Chapter 21
Electric Current and Direct-Current Circuit

Outline

21-3 Energy and Power in Electric Circuit

Deriving electric power in a circuit

If a charge ΔQ moves across a potential difference V, its electrical potential energy, U, changes by the amount

$$\Delta U = (\Delta Q)\, V$$

Since the power is the rate of the energy changes with time, we have

$$P = \frac{\Delta U}{\Delta t} = \frac{(\Delta Q) V}{\Delta t}$$

Electric Power

$$P = I\, V \qquad (21\text{-}4)$$

SI unit: watt, W

$$Also \quad P = \frac{V^2}{R}, \qquad P = I^2 R$$

Problem 21-20

A portable CD player operates with a current of 22 mA at a potential difference of 4.1 V.

(a) What is the power usage of the CD players?

(b) What is the electric energy the player used in 2 hours of time?

Solution:

(a) From Eq. (21-4):

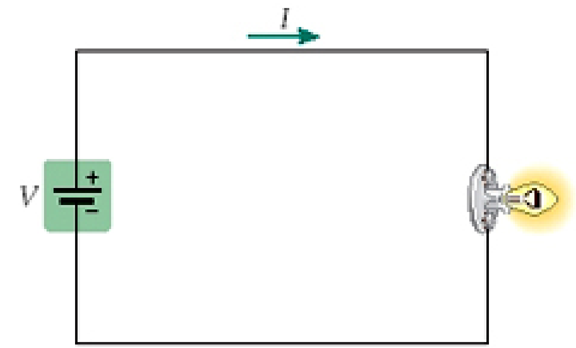
$$P = IV = (0.022\,\text{A})(4.1\,\text{V}) = \boxed{0.090\ \text{W}}$$

(b) The total energy in 2 hours is:

$$\Delta U = P \times \Delta t$$
$$= 0.09 \times (2 \times 3600) = 648 \qquad \textit{Joules}$$

CONCEPTUAL CHECKPOINT 21–2

A battery that produces a potential difference V is connected to a 5-W light bulb. Later, the 5-W light bulb is replaced with a 10-W light bulb. **(a)** In which case does the battery supply the greatest current? **(b)** Which light bulb has the greater resistance?



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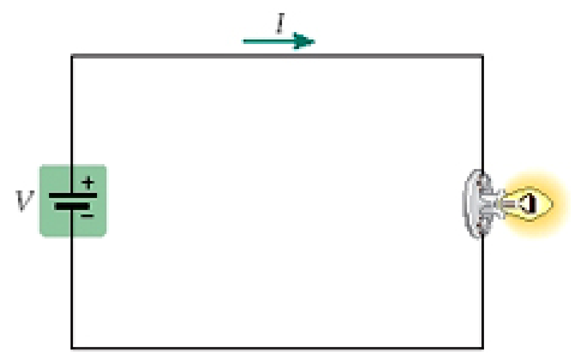


Reasoning and Discussion

(a) To compare the currents we need consider only the relation $P = IV$. Solving for the current yields $I = P/V$. When the voltage V is the same, it follows that the greater the power, the greater the current. In this case, then, the current in the 10-W bulb is twice the current in the 5-W bulb. **(b)** We now consider the relation $P = V^2/R$, which gives resistance in terms of voltage and power. In fact, $R = V^2/P$. Again, with V the same, it follows that the smaller the power the greater the resistance. Thus, the resistance of the 5-W bulb is twice that of the 10-W bulb.

Answer:

(a) When the battery is connected to the 10-W bulb it delivers twice as much current as when it is connected to the 5-W bulb. **(b)** The 5-W bulb has twice as much resistance as the 10-W bulb.

Energy Usage

1 kilowatt · hour (kWh) = (1000 W) (3600 s) = 3.6×10^6 J

1 J= $1/(3.6\times10^6)$ kWh

Problem 21-22

The current in a 120-V reading lamp is 2.3 A. If the cost of electrical energy $0.075 per kilowatt-hour, how much does it cost to operate the light for an hour?

