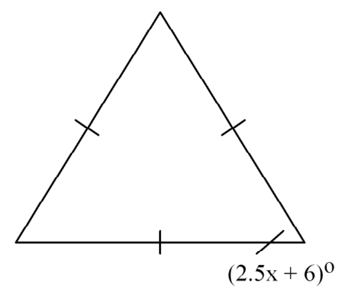
Name: ________________________ Class: ___________________ Date: __________ ID: A

Chapter 4 Study guide

Numeric Response

1. An isosceles triangle has a perimeter of 50 in. The congruent sides measure $(2x + 3)$ cm. The length of the third side is $4x$ cm. What is the value of x?

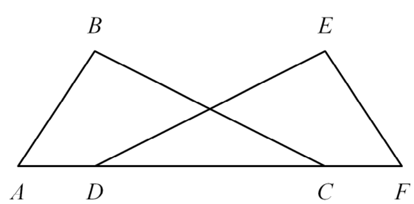
2. Find the value of x.



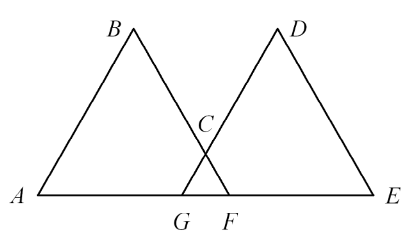
Short Answer

3. Two sides of an equilateral triangle measure $(2y + 3)$ units and $(y^2 - 5)$ units. If the perimeter of the triangle is 33 units, what is the value of y?

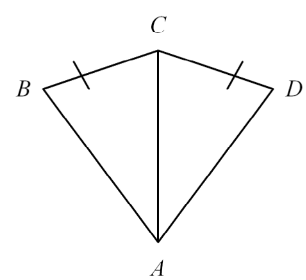
4. Find $m\angle DCB$, given $\angle A \cong \angle F$, $\angle B \cong \angle E$, and $m\angle CDE = 46°$.



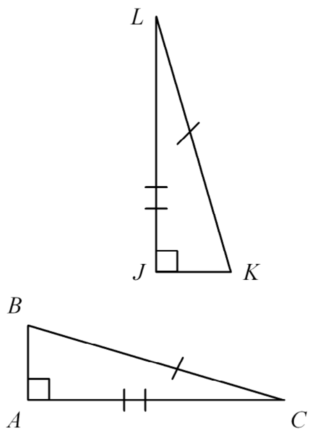
5. $\Delta ABF \cong \Delta EDG$. ΔABF and ΔGCF are equilateral. $AG = 21$ and $CG = \frac{1}{4}AB$. Find the total distance from A to B to C to D to E.



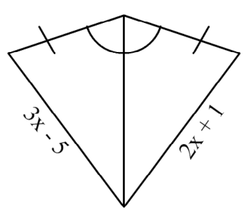
6. What additional information do you need to prove $\Delta ABC \cong \Delta ADC$ by the SAS Postulate?



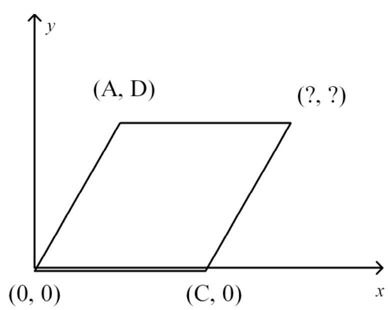
7. For these triangles, select the triangle congruence statement and the postulate or theorem that supports it.



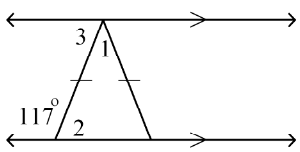
8. Find the value of x.



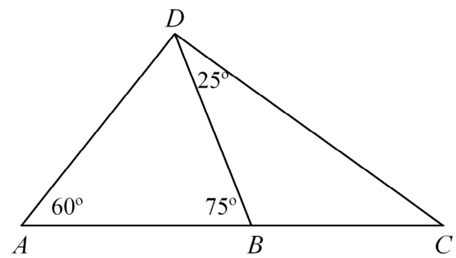
9. Find the missing coordinates for the rhombus.



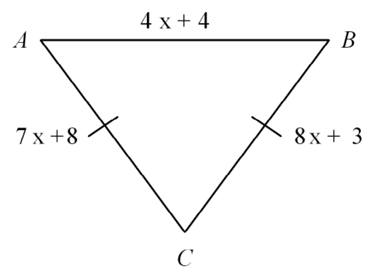
10. Find the measure of each numbered angle.



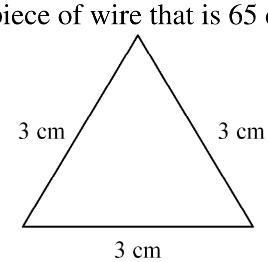
11. Classify ΔDBC by its angle measures, given $m\angle DAB = 60°$, $m\angle ABD = 75°$, and $m\angle BDC = 25°$.



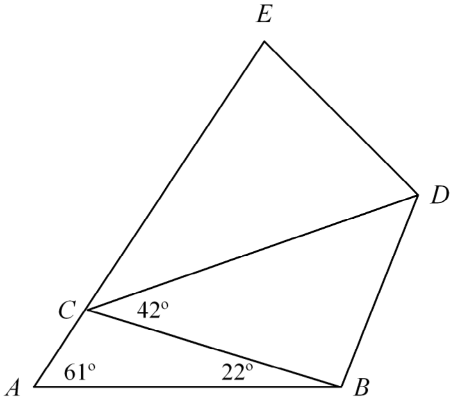
12. ΔABC is an isosceles triangle. $\overline{AB}$ is the longest side with length $4x + 4$. $BC = 8x + 3$ and $CA = 7x + 8$. Find AB.



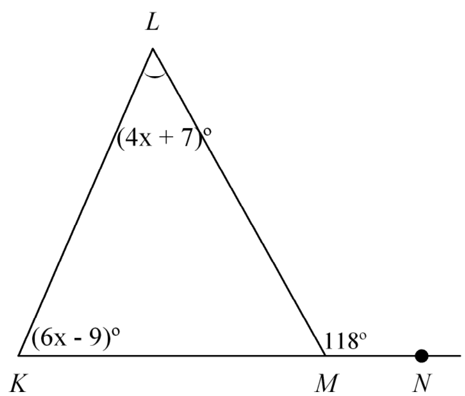
13. A jeweler creates triangular medallions by bending pieces of silver wire. Each medallion is an equilateral triangle. Each side of a triangle is 3 cm long. How many medallions can be made from a piece of wire that is 65 cm long?



