# Astronomy 109 Lab Manual 

Department of Astronomy<br>San Diego State University

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## Preface

"All truths are easy to understand once they are discovered; the point is to discover them."

- Galileo Galilei

In an effort to teach the most interesting and interactive astronomy labs possible, the San Diego State University Astronomy Department undertook a major revision of its lab course (Astronomy 109) during the summer of 2008. Following in that tradition, we have again substantially revised the manual in each subsequent summer, working to improve existing labs and build new labs combining ideas from several sources. The result is what you are now holding in your hands!
Of course, nothing is ever done in isolation, and our efforts relied heavily on those who came before. We acknowledge the work of Phil Rosenfeld, who led a prior revision of the manual that greatly improved it. We'd like to also thank those graduate student instructors who worked with some of us to revise the manual in past: David Baer, Gideon Bass, Alexander Burke, Shimonee Kadakia, David Krogsrud, K. Azalee Bostroem, Jim Davenport, J. Emilio Enriquez and Meredith Rawls. In addition, we'd like to acknowledge previous SDSU graduate student instructors Raj Pandya, Matt Davis, Tyler Mitchell, Tamara Reimer, Tiffany Borders, Abhijith Rajan, Andrew Serio, Mallory Vale, Lauren Brewer, Jay Franck, and MJ Jeffries for their earlier efforts on behalf of this course.
Everyone from SDSU's Astronomy Department hopes you will find this manual and course an introduction to life-long learning of science, and especially astronomy!

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## Part I

## Required Laboratory Exercises

## Lab 1

## The Size of the Earth

Written by Matt Davis and Jim Davenport Revised by Meredith Rawls and MJ Jeffries

### 1.1 Introduction

We begin our laboratory study of astronomy with the astronomical object most familiar to us all: the planet Earth. For us to measure and comprehend the distances to other planets, stars, and even galaxies we must first determine just how big our own planet is. This should help you begin to place the astronomically (no pun intended) large numbers that follow into a more human context. In this course we will use many lab tools and methods, but none will be more critical than your own logic and reasoning ability. For this reason this lab provides a refresher on critical thinking and some basic mathematics. If you feel unprepared or are confused by any step, please see your instructor for assistance! That is what they are there for! A helpful mathematics review can be found in Section III: Appendices, located at the end of this lab manual. The appendices are very useful; use them!

### 1.2 Historical Background

As children in the US, we are often told that Columbus discovered that the Earth is round by sailing across the Atlantic Ocean. The fact is that the ancient Greeks had previously determined that the Earth was round! More than 2300 years ago, a clever Greek librarian named Eratosthenes managed to estimate (with great accuracy) the circumference of the Earth using simple observations and some straightforward math similar to what you learned in high school. Today you will recreate his findings.

### 1.3 Procedure

Suppose that you are vacationing in Cabo San Lucas, Baja California Sur, Mexico. As you go outside at noon local time on June 21 (a clear, sunny day), you notice something quite peculiar: the flagpole outside your building is not casting a shadow, and neither is your building! How can this be?

You are so astounded by this discovery that you immediately call your friend back home. She goes outside and informs you that you are quite crazy - flagpoles and buildings are definitely casting shadows in San Diego.

1. How can objects cast no shadows in Cabo San Lucas, while at the same time objects in San Diego do? Have a look at Figure 1, and discuss your ideas with your lab partner. Write your explanation here. (You are encouraged to use the globes in the back of the classroom - just ask your instructor first.)


Figure 1. - A scale map showing both San Diego and Cabo San Lucas. Since Cabo San Lucas is barely south of the Tropic of Cancer, there are a few days each year (around June 21) when the sun will shine directly overhead at noon, casting no shadows. Image taken from http://www.map-of-mexico.co.uk/images/bajacalifornianorteenglish.gif.

At this point, you should have concluded that the best way to explain the observations is that the Earth's surface is curved. However, your scientific mind senses that there is more to be learned here. What if the Earth was not only curved, but was spherical? This is clearly allowed by your observations. If it were spherical, then you should be able to do an experiment to find its exact size - i.e., determine its circumference: the total distance once around the globe.

To begin, imagine that the Sun is so far away that light reaching Earth approaches in parallel rays, as shown by arrows in Figure 2.


Figure 2. - Geometry of sunlight striking the Earth. Two sticks are planted in the ground so that they are both pointed at the center of the (perfectly spherical) Earth, where they would intersect with an angle $\theta$.

As you can see, the pole in Cabo San Lucas casts no shadow because the Sun is shining directly down upon it, but the sunlight hits the pole in San Diego at an angle so that it casts a shadow. The angle formed by the shadow and the stick is equal to the angle formed by the two sticks at their (hypothetical) meeting point at the center of Earth, labeled $\theta$ in Figure 2.

Let us represent the total circumference of Earth as $C$, and let's call the distance from San Diego to Cabo San Lucas $x$. Knowing that there are 360 degrees in a full circle, you study Figure 2 and, assuming that it is due south from San Diego to Cabo San Lucas, you can see that the ratio of the angle $\theta$ to 360 degrees must be the same as the ratio of the arc length $x$ to the total circumference of Earth, $C$.

To put it another way, focus on the angle $\theta$ at the center of the Earth. Assuming Cabo San Lucas is due south of San Diego, you can divide $\theta$ into $360^{\circ}$ and it should give you the same number as when you divide the distance from San Diego to Cabo San Lucas, $x$, by the circumference of the Earth, $C$.

The above two statements can be written as an equation:

$$
\begin{equation*}
\frac{\theta}{360^{\circ}}=\frac{x}{C} \tag{1.1}
\end{equation*}
$$

By a theorem you may have seen in a geometry class, we know that the angle $\theta$ at the center of the Earth is equal to the other angle marked $\theta$ in Figure 2. The trick is parallel lines. Notice in Figure 2 that the stick in Cabo San Lucas is parallel to the shadow projection in San Diego. If you extend the stick in San Diego in both directions, you can convince yourself that the two angles marked $\theta$ must be equal. If you are confused, ask your instructor!
2. Solve $\frac{\theta}{360^{\circ}}=\frac{x}{C}$ (Equation 1.1) for the variable $C$ (that is, rearrange the equation so that $C$ is in the numerator and alone on one side).

You now need to find a value for the angle $\theta$. Suppose you are in Cabo San Lucas. While on the phone with your friend in San Diego, you ask her to help you carry out an experiment. She cuts a stick to be 20.0 cm long, sticks it vertically into the ground, and measures it to have a shadow 3.50 cm long.

- Using a ruler and graph paper, draw the 20.0 cm stick and its 3.50 cm shadow on the Earth (remember, the stick and the Earth are perpendicular to each other!). Use a pencil and draw the figure as accurately as you can. Now connect the tip of the shadow to the tip of the stick, thus forming a triangle.

3. Using a protractor, carefully measure the small angle that was formed by connecting the tip of the stick to the tip of the shadow - this angle corresponds to the angle $\theta$ from Figure 2. What is this angle in degrees? Using Appendix E as a guide, how precisely do you think you can measure $\theta$ with your protractor? (i.e., include the number of significant figures! There is a big difference in precision between $8^{\circ}, 8.0^{\circ}$, and $8.012^{\circ}$.) You need to always think about how precise your measurements are in this course!

Looking back at Equation 1.1, we still have two missing pieces: the distance from San Diego to Cabo San Lucas $(x)$ and the circumference of the Earth $(C)$. Using the map in Figure 1, answer the following questions.
4. Using a ruler, measure the length of the map scale (the line in the lower left corner that is labeled as 300 km ). Write down your value in cm here.
5. On the map, measure the distance from San Diego to Cabo San Lucas "as the crow flies." Write down that value in cm here.
6. Now, convert your measurement from the previous question into "real life" kilometers. Hint: set it up as a ratio. You should have $\frac{300 \mathrm{~km}}{\text { measured map scale }(\mathrm{cm})}=\frac{x}{\text { measured distance between cities ( } \mathrm{cm} \text { ) }}$, so solve for $x$ with your algebra skills!
$\rightarrow$ The ratio of a real distance to a measured distance on an image is called an image scale.
7. Finally, using your value for $x$ (from Question 6) and $\theta$ (from Question 3), calculate the circumference of the Earth using the reworked version of Equation 1.1 (from Question 2).

### 1.4 Further Analysis

8. In astronomy, we typically use the metric system (i.e., kilometers rather than miles). However, as a student in the US, you are probably more familiar with miles. Given that there are 1.61 km for every 1 mile, convert your answer for $C$ into miles. Hint: Set up a ratio. If you're stuck on how to do this, take a look at Appendix F!
9. Comment on how your result compares to the actual value of roughly 24,900 miles. Also, calculate your percentage error using the below equation.

$$
\text { Percent Error }=\left|\frac{C_{\text {measured }}-C_{\text {actual }}}{C_{\text {actual }}}\right| \times 100 \%
$$

10. Can you think of some sources of error in the procedure you used? Sources of error could include assumptions made and/or measurements taken that might have caused your final answer to be off from the actual value. Write down at least three and explain why they would lead to an error in your final calculation. Make a note of which one you think is the biggest contributor and why.
11. The formula for the circumference of a circle is $C=2 \pi R$, where $R$ is the radius of the circle. Using your value for $C$ from Question 8, solve for $R$ to find the radius of the Earth in miles.
12. Convert your answer for $R$ from the previous question into kilometers (km). (If you're still stuck on unit conversions, see Appendix F!!!)
13. The Moon is located roughly $3.84 \times 10^{5} \mathrm{~km}$ from Earth. How many times greater is the distance to the Moon than the radius of Earth?
14. Do you think you would get the same answer to the previous question if you converted $3.84 \times 10^{5} \mathrm{~km}$ to miles and also used your answer for $R$ in miles? Why or why not? You do not need to do any calculations to answer this question.

## Lab 2

## The Rotation of the Sun

## Adapted from a lab by the University of Washington Astronomy Department by Tina Güth and Jim Davenport Revised by Shimonee Kadakia

### 2.1 Background and Theory

The rotation period of an object is the time it takes it to spin on its axis once. For example, the rotation period of the Earth is 24 hours. In order to observe the rotation period of any rotating sphere, we can measure how many degrees of longitude (angular distance east or west on an object's surface from a given point) a surface feature moves in a given time interval. We then solve for the period of rotation by taking the ratio of one full rotation ( 360 degrees) to the observed angular motion, and multiply it by the observed time interval.

For example, if we observe that a given feature on an object moves 30 degrees of longitude in 2 hours, then the rotation period is:

$$
\begin{equation*}
\text { Period }=(2 \text { hours }) \times \frac{360^{\circ}}{30^{\circ}}=24 \text { hours } \tag{2.1}
\end{equation*}
$$

This method can be applied to the rotation of the Sun by observing active regions as they are carried across the disk by the Sun's rotation. One complication is that since the Sun is a ball of gas, it does not rotate like the Earth or other solid objects. In fact, the Sun's equator rotates faster than its poles!

- For something to travel around the Sun's equator once, it takes roughly 25 days.
- It takes about 35 days for something to travel once around the Sun's north or south pole.

This concept is called differential rotation, and, in fact, is largely responsible for the production of sunspots and other active regions on the Sun's surface. Like the Earth, the Sun possesses a magnetic field. The magnetic field lines of the Sun, however, are twisted due to the differential rotation, as can be seen in Figure 1. The twisting magnetic field lines are thought to be responsible for sunspots and other active regions on the Sun's surface, including flares, prominences, and coronal mass ejections, since all of these regions are found to have very strong magnetic fields.

Sunspots are relatively cool areas of the Sun, which stand out from the rest of the surface because they are darker. The temperature of a sunspot is "only" about 4200 K , compared to a temperature of about 5800 K for the rest of the Sun. If we could see the spots in isolation, they would actually appear bright red!

Solar flares occur in regions of the Sun where the magnetic field is twisted very tightly by the differential rotation of the Sun. This magnetic twisting releases energy. These active regions emit very high energy light, such as X-rays, and are quite hot.

The corona is the outer atmosphere of the Sun. It is very hot, but has a very low density. This layer can only be seen from Earth during a solar eclipse.

Prominences are spectacular phenomena which usually originate near sunspots. They are a huge clouds of relatively cool fairly dense gas suspended in the corona. They are extended from the Sun's surface and are often in a loop shape. When seen in contrast with the Sun's disk, these prominences appear as dark filaments, but when seen above the Sun's surface, they can be very bright. Figure 4 shows images of solar prominences and in Section 2.2.2 you will be asked to find the velocity of a prominence.

Coronal mass ejections are caused by the eruption of prominences, and are blasts of gas which move out through the Sun's corona into the rest of the solar system. We can usually measure how fast these energetic flares are moving away from the Sun. This has very practical applications, because if a coronal mass ejection heads in the direction of Earth, the radiation from the event can damage satellites, as well as be a hazard to astronauts and airline passengers. By studying the Sun, we can provide advance warning of these events and put the satellites into safe mode, minimizing the damage.


Figure 1. - The Sun's differential rotation. Magnetic field lines are indicated by arrows.
(Image from http://astronomy.nmsu.edu/astro/a1 10labs/labmanual/node11.html)


Figure 2. - A group of sunspots. (Image from http://astronomy.nmsu.edu/astro/a110labs/labmanual/node11.html)

### 2.2 Procedure

### 2.2.1 The Rotation Period of the Sun

- Examine the color version of Figure 3 provided by your instructor which shows pictures of the Sun taken one day apart.

1. Find the image scale for Figure 3. To get the image scale, measure the diameter of the Sun with a ruler (in cm ). Then, divide the actual diameter of the Sun ( 1.39 million km , or $1.39 \times 10^{6} \mathrm{~km}$ ) by the measured diameter from the picture (in cm ). Your units should be $\mathrm{km} / \mathrm{cm}$.
2. Choose one of the sunspots near the Sun's equator. Make sure that it is recognizable in both images. Use the images to find the distance $x$ (in cm ) that this feature moved between the two images, and record your measurement here.
3. Multiply $x$ (found in Question 2) by the image scale (found in Question 1) to find the actual distance, $Q$ (in km ), that the sunspot moved. (Use scientific notation as outlined in Appendix D)
4. Since the spots are not located exactly on the equator, measure the distance (in cm ) of the chord (the line from one edge of the Sun to the other, parallel to the equator) where the spots are located. Take this number and use your image scale to find the actual distance, $H$ (in km), of the chord.

- Spots on the Sun always move at a constant speed. One can use the distance a spot travels in a day to find the rotational period of the Sun by setting up a ratio that involves the total distance a spot travels vs. the total time it takes. If the spot were at the equator, the distance would be the circumference of the Sun, $\pi D$ (where $D$ is the diameter of the Sun, in km ). Since the spots in this lab are at a different latitude, the distance they travel would be $\pi H$ (where $H$ is the distance of the chord found in question 4, in km ). The ratio would then be:

$$
\frac{P}{T}=\frac{\pi H}{Q}
$$

$P$ is the rotational period in days, $T$ is the time between observations in days (the images you are using were taken one day apart), and $Q$ is the distance the sunspot moved in km as calculated in Question 3.
5. Calculate the rotational period, $P$, of the Sun in days. Note that $T=1$ day, since the images were taken one day apart.

- It is very important for a scientist to always state the precision with which a measurement has been made. Let's now estimate the uncertainty of your measurements (in cm ), and determine what effect it has on the precision of your final answer (in days) for the rotation period of the Sun.

6. Estimate the uncertainty in your measurement for $x$ made in Question 2. The measured uncertainty is the smallest unit you can confidently measure with your measuring instrument (ruler, protractor, etc.). Explain what this uncertainty represents and why you think that is a reasonable uncertainty.
$\rightarrow$ To learn more about measurements and uncertainties, check out Appendix E, Section 3.
7. Use the following formula to estimate the fractional error in your measurement:

$$
\text { fractional error }=\frac{\text { measured uncertainty }(\text { from the previous question })}{\text { measured value (from question } 2)},
$$

8. To find the corresponding error this measurement uncertainty had in your final answer for the rotation period of the Sun (in days), multiply the fractional error by the rotational period, $P$. This uncertainty tells you how far from the actual answer your answer could be due to the uncertainties in your original measurement even if you measured everything correctly. Write your final answer as period $\pm$ uncertainty days, e.g. $26 \pm 2.4$ days.
9. Using this information, explain whether the measured uncertainty in your rotational period is reasonable. To do this, compare your measured period and the given period of 27 days for that latitude. The difference between your period and the given period represents how far away from the actual value you were. Does the given value for the period fall within the range set by your measured uncertainty? Explain why the actual value should fall within your range. If the given period does not fall within the range provide a good reason why your measured period is off.

### 2.2.2 Solar Prominences

- Examine the color version of Figure 4 provided by your instructor. These images show the development of a solar prominence over time.
- For these pictures, you will need to find a new image scale. To do this, first note that each image shows only one quarter of the Sun, with the Sun's center in the very upper left corner of each image.

10. Determine the radius of the Sun in the images (in cm ) by measuring along the top edge of the first image from the corner to the edge of the Sun, and record your value here.
11. Divide the actual radius of the Sun (recall that the Sun's diameter is $1.39 \times 10^{6} \mathrm{~km}$ ) by the measured radius from the picture (in cm ). This will give you the image scale.
12. Measure the distance from the upper left hand corner (the center of the Sun) to the edge of the prominence in: (a) the upper left frame and (b) the lower right frame. To find how far the prominence has moved/grown, subtract the first distance from the second. Record the measurement (in cm ) here.
13. Convert this value into a real distance in km. Hint: Use the image scale!
14. The velocity of an object is the distance it travels in a certain amount of time (distance $=$ velocity $\times$ time). Find the velocity of this material as it left the Sun. Hint: You found the distance in Question 13, and you should look at the figures to find the time elapsed.
15. Convert this velocity from $\mathrm{km} / \mathrm{hr}$ to $\mathrm{mi} / \mathrm{hr}$, a scale that may be more familiar to you (note: $1.61 \mathrm{~km}=1 \mathrm{mile}$ ). Is this velocity fast or slow compared to the speeds of objects you are familiar with (for example, cars or jet planes)? Are you surprised by the answer? (NOTE — one more question on back page!)
16. Let's look at another example of the Sun's activity. In the most severe of solar storms, flares can reach velocities as fast as 2,000 kilometers per second! If the Earth is $1.50 \times 10^{8}$ kilometers from the Sun, how long (in days) would it take for this ejected material to reach the Earth?


Figure 3. - Two images of the Sun, taken one day apart.
(Image taken from: http://homepages.igrin.co.nz/astronomy/solar.html)


Figure 4. - Images showing the development of a solar prominence. Note that the date and time that each image was taken are indicated in the lower right corner.
(Image courtesy of SOHO (ESA \& NASA) EIT; taken from: http://soho.www.nascom.nasa.gov/gallery/images/eit003.html)

## Lab 3

## The Celestial Sphere

By Douglas Leonard

### 3.1 Introduction

If you go outside on a dark, clear night and look up, you can see hundreds (or thousands, if far away from city lights) of stars scattered across the dome of the sky. When you look at the stars, it appears as though these points of light are affixed to a sphere surrounding the Earth. If you watch for a few hours, you notice that the stars do not simply "stay put;" rather, the whole pattern of stars slowly rotates in the sky. New stars continually rise over your eastern horizon (note: "horizon" = the boundary that divides what you can see and can't see in the sky) and come into view, while others set below your western horizon, and disappear from view.

Our ancient ancestors looked up in wonder at this night sky and asked: How does it all work? What is responsible for the appearance and motions of objects (e.g., stars, the Sun) in the sky? To answer such questions, they constructed what, in science, is known as a conceptual scheme: a model used to explain observations. The specific model they chose is known today as the Celestial Sphere: an imaginary sphere centered on the center of the Earth, to which it appears the stars are affixed.

While many ancient societies (e.g., those of ancient Greece, about whom we have the most detailed information) truly believed that the "stars" were literally affixed to a crystalline sphere surrounding the Earth, today we know this to be incorrect. Stars are not attached to a physical sphere surrounding the Earth, but are actually all located at different distances from us. Nonetheless, for an Earth observer, it certainly does appear that the stars are affixed to such a sphere, and thus the Celestial Sphere model remains a very useful conceptual scheme to explain the appearance and motions of objects in our sky.

In this lab, you will be introduced to the Celestial Sphere model and shown how to use it to accurately predict the basic appearance and motions of the stars and Sun in the sky, from different locations on the Earth. In order to understand why the Celestial Sphere model works, there are five basic facts that you must be aware of:

1. The Earth is spherical in shape.
2. The Earth spins once per DAY on its axis.
3. The Earth orbits the Sun once per YEAR.
4. The Earth's axis is tipped relative to the plane of its orbit around the Sun.
5. Stars are very distant Suns. The stars that you see at night are just like our Sun but much farther away; the closest star, in fact, is over 250,000 times more distant than the Sun from us.

### 3.2 Procedure

### 3.2.1 Practice Using the Globes: Observer at the North Pole

Note: Specific questions that you must answer in your lab writeup are indicated by numbers.

- If not already done for you, set up your Celestial Sphere so that it looks like the diagram shown in Figure 1 (below), with the North Pole of the Earth pointing directly up. Setting the globe up in this orientation is useful to explain what an observer located at the North Pole sees in the sky.


Figure 1. - Celestial globe positioned for an observer located on Earth's North Pole.

- Familiarize yourself with all of the parts of the globe indicated in Figure 1. Note that we'll first be focusing on the "Celestial Globe." Later on in the lab we'll be considering the "Sunball," which indicates the location of the Sun relative to the Celestial Sphere - for now, ignore it.
In addition to the "Celestial Globe" perhaps the most important component to be familiar with is:
The horizon ring. This blue transparent ring that surrounds the Earth defines what an observer can and cannot see in the sky. When properly positioned, all of the stars that are above the horizon ring (actually, the projection of the horizon ring onto the Celestial Globe) are visible while those below it are not.
Note that star names are written to be readable by looking through the globe, from the inside, and are written in black. Constellation names are written similarly, but in blue. The size of the painted white circle for a star gives a relative indication of how bright that particular star is (bigger means brighter).
- Locate the star "Polaris" on the Celestial Globe, (Hint: look at the very top of the Celestial Globe!), and respond to the following:

1. For an observer located on Earth's North Pole, is Polaris visible in the sky (i.e., is it above the horizon)?
2. You should have answered "yes" to the last question - if not, ask your instructor for assistance! Now, describe as best you can exactly where in the sky Polaris is located for an observer standing at the North Pole.

Hopefully, for Question 2, you wrote something equivalent to saying that Polaris is located "straight up," or "at the highest point in the sky," for a North Pole observer. There are two more scientific ways of saying this same thing. First, you could say that Polaris is at this observer's zenith: The point directly overhead. A second way of stating Polaris' position in the sky is to give its elevation: The number of degrees above the horizon that an astronomical object is. In this case, you would say that Polaris' elevation is $90^{\circ}$ for a North Pole observer. Note that this point on the Celestial Sphere - directly above an observer located at the North Pole - is called the North Celestial Pole; similarly, the South Celestial Pole is at the zenith of an observer located on Earth's South Pole.

- Find the series of black, horizontal rings (circles) drawn onto the Celestial Globe - they are labeled $10^{\circ}, 20^{\circ}$, $30^{\circ}, 40^{\circ}, 50^{\circ}, 60^{\circ}, 70^{\circ}, 80^{\circ}, 90^{\circ}$. These circles (actually, called lines of "declination") will assist you in finding the elevation of stars in the sky for the rest of this lab.
- Find the bright star called "Capella" on the Celestial Globe. Hint: First find the constellation of Auriga (written in blue), and then locate Capella within it.

3. For an observer at the North Pole, what is the approximate elevation (in degrees) of Capella in the sky?

- Finally, locate the star "Sirius" on the Celestial Globe. Hint: It is a very bright star about $20^{\circ}$ below the Celestial Equator (the extension of Earth's equator onto the Celestial Sphere).

4. Can an observer at the North Pole see the star Sirius? Why or why not?

## The Spinning Earth

The Earth rotates once per day on its axis, and there are actually two equally valid (from the observational standpoint) ways to simulate the effects of the spinning Earth on our observations of the sky.

- Method 1: Pick up the Celestial Globe and turn the Earth Globe in a counter-clockwise direction (as viewed from above, looking down at the North Pole) by twisting the control knob for the Earth Globe. One complete turn of the Earth represents one day.
- Method 2: Pick up the Celestial Globe and, holding the control knob for the Earth Globe so that the Earth stays motionless, rotate the Celestial Globe in a clockwise direction (as viewed from above the North Pole). One complete turn of the Celestial Globe represents one day.

In terms of simulating what is observed throughout the course of a day, either of the above methods is valid: The fact that the stars appear to move in the sky could be due either to the fact that the Earth is rotating (Method 1) or that the Celestial Globe is rotating (Method 2). Remember, it is only a relatively recent discovery that it is actually the Earth, and not the Celestial Globe, that turns once per day; from an observational standpoint, both models produce the same result. Using either method to simulate the effects of a spinning Earth, answer the following two questions.
5. Does the elevation of Capella change during the course of a day for an observer at the North Pole? Explain how you know.
6. Describe as best you can what a North Pole observer sees if they watch the star Capella in the sky over the course of 24 hours (ignore the Sun for now). You should include a drawing to get your idea across.

### 3.2.2 Practice Using the Globes: Observer at the Equator

- Set up your Celestial Globe so that it looks like the diagram shown in Figure 2, and rotate the Earth so that the city Quito, Ecuador (located in South America, very close to the equator) is on top, facing directly up. Setting the globe up in this orientation demonstrates what an observer located at Earth's equator will see in the sky.
- Locate the star Polaris on the Celestial Globe, and then simulate the spinning Earth by holding the Earth still and rotating the Celestial Globe in a clockwise direction as viewed from above the North Pole (i.e., 'Method 2' from before).
$\rightarrow$ Notice that stars rise in the East and set in the West for an observer located at Earth's equator, and thus change their elevations throughout the course of a day. Stars achieve their maximum elevation when they cross an observer's meridian: the imaginary circle on the Celestial Sphere that passes through an observer's zenith and the North and South Celestial Poles.

7. For an observer on the equator, what is the elevation of Polaris? Does it change appreciably during the course of a day?


Figure 2. - Celestial globe positioned for an observer located on Earth's equator, in the city of Quito, Ecuador.

- Now locate the star Sirius on the Celestial Globe, and then simulate the spinning Earth again for a 24 hour period while observing how the location of Sirius changes with respect to the observers' horizon: Notice that the star is above the horizon for some part of the 24 hours, and below it for some part.

8. For an observer on the equator, what is the approximate elevation of Sirius above the horizon when it is highest in the sky? Explain how you arrived at your answer. Hint: The answer to this question is not $20^{\circ}$ !
9. Describe as best you can, and in as much detail as possible, what an observer located on the equator sees if they watch the star Sirius in the sky over the course of 24 hours (ignore the Sun for now). Use a drawing to get your ideas across. Explain how Sirius' movement in the sky is different from what the North Pole observer saw when observing Capella.

