# A User's Guide to Picat Version 2.4 

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## Preface

Despite the elegant concepts, new extensions (e.g., tabling and constraints), and successful applications (e.g., knowledge engineering, NLP, and search problems), Prolog has a bad reputation for being old and difficult. Many ordinary programmers find the implicit non-directionality and nondeterminism of Prolog to be hard to follow, and the non-logical features, such as cuts and dynamic predicates, are prone to misuses, leading to absurd codes. The lack of language constructs (e.g., loops) and libraries for programming everyday things is also considered a big weakness of Prolog. The backward compatibility requirement has made it hopeless to remedy the language issues in current Prolog systems, and there are urgent calls for a new language.

Several successors of Prolog have been designed, including Mercury, Erlang, Oz, and Curry. The requirement of many kinds of declarations in Mercury has made the language difficult to use; Erlang's abandonment of non-determinism in favor of concurrency has made the language unsuited for many applications despite its success in the telecom industry; Oz has never attained the popularity that the designers sought, probably due to its unfamiliar syntax and implicit laziness; Curry is considered too close to Haskell. All of these successors were designed in the 1990s, and now the time is ripe for a new logic-based language.

Picat aims to be a simple, and yet powerful, logic-based programming language for a variety of applications. Picat incorporates many declarative language features for better productivity of software development, including explicit non-determinism, explicit unification, functions, constraints, and tabling. Picat lacks Prolog's non-logical features, such as the cut operator and dynamic predicates, making Picat more reliable than Prolog. Picat also provides imperative language constructs for programming everyday things. The system can be used for not only symbolic computations, which is a traditional application domain of declarative languages, but also for scripting and modeling tasks.

Picat is a general-purpose language that incorporates features from logic programming, functional programming, and scripting languages. The letters in the name summarize Picat's features:

- Pattern-matching: A predicate defines a relation, and can have zero, one, or multiple answers. A function is a special kind of a predicate that always succeeds with one answer. Picat is a rule-based language. Predicates and functions are defined with pattern-matching rules.
- Intuitive: Picat provides assignment and loop statements for programming everyday things. An assignable variable mimics multiple logic variables, each of which holds a value at a different stage of computation. Assignments are useful for computing aggregates and are used with the foreach loop for implementing list and array comprehensions.
- Constraints: Picat supports constraint programming. Given a set of variables, each of which has a domain of possible values, and a set of constraints that limit the acceptable set of assignments of values to variables, the goal is to find an assignment of values to the variables that satisfies all of the constraints. Picat provides three solver modules: cp, sat, and mip. These three modules follow the same interface, which allows for seamless switching from one solver to another.
- Actors: Actors are event-driven calls. Picat provides action rules for describing eventdriven behaviors of actors. Events are posted through channels. An actor can be attached to a channel in order to watch and to process its events.
- Tabling: Tabling can be used to store the results of certain calculations in memory, allowing the program to do a quick table lookup instead of repeatedly calculating a value. As
computer memory grows, tabling is becoming increasingly important for offering dynamic programming solutions for many problems. The planner module, which is implemented by the use of tabling, has been shown to be a more efficient tool than ASP and PDDL for solving many planning problems.

The support of explicit unification, explicit non-determinism, tabling, and constraints makes Picat more suitable than functional and scripting languages for symbolic computations. Picat is arguably more expressive than Prolog for scripting and modeling. With arrays, loops, and list and array comprehensions, it is not rare to find problems for which Picat requires an order of magnitude fewer lines of code to describe than Prolog. Picat is more scalable than Prolog. The use of patternmatching rather than unification facilitates indexing of rules. Picat is also more reliable than Prolog. In addition to explicit non-determinism, explicit unification, and a simple static module system, the lack of cuts, dynamic predicates, and operator overloading also improves the reliability of the language. Picat is not as powerful as Prolog for metaprogramming and it's impossible to write a meta-interpreter for Picat in Picat itself. Nevertheless, this weakness can be remedied with library modules for implementing domain-specific languages.

The Picat implementation is based on the B-Prolog engine. The current implementation is already ready for many kinds of applications. It will also serve as a foundation for new additions, including external language interfaces with C, Java, and Python, an external language interface with MySql, threads, sockets, Web services, and language processing modules. Anybody is welcome to contribute. The C source code is available to registered developers and users. Please contact picat@picat-lang.org.

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The Picat implementation is based on the B-Prolog engine. It uses the following public domain modules: token. c by Richard O’Keefe; getline.c by Chris Thewalt; bigint.c by Matt McCutchen; Lingeling by Armin Biere; Espresso (by Berkeley). In addition, Picat also provides interfaces to GLPK by Andrew Makhorin and Gurobi by Gurobi Optimization, Inc.

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## Chapter 1

## Overview

Before we give an overview of the Picat language, let us briefly describe how to use the Picat system. The Picat system provides an interactive programming environment for users to load, debug, and execute programs. Users can start the Picat interpreter with the OS command picat.

OSPrompt picat
Once the interpreter is started, users can type a command line after the prompt Picat>. The help command shows the usages of commands, and the halt command terminates the Picat interpreter. Users can also use the picat command to run a program directly as follows:

$$
\text { OSPrompt picat File } \operatorname{Arg}_{1} \operatorname{Arg}_{2} \ldots \operatorname{Arg}_{n}
$$

where File (with or without the extension . pi) is the main file name of the program. The program must define a predicate named main/0 or main/1. If the command line contains arguments after the file name, then main/1 is executed. Otherwise, if the file name is not followed by any arguments, then main/ 0 is executed. When main/1 executed, all of the arguments after the file name are passed to the predicate as a list of strings.

### 1.1 Data Types

Picat is a dynamically-typed language, in which type checking occurs at runtime. A variable in Picat is a value holder. A variable name is an identifier that begins with a capital letter or the underscore. An attributed variable is a variable that has a map of attribute-value pairs attached to it. A variable is free until it is bound to a value. A value in Picat can be primitive or compound.

A primitive value can be an integer, a real number, or an atom. A character can be represented as a single-character atom. An atom name is an identifier that begins with a lower-case letter or a single-quoted sequence of characters.

A compound value can be a list in the form $\left[t_{1}, \ldots, t_{n}\right]$ or a structure in the form $\$ s\left(t_{1}, \ldots, t_{n}\right)$ where $s$ stands for a structure name, $n$ is called the arity of the structure, and each $t_{i}(1 \leq i \leq n)$ is a term which is a variable or a value. The preceding dollar symbol is used to distinguish a structure from a function call. Strings, arrays, maps, sets, and heaps are special compound values. A string is a list of single-character atoms. An array takes the form $\left\{t_{1}, \ldots, t_{n}\right\}$, which is a special structure with the name ${ }^{\prime}\{ \}^{\prime}$. A map is a hash-table represented as a structure that contains a set of key-value pairs. A set is a special map where only keys are used. A heap is a complete binary tree represented as an array. A heap can be a min-heap or a max-heap.

The function new_struct ( Name, IntOrList) returns a structure. The function new_map (S) returns a map that initially contains the pairs in list $S$, where each pair has the form $K e y=$ Val. The function new_set $(S)$ returns a map set that initially contains the elements in list
S. A map-set is a map in which every key is mapped to the atom not_a_value. The function new_array $\left(I_{1}, I_{2}, \ldots, I_{n}\right)$ returns an n-dimensional array, where each $I_{i}$ is an integer expression specifying the size of a dimension. An n-dimensional array is a one-dimensional array where the arguments are ( $\mathrm{n}-1$ )-dimensional arrays.

## Example

```
Picat> V1 = X1, V2 = _ab, V3 = _ % variables
Picat> N1 = 12, N2 = 0xf3, N3 = 1.0e8 % numbers
Picat> A1 = x1, A2 = '_AB', A3 = 'r % atoms
Picat> L = [a,b,c,d]
Picat> write("hello"++"picat")
% strings
[h,e,l,l,o,p,i,c,a,t]
Picat> print("hello"++"picat")
hellopicat
```

Picat> writef("\%s","hello"++"picat") \% formatted write
hellopicat
Picat> writef("\%-5d \%5.2f",2,2.0) \% formatted write
22.00
Picat $>S=\$$ point $(1.0,2.0) \quad \%$ a structure
Picat> $S=$ new_struct (point, 3) $\quad$ o create a structure
$S=p o i n t(\ldots 3 b 0, \ldots 3 b 4, \ldots 3 b 8)$
Picat $>A=\{a, b, c, d\} \quad \%$ an array
Picat> A = new_array (3) $\quad \%$ create an array
$A=\left\{\_3 \mathrm{~b} 0, \ldots 3 \mathrm{~b} 4, \ldots 3 \mathrm{~b} 8\right\}$
Picat $>M=$ new_map $([o n e=1$, two $=2]) \quad$ o create a map
$M=(\operatorname{map})[$ two $=2$, one $=1$ ]
Picat $>M=$ new_set $([$ one, two, three]) $\quad$ o create a map set
$M=$ (map) [two, one, three]
Picat $>X=1 . .2 .10 \quad$ \% ranges
$X=[1,3,5,7,9]$
Picat> $X=1 . .5$
$X=[1,2,3,4,5]$

Picat allows function calls in arguments. For this reason, it requires structures to be preceded
with a dollar symbol in order for them to be treated as data. Without the dollar symbol, the command S=point $(1.0,2.0)$ would call the function point $(1.0,2.0)$ and bind $S$ to its return value. In order to ensure safe interpretation of meta-terms in higher-order calls, Picat forbids the creation of terms that contain structures with the name ' .', index notations, array comprehensions, list comprehensions, and loops.

For each type, Picat provides a set of built-in functions and predicates. The index notation $X[I]$, where $X$ references a compound value and $I$ is an integer expression, is a special function that returns a single component of $X$. The index of the first element of a list or a structure is 1 . In order to facilitate type checking at compile time, Picat does not overload arithmetic operators for other purposes, and requires an index expression to be an integer.

A list comprehension, which takes the following form, is a special functional notation for creating lists:

$$
\left[T: E_{1} \text { in } D_{1}, \operatorname{Cond}_{1}, \ldots, E_{n} \text { in } D_{n}, \operatorname{Cond}_{n}\right]
$$

where $T$ is an expression, each $E_{i}$ is an iterating pattern, each $D_{i}$ is an expression that gives a compound value, and the optional conditions $\operatorname{Cond}_{1}, \ldots, \operatorname{Cond}_{n}$ are callable terms. This list comprehension means that for every tuple of values $E_{1} \in D_{1}, \ldots, E_{n} \in D_{n}$, if the conditions are true, then the value of $T$ is added into the list.

An array comprehension takes the following form:

$$
\left\{T: E_{1} \text { in } D_{1}, \operatorname{Cond}_{1}, \ldots, E_{n} \text { in } D_{n}, \operatorname{Cond}_{n}\right\}
$$

It is the same as:

$$
\text { to_array }\left(\left[T: \quad E_{1} \text { in } D_{1}, \operatorname{Cond}_{1}, \ldots, E_{n} \text { in } D_{n}, \operatorname{Cond}_{n}\right]\right)
$$

The predicate put ( $M a p, K e y, V a l$ ) attaches the key-value pair $K e y=V a l$ to the map $M a p$, where Key is a non-variable term, and Val is any term. The function get ( $\mathrm{Map}, \mathrm{Key}$ ) returns $V a l$ of the key-value pair $K e y=V a l$ attached to $M a p$. The predicate has_key ( $M a p, K e y$ ) returns true iff $M a p$ contains a pair with the given key.

An attributed variable has a map attached to it. The predicate put_attr ( $X, K e y, V a l$ ) attaches the key-value pair $K e y=V a l$ to $X$. The function get_attr ( $X, K e y$ ) returns $V a l$ of the key-value pair $K e y=V a l$ attached to $X$.

```
Example
Picat> integer(5)
yes
Picat> real(5)
no
Picat> var(X)
yes
Picat> X=5, var(X)
no
Picat> 5 != 2+2
yes
```

```
Picat> X = to_binary_string(5)
X = ['1','0','1']
Picat> L = [a,b,c,d], X = L[2]
X = b
Picat> L = [(A,I) : A in [a,b], I in 1..2].
L=[(a,1),(a,2),(b,1),(b, 2)]
Picat> put_attr(X,one,1), One = get_attr(X,one) % attributed var
One = 1
Picat> S = new_struct(point, 3), Name = name(S), Len = length(S)
S = point (_3b0,_3b4,_3b8)
Name = point
Len = 3
Picat> S = new_array (2, 3), S[1,1] = 11, D2 = length(S[2])
S = {{11,_93a0,_,93a4},{__938c,_,9390,__9394}}
D2 = 3
Picat> M = new_map(), put(M,one,1), One = get(M,one)
One = 1
Picat> M = new_set(), put(M,one), has_key(M,one).
```

Picat also allows OOP notations for calling predicates and functions. The notation $A_{1} . f\left(A_{2}, \ldots, A_{k}\right)$ is the same as $f\left(A_{1}, A_{2}, \ldots, A_{k}\right)$, unless $A_{1}$ is an atom, in which case $A_{1}$ must be a module qualifier for $f$. The notation $A . f$, where $f$ is an atom, is the same as the call $A . f()$.

## Example

```
Picat> X = 5.to_binary_string()
X = ['1','0','1']
Picat> X = 5.to_binary_string().length
X=3
Picat> X.put(one,1), One = X.get(one)
One = 1
Picat> X = math.pi
X=3.14159
Picat> S = new_struct(point, 3), Name = S.name, Len = S.length
S = point (_3b0,_3b4,_3b8)
Name = point
Len = 3
Picat> S = new_array (2,3), S[1,1] = 11, D2 = S[2].length
```

```
S = {{11,_,93a0,_93a4},{_938c,_9390,_9394}}
D2 = 3
Picat> M = new_map(), M.put(one,1), One = M.one.
One = 1
```


### 1.2 Defining Predicates

A predicate call either succeeds or fails, unless an exception occurs. A predicate call can return multiple answers through backtracking. The built-in predicate true always succeeds, and the built-in predicate fail (or false) always fails. A goal is made from predicate calls and statements, including conjunction ( $A, B$ and $A \& \& B$ ), disjunction ( $A ; B$ and $A \| B$ ), negation (not $A$ ), if-then-else, foreach loops, and while loops.

A predicate is defined with pattern-matching rules. Picat has two types of rules: the nonbacktrackable rule Head, Cond $=>$ Body, and the backtrackable rule Head, Cond ?=> Body. The Head takes the form $p\left(t_{1}, \ldots, t_{n}\right)$, where $p$ is called the predicate name, and $n$ is called the arity. When $n=0$, the parentheses can be omitted. The condition Cond, which is an optional goal, specifies a condition under which the rule is applicable. For a call $C$, if $C$ matches Head and Cond succeeds, meaning that the condition evaluates to true, the rule is said to be applicable to $C$. When applying a rule to call $C$, Picat rewrites $C$ into Body. If the used rule is non-backtrackable, then the rewriting is a commitment, and the program can never backtrack to $C$. If the used rule is backtrackable, however, the program will backtrack to $C$ once Body fails, meaning that Body will be rewritten back to $C$, and the next applicable rule will be tried on $C$.

## Example

```
fib(0,F) => F=1.
fib(1,F) => F=1.
fib(N,F),N>1 => fib(N-1,F1),fib(N-2,F2),F=F1+F2.
fib(N,F) => throw $error(wrong_argument,fib,N).
```

A call matches the head $f i b(0, F)$ if the first argument is 0 . The second argument can be anything. For example, for the call $f i b(0,2)$, the first rule is applied, since $f i b(0,2)$ matches its head. However, when the body is executed, the call $2=1$ fails.

The predicate $f i b / 2$ can also be defined using if-then-else as follows:

```
fib(N,F) =>
    if (N=0; N=1) then
        F=1
    elseif N>1 then
        fib(N-1,F1),fib(N-2,F2),F=F1+F2
    else
        throw $error(wrong_argument,fib,N)
    end.
```

An if statement takes the form if Cond then Goal $_{1}$ else Goal ${ }_{2}$ end. ${ }^{1}$ The then part can contain one or more elseif clauses. The else part can be omitted. In that case the else part is assumed to be else true. The built-in throw $E$ throws term $E$ as an exception.

[^0]
## Example

```
member(X,[Y|_]) ?=> X=Y.
member(X,[_|L]) => member(X,L).
```

The pattern [Y|_] matches any list. The backtrackable rule makes a call nondeterministic, and the predicate can be used to retrieve elements from a list one at a time through backtracking.

```
Picat> member(X, [1, 2, 3])
X=1;
X=2;
X=3;
no
```

After Picat returns an answer, users can type a semicolon immediately after the answer to ask for the next answer. If users only want one answer to be returned from a call, they can use once Call to stop backtracking.

The version of member that checks if a term occurs in a list can be defined as follows:

```
membchk(X,[X|_]) => true.
membchk(X,[_|L]) => membchk(X,L) .
```

The first rule is applicable to a call if the second argument is a list and the first argument of the call is identical to the first element of the list.

Picat allows inclusion of predicate facts in the form $p\left(t_{1}, \ldots, t_{n}\right)$ in predicate definitions. Facts are translated into pattern-matching rules before they are compiled. A predicate definition that consists of facts can be preceded by an index declaration in the form index $\left(M_{11}, \ldots, M_{1 n}\right)$ $\ldots\left(M_{m 1}, \ldots, M_{m n}\right)$ where each $M_{i j}$ is either + (meaning indexed) or - (meaning not indexed). For each index pattern ( $M_{i 1}, \ldots, M_{i n}$ ), the compiler generates a version of the predicate that indexes all of the + arguments.

## Example

```
index (+,-) (-,+)
edge (a,b).
edge (a,c).
edge (b, c).
edge (c,b).
```

For a predicate of indexed facts, a matching version of the predicate is selected for a call. If no matching version is available, Picat throws an exception. For example, for the call edge ( $\mathrm{X}, \mathrm{Y}$ ), if both $X$ and $Y$ are free, then no version of the predicate matches this call and Picat throws an exception. If predicate facts are not preceded by any index declaration, then no argument is indexed.

### 1.3 Defining Functions

A function call always succeeds with a return value if no exception occurs. Functions are defined with non-backtrackable rules in which the head is an equation $F=X$, where $F$ is the function pattern in the form $f\left(t_{1}, \ldots, t_{n}\right)$ and $X$ holds the return value. When $n=0$, the parentheses can be omitted.

## Example

```
fib(0)=F => F=1.
fib(1)=F => F=1.
fib(N) =F,N>1 => F=fib(N-1) +fib(N-2).
qsort([])=L => L=[].
qsort([H|T])=L => L = qsort([E : E in T, E=<H])++[H]++
    qsort([E : E in T, E>H]).
```

A function call never fails and never succeeds more than once. For function calls such as $f i b(-1)$ or fib (X), Picat raises an exception.

Picat allows inclusion of function facts in the form $f\left(t_{1}, \ldots, t_{n}\right)=E x p$ in function definitions.

## Example

```
fib(0)=1.
fib(1)=1.
fib(N) =F,N>1 => F=fib(N-1) +fib(N-2).
qsort([])=[].
qsort([H|T]) =
    qsort([E : E in T, E=<H])++[H]++qsort([E : E in T, E>H]).
```

Function facts are automatically indexed on all of the input arguments, and hence no index declaration is necessary. Note that while a predicate call with no argument does not need parentheses, a function call with no argument must be followed with parentheses, unless the function is modulequantified, as in math. pi.

The fib function can also be defined as follows:

```
fib(N) = cond((N=0;N=1),1,fib(N-1) +fib(N-2)).
```

The conditional expression returns 1 if the condition ( $\mathrm{N}=0 ; \mathrm{N}=1$ ) is true, and the value of fib $(N-1)+f i b(N-2)$ if the condition is false.

### 1.4 Assignments and Loops

Picat allows assignments in rule bodies. An assignment takes the form $L H S:=R H S$, where $L H S$ is either a variable or an access of a compound value in the form $X[\ldots]$. When $L H S$ is an access in the form $X[I]$, the component of $X$ indexed $I$ is updated. This update is undone if execution backtracks over this assignment.

## Example

```
test \(=>X=0, X:=X+1, X:=X+2\), write \((X)\).
```

In order to handle assignments, Picat creates new variables at compile time. In the above example, at compile time, Picat creates a new variable, say X 1 , to hold the value of X after the assignment $\mathrm{X}:=\mathrm{X}+1$. Picat replaces X by X 1 on the LHS of the assignment. It also replaces all of the occurrences of $X$ to the right of the assignment by X 1 . When encountering $\mathrm{X} 1:=\mathrm{X} 1+2$, Picat creates another new variable, say X 2 , to hold the value of X 1 after the assignment, and replaces the remaining occurrences of X 1 by X 2 . When write $(\mathrm{X} 2)$ is executed, the value held in X 2 , which is 3 , is printed. This means that the compiler rewrites the above example as follows:

```
test => X=0, X1=X+1, X2=X1+2, write(X2).
```

Picat supports foreach and while statements for programming repetitions. A foreach statement takes the form

```
foreach ( }\mp@subsup{E}{1}{}\mathrm{ in }\mp@subsup{D}{1}{},\mp@subsup{C}{ond}{1},\ldots,\ldots,\mp@subsup{E}{n}{}\mathrm{ in }\mp@subsup{D}{n}{},\mp@subsup{C}{ond}{n}
    Goal
end
```

where each iterator, $E_{i}$ in $D_{i}$, can be followed by an optional condition Cond $_{i}$. Within each iterator, $E_{i}$ is an iterating pattern, and $D_{i}$ is an expression that gives a compound value. The foreach statement means that Goal is executed for every possible combination of values $E_{1} \in$ $D_{1}, \ldots, E_{n} \in D_{n}$ that satisfies the conditions $\operatorname{Cond}_{1}, \ldots$, Cond $_{n}$. A while statement takes the form

```
while (Cond)
Goal
end
```

It repeatedly executes Goal as long as Cond succeeds. A variant of the while loop in the form of

```
do
    Goal
while (Cond)
```

executes Goal one time before testing Cond.
A loop statement forms a name scope. Variables that occur only in a loop, but do not occur before the loop in the outer scope, are local to each iteration of the loop. For example, in the following rule:

```
p(A) =>
    foreach (I in 1 .. A.length)
        E = A[I],
        writeln(E)
    end.
```

the variables I and $E$ are local, and each iteration of the loop has its own values for these variables.

## Example

```
write_map(Map) =>
    foreach (Key=Value in Map)
        writef("%w=%w\n",Key,Value)
    end.
sum_list(L)=Sum => % returns sum(L)
    S=0,
    foreach (X in L)
        S:=S+X
    end,
    Sum=S .
read_list=List =>
```

```
L=[ ],
E=read_int(),
while (E != 0)
    L := [E|L],
    E := read_int()
end,
List=L.
```

The function read_list reads a sequence of integers into a list, terminating when 0 is read. The loop corresponds to the following sequence of recurrences:

$$
\begin{aligned}
& L=[] \\
& L_{1}=\left[e_{1} \mid L\right] \\
& L_{2}=\left[e_{2} \mid L_{1}\right] \\
& \ldots \\
& L_{n}=\left[e_{n} \mid L_{n-1}\right] \\
& \text { List }=L_{n}
\end{aligned}
$$

Note that the list of integers is in reversed order. If users want a list in the same order as the input, then the following loop can be used:

```
read_list=List =>
    List=L,
    E=read_int(),
    while (E != 0)
        L = [E|T],
        L := T,
        E := read_int()
    end,
    L= [ ] .
```

This loop corresponds to the following sequence of recurrences:

```
\(L=\left[e_{1} \mid L_{1}\right]\)
\(L_{1}=\left[e_{2} \mid L_{2}\right]\)
\(L_{n-1}=\left[e_{n} \mid L_{n}\right]\)
\(L_{n}=\) [ ]
```

Loop statements are compiled into tail-recursive predicates. For example, the second read_list function given above is compiled into:

```
read_list=List =>
    List=L,
    E=read_int(),
    p(E,L,Lout),
    Lout=[].
p(0,Lin,Lout) => Lout=Lin.
p(E,Lin,Lout) =>
    Lin=[E|Lin1],
    NE = read_int(),
    p(NE,Lin1,Lout).
```

A list comprehension is first compiled into a foreach loop, and then the loop is compiled into a call to a generated tail-recursive predicate. For example, the list comprehension

```
List = [(A,X) : A in [a,b], X in 1..2]
```

is compiled into the following loop:

```
List = L,
foreach(A in [a,b], X in 1..2)
    L = [(A,X)|T],
    L := T
end,
L = [].
```


### 1.5 Tabling

A predicate defines a relation where the set of facts is implicitly generated by the rules. The process of generating the facts may never end and/or may contain a lot of redundancy. Tabling can prevent infinite loops and redundancy by memorizing calls and their answers. In order to have all calls and answers of a predicate or function tabled, users just need to add the keyword table before the first rule.

```
Example
table
fib(0) = 1.
fib(1) = 1.
fib(N) = fib(N-1)+fib(N-2).
```

When not tabled, the function call fib ( N ) takes exponential time in N . When tabled, however, it takes only linear time.

Users can also give table modes to instruct the system on what answers to table. Mode-directed tabling is especially useful for dynamic programming problems. In mode-directed tabling, a plussign (+) indicates input, a minus-sign (-) indicates output, max indicates that the corresponding variable should be maximized, min indicates that the corresponding variable should be minimized, and $n t$ indicates that the corresponding argument is not tabled. ${ }^{2}$

## Example

```
table(+,+,min)
edit([],[],D) => D=0.
edit([X|Xs],[X|Ys],D) =>
    edit(Xs,Ys,D).
edit(Xs,[Y|Ys],D) ?=> % insert
    edit(Xs,Ys,D1),
    D=D1+1.
edit([X|Xs],Ys,D) => % delete
    edit(Xs,Ys,D1),
    D=D1+1.
```

[^1]For a call edit (L1, L2, D), where L1 and L2 are given lists and $D$ is a variable, the rules can generate all facts, each of which contains a different editing distance between the two lists. The table mode table $(+,+, \min )$ tells the system to keep a fact with the minimal editing distance.

A tabled predicate can be preceded by both a table declaration and at most one index declaration if it contains facts. The order of these declarations is not important.

### 1.6 Modules

A module is a source file with the extension . pi. A module begins with a module name declaration and optional import declarations. A module declaration has the form:

```
module Name.
```

where Name must be the same as the main file name. A file that does not begin with a module declaration is assumed to belong to the global module, and all of the predicates and functions that are defined in such a file are visible to all modules as well as the top-level of the interpreter.

An import declaration takes the form:

```
import Name., ..., Namen.
```

where each $N a m e_{i}$ is a module name. When a module is imported, all of its public predicates and functions will be visible to the importing module. A public predicate or function in a module can also be accessed by preceding it with a module qualifier, as in $\mathrm{m} . \mathrm{p}()$, but the module still must be imported.

Atoms and structure names do not belong to any module, and are globally visible. In a module, predicates and functions are assumed to be visible both inside and outside of the module, unless their definitions are preceded by the keyword private.

## Example

```
% in file my_sum.pi
module my_sum.
my__sum(L) =Sum =>
    sum_aux(L, 0, Sum).
private
sum_aux([],Sum0, Sum) => Sum=Sum0.
sum_aux([X|L],Sum0,Sum) => sum_aux(L, X+Sum0,Sum).
% in file test_my_sum.pi
module test_my_sum.
import my_sum.
go =>
    writeln(my_sum([1, 2, 3, 4])).
```

The predicate sum_aux is private, and is never visible outside of the module. The following shows a session that uses these modules.

```
Picat> load("test_my_sum")
Picat> go
10
```

The command load (File) loads a module file into the system. If the file has not been compiled, then the load command compiles the file before loading it. If this module is dependent on other modules, then the other modules are loaded automatically if they are not yet in the system. ${ }^{3}$ When a module is loaded, all of its public predicates and functions become visible to the interpreter

The Picat module system is static, meaning that the binding of normal calls to their definitions takes place at compile time. For higher-order calls, however, Picat may need to search for their definitions at runtime. Several built-in modules are imported by default, including basic, io, math, and sys. For a normal call that is not higher-order in a module, the Picat compiler searches modules for a definition in the following order:

1. The implicitly imported built-in modules in the order from basic to sys.
2. The enclosing module of the call.
3. The explicitly imported modules in the order that they were imported.
4. The global module.

### 1.7 Constraints

Picat can be used as a modeling and solving language for constraint satisfaction and optimization problems. A constraint program normally poses a problem in three steps: (1) generate variables; (2) generate constraints over the variables; and (3) call solve to find a valuation for the variables that satisfies the constraints, and possibly optimizes an objective function. Picat provides three solver modules, including cp , sat, and mip.

```
Example
import cp.
go =>
    Vars=[S,E,N,D,M,O,R,Y], % generate variables
    Vars :: 0..9,
    all_different(Vars), % generate constraints
    S #!= 0,
    M #!= 0,
    1000*S+100*E+10*N+D+1000*M+100*O+10*R+E
            #=10000*M+1000*O+100*N+10*E+Y,
    solve(Vars), % search
    writeln(Vars).
```

In arithmetic constraints, expressions are treated as data, and it is unnecessary to enclose them with dollar-signs.

The loops provided by Picat facilitate modeling of many constraint satisfaction and optimization problems. The following program solves a Sudoku puzzle:

[^2]```
import cp.
sudoku =>
    instance(N,A),
    A :: 1..N,
    foreach(Row in 1..N)
        all_different(A[Row])
    end,
    foreach(Col in 1..N)
        all_different([A[Row,Col] : Row in 1..N])
    end,
    M=floor(sqrt(N)),
    foreach(Row in 1..M, Col in 1..M)
        Square = [A[Row1,Col1] :
                                    Row1 in (Row-1)*M+1..Row*M,
                                    Col1 in (Col-1)*M+1..Col*M],
            all_different(Square)
    end,
    solve(A),
    foreach(I in 1..N) writeln(A[I]) end.
instance(N,A) =>
    N=9,
    A={{5,3,_'_, 7,_\prime_'_'__},
        {6,_,_,1,9,5,_\prime_'_},
        {_,9,8,_'_'_'_,6,_},
        {8,_,_r_,6,_\prime_,_,3},
        {4,_\prime_,8,_,3,_r_,1},
        {7,_,_'_, 2,_\prime_,_,6},
        {_\prime_,_\prime_\prime_\prime_'_\prime_'__},
        {_\prime_,_'_\prime__\prime_'_'_\prime__},
        {_'_'_'_'_'_'_'_'__},
        {_'_'_,_'_,_,_'_'_} }.
```

Recall that variables that occur within a loop, and do not occur before the loop in the outer scope, are local to each iteration of the loop. For example, in the third foreach statement of the sudoku predicate, the variables Row, Col, and Square are local, and each iteration of the loop has its own values for these variables.

### 1.8 Exceptions

An exception is an event that occurs during the execution of a program that requires a special treatment. In Picat, an exception is just a term. Example exceptions thrown by the system include divide_by_zero, file_not_found, number_expected, interrupt, and out_of_range. The exception interrupt (keyboard) is raised when ctrl-c is typed during a program's execution. The built-in predicate throw Exception throws Exception.

All exceptions, including those raised by built-ins and interruptions, can be caught by catchers. A catcher is a call in the form: catch (Goal, Exception, Handler) which is equivalent to Goal, except when an exception is raised during the execution of Goal that unifies Exception.

When such an exception is raised, all of the bindings that have been performed on variables in Goal will be undone, and Handler will be executed to handle the exception.

The call call_cleanup (Goal, Cleanup) is equivalent to call (Call), except that Cleanup is called when Goal succeeds determinately (i.e., with no remaining choice point), when Goal fails, or when Goal raises an exception.

### 1.9 Higher-Order Calls

A predicate or function is said to be higher-order if it takes calls as arguments. The built-ins call, apply, and find_all are higher-order. The predicate call ( $S, \operatorname{Arg}_{1}, \ldots, \operatorname{Arg} g_{n}$ ), where $S$ is an atom or a structure, calls the predicate named by $S$ with the arguments that are specified in $S$ together with extra arguments $\operatorname{Arg} g_{1}, \ldots, \operatorname{Arg} g_{n}$. The function apply ( $S, \operatorname{Arg}, \ldots, \operatorname{Arg} g_{n}$ ) is similar to call, except that apply returns a value. The function findall (Template, $S$ ) returns a list of all possible solutions of call $(S)$ in the form of Template. Other higher-order predicates include call_cleanup/2, catch/3, count_all, freeze/2, not/1, maxof/2-3, maxof_inc/2-3, minof/2-3, minof_inc/2-3, once/1, time/1, time_out/3 and time2/1. All of these higher-order predicates are defined in the basic module, except for time/1, time2/1, and time_out/3, which are defined in the sys module. Higher-order calls cannot contain assignments or loops.

## Example

```
Picat> S=$member(X), call(S,[1, 2, 3])
X=1;
X=2;
X=3;
no
Picat> L=findall(X,member(X,[1,2,3])).
L=[1, 2, 3]
Picat> Z=apply('+',1,2)
Z=3
```

Among the higher-order built-ins, findall is special in that it forms a name scope like a loop. Local variables that occur in a findall call are not visible to subsequent calls in the body or query.

The meta-call apply never returns a partially evaluated function. If the number of arguments does not match the required number, then it throws an exception.

## Example

```
map(_F,[]) = [].
map(F,[X|Xs])=[apply(F,X)|map(F,Xs)].
map2(_F,[],[]) = [].
map2(F,[X|Xs],[Y|Ys])=[apply(F,X,Y)|map2(F,Xs,Ys)].
fold(_F,Acc,[]) = Acc.
fold(F,Acc,[H|T])=fold(F, apply(F,H,Acc),T).
```

A call that is passed to apply is assumed to invoke a definition in a pre-imported built-in module, the enclosing module in which apply occurs, an imported module of the enclosing module, or the global module. Due to the overhead of runtime search, the use of higher-order calls is discouraged. Whenever possible, recursion, loops, or list and array comprehensions should be used instead.

### 1.10 Action Rules

Picat provides action rules for describing event-driven actors. An actor is a predicate call that can be delayed, and can be activated later by events. Each time an actor is activated, an action can be executed. A predicate for actors contains at least one action rule in the form:

