Midterm I Exam

15-122 Principles of Imperative Computation
Frank Pfenning

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Name: **Sample Solution** Andrew ID: **fp** Section:

Instructions

- This exam is closed-book with one sheet of notes permitted.
- You have 80 minutes to complete the exam.
- There are 4 problems.
- Read each problem carefully before attempting to solve it.
- Do not spend too much time on any one problem.
- Consider if you might want to skip a problem on a first pass and return to it later.

	Mod.arith.	Search	Stacks	Queues	
	Prob 1	Prob 2	Prob 3	Prob 4	Total
Score	**20**	**30**	**25**	**25**	**100**
Max	20	30	25	25	100
Grader	tc	fp/jp	kbn	wjl/kbn	

1 Modular Arithmetic (20 pts)

In C0, values of type `int` are defined to have 32 bits. In this problem we work with a version of C0 called C8 where values of type `int` are defined to have only 8 bits. In other respects it is the same as C0. All integer operations are still in two's complement arithmetic, but now modulo 2^8. All bitwise operations are still bitwise, except on only 8 bit words instead of 32 bit words.

Task 1 (10 pts). Fill in the missing quantities, in the specified notation.

a. The minimal negative integer, in decimal: ___-128___

b. The maximal positive integer, in decimal: ___127___

c. -2, in hexadecimal: 0x___FE___

d. 19, in hexadecimal: 0x___13___

e. `0x45`, in decimal: ___69___

Task 2 (10 pts). Assume `int x` has been declared and initialized to an unknown value. For each of the following, indicate if the expression always evaluates to `true`, or if it could sometimes be `false`. In the latter case, indicate a counterexample in C8 by giving a value for `x` that falsifies the claim. You may use decimal or hexadecimal notation.

a. `x+1 > x` ___false, x = 127___

b. `(((x>>1)<<1) | (x&1)) == x` ___true___

c. `(x ^ (~x)) == 0` false, x = 0 (or any other x)

d. `x <= (1<<7)-1` ___true___

e. `x+x == 2*x` ___true___

2 Search Algorithms (30 pts)

We explore the application of ideas behind linear and binary search in a new context. We are given an array of integers subject only to the requirement that the first integer in the array is negative ($A[0] < 0$) and the last integer in the array is nonnegative ($A[n-1] \geq 0$). We cannot assume that the array is sorted. We want to find an index i in the array such that $A[i] < 0$ and $A[i+1] \geq 0$.

The following is a function to find such an index, searching through the array from left to right.

```
int find1(int[] A, int n)
//@requires 2 <= n && n <= \length(A);
//@requires A[0] < 0 && 0 <= A[n-1];
//@ensures 0 <= \result && \result < n-1;
//@ensures A[\result] < 0 && 0 <= A[\result+1];
{
  for (int i = 0; i < n-1; i++)
    //@loop_invariant 0 <= i && i <= n-1; /* answer line 1 */
    //@loop_invariant A[i] < 0;           /* answer line 2 */
    {
      if (0 <= A[i+1]) return i;
    }
  /* should never get here */
  //@assert false;
  return -1;
}
```

Task 1 (4 pts). Fill in loop invariants. Your loop invariants (together with the function preconditions) should be strong enough to guarantee the postconditions.

Task 2 (1 pts). What is the worst-case asymptotic complexity of the `find1` as a function of n, in big-O notation? You do not need to justify your answer.

$O(n)$

Task 3 (2 pts). Show that your loop invariants hold initially.

Initially, $i = 0$, so $0 \leq i$ and $i \leq n-1$ since $n \geq 2$.
Also, $A[i] < 0$ since $A[0] < 0$ by precondition.

Task 4 (5 pts). Show that your loop invariants are preserved by one iteration of the loop.

For the first invariant, we assume the invariant $0 \leq i \leq n-1$. Also, $i < n-1$ since the loop condition is true. In the iteration we set $i' = i+1$. Then $0 \leq i' = i+1$ since $0 \leq i$. Also, $i' = i+1 \leq n-1$ since $i < n-1$.

For the second invariant, we only get back to the loop test if $A[i+1] < 0$ (otherwise we return). But $i' = i+1$ so $A[i'] < 0$.

Task 5 (5 pts). Show that your loop invariants (together with the function preconditions) imply the postconditions. You may assume without proof that the statement after the loop is never reached, so that the final return statement can never be executed.