### 3.2.3 On Your Own: Observer in San Diego

- Using what you have learned thus far, do your best to set up your Celestial Globe so that it is in the appropriate orientation for an observer located in San Diego, CA. (Note that San Diego isn't marked on the Earth Globe, but Los Angeles is, so use it instead - it's close enough.) Hint: In your final setup, the horizon ring should be parallel to the floor, and Los Angeles should be at the top of the globe.
- When you think you have your Celestial Sphere set correctly for San Diego, simulate the spinning Earth by either (a) spinning the Earth in a counter-clockwise direction as viewed from above the North Pole; or (b) holding the Earth still and rotating the Celestial Globe in a clockwise direction as viewed from above the North Pole.

10. For an observer located in San Diego, what is the approximate elevation of Polaris? Does the elevation of Polaris change appreciably throughout the course of a 24 hour period?
11. Call your instructor over to check that you have gotten everything set up just right, and that you thoroughly understand how to set up the Celestial Sphere for any particular city - beware that at this point your instructor may ask you to individually set it up for some random city to make sure you've "got it."
When your instructor is satisfied that you are now a "Master of the Celestial Sphere," s/he will place her/his initials below, and you may move on to the next question.

### 3.2.4 A Curious Picture

Now that you are a Master of the Celestial Sphere, it's time for a "thought experiment."

- Together with your lab partner(s), discuss what you expect the outcome of the following experiment to be:

At night, somewhere in the Northern hemisphere (i.e., either at the equator, the North Pole, or some latitude in between), you take a camera, set it on a tripod (so that it can't be bumped for a long time!), aim it at the North Celestial Pole (i.e., Polaris), and then take a picture - but have the camera set so that the shutter stays open for two hours (instead of the usual $1 / 16$ of a second!). Thus, the camera will finish taking the picture two hours after you push the button. After two hours, you look at the picture.
12. What do you expect to see in the picture? How will the picture differ, depending on which latitude (i.e., the equator, North Pole, or somewhere in between) you take the picture at?
$\rightarrow$ Note that you won't be graded on whether your predictions are right or not, but you will be graded on how thoroughly you explain your reasons for believing what you do! You should include at least one illustration to help get your ideas across.

The procedure just described is actually a favorite activity for photographers. Turn the page now to see an actual picture taken in just such a manner. (You may also ask your instructor to see a nice, color image of this picture.)


Figure 3. - Photograph of the night sky, showing dozens of "star trails", representing an exposure of over two hours. (Image credit: Peter Michaud (Gemini Observatory), AURA, NSF)
13. Do you believe the photograph in Figure 3 was taken at the North Pole, the Equator, or at some latitude in between? Carefully explain your reasoning. How does this picture compare with the expectations you wrote about in response to Question 12?
14. In Figure 3, draw arrows on at least two of the star trails to indicate the direction that the stars would appear to be moving in the sky (i.e., are they moving clockwise or counter-clockwise?).

### 3.2.5 Adding the Sun to the Sky

Let's now investigate how including the Sun changes what an Earth observer sees in the sky.

- Set up your Celestial Globe so that it is again in the appropriate orientation for an observer located on the North Pole (i.e., Figure 1).
- Locate the "Sunball" inside your Celestial Globe.
- Twist the knob controlling the Sunball in a counter-clockwise direction. Notice that the location of the Sunball against the constellations in the Celestial Globe changes, tracing out a full circle. This apparent path of the Sun
through the Celestial Sphere is called the ecliptic. If you look closely at the Celestial Globe, you will find dates marked along the ecliptic, written in black; these dates indicate where the Sun will appear in the sky relative to the background stars on a particular day. For instance, on August 20, the Sun is 'in' the constellation of Leo. Notice also that the circle of the ecliptic is tipped relative to the "Celestial Equator." This is caused by the fact that the Earth's axis is tipped relative to the plane of its orbit around the Sun.
- Move the Sunball so that it aligns with the date June 21.

15. What is the approximate elevation of the Sun in the sky for an observer located at the North Pole on June 21? Does the Sun's elevation change throughout the course of a day (i.e., a 24 -hour period; spin the Earth - or Celestial Globe - one time) for an observer located at the North Pole?

- Rotate the Sunball so that it aligns with the date December 21 (six months after June 21).

16. Can an observer located at the North Pole see the Sun at any point during the "day" (i.e., a 24 -hour period) on December 21?
17. It is often said: "If you live at the North Pole, you get six months of sunshine and six months of darkness." How true do you think this statement is? Carefully explain your reasoning.
18. Based on what you have just observed, can you provide a logical explanation for why it is warmer at the North Pole in June than it is in December? How does this relate to the seasons in the northern hemisphere?

### 3.3 Putting it all Together!

Having now completed this exploration of the Celestial Sphere, reflect back on those five basic facts in Section 3.1, and see if you can put everything together by answering the following questions. For each question, be sure to both list the specific fact(s) and provide a more detailed explanation. Note that if more than one "fact" is responsible for the given observation, you should list all that apply!
19. Which basic fact(s) is/are responsible for the observation that stars complete one circuit around the celestial pole (i.e., the star Polaris in the Northern Hemisphere) per day? Explain.
20. Which basic fact(s) is/are responsible for the changing location of the Sun relative to the background stars throughout the course of a year? Discuss.
21. Which basic fact(s) is/are responsible for the observation that seasons (i.e., times of the year when it is hotter/colder than other times) exist on Earth? Explain.

## Lab 4

## Kepler's First Law: The Elliptical Orbits of Planets

By K. Azalee Bostroem<br>Edited and adapted by Douglas Leonard<br>Revised by Alex Burke and Jay Franck

### 4.1 Ellipses

### 4.1.1 Introduction to Ellipses

Until the 16th century in Europe, astronomy and astrology were intertwined. Ptolemy's great work, The Almagest, had survived from 150 AD and still ruled as the authoritative text that showed how the planets ("wandering stars") moved throughout time. By predicting where a planet would be at any given date, it told the future. The mathematical model used in ancient times to explain the planetary paths assumed the Earth had a very special place in the universe: the very center.

In 1543, The Almagest met its match. In that year, Nicolas Copernicus published a book that also accurately predicted the motions of the planets, but this time the center of the universe was our Sun. In the years immediately following its publication, it was often looked at simply as a mathematical trick - how clever that one could predict the motions of the planets as if they orbited the Sun - since it turned out that Copernicus' method was actually no more precise than that of The Almagest for predicting the future paths of planets. However, we now know that this is because Copernicus and Ptolemy both made the same mistake: they believed the motions of the planets were perfect circles, an ideal handed down by Plato from ancient times.

In 1608, Johannes Kepler discovered the source of the error. With the precise planetary charts that he inherited from the great Danish astronomer Tycho Brahe and a head full of mathematics, he discovered that if the planets were orbiting the Sun, they could be described more simply and precisely if their orbits were allowed to be ellipses, which are a type of "oval," or flattened circle.

Since ellipses do turn out to describe the orbits of planets, they are of extreme importance to astronomy. In this lab, you will first spend a little time investigating some properties of the ellipse. After this is done, you will derive the elliptical shape of Mercury's orbit, using the observations of Tycho Brahe and the method of greatest elongation.

### 4.1.2 Materials

- A loop of string
- A piece of cardboard
- A pencil
- Two pushpins
- A blank piece of paper
- A ruler
- A protractor


### 4.1.3 Procedure

- Place the piece of paper over the cardboard. Stick the pushpins far away from each other (but close enough that the string can still reach over both of them). Place the loop of string around both tacks.
- Place your pencil in the loop and pull the string taught. Draw the shape made by moving your pencil while maintaining the tautness of the string. See Figure 1. Label this ellipse "Ellipse A."


Figure 1. - Drawing an ellipse by using pushpins and a string.

- On the other side of the paper, stick the pushpins very close together. Place the string around the pushpins and draw another ellipse. Label this ellipse "Ellipse B." Note that the pushpin holes represent the foci (plural of focus) of the ellipse.

1. Which of the two ellipses that you have drawn is closest to being a circle?
2. Where would you expect the foci of a circle to be located?
3. Notice that the total length of your string never changed as you drew each ellipse. Now choose any random point on the edge of one ellipse, and label it "A." Measure the length from one focus to point "A." Then measure the length from the other focus to point "A." Add the lengths and record your result in the space below. Now, do this for two more points on the ellipse, labeling each of them "B" and "C," respectively.
4. What do you conclude is the relationship between every point on an ellipse and the distance to the foci? Check your answer to this question with your instructor before continuing.

- Remove the pushpins and string and draw a straight line connecting the foci holes. Extend the line in both directions until it touches the ellipse in two points on opposite sides. The line you have just drawn is the major axis (the longer axis) of the ellipse. Often you will see reference to the "semi-major axis" of an ellipse: this is just half of the major axis. Each focus is the same distance from the side along the major axis.
- The minor axis is a line that is perpendicular to the major axis and falls exactly halfway across the major axis. Use your protractor and ruler to draw in the minor axis. The "semi-minor axis" is just half of the minor axis.

Now that you have identified the parts of an ellipse, let's investigate an important property: eccentricity, $e$. The eccentricity of an ellipse describes how squashed or flattened it is. The eccentricity is defined to be the ratio of the distance between the foci to the length of the major axis. (Note: eccentricity is a unitless number since it is a distance divided by another distance.)

$$
e=\frac{\text { distance between foci }}{\text { major axis }}
$$

5. Calculate the eccentricity for each of your ellipses and record your answers here.
6. Which ellipse has a greater eccentricity?
7. What eccentricity would you expect a circle to have? Why?
8. What are the minimum and maximum values that the eccentricity of an ellipse can have? Why?
$\rightarrow$ Ellipses have many cool properties. To learn more, check out:
```
http://mathworld.wolfram.com/Ellipse.html
```


### 4.2 The Orbit of Mercury

### 4.2.1 Introduction

Mercury is the closest planet to the Sun. This makes it very difficult to spot in the sky, since it always appears very close to the Sun from Earth. Thus, you can only see Mercury shortly after sunset or shortly before sunrise. Tycho Brahe made many observations of Mercury and measured how many degrees away from the Sun it appeared to be at each observation. The geometry of the Earth-Mercury-Sun system is shown in Figure 2.


Figure 2. - The geometry of observing the planet Mercury from Earth as both planets orbit the Sun. Note especially the geometry of the Earth-Mercury-Sun system when Mercury is furthest from the Sun in the sky, at the moment of its greatest elongation.

As the Earth and Mercury orbit the Sun, the angular separation of Mercury and the Sun in the sky continually changes. On any given day from Earth, we will see the angular separation of Mercury from the Sun as $\angle b E d$ (where $E$ represents the Earth), as seen in Figure 2. This angle is known in astronomy as the elongation of Mercury. Notice, however, that line $\overline{E d}$ crosses Mercury's orbit in two different places. Thus measuring the elongation does not uniquely specify a single point on the orbit of Mercury. In other words, two different points in Mercury's orbit can both yield the same observed elongation, as shown in Figure 2. This makes it difficult to generate an orbit for Mercury, as its exact position is not known with certainty.

However, there is one very special point in Mercury's orbit when its exact position can be uniquely determined: at the moment of greatest elongation. This is when the Sun and Mercury appear to be furthest apart in the sky, so the line of sight is now tangent to the orbit of Mercury (illustrated in Figure 2 by line $\overline{E a}$ ). That is, the line of sight touches the orbit of Mercury at one point and one point only, the aptly named tangent point. The point of greatest elongation thus represents a very important observation, since only at this point will there be a $90^{\circ}$ angle between the Sun, Mercury, and the Earth ( $\angle E a b$ ), ensuring that Mercury's position is known exactly. Again, This is the only time when the angle observed from Earth corresponds to one unique place in Mercury's orbit.

In this part of the lab, you will use observations of the planet Mercury, taken by Tycho Brahe in the later part of the 16th century, to deduce the shape of Mercury's orbit. These observations of the exact date and size of Mercury's angle of greatest elongation can be used to construct lines of sight that are tangent to the orbit of Mercury, touching it at only one point. Since Mercury orbits the Sun in an ellipse it is sometimes closer and sometimes further from the Sun in its orbit. Thus, the angle of greatest elongation as viewed from the Earth will change depending on where Mercury is in its orbit. By measuring many angles of greatest elongation over many orbits, you will be able to map out the shape of Mercury's orbit around the Sun.

### 4.2.2 Procedure

- Take out the sheet "Determination of the orbit shape of an inferior planet by the use of greatest elongation" from the back of the lab manual. On this sheet is a scale drawing of Earth's orbit around the Sun. The Sun's position is represented by the dot near the center. The ring of dates, given in ten-day intervals, corresponds to Earth's position in its orbit on that date. Thus, on August 1 the Earth would be located at the August 1 tick mark. (Small tick marks have also been added for each observation day.)

The data tables in Table 4.1 list the angles of greatest elongation made by the planet Mercury and the dates on which they occurred, as recorded by Tycho Brahe. The table labeled East of Sun lists the angles Mercury made to the left of the Sun as viewed from Earth in the sky. The other table, labeled West of Sun, corresponds to the angles Mercury made to the right of the Sun.


Figure 3. - Example showing how to use the protractor for a hypothetical observation: July 1st, $30^{\circ}$ to the East (left) of the Sun. If this date were for an angle West (right) of the Sun, the protractor would be flipped around (it would look like a big smile).

- Follow this procedure to sketch out the angles of greatest elongation for Mercury using Tycho Brahe's observations given in Table 4.1.
- Locate and label the tick mark of the exact date of interest within Earth's orbit.
- Draw a light or dashed line connecting the date of interest to the Sun. Use a pencil, not a pen!
- Measure the angle to the right (if West of the Sun) or left (if East of the Sun) as demonstrated by your instructor and in Figure 3. If the protractor is oriented vertically on the page with the date of interest closest to you, the protractor should look like a "D" for angles right of the Sun, and a "C" for angles left of the Sun.
- Carefully draw a line roughly two-thirds of the way from the position of the Earth toward the other side of the circle $(\overline{E a})$.

It is important that you measure the angles and draw them in very carefully! Do this for all 19 dates.

| East (Left) of Sun |  |
| :--- | :---: |
| Date | Angle of Greatest Elongation |
| April 18, 1588 | $20^{\circ}$ |
| August 17 | $27^{\circ}$ |
| December 11 | $21^{\circ}$ |
| April 1, 1589 | $19^{\circ}$ |
| July 29 | $27^{\circ}$ |
| November 23 | $22^{\circ}$ |
| March 14, 1590 | $18^{\circ}$ |
| July 10 | $26^{\circ}$ |
| November 5 | $23^{\circ}$ |


| West (Right) of Sun |  |
| :--- | :---: |
| Date | Angle of Greatest Elongation |
| February 6, 1588 | $26^{\circ}$ |
| June 5 | $24^{\circ}$ |
| September 28 | $18^{\circ}$ |
| January 18, 1589 | $24^{\circ}$ |
| May 17 | $26^{\circ}$ |
| September 12 | $18^{\circ}$ |
| January 1, 1590 | $23^{\circ}$ |
| April 28 | $27^{\circ}$ |
| August 25 | $18^{\circ}$ |
| December 14 | $21^{\circ}$ |

Table 4.1: List of observations made by Tycho Brahe.

## Sketching in the Ellipse

- Since each of the lines you have drawn must touch the orbit of Mercury in one (and only one) location, the collection of lines that you have drawn represents an outline of Mercury's orbit. That is, since the line for each date represents a "tangent line" to Mercury's orbit, the orbit itself must be a symmetrical, smoothly-drawn curve that fits just inside the tangent lines, and ideally touches each line at only one point. Carefully sketch in this curve with a pencil.
If you did things correctly, you should immediately see that the shape you sketched in is not a circle, but is an ellipse!
$\rightarrow$ If your shape does not look like an ellipse, call your instructor over - you may need to redo your diagram.
Let's now examine the properties of this ellipse.


## Making Measurements on the Orbit of Mercury

- Step 1: Perihelion is the point in a planet's orbit where it gets the closest to the Sun. The point at which it is furthest from the Sun is called aphelion. These two points should lie on a straight line across from each other through the foci. Notice that the Sun is one focal point. Using a ruler, find and label the perihelion and aphelion points on Mercury's orbit. The major axis connects perihelion and aphelion in a straight line through the Sun. Draw and label the major axis of Mercury's orbit.
- Step 2: Find and label the minor axis of Mercury's orbit. Remember that it crosses the major axis at its midpoint - not at a focus.
- Step 3: Recall that the Sun is at one focus of Mercury's orbit. The other focus must lie on the major axis and be the same distance from the midpoint of the semi-major axis as the Sun is. Locate and label the two foci as F1 and F2 (Sun).


## Is the Orbit an Ellipse?

Figure 4 shows a diagram representing the elliptical orbit of a planet (e.g., Mercury).


Figure 4. - The elliptical orbit of a planet (not to scale). The eccentricity in this diagram is exaggerated.

- Step 4: On the figure that you have drawn to represent Mercury's orbit, select three random points along the orbit, and label each one with the letters $A, B$, and $C$ (like point $B$ in Figure 4, above). Measure and record in the table below the distances between F1 and the point and F2 and the point for each letter. Record also the sum of the two distances for each point in the table.

|  | F1 to the point | F2 to the point | (F1 to the point) + (F2 to the point) |
| :--- | :--- | :--- | :--- |
| Point A |  |  |  |
| Point B |  |  |  |
| Point C |  |  |  |

9. Are the sums of (F1 to the point) + ( F 2 to the point) all the same? If you drew the ellipse perfectly, they should be, but that is very doubtful given the constraints of this lab. What do you think may have caused the error? (If your answers are way off, discuss this with your instructor; you may need to redo the drawing in order to continue.)

### 4.2.3 Eccentricity of the Orbit

Use your diagram of Mercury's orbit to answer the following questions.
10. Measure the distance between the foci F1 and F2 (in mm).
11. Measure the length of the major axis (in mm).
12. As you learned earlier, the eccentricity of an ellipse is the ratio of the distance between F1 and F2 to the length of the major axis. It is a measure of the "flattening" of the ellipse. Calculate the eccentricity of Mercury's orbit.
13. The actual value of the eccentricity of Mercury's orbit is 0.205 . How does your value compare with this? Compute the percentage error (which is always a positive number due to the absolute value signs) made by your calculation according to the formula:

$$
\text { Percentage error }=\left|\frac{\text { measured value }- \text { true value }}{\text { true value }}\right| \times 100 \%
$$

where "measured value" is the eccentricity you measured in Question 12.
14. Earth's orbit has an eccentricity of 0.017 . Comparing the eccentricity of Earth with Mercury's, which orbit is more circular? How did you determine this?

### 4.3 Analysis

Congratulations - you have now shown that Mercury's orbit is, indeed, an ellipse by using just the observations of Tycho Brahe! Consider now everything you have learned in this lab to answer these final two questions.
15. Why did Tycho Brahe only make observations of Mercury near sunrise and sunset? Hint: Reread Section 4.2.1!
16. The planet Mars is located further away from the Sun than Earth. Do you think you could use the method from this lab to determine the size and shape of Mars’ orbit? Why or why not? Draw a picture to help explain your answer. Hint: Look at Figure 2 and consider the following: Is there any way to make a $90^{\circ}$ angle between the Earth, Mars, and the Sun?

## Lab 5

# Kepler's Third Law and Jupiter's Moons 

Adapted from CLEA: Contemporary Laboratory Experiences in Astronomy<br>Revised by Meredith Rawls and MJ Jeffries

### 5.1 Introduction

One of the most basic properties of any object is its mass: the total amount of "stuff" it contains. For objects on the Earth, it is relatively simple to determine the mass of an object - just put it on a scale! By doing this, we measure the force that Earth's gravity exerts on the object (called the object's weight, which is directly related to its mass). Unfortunately, astronomers are forever removed from the objects they study, and so the question becomes: How can we measure the mass of a distant object?

One of Isaac Newton's greatest discoveries was that we can measure the mass of a distant object if there is a second object orbiting it. The path taken by the second object is directly affected by the gravity of the first object, and the first object's gravity depends directly on its mass. The formula is simple, and is known as Newton's version of Kepler's Third Law. For a small object (e.g., a moon) orbiting a much more massive object (e.g., a planet):

$$
\begin{equation*}
M=\frac{a^{3}}{P^{2}} \tag{5.1}
\end{equation*}
$$

- $M$ is the mass of the planet (in units of solar masses, $M_{\odot}$ )
- $a$ is the length of the semi-major axis of the moon's orbit around the planet (in units of Astronomical Units, AU) $\rightarrow$ If an orbit is circular, the semi-major axis is equal to the radius.
- $P$ is the period of the orbit - how long it takes for the moon to orbit the planet once (in units of years)

Take note that the units for this equation are special. For example, if you tried plugging in meters for $a$ and seconds for $P$, you would get meaningless $\frac{\text { meters }^{3}}{\text { seconds }^{2}}$ for the units of $M$.

To see how this works, let's use Equation 5.1 to calculate the mass of the Sun based on the orbit of the Earth. Since the definition of an AU is the length of Earth's semi-major axis, we know $a=1 \mathrm{AU}$. We also know it takes one year for the Earth to fully orbit the Sun, which means $P=1$ year. Let's plug it in $\ldots$

$$
\begin{equation*}
M=\frac{a^{3}}{P^{2}}=\frac{(1 \mathrm{AU})^{3}}{(1 \mathrm{yr})^{2}}=1 M_{\odot} \tag{5.2}
\end{equation*}
$$

This is good - we should hope the Sun would have a mass of one solar mass! Hopefully now you can see why the units are especially important in this equation. If you are still curious about why it works out so neatly, ask your instructor to show you the full form of Newton's version of Kepler's Third Law.

Now that we have calculated the mass of the Sun based on the Earth's orbit, we can determine the mass of Jupiter the same way, by using the orbits of Jupiter's moons. In order to complete this lab, we have to make some very important assumptions:

- The orbits of the moons are all perfectly circular with unchanging radii
- The orbits do not speed up or slow down from one period to the next (i.e. the period does not change)
- The moons all orbit Jupiter at zero inclination (edge-on)
- Jupiter is much more massive than its moons ( $M_{\mathrm{Jup}} \gg M_{\text {moon }}$ )
- Every 12 hours we are able to take a picture of Jupiter

In this lab you will use a computer program to "observe" the four moons of Jupiter that Galileo saw through his telescope, known today as the Galilean moons. They are named Io, Europa, Ganymede, and Callisto, from closest to farthest distance from Jupiter. You will use these observations to estimate the mass of Jupiter.

### 5.2 Computer Instructions

### 5.2.1 Setup

- Double click on the Desktop icon for the CLEA program "Revolution of Jupiter Moons"
- Click on File . . . Login
- Enter your and your lab partner's names (leave the table number blank) and click OK
- Click on File ... Run
- Make sure today's date is displayed (leave the time alone) and click OK
- Click on File ... Timing
- Set "Observation Step (Hrs)" to 12 and click OK


### 5.2.2 Recording Data

You must record at least 23 observations, not including cloudy nights. A sample observation screen is shown in Figure 1. Warning: There is no "back" button. If you forget to record data for a particular night, you cannot go back to it.

- Click on a moon to see its location. On the right, you should see the name of the moon and its distance east (E) or west (W) of Jupiter in units of "Jupiter Diameters." The date and time are displayed on the left. You can change the magnification to zoom in or out if you wish.
- Click the "Record" button to enter the observational data into the computer. When the measurement window pops up, it will have location data for one moon only. To record data for the other three moons, click on each one so its name appears to the right and then click "Record."
- If a night is cloudy, hit "Next." Cloudy nights do NOT count toward your 23 required observations.
- Note: All four moons might not be observable on a given night. This is most likely due to the fact that a particular moon is behind Jupiter. This is not a bad thing, it still remains a valid data point. There will just not be a data point for this moon on this night. If you want to ensure that the moon is behind Jupiter ask your instructor to show you how to access the top view feature.
- Once you have recorded the position of all four moons in the computer, click OK for the computer to save your data and then "Next" to advance 12 hours in the future. Check off one of the boxes in Table 5.1, in order to keep track of the number of observations you have completed. Repeat the above steps until you have at least 23 observations.


Figure 1. - A sample observation of Jupiter and its moons. This is what Jupiter looks like through a medium-sized telescope!

Table 5.1: Use the below table to check off each observation night. Remember you need at least 23 observation points and cloudy nights do not count.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

### 5.2.3 Analysis of Observations

Next, you will make four plots showing the position of each of Jupiter's moons relative to Jupiter. An example of the type of graph you are trying to create is shown in Figure 2.


Figure 2. - This graph illustrates the period and amplitude of a sine curve. This particular sine curve has a period of about 14 days and an amplitude of about 3 Jupiter Diameters.

## How to Create a Plot

- Click File . . . Data . . . Analyze
- To select a moon in the analysis window, click on Data ... Select Moon $\rightarrow$ Hint: You may find it easier to start with Ganymede or Callisto.
- Click Data . . . Plot . . . Plot Type . . . Connect Points
- We want to fit a sine curve to the data, but we need to enter rough guesses before we begin tweaking the fit. We need to specify three values: the period, the amplitude, and a zero point called "T-Zero."
- Click Data . . . Plot . . . Fit Sine Curve . . . Set Initial Parameters
- To estimate T-Zero, locate a point on the graph where the line crosses the X -axis from negative to positive. Click here and note the X value in the lower left corner (the Y value should be about zero). This date is our initial guess for T-Zero. Enter this value in for T-Zero.
- Now determine your best guess for the period and amplitude of the curve, enter the values and press OK. These are just starting guesses and do not have to be perfect! See Figure 3 for reference.
- There are three scroll bars that change T-Zero, the period, and the amplitude. Use the scroll bars to adjust your sine curve so that it passes through all the data points.
- Once you have that, take a look at the RMS Residual value. Adjust the scrollbars one at a time to try to get it as small as possible. (This number is a measure of how well the sine curve fits your data points - an RMS of zero means there is zero difference between the curve and the points.) Remember that $9.0 \times 10^{-2}=0.090$ is a smaller number than $1.0 \times 10^{-1}=0.10$. The RMS turns red when it gets larger and green when it gets smaller. Keep tweaking the scrollbars until you have the best possible fit! See Figure 4 for reference.
- After your first graph is complete, check with your instructor to make sure it is OK before continuing. They will place their initials below when they are satisfied with your fit and you are ready to fit the other three moons:

> (Instructor's initials)

- Click Data . . . Print . . . Print Current Display to print a copy of the moon's graph.
- Record the amplitude and period of the curve in the appropriate box in Table 5.2. (If you were instructed not to print the graph, also record the RMS Residual value before continuing.)
- Repeat the above steps for the other three moons.


Figure 3. - Estimating initial parameters for the sine curve. This imaginary moon named CLEA appears to have a T-Zero point around 13.9 Modified Julian Date, a Period around 3.5 days, and an Amplitude around 4 Jup. Diams.


Figure 4. - Adjusting the T-Zero, period, and amplitude for a sine curve. Assuming the curve (not shown here) goes through every data point, the low RMS Residual of $8.847 \times 10^{-2}$ indicates a reasonably good fit.