```
Head, Cond, \(\{\) Event \(\}=>\) Body
```

where Head is an actor pattern, Cond is an optional condition, Event is a non-empty set of event patterns separated by ', ', and Body is an action. For an actor and an event, an action rule is said to be applicable if the actor matches Head and Cond is true. A predicate for actors cannot contain backtrackable rules.

An event channel is an attributed variable to which actors can be attached, and through which events can be posted to actors. A channel has four ports: ins, bound, dom, and any. An event pattern in Event specifies the port to which the actor is attached. The event pattern ins $(X)$ attaches the actor to the ins-port of channel $X$, and the actor will be activated when $X$ is instantiated. The event pattern event $(X, T)$ attaches the actor to the dom-port of channel $X$. The built-in post_event ( $X, T$ ) posts an event term $T$ to the dom-port of channel $X$. After an event is posted to a port of a channel, the actors attached to that port are activated. For an activated actor, the system searches for an applicable rule and executes the rule body if it finds one. After execution, the actor is suspended, waiting to be activated again by other events. Picat does not provide a built-in for detaching actors from channels. An actor fails if no rule is applicable to it when it is activated or the body of the applied rule fails. An actor becomes a normal call once a normal non-backtrackable rule is applied to it.

## Example

```
echo(X,Flag),var(Flag),{event(X,T)} => writeln(T).
echo(_X,_Flag) => writeln(done).
foo(Flag) => Flag=1.
```

When a call echo ( $\mathrm{X}, \mathrm{Fl}$ ag ) is executed, where Flag is a variable, it is attached to the dom-port of $X$ as an actor. The actor is then suspended, waiting for events posted to the dom-port. For this actor definition, the command

```
echo(X,Flag), post_event(X,hello), post_event(X,picat).
```

prints out hello followed by picat. If the call foo(Flag) is inserted before the second call to post_event, then var (Flag) fails when the actor is activated the second time, causing the second rule to be applied to the actor. Then, the output will be hello followed by done. Note that events are not handled until a non-inline call is executed. Replacing foo (Flag) by Flag=1 will result in a different behavior because Flag=1 is an inline call.

### 1.11 Prebuilt Maps

Picat has three kinds of prebuilt maps: heap maps, global maps, and table maps. Prebuilt heap maps are created on the heap immediately after the system is started. The built-in function get_heap_map $(I D)$ returns the heap map that is associated with $I D$, where $I D$ must be a ground term. If no heap map is associated with $I D$, then this function establishes an association between $I D$ and an unused heap map, and returns the map. A heap map is like a normal map. Users use put to add key-value pairs into the map. Users use get to retrieve a value that is associated with a key in the map. Changes to a heap map up to a choice point are undone when execution backtracks to that choice point. The built-in function get_heap_map () returns the heap map that is associated with a system-generated default identifier. There are an unlimited number of prebuilt heap maps.

Global maps are created in the global area when the Picat system is started. The built-in function get_global_map $(I D)$ returns the global map that is associated with $I D$, where $I D$ must be a ground term. If no global map is associated with $I D$, then this function establishes an association between $I D$ and an unused global map, and returns the map. A big difference between a global map and a heap map is that changes to the global map are not undone upon backtracking. When a key-value pair is added into the global map, the variables in the value term are numbered before they are copied to the global area. If the value term contains attributed variables, then the attributes of the variables are not copied, and are therefore lost. When retrieving a value that is associated with a key, the value term in the global area is copied back to the heap after all of the numbered variables are unnumbered. The built-in function get_global_map () returns the global map that is associated with a system-generated default identifier. The number of prebuilt global maps is 97 , and the system halts if a program requests more than 97 global maps.

Table maps are created in the table area when the Picat system is started. The built-in function get_table_map $(I D)$ returns the table map that is associated with $I D$, where $I D$ must be a ground term. If no table map is associated with $I D$, then this function establishes an association between $I D$ and an unused table map, and returns the map. Like the global map, changes to a table map are not undone upon backtracking. Unlike the global map, however, keys and values are hash-consed so that common ground sub-terms are not replicated in the table area. The builtin function get_table_map () returns the table map that is associated with a system-generated default identifier. The number of prebuilt table maps is 97 , and the system halts if a program requests more than 97 table maps.

The advantage of using prebuilt maps is that data can be accessed everywhere without being passed as arguments, and the disadvantage is that it affects locality of data and thus the readability of programs. In tabled programs, using prebuilt maps is discouraged because it may cause unanticipated effects.

## Example

```
go ?=>
    get_heap_map(h1).put(one,1),
    get_global_map(g1).put(one,1),
    get_table_map(t1).put(one,1),
    fail.
go =>
    if (get_heap_map(h1).has_key(one)) then
        writef("heap map h1 has key%n")
        else
            writef("heap map h1 has no key%n")
```

```
end,
if (get_global_map(g1).has_key(one)) then
    writef("global map g1 has key%n")
else
    writef("global map g1 has no key%n")
end,
if (get_table_map(t1).has_key(one)) then
    writef("table map t1 has key%n")
else
    writef("table map t1 has no key%n")
end.
```

For the call go, the output is:

```
heap map h1 has no key
global map g1 has key
table map t1 has key
```

The fail call in the first rule causes execution to backtrack to the second rule. After backtracking, the pair added to the heap map by the first rule is lost, but the pair added to the global map and the pair added to the table map remain.

### 1.12 Programming Exercises

Project Euler (projecteuler. net) is an excellent platform for practicing programming and problem solving skills. You can find Picat solutions to some of the problems at picat-lang. org/projects.html. Select five problems from the Project Euler problem set for which no solutions have been posted, and write a program in Picat for each of them.

## Chapter 2

## How to Use the Picat System

### 2.1 How to Use the Picat Interpreter

The Picat system is written in both C and Picat. The Picat interpreter is provided as a single standalone executable file, named picat.exe for Windows and picat for Unix. The Picat interpreter provides an interactive programming environment for users to compile, load, debug, and execute programs. In order to start the Picat interpreter, users first need to open an OS terminal. In Windows, this can be done by selecting Start->Run and typing cmd or selecting Start->Programs->Accessories->Command Prompt. In order to start the Picat interpreter in any working directory, the environment variable path must be properly set to contain the directory where the executable is located.

### 2.1.1 How to Enter and Quit the Picat Interpreter

The Picat interpreter is started with the OS command picat.

## OSPrompt picat

where OSPrompt is the OS prompt. After the interpreter is started, it responds with the prompt Picat>, and is ready to accept queries.

In general, the picat command takes the following form:
picat Options PicatMainFileName $A_{1} A_{2} \ldots A_{n}$
where PicatMainFileName can have the extension .pi and can contain a path of directories, and Options is a sequence of options of the following:

-     - 9 InitGoal: This option makes Picat execute a specified initial query InitGoal rather than the default main predicate.
- --help: Print out the help info.
- -log: The option -log makes the system print log information and warning messages.
- -path Directories: Directories is a semicolon-separated and double-quoted list of directories. This option sets the value of the environment variable PICATPATH before the execution of the program. The Picat system will look for PicatMainFileName and the related modules in these directories.
- -s Size: This option reserves Size words for the stack and the heap when the system is started.
- --v:
- --version: This option makes Picat print the version number.

Once the interpreter is started, users can type a query after the prompt. For example,

```
Picat> X=1+1
X=2
Picat> printf("hello"++" picat")
hello picat
```

The halt predicate, or the exit predicate, terminates the Picat interpreter. An alternative way to terminate the interpreter is to enter ctrl-d (control-d) when the cursor is located at the beginning of an empty line.

### 2.1.2 How to Use the Command-line Editor

The Picat interpreter uses the get line program written by Chris Thewalt. The getline program memorizes up to 100 of the most recent queries that the users have typed, and allows users to recall past queries and edit the current query by using Emacs editing commands. The following gives the editing commands:
ctrl-f Move the cursor one position forward.
ctrl-b Move the cursor one position backward.
ctrl-a Move the cursor to the beginning of the line.
ctrl-e Move the cursor to the end of the line.
ctrl-d Delete the character under the cursor.
ctrl-h Delete the character to the left of the cursor.
ctrl-k Delete the characters to the right of the cursor.
ctrl-u Delete the whole line.
ctrl-p Load the previous query in the buffer.
ctrl-n Load the next query in the buffer.
Note that the command ctrl-d terminates the interpreter if the line is empty and the cursor is located in the beginning of the line.

### 2.1.3 How to Compile and Load Programs

A Picat program is stored in one or more text files with the extension name pi. A file name is a string of characters. Picat treats both $' / \prime^{\prime}$ and ${ }^{\prime} \backslash^{\prime}$ as file name separators. Nevertheless, since ${ }^{\prime} \backslash^{\prime}$ is used as the escape character in quoted strings, two consecutive backslashes must be used, as in "c: <br>work $\backslash \backslash$ myfile.pi", if $\backslash^{\prime}$ is used as the separator.

For the sake of demonstration we assume the existence of a file named welcome.pi in the current working directory that stores the following program:

```
main =>
    print(" Welcome to PICAT's world! \n ").
main(Args) =>
    print(" Welcome to PICAT's world! \n"),
    foreach (Arg in Args)
        printf("%s\n", Arg)
    end.
```

- cl (FileName): A program first needs to be compiled and loaded into the system before it can be executed. The built-in predicate cl (FileName) compiles and loads the source file named FileName.pi. Note that if the full path of the file name is not given, then the file is assumed to be in the current working directory. Also note that users do not need to give the extension name. The system compiles and loads not only the source file FileName. pi, but also all of the module files that are either directly imported or indirectly imported by the source file. The system searches for such dependent files in the directory in which FileName.pi resides or the directories that are stored in the environment variable PICATPATH. For FileName.pi, the cl command loads the generated byte-codes without creating a byte-code file. For example,

```
Picat> cl("welcome")
Compiling:: welcome.pi
welcome.pi compiled in 4 milliseconds
```

- cl: The built-in predicate cl (with no argument) compiles and loads a program from the console, ending when the end-of-file character (ctrl-z for Windows and ctrl-d for Unix) is typed.
- compile (FileName) : The built-in predicate compile (FileName) compiles the file FileName.pi and all of its dependent module files without loading the generated bytecode files. The destination directory for the byte-code file is the same as the source file's directory. If the Picat interpreter does not have permission to write into the directory in which a source file resides, then this built-in throws an exception. For example,

```
Picat> compile("welcome")
Compiling::welcome.pi
welcome.pi compiled in 4 milliseconds
```

- load (FileName): The built-in predicate load (FileName) loads the byte-code file FileName. qi and all of its dependent byte-code files. For FileName and its dependent file names, the system searches for a byte-code file in the directory in which FileName. qi resides or the directories that are stored in the environment variable PICATPATH. If the byte-code file FileName. qi does not exist but the source file FileName.pi exists, then this built-in compiles the source file and loads the byte codes without creating a qi file.

```
Picat> load("welcome")
loading...welcome.qi
```


### 2.1.4 How to Run Programs

After a program is loaded, users can query the program. For each query, the system executes the program, and reports yes when the query succeeds and no when the query fails. When a query that contains variables succeeds, the system also reports the bindings for the variables. For example,

```
Picat> cl("welcome")
Compiling:: welcome.pi
welcome.pi compiled in 4 milliseconds
loading...
```

```
yes
Picat> main
    Welcome to PICAT's world!
yes
```

Users can ask the system to find the next solution by typing ' ; ' after a solution if the query has multiple solutions. For example,

```
Picat> member(X,[1,2,3])
X=1;
X=2;
X=3;
no
```

Users can force a program to terminate by typing ctrl-c, or by letting it execute the builtin predicate abort. Note that when the system is engaged in certain tasks, such as garbage collection, users may need to wait for a while in order to see the termination after they type ctrl-c.

### 2.1.5 How to Run Programs Directly

Programs that define the main/0 predicate or the main/1 predicate can be run directly as a OS command. For example,

```
$ picat welcome
    Welcome to PICAT's world!
$ picat welcome a b c
    Welcome to PICAT's world!
a
b
C
```

The ' $\$$ ' sign is the prompt of the OS. It is assumed that the environment variable PATH has been set to contain the directory of the executable picat (picat. exe for Windows), and the environment variable PICATPATH has been set to contain the directory of the welcome.pi file or the file is in the current working directory.

### 2.1.6 Creating Standalone Executables

It is possible to create a script that can be run as a standalone executable. For example, consider the following script wel come. exe for Linux:

```
#!/bin/bash
picat welcome.pi
echo " Finished!"
```

Once the environment variables PATH and PICATPATH are set properly, and the script is set to have the execution permission, it can be executed as follows:

```
$ welcome.exe
    Welcome to PICAT's world!
    Finished!
```


### 2.2 How to Use the Debugger

The Picat system has three execution modes: non-trace mode, trace mode, and spy mode. In trace mode, it is possible to trace the execution of a program, showing every call in every possible stage. In order to trace the execution, the program must be recompiled while the system is in trace mode. In spy mode, it is possible to trace the execution of individual functions and predicates that are spy points. When the Picat interpreter is started, it runs in non-trace mode. The predicate debug or trace changes the mode to trace. The predicate nodebug or notrace changes the mode to non-trace.

In trace mode, the debugger displays execution traces of queries. An execution trace consists of a sequence of call traces. Each call trace is a line that consists of a stage, the number of the call, and the information about the call itself. For a function call, there are two possible stages: Call, meaning the time at which the function is entered, and Exit, meaning the time at which the call is completed with an answer. For a predicate call, there are two additional possible stages: Redo, meaning a time at which execution backtracks to the call, and Fail, meaning the time at which the call is completed with a failure. The information about a call includes the name of the call, and the arguments. If the call is a function, then the call is followed by = and ? at the Call stage, and followed by = Value at the Exit stage, where Value is the return value of the call.

Consider, for example, the following program:

$$
\begin{aligned}
& p(X) \quad ?=>X=a . \\
& p(X) \quad \Rightarrow \quad X=b . \\
& q(X) \quad ?=>X=1 . \\
& q(X) \quad \Rightarrow \quad X=2 .
\end{aligned}
$$

Assume the program is stored in a file named myprog.pi. The following shows a trace for a query:

```
Picat> debug
{Trace mode}
Picat> cl(myprog)
{Trace mode}
Picat> p(X),q(Y)
    Call: (1) p(_328) ?
    Exit: (1) p(a)
    Call: (2) q(_378) ?
    Exit: (2) q(1)
X = a
Y = 1 ?;
    Redo: (2) q(1) ?
    Exit: (2) q(2)
X = a
Y = 2 ?;
    Redo: (1) p(a) ?
```

```
    Exit: (1) p(b)
    Call: (3) q(_378) ?
    Exit: (3) q(1)
X = b
Y = 1 ?;
    Redo: (3) q(1) ?
    Exit: (3) q(2)
X = b
Y = 2 ?;
no
```

In trace mode, the debugger displays every call in every possible stage. Users can set spy points so that the debugger only shows information about calls of the symbols that users are spying. Users can use the predicate

## spy \$Name/N

to set the functor $N a m e / N$ as a spy point, where the arity $N$ is optional. If the functor is defined in multiple loaded modules, then all these definitions will be treated as spy points. If no arity is given, then any functor of Name is treated as a spy point, regardless of the arity.

After displaying a call trace, if the trace is for stage Call or stage Redo, then the debugger waits for a command from the users. A command is either a single letter followed by a carriagereturn, or just a carriage-return. See Appendix B for the debugging commands.

## Chapter 3

## Data Types, Operators, and Built-ins

Picat is a dynamically-typed language, in which type checking occurs at runtime. A variable gets a type once it is bound to a value. In Picat, variables and values are terms. A value can be primitive or compound. A primitive value can be an integer, a real number, or an atom. A compound value can be a list or a structure. Strings, arrays, and maps are special compound values. This chapter describes the data types and the built-ins for each data type that are provided by the basic module.

Many of the built-ins are given as operators. Table 3.1 shows all of the operators that are provided by Picat. Unless the table specifies otherwise, the operators are left-associative. The as-pattern operator ( ( ) and the operators for composing goals, including not, once, conjunction (, and $\& \&$ ), and disjunction (; and |।), will be described in Chapter 4 on Predicates and Functions. The constraint operators (the ones that begin with \#) will be described in Chapter 12 on Constraints. In Picat, no new operators can be defined, and none of the existing operators can be redefined.

The dot operator (.) is used in OOP notations for calling predicates and functions. It is also used to qualify calls with a module name. The notation $A_{1} \cdot f\left(A_{2}, \ldots, A_{k}\right)$ is the same as $f\left(A_{1}, A_{2}, \ldots, A_{k}\right)$, unless $A_{1}$ is an atom, in which case $A_{1}$ must be a module qualifier for $f$. If an atom needs to be passed as the first argument to a function or a predicate, then this notation cannot be used. The notation A.Attr, where Attr does not have the form $\mathrm{f}(\ldots)$, is the same as the function call get $(A, A t t r)$. For example, the expression $S$. name returns the name, and the expression $S$.arity returns the arity of $S$ if $S$ is a structure. Note that the dot operator is left-associative. For example, the expression $X . f() \cdot g()$ is the same as $g(f(X))$.

The following functions are provided for all terms:

- copy_term $\left(\right.$ Term $\left._{1}\right)=$ Term $_{2}$ : This function copies Term $_{1}$ into Term $_{2}$. If Term $_{1}$ is an attributed variable, then $\mathrm{Term}_{2}$ will not contain any of the attributes.
- hash_code (Term) = Code: This function returns the hash code of Term. If Term is a variable, then the returned hash code is always 0 .
- to_codes (Term) = Codes: This function returns a list of character codes of Term.
- to_fstring (Format, Args...): This function converts the arguments in the Args... parameter into a string, according to the format string Format, and returns the string. The number of arguments in Args ... cannot exceed 10. Format characters are described in Chapter 10.
- to_string $($ Term $)=$ String: This function returns a string representation of Term.

Other built-ins on terms are given in Sections 3.5 and 3.8.

Table 3.1: Operators in Picat

| Precedence | Operators |
| :---: | :---: |
| Highest | . , @ |
|  | ** (right-associative) |
|  | unary +, unary -, ~ |
|  | *, /, //, /<, />, div, mod, rem |
|  | binary +, binary - |
|  | >>, << |
|  | / |
|  | - |
|  | \/ |
|  | -• |
|  | ++ (right-associative) |
|  | $\begin{aligned} & =,!=,:=,==,!==,=:=,<,=<,<=,>,>=,::, \text { in, notin } \\ & \#=, \#!=, \#<, \#=<, \#<=, \#>, \#>=, \text { @ }<, ~ @=<, ~ @<=, ~ @>, ~ @>= \end{aligned}$ |
|  | \# |
|  | \# / \} |
|  | \# ${ }^{\text {- }}$ |
|  | \# \/ |
|  | \# => (right-associative) |
|  | \#<=> |
|  | not, once |
|  | , (right-associative), \&\& (right-associative) |
| Lowest | ; (right-associative), \|| (right-associative) |

### 3.1 Variables

Variables in Picat, like variables in mathematics, are value holders. Unlike variables in imperative languages, Picat variables are not symbolic addresses of memory locations. A variable is said to be free if it does not hold any value. A variable is instantiated when it is bound to a value. Picat variables are single-assignment, which means that after a variable is instantiated to a value, the variable will have the same identity as the value. After execution backtracks over a point where a binding took place, the value that was assigned to a variable will be dropped, and the variable will be turned back into a free variable.

A variable name is an identifier that begins with a capital letter or the underscore. For example, the following are valid variable names:
$\mathrm{X1}$ - _ab
The name _ is used for anonymous variables. In a program, different occurrences of _ are treated as different variables. So the test ${ }_{-}==_{-}$is always false.

The following two built-ins are provided to test whether a term is a free variable:

- var $($ Term $)$ : This predicate is true if Term is a free variable.
- nonvar (Term) : This predicate is true if Term is not a free variable.

An attributed variable is a variable that has a map of attribute-value pairs attached to it. The following built-ins are provided for attributed variables:

- attr_var (Term) : This predicate is true if Term is an attributed variable.
- dvar (Term) : This predicate is true if Term is an attributed domain variable.
- bool_dvar (Term): This predicate is true if Term is an attributed domain variable whose lower bound is 0 and whose upper bound is 1 .
- dvar_or_int (Term) : This predicate is true if Term is an attributed domain variable or an integer.
- get_attr $(X, K e y)=V a l$ : This function returns the Val of the key-value pair Key=Val that is attached to $X$. It throws an error if $X$ has no attribute named Key.
- get_attr $(X, K e y, ~ D e f a u l t V a l)=$ Val: This function returns Val of the keyvalue pair $K e y=V a l$ that is attached to $X$. It returns DefaultVal if $X$ does not have the attribute named Key.
- put_attr $(X, K e y, V a l)$ : This predicate attaches the key-value pair $K e y=V a l$ to $X$, where Key is a non-variable term, and Val is any term.
- put_attr $(X, K e y):$ This predicate call is the same as put_attr $(X, K e y$, not_a_value).


### 3.2 Atoms

An atom is a symbolic constant. An atom name can either be quoted or unquoted. An unquoted name is an identifier that begins with a lower-case letter, followed by an optional string of letters, digits, and underscores. A quoted name is a single-quoted sequence of arbitrary characters. A character can be represented as a single-character atom. For example, the following are valid atom names:

```
x x_1 '_'' '\\' 'a\'亘\n' '__ab' '$%'
```

No atom name can last more than one line. An atom name cannot contain more than 1000 characters. The backslash character ${ }^{\prime} \backslash^{\prime}$ is used as the escape character. So, the name ' $\mathrm{a} \backslash^{\prime} \mathrm{b} \backslash \mathrm{n}^{\prime}$ contains four characters: $\mathrm{a},{ }^{\prime}, \mathrm{b}$, and $\backslash \mathrm{n}$.

The following built-ins are provided for atoms:

- atom $($ Term $)$ : This predicate is true if Term is an atom.
- atom_chars $($ Atm $)=$ String: This function returns string that contains the characters of the atom Atm. It throws an error if Atm is not an atom.
- atom_codes $($ Atm $)=$ List: This function returns the list of codes of the characters of the atom Atm . It throws an error if Atm is not an atom.
- atomic (Term): This predicate is true if Term is an atom or a number.
- char (Term): This predicate is true if Term is an atom and the atom is made of one character.
- chr (Code) $=$ Char: This function returns the UTF-8 character of the code point Code.
- digit (Term): This predicate is true if Term is an atom and the atom is made of one digit.
- len $($ Atom $)=$ Len: This function returns the number of characters in Atom. Note that this function is overloaded in such a way that the argument can also be an array, a list, or a structure.
- length $($ Atom $)=$ Len: This function is the same as len $($ Atom $)$.
- ord (Char) = Int: This function returns the code point of the UTF-8 character Char. It throws an error if Char is not a single-character atom.


### 3.3 Numbers

A number can be an integer or a real number. An integer can be a decimal numeral, a binary numeral, an octal numeral, or a hexadecimal numeral. In a numeral, digits can be separated by underscores, but underscore separators are ignored by the tokenizer. For example, the following are valid integers:

12_345 a decimal numeral
0.b100 4 in binary notation

007359 in octal notation
$0 \times f 7 \quad 247$ in hexadecimal notation
A real number consists of an optional integer part, an optional decimal fraction preceded by a decimal point, and an optional exponent. If an integer part exists, then it must be followed by either a fraction or an exponent in order to distinguish the real number from an integer literal. For example, the following are valid real numbers.

$$
12.345 \quad 0.123 \quad 12-\mathrm{e} 10 \quad 0.12 \mathrm{E} 10
$$

Table 3.2: Arithmetic Operators

| X ** $Y$ | power |
| :---: | :---: |
| +X | same as $X$ |
| -X | sign reversal |
| X | bitwise complement |
| $X * Y$ | multiplication |
| $X / Y$ | division |
| $X / / Y$ | integer division, truncated |
| X /> Y | integer division (ceiling $(X / Y)$ ) |
| X $1<Y$ | integer division (floor $(X / Y)$ ) |
| $X$ div $Y$ | integer division, floored |
| $X \bmod Y$ | modulo, same as $X$ - floor $(X \operatorname{div} Y) * Y$ |
| $X$ rem $Y$ | remainder ( $X-(X / / Y) * Y)$ |
| $X+Y$ | addition |
| $X-Y$ | subtraction |
| X >> Y | right shift |
| $X \ll Y$ | left shift |
| $X / \backslash Y$ | bitwise and |
| X ^ $Y$ | bitwise xor |
| $X \backslash / Y$ | bitwise or |
| From .. Step . . To | A range (list) of numbers with a step |
| From . . To | A range (list) of numbers with step 1 |
| $X=:=Y$ | pretty much (numerically) equal |

Table 3.2 gives the meaning of each of the numeric operators in Picat, from the operator with the highest precedence $(* *)$ to the one with the lowest precedence (..). Except for the power operator $\star *$, which is right-associative, all of the arithmetic operators are left-associative.

In addition to the numeric operators, the basic module also provides the following built-ins for numbers:

- between (From, To, $X$ ) (nondet): If $X$ is bound to an integer, then this predicate determines whether $X$ is between From and To. Otherwise, if $X$ is unbound, then this predicate nondeterministically selects $X$ from the integers that are between From and To. It is the same as member ( $X$, From. .To) .
- float (Term) : This predicate is true if Term is a real number.
- int (Term) : This predicate is true if Term is an integer.
- integer (Term) : The same as int (Term).
- $\max (X, Y)=$ Val: This function returns the maximum of $X$ and $Y$, where $X$ and $Y$ are terms.
- maxint_small() = Int: This function returns the maximum integer that is represented in one word. All integers that are greater than this integer are represented as big integers.
- $\min (X, Y)=V a l$ : This function returns the minimum of $X$ and $Y$, where $X$ and $Y$ are terms.
- minint_small() = Int: This function returns the minimum integer that is represented in one word. All integers that are smaller than this integer are represented as big integers.
- number $($ Term $)$ : This predicate is true if Term is a number.
- number_chars $(N u m)=$ String: This function returns a list of characters of Num. This function is the same as to_fstring ("\%d",Num) if Num is an integer, and the same as to_fstring("\%f",Num) if Num is a real number.
- number_codes $(N u m)=$ List: This function returns a list of codes of the characters of Num. It is the same as number_chars (Num). to_codes ().
- real (Term) : This predicate is the same as float (Term).
- to_binary_string (Int) = String: This function returns the binary representation of the integer Int as a string.
- to_float $(N S)=$ Real: This function is the same as $N S \star 1.0$ if $N S$ is a number, and the same as parse_term ( $N S$ ) if $N S$ is a string of digits.
- to_hex_string (Int) = String: This function returns the hexadecimal representation of the integer Int as a string.
- to_int $(A N S)=I n t$ : This function is the same as truncate $(A N S)$ in the math module if $A N S$ is a number, the same as ord ( $A N S$ )-ord ('0') if $A N S$ is a digit character, and the same as parse_term ( $A N S$ ) if $A N S$ is a string.
- to_integer $(A N S)=I n t$ : This function is the same as to_int $(A N S)$.
- to_number $(A N S)=N u m$ : This function is the same as $A N S$ if $A N S$ is a number, the same as ord $(A N S)$-ord (' $\left.0^{\prime}\right)$ if $A N S$ is a digit character, and the same as parse_term ( $A N S$ ) if $A N S$ is a string.
- to_oct_string (Int) = String: This function returns the octal representation of the integer Int as a string.
- to_radix_string (Int, Base) = String: This function returns the representation of the integer Int of the numeral Base as a string, where Base must be greater than 1 and less than 37. The call to_oct_string (Int) is the same as to_radix_string (Int, 8).
- to_real $(N S)=$ Real: This function is the same as to_float $(N S)$.

The math module provides more numeric functions. See Appendix A.

### 3.4 Compound Terms

A compound term can be a list or a structure. Components of compound terms can be accessed with subscripts. Let $X$ be a variable that references a compound value, and let $I$ be an integer expression that represents a subscript. The index notation $X[I]$ is a special function that returns the $I$ th component of $X$, counting from the beginning. Subscripts begin at 1 , meaning that $X$ [1] is the first component of $X$. An index notation can take multiple subscripts. For example, the expression $\mathrm{X}[1,2]$ is the same as $\mathrm{T}[2]$, where T is a temporary variable that references the component that is returned by $\mathrm{X}[1]$. The predicate compound (Term) is true if Term is a compound term.

### 3.4.1 Lists

A list takes the form $\left[t_{1}, \ldots, t_{n}\right]$, where each $t_{i}(1 \leq i \leq n)$ is a term. Let $L$ be a list. The expression $L$. length, which is the same as the functions get ( $L$, length) and length ( $L$ ), returns the length of $L$. Note that a list is represented internally as a singly-linked list. Also note that the length of a list is not stored in memory; instead, it is recomputed each time that the function length is called.

The symbol ' |' is not an operator, but a separator that separates the first element (so-called $c a r$ ) from the rest of the list (so-called $c d r$ ). The cons notation [ $H \mid T$ ] can occur in a pattern or in an expression. When it occurs in a pattern, it matches any list in which $H$ matches the car and $T$ matches the cdr. When it occurs in an expression, it builds a list from $H$ and $T$. The notation $\left[A_{1}, A_{2}, \ldots, A_{n} \mid T\right]$ is a shorthand for $\left[A_{1} \mid\left[A_{2} \mid \ldots\left[A_{n} \mid T\right] \ldots\right]\right.$. So [a,b, c] is the same as [a|[b|[c|[]]]].

The basic module provides the following built-ins on lists, most of which are overloaded for strings (3.4.2) and arrays (see 3.4.4).

- List $_{1}++$ List $_{2}=$ List: This function returns the concatenated list of List $_{1}$ and List $_{2}$.
- append $(X, Y, Z)$ (nondet): This predicate is true if appending $Y$ to $X$ can create $Z$. This predicate may backtrack if $X$ is not a complete list. ${ }^{1}$
- append $(W, X, Y, Z)$ (nondet): This predicate is defined as:

$$
\text { append }(W, X, Y, Z)=>\operatorname{append}(W, X, W X), \quad \operatorname{append}(W X, Y, Z) .
$$

[^3]- $\operatorname{avg}($ List $)=$ Val: This function returns the average of all the elements in List. This function throws an exception if List is not a list or any of the elements is not a number.
- delete (List, $X$ ) $=$ ResList: This function deletes the first occurrence of $X$ from List, returning the result in ResList. The built-in $!=/ 2$ is used to test if two terms are different. No variables in List or $X$ will be bound after this function call.
- delete_all $($ List, $X)=$ ResList: This function deletes all occurrences of $X$ from List, returning the result in ResList. The built-in $!=/ 2$ is used to test if two terms are different.
- first $($ List $)=$ Term: This function returns the first element of List.
- flatten (List) $=$ ResList: This function flattens a list of nested lists into a list. For example, flatten ([[1], [2, [3]]]) returns [1,2,3].
- head $($ List $)=$ Term: This function returns the head of the list List. For example, head ( $[1,2,3])$ returns 1 .
- insert (List, Index, Elm) = ResList: This function inserts Elm into List at the index Index, returning the result in ResList. After insertion, the original List is not changed, and ResList is the same as
List.slice(1, Index-1) $++[$ Elm $\mid$ List.slice (Index, List.length) $]$.
- insert_all (List, Index, AList) $=$ ResList: This function inserts all of the elements in AList into List at the index Index, returning the result in ResList. After insertion, the original List is not changed, and ResList is the same as
List.slice(1, Index-1) ++ AList ++ List.slice (Index, List.length).
- insert_ordered (List,Term) : This function inserts Term into the ordered list List, such that the resulting list remains sorted.
- insert_ordered_down (List,Term) : This function inserts Term into the descendantly ordered list List, such that the resulting list remains sorted down.
- last $($ List $)=$ Term: This function returns the last element of List.
- len $($ List $)=L e n$ : This function returns the number of elements in List. Note that this function is overloaded in such a way that the argument can also be an atom, an array, or a structure.
- length $(L i s t)=$ Len: This function is the same as len (List).
- list (Term) : This predicate is true if Term is a list.
- $\max ($ List $)=$ Val: This function returns the maximum value that is in List, where List is a list of terms.
- membchk (Term, List) : This predicate is true if Term is an element of List.
- member (Term, List) (nondet): This predicate is true if Term is an element of List. When Term is a variable, this predicate may backtrack, instantiating Term to different elements of List.
- $\min ($ List $)=$ Val: This function returns the minimum value that is in List, where List is a list or an array of terms.
- new_list $(N)=$ List: This function creates a new list that has $N$ free variable arguments.
- new_list $(N$, InitVal $)=$ List: This function creates a new list that has $N$ arguments all initialized to InitVal.
- nth (Index, List, Elem) (nondet): This predicate is true when Elem is the Index'th element of List. Counting starts at 1 . When Index is a variable, this predicate may backtrack, instantiating Index to a different integer between 1 and len (List).
- $\operatorname{prod}(L i s t)=V a l:$ This function returns the product of all of the values in List.
- remove_dups $(L i s t)=$ ResList: This function removes all duplicate values from List, retaining only the first occurrence of each value. The result is returned in ResList. Note that an $O\left(n^{2}\right)$ algorithm is used in the implementation. If List is large, then sort_remove_dups (List) may be faster than this function.
- reverse (List) $=$ ResList: This function reverses the order of the elements in List, returning the result in ResList.
- select ( $X$, List, ResList) (nondet): This predicate nondeterministically selects an element $X$ from List, and binds ResList to the list after $X$ is removed. On backtracking, it selects the next element.
- sort (List) $=$ SList: This function sorts the elements of List in ascending order, returning the result in SList.
- sort (List, KeyIndex) = SList: This function sorts the elements of List by the key index KeyIndex in ascending order, returning the result in SList. The elements of List must be compound values and KeyIndex must be a positive integer that does not exceed the length of any of the elements of List. This function is defined as follows:

```
sort(List,KeyIndex) = SList =>
    List1 = [(E[KeyIndex],E) : E in List],
    List2 = sort(List1),
    SList = [E : (_,E) in List2].
```

- sort_remove_dups (List) = SList: This function is the same as the following, but is faster.

```
sort(List).remove_dups()
```

- sort_remove_dups (List, KeyIndex) = SList: This function is the same as the following, but is faster.
sort (List, KeyIndex). remove_dups()
- sort_down (List) $=S L i s t$ : This function sorts the elements of List in descending order, returning the result in SList.
- sort_down (List, KeyIndex) = SList: This function sorts the elements of List by the key index KeyIndex in descending order, returning the result in SList.
- sort_down_remove_dups (List) = SList: This function is the same as the following, but is faster.