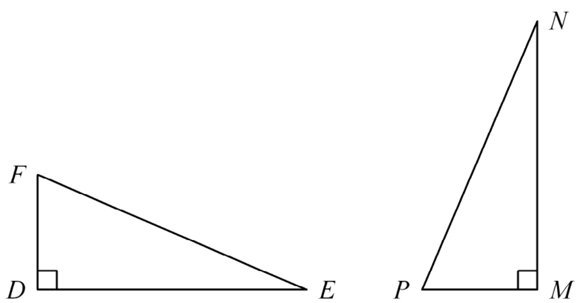
14. Daphne folded a triangular sheet of paper into the shape shown. Find $m\angle ECD$, given $m\angle CAB = 61°$, $m\angle ABC = 22°$, and $m\angle BCD = 42°$.



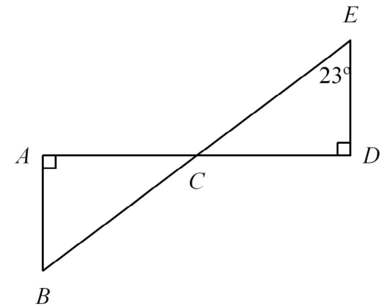
15. Find $m\angle K$.



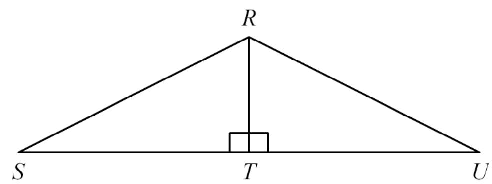
16. Find $m\angle E$ and $m\angle N$, given $m\angle F = m\angle P$, $m\angle E = (x^2)°$, and $m\angle N = (4x^2 - 75)°$.



17. Given that $\Delta ABC \cong \Delta DEC$ and $m\angle E = 23°$, find $m\angle ACB$.



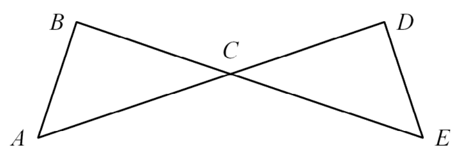
18. **Given**: $\overline{RT} \perp \overline{SU}$, $\angle SRT \cong \angle URT$, $\overline{RS} \cong \overline{RU}$. T is the midpoint of $\overline{SU}$.



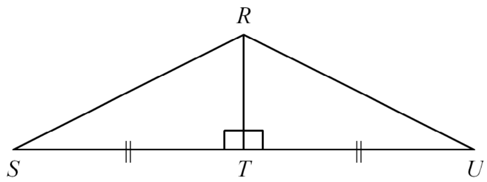
Prove: $\Delta RTS \cong \Delta RTU$

Complete the proof.

19. Tom is wearing his favorite bow tie to the school dance. The bow tie is in the shape of two triangles.
Given: $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{AC} \cong \overline{EC}$, $\angle A \cong \angle E$
Prove: $\Delta ABC \cong \Delta EDC$



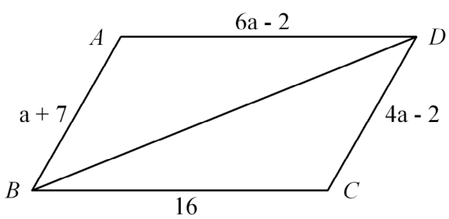
20. The figure shows part of the roof structure of a house. Use SAS to explain why $\Delta RTS \cong \Delta RTU$.



Complete the explanation.

It is given that [1]. Since $\angle RTS$ and $\angle RTU$ are right angles, [2] by the Right Angle Congruence Theorem. By the Reflexive Property of Congruence, [3]. Therefore, $\Delta RTS \cong \Delta RTU$ by SAS.

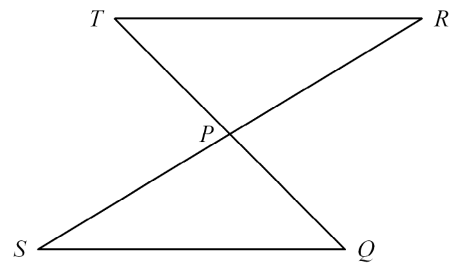
21. Show $\Delta ABD \cong \Delta CDB$ for $a = 3$.



Complete the proof.

$AB = a + 7 = [1] = 10$
$CD = 4a - 2 = [2] = 12 - 2 = 10$
$AD = 6a - 2 = 6(3) - 2 = 18 - 2 = [3]$
$CB = [4]$
$\overline{AB} \cong \overline{CD}$. $\overline{AD} \cong \overline{CB}$. $\overline{BD} \cong \overline{BD}$ by the Reflexive Property of Congruence. So $\Delta ABD \cong \Delta CDB$ by [5].

22. **Given:** P is the midpoint of $\overline{TQ}$ and $\overline{RS}$.
Prove: $\Delta TPR \cong \Delta QPS$

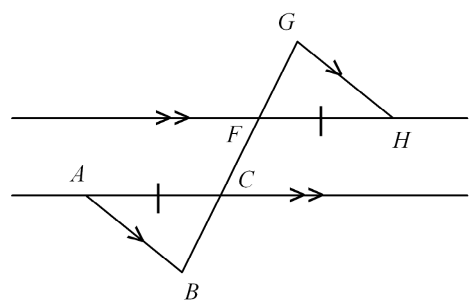


Complete the proof.

23. Using the information about John, Jason, and Julie, can you uniquely determine how they stand with respect to each other? On what basis?

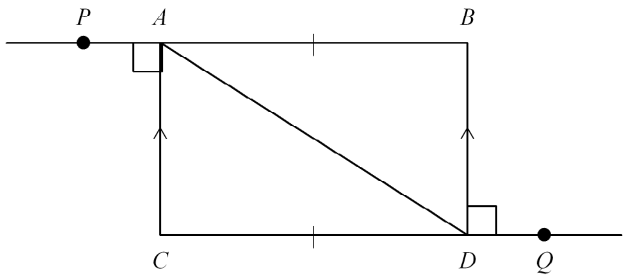
Statement 1: John and Jason are standing 12 feet apart.
Statement 2: The angle from Julie to John to Jason measures 31º.
Statement 3: The angle from John to Jason to Julie measures 49º.

