

MAIN IDEA

Identify similar polygons and find missing measures of similar polygons.

New Vocabulary

polygon
similar
corresponding parts
congruent
scale factor

Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz

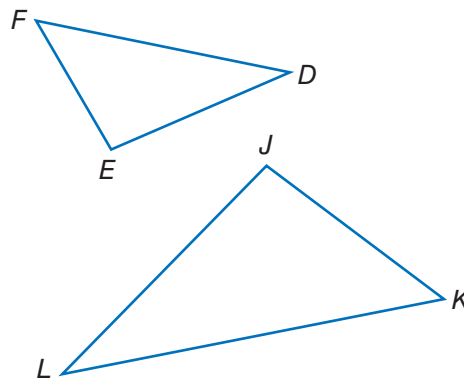
MINI Lab

Follow the steps below to discover how the triangles at the right are related.

STEP 1 Copy both triangles onto tracing paper.

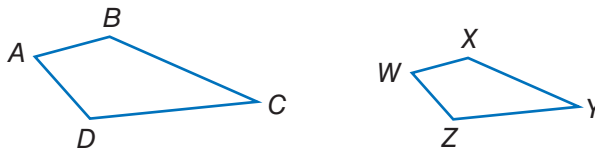
STEP 2 Measure and record the sides of each triangle.

STEP 3 Cut out both triangles.

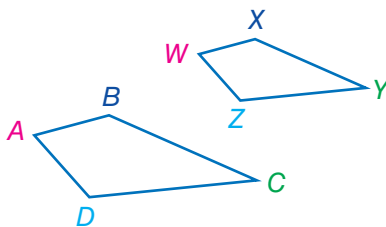


1. Compare the angles of the triangles by matching them up. Identify the angle pairs that have equal measure.
2. Express the ratios $\frac{DF}{LK}$, $\frac{EF}{JK}$, and $\frac{DE}{LJ}$ as decimals to the nearest tenth.
3. What do you notice about the ratios of these sides of matching triangles?

A **polygon** consists of a sequence of consecutive line segments in a plane, placed end to end to form a simple closed figure. Polygons that have the same shape are called **similar** polygons. In the figure below, polygon $ABCD$ is similar to polygon $WXYZ$. This is written as polygon $ABCD \sim$ polygon $WXYZ$.

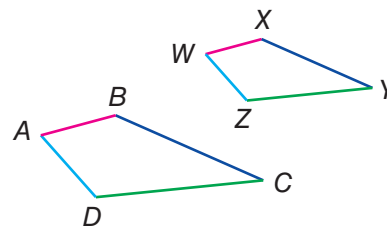


The parts of similar figures that “match” are called **corresponding parts**.



Corresponding Angles

$\angle A \leftrightarrow \angle W$, $\angle B \leftrightarrow \angle X$,
 $\angle C \leftrightarrow \angle Y$, $\angle D \leftrightarrow \angle Z$



Corresponding Sides

$\overline{AB} \leftrightarrow \overline{WX}$, $\overline{BC} \leftrightarrow \overline{XY}$,
 $\overline{CD} \leftrightarrow \overline{YZ}$, $\overline{DA} \leftrightarrow \overline{ZW}$

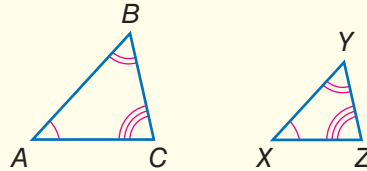
The similar triangles in the Mini Lab suggest the following.

Similar Polygons

Key Concept

- Words** If two polygons are similar, then
- their corresponding angles are **congruent**, or have the same measure, and
 - the measures of their corresponding sides are proportional.

Model



$$\triangle ABC \sim \triangle XYZ$$

Symbols $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$, and $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$

Reading Math

Congruence The symbol \cong is read *is congruent to*. Arcs are used to show congruent angles.

EXAMPLE Identify Similar Polygons

- 1 Determine whether rectangle $HJKL$ is similar to rectangle $MNPQ$. Explain.

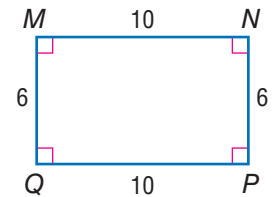
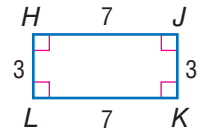
First, check to see if corresponding angles are congruent.

Since the two polygons are rectangles, all of their angles are right angles. Therefore, all corresponding angles are congruent.