```
sort_down(List).remove_dups()
```

- sort_down_remove_dups (List,KeyIndex) $=$ SList: This function is the same as the following, but is faster.

```
sort_down(List,KeyIndex).remove_dups()
```

- slice (List, From,To) $=$ SList: This function returns the sliced list of List from index From through index To. From must not be less than 1.
- slice (List, From) $=$ SList: This function is the same as the following.

> slice (List, From,List.length)

- $\operatorname{sum}($ List $)=$ Val: This function returns the sum of all of the values in List.
- tail $($ List $)=$ Term: This function returns the tail of the list List. For example, the call tail([1, 2, 3]) returns [2, 3].
- to_array (List) = Array: This function converts the list List to an array. The elements of the array are in the same order as the elements of the list.
- zip $\left(\right.$ List $\left._{1}, L_{i s t_{2}}, \ldots, L_{i s t_{n}}\right)=$ List: This function makes a list of array tuples. The $j$ th tuple in the list takes the form $\left\{E_{1 j}, \ldots, E_{n j}\right\}$, where $E_{i j}$ is the $j$ th element in $L_{i s t}$. In the current implementation, $n$ can be 2,3 , or 4 .


### 3.4.2 Strings

A string is represented as a list of single-character atoms. For example, the string "hello" is the same as the list $[h, e, l, l, 0]$. In addition to the built-ins on lists, the following built-ins are provided for strings:

- string (Term) : This predicate is true if Term is a string.
- to_lowercase (String) $=$ LString: This function converts all uppercase alphabetic characters into lowercase characters, returning the result in LString.
- to_uppercase (String) $=$ UString: This function converts all lowercase alphabetic characters into uppercase characters, returning the result in UString.


### 3.4.3 Structures

A structure takes the form $\$ s\left(t_{1}, \ldots, t_{n}\right)$, where $s$ is an atom, and $n$ is called the arity of the structure. The dollar symbol is used to distinguish a structure from a function call. The functor of a structure comprises the name and the arity of the structure.

The following types of structures can never denote functions, meaning that they do not need to be preceded by a $\$$ symbol.

```
Goals: (a,b), (a;b), not a,X = Y, X != 100,X > 1
Constraints: X+Y #= 100,X #!= 1
Arrays: {2, 3, 4},{P1, P2, P 3}
```

Picat disallows creation of the following types of structures:

| Dot notations: | math.pi, my_module.f(a) |
| :--- | :--- |
| Index notations: | $X[1]+2 \mathrm{X}[\mathrm{Y}[\mathrm{I}]]$ |
| Assignments: | $\mathrm{X}:=\mathrm{Y}+\mathrm{Z}, \mathrm{X}:=\mathrm{X}+1$ |
| Ranges: | $1 \ldots 10,1 \ldots 2 \ldots 10$ |
| List comprehensions: | $[\mathrm{X}: \mathrm{X}$ in $1 \ldots 5]$ |
| Array comprehensions: $\{\mathrm{X}: \mathrm{X}$ in $1 \ldots 5\}$ |  |
| If-then: | if $\mathrm{X}>\mathrm{Y}$ then $\mathrm{Z}=\mathrm{X}$ else $\mathrm{Z}=\mathrm{Y}$ end |
| Loops: | foreach $(\mathrm{X}$ in L$)$ writeln $(\mathrm{X})$ end |

The compiler will report a syntax error when it encounters any of these expressions within a term constructor.

The following built-ins are provided for structures:

- arity $($ Struct $)=$ Arity: This function returns the arity of Struct, which must be a structure.
- len $($ Struct $)=$ Arity: This function is the same as arity (Struct).
- name (Struct) $=$ Name: This function returns the name of Struct.
- new_struct (Name, IntOrList) $=$ Struct: This function creates a structure that has the name $N a m e$. If IntOrList is an integer, $N$, then the structure has $N$ free variable arguments. Otherwise, if IntOrList is a list, then the structure contains the elements in the list.
- struct (Term): This predicate is true if Term is a structure.
- to_list (Struct) $=$ List: This function returns a list of the components of the structure Struct.


### 3.4.4 Arrays

An array takes the form $\left\{t_{1}, \ldots, t_{n}\right\}$, which is a special structure with the name ${ }^{\prime}\{ \}^{\prime}$ and arity $n$. Note that, unlike a list, an array always has its length stored in memory, so the function length (Array) always takes constant time. Also note that Picat supports constant-time access of array elements, so the index notation A [ I ] takes constant time.

In addition to the built-ins for structures, the following built-ins are provided for arrays:

- $\operatorname{array}($ Term $)$ : This predicate is true if Term is an array.
- new_array $\left(D_{1}, \ldots, D_{n}\right)=$ Arr: This function creates an n-dimensional array, where each $D_{i}$ is an integer expression that specifies the size of a dimension. In the current implementation, $n$ cannot exceed 10 .

The following built-ins, which are originally provided for lists (see 3.4.1), are overloaded for arrays:

$$
\text { - } \text { Array }_{1}++ \text { Array }_{2}=\text { Array }
$$

- $\operatorname{avg}($ Array $)=V a l$
- first(Array) $=$ Term
- last (Array) $=$ Term
- len $($ Array $)=$ Len
- length (Array) $=$ Len
- $\max ($ Array $)=$ Val
- $\min ($ Array $)=$ Val
- nth (Index, List, Elem) (nondet)
- reverse (Array) = ResArray
- slice (Array,From,To) $=$ SArray
- slice (Array, From) $=$ SArray
- $\operatorname{sum}($ Array $)=$ Val
- $\operatorname{sort}($ Array $)=$ SArray
- sort (Array, KeyIndex) $=$ SArray
- sort_remove_dups (Array) $=$ SArray
- sort_remove_dups (Array, KeyIndex) $=$ SArray
- sort_down (Array) $=$ SArray
- sort_down (Array, KeyIndex) $=$ SArray
- sort_down_remove_dups (Array) = SArray
- sort_down_remove_dups (Array, KeyIndex) $=$ SArray

Note that many of the overloaded built-ins for arrays are not implemented efficiently, but are provided for convenience. For example, sort (Array) is implemented as follows:

```
sort(Array) = Array.to_list().sort().to_array().
```


### 3.4.5 Maps

A map is a hash-table that is represented as a structure that contains a set of key-value pairs. The functor of the structure that is used for a map is not important. An implementation may ban access to the name and the arity of the structure of a map. Maps must be created with the built-in function new_map, unless they are prebuilt (see Section 1.11). In addition to the built-ins for structures, the following built-ins are provided for maps:

- clear (Map): This predicate clears the map Map. It throws an error if Map is not a map.
- get $(M a p, K e y)=V a l$ : This function returns $V a l$ of the key-value pair $K e y=V a l$ in Map. It throws an error if Map does not contain the key Key.
- get (Map, Key, DefaultVal) = Val: This function returns Val of the key-value pair Key=Val in Map. It returns DefaultVal if Map does not contain Key.
- has_key (Map, Key) : This predicate is true if Map contains a pair with Key.
- keys $(X)=$ List: This function returns the list of keys of the pairs in Map.
- map (Term) : This predicate is true if Term is a map.
- map_to_list $(M a p)=$ PairsList: This function returns a list of Key=Val pairs that constitute Map.
- new_map (IntOrPairsList) = Map: This function creates a map with an initial capacity or an initial list of pairs.
- new_map $(N$, PairsList $)=M a p$ : This function creates a map with the initial capacity $N$, the initial list of pairs PairsList, where each pair has the form $K e y=V a l$.
- put (Map, Key, Val): This predicate attaches the key-value pair Key=Val to Map, where Key is a non-variable term, and Val is any term.
- put (Map, Key): This predicate is the same as put (Map, Key, not_a_value).
- values $($ Map $)=$ List: This function returns the list of values of the pairs in Map.
- $\operatorname{size}(M a p)=$ Size: This function returns the number of pairs in Map.

Most of the built-ins are overloaded for attributed variables.

### 3.4.6 Sets

A set is a map where every key is associated with the atom not_a_value. All of the built-ins for maps can be applied to sets. For example, the built-in predicate has_key ( Set, Elm) tests if Elm is in Set. In addition to the built-ins on maps, the following built-ins are provided for sets:

- new_set (IntOrKeysList) $=$ Set: This function creates a set with an initial capacity or an initial list of keys.
- new_set $(N$, KeysList $)=$ Set: This function creates a set with the initial capacity $N$ and the initial list of keys KeysList.


### 3.4.7 Heaps

A heap ${ }^{2}$ is a complete binary tree represented as an array. A heap can be a min-heap or a maxheap. In a min-heap, the value at the root of each subtree is the minimum among all the values in the subtree. In a max-heap, the value at the root of each subtree is the maximum among all the values in the subtree.

- heap_is_empty (Heap) : This predicate is true if Heap is empty.
- heap_pop $($ Heap $)=$ Elm: This function removes the root element from the heap, and returns the element. As the function updates the heap, it is not pure. The update will be undone when execution backtracks over the call.

[^4]- heap_push (Heap, Elm) : This predicate pushes Elm into Heap in a way that maintains the heap property. The update to Heap will be undone when execution backtracks over the call.
- heap_size (Heap) = Size: This function returns the size of Heap.
- heap_to_list (Heap) = List: This function returns a list of the elements in Heap.
- heap_top $($ Heap $)=$ Elm: This function returns the element at the root of the heap. If Heap is a min-heap, then the element is guaranteed to be the minimum, and if Heap is a max-heap, then the element is guaranteed to be the maximum.
- new_max_heap (IntOrList) = Heap: This function creates a max-heap. If IntOr List is an integer,then it indicates the capacity. Otherwise, if IntOr List is a list, then the maxheap contains the elements in the list in an order that maintains the heap property.
- new_min_heap (IntOrList) = Heap: This function creates a min-heap. If IntOr List is an integer,then it indicates the capacity. Otherwise, if IntOrList is a list, then the minheap contains the elements in the list in an order that maintains the heap property.


## Example

```
main =>
    L = [1,3,2,4,5,3,6],
    H = new_min_heap(L),
    N = H.heap_size(),
    S = [H.heap_pop() : _ in 1..N],
    println(S).
```


### 3.5 Equality Testing, Unification, and Term Comparison

The equality test $T_{1}==T_{2}$ is true if term $T_{1}$ and term $T_{2}$ are identical. Two variables are identical if they are aliases. Two primitive values are identical if they have the same type and the same internal representation. Two lists are identical if the cars are identical and the cdrs are identical. Two structures are identical if their functors are the same and their components are pairwise identical. The inequality test $T_{1} \quad!==T_{2}$ is the same as not $T_{1}==T_{2}$. Note that two terms can be identical even if they are stored in different memory locations. Also note that it takes linear time in the worst case to test whether two terms are identical, unlike in C-family languages, in which the equality test operator $==$ only compares addresses.

The unification $T_{1}=T_{2}$ is true if term $T_{1}$ and term $T_{2}$ are already identical, or if they can be made identical by instantiating the variables in the terms. The built-in $T_{1} \quad!=T_{2}$ is true if term $T_{1}$ and term $T_{2}$ are not unifiable. The predicate bind_vars (Term, Val) binds all of the variables in Term to Val.

## Example

```
Picat> X = 1
X = 1
Picat> $f(a,b) = $f(a,b)
yes
Picat> [H|T] = [a,b,c]
```

```
\(\mathrm{H}=\mathrm{a}\)
\(\mathrm{T}=[\mathrm{b}, \mathrm{c}]\)
Picat> \$f(X,b) = \$f(a,Y)
\(X=a\)
\(\mathrm{Y}=\mathrm{b}\)
Picat> bind_vars(\{X,Y,Z\},a)
Picat> X = \$f(X)
```

The last query illustrates the occurs-check problem. When binding $X$ to $f(X)$, Picat does not check if $X$ occurs in $f(X)$ for the sake of efficiency. This unification creates a cyclic term, which can never be printed.

When a unification's operands contain attributed variables, the implementation is more complex. When a plain variable is unified with an attributed variable, the plain variable is bound to the attributed variable. When two attributed variables, say $Y$ and $O$, where $Y$ is younger than $O$, are unified, $Y$ is bound to $O$, but $Y$ 's attributes are not copied to $O$. Since garbage collection does not preserve the seniority of terms, the result of the unification of two attributed variables is normally unpredictable.

### 3.5.1 Numerical Equality

The numerical equality test $T_{1}=:=T_{2}$ is true if term $T_{1}$ and term $T_{2}$ are pretty much the same numerical value. This means that $T 1$ and $T 2$ must both be numbers. Whereas the test $T_{1}==$ $T_{2}$ fails if one number is an integer and one number is a real number, the test $T_{1}=:=T_{2}$ may succeed. Consider the following examples.

## Example

```
Picat> 1 == 1.0
no
Picat> 1 =:= 1.0
yes
```

In the first query, 1 is an integer, while 1.0 is a real number, so the equality test fails. However, the second query, which is a numerical equality test, succeeds.

### 3.5.2 Ordering of Terms

Picat orders terms in the following way:

```
var < number < atom < structure and array < list and string
```

Variables are ordered by their addresses. Note that the ordering of variables may change after garbage collection. Numbers are ordered by their numerical values. Atoms are ordered lexicographically. Structures are first ordered lexicographically by their names; if their names are the same, then they are ordered by their components. Arrays are ordered as structures with the special name ' $\}$ '. Lists and strings are ordered by their elements.

- Term1 @< Term2: The term Term1 precedes the term Term 2 in the standard order. For example, a $\mathrm{a}<\mathrm{b}$ succeeds.
- Term $1 @=<$ Term 2 : The term Term 1 either precedes, or is identical to, the term Term 2 in the standard order. For example, $a \quad \varrho=<b$ succeeds.
- Term 1 @ $<=$ Term 2 : This is the same as Term $1 @=<$ Term 2 .
- Term1 @> Term 2 : The term Term 1 follows the term Term 2 in the standard order.
- Term1 @>= Term2: The term Term 1 either follows, or is identical to, the term Term 2 in the standard order.


### 3.6 Expressions

Expressions are made from variables, values, operators, and function calls. Expressions differ from terms in the following ways:

- An expression can contain dot notations, such as math.pi.
- An expression can contain index notations, such as $X[I]$.
- An expression can contain ranges, such as 1.. 2 . . 100 .
- An expression can contain list comprehensions, such as [X : X in 1..100].
- An expression can contain array comprehensions, such as $\{X: X$ in $1 . .100\}$.

A conditional expression, which takes the form cond (Cond, $\operatorname{Exp}_{1}, E x p_{2}$ ), is a special kind of function call that returns the value of $E x p_{1}$ if the condition Cond is true and the value of $E x p_{2}$ if Cond is false.

Note that, except for conditional expressions in which the conditions are made of predicates, no expressions can contain predicates. A predicate is true or false, but never returns any value.

### 3.7 Higher-order Predicates and Functions

A predicate or function is said to be higher-order if it takes calls as arguments. The basic module has the following higher-order predicates and functions.

- apply $\left(S, \operatorname{Arg} g_{1}, \ldots, \operatorname{Arg} g_{n}\right)=V a l: S$ is an atom or a structure. This function calls the function that is named by $S$ with the arguments that are specified in $S$, together with extra arguments $A r g_{1}, \ldots, A r g_{n}$. This function returns the value that $S$ returns.
- call $\left(S, \operatorname{Arg}_{1}, \ldots, \operatorname{Arg} g_{n}\right): S$ is an atom or a structure. This predicate calls the predicate that is named by $S$ with the arguments that are specified in $S$, together with extra arguments $\operatorname{Arg}_{1}, \ldots, \operatorname{Arg}_{n}$.
- call_cleanup (Call, Cleanup) : This predicate is the same as call (Call), except that Cleanup is called when Call succeeds determinately (i.e., with no remaining choice point), when Call fails, or when Call raises an exception.
- catch (Call, Exception, Handler): This predicate is the same as Call, except when an exception that matches Exception is raised during the execution of Call. When such an exception is raised, all of the bindings that have been performed on variables in Call will be undone, and Handler will be executed to handle the exception.
- count_all (Call) = Count: This function returns the number of all possible instances of call (Call) that are true. For example, count_all (member (X, [1, 2, 3])) returns 3.
- findall (Template, Call) = Answers: This function returns a list of all possible instances of call (Call) that are true in the form of Template. Note that Template is assumed to be a term without function calls, and that Call is assumed to be a predicate call whose arguments can contain function calls. Also note that, like a loop, findall forms a name scope. For example, in findall $(f(X), p(X, g(Y))), f(X)$ is a term even though it is not preceded with $\$ ; g(Y)$ is a function call; the variables $X$ and $Y$ are assumed to be local to findall if they do not occur before in the outer scope.
- find_all (Template, Call) = Answers: This function is the same as the above function.
- freeze ( $X, C$ all $)$ : This predicate delays the evaluation of $C$ all until $X$ becomes a nonvariable term.
- map (FuncOrList, ListOrFunc) = ResList: This function applies a given function to every element of a given list and returns a list of the results. One of the arguments is a function, and the other is a list. The order of the arguments is not important.
- map (Func, List1, List2) = ResList: Let List1 be $\left[A_{1}, \ldots, A_{n}\right]$ and List2 be [ $B_{1}, \ldots, B_{n}$ ]. This function applies the function Func to every pair of elements $\left(A_{i}, B_{i}\right)$ by calling apply $\left(F u n c, A_{i}, B_{i}\right)$, and returns a list of the results.
- maxof (Call, Objective): This predicate finds a satisfiable instance of Call, such that Objective has the maximum value. Here, Call is used as a generator, and Objective is an expression to be maximized. For every satisfiable instance of Call, Objective must be a ground expression. For maxof, search is restarted with a new bound each time that a better answer is found.
- maxof(Call, Objective, ReportCall): This is the same as maxof (Call,Objective), except that call (ReportCall) is executed each time that an answer is found.
- maxof_inc (Call, Objective): This is the same as maxof (Call,Objective), except that search continues rather than being restarted each time that a better solution is found.
- maxof_inc (Call, Objective, ReportCall): This is the same as the previous predicate, except that call (ReportCall) is executed each time that an answer is found.
- minof (Call, Objective): This predicate finds a satisfiable instance of Call, such that Objective has the minimum value.
- minof(Call, Objective, ReportCall): This is the same as minof(Call,Objective), except that call (ReportCall) is executed each time that an answer is found.
- minof_inc (Call, Objective): This predicate is the same as minof (Call,Objective), except that search continues rather than being restarted each time that a better solution is found.
- minof_inc (Call, Objective, ReportCall): This predicate is the same as the previous one, except that call (ReportCall) is executed each time that an answer is found.
- reduce (Func, List) = Res: If List is a list that contains only one element, this function returns the element. If List contains at least two elements, then the first two elements $A_{1}$ and $A_{2}$ are replaced with apply $\left(F u n c, A_{1}, A_{2}\right)$. This step is repeatedly applied to the list until the list contains a single element, which is the final value to be returned. The order of the arguments is not important, meaning that the first argument can be a list and the second one can be a function.
- reduce (Func, List, InitVal) = Res: This function is the same as reduce (Func, [InitVal|List]).


### 3.8 Other Built-ins in the basic Module

- acyclic_term (Term) : This predicate is true if Term is acyclic, meaning that Term does not contain itself.
- and_to_list $(\operatorname{Conj})=$ List: This function converts $\operatorname{Conj}$ in the form $\left(a_{1}, \ldots, a_{n}\right)$ into a list in the form $\left[a_{1}, \ldots, a_{n}\right]$.
- compare_terms $\left(\right.$ Term $_{1}$, Term $\left._{2}\right)=$ Res: This function compares Term $_{1}$ and Term $_{2}$. If $\operatorname{Term}_{1}<\operatorname{Term}_{2}$, then this function returns -1 . If $\operatorname{Term}_{1}==$ Term $_{2}$, then this function returns 0 . Otherwise, Term $_{1}>$ Term $_{2}$, and this function returns 1.
- different_terms $\left(\right.$ Term $_{1}$, Term $\left._{2}\right)$ : This constraint ensures that Term $_{1}$ and Term $_{2}$ are different. This constraint is suspended when the arguments are not sufficiently instantiated.
- get_global_map () = Map: This function returns the global map, which is shared by all threads.
- get_heap_map () = Map: This function returns the current thread's heap map. Each thread has its own heap map.
- get_table_map () = Map: This function returns the current thread's table map. Each thread has its own table map. The table map is stored in the table area and both keys and values are hash-consed (i.e., common sub-terms are shared).
- ground (Term) : This predicate is true if Term is ground. A ground term does not contain any variables.
- list_to_and $($ List $)=C o n j$ : This function converts List in the form $\left[a_{1}, \ldots, a_{n}\right]$ into a term in the form $\left(a_{1}, \ldots, a_{n}\right)$.
- number_vars $\left(\right.$ Term, $\left.N_{0}\right)=N_{1}$ : This function numbers the variables in Term by using the integers starting from $N_{0} . N_{1}$ is the next integer that is available after Term is numbered. Different variables receive different numberings, and the occurrences of the same variable all receive the same numbering.
- parse_radix_string (String, Base) = Int: This function converts a radix String of Base into a decimal integer Int, where Base must be greater than 1 and less than 37. For example, parse_radix_string ("101",2) returns 5, which is the same as parse_term("0b101").
- parse_term (String, Term, Vars): This predicate uses the Picat parser to extract a term Term from String. Vars is a list of pairs, where each pair has the form Name=Var.
- parse_term (String) = Term: This function converts String to a term.
- second (Compound) = Term: This function returns the second argument of the compound term Compound.
- subsumes $\left(\right.$ Term $_{1}$, Term $\left.m_{2}\right)$ : This predicate is true if Term $_{1}$ subsumes Term $_{2}$.
- variant (Term Th $_{1}$ Term $)_{2}$ : This predicate is true if Term $_{2}$ is a variant of Term ${ }_{1}$.
- $\operatorname{vars}($ Term $)=$ Vars: This function returns a list of variables that occur in Term.


## Chapter 4

## Predicates and Functions

In Picat, predicates and functions are defined with pattern-matching rules. Picat has two types of rules: the non-backtrackable rule

Head, Cond => Body.
and the backtrackable rule
Head, Cond ?=> Body.
Each rule is terminated by a dot (.) followed by a white space.

### 4.1 Predicates

A predicate defines a relation, and can have zero, one, or multiple answers. Within a predicate, the Head is a pattern in the form $p\left(t_{1}, \ldots, t_{n}\right)$, where $p$ is called the predicate name, and $n$ is called the arity. When $n=0$, the parentheses can be omitted. The condition Cond, which is an optional goal, specifies a condition under which the rule is applicable. Cond cannot succeed more than once. The compiler converts Cond to once Cond if would otherwise be possible for Cond to succeed more than once.

For a call $C$, if $C$ matches the pattern $p\left(t_{1}, \ldots, t_{n}\right)$ and $C o n d$ is true, then the rule is said to be applicable to $C$. When applying a rule to call $C$, Picat rewrites $C$ into Body. If the used rule is non-backtrackable, then the rewriting is a commitment, and the program can never backtrack to $C$. However, if the used rule is backtrackable, then the program will backtrack to $C$ once Body fails, meaning that Body will be rewritten back to $C$, and the next applicable rule will be tried on $C$.

A predicate is said to be deterministic if it is defined with non-backtrackable rules only, nondeterministic if at least one of its rules is backtrackable, and globally deterministic if it is deterministic and all of the predicates in the bodies of the predicate's rules are also globally deterministic. A deterministic predicate that is not globally deterministic can still have more than one answer.

## Example

```
append(Xs,Ys,Zs) ?=> Xs=[], Ys=Zs.
append(Xs,Ys,Zs) => Xs=[X|XsR], append(XsR,Ys,Zs).
min_max([H],Min,Max) => Min=H, Max=H.
min_max([H|T],Min,Max) =>
        min_max(T,MinT,MaxT),
```

```
Min=min(MinT,H),
Max=max (MaxT,H).
```

The predicate append ( $\mathrm{Xs}, \mathrm{Ys}, \mathrm{Zs}$ ) is true if the concatenation of Xs and Ys is Zs . It defines a relation among the three arguments, and does not assume directionality of any of the arguments. For example, this predicate can be used to concatenate two lists, as in the call

```
append([a,b],[c,d],L)
```

this predicate can also be used to split a list nondeterministically into two sublists, as in the call append $(L 1, L 2,[a, b, c, d])$; this predicate can even be called with three free variables, as in the call append (L1, L2, L3).

The predicate min_max (L, Min, Max) returns two answers through its arguments. It binds Min to the minimum of list $L$, and binds Max to the maximum of list $L$. This predicate does not backtrack. Note that a call fails if the first argument is not a list. Also note that this predicate consumes linear space. A tail-recursive version of this predicate that consumes constant space will be given below.

### 4.2 Functions

A function is a special kind of a predicate that always succeeds with one answer. Within a function, the Head is an equation $p\left(t_{1}, \ldots, t_{n}\right)=X$, where $p$ is called the function name, and $X$ is an expression that gives the return value. Functions are defined with non-backtrackable rules only.

For a call $C$, if $C$ matches the pattern $p\left(t_{1}, \ldots, t_{n}\right)$ and $C o n d$ is true, then the rule is said to be applicable to $C$. When applying a rule to call $C$, Picat rewrites the equation $C=X^{\prime}$ into (Body, $X^{\prime}=X$ ), where $X^{\prime}$ is a newly introduced variable that holds the return value of $C$.

Picat allows inclusion of function facts in the form $p\left(t_{1}, \ldots, t_{n}\right)=E x p$ in function definitions. The function fact $p\left(t_{1}, \ldots, t_{n}\right)=\operatorname{Exp}$ is shorthand for the rule:

$$
p\left(t_{1}, \ldots, t_{n}\right)=X \Rightarrow X=E x p
$$

where $X$ is a new variable.
Although all functions can be defined as predicates, it is preferable to define them as functions for two reasons. Firstly, functions often lead to more compact expressions than predicates, because arguments of function calls can be other function calls. Secondly, functions are easier to debug than predicates, because functions never fail and never return more than one answer.

## Example

```
qequation(A,B,C) = (R1,R2),
    D = B*B-4*A*C,
    D >= 0
=>
    NTwoC = -2*C,
    R1 = NTwoC/(B+sqrt(D)),
    R2 = NTwoC/(B-sqrt(D)).
rev([]) = [].
rev([X|Xs]) = rev(Xs)++[X].
```

The function qequation ( $A, B, C$ ) returns the pair of roots of $A * X^{2}+B * X+C=0$. If the discriminant $B * B-4 * A * C$ is negative, then an exception will be thrown.

The function rev (L) returns the reversed list of $L$. Note that the function rev (L) takes quadratic time and space in the length of $L$. A tail-recursive version that consumes linear time and space will be given below.

### 4.3 Patterns and Pattern-Matching

The pattern $p\left(t_{1}, \ldots, t_{n}\right)$ in the head of a rule takes the same form as a structure. Function calls are not allowed in patterns. Also, patterns cannot contain index notations, dot notations, ranges, array comprehensions, or list comprehensions. Pattern matching is used to decide whether a rule is applicable to a call. For a pattern $P$ and a term $T$, term $T$ matches pattern $P$ if $P$ is identical to $T$, or if $P$ can be made identical to $T$ by instantiating $P$ 's variables. Note that variables in the term do not get instantiated after the pattern matching. If term $T$ is more general than pattern $P$, then the pattern matching can never succeed.

Unlike calls in many committed-choice languages, calls in Picat are never suspended if they are more general than the head patterns of the rules. A predicate call fails if it does not match the head pattern of any of the rules in the predicate. A function call throws an exception if it does not match the head pattern of any of the rules in the function. For example, for the function call $r e v(L)$, where $L$ is a variable, Picat will throw the following exception:

```
unresolved_function_call(rev(L)).
```

A pattern can contain as-patterns in the form $V @ P a t t e r n$, where $V$ is a new variable in the rule, and Pattern is a non-variable term. The as-pattern $V @ P a t t e r n$ is the same as Pattern in pattern matching, but after pattern matching succeeds, $V$ is made to reference the term that matched Pattern. As-patterns can avoid re-constructing existing terms.

## Example

```
merge([],Ys) = Ys.
merge(Xs,[]) = Xs.
merge([X|Xs],Ys@[Y|_]) = [X|Zs], X<Y => Zs=merge(Xs,Ys).
merge(Xs,[Y|Ys]) = [Y|merge(Xs,Ys)].
```

In the third rule, the as-pattern Ys@[Y|_] binds two variables: Ys references the second argument, and $Y$ references the car of the argument. The rule can be rewritten as follows without using any as-pattern:

```
merge([X|Xs],[Y|Ys]) = [X|Zs], X<Y => Zs=merge(Xs,[Y|Ys]).
```

Nevertheless, this version is less efficient, because the cons [Y|Ys] needs to be re-constructed.

### 4.4 Goals

In a rule, both the condition and the body are goals. Queries that the users give to the interpreter are also goals. A goal can take one of the following forms:

- true: This goal is always true.
- fail: This goal is always false. When fail occurs in a condition, the condition is false, and the rule is never applicable. When fail occurs in a body, it causes execution to backtrack.
- false: This goal is the same as fail.
- $p\left(t_{1}, \ldots, t_{n}\right)$ : This goal is a predicate call. The arguments $t_{1}, \ldots, t_{n}$ are evaluated in the given order, and the resulting call is resolved using the rules in the predicate $p / n$. If the call succeeds, then variables in the call may get instantiated. Many built-in predicates are written in infix notation. For example, $X=Y$ is the same as $\prime^{\prime}={ }^{\prime}(X, Y)$.
- $P, Q$ : This goal is a conjunction of goal $P$ and goal $Q$. It is resolved by first resolving $P$, and then resolving $Q$. The goal is true if both $P$ and $Q$ are true. Note that the order is important: $(P, Q)$ is in general not the same as $(Q, P)$.
- $P \& \& Q$ : This is the same as $(P, Q)$.
- $P ; Q$ : This goal is a disjunction of goal $P$ and goal $Q$. It is resolved by first resolving $P$. If $P$ is true, then the disjunction is true. If $P$ is false, then $Q$ is resolved. The disjunction is true if $Q$ is true. The disjunction is false if both $P$ and $Q$ are false. Note that a disjunction can succeed more than once. Note also that the order is important: $(P ; Q)$ is generally not the same as $(Q ; P)$.
- $P \| Q$ : This is the same as $(P ; Q)$.
- not $P$ : This goal is the negation of $P$. It is false if $P$ is true, and true if $P$ is false. Note a negation goal can never succeed more than once. Also note that no variables can get instantiated, no matter whether the goal is true or false.
- once $P$ : This goal is the same as $P$, but can never succeed more than once.
- repeat: This predicate is defined as follows:

```
repeat ?=> true.
repeat => repeat.
```

The repeat predicate is often used to describe failure-driven loops. For example, the query

```
repeat,writeln(a),fail
```

repeatedly outputs ' $a$ ' until ctrl-c is typed.

- if-then: An if-then statement takes the form

```
    if Cond d then
        Goal1
    elseif Cond ( then
        Goal2
        \vdots
    elseif Cond}\mp@subsup{|}{n}{}\mathrm{ then
        Goaln
    else
        Goalelse
    end
```

where the elseif and else clauses are optional. If the else clause is missing, then the else goal is assumed to be true. For the if-then statement, Picat finds the first condition Cond $_{i}$ that is true. If such a condition is found, then the truth value of the if-then statement is the same as Goal ${ }_{i}$. If none of the conditions is true, then the truth value of the if-then statement is the same as Goal else . Note that no condition can succeed more than once.

- throw Exception: This predicate throws the term Exception. This predicate will be detailed in Chapter 6 on Exceptions.
- Loops: Picat has three types of loop statements: foreach, while, and do-while. A loop statement is true if and only if every iteration of the loop is true. The details of loops are given in Chapter 5.


### 4.5 Predicate Facts

For an extensional relation that contains a large number of tuples, it is tedious to define such a relation as a predicate with pattern-matching rules. It is worse if the relation has multiple keys. In order to facilitate the definition of extensional relations, Picat allows the inclusion of predicate facts in the form $p\left(t_{1}, \ldots, t_{n}\right)$ in predicate definitions. Facts and rules cannot co-exist in predicate definitions and facts must be ground. A predicate definition that consists of facts must be preceded by an index declaration in the form

```
index ( }\mp@subsup{M}{11}{},\mp@subsup{M}{12}{},\ldots,\mp@subsup{M}{1n}{})\ldots(\mp@subsup{M}{m1}{},\mp@subsup{M}{m2}{},\ldots,\mp@subsup{M}{mn}{}
```

where each $M_{i j}$ is either + (meaning indexed) or - (meaning not indexed). Facts are translated into pattern-matching rules before they are compiled.

## Example

```
index (+,-) (-,+)
edge (a,b).
edge (a,c).
edge (b, c).
edge (c,b).
```

The predicate edge is translated into the following rules:

```
edge(a,Y) ?=> Y=b.
edge(a,Y) => Y=c.
edge(b,Y) => Y=c.
edge(c,Y) => Y=b.
edge(X,b) ?=> X=a.
edge(X,c) ?=> X=a.
edge(X,c) => X=b.
edge(X,b) => X=c.
```


### 4.6 Tail Recursion

A rule is said to be tail-recursive if the last call of the body is the same predicate as the head. The last-call optimization enables last calls to reuse the stack frame of the head predicate if the frame is not protected by any choice points. This optimization is especially effective for tail recursion,
because it converts recursion into iteration. Tail recursion runs faster and consumes less memory than non-tail recursion.

The trick to convert a predicate (or a function) into tail recursion is to define a helper that uses an accumulator parameter to accumulate the result. When the base case is reached, the accumulator is returned. At each iteration, the accumulator is updated. Initially, the original predicate (or function) calls the helper with an initial value for the accumulator parameter.

## Example

