shear capacity.

$$\boxed{\text{Shear Ratio} = \frac{v_U}{\varphi v_v} = \frac{1.792}{1.127} = 1.59}$$

CSA A23.3-14 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

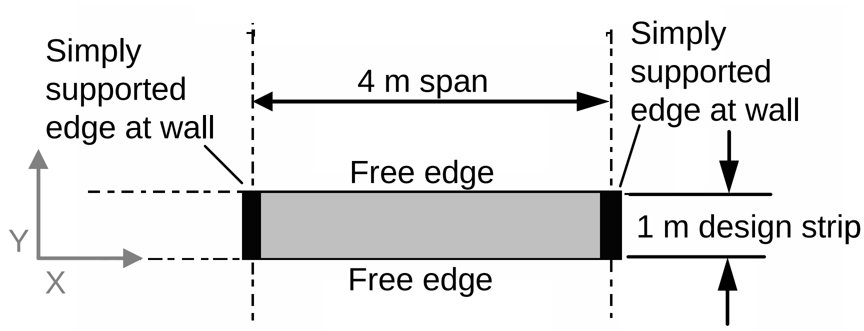


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the CSA A23.3-14 load combination factors, 1.25 for dead loads and 1.5 for live loads. The model is analyzed for these load cases and load combinations.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing the analysis, design is performed using the CSA A23.3-14 code by ETABS and also by hand computation. Table 1 show the comparison of the design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2x10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	ETABS	25.00	5.414
	Calculated	25.00	5.528

$A^+_{s,\min}$ = 357.2 sq-mm

COMPUTER FILE: CSA A23.3-14 RC-SL Ex001.EDB

CONCLUSION

The ETABS results show a very close comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

ϕ_c = 0.65 for concrete

ϕ_s = 0.85 for reinforcement

$$A_{s,\min} = \frac{0.2\sqrt{f'_c}}{f_y}\ b_w\ h = 357.2 \text{ sq-mm}$$

b = 1000 mm

$\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67 = 0.805$

$\beta_1 = 0.97 - 0.0025 f'_c \geq 0.67 = 0.895$

$$c_b = \frac{700}{700 + f_y} d = 75.43 \text{ mm}$$

$a_b = \beta_1 c_b = 67.5$ mm

For the load combination, w and M^* are calculated as follows:

$w = (1.25 w_d + 1.5 w_t)\ b$

$$M_u = \frac{w l_1^2}{8}$$

$A_s = \min[A_{s,\min}, (4/3)\ A_{s,\text{required}}] = \min[357.2, (4/3)540.63] = 357.2$ sq-mm

$= 0.22 \bullet (150/125)^2 \bullet 0.6 \bullet \text{SQRT}(30)/460 \bullet 100 \bullet 125$

$= 282.9$ sq-mm

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 12.5 kN/m

$M_{f\text{-}strip}$ = 25.0 kN-m

$M_{f\text{-}design}$ = 25.529 kN-m

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M_f\right|}{\alpha_1 f'_c \phi_c b}} = 13.769 \text{ mm} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_f}{\phi_s f_y \left(d - \frac{a}{2}\right)} = 552.77 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.528 \text{ sq-cm}$$

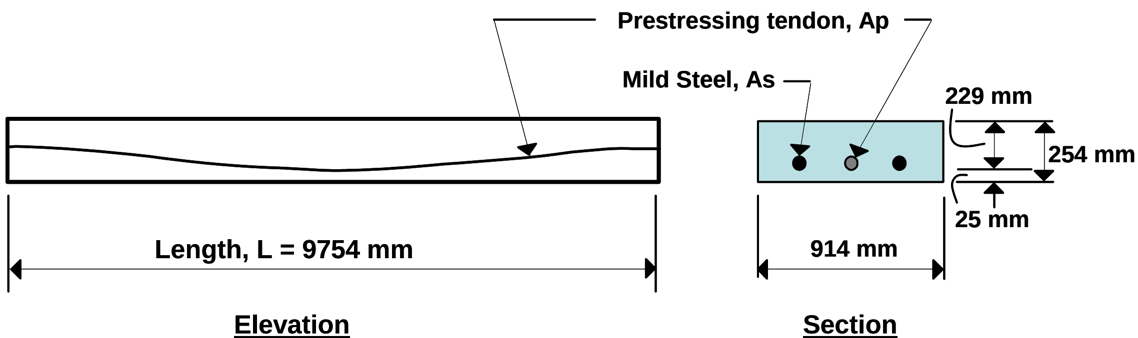
EN 2-2004 PT-SL EXAMPLE 001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm as shown in shown in Figure 1.



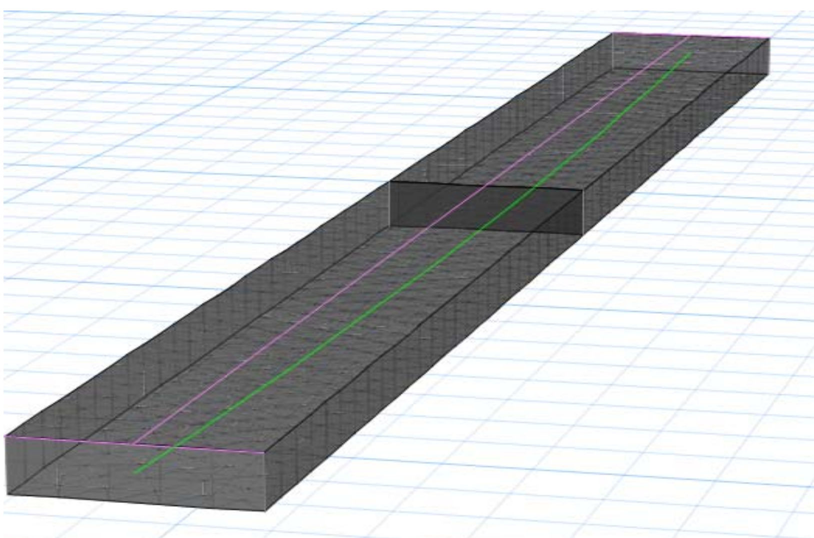


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, was added to the A-Strip. The self weight and live loads have been added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the ETABS results and summarized for verification and validation of the ETABS results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	KN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	KN/m^2
Live load	w_l	=	4.788	KN/m^2

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing, and slab stresses with independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	166.41	166.44	0.02%
Transfer Conc. Stress, top ($D+PT_I$), MPa	–5.057	–5.057	0.00%
Transfer Conc. Stress, bot ($D+PT_I$), MPa	2.839	2.839	0.00%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	–10.460	–10.467	0.07%
Normal Conc. Stress, bot ($D+L+PT_F$), MPa	8.402	8.409	0.08%
Long-Term Conc. Stress, top ($D+0.5L+PT_{F(L)}$), MPa	–7.817	–7.818	0.01%
Long-Term Conc. Stress, bot ($D+0.5L+PT_{F(L)}$), MPa	5.759	5.760	0.02%

Table 2 Comparison of Design Moments and Reinforcements

National Annex	Method	Design Moment (kN-m)	Reinforcement Area (sq-cm)
			A_s^+
CEN Default, Norway, Slovenia and Sweden	ETABS	166.44	15.39
	Calculated	166.41	15.36
Finland , Singapore and UK	ETABS	166.44	15.90
	Calculated	166.41	15.87
Denmark	ETABS	166.44	15.96
	Calculated	166.41	15.94

COMPUTER FILE: EN 2-2004 PT-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

Design Parameters:

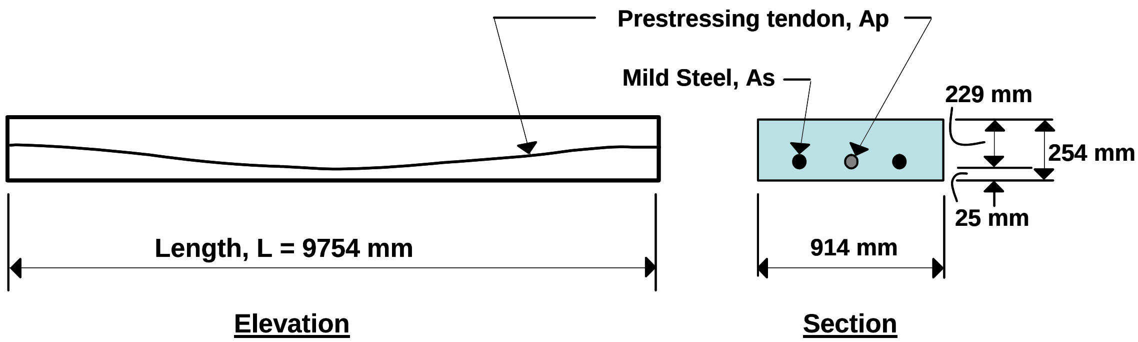
Mild Steel Reinforcing	Post-Tensioning
$f'c$ = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$

$\eta = 1.0$ for $f_{ck} \le 50$ MPa

$\lambda = 0.8$ for $f_{ck} \le 50$ MPa



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.35 = 8.078 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) × 1.50 = 7.182 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 15.260 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 15.260 kN/m^2 × 0.914 m = 13.948 kN/m

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8} = 13.948 \times (9.754)^2 / 8 = 165.9$ kN-m

Ultimate Stress in strand, $f_{PS} = f_{SE} + 7000d\left(1 - 1.36\frac{f_{PU}A_P}{f_{CK}bd}\right)\Big/ l$

$$= 1210 + 7000(229)\left(1 - 1.36\frac{1862(198)}{30(914)(229)}\right)\Big/(9754)$$

$$= 1361\text{ MPa}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 2(99)(1361)/1000 = 269.5\text{ kN}$

CEN Default, Norway, Slovenia and Sweden:
Design moment M = 166.4122 kN-m

Compression block depth ratio: $m = \dfrac{M}{bd^2\eta f_{cd}}$

$$= \frac{166.4122}{(0.914)(0.229)^2(1)(30000/1.50)} = 0.1736$$

Required area of mild steel reinforcing,

$$\omega = 1 - \sqrt{1-2m} = 1 - \sqrt{1-2(0.1736)} = 0.1920$$

$$A_{EquivTotal} = \omega\left(\frac{\eta f_{cd}bd}{f_{yd}}\right) = 0.1920\left(\frac{1(30/1.5)(914)(229)}{400/1.15}\right) = 2311\text{ mm}^2$$

$$A_{EquivTotal} = A_P\left(\frac{1361}{400/1.15}\right) + A_S = 2311\text{ mm}^2$$

$$A_S = 2311 - 198\left(\frac{1361}{400/1.15}\right) = 1536\text{ mm}^2$$

Finland, Singapore and UK:
Design moment M = 166.4122 kN-m

Compression block depth ratio: $m = \dfrac{M}{bd^2\eta f_{cd}}$

$$= \frac{166.4122}{(0.914)(0.229)^2(0.85)(30000/1.50)} = 0.2042$$

Required area of mild steel reinforcing,

$$\omega = 1 - \sqrt{1-2m} = 1 - \sqrt{1-2(0.2042)} = 0.23088$$

$$A_{EquivTotal} = \omega\left(\frac{\eta f_{cd} bd}{f_{yd}}\right) = 0.23088\left(\frac{0.85(30/1.5)(914)(229)}{400/1.15}\right) = 2362 \text{ mm}^2$$

$$A_{EquivTotal} = A_P\left(\frac{1361}{400/1.15}\right) + A_S = 2362 \text{ mm}^2$$

$$A_S = 2362 - 198\left(\frac{1361}{400/1.15}\right) = 1587 \text{ mm}^2$$

Denmark:

Design moment M = 166.4122 kN-m

Compression block depth ratio: $m = \dfrac{M}{bd^2 \eta f_{cd}}$

$$= \frac{166.4122}{(0.914)(0.229)^2(1.0)(30000/1.45)} = 0.1678$$

Required area of mild steel reinforcing,

$$\omega = 1 - \sqrt{1-2m} = 1 - \sqrt{1-2(0.1678)} = 0.1849$$

$$A_{EquivTotal} = \omega\left(\frac{\eta f_{cd} bd}{f_{yd}}\right) = 0.1849\left(\frac{1.0(30/1.45)(914)(229)}{400/1.20}\right) = 2402 \text{ mm}^2$$

$$A_{EquivTotal} = A_P\left(\frac{1361}{400/1.2}\right) + A_S = 2402 \text{ mm}^2$$

$$A_S = 2402 - 198\left(\frac{1361}{400/1.2}\right) = 1594 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination ($D+PT_i$) = 1.0D+0.0L+1.0PT_I

Tendon stress at transfer = jacking stress − stressing losses =1490 − 186 = 1304 MPa

The force in the tendon at transfer = $1304(197.4)/1000 = 257.4$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-257.4}{0.254(0.914)} \pm \dfrac{65.04 - 26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.109 \pm 3.948 \text{ MPa}$$
$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT$_F$) = 1.0D+1.0L+1.0PT$_F$

Tendon stress at normal = jacking − stressing − long-term=1490 − 186 − 94 = 1210 MPa

The force in tendon at normal = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT$_F$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$
$$f = -1.029 \pm 9.431$$
$$f = -10.460(\text{Comp})\max,\ 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: (D+0.5L+PT$_{F(L)}$) = 1.0D+0.5L+1.0PT$_F$

Tendon stress at normal = jacking − stressing − long-term = 1490 − 186 − 94 = 1210 MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+0.5L+PT$_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$
$$f = -1.029 \pm 6.788$$
$$f = -7.817(\text{Comp})\max,\ 5.759(\text{Tension})\max$$

EN 2-2004 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

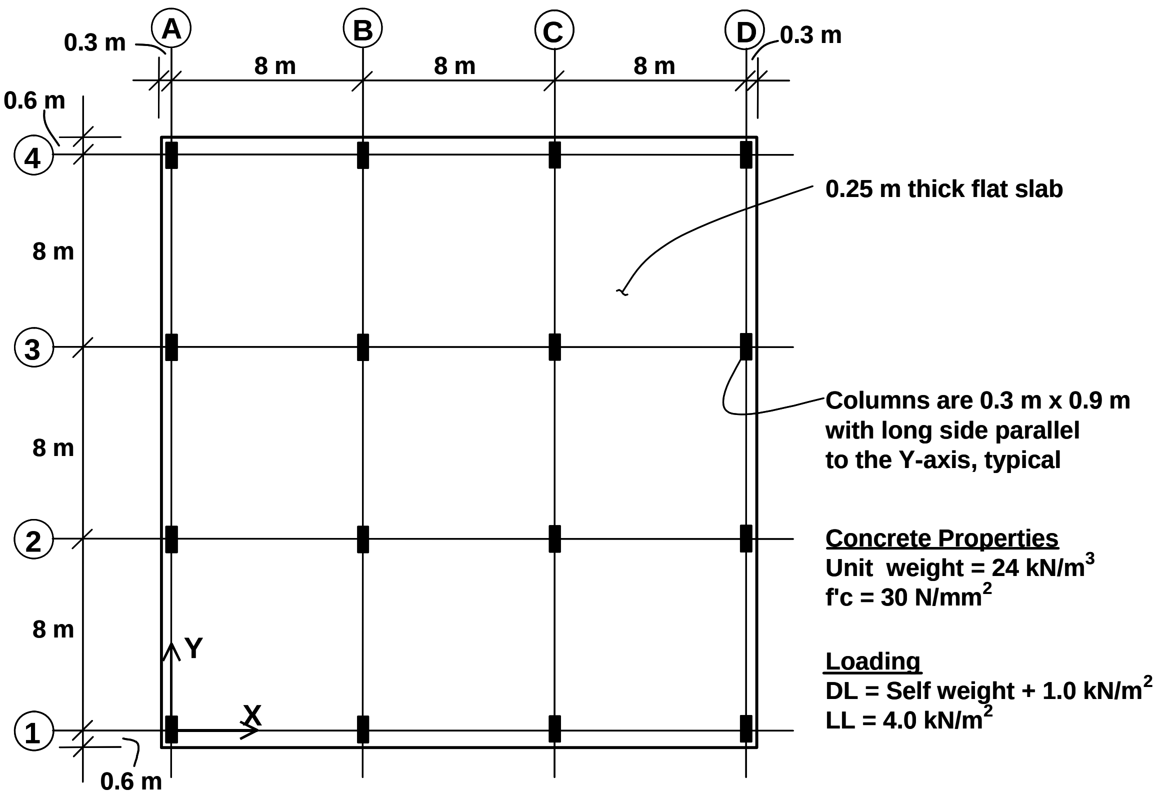


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. Thick shell properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a $f'c$ of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from ETABS with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

National Annex	Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
CEN Default, Norway, Slovenia and Sweden	ETABS	1.107	0.610	1.82
	Calculated	1.089	0.578	1.89
Finland, Singapore and UK	ETABS	1.107	0.612	1.81
	Calculated	1.089	0.5796	1.88
Denmark	ETABS	1.107	0.639	1.73
	Calculated	1.089	0.606	1.80

COMPUTER FILE: EN 2-2004 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Hand Calculation for Interior Column using ETABS Method

$d = [(250 - 26) + (250 - 38)]/2 = 218$ mm

Refer to Figure 2.

$u_1 = u = 2 \bullet 300 + 2 \bullet 900 + 2 \bullet \pi \bullet 436 = 5139.468$ mm

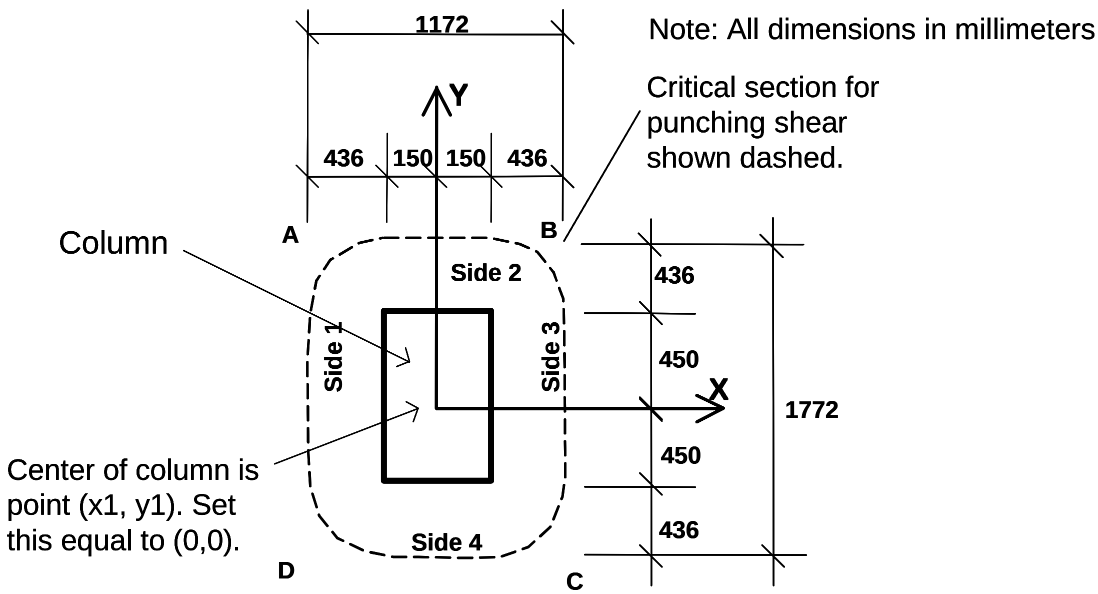


Figure 2: Interior Column, Grid B-2 in ETABS Model

From the ETABS output at Grid B-2:

$V_{Ed} = 1112.197$ kN

$k_2 M_{Ed2} = 38.933$ kN-m

$k_3 M_{Ed3} = 17.633$ kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{Ed} = \frac{V_{Ed}}{ud}\left[1 + \frac{k_2 M_{Ed,2} u_1}{V_{Ed} W_{1,2}} + \frac{k_3 M_{Ed,3} u_1}{V_{Ed} W_{1,3}}\right] \quad \text{(EC2 6.4.4(2))}$$

$$W_1 = \frac{c_1^2}{2} + c_1 c_2 + 4c_2 d + 16d^2 + 2\pi d c_1$$

$$W_{1,2} = \frac{900^2}{2} + 300 \bullet 900 + 4 \bullet 300 \bullet 218 + 16 \bullet 218^2 + 2\pi \bullet 218 \bullet 900$$

$$W_{1,2} = 2,929,744.957 \text{ mm}^2$$

$$W_{1,3} = 3\frac{900^2}{2} + 900 \bullet 300 + 4 \bullet 900 \bullet 218 + 16 \bullet 218^2 + 2\pi \bullet 218 \bullet 300$$

$$W_{1,2} = 2,271,104.319 \text{ mm}^2$$

$$v_{Ed} = \frac{V_{Ed}}{ud}\left[1 + \frac{k_2 M_{Ed,2} u_1}{V_{Ed} W_{1,2}} + \frac{k_3 M_{Ed,3} u_1}{V_{Ed} W_{1,3}}\right]$$

$$v_{Ed} = \frac{1112.197 \bullet 10^3}{5139.468 \bullet 218}\left[1 + \frac{38.933 \bullet 10^6 \bullet 5139.468}{1112.197 \bullet 10^3 \bullet 2929744.957} + \frac{17.633 \bullet 10^6 \bullet 5139.468}{1112.197 \bullet 10^3 \bullet 2271104.319}\right]$$

$$v_{Ed} = 1.089 \text{ N/mm}^2$$

Thus v_{max} = **1.089 N/mm²**

For CEN Default, Finland, Norway, Singapore, Slovenia, Sweden and UK:

$$C_{Rd,c} = 0.18/\gamma_c = 0.18/1.5 = 0.12 \quad \text{(EC2 6.4.4)}$$

For Denmark:

$$C_{Rd,c} = 0.18/\gamma_c = 0.18/1.45 = 0.124 \quad \text{(EC2 6.4.4)}$$

The shear stress carried by the concrete, $V_{Rd,c}$, is calculated as:

$$V_{Rd,c} = \left[C_{Rd,c} k \left(100 \rho_1 f_{ck}\right)^{1/3} + k_1 \sigma_{cp}\right] \quad \text{(EC2 6.4.4)}$$

with a minimum of:

$$v_{Rd,c} = \left(v_{min} + k_1 \sigma_{cp}\right) \quad \text{(EC2 6.4.4)}$$

$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 = 1.9578$ (EC2 6.4.4(1))

k_1 = 0.15. (EC2 6.2.2(1))

