

Full Length Research Paper

New approach of delay penalties formulation: Application to the case of construction projects in the Republic of Congo

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In the construction industry, delay penalties are usually applied at the end of the normal contractual period. Those penalties, daily in general, are of two types: linear monotonous delay penalties and gradual delay penalties. Usually penalties are fixed without considering the real size of the project, both by its initial overall cost and by its length. In the ordinary classic method, we focus on determining the threshold amount of delay penalties, which enables to determine the period of the contract cancellation. However, it is often possible to find cancellation date excessively elongated, exceeding in some cases the initial term of the project, in particular for relative low delay penalties. In this work, we propose a new approach for calculating delay penalties, which on the contrary focuses on ascertaining the cancellation date and then deducing the total amount of delay penalties. It is a gradual method which allows to estimate the threshold total amount of first level (stage 1) delay penalties which are low enough, then those of the second level (stage 2) associated with higher penalties, and if necessary those of the third level (stage 3) sometimes lower or higher than the previous ones. This gives us various possibilities of divisions which enable both the owner and the contractor to agree in advance on a given type of delay penalties variation on signing contract. We also indicate how to know if a delay penalty is too high or, conversely, too low by comparing the value of the daily delay penalty in stage 1 with that classical uniform delay penalty.

Key words: Delay penalties, cancellation date, delay threshold, construction project.

INTRODUCTION

The construction industry and public works sector is particularly important in the economic and social development of a country. It is an essential tool of public policy. It is required for the functioning of enormous resources regarding the extent of human, material and financial means which are employed. To secure the development sector, contracts relating thereto are subject to strict regulations regarding their execution. Generally, the contractor is mainly responsible for the delay in a

construction. To compel the contractor to complete the work on time according to the contract, a section on delay penalties was conceived. Unfortunately, the application of delay penalties is often subject to many problems. They are still not defined consistently. Sometimes the delay penalties are too low, sometimes they are too high. Remember that delay penalties apply to the time overrun context (Textes, 2006; Droit, 2007). The penalties are a fine charged to the contractor for delays in carrying out the work. In most countries the penalties for delay in construction projects are planned by regulations (Chicago Housing Authority, 2007; Mbani, 2007) including the specification of general administrative clauses or terms of

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clauses. The delay penalties are calculated from a fraction of the original contract price per calendar day of delay. This fraction is fixed at the contract signing. It can vary from 1/1,000 to 1/200 (Textes, 2006; Droit, 2007), without a sufficiently clear justification. When the total amount of penalties reaches 5 to 10% of the initial total cost of the construction project, the contract is then cancelled (Chicago Housing Authority, 2007; Al-Tabtabai Hashed, 1998). However, with this procedure, it may happen that the date of the contract's cancellation is excessively postponed. A major consequence that results from it is a great exceedance of the total duration of the construction project.

In the literature, there are very few researchers who were interested in the methods of calculating delay penalties in order to improve the existing formulae. Al-Tatabaï et al. (1998) attempted to bring some improvements in the case of Koweit. They proposed two formulae for calculating delay penalties applied after the expiration of the contractual period. The proposed improved formulae aim to: 1) Control the progression of work during its execution, 2) Reduce the weight of penalties during the first days of delay and Three) Allow the gradual evolution of the penalties curve depending on the duration. Other authors like D'Alpaos et al. (2009) think that the delay penalties costs depend closely on the "quality" of the judicial system. For the specific case of Italy, they showed that the optimal penalties should be higher than those applied by the current Italian law.

Given the importance of delay penalties in that they induce additional costs in a time overrun context (Louzolo et al., 2007; Moushki et al., 2005) it seems useful to develop a calculation method for determining more consistently the maximum allowed delay penalties and the amount of the penalties associated, according to the importance of the construction project. This should normally be suitable for both the owner and the contractor.

DIFFERENT TYPES OF DELAY PENALTIES AND THEIR APPLICATION MODES

Delay penalties applied in the Republic of the Congo

Delay penalties applied in the construction projects in the Republic of the Congo, like in most countries they are linear and can be classed in three categories:

- Category I: First-level monotonous delay penalties (MDP1);
- Category II: Second-level monotonous delay penalties (MDP2);
- Category III: Gradual delay penalties (GDP).

First-level daily monotonous delay penalties (MDP1)

This type of penalty applies uniformly. They are a fraction

(f) of the initial total cost (C_T) of the construction work. This fraction generally runs in the interval $f = 1/10,000$ to 1/200, hence the penalties:

$$C_T/10,000 \leq MDP1 \leq C_T/200 \tag{1}$$

The application field of the MDP1 is the following one:

$$D_T + 1 \rightarrow D_T + R \tag{2}$$

With:

- D_T: initial total duration of the project (day)
- R: period for reaching the cancellation date (day)

The cancellation is decided when the total amount of delay penalties reaches 5 to 10% of the initial cost, the percentage being determined during the contract signing. We have:

$$R_5 = \frac{5}{100} C_T \frac{1}{f C_T} = \frac{1}{20 f} \tag{3-a}$$

Or:

$$R_{10} = \frac{1}{10 f} \tag{3-b}$$

Formula (3) clearly shows that the cancellation date R (R₅ or R₁₀) is independent of both the initial total cost and the initial duration of the project. It is inversely proportional to fraction f for delay penalties of type MDP1.

The advantage of the (MDP1) lies in its ease application. Indeed, penalties are directly determined from the initial total cost of the project and apply uniformly during the concerned period. However, the MDP1 does not permit the contractor a certain margin allowing him to reduce the delay in a relatively short period, at lower cost.

Second-level daily monotonous delay penalties (MDP2)

These penalties differ from the MDP1 by the fact that the fraction f only applies on the remaining amount of the project construction. Generally, we take $1/5000 \leq f \leq 1/1000$ hence the penalties:

$$\frac{1}{5000} |(C_T - C_{TE})| \leq MDP2 \leq \frac{1}{1000} |(C_T - C_{TE})| \tag{4}$$

with:

- C_{TE}: total cost of activities performed.

The application field is:

$$D_T + 1 \rightarrow D_T + R$$

Here, we have:

$$R = \frac{1}{100} C_T \frac{1}{f |C_T - C_{TE}|} \quad (5)$$

This method seems advantageous for the contractor in so far the delay penalties will be even lower than the cost of the construction work already performed will approaching the initial total cost would still have to argue that construction work in tandem with expenses provided. But as the cost overruns are very common in the construction field, especially in developing countries, it could happen that we find a negative delay penalty, so the difference $C_T - C_{TE}$ will be given in absolute value. Note also that the cancellation date could be extended indefinitely when C_{TE} is similar to C_T , and in extreme cases, R would have no meaning. So the penalty of type MDP2 should be applied with great caution.

Gradual delay penalties (GDP)

These penalties apply in two steps;

a) Step one: First-level gradual delay penalties (GDP1)

For this first step, lasting up to 40 days, the penalties are lower. This "grace period" is intended to encourage the contractor to reduce the delay as soon as possible. The application domain of GDP1 lies in the range:

$$D_T + 1 \rightarrow D_T + N \quad (6)$$

with $20 \leq N \leq 40$

N is the number of days set for the first step.

The fraction determining the penalty is usually in the range $\frac{1}{10000} \leq f \leq \frac{1}{5000}$, hence the daily penalties:

$$\frac{1}{10000} C_T \leq GDP1 \leq \frac{1}{5000} C_T \quad (7)$$

during N days.

b) Step two: Second-level gradual delay penalties (GDP2)

Here the penalties that apply after the first N days are higher. The application field extends in the interval:

$$D_T + (N + 1) \rightarrow D_T + R \quad (8)$$

The fraction relating to the delay penalty is taken in the interval: $\frac{1}{3000} \leq f \leq \frac{1}{2000}$.