Next, check to see if corresponding sides are proportional.

$$\frac{HJ}{MN} = \frac{7}{10} \quad \frac{JK}{NP} = \frac{3}{6} \text{ or } \frac{1}{2} \quad \frac{KL}{PQ} = \frac{7}{10} \quad \frac{LH}{QM} = \frac{3}{6} \text{ or } \frac{1}{2}$$

Since $\frac{7}{10}$ and $\frac{1}{2}$ are not equivalent ratios, rectangle $HJKL$ is *not* similar to rectangle $MNPQ$.

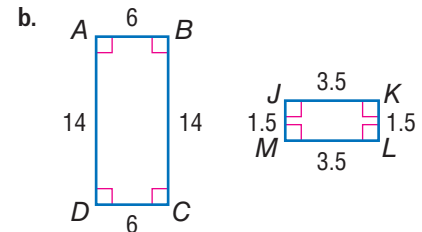
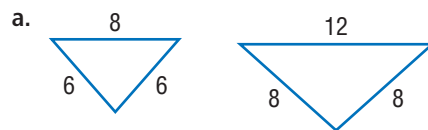


Study Tip

Common Error
Do not assume that two polygons are similar just because their corresponding angles are congruent. Their corresponding sides must also be proportional.

CHECK Your Progress

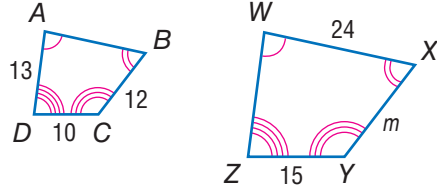
Determine whether these polygons are similar. Explain.



The ratio of the lengths of two corresponding sides of two similar polygons is called the **scale factor**. You can use the scale factor of similar figures or a proportion to find missing measures.

EXAMPLE Find Missing Measures

- 2 GEOMETRY** Given that polygon $WXYZ \sim$ polygon $ABCD$, find the missing measure.



METHOD 1 Write a proportion.

The missing measure m is the length of \overline{XY} . Write a proportion.

$$\begin{array}{l} \text{polygon } WXYZ \rightarrow \frac{XY}{12} = \frac{YZ}{10} \leftarrow \text{polygon } WXYZ \\ \text{polygon } ABCD \rightarrow \frac{BC}{12} = \frac{CD}{10} \leftarrow \text{polygon } ABCD \end{array}$$

$$\frac{m}{12} = \frac{15}{10} \quad XY = m, BC = 12, \\ YZ = 15, \text{ and } CD = 10.$$

$$m \cdot 10 = 12 \cdot 15 \quad \text{Find the cross products.}$$

$$10m = 180 \quad \text{Multiply.}$$

$$m = 18 \quad \text{Divide each side by 10.}$$

Reading Math

Segment Measure The measure of \overline{XY} is written as XY . It represents a number.

METHOD 2 Use the scale factor to write an equation.

Find the scale factor from polygon $WXYZ$ to polygon $ABCD$.

$$\text{scale factor: } \frac{YZ}{CD} = \frac{15}{10} \text{ or } \frac{3}{2} \quad \text{The scale factor is the constant of proportionality.}$$

Words A length on polygon $WXYZ$ is $\frac{3}{2}$ times as long as a corresponding length on polygon $ABCD$.

Variable Let m represent the measure of \overline{XY} .

Equation
$$m = \frac{3}{2} \cdot 12$$

$$m = \frac{3}{2}(12) \quad \text{Write the equation.}$$

$$m = 18 \quad \text{Multiply.}$$

Study Tip

Scale Factor
In Example 2, the scale factor from polygon $ABCD$ to polygon $WXYZ$ is $\frac{2}{3}$, which means that a length on polygon $ABCD$ is $\frac{2}{3}$ as long as a length on polygon $WXYZ$.

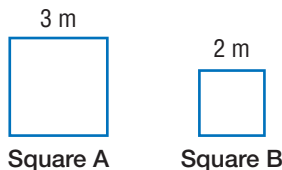
CHOOSE Your Method

Find each missing measure above.

c. WZ

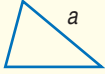
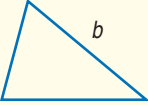
d. AB

Square A \sim square B with a scale factor of 3:2. Notice that the ratio of their perimeters is 12:8 or 3:2.



Square	Perimeter
A	12 m
B	8 m

This and other related examples suggest the following.

