guide to solve cryptic multiplication question

Monday, 9 July 2012

Guys in elitmus test you will be getting 3 questions(30 marks) on cryptic multiplication...damn sure!!!
This tutorial will be very helpful in solving those questions.I myself solved those 3 questions just by reading this tutorial.you won't find any help regarding this topic anywhere else so please read this tutorial.

WHAT IS CRYPTARITHMETIC?

Cryptarithmetic is the science and art of creating and solving cryptarithms.

A cryptarithm is a genre of mathematical puzzle in which the digits are replaced by letters of the alphabet or other symbols.

The invention of Cryptarithmetic has been ascribed to ancient China. This art was originally known as letter arithmetic or verbal arithmetic. In India, during the Middle Ages, were developed the arithmetical restorations or "skeletons" a type of cryptarithms in which most or all of the digits have been replaced by asterisks.

In 1864 the first cryptarithm appeared in the USA, in American Agriculturist.

The word cryptarithmetic ("cryptarithmie" in French) was introduced by M. Vatriquant, writing under the pseudonym Minos, in the May 1931 issue of Sphinx, a Belgian magazine of recreational mathematics published in French from 1931 to 1939.

A type of alphametic addition puzzle termed "doubly-true" was introduced in 1945 by Alan Wayne. It is made up of "number words" that, when read, also form a valid sum.

In 1955, J. A. H. Hunter coined the word alphametic to designate a cryptarithm whose letters form sensible words or phrases.

The world's best known alphametic puzzle is undoubtedly SEND + MORE = MONEY. It was created by H. E. Dudeney and first published in the July 1924 issue of Strand Magazine associated with the story of a kidnapper's ransom demand.

Modernization by introducing innovations such as computers and the Internet is making quite an impact on cryptarithmetic. If you are interested in knowing more about this revolution read the article Will cryptarithmetic survive innovation?

HOW TO SOLVE A PUZZLE

1. Preparation

Rewrite the problem, expanding the interlinear space to make room for
trial numbers that will be written under the letters.

For example, the puzzle SEND + MORE = MONEY, after solving, will appear like
this:

```
S E N D
            9 5 6 7
          + M O R E
            1 0 8 5
          ---------
          M O N E Y
          1 0 6 5 2
```

2. Remember cryptarithmetic conventions

- Each letter or symbol represents only one digit throughout the problem;
- When letters are replaced by their digits, the resultant arithmetical
  operation must be correct;
- The numerical base, unless specifically stated, is 10;

- Numbers must not begin with a zero;
- There must be only one solution to the problem.

3. See subtractions as "upside-down" additions

Ease the analysis of subtractions by reading them as upside-down additions. Remember that you can check a subtraction by adding the difference and the subtracter to get the subtrahend: it's the same thing. This subtraction:

```
C O U N T
           - C O I N
           ---------
             S N U B
```

must be read from the bottom to the top and from the right to the left, as if it were this series of additions:

```
B + N = T + C1
        U + I = N + C2
        N + O = U + C3
        S + C = O + C4
```

C1, C2, C3 and C4 are the carry-overs of "0" or "1" that are to be added to the next column to the left.

4. Search for "0" and "9" in additions or subtractions

A good hint to find zero or 9 is to look for columns containing two or three identical letters.
Look at these additions:

```
* * * A                 * * * B
  + * * * A               + * * * A
    -------                 -------
    * * * A                 * * * B
```

The columns A+A=A and B+A=B indicate that A=zero. In math this is called the "additive identity property of zero"; it says that you add "0" to anything and it doesn't change, therefore it stays the same. Now look at those same additions in the body of the cryptarithm:

```
* A * *                 * B * *
  + * A * *               + * A * *
    -------                 -------
    * A * *                 * B * *
```

In these cases, we may have A=zero or A=9. It depends whether or not "carry 1" is received from the previous column. In other words, the "9" mimics zero every time it gets a carry-over of "1".