Hence the penalties:

$$\frac{1}{3000} C_T \leq GDP2 \leq \frac{1}{2000} C_T \quad (9)$$

The GDP2 has the advantage of having two slopes. At the first slope, the delay penalties are low. The aim of this provision is to encourage the contractor to complete the construction work quickly, trying to minimize financial losses. The second slope is related to higher penalties, with the intention of forcing contractor to carry out the construction work much faster, otherwise a financial downfall might occur, and given the very high fees would have to pay.

Determination of the cancellation dates for some construction projects

The cancellation date corresponds to the deadline at which the total amount of penalties reaches the acceptable threshold. We present in Table 1 the results of calculating the cancellation date for a sample of ten construction projects, using different types of delay penalties.

It should be noted that the cancellation date R is determined for a total amount of penalties equal to either 5 or 10% of the initial cost of the project, according to the contract. We have carried out calculation on the basis of 25 working days per month.

We noted that for the projects P2, P3 and P6, the calculation of the threshold amount of penalties, leads to a cancellation date that is reached after 25 months, starting from the fact that the construction work has already been performed at 60% at the end of the initial contractual period. Finally, this cancellation date appears much higher than the original completion dates that are 12 months for the project P2 and 18 months for the projects P3 and P6, corresponding to respective exceedances of the total duration of 208% (P2) and 139% (P3 and P4). The fraction ($f=1/5,000$) which was set is too low and has practically no binding effect on the contractor. Moreover, it is also true for the project P4 ($f = 1/5,000$) where the cancellation date is reached in 10 months, while the initial deadline for completion of the project is 6 months, which results in a rate exceeding the period of 167%. For the project P9 ($f = 1/5,000$), we find a cancellation date of 10 months, close to the expected initial total duration which is 12 months, in other words an exceedance of 83%. However, for the project P7 ($f = 3/10,000$), the cancellation date seems too short, it is only of 1.3 month for a total completion duration of 10 months, that is to say an exceedance of 13% only. The fraction $f = 3/1,000$ set when signing the contract, makes delay

Table 1. Cancellation date of 10 construction projects in Republic of Congo (classical formulae).

Project code	Initial total cost: C_T ($\times 10^9$ F CFA')	Initial duration of the construction project : D_T (month)	Uniform daily delay penalties: ω_c ($\times 10^6$ F CFA)	Daily cost of the construction project: W ($\times 10^6$ F CFA/ day)	Attenuation factor: β	Fraction related to the type of delay penalty: f				Period for reaching the cancellation date: R_{day} (month)		Rate of exceeding the initial duration (%)
						$MDP1$	$MDP2$	$GDP1$	$GDP2$	Threshold at 5% C_T	Threshold at 10% C_T	
P1	52.055	46	5.205, 17.351	45.265	0.12, 0.38			1/10000	1/3000	175 (7)		15
P2	5.740	12	0.459	19.133	0.06			1/5000		625 (25)		208
P3	18.314	18	1.465	40.698	0.09			1/5000		625 (25)		139
P4	0.020	6	0.004	0.133	0.03	1/5000				250 (10)		167
P5	5.115	36	21.046	5.683	0.9			1/1000		125 (5)		14
P6	6.000	18	0.480	13.333	0.09			1/5000		625 (25)		139
P7	0.150	10	0.450	0.600	0.75	3/1000					33 (1,3)	13
P8	0.040	5	0.080	0.320	0.25	1/500				25 (1)		20
P9	0.039	12	0.008	0.130	0.06	1/5000				250 (10)		83
P10	0.050	6	0.050	0.333	0.15	1/1000					100 (4)	67

(*): 1 Euro = 656 F CFA, GDP1: penalties applicable during the early forty days of delay. GDP2: penalties applicable after the fortieth day of delay until the cancellation date. MDP1: penalties applicable uniformly from the first day of delay until the cancellation date. MDP2: delay penalties calculated by supposing about 60% the total cost of performed work before completion time. β : attenuation factor.

penalty apparently too strong. Based on the results (Table 1), we can classify the delay penalties according to the rate of exceeding the initial duration (REID) of the construction project from the contractual cancellation date. We get the four categories below:

- REID < 20% : penalties too high;
- 20% ≤ REID ≤ 30% : adequate penalties;
- 30% < REID < 60% : low penalties;
- REID ≥ 60% : penalties too low.

Delay penalties applied in some countries

Case of Kuwait

We rely here on the study of (Al-Tabtabaï et al., 1998).
The former formula used in Kuwait for the

calculation of delay penalties per calendar day was like:

$$\omega = \beta \frac{C_T}{D_T} \tag{10}$$

With:
 C_T : initial total cost of the project
 D_T : initial total duration of the project
 β : coefficient varying between 1/4 and 3/2

This calculation method does not compel enough the contractor to speed up work once the execution time is exceeded. This is mainly due to the nature of monotonous linear penalties. These are constant throughout the delay period, which cannot help to encourage the contractor to quickly reduce the delay.

Then a hybrid method has replaced the previous one. Before the expiry of the contractual period, delay penalties are applied per month according to the critical path method (CPM). If in the meantime the delay is caught up, the amount withheld shall be returned to the contractor. After the expiration of the contractual period, the penalty applies per calendar day, with a lower coefficient β . This hybrid method allows control of the progress of construction work as and when they run. However, it has weaknesses particularly with regard to the application of penalties as they remain constant during all the period of delayed work.

Providing a synthesis of the previous formulae, Al-Tabtabaï et al. (1998) wanted to introduce some improvements. First, following the critical path and before the expiry of the initial contractual period, the amount of delay penalties is declining

according to the progress of work fixed per intervals. With regard to delay penalties applied after the expiry of the contractual period, two formulae have been proposed. The first is a quadratic equation of the form:

$$\left(\frac{C_P}{C_T} \times 100 \right) = 25 \times 10^{-3} \times \left(\frac{D_R}{D_T} \times 100 \right)^2 \quad (11)$$

with:

C_P : penalties cost
 D_R : delay duration

The second formula is a cubic equation which has the expression:

$$\left(\frac{C_P}{C_T} \times 100 \right) = 125 \times 10^{-5} \times \left(\frac{D_R}{D_T} \times 100 \right)^3 \quad (12)$$

To establish these two formulae, the authors supposed that a maximum delay of 20% on the total project duration could correspond to a cost of delay penalties of 10% of the total initial project cost, which is the threshold amount of penalties resulting in the cancellation of the contract.

Case of Cambodia

The delay penalties in Cambodia can be classified as delay first-level monotonous delay penalties (first-level daily monotonous delay penalties and second-level daily monotonous delay penalties parts of this work). The daily penalty is expressed as follows (Droit, 2007).

$$MDP1 = \frac{1}{1000} C_T, \text{ or } MDP2 = \frac{1}{1000} (C_T - C_{TE}) \quad (13)$$

With C_{TE} : total cost of construction work already performed.

These penalties must not exceed 10% of the total cost.

Case of France

In France, first-level and second-level delay penalties are applied (Decret, 1994).

$$MDP1 = \frac{1}{3000} C_T, \text{ or } MDP2 = \frac{1}{3000} (C_T - C_{TE}) \quad (14)$$

Conclusion on the application mode of different types of delay penalties

Delay penalties which are applied only after the

exceedance of the completion time of work, particularly in case where they are low and monotonous, do not sufficiently encourage the contractor to accelerate the work in order to reduce rapidly the delay.

The penalties of type MDP1 and MDP2 show that for two different projects with the same fraction f and having reached the same level of works, there is always the same duration for the cancellation date regardless of the initial total cost and the initial total duration of the projects. In other words, the cancellation date does not depend on the importance of the project both in cost and duration. Therefore, penalties of type MDP1 and MDP2 are inappropriate for determining the cancellation date. In addition, this type of penalties unduly prolong the cancellation date particularly for low values of the fraction f and finally, the type of penalties that seems to suit the penalty delay of type GDP (gradual delay penalty).