24. Use AAS to prove the triangles congruent.
Given: $\overline{AB} \parallel \overline{GH}$, $\overleftrightarrow{AC} \parallel \overleftrightarrow{FH}$, $\overline{AC} \cong \overline{FH}$
Prove: $\Delta ABC \cong \Delta HGF$

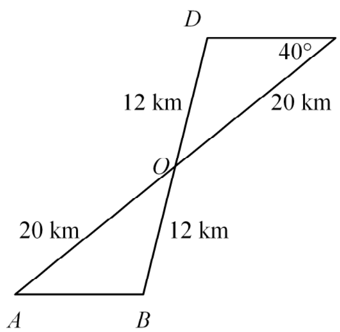


Write a two column proof.

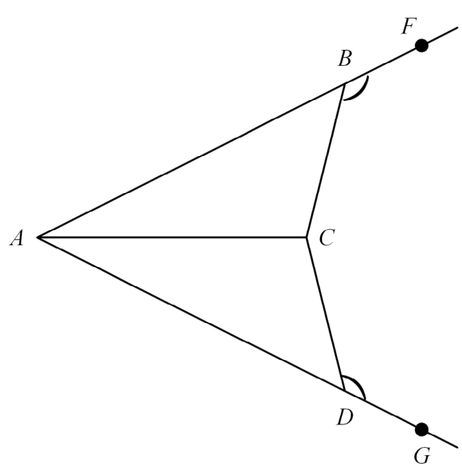
25. Determine if you can use the HL Congruence Theorem to prove $\Delta ACD \cong \Delta DBA$. If not, tell what else you need to know.



26. A pilot uses triangles to find the angle of elevation $\angle A$ from the ground to her plane. How can she find $m\angle A$?

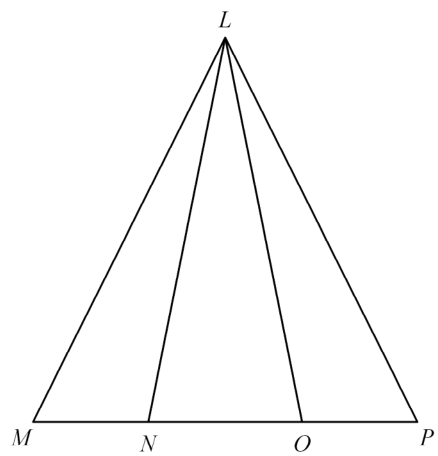


27. **Given:** $\angle CBF \cong \angle CDG$, $\overline{AC}$ bisects $\angle BAD$
Prove: $\overline{AD} \cong \overline{AB}$



Write a two column proof

28. **Given**: $\angle MLN \cong \angle PLO$, $\angle MNL \cong \angle POL$, $\overline{MO} \cong \overline{NP}$
Prove: ΔMLP is isosceles.



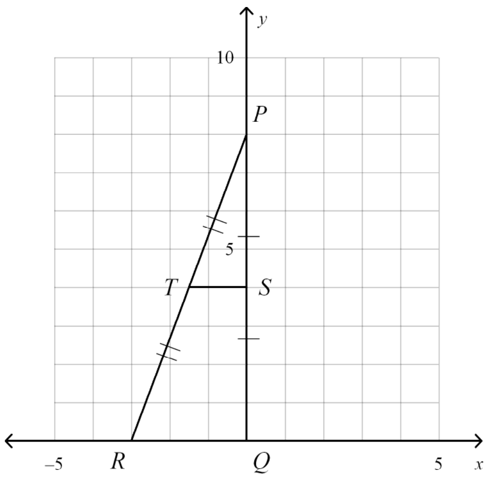
Complete a two column proof.

Proof:

29. **Given:** $A(3, –1)$, $B(5, 2)$, $C(–2, 0)$, $P(–3, 4)$, $Q(–5, –3)$, $R(–6, 2)$
Prove: $\angle ABC \cong \angle RPQ$

Complete the coordinate proof.

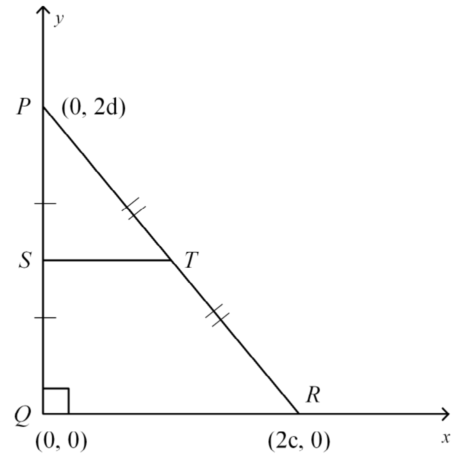
30. **Given:** ΔPQR has vertices $P(0, 8)$, $Q(0, 0)$, and $R(-3, 0)$. S is the midpoint of $\overline{PQ}$ and T is the midpoint of $\overline{RP}$.
Prove: The area of ΔPST is one fourth the area of ΔPQR.



Complete a coordinate proof.

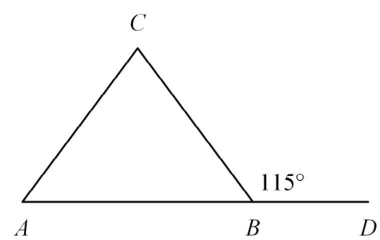
31. Position a right triangle with leg lengths r and $2s + 4$ in the coordinate plane and give the coordinates of each vertex.

32. **Given:** $\angle Q$ is a right angle in ΔPQR. S is the midpoint of $\overline{PQ}$ and T is the midpoint of $\overline{RP}$.
Prove: The area of ΔPST is one fourth the area of ΔPQR.



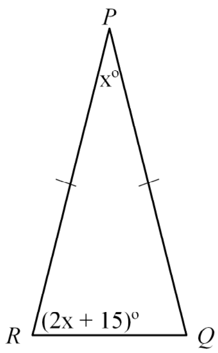
Complete a coordinate roof.

33. Two Seyfert galaxies, BW Tauri and M77, represented by points A and B, are equidistant from Earth, represented by point C. What is m$\angle A$?