Solution:

1. Calculate the power delivered to the lamp:

$$P = IV = (2.3\ \text{A})(120\ \text{V}) = \underline{\underline{280\ \text{W}}}$$

2. Multiply P by Δt to find :

$$\Delta U = P\,\Delta t = (0.28\ \text{kW})(1.0\ \text{h}) = 0.28\ \text{kWh}$$

3. Multiply by the cost per kilowatt-hour:

$$\text{cost} = (0.28\ \text{kWh})(\$0.075/\text{kWh}) = \boxed{\$0.021}$$

21-4 Resistors in Series and Parallel

Electric circuit often consist of a number of resistors in various ways. To make the circuit simple, a group of resistors can be expressed as an *equivalent resistor* that has the same resistance for the circuit.

Resistors in Series

Resistors are connected one after the other, and that they have the same current I through each resistor.

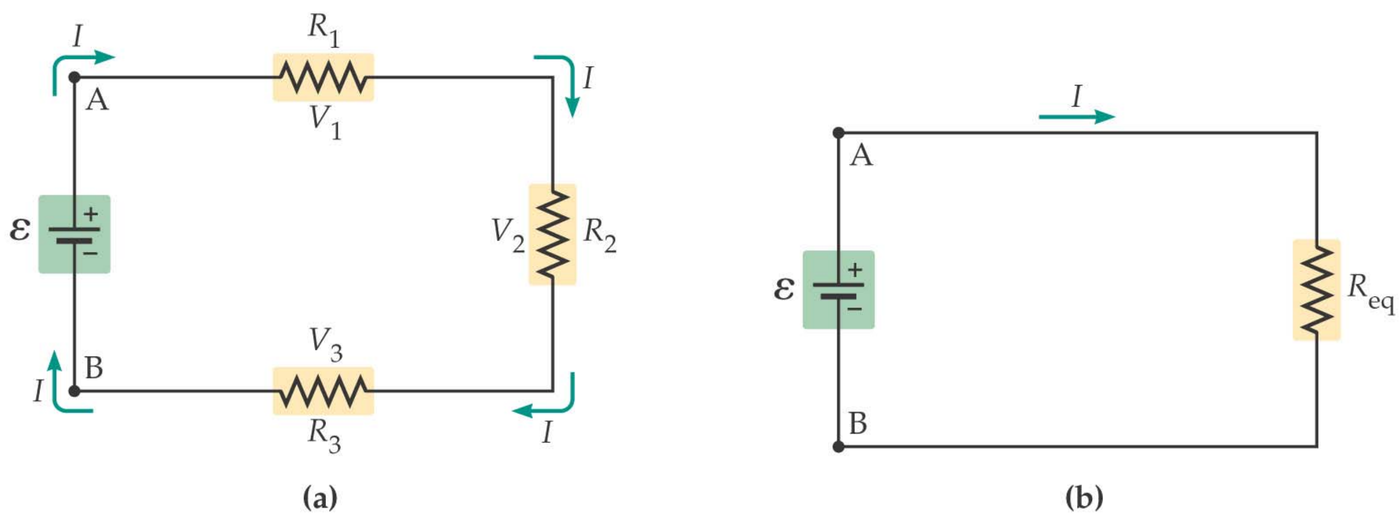


Figure 21-6
Resistors in Series

Deriving Equivalent Resistance

In Fig 21-6 (a), since the total potential difference from point A to point B must be equal to the emf of the battery

$$\begin{aligned} \varepsilon &= V_1 + V_2 + V_3 \\ &= IR_1 + IR_2 + IR_3 \\ &= I(R_1 + R_2 + R_3) \end{aligned}$$

Similarly, in Fig 21-6 (b), we have

$$\varepsilon = IR_{eq}$$

Compare the above two equations, we have

$$R_{eq} = R_1 + R_2 + R_3$$

Equivalent Resistance for Resistors in Series

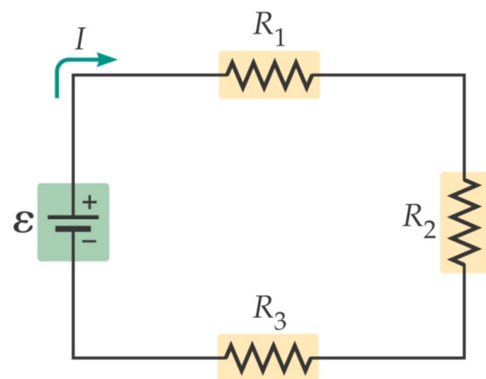
$$R_{eq} = R_1 + R_2 + R_3 + \ldots = \Sigma R$$

SI unit: ohm, Ω

Example 21-5 Three Resistors in Series

A circuit consists of three resistors connected in series to a 24.0 V battery. The current in the circuit is 0.0320 A. Given that R_1=250.0 Ω and R_2=150.0 Ω.