$\rho_1 = \frac{A_{s1}}{b_w d} \leq 0.02$

Area of reinforcement at the face of column for design strip are as follows:

For CEN Default, Norway, Slovenia and Sweden:

A_s in Strip Layer A = 9204.985 mm^2

A_s in Strip Layer B = 8078.337 mm^2

Average A_s = $(9204.985 + 8078.337)/2$ = 8641.661 mm^2

ρ_1 = $8641.661/(8000 \bullet 218)$ = 0.004955 ≤ 0.02

For Finland, Singapore and UK:

A_s in Strip Layer A = 9319.248 mm^2

A_s in Strip Layer B = 8174.104 mm^2

Average A_s = $(9319.248 + 8174.104)/2$ = 8746.676 mm^2

ρ_1 = $8746.676/(8000 \bullet 218)$ = 0.005015 ≤ 0.02

For Denmark:

A_s in Strip Layer A = 9606.651 mm^2

A_s in Strip Layer B = 8434.444 mm^2

Average A_s = $(9606.651 + 8434.444)/2$ = 9020.548 mm^2

ρ_1 = $9020.548/(8000 \bullet 218)$ = 0.005172 ≤ 0.02

For CEN Default, Denmark, Norway, Singapore, Slovenia, Sweden and UK:

$$v_{\min} = 0.035k^{3/2} f_{ck}^{\ 1/2} = 0.035\left(1.9578\right)^{3/2}\left(30\right)^{1/2} = 0.525 \text{ N/mm}^2$$

For Finland:

$$v_{\min} = 0.035k^{2/3} f_{ck}^{\ 1/2} = 0.035\left(1.9578\right)^{2/3}\left(30\right)^{1/2} = 0.3000 \text{ N/mm}^2$$

For CEN Default, Norway, Slovenia and Sweden:

$$v_{Rd,c} = \left[0.12 \bullet 1.9578\left(100 \bullet 0.004955 \bullet 30\right)^{1/3} + 0\right] = 0.5777 \text{ N/mm}^2$$

For Finland, Singapore, and UK:

$$v_{Rd,c} = \left[0.12 \bullet 1.9578\left(100 \bullet 0.005015 \bullet 30\right)^{1/3} + 0\right] = 0.5796 \text{ N/mm}^2$$

For Denmark:

$$v_{Rd,c} = \left[0.124 \bullet 1.9578\left(100 \bullet 0.005015 \bullet 30\right)^{1/3} + 0\right] = 0.606 \text{ N/mm}^2$$

For CEN Default, Norway, Slovenia and Sweden:

$$\text{Shear Ratio} = \frac{v_{\max}}{v_{Rd,c}} = \frac{1.089}{0.5777} = 1.89$$

For Finland, Singapore and UK:

$$\text{Shear Ratio} = \frac{v_{\max}}{v_{Rd,c}} = \frac{1.089}{0.5796} = 1.88$$

For Denmark:

$$\text{Shear Ratio} = \frac{v_{\max}}{v_{Rd,c}} = \frac{1.089}{0.606} = 1.80$$

EN 2-2004 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

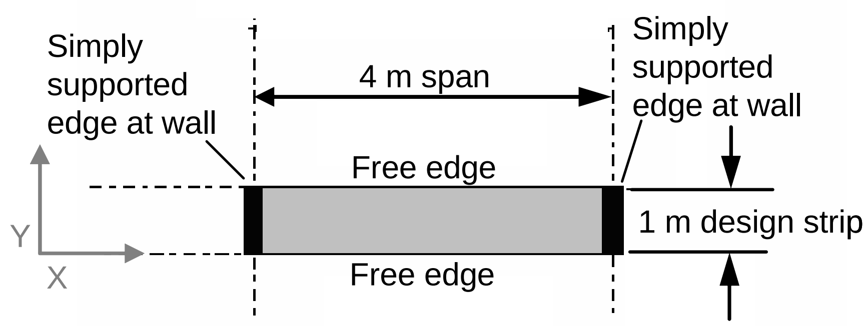


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the Eurocode 2-04 load combination factors, 1.35 for dead loads and 1.5 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. These moments are identical. After completing the analysis, design is performed using the Eurocode 2-04 code by ETABS and also by hand computation. Table 1 shows the comparison of the design reinforcements computed by the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_{ck}	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	$2x10^6$	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

National Annex	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
CEN Default, Norway, Slovenia and Sweden	ETABS	25.797	5.400
	Calculated	25.800	5.400
Finland , Singapore and UK	ETABS	25.797	5.446
	Calculated	25.800	5.446
Denmark	ETABS	25.797	5.626
	Calculated	25.800	5.626

$A^+_{s,min}$ = 204.642 sq-mm

COMPUTER FILE: EN 2-2004 RC-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$$\eta = 1.0 \text{ for } f_{ck} \le 50 \text{ MPa}$$

$$\lambda = 0.8 \text{ for } f_{ck} \le 50 \text{ MPa}$$

$$b = 1000 \text{ mm}$$

For the load combination, w and M are calculated as follows:

$$w = (1.35 w_d + 1.5 w_t)\, b$$

$$M = \frac{w l_1^2}{8}$$

$$A_{s,\min} = \max \begin{cases} 0.0013 b_w d \\ 0.26 \dfrac{f_{ctm}}{f_{yk}} bd \end{cases}$$

$$= 204.642 \text{ sq-mm}$$

COMB100

$$w_d = 4.0 \text{ kPa}$$

$$w_t = 5.0 \text{ kPa}$$

$$w = 12.9 \text{ kN/m}$$

$$M_{\text{-strip}} = 25.8 \text{ kN-m}$$

$$M_{\text{-design}} = 25.8347 \text{ kN-m}$$

For CEN Default, Norway, Slovenia and Sweden:

$$\gamma_{m,\, steel} = 1.15$$

$$\gamma_{m,\, concrete} = 1.50$$

$$\alpha_{cc} = 1.0$$

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta f_{cd}} = \frac{25.8347 \bullet 10^6}{1000 \bullet 125^2 \bullet 1.0 \bullet 1.0 \bullet 30/1.5} = 0.08267$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

$k_1 = 0.44$

$k_2 = k_4 = 1.25(0.6 + 0.0014/\varepsilon_{cu2}) = 1.25$

δ is assumed to be 1

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa} = 0.448$$

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.294$$

$$\omega = 1 - \sqrt{1 - 2m} = 0.08640$$

$$A_s = \omega\left(\frac{\eta f_{cd} bd}{f_{yd}}\right) = 540.024 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.400 sq-cm

For Singapore and UK:

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$

$\alpha_{cc} = 0.85$:

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta f_{cd}} = \frac{25.8347 \bullet 10^6}{1000 \bullet 125^2 \bullet 1.0 \bullet 0.85 \bullet 30/1.5} = 0.097260$$

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.48$$

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa} = 0.60$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

$k_1 = 0.40$

$k_2 = (0.6 + 0.0014/\varepsilon_{cu2}) = 1.00$

δ is assumed to be 1

$$\omega = 1 - \sqrt{1-2m} = 0.10251$$

$$A_s = \omega\left(\frac{\eta f_{cd} bd}{f_{yd}}\right) = 544.61 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.446 \text{ sq-cm}$$

For Finland:

$\gamma_{m,\, steel} = 1.15$

$\gamma_{m,\, concrete} = 1.50$

$\alpha_{cc} = 0.85$:

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta f_{cd}} = \frac{25.8347 \bullet 10^6}{1000 \bullet 125^2 \bullet 1.0 \bullet 0.85 \bullet 30/1.5} = 0.097260$$

$$m_{\lim} = \lambda\left(\frac{x}{d}\right)_{\lim}\left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 032433$$

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa} = 0.5091$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

$k_1 = 0.44$

$k_2 = 1.1$

$k_4 = 1.25(0.6 + 0.0014/\varepsilon_{cu2}) = 1.25$

δ is assumed to be 1

$$\omega = 1 - \sqrt{1-2m} = 0.10251$$

$$A_s = \omega\left(\frac{\eta f_{cd} bd}{f_{yd}}\right) = 544.61 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.446 \text{ sq-cm}$$

For Denmark:

$\gamma_{m,\ steel}$ = 1.20

$\gamma_{m,\ concrete}$ = 1.45

α_{cc} = 1.0

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta f_{cd}} = \frac{25.8347 \bullet 10^6}{1000 \bullet 125^2 \bullet 1.0 \bullet 1.0 \bullet 30/1.5} = 0.0799153$$

$$m_{\lim} = \lambda \left(\frac{x}{d}\right)_{\lim} \left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.294$$

$$\left(\frac{x}{d}\right)_{\lim} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa} = 0.448$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

$k_1 = 0.44$

$k_2 = k_4 = 1.25(0.6 + 0.0014/\varepsilon_{cu2}) = 1.25$

δ is assumed to be 1

$$\omega = 1 - \sqrt{1 - 2m} = 0.08339$$

$$A_s = \omega \left(\frac{\eta f_{cd} bd}{f_{yd}}\right) = 562.62 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.626 sq-cm

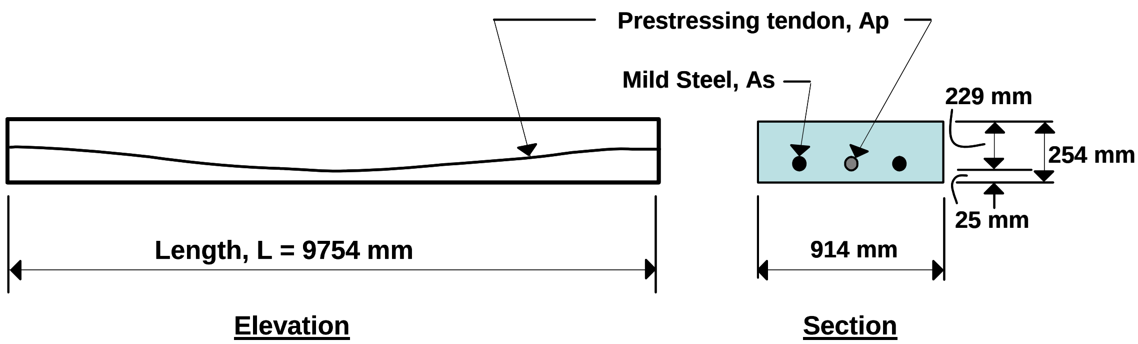
HK CP-2004 PT-SL EXAMPLE 001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.



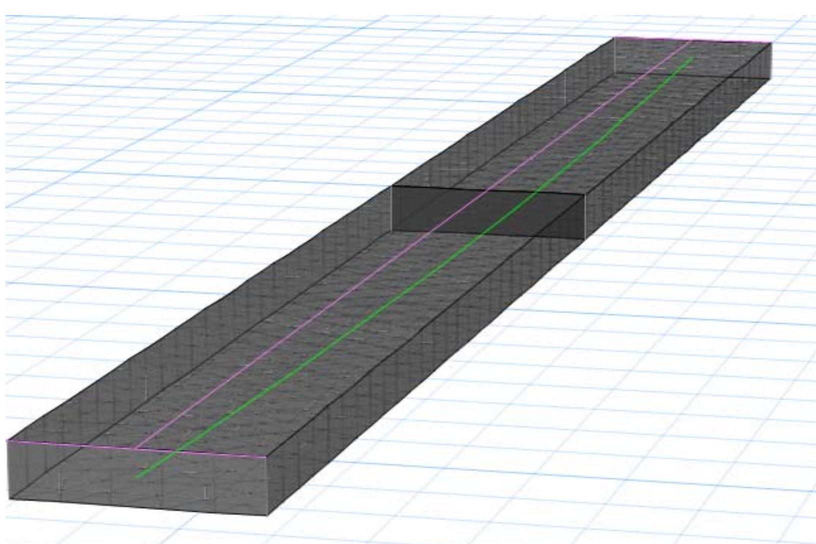


Figure 1 One-Way Slab

To ensure one-way action Poisson's ratio is taken to be zero. A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, was added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement and slab stresses are reported at the midspan of the slab. Independent hand calculations were compared with the ETABS results and summarized for verification and validation of the ETABS results.

Geometry, Properties and Loading

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	KN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	KN/m^2
Live load	w_l	=	4.788	KN/m^2

Technical Features of ETABS Tested

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

Results Comparison

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing, and slab stresses with independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (KN-m)	174.4	174.4	0.00%
Area of Mild Steel req'd, As (cm^2)	19.65	19.80	0.41%
Transfer Conc. Stress, top ($D+PT_I$), MPa	–5.056	–5.057	0.02%
Transfer Conc. Stress, bot ($D+PT_I$), MPa	2.836	2.839	0.11%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	–10.547	–10.467	-0.76%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	8.323	8.409	1.03%

COMPUTER FILE: HK CP-2004 PT-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

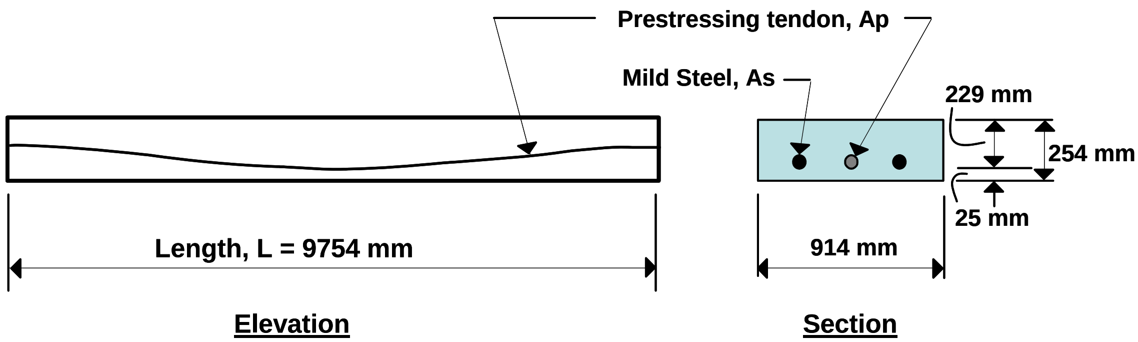
HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
fc = 30 MPa	f_{pu} = 1862 MPa
fy = 400 MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.4 = 8.378 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) × 1.6 = 7.661 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 16.039 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 16.039 kN/m^2 × 0.914 m = 14.659 kN/m

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8} = 14.659 \times (9.754)^2 / 8$ = 174.4 kN-m

Ultimate Stress in strand, $f_{pb} = f_{pe} + \frac{7000}{l/d}\left(1 - 1.7\frac{f_{pu}A_p}{f_{cu}bd}\right)$

$$= 1210 + \frac{7000}{9.754/0.229}\left(1 - 1.7\frac{1862(198)}{30(914)(229)}\right)$$

$$= 1358 \text{ MPa} \le 0.7 f_{pu} = 1303 \text{ MPa}$$

K factor used to determine the effective depth is given as:

$$K = \frac{M}{f_{cu}bd^2} = \frac{174.4}{30000(0.914)(0.229)^2} = 0.1213 < 0.156$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d = 192.2 \text{ mm}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197.4(1303)/1000 = 257.2$ KN

Ultimate moment due to PT, $M_{ult,PT} = F_{ult,PT}(z)/\gamma = 257.2(0.192)/1.15 = 43.00$ KN-m

Net Moment to be resisted by As, $M_{NET} = M_U - M_{PT}$

$$= 174.4 - 43.00 = 131.40 \text{ kN-m}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M}{0.87 f_y z} = \frac{131.40}{0.87(400)(192)}(1e6) = 1965 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination ($D+PT_i$) = $1.0D+0.0L+1.0PT_I$

Tendon stress at transfer = jacking stress − stressing losses = 1490 − 186 = 1304 MPa

The force in the tendon at transfer, = $1304(2)(99)/1000 = 258.2$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 258.2(101.6 \text{ mm})/1000 = 26.23$ kN-m

Stress in concrete, $f = \frac{F_{PTI}}{A} \pm \frac{M_D}{S} \pm \frac{M_{PT}}{S} = \frac{-258.2}{0.254(0.914)} \pm \frac{65.04}{0.00983} \pm \frac{26.23}{0.00983}$

where S = 0.00983 m^3

$$f = -1.112 \pm 6.6166 \pm 2.668 \text{ MPa}$$

$$f = -5.060(\text{Comp})\max,\ 2.836(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT$_F$) = 1.0D+1.0L+1.0PT$_F$

Tendon stress at normal = jacking − stressing − long-term = 1490 − 186 − 94 = 1210 MPa
The force in tendon at normal, = $1210(2)(99)/1000 = 239.5$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 239.5(101.6 \text{ mm})/1000 = 24.33$ kN-m

Stress in concrete for (D+L+PT$_F$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_D}{S} \pm \frac{M_{PT}}{S} = \frac{-258.2}{0.254(0.914)} \pm \frac{117.08}{0.00983} \pm \frac{24.33}{0.00983}$$
$$f = -1.112 \pm 11.910 \pm 2.475$$
$$f = -10.547(\text{Comp})\max, 8.323(\text{Tension})\max$$

HK CP-2004 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

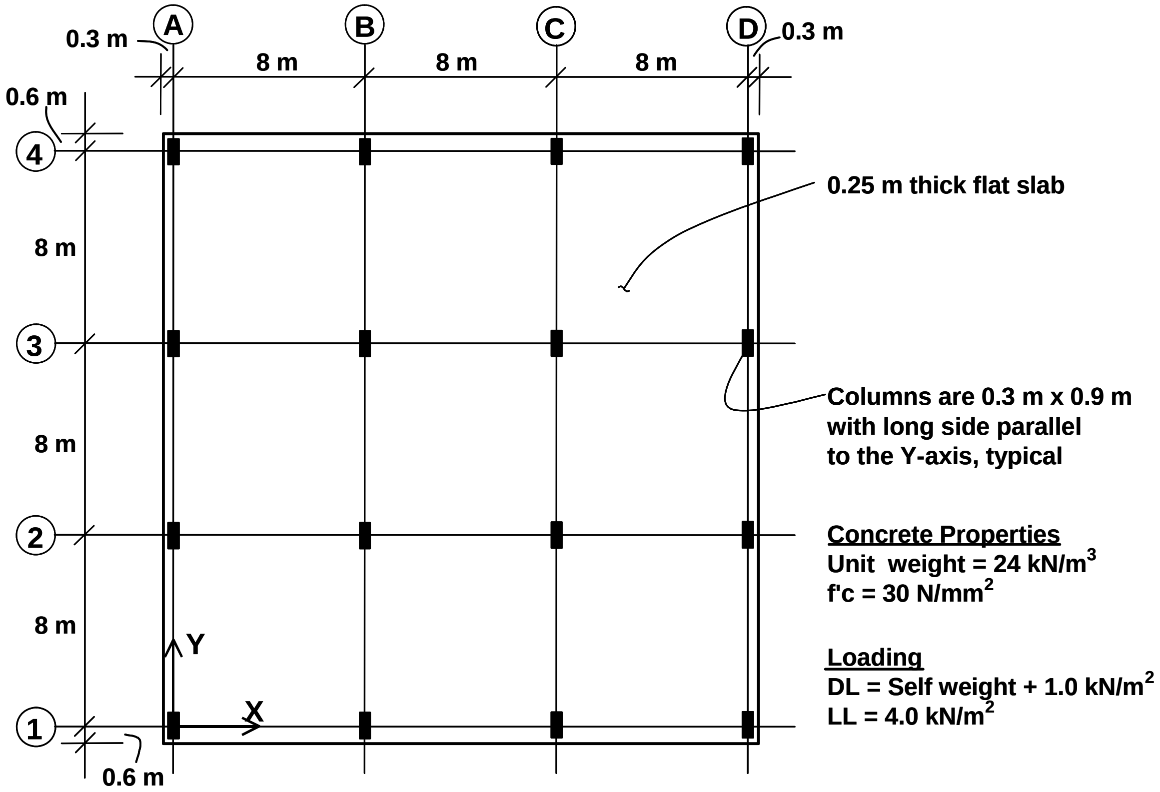


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f_{cu} of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from ETABS with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
ETABS	1.116	0.662	1.69
Calculated	1.105	0.625	1.77

COMPUTER FILE: HK CP-2004 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using ETABS Method

$d = [(250 - 26) + (250 - 38)]/2$ = 218 mm

Refer to Figure 1.

u = 954+ 1554 + 954 + 1554 = 5016 mm

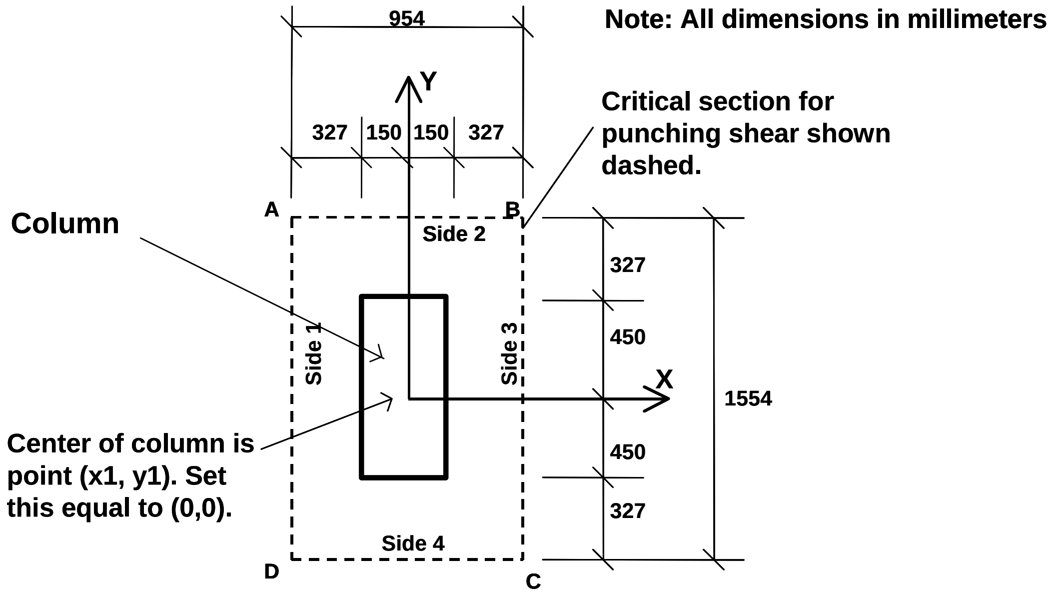


Figure 2: Interior Column, Grid B-2 in ETABS Model

From the ETABS output at Grid B-2:

V= 1126.498 kN

M_2 = 51.9908 kN-m

M_3 = 45.7234 kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{eff,x} = \frac{V}{ud}\left(f + \frac{1.5M_x}{Vy} \right)$$

$$v_{eff,x} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 51.9908 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 954} \right) = 1.1049 \text{ (Govern)}$$

$$v_{eff,y} = \frac{V}{ud}\left(f + \frac{1.5M_y}{Vx} \right)$$

$$v_{eff,y} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 45.7234 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 1554} \right) = 1.0705$$

The largest absolute value of *v* = **1.1049 N/mm²**

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} = 0.3568 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3} = \left(\frac{30}{25}\right)^{1/3} = 1.0627 > 1.0 \text{ OK}$$

$\gamma_m = 1.25$

$$\left(\frac{400}{d}\right)^{1/4} = 1.16386 > 1 \text{ OK.}$$

$f_{cu} \leq 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

Area of reinforcement at the face of column for design strip are as follows:

A_s in Strip Layer A = 9494.296 mm^2

A_s in Strip Layer B = 8314.486 mm^2

$$\text{Average } A_s = (9494.296 + 8314.486)/2 = 8904.391 \text{ mm}^2$$

$$\frac{100\ A_s}{bd} = 100 \bullet 8904.391/(8000 \bullet 218) = 0.51057$$

$$v_c = \frac{0.79 \bullet 1.0 \bullet 1.0627}{1.25} \bullet (0.51057)^{1/3} \bullet 1.16386 = 0.6247 \text{ MPa}$$

BS 3.7.7.3 yields the value of v = **0.625 N/mm²**, and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{v} = \frac{1.1049}{0.6247} = 1.77$$

HK CP-2004 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

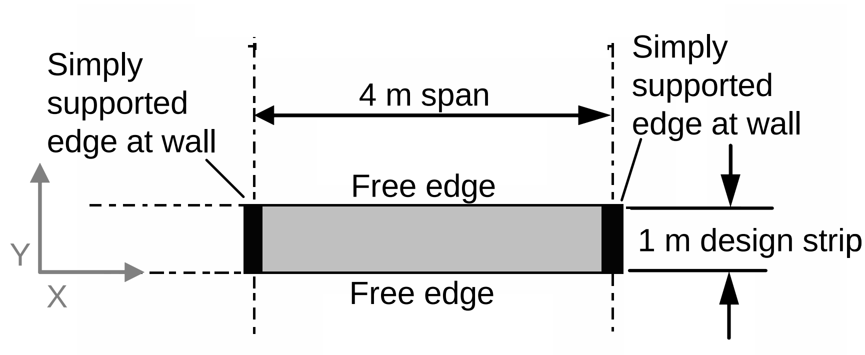


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the Hong Kong CP-04 load combination factors, 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing analysis, design is performed using the Hong Kong CP-04 code by ETABS and also by hand computation. Table 1 shows the comparison of the design reinforcements computed using the two methods.

PROGRAM NAME: ETABS
REVISION NO.: 0

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2×10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	ETABS	27.197	5.853
	Calculated	27.200	5.842

$A^{+}_{s,\min}$ = 162.5 sq-mm

COMPUTER FILE: HK CP-2004 RC-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\gamma_{m,\,steel}$ = 1.15

$\gamma_{m,\,concrete}$ = 1.50

b = 1000 mm

For the load combination, the w and M are calculated as follows:

$$w = (1.4w_d + 1.6w_t)\,b$$

$$M = \frac{wl_1^2}{8}$$

$$A_{s,\min} = 0.0013 b_w d$$

$$= 162.5 \text{ sq-mm}$$

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 13.6 kN/m

$M_{\text{-}strip}$ = 27.2 kN-m

$M_{\text{-}design}$ = 27.2366 kN-m

The depth of the compression block is given by:

$$K = \frac{M}{f_{cu} b d^2} = 0.05810 < 0.156$$

The area of tensile steel reinforcement is then given by:

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d = 116.3283$$

$$A_s = \frac{M}{0.87 f_y z} = 585.046 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.850 \text{ sq-cm}$$

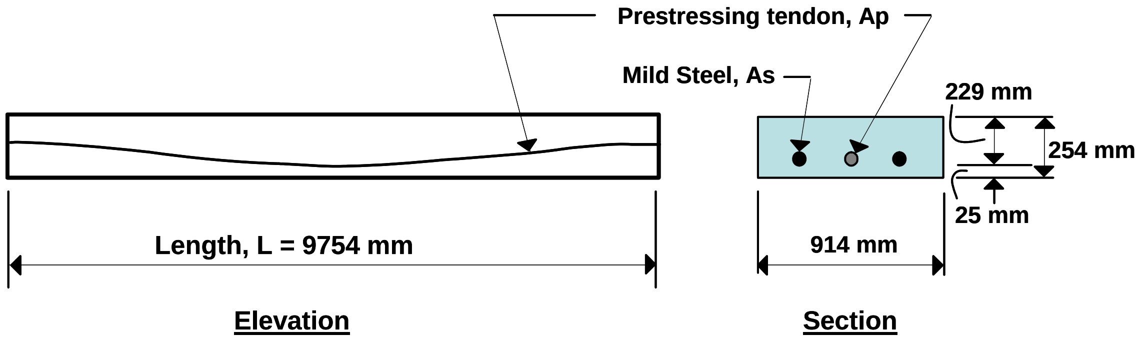
HK CP-2013 PT-SL EXAMPLE 001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.



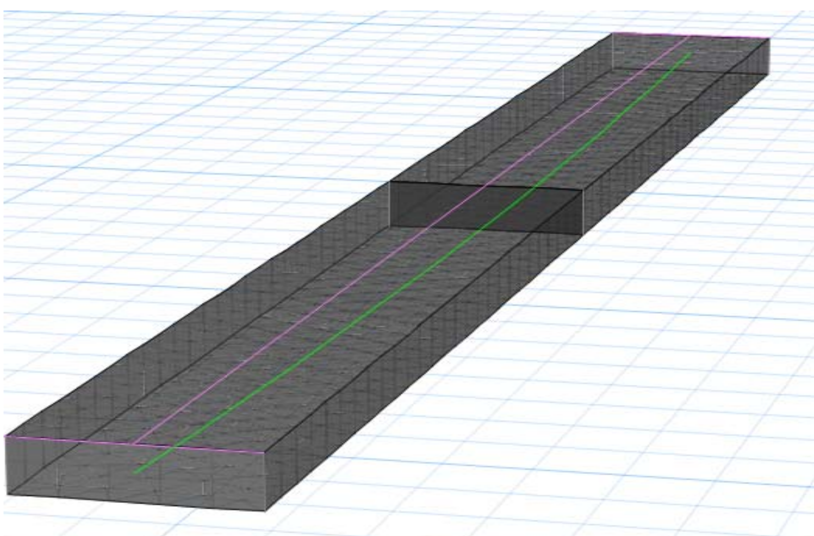


Figure 1 One-Way Slab

To ensure one-way action Poisson's ratio is taken to be zero. A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, was added to the A-Strip. The self weight and live loads were added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement and slab stresses are reported at the midspan of the slab. Independent hand calculations were compared with the ETABS results and summarized for verification and validation of the ETABS results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	KN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	KN/m^2
Live load	w_l	=	4.788	KN/m^2

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing, and slab stresses with independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (KN-m)	174.4	174.4	0.00%
Area of Mild Steel req'd, As (cm^2)	19.65	19.80	0.41%
Transfer Conc. Stress, top ($D+PT_I$), MPa	–5.056	–5.057	0.02%
Transfer Conc. Stress, bot ($D+PT_I$), MPa	2.836	2.839	0.11%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	–10.547	–10.467	-0.76%
Normal Conc. Stress, top ($D+L+PT_F$), MPa	8.323	8.409	1.03%

COMPUTER FILE: HK CP-2013 PT-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

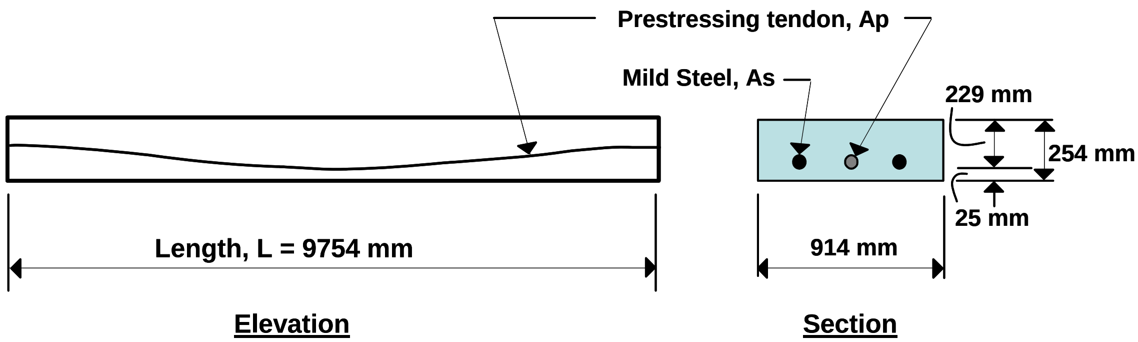
HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
fc = 30 MPa	f_{pu} = 1862 MPa
fy = 400 MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.4 = 8.378 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) × 1.6 = 7.661 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 16.039 kN/m^2 (D+L)ult

ω = 10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 16.039 kN/m^2 × 0.914 m = 14.659 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8} = 14.659 \times (9.754)^2 / 8$ = 174.4 kN-m

Ultimate Stress in strand, $f_{pb} = f_{pe} + \frac{7000}{l/d}\left(1 - 1.7\frac{f_{pu}A_p}{f_{cu}bd}\right)$

$$= 1210 + \frac{7000}{9.754/0.229}\left(1 - 1.7\frac{1862(198)}{30(914)(229)}\right)$$

$$= 1358 \text{ MPa} \le 0.7 f_{pu} = 1303 \text{ MPa}$$

K factor used to determine the effective depth is given as:

$$K = \frac{M}{f_{cu}bd^2} = \frac{174.4}{30000(0.914)(0.229)^2} = 0.1213 < 0.156$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d = 192.2 \text{ mm}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197.4(1303)/1000 = 257.2 \text{ KN}$

Ultimate moment due to PT, $M_{ult,PT} = F_{ult,PT}(z)/\gamma = 257.2(0.192)/1.15 = 43.00 \text{ KN-m}$

Net Moment to be resisted by As, $M_{NET} = M_U - M_{PT}$

$$= 174.4 - 43.00 = 131.40 \text{ kN-m}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M}{0.87 f_y z} = \frac{131.40}{0.87(400)(192)}(1e6) = 1965 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (D+PT_i) = 1.0D+0.0L+1.0PT_I

Tendon stress at transfer = jacking stress − stressing losses = 1490 − 186 = 1304 MPa

The force in the tendon at transfer, = $1304(2)(99)/1000 = 258.2$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04 \text{ kN-m}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 258.2(101.6 \text{ mm})/1000 = 26.23 \text{ kN-m}$

Stress in concrete, $f = \frac{F_{PTI}}{A} \pm \frac{M_D}{S} \pm \frac{M_{PT}}{S} = \frac{-258.2}{0.254(0.914)} \pm \frac{65.04}{0.00983} \pm \frac{26.23}{0.00983}$

where S = 0.00983 m^3

$$f = -1.112 \pm 6.6166 \pm 2.668 \text{ MPa}$$

$$f = -5.060(\text{Comp})\max,\ 2.836(\text{Tension})\max$$

Normal Condition, load combinations: ($D+L+PT_F$) = $1.0D+1.0L+1.0PT_F$

Tendon stress at normal = jacking − stressing − long-term = 1490 − 186 − 94 = 1210 MPa
The force in tendon at normal, = $1210(2)(99)/1000 = 239.5$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 239.5(101.6\text{ mm})/1000 = 24.33$ kN-m

Stress in concrete for ($D+L+PT_F$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_D}{S} \pm \frac{M_{PT}}{S} = \frac{-258.2}{0.254(0.914)} \pm \frac{117.08}{0.00983} \pm \frac{24.33}{0.00983}$$

$$f = -1.112 \pm 11.910 \pm 2.475$$

$$f = -10.547(\text{Comp})\max, 8.323(\text{Tension})\max$$

HK CP-2013 RC-PN EXAMPLE 001

Slab Punching Shear Design

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The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

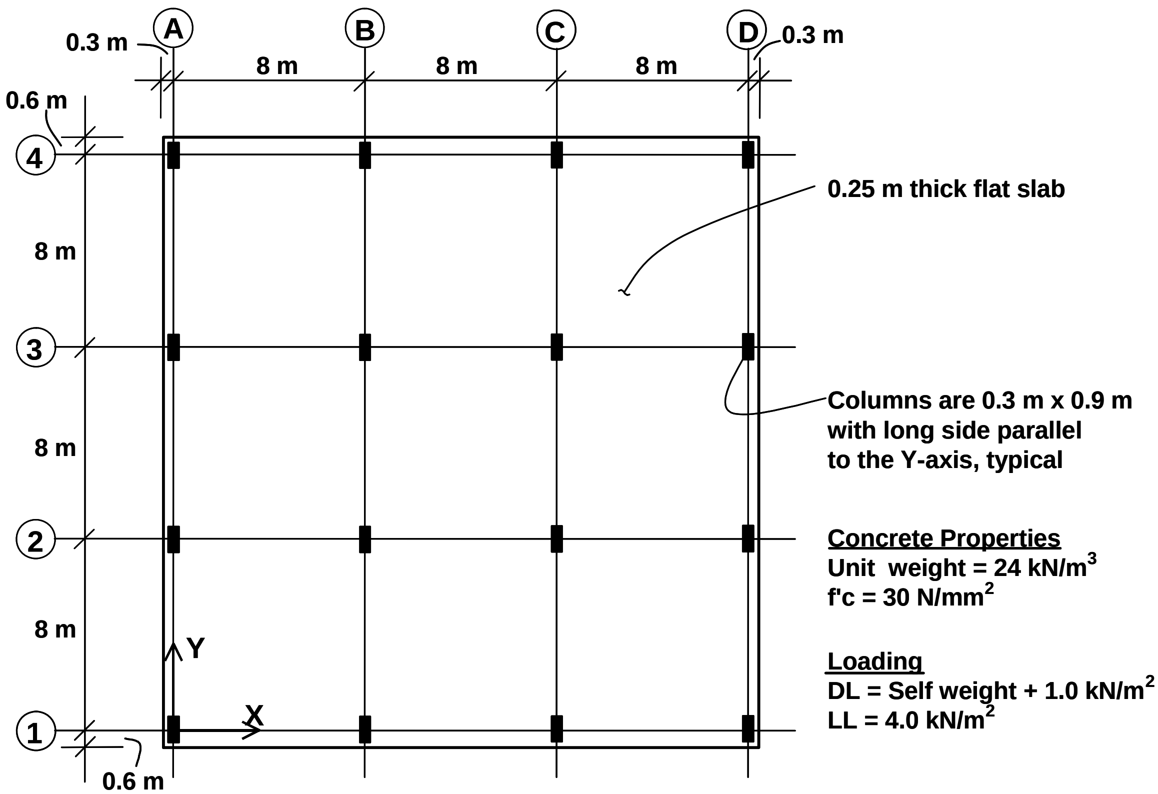


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f_{cu} of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from ETABS with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
ETABS	1.116	0.662	1.69
Calculated	1.105	0.625	1.77

COMPUTER FILE: HK CP-2013 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using ETABS Method

$d = [(250-26)+(250-38)]/2$ = 218 mm

Refer to Figure 1.

u = 954+ 1554 + 954 + 1554 = 5016 mm

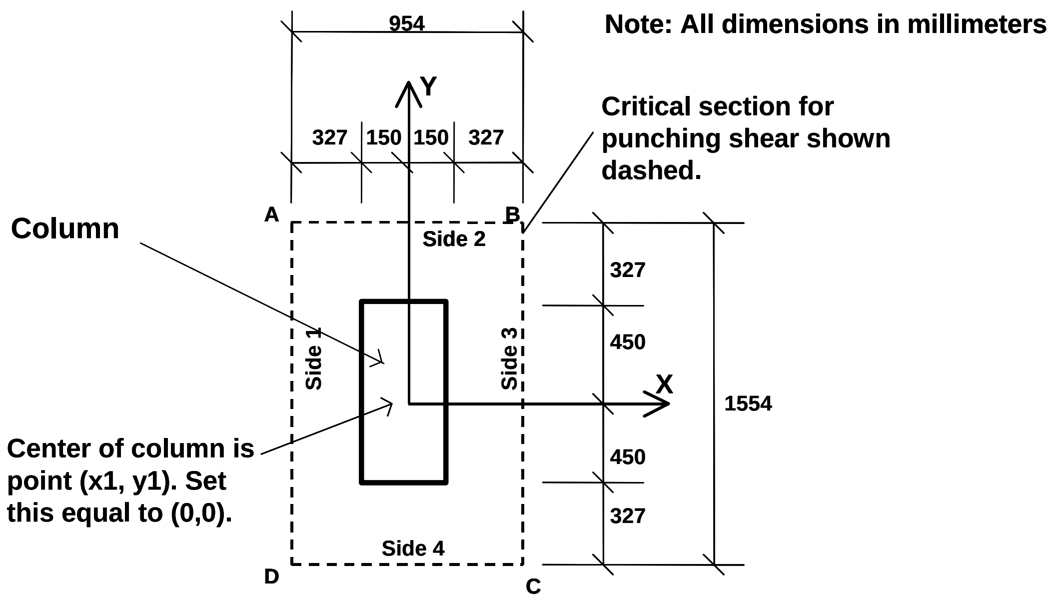


Figure 2: Interior Column, Grid B-2 in ETABS Model

From the ETABS output at Grid B-2:

V= 1126.498 kN

M_2 = 51.9908 kN-m

M_3 = 45.7234 kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{eff,x} = \frac{V}{ud}\left(f + \frac{1.5M_x}{Vy} \right)$$

$$v_{eff,x} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 51.9908 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 954} \right) = 1.1049 \text{ (Govern)}$$

$$v_{eff,y} = \frac{V}{ud}\left(f + \frac{1.5M_y}{Vx} \right)$$

$$v_{eff,y} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 45.7234 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 1554} \right) = 1.0705$$

The largest absolute value of v = **1.1049 N/mm²**

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.79 k_1 k_2}{\gamma_m}\left(\frac{100 A_s}{bd} \right)^{1/3}\left(\frac{400}{d} \right)^{1/4} = 0.3568 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{25} \right)^{1/3} = \left(\frac{30}{25} \right)^{1/3} = 1.0627 > 1.0 \text{ OK}$$

$\gamma_m = 1.25$

$$\left(\frac{400}{d} \right)^{1/4} = 1.16386 > 1 \text{ OK.}$$

$f_{cu} \leq 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

Area of reinforcement at the face of column for design strip are as follows:

A_s in Strip Layer A = 9494.296 mm^2

A_s in Strip Layer B = 8314.486 mm^2

$$\text{Average } A_s = (9494.296 + 8314.486)/2 = 8904.391 \text{ mm}^2$$

$$\frac{100\ A_s}{bd} = 100 \bullet 8904.391/(8000 \bullet 218) = 0.51057$$

$$v_c = \frac{0.79 \bullet 1.0 \bullet 1.0627}{1.25} \bullet (0.51057)^{1/3} \bullet 1.16386 = 0.6247 \text{ MPa}$$

BS 3.7.7.3 yields the value of v = **0.625 N/mm²**, and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{v} = \frac{1.1049}{0.6247} = 1.77$$

HK CP-2013 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

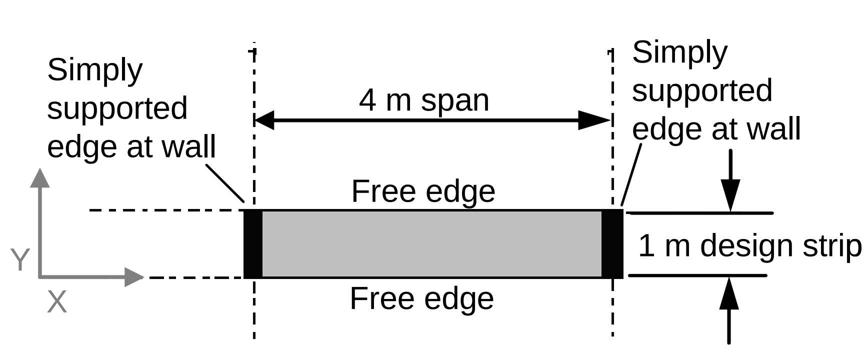


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the Hong Kong CP-04 load combination factors, 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing analysis, design is performed using the Hong Kong CP-04 code by ETABS and also by hand computation. Table 1 shows the comparison of the design reinforcements computed using the two methods.