Name: ________________________ **ID: A**

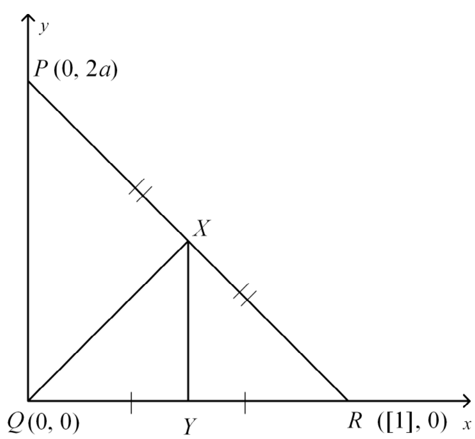
34. Find m$\angle Q$.



35. **Given:** $\angle Q$ is a right angle in the isosceles ΔPQR. X is the midpoint of $\overline{PR}$. Y is the midpoint of $\overline{QR}$.
Prove: ΔQXY is isosceles.

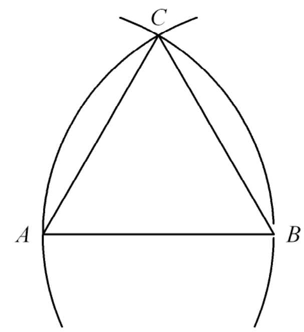
Complete the paragraph proof.

Proof: Draw a diagram and place the coordinates of ΔPQR and ΔQXY as shown.



complete a coordinate proof

36. **Given**: diagram showing the steps in the construction
Prove: m$\angle A = 60°$



Complete the paragraph proof.

Proof:
The same compass setting was used to create $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, so [1]. By the [2], ΔABC is equilateral. Since ΔABC is equilateral, it is also [3]. So $m\angle A + m\angle B + m\angle C = 180°$. Therefore, $m\angle A = 60°$.

Chapter 4 Study guide
Answer Section

NUMERIC RESPONSE

1. ANS: 5.5

   PTS: 1 DIF: Advanced NAT: 12.3.2.e STA: GE8.0
   TOP: 4-1 Classifying Triangles KEY: perimeter | isosceles

2. ANS: 21.6

   PTS: 1 DIF: Average NAT: 12.2.1.f STA: GE12.0
   TOP: 4-8 Isosceles and Equilateral Triangles KEY: equilateral | equiangular

SHORT ANSWER

3. ANS:
   $y = 4$
   The perimeter is 33 units and it is an equilateral triangle, so each side has length 11 units.
   Use this to solve for either side.

   $11 = 2y + 3$
   $8 = 2y$
   $4 = y$

   $11 = y^2 - 5$
   $16 = y^2$
   $4 = y$
   An answer of -4 does not apply here.

   PTS: 1 DIF: Advanced NAT: 12.2.1.h STA: GE12.0
   TOP: 4-1 Classifying Triangles

4. ANS:
   $m\angle DCB = 46°$
   The Third Angles Theorem states that if two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.

   It is given that $\angle A \cong \angle F$ and $\angle B \cong \angle E$. Therefore, $\angle CDE \cong \angle DCB$. So, $m\angle DCB = 46°$.

   PTS: 1 DIF: Advanced NAT: 12.3.3.f STA: GE12.0
   TOP: 4-2 Angle Relationships in Triangles

5. ANS:
98
ΔGCF is equilateral, so $CG = GF$.
ΔABF is equilateral, so $AB = AF$.
$\Delta ABF \cong \Delta EDG$, so $AB = DF$, $BC = CD$, and $CG = CF$.
The total distance from A to B to C to D to $E = AB + BC + CD + DE$.

Step 1 Find AB and DE by finding AF.
Since $CG = \frac{1}{4}AB$, use substitution to get $GF = \frac{1}{4}AF$.

$AF = AG + GF = AG + \frac{1}{4}AF$

$\frac{3}{4}AF = AG$

$\frac{3}{4}AF = 21$

$AF = 28$

Since ΔABF is equilateral, $AB = BF = 28$.
Since $\Delta ABF \cong \Delta EDG$, $AB = DE = 28$.

Step 2 Find BC and CD.
Since ΔGCF is equilateral and $CG = \frac{1}{4}AB$, $CG = CF = 7$.
So $BC = BF - CF = 28 - 7 = 21$.
Since $\Delta ABF \cong \Delta EDG$, $DC = BC = 21$.

Step 3 Substitute to find the distance from A to B to C to D to E.
$AB + BC + CD + DE = 28 + 21 + 21 + 28 = 98$.

PTS: 1 DIF: Advanced NAT: 12.3.3.f STA: GE5.0
TOP: 4-3 Congruent Triangles KEY: multi-step

6. ANS:
$\angle ACB \cong \angle ACD$
The SAS Postulate is used when two sides and an included angle of one triangle are congruent to the corresponding sides and included angle of a second triangle.
From the given, $\overline{BC} \cong \overline{DC}$.
From the figure, $\overline{AC} \cong \overline{AC}$ by the Reflexive Property of Congruence.
You have two pair of congruent sides, so you need information about the included angles.

Use these pairs of sides to determine the included angles.
The angle between sides $\overline{AC}$ and $\overline{BC}$ is $\angle ACB$.
The angle between sides $\overline{AC}$ and $\overline{DC}$ is $\angle ACD$.
You need to know $\angle ACB \cong \angle ACD$ to prove $\Delta ABC \cong \Delta ADC$ by the SAS Postulate.

PTS: 1 DIF: Advanced NAT: 12.3.5.a STA: GE5.0
TOP: 4-4 Triangle Congruence: SSS and SAS

7. ANS:
$\Delta ABC \cong \Delta JKL$, HL
Because $\angle BAC$ and $\angle KJL$ are right angles, ΔABC and ΔJKL are right triangles.
You are given a pair of congruent legs $\overline{AC} \cong \overline{JL}$ and a pair of congruent hypotenuses $\overline{CB} \cong \overline{LK}$.
So a hypotenuse and a leg of ΔABC are congruent to the corresponding hypotenuse and leg of ΔJKL.
$\Delta ABC \cong \Delta JKL$ by HL.

PTS: 1 DIF: Advanced NAT: 12.3.5.a STA: GE5.0
TOP: 4-5 Triangle Congruence: ASA AAS and HL

8. ANS:
$x = 6$
The triangles can be proved congruent by the SAS Postulate.
By CPCTC, $3x - 5 = 2x + 1$.

Solve the equation for x.
$3x - 5 = 2x + 1$
$3x = 2x + 6$
$x = 6$

PTS: 1 DIF: Advanced NAT: 12.3.2.e STA: GE5.0
TOP: 4-6 Triangle Congruence: CPCTC

9. ANS:
$(A + C, D)$
The horizontal sides are parallel, so the y-value is the same as in the point (A, D).
The missing y-coordinate is D.