```
min_max([H|T],Min,Max) =>
    min_max_helper([H|T],H,Min,H,Max).
min_max_helper([],CMin,Min,CMax,Max) => Min=CMin, Max=CMax.
min_max_helper([H|T],CMin,Min,CMax,Max) =>
        min_max_helper(T,min(CMin,H),Min,max(CMax,H),Max).
rev([]) = [].
rev([X|Xs]) = rev_helper(Xs,[X]).
rev_helper([],R) = R.
rev_helper([X|Xs],R) = rev_helper(Xs,[X|R]).
```

In the helper predicate min_max_helper (L, CMin, Min, CMax, Max), CMin and CMax are accumulators: CMin is the current minimum value, and CMax is the current maximum value. When L is empty, the accumulators are returned by the unification calls Min=CMin and Max=CMax. When L is a cons [H|T], the accumulators are updated: CMin changes to min (CMin, H), and CMax changes to max ( $\mathrm{CMax}, \mathrm{H}$ ). The helper function $r e v \_h e l p e r(L, R)$ follows the same idea: it uses an accumulator list to hold, in reverse order, the elements that have been scanned. When $L$ is empty, the accumulator is returned. When $L$ is the cons $[\mathrm{X} \mid \mathrm{Xs}$ ], the accumulator $R$ changes to $[X \mid R]$.

## Chapter 5

## Assignments and Loops

This chapter discusses variable assignments, loop constructs, and list and array comprehensions in Picat. It describes the scope of an assigned variable, indicating where the variable is defined, and where it is not defined. Finally, it shows how assignments, loops, and list comprehensions are related, and how they are compiled.

### 5.1 Assignments

Picat variables are single-assignment, meaning that once a variable is bound to a value, the variable cannot be bound again. In order to simulate imperative language variables, Picat provides the assignment operator $:=$. An assignment takes the form $L H S:=R H S$, where $L H S$ is either a variable or an access of a compound value in the form $X[\ldots]$. When $L H S$ is an access in the form $X[I]$, the component of $X$ indexed $I$ is updated. This update is undone if execution backtracks over this assignment.

## Example

test $=>\mathrm{X}=0, \mathrm{X}:=\mathrm{X}+1, \mathrm{X}:=\mathrm{X}+2$, write $(\mathrm{X})$.
The compiler needs to give special consideration to the scope of a variable. The scope of a variable refers to the parts of a program where a variable occurs.

Consider the test example. This example binds $X$ to 0 . Then, the example tries to bind $X$ to $X+1$. However, $X$ is still in scope, meaning that $X$ is already bound to 0 . Since $X$ cannot be bound again, the compiler must perform extra operations in order to manage assignments that use the $:=$ operator.

In order to handle assignments, Picat creates new variables at compile time. In the test example, at compile time, Picat creates a new variable, say $X 1$, to hold the value of $X$ after the assignment $\mathrm{X}:=\mathrm{X}+1$. Picat replaces X by X 1 on the LHS of the assignment. All occurrences of $X$ after the assignment are replaced by $X 1$. When encountering $X 1:=X 1+2$, Picat creates another new variable, say $X 2$, to hold the value of $X 1$ after the assignment, and replaces the remaining occurrences of X 1 by X 2 . When write ( X 2 ) is executed, the value held in X 2 , which is 3 , is printed. This means that the compiler rewrites the above example as follows:
test $\Rightarrow \mathrm{X}=0, \mathrm{X} 1=\mathrm{X}+1, \mathrm{X} 2=\mathrm{X} 1+2$, write (X2).

### 5.1.1 If-Else

This leads to the question: what does the compiler do if the code branches? Consider the following code skeleton.

## Example

```
if_ex(Z) =>
    X = 1, Y = 2,
    if Z > 0 then
        X := X * Z
    else
        Y := Y + Z
    end,
    println([X,Y]).
```

The if_ex example performs exactly one assignment. At compilation time, the compiler does not know whether or not $Z>0$ evaluates to true. Therefore, the compiler does not know whether to introduce a new variable for X or for Y .

Therefore, when an if-else statement contains an assignment, the compiler rewrites the if-else statement as a predicate. For example, the compiler rewrites the above example as follows:

```
if_ex(Z) =>
    X = 1, Y = 2,
    p(X, Xout, Y, Yout, Z),
    println([Xout,Yout]).
p(Xin, Xout, Yin, Yout, Z), Z > 0 =>
    Xout = X * Z,
    Yout = Yin.
p(Xin, Xout, Yin, Yout) =>
    Xout = Xin,
    Yout = Y + Z.
```

One rule is generated for each branch of the if-else statement. For each variable $V$ that occurs on the LHS of an assignment statement that is inside of the if-else statement, predicate $p$ is passed two arguments, Vin and Vout. In the above example, $X$ and $Y$ each occur on the LHS of an assignment statement. Therefore, predicate $p$ is passed the parameters Xin, Xout, Yin, and Yout.

### 5.2 Types of Loops

Picat has three types of loop statements for programming repetitions: foreach, while, and do-while.

### 5.2.1 Foreach Loops

A foreach loop has the form:

```
foreach ( }\mp@subsup{E}{1}{}\mathrm{ in }\mp@subsup{D}{1}{},\mp@subsup{C}{Cond}{1},\ldots,\mp@subsup{E}{n}{}\mathrm{ in }\mp@subsup{D}{n}{},\mp@subsup{C}{ond}{n}
    Goal
end
```

Each $E_{i}$ is an iterating pattern. Each $D_{i}$ is an expression that gives a compound value. Each Cond $_{i}$ is an optional condition on iterators $E_{1}$ through $E_{i}$.

Foreach loops can be used to iterate through compound values, as in the following examples.

## Example

```
loop_ex1 =>
    L = [17, 3, 41, 25, 8, 1, 6, 40],
    foreach (E in L)
        println(E)
    end.
loop_ex2(Map) =>
    foreach(Key=Value in Map)
        writef("%w=%w\n", Key, Value)
    end.
```

The loop_ex1 example iterates through a list. The loop_ex2 example iterates through a map, where $\mathrm{Key}=\mathrm{Value}$ is the iterating pattern.

The loop_ex1 example can also be written, using a failure-driven loop, as follows.

## Example

```
loop_ex1 =>
    L = [17, 3, 41, 25, 8, 1, 6, 40],
    ( member(E, L),
        println(E),
        fail
    ;
        true
    ).
```

Recall that the range Start..Step..End stands for a list of numbers. Ranges can be used as compound values in iterators.

## Example

```
loop_ex3 =>
    foreach(E in 1 .. 2 .. 9)
        println(E)
    end.
```

Also recall that the function zip $\left(\right.$ List $\left._{1}, L_{i s t_{2}}, \ldots, L i s t_{n}\right)$ returns a list of tuples. This function can be used to simultaneously iterate over multiple lists.

## Example:

```
loop_ex_parallel =>
    foreach(Pair in zip(1..2, [a,b]))
        println(Pair)
    end.
```


### 5.2.2 Foreach Loops with Multiple Iterators

Each of the previous examples uses a single iterator. Foreach loops can also contain multiple iterators.

## Example:

```
loop_ex4 =>
    L = [2, 3, 5, 10],
    foreach(I in L, J in 1 .. 10, J mod I != 0)
        printf("%d is not a multiple of %d%n", J, I)
    end.
```

If a foreach loop has multiple iterators, then it is compiled into a series of nested foreach loops in which each nested loop has a single iterator. In other words, a foreach loop with multiple iterators executes its goal once for every possible combination of values in the iterators.

The foreach loop in loop_ex 4 is the same as the nested loop:

```
loop_ex5 =>
    L = [2, 3, 5, 10],
    foreach(I in L)
        foreach(J in 1..10)
            if J mod I != 0 then
                printf("%d is not a multiple of %d%n", J, I)
            end
        end
    end.
```


### 5.2.3 While Loops

A while loop has the form:

```
while (Cond)
    Goal
end
```

As long as Cond succeeds, the loop will repeatedly execute Goal.

## Example:

```
loop_ex6 =>
    I = 1,
    while (I <= 9)
        println(I),
        I := I + 2
    end.
loop_ex7 =>
    J = 6,
    while (J <= 5)
        println(J),
        J := J + 1
    end.
loop_ex8 =>
    E = read_int(),
    while (E mod 2 == 0; E mod 5 == 0)
```

```
    println(E),
    E := read_int()
    end.
loop_ex9 =>
    E = read_int(),
    while (E mod 2 == 0, E mod 5 == 0)
        println(E),
        E := read_int()
    end.
```

The while loop in loop_ex 6 prints all of the odd numbers between 1 and 9 . It is similar to the foreach loop

```
foreach(I in 1 .. 2 .. 9)
    println(I)
end.
```

The while loop in loop_ex 7 never executes its goal. $J$ begins at 6 , so the condition $J<=5$ is never true, meaning that the body of the loop does not execute.

The while loop in loop_ex 8 demonstrates a compound condition. The loop executes as long as the value that is read into $E$ is either a multiple of 2 or a multiple of 5 .

The while loop in loop_ex 9 also demonstrates a compound condition. Unlike in loop_ex8, in which either condition must be true, in loop_ex9, both conditions must be true. The loop executes as long as the value that is read into $E$ is both a multiple of 2 and a multiple of 5 .

### 5.2.4 Do-while Loops

A do-while loop has the form:

```
do
        Goal
while (Cond)
```

A do-while loop is similar to a while loop, except that a do-while loop executes Goal one time before testing Cond. The following example demonstrates the similarities and differences between do-while loops and while loops.

## Example

```
loop_ex10 =>
    J = 6,
    do
        println(J),
        J := J + 1
    while (J <= 5).
```

Unlike loop_ex7, loop_ex10 executes its body once. Although J begins at 6 , the do-while loop prints $J$, and increments $J$ before evaluating the condition $J<=5$.

### 5.3 List and Array Comprehensions

A list comprehension is a special functional notation for creating lists. List comprehensions have a similar format to foreach loops.

$$
\left[T: E_{1} \text { in } D_{1}, \operatorname{Cond}_{1}, \ldots, E_{n} \text { in } D_{n}, \operatorname{Cond}_{n}\right]
$$

$T$ is an expression. Each $E_{i}$ is an iterating pattern. Each $D_{i}$ is an expression that gives a compound value. Each Cond $_{i}$ is an optional condition on iterators $E_{1}$ through $E_{i}$.

An array comprehension takes the following form:

$$
\left\{T: E_{1} \text { in } D_{1}, \operatorname{Cond}_{1}, \ldots, E_{n} \text { in } D_{n}, \text { Cond }_{n}\right\}
$$

It is the same as:

$$
\text { to_array }\left(\left[T: E_{1} \text { in } D_{1}, \operatorname{Cond}_{1}, \ldots, E_{n} \text { in } D_{n}, \operatorname{Cond}_{n}\right]\right)
$$

## Example

```
picat> L = [(A, I) : A in [a, b], I in 1 . . 2].
L = [ ( a , 1 ) , ( a , 2 ) , ( b , 1 ) , ( b , 2 ) ]
picat> L = {(A, I) : A in [a, b], I in 1.. 2}.
L}={(a,1),(a,2),(b,1),(b,2)
```


### 5.4 Compilation of Loops

Variables that occur in a loop, but do not occur before the loop in the outer scope, are local to each iteration of the loop. For example, in the rule

```
p(A) =>
    foreach (I in 1 .. A.length)
        E = A[I],
        println(E)
    end.
```

the variables I and E are local, and each iteration of the loop has its own values for these variables.
Consider the example:

## Example

```
while_test(N) =>
    I = 1,
    while (I <= N)
        I := I + 1,
        println(I)
    end.
```

In this example, the while loop contains an assignment statement. As mentioned above, at compilation time, Picat creates new variables in order to handle assignments. One new variable is created for each assignment. However, when this example is compiled, the compiler does not know the number of times that the body of the while loop can be executed. This means that the compiler does not know how many times the assignment $I:=I+1$ will occur, and the compiler is
unable to create new variables for this assignment. In order to solve this problem, the compiler compiles while loops into tail-recursive predicates.

In the while_test example, the while loop is compiled into:

```
while_test(N) =>
    I = 1,
    p(I, N).
p(I, N), I <= N =>
    I1 = I + 1,
    println(I1),
    p(II, N).
p(_, _) => true.
```

Note that the first rule of the predicate p (I, N) has the same condition as the while loop. The second rule, which has no condition, terminates the while loop, because the second rule is only executed if $I>N$. The call $\mathrm{P}(\mathrm{I1}, \mathrm{~N})$ is the tail-recursive call, with I1 storing the modified value.

Suppose that a while loop modifies a variable that is then used outside of the while loop. For each modified variable $V$ that is used after the while loop, predicate $p$ is passed two arguments, Vin and Vout. Then, a predicate that has the body true is not sufficient to terminate the compiled while loop. Instead, a predicate fact must be used, as in the next example.

The next example demonstrates a loop that has multiple accumulators, and that modifies values which are then used outside of the loop.

## Example

```
min_max([H|T], Min, Max) =>
    LMin = H,
    LMax = H,
    foreach (E in T)
        LMin := min(LMin, E),
        LMax := max(LMax, E)
    end,
    Min = LMin,
    Max = LMax.
```

This loop finds the minimum and maximum values of a list. The loop is compiled to:

```
min_max([H|T], Min, Max) =>
    LMin = H,
    LMax = H,
    p(T, LMin, LMin1, LMax, LMax1),
    Min = LMin1,
    Max = LMax1.
p([], MinIn, MinOut, MaxIn, MaxOut) =>
    MinOut = MinIn,
    MaxOut= MaxIn.
p([E|T], MinIn, MinOut, MaxIn, MaxOut) =>
    Min1 = min(MinIn, E),
```

```
Max1 = max(MaxIn, E),
p(T, Min1, MinOut, Max1, MaxOut).
```

Notice that there are multiple accumulators: MinIn and MaxIn. Since the min_max predicate returns two values, the accumulators each have an "in" variable (MinIn and Maxin) and an "out" variable (MinOut and MaxOut). If the first parameter of predicate $p$ is an empty list, then MinOut is set to the value of MinIn, and MaxOut is set to the value of MaxIn.

Foreach and do-while loops are compiled in a similar manner to while loops.

## Nested Loops

As mentioned above, variables that only occur within a loop are local to each iteration of the loop. In nested loops, variables that are local to the outer loop are global to the inner loop. In other words, if a variable occurs in the outer loop, then the variable is also visible in the inner loop. However, variables that are local to the inner loop are not visiable to the outer loop.

For example, consider the nested loops:

```
nested =>
    foreach (I in 1 .. 10)
        printf("Numbers between %d and %d ", I, I * I),
        foreach (J in I .. I * I)
            printf("%d ", J)
        end,
        nl
    end.
```

Variable I is local to the outer foreach loop, and is global to the inner foreach loop. Therefore, iterator $J$ is able to iterate from $I$ to $I \star I$ in the inner foreach loop. Iterator $J$ is local to the inner loop, and does not occur in the outer loop.

Since a foreach loop with $N$ iterators is converted into $N$ nested foreach loops, the order of the iterators matters.

### 5.4.1 List Comprehensions

List comprehensions are compiled into foreach loops.

## Example

```
comp_ex =>
    L [(A, X):A in [a, b], X in 1 .. 2].
```

This list comprehension is compiled to:

```
comp_ex =>
    List = L,
    foreach (A in [a, b], X in 1 .. 2)
        L = [(A, X) | T],
        L := T
    end,
    L = [].
```


## Example

```
make_list1 =>
    L = [Y : X in 1..5],
    write(L).
make_list2 =>
    Y = Y,
    L = [Y : X in 1..5],
    write(L).
```

Suppose that a user would like to create a list $[\mathrm{Y}, \mathrm{Y}, \mathrm{Y}, \mathrm{Y}, \mathrm{Y}]$. The make_list1 predicate incorrectly attempts to make this list; instead, it outputs a list of 5 different variables since $Y$ is local. In order to make all five variables the same, make_list2 makes variable Y global, by adding the line $Y=Y$ to globalize $Y$.

## Chapter 6

## Exceptions

An exception is an event that occurs during the execution of a program. An exception requires a special treatment. In Picat, an exception is just a term. A built-in exception is a structure, where the name denotes the type of the exception, and the arguments provide other information about the exception, such as the source, which is the goal or function that raised the exception.

### 6.1 Built-in Exceptions

Picat throws many types of exceptions. The following are some of the built-in exceptions: ${ }^{1}$

- zero_divisor (Source) : Source divides a number by zero.
- domain_error (Val,Source): Source receives a value Val that is unexpected in the domain.
- existence_error (Entity, Source) : Source tries to use Entity, such as a file, a function, or a solver, that does not exist.
- interrupt (Source): The execution is interrupted by a signal. For an interrupt caused by ctrl-c, Source is keyboard.
- io_error (ENo, EMsg, Source) : An I/O error with the number ENo and message EMsg occurs in Source.
- load_error (FName, Source) : An error occurs while loading the byte-code file named FName. This error is caused by the malformatted byte-code file.
- out_of_memory (Area) : The system runs out of memory while expanding Area, which can be: stack_heap, trail, program, table, or findall.
- out_of_bound (EIndex, Source) : Source tries to access an element of a compound value using the index EIndex, which is out of bound. An index is out of bound if it is less than or equal to zero, or if it is greater than the length of the compound value.
- syntax_error (String, Source) : String cannot be parsed into a value that is expected by Source. For example, read_int () throws this exception if it reads in a string "a" rather than an integer.
- unresolved_function_call (FCall): No rule is applicable to the function call FCall.

[^5]- Type_expected (EArg, Source): The argument EArg in Source is not an expected type or value, where Type can be var, nonvar, dvar, atom, integer, real, number, list, map, etc.


### 6.2 Throwing Exceptions

The built-in predicate throw Exception throws Exception. After an exception is thrown, the system searches for a handler for the exception. If none is found, then the system displays the exception and aborts the execution of the current query. It also prints the backtrace of the stack if it is in debug mode. For example, for the function call open ("abc.txt"), the following message will be displayed if there is no file that is named "abc.txt".

```
*** error(existence_error(source_sink,abc.txt),open)
```


### 6.3 Defining Exception Handlers

All exceptions, including those raised by built-ins and interruptions, can be caught by catchers. A catcher is a call in the form:

```
catch(Goal,Exception,RecoverGoal)
```

which is equivalent to Goal, except when an exception is raised during the execution of Goal that unifies Exception. When such an exception is raised, all of the bindings that have been performed on variables in Goal will be undone, and RecoverGoal will be executed to handle the exception. Note that Exception is unified with a renamed copy of the exception before RecoverGoal is executed. Also note that only exceptions that are raised by a descendant call of Goal can be caught.

The call call_cleanup (Call, Cleanup) is equivalent to call(Call), except that Cleanup is called when Call succeeds determinately (i.e., with no remaining choice point), when Call fails, or when Call raises an exception.

## Chapter 7

## Tabling

The Picat system is a term-rewriting system. For a predicate call, Picat selects a matching rule and rewrites the call into the body of the rule. For a function call $C$, Picat rewrites the equation $C=X$ where $X$ is a variable that holds the return value of $C$. Due to the existence of recursion in programs, the term-rewriting process may never terminate. Consider, for example, the following program:

```
reach(X,Y) ?=> edge(X,Y).
reach(X,Y) => reach(X,Z),edge(Z,Y).
```

where the predicate edge defines a relation, and the predicate reach defines the transitive closure of the relation. For a query such as reach $(a, X)$, the program never terminates due to the existence of left-recursion in the second rule. Even if the rule is converted to right-recursion, the query may still not terminate if the graph that is represented by the relation contains cycles.

Another issue with recursion is redundancy. Consider the following problem: Starting in the top left corner of a $N \times N$ grid, one can either go rightward or downward. How many routes are there through the grid to the bottom right corner? The following gives a program in Picat for the problem:

```
route(N,N,_Col) = 1.
route(N,_Row,N) = 1.
route(N,Row,Col) = route(N,Row+1,Col)+route(N,Row,Col+1).
```

The function call route $(20,1,1)$ returns the number of routes through a $20 \times 20$ grid. The function call route ( $\mathrm{N}, 1,1$ ) takes exponential time in $N$, because the same function calls are repeatedly spawned during the execution, and are repeatedly resolved each time that they are spawned.

### 7.1 Table Declarations

Tabling is a memoization technique that can prevent infinite loops and redundancy. The idea of tabling is to memorize the answers to subgoals and use the answers to resolve their variant descendants. In Picat, in order to have all of the calls and answers of a predicate or function tabled, users just need to add the keyword table before the first rule.

## Example

```
table
reach(X,Y) ?=> edge(X,Y).
```

```
reach(X,Y) => reach(X,Z),edge(Z,Y).
table
route(N,N,_Col) = 1.
route(N,_Row,N) = 1.
route(N,Row,Col) = route(N,Row+1,Col)+route(N,Row,Col+1).
```

With tabling, all queries to the reach predicate are guaranteed to terminate, and the function call route ( $\mathrm{N}, 1,1$ ) takes only $\mathrm{N}^{2}$ time.

For some problems, such as planning problems, it is infeasible to table all answers, because there may be an infinite number of answers. For some other problems, such as those that require the computation of aggregates, it is a waste to table non-contributing answers. Picat allows users to provide table modes to instruct the system about which answers to table. For a tabled predicate, users can give a table mode declaration in the form ( $M_{1}, M_{2}, \ldots, M_{n}$ ), where each $M_{i}$ is one of the following: a plus-sign (+) indicates input, a minus-sign (-) indicates output, max indicates that the corresponding variable should be maximized, and min indicates that the corresponding variable should be minimized. The last mode $M_{n}$ can be nt, which indicates that the argument is not tabled. Two types of data can be passed to a tabled predicate as an nt argument: (1) global data that are the same to all the calls of the predicate, and (2) data that are functionally dependent on the input arguments. Input arguments are assumed to be ground. Output arguments, including min and max arguments, are assumed to be variables. An argument with the mode min or max is called an objective argument. Only one argument can be an objective to be optimized. As an objective argument can be a compound value, this limit is not essential, and users can still specify multiple objective variables to be optimized. When a table mode declaration is provided, Picat tables only one optimal answer for the same input arguments.

## Example

```
table(+,+,-,min)
sp(X,Y,Path,W) ?=>
        Path = [(X,Y)],
        edge (X,Y,W).
sp(X,Y,Path,W) =>
    Path = [(X,Z)|Path1],
    edge(X,Z,Wxz),
        sp(Z,Y,Path1,W1),
        W = Wxz+W1.
```

The predicate edge $(X, Y, W)$ specifies a weighted directed graph, where $W$ is the weight of the edge between node $X$ and node $Y$. The predicate $\operatorname{sp}(X, Y, P a t h, W)$ states that Path is a path from $X$ to $Y$ with the minimum weight $W$. Note that whenever the predicate $s p / 4$ is called, the first two arguments must always be instantiated. For each pair, the system stores only one path with the minimum weight.

The following program finds a shortest path among those with the minimum weight for each pair of nodes:

```
table (+,+,-,min).
sp(X,Y,Path,WL) ?=>
    Path = [(X,Y)],
    WL = (Wxy,1),
```

```
    edge(X,Y,Wxy).
sp(X,Y,Path,WL) =>
    Path = [(X,Z)|Path1],
    edge(X,Z,Wxz),
    sp(Z,Y,Path1,WL1),
    WL1 = (Wzy,Len1),
    WL = (Wxz+Wzy,Len1+1).
```

For each pair of nodes, the pair of variables ( $W, L e n$ ) is minimized, where $W$ is the weight, and Len is the length of a path. The built-in function compare_terms $\left(T_{1}, T_{2}\right)$ is used to compare answers. Note that the order is important. If the term would be (Len, $W$ ), then the program would find a shortest path, breaking a tie by selecting one with the minimum weight.

The tabling system is useful for offering dynamic programming solutions for planning problems. The following shows a tabled program for general planning problems:

```
table (+,-,min)
plan(S,Plan,Len),final(S) => Plan=[],Len=0.
plan(S,Plan,Len) =>
    action(Action,S,S1),
    plan(S1,Plan1,Len1),
    Plan = [Action|Plan1],
    Len = Len1+1.
```

The predicate action (Action, $S, S 1$ ) selects an action and performs the action on state $S$ to generate another state, S1.

## Example

The program shown in Figure 7.1 solves the Farmer's problem: The farmer wants to get his goat, wolf, and cabbage to the other side of the river. His boat isn't very big, and it can only carry him and either his goat, his wolf, or his cabbage. If he leaves the goat alone with the cabbage, then the goat will gobble up the cabbage. If he leaves the wolf alone with the goat, then the wolf will gobble up the goat. When the farmer is present, the goat and cabbage are safe from being gobbled up by their predators.

### 7.2 The Tabling Mechanism

The Picat tabling system employs the so-called linear tabling mechanism, which computes fixpoints by iteratively evaluating looping subgoals. The system uses a data area, called the table area, to store tabled subgoals and their answers. The tabling area can be initialized with the following built-in predicate:

- initialize_table: This predicate initializes the table area.

This predicate clears up the table area. It's users' responsibility to ensure that no data in the table area are referenced by any part of the application.

Linear tabling relies on the following three primitive operations to access and update the table area.

Subgoal lookup and registration: This operation is used when a tabled subgoal is encountered during execution. It looks up the subgoal table to see if there is a variant of the subgoal.

```
go =>
    S0=[s,s,s,s],
    plan(S0,Plan,_),
    writeln(Plan.reverse()).
table (+,-,min)
plan([n,n,n,n],Plan,Len) => Plan=[], Len=0.
plan(S,Plan,Len) =>
    Plan=[Action|Plan1],
    action(S,S1,Action),
    plan(S1,Plan1,Len1),
    Len=Len1+1.
action([F,F,G,C],S1,Action) ?=>
    Action=farmer_wolf,
    opposite(F,F1),
    S1=[F1,F1,G,C],
    not unsafe(S1).
action([F,W,F,C],S1,Action) ?=>
    Action=farmer_goat,
    opposite(F,F1),
    S1=[F1,W,F1,C],
    not unsafe(S1).
action([F,W,G,F],S1,Action) ?=>
    Action=farmer_cabbage,
    opposite(F,F1),
    S1=[F1,W,G,F1],
    not unsafe(S1).
action([F,W,G,C],S1,Action) ?=>
    Action=farmer_alone,
    opposite(F,F1),
    S1=[F1,W,G,C],
    not unsafe(S1).
index (+,-) (-,+)
opposite(n,s).
opposite(s,n).
unsafe([F,W,G,_C]),W==G,F!==W => true.
unsafe([F,_W,G,C]),G==C,F!==G => true.
```

Figure 7.1: A program for the Farmer's problem.

If not, it inserts the subgoal (termed a pioneer or generator) into the subgoal table. It also allocates an answer table for the subgoal and its variants. Initially, the answer table is empty. If the lookup finds that there already is a variant of the subgoal in the table, then the record that is stored in the table is used for the subgoal (called a consumer). Generators and consumers are handled differently. In linear tabling, a generator is resolved using rules, and a consumer is resolved using answers; a generator is iterated until the fixed point is reached, and a consumer fails after it exhausts all of the existing answers.

Answer lookup and registration: This operation is executed when a rule succeeds in generating an answer for a tabled subgoal. If a variant of the answer already exists in the table, then it does nothing; otherwise, it inserts the answer into the answer table for the subgoal, or it tables the answer according to the mode declaration. Picat uses the lazy consumption strategy (also called the local strategy). After an answer is processed, the system backtracks to produce the next answer.

Answer return: When a consumer is encountered, an answer is returned immediately, if an answer exists. On backtracking, the next answer is returned. A generator starts consuming its answers after it has exhausted all of its rules. Under the lazy consumption strategy, a top-most looping generator does not return any answer until it is complete.

## Chapter 8

## The planner Module

The planner module provides several predicates for solving planning problems. Given an initial state, a final state, and a set of possible actions, a planning problem is to find a plan that transforms the initial state to the final state. In order to use the planner module to solve a planning problem, users have to provide the condition for the final states and the state transition diagram through the following global predicates:

- final $(S)$ : This predicate succeeds if $S$ is a final state.
- final ( $S$, Plan , Cost) : A final state can be reached from $S$ by the action sequence in Plan with Cost. If this predicate is not given, then the system assumes the following definition:

```
final(S,Plan,Cost) => Plan=[], Cost=0, final(S).
```

- action (S,NextS,Action, ActionCost): This predicate encodes the state transition diagram of the planning problem. The state $S$ can be transformed into $N e x t S$ by performing Action. The cost of Action is ActionCost. If the plan's length is the only interest, then ActionCost should be 1 .

A state is normally a ground term. As all states are tabled during search, it is of paramount importance to find a good representation for states such that terms among states can be shared as much as possible.

In addition to the two required predicates given above, users can optionally provide the following global procedures to assist Picat in searching for plans:

- heuristic $(S)$ : This function returns an estimation of the resource amount needed to transform $S$ into a goal state. This estimation is said to be admissible if it never exceeds the real cost. All heuristic estimations must be admissible in order for the planner to find optimal plans. This function is used by the planner to check the heuristic estimation before each state expansion.
- sequence ( $P$, Action) : This predicate binds Action to a viable action form based on the current partial plan $P$. Note that the actions in list $P$ are in reversed order, with the most recent action occurring first in the list, and the first action occurring last in the list. The planner calls sequence/ 2 to find an action for expanding the current state before calling action/4. For example,
sequence([move(R,X,Y)|_], Action) ?=> Action = \$move(R,Y,_).

```
sequence([move(R,_,_)|_], Action) ?=> Action = $jump(R).
sequence([move(R,_,_)|_], Action) => Action = $wait(R).
sequence(_, _) => true.
```

These sequence rules ban robots from moving in an interleaving fashion; a robot must continue to move until it takes the action jump or wait before another robot can start moving. The last rule sequence ( ${ }_{-}, \quad$ ) $\quad=>$ true is necessary; it permits any action to be taken if the partial plan is empty, or if the most recent action in the partial plan is not move.

### 8.1 Depth-Bounded Search

Depth-bounded search amounts to exploring the search space, taking into account the current available resource amount. A new state is only explored if the available resource amount is nonnegative. When depth-bounded search is used, the function current_resource () can be used to retrieve the current resource amount. If the heuristic estimate of the cost to travel from the current state to the final state is greater than the available resource amount, then the current state fails.

- plan (S,Limit, Plan, Cost) : This predicate, if it succeeds, binds Plan to a plan that can transform state $S$ to a final state that satisfies the condition given by final/1 or final/3. Cost is the cost of Plan, which cannot exceed Limit, which is a given nonnegative integer.
- plan $(S$, Limit, Plan $):$ If the second argument is an integer, then this predicate is the same as the plan/ 4 predicate, except that the plan's cost is not returned.
- plan $(S$, Plan, PlanCost $)$ : If the second argument is a variable, then this predicate is the same as the plan / 4 predicate, except that the limit is assumed to be 268435455.
- plan $(S, P l a n)$ : This predicate is the same as the plan/4 predicate, except that the limit is assumed to be 268435455, and that the plan's cost is not returned.
- best_plan (S, Limit, Plan, Cost) : This predicate finds an optimal plan by using the following algorithm: It first calls plan/ 4 to find a plan of 0 cost. If no plan is found, then it increases the cost limit to 1 or double the cost limit. Once a plan is found, the algorithm uses binary search to find an optimal plan.
- best_plan ( $S$, Limit, Plan) : If the second argument is an integer, then this predicate is the same as the best_plan/4 predicate, except that the plan's cost is not returned.
- best_plan ( $S$, Plan, PlanCost) : If the second argument is a variable, then this predicate is the same as the best_plan/4 predicate, except that the limit is assumed to be 268435455.
- best_plan $(S, P l a n)$ : This predicate is the same as the best_plan/4 predicate, except that the limit is assumed to be 268435455 , and that the plan's cost is not returned.
- best_plan_nondet ( $S$, Limit, Plan, Cost) : This predicate is the same as best_plan ( $S$, Limit, Plan, Cost) , except that it allows multiple best plans to be returned through backtracking.
- best_plan_nondet ( $S$, Limit, Plan) : If the second argument is an integer, then this predicate is the same as the best_plan_nondet/4 predicate, except that the plan's cost is not returned.
- best_plan_nondet ( $S$, Plan, PlanCost) : If the second argument is a variable, then this predicate is the same as the best_plan_nondet/4 predicate, except that the limit is assumed to be 268435455 .
- best_plan_nondet ( $S$, Plan) : This predicate is the same as the best_plan_nondet / 4 predicate, except that the limit is assumed to be 268435455, and that the plan's cost is not returned.
- best_plan_bb (S,Limit, Plan, Cost) : This predicate, if it succeeds, binds Plan to an optimal plan that can transform state $S$ to a final state. Cost is the cost of Plan, which cannot exceed Limit, which is a given non-negative integer. The branch-and-bound algorithm is used to find an optimal plan.
- best_plan_bb $(S$, Limit, Plan $)$ : If the second argument is an integer, then this predicate is the same as the best_plan_bb/4 predicate, except that the plan's cost is not returned.
- best_plan_bb ( $S$, Plan, PlanCost): If the second argument is a variable, then this predicate is the same as the best_plan_bb/ 4 predicate, except that the limit is assumed to be 268435455 .
- best_plan_bb $(S, P l a n)$ : This predicate is the same as the best_plan_bb/4 predicate, except that the limit is assumed to be 268435455 , and that the plan's cost is not returned.
- best_plan_bin ( $S$, Limit, Plan, Cost): This predicate, if it succeeds, binds Plan to an optimal plan that can transform state $S$ to a final state. Cost is the cost of Plan, which cannot exceed Limit, which is a given non-negative integer. The branch-and-bound algorithm is used with binary search to find an optimal plan.
- best_plan_bin $(S$, Limit, Plan $)$ : If the second argument is an integer, then this predicate is the same as the best_plan_bin/4 predicate, except that the plan's cost is not returned.
- best_plan_bin ( $S$, Plan, PlanCost): If the second argument is a variable, then this predicate is the same as the best_plan_bin/4 predicate, except that the limit is assumed to be 268435455 .
- best_plan_bin ( $S, P l a n)$ : This predicate is the same as the best_plan_bin/4 predicate, except that the limit is assumed to be 268435455 , and that the plan's cost is not returned.
- current_resource ()=Amount: This function returns the current available resource amount of the current node. If the current execution path was not initiated by one of the calls that performs resource-bounded search, then 268435455 is returned. This function can be used to check the heuristics. If the heuristic estimate of the cost to travel from the current state to a final state is greater than the available resource amount, then the current state can be failed.
- current_plan ()$=$ Plan: This function returns the current plan that has transformed the initial state to the current state. If the current execution path was not initiated by one of the calls that performs resource-bounded search, then [] is returned.
- current_resource_plan_cost (Amount, Plan, Cost) : This predicate retrieves the attributes of the current node in the search tree, including the resource amount, the path to the node, and its cost.
- is_tabled_state $(S)$ : This predicate succeeds if the state $S$ has been explored before and has been tabled.


### 8.2 Depth-Unbounded Search

In contrast to depth-bounded search, depth-unbounded search does not take into account the available resource amount. A new state can be explored even if no resource is available for the exploration. The advantage of depth-unbounded search is that failed states are never re-explored.

- plan_unbounded ( $S$, Limit, Plan, Cost) : This predicate, if it succeeds, binds Plan to a plan that can transform state $S$ to a final state. Cost is the cost of Plan, which cannot exceed Limit, which is a given non-negative integer.
- plan_unbounded ( $S$, Limit, Plan) If the second argument is an integer, then this predicate is the same as the plan_unbounded/4 predicate, except that the plan's cost is not returned.
- plan_unbounded ( $S$, Plan, PlanCost) : If the second argument is a variable, then this predicate is the same as the plan_unbounded/4 predicate, except that the limit is assumed to be 268435455.
- plan_unbounded $(S$, Plan $)$ : This predicate is the same as the above predicate, except that the limit is assumed to be 268435455.
- best_plan_unbounded ( $S$, Limit, Plan, Cost) : This predicate, if it succeeds, binds Plan to an optimal plan that can transform state $S$ to a final state. Cost is the cost of Plan, which cannot exceed Limit, which is a given non-negative integer.
- best_plan_unbounded ( $S$, Limit, Plan) If the second argument is an integer, then this predicate is the same as the best_plan_unbounded/4 predicate, except that the plan's cost is not returned.
- best_plan_unbounded ( $S$, Plan, PlanCost) : If the second argument is a variable, then this predicate is the same as the best_plan_unbounded/4 predicate, except that the limit is assumed to be 268435455 .
- best_plan_unbounded ( $S$, Plan): This predicate is the same as the above predicate, except that the limit is assumed to be 268435455.


### 8.3 Examples

The program shown in Figure 8.1 solves the Farmer's problem by using the planner module.
Figure 8.2 gives a program for the 15 -puzzle problem. A state is represented as a list of sixteen locations, each of which takes the form $\left(R_{i}, C_{i}\right)$, where $R_{i}$ is a row number and $C_{i}$ is a column number. The first element in the list gives the position of the empty square, and the remaining elements in the list give the positions of the numbered tiles from 1 to 15 . The function heuristic(Tiles) returns the Manhattan distance between the current state and the final state.