NEW FORMULATION OF DELAY PENALTIES

The inefficiency and the inadequacy of delay penalties result from the fact that they generally do not take into account the size of the project both in cost and in duration.

To help actors operating in the construction field to establish suitable delay penalties in procurement, we propose a new method for calculating penalties. It is an approach which associates the initial cost and the total duration of the construction project, taking into account the size of the project for determining the weight of the penalties.

The formula we try to establish is an improved version of the type GDP.

Determination of the attenuation factor

We know that the daily cost of the construction project is written:

$$W = \frac{C_T}{D_T} \quad (15)$$

According to formula (10), delay penalties per calendar day can be expressed as follows:

$$\omega = \beta W \quad (16)$$

In the classical form formulae (1 and 7), we have:

$$\omega_C = f C_T \quad (17)$$

We can then deduce, equalling formulae (16) and (17):

$$\beta = f D_T \quad (18)$$

Table 2. Value of β according to the importance of the project.

Daily cost of the construction work: $W (\times 10^6 \text{ F CFA/day})$	Attenuation factor β
0.001 - 0.050	1
0.051 - 0.500	1 - 0.75
0.101 - 0.500	0.75 - 0.50
0.501 - 1.000	0.75 - 0.50
1.001 - 2.000	0.50 - 0.25
2.001 - 3.000	0.50 - 0.25
3.001 - 4.000	0.25 - 0.20
4.001 - 5.000	0.25 - 0.20
5.001 - 10.000	0.20 - 0.15
> 10.000	0.15 - 0.10

with D_T in days.

We will call β the “attenuation factor” (or weight of delay penalties). However, Table 1 shows that the values of β are not related in proportion to the initial total cost C_T or the initial total duration D_T of the project.

If we assume it is the daily cost of the project that indicates its importance, we can then determine the values of the factor β (Table 2) so that the daily penalties are not excessively high.

Determination of the delay penalties thresholds

Considering the formula (16), we can determine the delay penalties thresholds (P_s), that is, the total amount of penalties causing the cancellation of the contract. We have:

$$P_s = \omega S \tag{19}$$

where S is the delay threshold, or in other words the number of maximum allowed delay days. We can calculate S from the following expression:

$$S = X D_T \tag{20}$$

where X is the percentage giving the delay threshold. The value of X is fixed at the signing of the contract.

According to the results of Table 1, we can reasonably set the values of X in the range $10\% \leq X \leq 30\%$. Al-Tabtabaī et al. (1998) for example, have allowed a maximum delay of 20% over the initial term of a project.

For their part, Sadi et al. (2006) conducted a study which showed that the average overrun time of a project is between 10 and 30% over the initial project duration.

To encourage the contractor to quickly reduce the delay, we define a first stage in which the penalties are relatively low. The duration of this stage is represented by

S_1 .

A second stage of higher penalties (S_2) is provided for exceeding the first stage, to compel the contractor to complete the work.

Finally, a third stage of penalties more or less moderate (S_3) occurs when exceeding the second threshold, until the cancellation of the contract.

Figure1 shows the process of applying delay penalties. The duration of the first stage S_1 is obtained from the following formula:

$$S_1 = X_1 D_T \tag{21}$$

where X_1 , expressed as a percentage, indicates the extent of the first stage of penalties.

Similarly, the duration of the second stage is written:

$$S_2 = X_2 D_T \tag{22}$$

and the third stage is:

$$S_3 = X_3 D_T \tag{23}$$

where X_2 and X_3 denote the extent of stages 2 and 3 in percentage.

The numbers X_1 , X_2 , and X_3 should verify the following condition:

$$X_1 + X_2 + X_3 = X \tag{24}$$

We express below the formulae for determining the threshold amounts of penalties in each of the three stages PS_1 , PS_2 and PS_3 respectively:

$$P_{S_1} = \omega_1 S_1 \tag{25-a}$$

$$P_{S_2} = \omega_2 S_2 \tag{25-b}$$

$$P_{S_3} = \omega_3 S_3 \tag{25-c}$$

with:

$$\omega_1 = \gamma \omega, \text{ and} \tag{26-a}$$

$$\omega_2 = (2 - \gamma) \omega \tag{26-b}$$

The quantities ω_1 , ω_2 , and ω_3 represent the daily delay penalties relative to the first, second, and eventually the third stage respectively.

The quantity γ represents the “bending factor” and chosen as:

$$\frac{1}{5} \leq \gamma < 1 \tag{27}$$

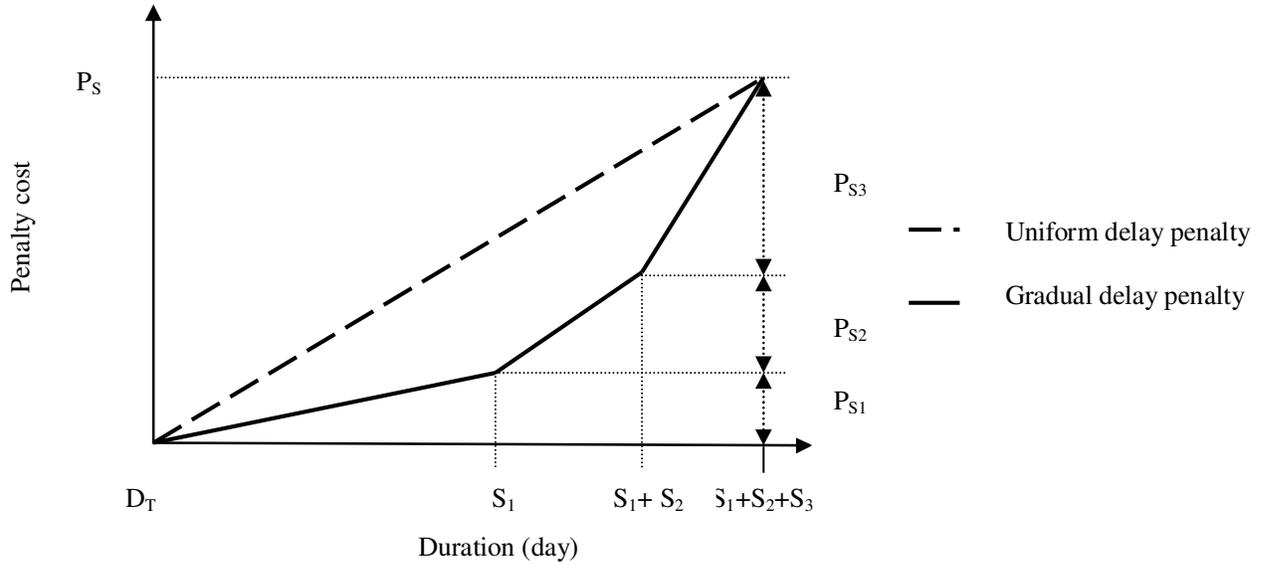


Figure 1. Diagram of gradual penalty.

So we always have $\omega_2 > \omega_1$.

The constraint (27) combined with (26), means that if the delay penalties in the first stage are low ($\gamma \rightarrow 1/5$), they will be very high in the second stage. Conversely, if the penalties are high in the first stage ($\gamma \rightarrow 1$), then they will be more moderate in the second stage.

The percentages X_1 and X_2 are set at the beginning and represent fractions of X , that is:

$$X_1 = aX \quad (28-a)$$

and

$$X_2 = bX \quad (28-b)$$

where a and b are fractions.