A rhombus has congruent sides, so the x-value is the same horizontal distance from $(C, 0)$ as the point (A, D) is from the point $(0, 0)$. This horizontal distance is A units.
The missing x-coordinate is $A + C$.

PTS: 1 DIF: Advanced NAT: 12.2.1.e STA: GE17.0
TOP: 4-7 Introduction to Coordinate Proof

10. ANS:
$m\angle 1 = 54°$, $m\angle 2 = 63°$, $m\angle 3 = 63°$
Step 1: $\angle 2$ is supplementary to the angle that is 117°.
$117° + m\angle 2 = 180°$. So $m\angle 2 = 63°$.

Step 2: By the Alternate Interior Angles Theorem, $\angle 2 \cong \angle 3$.
So $m\angle 2 = m\angle 3 = 63°$.

Step 3: By the Isosceles Triangle Theorem, $\angle 2$ and the angle opposite the other side of the isosceles triangle are congruent. Let $\angle 4$ be that unknown angle.
Then, $\angle 2 \cong \angle 4$ and $m\angle 2 = m\angle 4 = 63°$.
$m\angle 1 + m\angle 2 + m\angle 4 = 180°$ by the Triangle Sum Theorem.
$m\angle 1 + 63° + 63° = 180°$. So $m\angle 1 = 54°$.

PTS: 1 DIF: Advanced NAT: 12.3.2.e STA: GE12.0
TOP: 4-8 Isosceles and Equilateral Triangles KEY: multi-step

11. ANS:
obtuse triangle
$\angle ABD$ and $\angle DBC$ form a linear pair, so they are supplementary. Therefore $m\angle ABD + m\angle DBC = 180°$. By substitution, $75° + m\angle DBC = 180°$. So $m\angle DBC = 105°$. ΔDBC is an obtuse triangle by definition.

PTS: 1 DIF: Average REF: Page 216
OBJ: 4-1.1 Classifying Triangles by Angle Measures NAT: 12.3.3.f
STA: GE12.0 TOP: 4-1 Classifying Triangles

12. ANS:
$AB = 24$

Step 1 Find the value of *x*.

$$\begin{aligned} BC &= CA \\ 8x+3 &= 7x+8 \\ x &= 5 \end{aligned}$$

Step 2 Find *AB*.

$$\begin{aligned} AB &= 4x+4 \\ &= 4(5)+4 \\ &= 24 \end{aligned}$$

PTS: 1 DIF: Average REF: Page 217 OBJ: 4-1.3 Using Triangle Classification
NAT: 12.3.3.f STA: GE12.0 TOP: 4-1 Classifying Triangles

13. ANS:
7 medallions
The amount of silver needed to make one medallion is equal to the perimeter *P* of the equilateral triangle.
$P = 3(3) = 9$ cm
To find the number of medallions that can be made from 65 cm of silver, divide 65 by the amount of silver needed for one medallion.
$65 \div 9 = 7\frac{2}{9}$ medallions
There is not enough silver to complete an eighth triangle. So the jeweler can make 7 medallions from a 65 cm piece of wire.

PTS: 1 DIF: Average REF: Page 218 OBJ: 4-1.4 Application
NAT: 12.3.3.f STA: GE12.0 TOP: 4-1 Classifying Triangles

14. ANS:
$m\angle ECD = 41°$
Step 1 Find $m\angle ACB$.

$m\angle CAB + m\angle ABC + m\angle ACB = 180°$	Triangle Sum Theorem
$61° + 22° + m\angle ACB = 180°$	Substitute 61° for $m\angle CAB$ and 22° for $m\angle ABC$.
$83° + m\angle ACB = 180°$	Simplify.
$m\angle ACB = 97°$	Subtract 83° from both sides.

Step 2 Find $m\angle ECD$.

$m\angle ACB + m\angle BCD + m\angle ECD = 180°$	Linear Pair Theorem and Angle Addition Postulate
$97° + 42° + m\angle ECD = 180°$	Substitute 97° for $m\angle ACB$ and 42° for $m\angle BCD$.
$139° + m\angle ECD = 180°$	Simplify.
$m\angle ECD = 41°$	Subtract 139° from both sides.

PTS: 1 DIF: Average REF: Page 224 OBJ: 4-2.1 Application
NAT: 12.3.3.f STA: GE12.0 TOP: 4-2 Angle Relationships in Triangles

15. ANS:
$m\angle K = 63°$

$m\angle K + m\angle L = m\angle LMN$	Exterior Angle Theorem
$(6x - 9)° + (4x + 7)° = 118°$	Substitute $6x - 9$ for $m\angle K$, $4x + 7$ for $m\angle L$, and 118 for $m\angle LMN$.
$10x - 2 = 118$	Simplify.
$10x = 120$	Add 2 to both sides.
$x = 12$	Divide both sides by 10.

$m\angle K = 6x - 9 = 6(12) - 9 = 63°$

PTS: 1 DIF: Average REF: Page 225
OBJ: 4-2.3 Applying the Exterior Angle Theorem NAT: 12.3.3.f
STA: GE12.0 TOP: 4-2 Angle Relationships in Triangles

16. ANS:
$m\angle E = 25°$, $m\angle N = 25°$

$\angle E \cong \angle N$	Third Angles Theorem
$m\angle E = m\angle N$	Definition of congruent angles
$(x^2)° = (4x^2 - 75)°$	Substitute x^2 for $m\angle E$ and $4x^2 - 75$ for $m\angle N$.
$-3x^2 = -75$	Subtract $4x^2$ from both sides.
$x^2 = 25$	Divide both sides by –3.

So $m\angle E = 25°$.
Since $m\angle E = m\angle N$, $m\angle N = 25°$.