The first postcondition holds because $0 \leq i$ by the loop invariant, and $i < n-1$ by the loop test, and we return i.

The second postcondition holds because $A[i] < 0$ by the loop invariant and $0 \leq A[i+1]$ by the condition guarding the `return`, and we return i.

Task 6 (10 pts). Now apply the idea behind binary search to complete the following function to satisfy the same pre- and post-condition. You do not need to prove this, but we suggest that you reason it out to yourself to make sure your function and the invariants are correct.

```
int find2(int[] A, int n)
//@requires 2 <= n && n <= \length(A);
//@requires A[0] < 0 && A[n-1] >= 0;
//@ensures 0 <= \result && \result < n-1;
//@ensures A[\result] < 0 && 0 <= A[\result+1];
{ int lower = 0;
  int upper = n-1;
  while (upper-lower > 1)
    //@loop_invariant 0 <= lower && lower < upper && upper < n;
    //@loop_invariant A[lower] < 0 && 0 <= A[upper];
    { int mid = lower + (upper-lower)/2;
      if (A[mid] < 0)
        lower = mid;
      else
        upper = mid;
    }
  return lower;
}
```

Task 7 (2 pts). Does `find2` always return the same answer as `find1`? Explain your answer in one or two sentences.

No. `find1` will always return the index of the first sign change, `find2` might return another one. For example, for the array `{-1, 1, -1, 1, 2}` `find1` will return 0 while `find2` will return 2.

Task 8 (1 pts). What is the worst-case asymptotic complexity of `find2` as a function of n, in big-O notation? You do not need to justify your answer.

$O(\log(n))$

3 Stacks (25 pts)

In this problem we will implement a simple text buffer with some editing functions. A text buffer consists of a sequence of characters with a distinguished position called the *point*. A buffer with characters `abcdef` and the point between `b` and `c` is written as

```
a b|c d e f
```

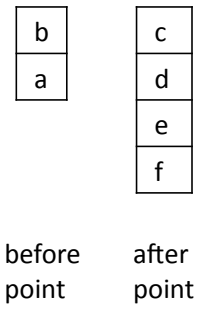
The abstract interface has operations to create a new buffer, move *point* forward or backward, and to insert or delete character at *point*. Each operation is explained in detail where you are asked to implement it. All operations should be constant time ($O(1)$).

```
typedef struct tbuf* tbuf;
tbuf tbuf_new();                    /* create new text buffer */
void insert_char(tbuf B, char c);   /* insert character at point */
void delete_char(tbuf B);           /* delete character before point */
void forward_char(tbuf B);          /* move point forward one char */
void backward_char(tbuf B);         /* move point backward one char */
// ... others elided
```

We are implementing a text buffer by two stacks, one with the characters before the *point*, and one with the characters after the *point*. The text buffer

```
a b|c d e f
```

above would be represented by the following two stacks:



The *before* stack has two elements, with `b` on top; the *after* stack has four elements with `c` on top.

The implementation will use stacks of characters according to the following interface as developed in lecture.

```
typedef char elem;

typedef struct stack* stack;
bool stack_empty(stack S);      /* O(1) */
stack stack_new();              /* O(1) */
void push(stack S, elem e);     /* O(1) */
elem pop(stack S)               /* O(1) */
//@requires !stack_empty(S);
  ;
```

As explained, a text buffer is represented by a struct containing two stacks, *before* and *after*.

```
struct tbuf {
  stack before;
  stack after;
};
```

The following function checks if B is a valid text buffer. Your functions must preserve this property.

```
bool is_tbuf(tbuf B) {
  if (B == NULL) return false;
  return (B->before != NULL && B->after != NULL);
}
```

Task 1 (5 pts). Write a function to create a new (empty) text buffer.

```
tbuf tbuf_new()
//@ensures is_tbuf(\result);
{
  stack before = stack_new();
  stack after = stack_new();
  tbuf B = alloc(struct tbuf);
  B->before = before;
  B->after = after;
  return B;
}
```

In response to the questions below, you do not need to write annotations, but you are free to do so if you wish. You may assume that all function arguments of type `tbuf` are valid text buffers (according to `is_tbuf`) and your functions should ensure that they remain valid.