From the equation (24), we deduce:

$$X_3 = [1 - (a + b)]X \quad (29)$$

with the condition

$$a + b < 1 \quad (30)$$

On the other hand, we can write:

$$P_S = P_{S1} + P_{S2} + P_{S3} \quad (31)$$

From the relations (19), (20), (23) and (24), expression (31):

$$\omega X = \gamma \omega X_1 + (2 - \gamma) \omega X_2 + \omega_3 X_3 \quad (32)$$

Thus, given equation s(28) and (29), equation (32) allows us to obtain the expression of ω_3 , namely:

$$\omega_3 = \left[\frac{(1 - 2b) - (a - b)\gamma}{1 - (a + b)} \right] \omega \quad (33)$$

As $\omega_3 > 0$, we must add the following condition:

$$b < \frac{1 - a\gamma}{2 - \gamma} \quad (34)$$

Summary of quantities to be determined at different stages

Total duration of penalties per stage

Stage 1:

$$S_1 = aXD_T \quad (35)$$

Stage 2:

$$S_2 = bXD_T \quad (36)$$

Stage 3:

$$S_3 = [1 - (a + b)]XD_T \quad (37)$$

Table 3. Value of parameters γ , a and b.

Parameters Delay threshold (%)	Bending factor: γ	Fraction of the percentage of delay related to stage1(a)	Fraction of the percentage of delay related to stage 2(b)
30	1/5	1/4	< 0.528
		1/2	< 0.500
		3/4	< 0.472
	9/10	1/4	< 0.705
		1/2	< 0.500
		3/4	< 0.295
20	2/5	1/3	< 0.542

Total cost of penalties per stage

Stage 1:

$$P_{S1} = \gamma\omega a X D_T \tag{38}$$

Considering formula (10):

$$P_{S1} = a\gamma\beta X C_T \tag{39}$$

Stage 2:

$$P_{S2} = (2 - \gamma)\omega b X D_T = b(2 - \gamma)\beta X C_T \tag{40}$$

Stage 3:

$$P_{S3} = [(1 - 2b) - (a - b)\gamma]\omega X D_T = [(1 - 2b) - (a - b)\gamma]\beta X C_T \tag{41}$$

Range of different stages of delay penalties

Stage 1:

$$[D_T + 1; D_T + S_1] \tag{42}$$

Stage 2:

$$[D_T + (S_1 + 1); D_T + (S_1 + S_2)] \tag{43}$$

Stage 3:

$$[D_T + (S_1 + S_2 + 1); D_T + (S_1 + S_2 + S_3)] \tag{44}$$

It should be noted that the two basic classical quantities set when signing the contract are:

C_T : initial total cost of the project;

D_T : initial total duration of the project.

Added to this are the new parameters as follows:

- X: maximum extent of tolerated delay in percentage
- a: fraction of the percentage of delay related to stage1
- b: fraction of the percentage of delay related to stage2
- β : attenuation factor.
- γ : bending factor.

APPLICATION TO TEN CONSTRUCTION PROJECTS

We reported in Table 3 some values of parameters γ , a and b, considering the constraints (30) and (34).

We give in Tables 4 to 7 the results of calculations relative to the threshold duration of the delay and the threshold amounts of delay penalties in the different stages, and the cancellation dates.

In the approach we propose, we can manipulate parameters γ and a. We then get various options for dividing the durations of delay thresholds. Thus, we have the following six options for dividing the three stages:

- a) $S_1 > S_2 > S_3$; b) $S_1 > S_3 > S_2$; c) $S_2 > S_1 > S_3$; d) $S_2 > S_3 > S_1$; e) $S_3 > S_1 > S_2$; f) $S_3 > S_2 > S_1$

It should be noted that if N is the number of divisions, then there will be N! possibilities of divisions.

The divisions will be designated by D_{ijk} , where i, j and k indicate the different delay threshold per stage. The order i, j and k give the decreasing extent of delay threshold. For example: D_{123} means that the stage 1 has a delay threshold larger than the stage 2 and that the stage 2 has a delay threshold greater than the stage 3, which is to say, $D_{123} = S_1 > S_2 > S_3$. Similarly, we have $D_{132} = S_1 > S_2 > S_3$, etc.

The possibility a) or division of type D_{123} refers to Table 4, the corresponding curve is in Figure 2.

Table 4. Delay threshold duration and penalty threshold total amount in various stage (X = 30%; $\gamma = 1/5$; a = 3/4; b = 1/5); type D₁₂₃

Project code	Constants				Initial total cost of project C _T (10 ⁶ F CFA)	Initial duration of the project D _T (month)	Daily cost of the project W (10 ⁶ F CFA/day)	Attenuation factor β	Delay threshold duration in various stage			Penalty threshold total amount in various stage			Cancellation date S (day)
	X	a	b	γ					Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3	
									S ₁ (day)	S ₂ (day)	S ₃ (day)	P _{S1} (10 ⁶ F CFA)	P _{S2} (10 ⁶ F CFA)	P _{S3} (10 ⁶ F CFA)	
P ₁	0.30	0.75	0.20	0.20	52054.647	46	45.265	0.25	259	69	17	585.615	1405.475	1913.008	345
P ₂	0.30	0.75	0.20	0.20	5740.302	12	19.134	0.50	68	18	5	129.157	309.976	421.912	90
P ₃	0.30	0.75	0.20	0.20	18314.078	18	40.698	0.25	101	27	7	206.033	494.480	673.042	135
P ₄	0.30	0.75	0.20	0.20	19.661	6	0.131	1.00	34	9	2	0.885	2.123	2.890	45
P ₅	0.30	0.75	0.20	0.20	5114.829	36	5.683	0.75	203	54	14	172.625	414.301	563.910	270
P ₆	0.30	0.75	0.20	0.20	6000.000	18	13.333	0.25	101	27	7	67.500	162.000	220.500	135
P ₇	0.30	0.75	0.20	0.20	150.000	10	0.600	1.00	56	15	4	6.750	16.200	22.050	75
P ₈	0.30	0.75	0.20	0.20	39.690	5	0.318	1.00	28	8	2	1.786	4.287	5.834	38
P ₉	0.30	0.75	0.20	0.20	38.987	12	0.130	1.00	68	18	5	1.754	4.211	5.731	90
P ₁₀	0.30	0.75	0.20	0.20	50.216	6	0.335	1.00	34	9	2	2.260	5.423	7.382	45

N.B. Values of S₁, S₂ and S₃ are rounded up to unit. It will be taken into account if verification of the results

Table 5. Delay threshold duration and penalty threshold total amount in various stage (X = 30%; $\gamma = 9/10$; a = 1/4; b = 7/10); type D₂₁₃.

Project code	Constants				Initial total cost of project C _T (10 ⁶ F CFA)	Initial duration of the project D _T (month)	Daily Cost of the project W (10 ⁶ F CFA/day)	Attenuation factor β	Delay threshold duration in various stage			Penalty threshold total amount in various stage			Cancellation date S (day)
	X	a	b	γ					Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3	
									S ₁ (day)	S ₂ (day)	S ₃ (day)	P _{S1} (10 ⁶ F CFA)	P _{S2} (10 ⁶ F CFA)	P _{S3} (10 ⁶ F CFA)	
P ₁	0.30	0.25	0.70	0.90	52054.647	46	45.265	0.25	86	242	17	878.422	3006.156	19.520	345
P ₂	0.30	0.25	0.70	0.90	5740.302	12	19.134	0.50	23	63	5	193.735	663.005	4.305	90
P ₃	0.30	0.25	0.70	0.90	18314.078	18	40.698	0.25	34	95	7	309.050	1057.638	6.868	135
P ₄	0.30	0.25	0.70	0.90	19.661	6	0.131	1.00	11	32	2	1.327	4.542	0.029	45
P ₅	0.30	0.25	0.70	0.90	5114.829	36	5.683	0.75	68	189	14	258.938	886.144	5.754	270
P ₆	0.30	0.25	0.70	0.90	6000.000	18	13.333	0.25	34	95	7	101.250	346.500	2.250	135
P ₇	0.30	0.25	0.70	0.90	150.000	10	0.600	1.00	19	53	4	10.125	34.650	0.225	75
P ₈	0.30	0.25	0.70	0.90	39.690	5	0.318	1.00	9	26	2	2.679	9.168	0.060	38
P ₉	0.30	0.25	0.70	0.90	38.987	12	0.130	1.00	23	63	5	2.632	9.006	0.058	90
P ₁₀	0.30	0.25	0.70	0.90	50.216	6	0.335	1.00	11	32	2	3.390	11.600	0.075	45

Table 6. Delay threshold duration and penalty threshold total amount in various stage ($X = 30\%$; $\gamma = 1/5$; $a = 1/4$; $b = 2/5$); type D_{231} .