5. Search for "1" in additions or subtractions

Look for left hand digits. If single, they are probably "1".
Take the world's most famous cryptarithm:

```
S E N D
         + M O R E
         ---------
         M O N E Y
```

"M" can only equal 1, because it is the "carry 1" from the column S+M=O (+10). In other words, every time an addition of "n" digits gives a total of "n+1" digits, the left hand digit of the total must be "1". In this Madachy's subtraction problem, "C" stands for the digit "1":

```
C O U N T
         - C O I N
         ---------
           S N U B
```

6. Search for "1" in multiplications or divisions

In this multiplication:

```
M A D
             B E
         -------
           M A D
         R A E
         -------
         A M I D
```

The first partial product is E x MAD = MAD. Hence "E" must equal "1". In math jargon this is called the "identity" property of "1" in multiplication; you multiply anything by "1" and it doesn't change, therefore it remains the same.Look this division:

```
K T
            --------
     N E T / L I N K
             N E T
             -------
             K E K K
             K T E C
             -------
               K E Y
```

In the first subtraction, we see K x NET = NET. Then K=1.

7. Search for "1" and "6" in multiplications or divisions

Any number multiplied by "1" is the number itself. Also, any even number multiplied by "6" is the number itself:

```
4 x 1 = 4
         7 x 1 = 7
         2 x 6 = 2 (+10)
         8 x 6 = 8 (+40)
```

Looking at right hand digits of multiplications and divisions, can help you spot digits "1" and "6". Those findings will show like these ones:

```
C B
                          ----------
     * * A         * * A / * * * * *
       B C                 * * * C
   ------                  ---------
  * * * C                    * * * *
 * * * B                     * * * B
 ---------                   -------
 * * * * *                     * * *
```

The logic is: if

```
C  x  * * A  =  * * * C
   B  x  * * A  =  * * * B
```

then A=1 or A=6.

8. Search for "0" and "5" in multiplications or divisions

Any number multiplied by zero is zero. Also, any odd number multiplied by "5" is "5":

```
3 x 0 = 0
          6 x 0 = 0
          7 x 5 = 5 (+30)
          9 x 5 = 5 (+40)
```

Looking at right hand digits of multiplications and divisions, can help you spot digits "0" and "5". Those findings will show like these ones:

```
C B
                          ----------
    * * A         * * A / * * * * *
      B C                 * * * A
  -------                 ---------
  * * * A                   * * * *
* * * A                     * * * A
---------                   -------
* * * * *                     * * *
```

The logic is: if

```
C  x  * * A  =  * * * A
   B  x  * * A  =  * * * A
```

then A=0 or A=5

9. Match to make progress

Matching is the process of assigning potential values to a variable and testing whether they match the current state of the problem. To see how this works, let's attack this long-hand division:

```
K M
           ----------
    A K A / D A D D Y
            D Y N A
            ---------
              A R M Y
```

```
            A R K A
            -------
                R A
```

To facilitate the analysis, let's break it down to its basic components, i.e., 2 multiplications and 2 subtractions:

```
I.    K  x  A K A = D Y N A

  II.  M  x  A K A = A R K A

  III.         D A D D
             - D Y N A
             ---------
                 A R M

  IV.          A R M Y
             - A R K A
             ---------
                   R A
```

From I and II we get:

```
K  x  * * A  =  * * * A
   M  x  * * A  =  * * * A
```

This pattern suggests A=0 or A=5. But a look at the divisor "A K A" reveals that A=0 is impossible, because leading letters cannot be zero. Hence A=5.Replacing all A's with "5", subtraction IV becomes:

```
5 R M Y
        - 5 R K 5
        ---------
              R 5
```

From column Y-5=5 we get Y=0. Replacing all Y's with zero, multiplication I will be:

```
K  x  5 K 5  = D 0 N 5
```

Now, matching can help us make some progress. Digits 1, 2, 3, 4, 6, 7, 8 and 9 are still unidentified. Let's assign all these values to the variable K, one by one, and check which of them matches the above pattern. Tabulating all data, we would come to:

```
K  x  5K5  =  D0N5
           ----------------------
             1     515      515
             2     525     1050
             3     535     1605
             4     545     2180
             6     565     3390
SOLUTION --> 7     575     4025 <-- SOLUTI
ON
             8     585     4680
             9     595     5355
            ----------------------
```

You can see that K=7 is the only viable solution that matches the current pattern of multiplication I, yielding:

```
K  x  A K A  =  D Y N A
   7     5 7 5     4 0 2 5
```

This solution also identifies two other variables: D=4 and N=2.