```
import planner.
go =>
    S0=[s,s,s,s],
    best_plan(S0,Plan),
    writeln(Plan).
final([n,n,n,n]) => true.
action([F,F,G,C],S1,Action,ActionCost) ?=>
    Action=farmer_wolf,
    ActionCost = 1,
    opposite(F,F1),
    S1=[F1,F1,G,C],
    not unsafe(S1).
action([F,W,F,C],S1,Action,ActionCost) ?=>
    Action=farmer_goat,
    ActionCost = 1,
    opposite(F,F1),
    S1=[F1,W,F1,C],
    not unsafe(S1).
action([F,W,G,F],S1,Action,ActionCost) ?=>
    Action=farmer_cabbage,
    ActionCost = 1,
    opposite(F,F1),
    S1=[F1,W,G,F1],
    not unsafe(S1).
action([F,W,G,C],S1,Action,ActionCost) =>
    Action=farmer_alone,
    ActionCost = 1,
    opposite(F,F1),
    S1=[F1,W,G,C],
    not unsafe(S1).
index (+,-) (-,+)
opposite(n,s).
opposite(s,n).
unsafe([F,W,G,_C]),W==G,F!==W => true.
unsafe([F,_W,G,C]),G==C,F!==G => true.
```

Figure 8.1: A program for the Farmer's problem using planner.
import planner.

```
main =>
    InitS = [(1,2), (2,2), (4,4), (1,3),
        (1,1), (3,2), (1,4), (2,4),
        (4,2), (3,1), (3,3), (2,3),
        (2,1), (4,1), (4,3), (3,4)],
    best_plan(InitS,Plan),
    foreach (Action in Plan)
        println(Action)
    end.
```

final(State) => State=[(1,1),(1,2),(1,3),(1,4),
$(2,1),(2,2),(2,3),(2,4)$,
$(3,1),(3,2),(3,3),(3,4)$,
$(4,1),(4,2),(4,3),(4,4)]$.
action([P0@(R0,C0)|Tiles],NextS,Action, Cost) =>
Cost = 1,
(R1 = R0-1, R1 >= 1, C1 = C0, Action = up;
R1 $=$ R0+1, R1 $=<4, C 1=C 0$, Action = down;
R1 = R0, C1 = C0-1, C1 >= 1, Action = left;
R1 = R0, C1 = C0+1, C1 =< 4, Action = right),
P1 = (R1,C1),
slide(P0,P1,Tiles,NTiles),
NextS = [P1|NTiles].
\% slide the tile at P1 to the empty square at P0
slide (P0,P1, [P1|Tiles],NTiles) =>
NTiles = [PO|Tiles].
slide (P0,P1,[Tile|Tiles],NTiles) =>
NTiles=[Tile|NTilesR],
slide (P0,P1,Tiles, NTilesR).
heuristic([_|Tiles]) = Dist =>
final([_|FTiles]),
Dist $=\operatorname{sum}([a b s(R-F R)+a b s(C-F C)$ :
$\{(R, C),(F R, F C)\}$ in zip(Tiles,FTiles)]).

Figure 8.2: A program for the 15-puzzle

## Chapter 9

## Modules

A module is a bundle of predicate and function definitions that are stored in one file. A module forms a name space. Two definitions can have the same name if they reside in different modules. Because modules avoid name clashes, they are very useful for managing source files of large programs.

### 9.1 Module and Import Declarations

In Picat, source files must have the extension name ".pi". A module is a source file that begins with a module name declaration in the form:

```
module Name.
```

where Name must be the same as the main file name. A file that does not begin with a module declaration is assumed to belong to the default global module. The following names are reserved for system modules and should not be used to name user's modules: bp, basic, cp, glb, io, math, mip, planner, ordset, sat, sys, os, and util.

In order to use symbols that are defined in another module, users must explicitly import them with an import declaration in the form:
import Name $_{1}, \ldots$, Name ${ }_{n}$.
where each imported $N a m e_{i}$ is a module name. For each imported module, the compiler first searches for it in the search path that is specified by the environment variable PICATPATH. If no module is found, the compiler gives an error message. Several modules are imported by default, including basic, io, math, and sys.

The import relation is not transitive. Suppose that there are three modules: $A, B$, and $C$. If $A$ imports $B$ and $B$ imports $C$, then $A$ still needs to import $C$ in order to reference $C$ 's symbols.

The built-in command $\mathrm{cl}($ " xxx ") compiles the file $\mathrm{xxx} . \mathrm{pi}$ and loads the generated code into the interpreter. The built-in command load ("xxx") loads the bytecode file xxx . qi. It compiles the source file xxx .pi only when necessary. The load command also imports the public symbols defined in the module to the interpreter. This allows users to use these symbols on the command line without explicitly importing the symbols. If the file xxx . pi imports modules, those module files will be compiled and loaded when necessary.

### 9.2 Binding Calls to Definitions

The Picat system has a global symbol table for atoms, a global symbol table for structure names, and a global symbol table for modules. For each module, Picat maintains a symbol table for the
public predicate and function symbols defined in the module. Private symbols that are defined in a module are compiled away, and are never stored in the symbol table. While predicate and function symbols can be local to a module, atoms and structures are always global.

The Picat module system is static, meaning that the binding of normal (or non-higher-order) calls to their definitions takes place at compile time. For each call, the compiler first searches the default modules for a definition that has the same name as the call. If no definition is found, then the compiler searches for a definition in the enclosing module. If no definition is found, the compiler searches the imported modules in the order that they were imported. If no definition is found in any of these modules, then the compiler will issue an warning ${ }^{1}$, assuming the symbol is defined in the global module.

It is possible for two imported modules to contain different definitions that have the same name. When multiple names match a call, the order of the imported items determines which definition is used. Picat allows users to use qualified names to explicitly select a definition. A module-qualified call is a call preceded by a module name and ' .' without intervening whitespace.

## Example

\% qsort.pi
module qsort.

```
sort([]) = [].
sort([H|T]) =
    sort([E : E in T, E=<H])++[H]++sort([E : E in T, E>H]).
% isort.pi
module isort.
sort([]) = [].
sort([H|T]) = insert(H,sort(T)).
private
insert (X,[]) = [X].
insert(X,Ys@[Y|_]) = Zs, X=<Y => Zs=[X|Ys].
insert(X,[Y|Ys]) = [Y|insert(X,Ys)].
```

The module qsort. pi defines a function named sort using quick sort, and the module isort defines a function of the same name using insertion sort. In the following session, both modules are used.

```
picat> load("qsort")
picat> load("isort")
picat> L=sort([2,1,3])
L = [1,2,3]
picat> L=qsort.sort([2,1,3])
L=[1,2,3]
picat> L=isort.sort([2,1,3])
L = [1,2,3]
```

[^6]As sort is also defined in the basic module, which is preloaded, that function is used for the command $\mathrm{L}=\operatorname{sort}([2,1,3])$.

Module names are just atoms. Consequently, it is possible to bind a variable to a module name. Nevertheless, in a module-qualified call M.C, the module name can never be a variable. Recall that the dot notation is also used to access attributes and to call predicates and functions. The notation $M . C$ is treated as a call or an attribute if $M$ is not an atom.

Suppose that users want to define a function named generic_sort ( $M, L$ ) that sorts list $L$ using the sort function defined in module M. Users cannot just call M. sort (L), since M is a variable. Users can, however, select a function based on the value held in $M$ by using function facts as follows:

```
generic_sort(qsort,L) = qsort.sort(L).
generic_sort(isort,L) = isort.sort(L).
```


### 9.3 Binding Higher-Order Calls

Because Picat forbids variable module qualifiers and terms in dot notations, it is impossible to create module-qualified higher-order terms. For a higher-order call, if the compiler knows the name of the higher-order term, as in findall $(\mathrm{X}, \operatorname{member}(X, L)$ ), then it searches for a definition for the name, just like it does for a normal call. However, if the name is unknown, as in $\operatorname{apply}(F, X, Y)$, then the compiler generates code to search for a definition. For a higher-order call to call/ N or $\operatorname{apply} / \mathrm{N}$, the runtime system searches modules in the following order:

1. The implicitly imported built-in modules basic, io, math, and sys.
2. The enclosing module of the higher-order call.
3. The explicitly imported modules in the enclosing module in the order that they were imported.
4. The global module.

As private symbols are compiled away at compile time, higher-order terms can never reference private symbols. Due to the overhead of runtime search, the use of higher-order calls is discouraged.

### 9.4 Library Modules

Picat comes with a library of standard modules, described in separate chapters. The function sys.loaded_modules () returns a list of modules that are currently in the system.

## Chapter 10

## I/O

Picat has an io module for reading input from files and writing output to files. The io module is imported by default.

The io module contains functions and predicates that read from a file, write to a file, reposition the read/write pointer within a file, redirect input and output, and create temporary files and pipes.

The io module uses file descriptors to read input from files, and to write output to files. A file descriptor is a structure that encodes file descriptor data, including an index in a file descriptor table that stores information about opened files. The following example reads data from one file, and writes the data into another file.

## Example

```
rw =>
    Reader = open("input_file.txt"),
    Writer = open("output_file.txt", write),
    L = read_line(Reader),
    while (L != end_of_file)
            println(Writer, L),
            flush(Writer),
            L := read_line(Reader)
    end,
    close(Reader),
    close(Writer).
```


### 10.1 Opening a File

There are two functions for opening a file. Both of them are used in the previous example.

- open $($ Name $)=F D$ : The Name parameter is a filename that is represented as a string. This function opens the file with a default read mode.
- open (Name, Mode) $=F D$ : The Mode parameter is one of the four atoms: read, write, or append. The read atom is used for reading from a file; if the file does not exist, or the program tries to write to the file, then the program will throw an error. The write atom is used for reading from a file and writing to a file; if the file already exists, then the file will be overwritten. The append atom is similar to the write atom; however, if the file already exists, then data will be appended at the end of the pre-existing file.


### 10.2 Reading from a File

The io module has at least one function for reading data into each primitive data type. It also has functions for reading tokens, strings, and bytes. Recall that strings are stored as lists of singlecharacter atoms.

The read functions in the io module take a file descriptor as the first parameter. This file descriptor is the same descriptor that the open function returns. The parameter can be omitted if it is the standard input file stdin.

- read_int $(F D)=I n t$ : This function reads a single integer from the file that is represented by $F D$. It throws an input_mismatch exception if $F D$ is at the end of the file or the next token at $F D$ is not an integer.
- read_int () = Int: This function is the same as read_int (stdin).
- read_real $(F D)=$ Real: This function reads a single real number from the file that is represented by $F D$. It throws an inputmismatch exception if $F D$ is at the end of the file or the next token at $F D$ is not a number.
- read_real () = Real: This function is the same as read_real(stdin).
- read_char $(F D)=$ Val: This function reads a single UTF-8 character from the file that is represented by $F D$. It returns end_of_file if $F D$ is at the end of the file.
- read_char ()$=$ Val: This function is the same as read_char(stdin).
- read_char $(F D, N)=$ String: This function reads up to $N$ UTF-8 characters from the file that is represented by $F D$. It returns a string that contains the characters that were read.
- read_char_code $(F D)=$ Val: This function reads a single UTF-8 character from the file that is represented by $F D$ and returns its code point. It returns -1 if $F D$ is at the end of the file.
- read_char_code () = Val: This function is the same as read_char_code (stdin).
- read_char_code $(F D, N)=L i s t$ : This function reads up to $N$ UTF-8 characters from the file that is represented by $F D$. It returns a list of code points of the characters that were read.
- read_picat_token ( $F D$, TokenType, TokenValue): This predicate reads a single Picat token from the file that is represented by FD. TokenType is the type and TokenValue is the value of the token. TokenType is one of the following: atom, end_of_file, end_of_rule, integer, punctuation, real, string, underscore, and var.
- read_picat_token (TokenType,TokenValue): This predicate reads a token from stdin.
- read_picat_token $(F D)=$ TokenValue: This function reads a single Picat token from the file that is represented by $F D$ and returns the token value.
- read_picat_token () = TokenValue: This function is the same as the above, except that it reads from stdin.
- read_term $(F D)=$ Term: This function reads a single Picat term from the file that is represented by $F D$. The term must be followed by a dot '.' and at least one whitespace character. This function consumes the dot symbol. The whitespace character is not stored in the returned string.
- read_term() $=$ Term: This function is the same as read_term(stdin).
- read_line $(F D)=$ String: This function reads a string from the file that is represented by $F D$, stopping when either a newline (' $\backslash r \backslash n$ ' on Windows, and ' $\backslash n$ ' on Unix) is read, or the end_of_file atom is returned. The newline is not stored in the returned string.
- read_line() = String: This function is the same as read_line(stdin).
- readln $(F D)=$ String: This function does the same thing as read_line.
- readln() $=$ String: This function is the same as readln(stdin).
- read_byte $(F D)=$ Val: This function reads a single byte from the file that is represented by $F D$.
- read_byte () = Val: This function is the same as read_byte(stdin).
- read_byte $(F D, N)=$ List: This function reads up to $N$ bytes from the file that is represented by $F D$. It returns the list of bytes that were read.
- read_file_bytes (File) = List: This function reads an entire byte file into a list.
- read_file_bytes () = List: This function reads an entire byte file from the console into a list.
- read_file_chars (File) = String: This function reads an entire character file into a string.
- read_file_chars() = String: This function reads an entire character file from the console into a string.
- read_file_codes (File) $=$ List: This function reads UTF-8 codes of an entire character file into a list.
- read_file_codes () = List: This function reads UTF-8 codes of an entire character file from the console into a list.
- read_file_lines (File) = Lines: This function reads an entire character file into a list of line strings.
- read_file_lines () = Lines: This function reads an entire character file from the console into a list of line strings.
- read_file_terms (File) = Lines: This function reads an entire text file into a list of terms. In the file, each term must be terminated by '.' followed by at least one white space.
- read_file_terms () = Lines: This function reads an entire text file from the console into a list of terms.

There are cases when the read_char $(F D, N)$, and read_byte $(F D, N)$ functions will read fewer than $N$ values. One case occurs when the end of the file is encountered. Another case occurs when reading from a pipe. If a pipe is empty, then the read functions wait until data is written to the pipe. As soon as the pipe has data, the read functions read the data. If a pipe has fewer than $N$ values when a read occurs, then these three functions will return a string that contains all of the values that are currently in the pipe, without waiting for more values. In order to determine the actual number of elements that were read, after the functions return, use length (List) to check the length of the list that was returned.

The io module also has functions that peek at the next value in the file without changing the current file location. This means that the next read or peek function will return the same value, unless the read/write pointer is repositioned or the file is modified.

- peek_char $(F D)=$ Val
- peek_byte $(F D)=$ Val


### 10.2.1 End of File

The end of a file is detected through the end_of_file atom. If the input function returns a single value, and the read/write pointer is at the end of the file, then the end_of_file atom is returned. If the input function returns a list, then the end-of-file behavior is more complex. If no other values have been read into the list, then the end_of_file atom is returned. However, if other values have already been read into the list, then reaching the end of the file causes the function to return the list, and the end_of_file atom will not be returned until the next input function is called.

Instead of checking for end_of_file, the at_end_of_stream predicate can be used to monitor a file descriptor for the end of a file.

- at_end_of_stream $(F D)$ : The at_end_of_stream predicate is demonstrated in the following example.


## Example

```
rw =>
    Reader = open("file1.txt"),
    Writer = open("file2.txt", write),
    while (not at_end_of_stream(Reader))
                L := read_line(Reader),
                println(Writer, L),
                flush(Writer)
    end,
    close(Reader),
    close(Writer).
```

The advantage of using the at_end_of_stream predicate instead of using the end_of_file atom is that at_end_of_stream immediately indicates that the end of the file was reached, even if the last read function read values into a list. In the first example in this chapter, which used the end_of_file atom, an extra read_line function was needed before the end of the file was detected. In the above example, which used at_end_of_stream, read_line was only called if there was data remaining to be read.

### 10.3 Writing to a File

The write and print predicates take a file descriptor as the first parameter. The file descriptor is the same descriptor that the open function returns. If the file descriptor is stdout, then the parameter can be omitted.

- write (FD, Term): This predicate writes Term to a file. Single-character lists are treated as strings. Strings are double-quoted, and atoms are single-quoted when necessary. This predicate does not print a newline, meaning that the next write will begin on the same line.
- write (Term) : This predicate is the same as write (stdout, Term).
- write_byte (FD, Bytes) : This predicate writes a single byte or a list of bytes to a file.
- write_byte (Bytes): This predicate is the same as write_byte (stdout, Bytes).
- write_char (FD, Chars): This predicate writes a single character or a list of characters to a file. The characters are not quoted. When writing a single-character atom Char, write_char (FD, Char) is the same as print (FD, Char), but write_char is faster than print.
- write_char (Chars) : This predicate is the same as write_char(stdout,Chars).
- write_char_code (FD, Codes) : This predicate writes a single character or a list of characters of the given code or list of codes to a file.
- write_char_code (Codes) : This predicate is the same as the above, except that it writes to stdout.
- writeln $(F D$, Term): This predicate writes Term and a newline, meaning that the next write will begin on the next line.
- writeln (Term) : This predicate is the same as writeln(stdout,Term).
- writef (FD, Format, Args...) : This predicate is used for formatted writing, where the Format parameter contains format characters that indicate how to print each of the arguments in the Args parameter. The number of arguments in Args ... cannot exceed 10.

Note that these predicates write both primitive values and compound values.
The writef predicate includes a parameter that specifies the string that is to be formatted. The Format parameter is a string that contains format characters. Format characters take the form \%[flags][width][.precision]specifier. Only the percent sign and the specifier are mandatory. Flags can be used for justification and padding. The width is the minimum number of characters that are to be printed. The precision is the number of characters that are to be printed after the number's radix point. Note that the width includes all characters, including the radix point and the characters that follow it. The specifier indicates the type of data that is to be written. A specifier can be one of the C format specifiers $\% \%$, $\% \mathrm{C},{ }^{1} \% \mathrm{~d}, \% \mathrm{e}, \% \mathrm{E}, \% \mathrm{f}, \% \mathrm{~g}, \% \mathrm{G}, \% \mathrm{i}, \% \mathrm{O}, \% \mathrm{~s}$, $\% u$, $\% x$, and $\% X$. In addition, Picat uses the specifier $\% \mathrm{n}$ for newlines, and uses $\% \mathrm{w}$ for terms. For details, see Appendix F.

[^7]
## Example

```
formatted_print =>
    FD = open("birthday.txt",write),
    Format1 = "Hello, %s. Happy birthday! ",
    Format2 = "You are %d years old today. ",
    Format3 = "That is %.2f%% older than you were last year",
    writef(FD, Format1, "Bob"),
    writef(FD, Format2, 7),
    writef(FD, Format3, 7.0 / 6.0),
    close(FD).
```

This writes"Hello, Bob. Happy birthday! You are 7 years old today. That is $1.17 \%$ older than you were last year".

The io module also has the three print predicates.

- print (FD, Term) : This predicate prints Term to a file. Unlike the write predicates, the print predicates do not place quotes around strings and atoms.
- print (Term): This predicate is the same as print (stdout,Term).
- println $(F D$, Term) This predicate prints Term and a newline.
- println(Term) This predicate is the same as println(stdout,Term).
- printf (FD, Format, Args...) : This predicate is the same as writef, except that printf uses print to display the arguments in the Args parameter, while writef uses write to display the arguments in the Args parameter.

The following example demonstrates the differences between the write and print predicates.

## Example

```
picat> write("abc")
[a,b,c]
picat> write([a,b,c])
[a,b,c]
picat> write('a@b')
'a@b'
picat> writef("%w %s%n", [a,b,c],"abc")
[a,b,c] abc
picat> print("abc")
abc
picat> print([a,b,c])
abc
picat> print('a@b')
a@b
picat> printf("%w %s%n", [a,b,c],"abc")
abc abc
```


### 10.4 Flushing and Closing a File

The io module has one predicate to flush a file stream, and one predicate to close a file stream.

- flush $(F D)$ : This predicate causes all buffered data to be written without delay.
- close $(F D)$ : This predicate causes the file to be closed, releasing the file's resources, and removing the file from the file descriptor table. Any further attempts to write to the file descriptor without calling open will cause an error to be thrown.


### 10.5 Standard File Descriptors

The atoms stdin, stdout, and stderr represent the file descriptors for standard input, standard output, and standard error. These atoms allow the program to use the input and output functions of the io module to read from and to write to the three standard streams.

## Chapter 11

## Event-Driven Actors and Action Rules

Many applications require event-driven computing. For example, an interactive GUI system needs to react to UI events such as mouse clicks on UI components; a Web service provider needs to respond to service requests; a constraint propagator for a constraint needs to react to updates to the domains of the variables in the constraint. Picat provides action rules for describing event-driven actors. An actor is a predicate call that can be delayed and can be activated later by events. Actors communicate with each other through event channels.

### 11.1 Channels, Ports, and Events

An event channel is an attributed variable to which actors can be attached, and through which events can be posted to actors. A channel has four ports, named ins, bound, dom, and any, respectively. Many built-ins in Picat post events. When an attributed variable is instantiated, an event is posted to the ins-port of the variable. When the lower bound or upper bound of a variable's domain changes, an event is posted to the bound-port of the variable. When an inner element $E$, which is neither the lower or upper bound, is excluded from the domain of a variable, $E$ is posted to the dom-port of the variable. When an arbitrary element $E$, which can be the lower or upper bound or an inner element, is excluded from the domain of a variable, $E$ is posted to the any-port of the variable. The division of a channel into ports facilitates speedy handling of events. For better performance, the system posts an event to a port only when there are actors attached to the port. For example, if no actor is attached to a domain variable to handle exclusions of domain elements, then these events will never be posted.

The built-in post_event ( $X, T$ ) posts the event term $T$ to the dom-port of the channel variable $X$.

The following built-ins are used to post events to one of a channel's four ports:

- post_event_ins $(X)$ : posts an event to the ins-port of the channel $X$.
- post_event_bound $(X)$ : posts an event to the bound-port of the channel $X$.
- post_event_dom $(X, T)$ : posts the term $T$ to the dom-port of the channel $X$.
- post_event_any $(X, T)$ : posts the event $T$ to the any-port of the channel of $X$.

The call post_event $(X, T)$ is the same as post_event_dom $(X, T)$. This means that the dom-port of a finite domain variable has two uses: posting exclusions of inner elements from the domain, and posting general term events.

### 11.2 Action Rules

Picat provides action rules for describing the behaviors of actors. An action rule takes the following form:

```
Head, Cond, \(\{\) Event \(\}=>\) Body
```

where Head is an actor pattern, Cond is an optional condition, Event is a non-empty set of event patterns separated by ', ', and Body is an action. For an actor that is activated by an event, an action rule is said to be applicable if the actor matches Head and Cond is true. A predicate for actors is defined with action rules and non-backtrackable rules. It cannot contain backtrackable rules.

Unlike rules for a normal predicate or function, in which the conditions can contain any predicates, the conditions of the rules in a predicate for actors must be conjunctions of inline test predicates, such as type-checking built-ins (e.g., integer $(X)$ and $\operatorname{var}(X)$ ) and comparison built-ins (e.g., equality test $X==Y$, disequality test $X \quad!==Y$, and arithmetic comparison $X$ $>Y$ ). This restriction ensures that no variables in an actor can be changed while the condition is executed.

For an actor that is activated by an event, the system searches the definition sequentially from the top for an applicable rule. If no applicable rule is found, then the actor fails. If an applicable rule is found, the system executes the body of the rule. If the body fails, then the actor also fails. The body cannot succeed more than once. The system enforces this by converting Body into 'once Body' if Body contains calls to nondeterministic predicates. If the applied rule is an action rule, then the actor is suspended after the body is executed, meaning that the actor is waiting to be activated again. If the applied rule is a normal non-backtrackable rule, then the actor vanishes after the body is executed. For each activation, only the first applicable rule is applied.

For a call and an action rule 'Head, Cond, $\{$ Event $\}=>$ Body', the call is registered as an actor if the call matches Head and Cond evaluates to true. The event pattern Event implicitly specifies the ports to which the actor is attached, and the events that the actor watches. The following event patterns are allowed in Event:

- event $(X, T)$ : This is the general event pattern. The actor is attached to the dom-ports of the variables in $X$. The actor will be activated by events posted to the dom-ports. $T$ must be a variable that does not occur before event $(X, T)$ in the rule.
- ins $(X)$ : The actor is attached to the ins-ports of the variables in $X$. The actor will be activated when a variable in $X$ is instantiated.
- bound $(X)$ : The actor is attached to the bound-ports of the variables in $X$. The actor will be activated when the lower bound or upper bound of the domain of a variable in $X$ changes.
- $\operatorname{dom}(X)$ : The actor is attached to the dom-ports of the variables in $X$. The actor will be activated when an inner value is excluded from the domain of a variable in $X$. The actor is not interested in what value is actually excluded.
- dom $(X, E)$ : This is the same as $\operatorname{dom}(X)$, except the actor is interested in the value $E$ that is excluded. $E$ must be a variable that does not occur before $\operatorname{dom}(X, E)$ in the rule.
- dom_any (X) : The actor is attached to the any-ports of the variables in $X$. The actor will be activated when an arbitrary value, including the lower bound value and the upper bound value, is excluded from the domain of a variable in $X$. The actor is not interested in what value is actually excluded.
- dom_any $(X, E)$ : This is the same as dom_any $(X)$, except the actor is interested in the value $E$ that is actually excluded. $E$ must be a variable that does not occur before dom_any $(X, E)$ in the rule.

In an action rule, multiple event patterns can be specified. After a call is registered as an actor on the channels, it will be suspended, waiting for events, unless the atom generated occurs in Event, in which case the actor will be suspended after Body is executed.

Each thread has an event queue. After events are posted, they are added into the queue. Events are not handled until execution enters or exits a non-inline predicate or function. In other words, only non-inline predicates and functions can be interrupted, and inline predicates, such as $X=$ $Y$, and inline functions, such as $X+Y$, are never interrupted by events.

## Example

Consider the following action rule:

```
p(X), {event (X,T)} => writeln(T).
```

The following gives a query and its output:

```
Picat> p(X), X.post_event(ping), X.post_event(pong)
ping
pong
```

The call $p(X)$ is an actor. After $X$. post_event (ping), the actor is activated and the body of the action rule is executed, giving the output ping. After X. post_event (pong), the actor is activated again, outputting pong.

There is no primitive for killing actors or explicitly detaching actors from channels. As described above, an actor never disappears as long as action rules are applied to it. An actor vanishes only when a normal rule is applied to it. Consider the following example.

```
p(X,Flag),
    var(Flag),
    {event(X,T)}
=>
    writeln(T),
    Flag=1.
p(_,_) => true.
```

An actor defined here can only handle one event posting. After it handles an event, it binds the variable Flag. When a second event is posted, the action rule is no longer applicable, causing the second rule to be selected.

One question arises here: what happens if a second event is never posted to $X$ ? In this case, the actor will stay forever. If users want to immediately kill the actor after it is activated once, then users have to define it as follows:

```
p(X,Flag),
    var(Flag),
    {event(X,O),ins(Flag)},
=>
    write(O),
    Flag=1.
p(_,_) => true.
```

In this way, the actor will be activated again after Flag is bound to 1 , and will be killed after the second rule is applied to it.

### 11.3 Lazy Evaluation

The built-in predicate freeze ( $\mathrm{X}, \mathrm{Goal}$ ) is equivalent to 'once Goal', but its evaluation is delayed until X is bound to a non-variable term. The predicate is defined as follows:

```
freeze(X,Goal),var(X),{ins(X) } => true.
freeze(X,Goal) => call(Goal).
```

For the call freeze ( $X, G o a l$ ), if $X$ is a variable, then $X$ is registered as an actor on the ins-port of $X$, and $X$ is then suspended. Whenever $X$ is bound, the event ins is posted to the ins-port of $X$, which activates the actor freeze ( $X, G o a l$ ). The condition $\operatorname{var}(X)$ is checked. If true, the actor is suspended again; otherwise, the second rule is executed, causing the actor to vanish after it is rewritten into once Goal.

The built-in predicate different_terms $\left(T_{1}, T_{2}\right)$ is a disequality constraint on terms $T_{1}$ and $T_{2}$. The constraint fails if the two terms are identical; it succeeds whenever the two terms are found to be different; it is delayed if no decision can be made because the terms are not sufficiently instantiated. The predicate is defined as follows:

```
import cp.
different_terms(X,Y) =>
    different_terms(X,Y,1).
different_terms(X,Y,B),var(X),{ins(X),ins(Y) } => true.
different_terms(X,Y,B),var(Y),{ins(X)} => true.
different_terms([X|Xs],[Y|Ys],B) =>
    different_terms(X,Y,B1),
    different_terms(Xs,Ys,B2),
    B #= (B1 #\/ B2).
different_terms(X,Y,B),struct(X),struct(Y) =>
    writeln(X), writeln(Y),
    if (X.name !== Y.name; X.length !== Y.length) then
        B=1
    else
        Bs = new_list(X.length),
        foreach(I in 1 .. X.length)
                different_terms(X[I],Y[I],Bs[I])
        end,
        max(Bs) #= B
    end.
different_terms(X,Y,B),X==Y => B=0.
different_terms(X,Y,B) => B=1.
```

The call different_terms ( $X, Y, B$ ) is delayed if either $X$ or $Y$ is a variable. The delayed call watches ins ( $X$ ) and ins ( $Y$ ) events. Once both $X$ and $Y$ become non-variable, the action rule becomes inapplicable, and one of the subsequent rules will be applied. If $X$ and $Y$ are lists, then they are different if the heads are different (B1), or if the tails are different (B2). This relationship is represented as the Boolean constraint $B \quad \#=(B 1 \# / B 2)$. If $X$ and $Y$ are both structures,
then they are different if the functor is different, or if any pair of arguments of the structures is different.

### 11.4 Constraint Propagators

A constraint propagator is an actor that reacts to updates of the domains of the variables in a constraint.The following predicate defines a propagator for maintaining arc consistency on X for the constraint $\mathrm{X}+\mathrm{Y} \quad \#=\mathrm{C}$ :

```
import cp.
x_in_c_y_ac(X,Y,C),var(X),var(Y),
    {dom(Y,Ey) }
    =>
    fd_set_false(X,C-Ey).
x_in_c_y_ac(X,Y,C) => true.
```

Whenever an inner element Ey is excluded from the domain of $Y$, this propagator is triggered to exclude $C-E y$, which is the support of $E y$, from the domain of $X$. For the constraint $X+Y \quad \#=C$, users need to generate two propagators, namely, $x_{-}$in_c_Y_ac ( $X, Y, C$ ) and $x_{-} i n_{-} c_{-} Y \_a c(Y, X, C)$, to maintain the arc consistency. Note that in addition to these two propagators, users also need to generate propagators for maintaining interval consistency, because dom (Y, Ey) only captures exclusions of inner elements, and does not capture bounds. The following propagator maintains interval consistency for the constraint:

```
import cp.
x_add_Y_eq_c_ic(X,Y,C),var(X),var(Y),
    {generated,ins(X),ins(Y),bound(X),bound(Y) }
=>
    X :: C-Y.max .. C-Y.min,
    Y :: C-X.max .. C-X.min.
x_add_y_eq_c_ic(X,Y,C),var(X) =>
    X = C-Y .
x_add_Y_eq_c_ic(X,Y,C) =>
    Y = C-X.
```

When both $X$ and $Y$ are variables, the propagator $x_{-}$add_y_eq_c_ic $(X, Y, C)$ is activated whenever $X$ and $Y$ are instantiated, or whenever the bounds of their domains are updated. The body maintains the interval consistency of the constraint $\mathrm{X}+\mathrm{Y} \quad \#=\mathrm{C}$. The body is also executed when the propagator is generated. When either X or Y becomes non-variable, the propagator becomes a normal call, and vanishes after the variable X or Y is solved.

## Chapter 12

## Constraints

Picat provides three solver modules, including cp , sat, and mip, for modeling and solving constraint satisfaction and optimization problems (CSPs). All three of these modules implement the same set of constraints on integer-domain variables. The mip module also supports real-domain variables. In order to use a solver, users must first import the module. In order to make the symbols defined in a module available to the top level of the interpreter, users can use the built-in load to load the module or any module that imports the module. As the three modules have the same interface, this chapter describes the three modules together. Figure 12.1 shows the constraint operators that are provided by Picat. Unless it is explicitly specified otherwise, the built-ins that are described in this chapter appear in all three modules. In the built-ins that are presented in this chapter, an integer-domain variable can also be an integer, unless it is explicitly specified to only be a variable.

Table 12.1: Constraint operators in Picat

| Precedence | Operators |
| :---: | :---: |
| Highest | $::$, notin, $\#=, \#!=, \#<, \#=<, \#<=, \#>, \#>=$ |
|  | $\# \sim$ |
|  | $\# / \backslash$ |
|  | $\# \wedge$ |
|  | $\# \backslash /$ |
| Lowest | $\#=>$ |

A constraint program normally poses a problem in three steps: (1) generate variables; (2) generate constraints over the variables; and (3) call solve to find a valuation for the variables that satisfies the constraints and possibly optimizes an objective function.

## Example

This program in Figure 12.1 imports the cp module in order to solve the $N$-queens problem. The same program runs with the SAT solver if sat is imported, or runs with the LP/MIP solver if mip is imported. The predicate Qs :: 1..N declares the domains of the variables. The operator \#! = is used for inequality constraints. In arithmetic constraints, expressions are treated as terms, and it is unnecessary to enclose them with dollar-signs. The predicate solve (Qs) calls the solver in order to solve the array of variables $\ell s$. For $c p$, solve ([ff], $\ell s$ ), which always selects a

```
import cp.
queens(N) =>
    Qs=new_array(N),
    Qs :: 1..N,
    foreach (I in 1..N-1, J in I+1..N)
        Qs[I] #!= Qs[J],
        abs(Qs[I]-Qs[J]) #!= J-I
    end,
    solve(Qs),
    writeln(Qs).
```

Figure 12.1: A program for N -queens.
variable that has the smallest domain (the so-called first-fail principle), can be more efficient than solve(Qs).

### 12.1 Domain Variables

A domain variable is an attributed variable that has a domain attribute. The Boolean domain is treated as a special integer domain where 1 denotes true and 0 denotes false. Domain variables are declared with the built-in predicate Vars : : Exp.

- Vars : : Exp: This predicate restricts the domain or domains of Vars to Exp. Vars can be either a single variable, a list of variables, or an array of variables. For integer-domain variables, Exp must result in a list of integer values. For real-domain variables for the mip module, Exp must be an interval in the form $L . . U$, where $L$ and $U$ are real values.

Domain variables, when being created, are usually represented internally by using intervals. An interval turns to a bit vector when a hole occurs in the interval. The following built-in predicate can be used to reset the range or access the current range.

- fd_vector_min_max (Min, Max): When the arguments are integers, this predicate specifies the range of bit vectors; when the arguments are variables, this predicate binds them to the current bounds of the range. The default range is -3200 . 3200 .

The following built-ins are provided for domain variables.

- Vars notin Exp: This predicate excludes values Exp from the domain or domains of Vars, where Vars and Exp are the same as in Vars : : Exp. This constraint cannot be applied to real-domain variables.
- fd_degree $(F D V a r)=$ Degree: This function returns the number of propagators that are attached to $F D V a r$. This built-in is only provided by cp.
- fd_disjoint (FDVar1, FDVar2): This predicate is true if $F D \operatorname{Var} 1$ 's domain and FDVar2's domain are disjoint.
- fd_dom (FDVar) = List: This function returns the domain of FDVar as a list, where $F D V a r$ is an integer-domain variable. If $F D V a r$ is an integer, then the returned list contains the integer itself.
- fd_false (FDVar, Elm): This predicate is true if the integer Elm is not an element in the domain of FDVar.
- fd_max $(F D V a r)=$ Max: This function returns the upper bound of the domain of FDVar, where FDVar is an integer-domain variable.
- fd_min $(F D V a r)=$ Min: This function returns the lower bound of the domain of $F D V a r$, where $F D V a r$ is an integer-domain variable.
- fd_min_max (FDVar, Min, Max): This predicate binds Min to the lower bound of the domain of FDVar, and binds Max to the upper bound of the domain of FDVar, where $F D V a r$ is an integer-domain variable.
- fd_next (FDVar, Elm) = NextElm: This function returns the next element of Elm in FDVar's domain. It throws an exception if Elm has no next element in FDVar's domain.
- fd_prev (FDVar, Elm) = PrevElm: This function returns the previous element of Elm in FDVar's domain. It throws an exception if Elm has no previous element in FDVar's domain.
- fd_set_false (FDVar, Elm): This predicate excludes the element Elm from the domain of FDVar. If this operation results in a hole in the domain, then the domain changes from an interval representation into a bit-vector representation, no matter how big the domain is. This built-in is only provided by cp .
- fd_size (FDVar) = Size: This function returns the size of the domain of FDVar, where $F D V a r$ is an integer-domain variable.
- fd_true (FDVar, Elm): This predicate is true if the integer Elm is an element in the domain of FDVar.
- new_dvar() $=$ FDVar: This function creates a new domain variable with the default domain, which has the bounds -72057594037927935 . . 72057594037927935 on 64-bit computers and -268435455 . . 268435455 on 32-bit computers.


### 12.2 Table constraints

A table constraint, or an extensional constraint, over a tuple of variables specifies a set of tuples that are allowed (called positive) or disallowed (called negative) for the variables. A positive constraint takes the form table_in (DVars, $R$ ), where DVars is either a tuple of variables $\left\{X_{1}, \ldots, X_{n}\right\}$ or a list of tuples of variables, and $R$ is a list of tuples of integers in which each tuple takes the form $\left\{a_{1}, \ldots, a_{n}\right\}$. A negative constraint takes the form table_notin ( $D V \operatorname{ars}, R$ ).

## Example

The following example solves a toy crossword puzzle. One variable is used for each cell in the grid, so each slot corresponds to a tuple of variables. Each word is represented as a tuple of integers, and each slot takes on a set of words of the same length as the slot. Recall that the function ord (Char) returns the code of Char, and that the function chr (Code) returns the character of Code.

```
import cp.
crossword(Vars) =>
    Vars=[X1,X2,X3,X4,X5,X6,X7],
Words2=[{ord('I'),ord('N')},
    {ord('I'),ord('F')},
    {ord('A'),ord('S')},
    {ord('G'),ord('O')},
    {ord('T'),ord('O')}],
Words3=[{ord('F'),ord('U'),ord('N')},
    {ord('T'),ord('A'),ord('D')},
    {ord('N'),ord('A'),ord('G')},
    {ord('S'),ord('A'),ord('G')}],
table_in([{X1,X2},{X1,X3},{X5,X7},{X6,X7}], Words2),
table_in([{X3, X4, X5},{X2,X4,X6}],Words3),
solve(Vars),
writeln([chr(Code) : Code in Vars]).
```


### 12.3 Arithmetic Constraints

An arithmetic constraint takes the form

## Exp1 Rel Exp2

where $E x p 1$ and $E x p 2$ are arithmetic expressions, and Rel is one of the constraint operators: $\#=, \#!=, \#<, \#=<, \#<=, \#>$, or \#>=. The operators \#=< and \#<= are the same, meaning less than or equal to. An arithmetic expression is made from integers, variables, arithmetic functions, and constraints. The following arithmetic functions are allowed: + (addition), - (subtraction), * (multiplication), / (truncated integer division), / / (truncated integer division), count, div (floored integer division), mod, $* *$ (power), abs, min, max, and sum. Except for index notations, array comprehensions and list comprehensions, which are interpreted as function calls as in normal expressions, expressions in arithmetic constraints are treated as terms, and it is unnecessary to enclose them with dollar-signs. In addition to the numeric operators, the following functions are allowed in constraints:

- cond (BoolConstr, ThenExp, ElseExp) : This expression is the same as BoolConstr *ThenExp+ (1-BoolConstr) *ElseExp.
- count ( $V, D V a r s$ ) : The number of times $V$ occurs in $D V a r s$, where $D V a r s$ is a list of domain variables.
- min (DVars) : The minimum of DVars, where DVars is a list of domain variables.
- max (DVars) : The maximum of DVars, where DVars is a list of domain variables.
- min $(E x p 1, E x p 2):$ The minimum of $\operatorname{Exp} 1$ and $E x p 2$.
- max $(E x p 1, E x p 2)$ : The maximum of Exp 1 and $E x p 2$.
- sum (DVars) : The sum of DVars, where $D \operatorname{Vars}$ is a list of domain variables.
- prod (DVars) : The product of DVars, where $D$ Vars is a list of domain variables.

When a constraint occurs in an arithmetic expression, it is evaluated to 1 if it is satisfied and 0 if it is not satisfied.

## Example

```
import mip.
go =>
    \(M=\{\{0,3,2,3,0,0,0,0\}\),
        \(\{0,0,0,0,0,0,5,0\}\),
        \(\{0,1,0,0,0,1,0,0\}\),
        \(\{0,0,2,0,2,0,0,0\}\),
        \(\{0,0,0,0,0,0,0,5\}\),
        \(\{0,4,0,0,2,0,0,1\}\),
        \(\{0,0,0,0,0,2,0,3\}\),
        \(\{0,0,0,0,0,0,0,0\}\}\),
    maxflow (M,1,8).
maxflow(M,Source,Sink) =>
    \(\mathrm{N}=\mathrm{M}\). length,
    X=new_array ( \(\mathrm{N}, \mathrm{N}\) ) ,
    foreach(I in 1..N, J in 1..N)
        X[I,J] : : O..M[I,J]
    end,
    foreach(I in 1..N, I!=Source, I!=Sink)
        sum([X[J,I] : J in 1..N]) \#= sum([X[I,J] : J in 1..N])
    end,
    Total \#= sum([X[Source,I] : I in 1..N]),
    Total \#= sum([X[I,Sink] : I in 1..N]),
    solve([\$max(Total)],X),
    writeln(Total),
    writeln(X).
```

This program uses MIP to solve the maximum integer flow problem. Given the capacity matrix $M$ of a directed graph, the start vertex Source, and the destination vertex Sink, the predicate maxflow (M, Source, Sink) finds a maximum flow from Source to Sink over the graph. When two vertices are not connected by an arc, the capacity is given as 0 . The first foreach loop specifies the domains of the variables. For each variable $X[I, J]$, the domain is restricted to integers between 0 and the capacity, $M[I, J]$. If the capacity is 0 , then the variable is immediately instantiated to 0 . The next foreach loop posts the conservation constraints. For each vertex I, if it is neither the source nor the sink, then its total incoming flow amount

```
sum([X[J,I] : J in 1..N])
```

is equal to the total outgoing flow amount

```
sum([X[I,J] : J in 1..N]).
```

The total flow amount is the total outgoing amount from the source, which is the same as the total incoming amount to the sink.

### 12.4 Boolean Constraints

A Boolean constraint takes one of the following forms:

```
\# ~ BoolExp
BoolExp \# / \BoolExp
BoolExp \# ^ BoolExp
BoolExp \# \/ BoolExp
BoolExp \#=> BoolExp
BoolExp \#<=> BoolExp
```

BoolExp is either a Boolean constant (0 or 1), a Boolean variable (an integer-domain variable with the domain $[0,1]$ ), an arithmetic constraint, a domain constraint (in the form of Var : Domain or Var notin Domain), or a Boolean constraint. As shown in Table 12.1, the operator \# ~ has the highest precedence, and the operator \# $<=>$ has the lowest precedence. Note that the Boolean constraint operators have lower precedence than the arithmetic constraint operators. So the constraint

$$
X \#!=3 \# / \backslash X \#!=5 \# \Leftrightarrow B
$$

is interpreted as

$$
((X \#!=3) \# / \backslash(X \#!=5)) \#<=>B .
$$

The Boolean constraint operators are defined as follows.

- \# ${ }^{\sim}$ BoolExp: This constraint is 1 iff BoolExp is equal to 0 .
- BoolExp1 \# / \ BoolExp2: This constraint is 1 iff both BoolExp1 and BoolExp2 are 1.
- BoolExp1 \#^ BoolExp2: This constraint is 1 iff exactly one of BoolExp 1 and BoolExp 2 is 1 .
- BoolExp1 \# \/ BoolExp2: This constraint is 1 iff BoolExp1 or BoolExp 2 is 1 .
- BoolExp1 \#=> BoolExp2: This constraint is 1 iff BoolExp 1 implies BoolExp 2 .
- BoolExp1 \#<=> BoolExp2: This constraint is 1 iff BoolExp 1 and BoolExp2 are equivalent.


### 12.5 Global Constraints

A global constraint is a constraint over multiple variables. A global constraint can normally be translated into a set of smaller constraints, such as arithmetic and Boolean constraints. If the cp module is used, then global constraints are not translated into smaller constraints; rather, they are compiled into special propagators that maintain a certain level of consistency for the constraints. In Picat, constraint propagators are encoded as action rules. If the sat module is used, then global constraints are translated into smaller constraints before being translated further into conjunctive normal form. If the mip module is used, then global constraints are decomposed into equality and disequality constraints.

Picat provides the following global constraints.

- all_different (FDVars): This constraint ensures that each pair of variables in the list or array $F D V$ ars is different. This constraint is compiled into a set of inequality constraints. For each pair of variables $V 1$ and $V 2$ in $F D V a r s$, all_different ( $F D V a r s$ ) generates the constraint $V 1$ \# ! = V2.
- all_distinct (FDVars) : This constraint is the same as all_different, but for the cp module it maintains a higher level of consistency. For some problems, this constraint is faster and requires fewer backtracks than all_different, and, for some other problems, this constraint is slower due to the overhead of consistency checking.
- all_different_except_0 (FDVars) : This constraint is true if all non-zero values in FDVars are different.
- assignment (FDVars1, FDVars2): This constraint ensures that $F D \operatorname{Vars} 2$ is a dual assignment of $F D V$ ars1, i.e., if the $i$ th element of $F D V a r s 1$ is $j$, then the $j$ th element of $F D \operatorname{Vars} 2$ is $i$. The constraint can be defined as:

```
assignment(Xs,Ys) =>
    N = Xs.length,
    (var(Ys) -> Ys = new_list(N); true),
    Xs :: 1..N,
    Ys :: 1..N,
    foreach(I in 1..N, J in 1..N)
            X[I] #= J #<=> Y[J] #= I
        end.
```

- at_least $(N, L, V)$ : This constraint succeeds if there are at least $N$ elements in $L$ that are equal to $V$, where $N$ and $V$ must be integer-domain variables, and $L$ must be a list of integer-domain variables.
- at_most $(N, L, V)$ : This constraint succeeds if there are at most $N$ elements in $L$ that are equal to $V$, where $N$ and $V$ must be integer-domain variables, and $L$ must be a list of integer-domain variables.
- circuit (FDVars) : Let FDVars be a list of variables [ $X_{1}, X_{2}, \ldots, X_{N}$ ], where each $X_{i}$ has the domain $1 . . N$. A valuation $X_{1}=v_{1}, X_{2}=v_{2}, \ldots, X_{n}=v_{n}$ satisfies the constraint if $1->v_{1}, 2->v_{2}, \ldots, \mathrm{n}->v_{n}$ forms a Hamiltonian cycle. This constraint ensures that each variable has a different value, and that the graph that is formed by the assignment does not contain any sub-cycles. For example, for the constraint
circuit ([X1, X2, X3, X4])
$[3,4,2,1]$ is a solution, but $[2,1,4,3]$ is not, because the graph $1->2,2->1$, $3->4,4->3$ contains two sub-cycles.
- count $(V, F D V a r s, R e l, N)$ : In this constraint, $V$ and $N$ are integer-domain variables, FDVars is a list of integer-domain variables, and Rel is an arithmetic constraint operator $(\#=, \#!=, \#>, \#>=, \#<, \#=<$, or $\#<=$ ). Let Count be the number of elements in $F D V$ ars that are equal to $V$. The constraint is true iff Count Rel $N$ is true. This constraint can be defined as follows:

```
count(V,L,Rel,N) =>
    sum([V #= E : E in L]) #= Count,
    call(Rel,Count,N).
```

- count $(V, F D V a r s, N)$ : This constraint is the same as count $(V, F D V a r s, \#=$, $N)$.
- cumulative (Starts, Durations, Resources, Limit): This constraint is useful for describing and solving scheduling problems. The arguments Starts, Durations, and Resources are lists of integer-domain variables of the same length, and Limit is an integerdomain variable. Let Starts be $\left[S_{1}, S_{2}, \ldots, S_{n}\right]$, Durations be $\left[D_{1}, D_{2}, \ldots\right.$, $D_{n}$ ], and Resources be [ $R_{1}, R_{2}, \ldots, R_{n}$ ]. For each job $i, S_{i}$ represents the start time, $D_{i}$ represents the duration, and $R_{i}$ represents the units of resources needed. Limit is the limit on the units of resources available at any time. This constraint ensures that the limit cannot be exceeded at any time.
- decreasing $(L)$ : The sequence (an array or a list) $L$ is in (non-strictly) decreasing order.
- decreasing_strict $(L)$ : The sequence (an array or a list) $L$ is in strictly decreasing order.
- diffn (RectangleList) : This constraint ensures that no two rectangles in RectangleList overlap with each other. A rectangle in an $n$-dimensional space is represented by a list of $2 \times n$ elements $\left[X_{1}, X_{2}, \ldots, X_{n}, S_{1}, S_{2}, \ldots, S_{n}\right]$, where $X_{i}$ is the starting coordinate of the edge in the $i$ th dimension, and $S_{i}$ is the size of the edge.
- disjunctive_tasks (Tasks) : Tasks is a list of terms. Each term has the form disj_tasks ( $S_{1}, D_{1}, S_{2}, D_{2}$ ), where $S_{1}$ and $S_{2}$ are two integer-domain variables, and $D_{1}$ and $D_{2}$ are two positive integers. This constraint is equivalent to posting the disjunctive constraint $S_{1}+D_{1} \#=<S_{2} \# \backslash / S_{2}+D_{2} \#=<S_{1}$ for each term in Tasks; however the constraint may be more efficient, because it converts the disjunctive tasks into global constraints.
- element $(I, L i s t, V)$ : This constraint is true if the $I$ th element of List is $V$, where $I$ and $V$ are integer-domain variables, and List is a list of integer-domain variables.
- exactly $(N, L, V)$ : This constraint succeeds if there are exactly $N$ elements in $L$ that are equal to $V$, where $N$ and $V$ must be integer-domain variables, and $L$ must be a list of integer-domain variables.
- global_cardinality (List, Pairs) : Let List be a list of integer-domain variables $\left[X_{1}, \ldots, X_{d}\right]$, and Pairs be a list of pairs [ $K_{1}-V_{1}, \ldots, K_{n}-V_{n}$ ], where each key $K_{i}$ is a unique integer, and each $V_{i}$ is an integer-domain variable. The constraint is true if every element of List is equal to some key, and, for each pair $K_{i}-V_{i}$, exactly $V_{i}$ elements of List are equal to $K_{i}$. This constraint can be defined as follows:

```
global_cardinality(List,Pairs) =>
    foreach($Key-V in Pairs)
            sum([B : E in List, B#<=>(E#=Key)]) #= V
            end.
```

- increasing $(L)$ : The sequence (an array or a list) $L$ is in (non-strictly) increasing order.
- increasing_strict $(L)$ : The sequence (an array or a list) $L$ is in strictly increasing order.
- lex_le $\left(L_{1}, L_{2}\right)$ : The sequence (an array or a list) $L_{1}$ is lexicographically less than or equal to $L_{2}$.
- lex_lt $\left(L_{1}, L_{2}\right)$ : The sequence (an array or a list) $L_{1}$ is lexicographically less than $L_{2}$.
- matrix_element (Matrix, $I, J, V$ ): This constraint is true if the entry at $\langle I, J\rangle$ in Matrix is $V$, where $I, J$, and $V$ are integer-domain variables, and Matrix is an twodimensional array of integer-domain variables.
- neqs (NeqList) : NeqList is a list of inequality constraints of the form $X \#!=Y$, where $X$ and $Y$ are integer-domain variables. This constraint is equivalent to the conjunction of the inequality constraints in NeqList, but it extracts all_distinct constraints from the inequality constraints.
- nvalue ( $N$, List) : The number of distinct values in List is $N$, where List is a list of integer-domain variables.
- regular $(L, Q, S, M, Q 0, F)$ : Given a finite automaton (DFA or NFA) of $Q$ states numbered $1,2, \ldots, Q$ with input $1 . . S$, transition matrix $M$, initial state $Q 0(1 \leq Q 0 \leq Q)$, and a list of accepting states $F$, this constraint is true if the list $L$ is accepted by the automaton. The transition matrix $M$ represents a mapping from $1 . . Q \times 1 . . S$ to $0 . . Q$, where 0 denotes the error state. For a DFA, every entry in $M$ is an integer, and for an NFA, entries can be a list of integers.
- scalar_product ( $A, X, P r o d u c t)$ : The scalar product of $A$ and $X$ is Product, where $A$ and $X$ are lists or arrays of integer-domain variables, and Product is an integer-domain variable. $A$ and $X$ must have the same length.
- scalar_product ( $A, X$, Rel, Product) : The scalar product of $A$ and $X$ has the relation Rel with Product, where Rel is one of the following operators: \#=, \#!=, \#>=, \#>, $\#=<(\#<=)$, and \#<.
- serialized (Starts, Durations) : This constraint describes a set of non-overlapping tasks, where Starts and Durations are lists of integer-domain variables, and the lists have the same length. Let $O s$ be a list of 1 s that has the same length as Starts. This constraint is equivalent to cumulative (Starts, Durations, Os, 1).
- subcircuit (FDVars) : This constraint is the same as circuit (FDVars) , except that not all of the vertices are required to be in the circuit. If the $i$ th element of FDVars is $i$, then the vertex $i$ is not part of the circuit.


### 12.6 Solver Invocation

- solve (Options, Vars): This predicate calls the imported solver to label the variables Vars with values, where Options is a list of options for the solver. The options will be detailed below. This predicate can backtrack in order to find multiple solutions. The cp module allows incremental labeling of variables, and some variables that occur in constraints but are not passed to solve may remain uninstantiated after a call to solve. The user is responsible for having all the variables that need to be instantiated passed to solve. In constrast, the sat and mip modules do not support incremental labeling of variables.
- solve(Vars): This predicate is the same as solve([], Vars).
- indomain (Var): This predicate is only accepted by cp. It is the same as solve([], [Var]).
- solve_all (Options, Vars) = Solutions: This function returns all the solutions that satisfy the constraints.
- solve_all(Vars) = Solutions: This function is the same as solve_all([], Vars).
- solve_suspended (Options): After solve(Vars) has successfully labeled Vars, some constraints may remain suspended and not completely checked because not all of the decision variables are included in Vars. The solve_suspended (Options) predicate labels all remaining variables in the suspended constraints. This predicate is only provided by the cp module.
- solve_suspended: This predicate is the same as solve_suspended([]).
- indomain_down(Var): This predicate is the same as solve([down], [Var]). It is only accepted by cp.


### 12.6.1 Common Solving Options

The following options are accepted by all three of the solvers. ${ }^{1}$

- \$min (Var) : Minimize the variable Var.
- $\$ \max (E x p)$ : Maximize the variable Var.
- \$report (Call): Execute Call each time a better answer is found while searching for an optimal answer. This option cannot be used if the mip module is used.


### 12.6.2 Solving Options for cp

The cp module also accepts the following options:

- backward: The list of variables is reversed first.
- constr: Variables are first ordered by the number of attached constraints.
- degree: Variables are first ordered by degree, i.e., the number of connected variables.
- down: Values are assigned to variables from the largest to the smallest.
- ff : The first-fail principle is used: the leftmost variable with the smallest domain is selected.
- ffc: The same as with the two options: ff and constr.
- ffd: The same as with the two options: ff and degree.
- forward: Choose variables in the given order, from left to right.
- inout: The variables are reordered in an inside-out fashion. For example, the variable list $[X 1, X 2, X 3, X 4, X 5]$ is rearranged into the list $[X 3, X 2, X 4, X 1, X 5]$.
- label (CallName) : This option informs the CP solver that once a variable $V$ is selected, the user-defined call CallName $(V)$ is used to label $V$, where CallName must be defined in the same module, an imported module, or the global module.
- leftmost: The same as forward.
- max: First, select a variable whose domain has the largest upper bound, breaking ties by selecting a variable with the smallest domain.

[^8]- min: First, select a variable whose domain has the smallest lower bound, breaking ties by selecting a variable with the smallest domain.
- rand: Both variables and values are randomly selected when labeling.
- rand_var: Variables are randomly selected when labeling.
- rand_val: Values are randomly selected when labeling.
- reverse_split: Bisect the variable's domain, excluding the lower half first.
- split: Bisect the variable's domain, excluding the upper half first.
- updown: Values are assigned to variables from the values that are nearest to the middle of the domain.


### 12.6.3 Solving Options for sat

- dump: Dump the CNF code to stdout.
- dump (File) : Dump the CNF code to File.
- seq: Use sequential search to find an optimal answer.
- split: Use binary search to find an optimal answer (default).
- \$nvars(NVars): The number of variables in the CNF code is NVars.
- \$ncls(NCls): The number of clauses in the CNF code is NCls.
- \$threads $(N)$ : Use $N$ threads to solve the generated CNF code. In the current implementation, the parallel version of lingeling, plingeling, will be used if this option is given.
- threads: The same as \$threads (8).


### 12.6.4 Solving Options for mip

- dump: Dump the constraints in CPLEX format to stdout.
- dump (File) : Dump the CPLEX format to File.
- glpk: Instruct Picat to use the GLPK MIP solver. Picat uses the following command to call the GLPK solver:
glpsol --lp -o SolFile TmpFile
where SolFile is a solution file, and TmpFile is a file that stores the CPLEX-format constraints. Picat throws existence_error if the command glpsol is not available.
- gurobi: Instruct Picat to use the Gurobi MIP solver. Picat uses the following command to call the Gurobi solver:

```
gurobi_cl ResultFile=SolFile TmpFile
```

where SolFile is a file for the solution, and TmpFile is a file that stores the CPLEXformat constraints. Picat throws existence_error if the command gurobi_cl is not available.

- tmp (File) : Dump the CPLEX format to File rather than the default file '_tmp.lp', before calling the mip solver. The name File must be a string or an atom that has the extension name '.lp'. When this file name is specified, the mip solver will save the solution into a file name that has the same main name as File but the extension name '.sol'.


## Chapter 13

## The os Module

Picat has an os module for manipulating files and directories. In order to use any of the functions or predicates, users must import the module.

### 13.1 The Path Parameter

Many of the functions and predicates in this module have a Path parameter. This parameter is a string or an atom, representing the path of a file or directory. This path can be an absolute path, from the system's root directory, or a relative path, from the current file location. Different systems use different separator characters to separate directories in different levels of the directory hierarchy. For example, Windows uses ' $\backslash$ ' and Unix uses ' $/$ '. The following function outputs a single character, representing the character that the current system uses as a file separator.

- separator() = Val


### 13.2 Directories

The os module includes functions for reading and modifying directories. The following example shows how to list all of the files in a directory tree, using a depth-first directory traversal.

## Example

```
import os.
traverse(Dir), directory(Dir) =>
    List = listdir(Dir),
    printf("Inside %s%n",Dir),
    foreach(File in List)
        printf(" %s%n",File)
    end,
    foreach (File in List, File != ".", File != "..")
        FullName = Dir ++ [separator()] ++ File,
        traverse(FullName)
    end.
traverse(_Dir) => true.
```

The following function can be used to read the contents of a directory:

- listdir $($ Path $)=$ List: This function returns a list of all of the files and directories that are contained inside the directory specified by Path. If Path is not a directory, then an error is thrown. The returned list contains strings, each of which is the name of a file or directory.
The above example also uses the directory predicate, which will be discussed in Section 13.4.


### 13.2.1 The Current Working Directory

The os module includes two functions that obtain the program's current working directory:

- cwd () $=$ Path
- pwd() = Path

The os module also includes two predicates to change the program's current working directory:

- cd (Path)
- chdir (Path)

If the cd and chdir predicates cannot move to the directory specified by Path, the functions throw an error. This can occur if Path does not exist, if Path is not a directory, or if the program does not have permission to access Path.

### 13.3 Modifying Files and Directories

### 13.3.1 Creation

The os module contains a number of predicates for creating new files and directories:

- mkdir (Path): This predicate creates a new directory at location Path. The directory will be created with a default permission list of [rwu, rwg, ro]. If the program does not have permission to write to the parent directory of Path, this predicate will throw an error. An error will also occur if the parent directory does not exist.
- rename (Old, New): This renames a file or a directory from Old to New. This predicate will throw an error if Old does not exist. An error will also occur if the program does not have permission to write to Old or New.
- cp (FromPath, ToPath) : This copies a file from FromPath to ToPath. This predicate will throw an error if FromPath does not exist or FromPath is a directory. An error will also occur if the program does not have permission to read from FromPath, or if it does not have permission to write to ToPath.


### 13.3.2 Deletion

The os module contains a number of predicates for deleting files and directories.

- rm (Path) : This deletes a file. An error will be thrown if the file does not exist, if the program does not have permission to delete the file, or if Path refers to a directory, a hard link, a symbolic link, or a special file type.
- rmdir (Path): This deletes a directory. An error will be thrown if the directory does not exist, the program does not have permission to delete the directory, the directory is not empty, or if Path does not refer to a directory.


### 13.4 Obtaining Information about Files

The os module contains a number of functions that retrieve file status information, and predicates that test the type of a file. These predicates will all throw an error if the program does not have permission to read from Path.

- readable $(P a t h)$ : Is the program allowed to read from the file?
- writable (Path): Is the program allowed to write to the file?
- executable (Path): Is the program allowed to execute the file?
- $\operatorname{size}($ Path $)=$ Int: If Path is not a symbolic link, then this function returns the number of bytes contained in the file to which Path refers. If Path is a symbolic link, then this function returns the path size of the symbolic link. Because the function size/1 is defined in the basic module for returning the size of a map, this function requires an explicit module qualifier os.size (Path).
- file_base_name $(P a t h)=$ List: This function returns a string containing the base name of Path. For example, the base name of "a/b/c.txt" is "c.txt".
- file_directory_name (Path) = List: This function returns a string containing the path of the directory that contains Path. For example, the directory name of "a/b/c.txt" is " $a / b /$ ".
- exists (Path) : Is Path an existing file or directory?
- file (Path): Does Path refer to a regular file? This predicate is true if Path is neither a directory nor a special file, such as a socket or a pipe.
- file_exists (Path): This tests whether Path exists, and, if it exists, whether Path refers to a regular file.
- directory (Path): Does Path refer to a directory?

The following example shows how to use a few of the predicates.

## Example

```
import os.
test_file(Path) =>
    if (not exists(Path)) then
        printf("%s does not exist %n",Path)
    elseif (directory(Path)) then
        println("Directory")
    elseif (file(Path)) then
        println("File")
    else
        println("Unknown")
    end.
```


### 13.5 Environment Variables

- env_exists (Name): This predicate succeeds if Name is an environment variable in the system.
- getenv (Name) = String: This function returns the value of the environment variable Name as a string. This function will throw an error if the environment variable Name does not exist.


## Appendix A

## The math Module

Picat provides a math module, which has common mathematical constants and functions. The math module is imported by default.

## A. 1 Constants

The math module provides two constants.

- $e=2.71828182845904523536$
- $\mathrm{pi}=3.14159265358979323846$


## A. 2 Functions

The math module contains mathematical functions that serve a number of different purposes. Note that the arguments must all be numbers. If the arguments are not numbers, then Picat will throw an error.

## A.2.1 Sign and Absolute Value

The following functions deal with the positivity and negativity of numbers.

- $\operatorname{sign}(X)=$ Val: This function determines whether $X$ is positive or negative. If $X$ is positive, then this function returns 1 . If $X$ is negative, then this function returns -1 . If $X$ is 0 , then this function returns 0 .
- abs $(X)=$ Val: This function returns the absolute value of $X$. If $X \geq 0$, then this function returns $X$. Otherwise, this function returns $-X$.


## Example

```
Picat> Vall = sign(3), Val2 = sign(-3), Val3 = sign(0)
Vall = 1
Val2 = -1
Val3 = 0
Picat> Val = abs(-3)
val = 3
```


## A.2.2 Rounding and Truncation

The math module includes the following functions for converting a real number into the integers that are closest to the number.

- ceiling $(X)=$ Val: This function returns the closest integer that is greater than or equal to $X$.
- $\mathrm{floor}(X)=$ Val: This function returns the closest integer that is less than or equal to X.
- round $(X)=$ Val: This function returns the integer that is closest to $X$.
- truncate $(X)=$ Val: This function removes the fractional part from a real number.
- $\operatorname{modf}(X)=($ IntVal,FractVal): This function splits a real number into its integer part and its fractional part.


## Example

```
Picat> Val1 = ceiling(-3.2), Val2 = ceiling(3)
Val1 = -3
Val2 = 3
Picat> Val1 = floor(-3.2), Val2 = floor(3)
Val1 = -4
Val2 = 3
Picat> Vall = round(-3.2), Val2 = round(-3.5), Val3 = round(3.5)
Val1 = -3
Val2 = -4
Val3 = 4
Picat> Val1 = truncate(-3.2), Val2 = truncate(3)
Val1 = -3
Val2 = 3
Picat> IF = modf(3.2)
IF = (3.0 , 0.2)
```


## A.2.3 Exponents, Roots, and Logarithms

The following functions provide exponentiation, root, and logarithmic functions. Note that, in the logarithmic functions, if $X \leq 0$, then an error is thrown.

- pow $(X, Y)=$ Val: This function returns $X^{Y}$. It does the same thing as $X * * Y$.
- pow_mod $(X, Y, Z)=V a l$ : This function returns $X^{Y} \bmod Z$. All of the arguments must be integers, and $Y$ must not be negative.
- $\exp (X)=$ Val: This function returns $e^{X}$.
- $\operatorname{sqrt}(X)=$ Val: This function returns the square root of $X$. Note that the math module does not support imaginary numbers. Therefore, if $X<0$, this function throws an error.
- $\log (X)=V a l:$ This function returns $\log _{e}(X)$.
- $\log 10(X)=$ Val: This function returns $\log _{10}(X)$.
- $\log 2(X)=$ Val: This function returns $\log _{2}(X)$.
- $\log (B, X)=V a l:$ This function returns $\log _{B}(X)$.


## Example