Find **(a)** the value of R_3, and **(b)** the potential different across each resistor.



Solution

Part (a)

According to Ohm's law, the equivalent resistor is

$$R_{eq} = \frac{\varepsilon}{I} = \frac{24.0V}{0.0320A} = 7.50\times10^{2} \quad \Omega$$

Since

$$R_{eq} = R_1 + R_2 + R_3$$

we have $R_3 = R_{eq} - R_1 - R_2 = 7.50\times10^{2} - 250.0 - 150.0 = 3.50\times10^{2}\ \Omega$

Part (b)

Find the potential difference at each resistor, that is current times resistor, respectively

$$V_1 = IR_1 = (0.0320A)(250.0\Omega) = 8.00 \quad V$$

$$V_2 = IR_2 = (0.0320A)(150.0\Omega) = 4.80 \quad V$$

$$V_3 = IR_3 = (0.0320A)(3.50\times10^{2}\Omega) = 11.2 \quad V$$

Resistors in Parallel

Resistors are connected in parallel, and they have the same potential difference.

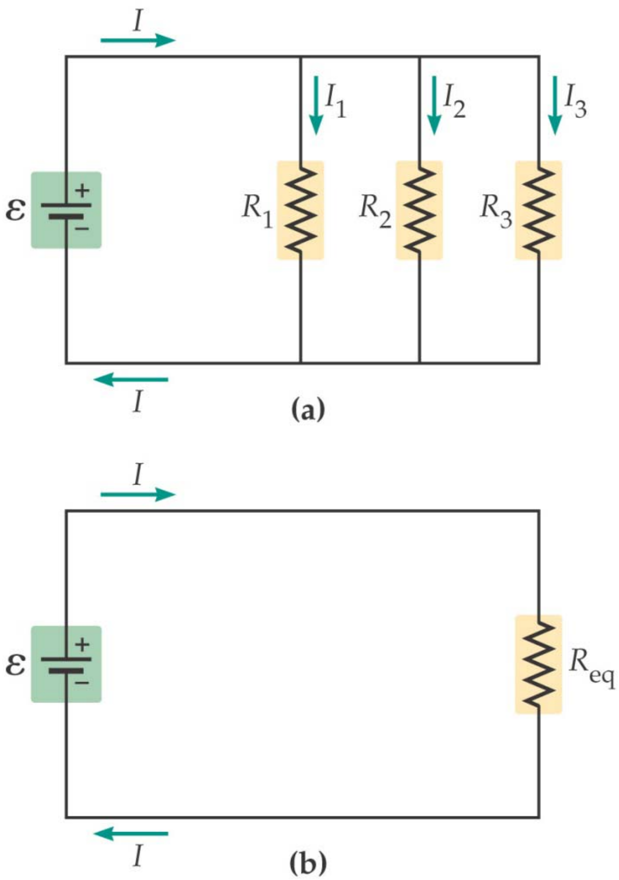


Figure 21-8
Resistors in Parallel

Deriving Equivalent Resistance

In Fig 21-8 (a), since the total current I is equal to the sum of the current through each resistor,

$$I = I_1 + I_2 + I_3$$

Since all resistors have the same potential difference,

$$\varepsilon = I_1 R_1 \qquad \varepsilon = I_2 R_2 \qquad \varepsilon = I_3 R_3$$

