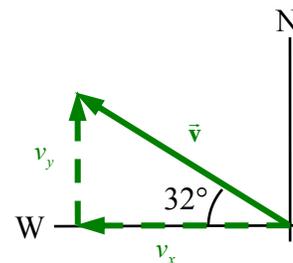


### Physics 100A Homework 3 – Chapter 4

4.2 A sailboat runs before the wind with a constant speed of 4.5 m/s in a direction 32° north of west.

**Picture the Problem:** The vector involved in this problem is depicted at right.

**Strategy:** Separate  $\vec{v}$  into  $x$ - and  $y$ -components. Let north be along the  $y$ -axis, west along the  $-x$  axis. Find the components of the velocity in each direction and use them to find the distances traveled.



**Solution: 1. (a)** Find  $v_x$  and  $v_y$  :  $v_x = -(4.2 \text{ m/s}) \cos 32^\circ = -3.56 \text{ m/s}$

$$v_y = (4.2 \text{ m/s}) \sin 32^\circ = 2.23 \text{ m/s}$$

2. Find the westward distance traveled:

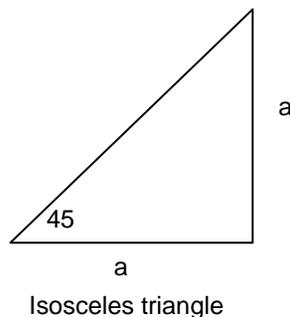
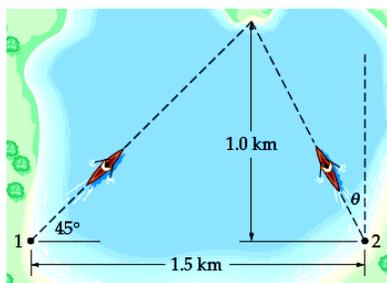
$$y = |v_x|t = (3.56 \text{ m/s})(25 \text{ min} \times 60 \text{ s/min}) = 5300 \text{ m} = \boxed{5.3 \text{ km}}$$

3. (b) Find the northward distance traveled:

$$x = v_y t = (2.23 \text{ m/s})(25 \text{ min} \times 60 \text{ s/min}) = 3300 \text{ m} = \boxed{3.3 \text{ km}}$$

**Insight:** The northward and westward motions can be considered separately. In this case they are each described by constant velocity motion.

4.7 Two canoeists start paddling at the same time and head toward a small island in a lake, as shown in the figure. Canoeist 1 paddles with a speed of 1.65 km/s at an angle of 45° north of east. Canoeist 2 starts on the opposite shore of the lake, a distance of 1.5 km due east of canoeist 1.



**Strategy:** Canoeist 1's 45° path determines an isosceles right triangle whose legs measure 1.0 km. So canoeist 2's path determines a right triangle whose legs measure 1.0 km north and 0.5 km west. Use the right triangle to find the angle  $\theta$ . Find the distance traveled by each canoeist and set the times of travel equal to each other to determine the appropriate speed of canoeist 2:

**Solution: 1. (a)** Find the angle  $\theta$  from the right triangle of canoeist 2

$$\theta = \tan^{-1}\left(\frac{0.5 \text{ km}}{1.0 \text{ km}}\right) = \boxed{27^\circ}$$

2. (b) Set the travel times equal to each other:

$$t_1 = t_2 \Rightarrow \frac{\Delta r_1}{v_1} = \frac{\Delta r_2}{v_2}$$

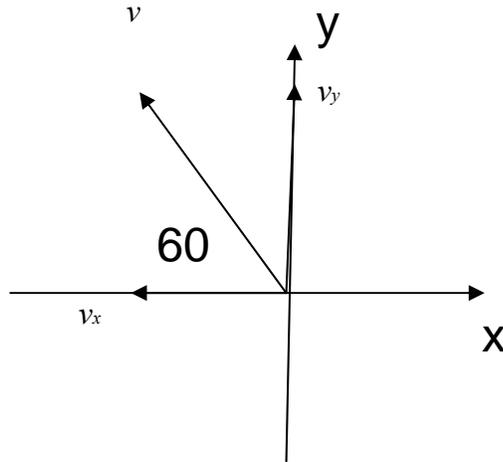
3. Use the resulting ratio to find the appropriate speed of canoeist 2:

$$v_2 = \frac{d_2}{d_1} v_1 = \frac{\sqrt{(0.5 \text{ km})^2 + (1.0 \text{ km})^2}}{\sqrt{(1.0 \text{ km})^2 + (1.0 \text{ km})^2}} \left(1.35 \frac{\text{m}}{\text{s}}\right) = \boxed{1.1 \text{ m/s}}$$

**Insight:** There are other ways to solve this problem. For instance, because the motions are independent, we could set the time it takes canoeist 1 to travel 1.0 km horizontally equal to the time it takes canoeist 2 to travel 0.5 km horizontally.