```
Picat> P1 = pow(2, 5), P2 = exp(2)
P1 = 32
P2 = 7.38906
Picat> S = sqrt(1)
S = 1.0
Picat> E = log(7), T = log10(7), T2 = log2(7), B = log(7, 7)
E = 1.94591
T = 0.845098
T2 = 2.80735
B = 1.0
```


## A.2.4 Converting Between Degrees and Radians

The math module has two functions to convert between degrees and radians.

- to_radians (Degree) = Radian: This function converts from degrees to radians.
- to_degrees (Radian) = Degree: This function converts from radians to degrees.


## Example

```
Picat> R = to_radians(180)
R = 3.14159
Picat> D = to_degrees(pi)
D = 180.0
```


## A.2.5 Trigonometric Functions

The math module provides the following trigonometric functions.

- $\sin (X)=V a l$ : This function returns the sine of $X$, where $X$ is given in radians.
- $\cos (X)=$ Val: This function returns the cosine of $X$, where $X$ is given in radians.
- $\tan (X)=$ Val: This function returns the tangent of $X$, where $X$ is given in radians. If the tangent is undefined, such as at pi / 2, then this function throws an error.
- $\sec (X)=V a l$ : This function returns the secant of $X$, where $X$ is given in radians. If $\cos (X)$ is 0 , then this function throws an error.
- $\csc (X)=V a l$ : This function returns the cosecant of $X$, where $X$ is given in radians. If $\sin (X)$ is 0 , then this function throws an error.
- $\cot (X)=$ Val: This function returns the cotangent of $X$, where $X$ is given in radians. If $\tan (X)$ is 0 , then this function throws an error.
- $\operatorname{asin}(X)=V a l$ : This function returns the arc sine of $X$, in radians. The returned value is in the range $[-\mathrm{pi} / 2$, pi / 2]. $X$ must be in the range $[-1,1]$; otherwise, this function throws an error.
- $\operatorname{acos}(X)=$ Val: This function returns the arc cosine of $X$, in radians. The returned value is in the range $[0, \mathrm{pi}] . X$ must be in the range $[-1,1]$; otherwise, this function throws an error.
- $\operatorname{atan}(X)=$ Val: This function returns the arc tangent of $X$, in radians. The returned value is in the range [-pi / 2, pi / 2].
- atan2 $(X, Y)=$ Val: This function returns the arc tangent of $Y / X$, in radians. $X$ and $Y$ are coordinates. The returned value is in the range [-pi, pi].
- $\operatorname{asec}(X)=$ Val: This function returns the arc secant of $X$, in radians. The returned value is in the range [0, pi]. $X$ must be in the range $(-\infty,-1]$ or $[1, \infty)$; otherwise, this function throws an error.
- $\operatorname{acsc}(X)=V a l:$ This function returns the arc cosecant of $X$, in radians. The returned value is in the range $[-\mathrm{pi} / 2$, pi $/ 2]$. $X$ must be in the range $(-\infty,-1]$ or $[1, \infty)$; otherwise, this function throws an error.
- $\operatorname{acot}(X)=$ Val: This function returns the arc cotangent of $X$, in radians. The returned value is in the range $[-\mathrm{pi} / 2$, pi / 2].


## Example

```
Picat> S = sin(pi), C = cos(pi), T = tan(pi)
S = 0.0
C = -1.0
T = 0.0
Picat> S = asin(0), C = acos(0), T = atan(0), T2 = atan2(-10, 10)
S = 0.0
C = 1.5708
T = 0.0
T2 = -0.785398
Picat> S = sec(pi / 4), C = csc(pi / 4), T = cot(pi / 4)
S = 1.41421
C = 1.41421
T = 1.0
Picat> S = asec(2), C = acsc(2), T = acot(0)
S = 1.0472
c = 0.5236
T = 1.5708
```


## A.2.6 Hyperbolic Functions

The math module provides the following hyperbolic functions.

- $\sinh (X)=$ Val: This function returns the hyperbolic sine of $X$.
- $\cosh (X)=$ Val: This function returns the hyperbolic cosine of $X$.
- $\tanh (X)=V a l:$ This function returns the hyperbolic tangent of $X$.
- $\operatorname{sech}(X)=$ Val: This function returns the hyperbolic secant of $X$.
- $\operatorname{csch}(X)=V a l$ : This function returns the hyperbolic cosecant of $X$. If $X$ is 0 , then this function throws an error.
- $\operatorname{coth}(X)=$ Val: This function returns the hyperbolic cotangent of $X$. If $X$ is 0 , then this function throws an error.
- $\operatorname{asinh}(X)=V a l$ : This function returns the $\operatorname{arc}$ hyperbolic sine of $X$.
- $\operatorname{acosh}(X)=V a l$ : This function returns the arc hyperbolic cosine of $X$. If $X<1$, then this function throws an error.
- atanh $(X)=$ Val: This function returns the arc hyperbolic tangent of $X . X$ must be in the range $(-1,1)$; otherwise, this function throws an error.
- $\operatorname{asech}(X)=$ Val: This function returns the arc hyperbolic secant of $X . X$ must be in the range $(0,1]$; otherwise, this function throws an error.
- $\operatorname{acsch}(X)=V a l$ : This function returns the $\operatorname{arc}$ hyperbolic cosecant of $X$. If $X$ is 0 , then this function throws an error.
- $\operatorname{acoth}(X)=$ Val: This function returns the arc hyperbolic cotangent of $X . X$ must be in the range $(-\infty,-1)$ or $(1, \infty)$; otherwise, this function throws an error.


## Example

```
Picat> S = sinh(pi), C = cosh(pi), T = tanh(pi)
S = 11.54874
C = 11.59195
T = 0.99627
Picat> S = sech(pi / 4), C = csch(pi / 4), T = coth(pi / 4)
S = 0.75494
C = 1.15118
T = 1.52487
Picat> S = asinh(0), C = acosh(1), T = atanh(0)
S = 0.0
C = 0.0
T = 0.0
```


## A.2.7 Random Numbers

The following functions provide access to a random number generator.

- random ( $)=$ Val: This function returns a random integer.
- random2 () = Val: This function returns a random integer, using an environment-dependent seed.
- rand_max () = Val: This function returns the maximum random integer.
- random $(S e e d)=$ Val: This function returns a random integer. At the same time, it changes the seed of the random number generator.
- random (Low, High) = Val: This function returns a random integer in the range Low..High.
- $\operatorname{frand}()=V a l$ : This function returns a random real number between 0.0 and 1.0 , inclusive.
- frand (Low, High) = Val: This function returns a random real number between Low and High, inclusive.


## A.2.8 Other Built-ins

- even $(N)$ : This predicate is true if $N$ is an even integer.
- $\operatorname{gcd}(A, B)$ : This function returns the greatest common divisor of integer $A$ and integer $B$.
- odd $(N)$ : This predicate is true if $N$ is an odd integer.
- prime $(N)$ : This predicate is true if $N$ is a prime number.
- $\operatorname{primes}(N)=$ List: This function returns a list of prime numbers that are less than or equal to $N$.


## Appendix B

## The sys Module

The sys module, which is imported by default, contains built-ins that are relevant to the Picat system. The built-ins in the sys module perform operations that include compiling programs, tracing execution, and displaying statistics and information about the Picat system.

## B. 1 Compiling and Loading Programs

The sys module includes a number of built-ins for compiling programs and loading them into memory.

- compile (FileName) : This predicate compiles the file FileName. pi and all of its dependent files without loading the generated byte-code files. The destination directory for the byte-code file is the same as the source file's directory. If the Picat interpreter does not have permission to write into the directory in which a source file resides, then this built-in throws an exception. If FileName.pi imports modules, then these module files are also compiled. The system searches for these module files in the directory in which FileName.pi resides or the directories that are stored in the environment variable PICATPATH.
- compile_bp (FileName) : This predicate translates the Picat file FileName.pi into a B-Prolog file FileName.pl. If the file is dependent on other Picat files, then those files are compiled using compile/1. The destination directory for the B-Prolog file is the same as the source file's directory. If the Picat interpreter does not have permission to write into the directory in which a source file resides, then this built-in throws an exception.
- load (FileName): This predicate loads the byte-code file FileName. qi and all of its dependent byte-code files into the system for execution. For FileName, the system searches for a byte-code file in the directory specified by FileName or the directories that are stored in the environment variable PICATPATH. For the dependent file names, the system searches for a byte-code file in the directory in which FileName. qi resides or the directories that are stored in the environment variable PICATPATH. If the byte-code file FileName.qi does not exist, but the source file FileName.pi exists, then this built-in compiles the source file and loads the byte codes without creating a qi file. Note that, for the dependent files, if the byte-code file does not exist, but the source file exists, then the source file will be compiled.
- cl (FileName) : This predicate compiles and loads the source file named FileName. pi. Note that the extension . pi does not need to be given. The system also compiles and loads all of the module files that are either directly imported or indirectly imported by the source
file. The system searches for such dependent files in the directory in which FileName.pi resides or the directories that are stored in the environment variable PICATPATH.
- cl: This predicate compiles and loads a program from the console, ending when the end-of-file character (ctrl-z for Windows and ctrl-d for Unix) is typed.
- cl_facts (Facts) : This predicate compiles and loads facts into the system. The argument Facts is a list of ground facts.
- cl_facts (Facts, IndexInfo): This predicate compiles and loads facts into the system. The argument Facts is a list of ground facts. The argument IndexInfo is a list of indexing information in the form $\mathrm{p}\left(M_{1}, M_{2}, \ldots, M_{n}\right)$. Each $M_{i}$ can either be + , which indicates that the argument is input, or - , which indicates that the argument is output.
- cl_facts_table (Facts): This predicate is the same as cl_facts/1, except that the facts are all tabled.
- cl_facts_table (Facts, IndexInfo): This predicate is the same as cl_facts/2, except that the facts are all tabled.


## B. 2 Tracing Execution

The Picat system has three execution modes: non-trace mode, trace mode, and spy mode. In trace mode, it is possible to trace the execution of a program, showing every call in every possible stage. In order to trace the execution, the program must be recompiled while the system is in trace mode. In spy mode, it is possible to trace the execution of individual functions and predicates. The following predicates are used to switch between non-trace mode and trace mode.

- trace: This predicate switches the execution mode to trace mode.
- notrace: This predicate switches the execution mode to non-trace mode.
- debug: This predicate switches the execution mode to trace mode.
- nodebug: This predicate switches the execution mode to non-trace mode.
- spy (Point) : This predicate places a spy point on Point, which is a function or a predicate, optionally followed by an arity. The creation of a spy point switches the Picat system to spy mode.
- nospy: This predicate removes all spy points, and switches the execution mode to non-trace mode.
- abort: This predicate terminates the current program. This can be used in all three execution modes.


## B.2.1 Debugging Commands

In trace mode, the system displays a message when a function or a predicate is entered (Call), exited (Exit), re-entered (Redo) or has failed (Fail). After a function or a predicate is entered or re-entered, the system waits for a command from the user. A command is a single letter followed by a carriage-return, or may simply be a carriage-return. The following commands are available:

-     + : create a spy point.
-     - : remove a spy point.
- $<$ : reset the print depth to 10 .
- < i : reset the print depth to i.
- a : abort, quit debugging, moving control to the top level.
- <cr> : A carriage return causes the system to show the next call trace.
- c: creep, show the next call trace.
- h : help, display the debugging commands.
- ? : help, display the debugging commands.
- I : leap, be silent until a spy point is encountered.
- n : nodebug, prevent the system from displaying debugging messages for the remainder of the program.
- r : repeat, continue to creep or leap without intervention.
- $s$ : skip, be silent until the call is completed (Exit or Fail).
- $t$ : backtrace, show the backtrace leading to the current call.
- t i: backtrace, show the backtrace from the call numbered i to the current call.
- u: undo what has been done to the current call and redo it.
- u i: undo what has been done to the call numbered i and redo it.


## B. 3 Information about the Picat System

The sys module contains a number of built-ins that display information about the Picat system. This information includes statistics about the system, including the memory that is used, and the amount of time that it takes to perform a goal.

## B.3.1 Statistics

The following built-ins display statistics about the memory that Picat system uses.

- statistics: This predicate displays the number of bytes that are allocated to each data area, and the number of bytes that are already in use.
- statistics (Key, Value): The statistics concerning Key are Value. This predicate gives multiple solutions upon backtracking. Keys include runtime, program, heap, control, trail, table, gc, backtracks, and gc_time. The values for most of the keys are lists of two elements. For the key runtime, the first element denotes the amount of time in milliseconds that has elapsed since Picat started, and the second element denotes the amount of time that has elapsed since the previous call to statistics/2 was executed. For the key gc , the number indicates the number of times that the garbage collector has been invoked. For the key backtracks, the number indicates the number of backtracks that have been done during the labeling of finite domain variables since Picat was started. For all other keys, the first element denotes the size of memory in use, and the
second element denotes the size of memory that is still available in the corresponding data area.
- statistics_all() = List: This function returns a list of lists that are in the form [Key, Value]. The list contains all of the keys that statistics/2 can display, together with their corresponding values.


## Example

```
Picat> statistics
    Stack+Heap: 8,000,000 bytes
        Stack in use: 1,156 bytes
        Heap in use: 28,592 bytes
        Program: 8,000,000 bytes
            In use: 1,448,436 bytes
        Symbols: 5,300
        Trail: 4,000,000 bytes
            In use: }936\mathrm{ bytes
        Memory manager:
            GC: Call(0), Time(0 ms)
            Expansions: Stack+Heap(0), Program(0), Trail(0), Table(0)
Picat> statistics(Key, Value)
        Key = runtime
        Value = [359947,66060]?;
        Key = program
        Value = [1451656,6548344]?;
        Key = heap
        Value = [34112,7964524]?;
        Key = control
        Value = [1360,7964524]?;
        Key = trail
        Value = [1496,3998504]?;
        Key = table
        Value = [0,4000000]?;
        Key = table_blocks
        Value = 1?;
        key = gc
        Value = 0?;
```

```
    Key = backtracks
    V = 0 ?;
    Key = gc_time
    Value = 0
Picat> L = statistics_all()
L = [[runtime, [359947,66060]], [program, [1451656,6548344]],
    [heap, [34112,7964524]], [control, [1360,7964524]],
    [trail, [1496,3998504]], [table, [0,4000000]],
    [table_blocks,1],[gc, 0], [backtracks, 0], [gc_time, 0]]
```


## B.3.2 Time

The following predicates display the amount of CPU time that it takes to perform a goal.

- time (Goal): This predicate calls Goal, and reports the number of seconds of CPU time that were consumed by the execution.
- time2 (Goal): This predicate calls Goal, and reports the number of seconds of CPU time that were consumed by the execution, and the number of backtracks that have been performed in labeling finite-domain variables during the execution of Goal.
- time_out (Goal, Limit, Result): This predicate is logically equivalent to once Goal, except that is imposes a time limit, in milliseconds, on the evaluation. If Goal is not finished when Limit expires, then the evaluation will be aborted, and Result will be unified with the atom time_out. If Goal succeeds within the time limit, then Result will be unified with the atom success. Note that time-out may be delayed or never occur because of the execution of an external $C$ function.


## B.3.3 Other System Information

- help: This predicate displays the usages of some of the commands that the system accepts.
- loaded_modules () = List: This function returns a list of the modules that are currently loaded in the Picat system. This list includes library modules and user-defined modules. By default, this function returns [basic, sys,io, math].
- nolog: This predicate turns off the logging flag.
- picat_path () = Path: This function returns the directories that are stored in the environment variable PICATPATH. If the environment variable PICATPATH does not exist, then this function throws an error.
- command $($ String $)=$ Int: This function sends the command String to the OS and returns the status that is returned from the OS.


## B. 4 Garbage Collection

Picat incorporates an incremental garbage collector for the control stack and the heap. The garbage collector is active by default. The sys module includes the following predicates for garbage collection.

- garbage_collect: This predicate starts the garbage collector.
- garbage_collect (Size): This predicate calls the garbage collector. If there are less than Size words on the control stack and heap after garbage collection, then it invokes the memory manager to expand the stack and heap so that there are Size words on the control stack and heap.


## B. 5 Quitting the Picat System

The following predicates can be used to terminate the Picat interpreter.

- exit
- halt


## Appendix C

## The util Module

The util module provides general useful utility functions and predicates. This module is expected to be expanded in the future. This module must be imported before use.

## C. 1 Utilities on Terms

- replace $($ Term, Old, New $)=$ NewTerm: This function returns a copy of Term, replacing all of the occurrences of Old in Term by New.
- replace_at (Term,Index, New) $=$ NewTerm: This function returns a copy of Term, replacing the argument at Index by New. Term must be a compound term.
- find_first_of (Term, Pattern) = Index: This function returns the first index at which the argument unifies with Pattern. If there is no argument that unifies with Pattern, then this function returns -1 . Term must be either a list or a structure.
- find_last_of (Term, Pattern) = Index: This function returns the last index at which the argument unifies with Pattern. If there is no argument that unifies with Pattern, then this function returns -1 . Term must be either a list or a structure.


## C. 2 Utilities on Strings and Lists

- chunks_of $($ List, $K)=$ ListOfLists: This function splits List into chunks, each of which has length $K$, and returns the list of the chunks. The last chunk may have less than $K$ elements if the length of List is not a multiple of $K$.
- drop $($ List, $K)=$ List: This function returns the suffix of List after the first $K$ elements are dropped, or [] if $K$ is greater than the length of List.
- find (String, SubString,From,To): This predicate searches for an occurrence of SubString in String, and binds From to the starting index and To to the ending index. On backtracking, this predicate searches for the next occurrence of SubString.
- find_ignore_case (String, SubString,From,To): This predicate is the same as find (String, SubString, From,To), except that it is case insensitive.
- join (Tokens) = String: This function is the same as join (Tokens," " ).
- join (Tokens, Separator) $=$ String: This function concatenates the tokens Tokens into a string, adding Separator, which is a string or an atom, between each two tokens.
- lstrip $($ List $)=$ List: This function is the same as lstrip (List," \t\n\r").
- lstrip $($ List, Elms $)=$ List: This function returns a copy of List with leading elements in Elms removed.
- rstrip $($ List $)=$ List: This function is the same as rstrip (List, " \t\n\r").
- rstrip $($ List, Elms $)=$ List: This function returns a copy of List with trailing elements in Elms removed.
- split $($ String $)=$ Tokens: This function is the same as split (String, " \t $\backslash n \backslash r ")$, which uses white spaces as split characters.
- split (String, Separators) = Tokens: This function splits String into a list of tokens, using characters in the string Separators as split characters. Recall that a string is a list of characters. A token is a string, so the return value is a list of lists of characters.
- strip (List) $=$ List: This function is the same as strip (List, " \t\n\r").
- strip $($ List, Elms $)=$ List: This function returns a copy of List with leading and trailing elements in Elms removed.
- take $($ List,$K)=$ List: This function returns the prefix of List of length $K$, or List itself if $K$ is greater than the length of List.


## C. 3 Utilities on Matrices

An array matrix is a two-dimensional array. The first dimension gives the number of the rows and the second dimension gives the number of the columns. A list matrix represents a matrix as a list of lists.

- matrix_multi $(A, B)$ : This function returns the product of matrices $A$ and $B$. Both $A$ and $B$ must be array matrices.
- transpose $(A)=B$ : This function returns the transpose of the matrix $A$. $A$ can be an array matrix or a list matrix. If $A$ is an array matrix, then the returned transpose is also an array. If $A$ is a list matrix, then the returned transpose is also a list.
- $\operatorname{rows}(A)=L i s t$ : This function returns the rows of the matrix $A$ as a list.
- $\operatorname{columns}(A)=$ List: This function returns the columns of the matrix $A$ as a list.
- diagonal1 $(A)=$ List: This function returns primary diagonal of the matrix $A$ as a list.
- diagonal2 $(A)=$ List: This function returns secondary diagonal of the matrix $A$ as a list.
- array matrix_to_list_matrix $(A)=$ List: This function converts the array ma$\operatorname{trix} A$ to a list matrix.
- array_matrix_to_list $(A)=$ List: This function converts the array matrix $A$ to a flat list, row by row.


## C. 4 Utilities on Lists and Sets

- permutation $(L i s t, P)$ : This predicate generates a permutation $P$ of List. This predicate is non-deterministic. On backtracking, it generates the next permutation.
- permutations (List) $=P s$ : This function returns all the permutations of List.
- nextto $(E 1, E 2, L i s t)$ : This predicate is true if $E 1$ follows $E 2$ in List. This predicate is non-deterministic.


## Appendix D

## The ordset Module

An ordered set is represented as a sorted list that does not contain duplicates. The ordset module provides useful utility functions and predicates on ordered sets. This module must be imported before use.

- delete $(O S$ et, Elm $)=$ OSet $_{1}$ : This function returns of a copy of OSet that does not contain the element Elm.
- disjoint ( OSet $_{1}$, OSet $_{2}$ ) : This predicate is true when $O$ Set $_{1}$ and $O$ Set $_{2}$ have no element in common.
- insert $(O S e t, E l m)=$ OSet $_{1}$ : This function returns a copy of OSet with the element Elm inserted.
- intersection $\left(\right.$ OSet $_{1}$, OSet $\left._{2}\right)=$ OSet $_{3}$ : This function returns an ordered set that contains elements which are in both $O S e t_{1}$ and $O S e t_{2}$.
- new_ordset (List) = OSet: This function returns an ordered set that contains the elements of List.
- ordset (Term) : This predicate is true if Term is an ordered set.
- subset $\left(O\right.$ Set $\left._{1}, O S e t_{2}\right)$ : This predicate is true if $O S e t_{1}$ is a subset of $O$ Set $_{2}$.
- subtract $\left(\right.$ OSet $_{1}$, OSet $\left._{2}\right)=$ OSet $_{3}$ : This function returns an ordered set that contains all of the elements of $O S e t_{1}$ which are not in OSet $_{2}$.
- union $\left(\right.$ SSet $\left._{1}, O S e t_{2}\right)=$ SSet $_{3}$ : This function returns an ordered set that contains all of the elements which are present in either $O$ Set $_{1}$ or OSet $_{2}$.


## Appendix E

## The datetime Module

Picat's datetime module provides built-ins for retrieving the date and time. This module must be imported before use.

- current_datetime() = DateTime: This function returns the current date and time as a structure in the form
date_time(Year, Month, Day, Hour, Minute, Second)
where the arguments are all integers, and have the following meanings and ranges.

| Argument | Meaning | Range |
| :--- | :--- | :--- |
| Year | years since 1900 | an integer |
| Month | months since January | $0-11$ |
| Day | day of the month | $1-31$ |
| Hour | hours since midnight | $0-23$ |
| Minute | minutes after the hour | $0-59$ |
| Second | seconds after the minute | $0-60$ |

In the Month argument, 0 represents January, and 11 represents December. In the Hour argument, 0 represents 12 AM , and 23 represents 11 PM . In the Second argument, the value 60 represents a leap second.

- current_date () = Date: This function returns the current date as a structure in the form date (Year, Month, Day), where the arguments have the meanings and ranges that are defined above.
- current_day () = WDay: This function returns the number of days since Sunday, in the range 0 to 6 .
- current_time () = Time: This function returns the current time as a structure in the form time (Hour, Minute, Second), where the arguments have the meanings and ranges that are defined above.


## Appendix F

## Formats

## F. 1 Formatted Printing

The following table shows the specifiers that can be used in formats for the writef, printf, and to_fstring.

| Specifier | Output |
| :--- | :--- |
| $\% \%$ | Percent Sign |
| $\% \mathrm{C}$ | Character |
| $\% \mathrm{~d}$ | Signed Decimal Integer |
| $\% \mathrm{e}$ | Scientific Notation, with Lowercase e |
| $\% \mathrm{E}$ | Scientific Notation, with Uppercase E |
| $\% \mathrm{f}$ | Decimal Real Number |
| $\% \mathrm{~g}$ | Shorter of $\% \mathrm{e}$ and $\% \mathrm{f}$ |
| $\% \mathrm{G}$ | Shorter of $\% \mathrm{E}$ and $\% \mathrm{f}$ |
| $\% \mathrm{i}$ | Signed Decimal Integer |
| $\% \mathrm{n}$ | Platform-independent Newline |
| $\% \mathrm{O}$ | Unsigned Octal Integer |
| $\% \mathrm{~S}$ | String |
| $\% \mathrm{u}$ | Unsigned Decimal Integer |
| $\% \mathrm{~W}$ | Term |
| $\% \mathrm{X}$ | Unsigned Lowercase Hexadecimal Integer |
| $\% \mathrm{X}$ | Unsigned Uppercase Hexadecimal Integer |

## Appendix G

## External Language Interface with C

Picat has an interface with C, through which Picat programs can call deterministic predicates that are written as functions in C. C programs that use this interface must include the file "picat . h" in the directory Picat/Emulator. In order to make C-defined predicates available to Picat, users have to re-compile Picat's C source code together with the newly-added C functions.

## G. 1 Term Representation

Picat's C interface provides functions for accessing, manipulating, and building Picat terms. In order to understand these functions, users need to know how terms are represented in Picat's virtual machine.

A term is represented by a word that contains a value and a tag. A word has 32 bits or 64 bits, depending on the underlying CPU and OS. The tag in a word distinguishes the type of the term.

The value of a term is an address, except when the term is an integer (in which case, the value represents the integer itself). The location to which the address points is dependent on the type of the term. In a reference, the address points to the referenced term. An unbound variable is represented by a self-referencing pointer. In an atom, the address points to the record for the atom symbol in the symbol table. In a structure, $f\left(t_{1}, \ldots, t_{n}\right)$, the address points to a block of $n+1$ consecutive words, where the first word points to the record for the functor, $f / n$, in the symbol table, and the remaining $n$ words store the components of the structure. Arrays, floating-point numbers, and big integers are represented as special structures. Picat lists are singly-linked lists. In a list, $[H \mid T]$, the address points to a block of two consecutive words, where the first word stores the car, $H$, and the second word stores the cdr, $T$.

## G. 2 Fetching Arguments of Picat Calls

A C function that defines a Picat predicate should not take any argument. The following function is used in order to fetch arguments in the current Picat call.

- TERM picat_get_call_arg(int i, int arity): Fetch the ith argument, where arity is the arity of the predicate, and i must be an integer between 1 and arity. The validity of the arguments is not checked, and an invalid argument may cause fatal errors.


## G. 3 Testing Picat Terms

The following functions are provided for testing Picat terms. They return PICAT_TRUE when they succeed and PICAT_FALSE when they fail.

- int picat_is_var(TERM $t)$ : Term $t$ is a variable.
- int picat_is_attr_var(TERM $t)$ : Term $t$ is an attributed variable.
- int picat_is_dvar (TERM $t)$ : Term $t$ is an attributed domain variable.
- int picat_is_bool_dvar (TERM t): Term t is an attributed Boolean variable.
- int picat_is_integer (TERM $t)$ : Term $t$ is an integer.
- int picat_is_float (TERM $t$ ): Term $t$ is a floating-point number.
- int picat_is_atom(TERM t): Term t is an atom.
- int picat_is_nil(TERM t): Term t is nil, i.e., the empty list [].
- int picat_is_list (TERM t): Term $t$ is a list.
- int picat_is_string(TERM $t)$ : Term $t$ is a string.
- int picat_is_structure (TERM $t$ ): Term $t$ is a structure (but not a list).
- int picat_is_array(TERM t): Term t is an array.
- int picat_is_compound(TERM t): True if either picat_is_list (t) or picat_is_structure (t) is true.
- int picat_is_identical(TERM t1, TERM t2): t1 and t2 are identical. This function is equivalent to the Picat call $t 1==t 2$.
- int picat_is_unifiable(TERM t1, TERM t2): t1 and t2 are unifiable. This is equivalent to the Picat call $\operatorname{not}(\operatorname{not}(t 1=t 2))$.


## G. 4 Converting Picat Terms into C

The following functions convert Picat terms to C. If a Picat term does not have the expected type, then the global C variable exception, which is of type Term, is assigned a term. A C program that uses these functions must check exception in order to see whether data are converted correctly. The converted data are only correct when exception is (TERM) NULL.

- long picat_get_integer (TERM t): Convert the Picat integer t into C. The term $t$ must be an integer; otherwise exception is set to integer_expected and 0 is returned. Note that precision may be lost if $t$ is a big integer.
- double picat_get_float (TERM t): Convert the Picat float $t$ into C. The term $t$ must be a floating-point number; otherwise exception is set to number_expected, and 0.0 is returned.
- (char *) picat_get_atom_name (TERM t): Return a pointer to the string that is the name of atom $t$. The term $t$ must be an atom; otherwise, exception is set to atom_expected, and NULL is returned.
- (char *) picat_get_struct_name (TERM t): Return a pointer to the string that is the name of structure $t$. The term $t$ must be a structure; otherwise, exception is set to structure_expected, and NULL is returned.
- int picat_get_struct_arity (TERM $t)$ : Return the arity of term $t$. The term $t$ must be a structure; otherwise, exception is set to structure_expected and 0 is returned.


## G. 5 Manipulating and Writing Picat Terms

- int picat_unify (TERM t1, TERM t2): Unify two Picat terms t1 and t2. The result is PICAT_TRUE if the unification succeeds, and PICAT_FALSE if the unification fails.
- TERM picat_get_arg(int i,TERM t): Return the ith argument of term t. The term $t$ must be compound, and i must be an integer that is between 1 and the arity of $t$; otherwise, exception is set to compound_expected, and the Picat integer 0 is returned.
- TERM picat_get_car (TERM $t$ ): Return the car of the list $t$. The term $t$ must be a non-empty list; otherwise exception is set to list_expected, and the Picat integer 0 is returned.
- TERM picat_get_cdr (TERM $t$ ): Return the cdr of the list $t$. The term $t$ must be a non-empty list; otherwise exception is set to list_expected, and the Picat integer 0 is returned.
- void picat_write_term(TERM t): Send term t to the standard output stream.


## G. 6 Building Picat Terms

- TERM picat_build_var(): Return a free Picat variable.
- TERM picat_build_integer (long i): Return a Picat integer whose value is i.
- TERM picat_build_float(double f): Return a Picat float whose value is $f$.
- TERM picat_build_atom (char *name): Return a Picat atom whose name is name.
- TERM picat_build_nil(): Return an empty Picat list.
- TERM picat_build_list(): Return a Picat list whose car and cdr are free variables.
- TERM picat_build_structure (char *name, int arity): ReturnaPicat structure whose functor is name, and whose arity is arity. The structure's arguments are all free variables.
- TERM picat_build_array (int n): Return a Picat array whose size is n. The array's arguments are all free variables.


## G. 7 Registering C-defined Predicates

The following function registers a predicate that is defined by a C function.

```
insert_cpred(char *name, int arity, int (*func)())
```

The first argument is the predicate name, the second argument is the arity, and the third argument is the name of the function that defines the predicate. The function that defines the predicate cannot take any argument. As described above, picat_get_call_arg (i, arity) is used to fetch arguments from the Picat call.

For example, the following registers a predicate whose name is "p", and whose arity is 2 .

```
extern int p();
insert_cpred("p", 2, p)
```

The C function's name does not need to be the same as the predicate name.
Predicates that are defined in C should be registered after the Picat engine is initialized, and before any call is executed. One good place for registering predicates is the Cboot () function in the file cpreds.c, which registers all of the C-defined built-ins of Picat. After registration, the predicate can be called. All C-defined predicates must be explicitly called with the module qualifier $b p$, as in bp. $p(a, X)$.

## Example

Consider the Picat predicate:

$$
\begin{aligned}
& p(a, X)=>X=\$ f(a) . \\
& p(b, X)=>X=[1] \\
& p(c, X)=>X=1.2 .
\end{aligned}
$$

where the first argument is given and the second is unknown. The following steps show how to define this predicate in C , and how to make it callable from Picat.

Step 1 . Write a C function to implement the predicate. The following shows a sample:

```
#include "picat.h"
p(){
    TERM a1, a2, a, b, c, f1, l1, f12;
    char *name_ptr;
    /* prepare Picat terms */
    a1 = picat_get_call_arg(1, 2); /* first argument */
    a2 = picat_get_call_arg(2, 2); /* second argument */
    a = picat__build_atom("a");
    b = picat_build_atom("b");
    c = picat_build_atom("c");
    f1 = picat_build_structure("f", 1); /* f(a) */
    picat_unify(picat_get_arg(1, f1), a);
    l1 = picat_build_list(); /* [1] */
    picat_unify(picat_get_car(l1), picat_build_integer(1));
    picat_unify(picat_get_cdr(l1), picat_build_nil());
    f12 = picat_build_float(1.2); /* 1.2 */
    /* code for the rules */
    if (!picat_is_atom(al))
        return PICAT_FALSE;
```

```
    name_ptr = picat_get_name(a1);
    switch (*name_ptr){
    case 'a':
        return (picat_unify(a1, a) ?
                                picat_unify(a2, f1) : PICAT_FALSE);
    case 'b':
        return (picat_unify(a1, b) ?
                            picat_unify(a2, l1) : PICAT_FALSE);
    case 'c':
        return (picat_unify(a1, c) ?
                        picat_unify(a2, f12) : PICAT_FALSE);
    default: return PICAT_FALSE;
    }
}
```

Step 2 . Insert the following two lines into Cboot () in cpreds.c:

```
extern int p();
insert_cpred("p", 2, p);
```

Step 3 . Modify the make file, if necessary, and recompile the system. Now, $\mathrm{p} / 2$ is in the group of built-ins in Picat.

Step 4 . Use bp.p (...) to call the predicate.