Substitute into the first Eq, we have

$$I = \frac{\varepsilon}{R_1} + \frac{\varepsilon}{R_2} + \frac{\varepsilon}{R_3} = \varepsilon\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Now, from Fig 21-8(b), according to Ohm's law, one has

$$I = \varepsilon\left(\frac{1}{R_{eq}}\right)$$

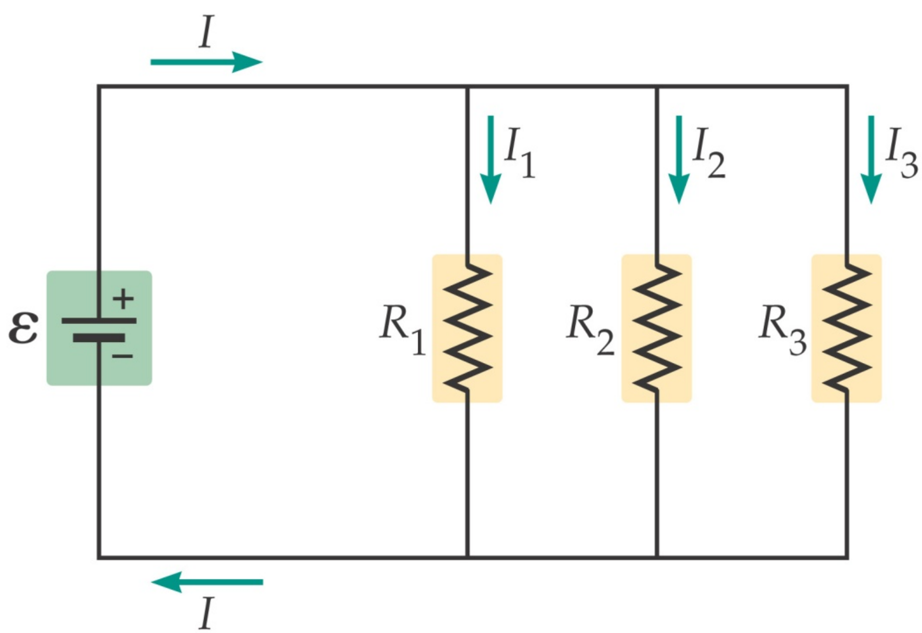
Equivalent Resistance for Resistors in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots = \Sigma \frac{1}{R}$$

SI unit: ohm, Ω

EXAMPLE 21-6 Three Resistors in Parallel

A circuit consists of three resistors, R_1=250.0 Ω, R_2=150.0 Ω, R_3=350.0 Ω and are connected in parallel with a 24.0 V battery. Find **(a)** the total current supplied by the battery and **(b)** the current through each resistor.



Example 21-6
Three Resistors in Parallel

Solution

Part (a)

Find the equivalent resistor

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{250.0\Omega} + \frac{1}{150.0\Omega} + \frac{1}{350.0\Omega} = 0.01352 \quad \Omega^{-1}$$

$$\mathrm{R}_{eq} = (0.01352)^{-1} = 73.96 \quad \Omega$$

So, the total current is

$$I = \frac{V}{R_{eq}} = \frac{24.0V}{73.96\Omega} = 0.325 \quad A$$

Part (b)

Find the currents at each resistor

$$I_1 = \frac{V}{R_1} = \frac{24.0V}{250.0\Omega} = 0.0960 \quad A$$

$$I_2 = \frac{V}{R_2} = \frac{24.0V}{150.0\Omega} = 0.160 \quad A$$

$$I_3 = \frac{V}{R_3} = \frac{24.0V}{350.0\Omega} = 0.0686 \quad A$$

$$I_1 + I_2 + I_3 = I\,?$$

CONCEPTUAL CHECKPOINT 21–3

Two identical light bulbs are connected to a battery, either in series or in parallel. Are the bulbs in series **(a)** brighter, **(b)** dimmer, or **(c)** the same brightness as the bulbs in parallel?

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Reasoning and Discussion

Both sets of light bulbs are connected to the same potential difference, V; hence, the power delivered to the bulbs is V^2/R_{eq}, where R_{eq} is twice the resistance of a bulb in the series circuit and half the resistance of a bulb in the parallel circuit. As a result, more power is converted to light in the parallel circuit.

Answer:

(b) The bulbs connected in series are dimmer than the bulbs connected in parallel.