Task 2 (5 pts). Write a function to insert a character into a text buffer before *point*. For example, if a text buffer B contains

```
a b|c d e f
```

then after `insert_char(B, 'w')`, it should have the form

```
a b w|c d e f
```

```
void insert_char(tbuf B, char c)
//@requires is_tbuf(B);
//@ensures is_tbuf(B);
{
  push(B->before, c);
}
```

Task 3 (5 pts). Write a function to delete the character before *point*. If the *point* is at the left end of the text buffer, the buffer should remain unchanged. For example, if a text buffer B contains

```
a b|c d e f
```

then after `delete_char(B)`, it should have the form

```
a|c d e f
```

```
void delete_char(tbuf B)
//@requires is_tbuf(B);
//@ensures is_tbuf(B);
{
  if (!stack_empty(B->before))
    pop(B->before);
}
```

Task 4 (5 pts). Write a function to move the *point* forward one character. If it is already at the right end of the text buffer, it should remain unchanged.

```
void forward_char(tbuf B)
//@requires is_tbuf(B);
//@ensures is_tbuf(B);
{
  if (!stack_empty(B->after))
    push(B->before, pop(B->after));
}
```

Task 5 (5 pts). Write a function to move the *point* backward one character. If the *point* is already at the left end of the text buffer, it should remain unchanged.

```
void backward_char(tbuf B)
//@requires is_tbuf(B);
//@ensures is_tbuf(B);
{
  if (!stack_empty(B->before))
    push(B->after, pop(B->before));
}
```

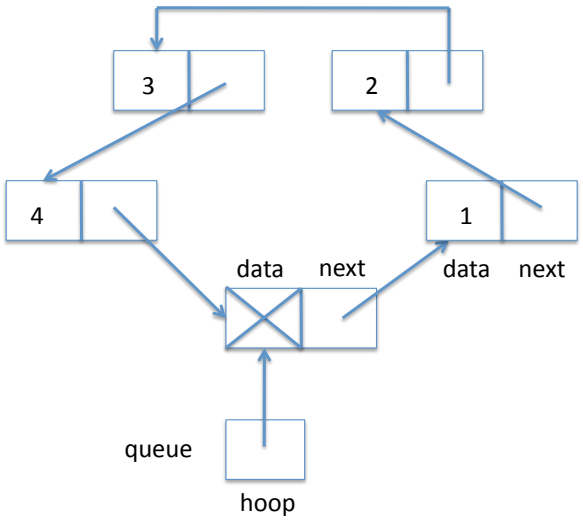
4 Queues and Linked Lists (25 pts)

Recall the definition of linked lists and the `is_segment` function that checks if a linked list beginning at `start` eventually arrives at `end`.

```
struct list {
  elem data;
  struct list* next;
};
typedef struct list* list;

bool is_segment(list start, list end);
```

A *hoop* represents a queue as a cyclic linked list with one extra node. For example, a queue with items $1, 2, 3, 4$ (in this order, 1 at the front of the queue and 4 at the back) would be represented as the following hoop:



We define

```
struct queue {
  list hoop;
};
typedef struct queue* queue;
```

As an example, here is a function that checks if the hoop is empty.

```
bool queue_empty(queue Q)
//@requires is_queue(Q);
{
  return Q->hoop == Q->hoop->next;
}
```

Task 1 (5 pts). Write a function that checks if a given queue is valid. [**Hint:** Use `is_segment`, and remember that pointers can be `NULL`.]

```
bool is_queue(queue Q) {
  if (Q == NULL) return false;
  if (Q->hoop == NULL) return false;
  return is_segment(Q->hoop->next, Q->hoop);
}
```

Task 2 (10 pts). Write a function to dequeue an element from the front of the queue. Your function should take constant time. [**Hint:** Draw a picture!]

```
elem deq(queue Q)
//@requires is_queue(Q);
//@requires !queue_empty(Q);
//@ensures is_queue(Q);
{
  elem e = Q->hoop->next->data;
  Q->hoop->next = Q->hoop->next->next;
  return e;
}
```

Task 3 (10 pts). Write a function to enqueue an element at the end of the queue. Your function should take constant time. [**Hint:** Draw a picture!]

```
void enq(queue Q, elem e)
//@requires is_queue(Q);
//@ensures is_queue(Q);
{
  list h = alloc(struct list);
  /* h->data is irrelevant */
  h->next = Q->hoop->next;
  Q->hoop->data = e;
  Q->hoop->next = h;
  Q->hoop = h;
  return;
}
```