PTS: 1 DIF: Average REF: Page 226
OBJ: 4-2.4 Applying the Third Angles Theorem NAT: 12.3.3.f
STA: GE12.0 TOP: 4-2 Angle Relationships in Triangles

17. ANS:

$m\angle ACB = 67°$	
$m\angle DCE + m\angle CED + m\angle EDC = 180°$	Triangle Sum Theorem
$m\angle DCE + 23° + 90° = 180°$	Substitution.
$m\angle DCE + 113° = 180°$	Simplify.
$m\angle DCE = 67°$	Subtract 113 from both sides.
$\angle DCE \cong \angle BCA$	Corresponding parts of congruent triangles are congruent.
$m\angle DCE = m\angle BCA$	Definition of congruent angles
$m\angle ACB = 67°$	Corresponding parts of congruent triangles are congruent.

PTS: 1 DIF: Average REF: Page 232
OBJ: 4-3.2 Using Corresponding Parts of Congruent Triangles NAT: 12.3.3.f
STA: GE5.0 TOP: 4-3 Congruent Triangles

18. ANS:
a
Proof:

Statements	Reasons
1. $\overline{RT} \perp \overline{SU}$	1. Given
2. $\angle RTS$ and $\angle RTU$ are right angles.	2. Definition of perpendicular lines
3. $\angle RTS \cong \angle RTU$	3. Right Angle Congruence Theorem
4. $\angle SRT \cong \angle URT$	4. Given
5. $\angle S \cong \angle U$	5. Third Angles Theorem
6. $\overline{RS} \cong \overline{RU}$	6. Given
7. T is the midpoint of $\overline{SU}$.	7. Given
8. $\overline{ST} \cong \overline{UT}$	8. Definition of midpoint
9. $\overline{RT} \cong \overline{RT}$	9. Reflexive Property of Congruence
10. $\Delta RTS \cong \Delta RTU$	10. Definition of congruent triangles

PTS: 1 DIF: Average REF: Page 232 OBJ: 4-3.3 Proving Triangles Congruent
NAT: 12.3.5.a STA: GE5.0 TOP: 4-3 Congruent Triangles

19. ANS:
a
Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{AC} \cong \overline{EC}$	1. Given
2. $\angle A \cong \angle E$	2. Given
3. $\angle BCA \cong \angle DCE$	3. Vertical Angles Theorem
4. $\angle B \cong \angle D$	4. Third Angles Theorem
5. $\Delta ABC \cong \Delta EDC$	5. Definition of congruent triangles

PTS: 1 DIF: Average REF: Page 233 OBJ: 4-3.4 Application
NAT: 12.3.5.a STA: GE5.0 TOP: 4-3 Congruent Triangles

20. ANS:
[1] $\overline{ST} \cong \overline{UT}$
[2] $\angle RTS \cong \angle RTU$
[3] $\overline{RT} \cong \overline{RT}$
It is given that $\overline{ST} \cong \overline{UT}$. Since $\angle RTS$ and $\angle RTU$ are right angles, $\angle RTS \cong \angle RTU$ by the Right Angle Congruence Theorem. By the Reflexive Property of Congruence, $\overline{RT} \cong \overline{RT}$. Therefore, $\Delta RTS \cong \Delta RTU$ by SAS.

PTS: 1 DIF: Average REF: Page 243 OBJ: 4-4.2 Application
NAT: 12.3.5.a STA: GE5.0 TOP: 4-4 Triangle Congruence: SSS and SAS

21. ANS:
[1] $3+7$
[2] $4(3)-2$
[3] 16
[4] 16
[5] SSS
$AB = a + 7 = 3 + 7 = 10$
$CD = 4a - 2 = 4(3) - 2 = 12 - 2 = 10$
$AD = 6a - 2 = 6(3) - 2 = 18 - 2 = 16$
$CB = 16$

$\overline{AB} \cong \overline{CD}$. $\overline{AD} \cong \overline{CB}$. $\overline{BD} \cong \overline{BD}$ by the Reflexive Property of Congruence. So $\Delta ABD \cong \Delta CDB$ by SSS.

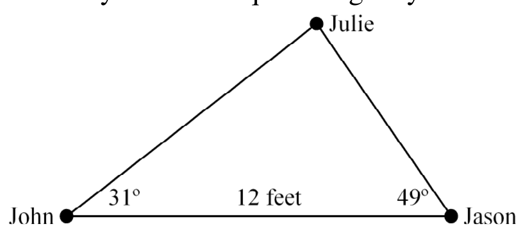
PTS: 1 DIF: Average REF: Page 244 OBJ: 4-4.3 Verifying Triangle Congruence
NAT: 12.3.5.a STA: GE2.0 TOP: 4-4 Triangle Congruence: SSS and SAS

22. ANS:
[1]. Definition of midpoint
[2] $\angle TPR \cong \angle QPS$
[3] SAS
Proof:

Statements	Reasons
1. P is the midpoint of $\overline{TQ}$ and $\overline{RS}$.	1. Given
2. $\overline{TP} \cong \overline{QP}$, $\overline{RP} \cong \overline{SP}$	2. Definition of midpoint
3. $\angle TPR \cong \angle QPS$	3. Vertical Angles Theorem
4. $\Delta TPR \cong \Delta QPS$	4. SAS

PTS: 1 DIF: Average REF: Page 244 OBJ: 4-4.4 Proving Triangles Congruent
NAT: 12.3.5.a STA: GE5.0 TOP: 4-4 Triangle Congruence: SSS and SAS

23. ANS:
Yes. They form a unique triangle by ASA.



Statements 2 and 3 determine the measures of two angles of the triangle.
Statement 1 determines the length of the included side.
By ASA, the triangle must be unique.

PTS: 1 DIF: Average REF: Page 252 OBJ: 4-5.1 Problem-Solving Application
NAT: 12.3.3.f STA: 7MR3.1 TOP: 4-5 Triangle Congruence: ASA AAS and HL

24. ANS:
1. Alternate Interior Angles Theorem
2. Alternate Exterior Angles Theorem
1. $\angle B$ and $\angle G$ are alternate interior angles and $\overline{AB} \parallel \overline{GH}$. Thus by the Alternate Interior Angles Theorem, $\angle B \cong \angle G$.

2. $\angle ACB$ and $\angle HFG$ are alternate exterior angles and $\overleftrightarrow{AC} \parallel \overleftrightarrow{FH}$. Thus by the Alternate Exterior Angles Theorem, $\angle ACB \cong \angle HFG$.