Project code	Constants				Initial total cost of the project C_T (10^6 F CFA)	Initial duration of the project D_T (month)	Daily cost of the project W (10^6 F CFA/day)	Attenuation factor β	Delay threshold duration in various stage			Penalty threshold total amount in various stage			Cancellation date S (day)
	X	a	b	γ					Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3	
					S_1 (day)	S_2 (day)	S_3 (day)	P_{S1} (10^6 F CFA)	P_{S2} (10^6 F CFA)	P_{S3} (10^6 F CFA)					
P ₁	0.30	0.25	0.40	0.20	52054.647	46	45.265	0.25	86	138	121	195.205	2810.951	897.943	345
P ₂	0.30	0.25	0.40	0.20	5740.302	12	19.134	0.50	23	36	32	43.052	619.953	198.040	90
P ₃	0.30	0.25	0.40	0.20	18314.078	18	40.698	0.25	34	54	47	68.678	988.960	315.918	135
P ₄	0.30	0.25	0.40	0.20	19.661	6	0.131	1.00	11	18	16	0.295	4.247	1.357	45
P ₅	0.30	0.25	0.40	0.20	5114.829	36	5.683	0.75	68	108	95	57.542	828.602	264.692	270
P ₆	0.30	0.25	0.40	0.20	6000.000	18	13.333	0.25	34	54	47	22.500	324.000	103.500	135
P ₇	0.30	0.25	0.40	0.20	150.000	10	0.600	1.00	19	30	26	2.250	32.400	10.350	75
P ₈	0.30	0.25	0.40	0.20	39.690	5	0.318	1.00	9	15	13	0.595	8.573	2.739	38
P ₉	0.30	0.25	0.40	0.20	38.987	12	0.130	1.00	23	36	32	0.585	8.421	2.690	90
P ₁₀	0.30	0.25	0.40	0.20	50.216	6	0.335	1.00	11	18	16	0.753	10.847	3.465	45

Table 7. Delay threshold duration and penalty threshold total amount in various stage ($X = 20\%$; $\gamma = 2/5$; $a = 1/3$; $b = 1/4$); type D_{312} .

Project code	Constants				Initial total cost of the project C_T (10^6 FCFA)	Initial duration of the project D_T (month)	Daily cost of the project W (10^6 FCFA/day)	Attenuation factor β	Delay threshold duration in various stage			Penalty threshold total amount in various stage (10^6 F CFA)			Cancellation date S (day)
	X	a	b	γ					Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3	
					S_1 (day)	S_2 (day)	S_3 (day)	P_{S1}	P_{S2}	P_{S3}					
P ₁	0.20	0.33	0.25	0.40	52054.647	46	45.265	0.25	77	58	96	347.031	1041.093	1214.608	230
P ₂	0.20	0.33	0.25	0.40	5740.302	12	19.134	0.50	20	15	25	76.537	229.612	267.881	60
P ₃	0.20	0.33	0.25	0.40	18314.078	18	40.698	0.25	30	23	38	122.094	366.282	427.328	90
P ₄	0.20	0.33	0.25	0.40	19.661	6	0.131	1.00	10	8	13	0.524	1.573	1.835	30
P ₅	0.20	0.33	0.25	0.40	5114.829	36	5.683	0.75	60	45	75	102.297	306.890	358.038	180
P ₆	0.20	0.33	0.25	0.40	6000.000	18	13.333	0.25	30	23	38	40.000	120.000	140.000	90
P ₇	0.20	0.33	0.25	0.40	150.000	10	0.600	1.00	17	13	21	4.000	12.000	14.000	50
P ₈	0.20	0.33	0.25	0.40	39.690	5	0.318	1.00	8	6	10	1.058	3.175	3.704	25
P ₉	0.20	0.33	0.25	0.40	38.987	12	0.130	1.00	20	15	25	1.040	3.119	3.639	60
P ₁₀	0.20	0.33	0.25	0.40	50.216	6	0.335	1.00	10	8	13	1.339	4.017	4.687	30

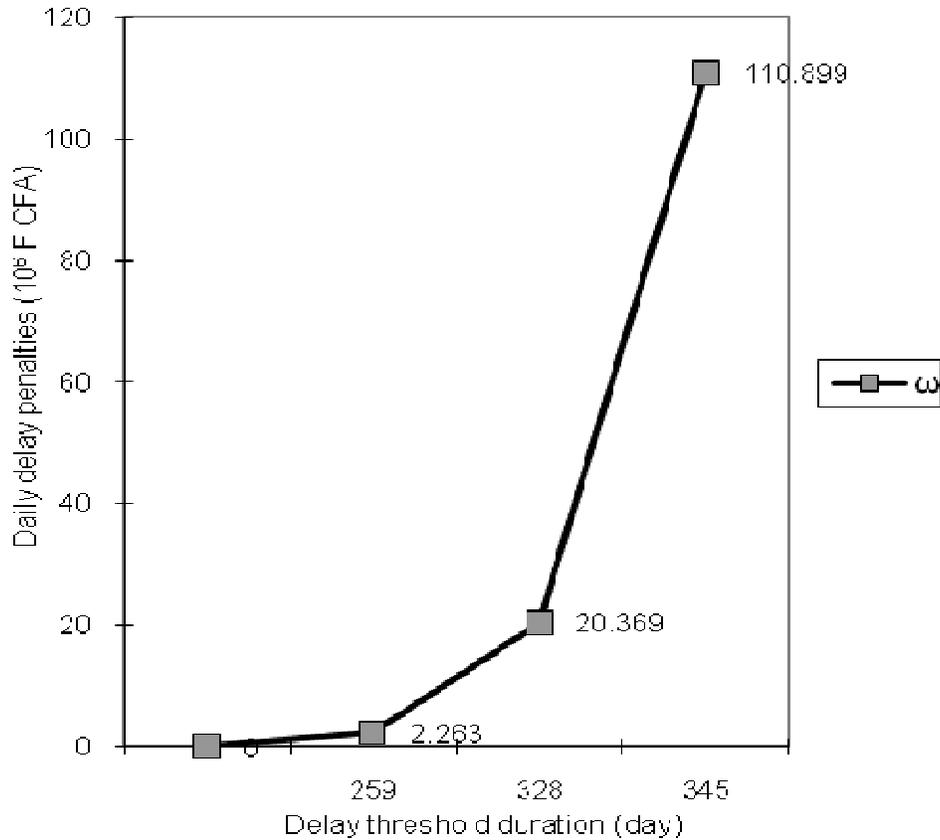


Figure 2. Division of type D_{123} (project P1). ω : Daily delay penalties ($\times 10^6$ F CFA). ω represents the slope of each segment. The value of ω is read on the second extremity of the segment. Example: $\omega_1 = 2.263 \times 10^6$; $\omega_2 = 20.369 \times 10^6$; $\omega_3 = 110.899 \times 10^6$.