Combination Circuits

A electric circuit may be more complex, which include resistors both in parallel and series. In this case, we can still applied the equivalent resistors in each part, individually

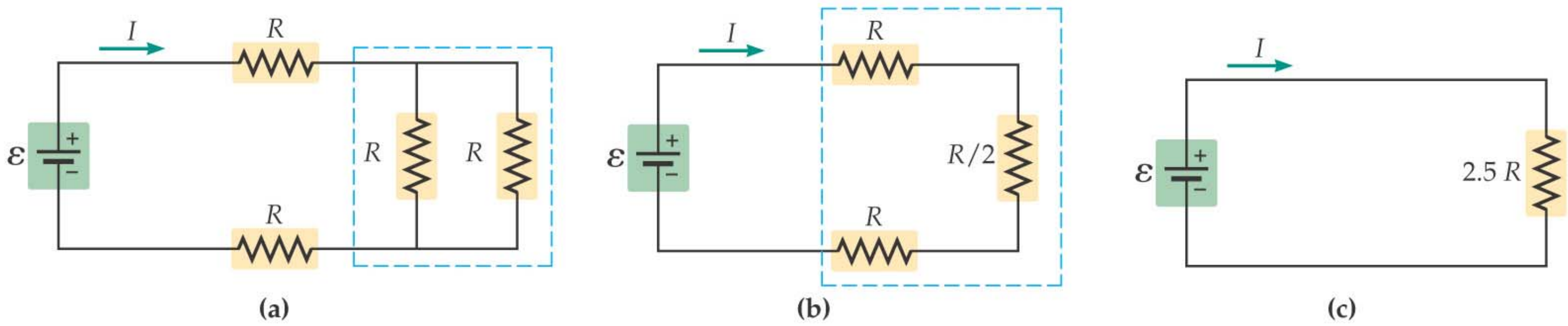
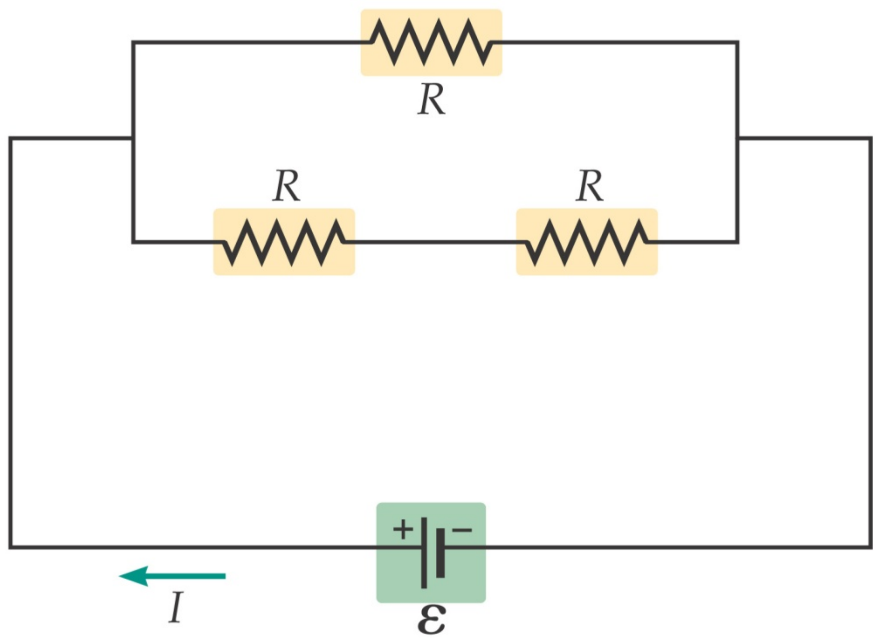


Figure 21-10
Analyzing a Complex Circuit of Resistors

Example 21-7 **Combination Special**

In a circuit shown in the diagram, the emf of the battery is 12.0 V, and all the resistors have a resistance of 200.0 Ω. Find the current applied by the battery to this circuit.



Example 21-7
Combination Special

Solution

1) Find the equivalent resistor of the two resistors

$$R_{eq,lower} = R + R = 2R$$

2) Find the equivalent resistor of the whole circuit

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R}$$

$$R_{eq} = \frac{2}{3}R = \frac{2}{3}(200.0\Omega) = 133.3 \quad \Omega$$

3) Find the current of the whole circuit

$$I = \frac{\varepsilon}{R_{eq}} = \frac{12.0V}{133.3\Omega} = 0.0900 \quad A$$

Summary

Energy and Power in Electric Circuit

$$P = \frac{\Delta U}{\Delta t} = \frac{(\Delta Q)V}{\Delta t}$$

$$P = I\,V \qquad (21\text{-}4)$$

Resistance in Series and Parallel

$$R_{eq} = R_1 + R_2 + R_3 + \ldots = \Sigma R$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots = \Sigma \frac{1}{R}$$

Exercise 21-2

A handheld electric fan operates on a 3.00-V battery. If the power generated by the fan is 2.24 W, what is the current supplied by the battery?