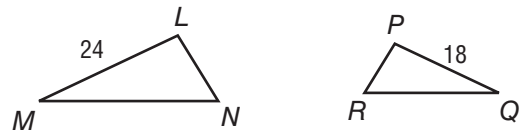
Ratios of Similar Figures		Key Concept
Words	If two figures are similar with a scale factor of $\frac{a}{b}$, then the perimeters of the figures have a ratio of $\frac{a}{b}$.	Model
		 
		Figure A Figure B

Test-Taking Tip

Similarity Statements In naming similar triangles, the order of the vertices indicates the corresponding parts. Read the similarity statement carefully to be sure that you compare corresponding parts.

TEST EXAMPLE

3 Triangle LMN is similar to triangle PQR . If the perimeter of $\triangle LMN$ is 64 units, what is the perimeter of $\triangle PQR$?



- A 108 units
- B 96 units
- C 48 units
- D 36 units

Read the Item

You know the measures of two corresponding sides and the perimeter of $\triangle LMN$. You need to find the perimeter of $\triangle PQR$.

Solve the Item

Triangle $LMN \sim$ triangle PQR with a scale factor of $\frac{24}{18}$ or $\frac{4}{3}$. The ratio of the perimeters of $\triangle LMN$ to $\triangle PQR$ is also $\frac{4}{3}$.

$$\left. \begin{array}{l} \text{perimeter of } \triangle LMN \rightarrow 64 \\ \text{perimeter of } \triangle PQR \rightarrow x \end{array} \right\} \frac{64}{x} = \frac{4}{3} \quad \text{Scale factor relating } \triangle LMN \text{ to } \triangle PQR$$

$$64 \cdot 3 = 4 \cdot x \quad \text{Find the cross products.}$$

$$192 = 4x \quad \text{Multiply.}$$

$$\frac{192}{4} = \frac{4x}{4} \quad \text{Divide each side by 4.}$$

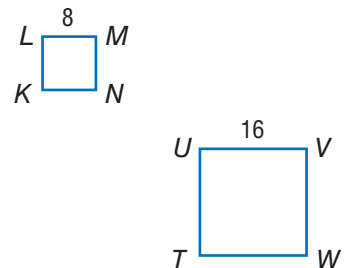
$$48 = x \quad \text{Simplify.}$$

The answer is C.

CHECK Your Progress

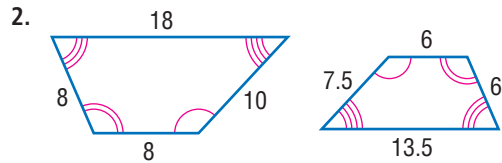
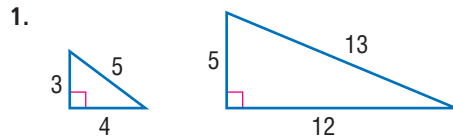
e. Square $KLMN$ is similar to square $TUVW$. If the perimeter of square $KLMN$ is 32 units, what is the perimeter of square $TUVW$?

- F 128 units
- G 96 units
- H 64 units
- J 40 units

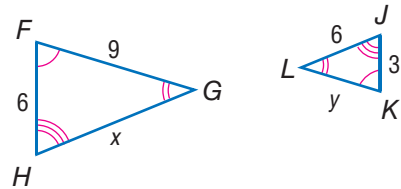


CHECK Your Understanding

Example 1 Determine whether each pair of polygons is similar. Explain.
(p. 219)

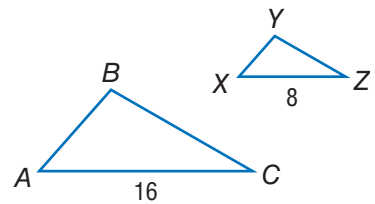


Example 2 In the figure at the right, $\triangle FGH \sim \triangle KLJ$. Write and solve a proportion to find each missing side measure.
(p. 220)



Example 3 **MULTIPLE CHOICE** $\triangle ABC$ is similar to $\triangle XYZ$. If the perimeter of $\triangle ABC$ is 40 units, what is the perimeter of $\triangle XYZ$?
(p. 221)

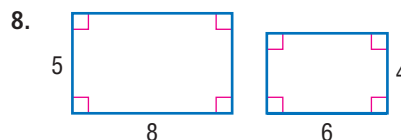
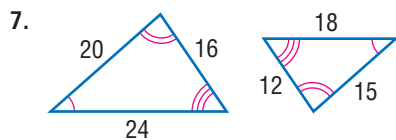
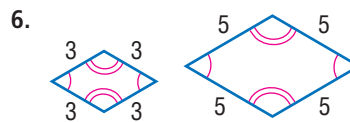
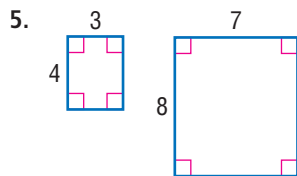
A 10 units C 40 units
B 20 units D 80 units