PTS: 1 DIF: Average REF: Page 254
OBJ: 4-5.3 Using AAS to Prove Triangles Congruent NAT: 12.3.5.a
STA: GE5.0 TOP: 4-5 Triangle Congruence: ASA AAS and HL

25. ANS:
Yes.
$\overline{AB} \parallel \overline{CD}$ is given. In addition, by the Reflexive Property of Congruence, $\overline{AD} \cong \overline{AD}$. Since $\overline{AC} \parallel \overline{BD}$ and $\overline{AC} \perp \overline{PB}$, by the Perpendicular Transversal Theorem $\overline{BD} \perp \overline{PB}$. By the definition of right angle, $\angle ABD$ is a right angle. Similarly, $\angle DCA$ is a right angle. Therefore, $\Delta ABD \cong \Delta DCA$ by the HL Congruence Theorem.

PTS: 1 DIF: Average REF: Page 255 OBJ: 4-5.4 Applying HL Congruence
NAT: 12.3.2.e STA: GE5.0 TOP: 4-5 Triangle Congruence: ASA AAS and HL

26. ANS:
$\Delta ABO \cong \Delta CDO$ by SAS and $\angle A \cong \angle C$ by CPCTC, so m$\angle A = 40°$ by substitution.
From the figure, $\overline{CO} \cong \overline{AO}$, and $\overline{DO} \cong \overline{BO}$. $\angle AOB \cong \angle COD$ by the Vertical Angles Theorem. Therefore, $\Delta ABO \cong \Delta CDO$ by SAS and $\angle A \cong \angle C$ by CPCTC. m$\angle A = 40°$ by substitution.

PTS: 1 DIF: Average REF: Page 260 OBJ: 4-6.1 Application
NAT: 12.3.2.e STA: GE5.0 TOP: 4-6 Triangle Congruence: CPCTC

27. ANS:
1. Congruent Supplements Theorem
2. $\angle CAB \cong \angle CAD$
3. Reflexive Property of Congruence
4. AAS
5. CPCTC
1a. By the Linear Pair Theorem, $\angle CBF$ and $\angle ABC$ are supplementary and $\angle CDG$ and $\angle ADC$ are supplementary.
1b. Given $\angle CBF \cong \angle CDG$, by the Congruent Supplements Theorem, $\angle ABC \cong \angle ADC$.

2. $\angle CAB \cong \angle CAD$ by the definition of an angle bisector.

3. $\overline{AC} \cong \overline{AC}$ by the Reflexive Property of Congruence

4. Two angles and a nonincluded side of ΔACB and ΔACD are congruent. By AAS, $\Delta ACB \cong \Delta ACD$.

5. Since $\Delta ACB \cong \Delta ACD$, $\overline{AD} \cong \overline{AB}$ by CPCTC.

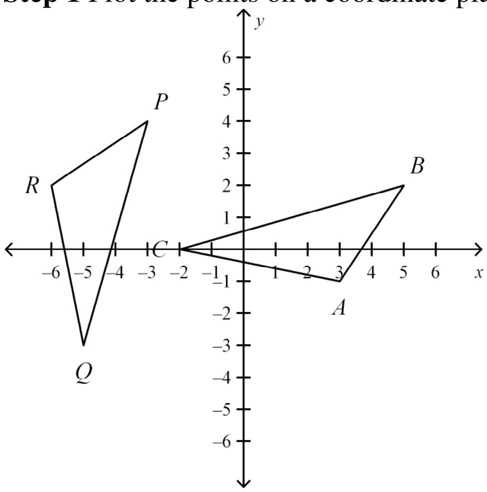
PTS: 1 DIF: Average REF: Page 260
OBJ: 4-6.2 Proving Corresponding Parts Congruent NAT: 12.3.5.a
STA: GE5.0 TOP: 4-6 Triangle Congruence: CPCTC

28. ANS:
[1] AAS
[2] CPCTC
[1] Steps 1 and 7 state that two angles and a nonincluded side of ΔMLN and ΔPLO are congruent. By AAS, $\Delta MLN \cong \Delta PLO$.

[2] Since $\Delta MLN \cong \Delta PLO$, by CPCTC, $\overline{ML} \cong \overline{PL}$.

PTS: 1 DIF: Average REF: Page 261 OBJ: 4-6.3 Using CPCTC in a Proof
NAT: 12.3.5.a STA: GE5.0 TOP: 4-6 Triangle Congruence: CPCTC

29. ANS:
[1] PQ
[2] $\overline{RP}$
[3] ΔRPQ
[4] SSS
[5] CPCTC
Step 1 Plot the points on a coordinate plane.



Step 2 Use the Distance Formula to find the lengths of the sides of each triangle.

$$D = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

$$AB = \sqrt{(5-3)^2 + \left(2-(-1)\right)^2} = \sqrt{4+9} = \sqrt{13}$$

$$RP = \sqrt{\left(-3-(-6)\right)^2 + (4-2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$BC = \sqrt{(-2-5)^2 + (0-2)^2} = \sqrt{49+4} = \sqrt{53}$$

$$PQ = \sqrt{\left(-5-(-3)\right)^2 + (-3-4)^2} = \sqrt{4+49} = \sqrt{53}$$

$$CA = \sqrt{\left(3-(-2)\right)^2 + (-1-0)^2} = \sqrt{25+1} = \sqrt{26}$$

$$QR = \sqrt{\left(-6-(-5)\right)^2 + \left(2-(-3)\right)^2} = \sqrt{1+25} = \sqrt{26}$$

$AB = RP = \sqrt{15}$, $BC = PQ = \sqrt{53}$, and $CA = QR = \sqrt{26}$. So $\overline{AB} \cong \overline{RP}$, $\overline{BC} \cong \overline{PQ}$, and $\overline{CA} \cong \overline{QR}$. Therefore $\Delta ABC \cong \Delta RPQ$ by SSS, and $\angle ABC \cong \angle RPQ$ by CPCTC.

PTS: 1 DIF: Average REF: Page 261
OBJ: 4-6.4 Using CPCTC in the Coordinate Plane NAT: 12.2.1.e
STA: GE5.0 TOP: 4-6 Triangle Congruence: CPCTC

30. ANS:
[1] a right
[2] 12
[3] Midpoint
[4] a right
[5] 3
[6] 12
ΔPQR is a right triangle with height PQ and base QR. The area of $\Delta PQR = \frac{1}{2}bh = \frac{1}{2}(3)(8) = 12$ square units. By the Midpoint Formula, the coordinates of $S = \left(\frac{0+0}{2}, \frac{0+8}{2}\right) = (0, 4)$ and the coordinates of $T = \left(\frac{-3+0}{2}, \frac{0+8}{2}\right) = \left(-\frac{3}{2}, 4\right)$. Thus ΔPST is a right triangle with height PS and base ST. So the area of $\Delta PST = \frac{1}{2}bh = \frac{1}{2}[0-(-\frac{3}{2})](8-4) = \frac{1}{2}(\frac{3}{2})(4) = 3$ square units. Since $3 = \frac{1}{4}(12)$, the area of ΔPST is one fourth the area of ΔPQR.