### 5.3 Calculate Jupiter's Mass

By now you have found the amplitude of a sine curve for each moon's orbit in Jupiter Diameters and the period in days. Use this information to answer the following questions. Remember to show ALL your work for full credit and to use scientific notation when appropriate (see Appendix D).

1. To find the mass of Jupiter, we need to use Equation 5.1 which calls for the radius $(a)$ and period $(P)$ of a moon's orbit. Explain in a sentence or two how a sine curve's amplitude and period correspond to the radius and period of an orbit. It may be useful to draw a picture.
2. To use Equation 5.1, we need to have the period in units of years rather than days. Use the fact that there are 365.25 days in one year to do a unit conversion, and record your answers in Table 5.2.
$\rightarrow$ Hint: Think of it as 1 year $=365.25$ days or $\frac{1 \text { year }}{365.25 \text { days }}$.
3. To use Equation 5.1, we need to have the radius (or amplitude) in units of AU rather than Jupiter Diameters. Something Galileo didn't know is that there are 1,050 Jupiter Diameters in 1 AU. With this information, convert your radii to units of AU and record your answers in Table 5.2.
$\rightarrow$ Hint: Think of it as $1 \mathrm{AU}=1050$ Jup. Diams. or $\frac{1 \mathrm{AU}}{1050 \text { Jup. Diams. }}$.
4. Use Equation 5.1 to calculate the mass of Jupiter based on the orbit of each of its moons, and fill out Table 5.2. Remember that the units of mass will be solar masses $\left(M_{\odot}\right)$.

Table 5.2

| Moon | Period (days) | Period (years) | Radius (Jup. Diams.) | Radius (AU) | $M_{\text {Jupiter }}\left(M_{\odot}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Io |  |  |  |  |  |
| Europa |  |  |  |  |  |
| Ganymede |  |  |  |  |  |
| Callisto |  |  |  |  |  |

5. Are your four estimates of Jupiter's mass about the same? Why or why not?
6. From these four estimates, calculate your best guess of the mass of Jupiter. That is, take the average of the four values to arrive at a final answer.
7. The accepted mass of Jupiter is $1.8986 \times 10^{27} \mathrm{~kg}$. What is the percent error of your measurement? Is it reasonable? Why or why not? $\left(1 M_{\odot}=1.99 \times 10^{30} \mathrm{~kg}\right)$

$$
\text { Percent Error }=\left|\frac{M_{\mathrm{J}, \text { measured }}-M_{\mathrm{J}, \text { actual }}}{M_{\mathrm{J}, \text { actual }}}\right| \times 100 \%
$$

8. Thinking back to the assumptions that we made in Section 5.1 on the top of page 36 , which assumptions are good and which are bad? For example, you should ask yourself, "Is it ok to assume that the orbits of the moons are all perfectly circular with unchanging radii?" for the first assumption. Be sure to explain why you made the classification for each assumption.

### 5.4 Applying Newton's Version of Kepler's Third Law

9. Earth's moon has an orbital period of 27.3 days at a radius of $2.56 \times 10^{-3} \mathrm{AU}$. What is the mass of the Earth?
10. How many times more massive is Jupiter than the Earth?
11. There are many moons of Jupiter beyond the orbit of Callisto. Will they have longer or shorter periods than Callisto? Use Newton's version of Kepler's third law (Equation 5.1) to explain.
12. Although Pluto was discovered in 1930, its moon, Charon, was not discovered until 1978. What previously unknown property of Pluto were astronomers able to determine when they finally discovered Charon?

## Lab 6

# Measuring Distances with Parallax 

By Jim Davenport and Douglas Leonard<br>Revised by Alexander Burke

### 6.1 Introduction

Today we will be discussing one of the most difficult and important questions in modern astronomy: how far away are the stars? Determining distances to even the closest stars is tough, and it wasn't successfully done until the middle part of the 1800 's. The technique that is used is called parallax.

The method of parallax is the beginning of the cosmic "distance ladder." It is (usually) the only method by which we can physically measure the distances to stars. Believe it or not, you actually already use parallax every day - it is the method your brain uses to determine distances to objects using your two eyes. Humans commonly call this effect "depth perception." Depth perception relies on the fact that each eye has a separate line-of-sight to a nearby object.

Unfortunately, since stars are so far away, astronomers can't simply use their eyes to measure astronomical distances. Instead, they use the orbit of the Earth around the Sun! We know the distance between the Earth and the Sun (1 AU). If we look at nearby stars over a 6-month interval, they will appear to change positions relative to far-away background stars. The angle that astronomers use to describe this shift is actually one half of the apparent angular shift, and is called the parallax angle, $P$. This is shown in Figure 1.


Figure 1. - A summary of how parallax works. The two bottom panels are what an observer on Earth would see when looking at the nearby star during those months. Note how the nearby star's position changes relative to more distant background stars.

In today's lab, you will first use your eyes to measure parallax here on Earth and estimate the distances to a few objects in the classroom. Rather than distant background stars, you will be using evenly spaced marks at the front of the classroom to make your measurements. You will then apply the parallax technique as used by astronomers to calculate distances to nearby stars.

### 6.2 Procedure

### 6.2.1 Using Your Eyes for Parallax

## Taking the Data

- Station 1: Get into groups of two. Very carefully measure the separation between your partner's eyes. Try to be as precise as possible, and make sure you measure from the centers of the pupils! Record below the separation of your own eyes, in centimeters (cm).

$$
\text { Separation of your eyes: } a=
$$

$\qquad$

- Station 2: Go to the back of the classroom and stand on the marked line behind the closest meterstick. Look at the meterstick with each eye independently (i.e., cover or close the other eye). You will see the meterstick appear to "jump" relative to the background lines! Make note of which background line is closest to the meterstick when viewed by each eye (you can use your partner to help with this). Measuring the number of lines will allow us to calculate the angle, $\theta$, that the meterstick appears to move when viewed by each of your eyes separately.


Figure 2. - An example for measuring the parallax of a meterstick in the classroom with your right and left eyes. In this case, you would record a jump of 9 lines.

- How many of the background lines separate the two "images" you took with each eye? Repeat this procedure three times for each of the metersticks, alternating which meterstick you observe. Record the number of lines for each meterstick in the table on the next page. Each person should make his or her own measurements. It is helpful if one lab partner stands at the front of the class to mark background lines for the observer.
Note: Since the metersticks are at different distances, you should never get the same number of lines!

Separation of your eyes: $a=$ $\qquad$

|  | Trial 1 | Trial 2 | Trial 3 | Average |
| :--- | :--- | :--- | :--- | :--- |
| Meterstick 1 (closest) |  |  |  |  |
| Meterstick 2 (middle) |  |  |  |  |
| Meterstick 3 (farthest) |  |  |  |  |

- Station 3: You must also measure the actual distances to each meterstick and record your answers below (in cm ). Use a measuring tape, and think about the precision of your measurements.

Meterstick 1 (actual distance): $D=$ $\qquad$

Meterstick 2 (actual distance): $D=$ $\qquad$

Meterstick 3 (actual distance): $D=$ $\qquad$

## Calculating Distances in the Classroom

- We must now convert the number of lines we measured from the back of the classroom to the proper unit: degrees. From the back of the classroom, each line is 0.25 degrees apart. How many degrees did each meterstick appear to move?

Meterstick 1: $\theta=$ $\qquad$

Meterstick 2: $\theta=$ $\qquad$

Meterstick 3: $\theta=$ $\qquad$

Now, let's calculate the distance to each meterstick. A little trigonometry yields the following formula for the distance $(D)$ to each meterstick in centimeters:

$$
D=\frac{a \times 180^{\circ}}{\theta \times \pi}
$$

where $a$ is the separation of your eyes and $\theta$ is the number of degrees each meterstick appeared to "jump."

- Record your estimates of the distance to each meterstick (in cm ) below.

Meterstick 1 (estimated distance): $D=$ $\qquad$

Meterstick 2 (estimated distance): $D=$ $\qquad$

Meterstick 3 (estimated distance): $D=$ $\qquad$

1. How close were your distances estimated using the parallax technique to the actual distances of the three metersticks? Be quantitative and find the percentage error for all three metersticks.

$$
\text { Percent Error }=\left|\frac{D_{\text {actual }}-D_{\text {measured }}}{D_{\text {actual }}}\right| \times 100 \%
$$

2. List at least three things about the experiment that creates error in the distance measurement. Which do you think is the biggest effect? How could you improve the setup? Think about the different measurements you made, and how the distances you calculated using parallax would be affected if your measurements were too small or large.

### 6.2.2 Calculating Distances to the Stars

Let's now determine the distance to the closest star, Alpha ( $\alpha$ ) Centauri, using the parallax technique. Just like we did with our eyes and metersticks in the classroom, we can use the Earth's orbit around the Sun in place of our eyes and a $\alpha$ Centauri in place of a meterstick. The diagram in Figure 3 shows the geometry.


Figure 3. - The geometry of the parallax measurement of $\alpha$ Centauri, showing the angle $P=0.75^{\prime \prime}$. This drawing is not even close to scale!

In astronomy, even the closest stars have tiny parallax angles. In fact, the closest star to our Sun, $\alpha$ Centauri, experiences a shift of less than $1 / 2000$ of a degree over the course of six months! To better represent such small angles, astronomers divide each degree up into 3600 equal units, called arcseconds (denoted in mathematical notation as " ). For reference, one arcsecond is smaller than the angle subtended by a single human hair at arm's length. Using this system of measurement, the parallax of $\alpha$ Centauri is $P=0.75^{\prime \prime}$.

Since it makes for a very easy equation, astronomers actually use one-half of the observed shift as the parallax angle to derive astronomical distances. That is, the parallax angle $P$ equals $\frac{1}{2} \theta$. To calculate distances, we can use the formula:

$$
\begin{equation*}
D=\frac{1}{P} \tag{6.1}
\end{equation*}
$$

where the parallax angle $P$ is in units of arcseconds and the distance $D$ is in units of parsecs. By definition, 1 parsec $(1 \mathrm{pc})$ is the distance to an object that has a parallax angle of $1^{\prime \prime}$. The parsec is a unit unique to astronomy: astronomers chose this unit for convenience when calculating distances to stars. Note: $1 \mathrm{pc}=3.262$ light years (lt-yr).
3. What is the distance from $\alpha$ Centauri to the Sun? Give your answer in units of parsecs (pc).
4. What is the distance from $\alpha$ Centauri to the Sun in kilometers? Note: $1 \mathrm{lt}-\mathrm{yr}=9.46 \times 10^{12} \mathrm{~km}$.
5. How many times further is the distance between the Sun and $\alpha$ Centauri than the distance between the Sun and the Earth? Note: $1 \mathrm{AU}=1.50 \times 10^{8} \mathrm{~km}$.
6. Determine the distances to the following stars in parsecs and then in light years, given their parallax angles:
(a) Procyon: $P=0.29^{\prime \prime}$
(b) Ross 780: $P=0.21^{\prime \prime}$
(c) Regulus: $P=0.04^{\prime \prime}$
(d) Sirius: $P=0.38^{\prime \prime}$

### 6.3 Analysis

You have now used your eyes to determine distances using the parallax technique and used measurements of nearby stars taken with a telescope over the course of six months to estimate their distances. Answer the following questions:
7. Do closer stars have greater or smaller parallax angles than far away stars? Why? A drawing may help.
8. When considering using your eyes versus the Earth to measure parallax distances to objects, what change in the Earth's position corresponds to switching your eyes? Draw and explain a picture illustrating your idea.
9. Ancient astronomers considered the possibility that the Earth revolved around the Sun. They recognized that if this were so, then they might be able to see the effects of parallax on nearby stars. Unfortunately, they were unable to measure parallax shifts to any stars, and thus (wrongly) concluded that the Earth was at rest in the universe. Why do you think they were unable to measure parallax shifts of nearby stars? Hint: Think about how many degrees the closest star, $\alpha$ Centauri, shifts.
10. Today, the smallest angle our best telescopes can measure is $P=2.4 \times 10^{-5}{ }^{\prime \prime}$, or 24 microarcseconds $^{1}$. If you measured a parallax angle of $P=2.4 \times 10^{-5}$ ' , how far away would that object be?
11. The nearest large galaxy to the Milky Way is the Andromeda galaxy. We know Andromeda is about $780,000 \mathrm{pc}$ away. Based on your answer for the last question, do you think parallax would be a useful distance finding method for very remote objects, such as other galaxies? Why or why not?
12. If we were patient observers who could make parallax measurements from the planet Jupiter, do you think we could make better or worse measurements than we do from Earth? Explain your answer, and draw a picture to help explain your reasoning. Hint: Jupiter is located about five times further away from the Sun than the Earth is, however this does not mean that Jupiter is five times closer to the stars than Earth.

[^0]
## Lab 7

# An Exploration of the Properties of Light and Atoms 

By K. Azalee Bostroem
Revised by Dave Baer, Alexander Burke, Jay Franck, and Eric McLaughlin

### 7.1 Introduction

Light is something that is very near and dear to every astronomer's heart. Without light, the existence of other stars (or our own Sun for that matter) would be hidden from us. Light leaves a star and travels for millions or even billions of years to reach our eyes on Earth. From light we can learn a lot about a star, including the elements it is made of and even its temperature. Thus, in order to learn about stars we must first understand the nature of light itself.

The idea of light brings to mind something bright that has some property called color. But what do we mean when we talk about color? What exactly is color? Before the time of Isaac Newton, color was thought to be an "impurity" introduced to white light by objects. For instance, a green shirt would somehow pollute "pure" white light with the color green. Newton, however, was able to show that, using a prism, white light could be dispersed into a full spectrum of color, proving that in fact white light is comprised of all colors of the rainbow. Newton devised an incorrect but insightful theory to explain this, explaining that light was a particle in nature.

However, in the 1800s, physicists began to question Newton's theory of light. Thomas Young, a British physicist (among other things) was extremely critical of the particle theory of light. He reasoned that a wave theory could explain phenomena exhibited by light much better than any particle theory could. In this lab, we will first follow in Young's footsteps and discover for ourselves what wave-like properties light exhibits.

### 7.2 Light as a Wave

Waves have two important properties: wavelength and speed (or velocity). The wavelength of light (typically denoted $\lambda$ ) is the distance over which one complete cycle or oscillation of the wave occurs. Figure 1 shows a wave with its wavelength indicated in two different places.


Figure 1. - Diagram of a wave, indicating its wavelength.

Different colors of visible light correspond to different wavelengths: red has the longest wavelength, about 700 nanometers, and violet light has the shortest wavelength, about 400 nanometers, where 1 nanometer (nm) is equal to a billionth of a meter ( $1 \mathrm{~nm}=10^{-9} \mathrm{~m}$ ).

Despite the progress made by Young, what kind of wave light was would not be understood until later. James Clerk Maxwell described light as an electromagnetic wave: it is comprised of perpendicular electric and magnetic field lines that oscillate in strength. The full spectrum of electromagnetic waves, or the full range in wavelengths of light, encompasses (from short wavelength to long): gamma rays, X-rays, ultraviolet (UV), visible light, infrared, microwave, and radio. Electromagnetic waves are unique because they do not require any material to propagate through. In other words, light can travel through the vacuum of space. While in a vacuum, light has a constant speed of about $3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}$, which in the physical sciences is represented by the constant $c$. We will only deal with visible wavelengths of electromagnetic radiation in this lab, and since the speed of light in air is about the same as that in a vacuum, we will use this same value for $c$.

So how will we explore the wave properties of light? First, let's test our wave hypothesis by setting up an experiment where the outcome will show whether light is a wave or not. One unique property of waves is that they diffract when passed through a narrow slit or gap. This means that a front of parallel waves will expand outwards when passed through some gap. If a single front of waves is passed through two gaps simultaneously, the waves that radiate from the gaps will interfere with themselves. The result is what is known as an "interference pattern." This phenomenon is illustrated in Figure 2.


Figure 2. - Waves diffracting through gaps in an impenetrable medium. Figure attributed to Niels Bohr.
The dark lines represent wave peaks (or crests) and the lighter lines in between are wave troughs. When two peaks line up, they add up to make a stronger peak. When a peak and a trough line up, they add up to zero.
Thus, when the waves impact upon some surface, the resulting pattern in wave intensity along the surface shows multiple peaks in intensity with areas of zero intensity in between. This is characteristic of all waves, from water waves to sound waves, and is known as an interference pattern. When a single wave is sent through slits that are close enough together, the interference pattern will look like a central, high intensity dot with a series of dimmer dots on either side of the central one.

In our lab today, we will replicate the spirit of Thomas Young's famous double slit experiment, where he passed light through two gaps to see whether they would display an interference pattern. We will use a diffraction grating, which is actually made up of many thousands of tiny slits, and a laser, which emits a powerful beam of monochrome (single color) light.

A geometric relationship exists among the distance of the grating from the table, the separation between the slits in the grating, the distance between the dots on the table, and the wavelength of light. Using this relationship we will determine the wavelength of the laser we are using, if we determine that light does indeed act like a wave. Figure 3 illustrates the experimental setup.


Figure 3. - Experimental setup of the laser and diffraction grating.

### 7.2.1 Procedure

DO NOT LOOK DIRECTLY AT THE LASER OR SHINE IT AT ANYONE'S EYES. EVEN SHORT EXPOSURE OF THE RETINA WILL LEAD TO PERMANENT DAMAGE!

- Place a piece of white paper on the table.
- Set the diffraction grating stand on top of the white paper.
- Set your red laser in the tripod so that the light beam shines straight down through the grating. You may also want to pinch the "on" button with the tripod holder to keep it on continuously. The laser should not be resting on the diffraction grating. It should be above the grating. It does not matter how high above, as long as it is perpendicular to the grating.

1. What do you observe? Does this suggest that light acts like a wave? Why not a particle? Hint: You can think of particles as tiny bullets. Think about how Figure 2 would look different if there was a gun, not light, firing bullets through the gaps.

- The diffraction grating should cause the laser point to appear in three places: once directly below the laser and then one on either side of the middle point creating a straight line (this is a linear diffraction grating, after all). Carefully mark on the piece of paper where the three dots appear.

2. Measure the distance between the center of the middle dot and the inside edge of the right dot. Now repeat the measurement from the center of the middle dot to the inside edge of the left dot and record both your measurements here. If they are not very close to equal, you should redo this part because you were holding something sideways! Once you have the two distances very close, take the average to arrive at a value for " $y$."

The formula relating the wavelength $(\lambda)$, the size of the diffraction grating (number of lines per mm: $d$ ), the distance from the diffraction grating to the paper ( $h$ ), and how far apart the diffracted laser points occur ( $y$ ) given in Figure 3 is:

$$
\begin{equation*}
\lambda=\frac{y}{d \times h} . \tag{7.1}
\end{equation*}
$$

3. What is the wavelength of the red laser in nanometers? Be sure to keep track of your units! Hint: The units on $d$ are effectively "per mm," or $1 / \mathrm{mm}$.
4. Repeat this experiment with the violet laser. If you see five dots, use only the three dots in the center for your calculations. What is its wavelength in nanometers? Which color light (red or violet) has the longer (greater) wavelength?

### 7.3 Light as a Particle

Until now, we have discussed light as a wave, and light was indeed thought of as a wave until the beginning of the 20th century. In 1905, however, Albert Einstein proposed (in a paper that won a Nobel prize) that light also behaves like a particle. It can be thought of as a little packet of energy traveling through space at the speed of light, $c$. Many experiments have been performed to try to prove that light is either a wave or a particle, but, in fact, we have come to conclude that it behaves like both. This idea is now referred to as "wave-particle duality."

Whether you get wave-like or particle-like behavior depends entirely on the type of experiment you conduct. This principle represented a dramatic shift in thinking about physics, and has other, more philosophical implications as well. Scientists therefore say that light is both a wave and a particle. Since we have just examined the wave properties of light, let's now look at its particle properties.

Think back to your high school science classes and the model of an atom you were presented. You probably pictured it as a nucleus surrounded by different circles, or energy levels, of electrons. This is a decent approximation, and we will make use of it. However, it is worth mentioning that this picture is also only an approximation. The reality of the atomic and sub-atomic world is far more rich and complex.

In this model, each electron is located in a different energy level or orbital around the nucleus. Electrons closest to the center are at a lower energy than electrons farther from the center. You can think of this as climbing a mountain: You need a lot more energy to climb to the top of the mountain than to sit at the bottom.

- These energy levels exist at very specific values.
- Different types of atoms (e.g., hydrogen, helium, mercury, etc.) have different sets of energy levels.
- In order for an electron to move from one energy level to the next it must gain or lose energy.

So how does this all relate to light? The main way electrons gain and lose energy is by absorbing or releasing little packets of light, called photons. These packets of light have the exact same energy as the difference in energy between the two levels that the electron is moving between. Specifically:

$$
\begin{equation*}
\Delta E=E_{\text {initial }}-E_{\text {final }} \tag{7.2}
\end{equation*}
$$

where $\Delta E$ is the energy difference between the two levels, typically measured in units called "electron volts" (eV).


Figure 4. - Basic model of an atom, with a nucleus (solid black dot) surrounded by electrons orbiting it.

Hydrogen's lowest energy level is -13.6 eV ; its next energy level (i.e., the second level) is -3.4 eV , and the next energy level (i.e., the third level) is -1.5 eV .
$\rightarrow$ Don't be confused by the negative signs here; they just note that the energies are increasing (i.e., getting closer to zero) the further the electron is away from the nucleus.

Here is an "energy level diagram" for the element hydrogen, showing the first three energy levels:


Figure 5. - Energy level diagram of hydrogen, showing the first three energy levels in which electrons can exist.
5. Looking back at Figure 5, suppose an electron moves from the 3 rd level $(n=3)$ to the 2 nd level $(n=2)$. How much energy is released in the light packet?

So why does this matter to astronomers? It is because the wavelength of the photon, or its "color" if it is visible light, corresponds directly to the amount of energy that photon has via the equation:

$$
\begin{equation*}
E=\frac{h c}{\lambda} \tag{7.3}
\end{equation*}
$$

where $h=4.1336 \times 10^{-15} \mathrm{eV} \times \mathrm{sec}$ and $c=3.0 \times 10^{10} \mathrm{~cm} / \mathrm{sec}$ (the speed of light). Note that $E$ will be in eV when $\lambda$ is in cm .
6. What is the wavelength corresponding to the energy transition found in Question 5 in nanometers (nm)? Note: you will need to convert from centimeters to nanometers. $1 \mathrm{~cm}=10^{7} \mathrm{~nm}$.
7. Given your earlier experiments with the laser, which color (red or violet) do you think the light that is emitted by this transition of hydrogen would be closest to? Why?
$\rightarrow$ The specific transition from the $n=3$ to $n=2$ energy states of hydrogen is very important in astronomy since there's lots of hydrogen in the universe. This is known as the $\mathbf{H} \alpha$ transition, characterized by its bright red color. It is one of the few hydrogen lines.

### 7.4 Using the Wave-like and a Particle-like Properties of Light

If a gas has energy added to it, many of its electrons are raised to the outer energy levels (i.e., far away from the nucleus) and then fall down, releasing photons at differing energies. When we look at this with our eyes, the gas appears to be glowing.

A common example of this is neon signs. When neon gas has electricity running through it, its electrons are excited to outer energy levels, and when they fall down to a lower energy level they must release energy, which we see as colored light. This light is actually made up of many different but very specific colors (hence energies) belonging to photons released from each specific transition. Each transition from one energy level to another lower energy level causes a light packet to be released at a different energy and therefore a different wavelength or color. If we separate the different wavelengths by using a diffraction grating (installed in a handheld spectroscope), then we can actually see the individual emission lines that result from each type of transition.

- Plug in the Power supply box. MAKE SURE THE SWITCH ON THE SIDE IS OFF.
- Place the helium tube into the metal holders. MAKE SURE NOT TO TOUCH THE METAL.
- Turn on the power supply by flipping the switch on the side.
- Look through the spectroscope and line up the glass tube with the slit. To the right you will see a scale with numbers. When aligned correctly, you will see stripes of color appear below the scale. Draw what you see in the provided box.


Helium Spectrum

- Now turn off your power supply and wait a minute. Be sure that the helium tube has cooled completely before taking it out a replacing it with the hydrogen gas tube.
- Turn on the power supply and look at the gas again through the spectroscope. Draw what you see below and label the colors.

- Turn off the power supply.

8. For your hydrogen spectrum, label what you think is the line due to the $\mathrm{H}_{\alpha}$ transition described earlier. Is this line consistent with your answers to questions 6 and 7? If not, you may want to revisit those questions.
9. Do the spectra of helium and hydrogen look the same? What are some similarities and differences? Do the wavelengths of the colors you observed roughlly correspond to your results from questions 3 and 4 ?

Suppose you took the spectrum of an astronomical object and find it to look like the spectrum shown below:


Figure 6. - Spectrum of a "mystery" astronomical object.
10. Which element (helium or hydrogen) would you conclude the object shown in Figure 6 is most likely made of? Why?

### 7.5 Analyzing the Light from Stars

Stars are hot, dense balls of gas that emit a tremendous amount of light at all wavelengths. Thus, if we are just looking at the dense part of a star, we would not see individual emission lines like we have for individual elements in the lab - rather, you see a continuous range of color, stretching from the shortest wavelengths to the longest.

Although the star itself produces this continuous range of color, there are individual atoms in its outer layers (the stellar atmosphere) that absorb some of the light emitted from the central region of the star. At the atomic level, this process is the opposite of what produces the emission lines we studied earlier: the electrons in the atoms absorb a
packet of light energy and jump up in energy. Whenever this happens, a little bit of the outgoing light from a star is removed. This process creates "absorption lines," also known as dark lines, in the spectrum of a star that correspond directly to the emission lines of a given element.

These dark lines can be used as "fingerprints" for identifying a particular element in a star. It is a tricky business, however, because stars are not entirely composed of any one element: they are made up of many elements in different quantities. To be sure an element is present in a star, an astronomer must see ALL of that element's absorption lines in the spectrum of a star.

Figure 7 shows the spectra of three fictional stars, called "Abba," "Babba," and "Cabba," along with the spectra of six elements measured in the laboratory.


Figure 7. - Spectra of three stars (Abba, Babba, Cabba), followed by the laboratory spectra of 6 elements. Note that violet light is on the left, red on the right. (Image taken from the University of Washington Astronomy Department Labs.)

Note that all three stars are made primarily of hydrogen and helium (just like all of the other stars in the sky). However, in this case, one of the stars has a large abundance of carbon in its atmosphere, another has a large abundance of nitrogen, and the third has a large abundance of oxygen. (Note: in terms of actual abundances, stars typically have about 10,000 times more hydrogen than, say, carbon.)
11. Which star has a high abundance of nitrogen in addition to hydrogen and helium?
12. Which star has a high abundance of oxygen in addition to hydrogen and helium?
13. Which star has a high abundance of carbon in addition to hydrogen and helium?
14. Do any of these three stars show evidence of having Xenon in their atmospheres? Explain your reasoning.