The possibility b) is a division of type D_{132} .
 The possibility c) or division of type D_{213} refers to Table 5, the corresponding curve is in Figure 3.
 The possibility d) or division of type D_{231} refers to Table 6, the corresponding curve is in Figure 4.
 The possibility e) or division of type D_{312} refers to Table 7, the corresponding curve is in Figure 5.
 The possibility f) is a division of type D_{321} .

In Tables 8 to 12, we showed the values of daily delay penalties for various stages.

Tables 8 to 12 allow a comparison of gradual daily delay penalties ω_i (calculated using the new formulae) with uniform delay penalties ω_c (obtained from the classic conventional method). It appears that:

- (i) for $\omega_c > \omega_i$, there are penalties too high (REID < 20 %);
- (ii) for $\omega_c < \omega_i/2$, the delay penalties are too low (REID \geq 60 %);
- (iii) for $\omega_i/2 < \omega_c < \omega_i$, delay penalties may be appropriate (20 % \leq REID \leq 30 %).

Table 13 shows the ranges of different stages of delay

penalties, namely: $[D_7; S_1]$; $[S_1; S_1 + S_2]$; $[S_1 + S_2; S_1 + S_2 + S_3]$.

Figures 2 to 5 help to visualise graphically the different types of divisions.

In each stage of delay thresholds variation, the daily delay penalty is constant and uniform. Its value is equal to the slope of segment (ω) in the range considered. In the interval $[0; S_1]$ the corresponding slope is denoted ω_1 , in the interval $[S_1; S_1 + S_2]$, the slope is ω_2 and in the interval $[S_1 + S_2; S_1 + S_2 + S_3]$, the slope is ω_3 .

Note that the product ω by S in a given interval, make it possible to calculate at all time the amount of delay penalties on this stage.

In Figures 3, 4, and 5 we see divisions that have practically the same slope. The slope in stage 3 is lower than in stage 2. We can therefore say in this case that the penalty in stage 3 remains lower than that of stage 2.

The increase of penalty in all three cases above is not continuously increasing per intervals, which is not the case in Figure 2 where the division shows a delay penalty which is accentuated by an interval to another. We can then say that division of type D_{123} is an "ideal" division.

For division of type D_{123} , we can obtain the relation

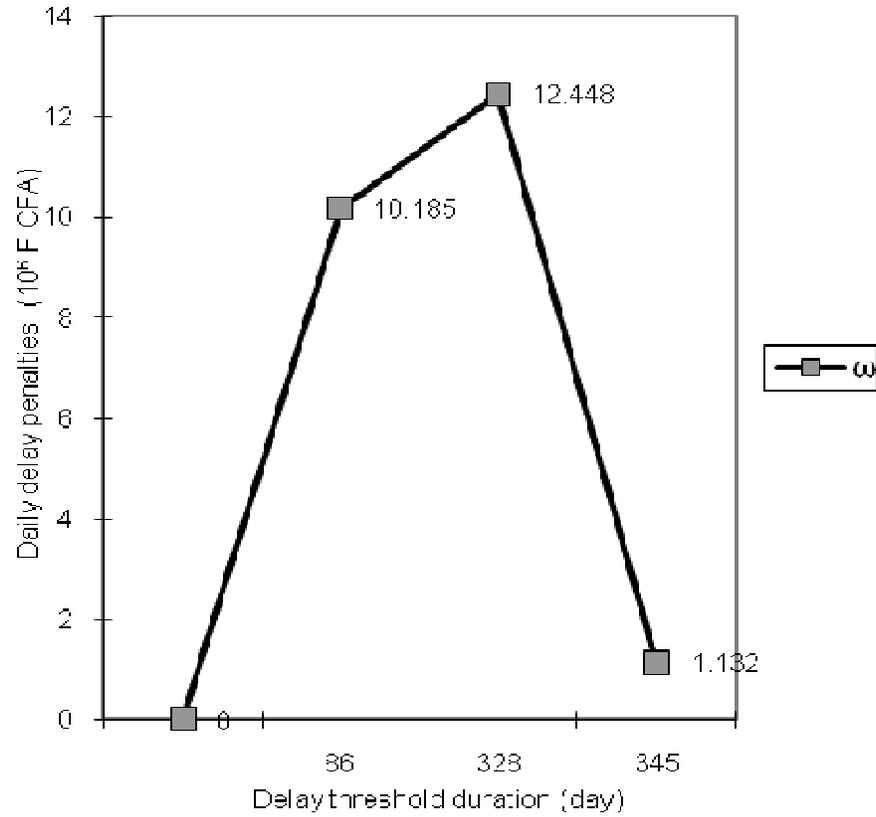


Figure 3. Division of type D₂₁₃ (project P1). ω: Daily delay penalties (× 10⁶ F CFA).

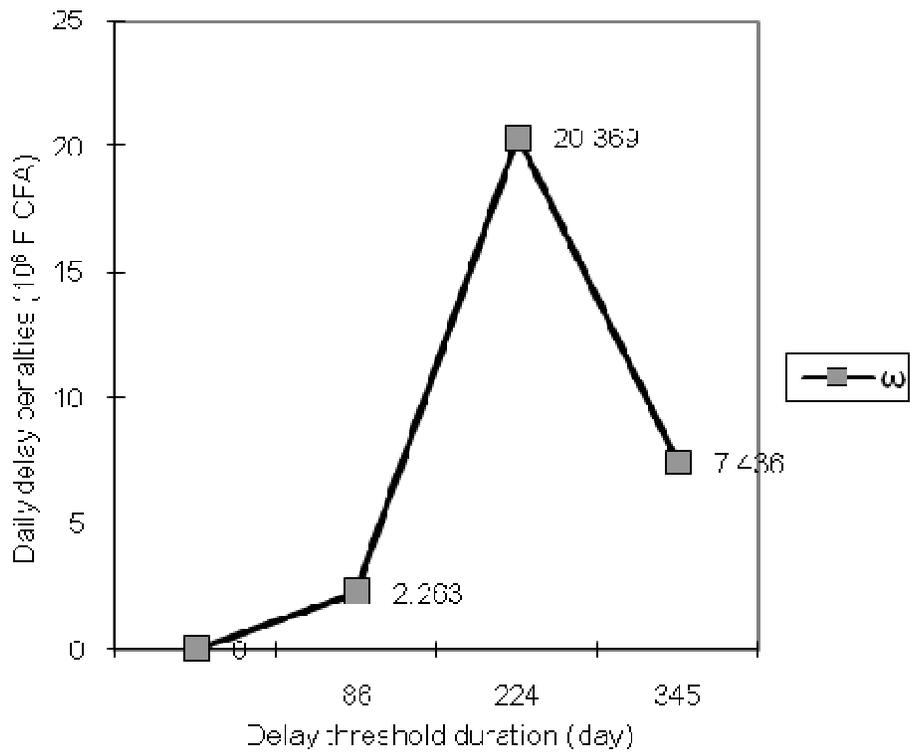


Figure 4. Division of type D₂₃₁ (project P1). ω: Daily delay penalties (× 10⁶ F CFA).