## Introduction to projectile motion.

Consider a particle with initial velocity  $\vec{v}$  that has magnitude of 12.0 m/s and is directed  $60.0^\circ$  above the negative x axis.



**Part A** What is the x component  $v_x$  of  $\vec{v}$ ?

$$v_x = -v \cos \theta = -12 \cos(60) = 6 \text{ m/s}$$

**Part B** What is the y component  $v_y$  of  $\vec{v}$ ?

$$v_y = v \sin \theta = 12 \sin(60) = 10.4 \text{ m/s}$$

**Part C** The vertical component exhibits motion with constant nonzero acceleration, whereas the horizontal component exhibits constant-velocity motion.

**Part D** How long does it take for the balls to reach the ground? Use 10 m/s for the magnitude of the acceleration due to gravity.

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$0 = 5 + 0 - \frac{1}{2}(10)t^2 \quad 5t^2 = 5 \quad t = 1 \text{ s}$$

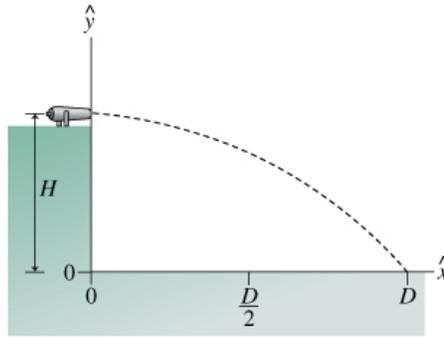
**Part E** Imagine that the ball on the left is given a nonzero initial velocity in the horizontal direction, while the ball on the right continues to fall with zero initial velocity. What horizontal speed  $v_{0x}$  must the ball on the left start with so that it hits the ground at the same position as the ball on the right?

From the applet we find that the balls are initially separated a distance of 3.0 m. The ball on the left must be able to cover this distance as it falls, so that both balls can reach the same point at the bottom.

$$x = v_{0x}t \quad v_{0x} = \frac{x}{t} = \frac{3}{1} = 3.0 \text{ m/s}$$

### Horizontal cannonball on a cliff.

A cannonball is fired horizontally from the top of a cliff. The cannon is at height  $H = 80.0 \text{ m}$  above ground level, and the ball is fired with initial horizontal speed  $v_0$ . Assume acceleration due to gravity to be  $g = 9.80 \text{ m/s}^2$ .



**Part A** Assume that the cannon is fired at time  $t=0$  and that the cannonball hits the ground at time  $t_g$ . What is the  $y$  position of the cannonball at the time  $t_g / 2$ ?

Starts with zero velocity in the  $y$ -direction and with height  $H$ . At the ground:

$$y = y_0 + v_{oy} - \frac{1}{2}gt^2 \quad 0 = H + 0 - \frac{1}{2}gt_g^2 \quad t_g = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2(80)}{9.8}} = 4.04 \text{ s}$$

At  $t_g/2 = 4.04/2 = 2.02 \text{ s}$

$$y = 80 + 0 - \frac{1}{2}9.8(2.02)^2 = 60 \text{ m}$$

**Part B** Given that the projectile lands at a distance  $D = 130 \text{ m}$  from the cliff, as shown in the figure, find the initial speed of the projectile  $v_0$ .

$$x = v_{0x}t \quad D = v_{0x}t_g \quad v_{0x} = \frac{D}{t_g} = \frac{130}{4.04} = 32.2 \text{ m/s}$$

**Part C** What is the  $y$  position of the cannonball when it is at distance  $D/2$  from the hill? If you need to, you can use the trajectory equation for this projectile, which gives  $y$  in terms of  $x$  directly:

$$y = H - \frac{gx^2}{2v_{0x}^2} = 80 - \frac{9.8(65)^2}{2(32.2)^2} = 60 \text{ m}$$

**4.8** Two divers run horizontally off the edge of a low cliff. Diver 2 runs with twice the speed of diver 1.

8. **Picture the Problem:** Two divers run horizontally off the edge of a low cliff.

**Strategy:** Use a separate analysis of the horizontal and vertical motions of the divers to answer the conceptual question.

**Solution: 1. (a)** As long as air friction is neglected there is no acceleration of either diver in the horizontal direction. The divers will continue moving horizontally at the same speed with which they left the cliff. However, the time of flight for each diver will be identical because they fall the same vertical distance. Therefore, diver 2 will travel twice as much horizontal distance as diver 1.