Lab 8

# A Universe of Galaxies and Dark Matter 

By Tina Güth and David Krogsrud<br>(Dark matter portion adapted from a lab by the University of Washington Astronomy Department) Revised by Gideon Bass

### 8.1 Background: Galaxies

In astronomy we find similar structures on many different scales. For instance, the Moon orbits the Earth, the Earth orbits the Sun, and the Sun orbits the center of our galaxy. Today we will be examining this last and largest scale the galactic scale.

A galaxy is a massive, gravitationally bound system that consists of a large number of stars along with gas, dust, and something called dark matter (more on dark matter in a bit). Interestingly, the word galaxy comes from the Greek root galaxias, meaning "milky circle." This is a reference to the Milky Way Galaxy, in which our Solar System resides. In our observable universe there are more than 100 billion galaxies! Some galaxies have only a few million stars, while others are gigantic with up to one trillion stars. Each star, like our Sun, orbits around the center of its galaxy, where there is often an extremely massive black hole - a region of space so dense that not even light can escape. In today's lab, you will study the shapes of galaxies and the evidence that a substantial fraction of their mass exists in a still unknown form, called dark matter.

### 8.2 The Shapes of Galaxies

### 8.2.1 Introduction

Galaxies are generally classified into one of three types: spiral, elliptical, or irregular. Spiral galaxies are flat and disk-shaped and have spiral "arms." About half of all spiral galaxies have a central bar-shaped structure and are thus called barred spiral galaxies; it is now known that our own Milky Way is a barred spiral. Elliptical galaxies are smooth and rather featureless, getting fainter from the center out. Irregular galaxies do not fit into either one of the other two categories due to a lack of coherent structure. They are often considered to have resulted from the disruption of neighboring galaxies.

Figure 1 displays the Hubble Classification Sequence for galaxies. In it, elliptical galaxies are located on the left. They are denoted by the letter E, followed by a number, which represents their ellipticity, or degree of flattening (recall your earlier study of ellipses from the Lab 4 on Kepler's First Law). The ellipticity increases from left to right in the diagram, from E0 (nearly circular) to E7 (highly flattened), respectively. On the right side of the diagram are the spiral galaxies, and they are split into two branches.

Spiral galaxies occupying the upper branch are denoted by the letter $S$, while those on the bottom branch are denoted SB, indicating barred spiral galaxies. Both types are further subdivided depending on the appearance of their spiral structure. This is done by adding a lower-case letter to the type. Spirals that have tightly-wound arms and a large, bright central bulge are denoted Sa (or SBa , if they are barred). $\mathrm{Sb}(\mathrm{SBb})$ galaxies have less tightly-wound arms and a fainter bulge. Galaxies with rather loosely wound arms and a smaller, fainter bulge are denoted Sc ( SBc ). Irregular galaxies do not have a place in the Hubble sequence because, as the name implies, they lack regular structure.


Figure 1. - The Hubble Classification Sequence of galaxies (A.K.A. the "Tuning Fork" Diagram). Image credit: Ville Koistinen.

In order to get a better idea of these different types of galaxies, you will be classifying some galaxies yourself. These images come from the Space Telescope Science Institute (STScI) Digitized Sky Survey website, which provides access to images of the sky taken with the Palomar and UK Schmidt telescopes. These images are of very high quality and readily available to the public.

### 8.2.2 Procedure

1. Take a look at the galaxies on the next page. Referring to Figure 1, classify each galaxy as spiral, elliptical, or irregular. Be sure to give both type and subtype (i.e. Sa, or E3) and give a description explaining how you determined the type. Use complete sentences.
(a) M 100
(b) M 87
(c) M 32
(d) NGC 4038
(e) M 104
(f) NGC 1300

(a) M 100

(c) M 32

(e) M 104

Figure 2. - Six galaxies for you to classify.

(b) M 87

(d) NGC 4038

(f) NGC 1300

### 8.3 Background: Dark Matter

You have now learned how to distinguish different types of galaxies. However, as it is often the case in astronomy, there is more to galaxies then meets the eye - literally! It turns out that a large fraction of a galaxy's mass is invisible. That is, there is only enough light coming from a typical galaxy to account for about $10 \%$ of the total mass of the galaxy. Because there is not enough light to account for the rest (about $90 \%$ ), astronomers have labeled the invisible material dark matter. Dark matter evidently does not interact with photons (particles of light), which means that it neither absorbs nor radiates light. This is why it is called "dark." Though it is not yet well understood (astronomers have no idea what dark matter actually consists of!), its existence can be inferred from its gravity. For the remainder of this lab, you will investigate the evidence for dark matter in the spiral galaxy NGC 2742.

The mass of a spiral galaxy is found by examining how quickly the objects within it are revolving around its center: The faster they orbit, the more mass there must be tugging on them. Figure 3 shows a rotation curve of NGC 2742. It plots the radial velocity (how fast material is moving in the line of sight direction in $\mathrm{km} / \mathrm{s}$ ) versus the distance from the galaxy's center (in kpc , where $1 \mathrm{kpc}=1000 \mathrm{pc}$ ).


Figure 3. - Rotation curve of NGC 2742 for the portion of the galaxy that is spinning away from us (redshifted spectral lines). Since the galaxy is tipped at an angle to us, the radial velocity that we detect is not the total velocity of matter orbiting around the center of NGC 2742. Thus, a correction has been made in the rotation curve to account for the inclination angle. Data is from Rubin, V. C., Burstein, S., Ford, W. K., \& Thonnard, N. 1985, ApJ, 289, 81.

### 8.3.1 Procedure

Remember back to the lab you completed earlier about Jupiter's moons. In that lab, you computed the mass of Jupiter based on the orbital speeds of its moons. In a very similar way, in today's lab you will calculate the mass of the galaxy NGC 2742 by examining the motions of the stars within it. To determine the gravitational mass profile of the galaxy (i.e., how much mass is contained within a given distance from the center), we will use Isaac Newton's gravity equation.

- From Figure 3, find the radii listed in the first column of Table 1 and record the corresponding radial velocity values in column two. Try to be as precise as possible.

| Radius (kpc) | Radial Velocity $(\mathrm{km} / \mathrm{s})$ | Total Mass $\left(M_{\odot}\right)$ | Luminosity inside $R\left(L_{\odot}\right)$ | Luminous Mass $\left(M_{\odot}\right)$ |
| :---: | :--- | :--- | :--- | :--- |
| 1.0 |  |  |  |  |
| 2.0 |  |  |  |  |
| 3.0 |  |  |  |  |
| 4.0 |  |  |  |  |
| 5.0 |  |  |  |  |
| 6.0 |  |  |  |  |
| 7.0 |  |  |  |  |
| 8.0 |  |  |  |  |
| 9.0 |  |  |  |  |
| 10.0 |  |  |  |  |

Table 8.1

- Determine the mass of the galaxy interior to each distance by using your values for the radius and the radial velocity. This can be done via the following equation, which is an extension of Newton's work on gravity and planetary orbits. It relates the radial velocity $v$ and position of an object to the mass of the object it orbits:

$$
\begin{equation*}
M=\frac{v^{2} R}{G} \tag{8.1}
\end{equation*}
$$

Here $G=4.31 \times 10^{-6} \frac{\mathrm{kpc} \mathrm{km}^{2}}{M_{\odot} \mathrm{s}^{2}}$ and $M$ is the mass contained inside of radius $R$. Record your values in Table 8.1.

Now that you have determined the mass of the galaxy interior to each radial distance based on its gravitational influence, let's find how much of this mass can be attributed to the luminous matter such as stars and gas, and how much to dark matter.

- Figure 4 is a plot of how much light NGC 2742 produces inside of the region encompassed by a given radius. Once again, the radius is the distance from the center of the galaxy in kpc. At each radius, determine the value of the luminosity of the galaxy using Figure 4, and record the values in Table 8.1.


Figure 4. - Amount of light (in solar luminosities, $L_{\odot}$ ) emitted inside a given radius from the center of NGC 2742.
Now that you have measured how much light is coming from NGC 2742, let's estimate the mass of the material that must be present to produce that light. To do this, we must estimate how much light a given amount of mass is expected to produce. For instance, our Sun produces $1 L_{\odot}$ of light and has a mass of $1 M_{\odot}$. This gives us a 1-to-1 ratio. However, not all stars are like the Sun. Some produce more light for their mass, and some produce less. If we consider a typical mix of stars found in a galaxy, astronomers have calculated that for every $1 L_{\odot}$ of light, there is approximately $2 M_{\odot}$ of luminous mass producing that light. In other words, the mass-to-light ratio would be 2-to-1, or $\mathbf{2} \mathrm{M}_{\odot} / \mathbf{1} \mathrm{L}_{\odot}$.

- Calculate how much mass must be present in the galaxy based on how much light we see and record this in the fifth column of Table 8.1.

2. Consider the furthest point from the center of the galaxy. What percentage of the total mass of NGC 2742 is luminous, or in other words, contained within stars and gas? What percentage of the total mass cannot be accounted for in the light observed?
3. Based on this, is there evidence for the existence of dark matter in NGC 2742? Clearly explain your answer.

- Using the blank graph on the last page, plot the total mass (as calculated from the rotation curve) versus radius. On the same graph, plot the luminous mass versus radius. Draw a line that goes through each point for both sets of data and clearly label them. Be sure to include axis labels, units, and a title on your graph. Note that on the $y$ axis, each tick mark is $0.1 \times 10^{10}=1.0 \times 10^{9}$.

Next we will be calculating the distribution of matter. The easiest way to do this is to look at the slope of the graphs you have made at various points. Remember, the slope between any two points can be calculated by dividing the "rise," how much the line increases in the $y$ direction between the two points, divided by the "run," the distance in the $x$ direction between the two points:

$$
\text { Slope }=\text { Rise } / \text { Run }
$$

The slope is a measurement of how quickly the quantity you have graphed is changing. A high slope in a particular area means that the quantity you are looking at is increasing rapidly in the area you are looking at. A low slope means there is not much of an increase in the quantity over the area you are looking at.
4. Calculate the slopes of the two fits you drew on your graph between 1 kpc and $3 \mathrm{kpc}, 4 \mathrm{kpc}$ and 7 kpc , and 8 kpc and 10 kpc for both the luminous mass and the total mass. Record your answers in Table 8.2 below.

| Radius Range (kpc) | Luminous Mass Slope $\left(M_{\odot} / \mathrm{kpc}\right)$ | Total Mass Slope $\left(M_{\odot} / \mathrm{kpc}\right)$ |
| :---: | :--- | :--- |
| $1.0-3.0$ |  |  |
| $4.0-7.0$ |  |  |
| $8.0-10$ |  |  |

Table 8.2: Slopes in different radii ranges for the luminous mass plot and the total mass plot.
5. How are the slopes you calculated above different from one another? What does this tell us about where mass is distributed in NGC 2742? Is it evenly distributed, or are the two types of masses concentrated in different parts of the galaxy? If it is not evenly distributed, where are the two types of masses concentrated?


## Lab 9

## Hubble's Law

By K. Azalee Bostroem and revised by Lauren Brewer (SDSU) Adapted from CLEA via David Vakil, El Camino College<br>Modified for IDEAS by William Welsh (2012 July)<br>Revised by David Jaimes (SDSU) 2016 August

### 9.1 Introduction

One of the most unexpected discoveries made about our universe was that almost all the galaxies in it (with the exception of a few nearby ones) are moving away from our own galaxy, the Milky Way. This curious fact was first discovered in the early 20th century by astronomers who noted that absorption lines in the spectra of most spiral galaxies had been stretched to longer wavelengths compared to those lines observed from stationary laboratory objects (i.e. the lines were located more towards the redder, or longer wavelength, part of the spectrum; such a shift is called a"redshift"). By realizing that the redshift was caused by the Doppler shift, astronomers concluded that the redshifted galaxies were all moving away from us. In the 1920s, Edwin Hubble measured distances to the galaxies for the first time, and when he compared (i.e., graphed) these distances against the velocities for each galaxy, he noted something quite remarkable: The further away a galaxy was from the Milky Way, the faster it was moving away. Such a relationship is exactly what is produced when an explosion occurs.

After some initial debate, astronomers came to interpret Hubble's relation as evidence of a universal expansion. The distance between galaxies in the universe is growing larger with time, like the distance between raisins in a rising loaf of bread or dots on a stretched rubber band or dots on an expanding balloon. An observer in any galaxy, not just our own, would see all the other galaxies traveling away, with the most distant galaxies traveling the fastest. This was a remarkable discovery. The expansion is believed today to be the result of a "Big Bang" which occurred roughly 13.7 billion years ago.

You will determine the rate of cosmic expansion by plotting the velocities of galaxies against their distances, and determine the slope of the graph, a number called Hubble's constant, or $H_{0}$. Hubble's constant tells us how fast a galaxy at a given distance is moving away from us. Hubble's discovery of the correlation between velocity and distance is fundamental in reckoning the history of the universe. Because of its importance, Hubble's experimentally determined relationship is now known as "Hubble's law."

The technique we will use in our simplified exercise to measure the expanding universe is very similar to one used by astronomers today. As astronomers continue to refine Hubble's original experiment, we have learned some further startling discoveries. The latest research indicates that not only is the universe expanding, but it is also speeding up, due to something called dark energy. We think this dark energy may be represented as something called the "cosmological constant." However, the discovery that the universe is accelerating is very recent. At this point, we simply don't know why it's happening.

### 9.2 Example: The Automobile Constant

Three cars leave San Diego at the same time. Car A travels at 25 mph , car B travels at 40 mph , and car C travels at 55 mph . If all three cars travel at a constant speed (they do not speed up or slow down) we find that car A covered a distance of 75 miles, car B covered a distance of 120 miles, and car C covered a distance of 165 miles, see Figure 9.1. Next, we can find the slope of all three points by fitting a line.


Figure 9.1: Three cars leaving San Diego at the same time. The origin represents zero time and zero distance.

1. In this case, the slope represents the "automobile constant" in mixed units like mph per mile. You can find the time each car has been traveling since leaving San Diego by taking the inverse of the slope. How long ago has it been since the cars departed from San Diego?
2. Now, three different cars left San Diego at the same time and with constant speed. Car A traveled a distance of 112 miles at 42 mph , car B traveled a distance of 152 miles at 57 mph , and car C traveled a distance of 264 miles at 99 mph . In a separate sheet, graph the three points.
3. Find the "automobile constant" and then find how long the cars have been traveling.

### 9.3 Procedure

To measure the expansion of the universe, we need to know the distances and velocities of far away galaxies. To simplify this exercise, we are going to use distances that are already given to us. (The distances can be obtained by using the brightness of the galaxy and knowing that the further away an object is, the fainter it is. The brightness decreases as an inverse-square law: $\frac{1}{d^{2}}$ where $d$ is the distance. Also, there are empirical methods for getting the true brightness based on things like the rotation and the angular size of the galaxy.)

To save time, we will look at only two galaxy clusters, UMa (Ursa Major) and the Coma Cluster, and use their velocities to estimate Hubble's constant. Astronomers would normally use many dozens or even hundreds of galaxies to be as precise as possible. But we can get the flavor of how it is done with just two galaxy clusters. To find the velocity, you need to measure the shift in wavelengths for some strong absorption lines in the spectrum, see Figure 9.2. The calcium K and H lines are excellent for this purpose, so we'll use those. You also need to know the speed of light, $c=300,000 \mathrm{~km} / \mathrm{s}$.

- Compute the velocities for both the K line and H line in the Ursa Major cluster galaxy. First measure the wavelength shift, then compute the velocity using the Doppler formula.
- Determine the average velocity from the two lines. Write your results in the Table 3.

Rest wavelengths:
$\rightarrow$ Ca-K line has $\lambda_{0}=3933.67 \AA$
$\rightarrow \mathbf{C a - H}$ line has $\lambda_{0}=3968.50 \AA$

## Doppler shift:

$\Delta \lambda=\lambda-\lambda_{0}$
Doppler's law:
$v=c \frac{\Delta \lambda}{\lambda_{0}}$

Table 1 - Measured Wavelengths from Spectra

| Table 1 - Measured Wavelengths from Spectra |  |  |
| :--- | :--- | :--- |
| Galaxy | Measured Ca-K $\lambda(\AA)$ | Measured Ca-H $\lambda(\AA)$ |
| UMa 1 |  |  |
| Coma 1 |  |  |


| Table 2 - Wavelength Shifts |  |  |
| :--- | :--- | :--- |
| Galaxy | $\Delta \lambda_{\mathrm{K}}(\AA)$ | $\Delta \lambda_{\mathrm{H}}(\AA)$ |
| UMa 1 |  |  |
| Coma 1 |  |  |


| Table 3 - Doppler Radial Velocity |  |  |  |
| :--- | :--- | :--- | :--- |
| Galaxy | $v_{\mathrm{K}}(\mathrm{km} / \mathrm{s})$ | $v_{\mathrm{H}}(\mathrm{km} / \mathrm{s})$ | $v_{\text {average }}(\mathrm{km} / \mathrm{s})$ |
| UMa 1 |  |  |  |
|  |  |  |  |
| Coma 1 |  |  |  |

### 9.3.1 Computing Hubble's Constant

When Edwin Hubble (and others who followed up on his work) graphed galaxies' velocities against their distances, he noticed that more distant galaxies receded faster than nearby ones according to a very simple straight-line relationship:

$$
\begin{equation*}
v=H_{0} d \tag{9.1}
\end{equation*}
$$

where $v$ is the velocity of recession, $d$ is the distance to the galaxy, and $H_{0}$ is a constant called Hubble's constant. The above equation is known as Hubble's Law. Complete the following five bullet points in a separate sheet to find Hubble's constant:

- Fill in the missing values in the first column of Table 4 using data from Table 3 for the UMa 1 galaxy, and from the measurement you made for the Coma 1 galaxy in the previous exercise.
- Using the values in the table, make a graph of velocity versus distance for the six galaxies. Don't forget to label the axes correctly!
- Using a ruler, estimate and draw in the best fitting line to the data. Make sure your line goes through the origin $(0,0)$.
- From your line, determine the slope.

| Table 4 - Velocity and Distance |  |  |
| :--- | :--- | :--- |
| Galaxy | $v_{\text {avg }}(\mathrm{km} / \mathrm{s})$ | Distance $(\mathrm{Mpc})$ |
| UMa 1 |  | 18.6 |
|  |  | 18.5 |
| UMa 2 | 1313 | 18.7 |
| UMa 3 | 1309 | 15.5 |
| UMa 4 | 1090 | 87 |
| Coma 1 |  | 102 |
| Coma 2 | 7180 | 96 |
| Coma 3 | 6800 |  |

- The slope of the line of velocity versus distance is $H_{0}$. What is your value for Hubble's constant $\left(H_{0}\right)$ ? (be sure to include the units!)


### 9.3.2 Estimating the age of the universe

Now that we have Hubble's constant ${ }^{1}$, we can use this to estimate the age of the universe. Assuming that the expansion of the universe has been at a constant rate, ${ }^{2}$ we can use that rate to get the age.
4. Why does taking the inverse of the Hubble constant yield the age of the universe? $\left(\right.$ age $\left.=1 / H_{0}\right)$
5. Find the conversion factor from megaparsecs (Mpc) to kilometers $(\mathrm{km})$. There are $3.09 \times 10^{13} \mathrm{~km}$ in one pc.
6. Re-write Hubble's constant without any distances in the units. The units should now be just ( $1 /$ seconds).
7. Take the inverse of your Hubble's constant. The units should now be seconds.
8. Convert your answer from seconds to years, and then to billions of years. There are $3.15 \times 10^{7}$ seconds in one year, and $10^{9}=1$ billion.
9. So according to your redshift measurements and slope, how long has it been since the Big Bang occurred? This number is commonly referred to by astronomers as "the age of the Universe."

[^1]


Figure 9.2: Two pedagogical example spectra of a galaxy in the Ursa Major cluster and another in the Coma cluster, zoomed in on the Ca II $\mathrm{K} \& \mathrm{H}$ lines. This data came from the NED database at https://ned.ipac.caltech.edu and the name of the galaxy is on the bottom right corner of each panel.

## Lab 10

# Astronomy Picture of the Day 

Conceived by Tyler Mitchell Edited and expanded by K. Azalee Bostroem

| Submit image selection to instructor |  |
| :--- | :--- |
| Submit slides to instructor |  |
| Presentation day and time |  |
| Paper due |  |

### 10.1 Introduction

Every day NASA posts an image to the following website: http://apod.nasa.gov/apod. The image is selected by a different astronomer each day who also includes a short paragraph explaining the picture. Included are links in the text to more information. If you click on archive at the bottom of the web page you can find images from 1995 to the present. These images encompass many different topics relevant to the field of astronomy. There are ancient astronomical instruments, supernova remnants, star clusters, black holes, and telescopes, just to name a few. For this project you will be researching a picture of your choice. This project is a chance for you to research something in astronomy that really interests you.

### 10.2 Assignment

NOTE: Your instructor may provide you with additional instructions that replace and/or supplement what is written here. Remember to read all directions given and ask your instructor if anything is unclear.

1. Select an image and submit it to your lab instructor. Images are assigned on a first come first serve basis, so it's best to get this done early! Keep in mind that you should not start your research until your instructor okays your image.
2. Next, find out all you can about this image. There many links in the explanation below the image are an excellent start. You will also want to look at a book on your subject (e.g., an Astronomy 101 textbook). You may also like to look for articles in a scientific journal. Some examples include the Astrophysical Journal, the Astronomical Journal, and Publications of the Astronomy Society of the Pacific. During your research, consider the following questions...

- Why were you drawn to this image?
- Who created the image, where, and when?
- Is there any relevant history associated with the image?
- What is the object classified as (planet, star, galaxy, etc.)?
- Have any scientific discoveries been made regarding your object?
- What does this image bring to the scientific community? e.g., it could be the discovery of a new object, an object we use for some purpose, or a new piece of equipment.
- What are the scientific processes concerning your object?
- What is still unknown about your object?
- Is there any current or planned research on your object or other objects similar to yours?

3. Write a paper about your image. Be sure you are not just re-stating what is printed in the APOD description. You should take various sources of information, interpret the information, and restate it in your own words. ${ }^{1}$ Your paper should be about 1,000 words. Images and quotes do not count toward the page requirement. Use a legible font (size 10-12 pt) and reasonable (roughly 1 inch ) margins. Your paper should be written as you would write any research paper or essay, with an introduction, body paragraphs, conclusion, and references. You should also include references and/or quotes from at least three sources. One of your sources must be a textbook or scientific paper. Use proper "MLA" formatting when citing your sources. Remember to cite all of your sources! Your instructor may have additional formatting guidelines for you to follow as well.
4. Plan a five minute oral presentation and practice it. The information you find does not have to be very technical, but make sure you present material that you understand. Speak clearly and loudly enough so that everyone can hear. It's OK to have some notes with you, but don't read your paper or slides word for word! Make eye contact, not just with the instructor but with the class as well. Show some enthusiasm! Convince the class that your object is interesting.
5. Give your presentation when scheduled and turn in your paper by the deadline.

### 10.3 Grading

As there is no final exam in Astronomy 109, this project is worth a substantial part of your course grade. You will be graded on both your presentation and your paper. Your grade will be based on, but not limited to:

- The timeliness of your work (image selection, presentation, and paper)
- The organization and length of both your paper and presentation
- The inclusion of relevant information
- Clarity (spelling, grammar, format, writing style/delivery) of your paper and presentation


## Good Luck! Feel free to contact your instructor if you have any questions.

[^2]
## Part II

## Supplemental Laboratory Exercises

## Lab 11

## Telescopes

By J. Emilio Enriquez and Raj Pandya
Revised by Dave Baer, Alexander Burke, Mallory Vale, and Eric McLaughlin

### 11.1 Introduction

One of the most useful observational tools astronomers have is the telescope. First used as an instrument of war for spying on enemies, Galileo Galilei constructed one from scratch and used it to observe the sky, revolutionizing astronomy as a science. Telescopes take rays of light and focus them using a lens or mirror, making dim objects appear bright and, to a lesser extent in astronomy, far objects appear nearer. Because nearly all of the information astronomers get about the stars comes in the form of light, telescopes are an extremely important tool. In the same way that advancements in computer technology allow us to access more information in a smaller amount of time, advancements in telescope technology allow astronomers to gather more information about the stars - that is, light - in smaller amounts of time.

1. If you had to design a bucket to collect as much rain as you could in as little time possible, what sort of a bucket would you design?

Just like a bucket collects rain, telescopes collect light. The human eye is the most basic example of a light collector, similar to a telescope. The light passes through your pupil to a lens in your eye, which focuses it to a point, projecting an image onto light-detecting nerves in the back of your eye. When you are in a dark room your pupils dilate, increasing the area of your eye's lens which can collect and focus more light. You can see better in a dim room with dilated pupils because you are able to collect more light in a given amount of time.

The same principle extends to modern telescopes: the greater the area over which light is collected, the dimmer the objects you can see. Just as you are able to see more detail in a dark room as your pupils dilate, larger telescopes are able to see more detail in the sky with larger mirrors and lenses. Thus the light-gathering power of a telescope is determined by the area of the lens or mirror that is focusing the light. So the greater the area of the lens or mirror, the greater the light-gathering power.

If we want to compare the light-gathering power of two detectors we can simply divide the area of one detector by the other to see how many times greater or by what fraction smaller the light-gathering power of one is over the other. Most detectors are circular, so the (light gathering) area is given by:

$$
\begin{equation*}
A=\frac{\pi d^{2}}{4} \tag{11.1}
\end{equation*}
$$

where $d$ is the diameter of the objective lens or mirror.

### 11.1.1 Types of Telescopes

There are two basic types of telescopes. One type uses a lens (like your eye) to gather and focus light rays through the process of refraction; the bending of light as it passes into a denser medium, such as glass. Hence, these are called refractors. The other type uses curved mirrors to bounce and focus light rays through the process of reflection; these are reflectors.

Refractors were the first telescopes built, mainly because the technology for creating lenses had already been developed to make glasses for people with poor vision. However, glass lenses suffer from a major problem: they gradually deform under their own weight. If you have ever seen a very old house (over 100 years old), you may have noticed that the windows are sometimes thicker at the bottom than at the top, making them appear to "sag." The heavier the piece of glass, the faster it will sag. Thus, there is an upper limit to how big a lens can be: any greater and the lens will sag so much it will distort the image it creates. Magnifying lenses also must be thick at the center and thin at the edges, making them susceptible to breakage if they are too large. Research telescopes are precise scientific instruments, so even slight deformations are very detrimental.

Isaac Newton saw a way around this: instead of using lenses to focus light, you could use a mirror. Mirrors have the advantage of being supported on more sides than a refractor, so they do not sag as much or break as easily. As a result, reflectors can be built with objective mirrors that are much larger than the lenses of the largest refractors. Reflectors also have another advantage: they can be made more compact. By positioning the mirrors in clever ways, they can collect the same amount of light as a refractor but take up only a fraction of the space.