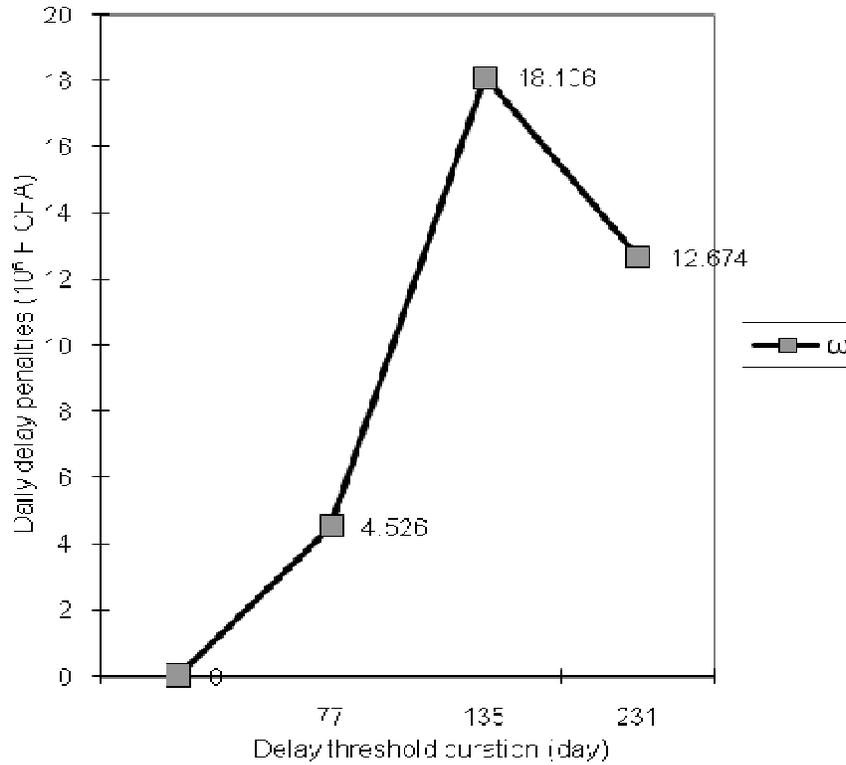


Figure 5. Division of type D_{312} (project P1). ω : Daily delay penalties ($\times 10^6$ F CFA).

$\omega_1 < \omega_2 < \omega_3$ (Table 8). In other words, the lowest delay penalty (2.263×10^6 F CFA, project P₁) applies during the most extended period (259 days, project P₁), which corresponds to stage 1 (Table 4). For the shortest period (17 days, project P₁) ending with the cancellation of the contract, the daily delay penalty becomes very high ($110,899 \times 10^6$ F CFA, project P₁). The division of type D_{123} perfectly illustrates the progressive application of gradual delay penalties.

For division of type D_{213} , $\omega_3 < \omega_1 < \omega_2$, we have the relation $\omega_1 < \omega_2$ (Table 9), that is to say that the daily delay penalty in stage 1 is lower than in stage 2, thus respecting the gradual nature of the penalty. We note that stage 1 is much shorter than stage 2 (Table 5). However, the lowest daily delay penalty is in stage 3 ($\omega_3 < \omega_1$) whose the extent is the smallest of the three stages.

Division of type D_{231} ($\omega_1 < \omega_3 < \omega_2$) imposes the relation $\omega_1 < \omega_2$ and $\omega_3 > \omega_1$ (Table 10). The goal is still respected here. Indeed, the delay penalty in stage 1 must be lower than that applied in stage 2. Like the type D_{213} , stage 2 is larger than stage 1. But we note here that stage 3 is larger than stage 1 (Table 6), with a higher daily delay penalty.

Finally, we have division of type D_{312} ($\omega_1 < \omega_1 < \omega_2$) with the relation $\omega_1 < \omega_2$ and $\omega_3 > \omega_1$ (Table 11), like division of type D_{231} above. However in contrast, stage 1

this time is larger than stage 2, without exceeding the stage 3 (Table 7).

CONCLUSION

In the usual approach, the focus is on determining the threshold amount of delay penalties, which make it possible to determine the cancellation date of the contract.

Unfortunately, it often results in a cancellation date greater than the initial period of project completion. This is particularly observed in cases where the penalties are relatively low or very moderate.

The new approach we propose, in contrast, focuses on determining the cancellation date, the threshold amount of penalties will be deduced afterwards.

The equations of the 2nd degree and 3rd degree proposed by Al-Tabtabaï et al. (1998) require us to calculate a new value of delay penalty every day, which does not seem practical in the real world.

The advantage to proceed with gradual delay penalties makes it possible to maintain the same value of the penalty in a definite time interval. This avoid studios calculation to make up for each delay day.

The value of the daily delay penalty for stage 1 (ω_1) is crucial. Compared with the usual uniform daily delay

Table 8. Daily delay penalties by stage for a division of type D_{123} ($X = 30\%$; $\gamma = 1/5$; $a = 3/4$; $b = 1/5$)

Project code	Constants			Daily cost of the project W (10^6 F CFA/day)	Attenuation factor β	Daily delay penalties by stage for various stage			Uniform daily delay penalties (Current formulae)
	a	b	γ			Stage 1	Stage 2	Stage 3	
							ω_1 (10^6 F CFA)	ω_2 (10^6 F CFA)	ω_3 (10^6 F CFA)
P ₁	0.75	0.20	0.20	45.265	0.25	2.263	20.369	110.899	17.351
P ₂	0.75	0.20	0.20	19.134	0.50	1.913	17.221	93.758	0.459
P ₃	0.75	0.20	0.20	40.698	0.25	2.035	18.314	99.710	1.465
P ₄	0.75	0.20	0.20	0.131	1.00	0.026	0.236	1.285	0.004
P ₅	0.75	0.20	0.20	5.683	0.75	0.852	7.672	41.771	2.046
P ₆	0.75	0.20	0.20	13.333	0.25	0.667	6.000	32.667	0.480
P ₇	0.75	0.20	0.20	0.600	1.00	0.120	1.080	5.880	0.450
P ₈	0.75	0.20	0.20	0.318	1.00	0.064	0.572	3.112	0.079
P ₉	0.75	0.20	0.20	0.130	1.00	0.026	0.234	1.274	0.008
P ₁₀	0.75	0.20	0.20	0.335	1.00	0.067	0.603	3.281	0.050

Table 9. Daily delay penalties by stage for a division of type D_{321} ($X = 30\%$; $\gamma = 9/10$; $a = 3/4$; $b = 1/5$)

Project code	Constants			Daily cost of project W (10^6 F CFA/day)	Attenuation factor β	Daily delay penalties for various stage			Uniform daily delay penalties (Current formulae)
	a	b	γ			Stage 1	Stage 2	Stage 3	
							ω_1 (10^6 F CFA)	ω_2 (10^6 F CFA)	ω_3 (10^6 F CFA)
P ₁	0.75	0.20	0.90	45.265	0.25	10.185	12.448	23.764	17.351
P ₂	0.75	0.20	0.90	19.134	0.50	8.610	10.524	20.091	0.459
P ₃	0.75	0.20	0.90	40.698	0.25	9.157	11.192	21.366	1.465
P ₄	0.75	0.20	0.90	0.131	1.00	0.118	0.144	0.275	0.004
P ₅	0.75	0.20	0.90	5.683	0.75	3.836	4.689	8.951	2.046
P ₆	0.75	0.20	0.90	13.333	0.25	3.000	3.667	7.000	0.480
P ₇	0.75	0.20	0.90	0.600	1.00	0.540	0.660	1.260	0.450
P ₈	0.75	0.20	0.90	0.318	1.00	0.286	0.349	0.667	0.079
P ₉	0.75	0.20	0.90	0.130	1.00	0.117	0.143	0.273	0.008
P ₁₀	0.75	0.20	0.90	0.335	1.00	0.301	0.368	0.703	0.050

penalty (ω_c), this value provides information on the weight of a given penalty, or it is too low, or too high. The suitable value of delay penalty could be in the range $\omega_1/2 < \omega_c < \omega_1$.

The threshold duration of delay will be prior negotiation between the owner and the contractor before signing the contract. Therefore, it is during this stage that the parameters γ , a , b , and X will

be fixed and adjusted to match one of six possible divisions for which the owner and the contractor will be agreed.

So the duration of each stage could be

Table 10. Daily delay penalties by stage for a division of type D_{231} ($X = 30\%$; $\gamma = 1/5$; $a = 1/4$; $b = 2/5$).