10. When stuck, generate-and-test

Usually we start solving a cryptarithm by searching for 0, 1, and 9. Then if we are dealing with an easy problem there is enough material to proceed decoding the other digits until a solution is found.
This is the exception and not the rule. Most frequently after decoding 1 or 2 letters (and sometimes none) you get stuck. To make progress we must apply the **generate-and-test** method, which consists of the following procedures:

- 1. List all digits still unidentified;
- 2. Select a base variable (letter) to start generation;
- 3. Do a cycle of generation and testing: from the list of still unidentified digits (procedure 1) get one and assign it to the base variable; eliminate it from the list; proceed guessing values for the other variables; test consistency; if not consistent, go to perform the next cycle (procedure 3); if consistent, stop: you have found the solution to the problem.

To demonstrate how this method works, let's tackle this J. A. H. Hunter's addition:

```
T A K E
                A
       +  C A K E
```

```
          ----------
             K A T E
```

The column AAA suggests A=0 or A=9. But column EAEE indicates that A+E=10, hence the only acceptable value for "A" is 9, with E=1. Replacing all "A's" with 9 and all "E's" with 1, we get

```
T 9 K 1
                9
       +  C 9 K 1
       ----------
          K 9 T 1
```

Letter repetition in columns KKT and TCK allows us to set up the following algebraic system of equations:

```
C1 + K + K = T + 10
       C3 + T + C = K
```

Obviously C1=1 and C3=1. Solving the equation system we get K+C=8: not much, but we discovered a relationship between the values of "K" and "C" that will help us later. But now we are stuck! It's time to use the **"generate-and-test"** method. Procedure 1: digits 2,3,4,5,6,7 and 8 are still unidentified;
Procedure 2: we select "K" as the base variable;
CYCLE #1, procedure 3: column TCK shows that T+C=K and no carry, hence "K" must be a high valued digit. So we enter the list obtained through procedure 1 from the high side, assigning "8" to the base variable "K".
Knowing that K+C=8, if K=8 then C=0. But this is an unacceptable value for "C", because the addend "CAKE" would become "0981" and cryptarithmetic conventions say that no number can start with zero. So, we must close this cycle and begin cycle #2.
By now, the addition layout and the table summarizing current variable data would look like this:

```
T 9 8 1   CYCLE  A   E   K   C   T
         9   ========================
+  0 9 8 1    #1    9   1   8  [0]
----------
   8 9 T 1
```

Conflicting values for variables are noted within square brackets.**CYCLE #2**, procedure 3: assigning "7" to the letter "K" we get C=1 because K+C=8. This is an unacceptable value for "C" considering that we have already fixed E=1. Again we have to close the current cycle and go to cycle #3, with the setup and table showing:

```
T 9 7 1   CYCLE  A   E   K   C   T
         9   ========================
+  1 9 7 1    #1    9   1   8  [0]
----------    #2    9   1   7  [1]
   7 9 T 1
```

CYCLE #3, procedure 3: assigning "6" to the letter "K" we get C=2 because K+C=8. Testing these values for "K" and "C" in the column TCK, we get C3+T+2+=6 making T=3.Now, testing T in column KKT, we would obtain C1+K+K=T+10 or 1+6+6=T+10, making T=3. This is an acceptable value for T, confirming the previous value T=3 we had already found.
So, we have got the final solution to the problem, stopping the routine **"generate-and-test"**.
The final layout and table would read

```
3 9 6 1   CYCLE  A   E   K   C   T
         9   ========================
+  2 9 6 1    #1    9   1   8  [0]
----------    #2    9   1   7  [1]
   6 9 3 1    #3    9   1   6   2   3
```

EXAMPLES WORKED OUT IN DETAIL BY MASTER PUZZLISTS

1. Geoffrey Mott-Smith

In "Mathematical Puzzles for Beginners & Enthusiasts"©

```
          S E N D
       +  M O R E
       -----------
        M O N E Y
```