In this lab, we'll compare these two basic types of telescopes with each other and with the human eye. The resolving power and magnification of telescopes will also be explored. We'll also look at how light from stars gets disturbed on its travel through space to us, and how this affects what stars looks like.

### 11.2 Comparing Detectors

2. The light-gathering power of a telescope is proportional to the area of its objective mirror. Let's compare the human eye, which has a circular lens about 0.5 cm in diameter, to a telescope with a objective circular lens 50 cm in diameter. By what factor is the light-gathering power of the telescope greater than that of the eye (how many times better)?
3. In general, if you triple the diameter of an objective lens, by what factor will the light-gathering power increase?

### 11.2.1 Refractors vs. Reflectors, and the $f /$

Now let's compare the largest refracting research telescope ever built, the Yerkes Refractor, built in 1897 and operated by the University of Chicago, with a reflecting telescope at Mount Laguna Observatory, operated by SDSU. The Yerkes refractor uses a $\sim 1 \mathrm{~m}$ lens as its objective lens. The primary research telescope at MLO (a medium size reflector) uses a $\sim 1 \mathrm{~m}$ curved mirror as its objective mirror.

Despite having the same diameters, these two telescopes have drastically different $f$-numbers, or $f /$ values. This number is the ratio between the focal length of the telescope, or the distance between the lens or mirror and the focal point, and the diameter of the objective lens or mirror:

$$
\begin{equation*}
f \text {-number }=\frac{\text { focal length }}{\text { objective diameter }} \tag{11.2}
\end{equation*}
$$

The Yerkes refractor has an $f$-number of 19 (typically written as $f / 19$ ), which means the focal length of the lens is about 19 times its diameter. The MLO reflector is $f / 3$, so the focal length of the mirror is about 3 times its diameter.

- On the last page of the lab, complete the schematic for both telescopes by drawing how parallel light rays from points "a" and "b" are focused through a point by the lens or mirror (remember, mirrors reflect light while lenses allow light to pass through them). This is a scale diagram, so be sure to measure accurately!
- For the refractor, find and label the focal length. Mark the focal point with a capital "F." Continue drawing the light rays "a" and "b" a little past the focal point and label where "a" and "b" end up on the image.

4. Should the light rays be in the same order (from left to right) or should they be reversed?

- For the reflector, also find and label the focal length and focal point. Draw the reflected light rays "a" and "b" past the focal point.

5. Do one of these telescopes have a better light-gathering power than the other? Why or why not? (It may help to review questions 2 and 3.)
6. Looking at your scale drawings, which type of telescope, refracting or reflecting, seems more appropriate and cost effective to build? Explain briefly why no large refracting telescopes were built in the twentieth century or are planned to be built.

### 11.3 Resolving Power

In addition to collecting more light from objects and making them look brighter, telescopes also resolve distant objects better than the eye. This means they have the ability to distinguish two objects from one another at a greater distance than the eye can, or that telescopes make objects look sharper, with more well defined edges.

The minimum resolvable angle, $\theta_{\min }$, is a measure of the resolving power of a telescope, and is described by the following equation:

$$
\begin{equation*}
\theta_{\min }=\frac{1.22 \times 206265^{\prime \prime} \times \lambda}{d}, \tag{11.3}
\end{equation*}
$$

where 1.22 is unitless factor and 206265 is in units of arcseconds $(")$. The smaller $\theta_{\min }$ is, the better the resolving power. It is dependent on the diameter of the objective lens or mirror, and on the wavelength $(\lambda)$ of the light coming from the object. Note that if $\lambda$ and $d$ have the same units, then $\theta_{\min }$ will be in units of arcseconds.
7. What is the minimum resolvable angle of the human eye ( $d=0.5 \mathrm{~cm}$, and $\lambda=550 \mathrm{~nm}$ for visible light) in arcseconds? (Recall that $1 \mathrm{~cm}=10^{7} \mathrm{~nm}$.)
8. Now let's consider visible light coming from the headlights of a far away car at nighttime. We'll assume that the car's headlights are 1.5 m apart. Draw a diagram of the situation from above (not to scale - just a sketch!). Label the following: the separation of the headlights, your location, $\theta$ (the angle created between one headlight, you, and the other headlight), and the distance to the car, $D$.
9. From the definition of tangent and using the law of small angles, it can be shown that:

$$
\begin{equation*}
D=\frac{206265^{\prime \prime} \times \text { distance between the headlights }}{\theta_{\min }} \tag{11.4}
\end{equation*}
$$

What is the farthest (in meters) from a car you can be at night and still make out the two headlights with one of your eyes? What is this distance in miles? Hint: $1 \mathrm{mi}=1610 \mathrm{~m}$.
10. What is $\theta_{\min }$ for a pair of binoculars with lenses 2.0 cm in diameter for visible light?
11. What is the farthest from a car you can be at night and still make out the two headlights with binoculars? Give your answer in meters and in miles.

### 11.4 Magnification

In both refractor and reflector telescopes, after light is gathered by the large objective lens or mirror, a small lens called an eyepiece allows for visual examination of the image. The combination of objective and eyepiece produces a magnified image of the object.

Magnification is likely the most commonly known property of telescopes. It is the increase in apparent size of far away objects compared to unaided visual observation. However, magnification is actually one of the least important characteristics of telescopes for professional astronomers. It is less important than both light-gathering power and angular resolution. This is because magnification makes all parts of the image appear bigger: the object you are interested in, and any distortions that may occur. If you have poor light-gathering power or resolution, magnification will only make the dim or blurry image bigger.

The way to calculate the magnification is:

$$
\begin{equation*}
\text { Magnification }=\frac{\text { focal length of the objective }}{\text { focal length of the eyepiece }} \tag{11.5}
\end{equation*}
$$

For instance, if the objective of a telescope has a focal length of 100 cm and the eyepiece has a focal length of 0.5 cm , then the magnification power of the telescope is:

$$
\text { Magnification }=\frac{100 \mathrm{~cm}}{0.5 \mathrm{~cm}}=200
$$

Or, as it is usually expressed:

$$
\text { Magnification }=200 \times
$$

While magnification may increase the amount of detail you can see to some extent, there is a limiting magnification for every telescope, after which images simply appear more and more distorted. Magnification, however, is very easy to control because it depends only on the focal lengths of the objective and eyepiece. While it may be difficult to change the focal length of the objective, changing eyepieces is exceedingly easy. Thus, magnification is not usually a primary design concern for telescopes.
12. Quantitatively, what happens to the magnification of a telescope if you:
(a) Double the focal length of the eyepiece?
(b) Double the focal length of the telescope?

### 11.5 The Disturbance of Starlight

As light travels from distant objects to us, it passes though different materials, such as the interstellar medium (consisting of dust and gas) and the Earth's atmosphere. Each time light passes from one transparent material to another, it is bent and distorted. Temperature can affect the density of materials, which will also affect how much light is bent as it passes through. Earth's atmosphere is made up of many layers of air with rapidly changing temperatures and densities. We see the distortion of starlight by the Earth's atmosphere as the "twinkling" of stars. The images of stars seen through a telescope, however, will appear somewhat blurry and less defined due to the distortion of light as it passes through different materials on its way to us.
13. Compare the two images on the last page of the lab. Both are of the same galaxy cluster. The panels on the left are taken from one of the most powerful telescopes on Earth, the 8 m Subaru Telescope in Mauna Kea, Hawaii. The right panels are from the 2.4 m Hubble Space Telescope (HST), orbiting 600 km above Earth. Describe which image looks clearer and more defined and why. Why do you think the HST can outdo the Subaru even though its mirror is almost four times smaller?

### 11.6 Comparing Two Telescopes

On the roof we have a few different types of telescopes. In this section we are going to compare a mini-telescope to one of the many Cassegrain telescopes the astronomy department houses on the roof. The mini-telescope has a 50 mm objective lens with a $f / 10$. The Cassegrain telescope has an 8 inch ( 203 mm ) mirror with a $f / 10$.
14. Without doing any calculations, which telescope will have the better light-gathering power? What about resolution?
15. The standard eyepiece on the mini-telescope gives a magnifications of $25 \times$, is this greater than or less than the magnification you would get if you put an eyepiece with a focal length of 40 mm on the 8 inch Cassegrain telescope? Please show all of your work.

## $\star$



Yerkes 1m
Refractor f/19
Lens

Scale: $.8 \mathrm{~cm}=1 \mathrm{~m}$


MLO 1m Reflector f/3



Image Credit: NASA, Mauro Giavalisco, Lexi Moustakas, Peter Capak, Len Cowie and the GOODS Team.

## Lab 12

# The Hertzsprung-Russell Diagram 

By K. Azalee Bostroem<br>Adapted from a lab by the University of Washington Astronomy Department Revised by Dave Baer and MJ Jeffries

### 12.1 Introduction

The Hertzsprung-Russell (H-R) diagram arose from roughly concurrent work done by two astronomers, Ejnar Hertzsprung and Henry Norris Russell. They used nearby stars with known distances to create a plot of the star's absolute magnitude (how bright the star would be if you were 10 parsecs away from it) and spectral class. This plot can also be made using luminosity and temperature. They found that almost all of the stars fell on a single curve across the diagram. This means that there exists a relationship between absolute magnitude (luminosity) and spectral type (surface temperature)! Theorists soon found a mathematical way to explain some of the trends observed on the H-R diagram. The following equation relates a star's luminosity to its radius and surface temperature.

$$
\begin{equation*}
L=C \times R^{2} \times T^{4} \tag{12.1}
\end{equation*}
$$

where $C$ is a constant, $R$ is the star's radius, and $T$ is the star's temperature. From the equation we can tell, for example, that a star with a greater surface area will be brighter, whereas one with a smaller surface area will be dimmer.

Astronomers use the H-R diagram to find the distances to stars that are too far away to use parallax. The spectral type and apparent magnitude (i.e., the brightness of a star as observed from Earth) can be observed. Using the spectral type and the H-R diagram, the absolute magnitude (i.e., the brightness of a star as if you were 10 parsecs away from it) can be found. Once you have apparent and absolute magnitude, the distance can be found by using the distance modulus equation:

$$
\begin{equation*}
d=10^{(m-M+5) / 5} \tag{12.2}
\end{equation*}
$$

where $d$ is the distance in parsecs, $m$ is the apparent magnitude, and $M$ is the absolute magnitude of the star. Note: the $(m-M+5) / 5$ part of the equation is an exponent!

The H-R diagram also provides information on the evolution of stars. H-R diagrams of star clusters of different ages will not look the same. Most of a star's life is spent burning hydrogen to helium via fusion in its core. During this part of a star's life, it can be found on the main sequence on a H-R diagram.

The more massive the star, the faster the fusion process. This means the most massive stars burn through their fuel the fastest, so they are the first ones to move off the main sequence. Very low-mass stars are still on the main sequence since they fuse hydrogen very slowly. In sum, high-mass stars are short-lived (a few million years), large in size, and high in luminosity, while low-mass stars are long-lived (billions of years), small in size, and low in luminosity.

After leaving the main sequence, a star begins to fuse heavier elements in its core, first fusing helium to produce carbon or oxygen. At this stage of a star's life, the star gets larger in size after using up the hydrogen in their core. These larger stars are called giants and supergiants. During the giant and supergiant phase, a star is still actively fusing elements in its core at a faster and faster pace since the temperature in the core is rising. Stars like our sun (relatively low-mass) wind up being small and very dense objects called white dwarfs. More massive stars go out with a bang exploding as a supernova and eventually forming a neutron star or black hole.

### 12.2 Procedure

### 12.2.1 Understanding the Diagram

Your first task is to understand the H-R diagram shown in Figure 1. There are roughly 200 stars plotted. The y-axis represents the absolute magnitude (or luminosity). Notice that the magnitude scale is backwards - that is, bright stars have smaller magnitudes than dim stars (i.e, a star with a magnitude of -5 is brighter than a star with a magnitude of 5). On this plot, the y-axis has been inverted so the dim stars are on the bottom and the bright stars are on the top.

The x-axis represents the spectral type. Spectral type is another way of expressing temperature. In order from hottest to coolest, the spectral types are: O B A F G K M. Each spectral type is further subdivided into numbers ranging from $0-9$ with 0 being the hottest. For example, in order from hottest to coolest, the spectral types can be written as: $\mathrm{O} 0-\mathrm{O} 9, \mathrm{~B} 0-\mathrm{B} 9, \mathrm{~A} 0-\mathrm{A} 9$, etc. The main sequence is a curve through the $\mathrm{H}-\mathrm{R}$ diagram on which a star spends almost all of its life.


Figure 1. - The Hertzsprung-Russell Diagram for roughly 425 stars. The data for the three categories are as follow: (1) the brightest stars as viewed from Earth came from http://www.astro.utoronto.ca/~garrison/oh.html, (2) the nearest stars to the Sun came from http://www.astro.wisc.edu/~dolan/constellations/extra/nearest.html, and (3) the white dwarfs came from Holberg, J.B., Oswalt, T.D., and Sion, E.M., 2002, ApJ, 571, 512-518.

1. It took the Sun about $3.0 \times 10^{7}$ years to evolve onto the main sequence. It will then spend a total of about $1.0 \times 10^{10}$ years on the main sequence, and then take $3.0 \times 10^{7}$ years to evolve off the main sequence and die. Given this information, what percent of its life will the Sun spend on the main sequence?
2. Once on the main sequence, stars generally do not move along it. This means that for most of its life, a star has a constant spectral type and absolute magnitude. Since stars spend most of their lives on the main sequence, this is the most populated area on the plot.

- Draw a smooth curve from the stars in the upper left corner to the stars in the bottom right corner in Figure 1. This is the main sequence.
- Describe how the characteristics of the stars change along the main sequence, as you progress from the upper left to the lower right on the H-R diagram (e.g., use words like hot, cold, bright, dim, etc.).

3. You will notice there are a few stars above the main sequence. These stars are in the process of dying and are called Giants or Supergiants. Notice that there are some $M$ type stars that are as bright as O type stars, even though M stars are much cooler than O stars. How is this possible? Consider what factors determine the luminosity of a star.
4. In addition, there are few stars below the main sequence. These stars have spectral types of $B, A$, and $F$ but have luminosities far lower than stars of similar spectral types on the main sequence. How is this possible? What are these objects called?

### 12.2.2 Using the H-R Diagram to Find Distance

Imagine you have just finished reducing data from an observing run. You have found the apparent magnitude and the spectral type of six main-sequence stars, as shown in the table below.

Use your main sequence best fit curve from earlier, and the spectral type of each star, to find each star's absolute magnitude. Then find the distance $D$ to each star (don't forget units) and record your results in the table.

| Star | Spectral type | $m_{\mathrm{v}}$ | $M_{\mathrm{v}}$ | $d$ (pc) |
| :--- | :---: | :---: | :---: | :---: |
| $\beta$ Cygni | B8 | 5.11 |  |  |
| $\xi$ Tauri | A7 | 4.22 |  |  |
| 61 Virginis | G6 | 4.75 |  |  |
| 22 Tauri | B9 | 5.75 |  |  |
| Ross 128 | M4 | 11.16 |  |  |
| 14 Herculis | K0 | 6.67 |  |  |

Using graph paper, make two plots with the same axis scale as in Figure 1.

- A plot of the apparent magnitude versus spectral type for all the stars listed in the table above.
- A plot of the absolute magnitude versus spectral type.

5. Which one of the two graphs you made looks similar to the H-R diagram in Figure 1? Explain why one plot displays a clear main sequence, but the other does not. Hint: Think about what makes absolute and apparent magnitudes different.

### 12.3 Clusters

Plotting an H-R diagram of a cluster of stars is especially useful. This is because the stars all formed at the same time and they are all at approximately the same distance from the Earth. This means that we can see relationships among age, spectral type, and temperature very simply.

There are two types of star clusters: open and globular. An open cluster has a few thousand stars that are loosely bound by gravity. A globular cluster can have up to a million or more stars that are tightly bound by gravity. Open clusters mainly have young, hot stars, while globular clusters have very old, cool stars. Figure 2 shows images of two nearby star clusters, 47 Tucanae ( 47 Tuc - a globular cluster) and the Pleiades (an open cluster).


Figure 2. - Images of two star clusters: 47 Tuc (left) and the Pleiades (right).
The oldest star clusters have actually been used to estimate the age of the universe. For this reason, finding the age of clusters is very important. We can use the H-R diagram of a cluster to find its age in the following way.

Bright, high mass stars burn through their fuel very quickly (astronomically speaking). When a star has finished burning hydrogen, it begins to move off the main sequence and becomes a giant. Therefore O stars are the first to leave, then B stars, etc.

- If you know the lifetime of the last star to leave the main sequence, then you know that the cluster is at least as old as that star.

The location of the hottest star still on the main sequence is called the cluster's "turn-off point." Take a look at the H-R diagrams of 47 Tuc (Figure 3) and the Pleiades (Figure 4) on the next page.


Figure 3. - (Top panel) The Hertzsprung-Russell Diagram for the stars in 47 Tuc. (Bottom panel) The Hertzsprung-Russell Diagram for the stars in the Pleiades.

- Sketch in the main sequence on the H-R diagram of each cluster. Remember that the main sequence consists of stars that are still burning hydrogen, and have NOT yet become giants. (Don't extend the main sequence beyond the turn-off point.) Label the turn-off point in both diagrams.

6. Why don't we see O-type stars on these diagrams?
7. Using the theoretical age predictions below and the turn-off points you found earlier, estimate the age of each cluster based on the following table. Explain how you arrived at your answer.

| Spectral Type | Main Sequence Lifetime (years) |
| :---: | :---: |
| O | $<10^{6}$ |
| B | $3 \times 10^{7}$ |
| A | $4 \times 10^{8}$ |
| F | $4 \times 10^{9}$ |
| G | $1 \times 10^{10}$ |
| K | $6 \times 10^{10}$ |
| M | $>10^{11}$ |

8. Notice that the cluster plots use apparent magnitude whereas the main H-R diagram, Figure 1, uses absolute magnitude. Why is it allowable to plot the apparent, rather than the absolute, magnitude for cluster stars? (Remember that this did not work for the stars back in Question 5!)

## Lab 13

## The Prism Spectrometer

### 13.1 Introduction

Astronomy is unique among the sciences because astronomers need to understand the largest of structures in the universe while the only data available comes from something that is still very mysterious: light.

Before Isaac Newton's famous experiment with them, prisms were thought to be magical because they produced a rainbow when exposed at certain angles to pure white light. Isaac Newton tried something different: he put two prisms together to start and then reversed the process, showing the world that it must be a property intrinsic to white light, not a magical prism, which produced a rainbow. White light is actually composed of all the colors of the rainbow. The colors that the human eye can see comprise the visible spectrum, but there are many other types of light that our eyes cannot sense. The combination of all wavelengths of light is known as the electromagnetic spectrum. By breaking white light up into its component colors, astronomers are able to extract a great deal of information about the light source, such as temperature and chemical composition.

In 1859, Robert Bunsen and Gustav Kirchoff found that each element they vaporized emitted light at discrete wavelengths, or colors. These sorts of sequences are called spectral lines. Kirchoff developed three laws to describe this new science of spectroscopy.

1. At a high enough pressure, any material will emit photons at all wavelengths - a continuous spectrum.
2. Hot diffuse gas will emit photons at specific energies (or wavelengths) depending on the elemental composition of the gas. The resulting spectrum is known as an emission spectrum.
3. If a cool diffuse gas is placed in front of a continuous spectrum source, an absorption spectrum will be produced. For example, the surface layer of the Sun (the photosphere) creates this effect from the continuous spectrum emitted from the solar interior.

In this lab, you will be making use of the second law. After measuring how your prism disperses light, you will be able to discover the visible wavelengths emitted by the element mercury.

### 13.1.1 The Prism Spectrometer

Light enters the spectrometer through the narrow slit, S , and rays of light are rendered parallel by the collimator, $\mathrm{L}_{1}$. The light then travels to a glass prism, P. The Prism bends light at a different angle depending on the wavelength or color of the light. The viewing telescope, T , focuses the light through the eyepiece, E , and is mounted on a swing arm which allows the amount of deviation to be measured for each individual line.


Figure 1. - Diagram of the Prism Spectrometer.

### 13.2 Procedure

- DO NOT touch the prism during the entire lab - if you do, it will drastically throw off your results.

With the power supply SWITCHED OFF and UNPLUGGED, place the helium tube firmly in the brackets. Plug in the power supply and switch it on.

- CAUTION: the power supply produces 5,000 Volts. DO NOT TOUCH ANY BARE METAL PARTS.

Maneuver the power supply (you may safely grip the base) so the helium tube is directly in front of the slit S , as close as possible. The telescope should be set so that bright colored lines are visible. If not, make sure it is positioned around $131^{\circ}$. You may also try to reposition the tube in front of the slit until the lines appear at maximum brightness. If you still can't see any lines, ask your instructor for help.

Loosen the screw below the prism and swing the telescope to the red line (B on the map) and lock it in place. Focus the eyepiece by gently pushing it in or pulling it out (use a twisting motion) and center the crosshairs on the line. You are now ready to start measuring the position of the spectral lines. If you cannot get both the crosshairs and the line in focus together, ask your instructor for assistance - further adjustments will be required before a good focus can be achieved. When you look at the lines in the deep violet, or in the red, the crosshairs will remain in sharp focus, but the spectral lines will be slightly out of focus.

### 13.2.1 The Helium Spectrum

The purpose of this lab is to observe the element mercury. So why do we first start with helium? Think of it like this: we want to measure the length of a table with a meterstick. Unfortunately, our meterstick has evenly spaced markings on it that do not correspond to centimeters or inches in any predictable fashion. So before we can measure the table, we first have to calibrate our meterstick with something we know is correct - a reference. To complete the analogy: the table length is the element mercury, the meterstick is the prism spectrometer, and the reference is helium.

Your first task is to calibrate your spectrometer to your prism (wavelengths in angstroms, or $\AA$, where $1 \AA=10^{-10} \mathrm{~m}$ ). Note that lines E and F are very close together (often seen as just one line).


Figure 2. - Spectral emission lines of helium.

| Line | $\lambda(\AA)$ |
| :---: | :---: |
| B | 7065 |
| C | 6678 |
| D | 5876 |
| E | 5048 |
| F | 5016 |
| G | 4921 |
| H | 4713 |
| K | 4471 |
| L | 4438 |
| M | 4388 |
| P | 4121 |

### 13.2.2 Measuring the Spectrum

Start with the extreme red end of the spectrum. Center the crosshair carefully on the line, and take the reading (R) of the telescope setting. Using the Vernier scale, you should be able to measure the setting to within one arcminute (recall $1^{\circ}=60^{\prime}$ ). Your instructor will explain the use of the Vernier. Enter your data in the data table provided at the end of this lab. You should measure at least eight lines.

## The Vernier Scale

Your instructor will provide a more complete explanation of the Vernier in class, but this is the basic idea. A Vernier scale is a small, movable scale placed next to the main scale of a measuring instrument. In this particular case, it allows you to make measurements to within $1^{\prime}$ (a small fraction of the smallest division on the main scale of the instrument). When you take a reading, you will see something like that shown in Figure 3.


Figure 3. - Example showing how to read the Vernier scale. The value displayed here is $131^{\circ} 13^{\prime}=131.217^{\circ}$.

Once you have the crosshair centered on a spectral line:

- First, locate a 0.5 degree window in which the small " 0 " falls. In the example in Figure 3, it is clearly somewhere between 131 and 131.5 degrees. In fact, you can narrow it down to somewhere between 131.2 and 131.3 degrees.
- Next, find where the Vernier scale tickmarks and the main scale tickmarks line up. In the example, it appears to be at 13 arcminutes. If two Vernier tickmarks are equally well lined up, choose the halfway point (e.g. if you thought 12 and 13 both looked good, you would record 12.5).
- Convert your Vernier scale reading into degrees and add it to the main scale value to arrive at your reading, R. The example result is $131^{\circ}+\left(\frac{13}{60}\right)^{\circ}=131.217^{\circ}$.

After you have obtained your reading for the "B" line of helium, proceed and measure the position of each of the lines in the helium wavelength table. Lines from red to blue should be easy to see and measure. When you get to the extreme violet, you may find it difficult to see the lines at all, much less measure their positions. You will have to be resourceful at screening out extraneous light in order to make the background as dark as possible. When you achieve this, however, it will be difficult to see the crosshairs.

Each member of your group should have a turn at measuring the positions of some lines, and be sure to check each other's measurements for accuracy.

- In the comments section, describe the color of each line and whether the line is relatively weak or strong.


### 13.2.3 The Calibration Curve

Once you have finished your observations, switch off and unplug the power supply.

- Using a piece of graph paper, plot the wavelength versus reading angle R for each line. Hint: Your axes don't need to start at zero! Choose your scale so that your calibration curve will fill the page. Look at the largest and smallest values you have for wavelength and for R.
- Next, draw a best-fit smooth curve through the data points - do NOT simply connect the dots. This is the calibration curve for your spectrometer. It is a graphical representation of the amount a given wavelength of light is bent by your prism. This curve is specific to your instrument, but is valid for any light source since every color bends at specific angles as mentioned before.


### 13.2.4 Identifying Spectra

In this last part of the lab, you will use the graph you just made for helium to identify the wavelengths of light emitted by mercury. Make sure the He tube is cool, then remove it from the clamps and place it in its cardboard container. Place the Hg (mercury) tube in the clamps, plug in the power supply, and switch it on.

Before measuring the reading angle, make sure you see more than three lines. Measure the reading angle of the lines shown in Figure 4.
$\rightarrow$ Note: The A/B and K/L lines can be measured as one line, and the G and $\mathrm{K} / \mathrm{L}$ lines may be faint!
Tabulate all of your results including comments as before. Using your helium calibration curve and the reading angle you just found, find the wavelengths of the Hg lines, i.e. find what wavelength corresponds to those found angles. Note: You are NOT creating a second graph using mercury - you are using the graph of helium to identify the wavelengths in mercury!


Figure 4. - Spectral emission lines of mercury.

## Check Your Results

Of course, in reality we already know the wavelengths of light emitted by mercury. You should compare your findings with the true values.

1. The most prominent mercury lines (H, F, E) have the following wavelengths: $4358 \AA, 5461 \AA$, and $5775 \AA$. How close did you come to finding them? You are encouraged to calculate the fractional error to quantify how close you are (see Lab 2 or Lab 4 for a refresher). Discuss the sources of error that led to any discrepancies.

| Helium Spectrum |  |  |  |
| :--- | :--- | :--- | :--- |
| Line | Wavelength ( $\AA$ ) | Reading R $\left(^{\circ}\right)$ | Comments on line appearance |
|  |  |  |  |
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| Mercury Spectrum |  |  |  |
| :--- | :--- | :--- | :--- |
| Line | Reading R $\left(^{\circ}\right)$ | Comments on line appearance | Wavelength (̊̊) |
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## Lab 14

## Distance Ladder

Adapted and modified by K. Azalee Bostroem Revised by David Krogsrud, Jay Franck, Haley Sharp and Hannah Rotter

### 14.1 Introduction

Determining the distance to objects in the night sky is a central concern for astronomers. The shape of our Galaxy, the knowledge of the existence of other galaxies, and much of cosmology rests on knowing the distance to celestial objects. How can we know the distance to something in space? Parallax worked well in Lab 6 when we found distances to our closest stellar neighbors. However, we also learned that parallax cannot be used to measure extragalactic distances, because more distant objects have smaller parallax angles. This limitation means that we need a new method for determining distance. Instead of parallax, which requires the measurement of very small angles, perhaps we could find something that is easier to measure...