Geometry, Properties and Loading

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2×10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

Technical Features of ETABS Tested

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

Results Comparison

Table 1 shows the comparison of the ETABS total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	ETABS	27.197	5.853
	Calculated	27.200	5.842

$A^+_{s,min}$ = 162.5 sq-mm

COMPUTER FILE: HK CP-2013 RC-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\gamma_{m,\ steel}$ = 1.15

$\gamma_{m,\ concrete}$ = 1.50

b = 1000 mm

For the load combination, the w and M are calculated as follows:

$$w = (1.4w_d + 1.6w_t)\ b$$

$$M = \frac{wl_1^2}{8}$$

$$A_{s,\min} = 0.0013 b_w d$$

$$= 162.5 \text{ sq-mm}$$

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 13.6 kN/m

$M_{\text{-}strip}$ = 27.2 kN-m

$M_{\text{-}design}$ = 27.2366 kN-m

The depth of the compression block is given by:

$$K = \frac{M}{f_{cu} b d^2} = 0.05810 < 0.156$$

The area of tensile steel reinforcement is then given by:

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d\ =116.3283$$

$$A_s = \frac{M}{0.87 f_y z} = 585.046 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.850 \text{ sq-cm}$$

IS 456-2000 PT-SL EXAMPLE 001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.

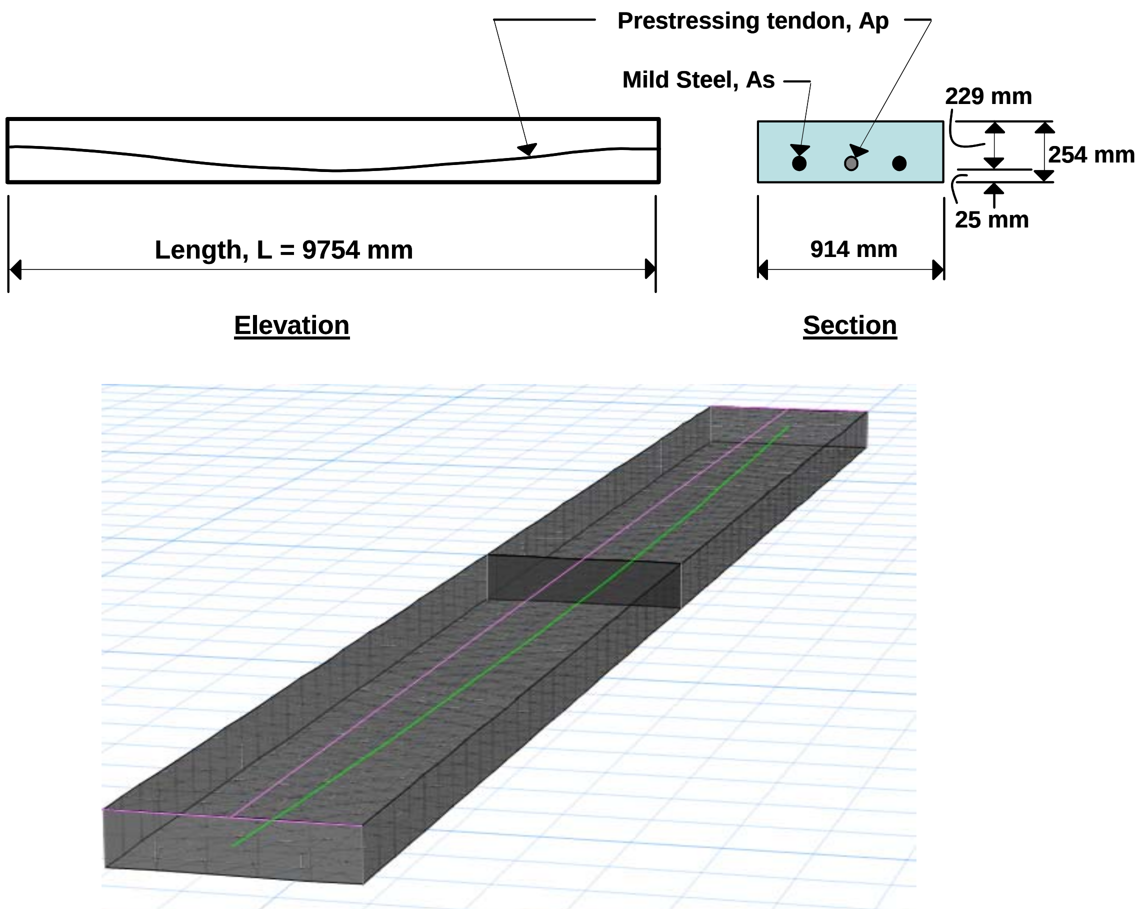


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads have been added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the ETABS results and summarized for verification and validation of the ETABS results.

Geometry, Properties and Loading

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	kN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	kN/m^2
Live load	w_l	=	4.788	kN/m^2

Technical Features of ETABS Tested

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads

Results Comparison

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	175.60	175.69	0.05%
Area of Mild Steel req'd, As (sq-cm)	19.53	19.775	1.25%
Transfer Conc. Stress, top $(D+PT_I)$, MPa	–5.058	–5.057	-0.02%
Transfer Conc. Stress, bot $(D+PT_I)$, MPa	2.839	2.839	0.00%
Normal Conc. Stress, top $(D+L+PT_F)$, MPa	–10.460	–10.467	0.07%
Normal Conc. Stress, bot $(D+L+PT_F)$, MPa	8.402	8.409	0.08%

COMPUTER FILE: IS 456-2000 PT-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
f_{ck} = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e =1210 MPa

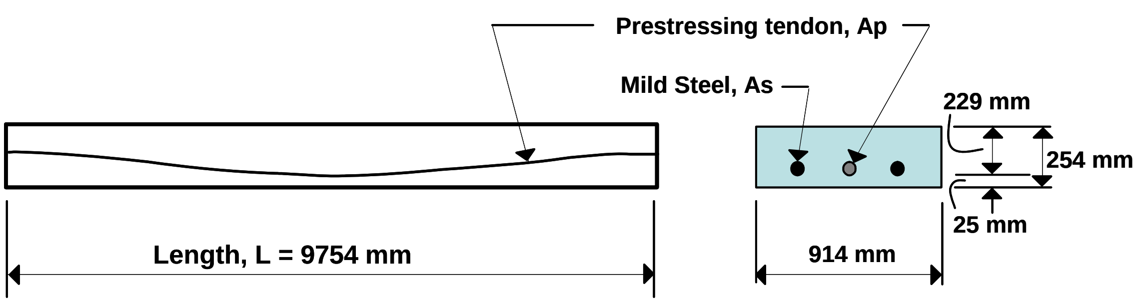
$\gamma_s = 1.15$

$\gamma_c = 1.50$

$\alpha = 0.36$

$\beta = 0.42 \qquad \frac{x_{max}}{d} = 0.53 - 0.05\frac{f_y - 250}{165} \quad \text{if} \quad 250 < f_y \leq 415 \text{ MPa}$

$$\frac{x_{u,max}}{d} = 0.484$$



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.50 = 8.976 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) × 1.50 = 7.182 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 16.158 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 16.158 kN/m^2 × 0.914 m = 14.768 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8} = 14.768 \times (9.754)^2 / 8$ = 175.6 kN-m

Ultimate Stress in strand, f_{PS} = from Table 11: f_p = 1435 MPa

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 197.4(1435)/1000 = 283.3$ kN

Compression block depth ratio: $m = \dfrac{M}{bd^2 \alpha f_{ck}}$

$$= \frac{175.6}{(0.914)(0.229)^2(0.36)(30000)} = 0.3392$$

Required area of mild steel reinforcing,

$$\frac{x_u}{d} = \frac{1-\sqrt{1-4\beta m}}{2\beta} = \frac{1-\sqrt{1-4(0.42)(0.3392)}}{2(0.42)} = 0.4094 \; > \; \frac{x_{u,\max}}{d} = 0.484$$

The area of tensile steel reinforcement is then given by:

$$z = d\left\{1-\beta \frac{x_u}{d}\right\} = 229\big(1-0.42(0.4094)\big) = 189.6 \text{ mm}$$

$$A_{NET} = \frac{M_u}{(f_y/\gamma_s)\,z} = \frac{175.6}{(400/1.15)189.6}(1e6) = 2663 \text{ mm}^2$$

$$A_s = A_{NET} - A_P\left(\frac{f_P}{f_y}\right) = 2663 - 198\left(\frac{1435}{400}\right) = 1953 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (D+PT$_i$) = 1.0D+0.0L+1.0PT$_I$

Tendon stress at transfer = jacking stress − stressing losses =1490 − 186 = 1304 MPa

The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-257.4}{0.254(0.914)} \pm \dfrac{65.04-26.23}{0.00983}$

where S=0.00983m^3

$$f = -1.109 \pm 3.948 \text{ MPa}$$

$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: $(D+L+PT_F) = 1.0D+1.0L+1.0PT_F$

Tendon stress at normal = jacking – stressing – long-term=1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, $= 1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for $(D+L+PT_F)$,

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$

$$f = -1.029 \pm 9.431$$

$$f = -10.460(\text{Comp})\max,\ 8.402(\text{Tension})\max$$

IS 456-2000 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

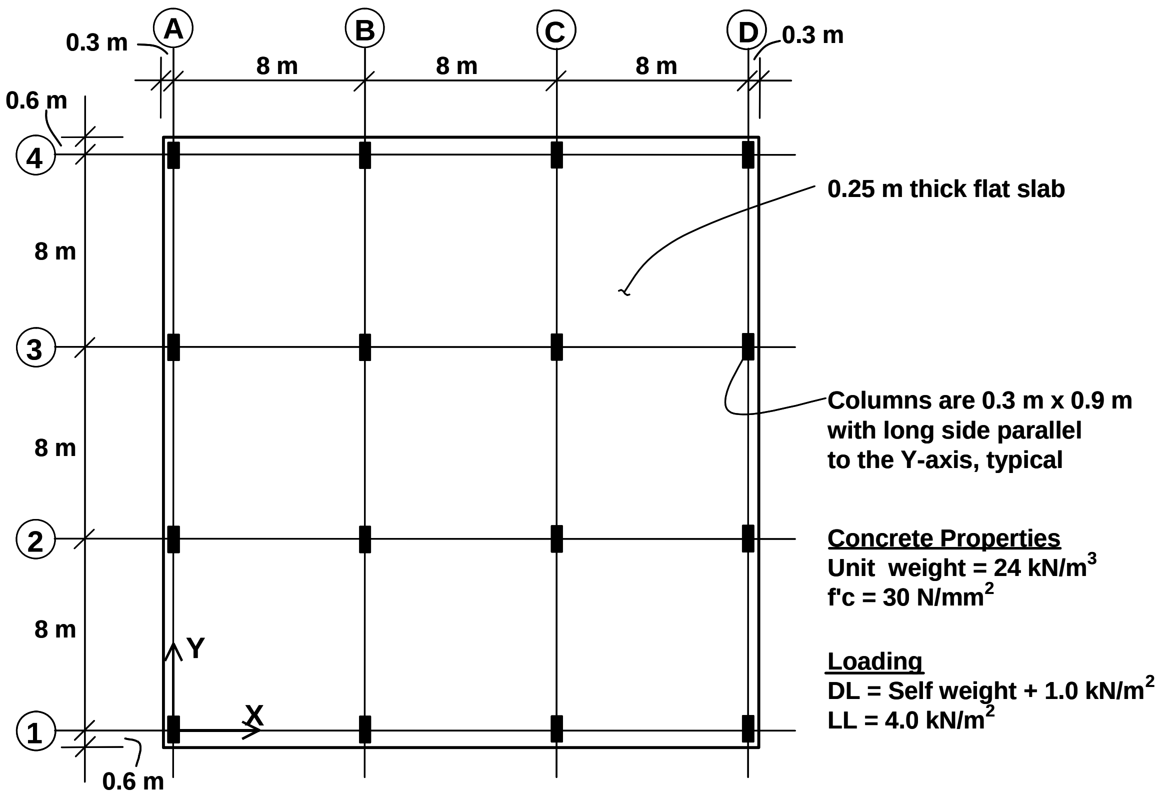


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a $f'c$ of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained in ETABS with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
ETABS	1.793	1.141	1.57
Calculated	1.792	1.141	1.57

COMPUTER FILE: IS 456-2000 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using ETABS Method

$d = [(250-26)+(250-38)]/2$ = 218 mm

Refer to Figure 1.

b_0 = 518+ 1118 + 1118 + 518 = 3272 mm

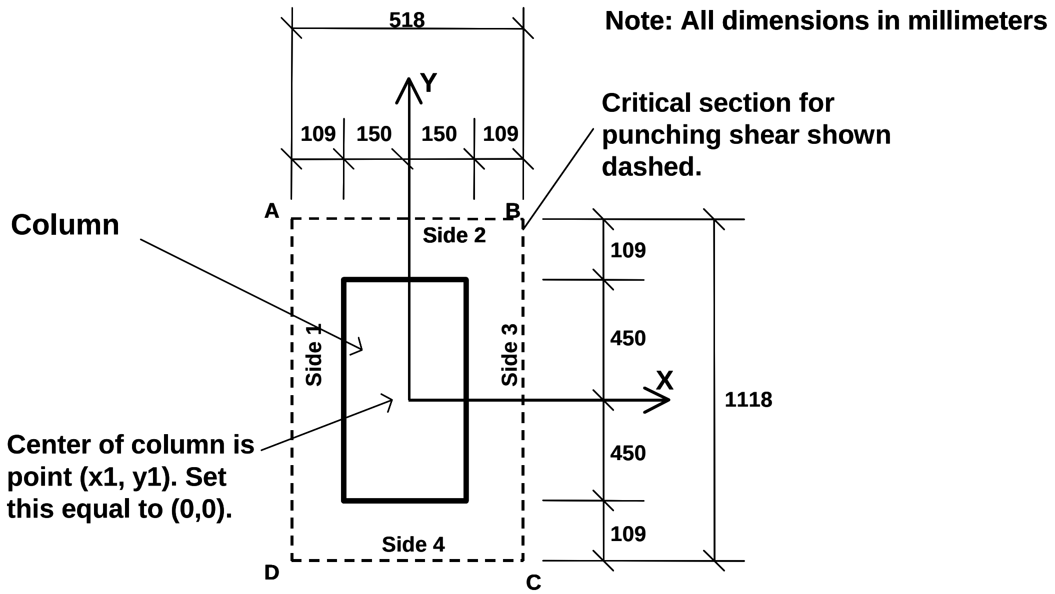


Figure 2: Interior Column, Grid B-2 in ETABS Model

$$\gamma_{V2} = 1 - \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{1118}{518}}} = 0.495$$

$$\gamma_{V3} = 1 - \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{518}{1118}}} = 0.312$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–259	0	259	0	N.A.
y_2	0	559	0	–559	N.A.
L	1118	518	1118	518	b_0 = 3272
d	218	218	218	218	N.A.
Ld	243724	112924	243724	112924	713296
Ldx_2	–63124516	0	63124516	0	0
Ldy_2	0	63124516	0	–63124516	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{713296} = 0\,mm$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{713296} = 0\,mm$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	1118	518	1118	518	N.A.
d	218	218	218	218	N.A.
$x_2 - x_3$	–259	0	259	0	N.A.
$y_2 - y_3$	0	559	0	–559	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	2.64E+10	3.53E+10	2.64E+10	3.53E+10	1.23E+11
I_{YY}	1.63E+10	2.97E+09	1.63E+10	2.97E+09	3.86E+10
I_{XY}	0	0	0	0	0

From the ETABS output at Grid B-2:

V_U = 1126.498 kN

$\gamma_{V2}\ M_{U2}$ = –25.725 kN-m

$\gamma_{V3}\ M_{U3}$ = 14.272 kN-m

At the point labeled A in Figure 2, x_4 = –259 and y_4 = 559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(559-0)-(0)(-259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(-259-0)-(0)(559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 – 0.1169 – 0.0958 = **1.3666 N/mm²** at point A

At the point labeled B in Figure 2, x_4 = 259 and y_4 = 559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(559-0)-(0)(259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(259-0)-(0)(559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 – 0.1169 + 0.0958 =**1.5582 N/mm²** at point B

At the point labeled C in Figure 2, x_4 = 259 and y_4 = –559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(-559-0)-(0)(259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(259-0)-(0)(-559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 + 0.1169 + 0.0958 = **1.792 N/mm²** at point C

At the point labeled D in Figure 2, x_4 = –259 and y_4 = –559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(-559-0)-(0)(-259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(-259-0)-(0)(-559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 + 0.1169 – 0.0958 = **1.6004 N/mm²** at point D

Point C has the largest absolute value of v_u, thus v_{max} = **1.792 N/mm²**

The shear capacity is calculated based on the minimum of the following three limits:

$k_s = 0.5 + \beta_c \le 1.0 = 0.833$ (IS 31.6.3.1)

$\tau_c = 0.25 = 1.127$ N/mm^2 (IS 31.6.3.1)

$v_c = k_s\ \tau_c = 1.141$ N/mm^2 (IS 31.6.3.1)

CSA 13.3.4.1 yields the smallest value of $v_c = 1.141$ N/mm^2, and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{v_c} = \frac{1.792}{1.141} = 1.57$$

IS 456-2000 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

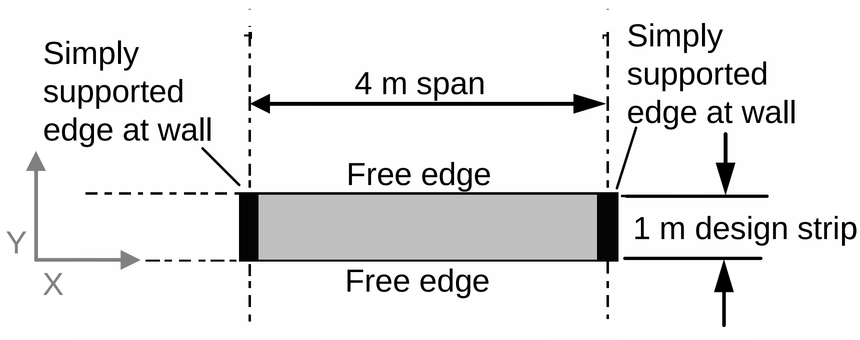


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the IS 456-00 load combination factors, 1.5 for dead loads and 1.5 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing analysis, design was performed using the IS 456-00 code by ETABS and also by hand computation. Table 1 shows the comparison of the design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2×10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm)	
			A_s^+	A_s^-
Medium	ETABS	26.997	5.830	--
	Calculated	27.000	5.830	--

$A^{+}_{s,min}$ = 230.978 sq-mm

COMPUTER FILE: IS 456-2000 RC-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\gamma_s = 1.15$

$\gamma_c = 1.50$

$\alpha = 0.36$

$\beta = 0.42$

$b = 1000$ mm

For the load combination, *w* and *M* are calculated as follows:

$w = (1.5w_d + 1.5w_t)\, b$

$$M = \frac{wl_1^2}{8}$$

$$A_{s,\min} = \frac{0.85}{f_y} bd$$

$= 230.978$ sq-mm

COMB100

$w_d = 4.0$ kPa

$w_t = 5.0$ kPa

$w = 13.5$ kN/m

$M_{\text{-strip}} = 27.0$ kN-m

$M_{\text{-design}} = 27.0363$ kN-m

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \le 250 \text{ MPa} \\ 0.53 - 0.05\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \le 415 \text{ MPa} \\ 0.48 - 0.02\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \le 500 \text{ MPa} \\ 0.46 & \text{if} \quad f_y \ge 500 \text{ MPa} \end{cases}$$

$$\frac{x_{u,\max}}{d} = 0.466$$

The depth of the compression block is given by:

$$m = \frac{M_u}{bd^2 \alpha f_{ck}}$$

$$= 0.16$$

$$\frac{x_u}{d} = \frac{1 - \sqrt{1 - 4\beta m}}{2\beta} = 0.1727488 < \frac{x_{u,\max}}{d}$$

The area of tensile steel reinforcement is given by:

$$z = d\left\{1 - \beta \frac{x_u}{d}\right\}. = 115.9307 \text{ mm}$$

$$A_s = \frac{M_u}{\left(f_y / \gamma_s\right) z}, = 583.027 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.830 \text{ sq-cm}$$

NTC 2008 PT-SL EXAMPLE 001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm as shown in shown in Figure 1.