We see at once that M in the total must be 1**,** since the total of the column SM cannot reach as high as 20**.** Now if M in this column is replaced by 1, how can we make this column total as much as 10 to provide the 1 carried over to the left below? Only by making S very large: 9 or 8. In either case the letter O must stand for zero: the summation of SM could produce only 10 or 11**,** but we cannot use 1 for letter O as we have already used it for M**.** If letter O is zero, then in column EO we cannot reach a total as high as 10**,** so that there will be no 1 to carry over from this column to SM. Hence S must positively be 9. Since the summation EO gives N, and letter O is zero, N must be 1 greater than E and the column NR must total over 10. To put it into an equation: E + 1 = N From the NR column we can derive the equation: N + R + (+ 1) = E + 10 We have to insert the expression (+ 1) because we don't know yet whether 1 is carried over from column DE. But we do know that 1 has to be carried over from column NR to EO. Subtract the first equation from the second: R + (+1) = 9 We cannot let R equal 9, since we already have S equal to 9. Therefore we will have to make R equal to 8; hence we know that 1 has to be carried over from column DE. Column DE must total at least 12**,** since Y cannot be 1 or zero. What values can we give D and E to reach this total? We have already used 9 and 8 elsewhere. The only digits left that are high enough are 7, 6 and 7, 5. But remember that one of these has to be E, and N is 1 greater than E. Hence E must be 5, N must be 6, while D is 7. Then Y turns out to be 2**,** and the puzzle is completely solved. ©

2. Steven Kahan

In "Take a Look at a Good Book"©

```
          E A T
+       T H A T
-----------
      A P P L E
```

Since every four-digit number is less than 10,000 and every three-digit number is less than 1,000, the sum of two such numbers is necessarily less than 11,000. This sum, though, is a five-digit number, hence is greater than 10,000. Consequently, A must be 1 and P must be 0. Further, we can conclude that T = 9. Otherwise, we would be adding a number less than 1,000 to one less than 9,000, leaving us short of the requisite total. The units column then produces E = 8 while generating a carryover of 1 into the tens column. Together with the previously found value of A, we learn from the tens column that L = 3. Finally, the hundreds column yields the equation E + H = P + 10, where the "10" is required to accommodate the needed carryover into the thousands column. When the values of E and P are substituted into this relationship, we get 8 + H = 10, from which it follows that H = 2. Therefore, the unique solution of the puzzle turns out to be 819 + 9219 = 10038.

3. J. A. H. Hunter

In "Entertaining Mathematical Teasers and How to Solve Them"©

```
        N O
      G U N
+       N O
---------
    H U N T
```

Obviously H = 1. From the NUNN column we must have "carry 1," so G = 9, U = zero. Since we have "carry" zero or 1 or 2 from the ONOT column, correspondingly we have N + U = 10 or 9 or 8. But duplication is not allowed, so N = 8 with "carry 2" from ONOT. Hence, O + O = T + 20 - 8 = T + 12. Testing for T = 2, 4 or 6, we find only T = 2 acceptable, O = 7. So we have 87 + 908 + 87 = 1082.

4. Maxey Brooke

In "150 Puzzles In Crypt-Arithmetic"©

```
      A B C
x       D E
---------
      F E C
    D E C
---------
    H G B C
```

In the second partial product we see D x A = D, hence A = 1. D x C and E x C both end in C, hence C = 5. D and E must be odd. Since both partial products have only three digits, neither can be 9. This leaves only 3 and 7. In the first partial product E x B is a number of two digits while in the second partial product D x B is a number of only one digit. Thus E is larger than D, so E = 7 and D = 3. Since D x B has only one digit, B must be 3 or less. The only two possibilities are 0 and 2. B cannot be zero because 7B is a two-digit number. Thus B = 2. By completing the multiplication, F = 8, E = 7, and G = 6. The answer is 125 x 37 = 4625

Dover Publications, Inc., New York, 1963.