Some stars vary in how much light they emit over relatively short periods of time. To understand how this is possible, let's first consider a stable, non-varying star:

- A balance exists in all stars, which keeps them from either collapsing or exploding. The two main forces are gravity, which pulls mass inward toward a common center, and pressure exerted over some area, which pushes the stellar material outward. This pressure comes from the energy being generated by the process of fusion in the center of the star. For most stars, the inward force due to gravity and the outward force due to pressure are exactly equal, meaning the star is stable. This is called hydrostatic equilibrium.

If there is not enough outward pressure to support the outer layers of a star, the inward force due to gravity will be stronger than the outward force due to pressure. As a result of this imbalance, the star will begin to collapse which causes the star to heat up. This collapse will occur until the outward pressure has built up enough to stop it from collapsing any further. At this point, the star will be so hot that it will need to expand in order to cool back down to its original temperature. If it continues to expand until there is not enough pressure to support the surface, it will begin to collapse again and the whole cycle will repeat itself. This process causes the star to pulsate, or let out more or less light over some time period due to its changing surface area and temperature.

Astronomers have known about variable stars like these for some time, but it was not until 1912 when Henrietta Leavitt at the Harvard College Observatory found a relationship between the period of the pulses and the absolute brightness (i.e., the luminosity) of the star. A star that follows this relationship is known as a "Cepheid Variable Star," or simply a "Cepheid," after the first of its kind which was found in the constellation Cepheus.

Leavitt was not a typical astronomer. In fact, she was a part of a group of women called "computers" who compiled and analyzed astronomical data. She did the tedious work of looking at hundreds of photographic plates to see if the size of a star on the plate changed over a period of time, which would mean the star's brightness was changing. This led her to realize something important:

- The time it takes a Cepheid Variable Star to change its luminosity is related to its average luminosity.

Herein lies the key to determining the distance to Cepheid stars. Recalling the inverse square law of light (i.e., that further objects appear dimmer), and knowing both an object's apparent and absolute brightness allows one to determine
the distance to that object. When dealing with light, we often work in the magnitude system. Thus, apparent brightness is related to apparent magnitude $\left(m_{\mathrm{V}}\right)$ and absolute brightness or luminosity is related to absolute magnitude $\left(M_{\mathrm{V}}\right)$ (In both cases, the V indicates that the brightness is measured in the visible part of the spectrum). This method of determining distance is known as the Cepheid period-luminosity relation.

A more recently found method of determining distances is with using a special type of star explosion called a "Type 1a Supernova" or also written as "SN 1a". These explosions reach a peak brightness very quickly and then slowly get dimmer over a few weeks. It is not quite known what kind of stars make these explosions, but what is important for finding distances is how bright they become, many even outshine their host galaxy! Since they are so bright they can be seen at extremely large distances.

In this lab you will be determining the distances to a Cepheid Variable and also a Type 1a Supernova. You should also be able to determine the maximum distances each of these can be seen with current equipment and understand why these are important tools in astronomy.

### 14.2 Procedure

### 14.2.1 Determining the Period-Luminosity Relation

Using the data provided in Figure 1 we can determine the Cepheid period-luminosity relation. Figure 1 shows the logarithm of the period $(\log P)$ of many different Cepheid stars versus their absolute magnitude $\left(M_{\mathrm{V}}\right)$. The line shown is a fit to the individual data points and the equation at the top of the graph is the equation that describes the line.

1. In the equation, $x$ and $y$ are related to measurable quantities. What are these measurable quantities?

### 14.2.2 Finding the Period of a Cepheid, Y Sagittarii

Figure 2 shows the observed brightness over time of one Cepheid star, called Y Sagittarii. Even at first glance, Figure 2 shows that this star not only changes in brightness over time, but that it does so in a repeatable manner.
2. Find the period of Y Sagittarii. Remember, the period is the amount of time it takes to complete one cycle (i.e. for the star to go from brightest to dimmest to brightest again).

### 14.2.3 Finding the Distances to Y Sagittarii and SN2014J

3. Using your calculator, find the logarithm of the period you just found for Y Sagittarii. If you need a quick refresher course on logarithms, refer to Appendix C in your lab manual.
4. Go back to Figure 1 and use the given equation that describes the trend line and the $\log P$ you just calculated to find the absolute magnitude of Y Sagittarii.
5. Go back to Figure 2. Find the minimum and maximum values of the apparent magnitude of Y Sagittarii and use these values to find the average apparent magnitude.
6. Now use the absolute and apparent magnitudes together with the following equation to find the distance to Y Sagittarii. Remember that in equations involving magnitudes, distance is measured in parsecs (pc).

$$
\begin{equation*}
\left(m_{\mathrm{V}}-M_{\mathrm{V}}\right)=5(\log D)-5, \text { where } D \text { is the distance. } \tag{14.1}
\end{equation*}
$$

solving for D , gives the following

$$
\begin{equation*}
D=10^{\left(m_{\mathrm{V}}-M_{\mathrm{V}}+5\right) / 5} \tag{14.2}
\end{equation*}
$$

7. We can use the same equation above to find the distance to the SN 1 a in Figure 3. The apparent magnitude ( $m_{\mathrm{V}}$ ) of SN2014J was measured to be 10.5. The average absolute magnitude ( $M_{\mathrm{V}}$ ) for a SN 1a is -19.3 . Calculate the distance to SN2014J.

### 14.2.4 Analysis

8. Now consider the possibility that the absolute magnitude you used in question six was incorrect for some reason. Calculate the distance to Y Sagitarii again, once using $M_{\mathrm{V}}$ increased by one, and once using $M_{\mathrm{V}}$ decreased by one. This should give you an idea of how the distance changes with $M_{\mathrm{V}}$.
9. Compare these new distances with the original distance you found to Y Sagittarii. How does the change in distance compare to the actual distance? How significant is this change? Why must astronomers be concerned that the Cepheid period-luminosity relation is calibrated correctly?
10. Interstellar dust dims the light from stars. Cepheids are usually found in the disk of our galaxy where the most dust is. What effect might this have on your determination of the distance to Y Sagittarii? In other words, if the light from the star is dimmed, how will the observed apparent magnitude and your calculated distance change?
11. The faintest apparent visual magnitude of a Cepheid star to which we can apply the Cepheid period-luminosity relation is about +24 . If the brightest Cepheid has an absolute visual magnitude of -6 , calculate the greatest possible distance of any Cepheid we can get reliable measurements for. That is, how far away can we place this Cepheid before it becomes too dim for us to see? Express your answer in megaparsecs. ( $1 \mathrm{Mpc}=10^{6} \mathrm{pc}$ )
12. What is the farthest possible distance we can detect a Supernova 1a if the average absolute magnitude is -19.3 and the faintest we can detect them is +24 ? Express your answer in megaparsecs.
13. Consider the distances you found in Questions 11 and 12 versus the maximum distance currently measurable by parallax (which we found in Lab 6 to be about 42 kpc , or $42,000 \mathrm{pc}$ ).
How many times further can you measure with the Cepheid distance scale than with parallax?
How many times further can you measure with SN 1a distance scale than with the Cepheid distance scale?
14. Our galaxy is about 30 kpc in diameter. Our local group of galaxies is about 3 Mpc in diameter. Is the Cepheid period-luminosity relation useful for measuring distances to the following objects: (A) Another star in the Milky Way? (B) A neighboring galaxy located within the local group? (C) A galaxy outside the local group? For each object, briefly explain your answer.
15. If you wanted to determine the distance to $\alpha$ Centauri, which method (parallax or the Cepheid period-luminosity relation) would you use and why? Why couldn't you use the other? Hint: $\alpha$ Centauri is the closest star to the Sun and is not variable.
16. Think about how the Cepheid period luminosity relationship was found in the first place. Leavitt would have had to know what the absolute magnitudes of the Cepheid Variables were before hand in order to determine that there was a relationship. Explain how she would have known the absolute magnitudes of these objects. (Hint: Use knowledge from past labs to help answer this question.)


Figure 1. - Plot of absolute magnitude vs. the logarithm of the period in days for a collection of Cepheid stars in the galaxy NGC 1637.


Figure 2. - Plot of apparent magnitude vs. time of observation in days for the Cepheid star Y Sagitarii.


Figure 3. - On the left is an image of the galaxy before the supernova went off and on the right is an image of after the supernova (called SN2014J) appeared.

## Lab 15

## The Moon

By Shimonee Kadakia<br>Revised by Gideon Bass and Lauren Brewer

### 15.1 Phases of the Moon

### 15.1.1 Introduction

Aside from the Sun, the Moon is the brightest and most noticeable object in our sky. It does not give off its own light, but instead reflects light from the Sun. It orbits the Earth about once every 27.33 days and is about 4.6 billion years old. The Moon is much less massive and smaller than the Earth. Its radius is only about $27 \%$ that of Earth's. Since it's smaller and less massive, the gravitational force on the Moon is only $1 / 6$ that of Earth, therefore if you were to go to the Moon you would weigh much less - that's a fast diet!

Since the Moon is Earth's constant companion, many cultures have used the Moon as a basis for their calendar. They noticed the different phases the Moon takes throughout the month and the different times the Moon is up in the sky. It is well known today that the Sun rises in the East in the morning and sets in the West in the evening. However, not many people talk about the Moon rising and setting.

As it turns out, you can go to the beach on certain nights and see a beautiful moonset over the ocean. Yes, the Moon rises and sets, but not at the same time every day - it varies throughout the lunar month. In fact, you can figure out what time it is just by looking at the Moon.

In the first part of this lab you will learn about the different phases of our Moon and figure out when different Moon phases rise and set. In the second part, you will see the role the Moon and Sun play during eclipses.

### 15.1.2 Procedure

Using Figure 1, do two things:

- Shade the part of the Moon that is NOT illuminated by the Sun.
- Outline the part that an observer on Earth WILL be able to see.

The center circle represents Earth, while the eight other circles are the Moon with the name of the phase next to each one. The line which passes through each Moon separates the light side from the dark side. As you can see, half of the Moon is always lit by the Sun.

The dashed line is the "boundary layer" used to determine what can or cannot be seen by the observer. Even though half the Moon is always lit up, that same portion is not always facing us. The Sun is located to the east and stays in that position for all the phases. One of the Moons has been done for you. By looking at what is outlined, it can be seen that an observer on Earth would be able to see just one quarter of the Moon, hence the phase name "Third Quarter."

Once you have completed the shading and outlining, the next step is to determine at what time that particular phase will be at the highest point in the sky. In order to do this, we will use the fact that the Third Quarter phase is at the highest point at 6 AM .

## First Quarter



Figure 1. - Diagram of the phases of the Moon.

- Label the Third Quarter phase to be highest in the sky at 6 AM on the line provided in Figure 1.
- Going in a counter-clockwise direction, add three hours to each previous time to calculate the other phases' highest points in the sky.

For example: The next phase would be the Waning Crescent and the time it would be highest would be $6 \mathrm{AM}+$ 3 hours $=9 \mathrm{AM}$.

Once you have done this, you should think about the fact that the Moon is only visible for 12 hours on any given day. Therefore, if the Moon is at its highest point, then it must have risen six hours earlier and will set 6 hours later. So for the Third Quarter, it will set at $6 \mathrm{AM}+6$ hours $=12$ noon, and it rose at $6 \mathrm{AM}-6$ hours $=12$ midnight.

Using your completed figure and newfound knowledge, you can answer the following questions.

### 15.1.3 Critical Thinking

1. At what time will the Third Quarter Moon be at its highest point in the sky?
2. At what time will the Full Moon set?
3. At what time will the Waxing Gibbous rise?
4. If you saw the Moon rising at 9 AM , what phase must it be in?
5. Say you were lost in the woods and you look into the sky at 1 AM and don't see the Moon at all. What phase(s) could the Moon have taken that day?
6. If the Moon sets at midnight, what phase was it in?
7. If you were on the Moon and your friend was on Earth, and you saw the Earth to be in the Waxing Crescent phase, what phase would your friend see you in?
8. Although the Moon rotates on its axis, observers from Earth only see one side of the Moon. How do you think it is possible to see just one side of the Moon even though it rotates?

### 15.2 Solar and Lunar Eclipses

### 15.2.1 Overview

The phases of the Moon are observations you can make every day (provided it is not cloudy!), but much rarer are eclipses.

There are two types of eclipses: lunar and solar. To have a lunar eclipse, the Earth needs to lie directly between the Sun and the Moon, so that Earth's shadow falls on the Moon. This is shown in Figure 2. For a solar eclipse, the Moon lies directly between the Earth and Sun, so that the Moon's shadow blocks the Sun. This is shown in Figure 3.

You may think that the alignment in Figure 2 (a lunar eclipse) should occur every month, but it doesn't. This is because the Moon's orbit is slightly inclined, by about $5^{\circ}$ to the ecliptic plane.


Figure 2.-Schematic of a lunar eclipse.

A solar eclipse is much rarer than a lunar eclipse. Solar eclipses are not only a beautiful sight, but they have greatly helped the scientific community. This is because during a solar eclipse, the Sun's brightest regions are blocked, allowing observers to see the diffuse, dim corona. The corona is very hot gas around the Sun which cannot be seen normally since it is very low density compared to the rest of the Sun.


Figure 3.- Schematic of a solar eclipse.

The angular diameter of an object is a way of discussing how big it appears in the sky. It is dependent both on how big the object is, and how far away it is. A larger objects appears bigger, but a more distant object appears smaller. Thus angular diameter depends both on the object you are looking at, and where you are looking at it from. For very distant objects, the angular diameter can be approximated as

$$
\begin{equation*}
\delta=d / D \tag{15.1}
\end{equation*}
$$

where $\delta$ is the angular diameter in a unit of angle called radians, $d$ is the diameter of the object you are looking at, and $D$ is its distance away from you.
9. Calculate the angular diameter of the Sun and the Moon from Earth. The Sun has a radius of $6.95 \times 10^{5} \mathrm{~km}$, and is $1.49 \times 10^{8} \mathrm{~km}$ away from us. The Moon has a radius of 1737 km , and is $3.72 \times 10^{5} \mathrm{~km}$ away.
10. What do you notice about the angular sizes determined for the Sun and the Moon? What does this mean if you were to see the Sun and Moon appear in the same place on the sky?
11. Calculate the angular diameter of the Earth from the Moon. Recall from Lab 1 that the Earth has a radius of 6300 km.
12. If you were on the Moon, draw a figure similar to Figure 2 to show how you could see the Sun being eclipsed BY the Earth. How would this solar eclipse look different than ones experienced on the Earth?
13. You are on the Moon, and you have a friend on Earth. You are on the side of the Moon facing the Earth, and he is on the side of the Earth facing the Moon. He calls you and tells you he is experiencing a total solar eclipse. What does the Earth look like from your point of view?

### 15.2.2 Critical Thinking

14. Can a person observe a solar eclipse at night? Explain your answer.
15. What phase is the Moon in during a solar eclipse? What about during a lunar eclipse?
16. Why is there not a lunar eclipse every month? Draw a picture to fully explain. Read Section 15.2 .1 for help.

### 15.3 Gravity on the Moon (and other Planets)

### 15.3.1 Overview

In the USA, when someone asks you how much you weigh, what is your response? Besides "none of your business," everyone replies with a number in the unit of pounds. If this class was being taught in Europe, the answer to the same question would be given in the unit of kilograms. What is the difference between these two different answers? If you remember back to your previous science classes you might remember a discussion of weight and mass. As it turns out, pounds are a unit of weight and kilograms are a unit of mass. Mass is how much "stuff" an object has. Weight is the force with which you are pulled downwards due to gravity. Therefore, it depends both on your mass, and on the strength of the gravitational force that is present on the particular celestial body you are standing on, in our case the Earth. If everybody is on the Earth, using weight as a measurement of mass is fine, because everyone is experiencing the same gravitational force. However, if you traveled to different planets, the force would change, even though your own mass does not.

Gravity depends both on the mass of the planet you are standing on and how much space it takes up. More massive planets have a stronger gravitational force, but if you take two planets with the same amount of mass and make one of them take up more space (have a larger volume), that one will have a smaller gravitational force on visitors. Mathematically, this argument can be seen through setting Newton's Second law (15.2) equal to Newton's Law of Gravitation (15.3), where $g$ is the acceleration of local gravity, $m$ is the smaller mass (you) that is affected by gravity of the larger mass $M_{p}$, and $R_{p}$ corresponds to the radius of the celestial object you are on. Setting (15.2) equal to (15.3) derives the local gravity for a body of a particular mass. Respectively:

$$
\begin{gather*}
F=m g  \tag{15.2}\\
F=G \frac{M_{p} m}{R_{p}^{2}} \tag{15.3}
\end{gather*}
$$

$$
\begin{align*}
m g & =G \frac{M_{p} m}{R_{p}^{2}} \\
g & =G \frac{M_{p}}{R_{p}^{2}} \tag{15.4}
\end{align*}
$$

So, when you answer the question about how much you weigh in the USA, and you give an answer in pounds, you are effectively giving the questioner the value for $F$ in Equation 15.2. Conversely, when answering this question in Europe, you are reporting the value (in kilograms) for $m$ in Equation 15.2. In other words, by answering in kilograms you are answering with a value that is independent of the local gravitational field, whereas pounds are dependent on the gravitational field. Therefore, your weight, in pounds, varies depending on the mass and radius of the celestial object you are standing on. Since we are dealing with the Moon in this lab, this leads to a simple question: What is your weight on the Moon?

- Note: if you do not feel comfortable completing this for your personal weight, you are more than welcome to use a fictional value.

17. In this equation, $F$ has units of pounds, and $m$ is in the unit of kilograms. Using Equation 15.5, solve for your mass on the Earth in kilograms.

$$
\begin{equation*}
F=2.2 \mathrm{lbs} / \mathrm{kg} \times m \tag{15.5}
\end{equation*}
$$

18. Using Equation 15.3, Newton's Law of Gravitation, find an expression that will give you your weight on the moon by plugging in the constants: $M_{\text {Moon }}=7.36 \times 10^{22} \mathrm{~kg}, R_{\text {Moon }}=1,737 \mathrm{~km}$, and $G=6.6726 \times$ $10^{-11} \mathrm{~m}^{3} \mathrm{~kg}^{-1} \mathrm{~s}^{-2}$, the Gravitational constant. (Hint: Make sure all of your units match!)
19. Convert the answer you got in Question 18 from the metric system to U.S. units using the conversion factor: $1 \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-2}=0.2248 \mathrm{lbs}$. How much do you weigh on the Moon?

## Lab 16

## Exoplanets

By Eliot Halley Vrijmoet<br>Partially adapted from NASA’s Planet Hunters Education Guide, April 2016

### 16.1 Introduction

Planets are a hot topic in astronomy these days, especially the study of planets outside the Solar System. These objects are known as extrasolar planets or exoplanets. Studying exoplanets has given us insight into the formation of our own Solar System, while studying our Solar System has given us insight into the more mysterious exoplanets.

How planets form is still very much a subject of debate, but in our Solar System there are actually a few clear trends:

- Most of the Solar System planets orbit in the same plane, with their orbital axes aligned within $10^{\circ}$ of each other. These observations suggest that planets form from a disk spinning around a young star. As matter in molecular clouds coalesces to create a new star, the leftover matter spins in a disk around the star; as they spin around in the disk, particles/grains stick together like snowflakes and grow into larger bodies. If these bodies get big enough, they become planets!
- The composition of the Solar System planets seems to be related to their distances from the Sun: the closest planets to the Sun are all rocky, with iron cores and distinct layers (like an onion) of metal and silicates, while all the planets past a certain distance are solid or gaseous or both. Jupiter and Saturn have extremely large gas layers, while Neptune and Uranus have more heavy elements.

It is useful to keep these Solar System trends in mind when studying exoplanets. While it is clear that most exoplanets form in a disk around their host stars, as our Solar System did, sometimes we find them in unexpected orbits. In particular, we have discovered numerous large gas planets orbiting very near their stars, in zones where we would expect to find rocky planets. One established hypothesis is that these gas giants formed at greater distances and then migrated inwards due to various other influences (such as gravitational forces from other objects nearby).

Prior to the launch of the Kepler space telescope in 2009, the primary method used for detecting extrasolar planets was Doppler spectroscopy. Using this method, an astronomer looks for evidence of a planet tugging on its host star by watching for tiny redshifts or blueshifts in the star's spectra. This method is good at catching massive planets orbiting close to their host stars, but it requires high precision and cannot easily be automated; stars must be examined one-by-one, making the procedure time-consuming for large data sets.

We can also detect planets by directly measuring the light from their host stars. Although most planets are lost in the glare of their host stars, we can measure the decrease in light that occurs when a planet crosses in front of a star. A planet moving across the face of a star is an event known as a planetary transit. Transits are a useful planet-finding method because drops in light levels are easy for a computer to recognize, especially now that we have space telescopes such as Hubble and Kepler that boast high precision. Precision is a huge advantage when detecting planetary transits-an Earth-sized planet crossing a Sun-sized star produces only a $0.0084 \%$ decrease in the star's light.

| HD 209458 |  | HD 209458 b |  |
| :---: | :---: | :---: | :---: |
| Parameter | Value | Parameter | Value |
| Star radius: $R_{\star}$ | $1.15 R_{\odot}$ | Orbital period: $P$ | 3.52 days |
| Star mass: $M_{\star}$ | $1.13 M_{\odot}$ | Orbital eccentricity: $e$ | 0.0 |
| Star temperature: $T_{\star}$ | 6000 K | Orbit inclination: $i$ | $86.6^{\circ}$ |

Table 16.1: Properties of the star HD 209458 and the orbit of its planet HD 209458 b.

The transit method has been one of the most successful planet-detecting methods to date; as of December 2015, many thousands more planets have been detected via transit than via Doppler spectroscopy ${ }^{1}$. Since planets that do transit their host star will usually do so periodically (once every $X$ days), transit detections can be easily automated and executed on a large scale. What's more, the shape of the dip in the star's light can tell us many things about the planet causing that dip-the transit duration tells us the speed of the planet, the transit depth suggests the planet's size relative to the star, and the rate at which the light falls and rises again gives information about the star's structure, among (many) other things.

We can find quite a bit of information about a planet using only the data obtained with these two methods (spectroscopy and transit). In this lab, we will use both methods to analyze light from the star HD 209458 to discover the properties of its planet, HD 209458 b. This planet is notable as the first exoplanet to be observed transiting its star and the first exoplanet known to have an atmosphere.

In the first section of this lab, we find the mass of the planet by looking at the velocity changes of its star. In the second section, we find the planet's radius by analyzing how much light is lost when it transits its star. In the last section, we consider the temperature of the planet and how pleasant it might (or might not) be to visit.

### 16.2 Procedure: Finding the planet mass from spectroscopy

1. Some useful facts about the star HD 209458 and its planet are given in Table 16.1. Take a close look-the star's radius, mass, and temperature are similar to those of our own Sun, but the planet's orbital period is only 3.52 days! How many orbits does the planet complete in one Earth year? $\quad(1$ year $=365.25$ days $)$

Figure 16.1 shows the radial velocity of the star HD 209458 as a function of the fraction of its orbital period, also known as orbital phase. Radial velocity is the part of the velocity along the line of sight between the observer and the star, so it is maximized if the star is moving straight toward us or away from us, and is zero if the star is moving horizontally across the sky (not changing its distance from us at all).

As the star and planet orbit their mutual center of mass, the star will periodically be approaching us and receding from us, so its radial velocity will show a sinusoidal pattern. The height (amplitude) of this curve depends on how much the planet is tugging on the star, which is determined by the distance between the planet and star and how massive the planet is compared to the star. Similarly, the planet's mass also dictates how far the star gets pulled away from the center of mass during orbit, or $a_{\star}$ (the semi-major axis of its orbit). So, if we find $a_{\star}$ and measure the amplitude $K$ of the radial velocity curve, we can ultimately find the planet's mass.

[^3]

Figure 16.1: Radial velocities of the star HD 209458 plotted against orbital phase, i.e., fractions of orbital period. The three different symbols represent data from the three observational programs used: HIRES, CORALIE, and ELODIE.

First, we assume that the star moves in a roughly circular orbit about the system's center of mass. Its orbital semimajor axis $a_{\star}$ is then the radius of this circle, which has a circumference of $2 \pi a_{\star}$. This circumference is the length of the path that the star travels in the course of one orbit.

The time it takes to complete that orbit is the orbital period, $P$. Since we have a distance $\left(2 \pi a_{\star}\right)$ and time $(P)$, we can mathematically find the star's speed $K$ by dividing the distance it travels (the circumference of its circular orbit) by the time it takes to go that distance:

$$
\begin{equation*}
K=\frac{2 \pi a_{\star}}{P} \tag{16.1}
\end{equation*}
$$

2. Rearrange Equation 16.1 so $a_{\star}$ is isolated on one side.
3. Use Figure 16.1 to measure the amplitude $K$ of the star's radial velocity curve. Do this by getting the radial velocity at the curve's maximum and subtracting the value at the average of the curve (indicated by the dotted line). Although each of these values is negative, your overall answer should turn out to be positive!
4. Substitute $K$ and $P$ into your equation from \#2 to get $a_{\star}$. Remember to convert $P$ to seconds first, so it matches the time units used in $K$.

A useful fact about two bodies orbiting each other is that the semi-major axes of their orbits about their mutual center of mass are related through their masses:

$$
\begin{equation*}
a_{\star} M_{\star}=a_{p} m_{p} \tag{16.2}
\end{equation*}
$$

Our goal is to get $m_{p}$, but we can't do that with this equation alone-we know the values of $a_{\star}$ and $M_{\star}$, but we don't know $a_{p}$. Fortunately, there are a couple more useful relationship that can help out:

- Firstly, the maximum separation of the star and planet $(a)$ is related to the semi-major axes of the star and planet's orbits through $a=a_{\star}+a_{p}$.
- Next, as long as the planet is much less massive than the star, the planet-star separation $a$ is related to the orbital period $P$ and star mass $M_{\star}$ through the simple form of Kepler's third law: $a^{3}=P^{2} M_{\star}$ (discussed in the context of Jupiter's moons in Lab 5). Note that this simple form is only valid if you use units of AU for $a$, years for $P$, and solar masses $\left(M_{\odot}\right)$ for $M$ !