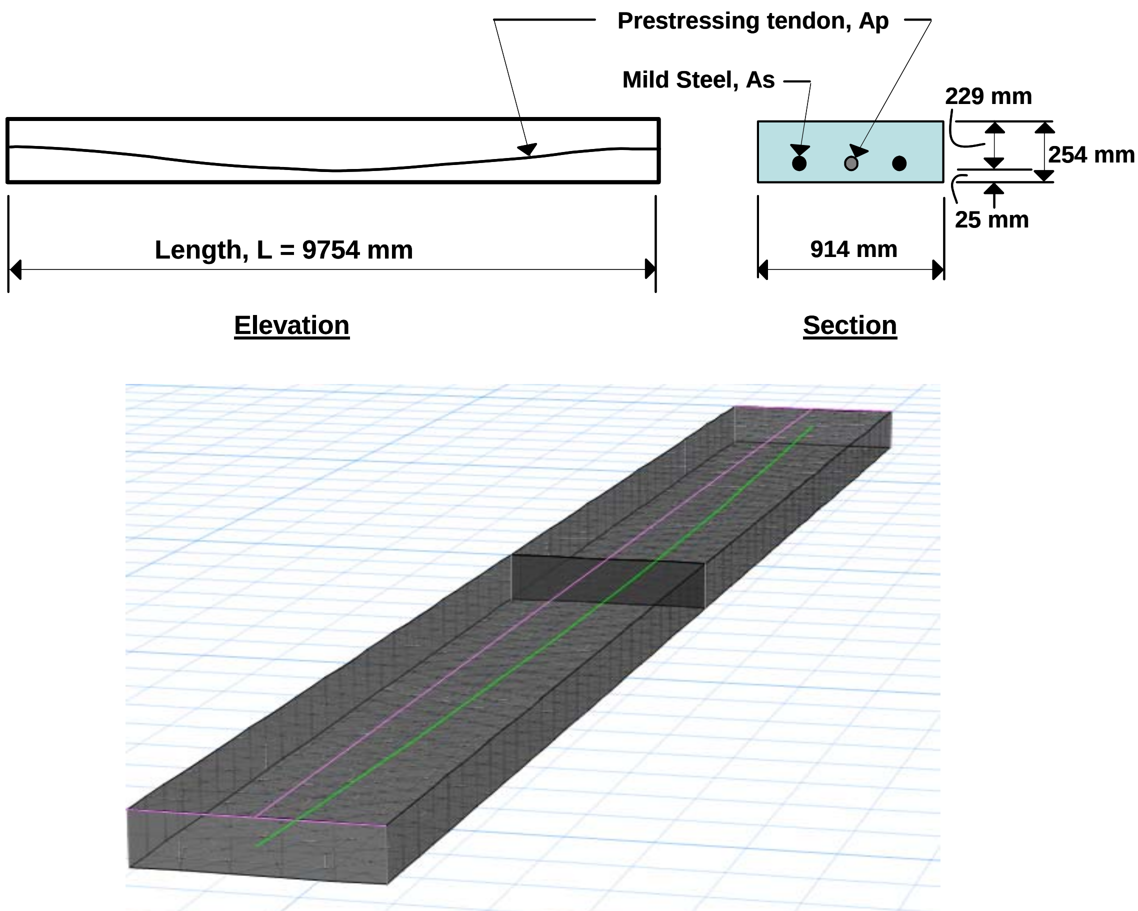


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, was added to the A-Strip. The self weight and live loads have been added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations were compared with the ETABS results and summarized for verification and validation of the ETABS results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	KN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	KN/m^2
Live load	w_l	=	4.788	KN/m^2

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing, and slab stresses with independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	165.90	165.93	0.02%
Transfer Conc. Stress, top $(D+PT_I)$, MPa	–5.057	–5.057	0.00%
Transfer Conc. Stress, bot $(D+PT_I)$, MPa	2.839	2.839	0.00%
Normal Conc. Stress, top $(D+L+PT_F)$, MPa	–10.460	–10.467	0.07%
Normal Conc. Stress, bot $(D+L+PT_F)$, MPa	8.402	8.409	0.08%
Long-Term Conc. Stress, top $(D+0.5L+PT_{F(L)})$, MPa	–7.817	–7.818	0.01%
Long-Term Conc. Stress, bot $(D+0.5L+PT_{F(L)})$, MPa	5.759	5.760	0.02%

Table 2 Comparison of Design Moments and Reinforcements

Method	Design Moment (kN-m)	Reinforcement Area (sq-cm)
		A_s^+
ETABS	165.9	16.40
Calculated	165.9	16.29

COMPUTER FILE: NTC 2008 PT-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

Design Parameters:

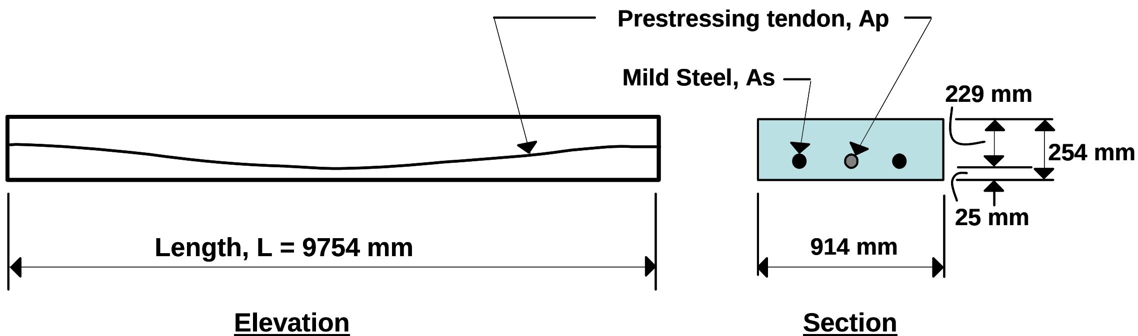
Mild Steel Reinforcing	Post-Tensioning
$f'c$ = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\gamma_{m,\,steel}$ = 1.15

$\gamma_{m,\,concrete}$ = 1.50

$\eta = 1.0$ for $f_{ck} \leq 50$ MPa

$\lambda = 0.8$ for $f_{ck} \leq 50$ MPa



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.35 = 8.078 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) × 1.50 = 7.182 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 15.260 kN/m^2 (D+L)ult

ω = 10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 15.260 kN/m^2 × 0.914 m = 13.948 kN/m

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8} = 13.948 \times (9.754)^2 / 8$ = 165.9 kN-m

Ultimate Stress in strand, $f_{PS} = f_{SE} + 7000d\left(1 - 1.36\dfrac{f_{PU}A_P}{f_{CK}bd}\right)\Big/ l$

$$= 1210 + 7000(229)\left(1 - 1.36\frac{1862(198)}{30(914)(229)}\right)\Big/(9754)$$

$$= 1361\ \text{MPa}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 2(99)(1361)/1000 = 269.5\ \text{kN}$

Design moment M = 165.9 kN-m

Compression block depth ratio: $m = \dfrac{M}{bd^2\eta f_{cd}}$

$$= \frac{165.9}{(0.914)(0.229)^2(1)(30000/1.50)} = 0.1731$$

Required area of mild steel reinforcing,

$$\omega = 1 - \sqrt{1 - 2m} = 1 - \sqrt{1 - 2(0.1731)} = 0.1914$$

$$A_{EquivTotal} = \omega\left(\frac{\eta f_{cd} bd}{f_{yd}}\right) = 0.1914\left(\frac{1(30/1.5)(914)(229)}{400/1.15}\right) = 2303\ \text{mm}^2$$

$$A_{EquivTotal} = A_P\left(\frac{1366}{400}\right) + A_S = 2311\ \text{mm}^2$$

$$A_S = 2303 - 198\left(\frac{1361}{400}\right) = 1629\ \text{mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (D+PT$_i$) = 1.0D+0.0L+1.0PT$_I$

Tendon stress at transfer = jacking stress − stressing losses =1490 − 186 = 1304 MPa

The force in the tendon at transfer = $1304(197.4)/1000 = 257.4$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102\ \text{mm})/1000 = 26.25$ kN-m

Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-257.4}{0.254(0.914)} \pm \dfrac{65.04 - 26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.109 \pm 3.948 \text{ MPa}$$
$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT_F) = 1.0D+1.0L+1.0PT_F

Tendon stress at normal = jacking − stressing − long-term=1490 − 186 − 94 = 1210 MPa
The force in tendon at normal = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT_F),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$
$$f = -1.029 \pm 9.431$$
$$f = -10.460(\text{Comp})\max,\ 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: (D+0.5L+$PT_{F(L)}$) = 1.0D+0.5L+1.0PT_F

Tendon stress at normal = jacking − stressing − long-term = 1490 − 186 − 94 = 1210 MPa
The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+0.5L+$PT_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$
$$f = -1.029 \pm 6.788$$
$$f = -7.817(\text{Comp})\max,\ 5.759(\text{Tension})\max$$

NTC 2008 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

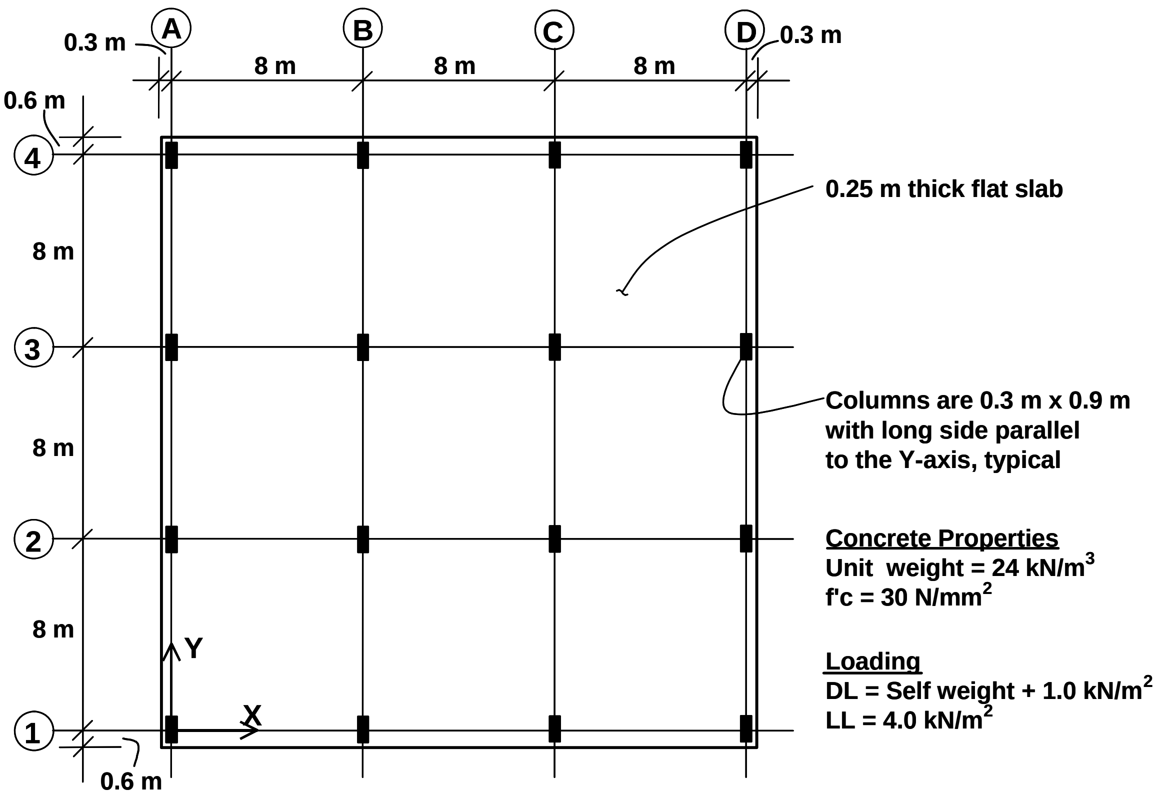


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. Thick shell properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a $f'c$ of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from ETABS with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
ETABS	1.117	0.611	1.83
Calculated	1.092	0.578	1.89

COMPUTER FILE: NTC 2008 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

Hand Calculation for Interior Column using ETABS Method

$d = [(250-26)+(250-38)]/2$ = 218 mm

Refer to Figure 2.

$u_1 = u = 2 \bullet 300 + 2 \bullet 900 + 2 \bullet \pi \bullet 436 = 5139.468$ mm

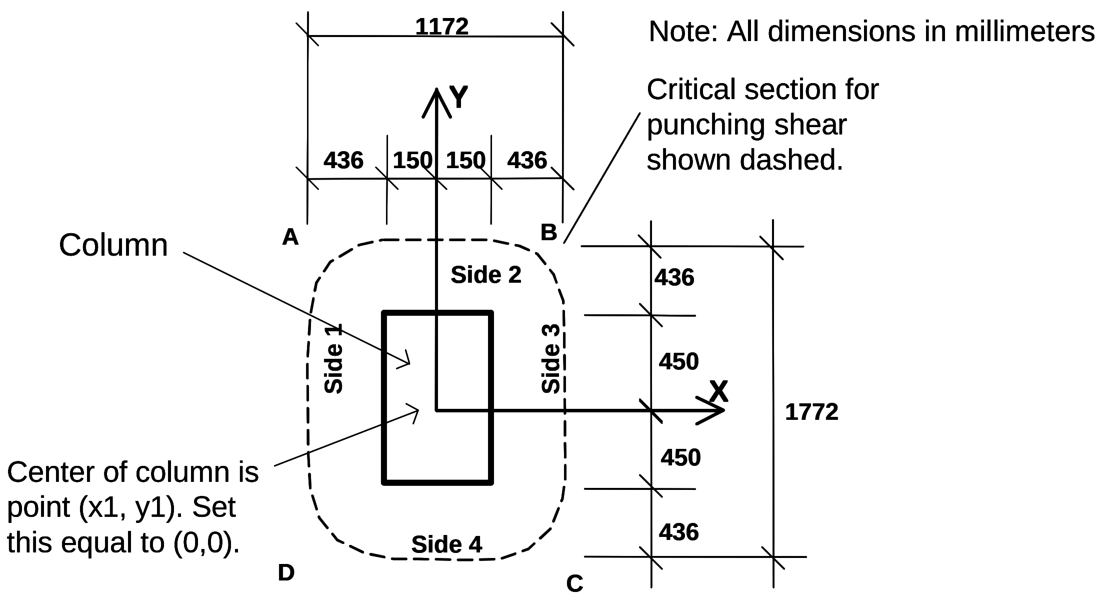


Figure 2: Interior Column, Grid B-2 in ETABS Model

From the ETABS output at Grid B-2:

V_{Ed} = 1112.197 kN

$k_2 M_{Ed2}$ = 38.933 kN-m

$k_3 M_{Ed3}$ = 17.633 kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{Ed} = \frac{V_{Ed}}{ud}\left[1 + \frac{k_2 M_{Ed,2} u_1}{V_{Ed} W_{1,2}} + \frac{k_3 M_{Ed,3} u_1}{V_{Ed} W_{1,3}}\right] \quad \text{(EC2 6.4.4(2))}$$

$$W_1 = \frac{c_1^2}{2} + c_1 c_2 + 4c_2 d + 16d^2 + 2\pi d c_1$$

$$W_{1,2} = \frac{900^2}{2} + 300 \bullet 900 + 4 \bullet 300 \bullet 218 + 16 \bullet 218^2 + 2\pi \bullet 218 \bullet 900$$

$$W_{1,2} = 2{,}929{,}744.957 \text{ mm}^2$$

$$W_{1,3} = 3\frac{900^2}{2} + 900 \bullet 300 + 4 \bullet 900 \bullet 218 + 16 \bullet 218^2 + 2\pi \bullet 218 \bullet 300$$

$$W_{1,2} = 2{,}271{,}104.319 \text{ mm}^2$$

$$v_{Ed} = \frac{V_{Ed}}{ud}\left[1 + \frac{k_2 M_{Ed,2} u_1}{V_{Ed} W_{1,2}} + \frac{k_3 M_{Ed,3} u_1}{V_{Ed} W_{1,3}}\right]$$

$$v_{Ed} = \frac{1112.197 \bullet 10^3}{5139.468 \bullet 218}\left[1 + \frac{38.933 \bullet 10^6 \bullet 5139.468}{1112.197 \bullet 10^3 \bullet 2929744.957} + \frac{17.633 \bullet 10^6 \bullet 5139.468}{1112.197 \bullet 10^3 \bullet 2271104.319}\right]$$

$$v_{Ed} = 1.089 \text{ N/mm}^2$$

Thus v_{max} = **1.089 N/mm²**

$$C_{Rd,c} = 0.18/\gamma_c = 0.18/1.5 = 0.12 \quad \text{(EC2 6.4.4)}$$

The shear stress carried by the concrete, $V_{Rd,c}$, is calculated as:

$$V_{Rd,c} = \left[C_{Rd,c} k \left(100\rho_1 f_{ck}\right)^{1/3} + k_1 \sigma_{cp}\right] \quad \text{(EC2 6.4.4)}$$

with a minimum of:

$$v_{Rd,c} = \left(v_{\min} + k_1 \sigma_{cp}\right) \quad \text{(EC2 6.4.4)}$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 = 1.9578 \quad \text{(EC2 6.4.4(1))}$$

$$k_1 = 0.15. \quad \text{(EC2 6.2.2(1))}$$

$\rho_1 = \dfrac{A_{s1}}{b_w d} \leq 0.02$

Area of reinforcement at the face of column for design strip are as follows:

A_s in Strip Layer A = 9204.985 mm^2

A_s in Strip Layer B = 8078.337 mm^2

Average $A_s = (9204.985 + 8078.337)/2$ = 8641.661 mm^2

$\rho_1 = \; 8641.661/(8000 \bullet 218)$ = 0.004955 ≤ 0.02

$v_{\min} = 0.035 k^{3/2} f_{ck}^{\;1/2} = \; 0.035(1.9578)^{3/2} (30)^{1/2}$ = 0.525 N/mm^2

$v_{Rd,c} = \left[0.12 \bullet 1.9578 (100 \bullet 0.004955 \bullet 30)^{1/3} + 0\right]$ = 0.5777 N/mm^2

$$\text{Shear Ratio} = \frac{v_{\max}}{v_{Rd,c}} = \frac{1.089}{0.5777} = 1.89$$

NTC 2008 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

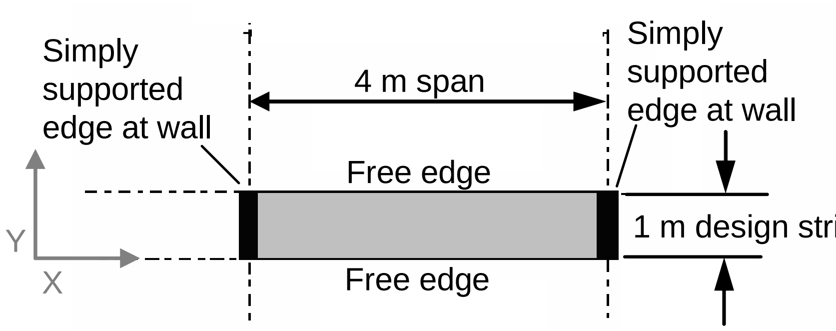


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the Italian NTC 2008 load combination factors, 1.35 for dead loads and 1.5 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. These moments are identical. After completing the analysis, design is performed using the Italian NTC 2008 code by ETABS and also by hand computation. Table 1 shows the comparison of the design reinforcements computed by the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_{ck}	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	$2x10^6$	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm)
		A_s^+
ETABS	25.797	5.400
Calculated	25.800	5.400

$A^+_{s,min}$ = 204.642 sq-mm

COMPUTER FILE: NTC 2008 RC-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$$\eta = 1.0 \text{ for } f_{ck} \leq 50 \text{ MPa}$$

$$\lambda = 0.8 \text{ for } f_{ck} \leq 50 \text{ MPa}$$

b = 1000 mm

For the load combination, w and M are calculated as follows:

$$w = (1.35w_d + 1.5w_t)\ b$$

$$M = \frac{wl_1^2}{8}$$

$$A_{s,\min} = \max \begin{cases} 0.0013 b_w d \\ 0.26 \dfrac{f_{ctm}}{f_{yk}} bd \end{cases}$$

$$= 204.642 \text{ sq-mm}$$

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 12.9 kN/m

$M_{\text{-strip}}$ = 25.8 kN-m

$M_{\text{-design}}$ = 25.8347 kN-m

$\gamma_{m,\ steel}$ = 1.15

$\gamma_{m,\ concrete}$ = 1.50

α_{cc} = 0.85:

The depth of the compression block is given by:

$$m = \frac{M}{bd^2 \eta f_{cd}} = \frac{25.8347 \bullet 10^6}{1000 \bullet 125^2 \bullet 1.0 \bullet 0.85 \bullet 30/1.5} = 0.097260$$

$$m_{\lim} = \lambda \left(\frac{x}{d}\right)_{\lim} \left[1 - \frac{\lambda}{2}\left(\frac{x}{d}\right)_{\lim}\right] = 0.48$$

$$\left(\frac{x}{d}\right)_{\text{lim}} = \frac{\delta - k_1}{k_2} \text{ for } f_{ck} \leq 50 \text{ MPa} = 0.60$$

For reinforcement with $f_{yk} \leq 500$ MPa, the following values are used:

$k_1 = 0.40$

$k_2 = (0.6 + 0.0014/\varepsilon_{cu2}) = 1.00$

δ is assumed to be 1

$$\omega = 1 - \sqrt{1 - 2m} = 0.10251$$

$$A_s = \omega\left(\frac{\eta f_{cd} bd}{f_{yd}}\right) = 544.61 \text{ sq-mm} > A_{s,\text{min}}$$

$A_s = 5.446$ sq-cm

NZS 3101-2006 PT-SL EXAMPLE 001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 915 mm wide and spans 9754 mm as, shown in shown in Figure 1.

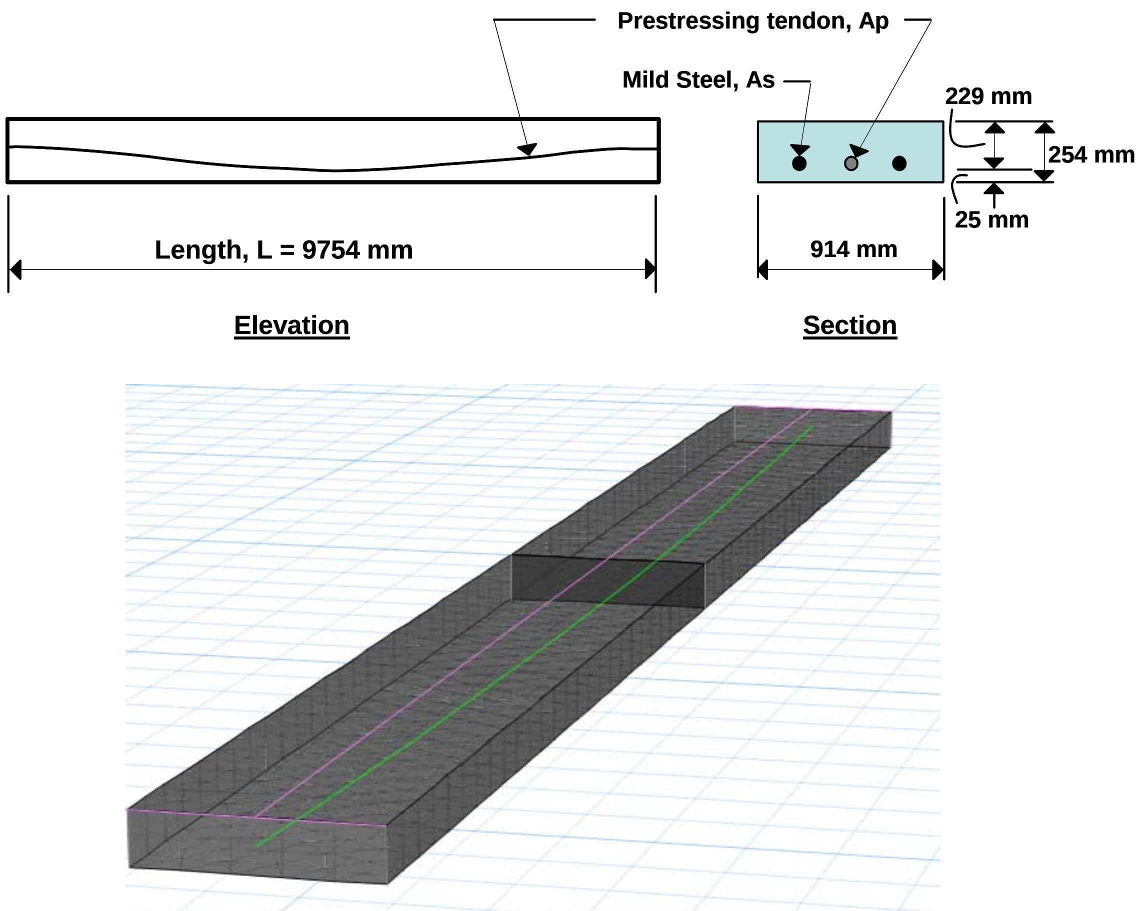


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads have been added to the slab. The loads and post-tensioning forces are as follows:

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the midspan of the slab. Independent hand calculations were compared with the ETABS results and summarized for verification and validation of the ETABS results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	kN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	kN/m^2
Live load	w_l	=	4.788	kN/m^2

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads.