**2. (b)** The best explanation (see above) is I. The drop time is the same for both divers. Statement II is true but not relevant. Statement III is false because the total distance covered depends upon the horizontal speed.

**Insight:** If air friction is taken into account diver 2 will travel less than twice the horizontal distance as diver 1. This is because air friction is proportional to speed, so diver 2, traveling at a higher speed, will experience a larger force.

**4.12** A diver runs horizontally off the end of a diving board with an initial speed of 2.25 m/s. If the diving board is 3.50 m above the water, what is the diver's speed just before she enters the water?

12. **Picture the Problem:** A diver runs horizontally off a diving board and falls down along a parabolic arc, maintaining her horizontal velocity but gaining vertical speed as she falls.

**Strategy:** Find the vertical speed of the diver after falling 3.00 m. The horizontal velocity remains constant throughout the dive. Then find the magnitude of the velocity from the horizontal and vertical components.

**Solution: 1.** Use equation 4-6 to find  $v_y$  :  $v_y^2 = v_{0y}^2 - 2g\Delta y = 0 - 2(9.81 \text{ m/s}^2)(0 \text{ m} - 3.00 \text{ m}) = \underline{\underline{58.9 \text{ m}^2/\text{s}^2}}$

2. Use the components  $v_x$  and  $v_y$  to find the speed:  $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(1.85 \text{ m/s})^2 + 58.9 \text{ m}^2/\text{s}^2} = \boxed{7.89 \text{ m/s}}$

**Insight:** Projectile problems are often solved by first considering the vertical motion, which determines the time of flight and the vertical speed, and then considering the horizontal motion.

### Arrow hits the apple

An arrow is shot at an angle of  $\theta = 45^\circ$  above the horizontal. The arrow hits a tree a horizontal distance  $D=220$  m away, at the same height above the ground as it was shot.

**Part A** Find, the time that the arrow spends in the air.

When the arrow hits the ground at the same height it was shot the magnitude of its final velocity is the same as that of its initial velocity. The y component of the velocity points opposite.

The x-equation

$$x = v_{0x}t \quad D = v_0 \cos(45)t \quad v_0 = \frac{D}{\cos(45)t}$$

The velocity y equation

$$v_y = v_{0y} - gt = -v_{0y} \quad -2v_{0y} = -gt \quad v_0 = \frac{gt}{2 \sin(45)}$$

We can now equate both equations (recall  $\cos(45)=\sin(45)$ , so these cancel)

$$\frac{gt}{2 \sin(45)} = \frac{D}{\cos(45)t} \quad t^2 = \frac{2D}{g} \quad t = \sqrt{\frac{2(220)}{9.8}} = 6.7 \text{ s}$$

**Part B** Suppose someone drops an apple from a vertical distance of 6.0 meters, directly above the point where the arrow hits the tree.

How long after the arrow was shot should the apple be dropped, in order for the arrow to pierce the apple as the arrow hits the tree?

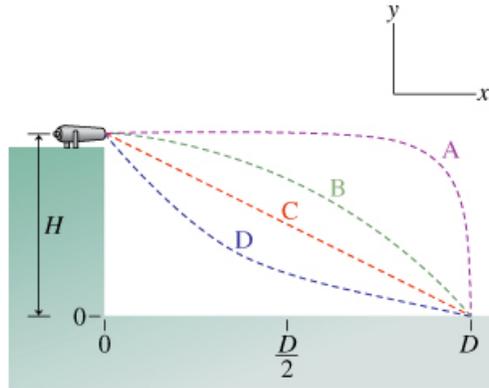
Since the arrow hits the tree at the ground, then we require that the apple hit the ground at the same moment.

The time for the apple to hit the ground from a height of 6.0 m having started from rest is:

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \quad 0 = 6 + 0 - \frac{1}{2}9.8t^2 \quad t = \sqrt{\frac{2(6)}{9.8}} = 1.1s$$

So if the arrow is in flight 6.7 s, the apple must be dropped 6.7-1.1=5.6s after the arrow is shot.