So, if we cube both sides of $a=a_{\star}+a_{p} \ldots$

$$
a^{3}=\left(a_{\star}+a_{p}\right)^{3}
$$

and substitute $a^{3}=P^{2} M_{\star}$ on the left-hand side...

$$
P^{2} M_{\star}=\left(a_{\star}+a_{p}\right)^{3}
$$

$\ldots$..we are left with an equation that relates the unknown $a_{p}$ to the known quantities $P, M_{\star}$, and $a_{\star}$. We solve it for $a_{p}$ by taking both sides to the $1 / 3$ power and isolating $a_{p}$ :

$$
\begin{aligned}
\left(P^{2} M_{\star}\right)^{1 / 3} & =\left(\left(a_{\star}+a_{p}\right)^{3}\right)^{1 / 3} \\
\left(P^{2} M_{\star}\right)^{1 / 3} & =a_{\star}+a_{p} \\
\left(P^{2} M_{\star}\right)^{1 / 3}-a_{\star} & =a_{p}
\end{aligned}
$$

Now we can get rid of $a_{p}$ in $a_{\star} M_{\star}=a_{p} m_{p}$ (Equation 16.2) by substituting the above expression, like so:

$$
\begin{aligned}
& a_{\star} M_{\star}=a_{p} m_{p} \\
& a_{\star} M_{\star}=\left[\left(P^{2} M_{\star}\right)^{1 / 3}-a_{\star}\right] m_{p}
\end{aligned}
$$

Divide both sides by the term in brackets to solve for $m_{p}$ :

$$
\begin{equation*}
m_{p}=\frac{a_{\star} M_{\star}}{\left(P^{2} M_{\star}\right)^{1 / 3}-a_{\star}} \tag{16.3}
\end{equation*}
$$

Now we have an equation that gives $m_{p}$ in terms of only things we already know! As you can see, the calculated mass of the planet depends quite a bit on how wide it stretches the star's orbit ( $a_{\star}$ ) and how massive the star is $\left(M_{\star}\right)$.
5. To make it easier to calculate, we will break Equation 16.3 into smaller pieces. First, evaluate $\left(P^{2} M_{\star}\right)^{1 / 3}$ using the star information in Table 16.1.
Important note: Since this expression comes from the simple form of Kepler's third law, use years for $P$ and $M_{\odot}$ (solar masses) for $M_{\star}$, so you can assume that the final result is in AU.
Hint: Do not hesitate to break this calculation into even smaller pieces. For example, you could find $P^{2}$ first, then multiply by $M_{\star}$ to get $P^{2} M_{\star}$, then take the cube root of that result for $\left(P^{2} M_{\star}\right)^{1 / 3}$.
6. Substitute your $a_{\star}$ (from \#4) and the results from \#5 into Equation 16.3 to get for the planet's mass $m_{p}$. Don't forget to convert $a_{\star}$ to AU first using the fact that there are $1.50 \times 10^{8} \mathrm{~km}$ in 1 AU .
7. Finally, to make this planet easier to visualize, convert its mass from $M_{\odot}$ (solar masses) to $M_{\oplus}$ (Earth masses). $1 M_{\odot}=332,949 M_{\oplus}$.
8. How many times more (or less) massive is it than Jupiter? $1 M_{\text {Jupiter }}=318 M_{\oplus}$.

### 16.3 Finding the planet radius from transits

Another useful property about a planet is its radius. The radius can tell us if a massive planet is very dense (lots of mass packed into a physically small planet) or actually just extremely large. The simplest way to infer a planet's radius is to watch the host star as the planet crosses its face (see Figure 16.2), known as a planetary transit. A tiny planet will block very little light during transit, while a larger planet will block more light. So, if we plot the light over time and note how much it falls during transit, we can estimate the size of the planet.

In astronomy, light levels are traditionally measured in units called flux. If the orbit is circular, then the light loss during transit, also called the transit depth $d$, is proportional to the ratio of the roughly circular areas of the two bodies on the sky:

$$
d=\frac{\pi r_{p}^{2}}{\pi R_{\star}^{2}}
$$

Simplifying the above equation gives us a useful relationship between the planet radius $r_{p}$, the radius $R_{\star}$ of the star it is transiting, and depth $d$ of the transit:

$$
\begin{equation*}
d=\frac{r_{p}^{2}}{R_{\star}^{2}} \tag{16.4}
\end{equation*}
$$

9. To make things easier for later problems, rearrange Equation 16.4 so $r_{p}$ is isolated (and not squared!) on one side.

In 2001, Timothy Brown, David Charbonneau, and their team used the Hubble Space Telescope to measure light during the transit of HD 209458 b. The resulting light curve is shown in Figure 16.3. Notice that the $y$-axis is relative flux: they have divided the entire light curve by the flux value at its maximum, so the light values are actually unitless and the maximum light level is 1.00 . This makes it easier to understand at a glance how deep a transit actually is.
10. Use Figure 16.3 to find $d$, the light lost during the transit of HD 209458 b, by subtracting the relative flux at the transit's lowest point from the out-of-transit relative flux. (Don't forget: relative flux is unitless.)
11. Calculate the radius of the planet by substituting the light loss during transit (your answer from \#10) for $d$ in your rearranged equation (from \#9). Refer back to Table 16.1 for the radius of the star HD 209458.


Figure 16.2: Top panel: Diagram (not to scale) of a planet (black circle) transiting a star (white circle).
Bottom panel: Basic diagram (also not to scale) of the drop in light that results when the planet transits the star. The in-transit and out-of-transit regions are labeled, and $d$ is the depth of the transit. This kind of graph of of light (or flux) versus time is also known as a light curve.


Figure 16.3: Transit of HD 209458 b. The dots represent data taken with the Hubble Space Telescope in April and May of 2000. The line is the computed model that best fits the data.
12. If you're like most astronomers, you used units of $R_{\odot}$ (solar radii) for $R_{\star}$, so your planet radius $r_{p}$ should have those same units. Convert $r_{p}$ to something more reasonable: Earth radii, a.k.a. $R_{\oplus}$.
(Don't forget: $1 R_{\odot}=109.12 R_{\oplus}$ )
13. Now that you have both its mass and radius, compute the density (mass per volume) of HD 209438 b in the standard units of $\mathrm{g} / \mathrm{cm}^{3}$. Assume that it is spherical, so its volume is $\frac{4}{3} \pi r_{p}^{3}$.
$1 M_{\oplus}=5.97 \times 10^{27} \mathrm{~g}$ and $1 R_{\oplus}=6.37 \times 10^{8} \mathrm{~cm}$.

Compare your result with the known densities given in Table 16.2, which includes some of some of the Solar System planets as well as a few interesting substances. Which substance or Solar System planet is it most similar to? What do you think HD 209458 b is mostly made of?

| Planet | Density $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ | Substance | Density $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ |
| :---: | :---: | :---: | :---: |
| Mercury | 5.43 | Liquid water | 1.00 |
| Earth | 5.52 | Solid rock (silicates) | 3.00 |
| Mars | 3.93 | Solid iron | 7.86 |
| Jupiter | 1.33 | Solid lead | 11.3 |
| Saturn | 0.687 | Liquid mercury | 13.6 |
| Neptune | 1.64 | Solid gold | 19.3 |

Table 16.2: Average densities of some Solar System planets and various substances (at 1 atm and $0^{\circ} \mathrm{C}$ ). Note that the large mass of the planets creates huge gravitational pressure, which compresses their contents so their density is much greater than those same materials would be at standard temperature and pressure on Earth.

### 16.4 Putting it all together: Temperature and habitability

In most cases, planets do not generate their own heat, so each planet's temperature depends entirely on its host star, the star-planet distance, and how well the planet absorbs and reflects light. A planet will be hotter if its star is hotter, or if it is closer to its star, or if it is better at absorbing starlight (or worse at reflecting starlight).

The equation that seems to best approximate a planet's temperature is

$$
T_{p}=\left(\frac{R_{\star}^{2} T_{\star}^{4}(1-A)}{4 a^{2}}\right)^{1 / 4}
$$

where $a$ is the maximum planet-star separation (semi-major axis of the planet's orbit) and $T_{\star}$ and $R_{\star}$ are the host star's temperature and radius.
$A$ is the albedo of the planet, or how reflective it is. Given as a unitless number, $A=1.0$ means the planet is perfectly reflective and absorbs zero light from its star-a good plan for staying cool, but physically impossible (try plugging in 1 for $A$ above and you'll get 0 Kelvin!). $A=0$ means the planet is perfectly absorptive and absorbs $100 \%$ of the light that hits it, so it gets as hot as possible.

Finding the albedo is no easy task, but astronomers have determined that for planets like HD 209458 b the most likely value is $A=0.52$.

To get the temperature, we will need the planet-star separation $a$. Earlier, we saw that $a=a_{\star}+a_{p}$ and that $a_{\star}+a_{p}=\left(P^{2} M_{\star}\right)^{1 / 3}$. This means that $a=\left(P^{2} M_{\star}\right)^{1 / 3}$. You may recall that we calculated this value earlier in the lab, in problem \#5. For convenience, copy it down again here:

$$
a=
$$

14. Based on the above information, what is the temperature of HD 209458 b? You will need your result from \#5 and the star information in Table 16.1. (Also useful: $1 \mathrm{AU}=214.94 R_{\odot}$ )
15. Convert the Kelvin temperature to Celsius using the fact that $T_{\text {Celsius }}=\left(T_{\text {Kelvin }}-273.15\right)^{\circ} C / K$.
16. To put this all into perspective for our American friends, convert the Celsius temperature to Farenheit:
$T_{\text {Fahrenheit }}=T_{\text {Celsius }} \times \frac{9^{\circ} F}{5^{\circ} \mathrm{C}}+32^{\circ} F$.
17. Considering the temperature you found for this planet and the conclusions you drew about it in \#13, what would be an appropriate name for planets of this type?
18. Finally, look back at all the information you've gathered about this planet. If you had a reliable, fast spaceship and a space suit that provided ample oxygen and protection from the elements, do you think you (or any of your fellow humans) could tolerate a 6-month stay on HD 209458 b? Provide at least two reasons to support your answer.
Hint: Table 16.1 and your answers to problems \#7, \#13, and \#15 or \#16 may be useful here.

Academic Sources for figures used in this lab:
Figure 16.1 is from Mazeh, T., Naef, D., Torres, G., et al. 2000, ApJ, 532, L55.
Figure 16.2 is adapted from Brown, T. M., Charbonneau, D., Gilliand, R.L̃., Noyes, R. W., \& Burrows, A. 2001, ApJ, 552, 699.
Figure 16.3 is from Brown et al. (2001).

## Part III

## Appendices

## Appendix A

## Equation Manipulations

## A. 1 Overview

The key to solving simple algebraic equations with a single unknown (e.g. $x+6=10$ ) is to realize that the equation is an equality. As long as you do the same mathematical operation to both sides, the equation is still an equality! This includes things like adding a constant, subtracting a constant, multiplying by a constant, and dividing by a constant. You can also square both sides, take the square root of both sides, take the logarithm of both sides, or raise both sides to a power.

- Both sides of an equation must always be equal. Whatever you do to one side, do the same to the other.

$$
\begin{aligned}
a & =b \\
a+c & =b+c \\
a c & =b c
\end{aligned}
$$

All of these equations are equivalent (as long as c is not zero in the third one). In these examples, $a, b$, and $c$ are variables or unknowns. They are used in place of a quantity (a number). Any letter - including Greek letters! can be used as a variable. Most commonly, you'll see $x$ for the variable.

Here is a table to summarize how to solve for $a$, how to isolate $a$, or how to get $a$ alone (three ways of saying the same thing):

| Equation | How to Isolate $a$ | Solve the Equation | Solution |
| :---: | :--- | :---: | :---: |
| $a-7=13$ | Add 7 to both sides | $a-7+7=13+7$ | $a=20$ |
| $a+6=25$ | Subtract 6 from both sides | $a+6-6=25-6$ | $a=19$ |
| $12 a=48$ | Divide both sides by 12 | $\frac{12 a}{12}=\frac{48}{12}$ | $a=4$ |
| $\frac{a}{3}=9$ | Multiply both sides by 3 | $\frac{a}{3} \times 3=9 \times 3$ | $a=27$ |
| $a^{2}=144$ | Take both sides to the $1 / 2$ power | $\left(a^{2}\right)^{1 / 2}=(144)^{1 / 2}$ | $a= \pm 12$ |

## A. 2 Fundamental Laws

There are also three fundamental laws to remember when you do algebra:

1. Commutative Laws

- $a+b=b+a$
- $a b=b a$

2. Associative Laws

- $(a+b)+c=a+(b+c)$
- $(a b) c=a(b c)$

3. Distributive Law

- $(a+b) c=a c+b c$

Here is an example that demonstrates some of these laws:

$$
\begin{aligned}
\frac{2 x+3}{4} & =\frac{x+7}{3} \\
(4)(3)\left[\frac{2 x+3}{4}\right] & =(4)(3)\left[\frac{x+7}{3}\right] \\
(3)(2 x+3) & =(4)(x+7) \\
6 x+9 & =4 x+28 \\
x(6-4) & =19 \\
2 x & =19 \\
x & =9.5
\end{aligned}
$$

When you solve equations, you should ALWAYS show your work! That way if you make a mistake, you can track down the source of your error more easily. Showing work also demonstrates to your instructor that you know what you are doing. As a note, it is usually best to do algebraic manipulations first, and plug in numbers later.

## A. 3 Practice

The distance you travel, $d$, is related to the velocity at which you move, $v$, and the amount of time you spend moving, $t$. That is, distance equals velocity times time:

$$
d=v t
$$

1. Solve for $v$.
2. Solve for $t$.
3. How many minutes does it take light (traveling at $3.0 \times 10^{5} \mathrm{~km} / \mathrm{s}$ ) to travel from the Sun to the Earth (a distance of $1.5 \times 10^{8} \mathrm{~km}$ )?
4. Here's a meaningless equation, just to get you used to switching variables around:

$$
\frac{a b}{c}=d e-f
$$

(a) Solve for $a$.
(b) Solve for $c$.
(c) Solve for $e$.
5. Here's another equation, known as Kepler's form of Newton's Third Law. This is the full form of an equation you'll see in one of the laboratory exercises.

$$
4 \pi^{2} a^{3}=G(M+m) P^{2}
$$

(a) Solve for $a$.
(b) Solve for $P$.
(c) Solve for $\frac{4 \pi^{2}}{G(M+m)}$.
6. Solve this equation for $x$ :

$$
\frac{7 x+2}{4}=4
$$

## Appendix B

## Exponential Functions

## B. 1 Overview

Exponential functions have a variety of uses in everyday life. Human population growth or decline, investment strategies, and earthquake magnitudes are some of the many concepts modeled with exponential functions. In astronomy, the flux of a star's light is related to the star's temperature through exponential functions. There are many uses for exponential functions in astronomy as well in your everyday life. Here's a quick explanation of how to make exponential functions tell you something meaningful.

Exponential functions are written as

$$
f(x)=b^{x} \quad \text { or } \quad y=b^{x}
$$

where $b$ is the base and $x$ is the exponent.
For example, in the equation $z=10^{y+3},(y+3)$ is the exponent and 10 is the base.
Let's find $z$ when $y=-1$ :

$$
\begin{aligned}
& z(y=-1)=10^{-1+3} \\
& z(y=-1)=10^{2} \\
& z(y=-1)=100
\end{aligned}
$$

Here are some useful properties of exponential functions:

| $x^{0}=1$ | $\frac{x^{a}}{x^{b}}=x^{(a-b)}$ |
| :---: | :---: |
| $x^{1}=x$ | $\left(x^{a}\right)^{b}=x^{a b}$ |
| $x^{-y}=\frac{1}{x^{y}}$ | $(x y)^{b}=x^{b} y^{b}$ |
| $x^{a} x^{b}=x^{(a+b)}$ | $\left(\frac{x}{y}\right)^{a}=\frac{x^{a}}{y^{a}}$ |

Exponents are also important when dealing with scientific notation or units (see Appendices D and F , respectively). For instance, scientific notation involves powers of $10: 10^{3}, 10^{9}, 10^{-6}$, etc. We can say it is "base $10^{\prime \prime}$ since $b=10$. When performing operations with the same base, the following rules apply:

1. When the operation involves multiplication, add the exponents.

- Example: $10^{3} \times 10^{4}=10^{3+4}=10^{7}$
- Example: $10^{5} \times 10^{2} \times 10^{-3}=10^{(5+2+(-3))}=10^{4}$

2. When the operation involves division, subtract the exponent in the denominator from the exponent in the numerator.

- Example: $10^{5} / 10^{3}=10^{5-3}=10^{2}$
- Example: $10^{7} / 10^{12}=10^{7-12}=10^{-5}$
- Example: $10^{8} / 10^{-3}=10^{(8-(-3))}=10^{11}$

3. When the operation involves powers, multiply the exponent by the power.

- Example: $\left(10^{5}\right)^{3}=10^{(5 \times 3)}=10^{15}$
- Example: $\left(10^{-7}\right)^{4}=10^{-(7 \times 4)}=10^{-28}$

When your numbers have units attached to them, there is one important rule:

## - Anything that you do to the number, do also to the unit.

This is a very important rule that is often overlooked. For instance, if you want to find the area $A$ of a circle that has radius $R=5.00$ meters, you would use the formula $A=\pi R^{2}$ :

$$
\begin{aligned}
& A=\pi\left(5.0 \text { meters }^{2}\right)^{2} \\
& A=78.5 \text { meters }^{2}
\end{aligned}
$$

If you don't square the unit along with the number, the answer makes no sense - you can't have an area with units of meters.

## B. 2 Practice

1. Simplify these exponential functions:
(a) $\left(x^{2}\right)\left(x^{3}\right)$
(b) $\frac{x^{9}}{x^{4}}$
(c) $\left(x^{2} y^{4}\right)^{5}$
(d) $\frac{x^{3} x^{2}}{x^{8}}$
(e) $\frac{y^{5}}{y^{2}} \times y^{7}$
(f) $\frac{x^{3}}{x^{0}}$
(g) $\frac{\left(x^{a}\right)\left(x^{e}\right)}{x^{f}}$
2. Solve these functions for $x(2)$ :
(a) $\left(x^{2}\right)\left(x^{2}\right)$
(b) $\frac{x^{5}}{x^{2}}$
3. Say that the human population on Europa (a moon of Jupiter) is given by this equation:

$$
f(x)=67.38(1.026)^{x}
$$

where $f(x)=$ Earth years after 27,000 .
(a) Plug zero into $f(x)$ to find the population that came off the space ship in 27,000 (Round to nearest whole number).
(b) Substitute 27 in for $x$ and find the population in 27,027 .
(c) Find the population in 27,054 and 27,081.
(d) What happens to the population every 27 years?
4. In the Earth year 27,056 an explosion at a power plant on Europa releases approximately 1000 kg of radioactive cesium-137 into the atmosphere. The expression

$$
f(x)=1000(0.5)^{\frac{x}{30}}
$$

describes the amount, in kg, of cesium-137 that remains in the general vicinity after $x$ years. Humans in the area must wear spacesuits if 100 kg or more are left in the atmosphere.
Find $f(80)$ and see if humans can stop wearing spacesuits in the year 27,136 ( 80 years after the explosion).

## Appendix C

## Logarithms

## C. 1 Overview

Logarithms are the inverse of exponential functions. They come in use when using large numbers. Astronomers, explaining huge distances and enormous energies, make heavy use of logarithms.

$$
\begin{array}{ccc}
y=\log _{b} x & \text { is the same as } & b^{y}=x \\
\text { (Logarithmic form) } & & \text { (Exponential form) }
\end{array}
$$

In this example, $y$ is the exponent and $b$ is the base.
In astronomy, and in this course, we typically use 10 as the base. In that case, we just write and assume the base is 10 , such as:

$$
y=\log _{10} x=\log x, \quad \text { which in its exponential form is } \quad 10^{y}=x .
$$

There is also a shorthand for the natural $\log$, written as $\ln$. It is a $\log$ with base $e$, where $e$ is a natural number (similar to $\pi$ ) approximately equal to 2.71 .

$$
y=\log _{e} x=\ln x, \quad \text { which in its exponential form is } \quad e^{y}=x
$$

Here are some useful properties of logarithms:

| $\log _{x} x=1$ | $\log _{b} M^{p}=p \log _{b} M$ |
| :---: | :---: |
| $\log _{x} 1=0$ | $\log _{b}(M \times N)=\log _{b} M+\log _{b} N$ |
| $\log _{b}\left(b^{x}\right)=x$ | $\log _{b} \frac{M}{N}=\log _{b} M-\log _{b} N$ |
| $b^{\left(\log _{b} x\right)}=x$ |  |

Here is a real life example involving logarithms.
The Richter scale is used to measure the intensity of earthquakes:

$$
R=\log \left(\frac{I}{I_{0}}\right)
$$

where $I_{0}$ is the intensity of a barely felt zero-level earthquake. In 1989, California had a 7.1 earthquake. How much more intense was it than a zero level?

Plug in $R=7.1$ :

$$
7.1=\log \left(\frac{I}{I_{0}}\right)
$$

Look back at the useful properties of logarithms and see that $b^{\left(\log _{b} x\right)}=x$. We can use this fact,

$$
10^{7.1}=10^{\log \left(\frac{I}{I_{0}}\right)}
$$

simplify it,

$$
10^{7.1}=\left(\frac{I}{I_{0}}\right)
$$

and rearrange to arrive at an answer:

$$
I=10^{7.1} I_{0}
$$

This means that the intensity of the 1986 earthquake was $10^{7.1}$ (or about $13,000,000$ ) times the intensity of a zero-level earthquake.

## C. 2 Logarithms in Astronomy

Astronomers quantify the brightness of a star by the magnitude, $m$, given by the equation

$$
m=-2.5 \log F
$$

where $F$ is the flux, or the amount of light that leaves the star measured at some distance. The brightness of an object depends on how far you are from it. This should make sense if you think about it. For example, a 100 Watt light bulb is still letting out 100 Watts even though it appears dimmer the further you are from it. This has to do with the inverse square law of light.

As it turns out, the difference in magnitudes of two different stars ( $m_{1}$ and $m_{2}$ ) is directly related to the ratio of their fluxes. This is hugely important in astronomy! To say the same thing in an equation:

$$
\begin{equation*}
m_{1}-m_{2}=-2.5 \log \frac{F_{1}}{F_{2}} \tag{C.1}
\end{equation*}
$$

For a star that is a perfect blackbody, the flux $F$ is given by:

$$
F=\frac{\sigma T^{4}}{4 \pi d^{2}}
$$

where $d$ is the distance to the center of the star (where the light is being emitted), $T$ is the effective temperature of the star, and $\sigma$ is a constant. You will use this information to answer questions 5-7 on the next page.

## C. 3 Practice

1. Write the following in exponential form:
(a) $2=\log _{5} x$
(b) $7=\log _{10} y$
(c) $x=\log _{3} 15$
(d) $13=\log _{a} 10$
2. Write the following in logarithmic form:
(a) $12^{2}=x$
(b) $\mathrm{e}^{y}=9$
(c) $b^{3}=8$
3. Solve for $x$ in the following equations:
(a) $\log _{10} 10-\log _{2.3} 1=x$
(b) $\log \frac{x}{3}=1$
4. After centuries of a successful settlement on Europa, a group of human explorers are sent to Jupiter's moon Io. Their mission is to place seismometers in several locations of Io's surface so Europan scientists can measure the intensities of ioquakes to see if humans could adapt to the environment. Due to Io's extreme proximity to Jupiter, the surface is considered the most active in the Solar System. The first data set sent back to Europa recorded a 13.1 ioquake (calibrated to Earth's Richter scale). How many times greater was this quake than a zero-level earthquake? Show your work and comment on your result.
5. If we combine fluxes and magnitudes, it is possible to relate the magnitude (something you can measure with a telescope) to the distance to the object.
We already know the flux of the star a distance $d$ parsecs away is

$$
F=\frac{\sigma T^{4}}{4 \pi d^{2}}
$$

and we can easily find the flux of the star $d=10$ parsecs away is

$$
F_{10}=\frac{\sigma T^{4}}{4 \pi(10 \text { parsecs })^{2}}
$$

Substitute $F$ and $F_{10}$ into Equation C. 1 and use $m_{2}=M=-2.5 \log F_{10}$ to solve for the distance. Show your work!
(You should find $m-M=5 \log \frac{d}{10}$. This is known as the distance modulus equation.)
6. Solve the distance modulus equation for $d$.
7. The quantity $(m-M)$ is used so often, it has its own name: the distance modulus. $m$ represents the apparent magnitude of a celestial object as measured by an observer from Earth. $M$ represents the absolute magnitude of a celestial object at a standard distance of 10 pc . Astronomers who use this equation regularly will often just say "the star has a distance modulus of 10 " and understand this to mean that the star is 1000 parsecs away. Verify that this is correct.

## Appendix D

## Scientific Notation

## D. 1 Overview

Scientific notation is the way that scientists handle very large numbers or very small numbers. For example, instead of writing 0.0000000056 , we write $5.6 \times 10^{-9}$. It is much less cumbersome than writing all those zeros out, and you will never screw up by forgetting a zero when you use scientific notation! So, how does it work?

A number in scientific notation is expressed as a number between 1 and 10 multiplied by some power of 10 . We can think of $5.6 \times 10^{-9}$ as the product of two numbers: 5.6 (the digit term) and $10^{-9}$ (the exponential term).

$$
\begin{array}{r}
6.4 \times 10^{5} \text { means } 640,000 \\
2.17 \times 10^{-3} \text { means } 0.00217
\end{array}
$$

The exponent can also be thought of as how many places to move the decimal point:

- For large numbers greater than 1 , count how many places you move the decimal point (to the right) and use that as the exponent on 10 .
- For small numbers less than 1 , count how many places you move the decimal point (to the left) and use the negative of that number as the exponent on 10 .

Here is a table of examples:

| $10000=1 \times 10^{4}$ | $24327=2.4327 \times 10^{4}$ |
| :---: | :---: |
| $1000=1 \times 10^{3}$ | $7354=7.354 \times 10^{3}$ |
| $100=1 \times 10^{2}$ | $482=4.82 \times 10^{2}$ |
| $10=1 \times 10^{1}$ | (leave as is) |
| $1=10^{0}$ | (leave as is) |
| $1 / 10=0.1=1 \times 10^{-1}$ | (leave as is) |
| $1 / 100=0.01=1 \times 10^{-2}$ | $0.053=5.3 \times 10^{-2}$ |
| $1 / 1000=0.001=1 \times 10^{-3}$ | $0.0078=7.8 \times 10^{-3}$ |
| $1 / 10000=0.0001=1 \times 10^{-4}$ | $0.00044=4.4 \times 10^{-4}$ |

It is important to know when to use scientific notation. In general, you never need to use $10^{-1}, 10^{0}$ or $10^{1}$. For anything much larger than 100 or much smaller than 0.1 , scientific notation is always necessary.

## D. 2 Manipulating Numbers in Scientific Notation

Here are some rules and examples for adding, subtracting, multiplying, dividing, and dealing with powers and exponentials when you work with scientific notation. Of course, your calculator can do this all for you, but it is still good to understand the fundamentals!