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	156.12	156.17	0.02%
Area of Mild Steel req'd, As (sq-cm)	14.96	15.08	0.74%
Transfer Conc. Stress, top $(D+PT_I)$, MPa	–5.058	–5.057	-0.02%
Transfer Conc. Stress, bot $(D+PT_I)$, MPa	2.839	2.839	0.00%
Normal Conc. Stress, top $(D+L+PT_F)$, MPa	–10.460	–10.467	0.07%
Normal Conc. Stress, bot $(D+L+PT_F)$, MPa	8.402	8.409	0.08%
Long-Term Conc. Stress, top $(D+0.5L+PT_{F(L)})$, MPa	–7.817	–7.818	0.01%
Long-Term Conc. Stress, bot $(D+0.5L+PT_{F(L)})$, MPa	5.759	5.760	0.02%

COMPUTER FILE: NZS 3101-2006 PT-SL EX001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
$f'c$ = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

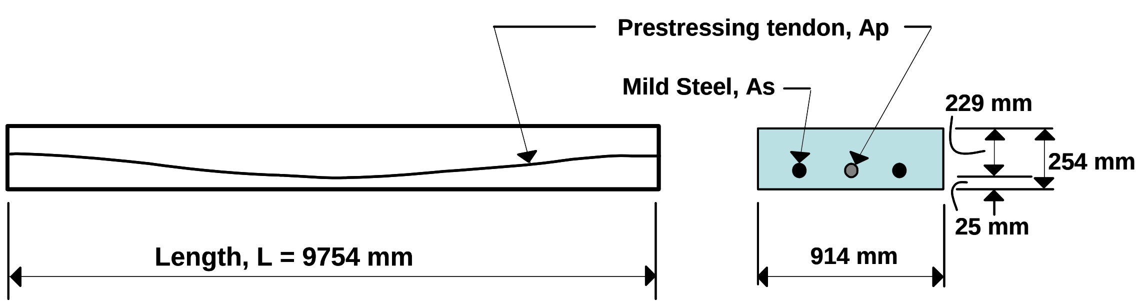
$\phi_b = 0.85$

$\alpha_1 = 0.85 \quad \text{for } f'_c \leq 55 \text{ MPa}$

$\beta_1 = 0.85 \quad \text{for } f'_c \leq 30,$

$$c_b = \frac{\varepsilon_c}{\varepsilon_c + f_y / E_s} d = 214.7$$

$a_{\max} = 0.75\beta_1 c_b = 136.8$ mm



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.2 = 7.181kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) × 1.5 = 7.182 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 14.363 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 14.363 kN/m^2 × 0.914 m = 13.128 kN/m

Ultimate Moment, $M_U = \frac{wl_1^2}{8}$ = 13.128 × (9.754)^2/8 = 156.12 kN-m

Ultimate Stress in strand, $f_{PS} = f_{SE} + 70 + \frac{f'c}{300\rho_P}$

$$= 1210 + 70 + \frac{30}{300(0.00095)}$$

$$= 1385 \text{ MPa} \le f_{SE} + 200 = 1410 \text{ MPa}$$

Ultimate force in PT, $F_{ult,PT} = A_P\left(f_{PS}\right) = 2(99)(1385)/1000 = 274.23 \text{ kN}$

Stress block depth, $a = d - \sqrt{d^2 - \frac{2M^*}{\alpha f'_c \phi b}}$

$$= 0.229 - \sqrt{0.229^2 - \frac{2(156.12)}{0.85(30000)(0.85)(0.914)}}(1e3) = 37.48 \text{ mm}$$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right)\phi = 274.23\left(229 - \frac{37.48}{2}\right)(0.85)/1000 = 49.01 \text{ kN-m}$$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 156.1 - 49.10 = 107.0 \text{ kN-m}$

Required area of mild steel reinforcing,

$$A_S = \frac{M_{net}}{\phi f_y (d - \frac{a}{2})} = \frac{107.0}{0.85(400000)\left(0.229 - \frac{0.03748}{2}\right)}(1e6) = 1496 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (D+PT$_i$) = 1.0D+0.0L+1.0PT$_I$

Tendon stress at transfer = jacking stress − stressing losses =1490 − 186 = 1304 MPa

The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4 \text{ kN}$

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04 \text{ kN-m}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25 \text{ kN-m}$

Stress in concrete, $f = \frac{F_{PTI}}{A} \pm \frac{M_D - M_{PT}}{S} = \frac{-257.4}{0.254(0.914)} \pm \frac{65.04 - 26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.109 \pm 3.948 MPa$$
$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT_F) = 1.0D+1.0L+1.0PT_F

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT_F),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$
$$f = -1.029 \pm 9.431$$
$$f = -10.460(\text{Comp})\max,\ 8.402(\text{Tension})\max$$

Long-Term Condition, load combinations: (D+0.5L+$PT_{F(L)}$) = 1.0D+0.5L+1.0PT_F

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+0.5L+$PT_{F(L)}$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+0.5L} - M_{PT}}{S} = \frac{-238.9}{0.254(0.914)} \pm \frac{91.06 - 24.33}{0.00983}$$
$$f = -1.029 \pm 6.788$$
$$f = -7.817(\text{Comp})\max,\ 5.759(\text{Tension})\max$$

NZS 3101-2006 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 8 m spans in each direction, as shown in Figure 1.

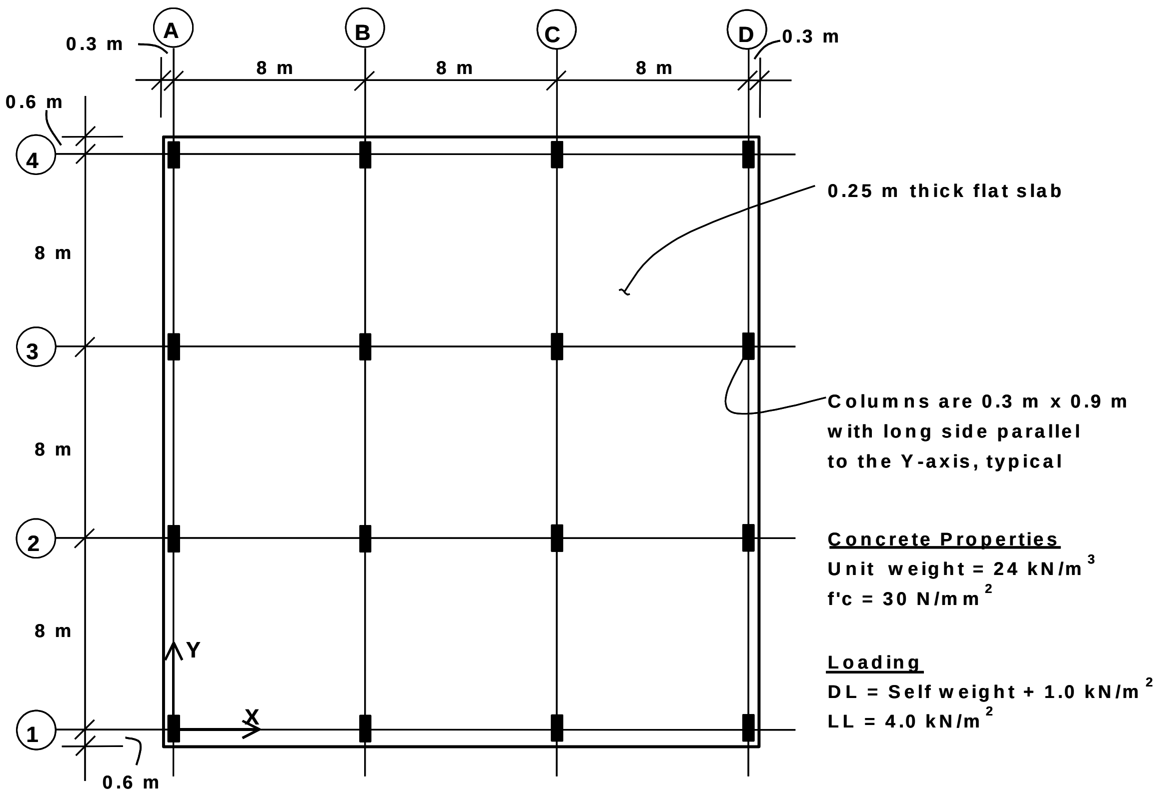


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a *f'c* of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from ETABS with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
ETABS	1.793	1.141	1.57
Calculated	1.792	1.141	1.57

COMPUTER FILE: NZS 3101-2006 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using ETABS Method

$d = [(259-26)+(250-38)]/2$ = 218 mm

Refer to Figure 2.

b_0 = 518+ 1118 + 1118 + 518 = 3272 mm

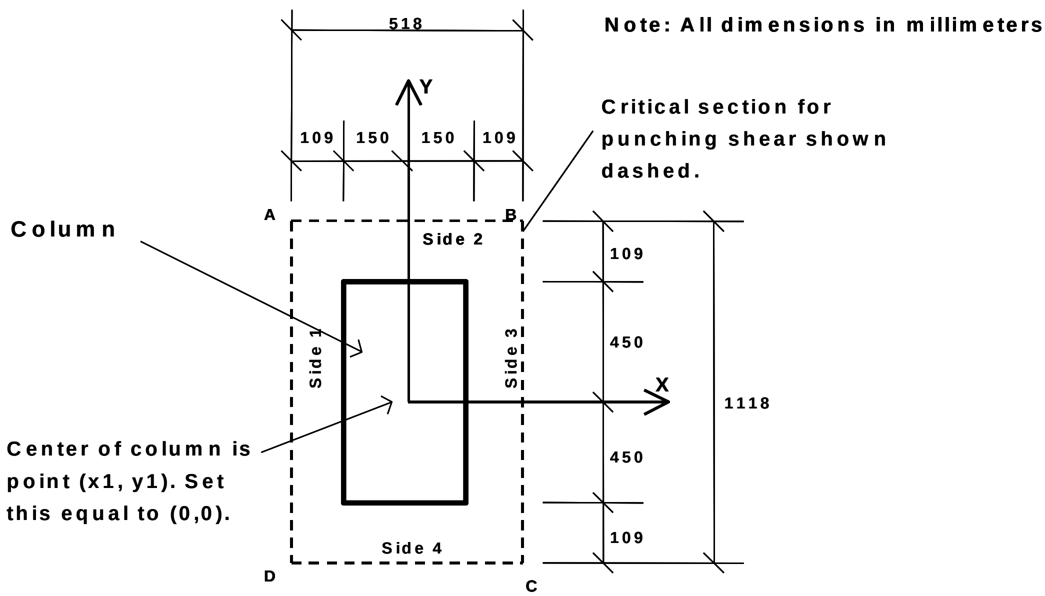


Figure 2: Interior Column, Grid B-2 in ETABS Model

$$\gamma_{V2} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{1118}{518}}} = 0.495$$

$$\gamma_{V3} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{518}{1118}}} = 0.312$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–259	0	259	0	N.A.
y_2	0	559	0	–559	N.A.
L	1118	518	1118	518	b_0 = 3272
d	218	218	218	218	N.A.
Ld	243724	112924	243724	112924	713296
Ldx_2	–63124516	0	63124516	0	0
Ldy_2	0	63124516	0	–63124516	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{713296} = 0\ mm$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{713296} = 0\ mm$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	1118	518	1118	518	N.A.
d	218	218	218	218	N.A.
$x_2 - x_3$	–259	0	259	0	N.A.
$y_2 - y_3$	0	559	0	–559	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b, 7	5a, 6a, 7	5b, 6b, 7	5a, 6a, 7	N.A.
I_{XX}	2.64E+10	3.53E+10	2.64E+10	3.53E+10	1.23E+11
I_{YY}	1.63E+10	2.97E+09	1.63E+10	2.97E+09	3.86E+10
I_{XY}	0	0	0	0	0

From the ETABS output at Grid B-2:

V_U = 1126.498 kN

$\gamma_{V2}\ M_{U2}$ = –25.725 kN-m

$\gamma_{V3}\ M_{U3}$ = 14.272 kN-m

At the point labeled A in Figure 2, x_4 = –259 and y_4 = 559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(559-0)-(0)(-259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(-259-0)-(0)(559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 – 0.1169 – 0.0958 = **1.3666 N/mm²** at point A

At the point labeled B in Figure 2, x_4 = 259 and y_4 = 559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(559-0)-(0)(259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(259-0)-(0)(559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 – 0.1169 + 0.0958 =**1.5582 N/mm²** at point B

At the point labeled C in Figure 2, x_4 = 259 and y_4 = –559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(-559-0)-(0)(259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(259-0)-(0)(-559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 + 0.1169 + 0.0958 = **1.792 N/mm²** at point C

At the point labeled D in Figure 2, x_4 = –259 and y_4 = –559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{25.725 \bullet 10^6 \left[3.86 \bullet 10^{10}(-559-0)-(0)(-259-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2} + \frac{14.272 \bullet 10^6 \left[1.23 \bullet 10^{11}(-259-0)-(0)(-559-0)\right]}{(1.23 \bullet 10^{11})(3.86 \bullet 10^{10})-(0)^2}$$

v_U = 1.5793 + 0.1169 – 0.0958 = **1.6004 N/mm²** at point D

Point C has the largest absolute value of v_u, thus v_{max} = **1.792 N/mm²**

The shear capacity is calculated based on the smallest of NZS 3101-06, with the b_o and u terms removed to convert force to stress.

$$\varphi v_v = \min \begin{cases} \frac{1}{6}\left(1 + \frac{2}{\beta_c}\right)\varphi\sqrt{f'_c} \\ \frac{1}{6}\left(1 + \frac{\alpha_s d}{b_0}\right)\varphi\sqrt{f'_c} \\ \frac{1}{3}\varphi\sqrt{f'_c} \end{cases} = 1.141\text{N/mm}^2 \text{ per} \qquad \text{(NZS 12.7.3.2)}$$

NZS 12.7.3.2 yields the smallest value of $\varphi v_v = 1.141$ N/mm^2, and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{\varphi v_v} = \frac{1.792}{1.141} = 1.57$$

NZS 3101-2006 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

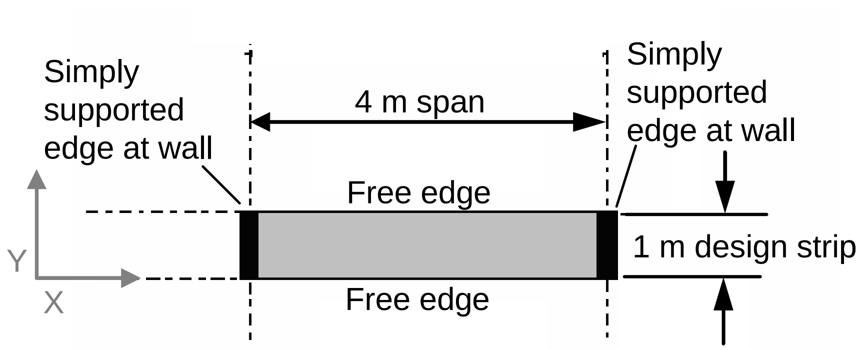


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 kN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the NZS 3101-06 load combination factors, 1.2 for dead loads and 1.5 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing analysis, design is performed using the NZS 3101-06 code by ETABS and also by hand computation. Table 1 shows the comparison of the design reinforcements computed using the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2×10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	ETABS	24.597	5.238
	Calculated	24.6	5.238

$A^+_{s,min}$ = 380.43 sq-mm

COMPUTER FILE: NZS 3101-2006 RC-SL Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

The following quantities are computed for the load combination:

$\phi_b = 0.85$

$b = 1000$ mm

$\alpha_1 = 0.85 \quad \text{for } f'_c \leq 55\text{MPa}$

$\beta_1 = 0.85 \quad \text{for } f'_c \leq 30,$

$$c_b = \frac{\varepsilon_c}{\varepsilon_c + f_y / E_s} d = 70.7547$$

$a_{\max} = 0.75\beta_1 c_b = 45.106$ mm

For the load combination, w and M^* are calculated as follows:

$w = (1.2w_d + 1.5w_t)\, b$

$$M_u = \frac{wl_1^2}{8}$$

$$A_{s,\min} = \max \begin{cases} \dfrac{\sqrt{f'_c}}{4 f_y} b_w d = 372.09 \text{ sq-mm} \\ 1.4 \dfrac{b_w d}{f_y} = 380.43 \text{ sq-mm} \end{cases}$$

$= 380.43$ sq-mm

COMB100

$w_d = 4.0$ kPa

$w_t = 5.0$ kPa

$w = 12.3$ kN/m

$M^*_{\text{-strip}} = 24.6$ kN-m

$M^*_{\text{-design}} = 24.6331$ kN-m

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2\left|M^*\right|}{\alpha_1 f'_c \phi_b b}} = 9.449 \text{ mm} < a_{\max}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M^*}{\phi_b f_y \left(d - \frac{a}{2} \right)} = 523.799 \text{ sq-mm} > A_{s,\min}$$

$$A_s = 5.238 \text{ sq-cm}$$

SS CP 65-99 PT-SL EXAMPLE 001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.

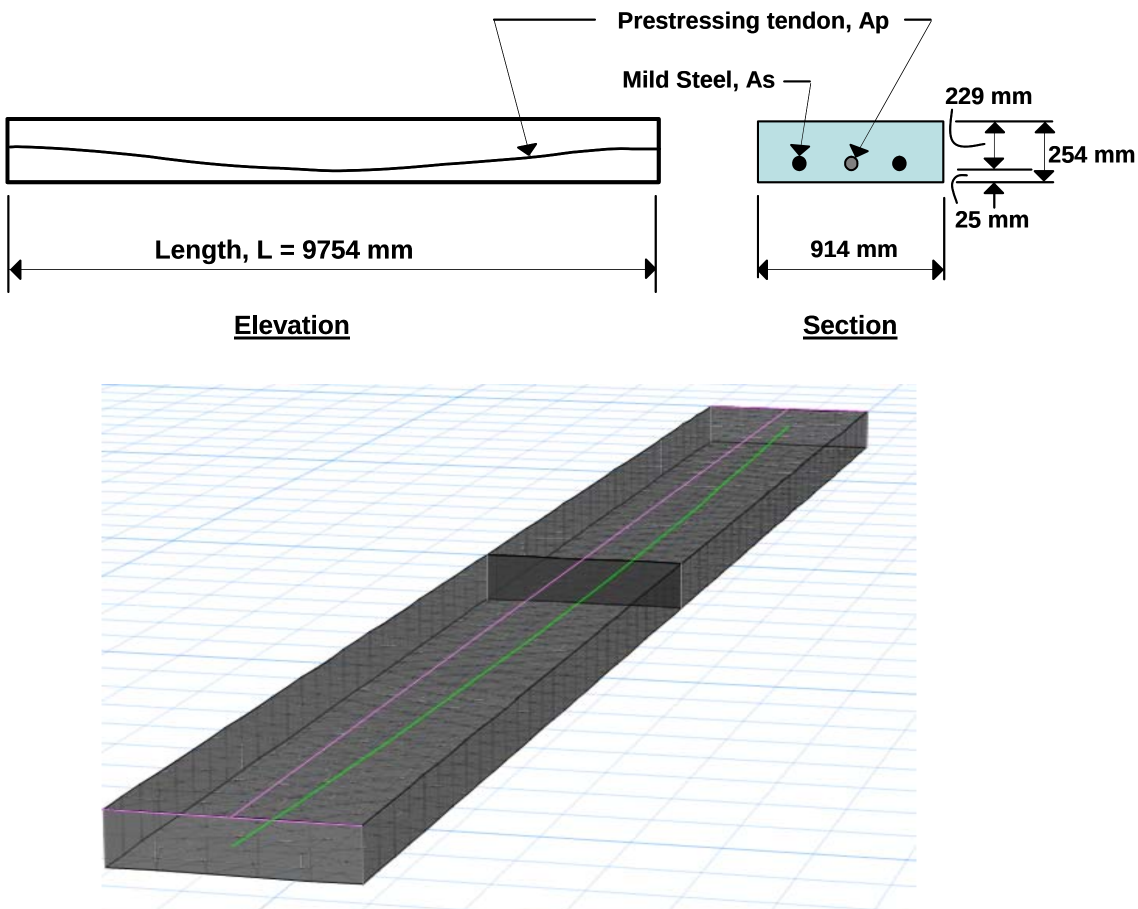


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads have been added to the slab. The loads and post-tensioning forces are as follows.