### Projectile Motion – Conceptual



**Part A** Based on the equations of motion the projectile follows a parabolic path, B

**Part B** The acceleration is always pointing downwards with a constant value of  $9.8 \text{ m/s}^2$

### Projectile Motion Ranking Task

**Part A** The case achieving the maximum height

**Part B** The case taking the longest time to hit the ground

In both cases the point is to find which case has the largest initial velocity in the  $y$  direction

1-  $v_{0y} = 15 \sin(60) = 13 \text{ m/s}$

2-  $v_{0y} = 10 \sin(60) = 8.7 \text{ m/s}$

3-  $v_{0y} = 10 \sin(90) = 10 \text{ m/s}$

4-  $v_{0y} = 15 \sin(0) = 0 \text{ m/s}$

5-  $v_{0y} = 15 \sin(30) = 7.5 \text{ m/s}$

6-  $v_{0y} = 15 \sin(45) = 10.6 \text{ m/s}$

The order from largest to smallest: 1, 6, 3, 2, 5, 4

**4.8** Three projectiles (A, B, and C) are launched with different initial speeds so that they reach the same maximum height, as shown in the figure. List the projectiles in order of increasing initial speed.

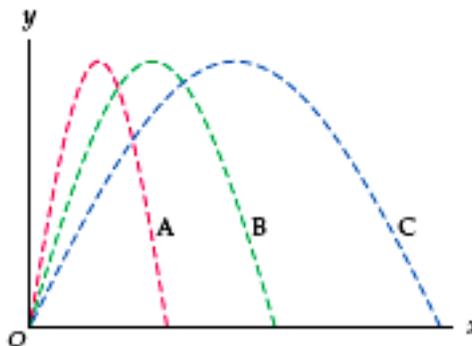
28. **Picture the Problem:** Three projectiles A, B, and C are launched with different initial speeds and angles and follow the indicated paths.

**Strategy:** Separately consider the  $x$  and  $y$  motions of each projectile in order to answer the conceptual question.

**Solution: 1. (a)** Since each projectile achieves the same maximum height, which is determined by the initial vertical velocity, we conclude that all three projectiles have the same initial vertical velocity. That means the larger the horizontal velocity, the larger the total initial velocity. The largest initial speed will therefore correspond with the longest range. The ranking is thus  $A < B < C$ .

**2. (b)** The flight time is longest for projectiles that have the highest vertical component of the initial velocity. In this case each projectile has the same maximum altitude and therefore the same initial vertical speed. That means they all have the same time of flight and the ranking is thus  $A = B = C$ .

**Insight:** Projectile C travels the farthest distance in the same amount of time because it has the highest speed.



**4.29** A second baseman tosses the ball to the first baseman, who catches it at the same level from which it was thrown. The throw is made with an initial speed of 18.0 m/s at an angle of  $35.0^\circ$  above the horizontal.

29. **Picture the Problem:** The ball travels along a parabolic arc, maintaining its horizontal velocity but changing its vertical speed due to the constant downwards acceleration of gravity.

**Strategy:** The given angle of the throw allows us to calculate the horizontal component of the initial velocity by using the cosine function. The vertical component of the velocity can be found by using the sine function. The time it takes the acceleration of gravity to slow down the vertical speed, bring it to zero, and speed it up again to its initial value is the same as the time the ball is in the air.

**Solution: 1. (a)** Find the  $x$  component of the initial velocity:  $v_{0x} = v_0 \cos \theta = (18.0 \text{ m/s}) \cos 37.5^\circ = \boxed{14.3 \text{ m/s}}$

**2. (b)** Find the  $y$  component of the initial velocity:  $v_{0y} = v_0 \sin \theta = (18.0 \text{ m/s}) \sin 37.5^\circ = \underline{\underline{11.0 \text{ m/s}}}$

**3.** Let  $v_y = -v_{0y}$ , and use equation 4-6 to find the time of flight:

$$t = \frac{v_y - v_{0y}}{-g} = \frac{-11.0 - (11.0 \text{ m/s})}{-9.81 \text{ m/s}^2} = \boxed{2.24 \text{ s}}$$

**Insight:** The flight of the ball is perfectly symmetric—the angle of the motion is  $37.5^\circ$  below horizontal at the instant it is caught, and the ball spends the same amount of time going upward to the peak of its flight as it does coming downward from the peak. This is only true if the ball is caught at the same level from which it was thrown.

**4.31** A cork shoots out of a champagne bottle at an angle of  $34.0^\circ$  above the horizontal.

If the cork travels a horizontal distance of 1.50 m in 1.35 s, what was its initial speed?

31. **Picture the Problem:** The cork travels along a parabolic arc, maintaining its horizontal velocity but changing its vertical speed due to the constant downward acceleration of gravity.

**Strategy:** Find the horizontal component of the initial velocity by dividing the horizontal distance traveled by the time of flight. Then use the cosine function to find the initial speed of the cork.