5. Joseph S. Madachy

In "Madachy´s Mathematical Recreations"©

```
(B E) (B E) = M O B
```

Here a 3-digit number is the product of a 2-digit number multiplied by itself. Basic knowledge of the laws of multiplication will immediately force the conclusion that B cannot be greater than 3. For if B is 4, and the lowest possible value, 0, is assigned to E then BE = 40. However, (40)(40) = 1,600, a 4-digit number, and the product in the puzzle to be solved has but 3 digits. Convention demands that the initial letters or symbols of alphametics cannot be 0, so B is either 1, 2, or 3. Another convention demands that 2 different letters cannot be substituted for the same digit. That is, if B turns out to be 3, then no other letter in this alphametic could stand for 3. Attention can be directed to E since much can be deduced from the fact that (E)(E) ends in B. If E equals 0, 1, 5, or 6, then the product would be a number ending in 0, 1, 5, or 6, respectively. Since the product, MOB, does not end in E, these numbers for E are eliminated. 2, 3, 4, 7, and 8 can also be eliminated as values for E, since they would yield the terminal digits of 4, 6, or 9 for MOB, and B has been established as being 1, 2, or 3. Only one value for E, 9, remains: (9) (9) = 81 so B = 1, and the alphametic is solved: (BE) (BE) = MOB is (19) (19) = 361.

6. C. R. Wylie Jr.

In "101 Puzzles in Thought & Logic"©

```
         A L E
     x   R U M
     ---------
       W I N E
     W U W L
   E W W E
  ------------
   E R M P N E
```

To systematize our work we first write in a row the different letters appearing in the problem:

```
A L E R U M W I N P
```

Over each letter we will write its numerical equivalent when we discover it. In the columns under the various letters we will record clues and tentative hypotheses, being careful to put all related inferences on the same horizontal line. In problems of this sort the digits 0 and 1 can often be found, or at least restricted to a very few possibilities, by simple inspection. For instance, 0 can never occur as the leftmost digit of an integer, and when any number is multiplied by zero the result consists exclusively of zeros. Moreover when any number is multiplied by 1 the result is that number itself. In the present problem, however, we can identify 0 by an even simpler observation. For in the second column from the right, N plus L equals N, with nothing carried over from the column on the right. Hence L must be zero. In our search for 1 we can eliminate R, U, and M at once, since none of these, as multipliers in the second row, reproduces A L E. Moreover E cannot be 1 since U times E does not yield a product ending in U. At present, however, we have no further clues as to whether 1 is A, I, N, P, or W. Now the partial product W U W L ends in L, which we know to be 0. Hence one of the two letters U and E must be 5. Looking at the units digits of the other partial products, we see that both M x E and R x E are numbers ending in E. A moment's reflection (or a glance at a multiplication table) shows that E must therefore be 5. But if E is 5, then both R and M must be odd, since an even number multiplied by 5 would yield a product ending in 0, which is not the case in either the first or third partial product. Moreover, by similar reasoning it is clear that U is an even number. At this point it is convenient to return to our array and list under U the various possibilities, namely 2, 4, 6, and 8. Opposite each of these we record the corresponding value of W as read from the partial product W U W L, whose last two digits are now determined since the factor A L E is known to be _05. These values of W are easily seen to be 1, 2, 3, and 4. From an inspection of the second column from the left we can now deduce the corresponding possibilities for R. As we have already noted, R must be odd; hence its value is twice W plus 1 (the 1 being necessarily carried over from the column on the right). The possible values for R are then 3, 5, 7, and 9, and our array looks like this:

```
  0 5
A L E R U M W I N P
      3 2   1
      5 4   2
      7 6   3
      9 8   4
```

Now in the third column from the left in the example the sum of the digits W, U, and W must be more than 9, since 1 had to be carried over from this column into the column on the left. The values in the first two rows of the array are too low for this, however, hence we can cross out both of these lines. A further consideration of the sum of the digits W, U, and W in the third column from the left, coupled with the fact that M is known to be odd, shows that in the third row of the array M must be 3 while in the fourth row it must be 7. This permits us to reject the third row of the array also, for it contains 3 for both M and W, which is impossible. The correct solution must therefore be the one contained in the fourth row. Hence R is 9, U is 8, M is 7, and W is 4. Substituting these into the problem it is a simple matter to determine that A is 6, I is 2, N is 3, and P is 1. This completes the solution.

POSTED BY GAURAV SINGLA AT 02:23

2 COMMENTS:

Rahul Sagar said... 7 August 2012 01:38

is there one problem and three questions based on that,or three different problems

Amit said... 11 August 2012 00:23

@rahul sagar there will be three questions based on one problem..i have posted the 2 such questions from elitmus...check them!

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