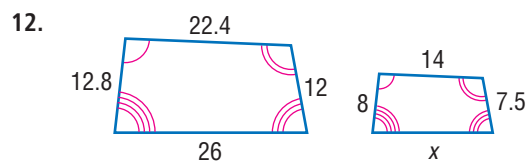
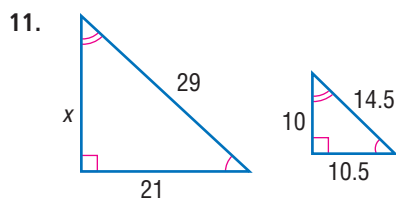
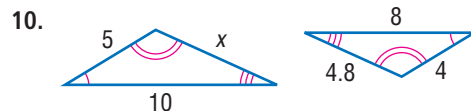
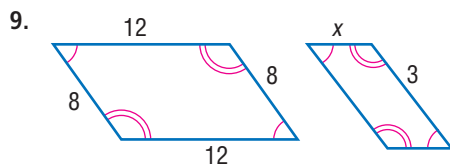
Practice and Problem Solving

HOMEWORK HELP	
For Exercises	See Examples
5–8	1
9–12	2
18, 19	3

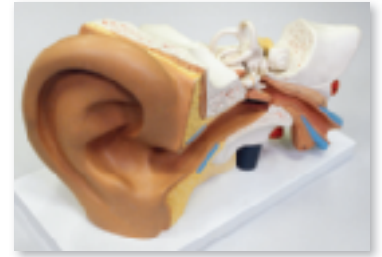
Determine whether each pair of polygons is similar. Explain.



Each pair of polygons is similar. Write and solve a proportion to find each missing side measure.



13. **LIFE SCIENCE** The scale factor from the model of a human inner ear to the actual ear is 55:2. If one of the bones of the model is 8.25 centimeters long, how long is the actual bone in a human ear?



14. **TELEVISION** The ratio of the length of a wide-screen TV to its width is 16:9. Find the width of a wide-screen TV if the length measures 28 inches. Round to the nearest tenth.

EXTRA PRACTICE

See pages 679, 703.

H.O.T. Problems

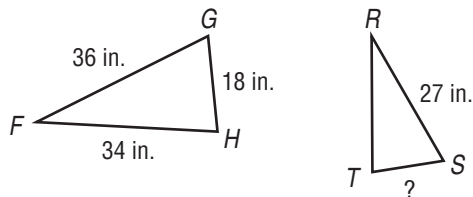
15. **CHALLENGE** Suppose two rectangles are similar with a scale factor of 2. What is the ratio of their areas? Explain.

WRITING IN MATH Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

16. Any two rectangles are similar. 17. Any two squares are similar.

TEST PRACTICE

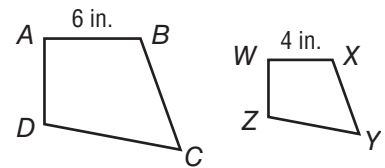
18. Triangle FGH is similar to triangle RST .



What is the length of \overline{TS} ?

- A $13\frac{1}{2}$ inches C 24 inches
B $22\frac{2}{3}$ inches D $25\frac{1}{2}$ inches

19. Quadrilateral $ABCD$ is similar to quadrilateral $WXYZ$.



If the perimeter of quadrilateral $ABCD$ is 54 units, what is the perimeter of quadrilateral $WXYZ$?

- F 13.5 inches H 27 inches
G 24 inches J 36 inches

Spiral Review

20. **ROCK CLIMBING** Grace is working her way up a climbing wall. Every 5 minutes she is able to climb 6 feet, but then loses her footing, slips back 1 foot, and decides to rest for 1 minute. If the rock wall is 30 feet tall, how long will it take her to reach the top? Use the *draw a diagram* strategy. (Lesson 4-6)

Solve each proportion. (Lesson 4-5)

21. $\frac{5}{4} = \frac{y}{12}$

22. $\frac{120}{b} = \frac{24}{60}$

23. $\frac{0.6}{5} = \frac{1.5}{n}$

GET READY for the Next Lesson

PREREQUISITE SKILL Graph and connect each pair of ordered pairs. (Lesson 3-6)

24. $(-2.5, 1.5), (1.5, -3.5)$

25. $(-2, -1\frac{1}{2}), (4, 3\frac{1}{2})$

26. $(-2\frac{1}{3}, 1), (2, 3\frac{2}{3})$