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations are compared with the ETABS results and summarized for verification and validation of the ETABS results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f'_c	=	30	MPa
Yield strength of steel	f_y	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	kN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	kN/m^2
Live load	w_l	=	4.788	kN/m^2

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	174.4	174.4	0.00%
Area of Mild Steel req'd, As (sq-cm)	19.65	19.80	0.76%
Transfer Conc. Stress, top (D+PT_I), MPa	–5.058	–5.057	-0.02%
Transfer Conc. Stress, bot (D+PT_I), MPa	2.839	2.839	0.00%
Normal Conc. Stress, top (D+L+PT_F), MPa	–10.460	–10.467	0.07%
Normal Conc. Stress, bot (D+L+PT_F), MPa	8.402	8.409	0.08%

COMPUTER FILE: SS CP 65-1999 PT-SL EX001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

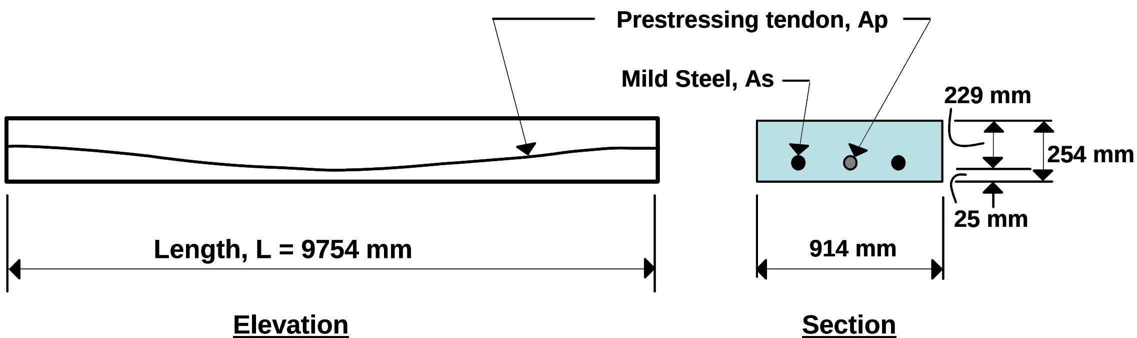
HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
$f'c$ = 30MPa	f_{pu} = 1862 MPa
fy = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\gamma_{m,\, steel} = 1.15$

$\gamma_{m,\, concrete} = 1.50$



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.4 = 8.378 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) × 1.6 = 7.661 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 16.039 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 16.039 kN/m^2 × 0.914 m = 14.659 kN/m

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8}$ = 14.659 × (9.754)^2/8 = 174.4 kN-m

Ultimate Stress in strand, $f_{pb} = f_{pe} + \frac{7000}{l/d}\left(1 - 1.7\frac{f_{pu}A_p}{f_{cu}bd}\right)$

$$= 1210 + \frac{7000}{9754/229}\left(1 - 1.7\frac{1862(198)}{30(914)(229)}\right)$$

$$= 1358 \text{ MPa} \leq 0.7 f_{pu} = 1303 \text{ MPa}$$

K factor used to determine the effective depth is given as:

$$K = \frac{M}{f_{cu}bd^2} = \frac{174.4}{30000(0.914)(0.229)^2} = 0.1213 < 0.156$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \leq 0.95d = 192.2 \text{ mm}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 2(99)(1303)/1000 = 258.0 \text{ kN}$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}(z)/\gamma = 258.0(0.192)/1.15 = 43.12 \text{ kN-m}$$

Net Moment to be resisted by As,

$$M_{NET} = M_U - M_{PT}$$

$$= 174.4 - 43.12 = 131.28 \text{ kN-m}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_{NET}}{0.87 f_y z_X} = \frac{131.28}{0.87(400)(192)}(1e6) = 1965 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination ($D+PT_i$) = $1.0D+0.0L+1.0PT_I$

Tendon stress at transfer = jacking stress – stressing losses = 1490 – 186 = 1304 MPa

The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4 \text{ kN}$

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04 \text{ kN-m}$

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25 \text{ kN-m}$

Stress in concrete, $f = \frac{F_{PTI}}{A} \pm \frac{M_D - M_{PT}}{S} = \frac{-257.4}{0.254(0.914)} \pm \frac{65.04 - 26.23}{0.00983}$

where S = 0.00983m^3

$$f = -1.109 \pm 3.948 \text{ MPa}$$

$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT_F) = 1.0D+1.0L+1.0PT_F

Tendon stress at normal = jacking – stressing – long-term = 1490 – 186 – 94 = 1210 MPa

The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN

Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m

Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m

Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102\text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT_F),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$

$$f = -1.029 \pm 9.431$$

$$f = -10.460(\text{Comp})\max, 8.402(\text{Tension})\max$$

SS CP 65-1999 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

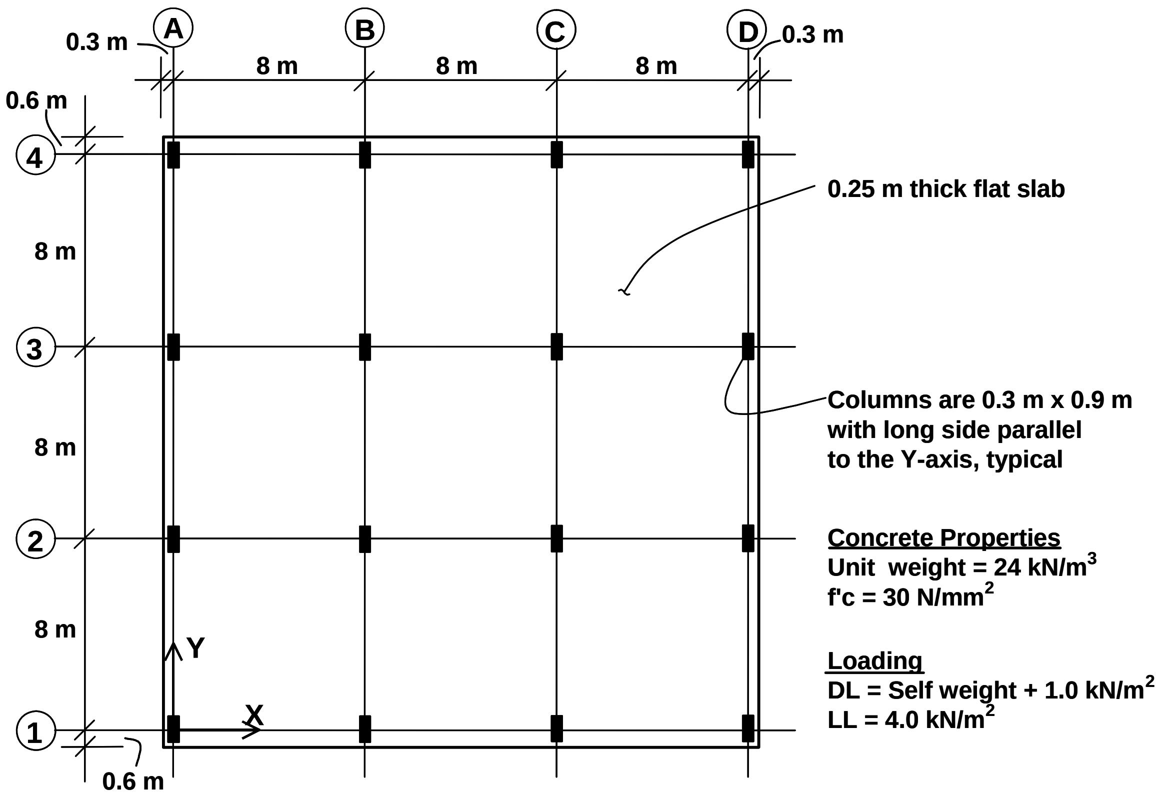


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f_{cu} of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from ETABS with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
ETABS	1.116	0.662	1.69
Calculated	1.105	0.620	1.77

COMPUTER FILE: SS CP 65-1999 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using ETABS Method

$d = [(250 - 26) + (250 - 38)]/2$ = 218 mm

Refer to Figure 1.

u = 954+ 1554 + 954 + 1554 = 5016 mm

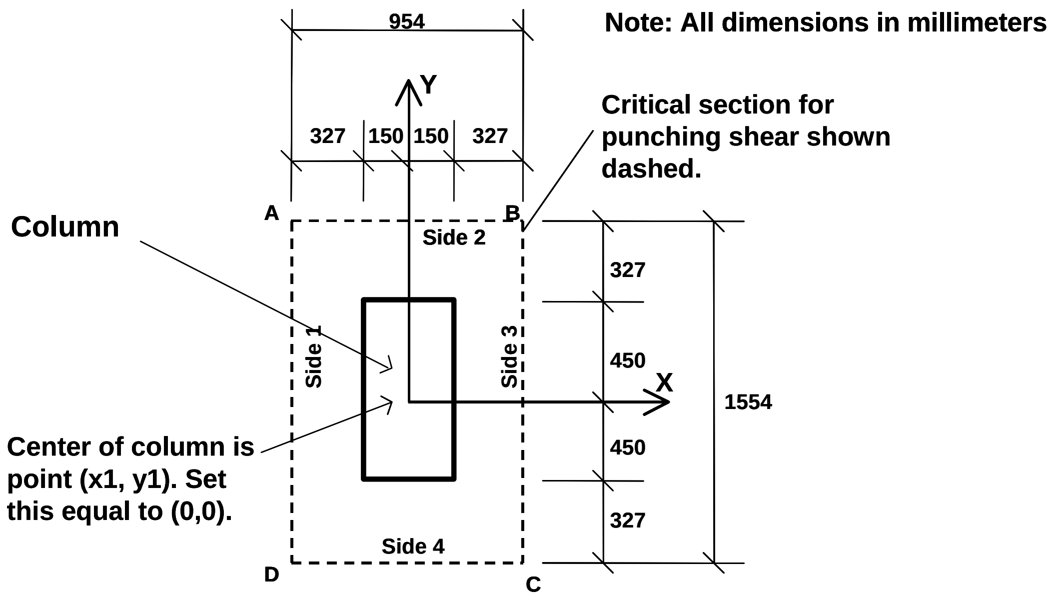


Figure 2: Interior Column, Grid B-2 in ETABS Model

From the ETABS output at Grid B-2:

V= 1126.498 kN

M_2 = 51.9908 kN-m

M_3 = 45.7234 kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{eff,x} = \frac{V}{ud}\left(f + \frac{1.5M_x}{Vy} \right) \quad \text{(CP 3.7.7.3)}$$

$$v_{eff,x} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 51.9908 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 954} \right) = 1.1049 \ \text{(Govern)}$$

$$v_{eff,y} = \frac{V}{ud}\left(f + \frac{1.5M_y}{Vx} \right)$$

$$v_{eff,y} = \frac{1126.498 \bullet 10^3}{5016 \bullet 218}\left(1.0 + \frac{1.5 \bullet 45.7234 \bullet 10^6}{1126.498 \bullet 10^3 \bullet 1554} \right) = 1.0705$$

The largest absolute value of v = **1.1049 N/mm^2**

The shear stress carried by the concrete, v_c, is calculated as:

$$v_c = \frac{0.79k_1k_2}{\gamma_m}\left(\frac{100A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} = 0.3568 \text{ MPa}$$

k_1 is the enhancement factor for support compression, and is conservatively taken as 1 .

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3} = \left(\frac{30}{25}\right)^{1/3} = 1.0627 > 1.0 \text{ OK}$$

$\gamma_m = 1.25$

$$\left(\frac{400}{d}\right)^{1/4} = 1.16386 > 1 \text{ OK.}$$

$f_{cu} \leq 40$ MPa (for calculation purposes only) and A_s is the area of tension reinforcement.

Area of reinforcement at the face of column for design strip are as follows:

A_s in Strip Layer A = 9494.296 mm^2

A_s in Strip Layer B = 8314.486 mm^2

Average A_s = (9494.296+8314.486)/2 = 8904.391 mm^2

$$\frac{100\,A_s}{bd} = 100 \bullet 8904.391/(8000 \bullet 218) = 0.51057$$

$$v_c = \frac{0.79 \bullet 1.0 \bullet 1.0627}{1.25} \bullet \left(0.51057\right)^{1/3} \bullet 1.16386 = 0.6247 \text{ MPa}$$

BS 3.7.7.3 yields the value of v = **0.625 N/mm^2,** and thus this is the shear capacity.

$$\text{Shear Ratio} = \frac{v_U}{v} = \frac{1.1049}{0.6247} = 1.77$$

SS CP 65-1999 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

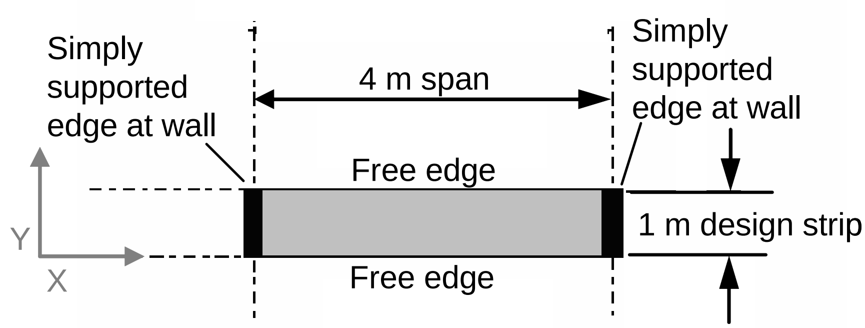


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 KN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the Singapore CP 65-99 load combination factors, 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing the analysis, design is performed using the Singapore CP 65-99 code by ETABS and also by hand computation. Table 1 shows the comparison of the design reinforcements computed by the two methods.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_c	=	30	MPa
Yield strength of steel	f_{sy}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2×10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm) A_s^+
Medium	ETABS	27.197	5.853
	Calculated	27.200	5.850

$A^{+}_{s,min}$ = 162.5 sq-mm

COMPUTER FILE: SS CP 65-1999 RC Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for all the load combinations:

$\gamma_{m,\,steel}$ = 1.15

$\gamma_{m,\,concrete}$ = 1.50

b = 1000 mm

For each load combination, the w and M are calculated as follows:

$$w = (1.4w_d + 1.6w_t)\,b$$

$$M = \frac{wl_1^2}{8}$$

$$A_{s,min} = 0.0013 b_w d$$

$$= 162.5 \text{ sq-mm}$$

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 13.6 kN/m

$M_{\text{-}strip}$ = 27.2 kN-m

$M_{\text{-}design}$ = 27.2366 kN-m

The depth of the compression block is given by:

$$K = \frac{M}{f_{cu} b d^2} = 0.05810 < 0.156$$

The area of tensile steel reinforcement is then given by:

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d \ = 116.3283$$

$$A_s = \frac{M}{0.87 f_y z} = 585.046 \text{ sq-mm} > A_{s,\min}$$

A_s = 5.850 sq-cm

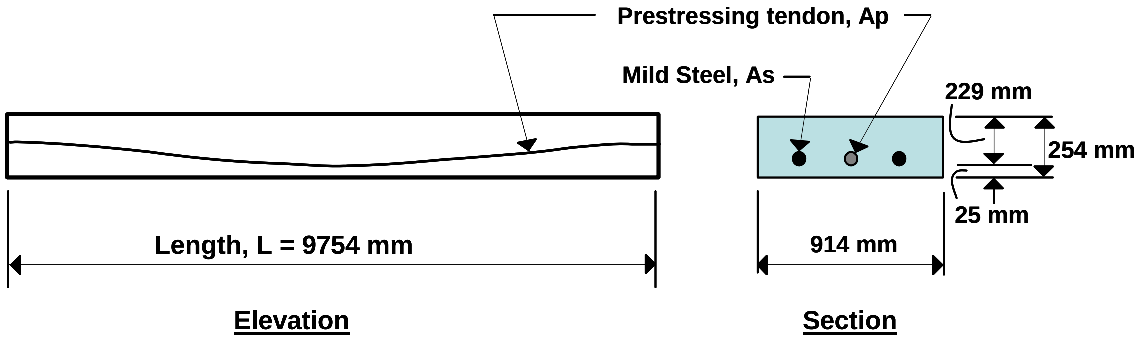
TS 500-2000 PT-SL EXAMPLE 001

Post-Tensioned Slab Design

PROBLEM DESCRIPTION

The purpose of this example is to verify the slab stresses and the required area of mild steel strength reinforcing for a post-tensioned slab.

A one-way, simply supported slab is modeled in ETABS. The modeled slab is 254 mm thick by 914 mm wide and spans 9754 mm, as shown in shown in Figure 1.



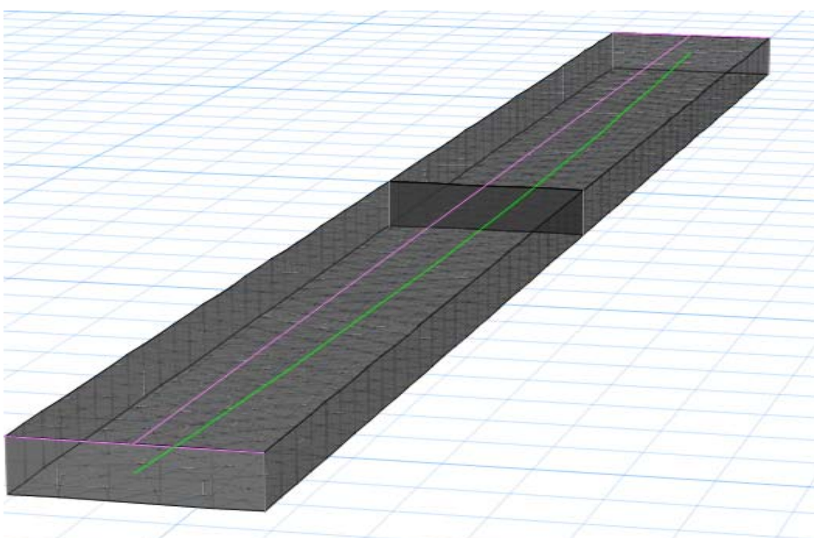


Figure 1 One-Way Slab

A 254-mm-wide design strip is centered along the length of the slab and has been defined as an A-Strip. B-strips have been placed at each end of the span, perpendicular to Strip-A (the B-Strips are necessary to define the tendon profile). A tendon with two strands, each having an area of 99 mm^2, has been added to the A-Strip. The self weight and live loads have been added to the slab. The loads and post-tensioning forces are as follows.

Loads: Dead = self weight, Live = 4.788 kN/m^2

The total factored strip moments, required area of mild steel reinforcement, and slab stresses are reported at the mid-span of the slab. Independent hand calculations are compared with the ETABS results and summarized for verification and validation of the ETABS results.

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	254	mm
Effective depth	d	=	229	mm
Clear span	L	=	9754	mm
Concrete strength	f_{ck}	=	30	MPa
Yield strength of steel	f_{yk}	=	400	MPa
Prestressing, ultimate	f_{pu}	=	1862	MPa
Prestressing, effective	f_e	=	1210	MPa
Area of Prestress (single strand)	A_p	=	198	mm^2
Concrete unit weight	w_c	=	23.56	kN/m^3
Modulus of elasticity	E_c	=	25000	N/mm^3
Modulus of elasticity	E_s	=	200,000	N/mm^3
Poisson's ratio	ν	=	0	
Dead load	w_d	=	self	kN/m^2
Live load	w_l	=	4.788	kN/m^2

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of the required flexural reinforcement
- Check of slab stresses due to the application of dead, live, and post-tensioning loads

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments, required mild steel reinforcing, and slab stresses with the independent hand calculations.