```
picat> bp.p(a,X)
X = f(a)
```


## Appendix H

## Appendix: Tokens

```
/* Picat lexical rules
    [...] means optional
    {...} means 0, 1, or more occurrences
    "..." means as-is
    /* ... */ comment
    Tokens to be returned:
    Token-type lexeme
    ======================
    ATOM a string of chars of the atom name
    VARIABLE a string of chars of the variable name
    INTEGER an integer literal
    FLOAT a float literal
    STRING a string of chars
    OPERATOR a string of chars in the operator
    SEPARATOR one of "(" ")" "{" "}" "[" "]"
*/
line_terminator ->
    the LF character, also known as "newline"
    the CR character, also known as "return"
    the CR character followed by the LF character
input_char ->
    unicode_input_char but not CR or LF
comment ->
    traditional_comment
    end_of_line_comment
traditional_comment ->
    "/*" comment_tail
comment_tail ->
    "*" comment_tail_star
    not_star comment_tail
comment_tail_star ->
    "/"
    "*" comment_tail_star
    not_star_not_slash comment_tail
not_star ->
    input_char but not "*"
    line_terminator
not_star_not_slash ->
    input_char but not "*" or "/"
    line_terminator
end_of_line_comment ->
```

"\%" \{input_char\} line_terminator
white_space ->
the SP character, also known as "space"
the HT character, also known as "horizontal tab"
the FF character, also known as "form feed"
line_terminator
token ->
atom_token
variable_token
integer_literal
real_literal
string_literal
operator_token
separator_token
atom_token ->
small_letter \{alphanumeric_char\}
single_quoted_token
variable_token ->
anonymous_variable
named_variable
anonymous variable ->
"_"
named_variable ->
"_" alphanumeric \{alphanumeric\}
capital_letter \{alphanumeric\}
alphanumeric ->
alpha_char
decimal_digit
alpha_char ->
underscore_char
letter
letter ->
small_letter
capital_letter
single_quoted_token ->
"'" \{string_char\} "'"
string_literal ->
"\"" \{string_char\} "\""
string_char ->
input_char
escape_sequence
integer_literal ->
decimal_numeral
hex_numeral
octal_numeral
binary_numeral
decimal_numeral ->
decimal_digit [decimal_digits_and_underscores]
decimal_digits_and_underscores ->
decimal_digit_or_underscore
decimal_digits_and_underscores decimal_digit_or_underscore
decimal_digit_or_underscore ->
decimal_digit
"_"

```
hex_numeral ->
    "0x" hex_digits
    "OX" hex_digits
hex_digits ->
    hex_digit [hex_digits_and_underscores]
hex_digits_and_underscores ->
    hex_digit_or_underscore
    hex_digits_and_underscores hex_digit_or_underscore
hex_digit_or_underscore ->
    hex_digit
    "_"
octal_numeral ->
    "00" octal_digits
    "Oo" octal_digits
octal_digits ->
    octal_digit [octal_digits_and_underscores]
octal_digits_and_underscores ->
    octal_digit_or_underscore
    octal_digits_and_underscores octal_digit_or_underscore
octal_digit_or_underscore ->
    octal_digit
    "_"
binary_numeral ->
        "Ob" binary_digits
        "OB" binary_digits
binary_digits:
        binary_digit [binary_digits_and_underscores]
binary_digits_and_underscores ->
        binary_digit_or_underscore
        binary_digits_and_underscores binary_digit_or_underscore
binary_digit_or_underscore:
        binary_digit
        " "
real_literal ->
        decimal_numeral "." decimal_numeral [exponent_part]
exponent_part ->
        exponent_indicator signed_integer
exponent_indicator ->
        "e"
        "E"
signed_integer ->
    [sign] decimal_numeral
sign ->
        "+"
        " -"
separator ->
        one of "(" ")" "{" "}" "[" "]"
operator ->
        one of
            "=" "!=" ">" ">=" "<" "<=" "=<" "..." "!"
            "," ";" ":" "::" "." ". " (dot-whitespace)
```

```
    "=>" "?=>" "==" "!==" ":=" "| " "$" "@"
    "/\" "\/" "~" "^" "<<" ">>"
    "+" "-" "\star" "**" "/" "/>" "/<" "^"
    "#=" "#!=" "#>" "#>=" "#<" "#<=" "#=<"
    "#/\" "#\/" "#~" "#^" "#=>" "#<=>"
    "@>" "@>=" "@<" "@<=" "@=<"
small_letter ->
    one of "a" "b" ... "z"
capital_letter ->
    one of "A" "B" ... "Z"
decimal_digit ->
    one of "0" "1" "2" "3" "4" "5" "6" "7" "8" "9"
hd ->
    hex_digit
hex_digit ->
    one of
        "0" "1" "2" "3" "4" "5" "6" "7" "8" "9"
        "a" "b" "c" "d" "e" "f" "A" "B" "C" "D" "E" "E"
octal_digit ->
    one of "0" "1" "2" "3" "4" "5" "6" "7"
binary_digit ->
    one of "0" "1"
escape_sequence ->
    "\b" /* \u0008: backspace BS */
    "\t" /* \u0009: horizontal tab HT */
    "\n" /* \u000a: linefeed LF */
    "\f" /* \u000c: form feed FF */
    "\r" /* \u000d: carriage return CR */
    "\"" /* \u0022: double quote " */
    "\'" /* \u0027: single quote ' */
    "\\" /* \u005c: backslash \ */
    unicode_escape
unicode_escape ->
    "\u" hd hd hd hd
    "\U" hd hd hd hd hd hd hd hd
```


## Appendix I

## Appendix: Grammar

```
/* Picat syntax rules
    [...] means optional
    {...} means 0, 1, or more occurrences
    (a | b) means choice
    "..." means a token
    %... one-line comment
    input tokens:
        atom
        variable
        integer
        float
        operator
            separator
            eor is "." followed by a white space or eof
*/
program ->
    [module_declaration]
    {import_declaration}
    program_body
program_body ->
    {predicate_definition | function_definition | actor_definition}
module_declaration ->
    "module" atom eor
import_declaration ->
    import import_item {"," import_item} eor
import_item ->
    atom
predicate_definition ->
    {predicate_directive} predicate_rule_or_fact {predicate_rule_or_fact}
function_definition ->
    {function_directive} function_rule_or_fact {function_rule_or_fact}
actor definition ->
    ["private"] action_rule {(action_rule
                                    | nonbacktrackable_predicate_rule)}
function_directive ->
    "private"
    "table"
predicate_directive ->
    "private"
    "table" ["(" table_mode {"," table_mode} ")" ]
    "index" index_declaration {"," index_declaration}
```

```
index_declaration ->
    "(" index_mode {"," index_mode} ")"
index_mode ->
    "+"
    "-"
table_mode ->
    "+"
    "-"
    "min"
    "max"
    "nt"
predicate_rule_or_fact ->
        predicate_rule
        predicate_fact
function_rule_or_fact ->
    function_rule
    function_fact
predicate_rule ->
        head ["," condition] ("=>" | "?=>") body eor
nonbacktrackable_predicate_rule ->
        head ["," condition] "=>" body eor
predicate_fact ->
    head eor
head ->
        atom ["(" [term {"," term}] ")"]
function_rule ->
    head "=" expression ["," condition] "=>" body eor
function_fact ->
        head "=" argument eor
action_rule ->
        head ["," condition] "," "{" event_pattern "}" => body eor
event_pattern ->
    term {',' term}
condition -> goal
body -> goal
goal ->
    disjunctive_goal
argument ->
        negative_goal
disjunctive_goal ->
        disjunctive_goal ";" conjunctive_goal
        conjunctive_goal
conjunctive_goal ->
        conjunctive_goal "," negative_goal
        negative_goal
negative_goal ->
    "not" negative_goal
    equiv_constr
equiv_constr ->
    equiv_constr "#<=>" impl_constr
```

```
    impl_constr
impl_constr ->
    impl_constr "#=>" or_constr
    or_constr
or_constr ->
    or_constr "#\/" xor_constr
    xor_constr
xor_constr ->
    xor_constr "#^" and_constr
    and_constr
and_constr ->
    and_constr "#/\" not_constr
    not_constr
not_constr ->
    "#~" not_constr
    enclosed_goal
enclosed_goal ->
    "if" goal "then" goal {"elseif" goal "then" goal} "else" goal "end"
    "foreach" "(" iterator {"," (iterator | condition)} ")" goal "end"
    "while" "(" goal ")" ["do"] goal "end"
    "do" goal "while" "(" goal ")"
    expression {bin_rel_op expression}
bin_rel_op ->
    "="
    "!="
    ":="
    "=="
    "!=="
    ">"
    ">="
    "<"
    "=<"
    "<="
    ": : "
    "in"
    "notin"
    "#="
    "#! = "
    "#>"
    "#>="
    "#<"
    "#=<"
    "#<="
    "@>"
    "@>="
    "@<"
    "@=<"
    "@<="
expression ->
    concat_expression
concat_expression ->
    range_expression ["++" concat_expression]
range_expression ->
    or_expression [".." or_expression [".." or_expression]]
or_expression ->
    xor_expression
    or_expression "\/" xor_expression
xor_expression ->
```

```
    and_expression
    xor_expression "^" and_expression % bit-wise xor
and_expression ->
    shift_expression
    and_expression "/\" shift_expression
shift_expression ->
    additive_expression
    shift_expr ( "<<" | ">>" ) additive_expression
additive_expression ->
    multiplicative_expression
    additive_expression "+" multiplicative_expression
    additive_expression "-" multiplicative_expression
multiplicative_expression ->
    unary_expression
    multiplicative_expression "*" unary_expression
    multiplicative_expression "/" unary_expression
    multiplicative_expression "//" unary_expression
    multiplicative_expression "/>" unary_expression
    multiplicative_expression "/<" unary_expression
    multiplicative_expression "div" unary_expression
    multiplicative_expression "mod" unary_expression
    multiplicative_expression "rem" unary_expression
unary_expression ->
    power_expression
    "+" unary_expression
    "-" unary_expression
    "~" unary_expression % bit-wise complement
power_expression ->
    primary_expression ["**" unary_expression]
primary_expression ->
    "(" goal ")"
    variable "[" argument ["," argument] "]" % subscript notation
    variable "@" term ["@"] % as-pattern
    variable
    integer
    float
    atom_or_call
    list_expression
    array_expression
    function_call
    term_constructor
    primary_expression "." atom_or_call % dot-notation
atom_or_call ->
    atom ["(" [argument {"," argument}] ")"]
list_expression ->
    "[]"
    "[" argument list_expression_suffix "]"
list_expression_suffix ->
    ":" iterator {"," (iterator | condition)} % list comprehension
    {"," argument} ["|" argument]
array_expression ->
    "{}"
    "{" argument array_expression_suffix "}"
array_expression_suffix ->
    ":" iterator {"," (iterator | condition)} % array comprehension
    {"," argument}
```

function_call ->
[primary_expression "."] atom "(" [argument \{"," argument\}] ")"
variable_list ->
"[" [variable \{"," variable\}] "]"
term_constructor -> "\$" goal ["\$"]
/* a term has the same form as a goal except that it cannot contain loops or if-then-else. Note that subscript notations, range expressions, dot notations, and list comprehensions are still treated as functions in term constructors */

## Appendix J

## Appendix: Operators

| Precedence | Operators |
| :---: | :---: |
| Highest | ., © |
|  | ** (right-associative) |
|  | unary +, unary -, ~ |
|  | *, /, //, /<, />, div, mod, rem |
|  | binary +, binary - |
|  | >>, << |
|  | / |
|  | - |
|  | \/ |
|  | -• |
|  | ++ (right-associative) |
|  | $\begin{aligned} & =,!=,:=,==,!==,=:=,<,=<,<=,>,>=,:: \text {, in, notin } \\ & \#=, \#!=, \#<, \#=<, \#<=, \#>, \#>=, ~ @<, ~ @=<, ~ @<=, ~ @>, ~ @>= \end{aligned}$ |
|  | \# |
|  | \# / \} |
|  | \# ${ }^{\text {}}$ |
|  | \# \/ |
|  | \# => (right-associative) |
|  | \#<=> |
|  | not, once |
|  | , (right-associative), \&\& (right-associative) |
| Lowest | ; (right-associative), \|| (right-associative) |

## Appendix K

## Appendix: The Library Modules

## Module basic (imported by default)

- $X \quad$ ! $=Y$
- $X$ ! $==Y$
- $X$ := $Y$
- $X<Y$
- $X<=Y$
- $X=Y$
- $X=<Y$
- $X=:=Y$
- $X==Y$
- $X>Y$
- $X>=Y$
- $X @<Y$
- $X$ @<= $Y$
- $X$ @ $=<Y$
- $X$ @ $>Y$
- $X$ @ $>=Y$
- Term $_{1}++$ Term $_{2}=$ List
- $[X: I$ in $D, \ldots]=$ List
- $L \ldots U=$ List
- L . . Step . . $U=$ List
- $-X=Y$
- $+X=Y$
- $X+Y=Z$
- $X-Y=Z$
- $X * Y=Z$
- $X / Y=Z$
- $X / / Y=Z$
- $X$ div $Y=Z$
- $X \quad /<Y=Z$
- $X />Y=Z$
- $X$ ** $Y=Z$
- $X \bmod Y=Z$
- $X$ rem $Y=Z$
- $\sim X=Y$
- $X \backslash / Y=Z$
- $X \wedge Y=Z$
- $X \wedge$ ^ $Y=Z$
- $X \ll Y=Z$
- $X \gg Y=Z$
- Var[ Index $_{1}, \ldots$, Index $\left._{n}\right]$
- Goal ${ }_{1}$, Goal $_{2}$
- Goal $_{1} \& \&$ Goal $_{2}$
- Goal ${ }_{1}$; Goal $_{2}$
- Goal $_{1}$ |। Goal ${ }_{2}$
- acyclic_term(Term)
- and_to_list $($ Conj) $=$ List
- append $(X, Y, Z)$ (nondet)
- append $(X, Y, Z, T)$ (nondet)
- $\operatorname{apply}\left(S, \operatorname{Arg}_{1}, \ldots, \operatorname{Arg}_{n}\right)=\operatorname{Val}$
- arity (Struct) $=$ Arity
- array (Term)
- atom (Term)
- atom_chars $($ Atm $)=$ String
- atom_codes $($ Atm $)=$ List
- atomic (Term)
- attr_var(Term)
- $\operatorname{avg}($ ListOrArray $)=$ Val
- between (From,To, X) (nondet)
- bind_vars (Term,Val)
- $\operatorname{call}\left(S, \operatorname{Arg}_{1}, \ldots, \operatorname{Arg}_{n}\right)$
- call_cleanup ( $S$, Cleanup)
- catch (S, Exception, Handler)
- char (Term)
- chr (Code) = Char
- clear (Map)
- compare_terms $\left(\right.$ Term $_{1}$, Term $\left._{2}\right)=$ Res
- compound (Term)
- copy_term $\left(\right.$ Term $\left._{1}\right)=$ Term $_{2}$
- count_all (Call) $=$ Int
- delete $($ List, $X)=$ ResList
- delete_all(List, X) = ResList
- different_terms $\left(\right.$ Term $_{1}$, Term $\left._{2}\right)$
- digit (Char)
- dvar (Term)
- dvar_or_int (Term)
- fail
- false
- find_all(Template, Call) $=$ List
- findall(Template,Call) = List
- first (Compound) $=$ Term
- flatten $($ List 1$)=$ List 2
- float(Term)
- $\operatorname{fold}(F, A C C$, List $)=$ Res
- freeze ( $X$, Goal)
- get $($ Map,Key $)=$ Val
- get (Map, Key, DefaultVal) = Val
- get_attr(AttrVar, Key) = Val
- get_attr(AttrVar, Key, DefaultVal) =Val
- get_global_map $(I D)=M a p$
- get_global_map() = Map
- get_heap_map $(I D)=M a p$
- get_heap_map() = Map
- get_table_map $(I D)=M a p$
- get_table_map() = Map
- ground (Term)
- handle_exception (Term,Term)
- has_key (Map, Key)
- hash_code (Term) = Int
- head (List) $=$ Term
- heap_is_empty (Heap)
- heap_pop $($ Heap $)=E l m$
- heap_push (Heap, Elm)
- heap_size (Heap) = Size
- heap_to_list (Heap) = List
- heap_top $(H e a p)=E l m$
- insert(List, Index, Elm) $=$ ResList
- insert_all(List,Index,AList) $=$ ResList
- insert_ordered (List,Term) $=R$
- insert_ordered_down(List,Term) $=R$
- int (Term)
- integer (Term)
- is (Exp,Exp)
- keys (Map) $=$ List
- last $($ Compound $)=$ Term
- len $($ Term $)=$ Len
- length $($ Term $)=$ Len
- list (Term)
- list_to_and (List) $=$ Conj
- lowercase (Char)
- map $($ Func, List1, List2 $)=$ ResList
- map $($ FuncOrList, ListOrFunc) $=$ ResList
- map (Term)
- map_to_list $(M a p)=$ List
- $\max ($ ListOrArray $)=$ Val
- $\max (X, Y)=$ Val
- maxint_small() = Int
- maxof (Call,Objective)
- maxof (Call,Objective, ReportCall)
- maxof_inc (Call,Objective)
- maxof_inc (Call,Objective, ReportCall)
- membchk (Term, List)
- member (Term, List) (nondet)
- $\min ($ ListOrArray $)=$ Val
- $\min (X, Y)=V a l$
- minint_small() = Int
- minof(Call,Objective)
- minof (Call,Objective, ReportCall)
- minof_inc (Call,Objective)
- minof_inc (Call, Objective, ReportCall)
- name (Struct) $=$ Name
- new_array $\left(D_{1}, \ldots, D_{n}\right)=\operatorname{Arr}$
- new_list $(N)=$ List
- new_list $(N$, InitVal) $=$ List
- new_map (Int, PairsList) $=$ Map
- new_map (IntOrPairsList) $=$ Map
- new_max_heap (IntOrList) $=$ Heap
- new_min_heap (IntOrList) $=$ Heap
- new_set (Int, ElmsList) $=$ Map
- new_set (IntOrElmsList) = Map
- new_struct (Name, IntOrList) $=$ Struct
- nonvar (Term)
- not Call
- nth (I, ListOrArray, Val) (nondet)
- number (Term)
- number_chars $(N u m)=$ String
- number_codes $(N u m)=$ List
- number_vars $\left(\right.$ Term, $\left.N_{0}\right)=N_{1}$
- once Call
- ord (Char) $=$ Int
- parse_radix_string (String, Base) = Int
- parse_term (String) $=$ Term
- parse_term(String,Term,Vars)
- post_event ( $X, E v e n t)$
- post_event_any ( $X, E v e n t)$
- post_event_bound $(X)$
- post_event_dom (X,Event)
- post_event_ins $(X)$
- $\operatorname{prod}($ ListOrArray $)=$ Val
- put (Map,Key)
- put(Map,Key,Val)
- put_attr (Var, Key)
- put_attr (Var, Key, Val)
- real (Term)
- reduce (Func,List) = Res
- reduce (Func, List,InitVal) $=$ Res
- remove_dups(ListOrArray) $=$ ResList
- repeat (nondet)
- reverse (ListOrArray) $=$ Res
- second (Compound $)=$ Term
- select ( $X$, List, ResList) (nondet)
- $\operatorname{size}(M a p)=$ Size
- slice (ListOrArray, From)
- slice (ListOrArray, From,To)
- $\operatorname{sort}($ ListOrArray) $=$ Sorted
- $\operatorname{sort}($ ListOrArray, KeyIndex) $=$ Sorted
- sort_down (ListOrArray) $=$ Sorted
- sort_down(ListOrArray, KeyIndex) = Sorted
- sort_down_remove_dups(ListOrArray) = Sorted
- sort_down_remove_dups(ListOrArray, KeyIndex) = Sorted
- sort_remove_dups (ListOrArray) $=$ Sorted
- sort_remove_dups(ListOrArray, KeyIndex) = Sorted
- sorted (ListOrArray)
- sorted_down (ListOrArray)
- string (Term)
- struct (Term)
- subsumes (Term ${ }_{1}$, Term $_{2}$ )
- $\operatorname{sum}($ ListOrArray $)=$ Val
- tail(List) $=$ Term
- throw $E$
- to_array $($ List $)=$ Array
- to_atom(String) = Atom
- to_binary_string(Int) $=$ String
- to_codes $($ Term $)=$ List
- to_fstring (Format, Args...) = String
- to_hex_string (Int) = String
- to_int(NumOrCharOrStr) $=$ Int
- to_integer(NumOrCharOrStr) = Int
- to_list (Struct) $=$ List
- to_lowercase (String) $=$ LString
- to_number(NumOrCharOrStr) = Number
- to_oct_string(Int) = String
- to_radix_string (Int, Base) $=$ String
- to_real (NumOrStr) = Real
- to_string (Term) = String
- to_uppercase (String) = UString
- true
- uppercase (Char)
- values $(M a p)=$ List
- var(Term)
- variant (Term Therm $_{2}$ )
- $\operatorname{vars}($ Term $)=$ Vars
- zip $\left(\right.$ List $_{1}$, List $\left._{2}\right)=$ List
- $\operatorname{zip}\left(\right.$ List $_{1}$, List $_{2}$, List $\left._{3}\right)=$ List
- $\operatorname{zip}\left(\right.$ List $_{1}$, List $_{2}$, List $_{3}$, List $\left._{4}\right)=$ List


## Module math (imported by default)

- $\operatorname{abs}(X)=V a l$
- $\operatorname{acos}(X)=V a l$
- $\operatorname{acosh}(X)=V a l$
- $\operatorname{acot}(X)=V a l$
- $\operatorname{acoth}(X)=V a l$
- $\operatorname{acsc}(X)=V a l$
- $\operatorname{acsch}(X)=V a l$
- $\operatorname{asec}(X)=V a l$
- $\operatorname{asech}(X)=V a l$
- $\operatorname{asin}(X)=$ Val
- $\operatorname{asinh}(X)=$ Val
- $\operatorname{atan}(X)=V a l$
- $\operatorname{atan} 2(X, Y)=V a l$
- $\operatorname{atanh}(X)=$ Val
- $\operatorname{ceiling}(X)=V a l$
- $\cos (X)=$ Val
- $\cosh (X)=$ Val
- $\cot (X)=$ Val
- $\operatorname{coth}(X)=V a l$
- $\csc (X)=V a l$
- $\operatorname{csch}(X)=$ Val
- $e()=2.71828182845904523536$
- even (Int)
- $\exp (X)=$ Val
- floor $(X)=$ Val
- frand () = Val
- $\mathrm{frand}($ Low, High $)=$ Val
- $\operatorname{gcd}(A, B)=$ Val
- $\log (X)=$ Val
- $\log (\mathrm{B}, X)=V a l$
- $\log 10(X)=V a l$
- $\log 2(X)=V a l$
- $\operatorname{modf}(X)=($ IntVal, FractVal $)$
- odd (Int)
- pi() $=3.14159265358979323846$
- $\operatorname{pow}(X, Y)=V a l$
- pow $\bmod (X, Y, Z)=V a l$
- prime (Int)
- $\operatorname{primes}($ Int $)=$ List
- rand_max () = Val
- random $=$ Val
- random(Low,High) $=$ Val
- $\operatorname{random}($ Seed $)=V a l$
- random2() = Int
- round $(X)=$ Val
- $\sec (X)=$ Val
- $\operatorname{sech}(X)=V a l$
- $\operatorname{sign}(X)=V a l$
- $\sin (X)=$ Val
- $\sinh (X)=$ Val
- $\operatorname{sqrt}(X)=V a l$
- $\tan (X)=V a l$
- $\tanh (X)=$ Val
- to_degrees $($ Radian $)=$ Degree
- to_radians $($ Degree $)=$ Radian
- truncate $(X)=V a l$


## Module io (imported by default)

- at_end_of_stream ( $F D$ )
- close (FD)
- flush (FD)
- flush()
- $\mathrm{nl}(F D)$
- nl()
- open (Name) $=F D$
- open(Name, Mode) $=F D$
- peek_byte $(F D)=V a l$
- peek_char $(F D)=$ Val
- print (FD,Term)
- print (Term)
- printf (FD, Format, Args...)
- println $(F D, T e r m)$
- println(Term)
- read_atom $(F D)=$ Atom
- read_atom() $=$ Atom
- read_byte $(F D)=$ Val
- read_byte $(F D, N)=$ List
- read_byte() $=$ Val
- read_char $(F D)=$ Val
- read_char $(F D, N)=$ String
- read_char() $=$ Val
- read_char_code $(F D)=$ Val
- read_char_code $(F D, N)=$ List
- read_char_code() = Val
- read_file_bytes $($ File $)=$ List
- read_file_bytes() = List
- read_file_chars (File) = String
- read_file_chars() = String
- read_file_codes $($ File $)=$ List
- read_file_codes() = List
- read_file_lines (File) $=$ List
- read_file_lines() = List
- read_file_terms (File) $=$ List
- read_file_terms() = List
- read_int $(F D)=I n t$
- read_int() $=$ Int
- read_line $(F D)=$ String
- read_line() = String
- read_number $(F D)=N u m b e r$
- read_number() = Number
- read_picat_token $(F D)=$ TokenValue
- read_picat_token (FD,TokenType,TokenValue)
- read_picat_token (TokenType,TokenValue)
- read_picat_token() = TokenValue
- read_real $(F D)=$ Real
- read_real() = Real
- read_term $(F D)=$ Term
- read_term() $=$ Term
- $\operatorname{readln}(F D)=$ String
- readln() = String
- write (FD,Term)
- write (Term)
- write_byte (Bytes)
- write_byte (FD, Bytes)
- write_char (Chars)
- write_char (FD, Chars)
- write_char_code (Codes)
- write_char_code (FD,Codes)
- writef (FD,Format, Args...)
- writeln (FD,Term)
- writeln(Term)


## Module ordset

- delete $($ OSet, Elm $)=$ OSet 1
- disjoint (OSet1,OSet2)
- insert (OSet,Elm) = OSet1
- intersection (OSet1,OSet2) =OSet3
- new_ordset (List)
- ordset (Term)
- subset (OSet1,OSet2)
- subtract (OSet $1, O \operatorname{Set} 2)=O$ Set 3
- union $($ OSet 1, OSet 2$)=O$ Set 3


## Module os

- cd (Path)
- chdir (Path)
- cp (FromPath,ToPath)
- cwd() = Path
- directory (Path)
- dir
- env_exists (Name)
- executable (Path)
- exists(Path)
- file(Path)
- file_base_name (Path) $=$ String
- file_directory_name(Path) $=$ String
- file_exists (Path)
- getenv(EnvString) = String
- listdir(Path) $=$ List
- ls
- mkdir (Path)
- pwd() = Path
- readable (Path)
- rename (Old,New)
- rm(Path)
- rmdir (Path)
- separator() $=$ Val
- $\operatorname{size}($ Path $)=I n t$
- writable (Path)


## Modules cp, sat, and mip

- $\#^{\sim} X$
- $X$ \#! $=Y$
- $X$ \# $\backslash Y$
- $X$ \#< $Y$
- $X$ \#<= $Y$
- $X$ \#<=> $Y$
- $X$ \#= $Y$
- $X$ \#=< $Y$
- $X$ \# $=>Y$
- $X$ \#> $Y$
- $X$ \# $>=Y$
- $X$ \# $\backslash / Y$
- $X$ \#^ $Y$
- Vars :: Exp
- Vars notin Exp
- all_different (FDVars)
- all_different_except_0 (FDVars)
- all_distinct (FDVars)
- assignment (FDVars1,FDVars 2 )
- at_least $(N, L, V)$ :
- at_most $(N, L, V)$ :
- circuit(FDVars)
- count $(V, F D V a r s, N)$
- count (V,FDVars, Rel, $N$ )
- cumulative (Ss,Ds,Rs,Limit)
- decreasing( $L$ )
- decreasing_strict ( $L$ )
- diffn(RectangleList)
- disjunctive_tasks (Tasks) (cp only)
- element (I, List,V)
- exactly $(N, L, V)$ :
- fd_degree $(F D V a r)=$ Degree (cp only $)$
- fd_disjoint (DVar1, DVar2)
- fd_dom(FDVar) = List
- fd_false (FDVar, Elm)
- fd_max $(F D V a r)=$ Max
- fd_min(FDVar) = Min
- fd_min max (FDVar, Min, Max)
- fd_next $(F D V a r, E l m)=$ NextElm
- fd_prev $(F D V a r, E l m)=$ PrevElm
- fd_set_false (FDVar, Elm) (cp only)
- fd_size(FDVar) = Size
- fd_true (FDVar, Elm)
- fd_vector_min_max (Min, Max)
- global_cardinality(List, Pairs)
- increasing(L)
- increasing_strict ( $L$ )
- indomain (Var) (nondet) (cp only)
- indomain_down (Var) (nondet) (cp only)
- lex_le $\left(L_{1}, L_{2}\right)$
- lex_lt $\left(L_{1}, L_{2}\right)$
- matrix_element (Matrix, $I, J, V$ )
- neqs ( NeqList) (cp only)
- new_dvar() = FDVar
- new_fd_var() $=F D V a r$
- regular $(X, Q, S, D, Q 0, F)$
- scalar_product ( $A, X$, Product)
- scalar_product ( $A, X$, Rel, Product)
- serialized(Starts, Durations)
- solve (Options, Vars) (nondet)
- solve(Vars) (nondet)
- solve_all(Options,Vars) = List
- solve_all(Vars) = List
- solve_suspended (cp only)
- solve_suspended (Options) (cp only)
- subcircuit(FDVars)
- table_in (DVars, $R$ )
- table_notin(DVars, $R$ )


## Module planner

- best_plan (S,Limit, Plan)
- best_plan (S, Limit, Plan, Cost)
- best_plan ( $S$, Plan)
- best_plan (S,Plan, PlanCost)
- best_plan_bb (S, Limit, Plan)
- best_plan_bb (S, Limit, Plan, Cost)
- best_plan_bb (S, Plan)
- best_plan_bb (S, Plan, PlanCost)
- best_plan_bin ( $S$, Limit, Plan)
- best_plan_bin (S,Limit, Plan, Cost)
- best_plan_bin (S, Plan)
- best_plan_bin (S, Plan, PlanCost)
- best_plan_nondet ( $S$, Limit, Plan) (nondet)
- best_plan_nondet ( $S$, Limit, Plan, Cost) (nondet)
- best_plan_nondet ( $S$, Plan) (nondet)
- best_plan_nondet ( $S$, Plan, PlanCost) (nondet)
- best_plan_unbounded ( $S$, Limit, Plan)
- best_plan_unbounded ( $S$, Limit, Plan, Cost)
- best_plan_unbounded ( $S$, Plan)
- best_plan_unbounded (S, Plan, PlanCost)
- current_plan () = Plan
- current_resource ()=Amount
- current_resource_plan_cost (Amount, Plan, Cost)
- is_tabled_state ( $S$ )
- plan (S, Limit, Plan)
- plan (S,Limit, Plan, Cost)
- plan (S, Plan)
- plan (S, Plan, PlanCost)
- plan_unbounded ( $S$, Limit, Plan)
- plan_unbounded ( $S$, Limit, Plan, Cost)
- plan_unbounded ( $S$, Plan)
- plan_unbounded (S, Plan, PlanCost)


## Module datetime

- current_datetime() = DateTime
- current_day() = WDay
- current_date() = Date
- current_time() = Time


## Module sys (imported by default)

- abort
- cl
- cl(File)
- cl_facts (Facts)
- cl_facts (Facts, IndexInfo)
- cl_facts_table (Facts)
- cl_facts_table (Facts,IndexInfo)
- command (String)
- compile (File)
- debug
- exit
- garbage_collect(Size)
- garbage_collect
- halt
- initialize_table
- load(File)
- loaded_modules()
- nodebug
- nospy
- notrace
- spy Functor
- statistics(Name, Value) (nondet)
- statistics
- time (Goal)
- time2 (Goal)
- time_out (Goal, Limit, Res)
- trace


## Module util

- array_matrix_to_list(Matrix) $=$ List
- array_matrix_to_list_matrix(AMatrix) $=$ LMatrix
- chunks_of(List,K) $=$ ListOfLists
- find (String, SubString, From,To) (nondet)
- find_first_of (Term, Pattern) $=$ Index
- find_ignore_case (String, SubString, From,To) (nondet)
- find_last_of(Term, Pattern) $=$ Index
- join(Words) $=$ String
- join(Words,Separator) $=$ String
- list_matrix_to_array_matrix(LMatrix) $=$ AMatrix
- lstrip(List) $=$ List
- lstrip $($ List, $E l m s)=$ List
- matrix_multi(MatrixA,MatrixB) $=$ MatrixC
- permutation (List, Perm) (nondet)
- permutations(List) $=$ Lists
- power_set (List) = Lists
- replace (Term,Old,New) = NewTerm
- replace_at (Term,Index,New) $=$ NewTerm
- rstrip(List) $=$ List
- rstrip(List,Elms) = List
- $\operatorname{split(List)}=$ Words
- split(List,Separators) $=$ Words
- $\operatorname{strip}($ List $)=$ List
- strip(List, Elms) = List
- take $($ List, K) $=$ List
- transpose (Matrix) $=$ Transposed


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[^0]:    ${ }^{1}$ Picat also accepts Prolog-style if-then-else in the form (If->Then;Else) and mandates the presence of the else-part.

[^1]:    ${ }^{2}$ An nt argument can carry some information that is dependent on the input arguments but useful for the computation.

[^2]:    ${ }^{3}$ Dependent modules must be in a path that is specified by the environment variable PICATPATH.

[^3]:    ${ }^{1}$ A list is complete if it is empty, or if its tail is complete. For example, $[\mathrm{a}, \mathrm{b}, \mathrm{c}]$ and $[\mathrm{X}, \mathrm{Y}, \mathrm{Z}]$ are complete, but $[a, b \mid T]$ is not complete if $T$ is a variable.

[^4]:    ${ }^{2}$ Note that a heap, as a data structure, is different from the heap area, in which data, including heap maps, are stored.

[^5]:    ${ }^{1}$ In the current implementation, exceptions thrown by B-Prolog are not compliant with the documentation.

[^6]:    ${ }^{1} \mathrm{~A}$ warning is issued instead of an error. This allows users to test incomplete programs with missing definitions.

[^7]:    ${ }^{1}$ The specifier $\% \mathrm{C}$ can only be used to print ASCII characters. Use the specifier $\% \mathrm{~W}$ to print UTF- 8 characters.

[^8]:    ${ }^{1}$ Note that the preceding dollar sign indicates that it is followed by a term, not a function call.