**Solution: 1.** Find the horizontal speed of the cork:

$$v_x = \frac{x}{t} = \frac{1.30 \text{ m}}{1.25 \text{ s}} = 1.04 \text{ m/s} = v_{0x}$$

2. Use the cosine function to find the initial speed:

$$v_0 = \frac{v_{0x}}{\cos \theta} = \frac{1.04 \text{ m/s}}{\cos 35.0^\circ} = \boxed{1.27 \text{ m/s}}$$

**Insight:** Because gravity acts only in the vertical direction, the horizontal component of the cork's velocity remains unchanged throughout the flight.

**4.39** The "hang time" of a punt is measured to be 4.50 s.

If the ball was kicked at an angle of  $61.0^\circ$  above the horizontal and was caught at the same level from which it was kicked, what was its initial speed?

39. **Picture the Problem:** The football travels along a parabolic arc, landing at the same level from which it was launched.

**Strategy:** The time of flight of a projectile that lands at the same level it is launched is determined by the time it takes the acceleration of gravity to slow down the vertical component of the initial velocity to zero and then speed it up again back to its original value. Thus upon landing the speed of the ball is  $v_y = v_{0y} = v_0 \sin \theta$ . Use these facts to determine the time of flight and then solve for  $v_0$ .

**Solution: 1.** Find the time of flight:

$$t = \frac{v_y - v_{0y}}{-g} = \frac{-v_0 \sin \theta - v_0 \sin \theta}{-g} = \frac{2v_0 \sin \theta}{g}$$

2. Solve for  $v_0$ :

$$v_0 = \frac{gt}{2 \sin \theta} = \frac{(9.81 \text{ m/s}^2)(4.50 \text{ s})}{2 \sin 63.0^\circ} = \boxed{24.8 \text{ m/s}}$$

**Insight:** The flight of the football would not be perfectly symmetric if air resistance were present or if it were caught at a different level from which it was kicked.

**4.40** In a friendly game of handball, you hit the ball essentially at ground level and send it toward the wall with a speed of 24 m/s at an angle of  $38^\circ$  above the horizontal.

**Part A** How long does it take for the ball to reach the wall if it is 3.4 m away?

40. **Picture the Problem:** The path of the ball is depicted at right.

**Strategy:** Use the horizontal component of the ball's velocity together with the horizontal distance  $d$  to find the time elapsed between the hit and its collision with the wall. Then use the time to determine the vertical position  $h$  of the ball when it collides with the wall.

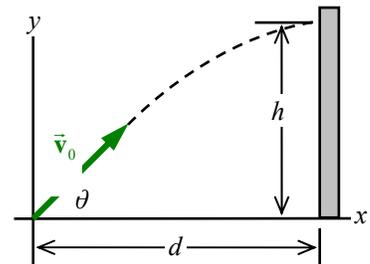
**Solution: 1. (a)** Use equation 4-10 to find the time:

$$t = \frac{d}{v_0 \cos \theta} = \frac{3.8 \text{ m}}{(18 \text{ m/s}) \cos 32^\circ} = \boxed{0.25 \text{ s}}$$

2. **(b)** Use equation 4-10 to find  $h$ :

$$h = (v_0 \sin \theta)t - \frac{1}{2}gt^2 = [(18 \text{ m/s}) \sin 32^\circ](0.25 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.25 \text{ s})^2 = \boxed{2.1 \text{ m}}$$

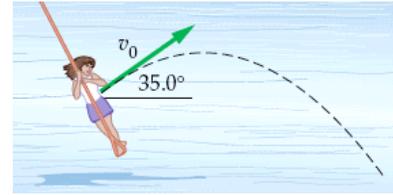
**Insight:** In many cases the vertical motion determines the time of flight, but in this case it is the horizontal distance between the point where the ball is struck and the wall that limits the time of flight.



**4.43** On a hot summer day, a young girl swings on a rope above the local swimming hole. When she lets go of the rope her initial velocity is 2.10 m/s at an angle of  $35.0^\circ$  above the horizontal.

If she is in flight for 0.616 s, how high above the water was she when she let go of the rope?

43. **Picture the Problem:** The trajectory of the girl is depicted at right.



**Strategy:** Use equation 4-10 and the given time of flight, initial speed, and launch angle to determine the initial height of the girl at the release point.

**Solution:** Use equation 4-10 to find the initial height of the girl at the release point. If we let the release height correspond to  $y = 0$ , then the landing height is:

$$\begin{aligned} y &= (v_0 \sin \theta)t - \frac{1}{2}gt^2 \\ &= (2.25 \text{ m/s})(\sin 35.0^\circ)(0.616 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.616 \text{ s})^2 \\ y &= -1.07 \text{ m} \end{aligned}$$

In other words, she was **1.07 m** above the water when she let go of the rope.

**Insight:** The girl's speed upon impact with the water is 5.10 m/s (check for yourself) or 11 mi/h. A fun plunge!