PTS: 1 DIF: Average REF: Page 268
OBJ: 4-7.2 Writing a Proof Using Coordinate Geometry NAT: 12.3.5.a
STA: GE17.0 TOP: 4-7 Introduction to Coordinate Proof

31. ANS:
Both a. and b.
Since the triangle has a right angle, place the vertex of the right angle at the origin and position a leg along each axis. This can be done in two ways: with the leg of length r along the x-axis or along the y-axis. In the first case, the three vertices are (0, 0), (r, 0), and (0, $2s+4$). In the second case, the three vertices are (0, 0), (0, r), and ($2s+4$, 0).

PTS: 1 DIF: Average REF: Page 268 OBJ: 4-7.3 Assigning Coordinates to Vertices
NAT: 12.3.4.d STA: GE17.0 TOP: 4-7 Introduction to Coordinate Proof

32. ANS:
[1] $2cd$
[2] Midpoint
[3] $\frac{1}{2}cd$
ΔPQR is right triangle with height $2d$ units and base $2c$ units. The area of $\Delta PQR = \frac{1}{2}bh = \frac{1}{2}(2c)(2d) = 2cd$ square units. By the Midpoint Formula, the coordinates of S are $\left(\frac{0+0}{2}, \frac{0+2d}{2}\right) = (0, d)$ and the coordinates of T are $\left(\frac{2c+0}{2}, \frac{0+2d}{2}\right) = (c, d)$. Thus ΔPST is a right triangle with height $2d - d = d$ units and base c units. So the area of $\Delta PST = \frac{1}{2}bh = \frac{1}{2}(c)(d) = \frac{1}{2}cd$ square units. Since $\frac{1}{2}cd = \frac{1}{4}(2cd)$, the area of ΔPST is one fourth the area of ΔPQR.

PTS: 1 DIF: Average REF: Page 269 OBJ: 4-7.4 Writing a Coordinate Proof
NAT: 12.3.5.a STA: GE17.0 TOP: 4-7 Introduction to Coordinate Proof

33. ANS:
$m\angle A = 65°$

BW Tauri and M77 are equidistant from Earth, so $\overline{AC} \cong \overline{BC}$. By the Isosceles Triangle Theorem, $\angle A \cong \angle CBA$. From the Angle Addition Postulate, $m\angle CBA = 65°$ and $m\angle A$=65°.

PTS: 1 DIF: Average REF: Page 274 OBJ: 4-8.1 Application
NAT: 12.3.3.c STA: GE12.0 TOP: 4-8 Isosceles and Equilateral Triangles

34. ANS:
$m\angle Q = 75°$

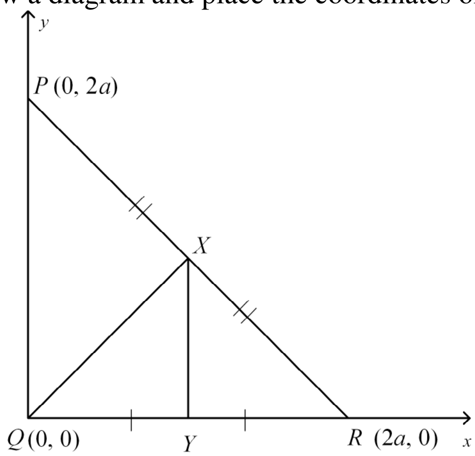
$m\angle Q = m\angle R = (2x + 15)°$	Isosceles Triangle Theorem
$m\angle P + m\angle Q + m\angle R = 180°$	Triangle Sum Theorem
$x + (2x + 15) + (2x + 15) = 180$	Substitute x for $m\angle P$ and substitute $2x + 15$ for $m\angle Q$ and $m\angle R$.
$5x = 150$	Simplify and subtract 30 from both sides.
$x = 30$	Divide both sides by 5.

Thus $m\angle Q = (2x + 15)° = [2(30) + 15]° = 75°$.

PTS: 1 DIF: Average REF: Page 274 OBJ: 4-8.2 Finding the Measure of an Angle
NAT: 12.3.3.f STA: GE12.0 TOP: 4-8 Isosceles and Equilateral Triangles

35. ANS:
[1] $2a$
[2] the Midpoint Formula
[3] a, [4] a
[5] the Distance Formula
[6] a, [7] a

Draw a diagram and place the coordinates of ΔPQR and ΔQXY as shown.



By the Midpoint Formula, the coordinates of X are $\left(\frac{0+2a}{2}, \frac{2a+0}{2}\right) = (a, a)$ and

the coordinates of Y are $\left(\frac{0+2a}{2}, \frac{0+0}{2}\right) = (a, 0)$.

By the Distance Formula,

$XY = \sqrt{(0-a)^2 + (a-a)^2} = a$ and

$QY = \sqrt{(0-0)^2 + (a-0)^2} = a$.

Since $XY = QY$, $\overline{XY} \cong \overline{QY}$ by definition. So ΔQXY is isosceles.

PTS: 1 DIF: Average REF: Page 275 OBJ: 4-8.4 Using Coordinate Proof
NAT: 12.3.5.a STA: GE17.0 TOP: 4-8 Isosceles and Equilateral Triangles

36. ANS:
[1] $\overline{AB} \cong \overline{BC} \cong \overline{CA}$
[2] definition of equilateral triangle
[3] equiangular

Proof:

The same compass setting was used to create $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, so $\overline{AB} \cong \overline{BC} \cong \overline{CA}$. By the definition of equilateral triangle, ΔABC is equilateral. Since ΔABC is equilateral, it is also equiangular. So $m\angle A + m\angle B + m\angle C = 180°$. Therefore, $m\angle A = 60°$.

PTS: 1 DIF: Average REF: Page 283
OBJ: 4-Ext.2 Proving the Construction of an Angle TOP: 4-Ext Proving Constructions Valid