## Addition and Subtraction:

- All numbers are converted to the same power of 10 , and the digit terms are added or subtracted.
- Example: $\left(4.215 \times 10^{-2}\right)+\left(3.2 \times 10^{-4}\right)=\left(4.215 \times 10^{-2}\right)+\left(0.032 \times 10^{-2}\right)=4.247 \times 10^{-2}$
- Example: $\left(8.97 \times 10^{4}\right)-\left(2.62 \times 10^{3}\right)=\left(8.97 \times 10^{4}\right)-\left(0.262 \times 10^{4}\right)=8.71 \times 10^{4}$


## Multiplication:

- The digit terms are multiplied in the normal way and the exponents are added. The end result is changed so that there is only one nonzero digit to the left of the decimal.
- Example: $\left(3.4 \times 10^{6}\right)\left(4.2 \times 10^{3}\right)=(3.4)(4.2) \times 10^{(6+3)}=14.28 \times 10^{9}=1.4 \times 10^{10}$ (to 2 significant figures)
- Example: $\left(6.73 \times 10^{-5}\right)\left(2.91 \times 10^{2}\right)=(6.73)(2.91) \times 10^{(-5+2)}=19.58 \times 10^{-3}=1.96 \times 10^{-2}$ (to 3 significant figures)


## Division:

- The digit terms are divided in the normal way and the exponents are subtracted. The quotient is changed (if necessary) so that there is only one nonzero digit to the left of the decimal.
- Example: $\left(6.4 \times 10^{6}\right) /\left(8.9 \times 10^{2}\right)=(6.4) /(8.9) \times 10^{(6-2)}=0.719 \times 10^{4}=7.2 \times 10^{3}$ (to 2 significant figures)
- Example: $\left(3.2 \times 10^{3}\right) /\left(5.7 \times 10^{-2}\right)=(3.2) /(5.7) \times 10^{3-(-2)}=0.561 \times 10^{5}=5.6 \times 10^{4}$ (to 2 significant figures)


## Powers or Exponentials:

- The digit term is raised to the indicated power and the exponent is multiplied by the number that indicates the power.
- Example: $\left(2.4 \times 10^{4}\right)^{3}=(2.4)^{3} \times 10^{(4 \times 3)}=13.824 \times 10^{12}=1.4 \times 10^{13}$ (to 2 significant figures)
- Example: $\left(6.53 \times 10^{-3}\right)^{2}=(6.53)^{2} \times 10^{(-3) \times 2}=42.64 \times 10^{-6}=4.26 \times 10^{-5}$ (to 3 significant figures)

To use scientific notation on your calculator, make sure that the number is put into your calculator correctly. Read the directions for your particular calculator. For most types of scientific calculators:

1. Punch the number (the digit number) into your calculator.
2. Push the EE or EXP button. Do NOT use the $\times$ (times) button!!
3. Enter the exponent number. Use the $+/-$ button to change its sign.
4. Voila! Treat this number normally in all subsequent calculations.

To check yourself, multiply $6.0 \times 10^{5}$ times $4.0 \times 10^{3}$ on your calculator. Your answer should be $2.4 \times 10^{9}$.

## D. 3 Practice

Write the following in scientific notation.

1. 23.650
2. 410
3. $7,410,000,000$
4. 0.0000000092
5. Place these numbers in order of greatest to least:
(a) 72
(b) $7.2 \times 10^{-5}$
(c) $7.2 \times 10^{-3}$
(d) $7.2 \times 10^{3}$

Write the following in (regular) decimal notation:
6. $3.8 \times 10^{2}$
7. $6 \times 10^{-4}$
8. $9.2 \times 10^{4}$
9. $2.4 \times 10^{0}$
10. $6.000001 \times 10^{10}$
11. $7.9 \times 10^{-1}$

Write the following in scientific notation:
12. 32,000
13. $638,000,000,000,000,000$
14. 64,000
15. 317
16. 0.0083
17. 0.00000000504

Simplify the following without using a calculator:
18. $\left(3 \times 10^{4}\right)\left(9 \times 10^{3}\right)$
19. $\left(2 \times 10^{-3}\right)\left(7 \times 10^{0}\right)$
20. $\frac{8.2 \times 10^{6}}{2.0 \times 10^{3}}$
21. $\frac{3.123 \times 10^{-6}}{3.123 \times 10^{-2}}$

## Appendix E

## Significant Digits

## E. 1 Overview

There are two kinds of numbers in the world ...

- Exact numbers:
- Example: There are exactly 12 eggs in one dozen.
- Example: Most people have exactly 10 fingers and 10 toes.
- Inexact numbers:
- Example: Any measurement.

If I quickly measure the width of a piece of notebook paper, I might get 220 mm ( 2 significant figures). If I am more precise, I might get 216 mm (3 significant figures). An even more precise measurement would be 215.6 mm ( 4 significant figures).

There are three simple rules to follow when you want to know how many significant digits a number has:

1. All non-zero digits are significant.
2. In a number without a decimal point, only zeros BETWEEN non-zero digits are significant (unless explicitly indicated otherwise).
3. In a number with a decimal point, all zeros to the right of the first non-zero digit are significant.

Examples:

| Number | Significant Digits | Explanation |
| :---: | :---: | :--- |
| 300 | 1 | These two zeros are placeholders. (You could <br> write it as $\left.3 \times 10^{2}.\right)$ |
| 311.0 | 4 | This zero isn't a placeholder, it's just as sig- <br> nificant as the 3,1, and 1. (In scientific nota- <br> tion, you'd write $\left.3.110 \times 10^{2}.\right)$ |
| 0.00123 | 3 | Zeros are placeholders. |
| 1.00123 | 6 | Zeros are part of the number. |

## E. 2 Precision versus Accuracy

- Accuracy refers to how closely a measured value agrees with the correct value.
- Precision refers to how closely individual measurements agree with each other.


Figure E. 1 - These four targets illustrate the distinction between the terms "accuracy" and "precision."

## E. 3 Measurements

In any measurement, the number of significant figures is critical. (Note that the terms significant digits, significant figures, and "sig figs" are all interchangeable.) The number of significant figures is the number of digits believed to be correct by the person doing the measuring. It includes one estimated digit. As it turns out, the concept of significant figures deals with precision, but NOT with accuracy. You'll see why if you read on ...

Let's look at an example where significant figures is important: measuring length or distance in the lab. Typically you'll be using a ruler for this (though you could use a meterstick or measuring tape depending on the lab!).

As a rule of thumb, if you are EXTREMELY CAREFUL when making your measurements, then you can usually record an answer accurate to $1 / 10$ or 0.1 of the smallest division. This means that the measurement uncertainty is $\pm 1 / 10$ or 0.1 of the smallest division on the ruler. If you are less sure of yourself, you may record $\pm 1 / 2$ or 0.5 of the smallest division. In general, the more carefully you make your measurements, the smaller your measurement uncertainty will be.

- Example: on most rulers, the smallest division is 1 mm . This means when you use a ruler, if you are as precise as you can possibly be, you may read the length to within $1 / 10$ of 1 mm , which is the same as saying within 0.1 mm or 0.01 cm .
- If the above is true, and I measure the length of a metal rod with a ruler, I would record (for instance) 7.89 cm . I would not record 8 cm , or 7.9 cm , or 7.895 cm - all of those values have the wrong precision and thus the wrong number of significant digits.

Hopefully you can see now that the number of significant figures is directly linked to a measurement. Further, we can see that significant digits only deal with precision, not with accuracy. Consider measuring the length of a metal rod several times with a ruler. You will get essentially the same measurement over and over again with a small measurement uncertainty equal to about $1 / 10$ of the smallest division on the ruler. You have determined the length with high precision. However, you don't know if the ruler was accurate to begin with. Perhaps it was a plastic ruler left in the hot sun and was stretched. You don't know the accuracy of your measuring device unless you calibrate it, i.e. compare it against a ruler you knew was accurate. Thankfully, for the purposes of this course, we will generally assume that our rulers and other measuring tools are accurate to begin with.

## E. 4 Working with Significant Digits

There are two rules to remember here - read them carefully!

1. For multiplication and division, the result should have as many significant figures as the measured number with the smallest number of significant figures.
2. For addition and subtraction, the result should have as many decimal places as the measured number with the smallest number of decimal places.

## Example:

$$
\frac{21}{3.00}
$$

Your answer is only as good as your least precise measurement. Let's reference Rule 1 above: 21 has two significant digits and 3.00 has three significant digits. Therefore, your answer should have two significant digits:

$$
\frac{21}{3.00}=7.0
$$

## Example:

$$
21.24-8.92
$$

This one is a bit trickier. Clearly, 21.24 has four significant digits and 8.92 has three. So let's look at our options here.

$$
\begin{gathered}
21.24-8.92=12.32 \\
\text { OR } \\
21.24-8.92=12.3
\end{gathered}
$$

Which is it? In this case, we have to use Rule 2 above: the result should have as many decimal places as the measured number with the smallest number of decimal places. Thus, the first answer is correct: 12.32 .

An important note: You only need to worry about significant digits in your final answer. Throughout your calculations, use all the digits you possibly can, then round the final values to use the right number of digits. This will avoid rounding errors. However, when showing your work during a calculation, feel free to round to a reasonable number of digits on paper while preserving the full number in your calculator.

Now that you understand significant digits, you will be expected to ALWAYS use them!

## E. 5 Notes on Rounding

When rounding off numbers to a certain number of significant figures or decimal places, do so to the nearest value.

- Example: Round to three sig figs: $2.3467 \times 10^{4}$
(Answer: $2.35 \times 10^{4}$ )
- Example: Round to two sig figs: $1.612 \times 10^{3}$
(Answer: $1.6 \times 10^{3}$ )
What happens if there is a $5 ? 5$ always rounds up.


## E. 6 Practice

How many significant digits are in the following?

1. 14530
2. 0.0000012
3. 32.02010
4. 30

Evaluate the following. Specify how many significant digits are in your final answer, and why.
5. $\frac{25.1}{3.32}$
6. $3.004 \times 1.95$
7. $7+3.99991$
8. $33.090-25.1$
9. $\sqrt{2.000}$
10. $4503+34.90+550$
11. $1.367-1.34$
12. $\left(1.3 \times 10^{3}\right)\left(5.724 \times 10^{4}\right)$
13. $\frac{6305}{0.010}$

## Appendix $\mathbf{F}$

## Units

## F. 1 Overview

The metric system took scientific notation to a new level. There is a name for nearly every factor of 10 . For example, $3 \times 10^{9}$ years is 3 billion years, 3 gigayears, and also abbreviated as 3 Gyr . Below is an abbreviated list so you can get the idea.

| Prefix | Symbol | Power of 10 | Equivalent |
| :---: | :---: | :--- | :---: |
| tera | T | $10^{12}=1,000,000,000,000$ | 1 Trillion |
| giga | G | $10^{9}=1,000,000,000$ | 1 Billion |
| mega | M | $10^{6}=1,000,000$ | 1 Million |
| kilo | k | $10^{3}=1,000$ | 1 Thousand |
|  |  | $100=1$ | $($ One $)$ |
| centi | c | $10^{-2}=0.01$ | 1 Hundredth |
| milli | m | $10^{-3}=0.001$ | 1 Thousandth |
| micro | $\mu$ | $10^{-6}=0.000001$ | 1 Millionth |
| nano | n | $10^{-9}=0.000000001$ | 1 Billionth |
| pico | p | $10^{-12}=0.000000000001$ | 1 Trillionth |

Another thing to notice: when you are talking about numbers less than 1 but more than 0 , you add a "th" to the ending. For example, $10^{9}$ is a billion and $10^{-9}$ is a billionth.

## F. 2 Unit Conversions

A common problem in astronomy is the conversion of a quantity measured in some units into a quantity based on other units. For example, we might know that Mars is 2 AU away from us, but need to know how many meters away it is. In everyday life similar situations arise: you are driving in Mexico and need to change the 100 km per hr speed limit into more familiar miles per hour, or perhaps you are shopping at Vons and need to compare the value of the 30 pound box of Corn Flakes with the 14 ounce box at Albertson's. In astronomy and elsewhere, you are almost guaranteed to get the wrong answer if you do not keep track of your units!

Let's start with the simple example of converting Mars' 2 AU distance into meters. Our problem is

$$
2 \mathrm{AU}=? ? ? \mathrm{~m}
$$

We need to somehow get rid of the AU on the left and change it into meters. What we have to work with is an equation that says

$$
1 \mathrm{AU}=1.5 \times 10^{11} \mathrm{~m}
$$

The first step is to divide both sides by 1 AU , and notice that of course 1 AU divided by itself equals 1 , so

$$
1=\frac{1 \mathrm{AU}}{1 \mathrm{AU}}=\frac{1.5 \times 10^{11} \mathrm{~m}}{1 \mathrm{AU}}
$$

Now if we take the distance to Mars of 2 AU and multiply it by 1 , we don't change the distance. So we can write

$$
\mathrm{AU} \times 1=2 \mathrm{AU} \times \frac{1.5 \times 10^{11} \mathrm{~m}}{1 \mathrm{AU}}=2 \times\left(1.5 \times 10^{11}\right) \frac{\mathrm{m} \mathrm{AU}}{\mathrm{AU}}=3 \times 10^{11} \mathrm{~m}
$$

Notice that we got rid of the AU by getting another AU on the bottom of the quantity, which allows us to cancel it out. And we are left with our quantity expressed in meters.

Next let's try a more complicated example: you are offered a full-time job paying \$40,000 per year and another paying $\$ 12$ per hour. Which should you take? Let's try to put these two salaries into the same units so we can compare them properly. Remember that "per" usually means "divided by'," so the first salary is

$$
40,000 \frac{\$}{\text { year }}=? ? ? \frac{\$}{\text { hour }}
$$

Using our knowledge that 1 year $=52$ weeks and that a full-time job entails 40 hours in 1 week, we write

$$
40,000 \frac{\$}{\text { year }}=40,000 \frac{\$}{\text { year }} \times\left(\frac{1 \text { year }}{52 \text { weeks }}\right) \times\left(\frac{1 \text { week }}{40 \text { hours }}\right)=\frac{40,000}{52 \times 40} \frac{\$}{\text { hour }}=19.23 \frac{\$}{\text { hour }}
$$

Notice that the two quantities in parentheses are equal to one, so we can multiply by them whenever we want without changing the salary. Then note that the "year" and "week" units were made to cancel out. Obviously this is a better job than the one paying $\$ 12 / \mathrm{hr}$.

Here is a more complicated example: How long would it take you to get to the Sun if you were to drive there in your car at 60 mph (miles per hour)?
First we remember that the equation for rates is

$$
\text { distance }=\text { rate } \times \text { time }
$$

and since the problem is asking for the time, we divide both sides by the rate to get

$$
\text { time }=\frac{\text { distance }}{\text { rate }}=\frac{1 \mathrm{AU}}{60 \frac{\text { miles }}{\mathrm{hr}}}=\frac{1}{60} \frac{\mathrm{AU} \mathrm{hr}}{\text { miles }}
$$

So the right-hand side gives the answer, but in some strange units that we cannot understand. We would like to get rid of the AU and the miles - perhaps we could try to change both of them into meters so that they will cancel out. We look in our metric conversion table to find 1 mile $=1.609$ kilometers, and then write

$$
\text { time }=\frac{1}{60} \frac{\mathrm{AU} \mathrm{hr}}{\text { miles }} \times\left(\frac{1.5 \times 10^{11} \mathrm{~m}}{1 \mathrm{AU}}\right) \times\left(\frac{1 \mathrm{mile}}{1.609 \mathrm{~km}}\right) \times\left(\frac{1 \mathrm{~km}}{1000 \mathrm{~m}}\right)=\frac{1.5 \times 10^{11}}{60 \times 1.609 \times 1000} \mathrm{hr}=1.55 \times 10^{6} \mathrm{hr}
$$

Note how we constructed various versions of 1 to get rid of the AU, miles, and km, then the meters canceled as well. The answer is not easy to comprehend, however - let's change the hours into years:

$$
\text { time }=1.55 \times 10^{6} \mathrm{hr} \times\left(\frac{1 \text { day }}{24 \mathrm{hr}}\right) \times\left(\frac{1 \mathrm{yr}}{365 \text { days }}\right)=\frac{1.55 \times 10^{6}}{24 \times 365} \mathrm{yr}=1.77 \times 10^{2} \mathrm{yr}=177 \mathrm{yr}!
$$

## F. 3 Sanity Check

Whenever you are dealing with numbers, you should stop and ask yourself two questions:

1. Does my answer make sense (i.e., is it the right order of magnitude)?
2. What, if any, units should my number have, and how can I be sure?

If you can think through these questions every time you do a calculation, ESPECIALLY a calculation involving many different units, you will be well on your way to success.

## F. 4 Practice

Convert the following. You may use outside resources to find the necessary conversion factors if absolutely necessary, but you must show all work! Be sure to keep the right amount of significant figures, and do sanity checks. Don't forget: $1 \mathrm{mi}=1.61 \mathrm{~km}$.

1. 16.0 miles to kilometers
2. $1.2 \times 10^{-2}$ miles to centimeters
3. 1600 meters to miles
4. 550 nm to meters
5. 26.2 miles to mm
6. $6.43 \times 10^{15}$ seconds to years
7. The speed of light is $3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}$. How many meters does light travel in a year? (This distance is known as a light year)

## Appendix G

## Graphing

One of the most useful ways to look for and analyze relationships between different physical parameters is to construct graphs. During the semester you will be asked to make several graphs to assist you in the analysis of your data. For the purposes of this class, we will be graphing on a Cartesian coordinate system (sometimes called rectangular coordinates).

If a mark is made on a sheet of graph paper, the position of that mark can easily be specified by its distance from the edges of the page. By convention the X-coordinates usually increase toward the right and the Y-coordinates usually increase toward the top of the page. However, in astronomy it is not uncommon to see these conventions reversed for a specific application.

It is rarely possible to obtain a complete set of coordinates that define some physical relationship. Most often, you will be provided with a set of data that is just a sample of a more complete data set. In order to use the data you will be required to make a best-fit approximation to the available data points. Sometimes this will mean that you draw a straight line through the data, which roughly represents the average of all of the points. Other times you will have to fit a smooth curve to the data. In any event, you should NOT simply connect the dots.

Shown below is an example of a graph made from a set of rectangular coordinates, and an appropriate best fit smooth curve.


Figure G. 1 - A sample graph showing several data points and an estimated best fit smooth curve.

Unfortunately, the graph on the previous page lacks a few things which your graphs should ALWAYS contain!

- Both the X and Y axes on the graph should have units. Does "Time into Lecture" represent seconds, minutes, or hours? You would probably guess minutes, but you should never have to guess.
- If the quantity "Student Alertness Factor" is a percentage that can never exceed 100, the graph should stop at a Y-value of 100 .
- The graph should have a title.
- The graph should have an accompanying description that explains how the data were recorded (e.g., how many students, what time of day, how was the alertness factor measured, etc.)

Below are the data that the graph was created from. If you are provided with both a graph and a table, it is ALWAYS best to read numbers straight from the table because you will not be estimating the location of a data point.

| Time (minutes) | Alertness Factor (\%) |
| :---: | :---: |
| 5 | 90 |
| 10 | 82 |
| 15 | 60 |
| 20 | 60 |
| 25 | 39 |
| 30 | 20 |
| 35 | 12 |
| 40 | 21 |
| 45 | 30 |
| 47 | 95 |
| 50 | 100 |

## G. 1 Practice

| Radius <br> (parsecs) | Surface Brightness <br> (ergs s $^{-1} \mathbf{c m}^{-2}$ ) |
| :---: | :---: |
| 0.0 | 12.0 |
| 0.5 | 7.1 |
| 1.0 | 5.3 |
| 1.5 | 4.7 |
| 2.0 | 4.5 |
| 2.5 | 3.6 |
| 3.0 | 3.2 |
| 3.5 | 2.8 |
| 4.0 | 2.4 |
| 4.5 | 2.3 |
| 5.0 | 2.1 |
| 6.0 | 1.7 |
| 7.0 | 1.3 |
| 8.0 | 1.1 |
| 9.0 | 0.8 |
| 10.0 | 0.6 |

1. The table of data to the left represents how the brightness of a Wolf-Rayet ring nebula behaves as the nebula expands. Plot the given points on a graph with Surface Brightness as the $y$-axis and Radius as the x -axis. Then, draw a best fit smooth curve through the data. Use the curve to answer Questions 2 and 3.
2. If it were impossible to see a ring nebula with a surface brightness fainter than $1.6 \operatorname{ergs~s}^{-1} \mathrm{~cm}^{-2}$, how big would the nebula be when it becomes invisible?
3. What would the surface brightness of a ring nebula be if it had a radius of 3.25 parsecs?

## Appendix H

## Important Documents

This section includes documents required for attending the Mount Laguna Observatory field trip. They must be signed and submitted to your instructor at least one class meeting (one week) prior to the field trip.

# Mount Laguna Observatory (MLO) Field Trip Potential Risks and Dangers 

No off-campus field trip is without risks or dangers. Below is a discussion of those related to the MLO field trip. In general, participants should exercise the use of common sense and good judgment. Participants are required to be registered students or appointed faculty or staff. Students should note that as with all university functions, the Standards for Student Conduct apply (see http://studentaffairs.sdsu.edu/SRR/conduct1.html).

Driving: The most dangerous part of the field trip involves transportation to and from the observatory. Obey traffic laws and be a courteous driver. Most of the route is on Interstate-8. As a rule, you should drive in the right-hand lane except to pass slow moving vehicles. Approximately two miles east of Pine Valley take Exit 47 and turn left onto Sunrise Highway (SD County S1) at the end of the exit ramp. Be aware of possible fast-moving vehicles on S 1 coming from the blind turn to your right.

The speed limit on S1 is 55 MPH , with yellow advisory signs as low as 30 MPH . The road is marked with a no-passing double yellow line the entire way to MLO. Turnouts are placed off the right side of the road at judicious places. If someone is following you closely, pull over at the next available turnout to let them and other pass you. Be careful to stay on your side of the double yellow line at all times to reduce the risk of head-on collisions at blind curves.

Animals on, or crossing the road are an every present danger. Excessive speed combined with last-second maneuvers to avoid collision frequently lead to accidents. Be aware that in California's open-range lands, the driver of a vehicle is at fault for hitting cattle, and must pay the rancher for damages and loss of income.

Observatory Grounds: You are to bring a small flashlight to illuminate your path when walking on or to observatory grounds. Furthermore, you are to remain on designated paths and roads for you personal safety. Personal injury from falls, or scrapes and cuts from tree limbs and brush can easily result from shortcutting paths. In addition, wildlife dangers do exist including scorpions, tarantulas, and rattlesnakes. Most rattlesnake bites result from people unknowingly stepping on a snake. Mountain lions, although rarely seen and usually not aggressive, are present in the surrounding forest. If challenged by a mountain lion, do not run! Stop, hold your ground, open your jacket to "appear" larger, and slowly back away.

I certify that I have read the above description of potential risks and dangers.

## RELEASE OF LIABILITY, PROMISE NOT TO SUE, ASSUMPTION OF RISK AND AGREEMENT TO PAY CLAIMS

Activity: $\qquad$

Activity Date(s) and Time(s):
Activity Location(s):
In consideration for being allowed to participate in this Activity, on behalf of myself and my next of kin, heirs and representatives, I release from all liability and promise not to sue the State of California, the Trustees of The California State University, California State University, San Diego State University, their employees, officers, directors, volunteers and agents (collectively "University") from any and all claims, including claims of the University's negligence, resulting in any physical or psychological injury (including paralysis and death), illness, damages, or economic or emotional loss I may suffer because of my participation in this Activity, including travel to, from and during the Activity.

I am voluntarily participating in this Activity. I am aware of the risks associated with traveling to/from and participating in this Activity, which include but are not limited to physical or psychological injury, pain, suffering, illness, disfigurement, temporary or permanent disability (including paralysis), economic or emotional loss, and/or death. I understand that these injuries or outcomes may arise from my own or other's actions, inaction, or negligence; conditions related to travel; or the condition of the Activity location(s). Nonetheless, I assume all related risks, both known or unknown to me, of my participation in this Activity, including travel to, from and during the Activity.

I agree to hold the University harmless from any and all claims, including attorney's fees or damage to my personal property, that may occur as a result of my participation in this Activity, including travel to, from and during the Activity. If the University incurs any of these types of expenses, I agree to reimburse the University. If I need medical treatment, I agree to be financially responsible for any costs incurred as a result of such treatment. I am aware and understand that I should carry my own health insurance.

I am 18 years or older. I understand the legal consequences of signing this document, including (a) releasing the University from all liability, (b) promising not to sue the University, (c) and assuming all risks of participating in this Activity, including travel to, from and during the Activity.

I understand that this document is written to be as broad and inclusive as legally permitted by the State of California. I agree that if any portion is held invalid or unenforceable, I will continue to be bound by the remaining terms.

I have read this document, and I am signing it freely. No other representations concerning the legal effect of this document have been made to me.

Participant Signature: $\qquad$ Date:

Participant Name (print): $\qquad$ Red ID (SDSU employee/student): $\qquad$

## If Participant is under 18 years of age:

I am the parent or legal guardian of the Participant. I understand the legal consequences of signing this document, including (a) releasing the University from all liability on my and the Participant's behalf, (b) promising not to sue on my and the Participant's behalf, (c) and assuming all risks of the Participant's participation in this Activity, including travel to, from and during the Activity. I allow Participant to participate in this Activity. I understand that I am responsible for the obligations and acts of Participant as described in this document. I agree to be bound by the terms of this document.

I have read this two-page document, and I am signing it freely. No other representations concerning the legal effect of this document have been made to me.

Signature of Minor Participant's Parent/Guardian

Name of Minor Participant’s Parent/Guardian (print)
Date

Minor Participant’s Name


[^0]:    ${ }^{1}$ Special note: Measuring an angle this small is comparable to measuring the width of a human hair from 1000 km away! This precision is achieved by the ESA's Gaia space observatory.

[^1]:    ${ }^{1}$ The most precise estimate of Hubble's constant today is $H_{0}=70.4_{-1.4}^{+1.3}(\mathrm{~km} / \mathrm{s}) / \mathrm{Mpc}$, coming from the WMAP analysis of the cosmic microwave background.
    ${ }^{2}$ Although the rate of expansion is not really constant, it is very close to being constant. This is known from actual measurements, not an assumption.

[^2]:    ${ }^{1}$ Please note that, as stated in the Course Syllabus, plagiarism is not tolerated in Astronomy 109. As defined by SDSU's General Catalog, "Plagiarism is formal work publicly misrepresented as original." Plagiarism is theft. If it is found that a student has copied more than 4 words in a row from any source (including any source on the World Wide Web) and has not properly referenced that source (i.e., put the words in quotes, with proper citation), then that student is committing plagiarism, and by University policy the incident must be reported to the Center for Student Rights and Responsibilities. Please see the Course Syllabus for details on the judicial process that follows when such a report is filed.

[^3]:    ${ }^{1}$ See http://www.exoplanets.eu or http://exoplanetarchive.ipac.caltech.edu/for an up-to-date list of planets and their discovery methods.