Project code	Constants			Daily cost of the project W (10^6 F CFA/day)	Attenuation factor β	Daily delay penalties for various stage			Uniform dailydelay penalties (Current formulae) $\omega_c(10^6$ F CFA)
	a	b	γ			Stage 1	Stage 2	Stage 3	
						$\omega_1(10^6$ F CFA)	$\omega_2(10^6$ F CFA)	$\omega_3(10^6$ F CFA)	
P ₁	0.25	0.40	0.20	45.265	0.25	2.263	20.369	7.436	17.351
P ₂	0.25	0.40	0.20	19.134	0.50	1.913	17.221	6.287	0.459
P ₃	0.25	0.40	0.20	40.698	0.25	2.035	18.314	6.686	1.465
P ₄	0.25	0.40	0.20	0.131	1.00	0.026	0.236	0.086	0.004
P ₅	0.25	0.40	0.20	5.683	0.75	0.852	7.672	2.801	2.046
P ₆	0.25	0.40	0.20	13.333	0.25	0.667	6.000	2.190	0.480
P ₇	0.25	0.40	0.20	0.600	1.00	0.120	1.080	0.394	0.450
P ₈	0.25	0.40	0.20	0.318	1.00	0.064	0.572	0.209	0.079
P ₉	0.25	0.40	0.20	0.130	1.00	0.026	0.234	0.085	0.008
P ₁₀	0.25	0.40	0.20	0.335	1.00	0.067	0.603	0.220	0.050

Table 11. Daily delay penalties by stage for a division of type D_{213} ($X = 30\%$; $\gamma = 9/10$; $a = 1/4$; $b = 7/10$).

Project code	Constants			Daily cost of the project W (10^6 F CFA/day)	Attenuation factor β	Daily delay penalties for various stage			Uniform dailydelay penalties (current formulae) $\omega_c(10^6$ F CFA)
	a	b	γ			Stage 1	Stage 2	Stage 3	
						$\omega_1(10^6$ F CFA)	$\omega_2(10^6$ F CFA)	$\omega_3(10^6$ F CFA)	
P ₁	0.25	0.70	0.90	45.265	0.25	10.185	12.448	1.132	17.351
P ₂	0.25	0.70	0.90	19.134	0.50	8.610	10.524	0.957	0.459
P ₃	0.25	0.70	0.90	40.698	0.25	9.157	11.192	1.017	1.465
P ₄	0.25	0.70	0.90	0.131	1.00	0.118	0.144	0.013	0.004
P ₅	0.25	0.70	0.90	5.683	0.75	3.836	4.689	0.426	2.046
P ₆	0.25	0.70	0.90	13.333	0.25	3.000	3.667	0.333	0.480
P ₇	0.25	0.70	0.90	0.600	1.00	0.540	0.660	0.060	0.450
P ₈	0.25	0.70	0.90	0.318	1.00	0.286	0.349	0.032	0.079
P ₉	0.25	0.70	0.90	0.130	1.00	0.117	0.143	0.013	0.008
P ₁₀	0.25	0.70	0.90	0.335	1.00	0.301	0.368	0.033	0.050

predetermined when signing the contract, and just as for the threshold amount of delay penalties at each stage until the contract cancellation.

The bending factor of delay penalty (γ)

introduces some flexibility depending on whether we want to intensify or attenuate the constraint on the early delay days at the stage1 (S1).

The choice of three different levels of daily delay

penalties, in particular the division of type D_{123} , allows the contractor, upon reaching the first delay penalties threshold (S1), to catch the stage S2 with penalties that are not too severe. If the

Table 12. Comparison of gradual daily delay penalties (proposed formula) with uniform daily delay penalties (current formula) for a division of type D_{312} ($X = 20\%$; $a = 1/3$; $b = 1/4$; $\gamma = 2/5$)

Project code	Daily cost of the project W (10^3 FCFA/day)	Initial duration of the project D_T (month)	Delay threshold duration in various stage			Daily delay penalties for various stage (proposed formula)			Uniform daily delay penalties (current formula) $\omega_c(10^6$ F CFA)
			Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3	
			S_1 (day)	S_2 (day)	S_3 (day)	ω_1 (10^6 FCFA)	ω_2 (10^6 F CFA)	ω_3 (10^6 F CFA)	
P ₁	45.265	46	77	58	96	4.526	18.106	12.674	17.351
P ₂	19.134	12	20	15	25	3.827	15.307	10.715	0.459
P ₃	40.698	18	30	23	38	4.070	16.279	11.395	1.465
P ₄	0.131	6	10	8	13	0.052	0.210	0.147	0.004
P ₅	5.683	36	60	45	75	1.705	6.820	4.774	2.046
P ₆	13.333	18	30	23	38	1.333	5.333	3.733	0.480
P ₇	0.600	10	17	13	21	0.240	0.960	0.672	0.450
P ₈	0.318	5	8	6	10	0.127	0.508	0.356	0.079
P ₉	0.130	12	20	15	25	0.052	0.208	0.146	0.008
P ₁₀	0.335	6	10	8	13	0.134	0.536	0.375	0.050

Table 13. Variation range of various stage of delay penalties for $X = 20\%$; $\gamma = 2/5$; $a = 1/3$; $b = 1/4$ (type D_{312})

Project code	Daily cost of project W (10^6 F CFA/j)	Initial duration of the project D_T (month)	Attenuation factor β	Delay threshold duration in various stage			Range of various stage of delay penalty (day)		
				Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3
				S_1 (day)	S_2 (day)	S_3 (day)	$[D_T + 1 ; D_T + S_1]$	$[D_T + (S_1 + 1) ; D_T + (S_1 + S_2)]$	$[D_T + (S_1 + S_2 + 1) ; D_T + (S_1 + S_2 + S_3)]$
P ₁	45.265	46	0.25	77	58	96	47-123	124-181	182-277
P ₂	19.134	12	0.50	20	15	25	13-32	33-47	48-72
P ₃	40.698	18	0.25	30	23	38	19-48	49-71	72-109
P ₄	0.131	6	1.00	10	8	13	7-16	17-24	25-37
P ₅	5.683	36	0.75	60	45	75	37-96	97-141	142-216
P ₆	13.333	18	0.25	30	23	38	19-48	49-71	72-109
P ₇	0.600	10	1.00	17	13	21	11-27	28-40	41-61
P ₈	0.318	5	1.00	8	6	10	6-13	14-19	20-29
P ₉	0.130	12	1.00	20	15	25	13-32	33-47	48-72
P ₁₀	0.335	6	1.00	10	8	13	7-16	17-24	25-37

second delay penalties threshold (S2) is reached, we topple over stage 3 where penalties are very

high until the cancellation of contract. However, the choice of a given type of division,

which specifies the amount of penalty to be applied for each stage and its duration, is a result

of the negotiation between the owner and the contractor.

ABBREVIATIONS

C_p, Penalty cost; **C_T**, initial total cost; **C_{TE}**, total cost of construction work already performed; **D_{ijk}**, type of division (i, j, k indicate the different delay threshold per stage); **D_R**, delay duration; **D_T**, initial duration of the project; **f**, fraction (of **C_T**) according to the type delay penalty; **P_S**, delay penalties threshold; **R**, period for reaching the cancellation date; **S**, delay threshold (cancellation date); **W**, daily cost of the construction project; **X**, percentage giving the delay threshold; **B**, attenuation factor; **γ**, bending factor; **ω**, delay penalties per calendar day; **ω_c**, uniform daily delay penalty; **GDP1**, gradual delay penalties for first level; **GDP2**, gradual delay penalties for second level; **MDP1**, monotonous delay penalty for first level; **MDP2**, monotonous delay penalty for second level; **REID**, rate of exceeding the initial duration.

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