Solution

Since P=IV, we have

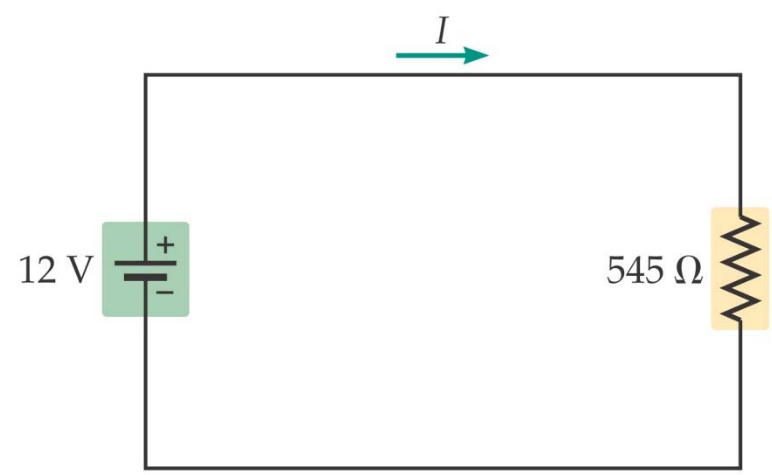
$$IV = P$$

$$I(3.00V) = 2.24W$$

$$I = \frac{2.24W}{3.00V} = 0.747 \quad A$$

Exercise 21-3 **Heated Resistance**

A battery with an emf of 12 V is connected to a 545 Ω resistor. How much energy is dissipated in the resistor in 65 s?



Solution

1) Find the power dissipated on the resistor:

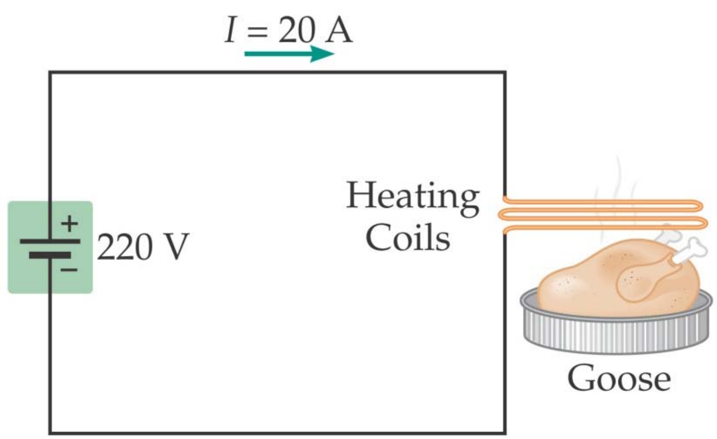
$$P = VI = V^2 / R = (12V)^2 / (546\ \Omega) = 0.26\ W$$

2) The energy dissipated is

$$\Delta U = P\ \Delta t = (0.26\ W)\ (65\ s) = 17\ J$$

Example 21-4

A holiday goose is cooked in the kitchen oven for 4.00 h. The oven current is 20.0 A and it operates at 220.0 V voltage. The cost of electrical energy is $0.048 per KWH. How much does it cost to cook your goose?



Solution

1) Find the power of the oven

$P = IV = (20.0\ \text{A})(220.0\ \text{V})$

$= 4.40 \times 10^3\ \text{W}$

2) The total energy in 4.00 h is

$\Delta U = P\,\Delta t = (4.40 \times 10^3\ \text{W})(4.00 \times 60 \times 60\ \text{s}) = 6.34 \times 10^7\ \text{J}$

$= 6.34 \times 10^7 / (3.6 \times 10^6)\ \text{kWh} = 17.6\ \text{kWh}$

3) The total cost is

Cost = (17.6 kWh) ($0.048/kWh) = $0.84