Table 1 Comparison of Results

FEATURE TESTED	INDEPENDENT RESULTS	ETABS RESULTS	DIFFERENCE
Factored moment, Mu (Ultimate) (kN-m)	174.4	174.4	0.00%
Area of Mild Steel req'd, As (sq-cm)	14.88	14.90	0.13%
Transfer Conc. Stress, top $(D+PT_I)$, MPa	–5.058	–5.057	-0.02%
Transfer Conc. Stress, bot $(D+PT_I)$, MPa	2.839	2.839	0.00%
Normal Conc. Stress, top $(D+L+PT_F)$, MPa	–10.460	–10.467	0.07%
Normal Conc. Stress, bot $(D+L+PT_F)$, MPa	8.402	8.409	0.08%

COMPUTER FILE: TS 500-2000 PT-SL Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

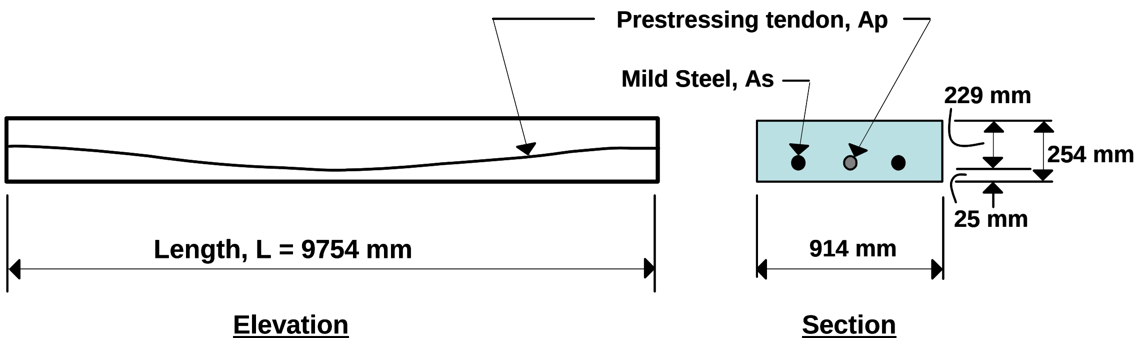
HAND CALCULATIONS:

Design Parameters:

Mild Steel Reinforcing	Post-Tensioning
f_{ck} = 30MPa	f_{pu} = 1862 MPa
f_{yk} = 400MPa	f_{py} = 1675 MPa
	Stressing Loss = 186 MPa
	Long-Term Loss = 94 MPa
	f_i = 1490 MPa
	f_e = 1210 MPa

$\gamma_{m,\,steel} = 1.15$

$\gamma_{m,\,concrete} = 1.50$



Loads:

Dead, self-wt = 0.254 m × 23.56 kN/m^3 = 5.984 kN/m^2 (D) × 1.4 = 8.378 kN/m^2 (D_u)

Live, = 4.788 kN/m^2 (L) × 1.6 = 7.661 kN/m^2 (L_u)

Total = 10.772 kN/m^2 (D+L) = 16.039 kN/m^2 (D+L)ult

ω =10.772 kN/m^2 × 0.914 m = 9.846 kN/m, ω_u = 16.039 kN/m^2 × 0.914 m = 14.659 kN/m

Ultimate Moment, $M_U = \dfrac{wl_1^2}{8}$ = 14.659 × (9.754)^2/8 = 174.4 kN-m

Ultimate Stress in strand, $f_{Pd} = f_{pe} + 7000d\left(1 - 1.36\dfrac{f_{PU}A_P}{f_{CK}bd}\right)\Big/ l$

$$= 1210 + 7000(229)\left(1 - 1.36\frac{1862(198)}{30(914)(229)}\right)\Big/(9754)$$

$$= 1361 \text{ MPa}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 2(99)(1361)/1000 = 269.5 \text{ kN}$

Stress block depth, $a = d - \sqrt{d^2 - \dfrac{2M_d}{0.85 f_{cd}\, b}}$

$$= 0.229 - \sqrt{0.229^2 - \frac{2(174.4)}{0.85(20000)(0.914)}}(1e3) = 55.816 \text{ mm}$$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}\left(d - \frac{a}{2}\right) = 269.5\left(229 - \frac{55.816}{2}\right)\Big/1000 = 54.194 \text{ kN-m}$$

Net ultimate moment, $M_{net} = M_U - M_{ult,PT} = 174.4 - 54.194 = 120.206$ kN-m

Required area of mild steel reinforcing,

$$A_S = \frac{M_{net}}{f_{yd}\left(d - \dfrac{a}{2}\right)} = \frac{120.206 \bullet 10^6}{(400)\left(229 - \dfrac{54.194}{2}\right)} = 1488.4 \text{ mm}^2$$

K factor used to determine the effective depth is given as:

$$K = \frac{M}{f_{cu}bd^2} = \frac{174.4}{30000/1.5(0.914)(0.229)^2} = 0.1819 < 0.156$$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) \le 0.95d = 192.2 \text{ mm}$$

Ultimate force in PT, $F_{ult,PT} = A_P(f_{PS}) = 2(99)(1303)/1000 = 258.0 \text{ kN}$

Ultimate moment due to PT,

$$M_{ult,PT} = F_{ult,PT}(z)/\gamma = 258.0(0.192)/1.15 = 43.12 \text{ kN-m}$$

Net Moment to be resisted by As,

$$M_{NET} = M_U - M_{PT}$$
$$= 174.4 - 43.12 = 131.28 \text{ kN-m}$$

The area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_{NET}}{f_{yd} z}_X = \frac{131.28}{0.87(400)(192)}(1e6) = 1965 \text{ mm}^2$$

Check of Concrete Stresses at Midspan:

Initial Condition (Transfer), load combination (D+PT$_i$) = 1.0D+0.0L+1.0PT$_I$

Tendon stress at transfer = jacking stress − stressing losses = 1490 − 186 = 1304 MPa
The force in the tendon at transfer, = $1304(197.4)/1000 = 257.4$ kN
Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m
Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 257.4(102 \text{ mm})/1000 = 26.25$ kN-m
Stress in concrete, $f = \dfrac{F_{PTI}}{A} \pm \dfrac{M_D - M_{PT}}{S} = \dfrac{-257.4}{0.254(0.914)} \pm \dfrac{65.04 - 26.23}{0.00983}$
where S = 0.00983m^3

$$f = -1.109 \pm 3.948 \text{ MPa}$$
$$f = -5.058(\text{Comp})\max,\ 2.839(\text{Tension})\max$$

Normal Condition, load combinations: (D+L+PT$_F$) = 1.0D+1.0L+1.0PT$_F$

Tendon stress at normal = jacking − stressing − long-term = 1490 − 186 − 94 = 1210 MPa
The force in tendon at normal, = $1210(197.4)/1000 = 238.9$ kN
Moment due to dead load, $M_D = 5.984(0.914)(9.754)^2/8 = 65.04$ kN-m
Moment due to live load, $M_L = 4.788(0.914)(9.754)^2/8 = 52.04$ kN-m
Moment due to PT, $M_{PT} = F_{PTI}(\text{sag}) = 238.9(102 \text{ mm})/1000 = 24.37$ kN-m

Stress in concrete for (D+L+PT$_F$),

$$f = \frac{F_{PTI}}{A} \pm \frac{M_{D+L} - M_{PT}}{S} = \frac{-238.8}{0.254(0.914)} \pm \frac{117.08 - 24.37}{0.00983}$$
$$f = -1.029 \pm 9.431$$
$$f = -10.460(\text{Comp})\max, 8.402(\text{Tension})\max$$

TS 500-2000 RC-PN EXAMPLE 001

Slab Punching Shear Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab punching shear design in ETABS.

The numerical example is a flat slab that has three 8-m spans in each direction, as shown in Figure 1.

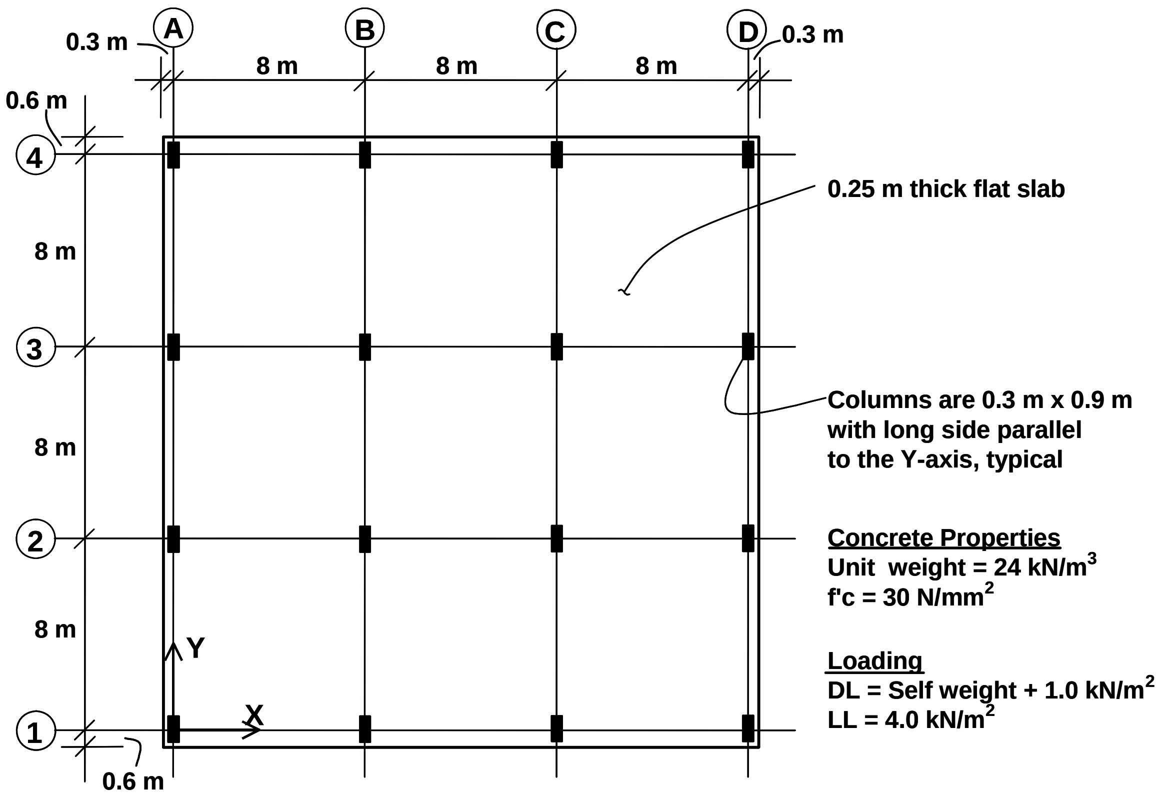


Figure 1: Flat Slab for Numerical Example

The slab overhangs beyond the face of the column by 0.15 m along each side of the structure. The columns are typically 0.3 m x 0.9 m with the long side parallel to the Y-axis. The slab is typically 0.25 m thick. Thick shell properties are used for the slab.

The concrete has a unit weight of 24 kN/m^3 and a f_{ck} of 30 N/mm^2. The dead load consists of the self weight of the structure plus an additional 1 kN/m^2. The live load is 4 kN/m^2.

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of punching shear capacity, shear stress and D/C ratio.

RESULTS COMPARISON

Table 1 shows the comparison of the punching shear capacity, shear stress ratio and D/C ratio obtained from ETABS with the punching shear capacity, shear stress ratio and D/C ratio obtained by the analytical method. They match exactly for this problem.

Table 1 Comparison of Design Results for Punching Shear at Grid B-2

Method	Shear Stress (N/mm^2)	Shear Capacity (N/mm^2)	D/C ratio
ETABS	1.695	1.278	1.33
Calculated	1.690	1.278	1.32

COMPUTER FILE: TS 500-2000 RC-PN Ex001.EDB

CONCLUSION

The ETABS results show an exact comparison with the independent results.

HAND CALCULATION

Hand Calculation For Interior Column Using ETABS Method

$d = [(250-26)+(250-38)]/2$ = 218 mm

b_0 = 518+ 1118 + 1118 + 518 = 3272 mm

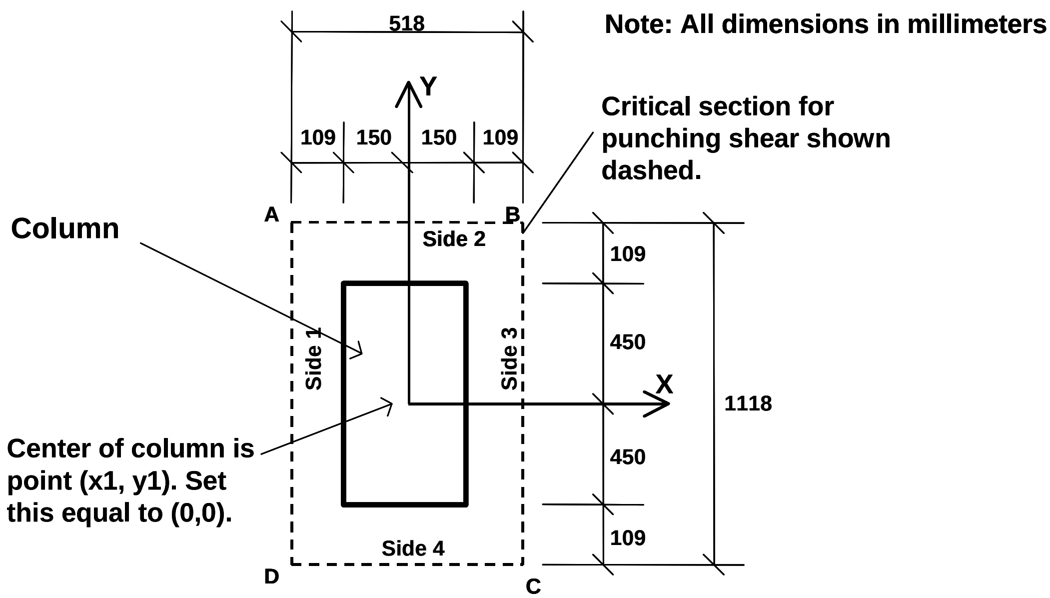


Figure 2: Interior Column, Grid B-2 in ETABS Model

$$\eta_2 = 1 - \frac{1}{1+\sqrt{\frac{1118}{518}}} = 0.595$$

$$\eta_3 = 1 - \frac{1}{1+\sqrt{\frac{518}{1118}}} = 0.405$$

The coordinates of the center of the column (x_1, y_1) are taken as (0, 0).

The following table is used for calculating the centroid of the critical section for punching shear. Side 1, Side 2, Side 3, and Side 4 refer to the sides of the critical section for punching shear as identified in Figure 2.

Item	Side 1	Side 2	Side 3	Side 4	Sum
x_2	–259	0	259	0	N.A.
y_2	0	559	0	–559	N.A.
L	1118	518	1118	518	b_0 = 3272
d	218	218	218	218	N.A.
Ld	243724	112924	243724	112924	713296
Ldx_2	–63124516	0	63124516	0	0
Ldy_2	0	63124516	0	–63124516	0

$$x_3 = \frac{\sum Ldx_2}{Ld} = \frac{0}{713296} = 0\,mm$$

$$y_3 = \frac{\sum Ldy_2}{Ld} = \frac{0}{713296} = 0\,mm$$

The following table is used to calculate I_{XX}, I_{YY} and I_{XY}. The values for I_{XX}, I_{YY} and I_{XY} are given in the "Sum" column.

Item	Side 1	Side 2	Side 3	Side 4	Sum
L	1118	518	1118	518	N.A.
d	218	218	218	218	N.A.
$x_2 - x_3$	–259	0	259	0	N.A.
$y_2 - y_3$	0	559	0	–559	N.A.
Parallel to	Y-Axis	X-axis	Y-Axis	X-axis	N.A.
Equations	5b, 6b	5a, 6a	5b, 6b	5a, 6a	N.A.
I_{XX}	5.43E+07	6.31E+07	2.64E+10	3.53E+10	1.23E+11
I_{YY}	6.31E+07	1.39E07	1.63E+10	2.97E+09	3.86E+10

From the ETABS output at Grid B-2:

V_d= 1126.498 kN

$0.4\eta M_{d,2}$ = -8.4226 kN-m

$0.4\eta M_{d,3}$ = 10.8821 kN-m

Maximum design shear stress in computed in along major and minor axis of column:

$$v_{pd} = \frac{V_{pd}}{u_p d}\left[1 + \eta\frac{0.4M_{pd,2}u_p d}{V_{pd}W_{m,2}} + \eta\frac{0.4M_{pd,3}u_p d}{V_{pd}W_{m,3}}\right], \qquad \text{(TS 8.3.1)}$$

At the point labeled A in Figure 2, x_4 = –259 and y_4 = 559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{8.423 \bullet 10^6 \left[3.86 \bullet 10^{10} \left(559 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)} + \frac{10.882 \bullet 10^6 \left[1.23 \bullet 10^{11} \left(-259 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)}$$

v_U = 1.5793 – 0.0383 – 0.0730 = **1.4680 N/mm²** at point A

At the point labeled B in Figure 2, x_4 = 259 and y_4 = 559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{8.423 \bullet 10^6 \left[3.86 \bullet 10^{10} \left(559 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)} + \frac{10.8821 \bullet 10^6 \left[1.23 \bullet 10^{11} \left(259 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)}$$

v_U = 1.5793 – 0.0383 + 0.0730 =**1.614 N/mm²** at point B

At the point labeled C in Figure 2, x_4 = 259 and y_4 = –559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{8.423 \bullet 10^6 \left[3.86 \bullet 10^{10} \left(-559 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)} + \frac{10.882 \bullet 10^6 \left[1.23 \bullet 10^{11} \left(259 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)}$$

v_U = 1.5793 + 0.0383 + 0.0730 = **1.690 N/mm²** at point C

At the point labeled D in Figure 2, x_4 = –259 and y_4 = –559, thus:

$$v_U = \frac{1126.498 \bullet 10^3}{3272 \bullet 218} - \frac{8.423 \bullet 10^6 \left[3.86 \bullet 10^{10} \left(-559 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)} + \frac{10.8821 \bullet 10^6 \left[1.23 \bullet 10^{11} \left(-259 - 0 \right) \right]}{\left(1.23 \bullet 10^{11} \right)\left(3.86 \bullet 10^{10} \right)}$$

v_U = 1.5793 + 0.383 – 0.0730 = **1.5446 N/mm²** at point D

Point C has the largest absolute value of v_u, thus v_{max} = **1.690 N/mm²**

The concrete punching shear stress capacity of a section with punching shear reinforcement is limited to:

$$v_{pr} = f_{ctd} = 0.35\sqrt{f_{ck}}/\gamma_c \qquad \text{(TS 8.3.1)}$$

$$v_{pr} = f_{ctd} = 0.35\sqrt{30}/1.5 = 1.278 \text{ N/mm}^2$$

$$\text{Shear Ratio} = \frac{v_{pd}}{v_{pr}} = \frac{1.690}{1.278} = 1.32$$

TS 500-2000 RC-SL EXAMPLE 001

Slab Flexural Design

PROBLEM DESCRIPTION

The purpose of this example is to verify slab flexural design in ETABS.

A one-way, simple-span slab supported by walls on two opposite edges is modeled using SAFE. The slab is 150 mm thick and spans 4 meters between walls. The walls are modeled as pin supports. The computational model uses a finite element mesh, automatically generated by SAFE. The maximum element size is specified as 0.25 meters. To obtain factored moments and flexural reinforcement in a design strip, one one-meter wide strip is defined in the X-direction on the slab, as shown in Figure 1.

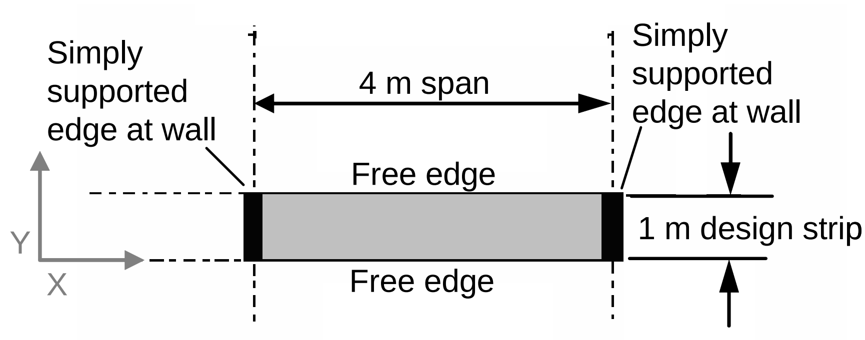


Figure 1 Plan View of One-Way Slab

One dead load case (DL4KPa) and one live load case (LL5KPa) with uniformly distributed surface loads of magnitudes 4 and 5 KN/m^2, respectively, are defined in the model. A load combination (COMB5kPa) is defined using the Turkish TS 500-2000 load combination factors, 1.4 for dead loads and 1.6 for live loads. The model is analyzed for both load cases and the load combination.

The slab moment on a strip of unit width is computed analytically. The total factored strip moments are compared with the ETABS results. After completing the analysis, design is performed using the Turkish TS 500-2000 code by ETABS and also by hand computation. Table 1 shows the comparison of the design reinforcements computed by the two methods.

PROGRAM NAME: ETABS
REVISION NO.: 0

GEOMETRY, PROPERTIES AND LOADING

Thickness	T, h	=	150	mm
Depth of tensile reinf.	d_c	=	25	mm
Effective depth	d	=	125	mm
Clear span	l_n, l_1	=	4000	mm
Concrete strength	f_{ck}	=	30	MPa
Yield strength of steel	f_{yk}	=	460	MPa
Concrete unit weight	w_c	=	0	N/m^3
Modulus of elasticity	E_c	=	25000	MPa
Modulus of elasticity	E_s	=	2×10^6	MPa
Poisson's ratio	ν	=	0	
Dead load	w_d	=	4.0	kPa
Live load	w_l	=	5.0	kPa

TECHNICAL FEATURES OF ETABS TESTED

- Calculation of flexural reinforcement
- Application of minimum flexural reinforcement

RESULTS COMPARISON

Table 1 shows the comparison of the ETABS total factored moments in the design strip with the moments obtained by the hand computation method. Table 1 also shows the comparison of the design reinforcements.

Table 1 Comparison of Design Moments and Reinforcements

Load Level	Method	Strip Moment (kN-m)	Reinforcement Area (sq-cm)
			A_s^+
Medium	ETABS	27.197	5.760
	Calculated	27.200	5.760

$A^+_{s,min}$ = 162.5 sq-mm

COMPUTER FILE: TS 500-2000 RC Ex001.EDB

CONCLUSION

The ETABS results show an acceptable comparison with the independent results.

HAND CALCULATION

The following quantities are computed for all the load combinations:

$\gamma_{m,\ steel} = 1.15$

$\gamma_{m,\ concrete} = 1.50$

$$f_{cd} = \frac{f_{ck}}{\gamma_{mc}} = \frac{30}{1.5} = 20$$

$$f_{yd} = \frac{f_{yk}}{\gamma_{ms}} = \frac{460}{1.15} = 400$$

$$c_b = \frac{\varepsilon_{cu} E_s}{\varepsilon_{cu} E_s + f_{yd}} d \ = 75 \text{ mm}$$

$$a_{\max} = 0.85 k_1 c_b \ = 52.275 \text{ mm}$$

where, $k_1 = 0.85 - 0.006\left(f_{ck} - 25\right) = 0.82$, $0.70 \le k_1 \le 0.85$

$$A_{s,\min} = \frac{0.8 f_{ctd}}{f_{yd}} bd \ = 325.9\, mm^2$$

Where $f_{ctd} = \dfrac{0.35\sqrt{f_{cu}}}{\gamma_{mc}} = \dfrac{0.35\sqrt{30}}{1.5} = 1.278$

For each load combination, the *w* and *M* are calculated as follows:

$$w = (1.4 w_d + 1.6 w_t)\, b$$

$$M = \frac{w l_1^2}{8}$$

COMB100

w_d = 4.0 kPa

w_t = 5.0 kPa

w = 13.6 kN/m

$M_{\text{-}strip}$ = 27.2 kN-m

$M_{\text{-}design}$ = 27.2366 kN-m

The depth of the compression block is given by:

The depth of the compression block is given by:

$$a = d - \sqrt{d^2 - \frac{2|M_d|}{0.85 f_{cd} b}} \qquad \text{(TS 7.1)}$$

$$a = 125 - \sqrt{125^2 - \frac{2 \bullet 27.2366 \bullet 10^6}{0.85 \bullet 20 \bullet 1000}} = 13.5518 \text{ mm}$$

If $a \le a_{max}$ (TS 7.1), the area of tensile steel reinforcement is then given by:

$$A_s = \frac{M_d}{f_{yd}\left(d - \frac{a}{2}\right)} = \frac{27.2366 \bullet 10^6}{400\left(125 - \frac{13.5518}{2}\right)} = 576 \text{ mm}^2$$

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